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## Extending the range of Plug-in Hybrid Electric Vehicles by CVT transmission optimal management

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### Abstract

Due to the ability to obtain energy from direct connection to the electricity grid, plug-in hybrid electric vehicles benefit both society and drivers environmentally and economically by reducing energy consumption and emissions. To extend the travel range, existing studies focus mostly on optimizing the power split of the internal combustion engine and battery without considering the impact of the transmission system. This paper deals with the optimization of a continuously variable transmission (CVT) operation for a parallel pre-transmission hybrid powertrain. The novelty of this paper is that the operation of both engine and CVT transmission are optimized together, resulting in a better fuel efficiency. Simulation work has been carried out to compare the performance of the optimized CVT transmission with a fixed transmission, showing that the fuel consumption is reduced and hence the range is increased for the optimized CVT, which makes the engine to operate in higher efficiency points. A key advantage of the formulation proposed is that the result of the optimization process may be expressed in terms of a 2D map that for any pair of vehicle speed and traction force gives the optimal value of the CVT transmission ratio. Such a map may be easily implemented in real vehicular applications.

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### 1. Introduction

As the need for more fuel-efficient and lower emission vehicles becomes more urgent, the demand for hybrid electric vehicles is rising, especially for Plug-in Hybrid Electric Vehicles (PHEV) [1, 2] which have the ability to store

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energy through direct connection to the electrical grid. An important determinant of fuel consumption and range for PHEVs is the energy management system [3], which improves the performance of the vehicle and its energy economy by directing the flow of energy to and from the battery and powertrain in the most efficient way. Various control techniques, ranging from rule-based [4, 5] to optimization-based [6, 7, 8], have been proposed in the literature for powertrain energy management to bring reductions in fuel consumption and noxious emissions with equal or improved vehicle performance. Optimization-based methods generally give optimal or near-optimal solutions of the energy management problem. However, one drawback of this kind of methods is the numerical complexity, which makes the problem difficult to solve. Due to the existence of the internal combustion engine (ICE) in PHEVs, a transmission is needed to make the ICE work at more efficient operating points. Hence, the transmission ratio is dynamically changed in regard to the vehicle speed, which introduces nonlinearity in the energy management problem for the HEV and increases the difficulty of solving the problem numerically. Hence, the influence of the transmission is mostly neglected in the literature about hybrid vehicle energy management [6] if the problem is formulated in an optimal control framework. To address this issue, a novel combined optimal ICE & CVT model is proposed in this paper.

## 2. Modeling of Plug-in Hybrid Electric Vehicles

The plug-in hybrid electric vehicle (PHEV) considered in this paper has a pre-transmission parallel powertrain topology, which is shown in Fig. 1. An internal combustion engine (ICE) with a fuel tank is connected to the driveshaft via a CVT transmission, while a permanent-magnet synchronous motor (PMSM) with a high voltage battery is connected to the same shaft in parallel.

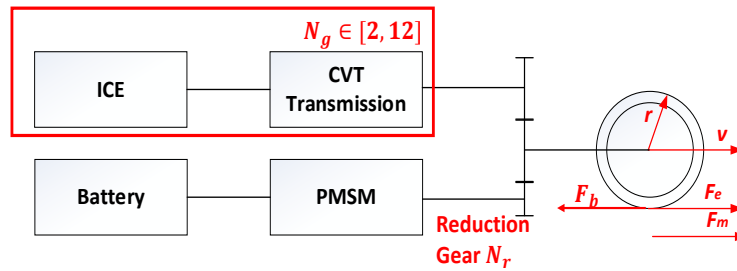


Fig. 1 Pre-transmission parallel Hybrid Electric Vehicle Powertrain

In the next section, the component models are presented. Instead of complicated dynamical models of the ICE and electrical traction system, static map-based models have been adopted with appropriate choices of inputs and outputs.

### 2.1. Vehicle Motion

The vehicle is taken as a mass obeying Newton's second law, so that the dynamics may be written as

$$\dot{s} = v \quad (1)$$

$$m\dot{v} = F_e + F_m + F_b - \frac{1}{2}\rho C_d A_f v^2 - C_{rr} m g \quad (2)$$

where  $m$  is the vehicle mass,  $s$  the travel distance,  $v$  the vehicle speed,  $F_e$  the wheel force due to the ICE,  $F_m$  the wheel force due to the electric motor, and  $F_b$  the braking force. The aerodynamic resistance force and the rolling friction force are considered as well, with  $\rho$  the air density,  $A_f$  the frontal area,  $C_d$  the coefficient of the drag and  $C_{rr}$  the coefficient of rolling resistance.

Based on the powertrain structure in Fig. 1, the engine speed  $\omega_e$  and torque  $T_e$  as well as the motor speed  $\omega_m$  and torque  $T_m$  can be calculated as a function of the CVT transmission ratio  $N_g$  as follows:

$$\omega_e = \frac{v}{r} N_g$$

$$\omega_m = \frac{v}{r} N_r$$

$$T_e = \frac{F_e r}{N_g}$$

$$T_e = \frac{F_m r}{N_r}$$

where  $N_g$  is the CVT ratio and  $N_r$  is the reduction gear for the motor.

## 2.2. Prime movers

The ICE model is implemented as a lookup table that maps torque demand and crankshaft speed values to a fuel consumption as shown in Fig 2. Hence we may consider the model as consisting of a single mathematical function:

$$\dot{m}_f = q_f(\omega_e, T_e) \quad (3)$$

where  $T_e$  is the torque demand at the crankshaft,  $\omega_e$  is the angular velocity of the crankshaft and  $\dot{m}_f$  is the rate of change of fuel mass in the fuel tank of the vehicle.

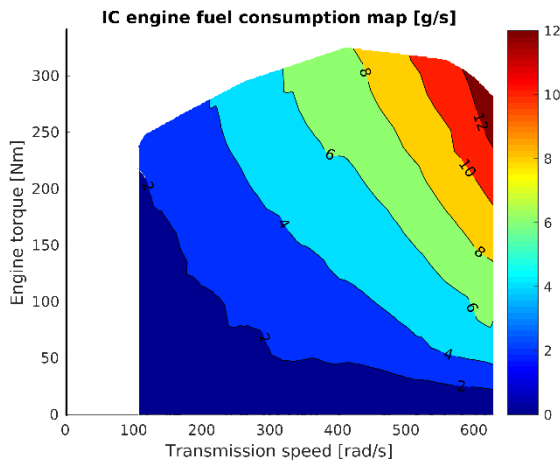


Fig. 2 Engine Fuel Consumption Map

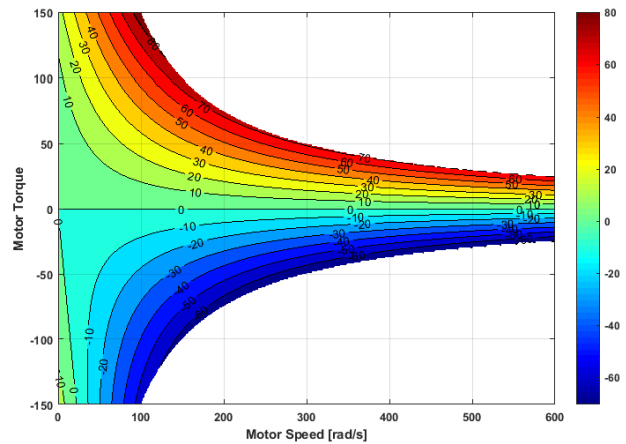


Fig. 3 Electric Traction System Battery Current Map

Similarly to the ICE, the electrical traction system of the vehicle, which includes the battery, DC/AC converters and the PMSM, is implemented as a lookup table mapping values of motor torque demand and motor speed to current consumption values at the battery as shown in Fig. 3.

$$\dot{Q}_b = i_b(\omega_m, T_m) \quad (4)$$

In this equation,  $Q_b$  represents battery charge,  $T_m$  is motor torque and  $\omega_m$  is the motor angular velocity. The map of battery current consumption is calculated using a MATLAB script that includes losses due to: stator resistance, eddy current losses, inverter and converter efficiencies. Then the battery state of charge (SoC) changes according to:

$$\dot{SoC} = -\dot{Q}_b / Q_{max} \quad (5)$$

## 3. Engine and CVT Transmission Optimization

For the pre-transmission hybrid powertrain considered in this paper, the CVT transmission does not affect the motor operation. Hence, it is reasonable to consider the combination of the ICE and CVT transmission in the model. The combined model is given by

$$\dot{m}_f = q_f(\omega_e, T_e) = q_f\left(\frac{v}{r}N_g, \frac{F_e r}{N_g}\right) \quad (6)$$

where the input of this model is vehicle speed  $v$ , wheel force due to engine  $F_e$ , and CVT ratio  $N_g$ . Then the combined model (6) is optimized to find the optimal fuel mass rate  $\dot{m}_{f0}$  for any possible combination of vehicle speed  $v$  and wheel force due to engine  $F_e$ . For any  $v \in [v_{min}, v_{max}]$ ,  $F_e \in [F_{min}, F_{max}]$ ,  $N_g \in [N_{min}, N_{max}]$ , the optimal fuel mass rate can be calculated as:

$$\dot{m}_{f0} = \min_{N_g \in [N_{min}, N_{max}]} q_f\left(\frac{v}{r}N_g, \frac{F_e r}{N_g}\right) = \bar{q}_f(v, F_e r) \quad (7)$$

While the optimal CVT ratio is given by:

$$N_{go} = \arg \min_{N_g \in [N_{min}, N_{max}]} q_f\left(\frac{v}{r}N_g, \frac{F_e r}{N_g}\right) = \bar{g}(v, F_e r) \quad (8)$$

In this way, the optimal fuel mass rate map and the corresponding optimal CVT ratio map is calculated by (7) and (8) after gridding up  $v$ ,  $F_e$  and  $N_g$  when  $N_g \in [2, 12]$ , which is shown in Fig 4.

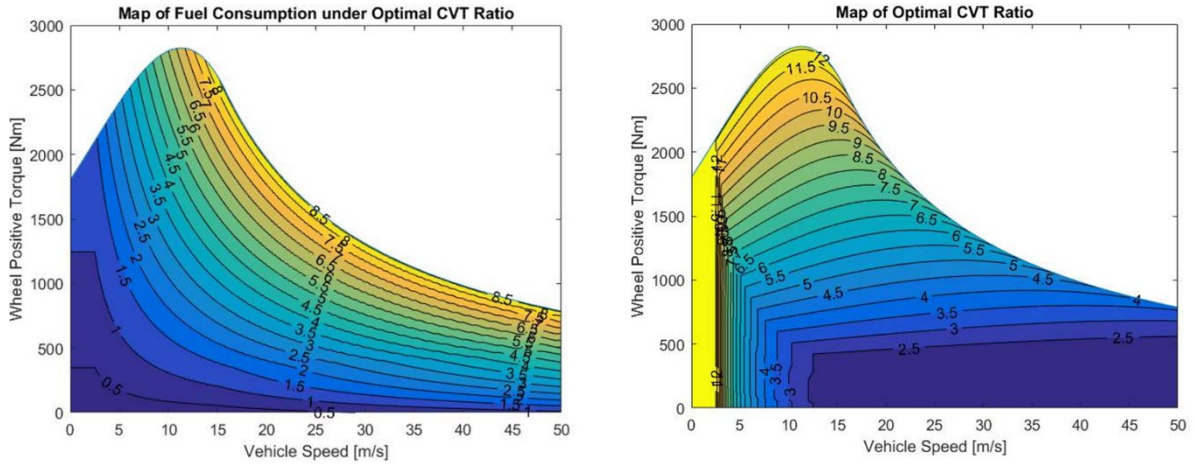


Fig. 4 Optimal Fuel Consumption Map and Optimal CVT Ratio Map for the engine and CVT combined model

#### 4. Power split optimization

The optimization of the CVT operation is just part of the energy management of the hybrid powertrain, which also requires the management of the power split between ICE and electric motor. Within the proposed approach, the power split may be optimized separately from the CVT. Indeed, for a given drive-cycle, the energy minimization problem is formulated for the PHEV as follows

$$\min_{\mathbf{u}(t)} J[\mathbf{x}, \mathbf{u}] = \int_0^T \left( \dot{m}_{f0} + w \frac{P_m}{LHV} \right) dt$$

$$\text{s.t. } \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}, t)$$

$$\mathbf{x} = [s \ v \ m_f \ SoC]^T, \ \mathbf{u} = [F_e \ F_m \ F_b]^T$$

Constraints on state  $\mathbf{x}$  and input  $\mathbf{u}$

where  $LHV$  is the lower heating value of petrol, and  $w$  is the weighting factor for the motor power. The cost function  $J[\mathbf{x}, \mathbf{u}]$  is the equivalent fuel consumption of the PHEV.

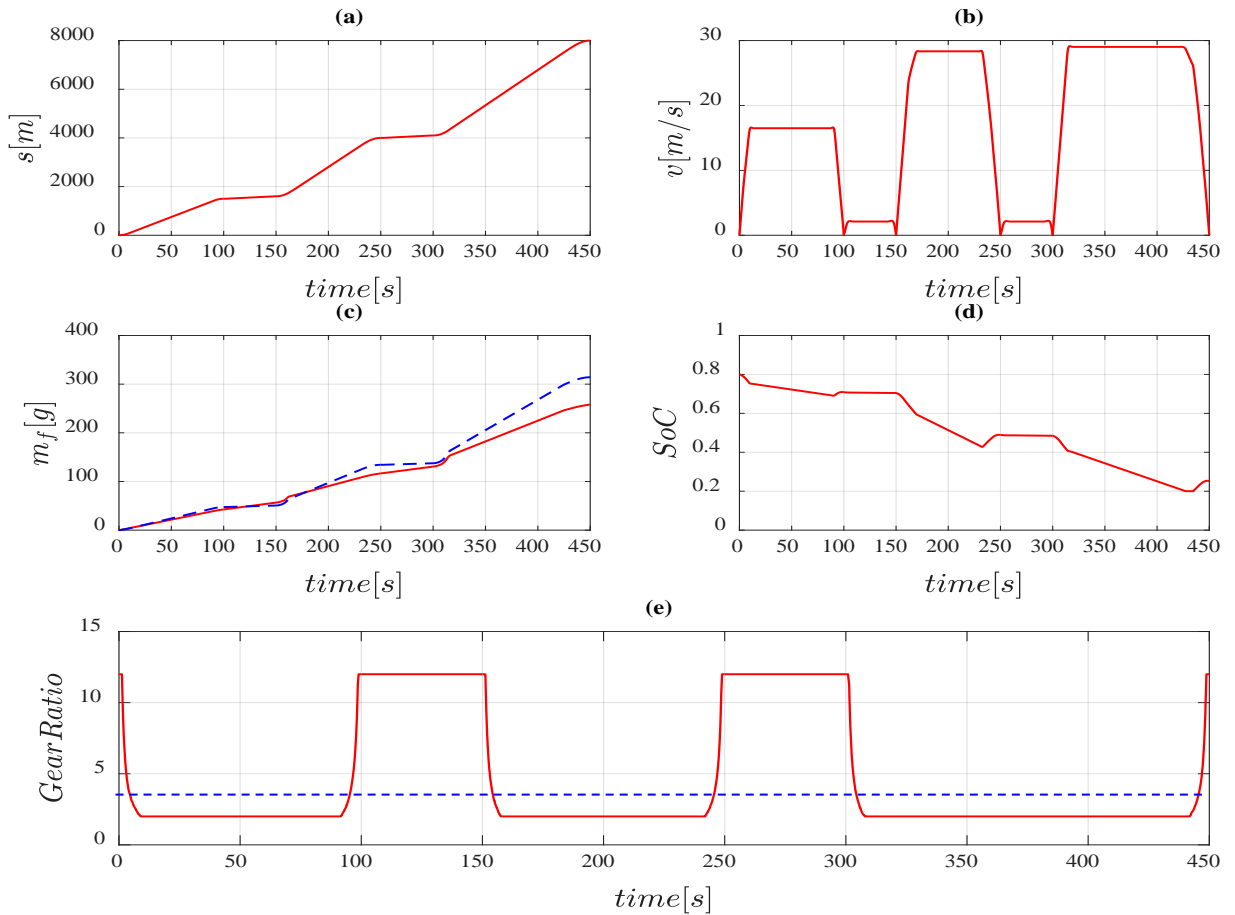


Fig. 5. Comparison of simulation results between fix gear (blue dashed line) and optimal CVT (Red solid line)

To demonstrate the merits of the pre-optimised ICE&CVT map, the vehicle is controlled to track a fixed driving cycle as shown in Fig. 5 (b), which lasts 450s with a distance of 8000m. The optimal control problem is solved by GPOPS II [9]. Two cases are compared:

- a) The CVT ratio  $N_g \in [2,12]$ , the optimized fuel mass rate map is used. The results is shown in red solid line in Fig. 5.
- b) A fixed gear box is considered, which mean  $N_{min} = N_{max}$ . The results is shown in blue dashed line in Fig. 5.

In order to compare the difference of the two case, the fixed gear case is used the same power split as the first case. Hence, as shown in Fig. 5. (d), the battery SoC is both cases are same while the fuel mass consumed by the engine is different as the fuel consumption model is different. For the whole journey,  $m_f(450) = 257 g$  with optimal ICE&CVT model, and  $m_f(450) = 314 g$  with a fixed gearbox, which is about 18% more. Therefore, the operation of the CVT transmission affects the fuel economy and by optimizing the CVT transmission, the travel range is extended as the overall energy consumption is reduced.

## 5. Conclusion

In this paper, the optimisation of the CVT operation for a parallel, pre-transmission hybrid electric vehicle has been discussed. The energy management problem is formulated as an optimal control problem to minimize the overall energy consumption of the PHEV. Simulation results demonstrated that the proposed modelling technique reduces energy consumption compared to the case which assumes the transmission as a fixed gearbox. The proposed method also simplifies the optimization problem since the transmission ratio is not modelled as a discrete state. The simplified problem can be easily solved by optimal control software, such as GPOPS II.

## Acknowledgements

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