An assessment of the optimum timing of coastal flood adaptation given sea-level rise using real options analysis

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Abstract

There is large uncertainty about the changing magnitude and occurrence of coastal flood events with sea-level rise, which poses significant challenges to adaptation planning. Recent decision-making employs flexible options that can be modified/adjusted over time to address uncertainty. This research analyses a real option based method for assessing adaptation investment timing under coastal flooding and sea-level rise. This method recognises and values the flexibility of waiting as an additional adaptation option under an uncertain future – i.e. the key question concerns when the option is best implemented? Real options are applied using a test-case in Lymington (UK), a floodprone coastal town. Our findings show that the option value (i.e. the net value of the overall benefit minus cost) grows with rising sea level to a maximum, which is the optimal time for the adaptation investment. The optimum investment time tends to occur at the same magnitude of sea-level rise (relative to 1990) across most sea-level rise scenarios for the same socio-economic scenario. Hence monitoring sea-level rise provides important information to plan adaptation. The analysis provides an analytical framework on how and when to implement the adaptation option given the various future scenarios which can be developed further.

Key words: climate change adaptation, real options, flexibility, uncertainty, flood risk analysis, sea-level rise, coastal flooding, cost-benefit analysis

1. Introduction

Over the last decade, the prospect of strong adverse effects of climate change and sea-level rise has elicited widespread concern. This has raised increasing interest in proactively adapting to climate change (e.g., Field et al., 2014). However, uncertainty about the magnitude of future climate change is large, which poses challenges to decisions about the selection and timing of adaptation measures. This highly uncertain future is likely to lead to either over- or under-adaptation and hence economically sub-optimal decisions (Hall et al. 2012). As a response to this challenge, a number of methods have emerged that focus on the reduction of risk in climate change adaptation under uncertainty, such as robust decision making, adaptation pathways or real options analysis (e.g. Haasnoot et al., 2013; Ranger et al., 2013; Woodward et al., 2014).

Adaptation pathways and real options analysis have commonalities in adjusting options or strategies depending on the realization of the future. Nowadays these concepts are increasingly used in designing or planning long-term adaptation options over many decades and longer (Neufville, 2003; Haasnoot et al., 2013; Ranger et al., 2013; Woodward et al., 2014; Nicholls et al., 2015). Flexible or adaptive options that can be adjusted in the future as new information emerges are more robust than a traditional approach made on a ‘now-or-never’ basis (Dixit and

Pindyck, 1994). Flexible options can spread investments over time by delaying, growing, shrinking or abandoning these investments. (Dixit and Pindyck, 1994; Park, 2002; Dobes, 2010; Woodward et al., 2014). These kinds of flexible options enable us to have time to manage unexpected cases where the investment are likely to be sunk, or the future risks are more severe than predicted (Ranger et al., 2013). Also, they allow us to learn via observation about the nature of the change we are facing – in this case climate change and sea-level rise (Ranger et al., 2013; Nicholls et al., 2015).

A few recent papers have focused on the application of real options analysis to climate change adaptation. These studies support an idea that having a set of flexible options which can be implemented or abandoned is a more robust strategy against uncertainty than having a single irreversible option implemented now (Dobes, 2010; Whitten et al., 2012; Jeuland and Whittington, 2014; Hino and Hall, 2017). Likewise, deferring options under uncertain conditions offers an additional decision that makes the original options flexible by allowing learning and more informed decisions in the future (Woodward et al., 2014). This idea is similar to the adaptation pathways applied for the Thames Barrier and defences (see Ranger et al., 2013). However, the key difference is that real option analysis estimates an actual value of such flexibility. Woodward et al. (2010, 2011, 2014) argues that real options including flexibility provides more benefit than a conventional option excluding flexibility, even though such a decision requires additional expenditure now to create the flexibility – for example, buying land adjacent to coastal defences or widening the base of coastal defences to allow possible future upgrade (Dobes, 2010).

These previous studies focus on assessing multiple stage adaptation options, which are called an ‘option to grow’ in the literature of real options analysis (Dixit and Pindyck, 1994). In contrast, this study addresses a different situation where a single deferrable adaptation option is considered under the uncertainty of sea-level rise, which is referred to as an ‘option to wait’ in real options analysis. Note that a single deferrable adaptation option cannot manage the current risk of coastal flooding. However, if any single deferrable adaptation option is implemented at the outset, the flexibility does not exist any longer for decision–makers. Thus, an option-to-wait case focuses on the option value and investment timing of an adaptation option under the uncertainty of future sea-level rise. This paper improves the previous approaches to evaluating real options in two ways. Firstly, we allow the investment time to continuously vary and, secondly, we consider a wide range of sea-level rise scenario from the UK climate projections 09 (UKCP 09) (Lowe et al., 2009). These are considered independently and real option values are provided for each sea-level rise scenario rather than a single combined value as in previous studies of the option to grow.

The aim of this paper is to assess the use of real options analysis to determine the best timing for investment in adaptation under the uncertainty of climate change – in this case, defence raising to stop increased coastal flooding from sea-level rise. The coastal area between Lymington and Milford-on-Sea, Hampshire (UK) (Figure 1) is selected as a case study area (henceforth referred to as Lymington). This is a good example of a coastal zone with urban areas that have been flooded several times in the past, and are vulnerable to coastal flooding and sea-level rise in the future (Ruocco et al., 2011; Wadey et al., 2013). While flood defences have been upgraded in the last 20 years, it is recognized that a further upgrade is needed in 30 to 60 years (NFDC, 2010). So, selecting an appropriate timing for the next phase of adaptation is already an issue.

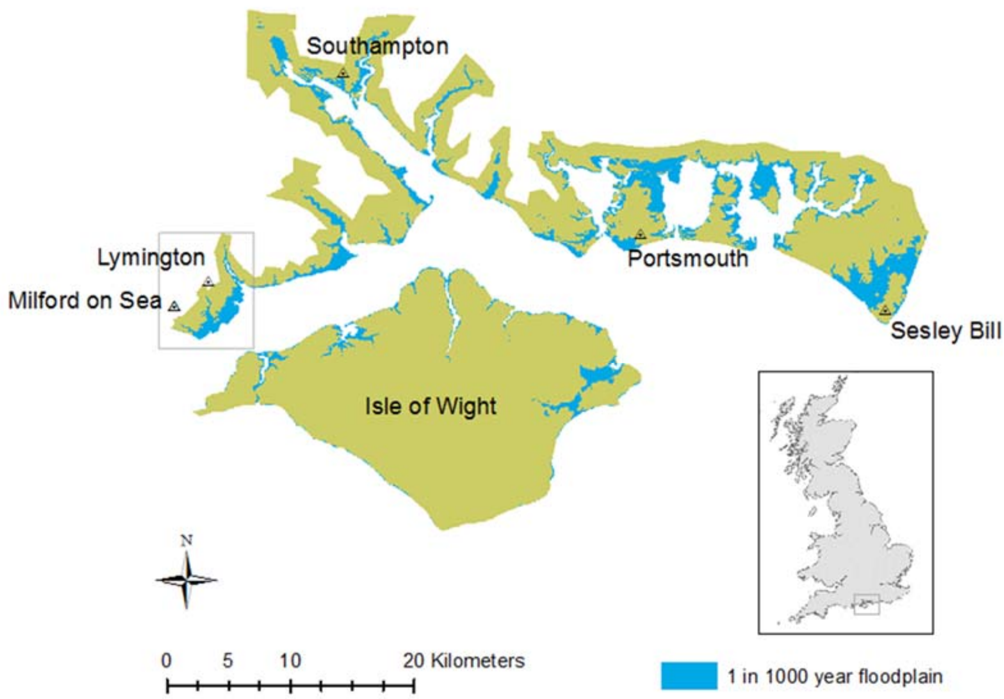


Figure 1. Lymington and the wider Solent Region on the English Channel coast, UK, including the 1-in-1000 coastal floodplain (Wadey et al., 2013).

This paper is structured as follows. The next section briefly describes how the method of real options analysis is adapted to deal with the problems of coastal flooding and sea-level rise. Sections 3 and 4 present the results of the flood risk analysis and real options analysis for raising the crest of coastal defences in Lymington, respectively. Lastly, section 5 concludes with the implications of the results for investment decision-making, and hence the application of real options analysis in practice.

2. Methodology

The framework for this research consists of two parts: uncertainty modelling and real options analysis. The uncertainty modelling examines how the risk of coastal flooding changes over time under different sea-level rise scenarios with a focus on the relation between climatic variables (e.g. extreme still water level or sea-level rise) and risks, the latter being hereafter represented by expected monetized flood damages. This study evaluates the investment opportunity of a deferrable option under the modelled uncertainty of climate change. The calculation of option values with varying investment time provides an understanding of optimum investment time for an adaptation option.

This analysis defines avoided risk as benefit from the adaptation option. In this paper, only direct economic risk – damage to property– is considered. This study assumes that a certain magnitude of coastal event corresponds to a certain amount of flood damage. The main focus is climate change, in particular, sea-level rise. We recognise that other variables, in particular socio-economic ones, play a considerable role in the evolution of risk, and economic or population growth is considered for a sensitivity analysis. To simplify the analysis, some factors are considered constant, including the storm characteristics and coastal morphology.

2.1 Uncertainty modelling

Coastal flooding is the consequence of the combining effects of storm surge events and astronomical tides (McMillian et al., 2011). Sea-level rise will increase the likelihood of extreme events and hence the risk of coastal flooding, all other factors being equal (Lowe et al., 2001; Linquiti and Vonortas, 2012). This study models the risk of coastal flooding by statistically integrating mean sea-level rise MSLR(t) to the pdf (probability density function) of extreme still water level (hereafter, referred to as ESWL) at the base year (i.e. 2008). The MSLR data are taken from UKCP 09 (Lowe et al., 2009). The effect of waves is also considered by adding the equivalent heights of wave overtopping volumes to extreme still water levels.

The extents and damages of coastal flood events associated with the possible magnitude of climatic variables (i.e. ESWL+MSLR+WAVE) are estimated assuming both for the current and upgraded defences each year. Hence, this provides a couple of damage curves and a benefit curve for the intervention measure over all the possible climatic variables. We have statistically estimated the expected flood damages for both cases and the expected benefit from the intervention each year. This process has been conducted for each MSLR scenario. The procedure is summarized in Figure 2.

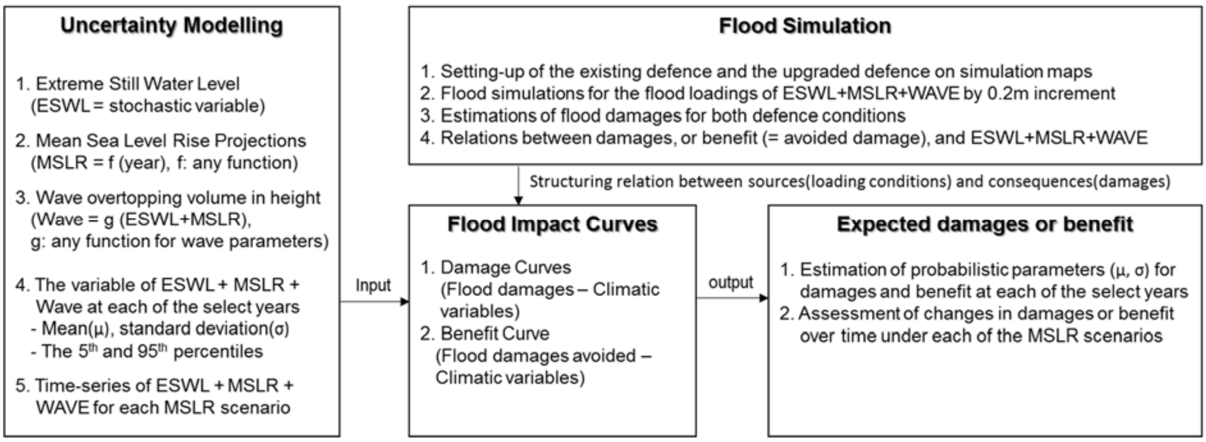


Figure 2. The framework for evaluating expected damages and benefits under rising sea levels.

2.2 Option evaluations

As long as option holders can defer an option, the choice, either to invest or to defer, remains flexible. Thus, the real options analysis assesses two option values: (1) a continuation value, which is an option value when the option is deferred, and (2) a termination value, which is an option value when the option is implemented. The real option value in any given year *t* can be defined by equation (1) (Bellman, 1952; Dixit and Pindyck, 1994).

$$F_t = max\left[F_{con,t}, F_{ex,t}\right] \quad (1)$$

Here, $F_{con,t}$ is a continuation value in year *t*, $F_{ex,t}$ is a termination value in year *t* and F_t is an option value in the year *t*, which is the highest of the two values. If a continuation value ($F_{con,t}$) is higher than a termination value ($F_{ex,t}$), it suggests waiting is preferable to implementation and vice versa (Bellman, 1952). The termination value ($F_{ex,t}$) at any year *t* is the net present value of the investment made at year *t*:

$$F_{ex,t} = \sum_{i=t+1}^{L+t} \frac{EAB_i}{(1+r)^i} - \frac{I}{(1+r)^t} \quad (2)$$

Where, EAB_i is the expected annual benefit of the project at year i

$$EAB_i = \sum_{m=1}^{M} p_i(x_{m,i}) \times B(x_{m,i}) \quad (3)$$

r is a discount rate, I is investment cost, and L is the number of project years, p_i is the probability density function at the year i, B is a benefit function of climatic variable $x_{m,i}$ representing a benefit curve (i.e. reduction in flood damages associated with the mth event), $x_{m,i}$ is the m-th event of climatic variable (i.e. ESWL+MSLR+WAVE) at year i, M is the number of discrete events adopted to describe the distribution of probabilty of coastal events.

On the other hand, when the investment is deferred from year t to year t+1, the valuation of the adaptation option at year t+1 uses the same method as in the year t. As a continuation value ($F_{con,t+1}$) and a termination value ($F_{ex,t+1}$) are values the option holder can expect in year t+1 by waiting, the continuation value at year t is the higher one of these two values discounted from year t+1 to year t:

$$F_{con,t} = \frac{1}{(1+r)} \times Max\left[F_{ex,t+1}, F_{con,t+1}\right] \quad (4)$$

This option evaluation process provides two decision criteria. Firstly, the overall benefits (B) during the project life should be higher than the overall costs (C). The second one concerns the investment timing for which termination and continuation values are compared every year. The calculation of the continuation and termination values starts from the end of the period of wait (denoted by T) using a backward induction method (Bellman, 1952; Dixit and Pindyck, 1994; Yang and Blyth, 2007). Figure 3 illustrates the process for the evaluation of real options.

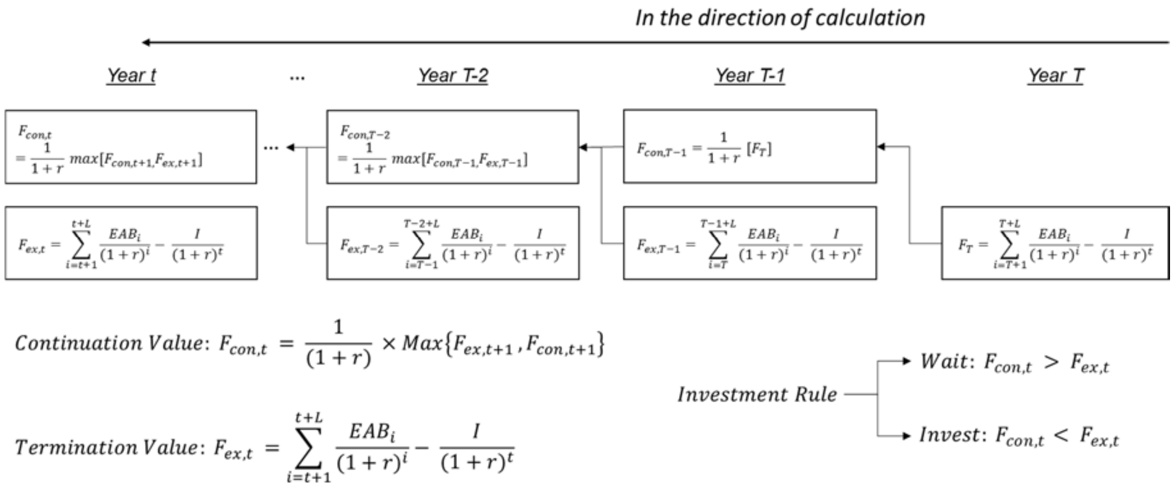


Figure 3. The concept and process of real options valuation based on dynamic programming (Bellman, 1952)

3. Flood risk assessment for uncertainty modelling

The methodology described in the previous sections is strongly based on the knowledge of the relations between climatic variables (i.e. ESWL+MSLR+WAVE) and flood damages, which need to be defined by a flood risk assessment. This section briefly describes the methodology used to assess the above relation in our case study of Lymington.

3.1 Case study description

Lymington is a coastal town in southern England that has experienced a number of coastal floods over the last 100 years, including 13-17 December 1989, 25 December 1999 and 10 March 2008 (Ruocco et al., 2011; Wadey et al., 2013). The coastal defences have been upgraded several times in response to these flood events and currently are at 2.5~3.0m (mAOD) elevation (OD approximates mean sea level). However, as sea level is rising, the risk of coastal flooding is growing. The Shoreline Management Plan (SMP) in Lymington has selected a Hold The Line (HTL) policy, involving coastal defence upgrade (NFDC, 2010). Thus, the timing of defence upgrade is of importance to Lymington.

3.2 The effect of sea level rise on coastal flooding

The distributions of extreme still water levels (ESWL) are available every 2km along the UK coastline from the Environment Agency for England and Wales (McMillian et al., 2011; Batstone et al., 2013). The locations of the data points around Lymington are shown in Figure 4a. We assume that the cumulative distribution function (CDF) of ESWL follows an exponential curve for the convenience of data fitting and calculation ($R^2 = 0.99$) (Figure 4b).

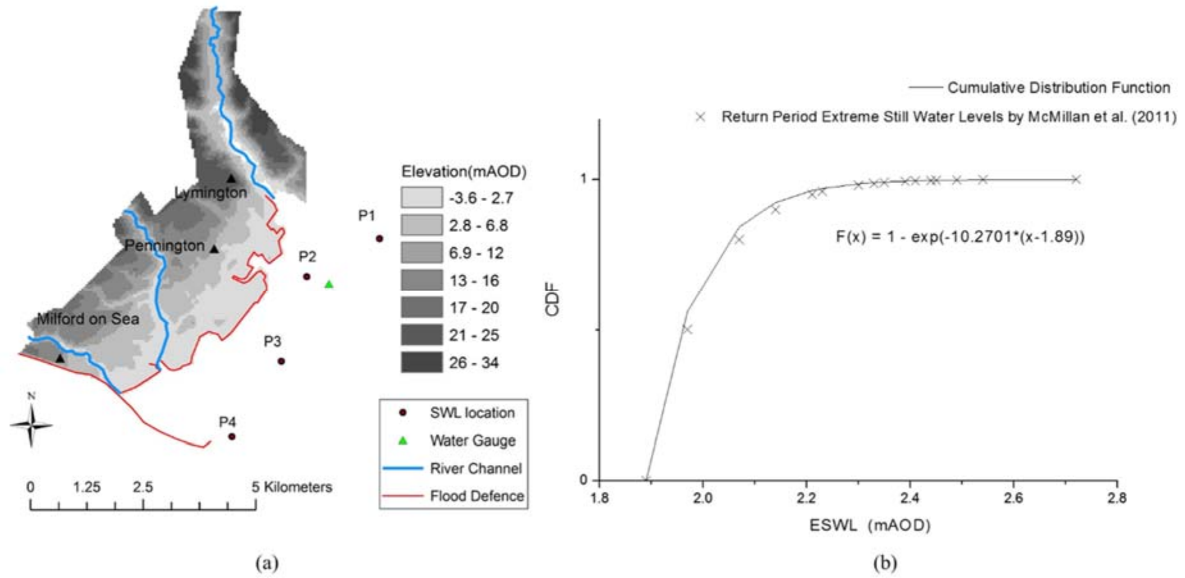


Figure 4. (a) Location of data points for extreme still water level around Lymington and (b) Cumulative Distribution Function at the point P1, after McMillan et al (2011) - For location, see Figure 1

The UKCP 09 data provides multiple scenarios of sea-level rise depending on greenhouse gas emissions from 1990 to 2100 as shown in Figure 5 (Lowe et al., 2009). This study includes the H++ sea level rise scenario as an unlikely, but extreme scenario. This is developed by scaling the High MSLR projection in Lymington with the ratio of the H++ MSLR scenario to the High MSLR scenario from the global scale data provided by Nicholls et al. (2014). For illustrative purposes, we also consider a slow-steady projection of historic trends of sea-level rise from the Southampton tidal gauge (Haigh et al., 2009)

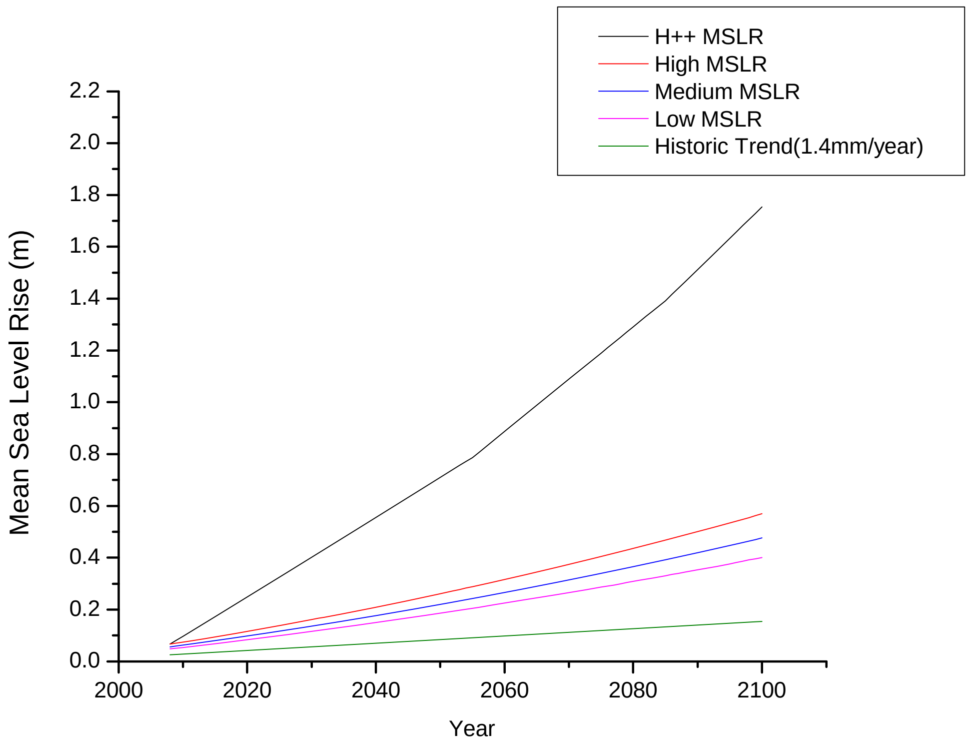


Figure 5. Mean sea-level rise (relative to 1990) scenarios for Lymington: 2008 to 2100 (Lowe et al., 2009) - the H++ MSLR is scaled from the global scale sea-level rise scenarios (Nicholls et al., 2014), and the historic trend of sea-level rise (1.4 mm/year) is from the Southampton tidal gauge (Haigh et al., 2009).

We statistically combine mean sea level rise MSLR(t) at the year t with ESWL(t_0), which is a stochastic variable representing extreme still water level at the base year (t_0=2008). Figure 6 shows four possible time-series of ESWL+MSLR that result from the combination of MSLR(t) and ESWL(t_0), within the probabilistic range of the corresponding 5th and 95th percentile lines. Sea-level rise shifts the exponential distributions displayed in Figures 6 (b) and (c) towards the right, leading to increases in frequency and magnitude of coastal flooding.

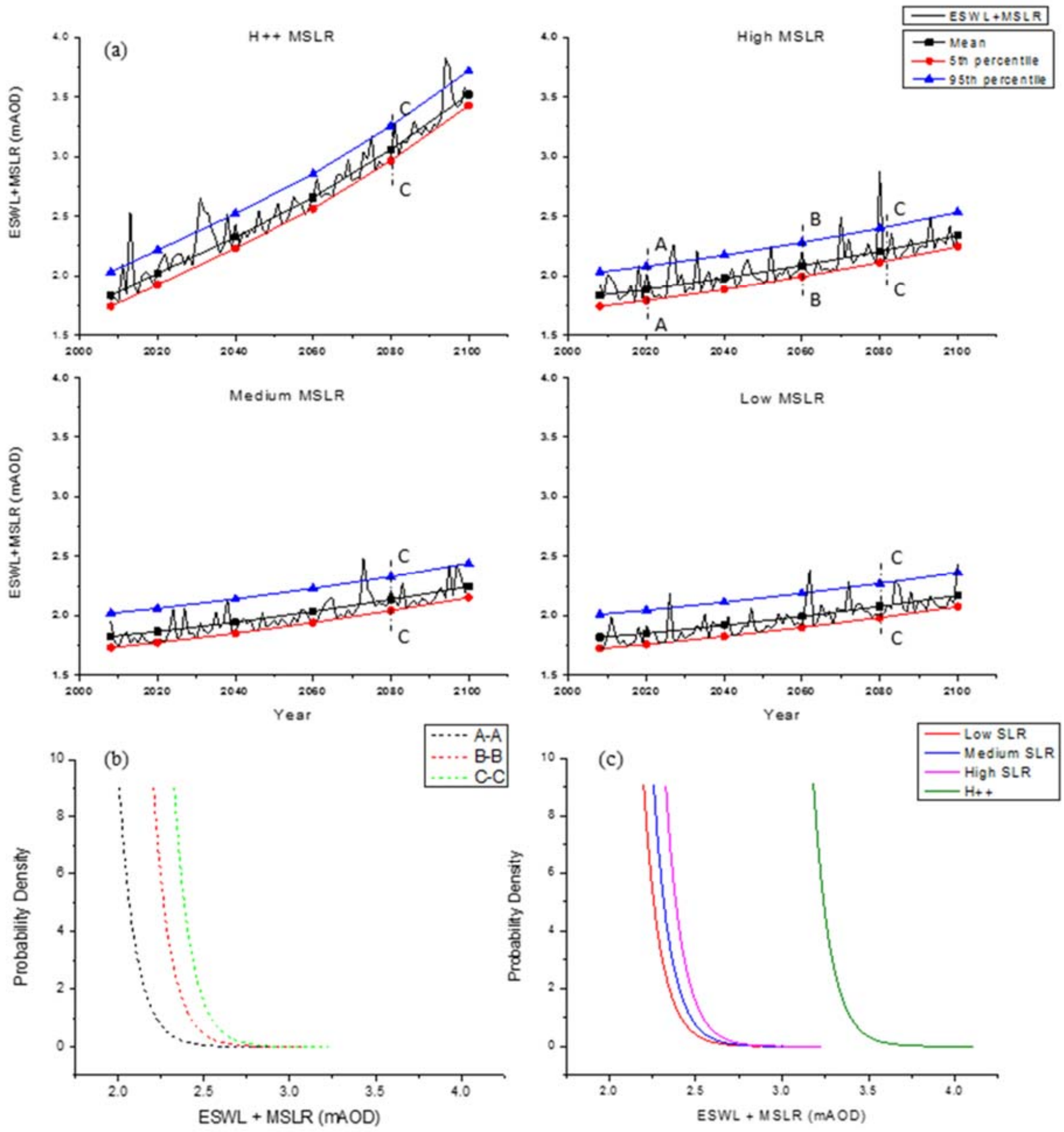


Figure 6. (a) Example of stochastically generated time-series of ESWL (annual maximum) for each MSLR scenario (b) Temporal change in probability density function (pdf) from the crosslines (A-A, B-B and C-C) for the High MSLR scenario (c) Probability density function (PDF) at 2080 from the crossline (C-C) for each MSLR scenario

3.3 Analysis of flood damages

A number of flood inundation simulations (15 times for each of 2 defence scenarios) were performed with a variety of the climatic variables (i.e. ESWL+MSLR+WAVE) for the existing coastal defences and the upgraded coastal defences, respectively. For simplicity, a single fixed adaptation option is considered in this study, which is to raise the crest of the coastal defences to 3.5m AOD with an investment cost of £64.2 millions as shown in

Figure 7 (NFDC, 2010). The possible values of ESWL+MSLR+WAVE were discretised in 0.2m increments from 1.4 to 4.0 mAOD. This covers all the likely range of coastal flood events through the 21st century in Lymington.

In order to estimate the performance of the upgraded coastal defence versus the existing coastal defence, a pair of flood risks for the existing defence and the upgraded defence are assessed, assuming overflowing and overtopping defence failures, for the same magnitude of coastal flood events, respectively. Breaching of coastal defence is not considered in this analysis, although breaching occurred in 1989 with a 2.10 mAOD of storm surge event in Pennington between Lymington and Milford-on-Sea (Ruocco et al., 2011; Wadey et al., 2012). In breach failure, flood damage will be significantly larger due to a large volume of seawater inflow through breaching points. To consider the risk of breaching failure for option evaluations, we needed to estimate the monetized damage of breaching failure and the probability of the occurrence of the breaching in a given year for both defence scenarios. However, the assessment of the risk of breaching failure is very complicated. For simplicity, the impact of breaching on the floodplains is excluded in the assessment of flood damages.

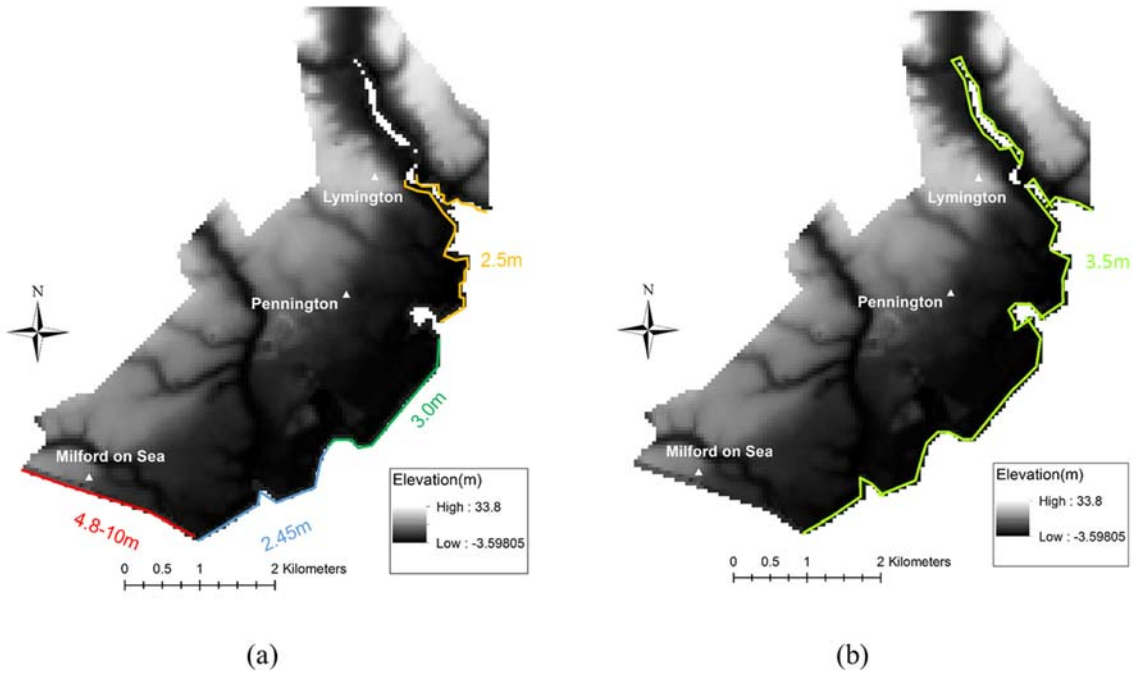
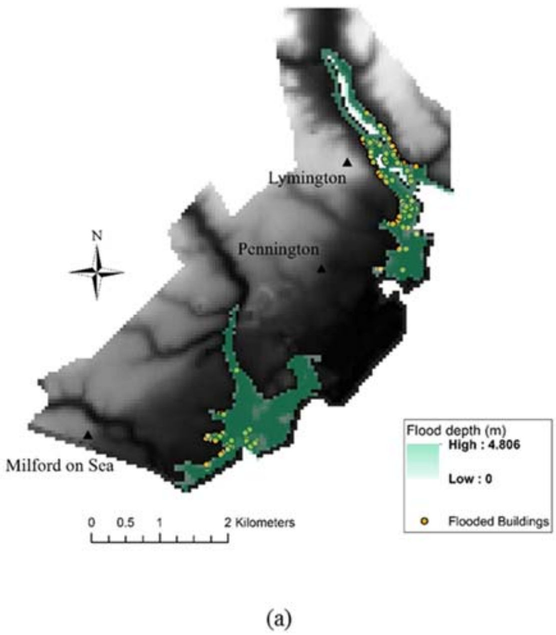


Figure 7. Location of the coastal defences and the amount raised in the upgrade scenarios for (a) existing coastal defence and (b) upgraded defence – The numbers shown on the maps are the crest level of coastal defence above the datum (unit: mAOD).

The standardized time series of water level per event has been based on the significant surge event on the 10 March 2008, which is the most recent flood event in Lymington (Wadey et al., 2013). The water level time-series during a flood event with a specific peak water level have been produced by offsetting the 2008 time-series by a difference between the peak water levels of the 2008 event and the peak water level of the considered flood event. The wave impacts were simulated by applying the wave height (H_s) and wave period (T_p) of 0.91m and 3.3s, respectively (which is recorded during the 2008 event). As overtopping volumes by waves highly depend on freeboard (i.e. the distance between the water level and the crest of coastal defence), we apply a single wave condition recorded in the 2008 flood event for the flood risk analysis, which ranges in overtopping volumes from 0.09m (min) to 0.20m (max) in equivalent height, depending on ESWL and MSLR. This assumption enables us to reduce the amount of calculations for wave impacts on coastal flooding, which are small compared to the impacts of ESWL and sea-level rise. The overtopping volumes over the crest of the coastal defence were calculated by an empirical formula for slope structures based on the previous studies (Owen, 1980; Wadey et al., 2012; EurOtop, 2016). The process of flood inundation in the floodplain was modelled using the LISFLOOD-FP hydrodynamic model (Bates et al., 2010; de Almeida et al., 2012).

The number of inundated properties and the corresponding inundation depth of each property has been determined from the model results. Figure 8 shows an example of flood inundation maps produced for both scenarios of the existing and upgraded defences. Based on property information provided by the Ordnance Survey (UK), the flood damages for each coastal flood event are monetized following the guidelines provided by the Environment Agency (2010) and Penning-Roswell et al. (2010; 2014). Figure 9 shows the damage (number of properties and economic losses) and the benefit curves over the variables of ESWL+MSLR+WAVE.



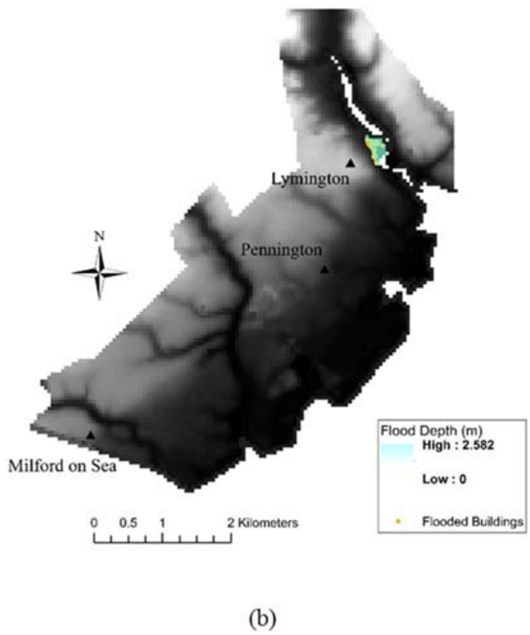
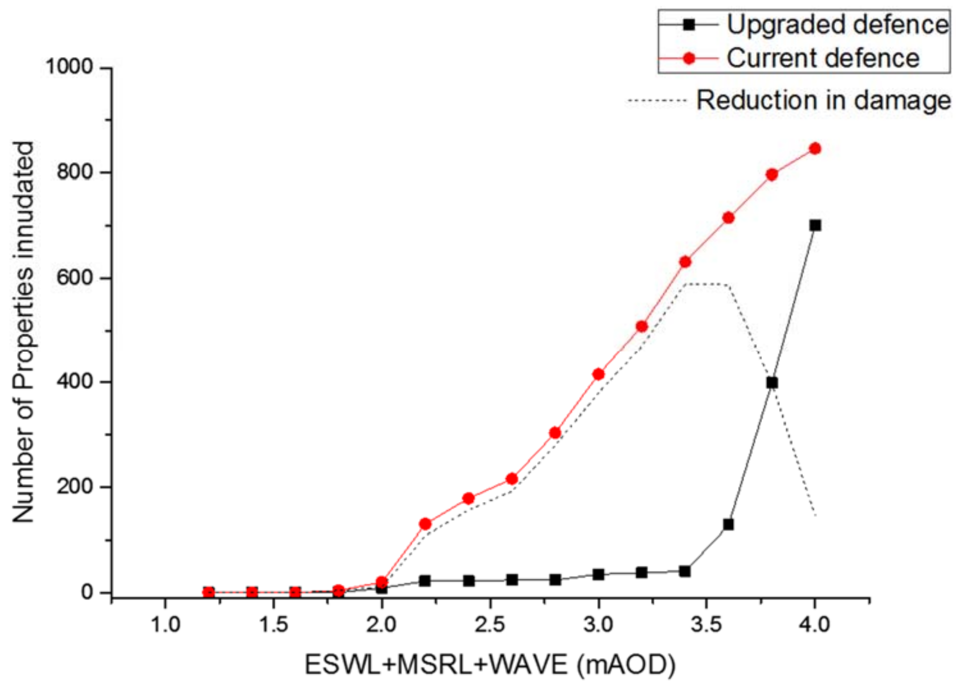
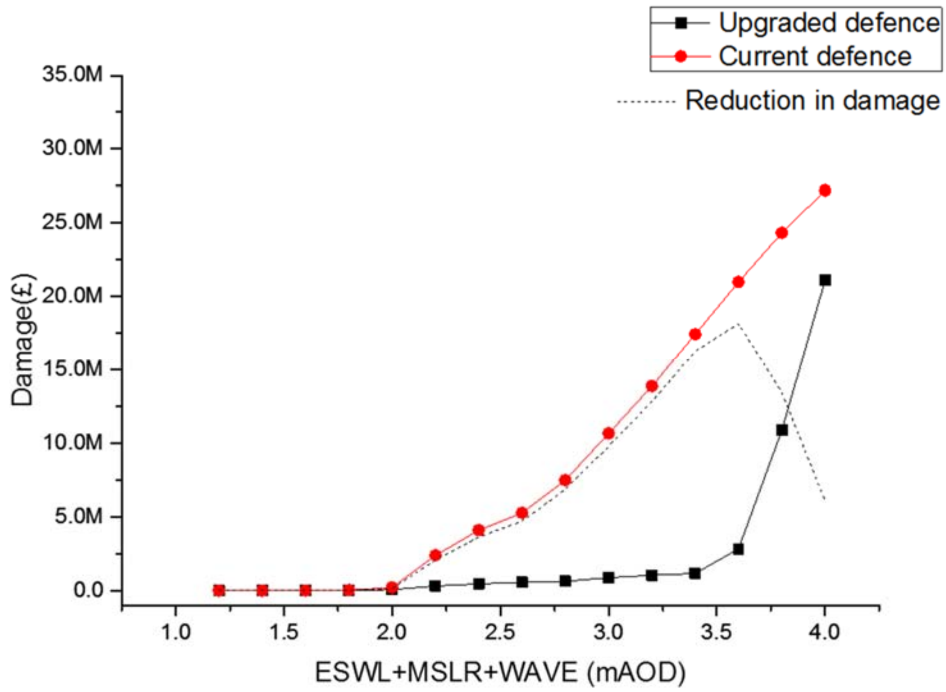


Figure 8. Example of flood maps including properties within the flooded area (blue-shaded areas) for (a) existing defences and (b) upgraded defences given a 1-in-200 year coastal flood event in 2008.



(a)



(b)

Figure 9. (a) The number of properties flooded by defence assumptions, and (b) Corresponding flood damages and reduction in the damages (£ in 2016)

3.4 Change in flood damages and benefits due to sea-level rise

The stochastic properties of the flood damages and benefits are estimated by a sampling method using a probability analysis programme (@Risk). In detail, the stochastic variables of ESWL+MSLR+WAVE sampled from the predefined probability (Figure 6) have iteratively been input into the established relations between the climatic variables and the flood damages (£) to construct the probability distribution of the damages. The mean and probabilistic range (5th and 95th percentiles) of the benefit value is estimated for some representative years (i.e. a base year (2008) and every 20 year from 2020) under each MSLR scenario (Figure 10). The figure also shows one particular realisation of the stochastic time series of the benefit.

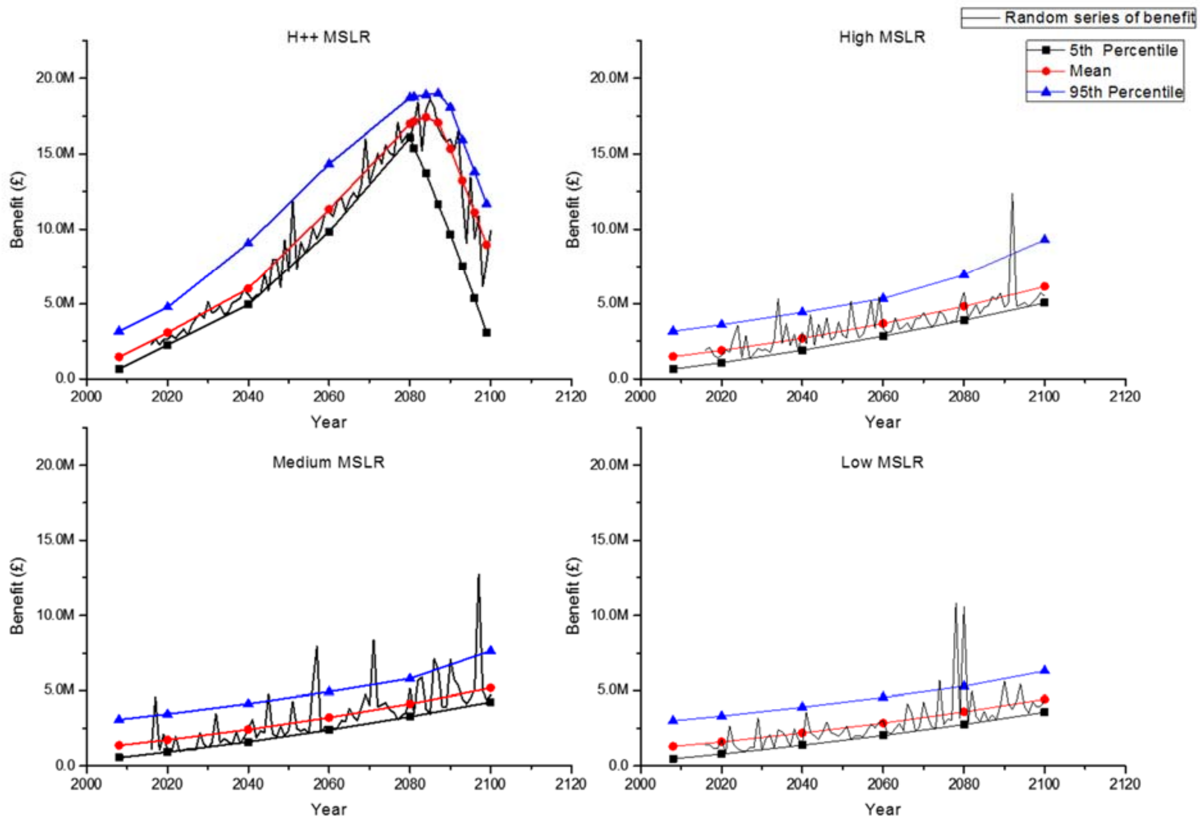


Figure 10. Changes in benefit due to the upgrade of coastal defences and its probabilistic range under each MSLR scenario

As sea level increases the damage of coastal flooding, it also leads to increase in the utility of the upgraded coastal defence as shown in Figure 10. However, for the H++ MSLR scenario, when ESWL+MSLR+WAVE exceeds 3.5m AOD (which would occur around 2080), the upgraded defence is less capable to protect Lymington. The annual benefit from the upgraded coastal defence will decline after 2080. Therefore, if sea-level rise follows the H++ MSLR scenario, the second upgrade of the costal defence needs to be considered before 2080, indicating a possible adaptation pathway for Lymington.

The mean value (red line) in Figure 10 plays an indicative role in option evaluation as it represents the expected annual benefit (hereafter, EAB), which is the integration of the avoided damages times the corresponding probabilities over all the possible events for any given year. As with flood risk analysis, we apply the concept of expected annual benefit (EAB) to estimate the effect of the intervention measure on reduction in flood damages in a given year. The utility of the upgraded defence has been estimated against all the possible flood events every year during the 21st century.

4. Option evaluation

4.1. Continuation and termination values

The coastal defence upgrade along the coastline between Lymington and Milford-on-Sea is planned for the period between 2030 and 2060 with an indicative cost of £ 64.2 million (the North Solent Shoreline Management Plan) (NFDC, 2010). Discount rates are taken from the Green Book (HM Treasury, 2011), which is a standard in project appraisals in the UK. Table 1 summarises the main parameters used in option evaluation.

Table 1. Conditions for real options analysis in Lymington

Investment conditions	Details
Investment cost	£ 64.20 M
Discount Rate (HM Treasury)	3.5% (0 – 30 years) 3.0% (31 to 75 years) 2.5% (after 75 years),
Project Life (L)	100 years
Sea Level Rise (average rates from 2090 to 2100) are assumed constant after 2100	H++ MSLR: 24.1 mm/year High MSLR: 6.8 mm/year Medium MSLR: 5.7 mm/year Low MSLR: 4.7 mm/year Historical MSLR: 1.4mm/year

The backward induction method requires a boundary value at the end of the time horizon for option evaluation (Dixit and Pindyck, 1994; Yang and Blyth, 2007). The analysis sets the end year at 2100, consistent with the period of the MSLR scenario projections from UKCP 09. Thus, this analysis assumes that option holders have the ability, but not the obligation, to defer the investment to 2100. The boundary value (F_T) is calculated at this end year. To obtain the boundary value for each MSLR scenario, the analysis has extrapolated sea-level rise from 2101 to 2200 following the linear rate of sea-level rise from the last 10-years of data from 2090 to 2100. The calculation and comparison of continuation and termination values at each year is conducted for each of the MSLR scenarios (Figure 11).

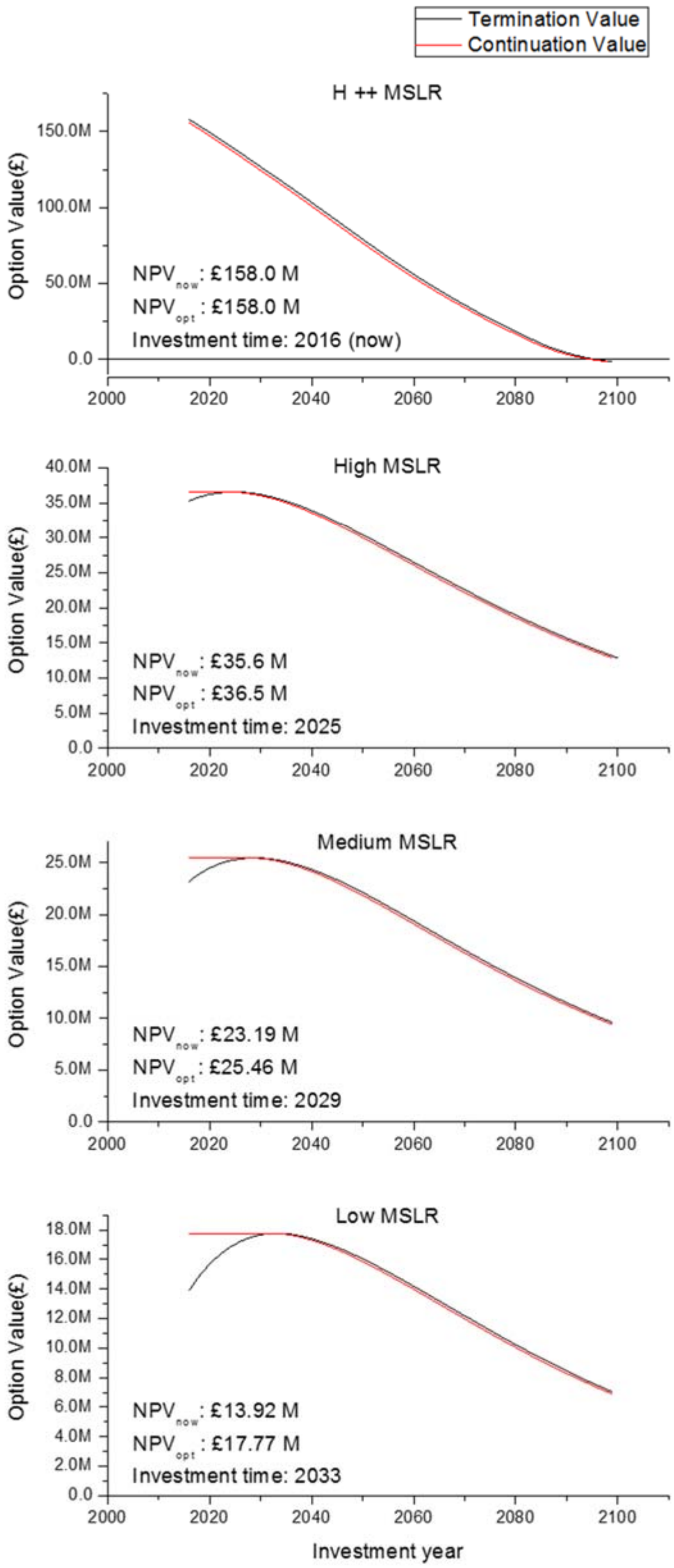


Figure 11. Option values indicated by the termination value and continuation value – all values are discounted to 2016. Note the different scales on the y-axis - Here, NPV_{now} is termed for the option value of an immediate investment while NPV_{opt} is for the option value of an optimal or timely investment, which is maximum during the period of wait.

The analysis shows how the option value for the upgrade of coastal defence changes according to the investment year in each MSLR scenario. As shown in Figure 11, the termination value (black line) is smaller than the continuation value (red line) at 2016 for all of the MSLR scenarios, except the fastest rise in sea level (the H++ scenario). As we move forwards in time, the termination value exceeds the continuation value, indicating the optimum time to invest. By this information, decision-makers can determine when the investment should be implemented in order to produce the largest net profits (i.e. the overall benefits minus the overall costs) given the sea-level rise information. After the optimum investment time, the option value keeps declining with time so that the deferrable adaptation option becomes less and less efficient in economic terms. Importantly, the more extreme the MSLR scenario, the earlier the optimum investment time occurs. Thus, for rapid sea-level rise, we need to react quickly, while for slower sea level rise we have more time to act (see Table 2). In addition, the difference between the maximum option value (NPV_{opt}) and the option value of the investment now (NPV_{now}) indicates how much more option holders can benefit from the delay of the option under the MSLR scenarios. The result for the historic trend of MSLR, which is a rate of 1.4mm/year in Southampton, is also included in Table 2.

Table 2. Results of real options analysis by different MSLR scenarios.

Scenarios	***H++***	***High***	***Medium***	***Low***	***Historic trend***
Investment cost	£ 64.20 M	£ 64.20 M	£ 64.20 M	£ 64.20 M	£ 64.20 M
Optimal Investment Time	2016 (i.e. now)	2025	2029	2033	2083
NPV_{now}	£ 158.0 M	£ 35.3 M	£ 23.19 M	£ 13.92 M	£ (-) 14.48 M
NPV_{opt}	£ 158.0 M	£ 36.5 M	£ 25.46 M	£ 17.77 M	£ 1.08 M
Δ(=NPV_{opt}- NPV_{now})	0 M	£ 1.2 M	£ 2.27 M	£ 3.85 M	£ 15.56 M

In the H++ MSLR scenario, sea-level rise will reach 1.75m (relative to 1990) in 2100. As expected, this scenario gives the highest NPV_{now} and NPV_{opt} and the earliest optimal investment time (2016) of all the MSLR scenarios (Table 2). In this case, the investment already gives a larger option value than the deferral of the option. Thus, the investment is optimal now in this most extreme scenario. (More generally, it is found that when the rate of sea-level rise exceeds 4.9mm/year, the optimal investment time occurs in 2016). There are two noteworthy points in the result of real options analysis in the H++ MSLR scenarios. Firstly, the upgrade of the coastal defence to the level of 3.5 mAOD will become less and less useful after 2080 under the H++ MSLR scenario in a statistical sense (See Figure 10). This implies that the coastal defence that will be upgraded after 2080 needs to be raised higher than the current plan (i.e. raising the crest level up to 3.5 mAOD) if sea-level rise follows the H++ MSLR scenario. The real options analysis is, thus, explicitly showing that the real option values – the continuation and termination values – are under zero after 2095 as shown in Figure 11. This means that the overall expected benefits after the investment at 2095 are less than the overall cost. Secondly, the option value in the H++ MSLR scenario rapidly declines from £158 million in 2016 if the investment is deferred. Thus, early detection of the rate of sea-level rise is important for implementing investment, especially if extreme sea-level rise is occurring.

4.4. The sensitivity of the optimal investment to the uncertainty of climatic variables

For sensitivity purposes, the probability distributions of option values have been constructed in cases where the investment is made now (2016), at the optimal investment time, and in the distant future (2060) for each of the MSLR scenarios. For the calculations, we have substituted the variable of EAB_i in equation (2) by the stochastic variable of B_i, which is obtained from the random variable of ESWL+MSLR+WAVE by the relation between the benefits and the climatic variables. This is a more realistic estimation of option values than by using the mean value of benefit (i.e. EAB). The probability distributions of option values at the different investment years have been constructed based on 10,000 realisations (Figure 12).

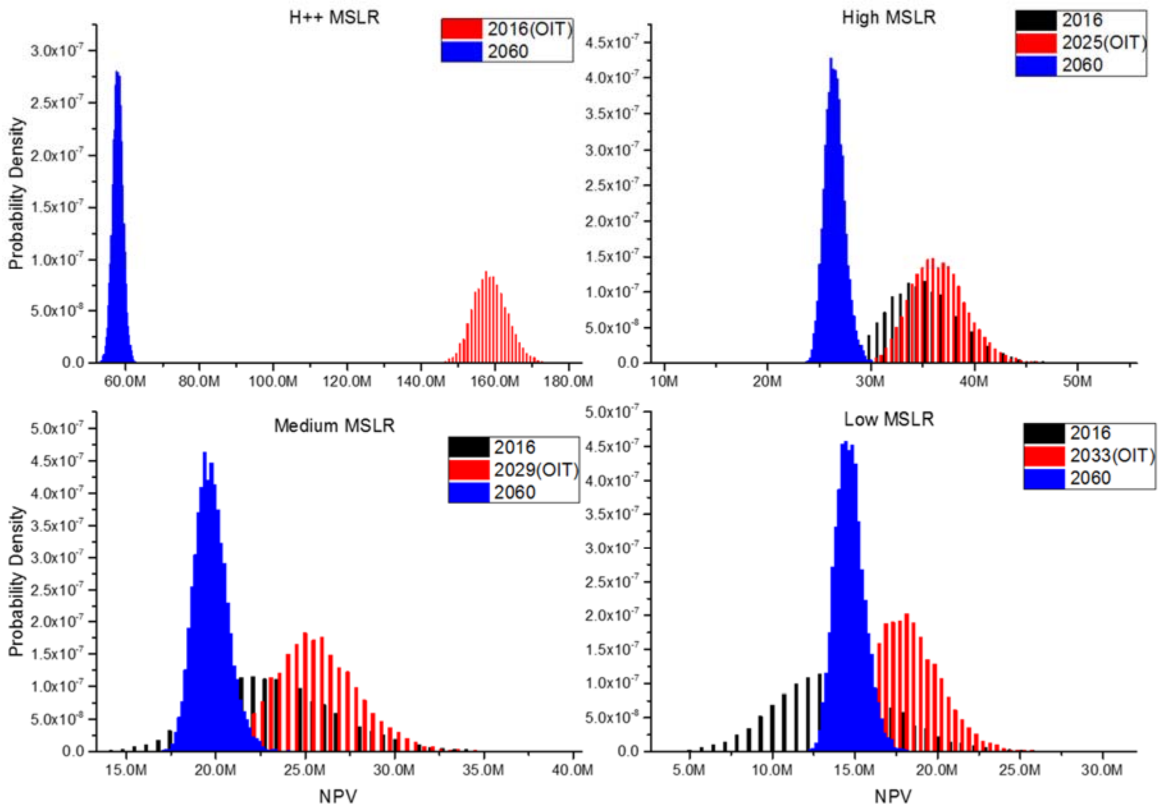


Figure 12. The distributions of option values at optimal investment time (OIT) and different investment years under random series of ESWL+MSLR+WAVE in each MSLR scenario. Note the different y axis scales.

The option value of the optimal investment time is likely to be higher than those of other investment times (Figure 12). If the investment is made earlier than the optimal time, its distribution (black bar) is in the lower range of option value with a high variance. If the investment is made later than the optimal time, its distribution (blue bar) again moves to the lower region, but with a smaller, narrow variance - The denominator $(1+r)^t$ increases significantly with the investment time so that the range of the option values becomes narrower. The sensitivity analysis shows that investment at the optimal investment time is likely to provide the largest option value under the uncertainty of magnitude and occurrence of coastal flooding.

4.5. The magnitude of sea-level rise at the optimal investment time

An interesting observation relates to the absolute sea-level rise at the optimal investment time for all the MSLR scenarios except the H++ MSLR scenario. MSLR at the optimal investment time is found at 13 to 14 cm relative to 1990 with EAB estimated around £ 2 million for all the MSLR scenarios except the H++ MSLR scenario (Table 3). As the expected annual benefit (EAB) is dependent on the stochastic properties of the climatic variables at a given time, MSLR at the optimal investment time accompanies the similar values of EAB (Refer to Appendix 1). The absolute sea-level rise has important implications for determining the optimal investment time in its application, as a single threshold value can be used to predict the investment timing for an adaptation option (Figure 13). By observing the rate and magnitude of sea-level rise at any time, rather than identifying the ongoing pattern of sea-level rise, we can decide when the upgrade of the coastal defence needs to be implemented.

However, the H++ MSLR scenario shows a different pattern in the occurrence of MSLR and EAB at the optimal investment time. The investment time for the H++ MSLR scenario occurs at the earliest time (i.e. 2016) among all the MSLR scenarios and the MSLR and EAB at this time is 19cm and £2.33 M, respectively. Even though the optimal investment time and MSLR were further traced to the past before 2016, the termination value was still higher than the continuation value. Thus, in the most extreme H++ MSLR scenario, the adaptation option should be implemented immediately. However, as sea-level rise does not yet reach 13cm (relative to 1990) in Lymington, the current trajectory of sea-level rise does not seem to be the H++ MSLR scenario.

Table 3. MSLR and EAB at optimal investment time in each of climate change scenarios

MSLR scenarios	Optimal Investment Year	MSLR (cm)	EAB (£ M)
H++	2016	19	1.49
High	2025	14	2.07
Medium	2029	13	2.03
Low	2033	13	1.99
Historical trend	2083	13	2.03
Critical rate (4.9mm/yr)	2016	13	2.00

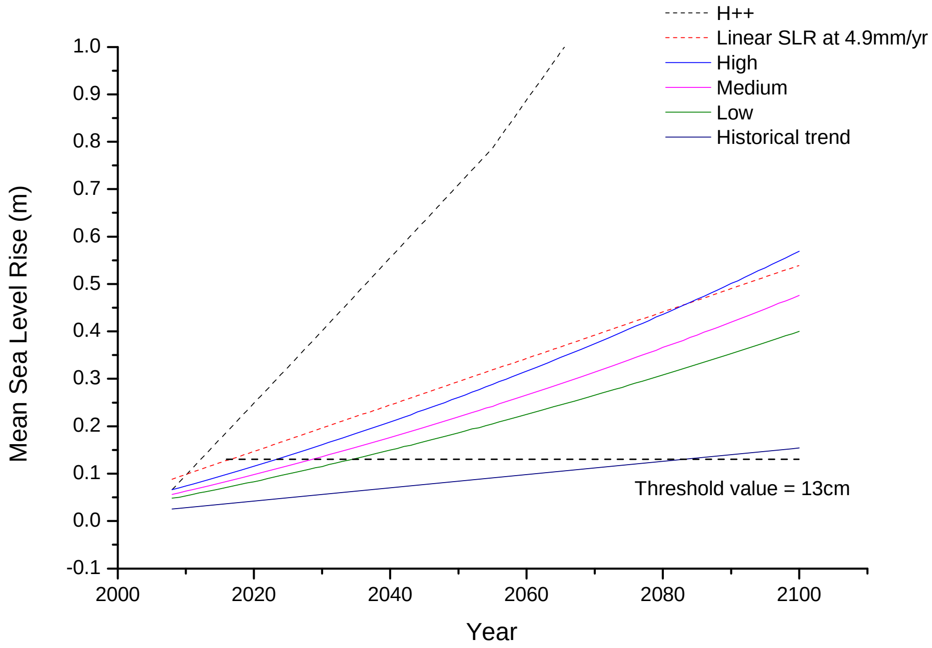


Figure 13. The optimal investment time for a threshold value of 13cm across all MSLR scenarios – The H++ scenario is only plotted within the y-scale. The linearly increasing sea-level rise (4.9mm/yr) is plotted to illustrate the maximum rate of sea-level rise to which the threshold value of 13cm is valid.

4.6. The effect of future development on option evaluation

The previous analysis estimated the optimal investment times and option values upon the assumption that there is no future development of properties. The increasing number of properties for economic or population growth will also have an influence on the expected annual benefit (EAB) due to the changing number and characteristic of the buildings and their values within the floodplains. For a sensitivity purpose, we extend the real options analysis to the cases of socio-economic developments at -0.5%, 1%, and 2% annual growth rates. This represents decline or growth in property stocks and values. We assume that the rate of the socio-economic change is constant over time. The values of EAB under different future growth scenarios are estimated by simply multiplying the EAB estimated in the no growth rate scenario by the growth rates. Figure 14 shows the results of EAB under different growth rate scenarios. Based on the projections of EAB, the backward induction approach has been applied from 2100 to 2016 for each growth rate scenario for each MSLR scenario as shown in Figure 15. The results are shown in Table 4.

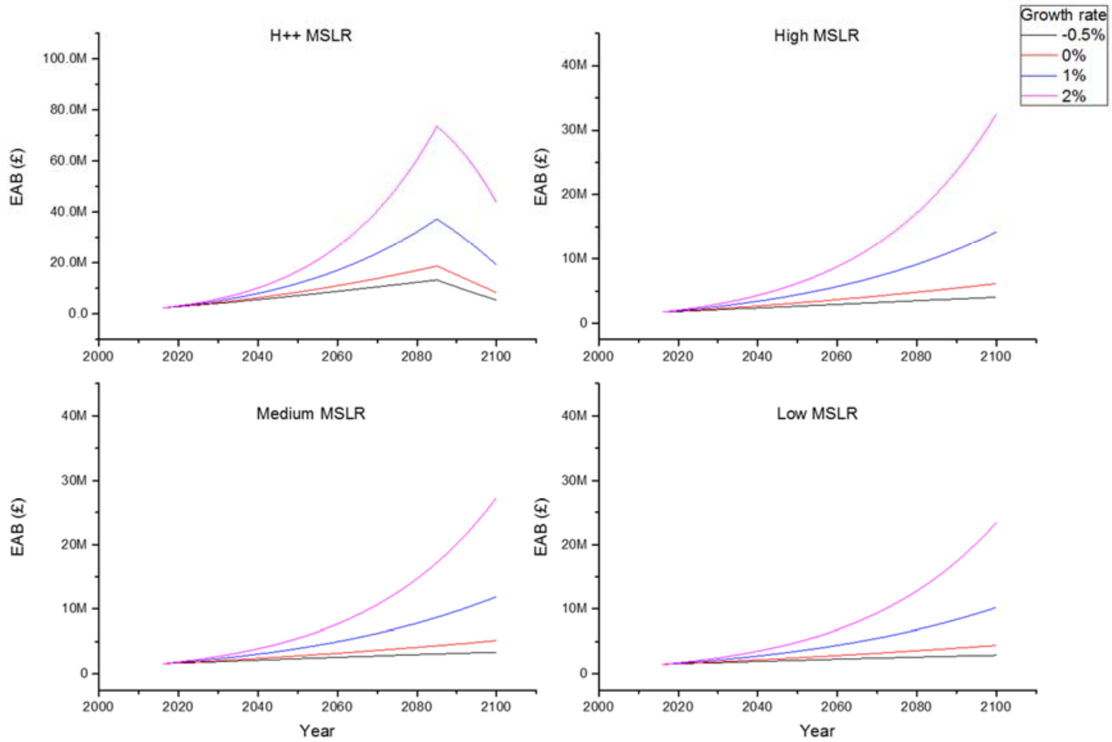


Figure 14. Changes in expected annual benefit by different growth rates of property development for each MSLR scenario. Note the different scale of y-axis for the H++ MSLR scenario - The projections of EAB under the different growth rates decline under the H++ scenario after 2080.

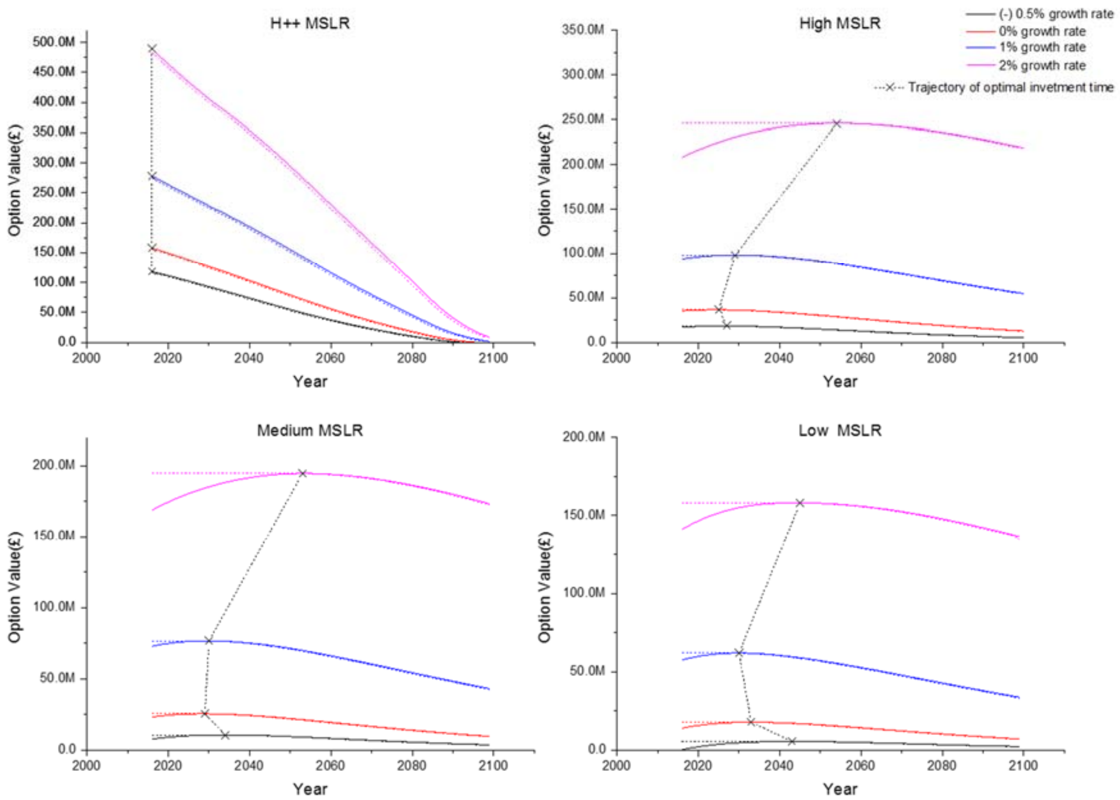


Figure 15. Option values and optimal investment times indicated by termination value (solid line) and continuation values (dash line) – Note the different scales on the y-axis.

Table 4. The results of option values and optimal investment times for different future growth rates in each MSLR scenario.

MSLR scenario	Future growth rate (%)	NPV_{now} (£ M)	NPV_{opt} (£ M)	Optimal Investment Time	MSLR at optimal investment time (cm)	EAB at optimal investment time (£ M)
H++	-0.5%	118.0	118.0	Now	19.0	2.39
	0%	158.0	158.0	Now	19.0	2.39
	1%	278.0	278.0	Now	19.0	2.39
	2%	489.0	489.0	Now	19.0	2.39
High	-0.5%	17.7	18.7	2027	14.7	2.03
	0%	35.3	36.5	2025	13.8	2.07
	1%	93.8	98.1	2029	15.6	2.54
	2%	207.8	245.4	2054	28.3	7.15
Medium	-0.5%	7.7	10.6	2034	15.2	2.02
	0%	23.2	25.5	2029	13.2	2.03
	1%	72.9	76.6	2030	13.6	2.37
	2%	168.7	194.7	2053	23.3	6.11
Low	-0.5%	0.25	5.5	2043	16.0	2.00
	0%	13.9	17.8	2033	12.6	1.98
	1%	57.5	61.9	2030	11.5	2.18
	2%	141.2	158.2	2045	16.8	4.19
Historical trend	-0.5%	-22.35	-1.06	No investment	-	-
	0%	-14.47	1.08	2083	13.0	2.03
	1%	9.96	20.47	2045	7.7	2.08
	2%	55.56	70.16	2044	7.6	2.69

As the positive future growth rates exponentially increase the expected annual benefit (EAB) during the project life (Figure 14), the option value of the investment now (i.e. NPV_{now}) and the optimal investment (i.e. NPV_{opt}) increases with the future growth rates as shown in Table 4. However, the optimal investment time for the high growth rates (i.e. 1 or 2 % growth rates) come later than that for the 0% growth rate in High, Medium and Low MSLR scenarios. This is because, while decisions in the cost-benefit analysis are based on a now-or-never basis, decisions based on the real options analysis are considered with the flexibility of wait. Thus, the real options analysis suggests that the wait under the high future growth scenarios may give a better opportunity to receive the increased expected benefits.

Nevertheless, when evaluating an adaptation option under various combinations of climate change scenarios and socio-economic scenarios, careful attentions need to be paid towards assuming future growth rate scenarios. The option value for any given investment year is always greater for scenarios with high growth rates. Thus, if a high growth rate is highly expected, the investment now (i.e. 2016) will be a strategic decision likely to provide high option value for any MSLR scenario. In this case, the timing is a less important factor than the future growth rate for the investment decision. However, if the future growth rate is uncertain, the investment decision needs time to wait and, then, take an appropriate future growth rate.

For example, the assumption of a 2% growth rate scenario under the high MSLR scenario will leave Lymington unprotected until 2054 with sea-level rise reaching 28cm. On the contrary, if we take the 0% growth rate scenario for investment timing, the investment will be made in 2025 with NPV_{2025} (which is termed for the investment at 2025) expected to be £ 36.5M or £ 225M under 0% or 2% growth rate, respectively (Figure 15). In this regard, climate change scenarios and future growth rate scenarios need to be properly combined in order to implement the climate change adaptation in a timely manner (Figure 16).

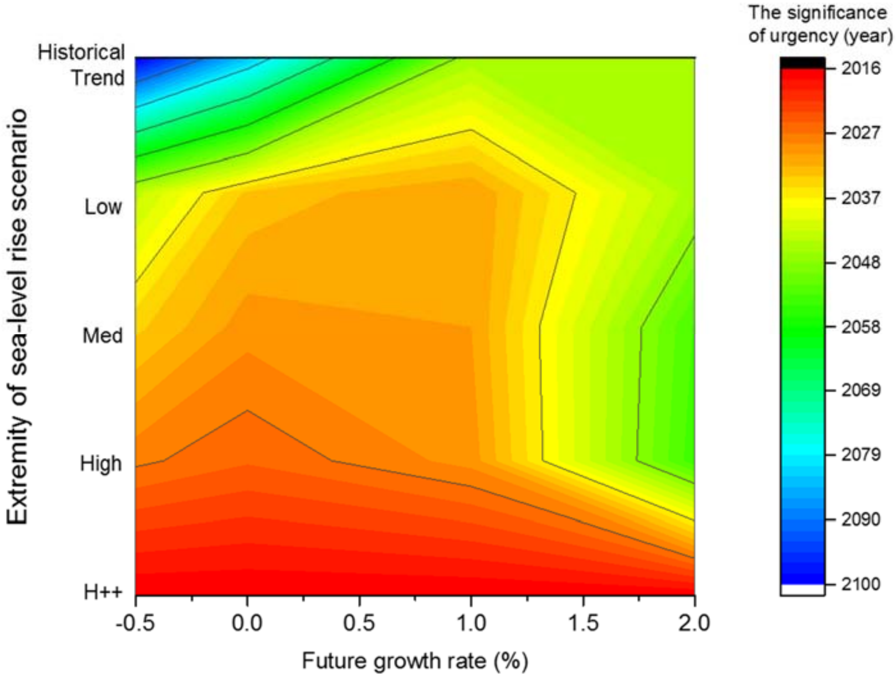


Figure 16. The urgency or priority of sea-level rise scenarios and future growth rate scenarios – The colour where the sea-level rise scenario and the growth rate scenario intersect indicates the optimal investment time. Conversely, the scenarios should be addressed in the sequential order of the optimal investment time. Note that the sea-level rise scenarios on the y-axis are equally distanced for the illustrative purpose.

5. Discussion/Conclusions

We have investigated how sea-level rise increases the coastal flood risk around Lymington and how a single deferrable adaptation option will manage this risk. Real options analysis is used to estimate the option value of the optimal investment with its investment time for the defence upgrade for a set of MSLR scenarios. For the UKCP 09 low to high scenarios, the option value increases over time to a maximum and then decreases subsequently. The analysis suggests that the investment in the adaptation measure for Lymington is worth deferring under all the MSLR scenarios up to 4.9mm/year in 2016. Above this, for the large H++ scenario, the investment should be made now to enhance protection of Lymington from coastal flooding.

It is demonstrated that the investment in the adaptation option at the optimal investment time is likely to give its maximum value with a high statistical confidence (Figure 12). However, it is highly uncertain what trajectory sea-level rise is actually following compared to the scenarios analysed here and accelerations in sea-level rise could take decades to be unambiguously detected (Church et al., 2013; Haigh et al., 2014). Subsequently, the effectiveness of real options approaches will rely on ability to measure the rate of sea-level rise in the real world. This analysis shows the value to adaptation of improving the accuracy of methods to quantify sea-level rise.

If we have to consider a lead time of construction work for the upgrade of coastal defences, determining the investment timing will be more complicated than in this study. This also affects the option value as the stream of benefits occurs right after the construction work is completed. If we are concerned of the economic efficiency of the investment, the construction work needs to be managed to be completed before sea-level rise reaches 13cm. This needs continuous observations on the ongoing pattern of sea-level rise during the construction work.

This research suggests that real options analysis in climate change adaptation requires two key steps: (1) a modelling process to establish the relation between the climatic variables of sea-level rise and the monetized benefits of adaptation measure; and (2) an observation process to detect a critical value (in this case, MSLR at the optimal investment time) to trigger the adaptation at the optimal investment time. These processes have the potential to improve the quality of our decision, especially investment time under uncertainty.

Different future growth rates also result in different investment time and option values. They change the optimal investment time and the critical value of sea-level rise, except for the H++ MSLR scenario. If we consider socio-economic changes in option evaluations, the estimation of optimal investment times and option values is more complicated. Nevertheless, the framework used in this paper helps to understand which combination of scenarios triggers the need for action over time.

In future work, other change factors such as coastal morphodynamics (generally erosion), changing waves and breaching of defences might be considered within the framework established in this paper. Hydrological and morphological conditions vary due to erosion and the loss of salt marsh by sea-level rise, leading to change in the relationship between climatic variables and flood damages avoided (Figure 9). Therefore, the validity of assuming no morphological and hydrological change needs to be further investigated by a long-term observation on sea-level rise and subsequent morphological changes in the marsh before the coastal defence.

Option to wait does not consider the current risk of coastal flooding in Lymington. If we invested in a significant flood defence upgrade in 2016, we might receive a return immediately if a major storm occurred. However, we also lose an investment opportunity in other sectors (e.g. health care or education). This balance needs to be addressed. Multiple-stage investment or growth option is a possible alternative strategy for the early protection of Lymington. In doing so, we can manage immediate risk within the time scale of adaptation planning. This type of adaptation option has been already addressed in the previous studies (Woodward et al., 2014; Hino and Hall, 2017). The framework presented here for the optimal investment time and option evaluation can be extended to the case of multiple stage adaptation in future research.

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(The number of words:5,961, exclusive of figures, tables, appendix and references)

Appendix 1. Derivation of a formulation for optimal investment time

This appendix is aimed to prove that sea-level rise at the optimal investment time appears within a close range independently of climate change scenarios. This supplementary analysis differentiates the option value of the investment at any given time *t* with respect to investment time *t*. The option value from the investment at any given year *t* can be obtained as shown in equation (1).

$$NPV_t = \sum_{i=t+1}^{L+t} \frac{EAB_i}{(1+r)^i} - \frac{I}{(1+r)^t} \tag{1}$$

Here, EAB_i is the expected annual benefit of the project at any year *i*, *r* is a discount rate, *I* is investment cost, and *L* is the number of project years.

In order to differentiate the equation (1) with respect to time *t*, this analysis transforms equation (1) to be an integral form. Thus, the investment time (*t*) is continuous. This derivation sets the time-discrete formula of NPV in equation (1) to be a continuous form in equation (2). We have demonstrated that EAB changes over time due to sea-level rise (See Figure 10 in the paper). Thus, EAB is also assumed to be a continuous function of time (*f=EAB(i)*) in this analysis. Therefore, the continuous form of NPV can be written with respect to the investment time (*t*).

$$NPV_t = \int_{t+1}^{t+L} EAB(i) * e^{-ri} di - I * e^{-rt} \tag{2}$$

Here, *r* is discount rate per time, $EAB(i)$ is the rate of benefit at given time *i* in money per time, t is investment time. If NPV_t in equation (2) becomes a maximum at any investment year *t'*, the year *t'* is the optimal investment time by definition. The derivative of NPV (= *d(NPV)/dt*) can be derived by directly differentiating the equation (2) using Leibniz's theorem as below.

$$NPV_t = \int_{t+1}^{t+L} EAB(i) * e^{-ri} di - I * e^{-rt}$$

$$\frac{d}{dt}(NPV_t) = EAB(t+L) * e^{-r(t+L)} \times \frac{d(t+L)}{dt} - EAB(t+1) * e^{-r(t+1)} \times \frac{d(t+1)}{dt}$$

$$- \int_{t+1}^{t+L} \frac{\partial}{\partial t}\left(\frac{EAB(i)}{(1+r)^i}\right) di + I * r * e^{-rt}$$

$$= EAB(t+L) * e^{-r(t+L)} - EAB(t+1) * e^{-r(t+1)} + I * r * e^{-rt} \tag{3}$$

As $d(NPV_t)/dt = 0$ at the optimal investment time (*t'*), we will have equation (4) for the optimal investment time.

$$\boldsymbol{EAB(t'+1) * e^{-r} - EAB(t'+L) * e^{-rL} = rI} \tag{4}$$

As $e^{-r} \approx 1/(1+r)$ and $e^{-rL} \approx 1/(1+r)^L$, the equation (4) can be rewritten as below.

$$\frac{\boldsymbol{EAB(t'+1)}}{\boldsymbol{1+r}} - \frac{\boldsymbol{EAB(t'+L)}}{\boldsymbol{(1+r)^L}} = \boldsymbol{rI} \tag{4'}$$

As can be seen from equation (4'), the first term on the left side is the expectation of benefit at the first year after the investment and the second term is the expectation of benefit at the end year of the project life (= the year $t+L$). Both values are discounted to the optimal investment year (t') by discount factors $1/(1+r)$ and $1/(1+r)^L$, respectively. On the left side, the term denotes an investment cost (I) times discount rate (r). Therefore, if the investment cost and discount rate are known for an adaptation option, the optimal investment time (t') satisfies the equation (4) or (4'). The first term is the expected benefit at the first year after the investment discounted by $1/(1+r)$. The second term is expected benefit at the end year of the project life discounted by $1/(1+r)^L$. However, $(1+r)^L$ becomes very large as the project year is 100 years. The second term becomes small compared to the first term. The equation (4') can be rewritten by ignoring the second term on the left-hand side as below.

$$\frac{EAB(t'+1)}{(1+r)} = r \times I \tag{5}$$

If the discount rate r and the investment cost I is known and fixed during the planning horizon, the formulation enables us to evaluate the critical value of *EAB* for the optimal investment time. Let EAB_c denote the critical threshold value of *EAB* for the optimal investment time (t') at which the investment provides a maximum NPV during the project life. Equation (5) can be written as below

$$EAB_c = (1+r) * r \times I \tag{6}$$

Equation (6) implies that we can determine the optimal investment time by seeing whether EAB reaches the critical threshold value (EAB_c). As *EAB* is projected over time in relation to sea-level rise scenario, this formulation enables us to find when the optimal investment time occurs by matching EAB_c with the projected EAB in climate change scenario. This equation also suggests that the optimal investment time (t') is independent of climate change scenarios if we can ignore the second term ($= \frac{EAB(t'+L)}{(1+r)^L}$) that is highly discounted by discount factor $1/(1+r)^L$.

This finding is identified in the case study of Lymington where the upgrade of coastal defence is planned with the investment cost of £64 million. Although the Greenbook approach takes different discount rates depending on the time period (3.5% for the first 30 years, 3.0% for the next 45 years and 2.5% afterwards), the values of sea-level rise at the optimal investment time for all MSLR scenario are found around 13cm in the analysis. Thus, the critical threshold value (SLR_c) for the optimal investment time are highly dependent on investment cost (I) and discount rate (r).

For the demonstration of the above relation, *EAB* at the first and end year of the project period and the optimal investment time and its sea-level rise for each of MSLR scenarios are summarized in Table 1. Here, the time period of the project life (L) is set to be 100 years. The investment cost is £64.2 mil with discount rate of 3.5% applied for this formulation. Note that different discount rates are applied for different periods in the UK project appraisal – 3.5% for the first 30 years, 3.0% for the next 45 years and 2.5% afterward.

Table 1. The comparisons of the investment cost (I) times discount rate (r) and the expected annual benefit discounted by $(1+r)$ at the optimal investment time for each MSLR scenarios

Scenario	Investment year (t')	Sea-level rise at year t'	$r*I$ (1)	EAB $(t'+1)$	EAB $(t'+L)$	$\frac{EAB(t'+1)}{(1+r)} - \frac{EAB(t'+L)}{(1+r)^L}$ (2)	(1) – (2)
High	2025	13.8cm	£2.21M	£2.11M	£8.79M	£1.76 M	£ 0.45 M*
Medium	2029	13.2cm	£2.21M	£2.07M	£7.06M	£1.77 M	£ 0.44 M
Low	2033	12.6cm	£2.21M	£2.01M	£5.72M	£1.75 M	£ 0.46 M
H++	2016	19cm***	£2.21M	£2.39M	0	£2.30 M	(-) £ 0.09 M**

* In this calculation by backward induction method, the value (1) and (2) are not exactly matched with each other because the function EAB is not continuous of time. In addition, the discount rates provided by the Greenbook (HM treasury, UK) are applied differently depending on the time periods of the project year. If we apply constant discount rate over the time periods, the value (2) is exactly matched with the value (1). Nevertheless, this result explicitly shows that the optimal investment time is approximately when the expectation of EAB at year i+1 reaches the value of $I \times r$.

** In the H++ scenario, the expectation of benefit value at the end of the project year (2116) is zero because the upgraded coastal defence cannot protect the floodplains from extreme still water level above 3.5m mAOD that are expected to occur at 2116. Thus, EAB cannot be defined as a continuous function of time in the H++ MSLR scenario.

*** In the H++ scenario, sea-level rise relative to 1990 already exceeded the critical threshold value of sea-level. This analysis conducted the calculation of option value between 2016 and 2100. Therefore, the sea-level rise at the optimal investment time is much higher than any other scenario in this extreme scenario.

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