

Modularity-based User-Centric Clustering and Resource Allocation for Ultra Dense Networks

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Abstract—A novel modularity-based user-centric (MUC) clustering is conceived for resource allocation in ultra dense networks (UDNs), in order to maximise the sum-rate per orthogonal resource block (RB). The idea of MUC clustering is to decompose the UDN into several sub-networks by exploiting the inherent group structure of user equipments (UEs). In particular, we propose a modified Louvain method for MUC clustering relying on efficient resource allocation heuristics. Our numerical results show the superiority of our MUC design.

Index Terms—Ultra dense networks, user-centric clustering, resource allocation, unsupervised learning.

I. INTRODUCTION

To meet the ever-increasing high data-rate demands required by each user equipment (UE), the user-centric (UC) clustering of ultra dense networks (UDNs) [1]–[4] has been found very promising. When jointly designing it with resource allocation, the UE’s data-rate may be further enhanced [5] [6]. For example, the authors of [7] and [8] focused their attention on the UC clustering design and resource allocation. However, given the large number of UEs in UDNs, the limited number of orthogonal resource blocks (RBs) requires sophisticated resource allocation algorithms to be carried out across all access points (APs) based on the full channel state information (CSI) of all UEs [9]. This approach is impractical and hence efficient counter-measures have to be developed to improve the exploitation of orthogonal RBs.

In this light, we propose to decompose the UDN into several sub-networks. This arrangement will lead to less information exchange and will support more efficient interference management. *Explicitly, our idea is to strictly rely on orthogonal RBs within the sub-networks, whilst loosening the requirement of orthogonal RBs amongst sub-networks.* Fundamentally, the concept of deriving sub-networks from a large network by discovering the inherent group structure has been widely

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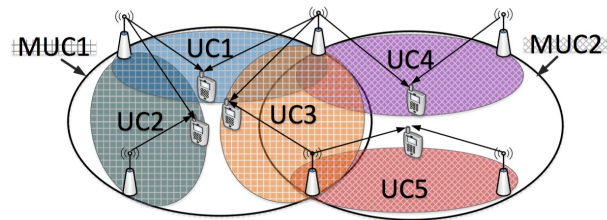


Fig. 1: An example of the MUC clustering in UDNs.

studied in the sociology, biology and computer science communities [10]–[12]. This concept has been recently adopted to device to device communications [13] [14], where the network is decomposed based on the UEs’ social behaviour, including locations, interests, or background. However, to the best of our knowledge, there has been no related work exploiting the group structure in the design of UC clustering and resource allocation for UDNs.

Against the above backdrop, we propose a novel modularity-based user-centric (MUC) clustering relying on the modified Louvain method and co-design it with the resource allocation, for improving the exploitation of orthogonal RBs in UDNs. To be specific, we first propose a novel MUC clustering framework for UDNs, by taking the UEs’ group structure into account specifically relying on their locations. Then, we formulate the joint design problem of MUC clustering as well as resource allocation and conceive a three-stage sequential solution relying on the modularity of the network. We show that the proposed design outperforms the state-of-the-art.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Notation

Matrices and column vectors are denoted by bold capital letters and bold lower-case letters, respectively. Scalar variables are denoted by italic symbols. $|\mathcal{A}|$ denotes the cardinality of a set \mathcal{A} . $\mathbb{C}^{N \times M}$ denotes the space of all $N \times M$ matrices with complex entries. Given a complex vector or matrix, $(\cdot)^H$ and $\|\cdot\|$ denote the conjugate transpose and norm, respectively.

B. System Model

We consider a downlink UDN that consists of a dense set of APs having the index set of $\mathcal{L} = \{1, \dots, L\}$ and a dense set of single-antenna UEs having the index set of $\mathcal{K} = \{1, \dots, K\}$. All APs are equipped with $M_A \geq 1$ antennas and they share the same spectrum, which is partitioned into multiple orthogonal RBs hosted in the index set of $\mathcal{N} = \{1, \dots, N\}$.

We assume that all APs and all UEs are uniformly distributed in the UDN, and the density of APs is comparable to that of the UEs. Let $\mathbf{h}_{j,k,n}$ denote the downlink channel gain spanning from AP j to UE k on RB n , including the path-loss and small-scale fading. In this paper, we follow the practical two-piece path-loss model of [15] [16], which includes the probabilistic combination of both line-of-sight (LoS) and non-line-of-sight (NLoS) connections. Additionally, the small-scale fading is modelled as identical independently distributed (i.i.d.) Rayleigh fading on each RB. Considering that each AP is capable of simultaneously serving several UEs, we assume that the AP-UE association can only be established, if the UE is within the AP's coverage distance of d_t . In our hypothesis, both the locations of APs and UEs as well as the channel characteristics are acquired. This process relies on the APs exchanging this information with the macro base station that manages the AP-UE association.

Let us first briefly review the concept of UC clustering. Each UE can be served simultaneously by multiple APs relying on joint transmission, where these APs constitute the UC cluster supporting each UE. An example of the UC clustering in UDN is illustrated in Fig. 1, where the UC clusters of UEs are represented as $\{UC1, \dots, UC5\}$. Considering the fact that in practical UDN the number of UEs is typically higher than the number of orthogonal RBs in UDN (i.e. $K > N$), we assume that only one RB can be assigned to each UC cluster and that the RB can be reused by several UC clusters. In order to enhance the UE's data-rate given the limited availability of orthogonal RBs, we propose a novel MUC clustering concept in UDN, by taking the UEs' group structure into account for decomposing a large number of UEs into groups. In this way, each MUC cluster will include a number of UC clusters, consisting of the union of the grouped UEs and their serving AP set, exemplified by $MUC1 = \{UC1, UC2, UC3\}$ and $MUC2 = \{UC4, UC5\}$ of Fig. 1. It is noteworthy that each UE is involved in one and only one MUC cluster, whilst each AP may simultaneously belong to multiple MUC clusters as a result of overlapped UC clusters [9] [17]. Hence, the interference will be stronger within the MUC clusters and weaker amongst MUC clusters. As a benefit, the resource allocation would be made more interference-conscious.

C. Problem Formulation

To formulate the problem, we define the serving AP set in the UC cluster of UE k as \mathcal{B}_k , the grouped UE set in the MUC cluster c as \mathcal{U}_c , and the group index that UE k belongs to as c_k , respectively. Then, we introduce $\mathbf{X} = [x_{j,k}]$ having $(L \times K)$ elements as the UC clustering matrix, where $x_{j,k} = 1$ if AP $j \in \mathcal{B}_k$, otherwise $x_{j,k} = 0$. Additionally, let $\boldsymbol{\alpha} = [\alpha_{k,n}]$ having $(K \times N)$ elements denote the RB allocation matrix, where $\alpha_{k,n} = 1$ if RB n is allocated to \mathcal{B}_k , otherwise $\alpha_{k,n} = 0$. Furthermore, we define $\mathbf{Y} = [y_{k,m}]$ having $(K \times K)$ elements as the common MUC cluster indicator matrix. Explicitly, if $c_k = c_m$, then $y_{k,m} = 1$, i.e. UE k and UE m are in the same group, otherwise we have $y_{k,m} = 0$. Finally, we let p_j denote the transmit power of AP j and we assume that the total power transmitted from

each AP is shared equally amongst all of its associated UEs. Hence, the power transmitted from AP j to its associated UE k is $p_{j,k} = p_j / \sum_{\kappa \in \mathcal{K}} x_{j,\kappa}$.

Accordingly, the instantaneous signal-to-interference-plus-noise ratio (SINR) of UE k on RB n is given by

$$\gamma_{k,n} = \frac{\sum_{j \in \mathcal{L}} p_{j,k} x_{j,k} \alpha_{k,n} |\mathbf{h}_{j,k,n}^H \mathbf{w}_{j,k,n}|^2}{\sigma^2 + \text{IF}_{k,n}}, \quad (1)$$

where we have

$$\begin{aligned} \text{IF}_{k,n} &= \sum_{m \in \mathcal{K} \setminus k} y_{k,m} \alpha_{m,n} \sum_{i \in \mathcal{L}} p_{i,m} x_{i,m} |\mathbf{h}_{i,k,n}^H \mathbf{w}_{i,m,n}|^2 \\ &+ \sum_{m \in \mathcal{K} \setminus k} (1 - y_{k,m}) \alpha_{m,n} \sum_{i \in \mathcal{L}} p_{i,m} x_{i,m} |\mathbf{h}_{i,k,n}^H \mathbf{w}_{i,m,n}|^2. \end{aligned} \quad (2)$$

Herein, σ^2 denotes the power of the additive white Gaussian noise (AWGN) at each UE, and $\mathbf{w}_{j,k,n}$ denotes the normalized beamforming vector transmitted from AP j to UE k on RB n . In this paper, we employ the classical Maximal Ratio Transmission (MRT), which is formulated as $\mathbf{w}_{j,k,n} = \mathbf{h}_{j,k,n} / \|\mathbf{h}_{j,k,n}\|$. Furthermore, we explicitly decompose (2) into a pair of items, where the first item is the intra-MUC cluster interference and the second item is the inter-MUC cluster interference.

This paper aims for jointly designing the MUC clustering and resource allocation for UDNs in order to maximise the exploitation of orthogonal RBs quantified in terms of the system's sum-rate per RB. Accordingly, our optimization problem may be formulated as

$$(P0): \max_{\mathbf{X}, \mathbf{Y}, \boldsymbol{\alpha}} \frac{\sum_{k=1}^K \sum_{n=1}^N \log_2(1 + \gamma_{k,n})}{\sum_{n=1}^N \text{sgn}(\sum_{k=1}^K \alpha_{k,n})} \quad (3)$$

$$\text{s.t. } x_{j,k} \in \{0, 1\}, \quad \forall j, \forall k, \quad (4)$$

$$y_{k,m} \in \{0, 1\}, \quad \forall k, \forall m, \quad (5)$$

$$y_{k,m} = y_{m,k}, \quad \forall k, \forall m, \quad (6)$$

$$\alpha_{k,n} \in \{0, 1\}, \quad \forall k, \forall n, \quad (7)$$

$$\sum_{n=1}^N \alpha_{k,n} \leq 1, \quad \forall k, \quad (8)$$

$$y_{k,m} \alpha_{m,n} = 0, \quad \text{if } \alpha_{k,n} = 1, \quad \forall k, \forall m, \forall n, \quad (9)$$

where the denominator item in (3) denotes the actual number of RBs in use. Additionally, (4)-(6) indicate the MUC clustering constraints, while (7)-(9) imply the resource allocation constraints. Explicitly, (9) confines the strict use of orthogonal RBs within MUC clusters, while no requirements are imposed for the use of orthogonal RBs amongst MUC clusters.

It becomes clear now that the MUC clustering problem and the resource allocation problem are coupled with each other in problem (P0). Therefore, the feasible way is to decouple the problem into a pair of independent sub-problems. To elaborate, we first consider how to construct the MUC clusters relying on the method of *community detection* [10] to determine \mathbf{Y} . The aim of community detection is to identify the modules as the gathering of vertices according to classic graph topology, which results in a number of UE groups. Based upon the MUC clustering results, we then proceed to design the associated resource allocation schemes. Since $\gamma_{k,n}$ relies on both \mathbf{X} and

the n -th column vector of α , (P0) is an integer non-linear programming problem even if \mathbf{Y} is pre-determined. At the time of writing, there have been no optimal algorithms to solve this NP-hard problem, even if the discrete variables within \mathbf{X} and α were relaxed to be continuous values. In this context, we will invoke heuristics to seek an efficient solution.

III. PROPOSED MUC CLUSTERING FRAMEWORK

In this section, we will introduce our proposed MUC clustering method and resource allocation heuristics for solving problem (P0). In terms of the MUC clustering, its formation relies on both the UC clustering results, i.e. \mathbf{X} , and on the group formation results, i.e. \mathbf{Y} . As for the resource allocation, i.e. α , the orthogonal RBs are allocated based on the MUC clustering results. Motivated by this, we propose a sequential operation as follows.

A. UC Cluster Construction

Since the objective function in problem (P0) is to maximise the utilisation of orthogonal RBs measured by the system's sum-rate per RB, the UC clustering process becomes that of maximising the benefits of AP cooperation. Since our focus is on MUC clustering, we adopt the coverage distance as the only criterion for creating the UC clusters, although other factors may also be considered for determining the topology. Hence, given the locations of all APs and all UEs, each UE will construct its UC cluster according to

$$x_{j,k} = \begin{cases} 1, & \text{if } d_{j,k} \leq d_t, \\ 0, & \text{otherwise,} \end{cases} \quad (10)$$

where $d_{j,k}$ is the coverage distance between AP j and UE k .

B. MUC Cluster Construction

Based upon the UC clustering results in \mathbf{X} , our next goal is to form UE groups and store them in \mathbf{Y} for constructing MUC clusters. The first solution that springs to mind is the classic K-means method. It discovers groups based on a pre-defined desired number of groups, namely the pre-set number of MUC clusters. However, the challenge is that the desired number of MUC clusters was to be known, but given the randomly evolving UE locations, it is generally unknown. Fortunately, the modularity-based unsupervised learning method of [12] is capable of appropriately constructing these groups, since *modularity* is by far the most used and best known quality-metric of group formation results. Hence, we treat the UE group formation as a problem of maximising the modularity of the network.

1) *Graph Construction*: In order to construct our MUC clusters, we first have to build the network topology graph. To this end, a graph model is constructed based on both the UE and AP locations, as well as on the channel characteristics. We define a graph consisting of the set of vertices \mathcal{K} and edges \mathcal{E} , denoted by $\mathcal{G} = (\mathcal{K}, \mathcal{E})$. More explicitly, the vertices correspond to the UC clusters, whilst the edges represent the interference amongst the UC clusters. Mathematically, an

Algorithm 1 Modified Louvain Based MUC Clustering and Resource Allocation for UDNs

- 1: **I. UC Cluster Construction**
- 2: $\forall k \in \mathcal{K}$, construct \mathcal{B}_k according to (10).
- 3: **II. MUC Cluster Construction**
- 4: Construct $\mathcal{G} = (\mathcal{K}, \mathcal{E})$ according to (11).
- 5: Initialize: $\forall k \in \mathcal{K}$, $c_k = \{k\}$.
- 6: **Sub-stage 1**: $\forall v \in \mathcal{K}$, find $k' \in \mathcal{K}$ where $e_{k,k'} = 1$ satisfies $c_{k'} = \operatorname{argmax}\{\Delta Q_{c_k=c_{k'}}\}$ and $|\mathcal{U}_{k'}| < N$. If $\max \Delta Q > 0$, $c_k = c_{k'}$, otherwise remain unchanged.
- 7: Repeat step 6 until $\max\{Q\}$ attained.
- 8: **Sub-stage 2**: Reconstruct \mathcal{G} according to $\{c_k\}_{k \in \mathcal{K}}$. Repeat step 6 until $\max\{Q\}$ attained.
- 9: Repeat step 6 and 8 until $\max\{Q\}$ attained.
- 10: **III. MUC RB Allocation**
- 11: $\forall c \in [1, C]$, sequentially assigning RBs from $\mathcal{N}_c = \{1, \dots, |\mathcal{U}_c|\}$ to \mathcal{U}_c .

edge between the UC cluster of UE k and that of UE m is established, i.e. $e_{k,m} = 1$, provided that we have:

$$\min_n \frac{\sum_{j \in \mathcal{L}} p_{j,k} x_{j,k} |\mathbf{h}_{j,k,n}^H \mathbf{w}_{j,k,n}|^2}{\sum_{i \in \mathcal{L}} p_{i,m} x_{i,m} |\mathbf{h}_{i,k,n}^H \mathbf{w}_{i,m,n}|^2} < \delta_g, \quad (11)$$

where δ_g is the threshold selected to reflect the severity of the interference between two UC clusters. Formally, the modularity quantifies the density of edges within groups as compared to edges between groups, which is defined in [12] as follows

$$Q = \frac{1}{2s} \sum_{k,m} \left[e_{k,m} - \frac{q_k q_m}{2s} \right] y_{k,m}, \quad (12)$$

where $e_{k,m}$ indicates the weight between vertex k and vertex m , while $q_k = \sum_m e_{k,m}$ is the sum of the weights of the edges incident to vertex k . Furthermore, $s = \frac{1}{2} \sum_k \sum_m e_{k,m}$ is the sum of the weights in the graph. Note that in this graph, all the weights of the edges are set to unity.

2) *Modified Louvain Method*: We adopt the powerful Louvain Method of [18] and modify it according to our problem, since we have to restrict each group within the pre-set maximum size in order to obey the strict use of orthogonal RBs. As a result, the size of the MUC clusters in our implementation should not be set higher than the number of orthogonal RBs. Our method is unsupervised, which only relies on the constructed graph. Based upon the above graph \mathcal{G} , we consider the problem of maximising the modularity Q of the network by iteratively evaluating the gain in modularity, denoted by ΔQ . The gain is obtained by moving an isolated node m into the MUC cluster c , which is given by

$$\Delta Q = \frac{q_{m,\text{in}}}{s} - \frac{\sum_1 q_m}{2s^2}, \quad (13)$$

where \sum_1 is the sum of the weights of the edges incident to vertices in MUC cluster c , and $q_{m,\text{in}}$ is the sum of the weights of the edges from vertex m to all vertices in MUC cluster c .

In our implementation, each vertex is initialized as a different MUC cluster based on \mathcal{G} , and the solution consists of

two sub-stages, where the iteration procedure of our modified Louvain method is as follows:

- 1) for each vertex, we evaluate ΔQ when removing this vertex from its present MUC cluster and adding it to the neighbouring MUC cluster in sequence. Herein, in order to satisfy (9), we judge the size of the neighbouring MUC cluster before adding the new vertex to it, which should be less than the number of RBs N . In this way, each vertex is incorporated into the new MUC cluster with the maximum value of ΔQ , while each MUC cluster is restricted within the maximum size N , after a number of iterations.
- 2) the MUC clusters found during the first sub-stage become the new vertices in the network. To this end, the weights of the edges between the new vertices are given by the sum of the weights of the edges between the old vertices in the corresponding two MUC clusters. Notice that the edges between vertices of the same MUC cluster lead to self-loops for this MUC cluster. Thus, it is possible that the first sub-stage will be revisited in the resultant weighted network and we continue to iterate. The whole process is iterated until there is no further improvement and a maximum value of Q is attained.

With regards to complexity, there is a paucity of accurate complexity analysis for the Louvain method, but it was shown empirically to be low when for millions of nodes and links [18]. Additionally, this method is guaranteed to converge as a result of having a finite number of vertices. Finally, based on the above process, a total of C MUC clusters are constructed, where each MUC cluster is a union of the UE group, as well as their serving AP sets.

C. Resource Allocation

Having stored the MUC clustering results in \mathcal{Y} , the resource allocation becomes vital in order to deal with both the intra-MUC cluster interference and the inter-MUC cluster interference. As a benefit of MUC clustering, the inter-MUC cluster interference would tend to be weaker than the intra-MUC cluster interference. Hence, we aim for mitigating the critical interference by imposing zero intra-MUC cluster interference as reflected in (9), while allowing different MUC clusters to reuse the same RB set. To this end, below we design a heuristic resource allocation solution. To be specific, by confining the maximum size of the MUC clusters, i.e. $|\mathcal{U}_c| \leq N, \forall c$, we sequentially assign orthogonal RBs to the UEs in \mathcal{U}_c from the RB set $\mathcal{N}_c = \{1, \dots, |\mathcal{U}_c|\}$, and finally the actual number of RBs in use becomes $\max\{|\mathcal{U}_c|\}, \forall c \in [1, C]$. In this way, we can achieve a higher exploitation of the orthogonal RBs to improve the system's sum-rate per RB. Note that more sophisticated resource allocation schemes may also be designed, but we leave this issue for our future work by focusing on MUC clustering.

IV. NUMERICAL PERFORMANCE

In this section, we provide numerical results for characterizing the proposed MUC clustering concept. For all simulations, we consider a $200 \text{ m} \times 200 \text{ m}$ network plane, where each AP's

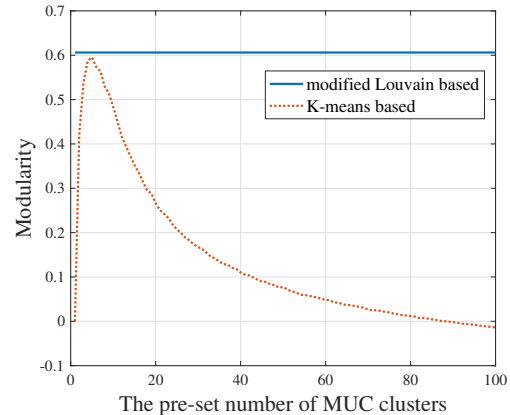


Fig. 2: The modularity value comparison.

coverage area is restricted within $d_t = 50 \text{ m}$. By default, we set $L = 80$, $K = 100$, $N = 50$, $M_A = 8$, $\delta_g = 5 \text{ dB}$ and $p_j = 30 \text{ dBm}$ ($\forall j$). The noise spectral density is -174 dBm/Hz with 5 dB noise figure and the RB bandwidth is 180 kHz . The parameters of the path loss model are adopted according to [9].

Fig. 2 compares the modularity value of both the modified Louvain based and of the K-means based MUC clustering solutions, which can be used for characterizing the partitioning of a network into groups. Having a higher degree of modularity is capable of supporting denser connections within the MUC clusters, but sparser connections amongst the different MUC clusters. Observe in Fig. 2 that in terms of the modularity, the K-means based MUC clustering solution performs worse than the modified Louvain based solution, regardless of the pre-set number of MUC clusters, since the modified Louvain based solution aims for directly maximizing the modularity. Again, the K-means based solution requires the knowledge of the pre-set number of MUC clusters, while the proposed modified Louvain based solution creates the number of MUC clusters adaptively based on the potential evolution of the network topology. Additionally, the K-means based solution fails to guarantee the strict use of orthogonal RBs within MUC clusters. When comparing to K-means based MUC clustering, we pre-set its number of MUC clusters to the highest possible modularity by enumerating all possible settings. We refer to this as modularity-optimal K-means based MUC clustering, namely to the ‘MUC w KM’. This benchmark is particularly biased for K-means, and we use it simply to demonstrate the superiority of our modified Louvain based MUC clustering solutions.

Let us now compare our proposed modified Louvain based MUC clustering to the ‘MUC w KM’ solution as well as to the UC clustering benchmarks (operating without UE grouping), namely to the ‘UC w RA’ solution (i.e. the UC clustering with random RB allocation) and the ‘UC w WP’ solution (i.e. the UC clustering with Welsh-Powell graph colouring algorithm [19]). Fig. 3 depicts the sum-rate per RB as a function of the AP density for the above-mentioned solutions, characterizing the orthogonal RBs. Firstly, we observe that there is an increase in the sum-rate per RB as the AP density increases for

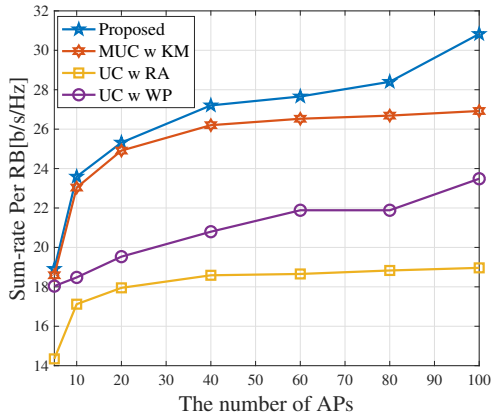


Fig. 3: Sum-rate per RB versus the AP density.

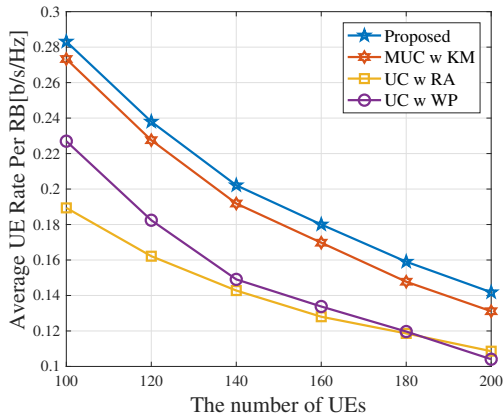


Fig. 4: Average UE Rate per RB versus the UE density.

all solutions with the RB allocation owing to the availability of more spatial resources. Secondly, it can be seen that the pair of MUC clustering solutions outperform the two conventional UC clustering solutions. This is owing to the fact that the RB allocation imposed for MUC clustering aims for mitigating the interference, which is an explicit benefit of MUC clustering. Finally, our proposed modified Louvain based MUC clustering philosophy exhibits superior performance over the modularity-optimal K-means based design alternative. This is a result of the strict use of orthogonal RBs as well as of the scheme’s agile adaptability to the network topology changes.

Another metric of quantifying the exploitation of orthogonal RBs is further investigated in Fig. 4 as a function of the UE density in terms of the average UE rate per RB, which is defined as the sum-rate per RB normalized by the number of UEs and formulated as

$$\frac{\sum_{k=1}^K \sum_{n=1}^N \log_2(1 + \gamma_{k,n})}{K \times \sum_{n=1}^N \text{sgn}(\sum_{k=1}^K \alpha_{k,n})}. \quad (14)$$

First of all, we observe that all the solutions exhibit a decaying trend, when the UE density increases. This is because an increased number of UEs share the fixed total transmit power at the AP side, as well as the limited availability of spatial resources. Additionally, we can see that the superiority of our MUC clustering solution sustained over the other three benchmarkers, regardless of the UE density.

V. CONCLUSIONS

In this paper, we proposed a novel MUC clustering technique for UDNs relying on a co-designed resource allocation scheme for maximising the RB exploitation. To solve the problem efficiently, we proposed a three-stage sequential solution relying on the proposed modified Louvain based MUC clustering and an efficient heuristic resource allocation solution. Explicitly, the proposed modified Louvain based MUC clustering solution outperforms the conventional UC clustering benchmarkers in terms of its sum-rate per RB.

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