

Benchmarking regions using a heteroskedastic grouped random parameters model with heterogeneity in mean and variance: applications to grade crossing safety analysis

Shahram Heydari (Corresponding author), PhD

Research Associate
Imperial College London
Centre for Environmental Policy
London, UK
Email: s.heydari@imperial.ac.uk

Liping Fu, PhD

Professor
Department of Civil and Environmental Engineering
University of Waterloo,
Waterloo, Canada
Email: lfu@uwaterloo.ca

Lalita Thakali, PhD

Postdoctoral Fellow
Department of Civil and Environmental Engineering
University of Waterloo
Waterloo, Canada
Email: lthakali@uwaterloo.ca

Lawrence Joseph, PhD

Professor
Department of Epidemiology, Biostatistics and Occupational Health
McGill University
Montreal, Canada
E-mail: lawrence.joseph@mcgill.ca

ABSTRACT

Comparing regions while adjusting for differences in characteristics of sites located in those regions is valuable since it identifies possible inter-regional dissimilarities in crash risk propensities according to specific safety performance measures (e.g., crash frequency of a specific type). This paper describes a framework to benchmark different regions (neighborhoods, provinces, etc.) in terms of a selected safety performance measure. To avoid issues relating to aggregated (macro-level) data, we use disaggregate (micro-level) data to draw inferences at a macro/region-level, which is often needed for developing large-scale transportation safety and planning programs and policies. To overcome unobserved heterogeneity, we employ a multilevel Bayesian heteroskedastic Poisson lognormal model with grouped random parameters allowing heterogeneity in both mean and variance parameters. The proposed approach is illustrated through a comprehensive study of highway railway grade crossings across Canada. The results indicate that the proposed model addresses unobserved heterogeneity more efficiently and provides more insight compared to conventional random parameters models. For example, we found that as traffic exposure increases, grade crossing safety deteriorates at a higher rate in the Canadian Prairies than in the other regions. Our benchmarking framework is also affected by different model specifications. The results indicate the need for further in-depth investigations, which could help to identify possible reasons for inter-region differences in terms of specific safety indicators. This study provides valuable guidelines to Canadian transportation authorities, revealing important underlying crash mechanisms at highway railway grade crossings in Canada.

1. Introduction

The presence of a vast railway network in Canada imparts some risk to road/rail users and to residents living around railway lines. As reported by the Transportation Safety Board of Canada, 11,736 rail accidents of various forms have been observed over a ten-year period from 2006 to 2015 ([Transportation Safety Board of Canada, 2015](#)). According to these data, around 17% of all rail accidents have occurred at highway-railway grade crossings. Despite improvements in the recent years, the number of grade crossing crashes remains high so that grade crossing safety is still a significant concern for transportation authorities and Canadian society as a whole. For example, the Transportation Safety Board of Canada reports 1,953 crossing accidents for the years 2006-2015, causing 219 fatalities and 267 serious injuries ([Transportation Safety Board of Canada, 2015](#)).

To address the safety concerns at grade crossings, Transport Canada initiated a funding program called “grade crossing improvement program” that provides contributes to the eligible costs of crossing improvements completed by railways and road authorities across Canada. More recently, another funding program, rail safety improvement program has started, which invests millions of dollars each year on various safety improvement programs across Canada. One of the emerging needs to support improved decision-making for this program is to develop knowledge and tools for fair distribution of funding among Canada’s vastly dispersed and different regions. This need calls for research to better understand the complex crash mechanisms at highway-railway grade crossings and develop robust methodology for benchmarking regions in terms of grade crossing safety.

1.1. Unobserved heterogeneity

A key issue in modeling crash data is addressing unobserved heterogeneity – which is caused by unobserved variables (Mannering and Bhat; 2014; Mannering et al., 2016). That is, there are factors (e.g., driver behavior) that contribute to the safety at a site; however, data on these factors may not be available in crash data sets. Consequently, differences between sites, which are similar in their known characteristics, with different crash frequencies may go unexplained by the data. Essentially, an important question that arises here is: could the effects of explanatory variables on safety vary across observations or groups of observations? The answer to this question is affirmative; therefore, approaches such as random parameters modeling and latent class modeling that allow model parameters to vary across sites (or groups of sites) are frequently used in traffic safety studies (Anastasopoulos and Mannering, 2009; El-Basyouny and Sayed, 2009; Dinu and Veeraragavan, 2011; Anastasopoulos et al., 2012; Venkataraman et al., 2014; Chen and Tarko, 2014; Barua et al., 2016; Coruh et al., 2015; Park et al., 2016; Bhat et al., 2017; Heydari et al., 2017a).

A number of traffic safety studies accounted for group effects (instead of site effects) in random effects, random parameters, and latent class modeling to capture variations in unknown/unmeasured factors that vary across groups (Wu et al., 2013; Heydari et al., 2014a; Heydari et al., 2016b; Sarwar et al., 2017; Fountas et al., 2018a and 2018b; Cai et al., 2018). A discussion on the importance of accounting for group (e.g., region) effects in traffic safety research is provided in Heydari et al., 2016b. For example, the latter study discusses how spatially and non-spatially related unobserved factors can be captured to some extent through accounting for group (e.g., region) specific effects. In this paper, we focus on random parameters (slopes) models. Interested readers are referred to

[Mannering et al. \(2016\)](#) for a comprehensive discussion on the unobserved heterogeneity problem and other statistical approaches that help mitigate this problem.

Recently, a few traffic safety studies have attempted to address the unobserved heterogeneity issue by employing random parameters models placing covariates on varying means and/or variances (so-called heterogeneity in means and/or variances models) mostly in crash injury-severity analysis ([Seraneeprakarn et al., 2017](#); [Behnood and Mannering, 2017](#); [Xin et al., 2017](#)). [Venkataraman et al. \(2014\)](#) developed a heterogeneity in mean count model for evaluating the effects of interchange type on heterogeneous influences of interstate geometrics on crash frequencies. The above studies highlight the advantages of the latter approach over conventional random parameters models. Basically, the heterogeneity in mean/variance approach models the within covariate variability (in both mean and variance) as a function of explanatory variables available in data. This could add robustness to the random parameters modeling approach while allowing the analyst to explain variability in model parameters through existing explanatory variables, shedding more light on the underlying mechanisms of traffic safety.

Most traffic safety studies, including those adopting a heterogeneity in mean and/or variance approach, have assumed homoskedasticity in their analysis. One could instead allow for heteroskedasticity that considers a heterogeneous variance. For example, [Hong et al. \(2016\)](#) considered heteroskedasticity in pedestrian exposure modeling. To accommodate heteroskedasticity, a potentially robust approach is to model the variance of the observational level error term as a function of explanatory variables. This allows the analyst to discover the source of heteroskedasticity or dispersion in a data set. Note that Bayesian methods could be extremely useful in terms of the ease of implementing heterogeneity in mean and/or variance models. The use of Bayesian statistics however has been rare if nonexistent in developing heterogeneity in mean/variance models in the extent of traffic safety literature.

While a general discussion on Bayesian methods and their advantages can be found in [Gelman et al. \(2004\)](#), here we provide a brief discussion. The Bayesian approach is promising not only in extending standard models to include more complex components including random parameters models, multilevel models, and multivariate models, but also easily enables combining these models together. While in frequentist statistics, new (often simulation-based) algorithms are needed to estimate more complex models, in Bayesian statistics standard Markov chain Monte Carlo (MCMC) algorithms are mostly used, without any need for developing tailored algorithms. In particular, using freely

available software such as WinBUGS ([Lunn et al., 2000](#)), complex extensions and their computation through standard MCMC methods is often straightforward. The Bayesian approach propagates uncertainty in all layers and across all parameters of a model better (more fully and more intuitively) than its classical counterpart. In fact, it provides full probability densities for model parameters, whereas the frequentist approach typically provides only point estimates and confidence intervals. Bayesian (credible) intervals directly imply the probability of a true parameter value being in an interval, given the data and any prior information included in the modelling process; however, the probability of the true parameter value belonging to any given confidence interval is either 0 or 1. So the Bayesian approach is more sensible when communicating uncertainty in traffic safety applications, which require a continued evaluation of risk and safety. Predictions made by a Bayesian model also tend to include all inherent uncertainty and are more easily communicated, presenting a more complete picture of uncertainties and traffic safety dynamics. Interested readers are referred to [Daziano et al. \(2013\)](#) and [Heydari et al. \(2014b\)](#) for a detailed discussion on potential advantages of the Bayesian framework in transportation research.

With regard to highway-railway grade crossing safety issues, several studies discuss various aspects of crossing safety ([Saccomanno et al., 2004](#); [Millegan et al., 2009](#); [Chaudhary et al., 2011](#); [Chadwick et al. 2014](#); [Tung et al., 2015](#); [Wang et al., 2016a](#); [Heydari et al., 2016b](#); [Haleem, 2016](#); [Liu and Khattak, 2017](#); [Hsu and Jones, 2017](#); [Sperry et al., 2017](#); [Zhang et al., 2017](#); [Larue et al., 2018](#); [Beanland et al., 2018](#)). Many grade crossing studies can be divided into two categories: (1) crash-frequency studies ([Hauer and Persaud, 1987](#); [Austin and Carson, 2002](#); [Saccomanno and Lai, 2005](#); [Park and Saccomanno, 2005](#); [Oh et al., 2006](#); [Yan et al., 2010](#); [Medina and Benekohal, 2015](#); [Heydari and Fu, 2015](#); [Lu and Tolliver, 2016](#); [Heydari et al., 2016a](#); [Heydari et al., 2017b](#); [Guadamuz-Flores and Agüero-Valverde, 2017](#)); and (2) crash-consequence studies ([Eluru et al., 2012](#); [Hao et al., 2015](#); [Ghomi et al., 2016](#); [Zhao et al., 2018](#)).

Relating to crash-frequency modeling at grade crossings, which is the focus of this paper, for example, [Heydari and Fu \(2015\)](#) investigated a Poisson Weibull model. Other researchers adopted alternative statistical models such as zero-inflated, hurdle, and generalized event count models to address issues such as the excess number of zero accidents and underdispersion ([Lee et al., 2004](#); [Oh et al., 2006](#); [Lu and Tolliver, 2016](#); [Ye et al., 2018](#)). Zero-inflated models have been successfully implemented in other traffic safety applications as well ([Anastasopoulos, 2016](#)). However, zero-inflated count models have been criticized due to their assumption of safe state, which seems to be unrealistic in the context of road safety analysis ([Lord et al., 2005](#); [Lord et al., 2007](#)). Previous studies

on grade crossings provide valuable insight; however, the use of random parameters models in the extent of grade crossing crash frequency modeling is rare. This is while advantages of using random parameters modeling are highlighted in other traffic safety applications. Note that unobserved heterogeneity could easily manifest itself in grade crossing crash data sets due to the complexity of crash mechanisms at crossings and lack of sufficient data.

1.2. Comparing sites: micro- and macro-level approaches

In traffic safety research, comparing road infrastructures in terms of their safety performances is usually achieved through a high crash location identification process. Most studies have focused on the identification of high crash locations at a micro-level (e.g., intersections or highway segments) (Persaud et al., 1999; Heydecker and Wu, 2001; Saccomanno et al., 2004; Miaou and Song, 2005; Cheng and Washington, 2005; Brijs et al. 2007; Miranda-Moreno et al., 2007; Elvik 2008; Montella, 2010; Heydari et al., 2013; Coll et al., 2013; El-Basyouny and Sayed, 2013; Qu and Meng, 2014; Jiang et al., 2014; Yu et al., 2014; Thakali et al., 2015; Sacchi et al., 2016; Lipovac et al., 2016; Fu, 2016; Fawcett et al., 2017; Debrabant et al., 2018). However, some recent studies have adopted a macro-level or macroscopic (e.g., region level) high crash location identification approach (Lee et al., 2015; Huang et al., 2016; Dong et al., 2016).

Micro-level models provide detailed information about safety of road infrastructure without the need for any substantial data aggregation. In contrast, macroscopic safety models (Hadayeghi et al., 2010; Ukkusuri et al., 2012; Wei and Lovegrove, 2013; Lee et al., 2014; Osama and Sayed, 2016; Wang et al., 2016b; Amoh-Gyimah et al., 2017; Lee et al., 2017) require a certain level of aggregation based on the spatial unit under investigation. While a macro-level approach can provide insights that are useful for planning and policy development (Washington et al., 2010), it may suffer from some shortcomings relating to the aggregation of data, which could, for instance, lead to ecological fallacy (Davis 2004). Previous work on grade crossing safety with a focus on making macro-level inferences is rare (Truong et al., 2016; Heydari et al., 2016b). To our knowledge, only Heydari et al. (2016b) draw macro-level inferences for grade crossings using disaggregates crossing-level crash data. Huang et al. (2016) discusses merits of both micro- and macro-level approaches. To avoid aggregation problems related to macroscopic analysis, the crash literature suggests using a micro-level approach and then summing up safety measurements (e.g., expected crash frequencies or differing injury-severity levels) of all sites located in each region to obtain an overall (macro-level) safety measure for each region (Huang et al. 2016). While allowing the identification of

macro level high crash locations, this method uses site specific characteristics at a micro-level, avoiding data aggregation problems.

Nevertheless, this approach may not be coherent for some purposes since regions with higher exposure or number of sites may appear at the top of the ranking list. In fact, these may not necessarily be the most dangerous regions in terms of crash risk because no adjustment is considered to account for differences in independent variables (contributing factors) that can exist across regions. If the purpose of a study is to simply identify regions with higher absolute numbers of crashes, then there is no need to adjust. For example, when the aim is to allocate ambulances to regions, the interest could mainly lie in the total injury frequency. However, such approaches are problematic when one is interested in fair comparison between different regions aiming to identify those with higher crash risk after differences in site-specific characteristics are accounted for. This could initiate further in-depth studies for understanding reasons for inter-region variations in terms of specific safety performance measures. For example, one could examine characteristics (sociodemographic, climate, etc.) of different regions - which were initially ignored - aiming at explaining revealed inter-region differences.

1.3. The current paper

This paper makes four contributions: (1) it introduces a method to benchmark regions according to pre-specified safety performance measures using disaggregate data, avoiding issues related to the aggregation of data; (2) to overcome unobserved heterogeneity, we employ and discuss a Bayesian heteroskedastic multilevel random parameters Poisson lognormal model with heterogeneity in mean and variance (to our knowledge, this is the first instance of such a model in traffic safety research); (3) it provides comparisons with conventional random parameters and random intercepts models with a focus on discussing the depth of insights provided by each model and investigating how our benchmarking is affected by different model formulations; and (4) the empirical work undertaken in this article focuses on a country-wide study of level crossing safety. This could result in an enhanced understanding of grade crossing safety mechanisms that lend itself to safety policy, resulting in more cost-effective countermeasures.

2. Canada-wide highway railway grade crossing data

A comprehensive data set containing almost all public highway railway grade crossings in Canada was used. The data contains crash counts at 16,549 highway railway grade crossings of different types (passive and active) located in eight major Canadian provinces (British Columbia (BC), Alberta (AB), Saskatchewan (SK), Manitoba (MB),

Ontario (ON), Quebec (QC), New Brunswick (NB), Nova Scotia (NS)), for the period 2008 to 2013. Passive crossings are those equipped with signs such as crossbucks, stop sign, etc., while active crossings are equipped with flashing lights and bells (FLB) or flashing lights, bells, and gates (FLBG) to advise road users of the presence of crossings and trains. Sixty percent of crossings are passive while the remaining forty percent are active crossings. The distribution of crossings in different provinces is shown in Fig. 1, and the spatial distribution of grade crossings across Canada is illustrated in Fig. 2 for both passive and active groups of grade crossings. The final dataset includes 860 grade crossing crashes for the above-mentioned period.

To prepare the data set, we combined two databases, namely RODS (railway occurrence database system) and IRIS (integrated railway information system). Data relating to the occurrence of grade crossing crashes across Canada are recorded in RODS; grade crossing characteristics are collected in IRIS. Variables such as daily train volume, vehicle volume, geometric and operational characteristics such as train speed (rail speed limit), road speed (road speed limit), and type of warning devices were available in the data set. We used total traffic exposure, the product of train volume and vehicle flow. The model with this exposure term was found to provide a better fit to the data than the model containing train volume and vehicle flow separately. Several interaction terms were also considered, but these were not found to be important in explaining safety at Canadian crossings. Due to high co-linearity, we could not include some variables in the model at the same time. Summary statistics of the dataset are provided in Table 1. Note that our data set has a hierarchical structure, that is, crossings are nested within different provinces. In other words, we deal with groups of crossings that are situated within distinct regions; i.e., a multilevel setting. We will see in Section 3 that our models accommodate such structure.

3. Methodology

In this section, first, we discuss three different multilevel approaches: a random intercepts model, a grouped random parameters model, and a heteroskedastic grouped random parameters model with heterogeneity in mean and variance. Second, we introduce our risk-adjusted approach that allows for benchmarking different regions in a fair manner. We then discuss the elicitation of priors for our MCMC scheme, the computation of the models, and the model performance framework in terms of replicating excess zero counts. In this research we adopted a Poisson lognormal model, which can accommodate overdispersion in crash count data (Winkelmann, 2008). Lord and Miranda-Moreno (2008) recommend Poisson lognormal models - over negative binomial (Poisson gamma) models - when a crash data set has a low mean value. For

more details related to the specification of Poisson lognormal models, see [Heydari et al., 2016a](#).

3.1. Standard random intercepts model

A standard multilevel random intercepts Poisson lognormal model can be defined as follows. Let y_j and θ_j be, respectively, the observed and expected crash frequencies for site j (e.g., intersection, highway segment). Let \mathbf{X} and $\boldsymbol{\gamma}$ represent the vectors of site attributes and their respective parameters. Let η_r denote the varying intercepts (here, region effects) that follow a normal density with the mean μ_r and the variance v_r for region r . Let ε_j be a normally distributed error term with the mean 0 and the variance v_ε at the site (here, grade crossing) level to account for extra variation that is not captured by explanatory variables. This model can then be written as

$$\begin{aligned}
 y_j | \mathbf{X}_j, \boldsymbol{\gamma}, \varepsilon_j, \eta_r &\sim \text{Poisson}(\theta_j) \\
 \theta_j &= \lambda_j * e^{\varepsilon_j} \\
 \log(\lambda_j) &= \eta_r + \boldsymbol{\gamma} \mathbf{X}_j \quad (1) \\
 \eta_r | \mu_\eta, v_\eta &\sim \text{normal}(\mu_\eta, v_\eta) \\
 \varepsilon_j | v_\varepsilon &\sim \text{normal}(0, v_\varepsilon)
 \end{aligned}$$

In the above model, random intercepts vary from one region (e.g., province) to another, reflecting between-region variations in unobserved or unmeasured factors. In Poisson lognormal models, e^ε is lognormally distributed, meaning that ε follows a normal density. As discussed by [Winkelmann \(2008\)](#), this specification is appealing from a theoretical standpoint because the sum of a large number of unobserved independent variables that affect an outcome of interest is expected to follow a normal density based on the central limit theorem.

3.2. Standard random parameters (slopes) model

A standard random parameter model that allows the effect of some covariates \mathbf{Z} vary across the population can be written as follows. In this model $\boldsymbol{\beta}$ is the vector of random parameters (varying across regions) associated with \mathbf{Z} . This is usually assumed to be normally distributed with the vectors of mean and variance $\boldsymbol{\mu}_\beta, \boldsymbol{v}_\beta$, respectively. The latter parameters are assumed to be fixed across observations or groups of observations.

$$\begin{aligned}
y_j | \mathbf{Z}_r, \mathbf{X}_j, \boldsymbol{\gamma}, \boldsymbol{\beta}_r, \varepsilon_j, \eta_r &\sim \text{Poisson}(\theta_j) \\
\log(\theta_j) &= \eta_r + \boldsymbol{\beta}_r \mathbf{Z}_r + \boldsymbol{\gamma} \mathbf{X}_j + \varepsilon_j \\
\eta_r | \mu_\eta, v_\eta &\sim \text{normal}(\mu_\eta, v_\eta) \\
\boldsymbol{\beta}_r | \boldsymbol{\mu}_\beta, \mathbf{v}_\beta &\sim \text{normal}(\boldsymbol{\mu}_\beta, \mathbf{v}_\beta) \\
\varepsilon_j | v_\varepsilon &\sim \text{normal}(0, v_\varepsilon)
\end{aligned} \tag{2}$$

3.3. Extension to heteroskedastic random parameters model with heterogeneity in mean and variance

The extension discussed in this section allows us to infer more detailed information on crash propensities and potential sources of variations and dispersion in crash data. To this end, the means and variances of the random parameters $\boldsymbol{\beta}$ are not fixed anymore and can, respectively, be modeled as a function of vectors of explanatory variables \mathbf{S}_μ and \mathbf{S}_v , which may or may not be a subset of principal model covariates \mathbf{X} or \mathbf{Z} . That is, for each varying mean μ (being the intercept's mean μ_η or the random parameters' means $\boldsymbol{\mu}_\beta$) and each varying variance v (being the intercept's variance v_η or the random parameters' variances \mathbf{v}_β), we can write

$$\begin{aligned}
\mu_r &= \alpha_0 + \boldsymbol{\alpha} \mathbf{S}_\mu \\
v_r &= \delta_0 + \boldsymbol{\delta} \mathbf{S}_v
\end{aligned} \tag{3}$$

where α_0 and δ_0 are intercepts; $\boldsymbol{\alpha}$ and $\boldsymbol{\delta}$ are the vectors of coefficients associated with explanatory variables in the mean and variance functions.

Note that in our model formulation, random parameters vary at the regional level r rather than observational level j . To allow for heteroskedasticity in the error term, we can allow the variance v_ε of the observational level error term to vary across observations j , where the varying variance (i.e., $v_{\varepsilon j}$) is modeled as a function of explanatory variables \mathbf{S}_ε with the intercept ω_0 and the vector of coefficients $\boldsymbol{\omega}$. Also, \mathbf{S}_ε may or may not be a subset of principal model covariates.

$$\begin{aligned}
\varepsilon_j | v_\varepsilon &\sim \text{normal}(0, v_{\varepsilon j}) \\
v_\varepsilon &= \omega_0 + \boldsymbol{\omega} \mathbf{S}_\varepsilon
\end{aligned} \tag{4}$$

3.4. Risk-adjusted comparison of regions

The idea here is to compare different regions after adding covariates (risk factors) to the model, to identify those with the highest risk of traffic accidents after adjustment for these risk factors. This adjustment is needed since, for example, regions (here, Canadian provinces) are often dissimilar in terms of traffic exposure of sites located in each region, the number of sites in each region, etc. Regions can be compared based on region effects η_r , which now represent adjusted crash risk. In comparison to macro-level models, an important advantage of our approach is that it provides detailed insight on factors (micro-level site characteristics) associated with crash frequencies.

In the above process of ranking or comparing based on region effects, caution should be taken since different sets of risk factors in the model may produce different results. To draw more reliable conclusions in this study, we first used several explanatory variables in the model to make sure no important variable is dropped from the model, affecting the adjusted risk drastically. Second, we computed probabilities that the adjusted crash risk in one region exceeds that of the other regions. Doing so, we were able to account for uncertainties associated with the estimated region effects. Consequently, our benchmarking framework is a fully probabilistic approach that allows us to account for uncertainties in our benchmarking framework.

By computing the above probabilities, we make pairwise comparisons between different regions. Under the Bayesian framework computing the above probabilities is straightforward. To this end, we create an $N \times N$ matrix of indicator variables $[I_{ik}]_{N \times N}$ comparing region i with region k at each iteration of our MCMC simulations based on their region effects η :

$$I_{ik} = \begin{cases} 0 & \text{if } \eta_i \leq \eta_k \\ 1 & \text{if } \eta_i > \eta_k \end{cases} \quad (5)$$

Averaging this indicator variable value over all iterations results in the probabilities of interest. We also compare regions summing up expected crash frequencies of sites nested within each region – as this is suggested in traffic safety literature discussed in Section 1.2. Recall that the latter approach (summing up) does not adjust for differences in site characteristics and the number of sites in each region. In contrast, the risk adjusted approach discussed in this section accounts for such differences, providing a fairer comparison.

3.5. Prior specification and posterior inference

We used non-informative priors across all model parameters. Specifically, gamma priors for the inverse of the variances and normal priors for other model parameters were used. We estimated the above models employing MCMC simulations using WinBUGS. The posterior inferences for model parameters are obtained from two chains with 30,000 iterations. The first 10,000 iterations were discarded to ensure convergence. Therefore, posterior inferences are drawn from a total of 40,000 samples. We ensured the sufficiency of the samples using Monte Carlo error estimates, history plots, and BGR diagrams based on the Gelman-Rubin statistic (Gelman and Rubin, 1992).

3.6. Model performance based on replicating excess zero counts

Due to the large number of zero counts in our data set, it is important to examine whether our model adequately predicts the observed number of zero-crash highway-railway grade crossings. We investigated this issue by computing a Bayesian p -value statistic based on the following algorithm.

- i. Using the estimated expected crash frequency for each grade crossing j , predict accidents for each crossing - at each iteration of the MCMC simulations.

$$y_{j(\text{predicted})} \sim \text{Poisson}(\lambda_j) \quad (6)$$

- ii. At each iteration verify whether the predicted and observed crash frequencies (for each crossing) are equal to zero, creating two indicator variables $I_{\text{predicted}}$ and I_{observed} .

$$I_{j(\text{predicted})} = \begin{cases} 1 & \text{if } y_{j(\text{predicted})} = 0 \\ 0 & \text{if } y_{j(\text{predicted})} \neq 0 \end{cases} \quad (7)$$

$$I_{j(\text{observed})} = \begin{cases} 1 & \text{if } y_{j(\text{observed})} = 0 \\ 0 & \text{if } y_{j(\text{observed})} \neq 0 \end{cases} \quad (8)$$

- iii. At each iteration, compute another indicator variable II_j for each crossing to compare $I_{\text{predicted}}$ and I_{observed} .

$$II_j = \begin{cases} 1 & \text{if } (I_{j(\text{predicted})} - I_{j(\text{observed})}) \geq 0 \\ 0 & \text{if } (I_{j(\text{predicted})} - I_{j(\text{observed})}) < 0 \end{cases} \quad (9)$$

- iv. Estimate the Bayesian p -value by averaging the above quantity II_j over all iterations and across all crossings in the data.

The above algorithm can be readily implemented in WinBUGS. A value of 0.5 indicates an ideal match between observed and predicted crash counts. See [Gelman et al. \(1996\)](#) for a detailed discussion.

4. Results and discussions

We discuss our findings in the following three sections, focusing on posterior estimates of the parameters, interpretation of the explanatory variables, potential explanations for our findings, and implications for benchmarking different regions. Since we analyze crash data covering a six-year period, temporal instability ([Mannering, 2018](#)) could affect our results, that is, the effect of covariates on safety could change over time. It would be interesting to investigate temporal instability; however, addressing this issue is beyond the scope of this paper. In this paper we did not include hierarchical-level (province-level) explanatory variables in our models because with an effective sample size of only 8 (i.e., our data are generated from 8 provinces), such coefficients would not be estimated accurately. Without loss of generality, one could include region-level variables in implementing our proposed approach when a data set allows. Also, the focus here is to show how to use disaggregate (micro-level) data to make inferences at a macro-level. As discussed in Section 1.1, macro-level inferences may be useful; for example, for planning large-scale transportation and safety policy so integrating macro and micro-level models is valuable.

4.1. Posterior estimates

Table 2 provides a summary of the estimated model parameters. Traffic exposure, train speed, road speed, and the number of roadway lanes have an increasing effect on grade crossing crash frequencies across Canada. In contrast, automated warning devices (FLB and FLBG) have a decreasing effect on crossing crash frequencies. In our random parameters setting, we found that the effect of traffic exposure (i.e., product of train and vehicle volumes at crossings) varies across provinces. We did not find any important support for a varying effect of other explanatory variables. Considering a random parameter for traffic exposure improved the model fit considerably: the deviance information criterion (DIC) ([Spiegelhalter et al., 2002](#)) reduces from 5796.19 in the random effects (intercepts) model to 5789.47 in the conventional random parameters model. We found that the standard deviation of the observational level error term reduces from 0.982 to 0.947, perhaps due to the fact that part of the extra variability is captured in the random parameter. Coefficient estimates obtained from the above standard models are similar.

When allowing for heterogeneity in mean and variance, we found that variation in the mean of the random parameter associated with traffic exposure can be explained by the variable Prairies (Alberta, Manitoba, and Saskatchewan) - which indicates the spatial location of grade crossings. Specifically, the results showed that the mean of the traffic exposure parameter in the Canadian Prairies is higher than in other provinces by a factor of 0.287. The mean of the traffic exposure parameter in the Prairies and other regions are, respectively, 1.159 and 0.872, a relatively large difference. This information on the association between traffic exposure and safety (this does not imply causality) could not be inferred from the standard models, which do not estimate the possibly varying effects of risk factor coefficients across sites. An empirical explanation of this finding is provided in Section 4.3. Interestingly, when using Prairies as a main explanatory variable in the model, it did not have any important effect on safety. But using such variable in the traffic exposure mean function provides more insights than just using it as a main regressor. Also, we are not directly entering any province as a main variable in the model since we use a hierarchical intercept term (province-effect), which allows for borrowing of strength across provinces, resulting in more efficient estimates. Therefore, it is redundant to also enter provinces as standard regression variables.

We found that the variance of the heteroskedastic error term can be explained by the variable urban. In particular, the variance for the Canadian highway railway grade crossings located in urban areas increases by a factor of 0.134 compared with non-urban areas. This means that grade crossing crash frequencies are more dispersed in urban areas. A detailed study (including data collection) is thus required to find the source of dispersion in urban areas. That said, one important advantage of the proposed model is that it allows targeting the exact point where improvements are needed to better overcome unobserved heterogeneity. We did not find any other variables that explained variations in the variance of the regression parameter associated with traffic exposure, nor did we find any variable that explained variations in the mean or variance of the random intercepts. Hence, variances for the latter parameters are fixed across provinces. Using a varying mean and a heteroskedastic error term seems to be able to capture most of the variability in random parameters variances. We also ran a model with randomly varying variances for random parameters (without specifying any model for them), but the model fit did not improve so in the favour of parsimony there is no need to have these variances as random variables in our study. In fact, in the presence of a heteroskedastic error term with variances varying at grade crossing level, fixed variances across provinces appear to be satisfactory.

Under the latter model, the average value of the standard deviation of the observational level error term reduces to 0.65 (average value) from 0.947 in the standard random parameters model, indicating that a significant portion of variability in the data is now captured through the heteroskedastic heterogeneity in mean/variance specification. Recall that the observational level error term represents the portion of data variability that cannot be explained or accounted for by the available variables. Interestingly, with the same set of covariates in the model, we were able to capture this variability partly, employing a more complex random parameters model. In other words, we are able to better account for unobserved heterogeneity without adding other explanatory variables to the model, so that the proposed model makes a more efficient use of the available data. The fit of the latter model improved relative to both standard models, with an estimated DIC of 5776.76. Following the procedure outlined in Section 3.6, an estimated Bayesian p -value of 0.58 was obtained. Therefore, the proposed model performs satisfactorily in replicating excess zero counts in our data.

Note that [Heydari et al., \(2016b\)](#), using a Dirichlet process mixture approach, found that the normality assumption in the random effects does not hold for Canadian grade crossings equipped with flashing lights and bells. Here, we did not find any evidence of non-normality, perhaps because in this paper we are analyzing all Canadian public grade crossings. This may indicate that the use of a larger set of data could help create a more homogeneous sample.

4.2. Interpretation of the estimated coefficients

With respect to the interpretation of explanatory variables and the magnitude of association between these variables and crash frequencies, we use elasticities for continuous variables and relative risk for categorical variables. The estimated values are reported in Table 3. For the continuous variables, the estimated coefficients correspond to elasticity for log-transformed explanatory variables: traffic exposure, number of lanes, train speed, and road speed. In general, elasticities of these variables are similar under both random intercepts and random parameters models. However, the interpretation based on the heteroskedastic random parameters model with heterogeneity in mean and variance is somewhat different. For example, 10% increase in road speed increases grade crossing crash frequencies by approximately 2.69% and 3.26% under the random intercepts model and the proposed random parameters model, respectively. Thus, the random intercepts model underestimates the association between road speed limit and crash frequencies for the Canadian highway-rail crossings.

The major difference emerges in the interpretation of traffic exposure under the heteroskedastic model with heterogeneity in mean and variance since, as explained above, the mean of the random parameter for traffic exposure varies across the population of grade crossings according to their geographical location, captured in the variable Prairies. Under the latter model, 10% increase in traffic exposure results in approximately 11.59% and 8.72% increase in the expected crash frequency in the Prairies and in other regions, respectively. In contrast, 10% increase in traffic exposure would, respectively, lead to approximately 4.86% and 4.66% increase in the expected crash frequency under the random intercepts and the conventional random parameters models, which underestimate the effect of traffic exposure. Note that the standard models cannot capture the difference between the Prairies and other regions.

For the ease of interpretation for categorical variables FLB and FLBG (warning device types), we estimated the relative risk by exponentiating their respective coefficients; i.e., $\exp(\beta)$. With respect to these categorical variables, we did not find any important difference between our three models. According to the proposed model, the expected crash frequency among FLB crossings is lower than passive crossings (reference group) by a factor of 0.471 ($= \exp(-0.752)$), meaning that the crash frequency is on average 52.9% ($100*(0.471-1)$) lower among FLB crossings relative to passive crossings. Similarly, the expected crash frequency is lower by a factor of 0.23 (i.e., 76.6% lower) for FLBG crossings in comparison to passive crossings (see Table 3).

4.3. Why does crossing safety deteriorate at a higher rate in the Canadian Prairies as traffic exposure increases?

Conducting a more detailed investigation, we found the following potential explanations for the fact that an increase in traffic exposure leads to a higher increase in grade crossing crash frequencies in the Prairies relative to other regions. By summarizing the data, we found that the average crash frequency in the Prairies provinces and other provinces are very similar (0.05 vs .053, respectively). Nevertheless, the average traffic exposures are very different: 5652.8 in the Prairies vs. 20492.2 in other regions. This is in accordance with what we have inferred from our proposed model. The next question is why does this happen?

We found that only 13% of crossings in the Canadian Prairies are located in urban areas; the remaining 87% are mostly in rural and remote areas. We can speculate that in non-urban areas (i) speeding and risk-taking behaviors are more frequent; (ii) illumination conditions often are not as suitable as in urban areas, limiting drivers' visibility; and (iii) road conditions in inclement weather remain less favorable (in comparison to urban

areas) from a traffic safety perspective, for example, due to the lack of adequate winter road maintenance in non-populated areas (see e.g. [Usman et al. \(2010\)](#) who studied the effect of winter road maintenance on traffic safety). We also found that the average road speed limit in proximity of crossings in the Prairies is higher than other regions (65.81 km/h vs. 59.91 km/h). These factors together could obviously result in a higher operating speed in the proximity of crossings in the Prairies than in other Canadian regions. In addition, we can observe that around 80% of crossings in the Prairies (analyzed in this study) are passive, being equipped only with simple road signs such as crossbucks - not flashing lights, bells and/or gates that advise road users of the presence of crossings and trains.

4.4. Implications for benchmarking different regions

We computed expected pairwise probabilities of adjusted crash risk in each province exceeding that of other provinces according to the algorithm discussed in Section 3.4. While complete results are provided in Appendix A, as an example, Fig. 3 clearly highlights the differences between the examined models in this study. It can be inferred from Fig. 3 that in terms of adjusted crash risk, the inferences drawn from the random intercepts model are different from both random parameters models. Fig. 3(a) indicates that the adjusted crash risk in Alberta exceeds that of other Canadian provinces with a probability of almost 1. However, Fig. 3(b) and Fig. 3(c), for instance, indicate that the adjusted crash risk in Alberta exceeds that of British Columbia with a probability smaller than 40%. Similarly, Alberta exceeds adjusted crash risk in Ontario with a probability around 60% under the proposed random parameters model (see Fig. 3(c)). This indicates that a simple random intercepts model may mask true inter-region variations in adjusted crash risks.

Differences in macro-level ranking criteria based on adjusted crash risk and total expected crash frequency are displayed in Fig. 4. Again, one can notice an obvious difference between the simple random intercepts model and both random parameters models when it comes to the risk adjusted benchmarking exercise. We found that comparing regions based on total expected crash frequencies of each region is not sensitive to model specification. As we discussed in Section 1.2, the latter approach falls short in drawing a full picture, neglecting differences in characteristics of road infrastructures located in each region. Based on the total expected crash frequency criterion, for instance, Ontario seems to be a higher crash province compared to British Columbia according to Fig. 4(c-2). After accounting for grade crossing differences (e.g., traffic exposure) and the number of crossings in both provinces, however, Ontario

appears to have a lower risk of grade crossing crashes than British Columbia (see Fig. 4(c-1)). Note that since we adjust for differences in regions, we can state which region has a higher crash risk relative to other region(s) under similar circumstances among sites nested within each region.

The results discussed in Section 4.4 could be particularly useful in terms of country-wide safety policy, highlighting the need for further investigation into the reasons for such differences in regional safety performances in Canada. Our findings suggest that further in-dept study is necessary for identifying the potential contributing factors such as sociodemographic characteristics, driver demography and behavior, traffic regulations, weather, and policies that may explain differences in crash risk propensities.

5. Summary

This study presented a method to compare different geographic areas (e.g., neighborhoods, provinces, etc.) in terms of a pre-specified safety performance measure such as crash frequency of a given type, integrating both micro- and macro-level approaches. The proposed method – being fully probabilistic – allows for the identification of macro-level high-crash-risk regions while adjusting for differences between sites (intersections, road segments, etc.) located in each region. In contrast to the usual macroscopic high crash location identification approaches, our model isolates the regional differences in site-specific characteristics such as variations in traffic exposure. An important advantage of our method is that, while we make between-region (macro-level) comparisons, we use detailed site-specific attributes that can explain safety. However, macro-level models use aggregated data and therefore cannot be used for such purpose at a disaggregate level.

We utilized a comprehensive country-wide highway railway grade crossing data as our empirical setting. The sample contains 16,549 public highway railway crossings located in eight major Canadian provinces. To overcome unobserved heterogeneity, we developed a heteroskedastic random parameters Poisson lognormal model with heterogeneity in mean and variance. We compared the proposed model with commonly used random intercepts and random parameters models. The results indicate that modeling the mean and/or the variance of the random parameters as a function of explanatory variables available in data could better capture the underlying structure of crash data. We found, for example, that the Canadian grade crossing crash frequencies are more dispersed among sites located in urban areas relative to non-urban areas. More interestingly, the results also revealed that safety at grade crossings located in the Canadian Prairies is more sensitive to traffic exposure. We provide a number of potential

explanations for the latter finding. This study indicates the need to open up a new line of inquiry that aims at explaining the above findings. Following the benchmarking exercise conducted in this paper, further research can be initiated to investigate reasons behind safety performance differences across Canadian provinces. This work could be useful in planning safety improvement programs at both micro- and macro-levels.

ACKNOWLEDGEMENT

We would like to thank the Natural Sciences and Engineering Research Council of Canada for partial funding of this study. We also wish to thank Transportation Safety Board of Canada for providing the data.

REFERENCES

- Amoh-Gyimah, R., Saberi, M., Sarvi, M., 2017. The effect of variations in spatial units on unobserved heterogeneity in macroscopic crash models. *Analytic Methods in Accident Research* 13, 28-51.
- Anastasopoulos, P.C., 2016. Random parameters multivariate tobit and zero-inflated count data models: addressing unobserved and zero-state heterogeneity in accident injury-severity rate and frequency analysis. *Analytic Methods in Accident Research* 11, 17-32.
- Anastasopoulos, P.C., Mannering, F.L., 2009. A note on modeling vehicle accident frequencies with random-parameters count models. *Accident Analysis and Prevention* 41(1), 153-159.
- Anastasopoulos, P.C., Shankar, V.N., Haddock, J.E., Mannering, F.L., 2012. A multivariate tobit analysis of highway accident-injury-severity rates. *Accident Analysis and Prevention* 45, 110-119.
- Austin, R., Carson, J., 2002. An alternative accident prediction model for highway-rail interfaces. *Accident Analysis and Prevention* 34(1), 31-42.
- Barua, S., El-Basyouny, K., Islam, M.T., 2016. Multivariate random parameters collision count data models with spatial heterogeneity. *Analytic Methods in Accident Research* 9, 1-15.
- Beanland, V., Grant, E., Read, G., Stevens, N., Thomas, M., Lenné, M.G., Stanton, N.A., Salmon, P.M., 2018. Challenging conventional rural rail level crossing design: evaluating three new systems thinking-based designs in a driving simulator. *Safety Science*. DOI: 10.1016/j.ssci.2018.03.002
- Behnood, A., Mannering, F., 2017. Determinants of bicyclist injury severities in bicycle-vehicle crashes: a random parameters approach with heterogeneity in means and variances. *Analytic Methods in Accident Research* 16, 35-47.
- Bhat, C.R., Astroza, S., Lavieri, P.S., 2017. A new spatial and flexible multivariate random-coefficients model for the analysis of pedestrian injury counts by severity level. *Analytic Methods in Accident Research* 16, 1-22.
- Brijs, T., Karlis, D., Van den Bossche, F., Wets, G., 2007. A Bayesian model for ranking hazardous road sites. *Journal of the Royal Statistical Society: Series A (Statistics in Society)* 170(4), 1001-1017.
- Cai, Q., Abdel-Aty, M., Lee, J., Wang, L., Wang, X., 2018. Developing a grouped random parameters multivariate spatial model to explore zonal effects for segment and intersection crash modeling. *Analytic Methods in Accident Research* 19, 1-15.
- Chadwick, S.G., Zhou, N., Saat, M.R., 2014. Highway-rail grade crossing safety challenges for shared operations of high-speed passenger and heavy freight rail in the US. *Safety Science* 68, 128-137.

Chaudhary, M., Hellman, A., Ngamdung, T., John, A., 2011. Volpe National Transportation Systems Center (U.S.). Railroad right-of-way incident analysis research. Federal Railroad Administration Office of Railroad Policy and Development, Washington, DC.

Chen, E., Tarko, A.P., 2014. Modeling safety of highway work zones with random parameters and random effects models. *Analytic Methods in Accident Research* 1, 86-95.

Cheng, W., Washington, S.P., 2005. Experimental evaluation of hotspot identification methods. *Accident Analysis and Prevention* 37(5), 870-881.

Coll, B., Moutari, S., Marshall, A.H., 2013. Hotspots identification and ranking for road safety improvement: an alternative approach. *Accident Analysis and Prevention* 59, 604-617.

Coruh, E., Bilgic, A., Tortum, A., 2015. Accident analysis with aggregated data: the random parameters negative binomial panel count data model. *Analytic Methods in Accident Research* 7, 37-49.

Davis, G.A., 2004. Possible aggregation biases in road safety research and a mechanism approach to accident modeling. *Accident Analysis and Prevention* 36(6), 1119-1127.

Daziano, R.A., Miranda-Moreno, L., Heydari, S., 2013. Computational Bayesian statistics in transportation modeling: from road safety analysis to discrete choice. *Transport Reviews* 33(5), 570-592.

Debrabant, B., Halekoh, U., Bonat, W.H., Hansen, D.L., Hjelmberg, J., Lauritsen, J., 2018. Identifying traffic accident black spots with Poisson-Tweedie models. *Accident Analysis and Prevention* 111, 147-154.

Dinu, R.R., Veeraragavan, A., 2011. Random parameter models for accident prediction on two-lane undivided highways in India. *Journal of safety research* 42(1), 39-42.

Dong, N., Huang, H., Lee, J., Gao, M., Abdel-Aty, M., 2016. Macroscopic hotspots identification: a Bayesian spatio-temporal interaction approach. *Accident Analysis and Prevention* 92, 256-264.

El-Basyouny, K., Sayed, T., 2009. Accident prediction models with random corridor parameters. *Accident Analysis and Prevention* 41(5), 1118-1123.

El-Basyouny, K., Sayed, T., 2013. Depth-based hotspot identification and multivariate ranking using the full Bayes approach. *Accident Analysis and Prevention* 50, 1082-1089.

Eluru, N., Bagheri, M., Miranda-Moreno, L.F., Fu, L., 2012. A latent class modeling approach for identifying vehicle driver injury severity factors at highway-railway crossings. *Accident Analysis and Prevention* 47, 119-127.

Elvik, R., 2008. Comparative analysis of techniques for identifying locations of hazardous roads. *Transportation Research Record: Journal of the Transportation Research Board* 2083, 72-75.

Fawcett, L., Thorpe, N., Matthews, J., Kremer, K., 2017. A novel Bayesian hierarchical model for road safety hotspot prediction. *Accident Analysis and Prevention* 99, 262-271.

Fountas, G., Anastasopoulos, P.C., Mannering, F.L., 2018a. Analysis of vehicle accident-injury severities: a comparison of segment-versus accident-based latent class ordered probit models with class-probability functions. *Analytic Methods in Accident Research* 18, 15-32.

Fountas, G., Sarwar, M., Anastasopoulos, P.Ch., Blatt, Majka, K., 2018b. Analysis of stationary and dynamic factors affecting highway accident occurrence: a dynamic correlated random parameters binary logit approach. *Accident Analysis and Prevention* 113, 330-340.

Fu, L., 2016. *GradeX – A decision support tool for improving safety at highway-railway grade crossings in Canada: technical background and user guide*, Transport Canada.

Fu, L., Saccomanno, F., Miranda-Moreno, L. F., Park, Y.J., Chen, J., 2006. *GradeX – a web-based decision support tool for hotspot identification and countermeasure analysis of Canadian highway-railway grade crossings*. Presented at the 9th international level crossing safety and Trespass Prevention Symposium, Montreal, Canada, September 10-14.

Gelman, A., Carlin, J.B., Stern, H.S., Rubin, D.B., 2004. *Bayesian Data Analysis*. 2nd ed.: Chapman & Hall/CRC.

Gelman, A., Rubin, D.B., 1992. Inference from iterative simulation using multiple sequences. *Statistical Science* 7(4), 457-472.

Ghomi, H., Bagheri, M., Fu, L., Miranda-Moreno, L.F., 2016. Analyzing injury severity factors at highway railway grade crossing accidents involving vulnerable road users: a comparative study. *Traffic Injury Prevention* 17(8), 833-841.

Guadamuz-Flores, R., Agüero-Valverde, J., 2017. Bayesian spatial models of crash frequency at highway–railway crossings. *Transportation Research Record: Journal of the Transportation Research Board* 2608, 27-35.

Hadayeghi, A., Shalaby, A.S., Persaud, B.N., 2010. Development of planning level transportation safety tools using geographically weighted Poisson regression. *Accident Analysis and Prevention* 42(2), 676-688.

Haleem, K., 2016. Investigating risk factors of traffic casualties at private highway-railroad grade crossings in the United States. *Accident Analysis and Prevention* 95, Part A, 274-283.

Hao, W., Kamga, C., Daniel, J., 2015. The effect of age and gender on motor vehicle driver injury severity at highway-rail grade crossings in the United States. *Journal of Safety Research* 55, 105-113.

Hauer, E., Persaud, B., 1987. How to estimate the safety of rail-highway grade crossings and the safety effects of warning devices. *Transportation Research Record: Journal of the Transportation Research Board* 1114, 131–140.

Heydari, S., Fu, L., 2015. Developing safety performance functions for railway grade crossings: a case study of Canada. *ASME. ASME/IEEE Joint Rail Conference, 2015 Joint Rail Conference*. DOI:10.1115/JRC2015-5768.

Heydari, S., Fu, L., Joseph, L., Miranda-Moreno, L.F., 2016a. Bayesian nonparametric modeling in transportation safety studies: applications in univariate and multivariate settings. *Analytic Methods in Accident Research* 12, 18-34.

Heydari, S., Fu, L., Lord, D., Mallick, B.K., 2016b. Multilevel Dirichlet process mixture analysis of railway grade crossing crash data. *Analytic Methods in Accident Research* 9, 27-43.

Heydari, S., Fu, L., Miranda-Moreno, L.F., Joseph, L., 2017(a). Using a flexible multivariate latent class approach to model correlated outcomes: a joint analysis of pedestrian and cyclist injuries. *Analytic Methods in Accident Research* 13, 16-27.

Heydari, S., Fu, L., Thakali, L., Joseph, L., 2017(b). Identifying areas of high risk for collisions: a Canada-wide study of grade crossing safety, 2017 4th International Conference on Transportation Information and Safety (ICTIS), Banff, AB, 640-644. DOI: 10.1109/ICTIS.2017.8047834

Heydari, S., Miranda-Moreno, L., Amador, L., 2013. Does prior specification matter in hotspot identification and before-after road safety studies? *Transportation Research Record: Journal of the Transportation Research Board* 2392, 31-39.

Heydari, S., Miranda-Moreno, L.F., Fu, L., 2014a. Speed limit reduction in urban areas: a before-after study using Bayesian generalized mixed linear models. *Accident Analysis and Prevention* 73, 252-261.

Heydari, S., Miranda-Moreno, L., Lord, D., Fu, L., 2014b. Bayesian methodology to estimate and update safety performance functions under limited data conditions: a sensitivity analysis. *Accident Analysis and Prevention* 64, 41-51.

Heydecker, B.G., Wu, J., 2001. Identification of sites for road accident remedial work by Bayesian statistical methods: an example of uncertain inference. *Advances in Engineering Software* 32(10-11), 859-869.

Hong, J., Shankar, V.N., Venkataraman, N., 2016. A spatially autoregressive and heteroskedastic space-time pedestrian exposure modeling framework with spatial lags and endogenous network topologies. *Analytic Methods in Accident Research* 10, 26-46.

Huang, H., Song, B., Xu, P., Zeng, Q., Lee, J., Abdel-Aty, M., 2016. Macro and micro models for zonal crash prediction with application in hot zones identification. *Journal of Transport Geography* 54, 248-256.

Hsu, C., Jones, E.G., 2017. Sensitivity analyses of stopping distance for connected vehicles at active highway-rail grade crossings. *Accident Analysis and Prevention* 99, Part A, 210-217.

Jiang, X., Abdel-Aty, M., Alamili, S., 2014. Application of Poisson random effect models for highway network screening. *Accident Analysis and Prevention* 63, 74-82.

Larue, G.S., Filtner, A.J., Wood, J.M., Demmel, S., Watling, C.N., Naweed, A., Rakotonirainy, A., 2018. Is it safe to cross? identification of trains and their approach speed at level crossings. *Safety Science* 103, 33-42.

- Lee, J., Abdel-Aty, M., Cai, Q., 2017. Intersection crash prediction modeling with macro-level data from various geographic units. *Accident Analysis and Prevention* 102, 213-226.
- Lee, J., Abdel-Aty, M., Choi, K., Huang, H., 2015. Multi-level hot zone identification for pedestrian safety. *Accident Analysis and Prevention* 76, 64-73.
- Lee, J., Abdel-Aty, M., Jiang, X., 2014. Development of zone system for macro-level traffic safety analysis. *Journal of Transport Geography* 38, 13-21.
- Lee, J., Nam, D., Moon, D., 2004. A zero-inflated accident frequency model of highway-rail grade crossing. In: *Proceedings of the Transportation Research Board Annual Meeting*. Washington, DC.
- Lipovac, K., Nešić, M., Tešić, M., Ross, A., Tubić, V., Marić, B., 2016. A comparative analysis of dangerous locations on the public roads in Serbia. *Safety Science* 84, 190-200.
- Liu, J., Khattak, A.J., 2017. Gate-violation behavior at highway-rail grade crossings and the consequences: using geo-spatial modeling integrated with path analysis. *Accident Analysis and Prevention* 109, 99-112.
- Lord, D., Miranda-Moreno, L.F., 2008. Effects of low sample mean values and small sample size on the estimation of the fixed dispersion parameter of Poisson-gamma models for modeling motor vehicle crashes: a Bayesian perspective. *Safety Science* 46(5), 751-770.
- Lord, D., Washington, S.P., Ivan, J.N., 2005. Poisson, Poisson-gamma and zero inflated regression models of motor vehicle crashes: balancing statistical fit and theory. *Accident Analysis and Prevention* 37(1), 35-46.
- Lord, D., Washington, S.P., Ivan, J.N., 2007. Further notes on the application of zero-inflated models in highway safety. *Accident Analysis and Prevention* 39(1), 53-57.
- Lu, P., Tolliver, D., 2016. Accident prediction model for public highway-rail grade crossings. *Accident Analysis and Prevention* 90, 73-81.
- Lunn, D.J., Thomas, A., Best, N., Spiegelhalter, D., 2000. WinBUGS—a Bayesian modelling framework: concepts, structure, and extensibility. *Statistics and Computing* 10 (4), 325-337.
- Mannering, F., 2018. Temporal instability and the analysis of highway accident data. *Analytic Methods in Accident Research* 17, Pages 1-13.
- Mannering, F.L., Bhat, C.R., 2014. Analytic Methods in Accident Research: methodological frontier and future directions. *Analytic Methods in Accident Research* 1, 1-22.
- Mannering, F.L., Shankar, V., Bhat, C.R., 2016. Unobserved heterogeneity and the statistical analysis of highway accident data. *Analytic Methods in Accident Research* 11, 1-16.
- Medina, J.C., Benekohal, R.F., 2015. Macroscopic models for accident prediction at railroad grade crossings: comparisons with US Department of Transportation accident

prediction formula. *Transportation Research Record: Journal of the Transportation Research Board* 2476, 85-93.

Miaou, S.P., Song, J.J., 2005. Bayesian ranking of sites for engineering safety improvements: decision parameter, treatability concept, statistical criterion, and spatial dependence. *Accident Analysis and Prevention* 37(4), 699-720.

Millegan, H., Yan, X., Richards, S., Han, L., 2009. Evaluation of effectiveness of stop sign treatment at highway-railroad grade crossings. *Transportation Safety and Security* 1(1), 46-60.

Miranda-Moreno, L.F., Labbe, A., Fu, L., 2007. Bayesian multiple testing procedures for hotspot identification. *Accident Analysis and Prevention* 39(6), 1192-1201.

Montella, A., 2010. A comparative analysis of hotspot identification methods. *Accident Analysis and Prevention* 42(2), 571-581.

Oh, J., Washington, S.P., Nam, D., 2006. Accident prediction model for railway-highway interfaces. *Accident Analysis and Prevention* 38(2), 346-356.

Osama, A., Sayed, T., 2016. Evaluating the impact of bike network indicators on cyclist safety using macro-level collision prediction models. *Accident Analysis and Prevention* 97, 28-37.

Park, B.J., Lord, D., Wu, L., 2016. Finite mixture modeling approach for developing crash modification factors in highway safety analysis. *Accident Analysis and Prevention* 97, 274-287.

Park, Y.-J., Saccomanno, F.F., 2005. Collision Frequency analysis using tree-based stratification. *Transportation Research Record* 1908, 119-121.

Persaud, B., Lyon, C., Nguyen, T., 1999. Empirical Bayes procedure for ranking sites for safety investigation by potential for safety improvement. *Transportation Research Record: Journal of the Transportation Research Board* 1665, 7-12.

Qu, X., Meng, Q., 2014. A note on hotspot identification for urban expressways. *Safety Science* 66, 87-91.

Sacchi, E., Sayed, T., El-Basyouny, K., 2015. Multivariate full Bayesian hot-spot identification and ranking. *Transportation Research Record: Journal of the Transportation Research Board* 2515, 1-9.

Saccomanno, F., Fu, L., Miranda-Moreno, L., 2004. Risk-based model for identifying highway-rail grade crossing blackspots. *Transportation Research Record: Journal of the Transportation Research Board* 1862, 127-135.

Saccomanno, F., Lai, X., 2005. A model for evaluating countermeasures at highway-railway grade crossings. *Transportation Research Record* 1918, 18-25.

Salmon, P.M., Read, G., Walker, G.H., Goode, N., Grant, E., Dallat, C., Carden, T., Naweed, A., Stanton, N.A., 2018. STAMP goes EAST: integrating systems ergonomics methods for the analysis of railway level crossing safety management. *Safety Science*. DOI: 10.1016/j.ssci.2018.02.014.

- Sarwar, M.T., Anastasopoulos, P.C., Golshani, N., Hulme, K.F., 2017. Grouped random parameters bivariate probit analysis of perceived and observed aggressive driving behavior: a driving simulation study. *Analytic Methods in Accident Research* 13, 52-64.
- Seraneeprakarn, P., Huang, S., Shankar, V., Mannering, F., Venkataraman, N., Milton, J., 2017. Occupant injury severities in hybrid-vehicle involved crashes: a random parameters approach with heterogeneity in means and variances. *Analytic Methods in Accident Research* 15, 41-55.
- Sperry, B.R., Naik, B., Warner, J.E., 2017. Current issues in highway–rail grade crossing hazard-ranking and project development. *Transportation Research Record: Journal of the Transportation Research Board* 2608, 19-26.
- Spiegelhalter, D.J., Best, N.G., Carlin, B.P., van der Linde, A., 2002. Bayesian measures of complexity and fit (with discussion). *Journal of the Royal Statistics Society, Series B* 64(4), 1–34.
- Thakali, L., Kwon, T.J., Fu, L., 2015. Identification of crash hotspots using kernel density estimation and kriging methods: a comparison. *Journal of Modern Transportation* 23(2), 93-106.
- Tung, L.W., Khattak, A., 2015. Distracted motor vehicle driving at highway–rail grade crossings. *Transportation Research Record: Journal of the Transportation Research Board* 2476, 77–84.
- Transportation Safety Board of Canada, 2015. Statistical summary, railway occurrences. <http://www.tsb.gc.ca/eng/stats/rail/2015/sser-ssro-2015.pdf> (Accessed April 2018)
- Truong, L.T., Kieu, L., Vu, T.A., 2016. Spatiotemporal and random parameter panel data models of traffic crash fatalities in Vietnam. *Accident Analysis and Prevention* 94, 153-161.
- Ukkusuri, S., Miranda-Moreno, L.F., Ramadurai, G., Isa-Tavarez, J., 2012. The role of built environment on pedestrian crash frequency. *Safety Science* 50(4), 1141-1151.
- Usman, T., Fu, L., Miranda-Moreno, L.F., 2010. Quantifying safety benefit of winter road maintenance: accident frequency modeling. *Accident Analysis and Prevention* 42(6), 1878-1887.
- Venkataraman, N., Shankar, V., Ulfarsson, G.F., Deptuch, D., 2014. A heterogeneity-in-means count model for evaluating the effects of interchange type on heterogeneous influences of interstate geometrics on crash frequencies. *Analytic Methods in Accident Research* 2, 12-20.
- Wang, X., Liu, J., Khattak, A.J., Clarke, D., 2016a. Non-crossing rail-trespassing crashes in the past decade: a spatial approach to analyzing injury severity. *Safety Science* 82, 44-55.
- Wang, X., Yang, J., Lee, C., Ji, Z., You, S., 2016b. Macro-level safety analysis of pedestrian crashes in Shanghai, China. *Accident Analysis and Prevention* 96, 12-21.

Washington, S.P., Van Schalkwyk, I., You, D., Shin, K., Samuelson, J.P., 2010. Forecasting the safety impacts of socio-demographic changes and safety countermeasures. NCHRP 8-44(2): Plansafe User Manual. Transportation Research Board, Washington, DC.

Wei, F., Lovegrove, G., 2013. An empirical tool to evaluate the safety of cyclists: community based, macro-level collision prediction models using negative binomial regression. *Accident Analysis and Prevention* 61, 129-137.

Winkelmann, R., 2008. *Econometric Analysis of Count Data*, 5th edition. Springer-Verlag Berline Heidelberg.

Wu, Z., Sharma, A., Mannering, F.L., Wang, S., 2013. Safety impacts of signal-warning flashers and speed control at high-speed signalized intersections. *Accident Analysis and Prevention* 54, 90-98.

Xin, C., Guo, R., Wang, Z., Lu, Q., Lin, P.S., 2017. The effects of neighborhood characteristics and the built environment on pedestrian injury severity: a random parameters generalized ordered probability model with heterogeneity in means and variances. *Analytic Methods in Accident Research* 16, 117-132.

Ye, Z., Xu, Y., Lord, D., 2018. Crash data modeling with a generalized estimator. *Accident Analysis and Prevention* 117, 340-345.

Yan, X., Richards, S., Su, X., 2010. Using hierarchical tree-based regression model to predict train-vehicle crashes at passive highway-rail grade crossings. *Accident Analysis and Prevention* 42(1), 64-74.

Yu, H., Liu, P., Chen, J., Wang, H., 2014. Comparative analysis of the spatial analysis methods for hotspot identification. *Accident Analysis and Prevention* 66, 80-88.

Zhang, Z., Trivedi, C., Liu, X., 2017. Automated detection of grade-crossing-trespassing near misses based on computer vision analysis of surveillance video data. *Safety Science*. DOI: 10.1016/j.ssci.2017.11.023

Zhao, S., Iranitalab, A., Khattak, A.J., 2018. A clustering approach to injury severity in pedestrian-train crashes at highway-rail grade crossings. *Journal of Transportation Safety and Security*. DOI: 10.1080/19439962.2018.1428257

Table 1. Summary statistics of the data

Variables	Mean	Std. Dev.	Min	Max
Crash frequency	0.05	0.26	0.00	4.00
Traffic Exposure (product of train and vehicle counts)	12,229.19	61,469.00	0.10	3,000,000.00
Number of tracks	1.18	0.50	1.00	9.00
Number of lanes	1.97	0.55	1.00	7.00
Angle (deviation from 90°)	19.57	19.43	0.00	87.00
Road speed (km/h) (road speed limit)	63.20	20.62	5.00	110.00
Train speed (km/h) (rail speed limit)	57.18	31.45	1.61	160.80
Dummy Variables	Frequency	Percent		
FLB (1 if equipped with flashing lights and bells)	4,202.00	25.39	-	-
FLBG (1 if equipped with flashing lights, bells and gates)	2,410.00	14.56	-	-
Passive (1 if passive; 0 otherwise)	9,937	60.05	-	-
Active (1 if active; 0 otherwise)	6,612	39.95	-	-
Urban (1 if urban area; 0 otherwise)	3,055	18.46	-	-
Whistle (1 if prohibited; 0 otherwise)	962	5.81	-	-
Prairies (1 if located in the Canadian Prairies; 0 otherwise)	9,215	55.68	-	-

Table 2. Posterior estimation summary

Random intercepts model		Mean	SD	95% Credible Intervals	
Province effect mean		-9.631	1.190	-11.040	-8.308
	<i>SD Province effect</i>	<i>0.564</i>	<i>0.235</i>	<i>0.265</i>	<i>1.150</i>
ln(exposure)		0.486	0.040	0.436	0.541
ln(train speed)		0.407	0.133	0.266	0.558
ln(road speed)		0.269	0.200	0.071	0.537
ln(lanes)		0.347	0.173	0.025	0.642
FLB		-0.702	0.150	-0.933	-0.476
FLBG		-1.459	0.250	-1.789	-1.145
	<i>SD obs. level error term</i>	<i>0.982</i>	<i>0.073</i>	<i>0.836</i>	<i>1.125</i>
Model fit (DIC)		5796.190	-	-	-
Random parameters model		Mean	SD	95% Credible Intervals	
Province effect mean		-9.530	0.623	-10.730	-8.277
	<i>SD Province effect</i>	<i>0.643</i>	<i>0.299</i>	<i>0.237</i>	<i>1.380</i>
ln(exposure)		0.466	0.049	0.369	0.561
	<i>SD ln(exposure)</i>	<i>0.106</i>	<i>0.038</i>	<i>0.056</i>	<i>0.201</i>
ln(train speed)		0.416	0.068	0.275	0.543
ln(road speed)		0.286	0.117	0.065	0.512
ln(lanes)		0.325	0.152	0.031	0.620
FLB		-0.730	0.114	-0.957	-0.509
FLBG		-1.439	0.153	-1.744	-1.143
	<i>SD obs. level error term</i>	<i>0.947</i>	<i>0.064</i>	<i>0.816</i>	<i>1.071</i>
Model fit (DIC)		5789.470	-	-	-
Heteroskedastic random parameters model with heterogeneity in mean and variance		Mean	SD	95% Credible Intervals	
Province effect mean		-9.651	0.696	-11.030	-8.308
	<i>SD Province effect</i>	<i>0.648</i>	<i>0.293</i>	<i>0.260</i>	<i>1.367</i>
ln(exposure):					
	<i>Constant</i>	<i>0.872</i>	<i>0.122</i>	<i>0.653</i>	<i>1.115</i>
	<i>Prairies</i>	<i>0.287</i>	<i>0.168</i>	<i>0.003</i>	<i>0.612</i>
	<i>SD ln(exposure)</i>	<i>0.090</i>	<i>0.036</i>	<i>0.047</i>	<i>0.180</i>
ln(train speed)		0.430	0.073	0.284	0.566
ln(road speed)		0.326	0.128	0.074	0.586
ln(lanes)		0.291	0.156	0.032	0.601
FLB		-0.752	0.115	-0.982	-0.529
FLBG		-1.454	0.155	-1.757	-1.153
Heteroskedastic error term variance:					
	<i>Constant</i>	<i>0.404</i>	<i>0.056</i>	<i>0.292</i>	<i>0.514</i>
	<i>Urban</i>	<i>0.134</i>	<i>0.079</i>	<i>0.012</i>	<i>0.293</i>
Model fit (DIC)		5776.760	-	-	-

Table 3. Approximate elasticities and relative risks

Random intercepts model	
<i>Continuous variables</i>	<i>Elasticities⁽¹⁾</i>
Traffic exposure	4.86%
Train speed	4.07%
Road speed	2.69%
Lanes	3.47%
<i>Categorical variables</i>	<i>Relative risk⁽²⁾</i>
FLB crossings	0.50
FLBG crossings	0.23
Random parameters model	
<i>Continuous variables</i>	<i>Elasticities</i>
Traffic exposure	4.66%
Train speed	4.16%
Road speed	2.86%
Lanes	3.25%
<i>Categorical variables</i>	<i>Relative risk</i>
FLB crossings	0.48
FLBG crossings	0.24
Heteroskedastic random parameters model with heterogeneity in mean and variance	
<i>Continuous variables</i>	<i>Elasticities</i>
Traffic exposure non-Prairies	8.72%
Traffic exposure Prairies	11.59%
Train speed	4.30%
Road speed	3.26%
Lanes	2.91%
<i>Categorical variables</i>	<i>Relative risk</i>
FLB crossings	0.47
FLBG crossings	0.23

⁽¹⁾Elasticities are based on a 10% increase in continuous variables

⁽²⁾Relative risks should be interpreted as indicated in Section 4.2 with respect to passive crossings (the reference group).

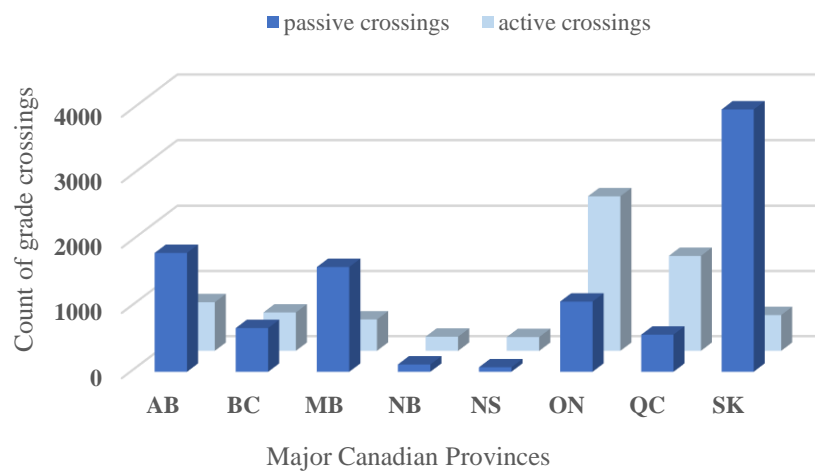


Figure 1. Distribution of passive and active crossings in different provinces (see Section 2 for description of provincial abbreviations)

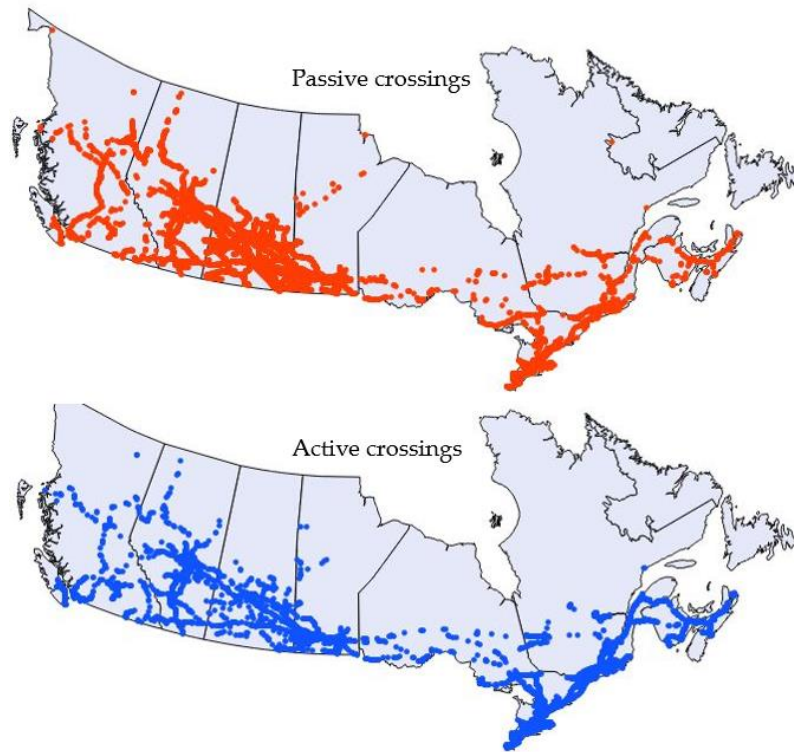


Figure 2. Spatial distribution of crossings across Canadian provinces

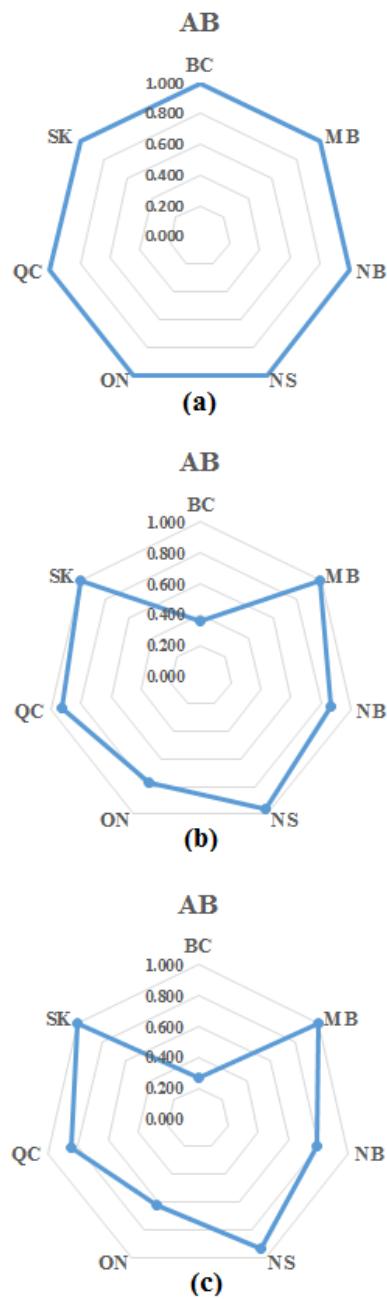


Figure 3. Expected exceeding probability of adjusted crash risk in Alberta vs. other major Canadian provinces: (a) Random intercepts model; (b) Standard random parameters model; and (c) Proposed random parameters model. (see Section 2 for description of provincial abbreviations)

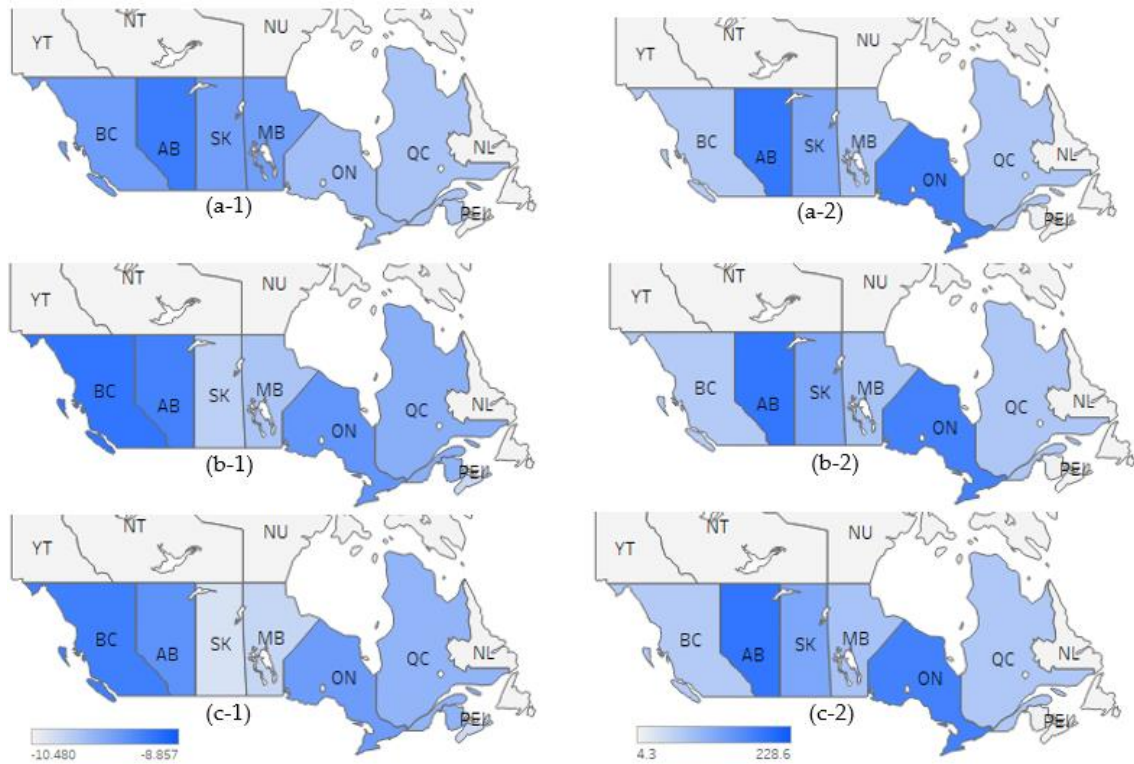


Figure 4. Log-scaled adjusted crash risk (1) vs. sum of expected crash frequency (2): (a) Random intercepts model; (b) Standard random parameters model; and (c) Proposed random parameters model
(see Section 2 for description of provincial abbreviations)

Appendix A.

Fig. 1.A displays a gray-scale plot of pairwise probabilities of exceeding adjusted crash risks, obtained under three different model formulations, for eight Canadian provinces. darker squares indicate larger probabilities of exceeding. For example, Fig. 1.A(c) indicates that adjusted grade crossing crash risk in Ontario is highly likely (with a probability greater than 70%) to be higher than crossing crash risk in New Brunswick, Nova Scotia, Manitoba, Saskatchewan, and Quebec. Similarly, the adjusted crash risk in Saskatchewan has a low probability of exceeding crash risk in other provinces. While estimated pairwise probabilities of exceeding are similar in both random parameter models (see Fig. 1.A(b) and 1.A(c)), these estimates are considerably different under the random intercepts model (see Fig. 1.A(a)). According to Fig. 1.A(c), British Columbia has the highest chance of grade crossing crash risk after adjusting for the effect of explanatory variables used in our study.

	AB	BC	MB	NB	NS	ON	QC	SK
AB	0.000	0.998	0.998	1.000	1.000	1.000	1.000	0.999
BC	0.002	0.000	0.567	0.975	0.999	0.990	0.997	0.577
MB	0.002	0.433	0.000	0.969	0.999	0.997	0.998	0.503
NB	0.000	0.025	0.031	0.000	0.897	0.315	0.427	0.028
NS	0.000	0.001	0.001	0.103	0.000	0.030	0.055	0.001
ON	0.000	0.010	0.003	0.685	0.970	0.000	0.730	0.001
QC	0.000	0.003	0.002	0.573	0.945	0.270	0.000	0.001
SK	0.001	0.423	0.497	0.972	0.999	0.999	0.999	0.000

(a)

	AB	BC	MB	NB	NS	ON	QC	SK
AB	0.000	0.362	0.991	0.872	0.964	0.769	0.925	1.000
BC	0.638	0.000	0.988	0.902	0.970	0.830	0.935	0.997
MB	0.009	0.012	0.000	0.322	0.624	0.063	0.266	0.665
NB	0.128	0.098	0.678	0.000	0.751	0.263	0.484	0.774
NS	0.036	0.030	0.376	0.250	0.000	0.093	0.231	0.476
ON	0.232	0.170	0.937	0.737	0.907	0.000	0.773	0.978
QC	0.075	0.065	0.735	0.516	0.769	0.227	0.000	0.850
SK	0.000	0.003	0.335	0.226	0.524	0.022	0.150	0.000

(b)

	AB	BC	MB	NB	NS	ON	QC	SK
AB	0.000	0.267	0.992	0.784	0.940	0.625	0.843	0.999
BC	0.733	0.000	0.992	0.882	0.970	0.804	0.923	0.999
MB	0.008	0.008	0.000	0.206	0.517	0.032	0.162	0.680
NB	0.216	0.118	0.794	0.000	0.765	0.292	0.509	0.879
NS	0.060	0.030	0.483	0.235	0.000	0.097	0.221	0.599
ON	0.375	0.196	0.968	0.708	0.903	0.000	0.760	0.993
QC	0.157	0.077	0.838	0.491	0.779	0.241	0.000	0.927
SK	0.001	0.001	0.320	0.121	0.401	0.007	0.073	0.000

(c)

Figure 1.A Gray-scale plot of pairwise probabilities of exceeding adjusted crash risks: (a) Random intercepts model; (b) Standard random parameters model; and (c) Proposed random parameters model. (see Section 2 for description of provincial abbreviations)