Hybrid Beamforming Design for Dual-Polarized Millimeter Wave MIMO Systems

K. Satyanarayana, Teodor Ivanescu, Mohammed El-Hajjar, Ping-Heng Kuo, Alain Mourad, Lajos Hanzo

Millimeter wave (mmWave) communications provides a favorable solution for the ever-increasing data rate demands because of its abundant bandwidth availability. The conventional wisdom about mmWave communication channel is that it always exhibits line-of-sight (LOS) propagation with directive antennas. However, it has been shown in the literature that in mobile communications, mmWave frequencies suffer from frequent LOS blockages and exhibit multipath propagation, which motivates the study and analysis of the polarimetric properties of the mmWave channel. While the exploitation of polarization is promising, the cross-polarization associated with it can be detrimental to the overall performance. Therefore, in this paper, we propose a hybrid precoder design which mitigates the cross-polarization by the joint design of a radio-frequency (RF) beamformer as well as the precoder and combiner used in the baseband. We demonstrate through simulations that the proposed design outperforms eigen-beamforming by 10 dB at a given rate, when considering polarization.

Introduction: Millimeter wave (mmWave) communication has recently gained attention due to its abundant spectral resources which can be harnessed to accommodate the increasing data rate demands of the mobile users [1]. However, given the high propagation losses due to the atmospheric absorption, foliage density and rain-induced fading, the signal-to-noise ratio (SNR) at the receiver would be typically low [1]. Therefore, to mitigate the propagation losses, directional transmission in conceived for increasing the SNR at the receiver. Directional transmission at mmWave frequencies would use hybrid beamforming, where the signals are processed both in the analog and digital domains relying on large $\lambda/2$-spaced antenna arrays. However, owing to the hardware and power requirements, employing large antenna arrays, especially at the mobile station would be challenging [2]. Therefore, a promising solution is to install a compact antenna array relying on a dual-polarization to leverage diversity in addition to beamforming gain. However, until recently, it has been believed that mmWave links always exhibit line-of-sight (LOS) due to using directive antennas and polarization matching, and hence the benefits of polarization have been underestimated in mmWave. Contrary to the popular belief, mmWave communications suffers from frequent LOS blockages and exhibit sparse multipath propagation, which motivates the study of polarization effects in the mmWave channel.

Song *et al.* [3] proposed a soft-decision beam-alignment scheme for exploiting the orthogonal polarization. More recently, Jo *et al.* [4] conducted empirical beamforming investigations by exploiting the dual-polarization diversity in mmWave MIMO channels. To dynamically exploit the polarization diversity and antenna array directivity, Wu [5] *et al.* proposed a reconfigurable hybrid beamforming architecture for dual-polarized mmWave channels.

Against this background, we propose a hybrid precoder design which mitigates the cross-polarization by the joint design of a radio-frequency (RF) beamformer as well as a precoder and combiner in the baseband. In this design, we first obtain the fully-digital precoder and combiner using an iterative algorithm which provides a locally-optimum solution. Then we perform the required matrix decomposition following the approach of [6], where the RF beamformers as well as precoders and combiners in the baseband are obtained. We demonstrate through our simulation results that the proposed design outperforms eigen-beamforming by about 10 dB at a given rate.

System Model: Consider a single user dual-polarized mmWave MIMO system shown in Fig. 1, where the transmitter and the receiver are equipped with $N_t/2$ and $N_r/2$ antennas, respectively.

Furthermore, the antenna arrays of both the transmitter and the receiver of Fig. 1 are dual-polarized with horizontal (H) and vertical (V) polarizations. The transmitter processes the input symbol vector $\mathbf{s}$ of size N_s using a digital transmit precoder (TPC) matrix $\mathbf{F}_{\text{BB}}$ of size $N_t^{\text{RF}} \times N_s$ in the baseband and the precoded symbols are then phase shifted using the beamformer matrix $\mathbf{F}_{\text{RF}}$ of size $N_t \times N_t^{\text{RF}}$ in the RF. Then the received

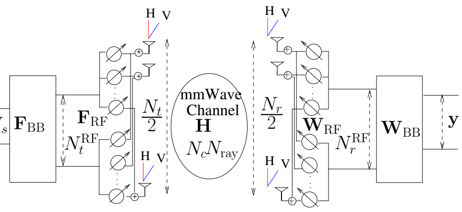

Fig. 1. *Dual-polarized hybrid beamforming architecture.*

signal vector $\mathbf{y}$ after RF and baseband combining is given by

$$\mathbf{y} = \sqrt{\rho}\mathbf{W}_{\text{BB}}^{\dagger}\mathbf{W}_{\text{RF}}^{\dagger}\mathbf{H}\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\mathbf{s} + \mathbf{W}_{\text{BB}}^{\dagger}\mathbf{W}_{\text{RF}}^{\dagger}\mathbf{n}, \tag{1}$$

where $\mathbf{W}_{\text{RF}}$ and $\mathbf{W}_{\text{BB}}$ are the RF and baseband combiner matrices of sizes $N_r \times N_r^{\text{RF}}$ and $N_r^{\text{RF}} \times N_s$, respectively. To elaborate further, $\mathbf{n}$ is the Gaussian noise with distribution $\mathcal{CN}(0, \sigma_n^2)$, while $\mathbf{H}$ is the dual-polarized mmWave channel matrix of size $N_t \times N_r$ and $\mathbb{E}[\|\mathbf{H}\|_F^2] = N_t N_r$. It is instructive to note that $\mathbf{F}_{\text{RF}}$ is used for analog beamforming, which is expressed as $\mathbf{F}_{\text{RF}} = \text{diag}(\mathbf{F}_H, \mathbf{F}_V)$, where $\mathbf{F}_H$ and $\mathbf{F}_V$ are the beamforming matrices of size $N_t/2 \times N_t^{\text{RF}}/2$ for horizontal and vertical polarizations, respectively. Similarly, $\mathbf{W}_{\text{RF}} = \text{diag}(\mathbf{W}_H, \mathbf{W}_V)$, where $\mathbf{W}_H$ and $\mathbf{W}_V$ are the beamforming matrices of size $N_r/2 \times N_r^{\text{RF}}/2$ for horizontal and vertical polarizations, respectively. Furthermore, the channel matrix $\mathbf{H}$ for a dual-polarized antenna array of Fig. 1, is given by [3][7][8]

$$\mathbf{H} = \left[\mathbf{X} \odot \left\{ \begin{bmatrix} e^{\mathrm{j}\angle\alpha_{\text{hh},n_c}^{n_{\text{ray}}}} & e^{\mathrm{j}\angle\alpha_{\text{hv},n_c}^{n_{\text{ray}}}} \\ e^{\mathrm{j}\angle\alpha_{\text{vh},n_c}^{n_{\text{ray}}}} & e^{\mathrm{j}\angle\alpha_{\text{vv},n_c}^{n_{\text{ray}}}} \end{bmatrix} \right\} \otimes \mathbf{H}'\right] \mathbf{G} = \begin{bmatrix} \mathbf{H}_{\text{HH}} & \mathbf{H}_{\text{HV}} \\ \mathbf{H}_{\text{VH}} & \mathbf{H}_{\text{VV}} \end{bmatrix},$$

where $\angle\alpha_{\text{xy},n_c}^{n_{\text{ray}}}$ is the initial random phase of n_c cluster and n_{ray} ray that departs from polarization 'x' and arrives in the polarization direction 'y', while $\mathbf{X}$ is the power-imbalance due to polarization and $\mathbf{H}'$ is the mmWave statistical channel model [3]. Furthermore, $\mathbf{G}$ is Givens' rotation that captures the difference in orientation between the transmitter and the receiver [7][8]. Note that $\odot$ and $\otimes$ represent Hadamard and Kronecker products, respectively.

Proposed Transceiver Design : In this section, we first present the fully-digital precoder and combiner design for the system model considered. Then after obtaining the fully-digital solution, we decompose the precoder and combiner matrices into the analog-digital hybrid design. Let us introduce $\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}} = \mathbf{F}$ and $\mathbf{W}_{\text{RF}}\mathbf{W}_{\text{BB}} = \mathbf{W}$. Furthermore, the system model in (1) can be decomposed into a sum of co-polarized and cross-polarized terms, which are given as

$$\mathbf{y}_{\text{H}} = \underbrace{\rho_{\text{hh}}\mathbf{W}_{\text{H}}^{\dagger}\mathbf{H}_{\text{HH}}\mathbf{F}_{\text{H}}\mathbf{s}_{\text{H}}}_{\text{co-polarization}} + \underbrace{\rho_{\text{hv}}\mathbf{W}_{\text{H}}^{\dagger}\mathbf{H}_{\text{HV}}\mathbf{F}_{\text{V}}\mathbf{s}_{\text{v}}}_{\text{cross-polarization}} + \underbrace{\mathbf{W}_{\text{H}}^{\dagger}\mathbf{n}}_{\text{noise}}, \tag{2}$$

$$\mathbf{y}_{\text{V}} = \underbrace{\rho_{\text{vv}}\mathbf{W}_{\text{V}}^{\dagger}\mathbf{H}_{\text{VV}}\mathbf{F}_{\text{V}}\mathbf{s}_{\text{V}}}_{\text{co-polarization}} + \underbrace{\rho_{\text{vh}}\mathbf{W}_{\text{V}}^{\dagger}\mathbf{H}_{\text{VH}}\mathbf{F}_{\text{H}}\mathbf{s}_{\text{H}}}_{\text{cross-polarization}} + \underbrace{\mathbf{W}_{\text{V}}^{\dagger}\mathbf{n}}_{\text{noise}}. \tag{3}$$

Now we aim for designing the precoders $\mathbf{F}_{\text{H}}, \mathbf{F}_{\text{V}}$ and combiners $\mathbf{W}_{\text{H}}, \mathbf{W}_{\text{V}}$ that minimize the cross-polarization for both horizontal and vertical polarizations. The cross-polarization power leakage from vertical to horizontal polarization plus noise for (2) is given by $\mathbf{C}_{\text{H}} = \text{Tr}\left(\mathbf{W}_{\text{H}}^H\mathbf{R}_{\text{H}}\mathbf{W}_{\text{H}}\right)$, where $\mathbf{R}_{\text{H}} = \rho_{\text{hv}}\mathbf{H}_{\text{HV}}\mathbf{F}_{\text{V}}(\mathbf{H}_{\text{HV}}\mathbf{F}_{\text{H}})^H + \sigma_n^2\mathbf{I}_n$, and $\mathbf{I}_n$ is the noise power. Our objective is to design $\mathbf{W}_{\text{H}}$ and $\mathbf{F}_{\text{H}}$ so that the cross-polarization leakage power $\mathbf{C}_{\text{H}}$ is minimized, while simultaneously preserving the signal dimensions, i.e. $\text{rank}\left(\mathbf{W}_{\text{H}}^H\mathbf{H}_{\text{HH}}\mathbf{F}_{\text{H}}\right) = N_{s_{\text{H}}}$.

The optimization problem can be formulated as,

$$\min_{\mathbf{W}_{\text{H}}} \text{Tr}\left(\mathbf{W}_{\text{H}}^{\dagger}\mathbf{R}_{\text{H}}\mathbf{W}_{\text{H}}\right) \tag{4}$$

$$s.t.\ \mathbf{W}_{\text{H}}^{\dagger}\mathbf{H}_{\text{HH}}\mathbf{F}_{\text{H}} = \alpha\mathbf{I}_{N_{s_{\text{H}}}},$$

where $\mathbf{R}_{\text{H}}$ is a positive definite matrix ($\mathbf{R}_{\text{H}} \succ 0$). Then we continue by forming the Lagrangian function by

$$\mathcal{L}(\mathbf{W}_{\text{H}}, z) = \left(\mathbf{W}_{\text{H}}^{\dagger}\mathbf{R}_{\text{H}}\mathbf{W}_{\text{H}}\right) + z\left(\mathbf{W}_{\text{H}}^{\dagger}\mathbf{H}_{\text{HH}}\mathbf{F}_{\text{H}} - \mathbf{I}_{N_s}\right). \tag{5}$$

Then, the Lagrangian conditions for this problem are

$$\nabla_{(\mathbf{W}_{\text{H}}^{\text{opt}})^{\dagger}}\mathcal{L} = 0 \tag{6}$$

$$z^*\left((\mathbf{W}_{\text{H}}^{\text{opt}})^{\dagger}\mathbf{H}_{\text{HH}}\mathbf{F}_{\text{H}} - \alpha\mathbf{I}_{N_s}\right) = 0. \tag{7}$$

Explicitly, (6) can be written as,

$$\nabla_{(\mathbf{W}_{\rm H}^{\rm opt})^\dagger}\ \text{Tr}\left((\mathbf{W}_{\rm H}^{\rm opt})^\dagger \mathbf{R}_{\rm H}\mathbf{W}_{\rm H}^{\rm opt}\right) + z^*\nabla_{(\mathbf{W}_{\rm H}^{\rm opt})^\dagger}\left((\mathbf{W}_{\rm H}^{\rm opt})^\dagger \mathbf{H}_{\rm HH}\mathbf{F}_{\rm H} - \mathbf{I}_{N_s}\right) = 0, \tag{8}$$

where ∇ is the gradient operation and z^* is the Lagrangian multiplier. By taking the derivative with the respect to $(\mathbf{W}_{\rm H}^{\rm opt})^\dagger$ in Equation (8), we obtain $\mathbf{R}_{\rm H}\mathbf{W}_{\rm H}^{\rm opt} + z\mathbf{H}_{\rm HH}\mathbf{F}_{\rm H} = 0$ and $\mathbf{W}_{\rm H}^{\rm opt} = -\mathbf{R}_{\rm H}^{-1}\mathbf{H}_{\rm HH}\mathbf{F}_{\rm H}z$.

Upon substituting $\mathbf{W}_{\rm H}^{\rm opt}$ into (7), we get

$$\mathbf{W}_{\rm H}^{\rm opt} = \alpha\mathbf{R}_{\rm H}^{-1}\mathbf{H}_{\rm HH}\mathbf{F}_{\rm H}\left[(\mathbf{H}_{\rm HH}\mathbf{F}_{\rm H})^\dagger\mathbf{R}_{\rm H}^{-1}(\mathbf{H}_{\rm HH}\mathbf{F}_{\rm H})\right]^{-1}, \tag{9}$$

where α is the normalization constant expressed as $\alpha = \frac{1}{\sqrt{\text{Tr}\left((\mathbf{W}_{\rm H}^{\rm opt})^H \mathbf{W}_{\rm H}^{\rm opt}\right)}}$.

Having designed the combiner matrix $\mathbf{W}_{\rm H}$, we now aim for designing the precoder matrix that minimizes the leakage of its own power into the orthogonal polarization V.

The cross-polarization power leakage induced by its own transmission into the vertical polarization (3) is given by $\mathbf{C}_{\rm V} = \text{Tr}\left(\mathbf{F}_{\rm H}^\dagger\mathbf{S}_{\rm V}\mathbf{F}_{\rm H}\right)$, where $\mathbf{S}_{\rm V} = \rho_{\rm vh}\left(\mathbf{W}_{\rm V}^\dagger\mathbf{H}_{\rm vh}\right)^\dagger\left(\mathbf{W}_{\rm V}^\dagger\mathbf{H}_{\rm vh}\right) + \mathbf{I}$.

Then, the optimization problem can be formulated as

$$\min_{\mathbf{F}_{\rm H}} \text{Tr}\left(\mathbf{F}_{\rm H}^\dagger\mathbf{S}_{\rm V}\mathbf{F}_{\rm H}\right) \tag{10}$$

$$s.t.\ \mathbf{W}_{\rm H}^\dagger\mathbf{H}_{\rm HH}\mathbf{F}_{\rm H} = \beta\mathbf{I}_{N_{s\rm H}}.$$

Following the same analysis for obtaining $\mathbf{W}_{\rm H}^{\rm opt}$, the locally-optimal solution for $\mathbf{F}_{\rm H}^{\rm opt}$ is given by

$$\mathbf{F}_{\rm H}^{\rm opt} = \beta\mathbf{S}_{\rm V}^{-1}\mathbf{H}_{\rm HH}^\dagger\mathbf{W}_{\rm H}^\dagger\left(\mathbf{W}_{\rm H}^\dagger\mathbf{H}_{\rm HH}\mathbf{S}_{\rm V}^{-1}\left(\mathbf{W}_{\rm H}^\dagger\mathbf{H}_{\rm HH}\right)^\dagger\right), \tag{11}$$

where β is the normalization constant expressed as $\beta = \frac{1}{\sqrt{\text{Tr}\left((\mathbf{F}_{\rm H}^{\rm opt})^H\mathbf{F}_{\rm H}^{\rm opt}\right)}}$.

Remark 1: The combiners $\mathbf{W}_{\rm H}$ and $\mathbf{W}_{\rm V}$ are designed to minimize the interference leakage from the opposite polarization, while the combiners $\mathbf{F}_{\rm H}$ and $\mathbf{F}_{\rm V}$ are designed to minimize the interference caused by its own transmission to the opposite polarization. In other words $\mathbf{W}_{\rm H}$ is designed for minimizing the cross-polarization in (2), while $\mathbf{F}_{\rm H}$ is designed for minimizing the cross-polarization because of its own transmission in (3).

Remark 2: The solution for combiner $\mathbf{W}_{\rm v}$ in (9) and the solution for $\mathbf{F}_{\rm H}$ in (11) are presented for horizontal polarization. However, the proposed derivation can be readily applied to vertical polarization to obtain the combiner $\mathbf{W}_{\rm V}$ and precoder $\mathbf{F}_{\rm V}$.

Having obtained the fully-digital solution, we now decompose the digital solution into hybrid product of $\mathbf{F}_{\rm RF}$ and $\mathbf{F}_{\rm BB}$. However, it is instructive to note that the beamformer matrix followed by the TPC imposes an important constraint to have constant modulus gain. Mathematically, this can be formulated as

$$\min_{\mathbf{F}_{\rm H_{RF}},\mathbf{F}_{\rm H_{BB}}} \|\mathbf{F}_{\rm H}^{\rm opt} - \mathbf{F}_{\rm H_{RF}}\mathbf{F}_{\rm H_{BB}}\|_F^2 \tag{12}$$

$$s.t. |\mathbf{F}_{\rm H_{RF}}(m,n)|^2 = 1. \tag{13}$$

Solving (12) is not straightforward, since the problem is non-convex. Therefore, we first obtain the solution using the least squares algorithm (LS), where we assume that $\mathbf{F}_{\rm H_{RF}}$ is constant to obtain $\mathbf{F}_{\rm H_{BB}}$, while we assume that $\mathbf{F}_{\rm H_{BB}}$ as constant to obtain $\mathbf{F}_{\rm H_{RF}}$. Then we invoke a specific proposition of [6] to meet the constraints imposed on the RF beamformer matrix. Note that this problem is solved iteratively.

Therefore, the LS solution for (12) in the $(k+1)^{\rm th}$ iteration is given by

$$\mathbf{F}_{\rm H_{BB}}^{k+1} = \left(\mathbf{F}_{\rm H_{RF}}^{\dagger k}\mathbf{F}_{\rm H_{RF}}^{k}\right)^{-1}\mathbf{F}_{\rm H_{RF}}^{\dagger k}\mathbf{F}_{\rm H}^{\rm opt}, \tag{14}$$

$$\mathbf{F}_{\rm H_{RF}}^{k+1} = \mathbf{F}_{\rm H}^{\rm opt}\mathbf{F}_{\rm H_{BB}}^{\dagger k+1}\left(\mathbf{F}_{\rm H_{BB}}^{\dagger k+1}\mathbf{F}_{\rm H_{BB}}^{\dagger k+1}\right)^{-1}. \tag{15}$$

Having obtained the LS solution, we now obtain the constrained RF beamformer solution by following the approach discussed in [6].

Similarly, the RF and baseband combiners $\mathbf{W}_{\rm H_{RF}}$ and $\mathbf{W}_{\rm H_{BB}}$ can be obtained for both the horizontal and vertical polarizations.

Simulation Results: In this section, we characterize the performance in terms of rate as well as of bit error ratio (BER) for both the proposed design and the eigen beamforming, where the right and left singular vectors of the corresponding channel matrix are used. To evaluate the performance, we have performed Monte Carlos simulations.

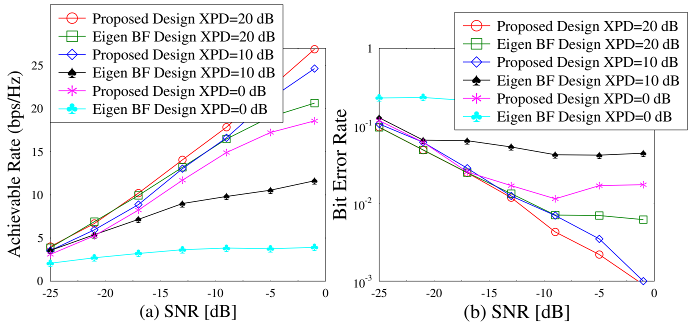

Fig. 2 *Performance of* 128×32 *dual-polarized antenna (a) achievable rate (b) bit error rate.*

Fig. 2 (a) shows the rate of the proposed design and of the eigen beamforming design for a 128×32 dual-polarized antenna array, where θ, ϕ are uniformly distributed from 0-to-2π and $N_t^{\rm RF}, N_r^{\rm RF}, N_s$ are set to 2 whilst $\mathcal{X}$ is varied from 0-to-20 dB, where $\mathcal{X}$ is defined as the ratio of co-polarization power ($\rho_{\rm xx}$) to cross polarization power ($\rho_{\rm xy}$). In this configuration, two streams are transmitted using two RF chains. It can be seen from Fig. 2 (a) that for lower values of $\mathcal{X}$, the eigen beamforming saturates at lower rates, while the proposed design outperforms eigen beamforming with SNR gain of more than 10 dB, especially when $\mathcal{X}$ is as low as 0 and 10 dB. Furthermore, when $\mathcal{X}$ is increased to 20 dB, the proposed design outperforms the eigen beamforming by about 5 dB.

Additionally, to understand the reliability of the system using cross-polarization, Fig. 2 (b) shows the BER of both the proposed as well as of the eigen beamforming designs. It is interesting to note that at SNR of -1 dB, the proposed design achieves a BER as low as 10^{-3} while the eigen beamforming produces an error floor for $\mathcal{X} = 10$ dB. By contrast, when the cross-polarization power leakage is high, i.e. $\mathcal{X} = 0$ dB, both produces error floor. However, the proposed design produces an error floor at a low BER. On the other hand, when the cross-polarization is low, i.e. $\mathcal{X} = 20$ dB, the proposed design outperforms the eigen beamforming by a significant margin.

Conclusion: In this paper, mmWave hybrid beamforming relying on dual-polarization is proposed. More explicitly, we proposed a hybrid precoder design which mitigates the cross-polarization by the joint design of RF beamformer as well as the precoder and combiner in the baseband. We demonstrated through our simulation results that the proposed design outperforms eigen-beamforming by more than 10 dB at a given rate.

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