

A new mathematical modeling method for four-stage helicopter main gearbox and dynamic response optimization

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A new mathematical modeling method, namely the finite element method and the lumped mass method (LMM-FEM) mixed modeling, is applied to establish the overall multi-node dynamic model of a four-stage helicopter main gearbox. The design of structural parameters of the shaft is the critical link in the four-stage gearbox, it affects the response of multiple input and output branches; however, only the meshing pairs were frequently shown in the dynamic model in previous research. Therefore, each shaft is also treated as a single node and the shaft parameters are coupled into the dynamic equations in this method, which is more accurate for the transmission chain. The differential equations of the system are solved by the Fourier series method, and the dynamic response of each meshing element is calculated. The sensitivity analysis method and parameter optimization method are applied to obtain the key shaft parameters corresponding to each meshing element. The results show that the magnitude of dynamic response in converging meshing pair and tail output pair are higher than that of other meshing pairs, and the wall thickness has great sensitivity to rotor shaft. In addition, the sensitivity analysis method can be used to select the corresponding shaft node efficiently and choose parameters appropriately for reducing the system response.

1. Introduction

The four-stage helicopter main gearbox has three input branches, they converge on the central gear, and goes one way to the rotor, and the other way to the tail chain. The transmission chain includes many parts, which makes its structure very complicated. The meshing pairs include internal excitations such as time-varying meshing stiffness and transmission error, in addition, external excitations like torques of the engine, rotor, and tail load also have important effect on the system. The dynamic response has a profound influence on the fatigue life of the system, therefore, it is necessary to carry out the dynamic analysis for the four-stage helicopter gearbox.

Regarding gearbox modeling, Kubur [1] established the dynamic model of a multi-shaft helical gear reduction system formed by flexible shafts, and studied the influence of some of the key system parameters under forced vibrations. Raclot [2] simulated the contributions of shape deviations and mounting errors to the dynamic behaviour of a multi-stage geared systems. Choy [3] presented a multi-stage multi-mesh gear transmission system, the individual modal component responses and the overall system

dynamics of the gear box were predicted. Dzitkowski [4] obtained the multi-stage gear mechanical characteristics by properly selecting the dynamical properties of the system based on using the active synthesis method. Chen [5] established the vibration model of a four-stage main transmission system in helicopter through lumped mass method and the influence of the torsional stiffness of shafts on the first five orders of the system's natural frequency was studied. However, in this dynamic model, the shaft were not regarded as a node and all the differential equations are related to the meshing pairs.

In the area of gear response analysis, Parker [6] analyzed the dynamic response of a helicopter planetary gear system under different range of operating speeds and torques based on finite element method, and focused on the gear contact conditions. Velex [7] calculated dynamic tooth loads and response on a planetary gear set by Ritz method, and compared the results with those given by direct integrations for highly reduced computation times. Chaari [8] compared dynamic response of healthy planetary gears with cracked planetary gears in both the time and frequency domains applied with the Wigner-Ville distribution method. Walha [9] investigated

dynamics of a two-stage gear system involving backlash and time- dependent mesh stiffness, and the decomposition of nonlinear system into some linear systems was solved by the Newmark iterative algorithm. Zhou [10] developed a coupled lateral-torsional nonlinear dynamic model with 16-degree-of-freedom (16-DOF) of gear-rotor-bearing transmission system considering the nonlinear features, and the mean load excitation had a complicated influence on the coupled system, he concluded that the torsional vibration was the dominate response in the geared system. Chen [11] detecting the key shafts of the four-stage helicopter gearbox and analyzed their sensitivity to each branch of the system through lumped mass method.

Gearbox parameter optimization plays an important role in helicopter design, Chen [12] demonstrated the effectiveness of the proposed mesh stiffness model under the influences of the tooth profile modification, not only applied in low contact ratio, but also in high contact ratio. Yang [13] presented the gearbox parameter optimization method by artificial bee colony algorithm when diagnosing the gear faults, and verified the theory through a two-stage parallel shaft gearbox. Bozca [14]–[16] studied gearbox geometric parameters optimization to reduce rattle noise in an automotive transmission based on a torsional vibration model, and the module, number of teeth, axial clearance, and backlash could be improved through this method. However, system dynamics optimization were mostly through gear parameters or profile modification, the shaft parameters were not regarded as a variables in these researches.

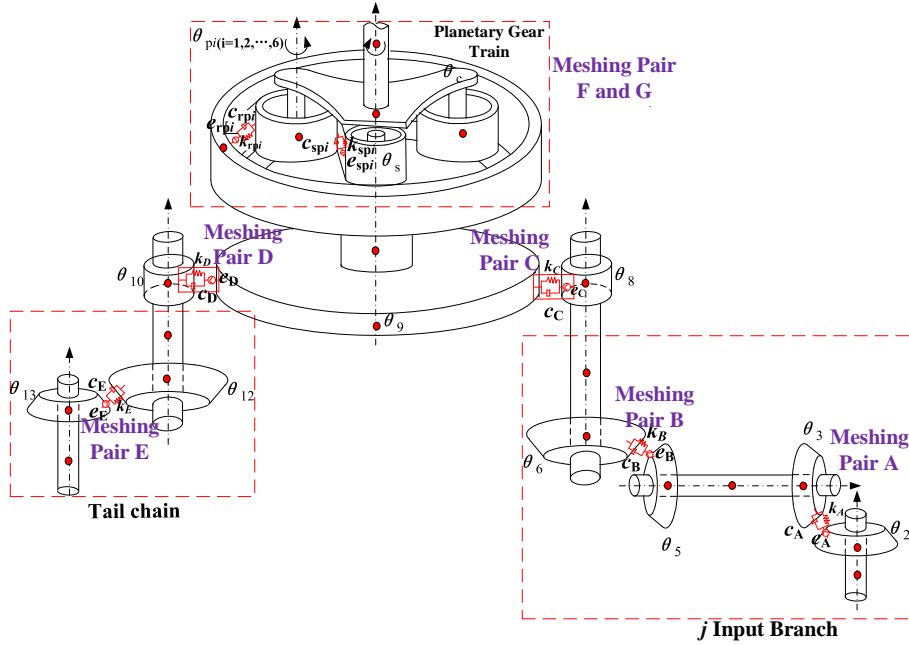
About the sensitivity analysis to system parameters, Lin [17] and Guo [18] investigated the natural frequency and vibration mode sensitivities to system parameters. Chen [19] studied the response sensitivity to system parameters like gear mesh stiffness, damping, diameter ratio, and gear mass unbalance in a coupled gear system. However, sensitivity analysis based on multi-node dynamic modeling hasn't been captured by these research.

In summary, the research of most scholars focuses on the planetary gear chain of the helicopter main gearbox. The dynamic equations also seldom reflect the characteristics of the shaft parameters as independent nodes, thus it is meaningful to improve the dynamic modeling and propose optimization method.

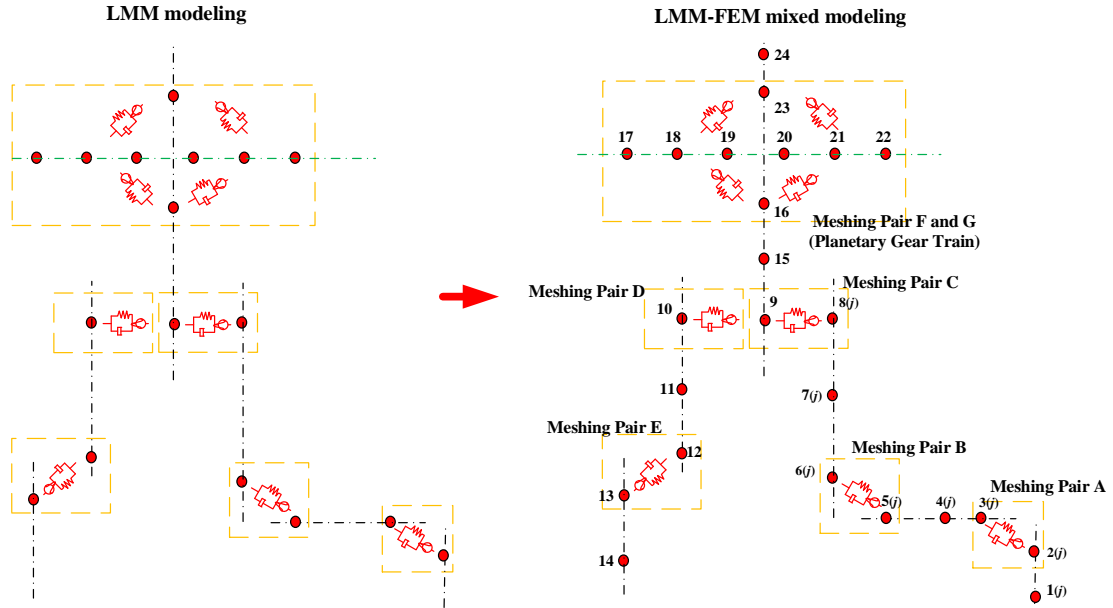
2. LMM-FEM mixed modeling

The dynamics modeling process of a typical four-stage helicopter main gearbox is shown in Fig.1 [20]. In the figure, Fig.1 (a) is the system dynamic model, there are seven meshing elements (A, B, C, D, E and F) in the system, and the elements F and G are internal and external meshing pairs in planetary system, which contains one sun gear, six planet gears and one carrier. θ is the rotational degree of freedom (DOF) of each node. Fig.1 (b) is a comparison of the LMM modeling method and the LMM-FEM mixed modeling method. In this new method, the gearbox is regarded as finite critical nodes, which include shaft and meshing pair, each DOF corresponds to each node of the system. The system's generalized DOF coordinate vector X is:

$$X = \{\theta_1^{(j)}, \theta_2^{(j)}, \theta_3^{(j)}, \theta_4^{(j)}, \theta_5^{(j)}, \theta_6^{(j)}, \theta_7^{(j)}, \theta_8^{(j)}, \theta_9, \theta_{10}, \theta_{11}, \theta_{12}, \theta_{13}, \theta_{14}, \theta_{15}, \theta_{16}, \theta_{17}, \theta_{18}, \theta_{19}, \theta_{20}, \theta_{21}, \theta_{22}, \theta_{23}, \theta_{24}\}^T$$



(a) Dynamic model



(b) Modeling comparison

FIGURE 1: Dynamic modeling for multi-node system.

1) Dynamic Modeling of the meshing pairs

For the seven meshing pairs (A, B, C, D, E, F, and G) highlighted in Fig.1, the time-varying meshing stiffness can be expressed in the Fourier series with meshing frequency ω : [21]

$$\begin{cases} k(t) = k_m + k_a \sin(\omega t + \beta) \\ k_{spi}(t) = k_{m,spi} + k_{a,spi} \sin(\omega t + \beta_{spi}) \\ k_{rpi}(t) = k_{m,rpi} + k_{a,rpi} \sin(\omega t + \beta_{rpi}) \end{cases} \quad (1)$$

where $k(t)$, $k_{spi}(t)$ and $k_{rpi}(t)$ are time-varying meshing stiffness of each gear pair; k_m and k_a are the

average and maximum variable meshing stiffness; β is the initial phase of meshing stiffness; ω is the fundamental meshing frequency.

The transmission errors are shown in the same way:

$$\begin{cases} e(t) = e_m + e_a \sin(\omega t + \varphi) \\ e_{spi}(t) = A_{spi} \sin(\omega t + \varphi_{spi}) + E_{pi} \sin(\omega_{pH} t + \varphi_{pi} + \alpha) \\ \quad + E_s \sin[\omega_{sH} t + \varphi_s - 2\pi(i-1)/N + \alpha] \\ e_{rpi}(t) = A_{rpi} \sin(\omega t + \varphi_{rpi}) + E_{pi} \sin(\omega_{pH} t + \varphi_{pi} - \alpha) \\ \quad + E_r \sin[\omega_{rH} t + \varphi_r - 2\pi(i-1)/N - \alpha] \end{cases} \quad (2)$$

here, $e(t)$, $e_{spi}(t)$, $e_{rpi}(t)$ are time-varying transmission

error of each gear pair; e_m and e_a are static and dynamic transmission error amplitude; φ is initial phase of transmission error; ω_{pH} , ω_{sH} and ω_{rH} are rotational frequency of planet gear, sun gear and carrier; α is pressure angle; N is the number of planet gear.

Tooth deflection along meshing line is defined as dynamic response, and presented as follows:

$$X(t) = \theta_d r_d - \theta_p r_p - e(t) \quad (3)$$

where θ_d and θ_p are rotational DOF of drive gear and driven gear; r_d and r_p are radius of base circle in drive gear and driven gear.

The dynamic forces of each gear pair $F(t)$ is defined as follows:

$$\begin{cases} F(t) = F^p(t) + F^d(t) \\ F^p(t) = k(t)X(t) \\ F^d(t) = c(t)\dot{X}(t) \end{cases} \quad (4)$$

here $c(t)$ is meshing damping; $\dot{X}(t)$ is relative velocity along the meshing line.

2) Differential equation of multi-node system

According to the dynamic modeling above, the differential equation of the multi-node dynamic model can be deduced through Newton's law:

Node 1 (Shaft node):

$$J_1 \ddot{\theta}_1^{(j)} + G_1 \frac{\pi D_1^4}{32 l_1} \left(1 - \frac{d_1^4}{D_1^4} \right) (\theta_1^{(j)} - T_{Ej}) \quad (5)$$

Node 2-3 (Meshing Pair A):

$$\begin{cases} J_2 \ddot{\theta}_2^{(j)} + [F_{2-3}^p(t) + F_{2-3}^d(t)] r_1 \\ + G_1 \frac{\pi D_1^4}{32 l_1} \left(1 - \frac{d_1^4}{D_1^4} \right) (\theta_2^{(j)} - \theta_1^{(j)}) = 0 \\ J_3 \ddot{\theta}_3^{(j)} - [F_{2-3}^p(t) + F_{2-3}^d(t)] r_1 \\ + G_4 \frac{\pi D_4^4}{32 l_4} \left(1 - \frac{d_4^4}{D_4^4} \right) (\theta_3^{(j)} - \theta_4^{(j)}) = 0 \end{cases} \quad (6)$$

Node 4 (Shaft node):

$$\begin{cases} J_4 \ddot{\theta}_4^{(j)} - G_4 \frac{\pi D_4^4}{32 l_4} \left(1 - \frac{d_4^4}{D_4^4} \right) (\theta_3^{(j)} - \theta_4^{(j)}) \\ + G_4 \frac{\pi D_4^4}{32 l_4} \left(1 - \frac{d_4^4}{D_4^4} \right) (\theta_4^{(j)} - \theta_5^{(j)}) = 0 \end{cases} \quad (7)$$

Node 5-6 (Meshing Pair B):

$$\begin{cases} J_5 \ddot{\theta}_5^{(j)} + [F_{5-6}^p(t) + F_{5-6}^d(t)] r_5 \\ + G_4 \frac{\pi D_4^4}{32 l_4} \left(1 - \frac{d_4^4}{D_4^4} \right) (\theta_5^{(j)} - \theta_4^{(j)}) = 0 \\ J_6 \ddot{\theta}_6^{(j)} - [F_{5-6}^p(t) + F_{5-6}^d(t)] r_6 \\ + G_7 \frac{\pi D_7^4}{32 l_7} \left(1 - \frac{d_7^4}{D_7^4} \right) (\theta_6^{(j)} - \theta_7^{(j)}) = 0 \end{cases} \quad (8)$$

Node 7 (Shaft node):

$$\begin{cases} J_7 \ddot{\theta}_7^{(j)} - G_7 \frac{\pi D_7^4}{32 l_7} \left(1 - \frac{d_7^4}{D_7^4} \right) (\theta_6^{(j)} - \theta_7^{(j)}) \\ + G_7 \frac{\pi D_7^4}{32 l_7} \left(1 - \frac{d_7^4}{D_7^4} \right) (\theta_7^{(j)} - \theta_8^{(j)}) = 0 \end{cases} \quad (9)$$

Node 8-10 (Meshing Pair C and D):

$$\begin{cases} J_8 \ddot{\theta}_8^{(j)} + [F_{8-9}^p(t) + F_{8-9}^d(t)] r_8 \\ + G_7 \frac{\pi D_7^4}{32 l_7} \left(1 - \frac{d_7^4}{D_7^4} \right) (\theta_8^{(j)} - \theta_7^{(j)}) = 0 \\ J_9 \ddot{\theta}_9 - \sum_{j=1}^3 [F_{8-9}^p(t) + F_{8-9}^d(t)] r_9 \\ + G_{15} \frac{\pi D_{15}^4}{32 l_{15}} \left(1 - \frac{d_{15}^4}{D_{15}^4} \right) (\theta_9 - \theta_{15}) \\ + [F_{9-10}^p(t) + F_{9-10}^d(t)] r_9 = 0 \\ J_{10} \ddot{\theta}_{10} - [F_{9-10}^p(t) + F_{9-10}^d(t)] r_{10} \\ + G_{11} \frac{\pi D_{11}^4}{32 l_{11}} \left(1 - \frac{d_{11}^4}{D_{11}^4} \right) (\theta_{10} - \theta_{11}) = 0 \end{cases} \quad (10)$$

Node 11 (Shaft node):

$$\begin{cases} J_{11} \ddot{\theta}_{11} + G_{11} \frac{\pi D_{11}^4}{32 l_{11}} \left(1 - \frac{d_{11}^4}{D_{11}^4} \right) (\theta_{11} - \theta_{12}) \\ - G_{11} \frac{\pi D_{11}^4}{32 l_{11}} \left(1 - \frac{d_{11}^4}{D_{11}^4} \right) (\theta_{10} - \theta_{11}) = 0 \end{cases} \quad (11)$$

Node 12 (Meshing Pair E):

$$\begin{cases} J_{12} \ddot{\theta}_{12} + [F_{12-13}^p(t) + F_{12-13}^d(t)] r_{12} \\ + G_{11} \frac{\pi D_{11}^4}{32 l_{11}} \left(1 - \frac{d_{11}^4}{D_{11}^4} \right) (\theta_{12} - \theta_{11}) = 0 \\ J_{13} \ddot{\theta}_{13} - [F_{12-13}^p(t) + F_{12-13}^d(t)] r_{13} \\ + G_{14} \frac{\pi D_{14}^4}{32 l_{14}} \left(1 - \frac{d_{14}^4}{D_{14}^4} \right) (\theta_{13} - \theta_{14}) = 0 \end{cases} \quad (12)$$

Node 14 (Shaft node):

$$J_{14} \ddot{\theta}_{14} + G_{14} \frac{\pi D_{14}^4}{32 l_{14}} \left(1 - \frac{d_{14}^4}{D_{14}^4} \right) \theta_{14} = T_t \quad (13)$$

Node 15 (Shaft node):

$$\begin{cases} J_{15} \ddot{\theta}_{15} + G_{15} \frac{\pi D_{15}^4}{32 l_{15}} \left(1 - \frac{d_{15}^4}{D_{15}^4} \right) (\theta_{15} - \theta_{16}) \\ - G_{15} \frac{\pi D_{15}^4}{32 l_{15}} \left(1 - \frac{d_{15}^4}{D_{15}^4} \right) (\theta_9 - \theta_{15}) = 0 \end{cases} \quad (14)$$

Node 16-23 (Meshing Pair F and G):

$$\left\{ \begin{aligned} & J_{16} \ddot{\theta}_{16} + \sum_{i=1}^6 \left[F_{spi}^p(t) + F_{spi}^d(t) \right] r_{16} \\ & + G_{15} \frac{\pi D_{15}^4}{32 l_{15}} \left(1 - \frac{d_{15}^4}{D_{15}^4} \right) (\theta_{16} - \theta_{15}) = 0 \\ & J_{pi} \ddot{\theta}_{pi} - \left[F_{spi}^p(t) + F_{spi}^d(t) \right] r_{pi} \\ & + \left[F_{cpi}^p(t) + F_{cpi}^d(t) \right] r_{pi} = 0 \\ & J_{23} \ddot{\theta}_{23} - \sum_{i=1}^6 \left[F_{cpi}^p(t) + F_{cpi}^d(t) \right] r_{pi} \\ & + G_{24} \frac{\pi D_{24}^4}{32 l_{24}} \left(1 - \frac{d_{24}^4}{D_{24}^4} \right) (\theta_{23} - \theta_{24}) = 0 \end{aligned} \right. \quad (15)$$

Node 24 (Shaft node):

where T_{Ej} is denoted as the torque of engine j ($j=1,2,3$); D and d are outer and inner diameters of the hollow shaft; G is shear elastic modulus of shaft; T_t is output torque of tail chain; T_r is output torque of rotor shaft.

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\} \quad (17)$$

The excitation $\{F\}$ could be expanded to Fourier series with the fundamental frequency as well, k^{th}

order excitation is:

The excitation causes the system to generate a response: [22]

here $\{B_1\}_k$ and $\{B_2\}_k$ could be solved by the following equation:

The dynamic response is linear superposition of the results corresponded by each order:

3. Numerical calculation

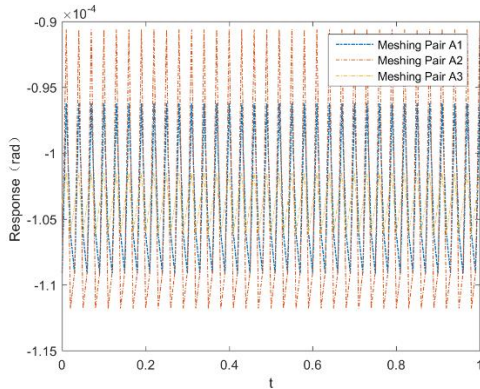
TABLE 1: Four-stage helicopter main gearbox parameters.

element								0.04	
Meshing element C, D	8(i)	65	3	0.42	8; 12; 10	$-\pi$; 0; $-\pi/6$	0; 0; 0	N/A	N/A
	9	144	3	0.42	8; 12; 10	$-\pi$; 0; $-\pi/6$	0; 0; 0	N/A	N/A
	10	70	3	0.42	10	$-\pi/6$	0	N/A	N/A
Shaft element	11	N/A	N/A	N/A	N/A	N/A	N/A	0.04/ 0.03	1.4
Meshing element E	12	40	3.5	0.47	12	$\pi/9$	0	N/A	N/A
	13	70	3.5	0.47	12	$\pi/9$	0	N/A	N/A
Shaft element	14	N/A	N/A	N/A	N/A	N/A	N/A	0.04/ 0.03	1.1
Shaft element	15	N/A	N/A	N/A	N/A	N/A	N/A	0.14/ 0.11	0.8
Meshing element F, G	16	68	5	0.4	13;8; 6;8; 12;5	$-4\pi/3; \pi/9$; $-\pi/7; -\pi/9$; $\pi/3; -\pi/9$	π ; 0; $4\pi/7$; 0; $-\pi/3$; 0	N/A	N/A
	17	37	5	0.4	13;8	$-4\pi/3$; $-\pi$	π ; $-\pi/3$	N/A	N/A
	18	37	5	0.4	8;9	$\pi/9$; 0	0; $4\pi/3$	N/A	N/A
	19	37	5	0.4	6;7	$-\pi/7$; $4\pi/3$	$4\pi/7$; $-\pi$	N/A	N/A
	20	37	5	0.4	8;6	$-\pi/9$; $-\pi/9$	0; $-4\pi/3$	N/A	N/A
	21	37	5	0.4	12;6	$\pi/3$; $\pi/9$	$-\pi/3$; 0	N/A	N/A
	22	37	5	0.4	5; 8	$-\pi/9$; $-5\pi/3$	0; π	N/A	N/A
	23	142	5	0.4	8; 9; 7 6; 6; 8	$-\pi$; 0; $4\pi/3; -\pi/9$; $\pi/9; -5\pi/3$	$-\pi/3$; $4\pi/3$; $-\pi$; $4\pi/3$; 0; π	N/A	N/A
Shaft element	24	N/A	N/A	N/A	N/A	N/A	N/A	0.12/ 0.09	2.0

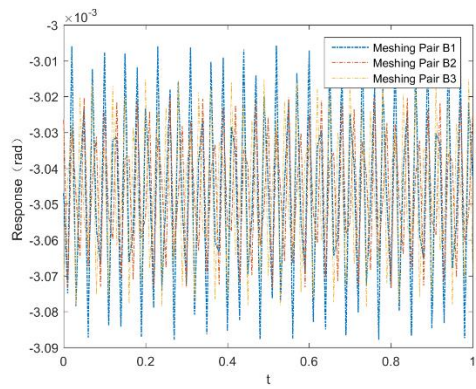
The calculation results can be seen from the Fig.2, the dynamic response exhibits periodic vibrations under multi-frequency excitation caused by time-varying meshing stiffness and transmission errors, thus the response amplitudes are different. The gear rotational frequency reduces with the transmission chain following to the output branch, in other words, the deceleration effect is obvious.

By comparison, it indicates that the dynamic

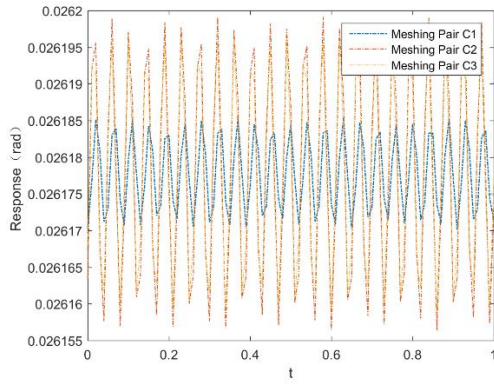
response of meshing pair E has the maximum amplitude due to the heavy tail output load. In addition, the magnitude of the response in the meshing pair C, especially pair C3, has relatively higher response as well, as a result of the three-input branches converging at the central gear. Regarding the planetary gear train, response of planetary gear 1 is greater than other planetary gears due to its largest transmission and manufacturing error.



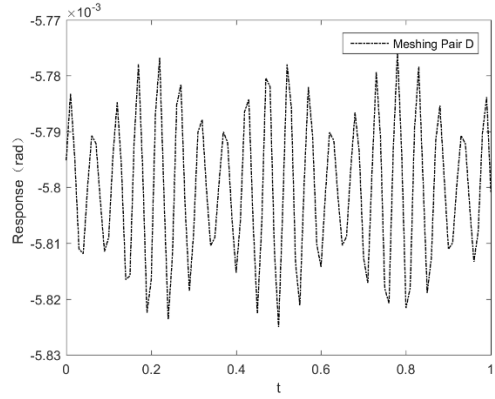
(a) Meshing pair A



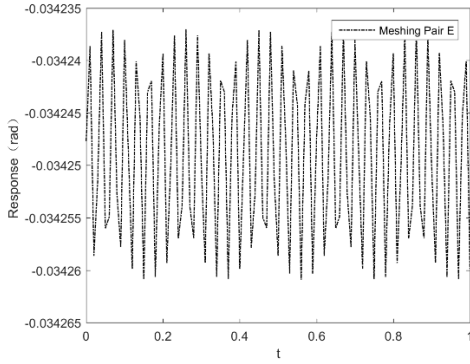
(b) Meshing pair B



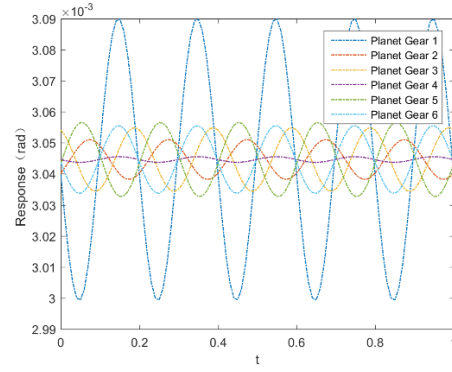
(c) Meshing pair C



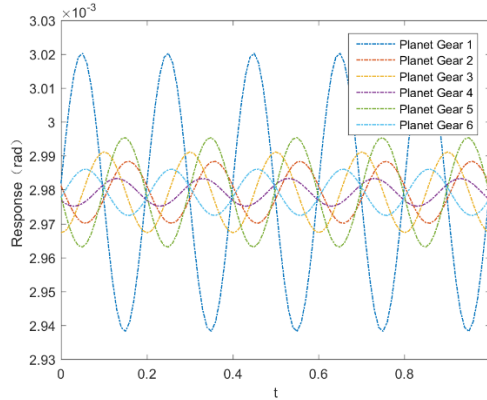
(d) Meshing pair D



(e) Meshing pair E



(f) Meshing pair F



(g) Meshing pair G

FIGURE 2: Dynamic response regarding tooth deflection in each gear pair.

3.2 Sensitivity analysis of responses influenced by shaft wall thickness Sensitivity analysis method is widely used to study the key variables under many uncertainties, which can provide a parametric reference for the design of four-stage helicopter gearbox. In this paper, the ratio of the inner and outer diameters of the shaft is regarded as a variable, the sensitivity coefficient of each pair is calculated, and the key shafts affecting the response characteristics are investigated.

The ratio of the inner and outer diameters of the shaft is defined as α , the sensitivity coefficient of the response $x(t)$ to α is defined as follows:

$$S_{\alpha}^x = \frac{|(x'_{\alpha'} - x_{\alpha}) / x_{\alpha}|}{|(\alpha' - \alpha) / \alpha|} \times 100\% \quad (22)$$

here, x and x' are the response amplitudes corresponding to the ratio of α and α' .

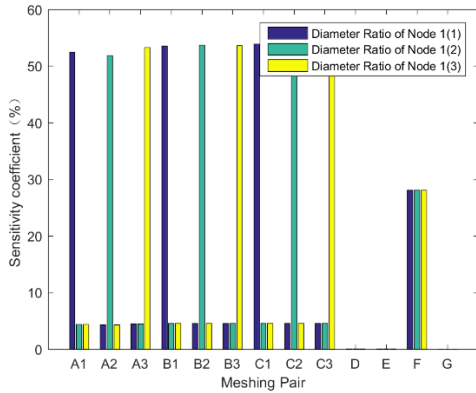
By changing the diameter ratio α , the sensitivity coefficient of each meshing pair is shown in Fig.3. According to Fig.3 (a), the sensitivity coefficients of Node 1(i) to meshing pair (A, B, and C) are greater than 50%, and the coefficient is gradually increased with the transmission chain. Therefore, the diameter ratio of Node 1 is the most sensitive to meshing pair C. In addition, Node 1 is also about 28% sensitive to the response of the planetary gear system, and the other meshing pairs are insensitive to this node.

According to Fig.3 (b), the sensitivity coefficient of Node 4 and meshing pair A is the highest, which directly affects the dynamic response

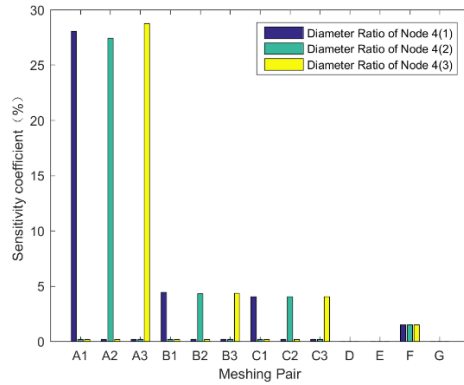
of the first-stage deceleration, and the sensitivity of the subsequent stages of is gradually reduced. The sensitivity of Node 7 in Fig.3 (c) has the similar influence law as that of Node 1, and the sensitivity coefficients are quite close as well. In Fig.3 (d), Node 11 has the highest sensitivity (14.5% and 38%) with its adjacent meshing pair D and meshing pair E. In Fig.3 (e), Node 14 are highly sensitive to the meshing pairs of the tail chain, which includes meshing pair D and E. In Fig.3 (f), the wall thickness of Node 15,

the input shaft of the sun gear, directly affects the response of the planetary gear train, thus it is the most critical node of the meshing pair F. Node 24 in Fig.3 (g), the rotor shaft, has the sensitivity of 17% to the meshing pair G, and the coefficient is much larger than that of other nodes.

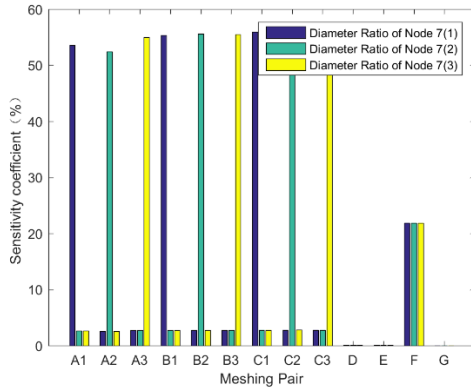
From these figures, the sensitivity correspondence and shaft selection method regarding wall thickness could be generalized in Table 2, which could be a reference for reducing the vibration shock.



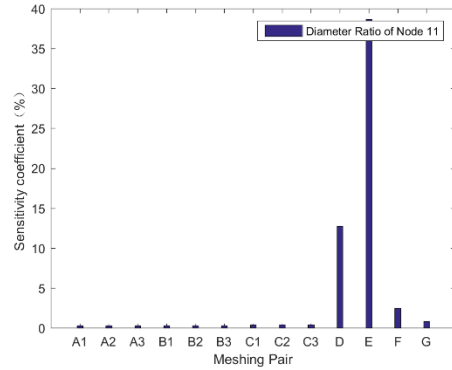
(a) Node 1



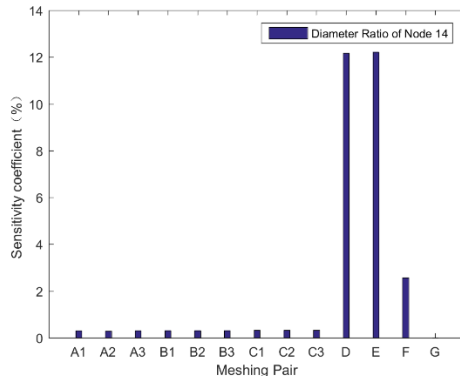
(b) Node 4



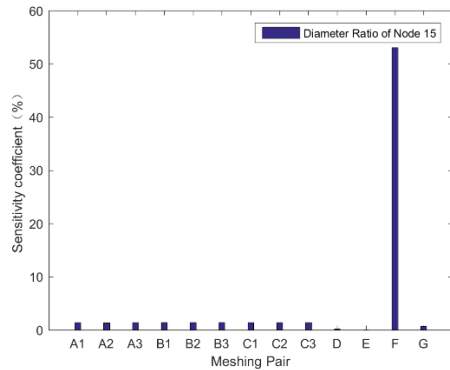
(c) Node 7



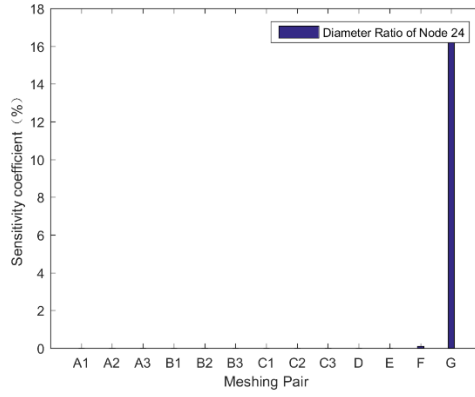
(d) Node 11



(e) Node 14



(f) Node 15



(g) Node 24

FIGURE 3: Sensitivity coefficient of each meshing pair influenced by α .

TABLE 2: Sensitivity Correspondence of wall thickness.

Meshing Pair	Most sensitive node
A, B, C	Node 1 and Node 7
D	Node 14
E	Node 11
F	Node 15
G	Node 24

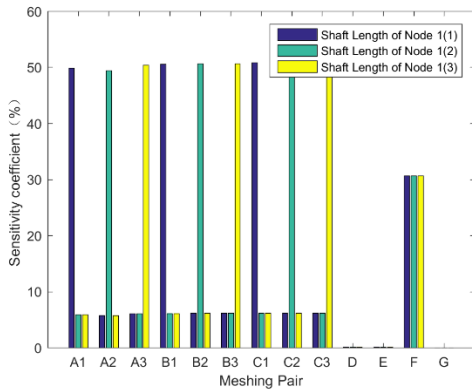
3.3 Sensitivity analysis of responses influenced by shaft length The sensitivity coefficient of the response $x(t)$ to shaft length is defined as follows:[23]

$$S_l^x = \frac{|(x_l' - x_l) / x_l|}{|(l' - l) / l|} \times 100\% \quad (23)$$

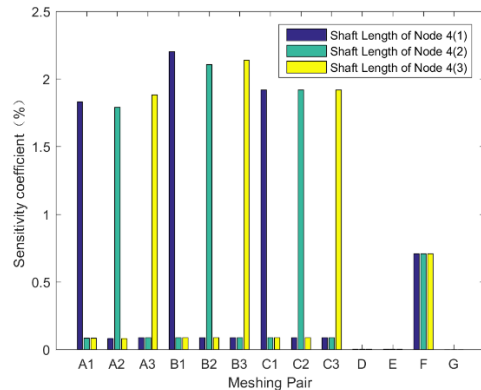
where x and x' are the response amplitudes corresponding to length l and l' .

By changing the only variable parameter shaft length l , and the sensitivity coefficient of each meshing pair is calculated in the same way, as is shown in Fig.4. From Fig.4 (a) to Fig.4 (c), it can be seen that the three meshing pairs of Node 1, Node 4, and Node 7 have the highest sensitivity to three meshing pairs in the input branch, but the sensitivity

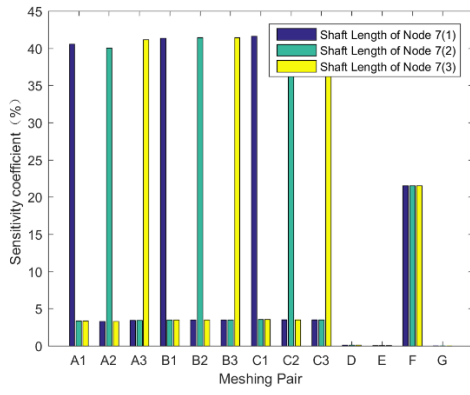
coefficient of Node 4 is relatively low. In Fig.4 (d) and Fig.4 (e), Node 11 and Node 14 are similar in the influence law, they both have about 10% sensitivity coefficient to meshing pair D and E. In Fig.4 (f), The coefficient of Node 15 to meshing pair F in the planetary gear system are 44%, with the coefficient depicted in Fig.4 (f), it could be concluded that the wall thickness and shaft length of Node 15 are critical parameters to internal meshing pair in the planetary gear train. In Fig.4 (g), although Node 24 is most sensitive to the external meshing pair of the planetary gear system, the sensitivity coefficient is only 0.12%. In addition, the highest response sensitivity correspondence regarding shaft length is generalized in Table 3.



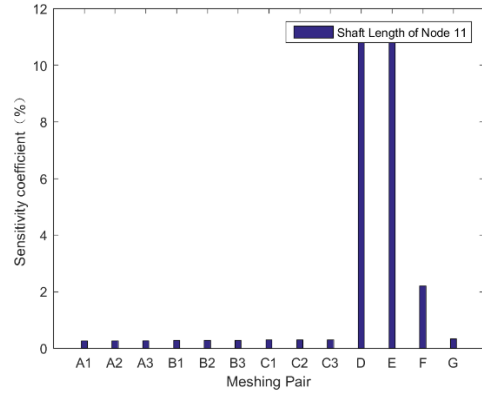
(a) Node 1



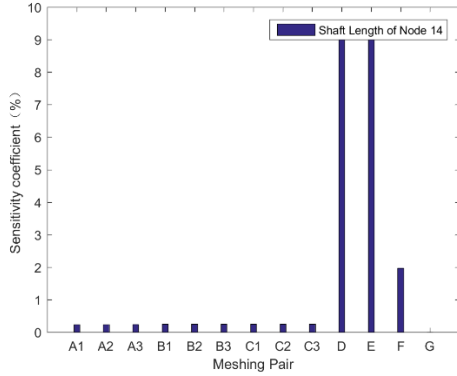
(b) Node 4



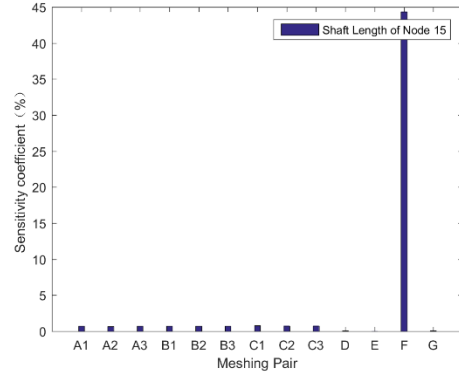
(c) Node 7



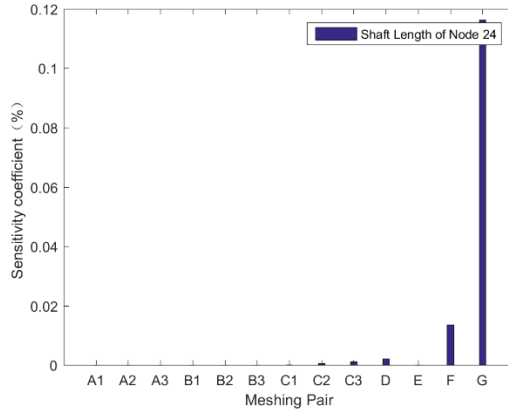
(d) Node 11



(e) Node 14



(f) Node 15



(g) Node 24

FIGURE 4: Sensitivity coefficient of each meshing pair influenced by shaft length.

TABLE 3: Sensitivity Correspondence of shaft length.

Meshing Pair	Most sensitive node
A, B, C	Node 1
D, E	Node 11
F	Node 15
G	None

3.4 Structural parameter optimization based on response sensitivity analysis By sensitivity analysis of the meshing pairs based on the wall thickness and

the shaft length, the most sensitive parameter has been investigated according to Table 2 and Table 3. From Fig.2, it could be noted that the maximum

response amplitude of the four-stage system is located at meshing pair E, according to the sensitivity analysis, the corresponding node is Node 11. The second largest response amplitude is situated at the meshing pair C3, and the corresponding node is Node 1.

Therefore, a parameter optimization method is provided by changing the structural parameters of the shaft node without considering other errors and phase changes. It is initially set that the design range of the shaft diameter ratio α of the Node 1 is 0.6 to 0.9, and the range of the shaft length l is 0.6 to 1.4m.

After going through the parameters in the range, the selected diameter ratio and shaft length is shown in Table 4 as improved group. Similarly, α_{11} is limited to 0.75~0.9, and l_{11} is 1.2~1.8m, the original group and improved group are selected in the same way. Calculate the dynamic response of the original group and the improved group separately, as shown in Fig.5. From the figure and table, it can be concluded that the response of meshing pair C3 is reduced by 34.7%, and that of meshing pair E is decreased by 36.1%, and the improvement effect is quite obvious.

TABLE 4: Parameter optimization of the hollow shaft.

Parameter	Original group	Improved group	Optimization effect
α_1	0.75	0.6	N/A
l_1	0.8	0.6	N/A
α_{11}	0.75	0.9	N/A
l_{11}	1.4	1.8	N/A
X_{C3}	0.0262	0.0171	-34.7%
X_E	0.0343	0.0219	-36.1%

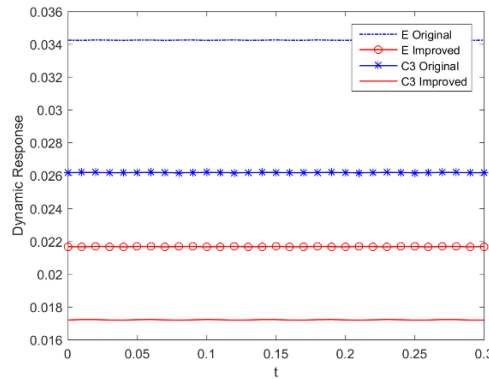


FIGURE 5: Dynamic response optimization comparison regarding tooth deflection.

4 Conclusion

Based on the new dynamic modeling method and the response characteristics analysis, the overall multi-node dynamic model of a four-stage helicopter main gearbox is established. By solving coupling differential equations, the sensitive laws of wall thickness and length of the hollow shaft are obtained. Therefore, the a structural parameter optimization method is proposed to improve the original parameter group. The results enable us to draw the following conclusions:

- 1) The magnitude of dynamic response in meshing pair E and C3 are higher than that of other meshing pairs due to the load torque and converging influence.
- 2) Each meshing pair has the most sensitive node according to the sensitivity analysis, parameter sensitivity correspondence tables are proposed for main gearbox design.
- 3) Regarding the rotor shaft (Node 24), the wall

thickness has great sensitivity to response, nevertheless the influence of shaft length is negligible.

- 4) In the design of the four-stage helicopter main gearbox, the shaft corresponding to Node 1 should be relatively short and thick, and the shaft corresponding to Node 11 should be comparatively long and thin, which can effectively reduce the response of the system.
- 5) The sensitivity analysis method can be applied to quickly select the corresponding shaft node and its parameters for reducing the response, which has a good auxiliary effect on system modification. The improvement effect is obvious according to the group comparison.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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