# Monitoring and repair of isolated trackbed defects on a ballasted railway

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## Abstract

Ballasted railway track suffers from a gradual loss of vertical geometry (level) as a result of settlements caused by trafficking and differences in support conditions. However, certain trackbed defects giving rise to a particularly rapid and severe loss of geometry are often localised, and possibly associated with zones of inadequate or variable support stiffness. Conventional line-tamping (usually by machine) may not be effective for these isolated trackbed defects. This paper demonstrates, with reference to three particular defect sites on a ballasted railway in the UK, the benefits in terms of both effectiveness and longevity of a more targeted repair strategy at such locations. First, track-based instrumentation was used to assess the nature and extent of the defect and to identify differential support stiffness conditions, which might also need to be addressed. The data were then used to inform repair strategy; for example, deflections measured during train passage were used to specify the thickness of shims placed between the rail pad and the sleeper. Finally, the track-based instrumentation was used to monitor the effectiveness and longevity of the repair, providing evidence that adequate support conditions had been restored. At the three defect sites investigated, the localised repairs are shown to be more effective and longer-lived than conventional line-tamping.

Railway track, Trackbed defect repair, Tamping, Lineside monitoring, Track stiffness, Track vibration

# 1. Introduction

All railway tracks deteriorate to some extent as they are trafficked by passing trains. Trackbed deterioration is particularly important for ballasted track, as differential settlement of the ballast and sub-ballast layers leads to irregular track geometry. Certain locations, often near transition zones (Hunt, 1997; Li and Davis, 2005; Coelho et al., 2011; Steenbergen, 2013; Le Pen et al., 2014; Paixão et al., 2014; Stark and Wilk, 2015), may experience accelerated rates of deterioration leading to unacceptable deviation from the required track geometry. These isolated trackbed faults or defects need localised maintenance to restore acceptable track geometry (British Standards Institution, 2017). They are often associated with poor support conditions, including voided sleepers (leading to non-linear support and increased loading; (Sussman et al., 2001; Lundqvist and Dahlberg, 2005), excessive track deflection and ballast attrition (Figure 1).

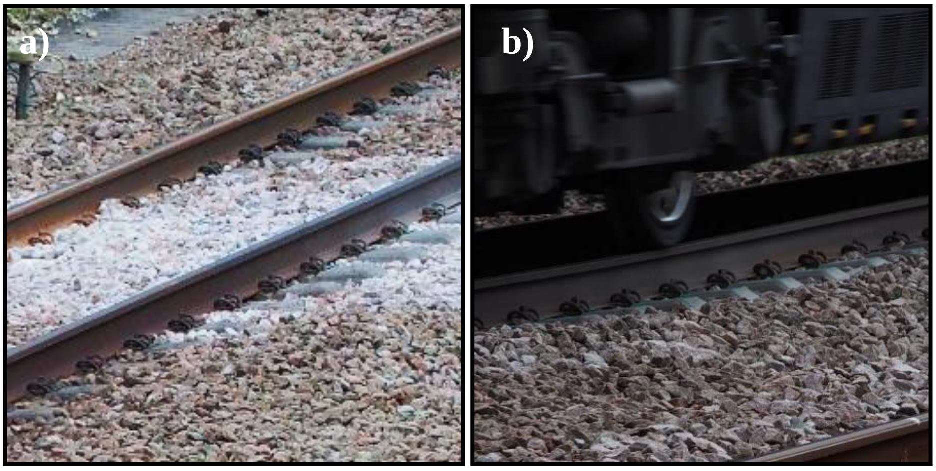


**Figure 1: a) Visible disturbance of the ballast, b) excessive displacement under load in a defect zone.**

Infrastructure managers may deploy tamping to restore acceptable track geometry at isolated trackbed defects (e.g. Selig and Waters, 1994; Cope and Ellis, 2001). However, such repairs are often not permanent, with infrastructure managers reporting geometry deterioration recurring within a matter of weeks, days or sometimes hours and requiring repeat repairs (as shown in Sections 3 and 4). More effective and sustainable repair methods for isolated trackbed defects need to be developed, tested and evaluated to prevent resources from being wasted on ineffective repairs and reduce the impact of excessive maintenance on the lifespan of trackbed components.

This paper demonstrates, with reference to three defect sites on a ballasted railway in the UK, how near-continuous lineside monitoring can be applied to inform and evaluate innovative and effective repair strategies for isolated trackbed defects. The repairs studied targeted the sleeper support conditions in defined defect zones, where mechanised general maintenance (tamping) had proved ineffective.

# 2. Monitoring and analysis techniques

A variety of lineside monitoring technologies may be used to measure low frequency (< 40 Hz) track vibrations from passing trains associated with the major trackbed movements and, following appropriate signal processing, to assess track performance in terms of track movement and stiffness (Milne et al., 2017). The monitoring techniques and the methods of analysis used in this research, together with typical sleeper deflections and track support system moduli for the railway line studied, will now be described.

## 2.1. Monitoring technologies

Multi depth deflectometers (MDDs) installed within the trackbed (Mishra et al., 2014), and high speed video of track mounted targets combined with digital image correlation (DIC) (Bowness et al., 2007; Le Pen et al., 2014; Murray et al., 2014; Wheeler et al., 2016), can be used to obtain track deflections directly. Inertial sensors such as geophones and accelerometers (Figure 2 (a & c)) measure track velocities and accelerations, from which track deflections can be deduced by filtering and integrating data once or twice respectively (Bowness et al., 2007; Cui et al., 2014). The installation of accelerometers and geophones is non-invasive and does not alter the behaviour of the track, allowing comparative measurements before and after a repair.

Recent improvements in low-cost Micro Electro Mechanical Systems (MEMS) accelerometers mean that affordable long-term, near continuous lineside monitoring is now possible (Milne et al., 2016). Multiple train passages can be analysed to provide a record of how the performance of the track changes as a consequence of maintenance, and whether any improvements are sustained under traffic.

Here, the three study sites were instrumented using low-cost MEMS accelerometers with a ±16 g range. They were placed on sleeper ends for extended periods of time, to inform and evaluate the repairs of the isolated trackbed defects. A Campbell Scientific CX9000 datalogger was deployed trackside, powered using batteries and solar panels. The systems were configured to record when increased vibration levels indicated an approaching train. Acceleration data were sampled at 500 Hz, stored in the logger memory and downloaded periodically. To obtain deflections, the acceleration signals were high- and low-pass filtered with cut-off frequencies of 2 Hz and 40 Hz, and integrated twice. Data were analysed and supplied to the infrastructure manager, to inform their specification of appropriate remedial work. Details of the instrumentation arrangements are given, together with the results from each site, in Section 4.

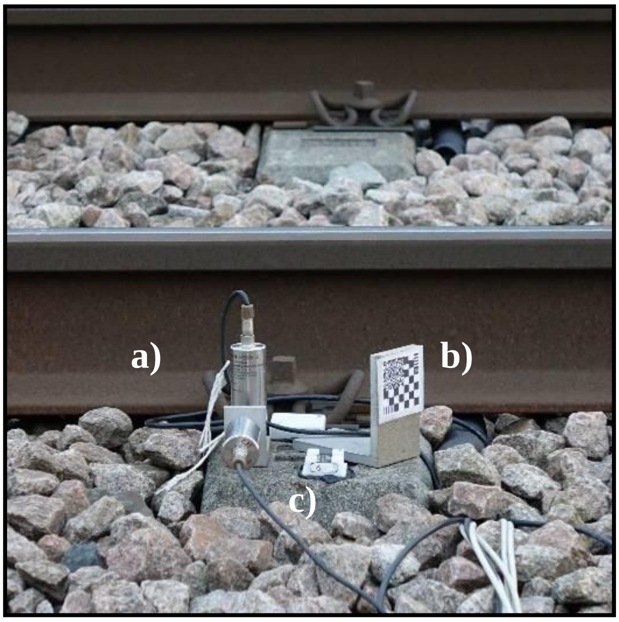


**Figure 2: a) Horizontal and vertical geophones, b) video target for DIC, c) MEMS accelerometer.**

Extended and extensive deployment of lineside monitoring generates large volumes of data requiring processing and interpretation. Previous studies have used the typical downward deflection beneath a wheel of a passing train, often referred to as the *characteristic deflection*, and the *track support system modulus* as measures of the performance and properties of the track. These parameters can be used to monitor and assess how the performance of the track changes over time. The analysis techniques used to obtain them algorithmically from the data are summarised in the following subsections.

## 2.2. Characterising deflection

Characterising dynamic track movements is a challenge because track vibration caused by a passing train is transient, and there is a degree of variability between different wheelsets. Furthermore, artefacts from the signal processing needed to obtain deflections from velocities or accelerations cause transients at the start and end of the train, and a shift in the signal level associated with the at-rest position of the track while the train is passing. This means that track deflections have traditionally been interpreted by inspection rather than automatically. (Milne et al., 2018) developed a statistical process, based on the cumulative distribution function for track deflection, to characterise automatically the typical track deflection below a given train. This is summarised for a well performing sleeper typical of the line studied in Figure 3.

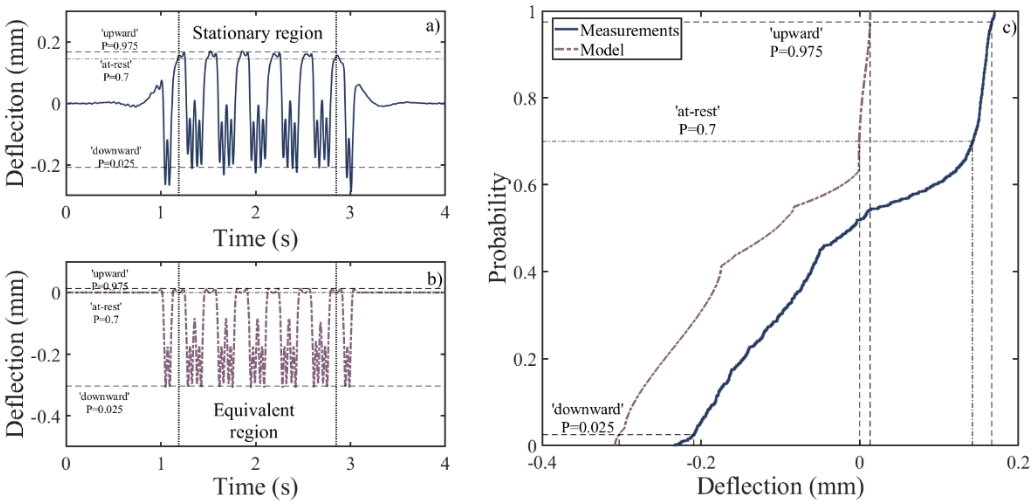


**Figure 3: a) Measured track deflections, b) reproduced using the beam on and elastic foundation model and c) cumulative distribution function for the stationary region from the measurements and the equivalent region in the model (after Milne et al. 2018).**

Provided the signal has been filtered appropriately, there should be a “stationary region” within the measured deflection signal (Figure 3(a)). Within an equivalent time period, the distribution of the measured data should be similar to that from a model for track deflection (Figure 3(b)). The cumulative distribution function for track deflection has a distinctive shape, which includes a steepening associated with the at-rest position. This occurs at zero deflection for the model but has apparently been shifted in the measurements (Figure 3(c)), owing to the use of a high pass filter.

The distribution can be used to characterise the typical deflection associated with the passage of a given train. Values are chosen to represent the typical extents of upward and downward movement and the at-rest position, based on the proportion of time P that the deflection signal is at or below the level in question. These values are then used to define the characteristic ranges of total and downward deflection. In this paper, values of P = 0.025, 0.7 and 0.975 were used to identify the maximum downward movement, the at-rest position and the maximum upward movement respectively. Previous measurements had indicated deflections typically between 0.5 mm and 1.0 mm for well- performing sleepers on this line (Le Pen et al., 2016).

### 2.3. Determining Stiffness

Previous studies have evaluated track stiffness under operational conditions from measurements of track deflection under a measured or assumed load, (Kerr, 2000; Priest and Powrie, 2009). (Le Pen et al., 2016) showed that the track stiffness could be determined from the spectrum of low frequency vibration using the properties of the Fourier transform of track deflection, velocity or acceleration, without knowledge of the applied wheel load. The frequency and magnitude of the dominant spectral peaks depend primarily on the train (or vehicle) geometry and the track stiffness (Auersch, 2006; Ju et al., 2009; Le Pen et al., 2016; Milne et al., 2017). In this paper, the ratio between the magnitudes of the 7$^{th}$ and 3$^{rd}$ dominant frequencies is used, following the method described by (Le Pen et al., 2016; Milne et al., 2017). Track system support moduli of between 20 MN/m$^2$ and 50 MN/m$^2$ had previously been measured at well- performing sleepers on this railway (Le Pen et al., 2016).

## 3. Defect remediation

This research was motivated by infrastructure managers reporting that the improvements in track geometry resulting from machine tamping through localised defect zones were often not sustained such that repeated remediation was often needed. Possible reasons for this were investigated by lineside monitoring and exploratory excavation into the ballast bed between sleepers, before some alternative strategies were considered. It is worth noting that other forms of mechanised intervention, e.g. stone blowing, rail vacuuming and dynamic track stabilisation following tamping of isolated track bed defects, are not currently practised on the railway studied when this study was carried out.

## 3.1. Tamping

Tamping is usually carried out by machine. The tamping vehicle lifts the track, and vibrating tines then compact the ballast beneath the sleepers (Figure 4(a)). Local manual tamping can be used for the same purpose: the track is lifted using jacks, and ballast is pushed underneath the sleeper using vibrating packing tools (Figure 4(b)). In the UK, tamping interventions are specified, controlled and accepted on the basis of the measured deformed track geometry, with no explicit consideration given to the support conditions in specifying the repair process.

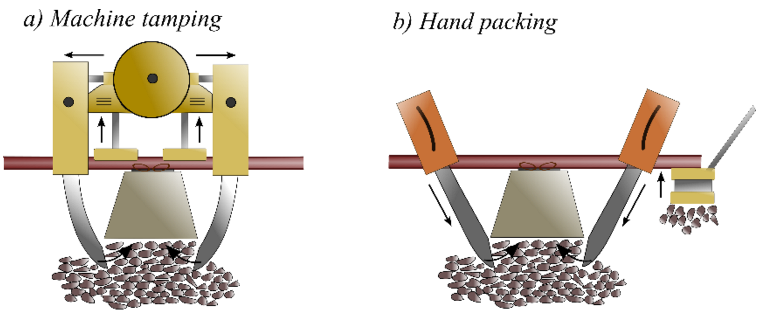


**Figure 4: a) Machine tamping and b) manual tamping.**

Figure 5 shows part of a deflection *vs* time history, obtained using high speed video and digital image correlation (DIC), for a sleeper within a defect zone the day before and the day after a machine tamping intervention. Before the intervention (Figure 5(a)), downward sleeper deflections were in excess of 5 mm, indicating poor performance and voiding. Sleeper deflections were reduced to about 2 mm after tamping (Figure 5(b)). However, this is still greater than the typical sleeper deflections of 0.5-1.0 mm expected on the railway studied; and significant disturbance of the ballast was reported a week following the tamping intervention. This suggests that adequate support conditions had not been fully restored by tamping.

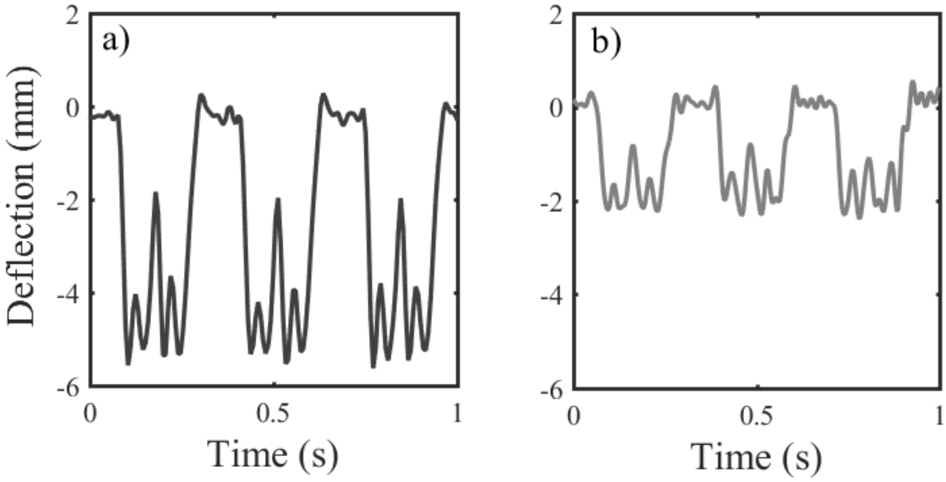


**Figure 5: Track deflections obtained using DIC with a defect zone a) the day before and b) the day after a tamping intervention.**

Figure 6 shows an exploratory excavation into the ballast bed between two sleepers within a defect zone. The ballast grains around the sleepers were generally large and easily removed. Deeper in the ballast bed at the base of the sleepers, the material appeared much more compacted, contained smaller grains (suggesting ravelling and / or ballast breakage), and was not easily removable manually. Machine tamping is unlikely to be effective where the trackbed material has undergone a segregation of different sized particles, resulting in a structure in which the larger grains overlie dense, more widely-graded and compactable material (Figure 7).

**Figure 6: Photograph of excavation into ballast bed below sleeper level.**

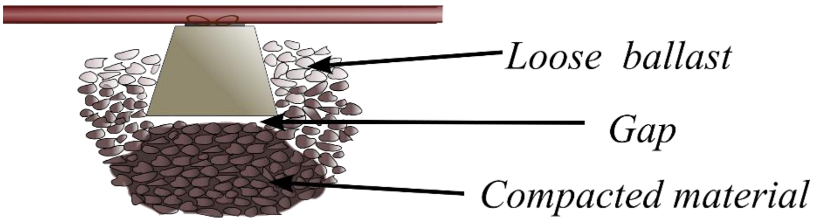


**Figure 7: Possible layering of trackbed within a defect zone.**

## 3.2. Targeted remediation

Lineside monitoring was used to inform and evaluate different repairs carried out at the three sites. The three defect sites studied encompass different features that may influence track performance, (Figure 8). Site 1 is at the end of a crossing. There is a change in sleeper type from mono-block within the crossing to duo-block on plain line, together with a shallow under track crossing (UTX) comprising a concrete culvert carrying services beneath the track (Figure 8(a)). The top of the culvert is in line with the bottom of the ballast layer which is about 0.5 m thick. At Site 2 there are two shallow UTX, of similar design and installation depth to that at Site 1, spaced at 4.8 m centres (Figure 8(b)). Site 3 is on a bridge in vicinity of a structural expansion joint between two bridge spans (Figure 8(c)). Here, continuous welded rail is supported by sleepers laid on a 0.5 m depth of ballast, placed directly onto the bridge deck. A steel plate spans the gap between the adjacent bridge decks.


**Figure 8: a) Change in sleeper type at Site 1, b) pair of UTX at Site 2 and c) bridge expansion joint at Site 3.**

The interventions carried out at each site are illustrated in Figure 9. These were intended to restore the support conditions rather than to correct the track geometry.

At Site 1, new sleepers fitted with under sleeper pads (USPs) were installed within the defect zone (Figure 9(a)). USP are conventionally used to reduce the impact of passing trains on the ballast (UIC, 2009; Schneider et al., 2011; Paixão et al., 2014; Abadi et al., 2016; Le Pen et al., 2017). Here, stiff pads were used to fill the gap and improve conformity between the sleeper and the compacted material, as well as mitigating the effect of a possible hard spot formed by the concrete UTX. Thus their purpose here was more as a shim / packer than as a conventional USP.

At Site 2, the sleepers within the defect zone were dug out, the compacted layer of the ballast bed below sleeper level was broken up manually to encourage better mixing with the returned and replacement ballast; and the ballast was compacted manually using vibrating tools to provide the required track geometry and contact between the ballast and the sleeper (Figure 9(b)).

At Site 3, additional stiff elastomer shims were installed in between the rail and the sleepers, using the adjustability of the fastening system (Figure 9(c)) to lower the sleeper into contact with the compacted underlying material.

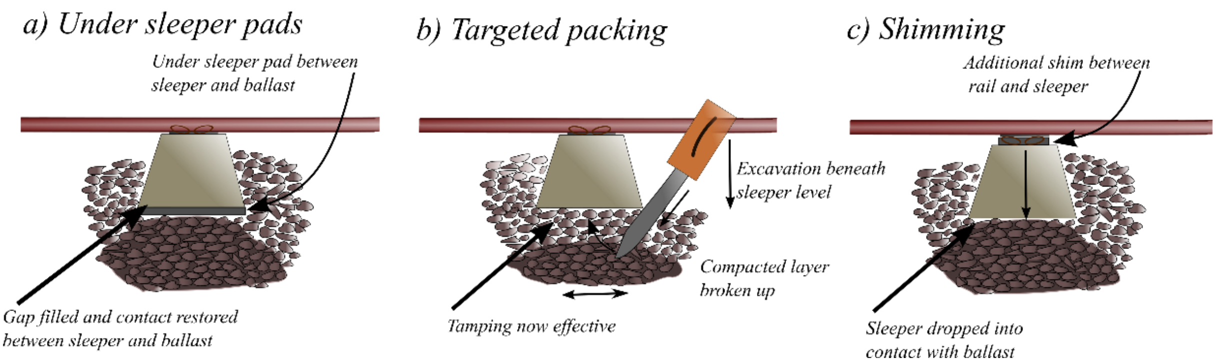


**Figure 9: Repair strategies for isolated defect zone studied during this research a) installation of under sleeper pads, b) targeted manual packing, after breaking up the compacted material in the defect zone, c) shimming between the rail and the sleeper.**

## 4. Results and Discussion

In this section the arrangements for instrumentation are presented, followed by results from continuous monitoring of selected sleepers, on a site by site basis. Each subplot in the results figure is for an individual sensor, and is labelled according to the associated instrumentation plan. Both deflection and stiffness data are shown. The deflection scale is on the left hand axis and the stiffness scale on the right. The dashed vertical lines indicate when maintenance was carried out. The track support system moduli were only determined post-intervention, when the track was expected to be adequately supported. Results are for a single train type; a passenger train comprising six near identical vehicles, typically travelling at 60 m/s. There are gaps in the data where the data acquisition system did not have sufficient energy to operate, but the trends in behaviour are clear.

## 4.1. Site 1

Site 1 was initially instrumented for approximately 18 months using three MEMS accelerometers as shown in Figure 10. Monitoring commenced following an ineffective tamping intervention in February 2015. Seven sleepers with stiff USPs were installed in April 2015 (Figure 9(a)). Monitoring continued until July 2016.

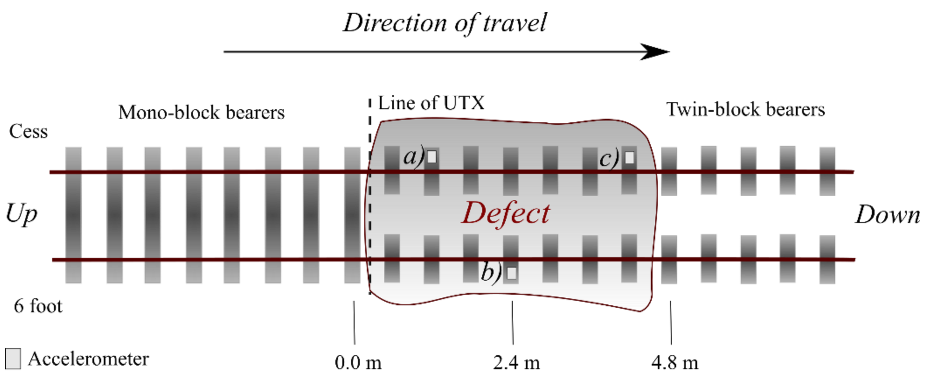


**Figure 10: Layout of instrumentation used at Site 1.**

Figure 11 shows sleeper deflections and stiffnesses at transducers a-c in Figure 10.

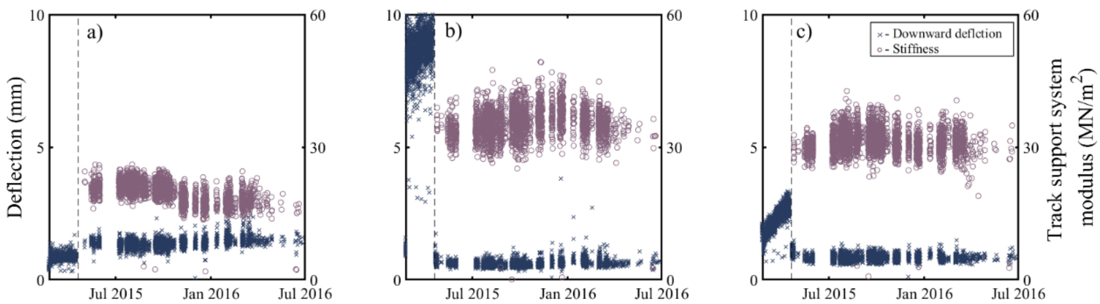


**Figure 11: Deflection and stiffness data from Site 1 from sensors *a, b, and c* between February 2015 and July 2016.**

Before the installation of USPs, indicated by the dashed line in Figure 11, performance within the defect zone was poor. Even after tamping, the sleeper deflections at *b* was in excess 8 mm, suggesting voiding between the sleeper and ballast. Towards the end of the defect zone at *c*, the sleeper deflections steadily increased from about 1.5 mm to 2.5 mm over a period of eight weeks.

Stiff, 9 mm thick USPs were selected based on the magnitudes of the sleeper movements. Following installation of the USPs, the sleeper deflections at the poorly performing sleepers *b*, and *c* reduced to about 0.6 mm and 0.8 mm respectively. Deflections then remained at these levels for the duration of monitoring. Sleeper deflections at *a* increased from about 1.0 mm to about 1.3 mm, as might be expected following the addition of an additional resilient element into the track system. The track support system moduli were restored to acceptable values in the range 20 MN/m$^2$ to 30 MN/m$^2$ after the intervention. There was some variation from one train to another, but generally the support moduli remained substantially constant over the ~14 months of continuous monitoring following the repair.

Further monitoring was carried out over five days in May 2017. Figure 12 shows that the sleeper deflections and support conditions had not changed since July 2016, indicating that the restoration of support conditions and the reduction in deflection achieved by the intervention had been sustained for over 2 years.

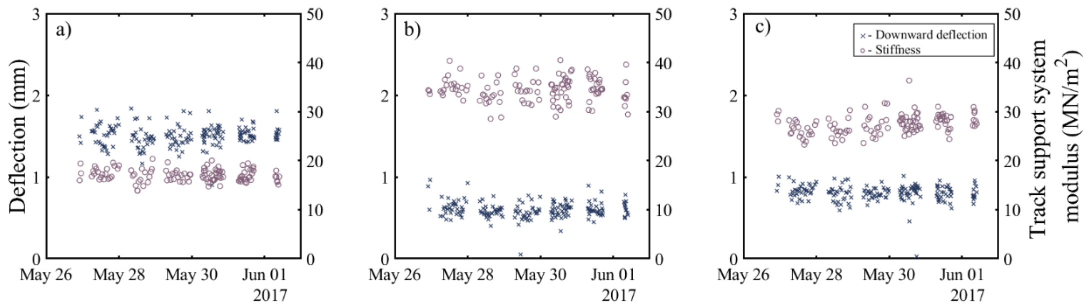


**Figure 12: Deflection and stiffness data from Site 1 from sensors *a*, *b* and *c* at the end of May 2017.**

## 4.2. Site 2

Site 2 was instrumented for approximately 4 months, using six MEMS accelerometers as shown in Figure 13. Monitoring started in June 2015. Initial results were used to inform a targeted packing intervention (Figure 9(b)) carried out in July 2015.

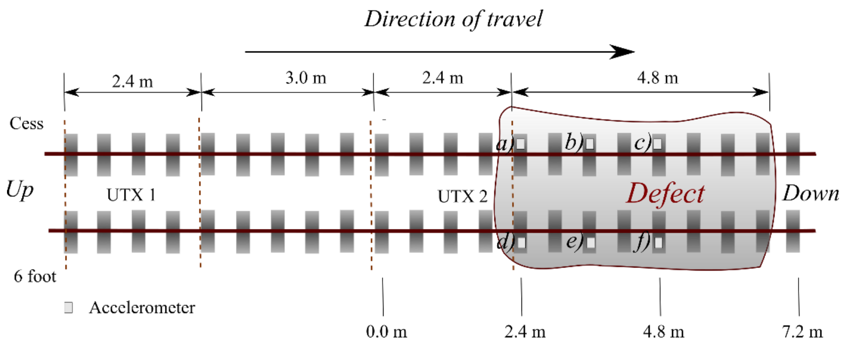


**Figure 13: Instrumentation layout used at Site 2.**

Figure 14 shows the sleeper deflection and track support system modulus data from transducers *a*-*f* indicated in Figure 13.

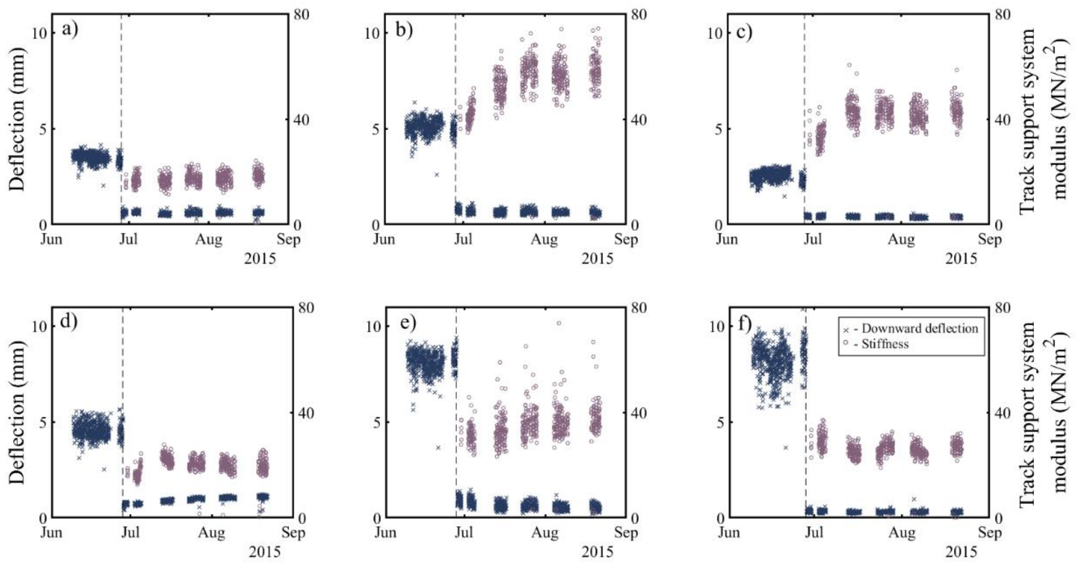


**Figure 14: Deflection and stiffness data from Site 2 at sensors *a* - *f* in Figure 14, between June and September 2015.**

Prior to the intervention, deflections were large, with a large range (variability) at each location. Locations *a*, *b* and *c* on the cess side were moving more than 3 mm, 5 mm and 3 mm respectively. Greater movements, in excess of 5 mm, 8 mm and 8 mm, were occurring at *d*, *e* and *f* on the six foot side, suggesting a twisting of the track as a train passed. Such large deflections indicate the possibility of voiding. Given the magnitude of the movements, the maintainer expected to (and did) find a trackbed condition similar to that shown in Figures 8 and 9, and specified excavation into the trackbed and breaking up of the compacted layers prior to tamping.  Further monitoring equipment was used to identify the extent of the defect. Additional ballast was placed as the track was repacked with the intention of raising the level of the trackbed to eliminate voiding.

After maintenance, sleeper deflections were reduced to typically between 0.5 mm and 1.0 mm. Following some initial bedding in, particularly at *b*, *c* and *d*, support conditions had been restored with support system moduli of 20 MN/m$^2$ to 50 MN/m$^2$, indicating some remaining variability along the site. Measured deflections and stiffnesses were stable and consistent at each location for the final two months of monitoring, with no indication of any further deterioration within the defect zone. Although labour intensive, excavation and breaking up of the compacted layer is now being carried out prior to manual packing or machine tamping to facilitate more effective and longer lasting remediation of these types of isolated defect zones.

## 4.3. Site 3

Site 3 was instrumented for approximately 4 months, using MEMS accelerometers placed on sleeper ends as shown in Figure 15, and a single sensor on the bridge parapet 3.6 m away from the structural expansion joint. Monitoring began in April 2016.

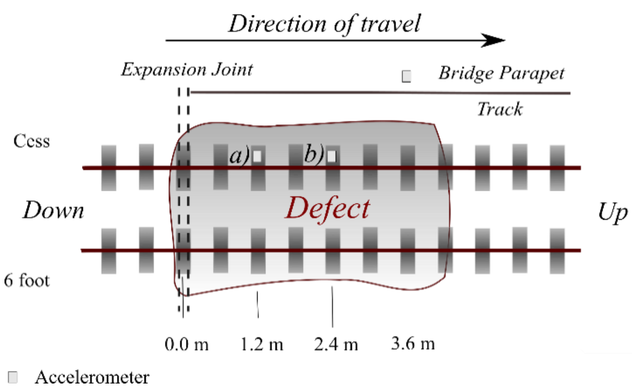


**Figure 15: Layout of instrumentation used at Site 3.**

Measurements at the bridge parapet confirmed that the deflection of the bridge was very small compared with the track movements at locations *a* and *b* (Figure 16).

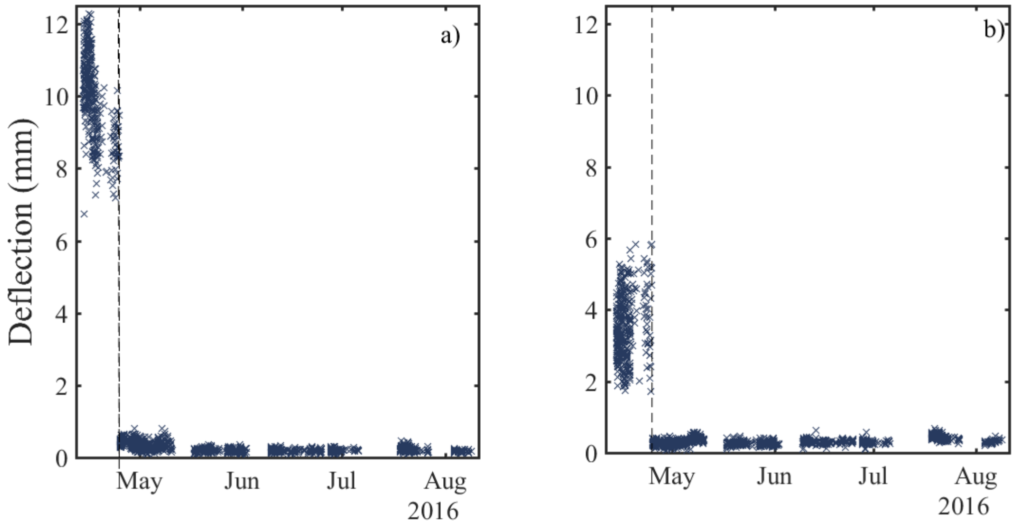


**Figure 16: Sleeper deflections from Site 3 at locations *a* and *b*, between April and August 2016.**

To repair this defect, additional, stiff, 4.5 mm thick elastomer shims were installed between the existing rail pad and the sleeper with the objective of lowering the sleeper into the void and restoring contact between the sleeper and the trackbed. Deflection data were used to specify the number of shims used; a single shim was used where the track movement was less than 4.5 mm, and two shims where the movement was greater than 4.5 mm. At locations *a* and *b*, the sleeper deflection was reduced to between 0.3 mm and 0.4 mm after the addition of shims. Track deflections remained at this level throughout the subsequent three months over which monitoring was carried out, indicating that the intervention had been successful. Track support system stiffnesses were not evaluated for Site 3, owing to the influence of the bridge.

## 5. Conclusions

Continuous lineside monitoring has allowed changes in track system performance over time to be investigated and quantified. The approach has been used to develop and evaluate repair strategies for isolated track defects where conventional interventions had proved ineffective, probably because the specification and acceptance of maintenance focused on attempting to correct track geometry rather than restoring adequate support conditions.

Although the detail of the repair differed for each site, all showed sustained improvements in terms of reduced sleeper deflections and, where relevant, plausible and consistent values for the track system support modulus. Common factors were that

- lineside monitoring was used to quantify and understand the track behaviour
- remediation work was then designed to restore the support conditions, informed by data from monitoring
- it was ensured that the sleepers were re-placed in sustained and sustainable contact with the trackbed, e.g. by introducing pads and shims to lower the sleeper soffit relative to rail level, and/or breaking up compacted material within the defect zone to encourage mixing of replacement / returned material prior to and during tamping.

Instrumentation combined with effective signal processing has been essential in

- informing an effective intervention
- providing both immediate and ongoing long-term feedback to verify that the intervention had achieved its intended purpose
- showing that the approach of targeting the support conditions, rather than the track geometry, when repairing isolated defects is effective over prolonged periods.

The use of continuous lineside monitoring also enables infrastructure managers to justify, trial and evaluate more innovative remediation techniques more widely across the network.

## 6. Acknowledgements

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Data supporting this study are openly available from the University of Southampton repository at https://doi.org/10.5258/SOTON/xxxxx.

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