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# An Evaluation of Structural Loop Analysis on Dynamic Models of Ecological and Socio-Ecological Systems

by

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ABSTRACT

FACULTY OF SOCIAL, HUMAN AND MATHEMATICAL SCIENCES
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## Abstract

This thesis evaluates a modelling analysis technique known as Loop Eigenvalue Elasticity Analysis for its utility and application to system dynamic models of ecological and socio-ecological systems. The technique acts as a structural analysis of the interactions within the system and is capable of identifying feedback loops as structural drivers of dynamic behaviour. With adverse behaviours of many ecological systems known to be driven by feedback mechanisms, Structural Loop Analysis could become an important method for increasing our understanding and control over the systems on which we so greatly depend. Within this thesis, a detailed account of the methodology and application of structural loop analysis to ecological dynamic models is undertaken. The focus of the thesis is an assessment of the technique for its ability to improve model design, to increase understanding of system behaviour and ultimately to evaluate if it could be used for the design and implementation of policy surrounding complex ecological and socio-ecological systems.

Dynamic system models are predominantly used for exploring the interactions which occur within and between systems. Dynamic system models are used across a wealth of academic fields and, much like the purpose of other models, allow the user to explore and manipulate a system where tests on its real-world equivalent would be impractical or unethical to carry out. Through the exploration of components interactions it is possible to learn about, observe and simulate endogenous drivers of systems as causes of dynamic behaviour and change. While the development and simulation of a dynamic system model can provide a wealth of information over a target system, model output alone can often generate more questions than were initially being asked. Converting a real world system to model format can often lead to black box models, where the combination of multiple system components and interactions between them generate unexpected dynamics, even when interactions at a local level are well understood. The complexity that is inherent to our worldly systems can often translate into the models used to represent them.

Within the fields of ecology and socio-ecology, the occurrence of black box models is common and seldom a surprise to the multi-disciplinary approach to system understanding. Ecological and socio-ecological systems are highly complex, naturally incorporating social aspects of human activity and decision making with the natural world, generating an array of human-environment interactions and forming multiple feedback mechanisms between the two spheres. Models of these systems can quickly become just as difficult to interpret as the real world systems, limiting our ability to run and understand sensitivity analysis, conduct meaningful scenario testing or use these models to reflect on policy implementation. Maintaining ecological systems in desirable states is key to developing a growing economy, alleviating poverty and achieving a sustainable future. While the driving forces of an environmental system are often well

known, the dynamics impacting these drivers can be hidden within a complex structure of causal chains and feedback loops.

It is important that we are always on the lookout for new modelling methods, developing and learning new ways to represent the dynamics and behaviours capable by our target system. Modelling analysis tools are an important step in the modelling process, able to extract additional information of a target system that is often unavailable from model output alone. Exploring analysis tools can bring new techniques and new understanding to our model systems which translates to a greater knowledge and understanding of the target system.

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# Chapter 1

# Introduction

This work takes interest in human interaction with ecological systems, focusing on the underlying structural drivers of system behaviour. The work takes a prime interest in our ability to understand complex system dynamic models. The system dynamic method has become a popular and resilient modelling technique to capture system complexity and dynamics (Forrester 2007a; Ford 1999b; Forrester 1994). The system dynamic method provides a unique perspective regarding system structure and interactions for the discussion of modern society concerns i.e. population growth and global resource management (Meadows 2008; Meadows et al. 1972).

Ecological and socio-ecological systems (SESs), sometimes referred to as social-environmental systems, are complex and dynamic whose coupled interactions and exogenous drivers we may never fully understand. As a society, we are an integrated part of these systems and are often a keystone species in multiple systems from local to global scale. Neglect of SESs can lead to a loss of ecosystem services, creating pressure on economic development and poverty alleviation (Carpenter et al. 2009). Examples include industrial waste contributing to coral bleaching, reducing tourism and driving away fish stocks and biodiversity loss driven by deforestation, impacting food security and heavily reducing the monetary value of an ecosystem's goods and services.

System dynamics, a modelling technique initially developed in the 1950s, has seen a wide use across ecological systems research. System dynamics is used to simulate potential non-linear behaviour, express multiple interactions and reveal emergent feedback structures of complex systems which can prove difficult to understand. In SESs, it has often been used to understand the interaction between human development and the environment. System dynamic models may prove to be as intractable as the real world system they seek to simulate. This work aims to gain insights into the structural processes and mechanisms underlying the behaviour of the model and the real world systems they represent. Identifying influential feedback structures of a system could have profound implications for the way that we approach model simulation, decision making and policy analysis.

This thesis takes seriously the eight problem areas in system dynamics posed by Richardson (1996). Through the exploration of a system dynamic analysis technique, this work takes direction from three of those problem areas for researchers of ecological systems: ‘understanding model behaviour’, ‘making models accessible’ and ‘widening the base’.

‘Understanding model behaviour’: This accounts for how a model’s output is generated by the system and what system components internally or externally drive its changes. Through the introduction of structural analysis techniques to a ecological context, this work investigates bridging the gap between our understanding and ability to learn from small and large system models. Small system models often contain drivers which are easy to identify, but contain dynamics and interactions which are relatively simple representations of real world complexity. On the other hand, large system models may feature more interactions and dynamics with the assumption that those elements are necessary to reproduce the behaviour of the system, but the identification of system drivers gets lost in a giant network of interactions causing the model’s output to appear as if it had been pulled from a ‘black box’.

‘Making models accessible’: Ecological and socio-ecological systems often express behaviour which quickly become global concerns. It is important to increase policy engagement with the modelling process in order for policy makers and governance to have a better understanding of ecological and socio-ecological systems through a greater understanding of the models which represent them. The study of ecological and socio-ecological systems is key to managing our social stability, but the techniques used to understand them are not straight forward and take time to understand. Through working on visual portrayal of system dynamics, clarity of modelling techniques within the context of ecological systems and suggestions on how to improve subject terminology, this work hopes to make system dynamic models easier for an experienced researcher to explain while simultaneously making the models easier to understand for impact relevance.

‘Widening the base’: This involves taking techniques which already exist, but perhaps have not been realised to their full potential and applying them to new systems and concepts with great effect. This work will show the benefits which a system dynamics approach can offer a study of ecological systems and the implications which this style of work can have on the way we generate policy and manage our systems.

The methods explored within this report have been selected for their ability to track the influential structures of system behaviour through time and rank this influence relative to other structures. The methods are used to highlight influential mechanisms at times of system instability and critical transitions between stable states.

## 1.1 Research Questions and Motivations

Each of the following questions help to structure discussion within the case studies of the thesis, but specific answers to each are held until the final discussion section at the end of the thesis. Each question is based on a drive to gain a greater understanding from our model systems for the effort that is put into building them. The questions take into account general weaknesses or failures of current modelling practices.

1. To what extent can the structure of a dynamic system model provide serviceable information about the behaviour of its output and therefore a real world system? The term 'serviceable information' has been created by the author to mean any information that the analysis generates which is both original (in that it cannot be gained, or is difficult to gain through other analytical means) and useful (that the output of the information can be used to enhance our ability to make informed policies, decision making and model scenario testing). Whether the output is original can be gained by comparison of structural output with potential output from alternative methods, but whether it is useful is at the discretion of the user and the questions relevant to the field of study.

2. Complex models (high order systems, with numerous interactions) of dynamic systems can be computationally demanding, as well as time consuming for model creation and validation. Can an intermediate level of analysis be conducted alongside the running of a simulation, which would provide insight into the behaviour of the model, thus increasing model efficiency?

3. Can anything be generalised across system dynamic models which may aid the understanding of model behaviour when a new model is produced or an old one is reconstructed?

## 1.2 The Immediate Audience

The content within this thesis is naturally multi-disciplinary, taking analysis tools from business and economics, which use principles founded in mathematics and attempting to apply them to ecological models to see if they have any utility for policy and societal needs. To that end, the overall approach and content of the thesis is primarily intended for an audience of model users and academics interested in ecological modelling. The material should have particular interest to model users who wish to understand how their model produces its behaviour and what internal mechanisms are driving it as it changes through time.

Each discussion section and the overall message of the thesis is tailored to help show the possibilities which the analysis has when used within the field of ecological and socio-ecological modelling. Upon reading some, if not all of the thesis material, ecological model users should be able to see how the analysis could benefit and enhance their own projects, while being able to determine if the analysis is appropriate for their model.

The case studies within each chapter are written assuming a general background knowledge in ecological phenomena, with more mathematically heavy material explained in a detailed manner for those not familiar with concepts such as Jacobian Matrices and Eigenvalues. Alternative sources for seeking information are referenced throughout the thesis whenever a deeper understanding of the mathematics is available, but is not necessary to understand the material within the thesis.

Model users interested in this study would benefit from prior knowledge of ecological systems, system dynamics, feedback loops and a general understanding of eigenvalues, but none of this is necessary as all concepts required to understand the material are laid out within the thesis.

*Ecological vs. Socio-Ecological System Modelling*

The models explored within this thesis are primarily ecological with relevance to societal interests such as social needs and policy. None of the models explored are strictly socio-ecological (that is that they incorporate aspects of social choice and decision making into theirinternal structure) nor have they been used in policy, but the work is still applicable and has significant potential within these fields, as discussed throughout the thesis.

It is important to distinguish between what makes a model an ecological systems model and what makes it a socio-ecological systems model. In the context of this thesis and the models which it explores there are two main distinctions: 1) Model structure and 2) Aspects of social choice. An ecological model refers to a model whose primary variables, interactions and dynamics are derived from natural phenomena. While ecological models may have social variables as part of their structure, they are often separated from the ecological variables and are not integrated as part of the ecological feedback structures. The models within this thesis do not incorporate aspects of human choice, decision making or agency of an individual, which are all features one would expect to find within a socio-ecological model.

As mentioned above, the models explored within this thesis primarily sit within the category of ecological models, but which have relevance to societal concerns such as pollution, overpopulation, overfishing and coral reef degradation. Discussions within the thesis surrounding why we should model, why we should care and utility of the analysis' explored are therefore relevant to the fields of both ecological and socio-ecological systems.

## 1.3 Outline of Thesis

1. Introduction
2. Literature Review
3. Methodology
4. The PLUM model
5. PLUM extension models
6. Sensitivity Analysis
7. Coral reef model
8. Discussion
9. Conclusion

1. Introduction: This introduction looks at the current practice of modelling ecological and socio-ecological systems (SES) and asks why we need to model in the context of current environmental concerns. The benefits of ecological and socio-ecological modelling are discussed alongside current issues which can be faced when attempting to relate societal needs to environmental thresholds. A technique known as Loop Eigenvalue Elasticity Analysis (LEEA) is introduced within this section as an analysis tool with the potential to greatly increase the information and control we are able to obtain from our dynamic models, but which has seen infrequent use in the field of ecology. The introduction concludes with a note on language terms and term consistency throughout the thesis.

2. Literature Review: Socio-ecology is a multi-disciplinary field of study, even prior to discussion of any modelling practices or analysis techniques. For this reason the literature review is subdivided into three main sections:

   - A review on the field of socio-ecology looking at current issues, concerns and interests among the scientific community.
   - A review of system dynamics, a tool commonly used in studies of SES which is used throughout this thesis as a modelling technique. System dynamics greatly compliments the study of system structure and the analysis tools investigated within this study. The review explores the origins of system dynamics, why it is important to study system structure and how system dynamics is well suited for the field of socio-ecology. System dynamics is also reviewed for its limitations in the context of other available modelling techniques.

- A review on current structural analysis techniques available to a system modeller. This sections acts as a compilation of the analysis techniques whose techniques focus on extracting information from system structure. Each analysis is reviewed for its benefits and limitations from the accessibility of its approach to the breadth of information it is able to provide the user. The section identifies why LEEA, among all other structural analysis techniques is pursued in the modelling works of this study and for its potential utility in the field of socio-ecology.

The literature review concludes with notes taken during a conversation with one of the lead authors of structural loop analysis, Associate Professor Christian Erik Kampmann of Copenhagen Business School. The conversation, conducted at early stages of the thesis showed great promise for the technique being appropriate to the structural properties and dynamic behaviours commonly found in SES, despite seeing little use in the field.

3. Methodology: LEEA is used to varying extents throughout each study of the thesis. While some chapters are able to act as standalone studies and thus include the basic concepts of LEEA, it is important that the overall methodology is uniformly available for each study. To account for this, the methodology section includes a step by step guide to carrying out LEEA and obtaining final outputs.

   The methodology section also demonstrates the conversion of dynamic equations to system dynamic format and vice versa. This practice is an important step in model construction for LEEA and understanding of how model structure takes shape from dynamic properties of a system.

   Finally the methodology section covers how to interpret information extracted from LEEA at each stage of the analysis as well as examples of common outputs of loop influence and elasticity plots and how to interpret each.

4. The PLUM model: The PLUM model chapter acts as a primary introduction to Loop Eigenvalue Elasticity Analysis when conducted on a three stock system dynamics model. The chapter focuses on how LEEA is conducted and what the output looks like, and how it can be interpreted. The chapter also doubles as an introductory paper for anyone unfamiliar with the technique.

   The PLUM model is based on the dynamics which can be observed in a shallow lake model being subject to high nutrient intake and undergoing eutrophication. The model was chosen to evaluate LEEA's utility and ability to produce servicable information on a relatively simple model where the dynamics of forward and reverse critical transitions of eutrophic lakes are relatively well known and observed in the real world.

Alongside acting as a primary introduction to LEEA, the study is used to investigate whether serviceable information can be gained from LEEA analysis of an SES model.

5. PLUM extension models: The PLUM extension chapter expands on the work and findings of the PLUM model. The study critically evaluates LEEA's application to larger and more complex models.

   The PLUM extensions subject the PLUM model to two expansions of its structure and internal dynamics, each extension acting as a development on the last with a final comparison of the three order PLUM model (three stock model based on three dynamic equations) with the seven order PLUMPlus model (seven stock model based on 7 dynamic equations). The extension models are used to explore the evolving difficulty of conducting and interpreting LEEA output with evolving model complexity. The chapter's discussion highlights both advantages and disadvantages of using LEEA with higher order models and discusses some of LEEA's limitations regarding computational cost and the high volume of graphical ouput it generates.

6. Sensitivity Analysis of Loop structures: Sensitivity analysis is a method used to investigate how a system will react to exploration of the parameter space. This study is not simply interested in manipulation of the parameter space, but also the structural sensitivity of dominant feedbacks when we try to manipulate entire feedback loops. Little work has been conducted on how loop dominance will react to such changes.

   This chapter acts as a meta-analysis conducted on LEEA, assessing how much a system's sensitive components must be altered in order to affect the loop influence outputs of LEEA. The work conducts Dynamic Decomposition Weights Analysis (DDWA), an extension of LEEA, used to link parameter values to their impact on individual model variables. DDWA is compared and contrasted to sensitivity analysis (SA) conducted on the same model system, seeking the most sensitive parameters of the system on which to base the meta-analysis of LEEA.

   With the results of DDWA, SA and the sensitivity of LEEA, the analyses are considered in the context of identifying system leverage points and thus the application to policy design, adaption and implementation.

7. Coral Reef Model: The coral reef chapter was initialised on the grounds of showing LEEA being used on an entirely separate system from the shallow lakes model of PLUM. The coral reef chapter not only acts as a secondary example of LEEA being utilised and showing promise within an SES context, it also provides a level of insight and understanding of the model system which the original study was unable to do. The study serves to strengthen the application and relevance of structural analysis to SES theory and modelling.

The coral reef model chapter works as a good example for the importance of acknowledging and understanding the presence of feedback loop interplay.

8. Discussion: In the discussion section the information gained from the main studies of the thesis are brought together, overviewing LEEA's potential as an analysis tool in the field of socio-ecology. LEEA's outputs, results, novelty and limitations are all discussed in the wider context of SES modelling, reflecting on the aims and motivations set out in the thesis introduction.

   It is then considered whether there is merit in using LEEA in the context of qualitative research, model design, policy implementation and system maintenance. Each of these topics make up key parts of SES research and are part of strong project design and direction. The discussion considers the extent to which LEEA could benefit each of these topic areas as an integrated part of a dynamic systems modelling approach.

   The discussion concludes by regarding the importance of accessibility and general terminology used across LEEA and general SES modelling. The importance for SES research and modelling tools to be accessible to the general public and a political audience is stressed.

9. Concluding statements: Concluding statements primarily cover the aims and objectives set out in the introduction. The conclusion highlights key findings of the thesis, overviewing what LEEA and the associated structural analysis tools could bring to past and future studies of SES. The conclusion ends with recommendations for the future development and use of LEEA, both as an analysis tool and those considering the analysis for future projects, primarily directed at those associated with socio-ecology.

10. References

11. Supplementary Information: Contains material from the main projects of the thesis which was not included within the main write-ups.

## 1.4 Concepts and Considerations

### 1.4.1 In Need of a Second Planet

Our world is a complex place and the human race is neither an exogenous driver, nor a catalyst which is unaffected by the reactions taking place. The human race is an integrated part of the Earth system and it is important that our way of thinking, our models and our policies reflect that fact. Viewing the Earth and all its inhabitants as a single entity is not a novel concept. Spanning from early concepts of Gaia, the Biosphere and the Nooshpere Vernadsky (1967), it has long been recognised that the actions of the human race cannot be detached from the reactions that we see from the Earth system. Instead humans and the environment form a network of dynamic interactions which are all connected and feedback into one another.

In recent decades, there has been a growing awareness of the increasing impact that human action is creating on the environment. Human pressures on environmental systems occur at different scales, from local ecosystems to global climate and over many temporal scales, from immediate effects of deforestation to habitat and biodiversity loss (Brooks et al. 2002) to pollution of the upper atmosphere (Seinfeld and Pandis 2016).

Arguably starting at the rise of human civilisation, the development of agricultural practices and reinforced at the beginning of the industrial revolution, our impact on the world around us has only been increasing. Growing economies and populations worldwide demand quick and easy solutions from industry and agriculture, often in the form of non-renewable energy sources and short term policies which can be implemented quickly and take little consideration for long term sustainability (Costanza 1987).

Understanding our impacts on the environment is key to ensuring a sustainable, just and safe future for the inhabitants of the planet, but awareness of the issues alone will not solve our problems (Beddington 2009). It is important for us to learn not just that we are affecting the planet, but how, where and to what extent. What issues need immediate attention and what is the best course of action to tackle these issues? How do the challenges that we face connect to each other? For example, can we truly expect to be able to tackle water, energy and food security without having any impact on energy security? Can we begin to tackle energy security while expecting no impact on Biodiversity loss?

There are many challenges for the future of the human race existing on a global scale (Biodiversity, Climate change, Energy Security, Food security, Governance, Health/Disease and Population) and we are at risk of transgressing many of the Planetary Boundaries (Climate change, Biodiversity loss, Biochemical, Ocean acidification, Land use, Freshwater, Ozone depletion, Atmospheric aerosols and Chemical Pollution (Rockström et al. 2009). It is unfortunate that no-one has yet created an alternate world for us to experiment with (at least not to the knowledge of the author) and that human impact, while predictable has not ever been seen at our current levels before. That is to say, the future is uncertain, but not uncertain enough for us to be able to go through life with blind ignorance. We must always explore ways in which we can monitor and improve our impact on the environment which we are such an integral part of and depend upon for our everyday lives.

### 1.4.2 All models are wrong, but some are useful ...

We live in a world where homosapiens have become a keystone species to the environmental systems that it inhabits. It is important that our numerical methods to learn about human impacts reflect the interconnected nature of the socio-ecological systems which we intend to model. Socio-ecological refers to the interactions held between society and the environment, where each should be viewed as an integrated component of the other, neither being able to impact the other without imposing feedback responses on themselves.

One approach to studying complex ecological dynamics is by building mechanistic, process based models which allow us to simulate and experiment with human-environment interactions in a controlled systematic manner. These models come in many forms, from agent based models, to network and graph theory, global climate models and time series plotting to GIS mapping. All are created to help us understand the extent of change that is occurring to the environment and what mechanisms are at play creating the change.

While modelling is not perfect and it is often inadvisable to model a socio-ecological system in its entirety, it is certainly better than experimenting on the real thing, which can be detrimental to all involved if not impractical. Process based models in particular allow the user to simulate behavioural trends in ecosystems and attempt to learn ways in which we might hope to control them.

Socio-ecological systems are capable of expressing numerous behavioural trends, many of which we may never truly understand. Examples include exponential growth and decay, dampening and levelling out, oscillations, sigmoidal behaviour, regime shifts, critical transitions and tipping events between stable states. While many other types of systems are capable of expressing these outputs, it is the social aspect of decision making, judgement, policy implementation and human perception that exists within the Anthropocene that makes socio-ecological behaviour trends so difficult to predict and model accurately.

Using regime shifts as one example: regime shifts occur within systems that are capable of abrupt transitions between contrasting stable states. Examples include the eutrophication of a shallow lake, or a forest transitioning into a scrub land. While inducing a regime shift in the real world can create negative impacts on both social and environmental factors alike, they can also be difficult to reverse. Modelling of such events instead allows us to investigate mechanistic drivers of the system with no risk to the real world and with enough model accuracy, improving our ability to reverse such events or prevent them in the future.

There are reasons to believe that many socio-ecological systems which we depend on exhibit critical transitions between stable states (Scheffer and Carpenter 2003, Steffen et al. 2015). Maintaining socio-ecological systems in desirable states, understanding why their behaviours change and how their trajectories are affected through time is fundamental for economic growth, poverty alleviation and general wellbeing. One approach to understanding these systems is through system dynamics; a technique which concentrates how a system's structure of interactions, feedback loops and causal chains can control a system's behaviour.

System dynamics is a methodology designed to help simulate such behaviours. This study uses system dynamics as a numerical integration scheme for representing a system's structure and generating simulation models. It is grounded on the basis that complex behaviour is generated endogenously from the interactions between its components and internal structure. It helps to show how phenomena such as non-linearity, emergence and self-organisation can arise purely as a consequence of internal feedback mechanisms, with no influence from exogenous drivers. System dynamics is therefore suitable for investigating ecological and socio-ecological systems, if a whole system's perspective is desirable. While reductionism has its uses it often focuses on just one part of a system at any one time and is unable to reflect the more complex dynamics and interactions that take place within a system in the same way that system dynamics can. While system dynamics uses a unique language of stocks and flows to model a system, the basic principles are the same, investigating changes within a system and the mechanisms which generate that change.

### 1.4.3 Current System Dynamic Models in Ecology and Socio-Ecology

A wealth of system dynamic models for environmental and ecological systems can be found throughout the academic literature. Some attempts have been made to catalogue these models, e.g. metasd.com (MetaSD 2016) but have not been kept up to date, or the material is no longer available. A catalogue of readily available system dynamic models can be found at forio.com/simulate (Forio 2016). The library holds a range of models based around system dynamic theory including: predator-prey interactions, population, fish stocks, the U.S. health system and even J. Forrester's original Urban Dynamics model (Forrester 1970). The models come in an array of sizes and complexity and provide a glimpse of the vast applications that system dynamics provides to socio-ecological studies.

Uses of system dynamics for environmental modelling have been documented by El-Sawah et al. (2012). These include modelling land use and population dynamics in Germany, a MedAction policy support, Groundwater decision support in Central Texas, communication of water resource management in Australia, Modular ecosystem modelling and a decision support tool for groundwater management in Western Australia. The report concludes that system dynamics strengthens the modelling process for both the modeller and the end user and provides concepts of stocks and flows which are highly transferable to other applications, giving system dynamics a unique selling point against other modelling practices.

Other examples from the literature include Guo et al. (2001) who presents a highly complex model for the study of Lake Erhai in China: a lake known to have undergone eutrophication. The model consists of seven sections: population, water resources, industry, pollution, agriculture, tourism, water-quality, each with between three to seven subsections and a system dynamic model to represent each and around 144 variables which must be validated separately. On a model of this scale, it can quickly become difficult to understand which drivers are perturbing the system.

While system dynamics is used across a wide range of socio-ecological models, there are still many opportunities and projects where it sees limited to no use. There is a distinct lack of system dynamics at an educational level and the field has seen no introduction to curriculum at secondary school, or university level. This leads to a simple lack of awareness of system dynamic theory, despite the potential it shows to improve many projects on both a qualitative level through systems thinking and quantitative level through simulation.

### 1.4.4 The Ambiguity of Model output

System dynamics builds models of coupled partial differential equations allowing users to construct highly complex models of socio-ecological systems through the mapping of their internal structure and dynamics. While this toolset has allowed for great advancements in the field of socio-ecological modelling, it has also lead to many models which are equally as complex as the systems they are trying to represent. These 'black box' models are capable of generating output via simulation, but what that output means, how it is generated and how to relate it to practical solutions is often unclear. This disparity between results and applications arises as sets of differential equations within system dynamics do not have to be solved analytically, but rather use computational methods. The only limitations of model complexity become computational resources. This 'black box' outcome is especially true in the field of socio-ecology, where social aspects such as choice and perception impose qualitative data on what would ideally be a fully quantitative model.

Given the complexity of socio-ecological systems in the real world, models of ecological and socio-ecological systems can quickly become complex and difficult to interpret, limiting our ability to run sensitivity analysis and understand causal drivers of undesirable behaviour.

While system dynamic models can be simplified to only include components where there is reliable data available, this can eliminate essential dynamics to the system and quickly become unrepresentative of the real world, requiring sufficient model complexity to explicitly represent the dynamics of the target system. Conversely, trying to build a model which includes every piece of empirical data able to be collected can quickly generate models where the essential drivers of system behaviour are lost among a spaghetti-like network of un-impactful connections and components. Simple models can assume away key dynamics, while including them generates a level of complexity which makes system output hard to interpret. Which leads to the question: How can we achieve tractable analysis from system dynamic models? This is a question that applies to all model building activities, so arguably science itself: what is the appropriate representation of the phenomena we are trying to understand?

### 1.4.5 Working towards a solution: A Metamodeling approach

There is a need for an intermediate level of analysis which is capable of highlighting behavioural drivers from a system model, regardless of model size or complexity, allowing the user to focus on the important dynamics with respect to the research questions of the system without concern for whether the size and complexity of the model will restrict the user's ability to gain serviceable information from it.

This work investigates modelling analysis techniques that are not traditionally considered within the fields of ecology and socio-ecology, but that could have profound impacts for the way that we view causality, and address policy making and scenario testing for ecological system models.

This study structurally analyses system dynamic models to determine whether serviceable information can be extracted from changes of structural feedback loop dominance in conjunction with model output. The technique explored in this initial study is known as Loop Eigenvalue Elasticity Analysis (LEEA).

### 1.4.6 Loop Eigenvalue Elasticity Analysis (LEEA): A Structural Analysis Technique to be Tested on Ecological models

Within this thesis, a structural model analysis technique known as Loop Eigenvalue Elasticity Analysis (LEEA) is tested for its viability and utility within the field of ecological systems modelling. Primarily the study focuses on models that are used to represent complex ecological real world counterparts, analysing feedback loops within their system for their structural and dynamic properties.

Structurally models are analysed for the number of feedback loops they contain and the polarity of the loops (whether they are positive or negative feedbacks). Dynamically the analysis establishes a hierarchy of influencial feedback loops and what dominance they have over the system's behaviour and how this changes over time.

The thesis contains four main studies; two studies, the first and fourth, apply LEEA to dynamic models from the ecological field, first of a eutrophic lake system undergoing critical transitions found in Chapter 4 and second a coral reef model looking at mono vs. bistability in Chapter 7, in order to evaluate the technique's utility and potential. The second study found in Chapter 5, investigates limitations of the technique. While the model studies of Chapter 4, 5 and 7 investigate the application process, interpretation and limitations of LEEA, the third study of Chapter 6 assesses the sensitivity of LEEA's output, investigating the extent to which LEEA's results can change with perturbations to the model system and its potential for identifying system leverage points.

The models within this thesis were selected because their topics of lake eutrophication and coral reef regime shift and their dynamics concerning critical transitions and hysteresis are relevant to many modern studies and concerns surrounding socio-ecological systems. The models chosen are relatively simple and are easily repeatable. The data and values assigned to each variable were readily available in order to recreate the authors' original results. Recreating the original models allowed the results of LEEA to be compared against a well-established discussion which was important for identifying whether LEEA was capable of adding anything new to the existing knowledge surrounding these models. The authors and co-authors of the models have many published works on critical transitions in ecological transitions, either theoretical studies or ones backed by empirical data. The point being that the models and dynamics explored within this study have been designed with a high level of existing knowledge over the target system and their dynamic behaviours. Overall this thesis evaluates models which were deemed by the author to have interesting dynamics to explore using LEEA, but that were easy to follow. It was important that the results of the model outputs, the results of LEEA and the motivations for introducing LEEA into the field of socio-ecology were easily understood by the wider scientific community and not just the user.

The thesis also presents minor studies to aid discussion. These studies apply LEEA to models of varying sizes and complexity, addressing the relevance of the technique. The first found within Chapter 5, known as the Yulin City model which discusses model scenarios where LEEA may not be so useful. The second minor study is incorperated into the final discussion of the thesis, based on work conducted by the author in the U.S. in association with Florida University's Harbor Branch Oceanographic Institute which discusses the applications of qualitative research in light of model conceptualisation and building.

Each model within this study is directly taken from or stems from the published literature in the field of socio-ecology. Save the model designed for the Harbor Branch project, each model presented within this study originates from peer reviewed models, the dynamic properties of which are well established within the field. This allowed the focus of the thesis to remain fixed on the utility and application of LEEA, and meant that each study could concentrate on the analysis, rather than the validation and justification for the models. This also ensured that there was no bias held when interpreting the analysis results caused by having built the models. LEEA could be conducted and interpreted from an outsider's perspective of the model system, meaning that no interpretation was made from an advanced knowledge over the target system's design.

With the potential for LEEA to increase the tools and analysis available to researchers of socio-ecology, extending existing knowledge surrounding model structure and behavioural drivers, LEEA may be utilised to understand more about how to influence the real world systems behind our models.

Alongside the thesis' main discussion, smaller discussion points which are deemed relevant to the fields of both ecological and socio-ecological modelling and analysis are included. These sections include the uses of qualitative research during model conceptualisation and construction and an insight into the language surrounding feedback loops and how it may not be providing the transparent, easy to understand rhetoric that the field requires in order to be accessible to the public and policy makers alike.

### 1.4.7 Socio-Ecology vs Social-Ecology: a note from the author

Throughout the literature of human-environmental systems, two phrases are commonly used to describe the field of human-environment interactions and are used almost interchangeably. These terms are socio-ecological systems and social-ecological systems. While some authors insist on one being correct over the other, the two have never been used consistently through time and which one is used seems to only fluctuate by which is popular at the time, or used by a more popular author.

It is the understanding of the author that the terms socio-ecological and social-ecological can in effect be used interchangeably. However, there is a key difference between similar terms of socio-ecology and social-ecology:

Socio-ecology is a field of study which focuses on the biophysical and social interactions held between humans and the environment. These systems are both complex and adaptive. Further definitions can be found in Redman et al. (2004).

Social-ecology refers to the critique of current social, political and anti-ecological trends, namely problems which arise as a result of hierarchical social-organisation.

While it is understandable that these two terms could easily be misplaced as the field of human-environment interactions is often concerned with policy implementation, this work will try to remain consistent by only using the term socio-ecology or socio-ecological models when referring to the models created for the purpose of investigating human-environment interactions and feedback.

# Chapter 2

# Literature Review

Reviewing structural analysis within the context of dynamic system model and ecological systems requires knowledge to be extracted from multiple disciplines in order to understand and justify the context with which the analysis is being examined. This literature review acts as a vital platform with which the integrated studies, practical approaches, aims and discussions of the thesis are based. The following review is an in depth critique of modelling practices, analytical techniques, and concerns known to exist within the fields of ecology and socio-ecology. The literature stems from basic numerical principles in graphical and model theory and dynamic properties of environmental habitats to the causality of social policy and the complexity of system interactions.

The review is split into three main subject areas:

- System Dynamics: A Modelling Tool Appropriate For Dynamic Systems Modelling
- Complex Ecological and Socio-Ecological Systems: An overview
- Analytical Techniques

## 2.1 System Dynamics: A Modelling Tool Appropriate For Dynamic Systems Modelling

### 2.1.1 What is System Dynamics?

System dynamic models are structural representations of dynamic real world systems. They can be conceptual, using qualitative data and mind models to better understand properties of a system such as feedback structure and internal connectedness. They can also be built upon sets of partial differential equations (PDEs), using empirical data to run simulations and gain insights into system behavioural trends.

System dynamics combats an open loop (linear) way of thinking; that taking action on a problem will give a result and nothing more will need to be done. The open-loop concept is one in which most discussions are debated in the press, business and government (Forrester 2009). The term 'open loop' refers to the user taking no account for feedback mechanisms that are inherent to the system's internal structure.

Jay Wright Forrester, born 1918 and the founder of system dynamics stated that system dynamic modelling can:

*"... organize the descriptive information, retain the richness of the real processes, build on the experiential knowledge of managers, and reveal the variety of dynamic behaviours that follow from different choices of policies."* (Forrester 1995).

System dynamics is a method for enhancing our understanding of complex systems and improve our ability to design effective policies. System dynamics is a fundamentally interdisciplinary approach, drawing together social practice, economics, social psychology, organisation theory and natural fluctuations in the environment. Generic structures within system dynamics often apply to a multitude of different scenarios and models. Understanding basic structures which are part of one dynamic system can help to quickly gain a basic framework necessary for another. Forrester (2009) uses the concepts of generic structures to compare the oscillations of a swinging pendulum to that of inventory and employment in a business.

System dynamic models take a resource based view of the world, characterising a system through a set of stocks and flows in order to represent its structure: a system's structure in turn, being what controls system behaviour. Stocks are usually, but not restricted to, material goods, and flows are pathways of material between stocks.

Stocks represent physical or imaginary parts of a system that can be measured - *"An accumulation of material or information that has built up in a system over time."* Meadows (2008). Examples include fish in a lake, patients in a hospital, happiness of children at school, content of clients of a business and predator or prey populations. Stocks are the main building blocks which make up a system and are represented in system dynamic model diagrams with square boxes.

Flows are what control the amount of material within a stock - *"Material or information that enters or leaves a stock over a period of time."* Meadows (2008). Often, flows are determined by a statement which reflects on the value of a stock in relation to a goal. Examples include birth or death rates in a population, inflow and outflow of water from a lake system, new employees entering a business. Flows are represented with large double lined arrows (often black) entering or leaving a stock and are often visualised with tap symbols to represent that the flow is capable of changing intensity and not set at a static rate.

System dynamic models also consist of auxiliary variables, constants, sources and sinks. Auxiliary variables are variables of the stock which are subject to change through time, often given their own space on a system dynamics diagram and sometimes, but not always, represented within circles. Sources and sinks are places outside the scope of the model where the material for a stock flows from and to respectively. Sources and sinks are represented with cloud symbols. Constants are values which do not change throughout a simulation, which are usually integral parts of dyamic equations acting as scalar values. As constants do not change throughout simulation, they can either be incorporated into the algebra of other variables or given their own space on the model. Whether constants are represented visually usually depends on visual simplicity and visual clarity required by the user.

A simple example of a system dynamic model can be seen in Figure 2.1.

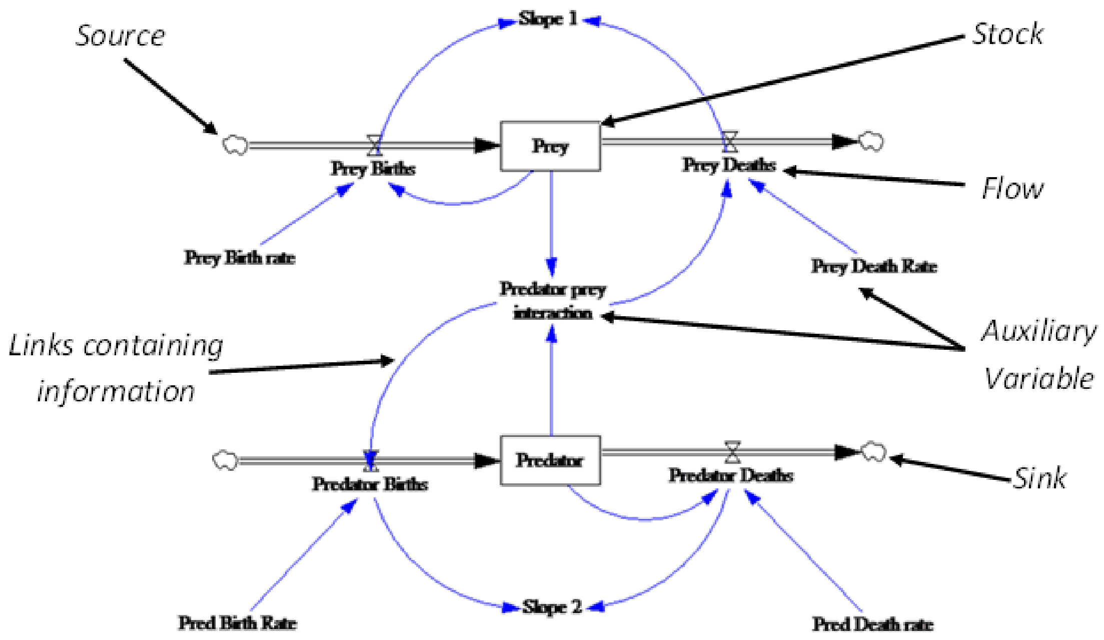



Figure 2.1: An example of a system dynamic model using a classic Lokta-Volterra system. The diagram shows examples of stocks, flows, auxiliary variables, sources, sinks, and links which carry information between system components.

While the output and drivers of simple system dynamic models are often easy to interpret, models of real world systems become increasingly complex when multiple stocks and flows are linked together to form extensive flow chains and multiple feedback loops.

The diagram in Figure 2.1 shows a second-order, nonlinear dynamic system as it is derived from two stocks and therefore two differential equations. Increasing order of a system is determined by the number of stocks it contains and gives a rough idea of the complexity of the system, as a higher order system will generally contain more variables, links and feedback loops. While simple loop models are easy to navigate, they often do not represent the complexity of the system they are based on leading to misleading results and teaching the wrong lessons (Forrester 2009).

In order to understand the value of system dynamics through the basis of its methodology and way of thinking, it is important to learn its origins and early development.

### 2.1.2 The Origin and Development of System Dynamics

System dynamics was first developed in the 1950s-60s when Jay W. Forrester began to investigate employment fluctuations at a General Electric plant in Kentucky. Using initial conditions and policy implemented for the company's inventory, backlogs, employees, orders and production rate, Forrester developed a system whereby each line of the model was computed from the preceding line. It became apparent that the system was unstable and able to undergo oscillatory trends generated purely from internal dynamics without any external forces (Forrester 2007a). Other early contributors to the field of system dynamics included Richard Bennett, J. L. Enos, Willard Fey and Edward Roberts.

Forrester's research and therefore the foundation of system dynamics developed from a background in feedback systems. At Massachusetts Institute of Technology (MIT), Forrester studied under the supervision of Gordon S. Brown, a pioneer in feedback Control Systems. This is perhaps why the main emphasis of system dynamics is how internal structures of a system can generate emergent, non-linear behaviour.

The development of system dynamics has not come without its opposition. Jay Forrester's early works using system dynamics involved the development of Urban Dynamics (Forrester 1970) which was written to study major urban policies in the United States which were shown to present neutral to highly detrimental properties on the city. Among the results, Forrester was able to show that the construction of low-cost housing within the city was a process generating high levels of poverty. Forrester (1970) explained how low-cost housing was being created in places where jobs could have been established, but instead was bringing in more people who needed them. Opposition to this highly controversial statement included a professor of social sciences at MIT who Forrester recalls anonymously, saying "I don't care whether you are right or wrong, the results are unacceptable." (Forrester 2007a). Forrester (2007b) explains how system dynamics has often been used to generate results to appease a client, rather than show the true dynamics of a system, of which the results can be both unwise and impossible to achieve. Despite its opposition and misguided uses, Urban Dynamics became the foundation for projects including World Dynamics (Forrester 1971) and Limits to Growth (Meadows et al. 1972).

While system dynamics can produce interesting results, increasing the understanding of a system much further than a mental model, its methods have not been open to public use. The components of system dynamic models must be defined and thus each model is constructed from a multitude of differential equations. In addition, the interesting messages which system dynamics can offer the user are almost always output in graphical format which are not always easy to interpret. Despite this, Forrester recounts that his first publication which received major public attention was World Dynamics (Forrester 1971), which saw reviews from The London Observer, Wall Street Journal, Christian Science Monitor and even a full length article in Playboy (Forrester 2007a).

The biggest step for system dynamics came in the 1980s with the development of system dynamic user friendly software including STELLA, Vensim and Powersim. These software packages provide an advanced user interface which visualise and compute the dynamic structure of system dynamic models. These programmes are still widely used today and are tied to a wealth of helpful introductory tutorials and user support groups.

Despite recent advancements in computational power and the creation of modelling tools for system dynamics, which are assumed to be beneficial for the field of system dynamics, Coyle (1998) speculates an alternative reasoning. By creating machines that are able to run and process information much faster than when system dynamics was first developed, it could mean that model developers do not have to spend as much time with rigorous inspection of their models and parameters before a model is sent to run (Coyle 1998). On the other hand, more computational power allows modellers to run many more iterations and scenarios over a short time period.

### 2.1.3 Construction of a System Dynamic Model

System dynamic models are often built from mental models, an image of a system built from the users pre-existing knowledge. Mental models are good at knowing what information is available, what variables to connect to each other and what the end goals are of each participant of the system. However, the human mind is unable to reliably determine the complex behavioural dynamic changes that would be generated when all of these factors are merged together in one system.

The transition from a mental model to a system dynamic model is not always simple and certain factors must be considered during the transition from one to the other. The following factors were detailed in J. W. Forrester's explanation of working with computer models (Forrester 2009):

1. All variables within a system dynamic model must be defined. All stocks must be associated with an unambiguous equation which explains how that stock accumulates.

2. System dynamic models are capable of generating unrealistic results such as negative water in a bath or millions of degrees of heat within a kettle. These results arise when sections of the model are not expanded to reflect their true nature, at which point that section of the model must be redesigned.

3. System dynamic models are capable of generating surprising behaviour purely as a result of endogenous dynamics. These dynamics may include a job training programme in a city leading to more unemployment or a company growing so rapidly that it leads to its own collapse (Forrester 2009). More often than not these results do not mean the model is wrong, but instead the internal dynamics of a system have generated a behaviour unable to be forecast from a mental model.

Forrester (2007a) explains that it is from simulations of system dynamic models that inconsistencies within our mental models are revealed. Mental models become inconsistent both in one's own thoughts and between individuals. System dynamics provides a platform for models to have a consistent and structured framework, which everyone can work from.

### 2.1.4 Issues associated with system dynamics and alternative methods

Issues with System Dynamic modelling reportedly occur when it is used to try to solve issues for which it was not designed (Barlas 2007; Forrester 2007b). System Dynamics' primary function is to explore "problematic behavior patterns caused primarily by the feedback structure of the setting" (Barlas 2007). System dynamics excels at simulating endogenous drivers of complex behaviour, while it has been criticised for confidence levels in model validation through the use of historical data, taking a reductionist perspective and for the way it addresses plurality and social ethics (Featherston et al. 2012).

A main criticism of system dynamics occurs when it is used to explore a problem which is fundamentally exogenous to the system. This can create problems of model size and complexity as the model user attempts to incorporate an exogenous driver into the internal feedback structure of a system (Forrester 2007b). There are very few examples of this practice within the literature, as many of the models fail to make publication.

A second criticism of system dynamics is an inability to mimic reality for the purpose of prediction (Hayden 2006). This criticism reflects that system dynamics is rarely used for producing specific values for any given time. While this limitation makes system dynamics limited at short term predictions, its ability to portray behavioural patterns have been shown to maintain accuracy to the system on much longer timescales (Meadows et al. 2004). Despite its long term appeal, this particular limitation does hinder system dynamics when it comes to model validation and use in policy, both of which often require specific values in order to cross reference with empirical data, or produce accurate predictions for policy briefs.

Another critique of system dynamic modelling includes its inability to deal with 'social messes', referring to it as a reductionist method because of its focus on the interactions between system parts, rather than the actions of individual agents (Keys 1990). This criticism also links to the idea of Pluralism which has two separate parts: 1) that different perspectives may be held regarding the system's complex problems and 2) that individuals may behave differently within a system. Finally, system dynamics has been critiqued for its general dehumanisation of people within a system. Jackson (1992) likens system dynamics treatment of humanity to mere 'cogs in a system', taking a deterministic view of a system and removing free will which dehumanises any topic which it is used to model. Again, these criticisms must be taken into consideration when using system dynamics for ecology, social-ecology and policy as the outputs and solutions which are produced from a system dynamic model may not take ethical, moral or environmental principles into account, unless they are specifically incorporated within the model.

Coyle (1998) explores two reasons why there can be drawbacks to investigating systems in terms of their feedback loops: 1) The smallest models can contain thousands of loops, 2) experimentation of these structures can never be completed because there is always something else that could be tested.

It is possible for the construction of system dynamic models to get out of hand, when the expansion of a model from its mental counterpart is never constrained or questioned. This can result in models which are initialised as simple representations of a system behaviour, but end up containing dozens to hundreds of uneccessary components. Regarding concern 1: Coyle (1998) recounts a time when he was shown a model with 20 000 variables, requiring 199 pages to print fully, which he noted unsurprisingly was found to have technical errors in its equations.

An alternative approach to system dynamics which addresses similar questions of behavioural causality is Field Anomaly Relaxation (FAR). FAR constructs system trees, where each branch of the tree leads to a possible future. The trees are based on bifurcations of dynamic transitions between states.

A further alternative method to system dynamics which has recently been used to investigate socio-ecological systems is Generalized Modelling. The method has seen relative success and provides a promising alternative to stock and flow modelling techniques. Generalised Modelling is a type of dynamic modelling, similar to stock and flow diagrams. Lade and Niiranen (2017) use Generalized Modelling to study a Baltic Cod fishery. The main advantage of Generalized modelling is that it functions well with qualitative dynamics, such as regime shifts and attractors, which are a prominent feature in many socio-ecological studies. The approach provides a mathematical approach to systems where empirical data is not available for all components and extensive knowledge is limited.

Generalized modelling is used to bridge the gap between specific case models where all data is available and theoretical models whereby data may not be so easy to acquire, if it is used at all. It is traditionally used to investigate the sensitivity of fixed points, but in a socio-ecological context can be used to study the sensitivity of a basin of attraction, in which a system may reside. Similar to stock and flow diagrams, many structural analysis techniques can be conducted on a generalized model. Upon conducting structural feedback loop anaysis, Lade et al. state: *"Meaningful results were obtained in the ecological and socio-ecological analysis, but the results for the social system were harder to interpret due to the model's structure"* Despite this, Lade and Niiranen (2017) still refer to feedback loop analysis as a promising tool for the study of qualitative dynamics in SESs.

Advantages of Generalized Modelling include:

- Able to be used on empirical systems which still contain knowledge gaps.
- Able to be used alongside a variety of model analysis techniques, including many structural analysis techniques.
- A solid method for SESs, which often express qualitative dynamics.

While Disadvantages include:

- Similar to 'Stock and Flow' diagrams, large model structures still pose a potential problem to interpretation of dominant drivers.
- Relatively time consuming: makes use of Jacobian matrix, which must be derived manually with limited automation.

### 2.1.5 Why Study System Structure?

System dynamic model analysis has developed over the last three decades in an attempt to create a governing strategy for investigating system behaviour. The premise behind structural analysis techniques is that structural features are the root cause of behavioural outputs. Several methods have been developed and tested for structural analysis, but none have yet seen common practice within the system dynamics method and fewer tested within the realm of socio-ecological systems.

Investigating system structure can produce important information of a target system including heavily connected variables, pathways and feedback loop structures, much of which can be attained from Graph Theory (West et al. 2001). Despite the benefits the application of Graph Theory can bring to system understanding, Kampmann and Oliva (2008) state: *"We will never be able to tell the behaviour that a model will produce simply by investigating its structure"*.

To purely study system structure is to remove the time variation between elements and thus neglect the dynamic interactions between them. However, identifying structural dominance change through time in conjunction with running the model simulation has proven to be an effective method for identifying structural drivers on observed behavioural output (Kampmann and Oliva 2006). Feedback loops create reinforcing and balancing behaviour, but can often be unpredictable due to their interconnected nature, which we observe within system dynamic models. Feedback loops are therefore seen as the key to understanding system behaviour.

### 2.1.6 Complexity within systems

Complexity in systems takes on different forms: 1) Complexity in terms of number of components 2) Combinatorial complexity - finding the best solution through an infinite number of possible approaches, 3) Dynamic complexity - counterintuitive behaviour that arises in systems through time.

Sterman (2001) describes ten reasons why dynamic, complex behaviour can occur within systems: constantly changing, tightly coupled, governed by feedback, nonlinear, history-dependent, self-organising, adaptive, characterized by trade-offs, counterintuitive, policy resistant.

Most of these factors are self-explanatory. Characterised by trade-offs refers to delays within systems which inhibit our ability to test hypothesis and learn as they alter response time of system components.

Just because a model is a simplified version of a real system, does not mean that the model itself is simple. Complexity and size of system dynamic models does not just depend on a model's order (the number of differential equations which define its dynamics). A system with only two stocks (for example) can have hundreds of auxiliary variables which determine the state of the stocks. These auxiliary variables contribute to feedback loops of their own and a much more complex web of interactions than those held between stocks.

### 2.1.7 Delays in feedback systems

Kampmann (2012) highlights the issues with simply running multiple model simulations by misinterpreting important structures. Despite reinforcing loops and balancing loops having specific behaviours that they generate (exponential growth/decay and oscillations/dampening respectively), behavioural output of systems is also heavily dependent on system delays.

Delays within feedback loops can form from lack of information between two stocks, a slow reaction time for a natural process or from a criteria that must be met before the material/information is passed on known as 'conditional elements'. Delays can be responsible for oscillations and overshoots of system trajectories. They can cause a balancing loop to act as a reinforcing loop and vice versa. Delays between system stocks create an extra level of dynamics within these systems. Delays are a prime reason why investigating structural behaviour is not a simple process.

### 2.1.8 System Dynamics and Policy

When the results of well-intentioned decisions do not produce the intended outcome, produce no results at all, or in fact have the reverse intended effect, it is known as policy resistance. A definition of *policy resistance* is as follows:

*"the tendency for interventions to be defeated by the response of a system to the intervention itself."* Sterman (2001). That is to say that you cannot alert one part of a system and expect a linear response to a desired outcome without affecting multiple components along the way.

Sterman (2001) provides several examples of policy resistance, a few of which include:

- Lowering the nicotine within cigarettes, intended to help people cut down on their addiction actually causing people to smoke more cigarettes in order to satisfy their craving.
- A U.S. policy to reduce forest fires leading to larger, more severe fires. As suppression of small frequent fires lead to a build-up of deadwood allowing fires to become hotter and more intense.

- The development of widely available and cheap antibiotics leading to the evolution of drug resistant pathogens.

The power inherent in system dynamics comes from its ability to connect policy to dynamic behavioural changes. It is able to show how low-leverage policies are doomed to fail and that the most strongly defended policies are often the ones causing the greatest issues. System dynamics allows for the identification of what Forrester (2007b) calls better-before-worse policies: policies which have short term benefits, but fail to solve long term issues; policies which are arguably an ingrained part of many established governing systems.

Many models, especially those concerned with modelling environmental systems are created for the purpose of forecasting by predicting future events and behaviour (i.e. Rodríguez et al. 2007; Ross et al. 2000). A common approach to forecasting is through curve fitting a model's output to past data (Motulsky and Christopoulos 2004). The closer the model curve fits to the empirical data, the more reliable the model is assumed to be at predicting a system's behaviour. Issues occur in model parameterisation and forecasting when a model is over fitted to a data set. Overfitting occurs when the model output is manipulated to match data to such a degree as to be detrimental to the practice of model construction. Issues include unnecessary parameterisation within the model and a loss of model generalisation as the model's purpose moves away from modelling behaviour trends and towards matching an individual scenario.

System dynamics should not be used for forecasting; at least, not in the form of curve fitting past events to infer future scenarios. Forrester (2007b) explains that the reasons forecasting of this nature fail are intrinsic to the nature of the systems in question. While curve fitting may impress a client, a model with enough parameters can be manipulated in such a way as to have a multitude of scenarios which match the data, often many of which demand unrealistic values from system variables. The type of forecasting that system dynamics is capable of involves the simulation of different system behaviour, based on policy decisions.

### 2.1.9 The role of System Dynamics to Ecological and Socio-Ecological Systems

Ecological and socio-ecological systems frequently change through time, known to express non-linear, emergent and self-organising behaviour (Young 2012). System dynamic modelling has developed into a powerful tool for helping to illustrate the complexity of such systems as they recreate behavioural trends through time.

The scientific approach has often taken to reductionism, breaking down complex problems into small, manageable and experimental components. This approach has proven sufficient time and time again. However, the development of complex systems science showed that systems must be considered in their entirety in order to understand the nature of system interactions when acting as a single interconnected body. Whereas reductionism attempts to break down a system into parts, system thinking looks at systems as a whole, allowing emergent behaviour to occur purely as a result of internal system properties.

*"Systems thinking keeps people in touch with the wholeness of our existence.It helps to keep in mind that human thought is not capable of knowing the whole, but it is capable of knowing that we don't know."* Flood (2010).

Dating back to the mid-20th century, system thinking was first applied in a socio-ecological systems context by The Tavistock Institute of Human Relations (Trist and Bamforth 1951) who were attempting to understand new social techniques within the coal mining industry leading to a more positive workforce environment. Trist and Bamforth (1951) found that it was not the attitude of the workers, nor the work itself that needed to be revised, but the structure of the system that could be manipulated in order to avoid bad practice and unrest in the mining community.

The complex network of connections and feedback mechanisms which are often expressed in SESs are too difficult to solve analytically, particularly as the behavioural dominance of the structures is as dynamic as the systems themselves. System dynamics has proven to be an effective way to visualise a system in its entirety, while still accounting for the unique properties of system variables. Many applications of system dynamic models to socio-ecological systems can be found in Ford's 'Modelling the Environment'(Ford 1999b and Ford 2010).

The system dynamics technique promotes scenario and strategy testing, otherwise known as 'what if' modelling. For example, increasing the fish stock of a lake does not just have to be done by the addition of more fish. It could be done by the restriction of fishing, the promotion of juvenile fish sanctuaries and the improvement of water quality to ensure that more fish survive to adulthood. This reflects on the multiple interactions which occur simultaneously to real world systems.

System dynamic models have been used as a decision support system (Chang et al. 2008) where they have helped to develop management strategies for coral reef systems. Examples where system dynamics has been used to explore ecological and socio-ecological issues include: Alternative Energy sources, Epidemics (Cholera and Mexican Flu), Population Collapse of Easter Island, Eutrophication of lakes, Fish stocks rise and decline, Food vs. Energy, Predator-Prey interactions, Populations dynamics (from societal ageing to muskrat plagues), Flooding and sickness through weather forecasting and most famously Urban Dynamics by J. Forrester (Forrester 1970). Replicas of many of these models can be found at http://Forio.com (Forio 2016) and there are many more that exist in the literature: Eutrophication of Lake Erhai (Guo et al.

2001), Waste Generation in Urban Regions (Dyson and Chang 2005), Environmental sustainability in agriculture (Saysel et al. 2002) and civilisation collapse among the Maya population (Hosler et al. 1977). An overarching report on the capabilities of System dynamics in relation to SES can also be found in ElSawah et al. (2012).

### 2.1.10 The role of Feedback Loops in System Dynamics

Dynamics within a system arise from two loop structures: positive (reinforcing) or negative (stabilizing). Behaviour of a system depends on what loops are incorporated, how many there are, how the loops are connected and which are dominating at a particular point in time. If positive feedback loops dominate then the system will tend to experience runaway behaviour either into exponential growth or exponential decline. If negative feedback loops dominate, the system will tend to oscillate around a stable position or head towards a state of equilibrium.

Positive loops amplify current system behaviour, while negative loops counteract or oppose change (Sterman 2001). Positive feedback loops may be referred to as reinforcing or run-away loops, and negative feedback loops can also be referred to as balancing or stabilizing loops. A reinforcing loop drives the system to alternative states, while balancing feedback always attempts to maintain a system at a constant state.

The impact with which a feedback loop acts on a system can be subtle or highly expressive. They come in the form of reinforcing and balancing mechanisms capable of generating multiple system behaviours, including exponential growth, decay, oscillation, overshoots and dampening. These behaviours have been shown to be at the source of system traps (Wolstenholme 2003, Meadows 2008) and can be the dominant features seen during system critical transitions and regime shifts (Scheffer 2009).

While feedback loops occur naturally within ecological and socio-ecological systems, they can arise as coincidental structures during model construction, having rarely been implemented on purpose. Despite their often unintended creation, feedback mechanisms become main drivers for emergent system behaviour (Ford 2010, Kampmann 2012). Feedback loops are capable of creating reinforcing and balancing behaviour within a system, but it is their interconnected nature which makes the behaviour they generate unpredictable. Phenomena including rising magnitude of fluctuations in a system, system shocks or a decrease in system resilience are often associated with a weakening of stabilizing feedback structures (Carpenter and Brock 2006, Scheffer 2009).

In system dynamic diagrams, feedback types are identified by the polarity of the links between stocks and variables. Symbols '+' and '-' are used to represent the flow within these links. A '+' symbol means that there is a positive relationship between two variables and whatever happens to one, will also happen to the other. An increase in value of one variable will cause an increase in value of the second. A decrease in value of one variable will cause a decrease in value of the second. Likewise, a '-' means that there is a negative relationship between two variables and whatever happens to one will create the opposite effect on the other. An increase in value of the first variable will cause a decrease in value of the second, while a decrease in the first will cause an increase in the second.

Often we attribute detrimental behaviour to faults of an individual or group, rather than the structure of the system which they are part of. This is so common that there is even a phrase for it in psychology, 'fundamental attribution error' (Ross 1977).

## 2.2 Ecological and Socio-Ecological Systems: An Overview

Ecological and Socio-ecological systems (SESs) encompass the dynamic interactions between human and environmental components of our planet. SESs are found at all planetary scales from small lakes, fishing ports and forests to oceans and the Earth's climate, all interconnecting to form a global scale living system. With the stability and resilience of our planet being vital to economic development, poverty alleviation and sustainable living (UN Millennium Goals 2016), it is within our best interests to understand as much about our planet's socio-ecological systems as possible.

Socio-ecological systems are complex, naturally requiring a multi-disciplinary approach to understand the social and environmental components and the multitude of interactions within and between them. The social aspect of socio-ecological systems causes uncertainty, particularly causing difficulty in predictive modelling as human behaviour can often be unpredictable and assumptions or generalisations of human activity can generate bias in a systems model. The extent to which human activity plays a role within socio-ecological systems is often uncertain (Walker et al. 2012) as these are effectively open systems; human influence can occur at a local (direct interaction), regional (within the same catchment) or global scale (i.e. through climate change impacts).

The study of resilience, vulnerability and stability has moved away from quantitative methods and towards a more descriptive and dynamic analysis of a system's interacting components (Young et al. 2006). The need for a more flexible approach was identified by Zadeh (1973), who described the relationships between system components using 'fuzzy conditional statements', showing that a purely analytical approach to system analysis was not sufficient to portray their complex behaviour.

In 2009, Ostrom developed a general framework to represent the vast range of socio-ecological systems, taking into account the users, governance, resources and interactions which collectively generate the outcomes we see from our ecosystems (Ostrom

2009). Today, the principles behind this framework are used to represent every socio-ecological system and help to justify the simulations behind each model scenario.

While many environmental studies model a natural system with human action as an external driver to a problem, social science models prioritise human behaviour and choice, with their environment considered as an external factor; socio-ecological systems view humans as a keystone species to any ecological system which has an integrated role within its environment. Human activity increases in dominance within ecosystems in both space and time (O'Neill and Kahn 2000) and should not be considered as separate entities from these systems.

### 2.2.1 Why Study Ecological and Socio-Ecological Systems?

Socio-ecological systems have developed from human integration into a biophysical world through their short term adaptability generating profound impacts on system resilience, robustness, vulnerability and adaptability (Young et al. 2006). Ecosystems are known to express unpredictable behaviour created by structural complexity, the unpredictable nature of the environment and ever changing impacts of human activity governed by local policy (Ostrom 2009).

The stochastic nature of ecosystems are exaggerated as systems are able to undergo regime shifts (the change of a system from one stable state to another), tipping points (sudden moments of state change) and express hysteresis (the ability for a system to be stable in more than one stable state under the same environmental conditions in the same space at a different time) (Scheffer 2009, Scheffer and Carpenter 2003). The driving forces behind these changes are often hidden beneath a complex network of social-environmental interconnections and feedback loops.

It can be argued that unpredictable behaviour only forms when there is a distinct lack of understanding in how these systems operate. However, despite a deterministic nature, ecosystems are far from predictable. Ecosystems by nature occur at all scales and interlink to form countless components of our living planet. By accepting the Earth as a global system, made up of countless regional systems with instabilities, limitations of resource and planetary boundaries (Rockström et al. 2009, Steffen et al. 2015), we can gain a greater understanding for ecological stability.

While much is understood about our planet's ecosystems, there is a need to find and develop more ways to communicate the complex behaviour of these systems in order to make the risk and uncertainty associated with the change and collapse of them more transparent for both the public and policy makers alike (Dearing et al. 2012). Some tools fail to capture the relevant complexities of the real world, falling short on either an observational, modelling or communication level. Modelling approaches which fail to acknowledge system feedbacks, are unable to facilitate system tipping points, or link ecosystem services to human wellbeing are likely to result in dangerous policy recommendations (Nicholson et al. 2009, Dearing et al. 2012).

While the scientific method often entails breaking a system down into smaller, more observable components, there is a risk of losing much of the complexity of a system which stems from the interactions between components. Scientists can also be under pressure to use their models for predictive simulation, mistaking system models for sources of fact (Dearing et al. 2012). Models of socio-ecological systems are difficult to use for prediction without solid evidence of what impact social choices will generate on the system and what drivers will be dominating the ecological processes. Instead, system models of socio-ecological systems are best used to explore the potential capabilities of a system under a set of given conditions, using high computer power to run multiple scenarios in a short timescale, on which the best course of action to take can be inferred and informed political decisions can be based.

While scenario testing provides a systematic, logical means to view potential futures of a system, it has one major flaw; systems run under the same scenario are subject to the same base set of rules, and dynamics. Scenario testing rarely accounts for emergent properties in the system which arise from complex system interactions generating unpredictable dynamics (Dearing et al. 2012). The inability to account for emergent system properties produces a knowledge gap between the capabilities of scenario testing and the needs of effective policy making. To account for this, users should first and foremost stay open-minded and upfront about the capabilities of their models. Understanding the capabilities or lack thereof within a model can often be attained by undergoing multiple iterations of model building/adaption, testing, analysis and scrutiny.

### 2.2.2 Dynamic Behaviors of Ecosystems

Dynamic behaviours which occur in ecological and socio-ecological systems include: non-linearity, emergence (the ability for a behaviour, pattern, or regularity to arise within a system as a consequence of component interactions, despite none of the components being able to display this behaviour themselves and with no external input), scale (not only local to global, but also through multiple layers of interconnected systems) and self-organisation (the tendency to fall back to stable states, but not necessarily desirable ones), see basins of attraction, Scheffer (2009).

Complex, nonlinear behaviour can arise endogenously within systems (Güneralp 2006) i.e. arising purely as a consequence of internal dynamic properties without pressure from external drivers. These behaviours occur when social demands meet ecological limits (Scheffer 2009). Stable states and behavioural trajectories can appear stochastic in nature (Meadows 2008) and therefore appear difficult to manage. Examples include lake eutrophication (Vezjak et al. 1998, van den Berg et al. 1998,Carpenter 2005), coral reef bleaching (Hoegh-Guldberg et al. 2007, Nyström et al. 2012), desert formation (Zhu et al. 2015; Ackland et al. 2003), and arctic ice retreat (Winton 2006, Serreze 2011): all of which have been observed either undergoing sudden regime shifts or driven into alternative states.

Dynamics within real world systems often favour the social sphere over the environmental sphere due to human needs. Oftentimes this can be in the form of economic growth and poverty alleviation, but at the expense of a loss of ecological resilience (i.e. Raudsepp-Hearne et al. 2010). This resilience loss can often lead to system collapse or the system undergoing a tipping point to a less desirable regime whereby the human wellbeing is improved short term, but suffers on longer timescales which is usually difficult to recover from due to the hysteresis effects of the natural system.

A series of evidence can be used to investigate systems expected to have undergone or becoming close to experiencing a tipping point (Lenton 2011). Four lines of evidence include: long term trends in ecosystem service degradation; changes in connections between system elements; increased system sensitivity to drivers and policy that maintains long term relationships between drivers and responses. All four of these lines of evidence can be linked to feedback loops. 1) Long term degradation can be caused by dominating positive feedbacks which continually reinforce detrimental behaviours. 2) connection changes can result in the loss or creation of new feedback loops, the relationship of which (no. of links to no. of feedback loops) is far from linear (Kampmann 2012). 3) increased system sensitivity can be caused by feedback loops switching dominance within the system where the sensitive variable is now more prevalent and 4) policy maintaining long term relationships implies a direct link of policy application to endogenous connections within a system.

Current practices for the identification of tipping points include identifying a signal of instability leading up to the transition. This signal is known as an early warning signal to the tipping event and is based on critical slowing down, whereby the ability for a system to recover from a disturbance slows down as it nears a tipping point or flickering theories, which are based on increased variance occurring at the tipping point (Dakos et al. 2012; Dakos et al. 2010; Scheffer et al. 2009). Policy must be based on social and ecological interactions which are inherent to the evolutionary policy making put forward by Ostrom (2009).

As the scale and patterns of human activities change, the structure and dynamics of the system are affected (Zurlini et al. 2006). SES behaviour is complex and can be easier to conceptualise than what can be understood from empirical data (Young et al. 2006). The complexity of SES arises in part due to human behaviour being reflexive; it is able to observe the system in which it resides and adapt its behaviour. The self-awareness; ability to learn; individual response to risk; ability to communicate and ability to influence over long distances are all aspects of how the social side of SES make system modelling and predictability a challenging process. Human action, however, is not completely unpredictable as it is often constrained by ideologies; large political economies, assigning values to natural resources and development of institutions whose sole task is to address persistent or novel problems (Redman et al. 2004).

Complexity from ecological systems stems from stochastic variation, adaptive change on long term and short term cycles, and a highly integrated network of internal and external interactions and knock on effects.

Change within SES should not be assumed continuous or chaotic, but occur as a consequence of the interactions between fast and slow acting system variables (Redman et al. 2004).

The ecological side of the systems are often measured through primary production, populations representing trophic structures, organic matter accumulation, movement of inorganic inputs and frequency and patterns of site disturbances. The social side is often determined by demography of the human population, technological advances, economic growth, political agendas, culture, existing knowledge and information exchange (Redman et al. 2004).

SESs are used to explain concepts of both macro and micro scale interactions within the social community such as governance over a region (macro) and individual behaviour (micro) (Binder et al. 2013), with a continual influence of macro processes governing the micro decisions and micro processes changing the macro structure. SESs then take this macro-micro approach and conceptualise them within the context of social - environmental interactions. This SES approach considers both how anthropogenic activity can drive behaviour of ecological systems, but also how ecological responses can reflect back on human lifestyle and wellbeing, ultimately focusing on the feedback loops which are inherent to SES system structures.

How to approach the study of a socio-ecological system and which framework to use can be determined by the relationship that exists between the social and environmental components and whether the internal processes are uni or bidirectional (Binder et al. 2013). A framework in this context is defined as a domain specific language based on a set of assumptions, concepts, values and practices whose purpose is to facilitate discussion within that domain in a clear and unambiguous manner. Research approaches are determined by the motivations for the research and whether they stem from an anthropogenic or economic perspective of the ecosystem with the end goal being either action or analysis orientated (Binder et al. 2013).

Frameworks which are analysis-orientated seek to develop research questions from a platform of greater understanding for the system, while action-orientated frameworks are designed with the end-goal of manipulating the real-world SES.

The modelling techniques, analysis and theory pursued within this study are applicable to many of these frameworks with the intention to help facilitate the clear and unambiguous discussion which these frameworks are designed for. The frameworks with which this study can be relevant to include Driver, Pressure, State, Impact, Response (DPSIR), Ecosystem Services (ES), Human Environmental Systems (HES), Material and Energy Flow Analysis (MEFA) and Social-Ecological Systems Framework (SESF) (Binder et al. 2013 and references therein). Of these frameworks, the socio-ecological systems and their associated models within this study are best suited to the SESF as described within Ostrom (2007) and Ostrom (2009) which organises system variables by relevance to a problem or question, able to incorporate interactions on both a macro and micro scale and bilaterally across the social and environmental spheres, with a primary focus on systems at a local or regional level.

Applying the motivation behind these frameworks to the structural based modelling technique, system dynamics, we see that both an analysis-orientated and action-orientated framework can be developed. The action of building a system's structure, studying the system's interactions and implementing the stocks and flows of a system, even with limited empirical data can help to develop a greater understanding of a SES and therefore be a useful step in analysis-orientated research. Likewise these structural techniques can be a crucial step in an action-orientated framework, as they can help in the policy management and planning of an SES system through the process of model building, data collection and scenario testing. Analysis techniques, used to further the information gained from the output of the models, such as structural loop analysis, sensitivity analysis and graph theory, can help to improve action-orientated frameworks by analysing the drivers of system behaviour and through the identification of leverage points (Ostrom 2009) which can be used to manipulate the real world system.

The role of feedback loops play across all scales of SES, from local to regional to global systems. Rockström et al. (2009) discuss that much of the uncertainty associated with planetary boundaries exists due to our lack of knowledge regarding the feedback mechanisms that play a role in the biophysical processes and control variables of the Earth processes. The understanding and knowledge gained from the study of feedback mechanisms is not limited to a subset of SES, with much to be gained from studying feedback loops across all levels of the Earth system. Feedback loops exist regardless of whether we acknowledge them or not and are inherent in almost every SES.

While the planetary boundaries act on a global scale, human action, social norms and policy does not and while some planetary boundaries may be in a safe space globally, i.e. ocean acidification Rockström et al. (2009), this may not be the case regionally. A path to start addressing our planetary boundaries may be from a bottom up approach, influencing systems at a regional scale by learning about their feedback loops and drivers in order to generate change at scales which society naturally operates.

### 2.2.3 Current practices for investigating ecological and socio-ecological systems

Ecosystem modelling benefits greatly from interdisciplinary research as the field naturally demands scientists from ecological, biophysical and social science backgrounds, while many of the techniques used to analyse the field stem from mathematic, economics and network science. Using a variety of tools and data based techniques including Geographic Informations Systems (GIS), remotely sensed data and concentrating research on specific regions, is believed to bring researchers together and promote interdisciplinary research as reviewed in Redman et al. (2004). Modelling techniques and graphical approaches aid in interdisciplinary research when the subject area requires understanding of the system on multiple scales and the real-world counterpart cannot be subject to experimentation without consequences. Zurlini et al. (2006) investigate socio-ecological system disturbance patterns through pixel mapping. This approach is another practice which addresses the complexity which human action creates on ecosystems across multiple scales, both temporally and spatially.

Young et al. (2006) were concerned with the effect that globalism has on properties of socio-ecological systems: resilience, vulnerability and adaptability. These three properties were analysed for different effects which globalism may have on a system including increased connectedness, speed, spatial stretching and declining diversity. Among their findings was that positive feedbacks played a large role in intensifying the effects of globalisation.

The language which we use to describe a system is important for our understanding of exactly what state the system is in and where its strengths and weaknesses reside. Young et al. (2006) offers a collection of key terms and definitions, including: adaption, adaptedness, adaptability, robustness, resilience and vulnerability. Many of these terms refer to the structural properties or dynamics of an ecosystem and are therefore connected to properties of feedback structures of the system. It is therefore deemed important to clarify their meaning within this study should they be relevant to the topic of feedback mechanics 2.1. The terms can be seen listed with their definition as highlighted from Young et al. (2006) and any references where these terms were defined.

Within the field of SES, and when conducting interdisciplinary research, often involving a varying range of knowledge and skill base, there are several factors which are implicit to SES which must be recognised. The following list is adapted from Redman et al. (2004) in their discussion on multiple scale approaches: 1) Scale and resolution of the target system will differ determined by the motivations of the research. 2) Timelags, non-linearity, and multiple responses to a perturbation across both the ecological and social spheres are inherent in SESs. 3) Boundary and threshold conditions play an important role in the behaviour of SESs. 4) SESs are subject to large

| **Key−term** | **Definition** | **References** |
|---|---|---|
| *Adaption* | "The process of structural change in response to external circumstances." | White (1949) and Steward (1972) |
| *Adaptedness* | "The extent to which a particular dynamic structure is effective in dealing with its environment." | White (1949) and Steward (1972) |
| *Adaptability* | "The capacity to adapt to future changes in the environment of the system concerned." | White (1949) and Steward (1972) |
| *Robustness* | "Structural and other properties of a system that allow it to withstand the influence of disturbances without changing structure or dynamics." | Anderies et al. (2004) |
| *Resilience* | "The capacity of a system to absorb and utilize or even benefit from perturbations and changes that attain it, and so persist without a qualitative change in the system's structure." | Holling (1973) |
| *Vulnerability* | "Situations in which neither robustness nor resilience enables a system to survive without structural changes. The system either has to adapt structurally or it is driven to extinction." | Young et al. (2006) |

Table 2.1: Terms associated with ecosystem dynamics and clarification of their meaning when used within the context of structural analysis.

and small scale processes, driven by fast and slow drivers, all of which can make large impacts on a system's stable state. Both large and small scale data sets can be used to identify varying scales of behavioural phenomena, but dynamical properties of systems can be difficult to determine from data alone without further analysis.

### 2.2.4 Modelling of Ecological systems

Our ability to design accurate models of ecological systems can run into difficulties by trying to represent an open real world system within a closed system model. In the construction of an ecosystem model, there is a risk of continually integrating new components and interactions in an attempt to incorporate every factor capable of influencing and driving the system.

To overcome this, modelling construction must be a well-designed and iterative process, with a clear goal of the dynamics which are to be investigated within the model. Models must be designed with a clear focus shared across every individual within the modelling team in order to reduce confusion and unnecessary model complexity.

Many system models simply start as an idea, or a concept which translates to a mental model of the system. Every individual's mental model of the same system will be different (Carpenter 2005) determined by their background knowledge and experiences, which is why it is so important to have a focal point within a modelling team.

In system models, the dynamics are determined by the state variables (the stocks) and the interactions which occur between these variables (Walker et al. 2012). In socio-ecological systems this equates to the endogenous interactions between variables, whose scope of potential dynamic behaviour is increased by the interactions joining together to form causal chains or feedback loops of reinforcing or stabilising behaviour.

Deciding which variables are outside the scope of a model system can also become a challenge. Walker et al. (2012) describe a scenario within an agricultural model where water availability may be considered as an external factor supplied by a local authority, or an internal variable determined by the needs of the farmers. In dynamical terms, this could mean the difference between inputting water availability as a constant value which acts as a linear input, or as an auxiliary variable, integrated into a feedback loop of agricultural output and individual decision making and needs. The difference between these two model scenarios could mean a massive difference to the output of the model and perceived success of the system.

A main distinction to make between external and internal system variables is determined by their connection to feedback structures. Feedback loops play a heavy role in the behaviour of a system and there are no feedbacks to external drivers in a model (Walker et al. (2012)).

Constructing ecological systems as dynamic models is not an easy task. Models of ecosystems are capable of taking multiple structures because they are not closed systems. There is never an overriding model structure which will represent a system because multiple structures could reflect the properties of the system given ample evidence (Cariboni et al. 2007; Konikow and Bredehoeft 1992; Oreskes et al. 1994; Aronica et al. 1998). The structure of an ecosystem model is also determined by the knowledge of the user, the goal of the model and the resources which the user pulls to construct the model.

Sometimes during model construction the parameters deemed to be necessary in order to accurately reflect the real world counterpart can get out of hand where hundreds of variables are introduced to the model, many of which make little or no difference to the model dynamics or output. Bigger models are not always better models; increasing the amount of information input into a model does not make it more relevant to the task the model is built to serve. The more variables within a model, the more degrees of freedom that model is capable of having and therefore any desired behaviour can be extracted from the model given enough manipulation and this can often be backed up with plausible evidence and parameter values which sit within the range of what would occur in the natural system (Hornberger and Spear 1981).

## 2.3 Analytical Techniques

As the role of system dynamic models has developed over the last 50 years, so too have analytical techniques attempting to determine the root cause of a system's output. Each technique has been developed assuming that the study of system structure, can help us to understand system behaviour. The integrated nature of feedback loops, causal links and causal chains create the behaviours that we see in model output.

Within this thesis, the model systems being examined take the form of system dynamic models concerning the stock and flow diagrams. Specifically we are interested in analyses which can increase our understanding of dynamic behaviour in ecological system models and have the potential to benefit current questions and concerns within the fields of ecology and socio-ecology. With that in mind, it is important that the analysis is able to have utility and application to a wide range of system behaviours, with a methodology that is centred around the numerous interactions which occur within and between society and the environment.

The following section reviews seven analytical techniques, which have been selected for their ability to analyse model structure and identify them as drivers of model behaviour. Each technique is capable of being used, but is not limited to the structural analysis of system dynamic models. Each analysis is reviewed for the theory which it is based upon and the technique which it uses to identify influential structures within a system. Each technique is then summarised listing its advantages and disadvantages, focusing on: applicability (The ability to be used across a wide range of systems and dynamic behaviours); novelty (The ability to provide novel information about system behaviour that cannot be gained through observation of model output or during model construction) and utility (The extent to which the analysis can benefit a study and effort required from the user).

Through the comparison of each technique, it is possible to identify the analysis best suited and most likely to benefit the study of an SES dynamic system model. The section concludes with general comments by authors who have stressed the importance of being able to correctly identify dominant structures of system behaviour. A short example is provided of how misunderstanding a system's main drivers can lead to ineffective policy and escalate detrimental system behaviour.

Techniques used to investigate the role of a dynamic system's structure include:

1. Traditional Control theory (Graham 1977)

2. Causal Loop Diagrams and System Archetypes (Wolstenholme and Coyle 1983)
3. Ford's Behavioural Approach (Ford 1999a)
4. Pathway Participation Metrics (PPM) (Mojtahedzadeh 1997)
5. Eigenvalue Elasticity Analysis (EEA) (Forrester 1982)
6. Loop Eigenvalue Elasticity Analysis (LEEA) (Forrester 1983)
7. Eigenvector Approach (EVA) & Dynamic Decomposition Weights Analysis (DDWA) (Saleh et al. 2010, Saleh et al. 2010)

Each technique has been successful to an extent, but none have yet become common practice as part of the system dynamics method. This is, in part, due to the limitations of each technique preventing them from being applicable to all dynamic systems models.

Güneralp (2006) notes that the established practice for system dynamic simulation does not involve structure-behaviour analysis: build model, verify model and run multiple simulations under different scenarios. Not only would structure-behaviour analysis aid output interpretation, it would also be well suited for scenario testing and in policy analysis by identifying key areas to concentrate research and decision making (Kampmann and Oliva 2008).

This review will address each technique, highlighting advantages and disadvantages of each with an extended section on the development of Loop Eigenvalue Elasticity Analysis. This review acts as a descriptive base for each method, but does not intricately analyse the mathematics behind each approach. Other breakdowns of these methods and more analytical descriptions of each method can be found in Kampmann and Oliva (2008), Güneralp (2006) and Richardson (1986).

### 2.3.1 Traditional Control Theory

The first method worked on by Graham (1977), examines the amplification, dampening and delays imposed on a disturbance as it travels around a loop set. Graham suggested calculating the gain of a loop. A gain value greater than 1 infers exponential growth, while a gain of less than 1 infers exponential decay or dampening, as explained in Kampmann and Oliva (2008). This method demonstrates how behaviour can emerge from within the system, without any external drivers.

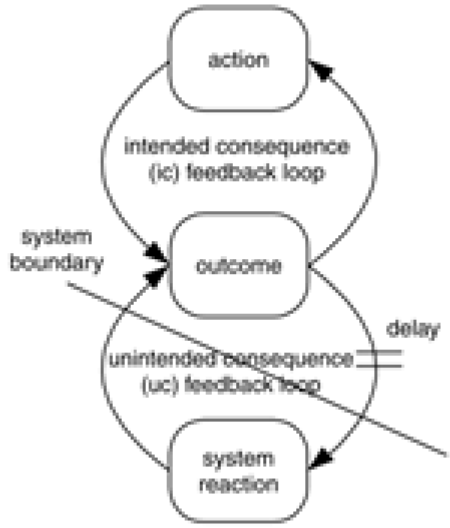



FIGURE 2.2: Extracted from Wolstenholme (2003) displaying the basic form of a system archetype.

### 2.3.2 Causal Loop Diagrams and System Archetypes

Causal Loop Diagrans (CLDs) are visual representations of a system's components and all of its interections in order to portray an overview of its structure. Following early proposals to use system dynamics to study systematic behaviour through time (Meadows 1982; Forrester 1994), system archetypes were developed attempting to isolate system structures responsible for generating specific behaviours (Wolstenholme and Coyle 1983; Senge 2006; Wolstenholme 1990; Kim 1992). Despite coming under much scrutiny in their applicability to real world systems (Richardson 1986), Causal Loop Diagrams (CLDs) are still used as simplistic teaching methods and explanations to the role of feedback loops.

System Archetypes consist of small sets of positive and negative feedback loops which, when positioned into particular structures, have consistent behavioural patterns. They were created to make the systems dynamic approach more accessible to a wide audience.

In 2003, Wolstenholme imposed a set of four system archetypes, which when modified could explain any system behaviour (Wolstenholme 2003). The diagrams consisted of an intended consequence feedback loop, an unintended consequence feedback loop, a delay before the unintended consequence manifests, and an organisational boundary, which hides the unintended consequence from view. This method proved useful for explaining small system behaviours and system traps, but the diagrams never extended past a three stock system. The behaviours are usually detrimental to at least one member, if not all of the system involved i.e. tragedy of the commons and shifting the burden (see Meadows 2008). The general form can be seen in figure 2.2.

Generally causal loop diagrams are used for visual and structural data only (i.e. connectedness or no. of feedbacks) while the shortfall of system archetypes is that the small diagrams cannot guarantee to produce similar behaviour when built as part of much larger systems. This happens because their connections to other loop systems create interference with the associated behaviour.

### 2.3.3 Ford's Behavioural Approach

Ford's Behavioural Approach (Ford 1999b) is a systematic method which involves the removal of individual loop structures at key stages of a system's behaviour i.e. around a behavioural change from exponential growth to decay. The more dramatic the change of behaviour after the removal of a loop, the greater impact that loop is assumed to hold at that point in time.

Ford's loop deactivation method is tested by Keijser et al. (2012), concluding that it requires further development on medium to large size models as the method quickly becomes exponentially inefficient with model size.

### 2.3.4 Pathway Participation Metrics (PPM)

Pathway Participation Metrics was developed in 1996 by Mojtahedzadeh (Mojtahedzadeh 1997) and extended by Mojtahedzadeh in 2004 (Mojtahedzadeh et al. 2004). The method is based on the works of Richardson (1984), identifying the significance of loop polarity and loop dominance within dynamic systems.

Loop polarity works on the premise that a positive feedback loop represents a positive polarity and a negative feedback loop, a negative polarity. When multiple loops interconnect, the system polarity is chosen by whichever loop holds dominance. As a system changes through time, loop systems can undergo a change in dominance and therefore, are capable of switching polarity.

Pathway Participation Metrics can be used to express whether a system's behaviour is dominated by positive or negative loops. PPM identifies dominant causal chains or pathways within a system's structure until it encounters a pathway which has already been counted, in which case, it identifies a loop.

Unlike previous methods, PPM is able to follow a single variable through time, rather than operating in the frequency domain. It also only identifies a single path of dominance at any one time, therefore the PPM method does not account for model-wide dynamics (Güneralp 2006).

### 2.3.5 Eigenvalue Elasticity Analysis (EEA)

Eigenvalue Elasticity Analysis was first developed by Forrester (1982) for uses in economic stabilisation policy. It is based on the idea that at any given time along a time series, a system's behaviour can be described through eigenvalues and eigenvectors, once the model has been linearised. It has been developed from tools of modern linear systems theory. EEA represents system models by converting them into matrix form in a similar way to methods used in graph theory.

Eigenvalues represent behavioural modes (possible states of the system at a given time), reflecting any changes which occur to the system. Perturbations of variables which occur within the system's structure which generate large changes in an eigenvalue have a high influence on the current behaviour. Perturbations which produce small changes reflect a small change in eigenvalues and have little to no influence on the current behaviour.

Güneralp (2006) describes how EEA can be used to reduce complex models to simpler versions by eliminating structures which, while introduced during model construction, make little or no contribution to behavioural output.

The elasticity of an eigenvalue $\lambda$ is measured with respect to a gain parameter $g$, which represents the strength of a connection, or link between two variables, in the form:

$$\varepsilon = \frac{\partial \lambda}{\partial g} . \frac{g}{\lambda} \tag{2.1}$$

where $g$ = the gain of each link and  = the eigenvalues. The partial derivative $\frac{\partial \lambda}{\partial g}$ can be calculated algebraically using Mathematica as generally it handles the partial derivative much better than Matlab.

This expresses the fractional change in the eigenvalue relative to the fractional change of the link (Kampmann and Oliva 2008). Alternatively the influence of the parameter can be measured with a dimension of 1/time and is therefore dependent on the time scale units:

$$\mu = \frac{\partial \lambda}{\partial g} . g \tag{2.2}$$

Measuring the influence, rather than the elasticity is considered easier to interpret and does not run into difficulties at low values of eigenvalue.

The Eigenvalue Elasticity method has been developed into multiple forms of use (EEA-/LEEA/EVA). This means that the method can be determined by the projects requirements and each method can overcome some of the downfalls posed by the others.

### 2.3.6 Loop Eigenvalue Elasticity Analysis (LEEA)

Loop Eigenvalue Elasticity Analysis (LEEA) is a structural analysis technique which identifies the influence of independent loop structures with respect to a system's behaviour at any point in time. LEEA was developed by Kampmann (2012), who extended the work of EEA to monitor the influence of entire feedback structures. Using the elements of a matrix $J$ to store information, the strength, or 'gain' of any feedback loop in the model can be calculated as the sum of all link gain that connected together to form that loop. A methodical breakdown of implementing LEEA can be found in Güneralp (2006). Similar to EEA, interpretation of LEEA's results must be undertaken with care as eigenvalues do not relate to the observed behaviour of individual variables.

A full explanation of the limitations of the LEEA technique, along with many solutions to these limitations have been addressed by Güneralp (2006).

### 2.3.7 Eigenvector Approach (EVA) & Dynamic Decomposition Weights Analysis (DDWA)

The eigenvector approach is an extension of the EEA/LEEA method and is the most recent approach to be developed. DDWA improves on LEEA by considering how much a change within an eigenvalue is weighted to an individual variable of the system.

As shown in Figure 2.3, extracted from Kampmann and Oliva (2008), it is the combination of eigenvalues and eigenvectors, which express the weighting of a behavioural mode with respect to state variables.

In this combined method, eigenvalues show the behavioural modes and eigenvector analysis provides the weights.

DDWA's method involves a state element (a stock of the system) being assigned weights coinciding with the extent to which it is affected by each eigenvalue. The sum of the weights produces a weighted measure of that element's overall significance (Güneralp 2006) as shown in Equation 2.3:

$$\ddot{x}(t_0) = \omega_1\lambda_1 + \cdots + \omega_n\lambda_n \tag{2.3}$$

Where $\ddot{x}(t_0)$ is the element's significance, $\omega_n$ is the weight of the behavioural mode and $\lambda_n$ is the behaviour mode. Güneralp (2006) extended this method by normalising elasticity values of system elements to vary between -1 and +1. However, whether this normalisation adds anything to the clarity of the method is questionable (Kampmann and Oliva 2008).

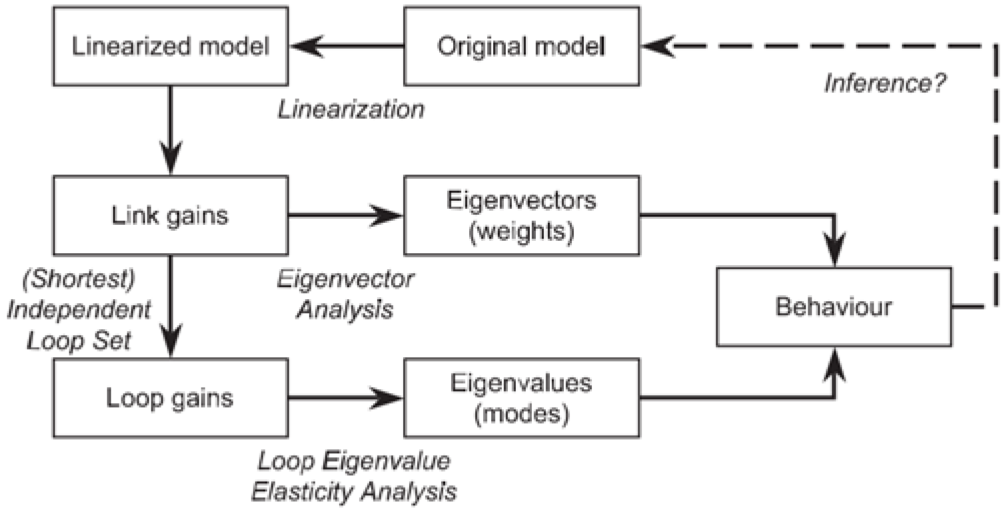



FIGURE 2.3: Extracted from Kampmann and Oliva (2008) showing a visual representation of procedures and order of operations to carry out LEEA and DDWA

Saleh et al. (2010) use this method on two business cycle models to highlight that the change in a parameter will affect, not only a behavioural mode, but simultaneously its influence on a system variable.

In 2009, Gonalves demonstrated that eigenvalues can be used to highlight long-term behavioural trends while eigenvectors highlight short-term behaviours (Gonçalves 2009).

| **Analytical Techniques of System Dynamics** | **Advantages** | **Disadvantages** |
|---|---|---|
| **2.3.1 Traditional Control Theory** | • Simple to carry out.<br>• Easy to teach beginners of systems theory. | • Applicable only to simple systems with very few variables. |
| **2.3.2 Causal Loop Diagrams and System Archetypes** | • Essential system components and interactions can be quickly represented.<br>• Simple, often used for concept building.<br>• Easily applicable to teaching and explanation | • CLD obscures the ′stock and flow′ structure that is common practice in system dynamics (Richardson 1986).<br>• System archetype diagrams cannot guarantee to produce similar behaviour when integrated as part of larger systems.<br>• The role of accumulation in stocks is lost as feedback structures are focused on.<br>• Makes no distinction between information links and rate-to-level links, creating false characterisation of polarities (Richardson (1986)).<br>• The position of organisational boundaries and delays in much larger systems are harder to determine. |

| | | |
|---|---|---|
| **2.3.3 Ford's Behavioural Approach** | • Easy method to carry out<br>• Very accessible - easy to comprehend. | • Time to carry out the method increases exponentially with the size of the model.<br>• Method only considers one loop structure at a time, and does not account for a behaviour potentially being the result of a combined output.<br>• Assumes a change in structure is always accompanied by a change in behaviour.<br>• Assumes that individual loops can be removed without disturbance of other structures. |
| **2.3.4 Pathway Participation Metrics (PPM)** | • Computationally simple.<br>• Numerically simple - Does not make use of eigenvalues.<br>• Makes direct connections between observed behaviour and structure.<br>• Able to identify how dominant structures change over time.<br>• Has been implemented into software package Digest. | • Method is not suitable for oscillatory systems<br>• Only identifies a single feedback loop to account for the behaviour at any one time.<br>• The method does not account for structures where behaviour may arise from more than one dominant pathway at a time.<br>• Only considers partial system structures, rather than global.<br>• Software programme still has limited use. |

| | | |
|---|---|---|
| **2.3.5 Eigenvalue Elasticity Analysis (EEA)** | • Analytically the strongest method.<br>• Elasticity values are dimensionless, the units of the system components are irrelevant. The method can theoretically be applied to all types of system dynamic models.<br>• Once the system is converted to the correct analysable form, the method is simply a matter of running multiple functions.<br>• Due to a matter of just running pre-built functions, the method is reasonably easy to use. | • The behavioural mode from a single eigenvalue does not represent the behaviour of a single variable, but of the entire system.<br>• Computationally demanding: at each time step the system must be linearised, the Jacobian Matrix must be generated and eigenvalues must be calculated.<br>• Analysis of results gets increasingly difficult with increasing model size.<br>• Interpretation of results is difficult requiring a refined knowledge of the technique and practice.<br>• Requires design of an implementation framework to consistently carry out the method. |

| | | |
|---|---|---|
| **2.3.6 Loop Eigenvalue Elasticity Analysis (LEEA)** | • Visualisation of loop influence over time produced as a (semi-)continuous graph.<br>• Encompasses the entire feedback structure at once and accounts for all system eigenvalues.<br>• Shortest Independent Loop Set (SILS) presents optimistic grounds for running LEEA on large system models.<br>• Same advantages as EEA (Section 2.3.5) | • Use of abstract mathematical terminology (eigenvalues & eigenvectors).<br>• The inability to identify structural influence on individual state variables.<br>• Eigenvalues can express similar levels of dominance simultaneously, reducing one's ability to specify an individual behavioural driver.<br>• The ability to interpret results becomes difficult as a greater number of variables are added to the model. |
| **2.3.7 Eigenvector Approach (EVA) & Dynamic Decomposition Weights Analysis (DDWA)** | • When combined with LEEA, DDWA is capable of relating the influence of loop structures to individual variables.<br>• Majority of the work is achieved through functions. | • Eigenvectors cannot provide a standalone method for loop dominance as they are not calculated as a function of loop gains.<br>• General EVA method encounters issues with oscillations, similar to PPM (section 2.3.4).<br>• Requires the system to be linearized at each time step. |

### 2.3.8 An Analysis Best Suited for Structural Analysis of Dynamic Systems

Each technique reviewed within this section has the capability to increase the knowledge and understanding gained from a system's model. However, the extent to which novel information can be gained and the potential of the application can vary heavily between techniques depending on the properties of the model system and what the project requires.

From the seven analysis techniques listed above, Causal Loop Diagrams (CLD), Loop Eigenvalue Elasticity Analysis (LEEA) and Dynamic Decomposition Weights Analysis (DDWA) have been selected as three techniques to pursue and assess for the advancement of SES models. The most recently developed technique is a combination of Loop Eigenvalue Elasticity Analysis (LEEA) and the Eigenvector Approach (EVA) (Kampmann and Oliva 2006, Kampmann and Oliva 2008). Collectively these techniques focus on feedback loops as endogenous drivers of system behaviour and allow the user to investigate the influence of loop structures on individual state variables (stocks) as they change through time.

Among all of the structural analysis techniques reviewed, LEEA removes the 'trial and error' component to identify influencial loops which are used by many of the other techniques. This means LEEA is efficient, especially when the system structure changes between design iterations or between simulations.

With the primary focus on LEEA, this work will conduct a meta-analysis of structural loop analysis across all stages of model design, implementation, output and application to policy. LEEA will be explored in the context of model construction alongside causal loop diagrams (CLDs) and in the context of policy design and implementation alongside DDWA.

### 2.3.9 Comments & Reflections

Kampmann and Oliva (2008) state that for policy analysis, both the eigenvalue and eigenvector approach will need to be used. Kampmann and Oliva (2008) states that we will "*always have to rely on simulation to discover the dynamics implied by a structure.*" However, we can aim for the identification of dominant structures within models to focus our efforts, such as key feedback loops and links.

Saleh et al. (2010) describes the danger of falsely allocating behaviours to the wrong feedback mechanisms. Intervention of a system after the misinterpretation of a feedback mechanism could create no impact at all, wasting valuable time and resources, or worse, drive a system to collapse at a faster rate.

The following example illustrates the dangers of misunderstanding the mechanisms at play within a system:

In the Yunnan Province, China, regional agriculture was increased to support the developing population and migration into the area requiring a greater amount of food. To increase output, farmers occupied more land for agriculture and were allowed to use more fertilizer. Misunderstanding the feedback mechanisms at play within the nearby lake, the excess nutrients, applied over a series of decades, caused the local lake to turn eutrophic, crippling the fishing and tourist industry of the region and ultimately leading to mass human migration away from the area. The critical transition which occurred was investigated by Wang et al. (2012).

LEEA, and other structural analysis techniques have only been used on small ecological models to act as examples of the technique's progression or to investigate small agricultural practices (Bueno 2012, Bueno 2013). Despite being shown as highly applicable to socio-ecological systems, structural analysis techniques have yet to be used extensively or see common practice within the field. Many behaviours known to act in SESs (i.e. tipping points, overshoots, oscillations, flickering, and critical slowing down, as shown in Scheffer (2009) & Wang et al. (2012)) have not been explored using structural analysis, despite acknowledging the multitude of feedback mechanisms at play.

In 2012, N. P. Bueno contributed to bridging this gap, aiming to find a common practice of measuring degrees of resilience through system dynamics and dominant loop polarity investigation. Bueno (2012) states that if the behaviour of a low resilience system is dominated by a feedback loop, then small changes in the strength of that loop can lead to large changes in the system's behaviour and therefore stability.

### 2.3.10 Personal communications with Prof. C. E. Kampmann (2015)

Professor. C. E Kampmann is one of the early developers and testers of Loop Eigenvalue Elasticity Analysis along with its multiple counterparts, EEA & DDWA. In the early stages of applying LEEA to ecological system models, a conversation was held with Professor Kampmann to discuss the technique's capabilities and whether there was any merit to applying LEEA to other worldly systems, other than the industrial and business models which it had been applied to previously. This section highlights extracts from that conversation and the initial afterthoughts. The content within this section is based on the opinions and experiences of Prof. Kampmann. This section has been separated from the main content of the literature review as the points made have not been established from peer review material, despite this, the experience of Prof. Kampmann is considered a valuable asset when determining the applicability of LEEA to the socio-ecological field.

*Discussion point 1: Concerning dynamic behaviours*

In Kampmann and Oliva (2008), three system models of varying sizes and dynamics were analysed using LEEA in order to access the application of structural loop analysis to varying business models. To lead on from the information gained from that study, one of the initial questions posed during personal communications with Prof. Kampmann was what models and system dynamics he found LEEA analysis most applicable to in terms of information gained and understanding of the system. Prof. Kampmann's response has been summarised:

*"... where the analysis really shines is where the system has some kind of oscillatory mode going on, or systems where there is a single transient."*

Prof. Kampmann then gave Limits to Growth by Meadows et al. (1972) as an example of a system which is capable of experiencing an overshoot, a collapse or a transition between states. This part of the conversation gave great promise to the application of LEEA to socio-ecological systems. Without prompt or mention of socio-ecological systems, Prof. Kampmann mentioned multiple dynamic behaviours that elicit large topics of discussion within the SES community. Many systems of SES are speculated for their ability to experience multiple stable states, with overshoots and collapses being common dynamics in population growth and ecosystem services.

With this in mind, the prospects of using LEEA in conjunction with SESs seemed positive. The question which naturally follows is what utility or additional value can the technique bring to systems which experience these behavioural dynamics? Limits to Growth is a reasonably simple model to understand as the properties of the model which cause it to undergo collapse and overshoots are well known. For LEEA to be considered as an appropriate analysis tool to SESs, it must be capable of adding to our existing knowledge of the systems dynamics within a socio-ecological context. From this section of the conversation one of the main aims of the thesis was formulated: Is LEEA able to provide us with novel information about our system's dynamic behaviours through the analysis of system structure?

*Discussion point 2: Models where LEEA may be inapproariate*

Prof. Kampmann expressed that in his experience the utility of the method (meaning LEEA) depends on the kind of system that you are dealing with. Systems which express chaotic behaviour will show little information in the output of the analysis as the analysis output will be just as complex to understand as the output of the model itself.

It must be accepted that this analysis will not always be appropriate for every system. The utility and information the analysis is able to provide will largely depend on the complexity, size and internal dynamics of the model. On the reverse side of model complexity, if the model system is relatively simple and well known, then loop analysis may not offer any new insight. Systems which are relatively simple, for example only carrying two stocks with very few feedback mechanisms can often be investigated using simple graph theory or using system archetypes. Other systems where this technique will not be appropriate are those whose stocks variables are not included as part of multiple loops as their dynamics will not be accounted for within the Jacobian matrix, where each cell represents an interaction between two stocks. Models which include stock-less feedback loops may have to be reconfigured structurally, or the influence and elasticity outputs of the analysis may not accurately represent the entire loop set, therefore highlighting a disadvantage of this technique.

*Discussion point 3: Future development of the method*

The third section of the discussion overviewed the general accessibility of LEEA, where it currently stands in development and limitations of the technique which Prof. Kampmann was particularly aware of. The accessibility of LEEA was of high concern when attempting to introduce LEEA to an entirely new field of research. Current socio-ecological studies do not utilise LEEA: not only is the analysis tool an unfamiliar one to the field of SES, but SES research does not inherently involve system dynamic modelling or involve in depth knowledge of feedback loops and their ability to drive system behaviour.

Prof Kampman's response on this issue stressed that LEEA as an analysis tool is still within its infancy and is still undeveloped as a method. In order to be capable of carrying out LEEA and fully appreciate the results, there is base level of mathematical foundation required. To get the most out of LEEA, it is helpful to have at least a basic understanding of system dynamic models and how to manipulate/extract data from them, linear stability theory, graph theory and feedback loop behaviour. While this may seem like a daunting process to a new user, Naumov & Oliva (2017) have developed online tools and software packages to help streamline the process. While some manual data manipulation and basic understanding of Jacobian matrices and eigenvalues is required in order to understand the information LEEA outputs, many of the calculation steps are taken care of within packages run through Mathematica and are in open access, available online.

Regarding analysis development and what was known of the technique, Prof. Kampmann inferred that there were still unsettled questions surrounding LEEA's capabilities and what niche it could occupy with future development. It is known that one of the main limitations of LEEA is its ability to be applied to large complex models, as the data produced by the analysis becomes increasingly as large and convoluted to interpret. The extent to which LEEA's application is still considered practical and useful is unknown. At the current stage in LEEA's development, the best method for discovering its uses and its limitations is just to try it and test it.

To a new user wishing to explore and utilise LEEA as part of their research there is an inherent risk of using pre-developed software for an analysis which is in its relative infancy. Attempting to use Naumov and Oliva's software packages without first learning the basic theory and steps involved in carrying out LEEA risks treating the calculations as A) gospel and B) 'black box' and thus losing a lot of information. Information gained from LEEA is obtained from multiple sections throughout the analysis process and is not all concentrated as a single output, although this could be a direction to take LEEA in order to make the processing more streamline and accessible.

*Afterthoughts*

LEEA has already been used successfully and shown to provide novel information to system models surrounding business and industry. With LEEA's strengths and most noteworthy application to systems which are prone to oscillations, critical transitions and collapses, there is also a great deal of potential for LEEA to help us understand more about our human-environment interactions which are recorded to experience these dynamic behaviours all the time. LEEA could bring an alternative way to study structural drivers of our socio-ecological models. In order to find its niche amongst current analysis tools, if it has a place at all, it must first be tried and tested in a variety of system models, which cover a wide range of sizes, dynamics and represent a multitude of real-world systems. While this requirement and amount of testing is too great a feat to achieve in one thesis, one can, for want of a better phrase, get the ball rolling. In light of this, this thesis takes an initial look at LEEA in the context of SES models. An overview of LEEA's methodology and how to carry it out is provided assuming little to no prior knowledge of system dynamics or structural analysis. LEEA is critiqued for its current accessibility, utility, limitations and what it could bring to an SES study.

## 2.4 Model and Analysis Validation

Validation in the context of ecological modelling refers to the extent that a model is fit for purpose to the job and initial goals for which it was designed. In order to achieve model validation, the user must first have decided upon: 1) the purpose of the model, 2) the performance criteria and 3) the context of the model (Rykiel Jr 1996). Validation is then split into operation, theory and data, where validation can be undertaken to the operation and data parts of a model and simulation, but not on the theory (Rykiel Jr 1996).

Rykiel Jr (1996) emphasises that the most common downfall of ecological model validation occurs with failure to state what the validation criteria are to be. Stating the validation criteria of a model is critical as there are no universal standards when it comes to modelling the environment. The most common validation criterion chosen is to be able to match simulation output with observed data, otherwise known as the 'goodness of fit'. Richardson and Pugh III (1981) enforce the idea that validation should be a continuous effort to build confidence within the model and not simply a task that must be undertaken as a stepping stone between model completion and policy analysis.

The ‘goodness-of-fit’ approach is a validation technique used as a correlation based measurement between simulation and data and has been applied to multiple ecological models, notably for hydrologic and hydroclimatic modelling with limited use (Legates and McCabe Jr 1999). Alternative approaches include coefficient of efficiency and index of agreement which are reportedly able to cover many of the limitations found when conducting correlation based measurements (Legates and McCabe Jr 1999). The original goodness of fit approach is called the Coefficient of Determination. This describes the proportion of variance within the real-world data set which can be explained by the model. The Coefficient of Efficiency and Index of Agreement both make use of a parameter known as the ratio of mean square error, where higher values indicate a better agreement between the model and observed data variance (Wilcox et al. 1990; Willmott 1981).

The ‘goodness-of-fit’ approaches provide model validation to an extent, but their ability to estimate and calibrate model performance does not guarantee a models ability to predict future events (Power 1993). If the purpose of a model is to be used for prediction then it can be compared and contrasted against other models built for the same purpose for statistical adequacy. Statistical adequacy stems from econometric and statistical forecasting literature and involves the use of a statistical checklist to rank the properties of a model (Power 1993).

Another form of model validation stems from a statistical comparison of a model’s multiple predictions against measurements of a system’s real world behaviour. This uses the hypothesis of ‘no difference’; a null-hypothesis that the model is acceptable (Robinson and Froese 2004). A null-hypothesis is a statistical test with the hypothesis that there is no significant difference between a model’s output and the observed data set with any variations being due to sampling or experimental error alone. Robinson and Froese (2004) work on an empirical forest growth model and use an approach known as the hypothesis of dissimilarity, a null hypothesis that the model is unacceptable, arguing that it is a more robust measurement of model validation.

### 2.4.1 Validation of LEEA and its outputs

In the context of validating LEEA, system dynamic models are used less for prediction purposes and more for simulation and scenario comparison, leaning on the strengths of system dynamic modelling which focus on system structure and behavioural trends, rather than specific output values. Validation can come in the form of an iterative process comparing model simulation to observable data, much like 'goodness-of-fit' approaches. However, this is for validation of the model, but not of the outputs of LEEA analysis itself.

Validation of LEEA's outputs comes in two parts, an internal validation and an external validation. The internal validation is a reflection of LEEA being a purely analytical technique with a set procedure to follow, meaning that its output is completely reliant on the structure, dynamics and values assigned to a model. LEEA's internal validation is therefore only as good as the validity of the model it is being used on.

External validation takes into account whether the information LEEA produced regarding feedback loop dominance is reflected in real world feedback mechanisms. This would require the ability to manipulate leverage points and feedback loops within the real world system to see if LEEA's outputs are reliable. If LEEA identifies a highly influential loop structure within the model, but upon manipulation of the real world system, nothing changes, it could mean there is a disconnect between the real world system and the structure or dynamics of the model LEEA is being used to analyse.

# Chapter 3

# Methodology

## 3.1 Loop Eigenvalue Elasticity Analysis (LEEA)

Loop Eigenvalue Elastcity Analysis is used throughout and is integral to the material within each chapter of this thesis. The following chapter is a breakdown of LEEA's methodology. Almost all calculations regarding LEEA and generating its output have been automated (Naumov & Oliva 2017), yet it is important that at each step of the methodology, what is being calculated and why, is understood, in order to make informed interpretations of the results.

Loop Eigenvalue Elasticity Analysis (LEEA) was developed in 1996 by Kampmann (2012) who extended the works of Forrester (1982) regarding Eigenvalue Elasticity Analysis (EEA) from 1982-1983. While the premise of using system eigenvalues to describe the behaviour of a linear system was well established in classical control theory, N.B. Forrester was the first to use this method on system dynamic models to develop notions of eigenvalue elasticity and link them to model parameters ( Forrester 1983, Forrester 1982). Eigenvalue elasticity was a way to measure the relative change occurring to a system eigenvalue with respect to a relative change to a system parameter. Forrester used this technique primarily in economic stabilization policy. Kampmann (2012) extended the theory behind EEA, allowing for entire feedback loop structures to be compared against the system's eigenvalues and thus formed the basis for LEEA.

EEA and LEEA were initially very manually intensive, taking a lot of preparation time before any computer simulations could be run. LEEA was later extended and made to be more automated (Kampmann 2012, Güneralp 2006, Kampmann and Oliva 2006), with the entire process being made much simpler, more automated and user friendly in recent years (Naumov & Oliva 2017, Oliva 2015). Structural Loop analysis has been used extensively on economic and industry modelling as well as to simply expand the capabilities and accessibility of the technique (Saleh et al. 2010, Güneralp 2006, Kampmann and Oliva 2006, Guneralp et al. 2005).

Loop Eigenvalue Elasticity Analysis uses system dynamic theory (Forrester 1971), where a system can be represented as a series of stocks and flows, based on the idea that a system's behaviour is driven by properties of its internal structure (Forrester 2007a). LEEA identifies a set of feedback loops within a system's structure known as the Shortest Independent Loop Set or SILS (Kampmann 2012, Gonçalves 2009, Oliva 2004), which are collectively responsible for driving the system's behaviour. LEEA then structurally analyses the loop set, identifying which feedback loops are dominating the system's behaviour at any point in time.

LEEA is based on the ability to describe a system's behaviour through a set of eigenvalues at any point along that system's time series and link them to the driving behaviour of feedback loops of the system's internal structure. The approach is taken from linear theory and is applied to non-linear systems by linearizing the system at every point in time (Gonçalves 2009). Each eigenvalue of the system represents a single behavioural mode which reflect the behaviour patterns of the system. Perturbations which occur within the system's structure (i.e. the system's feedback loops) are able to generate changes within the eigenvalues. Perturbations from feedback loops, which create large changes to an eigenvalue indicate high influence on the current behaviour, while perturbations which produce small changes to an eigevalue indicate little to no influence on the current behaviour. LEEA is able to identify how much an individual feedback loop is dominating the current system behaviour, thus allowing the user to identify a hierarchy of loop dominance at any point in time.

Despite multiple sources expressing the benefits that the LEEA technique can bring to an understanding of system behaviour (Guneralp et al. 2005, Kampmann and Oliva 2008, Kampmann 2012) and several attempts to make the technique more accessible, (Naumov & Oliva 2017, Oliva 2015, Güneralp 2006) the method has only seen limited use in the field of socio-ecological systems (Lade and Niiranen 2017, Bueno 2012, Bueno 2013). The purpose of this initial study is to assess the benefits and limitations which LEEA can bring to the field of socio-ecological and complex ecological modelling, with a focus on, but not limited to, systems able to undergo rapid regime shifts between stable states.

Calculating Loop Elasticities and Influence values is a step process to which the following must be calculated:

1. Constructing Differential Equations (3.1.1)
2. Jacobian Matrix of the system dynamic model (3.1.2)
3. Eigenvalues of the Jacobian matrix (Behavioural modes) (3.1.3)
4. Identifying the feedback loops in the Shortest Independent Loop Set (SILS) (3.1.4)

5. Loop gains (3.1.5)

6. Loop Eigenvalue Elasticity (3.1.6)

7. Loop Influence (3.1.7)

### 3.1.1 Constructing Differential Equations

A key step in the process of determining system eigenvalues, required for LEEA and the comparison to system feedback loops is the generation of a system's Jacobian matrix. The Jacobian matrix requires the system to be described as a set of differential equations. In order to produce analytical differential equations, all variables must be in analytical form.

System models developed using system dynamics are often created using a set of predetermined differential equations which describe the behaviours expressed by the system. This can often be the case in ecology and socio-ecology where dynamic behaviours i.e. populations growth or system collapse, have well establised analytical formulae associated with them. In many other cases, system dynamic models are not created using predetermined formula and are built from a foundation of model components and simple interactions, built up through multiple model iterations. In either circumstance, it is possible to use the form of a system dynamic model to gain a set of differential equations which describe the system.

In order to construct differential equations from a system dynamic model (sometimes referred to as a stock and flow model), the components of the model must first be categorised into its stocks, flows, auxiliary variables and constants. Each stock variable represents a separate differential equation used to describe the model system, i.e. three model stocks infers that the system can be described using three differential equations. From this, each stock variable is expanded to represent its inputs and outputs. Below is a very simple example of equating a system dynamic model stock to a differential equation. Any input of a stock corresponds to an addition (+) in its differential equation form and any outputs of a stock represent a subtraction (-). Examples can be seen below in figure 3.1.

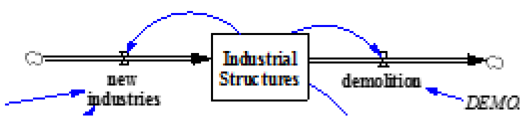



FIGURE 3.1: A visual respresentation of a stock variable (black box), its flows (double lined black arrows), sources and sinks (cloud shapes). The image is extracted from a Vensim model and translates to the following dynamical equation: $\frac{\delta IS}{\delta t} = newindustries - demolition$

### 3.1.2 The Jacobian Matrix

The structure of any linear system model can be represented within a square, '$nxn$' matrix (Saleh et al. 2005). The Jacobian matrix or 'Gain Matrix' is a representation of all the links between every stock in a system in their most compact form (Guneralp et al. 2005). The Jacobian is equivalent to writing down each differential equation of a system and taking the partial derivatve of each with respect to each stock of the system while all other variables are treated as constants. Each partial derivative generated equates to one entry or one coefficient of the Jacobian Matrix. Each element of the matrix represents how one stock affects another's trajectory (i.e. how it changes over time), when all other stocks are kept constant.

The Jacobian Matrix can also be referred to as the Gain Matrix because its entries are made up of the 'gains' of the links that exist between stocks. The concept of link gains and loop gains are expanded upon in section 3.1.5 entitles 'Loop Gains'.

To calculate the Jacobian Matrix, the differential equations for each stock of the system must be known. A Jacobian Matrix takes the generalised form:

$$J = \begin{vmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \cdots & \frac{\partial \dot{x}_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial \dot{x}_n}{\partial x_1} & \cdots & \frac{\partial \dot{x}_n}{\partial x_n} \end{vmatrix}. \tag{3.1}$$

Where $x$ represent stocks, $\dot{x}$ represents the rate of change of that stock or $\frac{\delta x}{\delta t}$ and $n$ represents the number of stocks. So $x_2$ refers to the second stock of the model. Note it does not matter which number is assigned to which stock so long as the order is kept consistent. Each entry of the Jacobian, i.e. $\frac{\partial \dot{x}_i}{\partial x_j}$ represents the change in net rate of stock variable $i$ as a response to an infintesimal change in the value of stock variable $j$ (Saleh et al. 2005).

The calculation of a Jacobian matrix requires the system to be linearised. By calculating the entries of the matrix at any point in time, all non-linear inputs can be taken as constants and the model can be linearised at that individual point in time (Kampmann 2012; Güneralp 2006; Forrester 1983). If the model is to be linearised at each time step, then the time steps or changes in the system between time steps, must be very small to assume linearity. "... the dynamics of the original model can be approximated through time by the behaviours of a series of linear models with varying entries in their gain matrix" (Guneralp et al. 2005, Güneralp 2006).

While the Jacobian Matrix can be calculated by hand, it can often get extremely complex, especially as systems get larger and stock number increases. Key functions in Matlab or Mathematica for this procedure include 'jacobian ([f(x),f(y)],[x,y])', eig(J) and subs(e) for the creation of the Jacobian matrix, its eigenvalues and substitution of data into those eigenvalues. The calculation of the Jacobian Matrix is done automatically within LEEA's online support material (Naumov & Oliva 2017), but it is calculated in the background code and is not seen by the user.

### 3.1.3 Calculating Eigenvalues

Eigenvalues are calculated from a system's Jacobian matrix and are an integral part of interpreting LEEA's output and the calculation of loop eiegnvalue elasticity. Since the Jacobian is an $nxn$ matrix, were $n$ is equal to the number of stocks in the model, the number of eigenvalues of a system is equal to the number of stocks in that system.

Eigenvalues are a set of scalars associated with the linear system equations and provide valuable insight into the properties of the Jacbian matrix. Eigenvalues have multiple uses across different fields (i.e. mechanical engineering, image processing, physics), but they are used in linear systems as they have a common application for stability analysis. Generally eigenvalues are able to characterise the underlying linear transformations of a system's behaviour in a much simpler manner than $n^2$ entries of a square matrix (Saleh et al. 2010).

Eigenvalues ($\lambda$) are calculated for an $nxn$ matrix ($J$) by satisfying the equation: $det(J-\lambda I) = 0$, where $I$ is an $nxn$ identity matrix. From linear stability theory, eigenvalues can be used to determine if a system at a fixed point is stable or unstable (Glendinning 1994). Eigenvalues are capable of being real, or complex numbers with a real and imaginary part. If the real part of all eigenvalues are negative, then the fixed point is stable. If the real parts of any of the eigenvalues positive, then the fixed point is unstable. The properties of these eigenvalues tell us a lot about what kind of behaviour the system will be expressing:

- If $\lambda$ = Positive and real: system is expressing exponential growth.
- If $\lambda$ = Negative and real: system is expressing exponential decay.
- If $\lambda$ = complex conjugate (has a real and imaginary part) with a positive real part: system is expressing oscillations which are expanding.
- If $\lambda$ = complex conjugate with negative real part: system is expressing oscillations which are dampening.

Like the majority of steps required for LEEA, many of the calculations are automated within Naumov & Oliva (2017). Otherwise, the calculation of eigenvalues can largely be achieved through pre-existing functions available in software such as Matlab or Mathematica. Using eig(J) Matlab can calculate the eigenvalues for a matrix $J$. The software is capable of calculating eigenvalue with algebraic symbols which can then be substituted for time-series data.

*Prioritising Eigenvalues*

Eigenvalues are prioritised during the interpretation of LEEA in order to filter out much of the enormous amount of data it is capable of producing which is not relevant to the questions and goals of a project. Since the choosing of eigenvalues and interpretation of loop influence plots is not an automated procedure and must be conducted by the user, it is the part of the analysis which can be the most time consuming. The following provides a short guide on how to choose eigenvalues that should be focused on during LEEA. An example has been provided to help illustrate this procedure.

The eigenvalues which should be focused on are based on three properties which should be prioritised in the following order:

1. *Are any eigenvalues positive?*

   Positive eigenvalues are associated with runaway, exponential and destabilizing behaviour. They indicate that a system is unstable, capable of transitioning away from their current state with only a small perturbation. Feedback loops dominating in positive eigenvalues can therefore represent long term behavioural trends in a system such as exponential growth or decay that takes time to build up. To that end, positive eigenvalues which hold relatively small magnitudes can be as, if not more important to focus on than negative eigenvalues with relatively large magnitudes.

2. *Which eigenvalues hold the highest magnitudes?*

   Eigenvalues which hold high magnitudes relative to other eigenvalues are seen as having more representation over the system's current behaviour. High magnitude, negative eigenvalues represent a steeper and faster pull of a system back to a stable state compared to low magnitude, negative eigenvalues. High magnitude, positive eigenvalues represent a steeper, faster pull away from a stable state compared to small magnitude, positive eigenvalues.

   Eigenvalues which hold relatively similar magnitudes must be compared against each other simultaneously in order to determine highly influential loops. The loop influences values associated with two different eigenvalues can often differ greatly, even when the eigenvalues hold a similar magnitude and the same polarity. When this occurs, dominant feedback loops within both eigenvalues should not be ignored, remembering that eigenvalues represent different long term and short term behaviours of a system. The dominant feedback loops across the differing eigenvalues will be representing alternate behaviours across potentially different timescales. The user should primarily go back to the variables, values and the equations of the interactions within those feedback loops in order to work out why the feedbacks are simultaneously registering as highly influential, but for different short term/ long term behaviours.

3. *How are the eigenvalues changing through time?*

   While the priority for choosing eigenvalues should consider the polarity and magnitude at individual points in time because that is how the analysis linearizes the model, the user should not completely ignore the trajectory of an eigenvalue as it changes through time. Eigenvalues which become progressively less negative may be indicative of behaviour within the system generating a decreasing amount of stability. Eigenvalues which sit at zero or are primarily negative may transition into positive values, where they must now be prioritised for information regarding the system's long term behaviours.

The best scenario is if a user is able to keep track on loop influence values associated with every eigenvalue of a system. However, this is often not practical if there are a high volume of eigenvalues and feedback loops and often not necessary if many of the eigenvalues hold relatively low magnitudes for the entire simulation.

If an eigenvalue is a complex number with real and imaginary parts, separate the two into different plots. Primarily use the real part to determine which eigenvalues to prioritise, but use the imaginary part to identify complex conjugate pairs and to acknowledge when the system is expressing oscillatory behaviour, rather than exponential or dampening behaviours.

When eigenvalue plots begin to look complicated, usually due to a high number of stocks being present in the system and some interesting dynamic behaviours occurring, the best thing to do is to break the output down into sections. An example has been selected to showcase how a user might go about interpreting an eigenvalue plot:

First things to note: 1) The system has seven eigenvalues. 2) The eigenvalues are complex numbers (this example will focus on the real part). 3) The system appears to undergo some cyclic behaviour shown by the periodic spiking of eigenvalue 1. 4) The eigenvalues change constantly throughout the time line, choosing which to prioritise is not a trivial process.

Since there is no clear eigenvalue to focus on, the user's best option is to break the plot into sections (Figure 3.3).

Now examine each section separately. While this may be a time consuming process, it will ensure the most understanding about the system across each point within the simulation. If the sections are inspected in the order in which they occur it may be easier to explain transitions and changes between sections which reflect in the system's overall behaviour.

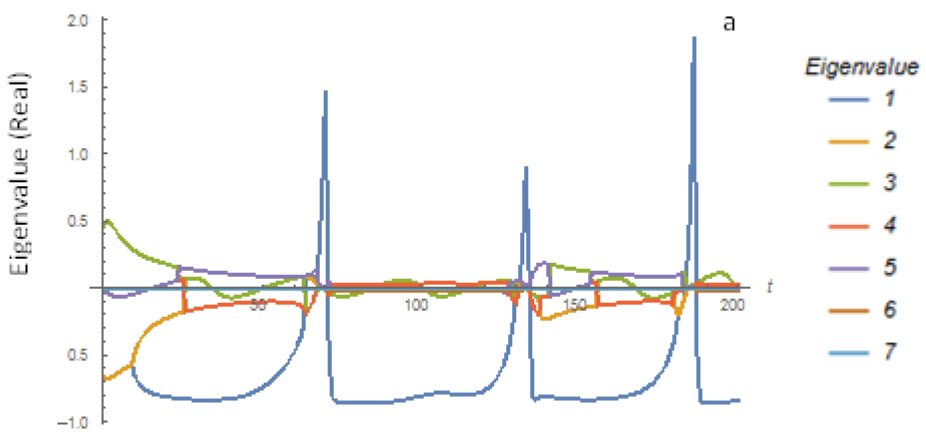



FIGURE 3.2: An eigenvalue plot produced from LEEA of a seven stock system. Each line represents a different eigenvalue of the system as it changes through the simulation.

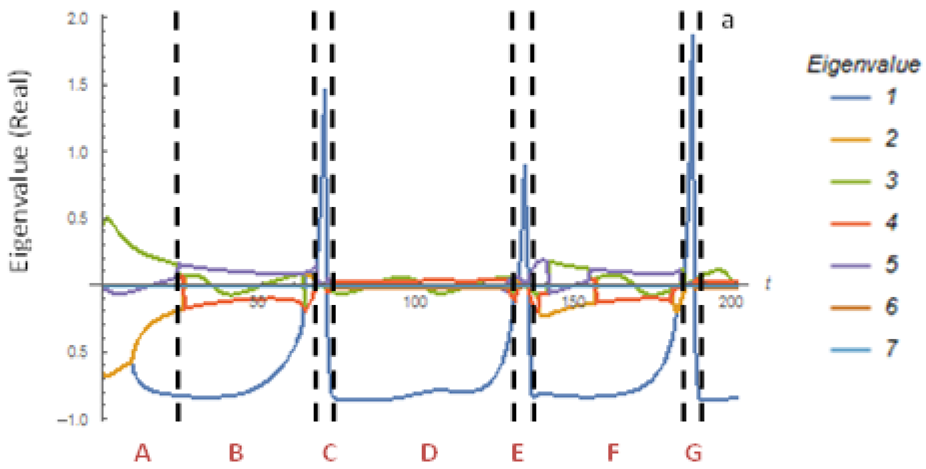



FIGURE 3.3: An eigenvalue plot of a seven stock system sectioned into managable parts fo analysis.

Section A:

- Section A appears not to follow the cyclic behaviour of the rest of the model simulation and so might be a remnant of model spin-up, largely being able to be ignored, otherwise
- Eigenvalue 3 holds the highest magnitude positive value which should be prioritised as a reflection of instability within the system, while eigenvalues 2 and 1 should be inspected for sources generating stability.

- Note that despite loop 3 holding a high positive value relative to loops 4-7 which are sitting closer to zero, the value of eigenvalues 2 and 1 still hold higher negative magnitudes so should not be ignored.

Sections B, D and F:

- These sections are the most complex of the time series.
- Each holds a relatively high negative eigenvalue which should be inspected for feedback loops generating stability within the system.
- Multiple eigenvalues appear to cross into positive values, but their trajectories change a lot within the section. Here the best thing to do for each of these sections is to plot each time section separately, potentially not plotting eigenvalue 1 so that a closer inspection of dynamics around zero can be achieved.
- The eigenvalues around zero all appear to hold relatively similar influences, so any which cross into positive values should be inspected for sources of instability.
- Feedback loops generating instability within these sections should be compared against those in eigenvalue 1, especially in the knowledge that eigenvalue 1 is about to spike into high positive values.

Sections C, E and G:

- These sections are much simpler to process. Eigenvalue 1 dominates these sections with relatively high spikes in positive values and so can be prioritised for inspection over all others.
- Note Feedback loops responsible for the exponential build up from the previous sections of these spikes.
- Be sure to actively compare loop influences within this eigenvalue against the spike. Do the loop influences spike at the same time and do some drop in magnitude in respect to the spike?

The interpretation of eigenvalues and feedback loop influences will always bring new challenges for interpretation of LEEA with each new model it is used upon. The best practice is to follow a structured guide such as the three steps mentioned above in order to maintain consistency and to acknowledge that careful interpretations can take time, but are often worth the extra understanding that is gained over the systems dynamics and behaviour.

### 3.1.4 Identifying the feedback loops in the Shortest Independent Loop Set (SILS)

LEEA requires all individual feedback loops of the model in question to be identified for analysis. As complexity of the model increases (increased by a greater number of components added to the model bringing more interactions) the task of identifying every feedback loop becomes increasingly difficult. The basic principles of LEEA, in calculating eigenavlue elasticity also require every loop to be treated as an individual structure which can be isolated for its influence over the system. However, it often occurs within system dynamic models that many interactions (also referred to as links, edges or connections) are part of the structure of more than one loop. This means that a change which occurs within a link would affect more than one loop simultaenously. To combat this problem, Kampmann (2012) devised a way of identifying a set of loops known as an Independent Loop Set (ILS), which were capable of describing the full structure of the system, without needing to include every possible feedback loop within the system. “The ILS, while not unique, is a complete description of the feedback complexity of a graph” (Oliva 2004). The ILS gave meaning to the relative importance of a loop within the context of that specific loop set. The issue then became that the ILS could take on many different forms to describe the same system. In order to ensure that the feedback loops within the loop set would be consistent between users, Oliva (2004) proposed the Shortest Independent Loop Set (SILS), which ensured that the loops idenitified for LEEA would be the same between users.

To identify the SILS of a system, the first step requires all causal links and pathways between stocks to be identified. The difference between the two is that a causal link occurs from one system component to another (direct neighbours), while pathways are a series of causal links which always start and end at a stock (it can be the same stock where the pathway starts and ends). Connections linking constants are not counted in the identification of causal links. This step is necessary as the causal links are used in calculating the SILS and pathways are used to calculate the elasticity eigenvalues of loop structures.

Oliva (2004) then created an algorithm with which to form the loop stucture of any model using the list of causal links and pathways in a consistent manner. In his description of the algorithm created to identify the SILS, Oliva (2004) states that the SILS is only capable of identifying the same loop set, even if others do exist. This means that the algorithm will be consistent for the same model structure, unlike ILS. It also ensures that every causal link is included in the list allowing for a full cycle partition to be recreated from the loop list. A cycle partition is a strongly connected graph with a directed path from each node to every other node. Oliva (2004) elaborates “The main advantage of a cycle partition is that it identifies the set of strongly connected elements that contain all the feedback complexity of a model structure.”

Cycle partitions are visually similar to system dynamic models, but include no dynamical properties of the stocks and each component is represented in the same way rather than specifying stocks, auxilary variables and constants.

### 3.1.5 Loop Gains

Loop gain is required to compare perturbations within the system's SILS feedback loops against the system's eigenvalues. Gain can be thought of as the strength of a flow around a feedback loop or through a link. The gain of a feedback loop or a link is a measure of the relative change caused to an output with respect to a change of its input. In order to calculate the gain of a loop, the gain of each causal link within that loop must first be determined. A causal link refers to the link (edge) between a predecessor, or input variable and its successor, or output variable. From any successor variable in the model, the gain of the link from its predecessor variable can be calculated. The gain of a causal link ($a$) is calculated by the partial derivative of a successor variable $v_1$ with respect to a predecessor variable $v_2$:

$$a_{v_1 v_2} = \frac{\partial successor}{\partial predecessor} = \frac{\partial v_1}{\partial v_2} \tag{3.2}$$

The gain of a loop ($g$) is then calculated through the product of all causal link gains which join together to form that loop (Kampmann 2012):

$$g = a_{v_1 v_2}.a_{v_2 v_3}.a_{v_3 v_4}.\ldots a_{v_n v_1} = \frac{\partial v_1}{\partial v_2}.\frac{\partial v_2}{\partial v_3}.\frac{\partial v_3}{\partial v_4}.\ldots\frac{\partial v_n}{\partial v_1} \tag{3.3}$$

where $v_{1,2,3,4n}$ represent individual variables which link together to form a feedback loop.

### 3.1.6 Loop Eigenvalue Elasticity

Loop Eigenvalue Elasticity is a measure of the fractional change in a system eigenvalue relative to a fractional change in the gain of a feedback loop (Kampmann and Oliva 2008, Kampmann and Oliva 2006). It is calculated as the partial derivative of the eigenvalue with respect to the loop gain, normalised for the size of the loop gain and size of the eigenvalue in order to isolate the effect of changes in size (value) (Gonçalves 2009). Thus loop eigenvalue elasticity indicates how much a loop contributes to changes within an eigenvalue. Loop Elasticity , is defined with respect to the loops gain parameter, $g$, in the form:

$$\varepsilon = \frac{\partial \lambda}{\partial g}.\frac{g}{\lambda} \tag{3.4}$$

where $\lambda$ is an eigenvalue of the Jacobian matrix and $g$ the gain of the current loop in question.

If the eigenvalue is a complex number:

$$\lambda = re^{i\theta} \tag{3.5}$$

where $r = |\lambda|$ is the absolute value of the eigenvalue and is known as the natural frequency, $i$ is the imaginary value and $cos\theta$ is known as the dampening ratio, then the loop eigenvalue elasticity will also be complex, holding real and imaginary parts:

$$Re\{\varepsilon\} = \frac{dr}{dg}.\frac{g}{r} \tag{3.6}$$

$$Im\{\varepsilon\} = \frac{d\theta}{dg}.g \tag{3.7}$$

It is possible to measure the real and imaginary parts of elasticity separately as shown in Kampmann (2012):

$$\varepsilon_1 = \frac{dRe\{\lambda\}}{dg}.\frac{g}{|\lambda|} \tag{3.8}$$

$$\varepsilon_2 = \frac{dIm\{\lambda\}}{dg}.\frac{g}{|\lambda|} \tag{3.9}$$

These calculations are possible because the eigenvalues and loop gains of a system are inherently connected via the system's characteristic polynomial ($P(\lambda)$). The characteristic polynomial of a system takes the form:

$$P(\lambda) = |\lambda I - J| \tag{3.10}$$

Where $I$ is an identity matrix, $\lambda$ is the system eigenvalues and $J$ the system's Jacobian Matrix. Eigenvalues of a system are determined as the roots of the characteristic polynomial and as it turns out, the coefficients of the characteristic polynomial can be expressed in terms of the feedback loop gains. Kampmann (2012) provides an example and helpful breakdown of this within Theorem 2 of his article 'Feedback loop gains and system behaviour (1996)'. This direct link between the eigenvalues as roots of the characteristic polynomial and loop gains as the coefficients of the characteristic polynomial allows one to attribute the change in eigenvalues directly to changes in individual feedback loops by implicit differentiation of the equation; $P(\lambda, g_i) = 0$, i.e.

$$\frac{d\lambda}{dg_i} = -\frac{\partial P(\lambda, g_i)}{\partial g_i} . \left( \frac{\partial P(\lambda, g_i)}{\partial \lambda} \right)^{-1} \tag{3.11}$$

Where $g_i$ is the gain associated with one feedback loop, $i$. As noted by Kampmann (2012), this formula can only be used and is valid if the change in the gain of a link can be attributed to the gain of one feedback loop independently of all others. This requirement is prevented by individual links potentially being part of multiple feedback loops at once. To combat this, the user must choose a particular loop set with which to describe the system (an Independent Loop Set, ILS) in order to give meaning to the relative importance of a loop within the context of the selected loop set. In order to make the loop sets consistent between users (because the ILS of a system can have many varieties), the Shortest Independent Loop Set (SILS) can be used which is always the same (Oliva 2004).

When loop elasticity is calculated for all feedback loops within a loop set (SILS), it allows a hierarchy of loop dominance over the system's behaviour to be established. Feedback loops which express high absolute levels of loop eigenvalue elasticity relative to others have greater dominance over the system behaviour. Feedbacks whose loop eigenvalue elasticity sit at or close to zero show little or no dominance over the current behaviour.

### 3.1.7 Loop Influence

Alternatively loop influence $\mu$, of a loop can be calculated through:

$$\mu = \frac{\partial \lambda}{\partial g} . g \tag{3.12}$$

From loop influence it is possible to determine the type of contribution a loop is having over an eigenvalue. A positive value of loop influence indicates a loop generating instability within an eigenvalue, while negative values indicate the generation of stability (Kampmann and Oliva 2006). Similar to loop elasticity, the greater the absolute value of a loop's influence, the greater the contribution that loop makes to the system's current behaviour.

Both Elasticity and Influence have reportedly the same properties (Kampmann 2012). However, Elasticity runs into troubles at eigenvalues at or close to zero and while Loop Influence values do not, they are dependent on the timescale of the model. In Elasticity, a positive Elasticity value indicates a drive for growth from the loop in the system, whereas negative elasticity values indicates a drive for decay. In loop influence, a negative value is associated with stabilizing influence of a loop and a positive value is associated with destabilizing influence. Generally loop influence is considered easier to interpret and its time dependency is seen only as a minor drawback (Saleh et al. 2010). A methodical breakdown of implementing LEEA can be found in Guneralp et al. (2005) and Güneralp (2006).

*Determining which eigenvalues to focus on:*

In EEA (Forrester 1982), in order to determine dominant structures within a model, the general approach was to select a dominant eigenvalue with which to focus analysis as this inherently would hold the most influential structures of system behaviour (Güneralp 2006). Eigenvalues are used in an extensive number of fields for a variety of reasons (i.e. mechanical engineering, physics, image processing, linear systems) and generally across all fields a dominant eigenvalue from a set is the one which holds the highest absolute value.

As an example, considering five eigenvalues with the values -10, -4, 0, 2 and 9, the eigenvalue holding -10 would be dominant. Intuitively this would mean that for loop dominance analysis the -10 eigenvalue would be prioritised for its influential loop structures in order to monitor which loops are influencing the behaviour expressed by the most dominant eigenvalue.

However, in linear systems and particularly in matters of system stability and exponential growth/decay, this concept of a dominant eigenvalue only works while all eigenvalues are negative. If any eigenvalue within a linear system holds a positive value, no matter how small its magnitude, it will eventually dominate the system's behaviour (Guneralp 2004; Franklin et al. 1994).

In the context of influential feedback structures, a shift in eigenvalue polarity often infers a change has occurred to which feedback loop is dominating within that eigenvalue. Changing from a dominant negative feedback to a dominant positive feedback can lead to a reinforcing or runaway behaviour, which eventually destabilizes the system. With regards to system stability, eigenvalues which sit at or close to zero and eigenvalues which hold large real positive magnitudes can be seen as more important than eigenvalues with larger negative magnitudes.

The eigenvalue of most interest regarding system stability and therefore the one whose loop dominance properties should be focused on, is determined by which holds the highest positive real value at any point in time, even if there are negative eigenvalues with greater absolute values. If all eigenvalues are negative then the dominant eigenvalue is taken to be the one whose real part holds the largest absolute value. The real part of the eigenvalue is used to determine eigenvalue dominance regardless of whether it is a complex number or not. Loop eigenvalue elasticity and loop influence in the dominant eigenvalue represent the most dominant structural drivers of the system. If more than one eigenvalue shows high dominance, then behavioural drivers must be determined from a comparison of loop dynamics across those eigenvalues.

## 3.2 A Worked Example: Forming the Jacobian Matrix and its connections to Feedback Loops

The following example uses Lokta-Volterra equations to showcase how the first principles used in LEEA work.

Lokta-Volterra or the predator-prey interactions take the following equations:

$$\frac{dx}{dt} = \alpha x - \beta xy \tag{3.13}$$

and

$$\frac{dy}{dt} = \delta xy - \gamma y \tag{3.14}$$

where $\frac{dx}{dt}$ is ho the population od pref $x$ changes over time, $\frac{dy}{dt}$ is how the population of predators $y$ changes over time, $\alpha$ is the birth rate of prey, $\beta$ is the rate of predation of the predators on the prey, $\delta$ is the growth rate factor of the predators, with the growth rate of predators, $\delta xy$, reliant on the population of prey and $\gamma$ is the natural death rate of predators.

In system dynamic form, these two equations form figure 3.4:

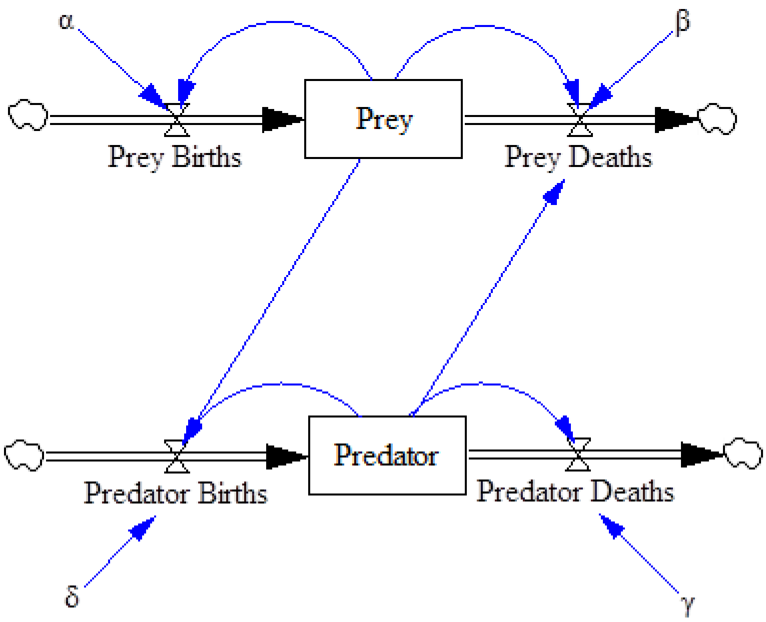



FIGURE 3.4: The system dynamic form of the predador- prey equations. This diagram is an example of how one might find an system dynamic model, with variables labelled as opposed to their equation equivalents.

This next image shows the same diagram with all variables written out for each component, and all causal links which are part of feedback loops assigned their link gain in algebraic form (figure 3.5):

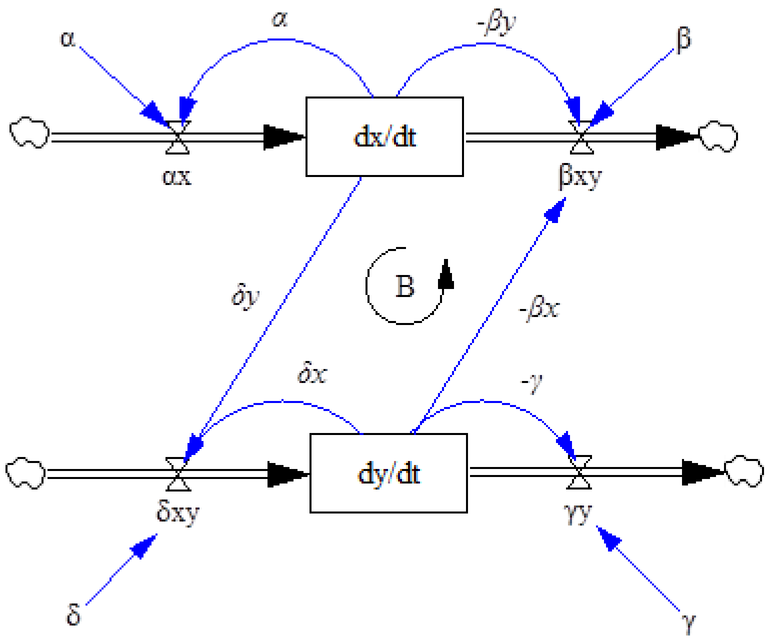



FIGURE 3.5: The system dynamic form of the predador- prey equations. This diagram shows how all of the equations of Lokta-Volterra fit into system dynamic model form.

Now the Jacobian Matrix for the lokta-volterra equations takes the general form:

$$J = \begin{vmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{vmatrix}. \tag{3.15}$$

We get:

$$J = \begin{vmatrix} \alpha - \beta y & -\beta x \\ \delta y & \delta x - \gamma \end{vmatrix}. \tag{3.16}$$

The Jacobian Matrix is a representation of all of the interactions that occur between the stocks (in this case, between the predators and prey) within the system. Note that each component of the Jacobian Matrix can be made using the gains of the links within the system dynamic diagram. The components of the Jacobian Matrix, which is used to calculate the system's eigenvalues, is made up of the link gains of the system. Intuitively, the gain of a feedback loop of the system can also be calculated as the product of the links that form their structure.

Thus, the gain of the balancing feedback loop which connects the predator and prey populations, labelled 'B' in the diagram structurally is formed of the link from prey population to predator births which holds the link gain $\delta y$ and from predator population to prey deaths which holds the link gain $-\beta x$. So the balancing feedback loop holds a loop gain of $-\beta\delta xy$.

When a change occurs within the system, it will cause one if not more of the variables to change in value. This value change gets reflected in the gain of any loop which contained that variable as well as changing the values within the systems Jacobian Matrix and therefore its eigenvalues. It is because of this that any changes which occur within the eigenvalues of the system's eigenvalues can be linked to the changes which are occurring within the feedback loops, and the relative influence of each feedback loop on the system can be calculated within LEEA.

## 3.3 Interpretation of Outputs

The following section overviews a series of typical outputs which a user might expect to gain from conducting LEEA on a system model. The outputs are largely extracts from the Naumov & Oliva (2017) online support material and software package for running LEEA. However, to aid understanding of what happens at every step, this section also provides examples of what outputs look like, which are calculated in the background of the online software and helps to explain how to interpret them.

Outputs of the system dynamic model, eigenvalues, loop gains, loop eigenvalue elasticities and loop influence values are all output within the online software package (Naumov & Oliva 2017. Their outputs are predominantly graphical, but eigenvalues and loop gains can also be output in table format, displaying specific values. The eigenvalue and loop gain data series can be used to acquire specific values for each time step in a system's model output. These time steps correspond 1 to 1 with the time steps of the system dynamic model.

Figure 3.6 shows a section of the data series of 21 time steps from the eigenvalue calculation of the LEEA code. The first column shows time steps and the following columns show the values of each eigenvalue of the system. This particular system had three stocks and therefore three eigenvalues, so three columns of data were produced. As eigenvalues can take the form of complex numbers, figure 3.6 shows the values of the real numbers and figure 3.7 shows the imaginary values across this section. As shown in figure 3.7, eigenvalues 1 & 2 express imaginary values for 3 timesteps showing that the system experiences oscillatory dynamics at this point in time.

Each time step of these data series can then be plotted in graphical form, where each eigenvalue is shown for its real vs. imaginary values. The following plot shows time step 78 of the above data (Figure 3.8).

| | | | |
|---|---|---|---|
| 60 | −0.7503492362727262 | −0.003128108078023554 | −0.0011166683866438024 |
| 61 | −0.73827128013495 | −0.003390059803650167 | −0.0011331601992600534 |
| 62 | −0.7247976306943756 | −0.0036910264532097363 | −0.0011517657465997947 |
| 63 | −0.7097502420986398 | −0.004038957166262995 | −0.001172769953420985 |
| 64 | −0.6929176544012422 | −0.004444230663821384 | −0.0011965116397718654 |
| 65 | −0.6740646734551091 | −0.004920163776548121 | −0.0012233695816999894 |
| 66 | −0.6528739223506183 | −0.005485754457113775 | −0.0012538511065109995 |
| 67 | −0.6290117189203798 | −0.00616586361582868 | −0.001288493094033326 |
| 68 | −0.6020213607014909 | −0.006997477177509921 | −0.001328023353929267 |
| 69 | −0.5713415346738975 | −0.0080352022321256 | −0.001373330815599203 |
| 70 | −0.536235004775551 | −0.009364781367446713 | −0.001425568203843124 |
| 71 | −0.49572348707779396 | −0.011128357201585122 | −0.0014862321408266438 |
| 72 | −0.44844936556411624 | −0.013581514059739286 | −0.0015573200565796635 |
| 73 | −0.3923024082627592 | −0.017247073061152567 | −0.0016417375482838121 |
| 74 | −0.32377632145842533 | −0.023395518478043168 | −0.001743650953332349 |
| 75 | −0.23505558520831427 | −0.03644588620862593 | −0.0018695067820319773 |
| 76 | −0.08848246522022202 | −0.08848246522022202 | −0.0020301216746087857 |
| 77 | −0.026877865525319296 | −0.026877865525319296 | −0.0022451420280323315 |
| 78 | 0.05886596975592401 | 0.05886596975592401 | −0.002555673639398713 |
| 79 | 0.3290405194399839 | 0.05418611352787055 | −0.0030657760879540057 |
| 80 | 0.8397373717290068 | 0.03062903966942411 | −0.004135242765374472 |

FIGURE 3.6: Data series from an eigenvalue calculation (real part) showing 21 time steps from 60-80. Columns 2-4 represent eigenvalues 1 2 and 3 of the system.

| | | | |
|---|---|---|---|
| 60 | 0 | 0 | 0 |
| 61 | 0 | 0 | 0 |
| 62 | 0 | 0 | 0 |
| 63 | 0 | 0 | 0 |
| 64 | 0 | 0 | 0 |
| 65 | 0 | 0 | 0 |
| 66 | 0 | 0 | 0 |
| 67 | 0 | 0 | 0 |
| 68 | 0 | 0 | 0 |
| 69 | 0 | 0 | 0 |
| 70 | 0 | 0 | 0 |
| 71 | 0 | 0 | 0 |
| 72 | 0 | 0 | 0 |
| 73 | 0 | 0 | 0 |
| 74 | 0 | 0 | 0 |
| 75 | 0 | 0 | 0 |
| 76 | 0.0446770421786636 | - 0.0446770421786636 | 0 |
| 77 | 0.10382166902803744 | - 0.10382166902803744 | 0 |
| 78 | 0.10219882449831474 | - 0.10219882449831474 | 0 |
| 79 | 0 | 0 | 0 |
| 80 | 0 | 0 | 0 |

FIGURE 3.7: Data series from an eigenvalue calculation (imaginary part) showing 21 time steps from 60-80. Columns 2-4 represent eigenvalues 1 2 and 3 of the system.

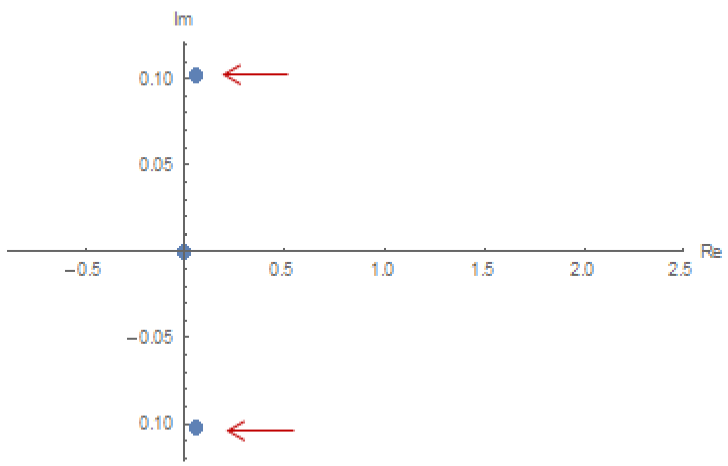



FIGURE 3.8: Real and Imaginary eigenvalues presented in graphical form. The x axis is the real part and y axis the imaginary part. Eigenvalues are represented with blue dots. This graphical form shows just one time step in the data series.

From the real vs imaginary eigenvalue plot a couple of points can be taken, two of the eigenvalues, indicated with the red arrows, are showing complex conjugate values, the real part of which holds a positive value. This shows that the system is unstable at this point in time, because at least one eigenvalue is expressing positive real values and the system, under these eigenvalues, is expressing oscillations which are expanding. The eigenvalue reference plot and data series, can be used to determine which eigenvalues hold the largest real values at any specific point in time in order to identify which eigenvalue to focus on and inspect for loop dominance over current behaviour.

Figure 3.9 is the line plot of the eigenvalues' data series for the real part of the complex values shown above. The corresponding time steps of the data series lie between time step 60 and 80, which can be seen outlined by a red box.

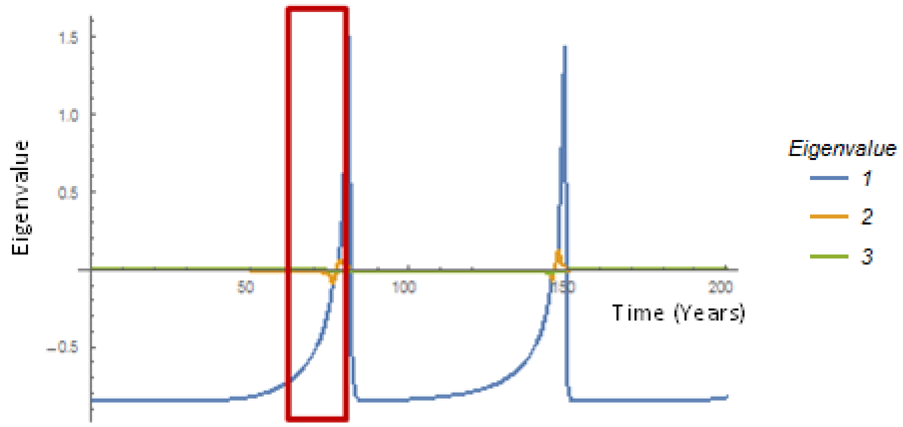



FIGURE 3.9: Real Eigenvalues plotted through time, each line represents the change in one eigenvalue and the red box highlights timesteps 60 to 80.

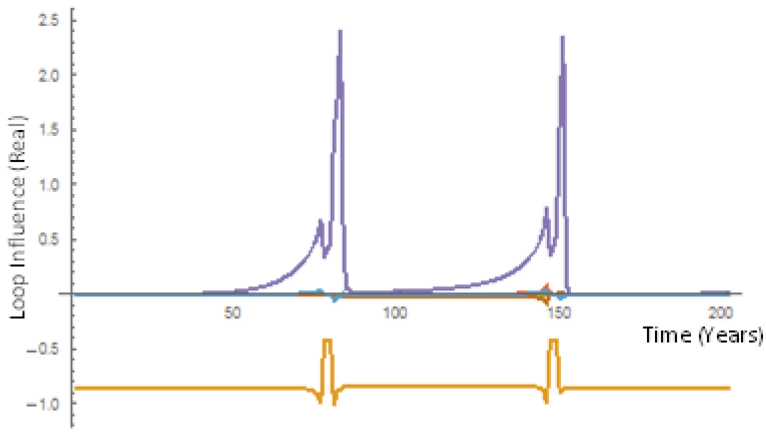
The line plot is particularly useful for seeing changes which occur to the eigenvalues through time and more notably identifying points in time where at least one eigenvalue holds a positive value as this infers unstable behaviour (i.e. exponential growth or expanding oscillations) developing in the system. Is this particular example, the line plot makes the ramp up and spike of eigenvalue 1 very clear, which may not have been picked out as easily from the data series within the eigenvalue table.

The main disadvantage of the line plot is that dynamics occurring in eigenvalues or loop gains which are not dominating the system may be overshadowed. It is therefore important to use both the data series, the individual time step plots and the line plot to investigate system behaviour. When inspecting line plots, it can often be worth producing multiple line plots which isolate each eigenvalue or each loop in order to view one trajectory at a time. Particular care must be taken when inspecting eigenvalues which sit close to zero. Small changes which occur to these eigenvalues, such as changing to positive values are deemed very important when regarding system stability, but may be overshadowed by larger scale changes which occur to greater negative eigenvalues on the same plot.

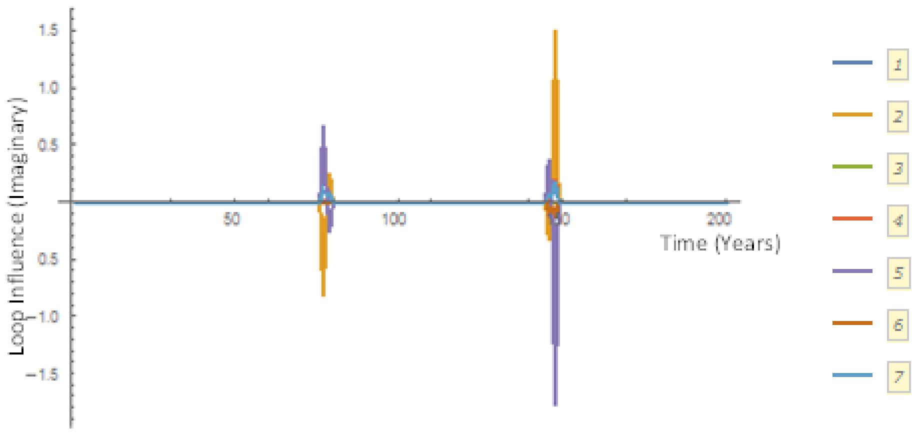
*Influence value plots*

Within this thesis, the majority of LEEA output used to gain information about the system's feedback loops and their dominance over the system's behaviour is extracted from loop influence line plots.

Due to eigenvalues being capable of holding complex values and loop influence values being derived from system eigenvalues, the loop influences can hold both real and imaginary values too, which can be plotted separately as shown in figure 3.10a and Figure 3.10b.




(A) Feedback Loop Influence - Real part




(B) Feedback Loop Influence - Imaginary part

FIGURE 3.10: Real and Imaginary results of feedback Loop influence plotted through time in graphical form, where each line represents a separate feedback structure of the system.

Within these plots each line represents a feedback loop from the SILS of the system. Each eigenvalue will correspond to a separate pair of loop influence plots which must be interpreted separately.

From these plots, the lines which hold the highest values correspond to the feedback loops which hold the highest influence over the eigenvalue and these are often the feedback loops which are paid most attention when concerned with the leading causes of system behaviour. As noted from the Eigenvalue Line plot (figure 3.9), the dynamics occurring in some of the loop influence values can be overshadowed by feedback loops expressing much higher values. To compensate for this, the graphs can either be plotted with logarithmic scales, the feedback influence values can be plotted separately, or the plot range can be altered as shown in figure 3.11.

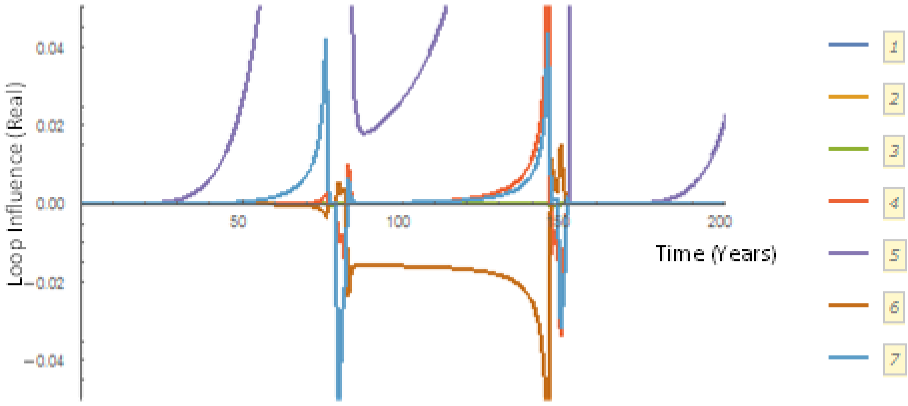



FIGURE 3.11: Feedback loop influence showing subsection of the real part with a specified shorter range for closer interpretation of results.

To identify the most influential loop structures in comparison to one another, absolute values may also be plotted, figure 3.12.

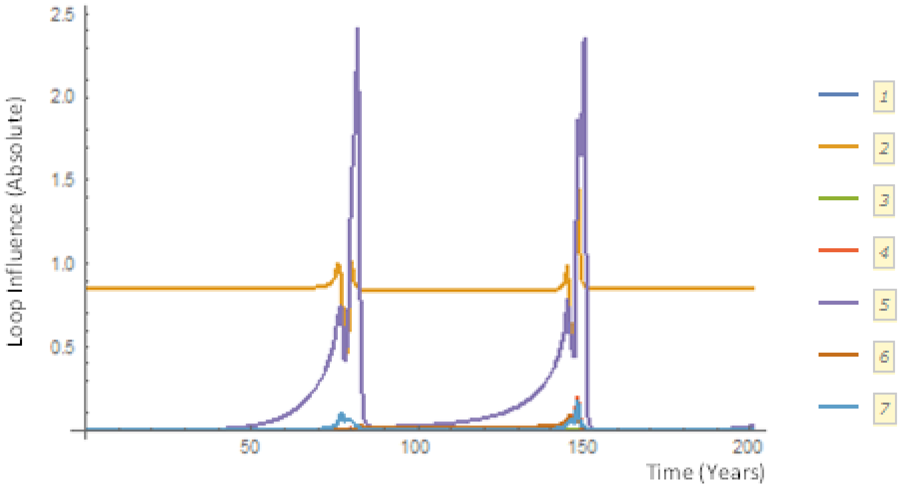


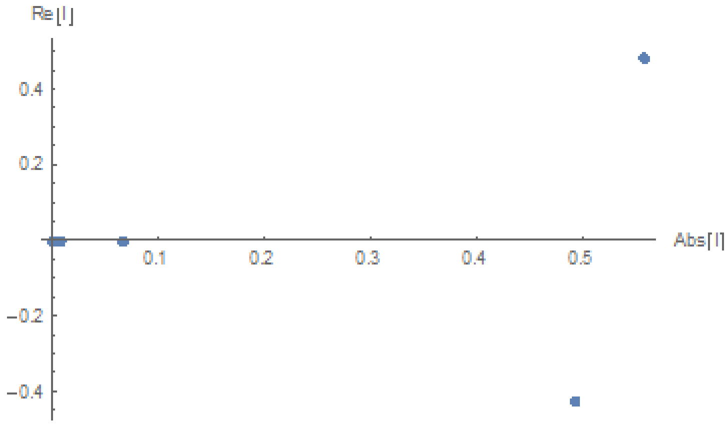

FIGURE 3.12: Feedback loop influence showing absolute values.

In figure 3.12 we see feedback loop 2, (orange line) dominating the system with all other loops sitting at or close to zero, appart from two spikes of feedback loop 5 (purple line) which can be seen building influence exponentially over time until it overtakes the influence of loop 2, only to quickly settle back down to zero.

If an individual time step for loop influences needs to be isolated, a graphical output similar to that available for the Eigenvalue data can be plotted, as shown in Figure 3.13.

| ? | 5 | 2 | 7 | 4 | 6 | 3 | 1 |
|---|---|---|---|---|---|---|---|
| Abs | 0.558 | 0.493 | 0.067 | 0.008 | 0.005 | 0.001 | 0.000 |
| Re | 0.484 | - 0.425 | 0.000 | - 0.000 | 0.000 | - 0.001 | - 0.000 |

(A)




(B)

FIGURE 3.13: A) Feedback loop influence values of seven loop structures for a single point in time of a simulation. The values are displayed as absolute values and their real part. B) Feedback loop influence values plotted in graphical form

Figure 3.13 shows time step 78, chosen here to show the loop influence output at the same time step as that chosen for the eigenvalue plot in figure 3.9. In the loop influence time step plots, each dot represents a feedback loop within the system. Unlike the eigenvalue plot, the y axis now represents the real part of the complex pair and the x axis represents the absolute values of the real and imaginary parts combined. The main purpose of this plot is to easily identify the order of hierarchy which the loops hold over the system at that point in time.

## 3.4 Regarding Error in LEEA

In order to make use of the Jacobian Matrix and its Eigenvalues, LEEA requires the system to be linearized at each point in time. LEEA therefore treats each time step as an isolated point in time and any points between the chosen time steps when eigenvalues or loop influence plots are plotted through time are subject to error because the plotting process simply assumes a linear trend between points. The extent of this error will depend on the model being analysed and the time step in question. The more complex the model's dynamics, the more frequent the models dynamics, and the larger the time step between points calculated by LEEA, the more likely the output of

LEEA is to be misinterpreted. There are two simple solutions to reduce this potential misinterpretation:

1. Reduce the time steps within the model simulation at which LEEA is calculated (i.e. from 1 to 0.125).
2. At times where there are lots of dynamics occurring within the systems output only refer to LEEAs outputs at individual time steps, prioritising individual time plots, over plots which hold the entire time series.

## 3.5 When to use and when not to use LEEA

When deciding whether or not to use LEEA on a prebuilt model system, there are a few properties of the models structure which the analyst should look for. Here is a quick guide to provide a model user with a rough idea as to whether LEEA is an appropriate analysis for a model. This guide assumes that the user has a system dynamic model in front of them and will help to determine LEEAs applicability from structural properties of the model alone.

1. *Proportion of the model included within feedback structures (Are all the stocks connected through feedback loops?)*

   LEEA is optimised when all stocks are connected in some way through feedback loops. The more stocks that are connected in this way, the more LEEA will be able to analyse behaviours occurring across the entire system. While this will not always be the case as many systems, particularly in socio-ecology, are key topics of research and not functioning as intended due to lack of feedback between key areas.

2. *Number of stocks (user should be wary if stock no. is >13)*

   One of the limitations of LEEA discussed within this thesis is the vast amounts of data that LEEA produces which is not needed to determine key drivers of the system. The number of eigenvalues needed to be interpreted within a system increases at a 1:1 ratio with the number of stocks. As the stock number increases, data output increases and interpretation takes more time. Currently the largest model where LEEA was still comfortably interpreted and had utility held 13 stocks (Oliva 2016). While LEEA will still comfortably analyse anything with >13 stocks, it is up to the user how much time and effort they are willing to spend interpreting all the data.

3. *Number of loops (the greater the ratio of loop no. to stock no. the better, within reason)*

> The number of feedback loops within a system can vary dramatically for a relatively small number of stocks, purely based on the number of interactions held between them. There is no general rule for the number of feedback loops a user of LEEA should look for in a model. However, generally speaking, the more feedback loops there are compared to the number of stocks, the more use an analyst will get out of LEEA, while the closer the ratio of loops to stock number is to 1:1, the less information LEEA will provide. The benefit of having a high ratio of feedback loops to stock no. will, of course, have an upper limit, where a model with too many feedback loops compared to its stock number may be inappropriate for the task it was intended for. The user is advised to use common sense and good modelling practice when considering LEEA, whereby often the best model is the simplest one.

If there were to be a ‘sweet spot’ for models where LEEA would have the most utility it would be on models with up to 13 stocks (as shown so far by Oliva (2016)), where the loop to stock ratio sits somewhere between 2:1 and 3:1 (of the ratios explored within this thesis) and where each stock of the system is connected to at least one other stock by a minimum of one feedback loop.

As models get larger, the best approach is initially to determine which data LEEA produces is going to be appropriate for the questions being asked. This can be achieved by taking the time to carefully inspect the eigenvalue and loop influence plots of the system, breaking it down into sections if necessary and producing new plots where the dynamics become complex.

## 3.6 Regarding LEEA’s ability to analyse system’s structure inside vs. outside a loop.

In LEEA’s analysis of a system, it is only able to account for structures that are inherently part of feedback loop structures. While LEEA successfully identifies which feedback loops are driving system behaviour, it does not directly identify which parts of a system are driving and/or kick starting those dominant loops.

The example in figure 3.14 shows a simplified version of processes involved in lake eutrophication. This structure would be part of a larger model, with many more feedback processes. The important thing to note structurally is that there is a long causal chain (multiple causal links connected together) leading up to what can be assumed to be one of the lake system’s most dominant feedback loops. Here, LEEA would pick up on the dominant reinforcing feedback loop on the system (labelled R), giving rise to the conclusion that this loop is driving system behaviour and must become the focus of an operation if the behaviour of this system is to be changed. However, LEEA is not able to account for the causal chain leading into this feedback loop, because it itself is not part of a feedback structure. There is a long line of events and system components represented here by letters a to f, which feed into the loop's structure. In this particular example, we know that crop fertilizer has a large influence on and would be a likely leverage point for this system taking into account practical and economic viability.

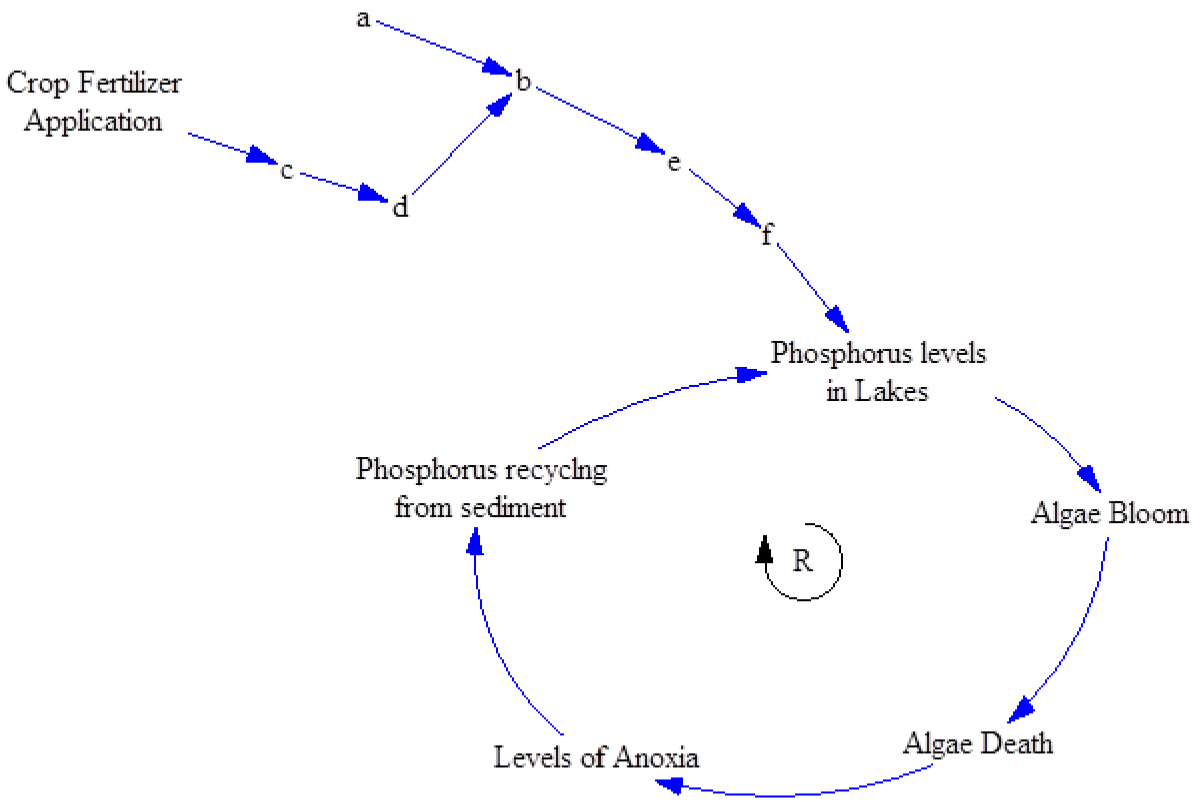



FIGURE 3.14: Example of a chain and loop structure connected to form part of a larger model structure. Letters a to f represent non specified auxillary variables within a causal chain leading into the feedback structure.

The point is that it is important to use the information gained from LEEA as a means to further explore and understand the system and not just take the most dominant loop structures as the be all and end all of system control and manipulation. When we are considering system leverage points, the best way to influence a dominant loop structure might not lie within the loop itself, but within the system components and causal links leading into that loop.

# Chapter 4: Chapter Preface

Chapter 4 initiates a series of case studies used to examine and access Loop Eigenvalue Elasticity Analysis (LEEA). The assessments primarily focus on feedback loop dominance within complex ecological and socio-ecological systems, investigating LEEA's ability to provide novel information surrounding model structure and system behaviour that cannot be gained from model conception or output alone. Each chapter from 4 to 7 represents a separate case study, with chapters 4-6 primarily based on dynamics found within lake ecology and chapter 7 based on coral reef ecology. Chapters 4, 6 and 7 are capable of acting as stand-alone case studies, each of which investigates LEEA in a separate context, with chapter 5 acting as an extension to the findings of chapter 4. Collectively the chapters act as an assessment of LEEA's application, novelty and utility, extending and reinforcing the knowledge gained in each previous chapter.

Chapter 4 begins with LEEA's application to a small system dynamic model based around a shallow lake system which undergoes eutrophication. It has previously been expressed and shown by the developers of LEEA that the technique excels at analysing collapsing or regime shifting behaviour within systems. In Chapter 4, a small eutrophic lake model is chosen to explore the capabilities of LEEA in order to show how it performs at analysing dynamic behaviours that are well known within the field of lake ecology. While LEEA's ability to analyse different dynamic behaviours is well established within the literature, it is the context of its application to complex ecological and socio-ecological models and the implications that the success of the technique could have for system understanding, modelling practices and policy design that makes this application of LEEA so important.

# Chapter 4

# Structural Loop Analysis of Complex Ecological Systems

## 4.1 Abstract

Ecosystems are complex systems that can be challenging to understand. We urgently need to assess human impacts on ecosystems which cause changes in structural feedbacks producing large, hard to reverse changes in state and functioning. System dynamics has proven to be a useful and versatile methodology for modelling complex systems given the comparative ease with which feedback loops can be modelled. However, a common issue arises when models become too large and structurally complex to understand the causal drivers of system behaviour. There is a need for an intermediate level of analysis capable of identifying causal driving structures and dynamics, regardless of model complexity. This study investigates Loop Eigenvalue Elasticity Analysis, a structural analysis technique commonly used in business and economic system dynamics models, and evaluates its utility for identifying feedback loop structures responsible for behavioural changes in complex ecological systems. The approach is demonstrated by analysing a simple lake system model that has been extensively studied in the past for its capacity to undertake critical transitions between alternative stable states. We show how the dominance of feedback loops can be tracked through time building influence over the system's behaviour decades prior to the actual collapse in the system. We discuss our findings in the context of the study of complex ecosystems, and socio-ecological systems.

**Keywords:** Complexity, Ecosystem, Socio-Ecology, Structural Analysis, Feedback Loop, Critical Transition, Modelling.

## 4.2 Introduction

Socio-ecological system models represent the interconnected nature of society and the environment. These systems are complex, able to exhibit emergence and self-organisation, with behaviours arising endogenously through non-linear dynamics (Güneralp 2006). A defining characteristic of socio-ecological systems is multiple feedback loops which collectively form the internal structure of the system (Meadows 2008), but disentangling and prioritising those feedbacks in order to understand system behaviour and develop effective policy is no simple task. Indeed, a necessary condition for labelling a system a socio-ecological system is that of a feedback loop operating between social and ecological elements. It is such feedback loops that are often the primary drivers of emergent system behaviour (Sterman 2001). Here use of the term 'driver', in the context of structural feedback loops, means the main endogenous cause of a system's behaviour. Systems may exhibit strong non-linear dynamics which are explored with the concepts of critical transitions between alternative stable states, regime shifts, and tipping points with potentially hard or effectively impossible to reverse changes in state due to properties of hysteresis (Scheffer 2009; Carpenter 2005). Consequently socio-ecological systems are hard to understand, hard to predict and difficult to manage (Meadows 2008). Maintaining socio-ecological systems in desirable states and understanding why their behaviours change through time is fundamental for economic growth, poverty alleviation and general wellbeing (United Nations 2015 2015; Scheffer 2009).

Process based, mechanistic, bottom up modelling has been used to understand socio-ecological systems (Verburg et al. 2016). System dynamics is one methodology that can be used to increase our understanding of such systems. System dynamics models are structural representations of dynamic real world systems. They take a resource based view of the world, characterising a system through a set of stocks and flows in order to represent its structure. Stocks are often, but not only, material goods, and flows are pathways of material between stocks (Ford 2010).

System dynamics has an established track record of being applied to ecological and socio-ecological modelling (e.g. Ford 2010; Meadows 2008; Dyson and Chang 2005; Saysel et al. 2002; Vezjak et al. 1998). Powerful and intuitive software packages such as Vensim (Ventana Systems Inc. 2006) and STELLA (isee systems 2016) and the exchange of established models and modelling libraries allows potentially very complex systems to be represented with models that produce output via computationally efficient numerical integration schemes. Whilst this has allowed a wide range of system dynamic models to be developed, it has, at times, produced models that are difficult to assess with regards to their overall utility in increasing our understanding of the real world systems. Such models can be challenging to parameterise, validate and interpret (Voinov and Shugart 2013). The risk is that some system dynamics models are essentially black box representations of the target system making them effectively as hard to interpret (Voinov and Shugart 2013).

In this study, we investigate a methodology that could increase our understanding, and potentially prediction, of large changes in system structure and functioning through a quantitative analysis of feedback loops as endogenous drivers of system behaviour. Rather than searching system-level properties and variables for statistical properties of impending critical transitions (Scheffer 2009; Scheffer and Carpenter 2003), we instead focus on the structural properties of the system which drives such behaviour.

We are motivated to understand how these sub-processes function collectively in producing system behaviour. One analogy is that if the system dynamics model is an organism that we can observe via its output, then we seek to understand the processes that drive such behaviour by peering within the model in order to identify 'organs' and 'physiological processes'. This analysis can be used in conjunction with evaluation of the system output, its stability, and identification of the most important individual components with respect to specific behaviours. In this study we investigate the mechanisms responsible for generating stability and instability within a system, how these change through time, whether stability or instability is dominated by an individual driver or generated by several, and how these drivers change in dominance as a system undergoes a phase shift, or transitions between stable and unstable states.

The technique explored within this study is known as Loop Eigenvalue Elasticity Analysis (LEEA). LEEA expands on the knowledge gained from linear stability analysis and graph theory, identifying a set of feedback loops within a system's structure known as the Shortest Independent Loop Set or SILS (Oliva 2015; Oliva 2004), which are collectively responsible for generating stability and instability within the system. A description of SILS, what it does and why it is necessary can be found within the Supplementary Information section 1 of this paper, under 'Loop Eigenvalue Elasticity'. LEEA then structurally analyses the loop set, identifying which feedback loops are dominating the system's behaviour at any point in time, generating a hierarchy of the influential feedback loops of the system.

Exploring ecosystem dynamics through the study of feedback loops has already shown potential to improve our mechanistic understanding of critical transitions and stability within lake systems (Kuiper et al. 2015). While the methodology of Kuiper et al. (2015) focusses primarily on food webs, their motivations of finding feedback loops within a lake ecosystem in order to determine stability and critical transitions between two regimes is similar to this study.

Previous research has demonstrated that LEEA can increase understanding of system behaviour and causal drivers across a range of model systems (Oliva 2015; Kampmann 2012; Gonçalves 2009; Kampmann and Oliva 2008; Güneralp 2006; Guneralp et al. 2005). Thus far the method has only seen limited use in the field of socio-ecology in the context of agriculture (Bueno 2013; Bueno 2012) and the Baltic cod fishery as a potential practice to be undertaken after conducting generalized modelling (Lade and Niiranen 2017). Here we extend this work and evaluate LEEA in the context of critical transitions and regime shifts, implementing loop analysis of a small lake model which can undergo critical transitions between clear and turbid states as a consequence of human drivers.

A full explanation of the limitations of the LEEA technique, along with many solutions to these limitations have been addressed by Güneralp (2006). Efforts to make the technique more automated have been conducted by Sergey Naumov and Rogelio Oliva and can be found online (Naumov & Oliva 2017).

*The model*

The model chosen to demonstrate the application of LEEA has been developed from Carpenter (2005) which formulated a simple model of a shallow lake, Lake Mendota in Wisconsin, USA, using empirical data for soil, lake and sediment phosphorus levels. The model was bistable as increasing phosphorus input in the lake produced a critical transition with a sudden shift from a clear to a turbid state. Shallow lakes are classic examples of bistable systems, capable of discrete transitions from clear to eutrophic conditions (Wang et al. 2012) and their properties are relatively well known (Carpenter et al. 2011; Carpenter 2005; Scheffer 2004; Ludwig et al. 2003; Scheffer and Carpenter 2003) with current theories attributing many eutrophic tipping events to large influxes of phosphorus through anthropogenic activity such as fertiliser runoff from farms in the lake catchment area. The model has been chosen for two principle reasons: 1) The main focus of the model's dynamic behaviour is a regime shift, allowing for an investigation of feedback loop behaviour around the point of a critical transition. 2) The model is relatively simple, allowing for a quantitative account of LEEA to be presented, and assessment of LEEA's utility for the analysis of such systems.

## 4.3 Background

*Lake Eutrophication*

Lake Mendota is a shallow freshwater lake surrounded by agricultural fields which receive ample supplies of phosphorus fertiliser. Soil erosion leads to excess phosphorus from the fertiliser, not taken up by vegetation, to be washed into surrounding streams and rivers, eventually leading to the lake. This process concentrates phosphorus runoff from the lake's catchment area into lake water where the excess of nutrients causes algal blooms to form. The formation of these blooms leads to plant death by blocking sunlight, fish death through generating anoxic conditions and phosphorus recycling from the lake sediment, which reinforces the high levels of phosphorus in the system

(Scheffer 2009; Scheffer 2004; Scheffer and Carpenter 2003). Combined, these events can cause a lake to undergo a critical transition from a nutrient poor, high biodiversity, clear water state to a nutrient rich, low biodiversity, eutrophic state.

Lake eutrophication is not only detrimental for the lake biota and biodiversity, it can have adverse effects on the system's provision of ecosystem services. Provisioning services are impacted primarily through the collapse of fisheries. Cultural services such as recreation and tourism can be affected (Dodds et al. 2008; Scheffer 2009). Attempts to return a eutrophic lake to its previous clear conditions require the levels of phosphorus in the lake to be reduced but these systems can have large hysteresis loops, making them challenging to recover (Carpenter et al. 1999b). If nutrient levels are reduced sufficiently, the lake becomes capable of undergoing a reverse critical transition, returning to its former low nutrient, clear state, however such a reverse in conditions may take many years (McCrackin et al. 2017; Wang et al. 2012).

*Feedback Loops*

Feedback loops can emerge in systems as coincidental structures when multiple interactions form between components that link an output back to its original source. Feedback loops are capable of existing in one of two forms, positive or negative. Positive feedback loops are known for generating reinforcing behaviour in a system and are associated with exponential growth or collapse, and are often found to be the causes of system instability. Negative feedback loops generate balancing behaviour within a system; they are associated with oscillatory trends or dampening and are often found to be the cause of system stability (Ford 2010). While the premise of positive and negative feedback can be easily understood, it is the interconnected nature of multiple feedback structures together that makes the behaviour they generate difficult to predict.

Feedback loops can be represented in models by constructing connections between any two system variables that interact with each other, where multiple connections join together to form a loop. These can be represented as coupled ordinary equations. Here we focus on the structural properties of the model system and the feedback loops which drive their behaviour. In 1982, Forrester (1982) first used concepts established within classical control theory, using eigenvalues to describe the behaviour of linear systems, to decompose the behaviour of linearized system dynamic models into simple reference modes. Forrester (1983) & Forrester (1982) then used this method to develop the concept of eigenvalue elasticity to model parameters which became known as Eigenvalue Elasticity Analysis (EEA). Loop Eigenvalue Elasticity Analysis (LEEA) was introduced in 1996, by Kampmann (2012) who extended the work of EEA, formally linking the strength of individual feedback loop structures to system eigenvalues. LEEA has since been applied to economic and industry modelling, with methodological refinements being made over this period (Saleh et al. 2010; Güneralp 2006;

Kampmann and Oliva 2006). LEEA is based on the ability to describe a system's behaviour through a set of eigenvalues at any moment of time and relate changes in output to influential feedback loop structures within the system.

The strength and sometimes even sign of feedback loops can alter in response to external driving. Changes which occur to a system can generate changes to the strengths (i.e. the change in output from a change in input) of individual links (the structural connection held between two system variables). Changes to the strength of individual links consequently change the strength of feedback loops, which as a result are able to generate changes to the system's eigenvalues. The 'strength' of a link may be referred to as the link gain and likewise the 'strength' of a feedback loop may be referred to as the loop gain, further explained in the methodology section (see below) (Güneralp 2006). Changes in feedback loop gain which create large changes to eigenvalues indicate high influence of that feedback loop on the current behaviour, while changes in loop gain which produce small changes to eigenvalues indicate little to no influence of that feedback loop on the current behaviour. LEEA is therefore able to identify how much an individual feedback loop has dominance over the current system behaviour, allowing the user to identify a hierarchy of loop influence.

## 4.4 Methodology

The following section gives a high level overview of the steps required to undertake LEEA. The formalism and expansions on the following steps can be found in the Supplementary Information section 1.

Calculation of loop eigenvalues and loop influence values is conducted via the following process:

1. The Jacobian Matrix of the linearized dynamical system model
2. Eigenvalues of the Jacobian matrix
3. Loop Gain
4. Loop Elasticity & Loop Influence

1. The Jacobian Matrix is an $nxn$ square matrix where each element of the matrix is a partial differential equation that represents how one stock affects another stock's derivative, when all other stocks are kept constant. The matrix therefore represents the links that exist between stocks (Gonçalves 2009).

2. *Eigenvalues* are calculated from a system's Jacobian matrix and therefore the number of eigenvalues of a system is equal to the number of stocks in that system. From linear stability theory, eigenvalues can be used to determine if a system at a fixed point is stable or unstable (Glendinning 1994). Eigenvalues are capable of being real, or complex numbers with a real and imaginary part. An eigenvalue with a real, negative value is associated with the system converging towards a fixed point. If the real part of all eigenvalues are negative, then the fixed point is stable. An eigenvalue with a real, positive value is associated with the system diverging away from a fixed point. Therefore, if the real parts of any of the eigenvalues are positive, then the fixed point is unstable. Eigenvalues which are complex are associated with oscillatory behaviour and mean the system will express either sustained oscillations, expanding oscillations or dampening oscillations if the real part of the complex eigenvalue is zero, positive or negative respectively.

   Eigenvalues which sit at or close to zero play a vital role in determining the system's state of stability. A system is determined to be unstable for any given time period while at least one eigenvalue holds a real positive value. This means that large changes in negative eigenvalues which remain negative, portray less information about a system's state of stability than small changes which happen to eigenvalues at or around zero.

   When interpreting output from LEEA, it is possible to gain a different hierarchy of loop dominance from each eigenvalue within a system as different elements will be responsible for generating each eigenvalue (Oliva 2016). This can lead to multiple outputs with apparent contrasting results as different feedback loops are capable of expressing dominance across different eigenvalues. To reduce confusion, eigenvalues holding the largest real positive values may be prioritised first for interpretation. The greater value of an eigenvalue, the more it determines the system's current behaviour. While this allows the elasticity and influence plots of some eigenvalues to be prioritised and others largely ignored, attention must be paid to eigenvalues switching dominance and to eigenvalues at or close to zero as this would change which eigenvalues must be prioritised.

3. *Loop Gain* reflects the overall strength of a feedback loop and is the product of the link gains whose associated links join together to form a loop. Link gains reflect the strength of influence of one variable upon another. An increase in an output with a fixed input infers an increase of gain within that link. Loop Gain is required to compare changes within the system's feedback loops against the system's eigenvalues.

4. *Loop Elasticity* is the change in eigenvalue relative to change in loop gain (Kampmann and Oliva 2008; Kampmann and Oliva 2006). Thus loop elasticity indicates how much a loop contributes to changes within an eigenvalue. Loops with

high absolute values of elasticity contribute the most to changes within an eigenvalue.

Loop Elasticity derives from an inherent connection that exists between system eigenvalues and loop gain through a system's characteristic polynomial ($P(\lambda)$) (Kampmann 2012). A characteristic polynomial takes the form $P(\lambda) = |\lambda I - J|$, where I is an Identity Matrix and J, the system's Jacobian Matrix. Loop gains make up the coefficients of the characteristic polynomial, while eigenvalues are determined as its roots. This connection between the two means that changes which occur in the gain of a loop can have a direct impact on the system's eigenvalues and the extent of this impact can be measured in the form of Loop Elasticity.

*Loop Influence* values determine the type of contribution a loop is having over an eigenvalue. A positive value of loop influence indicates a loop generating instability within an eigenvalue, while negative values indicate the generation of stability (Kampmann and Oliva 2006). Similar to loop elasticity, the greater the absolute value of a loop's influence, the greater the contribution that loop makes to the system's current behaviour.

## 4.5 Results

*The PLUM model*

Carpenter's study of Lake Mendota (Carpenter 2005) is represented using system dynamic terms in the form of the PLUM model ((P)Phosphorus (L)Loops in (U)Soil and (M)Sediment) (Figure 4.1).

System dynamic modelling is carried out using Vensim, which effectively numerically integrates Carpenter's series of ordinary differential equations. The model allows the user to visualise the number, polarity, position and interaction of the system's feedback loops alongside providing the ability to easily implement and manipulate the system's structure and dynamics. While representing the system's internal structure in this manner is not necessary for LEEA as it can be computed purely numerically; it is easy to do the computation in a software such as Vensim, so it is worth the effort of building a system dynamic model. Vensim is not necessary for this analysis, but it allows for easy manipulation of loop structures and numerical solutions, which facilitates model construction. Vensim also compliments the online materials Naumov & Oliva (2017), which have been designed to streamline the analysis process, including the ability to extract time series data from each simulation quickly and effectively. The packages used to calculate the results within this article were from the 2017 online material and require Vensim and Mathematica to reproduce, a link to the PLUM model and Mathematica codes used to generate the results can be found in the Supplementary Information, section 2.

The Carpenter (2005) study describes dynamic properties of phosphorus levels within a lake system as a set of coupled differential equations. This simple model, which is typically parameterised to empirical data, relates changes in the concentration of phosphorus in the water as phosphorus input into the lake increases over time. These equations model changes of phosphorus within the soil of the land (U), the water of the lake (P) and the lake sediment (M). A table listing all terms, units and descriptions is given in Table 4.1. The change of soil phosphorus over time if defined thus:

$$\frac{dU}{dt} = W + F - H - cU \tag{4.1}$$

with the runoff parameter $c$ being important in driving the dynamics of concentrations of phosphorus in the water:

$$\frac{dP}{dt} = cU - (s+h)P + rM\frac{P^q}{m^q + P^q} \tag{4.2}$$

Phosphorus can be stored or released in the lake sediment thus:

$$\frac{dM}{dt} = sP - bM - rM\frac{P^q}{m^q + P^q} \tag{4.3}$$

Derivation of these values can be found in Carpenter (2005). The equations themselves have been developed from previous studies of lake dynamics (Carpenter et al. 2003; Ludwig et al. 2003; Carpenter et al. 1999b).

Phosphorus recycling involves stored phosphorus, which has been accumulating in the lake sediment over year to decadal timescales, being released back into lake water as a consequence of chemical reactions driven by anoxic conditions at the bottom of the lake. The anoxic conditions are generated by populations of bacteria, which deplete oxygen during their decomposing of organic matter. The increase in organic matter, and so bacterial decomposition and bottom water anoxia, is a consequence of increased populations of algae and other lake biota taking advantage of the increasing levels of phosphorus in the lake water (Scheffer 2009; Carpenter 2005; Scheffer 2004; Ludwig et al. 2003). The sigmoidal function, the last term of equation 4.2 and 4.3 represents high levels of phosphorus being reintroduced to the system from the sediment through phosphorus recycling, as phosphorus levels in the water column become enriched from agricultural and non-agricultural sources.

Numerical solutions for the model were computed for 1000 years to recreate Scenario 1 from Carpenter's model (Carpenter 2005). Scenario 1 undergoes the following changes:

| Symbol | Definition | Units | Nominal Value |
|---|---|---|---|
| $b$ | Permanent burial rate of sediment P | $y^-1$ | 0.001 |
| $c$ | P runoff coefficient | $y^-1$ | 0.00115 |
| $F$ | Annual agricultural import of P to the watershed per unit lake area | $g.m^-2.y^-1$ | 31.6 |
| $h$ | Outflow rate of P | $y^-1$ | 0.15 |
| $H$ | Annual export of P from the watershed in farm products, per unit lake area | $g.m^-2.y^-1$ | |
| $m$ | P density in the lake when recycling is 0.5 r | $g.m^-2$ | 2.4 |
| $r$ | Maximum recycling rate of P | $g.m^-2.y^-1$ | 0.019 |
| $q$ | Parameter for steepness of f(P) near m | Unitless | 8 |
| $s$ | Sedimentation rate of P | $g.m^-2.y^-1$ | 0.7 |
| $W$ | Nonagricultural inputs of P to the watershed prior to disturbance, per unit lake area | $g.m^-2.y^-1$ | 0.147 |
| $W_D$ | Nonagricultural inputs of P to the watershed after disturbance, per unit lake area | $g.m^-2.y^-1$ | 1.55 |

Table 4.1: Modified from Carpenter (2005). Model parameters, nominal values and units used to implement PLUM.

| | |
|---|---|
| 0-100 years | presettlement conditions, $F = H = 0$ and $W$ is set to undisturbed conditions. |
| 100-200 years | advent of agriculture, $W$ changes to its disturbed value ($W_D$). |
| 200-250 years | intensive industrialized agriculture, $F > H$, values shown in table 1. |
| 250-1000 years | management to balance phosphorus budget of agriculture, $F = H$ and $W$ maintained at $W_D$. |

Table 4.2: Modified from Carpenter (2005). Changes to phosphorus input levels within the PLUM model for the first scenario, 0 to 1000 years.

The system has a single steady state value of lake phosphorus concentrations providing that non-agricultural inputs lie between $W$ and $W_D$ and agricultural inputs do not exceed the annual export of phosphorus through farming products ($F = H$). Beyond this regime, the system becomes bistable, having two stable equilibrium between an unstable equilibrium, with a single steady state value of much higher lake water phosphorus concentrations being found when $F$ is 1.7 times greater than $H$. Analytical solutions and graphical representation of the system's bistability can be found in Carpenter (2005).

Assuming a constant input of additional phosphorus over time, the lake system undergoes a critical transition at $\tilde{4}50$ years. Critical transition theory and resilience research often associate a change in system states with a shift in dominant feedback loops, where positive feedbacks drive a system into an alternative state through reinforcing behaviour, but due to the often complex dynamics involved, little has been done to actively show this happening (Scheffer 2009; Scheffer and Carpenter 2003). The mechanism driving the critical transition within this system is phosphorus recycling, which reinforces the amount of phosphorus within the lake water and is capable of causing large influxes of phosphorus to enter the water column long after attempts to control phosphorus input through the soil have been in place (Scheffer 2009).

Carpenter (2005) provides analytical solutions showing that the system is bistable, caused as a consequence of phosphorus recycling from the sediment. It is within its bistable state that the lake is capable of transitioning from its clear-water to turbid-water attractors following a perturbation. The bistable state also allows for a reverse transition from turbid-water to clear-water conditions to be possible but this recovery is slowed by the steady release of phosphorus from the lake's sediments. Carpenter notes that in order to induce a transition back to a clear-water attractor, either a stochastic event or deliberate management of phosphorus sediment recycling would have to take place.

A reverse transition was achieved within the model system by maintaining $F = H$ and converting $W$ back to its undisturbed conditions at 1000 years. Overall, this reduces the amount of phosphorus entering the lake water, creating a net output of phosphorus from the system over time. The reductions in phosphorus within the lake water undergo a linear decrease through time. However, a critical transition back to a clear water state does not occur until much later, around year 2050, when phosphorus levels have reduced well below levels of the original forward tip. The disparity between the point at which the forward and reverse critical transition occurs is known as hysteresis. In shallow lakes the hysteresis effect is often a result of different levels of nutrient loading (Scheffer 2009) i.e. how much phosphorus is able to enter the system within a given period of time. Results of the reverse critical transition are shown in figure 4.1 c. LEEA was then used to investigate which loop structures were generating stability and instability prior to, at and post these critical transitions

The model's three stocks U, P and M, shown as black boxes within figure 4.1a, relate to differential equations 1, 2 and 3 of the system. Each stock has an inflow and outflow (i.e. Uinput or Uoutput) shown by black double lined arrows which determine the level of material within the stock, which in this case is phosphorus density. Cloud shapes within the figure represent sources and sinks of the phosphorus, the specific locations of which are determined to be outside the scope of the model. Finally, blue arrows represent the interactions held between all model components which link together to form the feedback loops that occur between stocks. The model holds a total of seven feedback loop structures, six of which have been identified by the Shortest Independent Loop Set (SILS) and are labelled in figure 4.1a. The seventh loop is formed

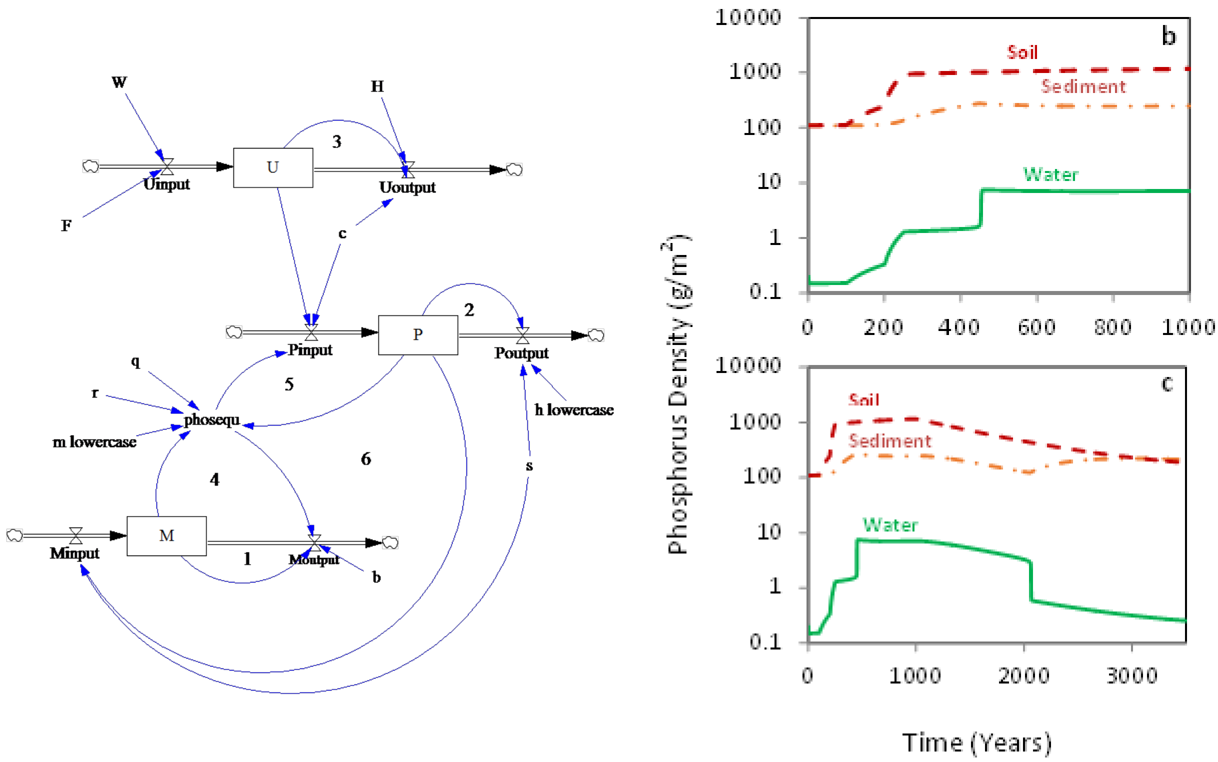



FIGURE 4.1: a. The PLUM model a structural representation of Carpenter's phosphorus equations (Carpenter 2005) containing the stocks (black boxes) water phosphorus density (P), soil phosphorus density (U) and sediment phosphorus density (M). The figure also contains system flows (black double arrows) going into and out of each stock, sources and sinks (cloud shapes) and component interactions (blue arrows). . The recreation of Scenario 1 can be seen in figure b. The extension to the Carpenter simulation, to invoke a reverse critical transition in the system can be seen in c.

from loops 4 and 5 connecting together. The seventh loop is not included within the SILS as its structure is fully accounted for within the loop set when loops 4 and 5 are calculated. A breakdown of the components within each of the SILS loops can be seen in table 4.3.

| Loop No. | Structure | Feedback Type |
|---|---|---|
| 1 | M $\rightarrow$ Moutput $\rightarrow$ M | Negative |
| 2 | P $\rightarrow$ Poutput $\rightarrow$ P | Negative |
| 3 | U $\rightarrow$ Uoutput $\rightarrow$ U | Negative |
| 4 | M $\rightarrow$ phosequ $\rightarrow$ Moutput $- >$ M | Negative |
| 5 | P $\rightarrow$ phosequ $\rightarrow$ Pinput | Positive |
| 6 | M $\rightarrow$ phosequ $\rightarrow$ Pinput $\rightarrow$ P $\rightarrow$ Minput $\rightarrow$ M | Positive |

TABLE 4.3: Loop numbers, structures and loop type within the PLUM model.

Eigenvalues, loop gains and loop influences were calculated once per year. For the forward critical transition, LEEA was calculated from the years 250-999 and for the reverse critical transition, LEEA was calculated from years 1000-2250. Outputs for the forward critical transition are shown in figure 4.2 a, b, c & d and reverse critical transition outputs are shown in figure 4.3 a, b, c, d & e.

In the forward and reverse transition of the PLUM model, we are concerned with the feedback loops responsible for maintaining the system's stable behaviour before and after the critical transition, while also understanding what changes occur to dominant feedback loops which lead to the change of state. It is therefore important that all three eigenvalues are considered throughout the time series, for those dominating at times of stability and those changing polarity leading up to and during critical transition of the system.

*Forward critical transition*

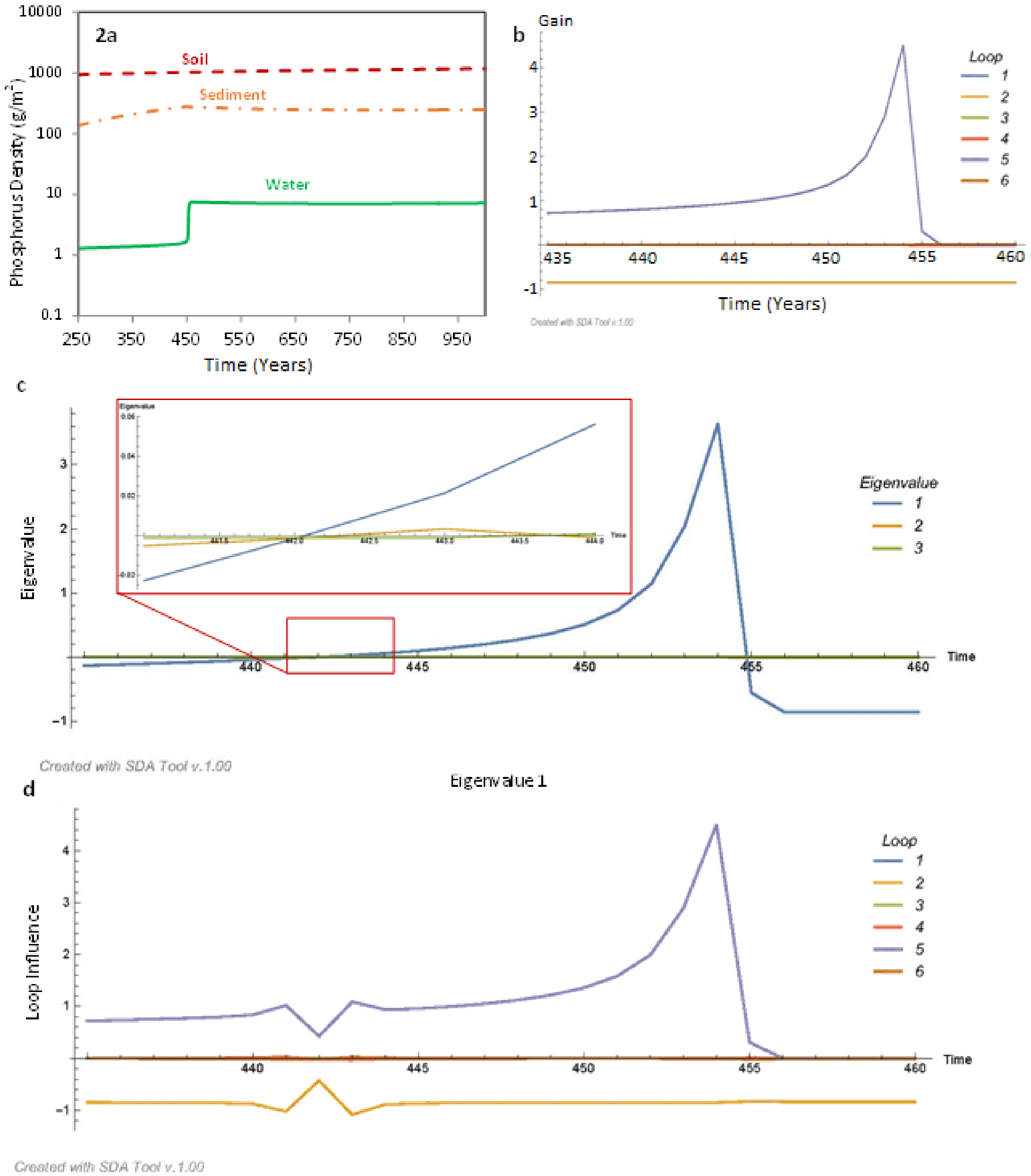



FIGURE 4.2: a) Shows the forward critical transition from years 250-999 from Scenario 1 of Carpenter (2005). b) Loop Gain values for the forward critical transition between years 435-460. c) The system's three eigenvalues calculated and plotted through time for the forward critical transition. The red box shows a zoom of the eigenvalues changing to positive polarity. d) Loop influence values within Eigenvalue 1 for years 435-460. Plots b,c and d are generated from Naumov & Oliva (2017) online material.

Figure 4.2a shows the lake system after 250 years, whereby sufficient phosphorus has been added to the system to produce a regime shift from a clear to turbid water. The transition is sharp and represented by a sudden increase of phosphorus density within the lake water at $\tilde{4}50$ years. After the transition has occurred, there is nothing within the system to reduce phosphorus input so the lake remains in a turbid state for the remainder of the simulation. As seen in figure 4.2a, there is little indication from the data that this transition is about to occur until a very narrow time window prior to the transition.

System loop gains for the six feedback loops of the forward critical transition between the years 435 to 460 can be seen in figure 4.2b. Loop gain values can give an initial impression of how the loop structures may influence the behaviour of the system, but cannot be used to directly infer drivers of system behaviour without first linking them to the system's eigenvalues.

In the forward critical transition, the real part of Eigenvalue 1 can be seen holding the highest negative real value for the first 192 years ($t = 250 - 442$) with the real part of eigenvalue 2 and eigenvalue 3 sitting just below zero at -0.00427 and -0.00115 respectively. The imaginary parts for all three eigenvalues sit at zero. In this time, the feedback loops producing the most influence within each eigenvalue are loop 2 (phosphorus water output) in eigenvalue 1, loop 3 (phosphorus soil output) in eigenvalue 2 and loop 1 (phosphorus sediment output) in eigenvalue 3. Loops 2, 3 and 1 all represent negative feedback loops within the system and express the highest values of loop influence which are all negative between 250-442 years, showing that these loops are generating stability within the system at this time. Note that during this time, the value of the real part of eigenvalue 1 has been getting less negative, exponentially increasing towards zero, with the influence of feedback loop 5 exponentially increasing in positive values, indicating an increasing level of instability building from this feedback structure.

At year 442, eigenvalue 1 switches to positive values, with eigenvalues 2 and 3 switching to positive values between years 442-443 and 444-445 respectively, as shown in figure 4.2c. This switch indicates all eigenvalues now representing exponential growth within the system, but note that this is still roughly 10 time steps prior to the critical transition being expressed within the water phosphorus of the system beginning around year 453.

The dominant loops within all three eigenvlaues, while they hold positive values within the system, are consistently loop 5 (phosphorus recycling linked to phosphorus levels in the water) and loop 2 (phosphorus water output). Loop 5 always holds positive influence values showing it is generating instability within the system and loop 2 always holding negative values showing it is acting to keep the system stable. Note that the positive values held by eigenvalues 2 and 3 are only brief and are lower in real positive value in comparison to eigenvalue 1 (figure 4.2c, red outline). The values of eigenvlaues 2 and 3 also drop back below zero to negative values at years 444 and 445 respectively, before the critical transition, while eigenvalue 1 remains positive until year 454, right up to the critical transition of the system. The build-up to and the forward critical transition of the system is therefore largely represented by eigenvalue 1, with feedback loop 5 being a dominant influence over the system's instability across all eigenvalues leading up to the transition and loop 2 attempting to act as a counterbalance during this time (figure 4.2d). As the system reaches its critical transition, Loop 5 can be seen spiking in positive values, thus becoming the most dominant feedback structure within the system at the point of transition.

During the critical transition (years 445  458) and post the transition (458  999) we see all eigenvalues dropping to negative values as the system stabilises into its new eutrophic state. In this new state, eigenvalue 1 falls to the largest negative values of the three eigenvalues once more, being dominated by feedback loop 2 with all other feedback influence values sitting at or close to zero showing little to no influence within this mode. Eigenvalue 2 shows some instability generated from loop 6, but not enough to override the stabilizing influence from loops 4 and 2, which are both negative feedbacks. Eigenvalue 3 becomes solely dominated by the stabilizing influence of negative feedback loop 3 with all other feedbacks sitting at or close to zero.

*Reverse critical transition*

Figure 4.3a shows how an overall decline in phosphorus levels into the water over time eventually leads to a reverse critical transition, which can be seen occurring at 2050 years. Similar to the forward critical transition, there is little indication that the reverse critical transition is going to take place from the simulated data up until a small time window beforehand. Once the reverse critical transition occurs, the system remains in its new clear state for the remainder of the simulation as phosphorus levels in the water continue to decrease. System loop gains for the six feedback loops of the reverse critical transition between years 2040 to 2065 can be seen in figure 4.3b.

The reverse transition tells a similar story in terms of the feedback loops which are influencing the transition. In the reverse transition, the critical transition back to a clear water state occurs just after year 2050, despite phosphorus input being cut from the system back in year 1000. The loop influence of eigenvalue 1 between years 1000-2049 is dominated by loop 2 with all other loops sitting at or near zero, eigenvalue 2 has high influence from loop 6 holding positive values, but being dominated by loops 2 and 4 with negative values and eigenvalue 3 has only loop 3 influencing it holding a constant negative value. At year 2049 eigenvalues 1 and 2 first show a positive real value (figure 4.3c), prior to this only negative feedback loops dominate the system.

Between years 2049 and 2052 both eigenvalues 1 and 2 transition into positive values holding the same value and therefore their loop influences must be looked at simultaneously to determine dominant structural drivers. In this period, similar to the forward transition, eigenvalue 1 is dominated by a stabilising influence of loop 2 and a growing destabilising influence of loop 5 (figure 4.3d). At the same time, eigenvalue 2 is also being dominated by loop 2 and loop 5 (figure 4.3e). At year 2052, the positive

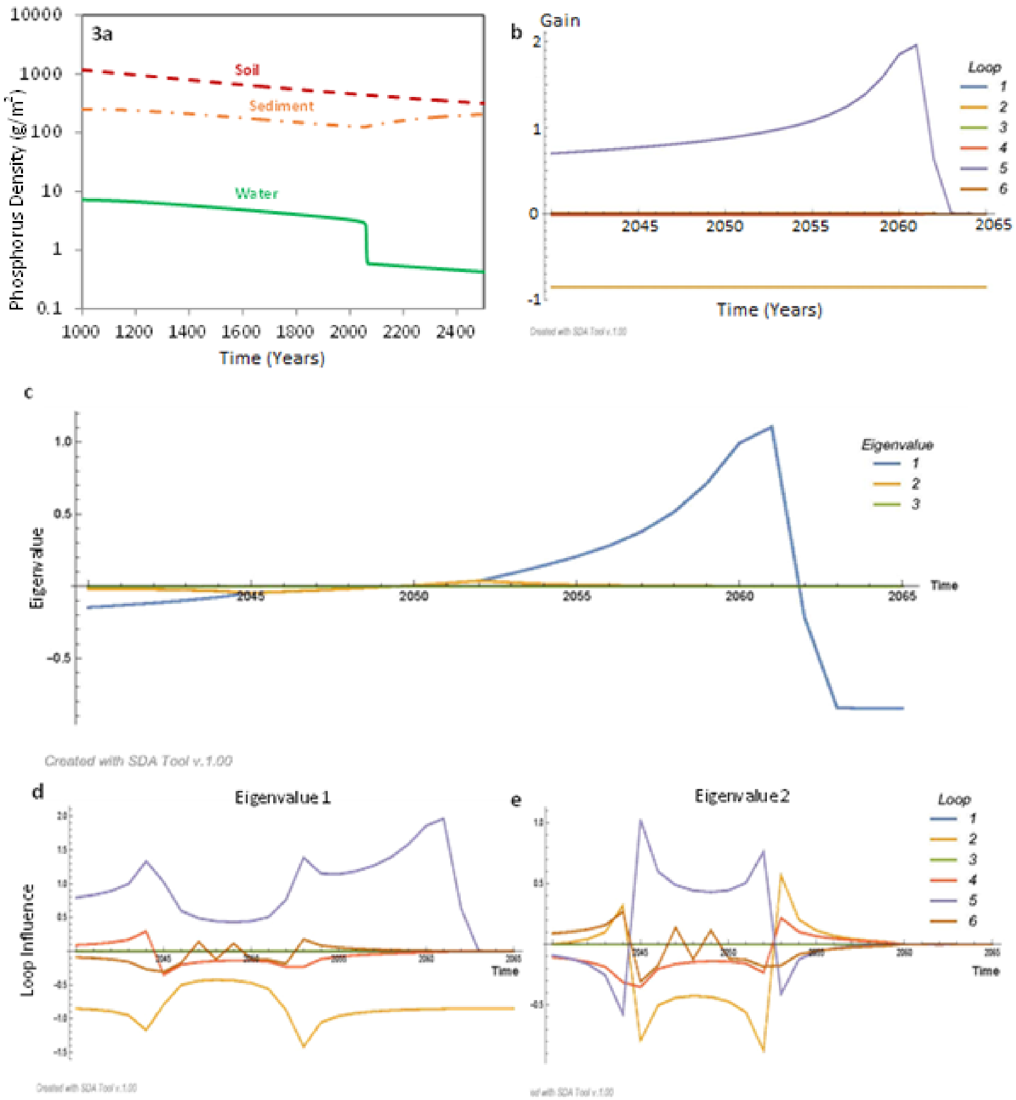



FIGURE 4.3: a) Shows the reverse critical transition from years 1000-2500 from the extension to Scenario 1 of Carpenter (2005). b) Loop Gain values for the reverse critical transition between years 2040 and 2065. c) The system's three eigenvalues calculated and plotted through time for the reverse critical transition. d) Loop influence values within Eigenvalue 1 for years 2040 to 2065. e) Loop influence values within Eigenvalue 2 for years 2040 to 2065. Plots b, c, d and e are generated from Naumov & Oliva (2017) online material.

value of eigenvalue 1 begins to increase, while eigenvalue 2 falls back to negative values and eigenvalue 3 only takes on positive values at year 2059, when eigenvalue 1's real value is already much greater. Eigenvalue 1 can therefore be concentrated on for the remainder of the reverse transition as it continues to be the dominant eigenvalue within the system. The growing influence of feedback loop 5 continues in eigenvalue 1 until year 2061, just at the start of the transition of the system back into a clear water state, whereby its influence then falls down to zero upon the system stabilizing into its new state.

From years 2061 onwards all eigenvalues drop back into negative values with eigenvalue 1 holding the highest negative value. From 2061 to the end of the simulation, eigenvalue 1 is dominated by loop 2, eigenvalue 2 is dominated by loop 3 and eigenvalue 3 is dominated by loop 1 all of which hold constant negative values throughout the clear state.

*Overview:*

Overall across both the forward and reverse transitions of the system, the behaviour of the system leading up to and during the transitions can largely be seen expressed by eigenvalue 1, which is primarily dominated by the stabilizing influence of negative feedback loop 2 (phosphorus output from the water). However, leading up to both critical transitions, positive feedback loop 5 can be seen growing in influence, overtaking the stabilizing influence of loop 2 years prior to both critical transition events. In both scenarios, once the critical transition has started its relatively sudden shift into the alternative state, the influence of loop 5 undergoes dramatic decline back to zero, whereby feedback loop 2 begins to dominate the system once again as it settles into its alternative state.

These results show that while the system is in either of its stable states it is largely dominated by the stabilizing influence of negative feedback loops. However as the system nears either its forward or reverse transition, the stabilizing influences become overtaken by the destabilizing behaviour of a positive feedback loop. The instability being generated by the positive feedback loop eventually spikes, at which point the system undergoes a sudden shift into its alternate state and the influence of the positive feedback loop drops.

*Loop connectedness*

Within the Loop influence plots of both the forward and reverse transitions (figures 4.2d, 4.3d and 4.3e), loops 2 and 5 can sometimes be seen having equal, but opposite values. The output appears to suggest that the two loops are linked somehow and are acting to counterbalance each others impact. This may be what Kampmann and Oliva (2006) refer to as 'ghost loop' phenomena, where the seemingly high dominance of a feedback loop is counteracted by an equal yet opposite feedback loop which arises as an artifact of model design as the two loops share similar dynamics. Feedback loops 2 and 5 are both connected to the Pwater stock and so will innately share similar components within the Jacobian Matrix. Although they are linked in this way the other dynamics within their loops structures still make a simple, yet fundamental difference to how they react to changes within the Pwater stock: Loop 5 is a positive feedback loop, meaning changes within Pwater get reinforced, while Loop 2 is a negative feedback loop, meaning changes get opposed. This may give some explanation as to why the two loops can be seen showing equal yet opposite dynamics within the loop influence plots.

All eigenvalues and loop influence plots can be recreated using Naumov & Oliva (2017) online material using the PLUM forward and reverse transition Vensim models, the locations of which can be found in Supplementary Information section 2.

## 4.6 Discussion

Overall the results of LEEA between the forward and reverse critical transition present similar results. The instability that is seen building up in the system years prior to the critical transition events can largely be attributed to Loop 5, a positive feedback loop (figure 4.4a) containing phosphorus recycling, while stability in the system is maintained by Loop 2, a negative feedback loop (figure 4.4b) responsible for the outflow of phosphorus from the lake.

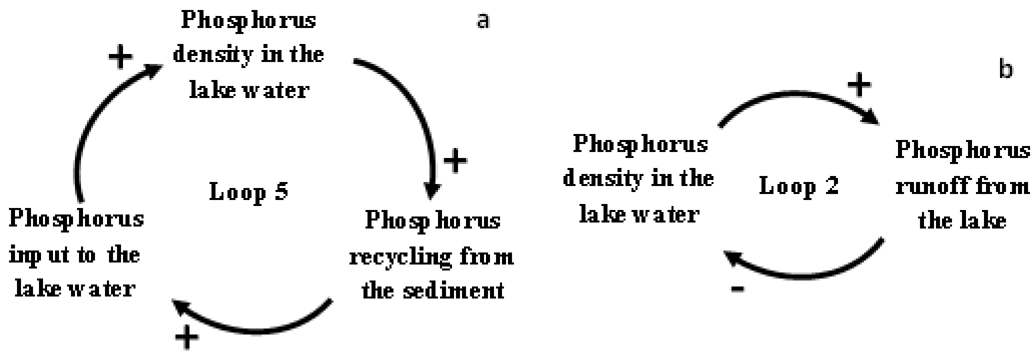



FIGURE 4.4: A) Shows Loop 5, the positive feedback loop shown from LEEA to generate instability within the system leading up to the critical transitions. B) Shows Loop 2, a negative feedback loop responsible for most of the stability within the system.

In this study, LEEA has been shown to reinforce existing knowledge about phosphorus recycling as a key driver of critical transitions during system bistability through the analysis of a shallow lake model from Carpenter (2005). The analysis is able to show instability being generated within the system years prior to any indication of a critical transition from the simulated data, shown in figures 4.2c and 4.3c. Of particular interest was the ability to track the influence and dominance of the phosphorus recycling loop (Loop 5) through time as the system nears its forward and reverse critical transition. While instability within this system and its change of state are known to be driven by phosphorus recycling activating from the lake sediment, LEEA allowed the observation of the growing influence of this key feedback loop through time, with respect to all other feedback mechanisms in the model. Loop 5 represents a latent self-reinforcing mechanism which always exists within the system, but only becomes dominant within the system near its transition. Using LEEA, the dominance of the phosphorus recycling loop can be tracked simultaneously with the output of the model, showing the exact point in time at which phosphorus recycling becomes the dominant mechanism within the system overriding the stability generated through phosphorus outflow.

Here we have shown LEEA to work on a relatively simple lake model, but the technique could help us to understand many other ecosystems and socio-ecological systems. Some examples of classic models where LEEA could be applied include Wonderland (Milik et al. 1996) and subsections of WORLD3 (Meadows et al. 2004), alongside Early Warnings of Catastrophe (Boerlijst et al. 2013). LEEA could be used to investigate structural drivers in systems which experience alternative stable states, sudden phase shifts or oscillations, or be used on stable systems to identify the mechanisms responsible for maintaining the system's stability. LEEA is capable of being used in parallel with monitoring efforts of real world systems, allowing for field monitoring of highly influential system components to be prioritised and policy interventions to be designed around a greater understanding of system drivers. While LEEA is good at showing the user how a system is changing and what is driving that change, it is not able to show what a specific change will lead to and cannot therefore be used as an early warning system. LEEA would be best used alongside early warning signals to promote when they should be used and help determine control mechanisms of an evolving system.

In alternative studies, LEEA has also shown high potential for use in system management and policy implementation through the identification of system leverage points using an extension to the base analysis known as Dynamic Decomposition Weights Analysis (DDWA) (Oliva 2015; Saleh et al. 2010). Identifying leverage points is thought to be a key practice in order to control the behaviour of ecological systems (Meadows 2008) and is of particular interest when concerned with economic development and ecological sustainability. Future exploration of socio-ecological dynamics using both LEEA and DDWA for the purpose of quantitative identification of leverage points will be a natural and justified progression to the work presented within this study.

A strength of LEEA is that it can be used across a range of existing approaches. Consider the following scenarios:

Scenario 1: Ordinary or partial derivative equations that fully describe the system are known and solutions can be found analytically (i.e. Carpenter 2005). Scenario 2: There is initially no mathematical representation of the system. Systems dynamics software such as Vensim is used to produce graphical functions that represent system variables and their interactions. Ordinary and partial differential equations can then be derived from these graphical functions. Scenario 3: Ordinary or partial derivative equations that fully describe the system are known but analytical solutions cannot be found. Numerical computation is used to determine solutions.

For all three scenarios, LEEA proceeds on the basis of computing eigenvalues and loop gain, dominance, and elasticity using these equations.

From Scenario 1, key system components and how they interact with each other are well known. LEEA can be used to understand which feedback mechanisms within the system are contributing most to system output and driving dynamic behaviours at any given time. This information cannot be understood from only viewing the system's equations or finding solutions. That is, identification of feedback loops may be possible via inspection with relatively simple models, but the influence or dominance that these loops have on system behaviour is not trivial. Scenario 1 is ideal for executing steps 1-4 of LEEA. Having a system's dynamic behaviour described through a series of differential equations means that the system's feedback loops are already described mathematically through the interactions within and between the system stocks. Using this information, a model can be built to immediately visualise the feedback loops and system eigenvalues and loop gains can be calculated.

In Scenario 2, the user may be less concerned with system output and more concerned with which model components to consider and the interactions between them. In this scenario, the functional forms of the system are not specified and so the model may start as a simple structural skeleton of variables and interactions, containing no formulae. Steps 1-4 of LEEA cannot be calculated until each system interaction has been described mathematically. Once this has been achieved, LEEA can be used to explore which structures within the model are generating the most influence over the system, but also identify structures within the model producing little to no impact. The user can use this knowledge to increase model efficiency prioritising model development on the structures generating the most influence over the system output.

In Scenario 3, the benefits of using LEEA come from the combined benefits of scenarios 1 & 2. With the system being relatively well known but system components and interactions still being explored, in order to reach a simple, yet accurate representation of the target system, LEEA can be used to both increase model efficiency and used to increase understanding of mechanisms driving key behaviours within the system output.

*A breakdown of LEEA Benefits and Limitations*

| **Benefits** | **Limitations** |
|---|---|
| • Tracks a system's behavioural drivers through time, with potential to inform policy implementation, scenario testing and overall control of the system.<br>• Increases structural understanding of model through identification of feedback loops, loop lengths, locations and components.<br>• Can be used with empirical data input into the system dynamic model to track system change in parallel with a model's real world counterpart.<br>• Alongside identifying key structures, LEEA is able to identify structures not contributing to system behaviour, with potential to improve model efficiency by identifying unnecessary model components and structures.<br>• Able to show changes in behavioural drivers prior to a tipping point when no changes are expressed in model output.<br>• Technique can be applied to both whole system models and simple models alike as it is not affected by subject matter. | • Computationally demanding: At each time step the system must be linearized, the Jacobian Matrix must be generated and eigenvalues must be calculated.<br>• Analysis of results gets increasingly difficult with increasing model size as eigenvalue number increases 1:1 with model stock number (Kampmann and Oliva 2006).<br>• A lack of automation or availability as part of a software programme. This is being improved with each iteration of LEEA's associated algorithms (see Analysis of more complex models below).<br>• Eigenvalues can express similar levels of dominance simultaneously, reducing one's ability to specify individual behavioural drivers. |

LEEA's primary utility is its ability to identify dominant loop structures driving a system's behaviour. Unfortunately, LEEA's main limitation is that its output becomes harder to interpret with increasing model complexity (Oliva 2016; Kampmann and Oliva 2006). This occurs as the number of eigenvalues that the system possesses increases in direct proportion to the number of stocks in the model. In turn this increases the potential number of eigenvalues that must be compared when interpreting loop elasticity and loop influence. This raises potential issues with models of socio-ecological systems such as Zhu et al. (2015) and Guan et al. (2011), as these models hold a greater number of stocks (6-8 stocks) and dozens more auxiliary variables per stock than the PLUM model explored in this study. Following on from a consideration of Scenario 2 above, LEEA can be used in conjunction with incremental increases in model complexity or as a technique to evaluate alternative models. For example, if adding further stocks and interactions to an existing model do not affect loop dominance, then there may be limited utility in including such elements into an existing model. Only where there are changes to the feedback structures, should additional model components and interactions justify the additional computation costs.

*Analysis of more complex models*

In the early stages of LEEA's development, analysis automation was low and output generation time was a serious limitation of the technique. Algorithms specifically designed to help with LEEA's automation greatly improved the techniques accessibility (Oliva 2015), but the computational power and run time, particularly concerning large, complex models (>10 stocks), limited the LEEA's utility to an extensive range of system models. In recent years, advancements made to the algorithms associated with LEEA and its companion technique, DDWA, have made vast improvements to the speed of execution (Naumov & Oliva 2017). Development of better heuristics and display formatting have also been introduced to support the interpretation of the output, with recent studies showing the techniques successfully being executed and interpreted on models of 13 and 10 stocks (Oliva 2016; Oliva 2015).

## 4.7 Conclusion

This study has shown how a structural loop analysis technique known as Loop Eigenvalue Elasticity Analysis (LEEA) can be applied to analyse the dynamics of a simple model of a shallow lake system that undergoes critical transitions between clear and turbid states. Analysing a system which contains feedback loops can help reveal significant loop structures operating within a system's behaviour: a non-trivial problem even with apparently simple systems. LEEA is able to show how feedback loops can be identified and classified in terms of their dominance, stabilising, and destabilising contributions. The results of this particular study show how growing levels of instability within the system can be largely attributed to an individual feedback loop, where the instability began to grow years prior to a critical transition. There is global significance for model scenario testing, policy implementation and ecological conservation to enhancing our understanding of socio-ecological systems, and the mechanisms for which they can be managed. LEEA was shown to be capable of providing benefits across multiple levels of prior system knowledge, allowing the user to identify key structural drivers of system stability and dynamic behaviour. Careful interpretation of LEEA output is required for more complex models. This can be regarded as a limitation of the technique, or alternatively a more robust and effective methodology for the analysis of complex models given that there are often no simple explanations for system's behaviour under such situations.

## Chapter 5: Chapter Preface

In the thesis so far, Loop Eigenvalue Elasticity Analysis (LEEA) has been selected against multiple alternative structural analysis techniques as having the greatest potential to be utilised and improve our understanding of socio-ecological dynamic models. In chapter four, LEEA was carried out on a small (three stock) system dynamic model of a shallow lake system capable of expressing hysteresis. LEEA was not only shown to be compatible with the model's dynamics, but able to expand on the structural information gained from the model as feedback loops were analysed for their influence over system behaviour. Chapter 4 evaluated LEEA's applicability to analyse a complex ecological system model, not just in ability to provide novel information, but showing that LEEA's methodology was well suited to exploring the critical transitions of an ecological system.

Alongside exploring LEEA's potential benefits, the limitations of the analysis were also accessed with a focus drawn to LEEA's ability to perform and be interpreted on larger, more complex models. In this chapter, the limitations which are inherent to LEEA are explored further. When examining the full extent of LEEA's applicability, novelty and utility, it is important to understand the extent to which its strengths and weaknesses may affect choices made by the model user. Choices surrounding LEEA may include whether it is the right analysis tool to use for the dynamics of the system in question. Chapter 5 explores the limitations which a model user of LEEA may come across and discusses the extent to which LEEA is applicable to all dynamic models of complex ecological and socio-ecological systems.

# Chapter 5

# PLUM Model Extensions

## 5.1 Abstract

Loop Eigenvalue Elasticity Analysis (LEEA), has shown to both support and be tangible with a small, yet complex ecological model and provide novel information regarding dynamic system drivers. While LEEA is still in its relative infancy, the identification, tracking and quantification of feedback loops as drivers of complex behaviour has shown great promise, but the application of LEEA to the field of socio-ecology is rare. Now that structural loop analysis has shown utility within a small ecological model, it is important to explore the performance of the analysis when applied to models of increasing size and complexity. Models within the realm of ecology and socio-ecology take many different forms, complexity and sizes which are largely determined by the goals of the project, the environment they represent and social interaction known to take place within the system. It is important to investigate the utility and limitations of LEEA with increasing model size and dynamic complexity for it to be considered within the wider context of socio-ecological systems.

Among the known limitations of Loop Eigenvalue Elasticity Analysis, its ability to be applied to large dynamic models and the user's ability to interpret its output is known to become increasingly difficult with model size. There have only been a few studies which investigate this limitation, all of which cross compare entirely separate systems and none of which investigate models within an ecological or socio-ecological context. This study acts as an extension to the modelling and analysis work conducted on the PLUM model in Chapter 4. LEEA's limitations, when used on larger models, is investigated by incrementally increasing the complexity of the PLUM model through the addition of stock variables and feedback mechanisms on the base structure, which leads to a cross comparison of LEEA outputs for the effort required to run and interpret the analysis' output. The work concludes that while it is physically possible to run LEEA on any system dynamic model, the practical application will not always

provide useful information. In its current stage of development, LEEA works best when used on small (>3 stock) to medium (10-13 stock) models. Conducting LEEA on much larger (>13 stock) models will result in greater time spent on preparation, computer simulation and code runs. More effort and time will also be required for user interpretation of analysis output, which may be just as complex if not more so to understand than the original system.

## 5.2 Introduction

Socio-ecological systems are complex by nature, connecting social structure to natural habitats and expressing dynamic behaviours determined by an interconnected network of feedback mechanisms. Simulating these complex dynamics through modelling techniques helps us to understand and explain the properties of our worldly systems at a level where they can be tried and tested with no detriment to their real world counterparts. It is important to keep exploring and developing new techniques to gain greater understanding over our models and model outputs to ensure that we are in the best position possible to translate model simulation back to real world action and decision making.

As part of the meta-analysis of Loop Eigenvalue Elasticity Analysis (LEEA) and its application to ecological and socio-ecological dynamic models, it is important to acknowledge and explore its limitations. The limitations associated with a technique can determine whether it is worth pursuing for a specific model or project aim and largely determine whether it has value within the wider context of resilience, sustainability and conservation.

This work directly follows modelling and structural loop analysis of the PLUM (Phosphorus Loops in lake water, soil(U) and sediment(M)) model, originating from the works of Carpenter (2005) and built upon within Chapter 4. The PLUM model was generated in order to test the practicality and applicability of LEEA on a small, yet complex ecological model. The application of LEEA to the PLUM model was achieved with relative success, with results showing how the dominance of feedback structures could be tracked leading up to a critical transition of the system, which had not been done before. Despite the technique showing potential to be explored in a wider context of ecological dynamics, it has also been noted that the results of LEEA become increasingly difficult to interpret with increasing system size, particularly for users unfamiliar with the technique (Kampmann and Oliva 2006; Kampmann 2012).

To explore how model size and complexity can impact a user's ability to make use of LEEA's output, the PLUM model has been extended, generating two further model iterations, PLUMGov and PLUMPlus. Here model size is determined by the number of variables (including stock, auxillary and constants) and model complexity is determined by the dynamic properties of the variables and amount of interactions held between all the variables. Overall, each model represents a greater number of feedback loops capable of driving the system's behaviour than the last. Feedback structure, model size and dynamic behaviours have all been increased from the original PLUM model in order to compare and contrast analysis output across a range of developing complexity.

The extensions of the PLUM model maintain critical transitions as their primary dynamic of interest within the system. By running LEEA on all three iterations of the PLUM model, it will be possible to compare these critical transition events in terms of loop influence and speculate whether drivers of the transition are as easy to interpret within the PLUM model as the number of model components, feedback loops and overall model complexity increases.

The main difference between the PLUM model and its extensions is that structurally, the PLUM model is classified as an ecological system as all social driving pressures act unilaterally into environmental variables and all feedbacks are internal to the environmental side of the model. Alternatively, the structures of the two extension models are much closer representations of socio-ecological models as they also incorporate feedbacks from the environmental components back to the social components.

In addition to the three PLUM models, LEEA is also carried out on the 'Yulin City Model', originally developed by Wang et al. (2011). The Yulin city model is not connected to the PLUM model, but is analysed as it represents a set of much larger system dynamic models which hold a relatively low number of feedback loops for the number of stocks. Exploring LEEA across a range of model systems helps to determine where LEEA can be used to great effect, why its application can be limited and where its methodology would be preferred over other structural analysis techniques.

All versions of the PLUM model simulate the concentration of phosphorus in lake water, containing endogenous system variables which directly and indirectly impact phosphorus density. Each model is a development on the last, simulating critical transitions between clear and eutrophic states. Therefore each model, ordered PLUM, PLUMGov and finally PLUMPlus, encompasses more system variables and interactions than the last and generates a greater number of feedback loops in the process. PLUMPlus and the Yulin city model also incorporates a greater number of model stock variables, (7 stock and 19 stock respectively) than the original PLUM model.

The study looks at LEEA's output when used on a dynamic behaviour (a critical transition) that remains consistent between models, but exists around varying levels of complexity. The aim is to investigate how effective LEEA is at identiying a key behaviour among multiple dynamics and an increasing number of feedback mechanisms.

## 5.3 Background

It has long been acknowledged that system structure is capable of driving system behaviour through feedback loops and causal links, without the need for exogenous driving pressures (Richardson 1996). However, it has also been made clear that there is a distinct lack of tools with which to explore and analyse system structure in system dynamic applications (Güneralp 2006), particularly when it comes to the field of socio-ecology (Lade and Niiranen 2017).

Kampmann (2012) explores how feedback loops are key drivers behind system behaviour and how they are not independent structures from one another. While there are no general formula for how many feedback loops a system of certain size should contain, Kampmann explores that within a maximally connected system, the number of feedback loops in a system with $n$ state variables and $p$ auxiliary variables should increase by $(n-1)!(n+1)^p$ (Kampmann 2012). Simply put, the number of feedback loops in a system increases at a faster rate than any new links or variables that are introduced. A single new link could lead to multiple new feedback loops being generated. With this in mind a relatively high number of feedback loops, and therefore high levels of complexity may be established in models without introducing an excessive amount of new variables and variable interactions.

In their review of LEEA on three separate models, Kampmann and Oliva (2006) state that the utility of LEEA results, and therefore our ability to acquire serviceable information from LEEA, is dependent on the nature of the system's dynamics (Kampmann and Oliva 2006). They also state that LEEA results can become increasingly difficult to interpret with model complexity. The PLUMGov and PLUMPlus models have been designed to introduce both more dynamics and higher levels of complexity into the PLUM model. While the output of these models is abstract compared to the PLUM model, they are used to explore the difficulties which could be encountered while running LEEA analysis.

LEEA's limitations associated with larger models is not primarily a limitation of the ability to carry out the technique or generate results. The technique's algorithms have been improved in recent years to cope with model compatibility, run speed and data visualisation (Naumov & Oliva 2017). The prime limitation comes from the user's ability to interpret the results.

Previous accounts of using LEEA have shown the method to be successful in complex models of stock size up to and including 10 and 13 stocks (Oliva 2016, Oliva 2015) and automation and algorithms surrounding LEEA have been improved within the last decade (Naumov & Oliva 2017). To build on from these findings and from that of the previous chapter, we situate our approach by applying LEEA to two extensions of the PLUM model, originated from Carpenter (2005) and the Yulin city model from Wang et al. (2011), each progressively larger and more complex than the last. Not only does this allow for the application of LEEA to be examined on a larger model, but on three models of comparable dynamics and evolving complexity, an exercise which has not been conducted in previous studies.

## 5.4 Methodology

Two system dynamic models, PLUMGov and PLUMPlus have been created in order to explore some of the main limitations of Loop Eigenvalue Elasticity Analysis (LEEA). The models have been constructed using Vensim software (Ventana Systems Inc. 2006) and are extensions of the PLUM model, based of equations from Carpenter (2005). The method for carrying out LEEA is exactly the same as described in the methodology section and Chapter 4 of this thesis alongside Kampmann (2012) where the main calculations are Loop Elasticity $\frac{\partial\lambda}{\partial g}\frac{g}{\lambda}$ and Loop Influence $\frac{\partial\lambda}{\partial g}.g$, where $\lambda$ is the eigenvalue and $g$ the loop gain.

The PLUM model is a construct of three differential equations describing phosphorus levels in the water of the lake, soil of the land and sediment of the lake bed developed within Carpenter (2005).

In order to test LEEA on a model set with increasing complexity, the base PLUM model could be taken in countless directions with the addition of extra auxiliary variables, constants, parameter interactions, feedback loops, causal chains and stocks. Increasing the complexity of the original PLUM model has been approached by maintaining the critical transition of the system as the main focus of dynamic behaviour. The dynamics which are implemented to increase the model's complexity are chosen primarily as drivers which act upon or are influenced by the level of phosphorus within the system, which is known to be a key driver of lake algal blooms and critical transitions (Carpenter 2005, Scheffer 2009).

While the PLUM model used dynamics and empirical data specific to Lake Mendota, Wisconsin USA (Carpenter 2005), the dynamics implemented within the PLUMGov model and PLUMPlus model are not based on a specific real world system, or existing models. PLUMGov and PLUMPlus are designed purely for the exploration of LEEA's limitations, making use of generic dynamic functions associated with lake systems and arbitrary values for their variables. Variables and dynamics incorporated to extend the PLUM model are based on social and ecological components known to affect lakes at a local to regional level, including a local growing human population, migration and organisms present in a typical shallow lake food web. The dynamics of these properties have been implemented based on dynamic equations able to reflect a behaviour of that parameter i.e. the human population is implemented to experience exponential growth, while the organisms of the lake express Lokta-Volterra (predator prey interactions).

For the purpose of model construction, model simulation and results, the running of LEEA and analysis evaluation, PLUMGov and PLUMPlus have been separated into two sections.

## 5.5 PLUMGov Model

The PLUMGov model has been designed to simulate both a forward critical transition and a reverse critical transition in lake water phosphorus levels as in the PLUM model. The structure includes seven loops in total and addresses the lack of feedback between the lake water phosphorus levels ($P$) and human activity driving the soil phosphorus levels ($U$).

PLUMGov represents a relatively small increase in complexity from the original PLUM model with the addition of a single variable and two new interactions, but no new stocks. As a consequence of the new features, the PLUMGov model holds one additional feedback loop from the original six within the PLUM model. More importantly, the stock '$U$', which represents the levels of phosphorus within the soil of the lake catchment, becomes an intergrated part of the model's interconnected feedback structures. With no stock variables being added, the PLUMGov model still only has an order of three, meaning that it is based on three dynamic equations. The new component added to the PLUMGov model acts as a governmental influence, able to respond to rising levels of phosphorus within the lake system internally. Government influence over the system's phosphorus levels is implemented as a relatively simple sigmoidal function, whose role is to reduce agricultural phosphorus input when the lake water phosphorus levels become too high.

In the original PLUM model, the reverse critical transition was implemented with a STEP function which reduced the non-agricultural phosphorus in the system suddenly and at a set time at the discretion of the model user. In the PLUMGov model, the reduction of phosphorus is now controlled by a feedback loop, where a governing body responds when phosphorus levels exceed a threshold. It is the structural differences between these two models, which are capable of expressing the same model output that will be of interest when accessing LEEA.

Government influence takes the form $\frac{1}{(1+xP^y)}$, where $x$ represents the extent of government interaction determined by the amount of phosphorus and $y$ represents the slope of the sigmoidal curve and therefore the speed at which the government action would take place. These values were chosen to directly counteract to the sigmoidal curve of phosphorus recycling and are implemented between phosphorus levels in the lake water ($P$) and agricultural input of phosphorus ($F$), thus generating the system's seventh feedback loop identified by the Shortest Independent Feedback Loop algorithm (SILS) and connecting the stock of Usoil to the current network of loops (Figure 5.1). The output of the first 200 years of this system can be seen in Figure 5.1.

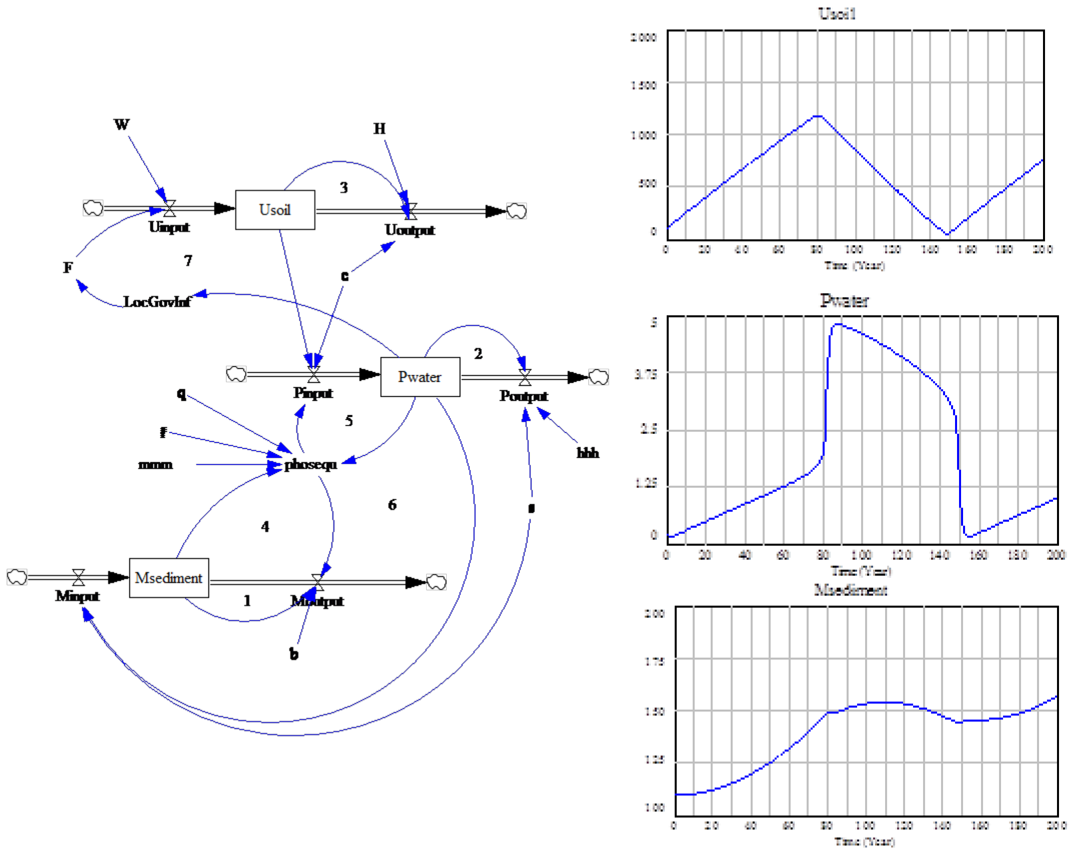



FIGURE 5.1: The PLUMGov Model. An extension of the PLUM model by inclusion of Government intervention which activates when the phosphorus levels get too high.

The addition of local government action creates a balancing feedback within the system, preventing runoff from the soil from being a simple linear input. While the output of the PLUMGov model is a coarse approach, the non-linearity of Usoil and inclusion of a Usoil feedback loop are aspects which were lacking in the PLUM model for Usoil to be considered as a non-linear endogenous driver. In the PLUM model, the stock Usoil only had a linear influence over the phosphorus in the water and therefore could technically be represented as a external input and discounted as a key component of the system. In PLUMGov, every stock is an integral part of the system's loop structure and is therefore taken into account during structural loop analysis.

Note that before hierarchy of loop dominance is considered, the introduction of government intervention has driven this system into a cyclic trajectory. This is because at high levels of phosphorus, the government intervenes to reduce phosphorus allowed to enter the system, but there is nothing in place to maintain the low levels of phosphorus input once a reverse transition occurs. The sigmoidal dynamics applied by the government influence variable therefore resets back to inactive levels; once the system is restored to low levels of phosphorus and the agricultural input increases to its original level, the cycle starts again (Figure 5.2).

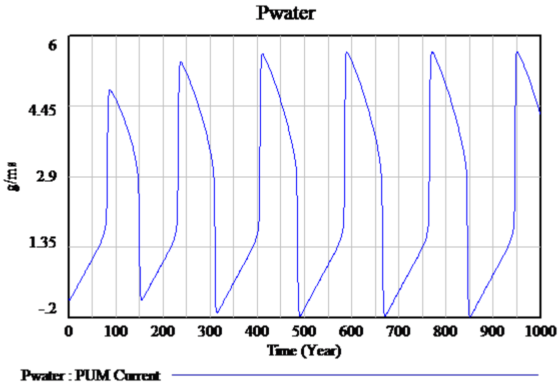



FIGURE 5.2: Water phosphorus output of the PLUMGov model showing forward and reverse critical transitions across a 1000 year time period.

Conducting LEEA on the PLUMGov model, it is possible to see how much the addition of one feedback loop has on the analysis outputs. It is of interest whether the hierarchy and dominance of loop structures surrounding the forward and reverse transition within the system will have changed, if at all, now that reductions in phosphorus after the forward tip are controlled by an internal sigmoidal dynamic as opposed to a user implemented STEP function. Arguably one of the most unique outputs from the PLUM model analysis was the ability to see loop influence building within the system prior to the forward and reverse critical transitions and that these phenomena were largely controlled by only one loop. In the analysis of PLUMGov, a comparison can be made to see if the same qualities arise from the loop influence plots now that Usoil is integrated into the feedback system.

## 5.6 PLUMGov Results

PLUMGov holds three eigenvalues and can be seen in figure 5.3. At the beginning of the simulation, all eigenvalues are negative, with eigenvalue 1 holding the largest magnitude, therefore having the most contribution to the system's current behaviour. Eigenvalue 1 is therefore inspected for its loop influence, but similar to the PLUM model particular attention must be paid to all three eigenvalues transitioning from negative to positive as the system nears its critical transition. In PLUMGov complex numbers have arisen in the eigenvalue 2 and 3 between critical transitions. Although the PLUM model's eigenvalues also held complex numbers, they only occurred momentarily at the points of critical transitions. In the PLUMGov model, we see that the addition of the 7th loop, formed to regulate phosphorus levels endogenously within the system, has generated oscillatory dynamics which means complex eigenvalues will occur throughout the simulation inferring that loop eigenvalue elastcitiy and loop influence values will also hold complex numbers. The eigenvalues have been separated into their real and imaginary pairs as shown in figure 5.3 a and b respectively. Determining which eigenvalues are most important to understand the system's current behaviour is done through an inspection and comparison of their real part, even if they hold complex values. However, during interpretation the presence of imaginary values changes the behaviour being expressed (i.e. oscillations rather than exponential growth/decay).

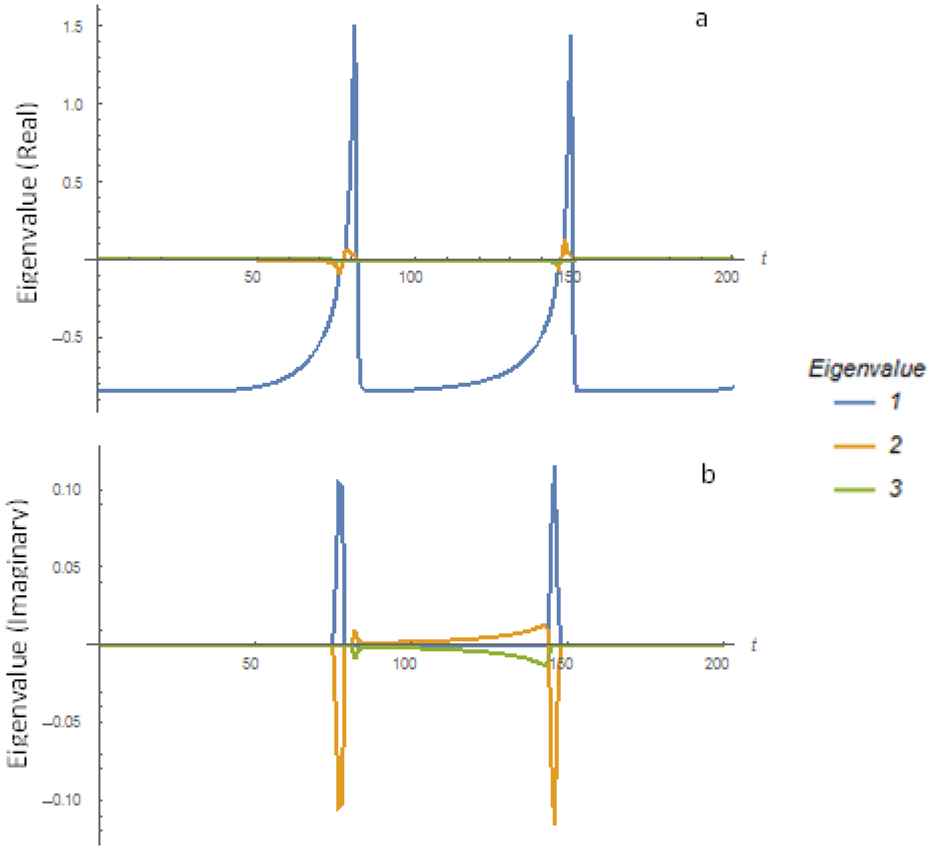



FIGURE 5.3: Eigenvalues from PLUMGov. 2a shows real values.2b shows imaginary values.

Output for PLUMGov's loop gains can be seen in figure 5.4. These give an initial indication into which loops may be dominating the system, before loop elasticity or loop influence is calculated. The PLUMGov model has been designed to largely express the same dynamics and a similar trajectory as the PLUM model, it is therefore no suprise that the loops with the strongest gain (loop 5 and loop 2) are the same across both models.

The interpretation of complex pairs in structural loop anlaysis is as follows: The positive and negative real part of the complex number indicate rate of expansion and contraction of oscillations respectively, while the variation in the imaginary part control the frequency of those oscillations (Forrester 1982, Kampmann and Oliva 2006,

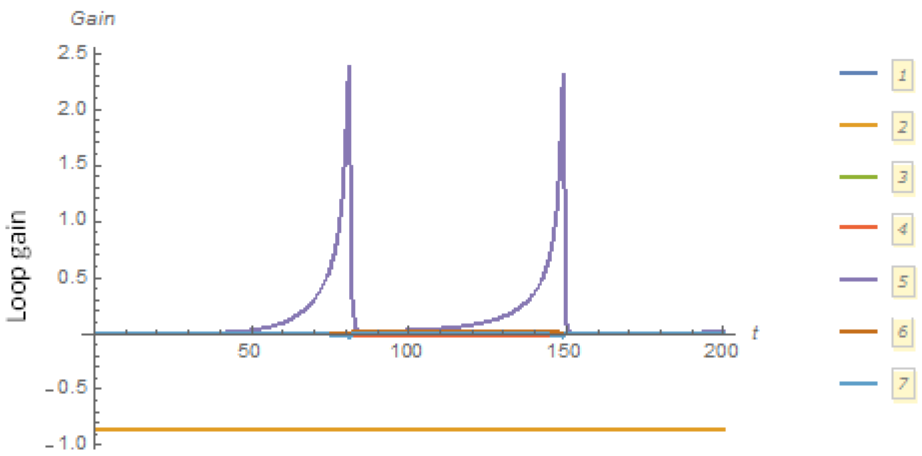



FIGURE 5.4: Loop gain values of PLUMGov

Güneralp 2006). Note that the oscillatory behaviour of the system's loops is concentrated specifically during and between the critical transitions.

*Impact from the additional dynamics*: The addition of the seventh loop does not appear to have a massive effect on the order of loop dominance throughout the simulation. The most noticeable change in the system which the dynamics in the 7th loop cause is the cyclic nature that the system now undergoes between forward and reverse tipping events as government influence switches between high and low values. While the presence of a seventh loop has made little impact on the large scale dynamics within eigenvalue 1 (figure 5.5) a huge change can be seen in the loop influences impacting eigenvalue 3. Figures 5.6 and 5.7 show the loop influence outputs of eigenvalue 3 in the build up to and during the critical transition in the PLUM model and the PLUMGov model respectively.

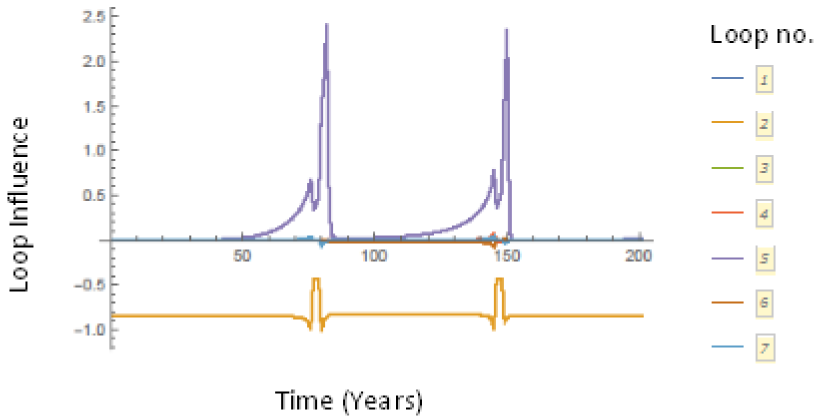



FIGURE 5.5: PLUMGov eigenvalue 1 loop influence values (real part). Each line on the plots represents 1 of the 7 loops within the system.

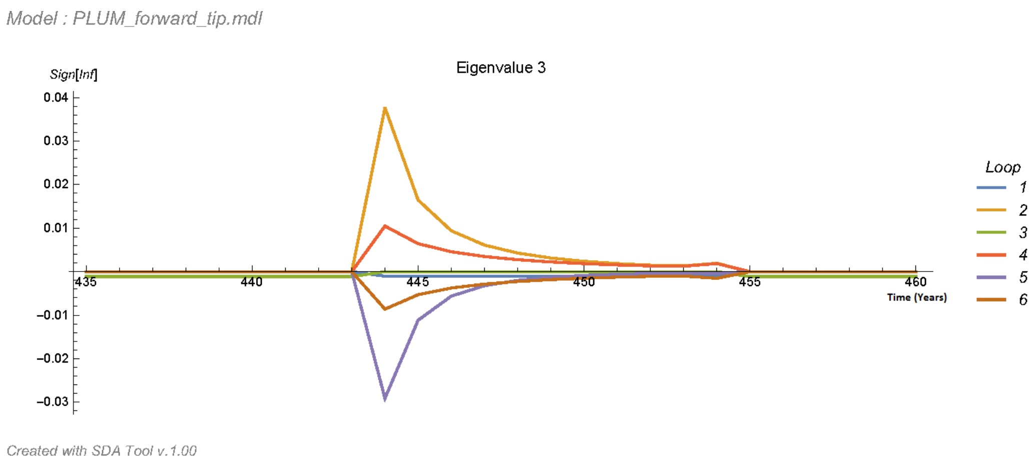



FIGURE 5.6: Loop Influnece values of eigenvalue 3 in the PLUM model leading up to the forward critical transition.

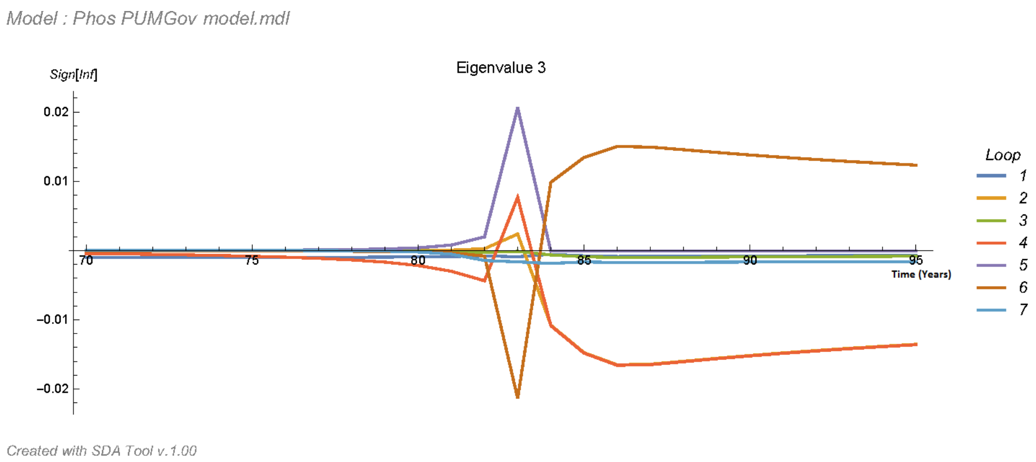



FIGURE 5.7: Loop Influnece values of eigenvalue 3 in the PLUMGov model leading up to the forward critical transition.

The results of the loop influence graph for eigenvalue 1, figure 5.5, express similar results to that of the PLUM model: The critical transitions of both the forward and reverse events are largely controlled by a single loop (loop 5) which undergoes a build-up of influence prior to the actual transition. As in the PLUM model, loop 2 refers to the outflow of water from the lake and is therefore partially responsible for removing phosphorus from the lake waters and therefore a stabilizing influence on eutrophication. Loop 5 relates to the phosphorus recycling loop, which is a main source of dynamic phosphorus input and therefore a driver of eutrophication, and instability within the system.

Unlike eigenvalue 1, the change in loop influences within eigenvalue 3 has undergone dramatic changes, expecially leading up to and during the forward and reverse critical transition events. Focusing primarily on the forward transition of the system to provide an example: eigenvalue 3 in the PLUM model holds positive values for a short window prior to the forward transition, around 1 year (444-445 years), while in the

PLUMGov model, eiegnvalue 3 never holds positive values and instead gains greater negative values leading up to the transition. The main influencial loops of eigenvalue 3 also change between models prior to the forward transition. Eigenvalue 3 of the PLUM model shows loops 2 and 5 to hold the most influence leading up to the forward critical transition (figure 5.6), while eigenvalue 3 of the PLUMGov model shows loops 4, 5 and 6 to hold the highest influence leading up to the forward critical transition, with loops 4 and 6 maintaining relative high influence levels during the transition.

It can be seen with a comparison between models that the simple addition of an additional feedback loop has completely changed how one of the eigenvalues of the system is to be interpreted, even across two system models where the rest of the model structure, initial conditions and overarching dynamic has been kept the same.

From observing loop elasticity and influence plots from eigenvalues 2 and 3, it is clear that the loops which appear to dominate the system's behaviour within these modes and the number of loops contributing to that behaviour, heavily depend on which eigenvalue is being analysed. The results across the eigenvalues now differ dramatically, which emphasises the need to identify which eigenvalue is contributing the most to current behaviour at an early stage in the analysis process. Even though from the Eigenvalue plot in figure 5.3, the real parts of eigenvalues 2 and 3 make little contribution to the system's behaviour, there are clearly still some active and interesting dynamics going on between the loops in these modes (figure 5.7).

In some of the results, notably the influence plot of eigenvalue 3, between years 85-100, loops 4 and 6 may be expressing a phenomena which Kampmann and Oliva (2006) refer to as 'artificial loops' or 'ghost loops' (figure 5.7). These artificial loops occur when the output of LEEA shows the two loops expressing high levels of dominance in the system, but in reality their dynamics act to counteract each other's contribution as their values are equal, but take opposite polarities.

Despite the changes in interpretation which the additional loop has created on the results of the system, the outputs of LEEA are still useful. LEEA may even be considered more valuable on PLUMGov than it was on PLUM as the stock of Usoil becomes an integrated part of the system's loop structures, making PLUMGov more difficult to understand with current analysis tools such as stability analysis or principle component analysis.

In order to investigate structural loop analysis even further, the PLUM model was extended once more to create PLUMPlus, incorporating more stocks into the system in order to determine how difficult the analysis might become to interpret when the system is associated with even more eigenvalues.

## 5.7 PLUMPlus Model

The PLUMPlus model represents an even greater increase of complexity to the original PLUM model. The PLUMPlus model supports four new stock variables, meaning there are now a total of seven stocks, therefore seven differential equations which represent this system's dynamics and seven eigenvalues to interpret during LEEA. In addition, these stocks bring with them 23 new variables and 32 new interactions from the PLUM model. In total, the PLUMPlus model contains 17 feedback structures, an additional 11 feedbacks from the original 6. The four stocks introduced to the PLUMPlus model take the form of a human population, human migration, lake algae and lake biota.

The PLUMPlus model includes actions of a local government as per PLUMGov, alongside a growing population, migration and predator - prey interactions between lake biota and algae; all of which can be seen in system dynamic form in figure 5.8. The output for phosphorus levels in the soil of the land, water of the lake and sediment of the lake can all be seen below in figure 5.9. The extended model output is abstract and has been generated for the purpose of demonstrating how LEEA analysis becomes more complex with larger models. The extended model combines bistable phosphorus dynamics with exponential growth of a local population and oscillatory trends of a predator-prey relationship within the lake. The local population creates pressures on the agricultural and non-agricultural uses of phosphorus, and the predator-prey relationship creates an additional feedback between levels of nutrients (phosphorus) in the water and levels of phosphorus recycling from the sediment by generating anoxic conditions (Scheffer 2009). These additional elements make explicit some of the social and biogeochemical processes implicit in the Carpenter (2005) model.

The human population represents the population which live within the lake catchment and depend on the agricultural and non-agricultural practices within the region. To represent this in the model, the human population is set to grow at a steady exponential rate and is linked to the original model by determining the productivity demanded by both the agricultural and non-agricultural variables of the model. As population increases, the demands on these practices also increases, causing a greater in-balance between phosphorus levels entering vs. leaving the system.

Migration is implemented as a control variable to the region's amassing population, but is also capable of reinforcing the population's effect. Migration acts as a linear feed which adds to or detracts from the population within the area at each time step. Migration is set to naturally feed into the population, but changes depending on the water quality of the region. The water quality is an arbitrary factor determined by the amount of phosphorus found within the lake water. As phosphorus concentration increases within the lake water, the water quality decreases and causes more people to migrate away from the region.

Lake algae and Lake biota are implemented to represent two sets of organisms which reside in the lake. The two are implemented as having a predator-prey relationship, causing oscillatory behaviour to be implemented into the model. The stocks and flows for lake algae and lake biota are a simplified representation for an enormous food web of the lake system. The connection of lake algae and lake biota to the PLUM model acts as an extension to the phosphorus recycling feedback loop, which is an important component of self-reinforcing phosphorus levels in a eutrophic lake (Carpenter 2005, Scheffer 2009). Lake algae are fed by the phosphorus within the lake water and connect to the birth rate of lake biota which is dependent on the number of algae within the system. As the lake biota die off they contribute to the anoxia levels building up within the lake system. In turn, the increased anoxia levels within the hypolimnion layer (the bottom waters) causes trapped phosphorus within the sediment to be released via phosphorus recycling. Overall these processes cause a positive feedback loop between phosphorus build-up within the lake waters and productivity of the lake biota as well as multiple feedbacks which are present within the predator-prey interactions. Against a real world lake system, the stocks of lake algae and lake biota represent a simplified version of the dynamic behaviour of lake organisms within the system. However, for the purposes of investigating LEEA under increasingly complex models, the stocks serve an important purpose, adding complexity to the system both structurally, and through the dynamics which they impose on an already dominant part of the system's behaviour.

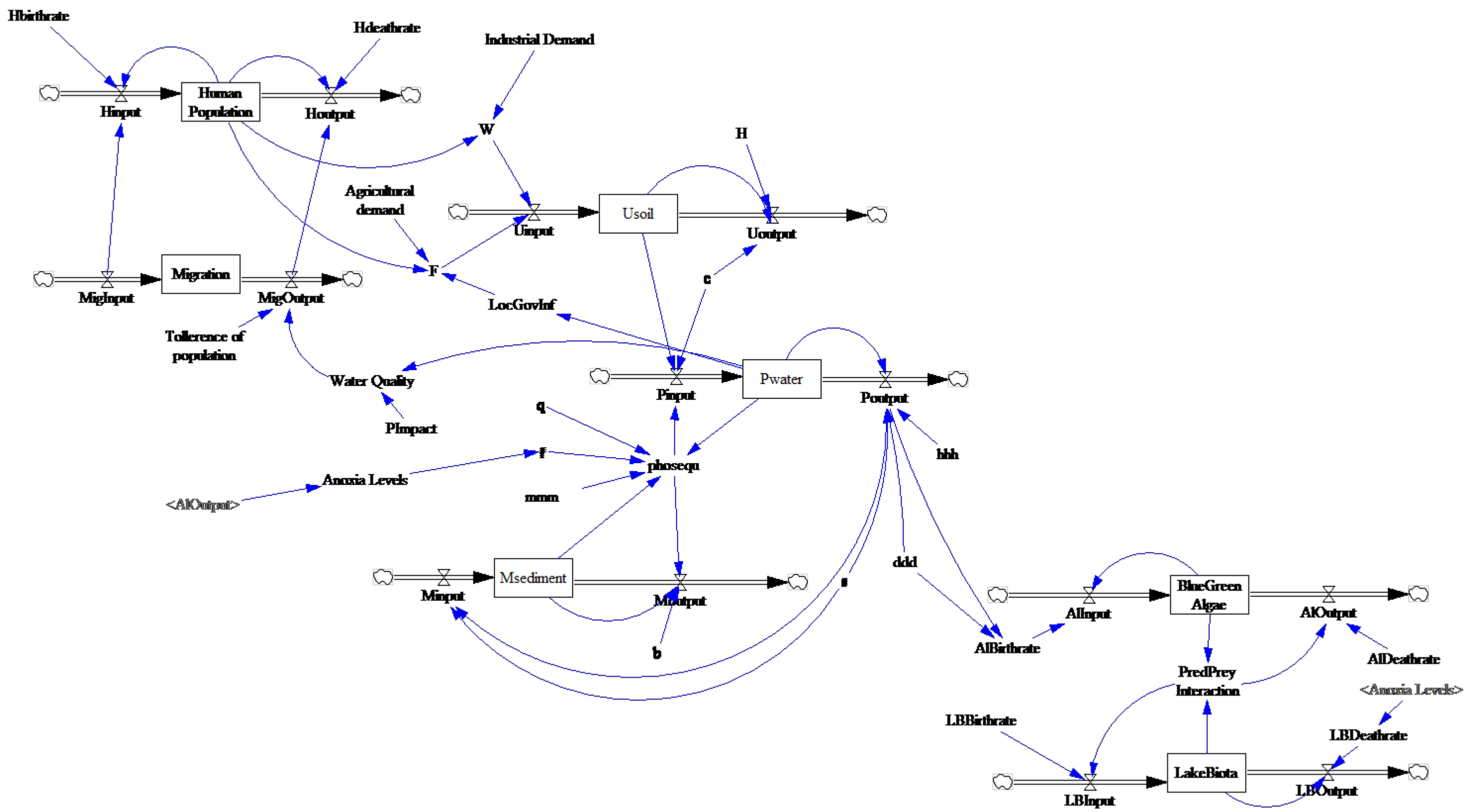



FIGURE 5.8: The system dynamic model of PLUMPlus. An extension to the PLUM model by the addition of a local government influence, a local population, migration into and out of the area and predator prey interactions between Lake Biota and Blue Green Algae.

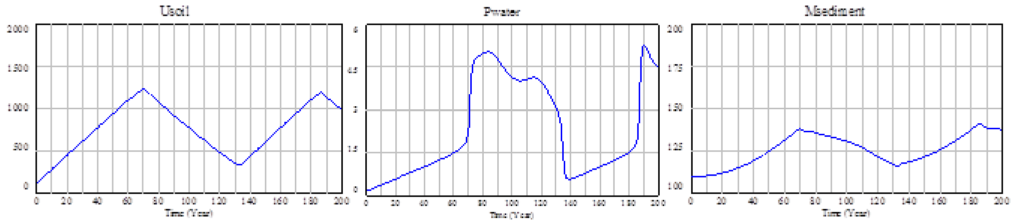



FIGURE 5.9: PLUMPlus Stock Outputs.

The four new stocks take the following forms:

Human Population (HP):

$\frac{dHP}{dt} = HPbirthrate.HP + MigInput - HPdeathrate.HP + MigOutput$

where $HPbrithrate$ and $HPdeathrate$ are constants which determine the growth or decay of the human population overtime. When $HPbirthrate$ is higher than $HPdeathrate$, the populations experiences exponential growth as is set in the model at 0.02 and 0.01 respectively. $MigInput$ and $MigOutput$ represent migration in and out of the lake catchments population, affecting the Population stock linearly.

Migration (Mig):

$\frac{dMig}{dt} = MigInput - TolerenceofPopulation.WaterQuality$

In migration, the migration into the lake catchment ($MigInput$) is set at a constant rate, but the migration out of the lake catchment is determined by the water quality lake and the tolerance of the population set by the user.

Algae(Alg):

$\frac{dAlg}{dt} = Algbirthrate.Alg - Algdeathrate.PredPreyInteraction$

$Algbirthrate = 0.2 + d * Poutput$

The birthrate of the algae population is largely controlled by the current algae population and birthrate of the algae which fluctuates determined by an uptake factor, $d$, the levels of phosphorus in the lake water. The deathrate of Algae is subject to an interaction parameter with lake biota which predate on the algae. It is the death rate of algae that contributes to the levels of anoxia and therefore the amount of phosphorus recycling occurring in the lake sediments.

Lake Biota(LB):

$\frac{dLB}{dt} = LBbirthrate.PredPreyInteraction - LB.LBDeathrate$

The numbers of Lake Biota is determined by the number of interactions they have with the population of algae in order to support their growth and constant set death rate which scales with the population of Lake Biota. Together with the population of algae, it is the predator-prey interaction which generates the Lokta-Volterra dynamics expressed by these two stocks.

While PLUM and PLUMGov hold only three stocks, with a total of six and seven loops found within their respective Shortest Independent Loop Sets (SILS), PLUMPlus holds seven stocks, with a total of 17 loops formed by the interactions between the new stocks and auxiliary variables of the system. An increased number of stocks within the system means that it has a larger Jacobian matrix to describe the interactions between components. In turn this leads to a greater number of eigenvalues generated for the system which could at any point in time be steering the system's behaviour.

Similar to the PLUM model, in the first 60 years of the model output there is nothing but linear trends. However, there are many dynamics occurring within the system's structural drivers which there is no indication of from simple model output. Unlike the PLUM model, the results are much harder to interpret: not only do more eigenvalues need to be taken into account, but also more feedback loops potentially driving the system's behaviour.

*Impact from the additional dynamics*: In PLUMPlus, while a forward and reverse critical transition are still simulated, both events occur in shorter time periods than in the previous models. This is due to the introduction of a human population applying pressure on agricultural and industrial inputs causing their values to change throughout the simulation, rather than remain constant. As the living demands from the population increases, the forward transition of the system takes less time to occur and the less time it takes for reverse transition to occur thereafter following government intervention. If the population is allowed to get too high, or is initially set too high, then a reverse transition is never able to occur in the system despite government influence and phosphorus levels begin to grow exponentially to match the population growth.

In between the forward and reverse transition events there is oscillation of the phosphorus levels within the lake water, caused by the oscillatory dynamics incorporated into the Predator-Prey interactions having knock-on effects to the phosphorus recycling of the system. Algal death rates of the system have been linked to the amount of phosphorus recycling, based on the anoxia levels which their activity and death events create in the lake. Anoxia levels in turn feed back into the death rates of lake biota as higher anoxia levels generate harsh living conditions for species that inhabit the lake (Carpenter 2005).

## 5.8 PLUMPlus Results

Seven eigenvalues were generated from PLUMPlus as seen in figure 5.10. The plots shown in this results section have been chosen specifically to highlight certain features of LEEA. Note that Eigenavalue 7 has not been plotted as its eigenvalues were zero throughout its time series and therefore did not express any loop dominance.

Similar to PLUMGov, the eigenvalues of PLUMPlus come in the form of complex numbers and therefore can be plotted in terms of their real and imaginary parts.

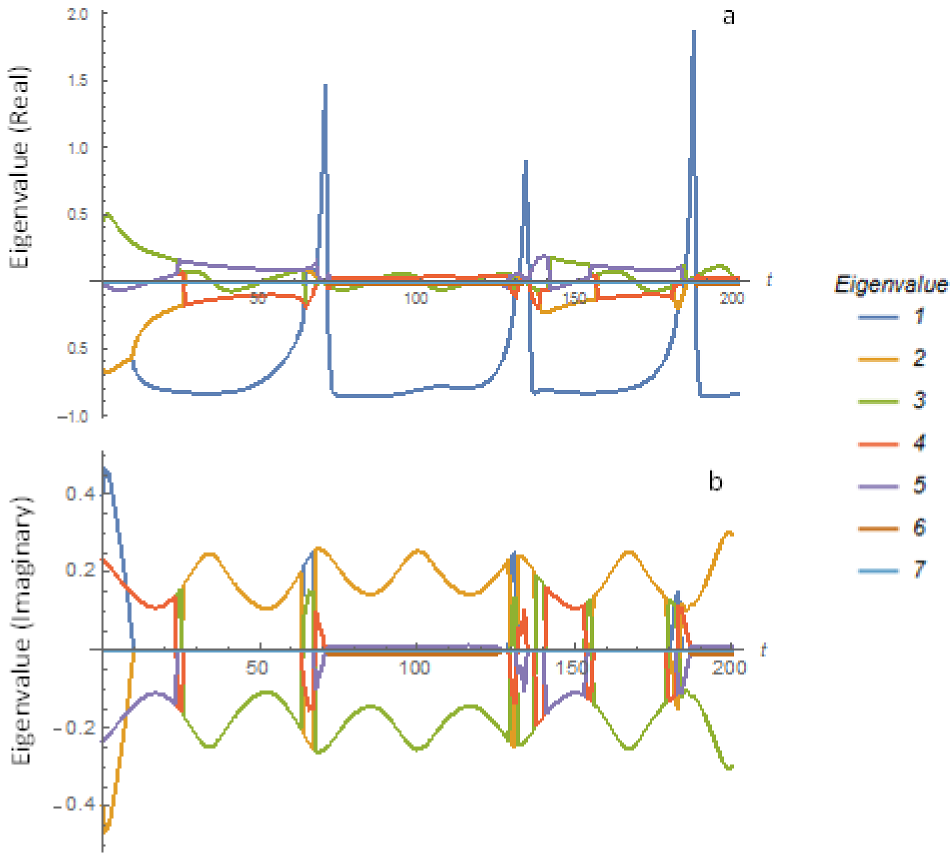



FIGURE 5.10: Seven behavioural modes from PLUMPlus. a shows real values, b shows imaginary values.

Straight away it can be seen that selecting dominant eigenvalues with which to focus on in the eigenvalues real part (figure 5.10 a) has increased in difficulty. Eigenvalue dominance remains somewhat constant throughout the simulation, with eigenvalue 1 usually holding the greatest absolute values at any point in time, but the stability of the system and polarity of eigenvalues constantly changes throughout the simulation, especially between critical transition events of the system. Of note, the system can no longer be considered to be governed by stabilizing behaviours in between critical transition events as at least one eigenvalue holds a positive value, but exactly which one and to what extend changes depending on the time step of interest.

Alongside the extra level of complexity in the real parts, the strong oscillatory dynamics within the model are also reflected by the eigenvalues holding complex values, the imaginary part of which is just as complex to interpret as the real part, if not more so around critical transition events, where many eigenvalues appear to spike simultaneously (figure 5.10 b).

*Model spin-up*

The PLUMPlus model undergoes cyclic behaviour, through a series of forward and reverse critical transitions. The cycle resets roughly every 120 years after each reverse transition upon the phosphorus water density reaching 0.6 $g/m^2$ (figure 5.10 a. This cyclic behaviour can also be seen in eigenvalue plot, signposted by the spikes of eigenvalue 1. The model does however undergo model spin-up in its first 30 years, which is not part of the cyclic behaviour. This has implications for the eigenvalue plot and therefore dominant loops registering with high values in the loop influence plots. A model user of LEEA must be conscious of the presence of model spin-up within their system as LEEA will not be able to distinguish it from the rest of the simulation. Feedback loops dominating the model's behaviour are likely to be different during model spin-up as the system takes time to reach a steady state from its initial conditions. Feedback loops which primarily control a systems steady state may not be the same as the loops which were dominating to drive it there. This is not a huge problem, but definitely something to keep in mind when starting a loop analysis interpretation.

*PLUMPlus loop gain plot*

The plot for PLUMPlus' loop gains can be seen in figure 5.11. The system holds 17 loops in total and loop gain will give the first impression of where loop dominance is stemming from. What is interesting from the loop gain plot is that despite there being 17 loops within the system, only 7 of them are showing high levels of strength within the system. The rest all sit around zero inferring that they will likely make little contribution to the behaviour of the system over this time period. Whether this is true of the real world lake system would require an accurate model to be created and analysed in order for the results to be justified.

Critical transitions which occur within the PLUMPlus model are still indicated by LEEA, shown by the peaks within figure 5.12 at time steps 75, 150, 195 and 270, but overall output is much more complex and identification of key behavioural drivers has become harder to interpret than within PLUM. Additionally, Figure 5.12 shows the real part of loops influence outputs from eigenvalue 3, only one of fourteen loop influence outputs which must be compared and contrasted across 7 different eigenvalues from the analysis. In some respects this represents a limitation of LEEA as output requires careful interpretation. However, the dynamics operating are complex, and consequently a simple, low dimensional representation of these dynamics is not a robust or reliable indicator of what is driving behaviour.

*Observations and comments from LEEA results:*

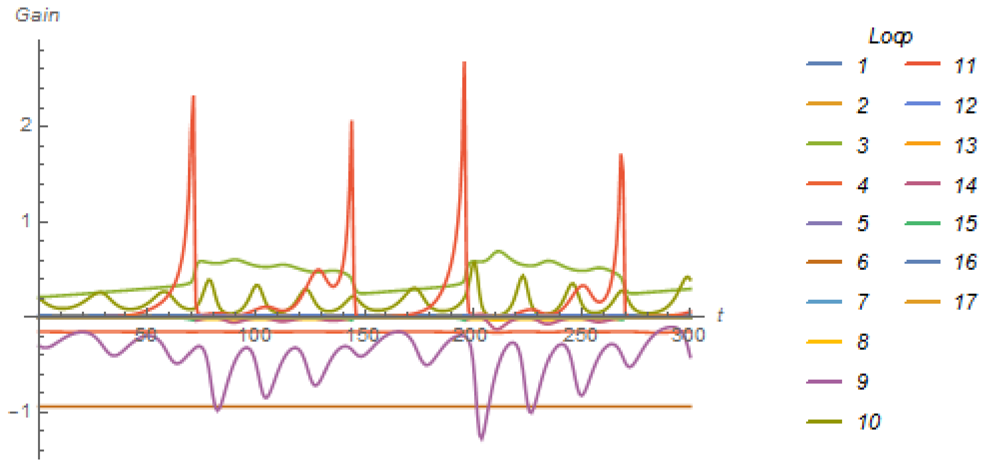



FIGURE 5.11: Loop gains of PLUMPlus including 17 loop total.

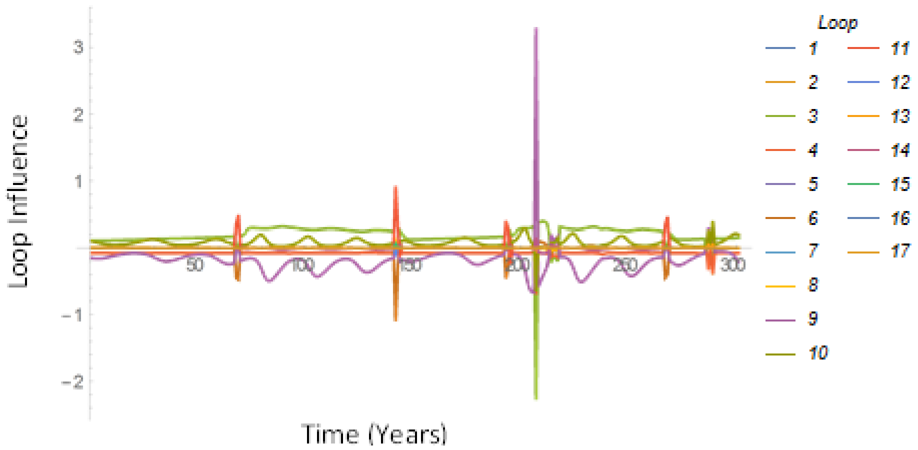



FIGURE 5.12: LEEA output for the PLUMPLus model, showing Loop influence output from eigenvalue 3. The output includes 17 structural feedback loops, each of which is represented by a single line within the plot. The plot is one of fourteen which would have to be compared in order to determine the dominant loop structures of the system.

As a more complex model than PLUM and PLUMGov, LEEA results of PLUMPlus have become even more difficult to interpret. Not only are there more eigenvalues to consider due to the addition of four new system stocks, but there is no longer an obvious single behavioural mode which all system behaviour can be attributed to. Similar to the PLUMGov model, the eigenvalues have formed complex pairs and so results have to be split into their real and imaginary parts.

From the real Elasticity values of eigenvalues 1, 3 and 6, it can easily be seen that there are dramatic differences in loop dominance depending on which eigenvalue is being analysed (figure 5.13).

Comparisons between Loop Elasticity and Loop Influence must be undertaken with care as shown in eigenvalue 3 in figure 5.14. Around critical transitions in both Elasticity and Influence values, loops 6 (brown) and 11 (red) both show higher dominance,

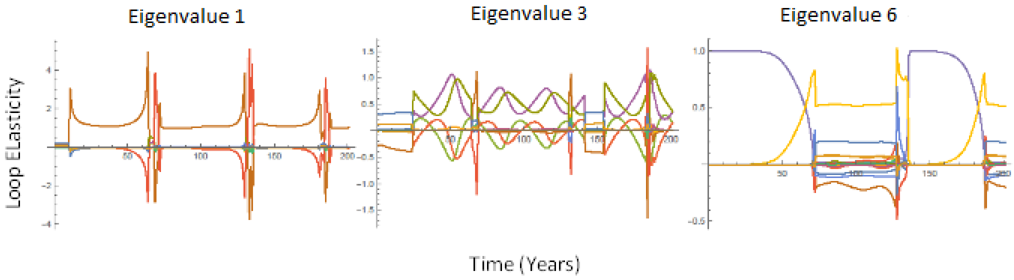



FIGURE 5.13: Showing how the results of Loop Elasticity can differ so much across eigenvalues.

but their polarities are switched between plots (Figure 5.14). This is fine as Elasticity values and Influence values mean different things, but care must be taken not to get the two mixed up during analysis.

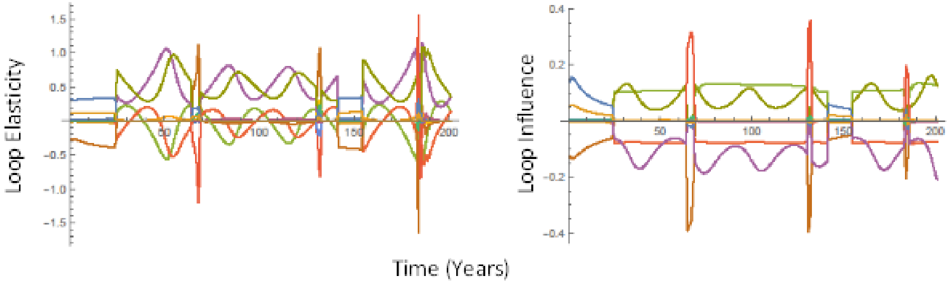



FIGURE 5.14: Showing the difference between Loop Elasticity Results and Loop Influence results.

## 5.9 Yulin City Model

The Yulin City model was used as a short study, adapting a water resource model from Wang et al. (2011) to test LEEA on a much larger system where the analysis did not prove as efficient or effective as in earlier cases. The model contains 19 stocks, 103 variables, 135 connections and 25 feedback loops (figure 5.15) and is therefore the largest model presented within this thesis.

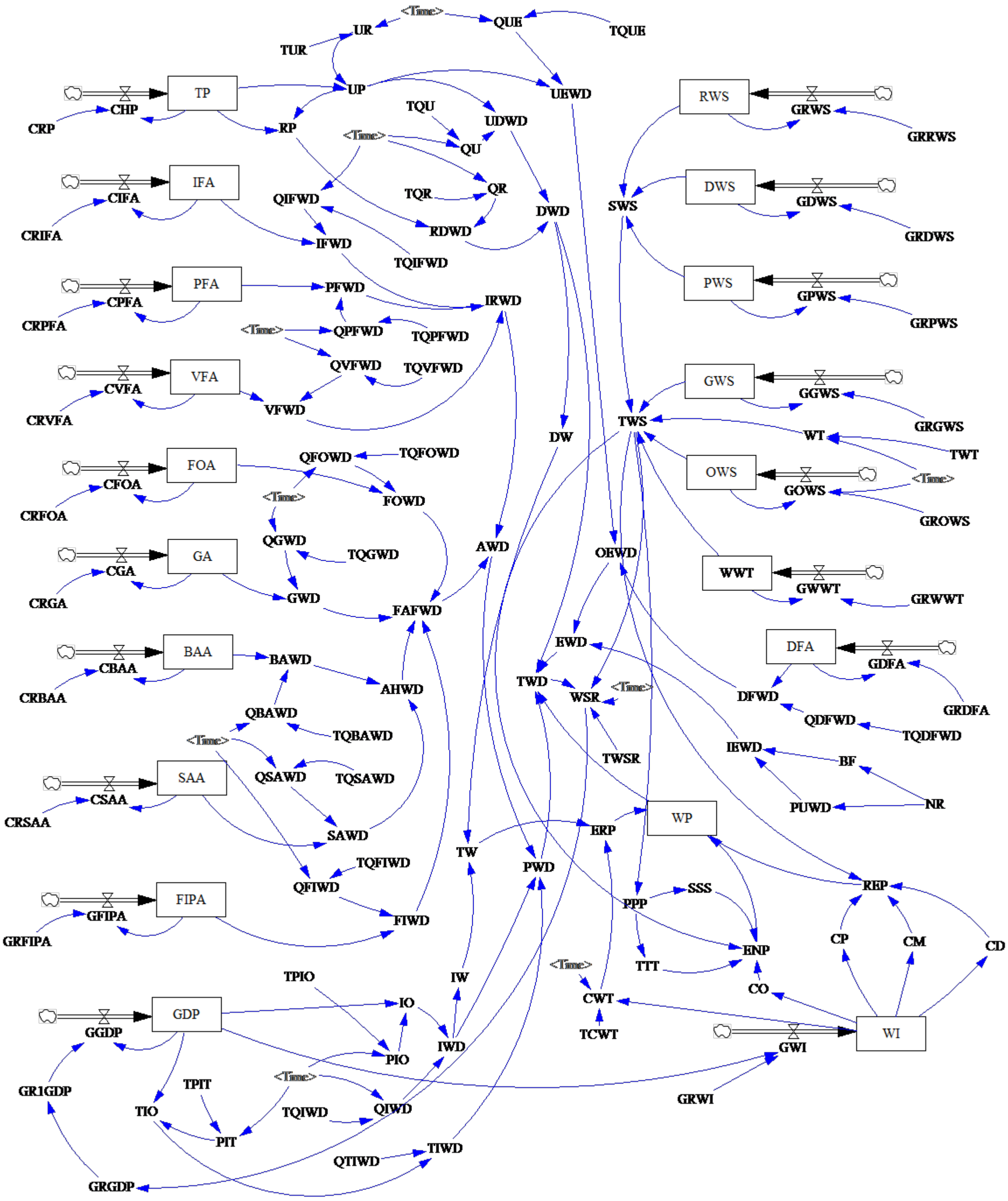



FIGURE 5.15: Structural representation of the Yulin city model adapted from Wang et al. (2011)

LEEA was difficult to implement as many dynamical functions, including 'IF THEN ELSE' statements within the model were incompatible with LEEA's automated software and had to be rewritten using graphical alternatives. Some of the Yulin model's structure had to be redesigned with the addition of interations to compensate for the alternative functions. Alongside the effort required to make LEEA and the Yulin City model compatible, the results showed limited use.

After processing, the graphical analysis section of LEEA identified that the loop to stock ratio, 25:19, was low compared to other models were LEEA has proven successful. Within the Yulin City model 16 out of 19 stocks were not linked through any kind of feedback to the rest of the system, effectively making them linear inputs to a much smaller set of feedbacks. As shown from the loop gain plot in figure 5.16, only one of the 25 loop structures showed some form of spiking dynamic during the first seven years of the simulation, with little changing in the rest feedback loops for the remainder of the time series.

LEEA can only account for the dynamics of a model which are taking place within feedback loop structures. Since only three stocks of the model were intergral parts of the main body of feedback structures, the information gained from LEEA could only be attributed to the behaviour of 3 of the 19 stock variables. The output of the analysis therefore did not reflect the dynamics within the majority of stocks within the model.


FIGURE 5.16: Loop Gain output from the Yulin City Model giving a first impression of the dynamics and dominance of the 25 loop structures present within the system prior to calculation of loop influence.

It is important to consider the volume of graphical output LEEA would produce for the Yulin City model, despite only representing dyamics in 3 of the 19 model stocks. Since there is a direct relationship of model stock number and eigenvalues calculated for the analysis, the Yulin City model would have at the very least, 40 graphical outputs (1 eigenvalue plot, 1 loog gain plot, 19 loop elasticity plots and 19 loop influence plots) needing to be prioritised for interpretation. This is a high volume of data considering it does not account for roughly 80% of the model's stocks.

LEEA does have the potential to be used on models of high order, size and complexity, but if such a model is not being built with LEEA in mind, then the user must be ready to partially rewrite the model in order to run the analysis. The size of the Yulin City model means that LEEA produces a large volume of graphical output, meaning interpretation of the analysis could be time consuming for the amount information gained. The low loop to stock ratio of the model means that the output of LEEA would likely be as much of a 'black box' to understand as the inner workings of the model itself.

## 5.10 Discussion

*Findings from the analysis of PLUMGov, PLUMPlus and Yulin models*

The purpose of constructing and analysing the two extensions to the PLUM model and of the analysis of the Yulin model was to investigate how interpretation of LEEA's output may become more complex with larger model size. The following discussion points therefore concentrate on the interpretation of the analysis outputs more than the specific dynamics expressed within the plots.

The PLUMGov model was created to show how the simple addition of an extra feedback loop could affect the interpretation of LEEA. While results remained relatively simple to interpret with one dominant eigenvalue containing only two influential loop structures, the addition of a balancing loop to counteract the system's primary mechanism of instability generated oscillatory trends across the simulation and therefore created complex numbers within the eigenvalues at and between the forward and reverse critical transitions which were not prominent within the original PLUM model. The difference was dependent on whether reductions of phosphorus after the forward critical transition were controlled by the active input of a user via a step fucntion at a selected time step (as in the PLUM model) or by a reactive endogenous feedback loop which contained a sigmoidal function (as in the PLUMGov model).

The PLUMPlus model was constructed in order to introduce a variety of new stocks to the system alongside several new dynamics which added to and manipulated feedback loops currently operating within the PLUM model. Overall interpretation of the results was still manageable, but addition of new stocks to the model brought with them a multitude of new graphical outputs to be examined. In the original PLUM model, three stocks meant that at most, 8 plots would have to be examined (1 eigenvalue plot, 1 loop gains plot, 3 loop elasticity plots & 3 loop influence plots), each with 6 feedback loops with which to identify dominant drivers of system behaviour. In comparison, the PLUMPlus model had seven stocks and its oscillatory trends meant that a total of 31 plots would have been examined (1 Eigenvalue plot [real part], 1 eigenvalue plot [imaginary part], 1 loop gains plot, 7 elasticity plots [real part], 7 elasticity plots [imaginary part], 7 influence plots [real part] & 7 influence plots [imaginary part]), each with 17 different feedback loops with which to identify dominant drivers of system behaviour.

An issue that might arise from a high number of system eigenvalues can be seen in figure 5.13, where different dominant eigenvalues expressing completely different feedback loops are seen influencing the system's behaviour. Both the PLUMGov outputs and PLUMPlus outputs also show signs of 'artificial loops' or 'ghost loops', where seemingly high influential loop structures are actually being counteracted by the dynamics of another loop and therefore are not contributing to the current behaviour of the system.

The Yulin city model was the third example used within this study to evaluate a model where LEEA may not be so appropriate. The Yulin city model's structure is largely designed around unidirectional connections between stocks which form a relatively low number of feedback loops for the high number of stocks that it contains. This means that any analysis obtained from LEEA would only be representative of a small section of the model. In this case other analysis methods such as sensitivity analysis may be more appropriate to identify key variables. The Yulin city model also took a long time to adapt manually in order to carry out LEEA as many of the functions were not compatible with the algorithms used to perform loop analysis. If LEEA is to be used successfully on models of this size (19 stocks) it may save time for the model to be designed with the application of LEEA in mind, though this is not usually the order in which model construction and analysis tools are considered.

*Physical limitations of the technique*

One aspect of socio-ecological systems and the models which represent them is an aspect of choice and decision making that exists within the social side of the models. Generally speaking, the idea of choice, or individuals within a system being able to make their own, sometimes irrational, decisions is an integral part of an accurate and justifiable SES model. In terms of running LEEA on such models, the analysis output could still prove useful, helping to examine which feedback loops drive certain decisions or sway a deciding vote. However, attention must be paid to the way that the aspect of choice has been implemented within the equations of a model as this could impact the extent to which LEEA could be utilised, or impact the manual labour time required to adapt the model to be compatible with LEEA. As an example, IF and IF THEN ELSE statements are often used to segregate a population between alternative decisions, but these statements are not compatible with current versions of the LEEA online utility algorithms (Naumov & Oliva 2017).

In order to address the lack of automation of the technique, Naumov & Oliva provide a worksheet in Mathematica to help users through the main steps on the analysis (Naumov & Oliva 2017). This has greatly streamlined the mechanical running of LEEA. However work must still go into preparing a system's model and its data output in order to use the worksheet and the code must be manipulated in order to gain Elasticity and Influence output plots.

*Validation of results*

As stated previously within this case study, values for system components of the PLUMGov and PLUMPlus models either stem from the original Carpenter model, or have been chosen arbitrarily for the purpose of analysis exploration. When working with a real system and in order to get the most out of loop analysis, all system components must be validated empirical data if any comparisons are to be made back to the real world system from which the model derives. However, the validation of system components and its output is a universal issue, with the field of ecology being no exception. Data collection for key system components can be impractical or economically unviable to acquire, making validation fitting techniques difficult to achieve. Validation of model outputs may also be impractical as they require manipulation of the real world system in order to test results, which may not be feasible.

*Interpretation error*

The dynamics within PLUMGov and PLUMPlus are interesting for their loop influence plots because the behaviour, system mechanisms and build-up to each critical transition can be observed changing through time leading up to each event. Observing how the trajectories of eigenvalue and loop influences changes through time is an important part of the analysis process as it can give clues as to the change of an eigenvalues polarity or the dominance a loop structure might gain further along the time series. However, there is an interpretation error associated with eigenvalue and loop influence line plots which increases as the time step between the points calculated for LEEA increases. The linearization process for calculating the system's Jacobian Matrix treats each point in time as a completely separate event so making interpretations between these points can lead to inaccuracies in interpretation. To overcome this, when possible, the time steps taken for LEEA can be reduced, decreasing the time between calculations and increasing the number of individual events calculated within the analysis. The user can also inspect each point in time on eigenvalue and loop influence plots individually, which is more time consuming, but completely removes this type of interpretation error.

*Regarding safe and just operating space*

Working within a safe and just operating space is a primary concern within the fields of sustainability, resilience, conservation and recovery, where many ecological and socio-ecological models stem from. LEEA is purely based on numerical techniques and mechanical responses; it therefore has no regard for operating within a socially acceptable and economically viable space, unless these concepts are integrated as part of the analysed model. It would therefore be discouraged to base policy changes and local decisions on structural analysis alone as its output should be considered alongside what is safe and just for the system and its inhabitants.

*Future work*

While LEEA has proven to be a promising technique for understanding socio-ecological dynamics, more experimentation needs to be conducted on it in order to understand its capabilities. By utilising structural loop analysis, sensitivity analysis could be run to investigate changes in loop magnitude, loop polarity and the order of loop dominance. This would not only identify which system components are having the greatest impact on behaviour, but also which mechanisms present in the model are allowing them to do so. This would mean measuring complexity not just by the number of variables, but the loops that they create. Using SILS to analyse loop structures would allow for the presence and abundance of feedback loops to be considered as a quantifiable level of complexity within system dynamic models in a similar way to how centrality, node connectedness and clustering are all concepts of complexity in network modelling (Costa et al. 2007; Dunne et al. 2002). Making loop analysis techniques a common practice through further development and automation, could have profound implications for the way that we approach model simulation, while also increasing model efficiency and understanding.

## 5.11 Concluding Statements

LEEA, in its current stage of design, is not an appropriate method for all dynamic system models. Its main limitations prevent it from lifting the black box away from large system models, outputting results which can be just as incomprehensible as the system which it is attempting to analyse. While the technique and methodology behind LEEA may improve with future iteration of the technique, its design is based around the analysis of feedback structures, meaning that there will be some system models where its practical application will never provide the user with novel information about the target system.

While feedback loops act as key driving mechanisms in practically every known system, models do not have to be designed in a way to best express dynamics via feedback structures, thus limiting LEEA's ability to analyse the link between system structure and system behaviour. With regards to model size, LEEA is physically capable of being carried out on any system model it has been presented with, but it is the user's ability to interpret LEEA's results that can run into difficulty. Using LEEA on large, highly complex models becomes less of a pragmatic problem, if the user is 1) familiar with the outputs of LEEA and how to interpret them, 2) familiar with the target system, able to apply LEEA's outputs back to the model, 3) has time available to review all of the output LEEA is able to produce from a single simulation.

# Chapter 6: Chapter Preface

Loop Eigenvalue Elasticity Analysis (LEEA) has been shown to have utility on a small dynamic lake model as well as being effective at analysing structural feedback loops driving critical transitions. Alongside investigating limitations of LEEA it is important to gain a greater understanding of how model properties (dynamics, structure and values) relate to LEEA output and vice versa. Determining the most dominant feedback loops in a system is a valuable asset to a model analysis, but understanding how sensitive a loop's dominance is to changes in model parameters can also be valuable when attempting to maintain or manipulate a behaviour or outcome.

Within this chapter, the meta-analysis of LEEA is continued to examine LEEA in the context of analysis sensitivity. Here, the extent to which a model parameter's value must be changed in order to cause change within the loop dominance outputs of LEEA is assessed. Conducting sensitivity analysis, not just on a model system, but on the analysis outputs allows a greater connection between model properties and LEEA results to be understood. Investigating a system's sensitive parameters in the context of dominant loop structures gives a greater understanding of potential leverage points within the system, which allows a connection to be drawn between LEEA implementation and policy design based on structural drivers of system behaviour.

# Chapter 6

# Accessing LEEA's Niche: An Analysis Comparison in the Context of Leverage Points

## 6.1 Abstract

System dynamic models are not commonly used for their predictive capabilities, nor to output exact values of given parameters. Instead, their strengths lie in simulating trends in model behaviour and can be used as a base with which to test the implementation of new policies, before they are implemented on a real ecosystem. Combined with the methodology from Loop Eiganvalue Elasticity Analysis (LEEA), system dynamic models could be used to identify leverage points of a system, identifying structures, or variables with the greatest ability to induce change to a system's behaviour when perturbed. The identification of leverage points is considered a valuable output of model analysis in light of environmental conservation, control and recovery.

Structural loop analysis of dynamic socio-ecological models has been shown to be a unique tool for quantifying a feedback loop's influence over a system's behaviour. The output of LEEA allows the user to identify dominant loop structures, controlling the system's current behaviour and rank their influence against other feedback structures within the model. Identifying dominant structures of model behaviour is a valuable tool for testing and designing model scenarios, but LEEA also has potential for use in policy design and implementation when used alongside its partnering analysis Dynamic Decomposition Weights Analysis (DDWA).

This study discusses the ability for structural loop analysis to aid in the identification of leverage points within a dynamic model system. As part of the discussion, LEEA

is assessed for its sensitivity to changes in model parameters and as part of a meta-analysis of LEEA, we test how liable the leverage points of a system are to change with different parameter starting values and trajectories through time. The discussion considers whether LEEA can be an appropriate method to help guide policy as an individual technique or as part of a toolset combined with other analytical methods. The results show the application of structural loop analysis to leverage point identification will question our ability to manipulate system variables over our ability to manipulate entire feedback structures within socio-ecological systems.

## 6.2 Introduction

The use of system dynamic models to portray ecological and socio-ecological systems (SES) alongside the results of Loop Eigenvalue Elasticity Analysis (LEEA) allow us to explore and understand a system's internal structures as drivers of endogenous dynamics, forming a cognitive link between system structure and system-wide behaviour. In order to further utilise these techniques in the context of modern requirements, such as system maintenance or system recovery, we must be able to utilise the analysis results in a way that is able to enhance and inform environmental policy.

A natural step in the process of understanding a socio-ecological system is how we can use knowledge over the system to adapt current policy and future legislation. By utilizing LEEA, whose analysis gives us greater information about behavioural influences and system drivers, there is great opportunity to design policy based on targeting specific feedback mechanisms with the intent of having the greatest impact for the time and effort put in.

The purpose of this chapter is to continue the meta-analysis of LEEA from previous chapters with an extension known as Decomposition Weights Analysis (DDWA) (Oliva 2015; Saleh et al. 2010) and combine the two techniques with that of a commonly used analysis tool within system dynamics and socio-ecological systems, Sensitivity Analysis (SA). While the analysis tools have similar purposes; to identify and rank important features within a system, they go about it in different ways. Sensitivity Analysis considers system parameters as individual sights for system sensitivity, while LEEA and DDWA focus on the interactions between components.

The theory and application of all three methods, LEEA, DDWA and SA, is used to bridge the gap between sensitive model components, structural drivers and policy implementation. By a comparison which takes into account all three analysis techniques, this study opens up a discussion as to how such techniques can be used to identify leverage points over a system's behaviour. It is speculated whether it is possible to gain a deeper understanding of system drivers and a stronger ability to manipulate a system in an effective way when these techniques are combined together, as opposed to viewing them as separate choices in model analysis.

In this study LEEA, DDWA and SA are carried out on the same model system. Each analysis is connected by their ability to identify and rank the influence or sensitivity of system components, be they in the form of individual variables or loop structures. LEEA and DDWA are linked further as they both stem from the same line of graph and linear analysis theory. LEEA can be used to identify loop structures driving the overarching behaviour of a system, while DDWA links parameter change to individual system stocks. Sensitivity analysis is a more commonly known technique, used to identify parameters of a system which the output is most sensitive to and can be applied at a local (one parameter at a time) or regional (multiple parameters simultaneously) scale. The techniques are compared and contrasted for the components which they identify as the most important to a system's behaviour.

This experiment has multiple purposes:

1. To investigate whether there is a link between sensitive variables identified within SA and influential loop structures identified within LEEA. Do the analysis techniques agree or complement each other and would it be beneficial to use both of these techniques during model analysis of future modelling practices?
2. Do variables which are contained within the most influential loop structures output as highly influential variables when analysed at an individual level?
3. Sensitivity degrees of system parameters reflect on the parameter's influence on the overall system's or a stock's output, but will changes to parameters with high sensitivity degrees also reflect a high impact on loop dominance within the system?
4. Bridging the gap between model analysis and policy implementation: what does LEEA provide within the context of current analysis techniques and how applicable is it to reflecting on real world systems, identification of leverage points and policy implementation over these systems? Does LEEA perform well within a niche that is currently lacking within the ecological modelling process?

Using the three techniques in tandem, on the same model system and comparing results will allow us to test which parameters and loop structures within the system individual stocks are most sensitive to. Identifying which parameters each technique outputs as a key driver of model behaviour will allow for a critical analysis of LEEA against other techniques. The study will also provide a base with which to test the sensitivity of LEEA's outputs and a platform with which to discuss the identification of system leverage points and incorporate LEEA as part of a policy design process.

## 6.3 Background

The following section breaks down current knowledge and practices surrounding sensitivity analysis and system leverage points.

*System Leverage points*

Within this study, LEEA is examined for its potential use in policy. In order to bridge the gap between the graphical outputs of LEEA and the practice of making effective policy, LEEA's output is justified in the context of system leverage points. Meadows (2008) states that the applications of model output and analysis to inform policy should be conducted through the discussion and identification of leverage points.

Leverage points are defined as *"Places within a complex system(a corporation, an economy, a living body, a city, an ecosystem) where a small shift in one thing can produce big changes in everything."* (Meadows 1999). Leverage points are often linked to feedback loops generating behaviour and can be sought after to address detrimental system behaviour. Meadows talks about using leverage points to influence our policies and LEEA could provide a method to identify a system's most influential components.

Leverage points are not a recent or novel idea and have been embedded into folklore and legends for centuries as the single hero, villain or one defining act that turns an entire story on its head. In the context of complex and dynamic systems, leverage points come in multiple forms, all of which are identified as a means to generate significant change in the target system. Meadows (1999) identifies twelve possible areas within a complex system which can be identified as leverage points. Two of those twelve are directly related to system feedback loops and our ability to manipulate them:

Leverage point no. 8; *"The strength of negative feedback loops relative to the impacts they are trying to correct against."* (Meadows 1999)

Leverage point no. 7; *"The gain around driving positive feedback loops."* (Meadows 1999)

Despite it being known for decades that feedback loops are one of the key components for manipulating and controlling system behaviour, the identification of the loop structures within systems and the significance of each loop has been difficult to ascertain, until now. LEEA has the potential to idenitfy feedback loops which, in the context of Meadow's points 7 and 8, could be leverage points to the system.

Caution must be taken when identifying leverage points as they cannot be considered the be all and end all of informed political decisions. Leverage points may not always be accessible, even if they are known and we are informed enough to know which way to push them (Meadows 1999). Leverage points must be identified for their effectiveness and efficiency at manipulating the desired system or a desired parameter as potentially all parts of a system could be regarded as a point of leverage. Some will be more effective than others.

The identification of leverage points in a system should be of high importance to any model analysis where the end goal of the user is to manipulate the system. While leverage points within a model system could mean the deliberate change to a parameter's value, they may also come in the form of an entire feedback structure which holds dominance over the system. While LEEA allows us to identify dominant feedback loops within a system structure, it is limited in its ability to offer knowledge of how to best manipulate and lever those loop structures.

*The multiple uses of sensitivity analsyis:*

Sensitivity analysis, when used within an ecological context, can determine the relative importance of parameter input values on the output of the model. Cariboni et al. (2007) discuss a range of questions which sensitivity analysis can be used to answer. A breakdown of Cariboni et al.'s discussion on the uses and capabilities of SA are listed below:

- For determining the influence of uncertain variables where the evidence surrounding them may not be strong.
- Identify variables which the system is highly sensitive to while also identifying highly uninfluential variables. It is therefore possible to use SA at various steps along the modelling process. It can help to determine inputs which could be eliminated as they contribute little to no input to the variance of the model output, simplifying model structure.
- Identify areas of the model which may wish to be expanded upon, if the analysis identifies a variable which the model is particularly sensitive to, it could indicate some key dynamics/ variables missing from that part of the model.
- Able to identify if there are ranges of values which a variable can be changed within without affecting the model results.
- Able to identify input combinations which could be classified as high risk within the output parameter space.
- Used to identify interactions between variables.
- Help determine specific goals of research.
- As a method to measure whether the model accurately reflects observations and data from their real-world ecosystems.

One particular use of sensitivity analysis, known as factor prioritisation, is performed post model construction, once the model has been validated and all values are assumed to be true or lie in a range that reflects the system of the real world ecosystem. The outcome of SA in this context allows the user to identify the most influential factors corresponding to the model's output. In factor prioritisation, factors are identified and ranked in order of most deserving to re-measure in order to reduce model variance (the spread of a set of data from its average) (Cariboni et al. 2007).

A second, or rather continuation of factor prioritisation allows for the identification of highly uninfluential factors. Cariboni et al. (2007) relay uninfluential factors of a model as ones who can be given any fixed value within their domain without having any impact on the model output i.e. without contributing to a reduction in model variance.

From the above examples, it is easy to draw similarities between SA and LEEA. Both SA and LEEA can take on similar roles in the identification of highly influential and non-influential model components. Both techniques are also able to rank the influence of their corresponding findings in order to produce a priority list for system parameters. The ranking system produced from either method can also be used to increase the efficiency of carrying out model simulation, or re-building by identifying low influence factors which can largely be ignored, or should be considered to remove from the model in order to reduce model size, and increase efficiency.

The main difference between SA and LEEA is what they identify and class as influential. SA mainly concentrates on factors at an individual level or multiple factors together that don't necessarily contain a link between them, while LEEA is designed to only focus on feedback loop structures. SA can be run on every variable within a model to test its input vs the sensitivity of model output, while LEEA will only consider a particular variable if it is an integral part of a feedback loop. Both analyses have their respective advantages and disadvantages. SA, when performed at a local level (one-at-a-time) does not take into account any dynamics within a model as it solely focuses on the input of an individual variable. At a global level in SA, when multiple factors are tested simultaneously, links between variables can be inferred, hinting at some dynamics properties of the model, but this does not equate to the identification of entire feedback structures, which LEEA specialises at analysing. However, LEEA is incapable of measuring the impact of variables which act exogenously to the dynamics of a system's feedback structure.

It is within these similarities and differences between LEEA and SA that the study will cross compare the technique outputs. As SA is already an established method within the field of socio-ecological studies (Kioutsioukis et al. 2004; EPA 2003; Zaldivar and Campolongo 2000) it is seen as a beneficial exercise to compare LEEA results in order to show its relevance to the field and ground it with context against a well-known and well used method.

Examples of SA being used within the field of socio-ecological systems include Chapman and Darby (2016) and Zhang et al. (2008), who use sensitivity analysis to test their system's response to changes in variable input values. Chapman and Darby (2016) use sensitivity analysis on five system parameters whose attributed values are "notably weak evidence based", meaning that the sources of their values were not as reliable or validated to the same extent as others within the model. Chapmann and

Darby used sensitivity analysis to explore the system's sensitivity to these five parameters and in doing so evaluate their confidence in the model.

## 6.4 Methodology

The methods section covers the analysis techniques, DDWA and SA, to be used and compared alongside LEEA.

Dynamic Decomposition Weights Analysis (DDWA)

Associated with LEEA is a technique known as Dynamic Decomposition Weights Analysis (DDWA) (Saleh et al. 2010), which has been designed to increase the relevance of LEEA's output to policy. While LEEA provides a method with which to develop an understanding of system behaviour which is based on endogenous system drivers, DDWA has been developed for more policy-orientated requirements (Oliva 2016). From linear systems theory, the behaviour of a given stock can be expressed as the sum of each eigenvalue within the system, each carrying a different weight which they have over that stock at any point in time (Saleh et al. 2010). A higher weight infers a greater expression of the eigenvalue's output within the behaviour of the system stock of interest. DDWA analyses what happens to the weights of eigenvalues when parameter values within the models change through time. Unlike LEEA, the weights within DDWA are specific to each stock and help to gain an idea of which parameters are holding the greatest impact on the system at specific points in time. The results of DDWA will also be conducted within this study and discussed alongside the results of SA and LEEA.

The main role of both LEEA and DDWA is to assess the role of endogenous drivers of a system. DDWA acts as an extension to the methodology conducted within LEEA. Where LEEA analyses the influences which feedback loops have on the entire system, DDWA is able to relate the influences on stock variables to individual parameter values. In doing so, DDWA allows system drivers to be identified at a parameter based level, a level on which policy discussion and implementaion is usually based. Alongside LEEA, the output from DDWA allows not only for highly influential loop structures to be identified, but determines which parameters within those loop structures are best to target if the behaviour of a desired stock is to be changed.

A system dynamic model can be translated to a pure mathematical form as a set of nonlinear differential equations. LEEA and DDWA both rely on the system they are analysing to be linearized, but it is rare to find a model system, particularly in the field of SES, that can simulate the behaviour of its target system without using non-linear equations. In order to accommodate for this, a model's behaviour may be approximated around any one point in time as a set of time-invariant linear differential equations as described in Saleh et al. (2010).

The weights within DDWA are calculated as a function of the system's eigenvectors. Dynamic Decomposition Weight Analysis (DDWA) was originated within Saleh et al. (2010) developed from the works of Gonçalves (2009), who extended the eigenvalue elasticity approach to focus on the trajectory of individual stocks. When focusing solely on the endogenous dynamics within the system, the exogenous variables can be set to zero or constant. With no exogenous drivers influencing system behaviour, the resulting output of each stock can be written as a weighted sum of all the behaviour modes (eigenvalues) within that system:

$$x_i(t) = w_{i,0} + w_{i,1}e^{\lambda_1 t} + \cdots + w_{i,n}e^{\lambda_n t} \tag{6.1}$$

Where $x_i(t)$ is the resulting behaviour of the state variable, $w$ are the weights associated with each eigenvalue which are constants dependent on the eigenvectors and the initial conditions of the system and $\lambda$ are the eigenvalues of the Jacobian matrix (see Saleh et al. 2010 for the derivation). DDWA is concerned with the weights $w$, of the above equation as the system changes through time. Similar to the output of LEEA, the relationship between the changes made to individual elements, $a$, of the system matrix and the weights, $w$, can be calculated either as elasticity measurements $\varepsilon_w$, or as influence $u_w$.

$$\varepsilon_w = \frac{\partial w}{\partial a}.\frac{a}{w} \quad \text{and} \quad u_w = \frac{\partial w}{\partial a}.a \tag{6.2}$$

A major difference between LEEA and DDWA is that LEEA only takes into account parameters and connections that form part of loop structures, while DDWA analyses all links within a system's structure as every link has potential relevance.

The weight of an eigenvalue to any given stock is not static and is able to vary with changes which occur in other parameters of the model. The connection between a changing parameter and the elasticity of an eigenvalue's weight can be assigned a value. The magnitude of the value (as an absolute number) reflects the level of impact that the parameter will have on the weight of an eigenvalue and by knowing this, it is possible to link parameter change with eigenvalue weight and therefore parameter change with stock behaviour. The polarity of the elasticity indicates whether the parameter will increase or decrease the weight of the mode for that stock. If the elasticity value is positive then an increase in the parameter corresponds to an increase in the weight. If the elasticity value is negative then an increase in the parameter corresponds to a decrease in weight.

Saleh et al. (2010) point out that DDWA allows for the user to rank the influence of each parameter within the model in the search for potential leverage points, without the need for multiple scenario testing. This makes the DDWA efficient, especially when used on larger models where scenario testing and model runs can be time consuming.

*Sensitivity Analysis*

System dynamic models are made of four components, stocks, flow rates, auxiliary variables and constants. Of those, stocks are determined by input and output flow rates and rates are determined by auxiliary variables and constants. Constants and auxiliary variables in the model can be run under sensitivity analysis. The values associated with these variables are common places where uncertainty can occur and are areas for potential error within a model. Sensitivity analysis on these variables can be used to test how sensitive a model output, or an individual model parameter (i.e. a system stock) will be to a change in these variables. Sensitivity analysis can not only be used to identify how sensitive an individual parameter is, but also the whole model's output. This sensitivity associated with each variable is given a number called its sensitivity degree.

A parameter's sensitivity degree can be calculated manually, through dynamic modelling software STELLA or Vensim PLE Plus. Vensim PLE Plus requires a license, but whose cost is reduced if being used for academic purposes. Any variable with a sensitivity degree of $> 0.1$ is seen as a highly sensitive variable. Using sensitivity analysis the user is able to identify 'system sensitive parts' (Zhang et al. 2008) i.e. components of the system whose output is affected the most by the change of a variable's input.

A common way to quantify sensitivity analysis within system dynamic models is by calculating the sensitivity degree associated with each variable. The sensitivity degree is how sensitive a parameter (stock) or the system as a whole is to that variable. Sensitivity degree ($S_Q$) takes the following form:

$$S_Q = |\frac{\Delta Q_{(t)}}{Q_{(t)}} . \frac{X_{(t)}}{\Delta X_{(t)}}| \tag{6.3}$$

where $t$ is time; $Q_{(t)}$ is the model output of a stock at time $t$; $X_{(t)}$ is the input of the variable being tested for sensitivity at time $t$.

When a sensitivity degree can be gained for each stock in the system, an average of these degrees taken across all stocks will give the general sensitivity degree. The general sensitivity degree of a variable shows how sensitive all system stocks are combined to that variable which is another way of saying how sensitive is the system as a whole to that variable. For $n$ system stocks, $(Q_{(1)}, Q_{(2)}, Q_{(3)} \ldots Q_{(n)})$, the general sensitivity degree of a variable is calculated by the following:

$$S = \frac{1}{n} . \sum_{i=1}^{n} S_{Qi} \tag{6.4}$$

Where $S$ is the general sensitivity degree of $n$ stocks to the variable being tested; $S_{(Q_i)}$ is the sensitivity degree of the variable being tested to stock $Q_{(i)}$ as seen in equation 6.3.

*Meta Analysis Breakdown:*

1. DDWA is run on the PLUM model to identify which parameters each stock is influenced by the most throughout the simulation.
2. Sensitivity analysis is run on each individual variable within the PLUM model system to identify which register as highly sensitive or critically sensitive variables via their sensitivity degree.
3. Results are compared to that of LEEA from previous chapters to see if the variables picked out by the sensitivity analysis are the same variables that are part of the most influential loop structures from structural loop analysis (LEEA).
    - If the same variables appear sensitive as the variables involved within the influential loops of LEEA then a comparison can be drawn as to whether the analysis methods complement one another.
    - If results are different, it is possible that local SA is not able to pick out the interactions between variables and therefore is not representing the dynamic properties that are present within the model.
4. A set of variables are determined based on the results of SA and DDWA. The results are used to design multiple scenarios, testing changes within model variables against the output of LEEA and the influential loop structures. Scenarios which then produce results of particular interest are selected to evaluate within the results. Questions during scenario testing include: Is there any way of causing the hierarchy of loop dominance to change regarding a key behavioural dynamic such as a critical transition? Will the change required to force a change in a dominant feedback sit within a realistic spectrum of natural phenomena or human ability?

The goal is to investigate how much impact on the model's most sensitive variables is required to change the influence of loops within the system and investigate whether there is a link between sensitive variables and dominant loop structures. In order to achieve this, an individually sensitive or multiple sensitive variables are selected from SA and changed by orders of 10-40% to generate different starting conditions for the PLUM model system. Variables which are not selected to be changed are kept at their values from the base scenario. Each new set of starting conditions will generate different scenarios and model outputs. Each of these scenarios is then analysed using LEEA in order to calculate the influence of loop structures within the new scenario. This process will also be tested on variables which were identified as uninfluential or having a low sensitivity degree. The outputs of LEEA will be used to compare and contrast any changes which occur from the original scenario of the PLUM models forward critical transition, as explored within chapter 4. This experiment investigates how much the initial conditions of the PLUM model must be manipulated to start seeing a difference in output plots of loop influence, giving some indication on the relationship between parameter starting values and the consistency of the outputs we might expect from LEEA.

*Local SA for PLUM model*

Sensitivity analysis has been conducted on the PLUM model. An initial sensitivity analysis of the PLUM model was conducted over the region of 250-1000 years. Starting at 250 years, each parameter within the model that acts as either a constant or an auxiliary variable was set to vary by  10% by the end of the simulation. The analysis was conducted at 250 time steps to avoid any changes which were introduced to the system regarding agricultural and non-agricultural changes prior to the critical transition, avoiding the manually implemented step function within the dynamics as this is incompatible and poses a physical limitation of the online software of Naumov & Oliva (2017).

Time steps for calculating a variable's sensitivity degree are chosen relative to the total number of time steps i.e. 750 time steps between 250 and the end of the simulation at 1000 is broken into points at 250, 400, 550, 700, 850 and 1000. Between each section (i.e. 250 to 400) the value of one parameter of the system increased by a set amount, 2%. For each section after, the same parameter is increased by a further 2% from the last to create a linear increase of the parameter through time, up to a total of 10% increase from the original.

A sensitivity degree was then recorded for each value at a 150 time step interval for both the system's manually changing input parameter $X$ and the affected output stock parameters $Q$, (equ. 6.3). This calculates the relative change in parameter Q in response to changes in X. A sensitivity degree value can then be calculated from each time section through equation 6.3. A general sensitivity degree can then be calculated as an average across all time sections.

## 6.5 Results

The results section breaks down the meta-analysis into three sections;

The first concerns DDWA conducted on the PLUM model, investigating which system variables are identified as highly influential to the stock concerning phosphorus density within the lake water ($P$). The stock $P$ is focused on as this is where the system's critical transition occurs within the model.

The second section shows SA conducted on the PLUM model, primarily at a local level, identifying which variables of the model have a high sensitivity degree. In both sections, once each of the system variables have been assigned a value representing its influence, or sensitivity degree, the levels of importance are cross compared with the most influential loop structures identified with LEEA.

The third section invokes changes to the base PLUM model's forward critical transition, where the system is manipulated based on the influential or sensitivity results gained from DDWA and SA. Each new version of the model is then run through LEEA in order to test the sensitivity of LEEA's outputs to changes in the model system. The outputs of LEEA on the new model simulations are displayed in graphical form. The changes which are highlighted from each output are based on the most important information which LEEA can provide to a user about their system, namely:

1. How many Loops display as influential and therefore hold some form of control over the system's behaviour.
2. How many loops are generating stability and how many are generating instability, which loops are they and how do they compare across all model scenarios?
3. Which loops generate the highest absolute values of system influence and are therefore deemed as the ones with the most contribution to the system's behaviour? Are the highest influential loops consistent regardless of the system's initial conditions provided it always expresses the same dynamics?

*Results: Dynamic Decomposition Weights Analysis (DDWA)*

DDWA is able to weight each eigenvalue to a specific stock within the system. From the PLUM model each stock can be plotted regarding the order which eigenvalues influence that stock. DDWA is conducted at individual points along a time series to identify which parameter values are most influential to a stock at that point in time. In the PLUM model, the main concern of the system is the critical transition which it undergoes into undesirable eutrophic conditions. DDWA has been conducted prior to the forward critical transition of the system in order to assess which parameters are most influential on the stock 'Pwater' before the system undergoes a change of state. The stock 'Pwater' is chosen as the stock of interest as it represents the levels of phosphorus within the water level, which are tied to the eutrophic transition and have been shown to be part of the most influential positive feedback loop within the system from chapter 4. The following plot (6.1) shows the weightings of each eigenvalue through time as a projection of the system's conditions from t=250 which correspond to the stocks Usoil, Pwater and Msediment. The relative influence of each eigenvalue have been normalised against a constant term (shown by the blue dashed line) in order to make the influences comparable.

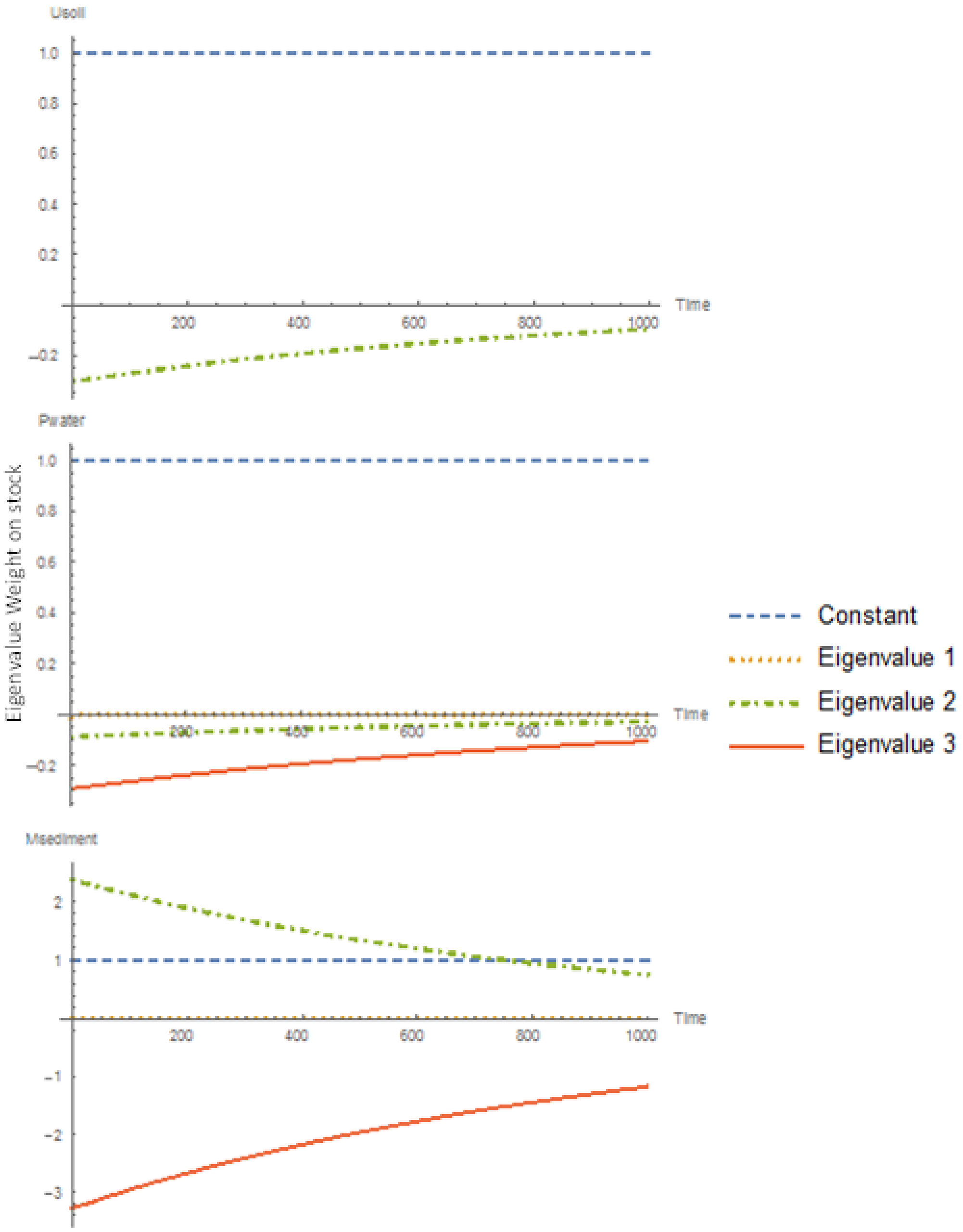



Figure 6.1: Eigenvalues weights from DDWA of the PLUM model at t=250

In each plot above, the relative influence of each eigenvalue on each individual stock can be seen through time by its associated absolute value. Unlike the entire system, whose overall behaviour was largely described within eigenvalue 1 (dotted orange line), we see that the behaviour of the 'Pwater' stock at this point in time is largely governed by eigenvalue 3 (solid orange). From LEEA, we see that at this point in time, eigenvalue 3 is dominanted by the stabilizing influence of loop 1, concerning the burial rate of phosphorus (see figure 6.2). It is therefore no suprise that the most impactful parameter at this point in time is parameter $b$, the permanent burial rate of sediment phosphorus ($years^{-1}$) (see table 6.1). It is using this information, that a deeper connection between individual model parameters and stocks can be established.

The elasticity of DDWA weight values can be calculated for each system parameter. The following table (6.1) expresses the estimated effect which each system parameter has on the elasticity of eigenvalue 3 for Pwater at time t=250. The parameters are ordered in descending order of elasticity. The values reflect the extent and manner which a change in the parameter will generate a change in the associated stock.

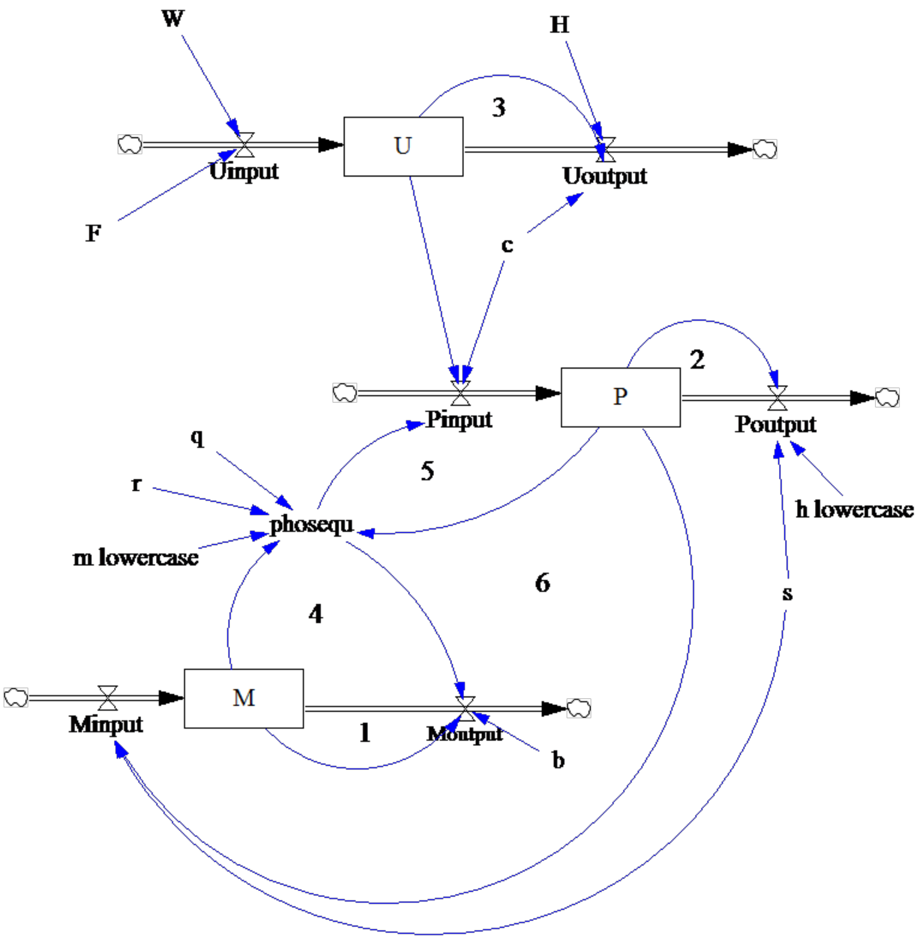



FIGURE 6.2: The PLUM model in system dynamic form

To highlight what DDWA is able to show us figure 6.3, below, shows what the elasticity values assigned to each variable mean when they are related back to a model

| | | | | |
|---|---|---|---|---|
| Stock | Pwater | | | |
| Eigenvalue | -0.00102 | Sorted By | Elasticity Re[Eigenvalue 3] | |
| | | | | |
| **Parameter** | **Re[EV] Elasticity** | **Im[EV] Elasticity** | **Re[EV] Influence** | **Im[EV] Influence** |
| **b** | 0.978065581 | Indet. | -0.00099988 | 0 |
| **m** | -0.19458945 | Indet. | 0.000198929 | 0 |
| **q** | -0.12210226 | Indet. | 0.000124826 | 0 |
| **r** | 0.024622396 | Indet. | -2.51716E-05 | 0 |
| **s** | -0.020280207 | Indet. | 2.07325E-05 | 0 |
| **h** | 0.017610751 | Indet. | -1.80035E-05 | 0 |
| **W** | 0 | Indet. | 0 | 0 |
| **H** | 0 | Indet. | 0 | 0 |
| **F** | 0 | Indet. | 0 | 0 |
| **c** | 0 | Indet. | 0 | 0 |
| | | | | |
| Modified from SDA Tool v.1.00 | | | | |

Table 6.1: DDWA elasticity values for each variable within the PLUM model of Chapter 4, for the Pwater stock (phosphorous density in lake water)

stock. In the Plot, parameter $m$ (2nd highest elasticity, negative polarity parameter) has been compared against parameter $r$ (4th ranked elasticity, positive polarity) whose elasticity is a magnitude lower. The absolute value of the elasticity shows the relative impact of that parameter. As can be seen on the plot, increasing or decreasing $m$ by 10% causes a much greater impact to the occurrence and level of the critical transition in stock P than increasing or decreasing $r$ by 10%. The polarity of m is negative, meaning that an increase in the parameter should decrease the weight of the eigenvalue, in this case , meaning the critical transition occurs much later in the time series. On the reverse, $r$ has a positive polarity, meaning that an increase to $r$ will increase the effective weight of the behaviour mode, which in Pwater causes the critical transition to occur much sooner.

As the system gets closer and closer to the forward critical transition, the weights of the eigenvalues and their projections change, depending on the desired time chosen. The critical transition of the system occurs between t=450 and t=460. At timestep 400, weights of the eigenvalues and their projections of stock Pwater have changed. At t=400, the weight of eigenvalue 2 on the Pwater stock has increased to positive values and is impacted purely by parameter $c$ carrying an elasticity value of 1, while the order of impact from parameters in eigenvalue 3, which still holds a large negative weight ranks in the following order (descending impact): $m$ (-1.8262), $b$ (0.8874), $q$ (-0.7971), $r$ (0.2380), $s$ (-0.1957), $h$ (0.0709) and $W, H, F$ and $c$ (0). If this is the desired time of interest, both eigenvalues 2 and 3 should be considered for their parameter elasticity values as they are weighted roughly the same. This exercise can be conducted as any point along the time series, which in reality, would largely be determined when requirements of manipulating the system with use of policy change would first be realised.

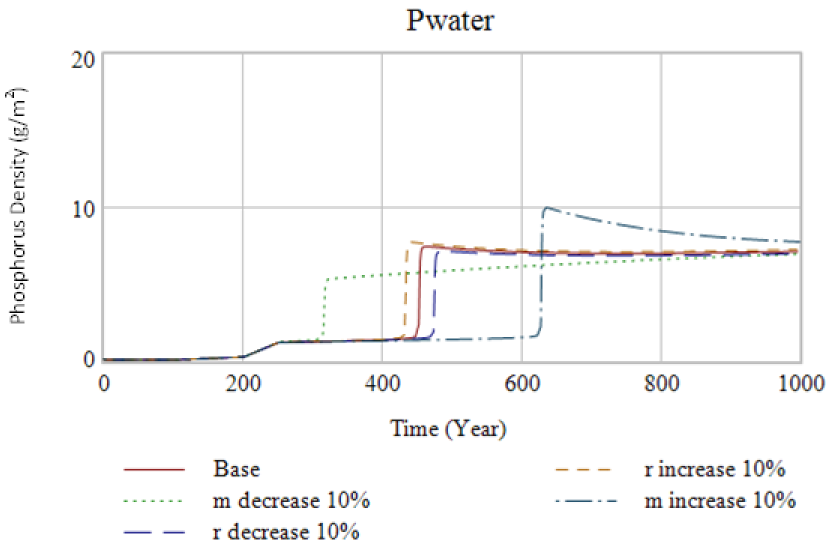



FIGURE 6.3: An example of how elastcity values from DDWA reflect back on model output, represented using variables $m$ and $r$. Variable $m$ has a high elasticity value which is negative so a 10% change can makes large differences to the output, while variable $r$ has a low elasticity, which is positive so a 10% change makes a relatively small difference to model output.

LEEA allows for important loop structures to be idenitified from a structural model, while DDWA allows the user to identify which parameters within those loop structures are generating the greatest impact to a corresponding stock. In the analysis of the PLUM model, we know from LEEA that loop 5 (the phosphorus recycling loop) was a key loop in the critical transition of this system. DDWA shows that as the parameters directly connected to that loop structure change and the system nears its critical transition, the influence of each paramter can be determined by the absolute value and polarity assigned to their elasticity. For parameters directly connected to the phsophorus recycling loop (loop 5), parameters $m$, $c$ and $q$ usually correspond to a negative polarity and $r$ corresponds to a positive polarity. This information can be used to infer which parameters of a dominating loop structure are most to least capable of generating change to any given stock.

*SA Results*

Calculating the sensitivity of a model to each of its parameters often requires a base case which the effect of increasing or decreasing a parameter value can be compared against. The PLUM model as explored within Chapter 4, using values from Carpenter (2005) is used as a base scenario with which to calculate the sensitivity of each parameter.

The results of calculating local sensitivity degrees are displayed in table 6.2 below. Each value was calculated by increasing the parameter's base value by 10% across 750 timesteps.

Table 6.2: Outputs of variable sensitivity within the PLUM model calculated relative to each system stock. Values highlighted in red register above a sensitivity of 1 and are therefore deemed the most sensitive components to the system.

| Sensitivity Variable | Usoil sensitivity | Pwater sensitivity | Msed sensitivity | Global sensitivity degree |
|---|---|---|---|---|
| W | 0.28 | 0.19 | 0.12 | 0.19 |
| F | 2.72 | 2.20 | 1.31 | 2.08 |
| H | 4.92 | 70.02 | 5.12 | 26.69 |
| c | 0.22 | 0.75 | 0.43 | 0.47 |
| q | 0.00 | 0.37 | 0.35 | 0.24 |
| r | 0.00 | 0.32 | 0.46 | 0.26 |
| m | 0.00 | 1.01 | 1.05 | 0.69 |
| h | 0.00 | 0.40 | 0.32 | 0.24 |
| s | 0.00 | 0.71 | 1.12 | 0.61 |
| b | 0.00 | 0.09 | 0.12 | 0.07 |

As shown from table 6.2, the highest sensitivity degree of any parameter within the model is held between $F$ and $H$. The sensitivity degrees of $F$ and $H$ vary from 2-26 and are much greater than any other parameter within the system. The huge difference between their degrees is determined by whether the degree is calculated with +10% or -10%. A 10% decrease in $F$ or a 10% increase in $H$ causes the system to stabilise throughout the time series and not undertake a critical transition. This causes the change in stock parameter P from the original system to be huge and therefore gain a large value for sensitivity. This shows that the system is more sensitive to increases in $H$ than it is to decreases in $H$, whereas changes in the parameter $b$, for example, it is equally sensitive to positive or negative changes.

Note that every parameter, with the exception of parameter $b$ holds a sensitivity degree which is $> 0.1$ meaning they are classed as 'highly sensitive' to the system. Changes to any parameter within a 10% range will generate significant changes to the model's output. This is most likely due to the system being a small enclosed system, where the stocks are closely linked and each parameter acts as an integral part of a stock, or dominant feedback loop.

The effect of local sensitivity, changing each parameter by 10% while maintaining all other parameters at their value within the base scenario, can be seen in the following sensitivity plots. Each plot was calculated using the system dynamic modelling software VENSIM PLE Plus (Ventana Systems Inc. 2006). Each parameter was set +/- 10% of its base value and simulated for 200 runs. Each parameter has 2 associated plots, 1) the output of the 200 runs and 2) the distribution of runs as 50, 75, 95 and 100 percentiles. For $H$, $F$ and $W$, the first 250 years has been cut to avoid the STEP functions interfering with the sensitivity analysis software package as the STEP fucntion occurred within the stock parameter U, which they were all directly linked to.

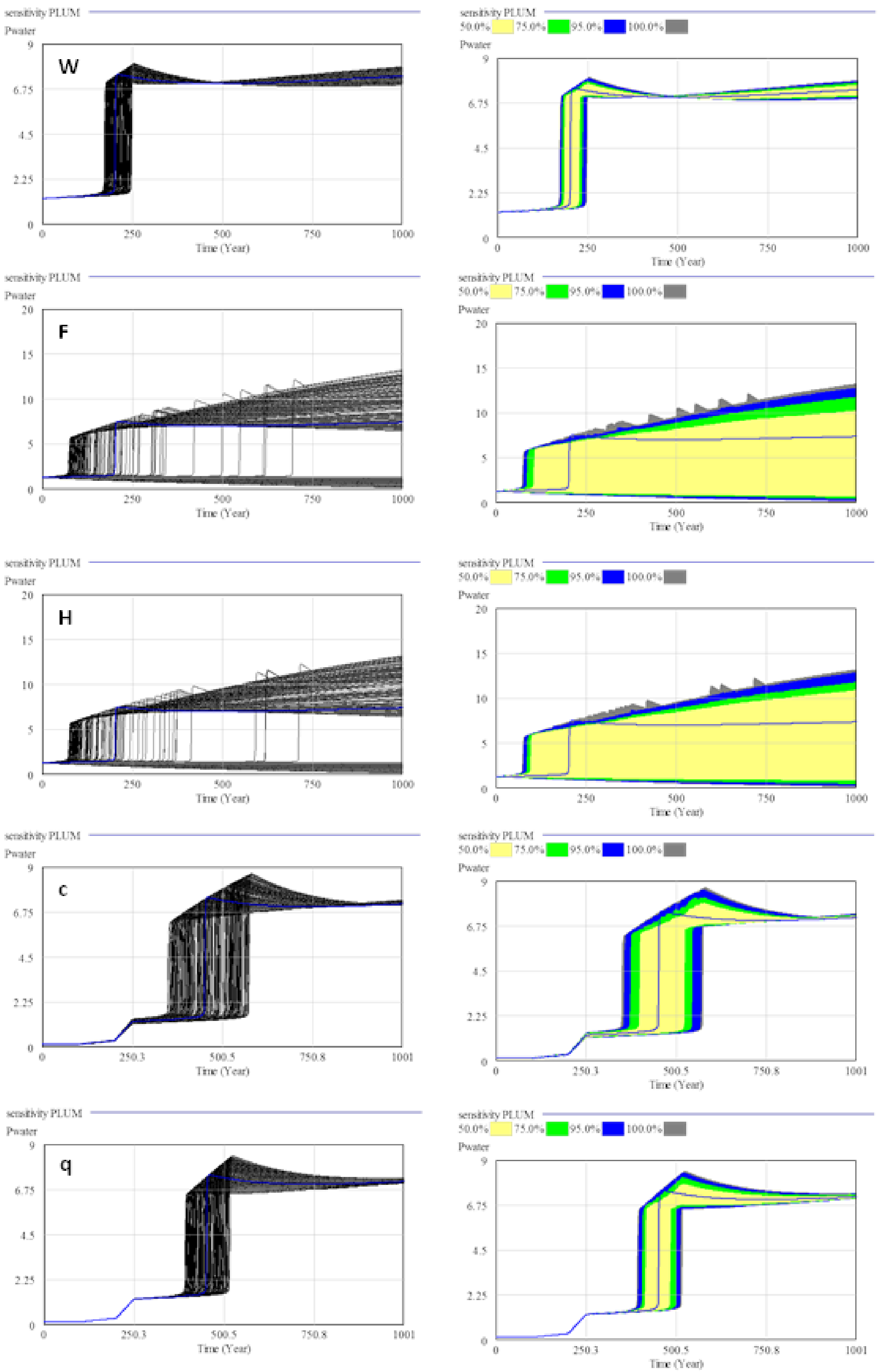



FIGURE 6.4: Sensitivity outputs for variables $W, F, H, c$ and $q$. Black line plots on the left hand side show each of the 200 runs and coloured plots on the right hand side show the range of values within the 50, 75, 95 and 100 percentiles.

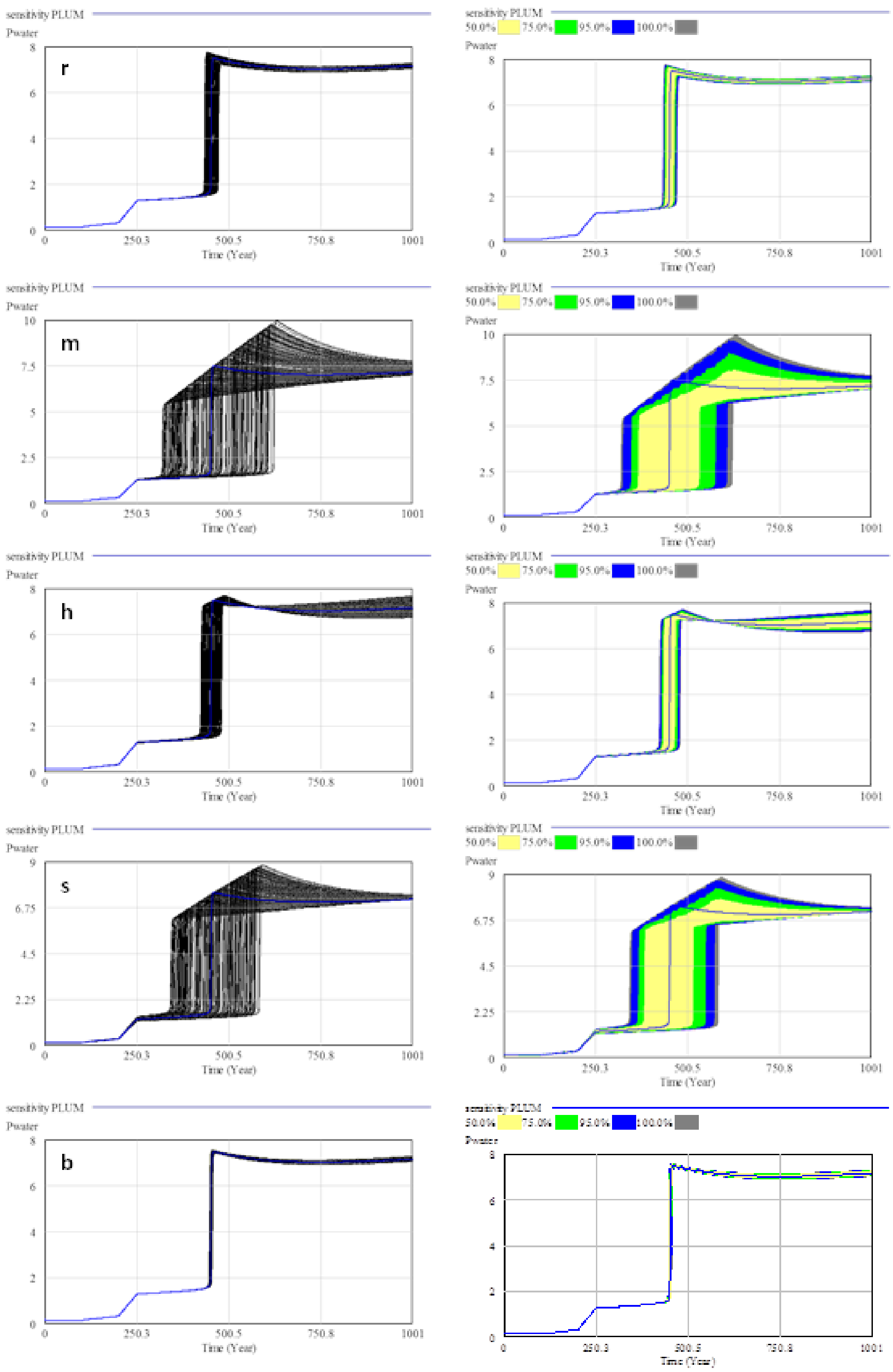



FIGURE 6.5: Sensitivity outputs for variables $r, m, h, s$ and $b$. Black line plots on the left hand side show each of the 200 runs and coloured plots on the right hand side show the range of values within the 50, 75, 95 and 100 percentiles.

From the sensitivity degree calculations and simulation runs it would appear that the model is most sensitive to parameters $F$ and $H$ as the fluctuation of these paramaters within a +/- 10% margin can determine if the system even undergoes a tipping point or not. The interesting result is that these parameters are not directly linked to the feedback loops which LEEA identifies as the most influecial loops within the system. Could it be that the failing of local sensitivity analysis to account for the dynamics in the model and the inability of LEEA to account for variables outside of feedback loop structures is making the results of influential system components across analysis tools so different?

*Manipulation of loop influence*

Taking the results of both DDWA and SA into account, the properties of the base PLUM model were manipulated to explore the extent to which loop influence surrounding the critical transition could be affected by changes in value of sensitive parameters.

Multiple scenarios were tested and ran through LEEA of which four have been selected and presented for their novel insight within the meta-analysis. The four scenarios presented include 1) increasing parameter $H$, delaying the critical transition, 2) increasing parameter $H$, preventing the critical transition, 3) increasing multiple parameters within realistic boundaries to achieve maximum phosphorus input to lake water, 4) Changing multiple parameters identified from DDWA to achieve a purposeful reduction in PLUM's most dominant loop structure (feedback loop 5).

The outputs have been arranged in a grid structure displaying the output of the water phosphorus stock (P) in each new scenario, the eigenvalues of the system and the output of the most dominant eigenvalue with regards to the whole system (figure 6.3).

Each scenario is compared against the forward critical transition within PLUM (Table 6.3):

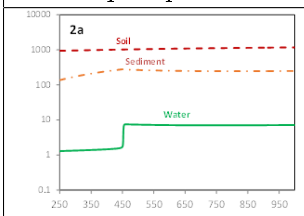
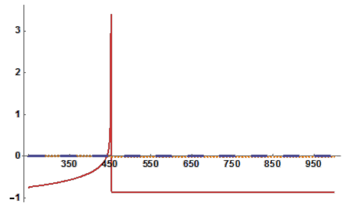
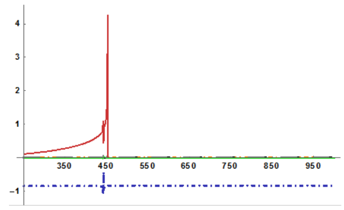
<table>
<tr><th>Water phosphorus density</th><th>Eigenvalue</th><th>Loop Influence (Eigevalue 1)</th></tr>
<tr><td>
</td><td>
</td><td>
</td></tr>
</table>

Table 6.3: The critical transition, eigenvalue plot and loop influence output of eigenvalue 1 from the original PLUM model.

1) $H$ increase with critical transition

This scenario explores the smallest change from the base model. Parameter $H$ has been identified by SA as having one of the highest sensitivity degrees within the system. $H$ represents phosphorus uptake by vegetation within the soil of the catchment, so increasing $H$ means that less phosphorus is capable of run off into the lake waters.

$H$ has been increased to 19 $g.m^-2.y^-1$ from 18.6, causing it to outweigh the 18.6 of agricultural Input ($F$). Parameter $H$ was given this specific value so that the phosphorus in the system is decreased more than that of the base system, but so that we still experience a critical transition (figure 6.4).

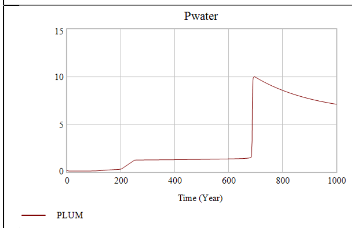
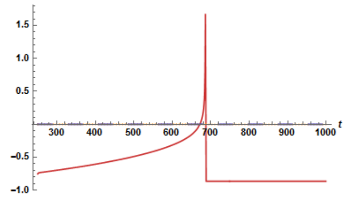
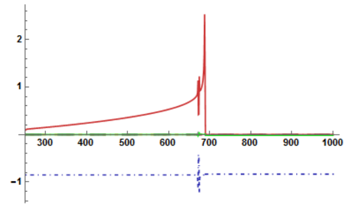
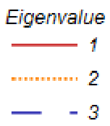
| Water phosphorus density | Eigenvalue | Loop Influence (Eigevalue 1) |
|---|---|---|
|  |  |  |
| |  |  |
| What the system outputs: System undergoes critical transition, but 200 years later than base scenario. | What the eigenvalues show: Eigenvalue 1 dominates the critical transition. The spike correlates with transition. | What loop influence shows: Loops 5 and 2 still dominate the system. Little change from base scenario. |

Table 6.4: The model output of the Pwater stock, eigenvalue plot and loop influence output of scenario 1: $H$ increase with critical transition.

Increasing $H$, to a point where the system still undergoes a critical transition, simply causes the spike build up and spike in loop influence of feedback 5 to occur relative to the critical transition within the data, but makes no difference to the dominant loops, or loop hierarchy (Table 6.4).

2) $H$ increase without critical transition

There are almost infinite possible ways of preventing the critical transition from occurring within the model version of this system through the manipulation of system values. This scenario is one of the simplest ways while still manipulating one of the system's most sensitive variables (figure 6.5).

$H$ has been increased to 19.2 ($g.m^-2.y^-1$) from 18.6. This not only outweighs 18.6 ($g.m^-2.y^-1$) of $F$, but now removes enough phosphorus from the system to stop a critical transition from occurring.

Running the simulation for 10000 time steps, the system did not undergo a critical transition. The system remains stable throughout simulation which can be seen by the constant stability from feedback loop 2 and the flattening out of instability from loop 5. No change has occurred to the dominant feedbacks loop of eigenvalue 1 as the system is still dominated by the same loop structures (loop 2 and loop 5) as before.

3) Maximum phosphorus input

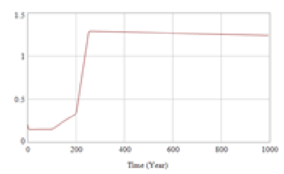
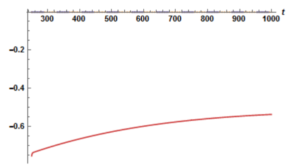
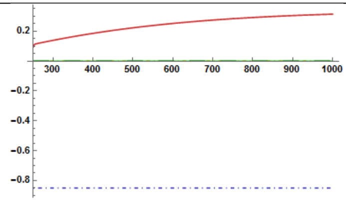
| Water phosphorus density | Eigenvalue | Loop Influence (Eigevalue 1) |
|---|---|---|
|  |  |  |
| System no longer undergoes a critical transition. | All eigenvalues either 0 or negative showing a stable system, Eigenvalue 1 dominates. | Loops 5 and 2 still dominate the system. Relative influence values of the loops heavily decreased with respect to other loops. |

Table 6.5: The model output of the Pwater stock, eigenvalue plot and loop influence output of scenario 2: $H$ increase without critical transition.

This scenario consisted of decreasing parameter $H$ and increasing $F$ to provoke an early critical transition and to maximise phosphorus input to the system (figure 6.6). $H$ and $F$ were identfied by SA as two parameters in the system with the highest degrees of general sensitivity. Phosphorus input was increased to its maximum possible within the system, while maintaining within the values of Carpenter (2005) by increasing $F$ to 31.6, and reducing $H$ to 0.

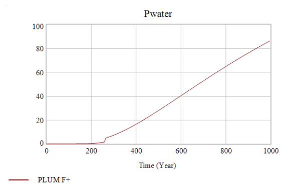
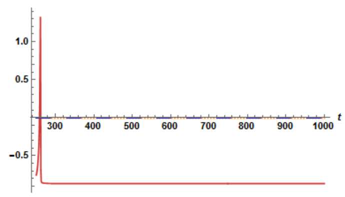
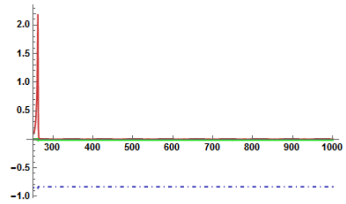
| Water phosphorus density | Eigenvalue | Loop Influence (Eigevalue 1) |
|---|---|---|
|  |  |  |
| System undergoes a critical transition early then rapidly increases with time. | Eig. 1 dominates, spike correlates with transition. | Loop 5 dominates the tip, loop 2 dominates the stability within the system in alternative state. |

Table 6.6: The model output of the Pwater stock, eigenvalue plot and loop influence output of scenario 3: Maximum phosphorus output.

The simulation makes the critical transition of the lake happen almost immediately, but makes no change to the loop hierarchy (Table 6.6). It appears that so long as a critical transition occurs within the system, loop 5 will always dominate the system in accordance to this transition.

4) Purposeful reduction in loop influence

This scenario manipulates five of the most impactful factors identified within DDWA leading up to the forward critical transition. Three of the parameters chosen ($m$, $r$

and $q$) are directly connected to the system's most impactful positive feedback loop (loop 5, concerning phosphorus recycling of the lake sediment). Parameter values of $m$, $c$, $q$, $b$ and $r$ have been manipulated in a direct attempt to change the loop hierarchy within the system's dominant eigenvalue (see table 6.7). The scenario was implemented by reducing parameters directly linked to loop 5 and increasing parameter $b$, which is linked to two stabilizing (negative feedback) loops, 1 and 4.

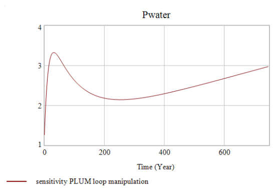
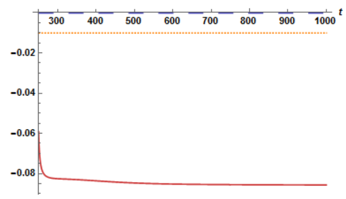
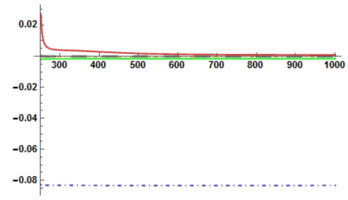
| Water phosphorus density | Eigenvalue | Loop Influence (Eigevalue 1) |
|---|---|---|
|  |  |  |
| System initially overshoots, reduces then recovers. | Eig 1 still the dominant eigenvalue but heavily reduced with respect to other eigenvalues. | Loops 5 and 2 still dominate this behaviour mode, but their influence is heavily reduced compared to other loops. |

Table 6.7: The model output of the Pwater stock, eigenvalue plot and loop influence output of scenario 4: Purposeful reduction in loop influence.

With this change all eigenvalues are negative showing that the system is heavily dampened. Of note, eigenvalue 1 is not as prevalent within the system as in other scenarios, but still holds dominance over the system.

From loop influence plot within Eigenvalue 1 (table 6.7) we can see that despite the change shown within the loop gain plot, there has been little change in the loop order of loop influence within the dominant eigenvalue. The influence of loops 2 and 5 have decreased with respect to the other loops, but they still dominate the stability and instability generated within this mode.

This scenario has been partially successful in reducing the influence of loops 2 and 5 within the system. The reduction in loop influence was achieved through the manipulation of their directly connected parameters. However, to achieve this outcome, variables in question had to be manipulated to an unrealistic extent, where values associated within the most influential loops were dropped to only 10% of their base model values which is unrealistic for the real world system. The difference made with regard to influential loop structures and influence hierarchy is minimal as the most influential loops within the dominant mode remain to be loops 2 and 5.

Despite this deliberate attempt to manipulate the hierarchy of feedback loop influence within the system, loops 2 and 5 still dominate over the system in all four scenarios. The main difference between each scenario is how dominant the loops appear with respect to other loops and whether their influence increases or decreases over time.

While the results of this test cannot be extrapolated to infer anything of other ecosystem models, it has shown within this model how prevalent the influential dynamics are within the system and how they continue to dominate as key dynamical mechanisms, despite deliberate parameter manipulation.

## 6.6 Discussion

*Reflecting on the analysis techniques*

Parameters $H$ and $F$ are identified by SA which the system was most sensitive to, yet they are not directly connected to the most influential loops in the system. This infers that there is a disconnect between the most influential loop structures and individual parameters the system is most senistive to. This disconnect between the two analysis techniques has occurred for the following two reasons 1) Local sensitivity analysis (one-at-a-time approach) is not able to account for the dynamics that occur within the model. 2) Loop analysis is unable to account for sections of the model which are not defined by loop structures.

Despite their different results, these methods are not necessarily in disagreement as to the most important and influential parts of the system. It is important to distinguish the difference between parameters which the system is highly sensitive to and the dynamics which are most influential over the system's behaviour because these two things are evidently different.

Feedback loops are responsible for the behaviour that we see in the model. So dominant feedback within the system determines which behaviours we can expect to get out of the system. Parameters which the system is the most sensitive to determine the extent to which we see that behaviour being expressed and at what point in time we see it occurring within the data.

In the PLUM model the presence and dominance of loop 5 is associated with the critical transition within the system. That is to say that whenever Loop 5 becomes dominant, the system experiences a critical transition as that is the dynamic behaviour associated with the dominance of that loop. When and if loop 5 becomes dominant is heavily determined by the initial conditions of the system.

*Reflecting on the system's dominant loop structures*

Regarding the four scenarios explored by conducting sensitivity tests on LEEA's output, it would appear that no matter how much the system's most sensitive variables are tweaked to avoid the critical transition dynamic or impede the dominance of the main two loops, the same two loops remain as highly influential structures regardless.

This raises interesting questions regarding the relationship between critical transition dynamics and feedback structures.

When $H$ is adjusted so that the critical transition no longer occurs, the system remains stable in its original state and the same stabilising loop (loop 2) dominates throughout the entire simulation. No amount of changing this highly sensitive parameter within the bounds of +/-10% affects the hierarchy of loops which dominate the system's dynamics.

It seems that so long as the critical transition occurs at some point within the simulation, loop 5 will spike relative to that transition, otherwise loop 2 keeps the system stable. In scenarios generated where the critical transition is unable to occur, regardless of parameter manipulation, loop 2 will always dominate the system's stability. It could be that within this model, there are simply not enough dynamics to compete with the influence of loop 2's negative feedback or loop 5's positive feedback during a transition.

A major question therefore is, does it take an entirely new behavioural dynamic within the system to be expressed before different loops come into play? Intuitively the answer is yes, so long as the same loops always dominate, the same dynamics will always be expressed within the system. However, we can see from scenario 2 that this is not inherently true, as feedback loop 5 is the second most dominant feedback loop within the system, but its influence plateaus, rather than grows exponentially, meaning that the system never experiences a critical transition. This means that not only the build-up of influence is important to recognise within these systems, but the trajectory by which it does so. In the case of PLUM, exponential build-up of loop 5's influence leads to a critical transition within the system, but a plateauing trajectory means the conditions for a critical transition are never achieved because the instability of loop 5, never overtakes the stability of feedback loop 2.

One of the advantages that LEEA holds over Sensitivity Analysis is its ability to display the change of loop influence through time. When interpreting a loop influence plot from LEEA, any point may be chosen along a time series and a hierarchy of loop influence may be determined. Sensitivity analysis, particularly local SA, tends to attribute a single value to each tested variable, its sensitivity degree, which represents the system's sensitivity to that variable for the whole simulation meaning the importance of a variable is assumed to be uniform across an entire simulation. This occurs because the interactions between the variables (and therefore the dynamic properties) within the system are not accounted for at a local level.

LEEA allows the user to provide good explanations of system-wide behaviour, with DDWA allowing the user to infer the causes of stock behaviour from individual links and parameters contained within loop systems. However, the interpretation of potential leverage points in this manner can run into difficulties. Suppose a scenario where

LEEA has identified two highly influential loops within a system, one generating high instability in the system and one high stability. Now suppose that these two loops share a link as part of their chains. LEEA may allow you to conclude that this link is a major factor in the behaviour of this system, but it is undermined by the extent to which inducing a change within this link will impact either loop. This scenario can be seen occurring within a model system from Saleh et al. (2010).

*Implications for leverage point identification and connection to policy*

With a dominant driving structure over a system's behaviour known, it is valuable to understand what control is held over that structure for conservation, manipulation and recovery purposes. Extrapolating this knowledge to possibilities for manipulating a target system via manipulation of its feedback loops, it might be tempting to conclude that manipulation of the most dominant feedback loop is the answer to steering the target system. There are however, multiple caveats within this line of thought;

1. The interconnected nature at which feedback loops occur means that feedback loops are seldom capable of changing independently and the manipulation of one will cause change to others (luckily this is why we have model simulation.)
2. When a loop displays as dominant over a target system, it is not easy to determine through LEEA alone, how sensitive the dominance of that loop is to changes within the individual parameters of the system. This is important with regards to loop manipulation.
3. With the resources and physical ability to influence the system, it must be appreciated that it may not always be possible to affect any of the components that are integrated within the most dominant feedback loop. Solutions for invoking change on the target system would therefore have to be achieved by considering alternative feedback loops or variables with less dominant influences.
4. It is possible that the manipulation of a system's dominant feedback loop would never result in the desired change to the system's behaviour because the dynamics inherent in that feedback prevent the desired behaviour from ever being achieved within our physical or economic capabilities. It may be that the only way to generate positive and effective change within a system is to totally remove that feedback loop from the system.

It is false to assume that we could strengthen or weaken the impact of a specific loop by altering the parameter values directly linked to those loops. This has profound implications regarding leverage point identification and feedback loop manipulation; namely finding the system's most dominant feedback loop does not go hand in hand with the identification of a system's most effective leverage points.

Relating this to policy, suppose that the goal of system management is making sure that that critical transition never happens on another lake system, similar to the one simulated within PLUM. Leverage points from this system can come from two angles A) focus on the parameters which the system is most sensitive to (i.e. in this case increasing $H$ or reducing $F$) or B) focus on altering the system in such a way that another, stabilizing feedback loop becomes dominant.

A) Assumes that the dominance and hierarchy of feedback loops and therefore dynamic properties of the system are inevitable or too difficult to change. B) Assumes that changing the most sensitive parameters in the systems is not going to make any difference because the system will inevitably undergo the behaviour anyway. A) Is theoretically easier to do because you are only having to affect one parameter B) May take more work, but in the end will yield a much better result as you have prevented the dynamics responsible for the undesirable behaviour from dominating within the system.

An example of this difference can be explained using real life examples of attempts to recover lake systems from eutrophic states back to clear conditions.

In Lake Erhai China, (Guo et al. 2001) one of the practices imposed to try and restore the lake from its eutrophic transition was to restrict the amount of runoff nutrients entering the water system. This is equivalent to policy scenario A and reducing parameter $F$ within the system.

In Lake Trehrningen, Sweden (Annadotter et al. 1999; Ryding 1982) the practice imposed was to dredge the lake of all of its high phosphorus sediment, thus removing the mechanism of phosphorus recycling from the lake and stopping the dynamics maintaining high levels of phosphorus in the system. This is equivalent to policy scenario B and changing the hierarchy of feedback loops so that the loop 5 would no longer dominate the system.

In order to control a system, an important step is to identify which parts of the system you can physically impact with regards to physical, ethical and economic constraints. This first step can largely be achieved through qualitative research at the beginning of a project, bringing together agents from all aspects on the system from those who use it for business, those who rely on it for wellbeing, those who exploit it for resources etc.

It is also important to acknowledge what the goal of the policy is i.e. what is the behaviour/component of the system that you wish to influence. Once these have been established a comparison can then be made of which parts you are knowingly able to impact and those that the analysis identifies as influential system components.

If both the influential components of the system and the parts of the system able to be affected by human intervention are the same, then policies can be built around the ability to directly affect an influential structure within the system. If the two are different, then a discussion must arise; can a knock on effect be implemented within the system whose end goal is to impact an influential structure which is otherwise unchangeable.

*The propagation of error*

The sensitivity analysis within this chapter can give some idea of how error associated with a variable within a system might propagate through the model output and impact the results of LEEA. As this chapter has already shown, a change of +/- 10% of a variable's initial conditions can have profound effects on system behaviour. If the error associated with a variable happened to be a key component of a dominant feedback loop, this would filter down through the Jacobian Matrix, into the eigenvalues and could completely change where a feedback loop registers within the influence plot. Although not directly addressed within this thesis, a useful study would be to investigate how error associated with model variables could propagate through the analysis process.

## 6.7 Conclusion

This study examines Loop Eigenvalue Elasticity Analysis (LEEA) in the context of identifying system leverage points and questions whether LEEA on its own is substantial enough to identify system leverage points to the extent where policy decision making and implementation can be based of the results. LEEA is also compared alongside sensitivity analysis, a technique commonly used to identify key drivers of system behaviour. The results highlight the importance of considering our ability to manipulate different parts of a system. The manipulation of a dominant feedback loop may prove more of a challenge than a single variable within that loop. The results of the chapter also highlight that analyses can produce different outputs of which variables are considered the key drivers, depending on whether they are analysed as individual variables or as part of feedback structures and this has consequences for the way we might address which parts of a system to target in policy. Loop Eigenvalue Elasticity Analysis might be exactly the right tool needed for the identification of leverage points within complex dynamic systems.

## Chapter 7: Chapter Preface

LEEA has now been successfully applied and tested on a model system of a shallow lake, alongside exploring its limitations and potential utility in identifying system leverage points. While the application of LEEA has proved it can be applicable to dynamics of lake hysteresis, it is important to explore the analysis in other socio-ecological contexts, if for no other reason than to show this analysis is not just applicable to lake ecology.

In the next chapter, LEEA is applied to an alternative socio-ecological system concerning the mono vs. bistability of coral reef systems. Not only does this chapter evaluate LEEA on an entirely different model system, it shows that LEEA can be used to great effect with an entirely different agenda of the model. In the PLUM model, LEEA was used to explore which feedback loops are dominating a critical transition, but in this chapter, LEEA is used to investigate the difference in the role of feedback loops in a monostable coral reef vs. a bistable coral reef.

# Chapter 7

# LEEA's Application to a Coral Reef System Capable of Alternative States - Multiple Feedback Analysis

## 7.1 Abstract

Coral reef degradation caused by anthropogenic activities is a major issue to coral reef biodiversity and ecosystem services worldwide. Major species of the reef including coral, macroalgae and herbivores are influenced by feedback mechanisms that act within and between the ecological and social spheres. Understanding these feedbacks, how and where they operate and the extent to which they drive and control the state of the reef has an important role in controlling and preventing ecosystem collapse, while reinforcing recovery. It is unconfirmed through empirical evidence whether hysteresis and alternative stable states exist within coral reef systems (as they have been proven to in shallow lakes), but it is important that we study and understand these phenomena within reef systems, as they have implications on reef restoration and marine conservation.

This study develops the analysis of a theoretical coral reef model. The model was originally created to explore how hysteresis might be generated in a reef system as a consequence of multiple feedback mechanisms reinforcing one another's behaviour. The extension conducts structural analysis on the model system, quantifying the influence which each feedback loop has over the system's behaviour at times of mono and bistability. The purpose is to explore how the study of feedback loops within ecosystems may be extended and used on reef models in the future to help us understand

endogenous system drivers and interpret their impact on system behaviour. The results of this study show that the feedback structures responsible for generating hysteresis within the system do not have a high level of impact directly to the system's behaviour, but instead amplify the contribution which every other feedback loop in the system produces. System hysteresis is shown to be generated through the amplification of every feedback loop within the system, rather than a small set of loops which dominate the system's behaviour.

## 7.2 Introduction

Coral reef degradation has become a large threat to the biodiversity and ecosystem services which coral reefs are able to provide. Pollution, nutrient loading, overfishing and poor boating practices are just some of the anthropogenic drivers which act to harm the reef ecosystem (Zaneveld et al. 2016). The practice of coral reef conservation is concerned with large system drivers on both short (nutrient and sediment loads) and long (climate change) timescales. Coral reefs can come under threat from multiple sources simultaneously, where system recovery can become impossible as previous stable attractors cease to exist and fast adaption into new niches becomes the only hope. In order to protect, conserve and restore coral reefs, it is important for us to know the extent to which human activities can drive these systems away from coral rich states and how we can focus our efforts should the reefs undergo undesirable regime shifts to conditions where the corals and fish species are not able to thrive.

A major concern for reef systems is whether they are capable of hysteresis, undergoing critical transitions between coral-dominanted and macro-dominanted stable states. Despite a lack of empirical evidence for alternative stable states being present within reef systems, alternative stable states have been observed within other ecosystems (Hughes et al. 2003, Scheffer and Carpenter 2003, Scheffer and Jeppesen 1998) and their potential presence has profound implications for coral reef restoration. Investigating the potential existence of alternative stable states has importance, not only for current theories of coral reef states, reef maintenance and recovery, but also on the capabilities of future models. We cannot predict a future transition within a model simulation, if the dynamics are not present in the model to allow it to happen.

Van de Leemput et al. (2016) investigates how hysteresis and therefore alternative stable states can be generated within a model coral reef system when feedback mechanisms, which individually hold negligible effect over the system, combine to reinforce each other's behaviour. Whether real world coral reefs are capable of hysteresis is vital to understand with regards to system stability, system recovery and species conservation (Scheffer 2009, Knowlton and Jackson 2008). Feedback loops play a vital role within system stability where negative feedbacks act to maintain a system within a stable equilibrium and positive feedbacks act to drive systems out of stable equilibriums.

Van de Leemput et al. (2016) model dynamics within a generalised coral reef system to investigate potential causes or drivers of alternative stable states being present. The dynamics within their model focus on three main community groups which act upon the reef: corals, macroalgae and herbivores. The dynamics present the co-operation and competition that occurs between the three groups. Alternative stable states are explored through the addition of feedback mechanisms which introduce further connectivity between the communities and eventually lead to the reef experiencing hysteresis across certain fishing levels. Van de Leemput et al. (2016) argue and demonstrate that we need to have a greater understanding regarding feedback mechanisms which produce alternative stable states.

Monostability vs. bistability of a reef system is explored within Van de Leemput et al. (2016) by presenting five scenarios of coral reef stability, each one different from the last determined by the feedback mechanism introduced to the system. Four of the five scenarios show the reef undergoing phase shifts (the movement of a system within a single attractor) with only one stable state across all levels of fishing (the anthropogenic stressor). The final scenario, which includes all feedback structures at once, shows the reef undergoing hysteresis and therefore being capable of bistability. The overriding concept is that under set conditions the reef requires multiple feedback mechanisms which reinforce each other's behaviour in order for the reef to experience multiple stable states.

This chapter acts as an extension to the work of Van de Leemput et al. (2016) by running a structural loop analysis known as Loop Eigenvalue Elasticity Analysis (LEEA) (Kampmann and Oliva 2006; Kampmann 2012) on the coral reef hysteresis model. The purpose is twofold: 1) Using LEEA to understand how feedback loops generate hysteresis within the system that is not found within van de Leemput's original study. 2) What information does LEEA provide us with regarding feedback mechanisms that could be useful in future reef models and socio-ecosystems in general?

Similar to the work of Nyström et al. (2012) and Van de Leemput et al. (2016), the purpose of this work is not to prove nor deny the existence of alternative stable states in coral reef systems, but rather to investigate the processes that may allow coral reefs to be capable of regime shifts through the study and analysis of a model system.

The results presented within Van de Leemput et al. (2016) were able to show how multiple feedback structures could generate hysteresis within a system, but the outputs of the system's behaviour alone gave little to no information as to how and why the system was changing. The results presented within this chapter extend the work of Van de Leemput et al. (2016), showing that bistability within the system is generated by the amplification of multiple feedback loops as a response to dynamic and structural properties and that the state of the system during a point of bi-stability is determined by a specific hierarchy of loop influence within the system.

The following section explores the consequences of coral reef degradation. It is important for us to understand whether the shift between coral and algae dominated states experienced by coral reefs can be characterised by examples of bistable, hysteric systems or whether the changes that have been observed are best approximated as phase shifts across a continuous surface.

Ecosystems which have previously been recorded degrading due to anthropogenic activity have been shown capable of transitioning between alternative stable states (expressing hysteresis), driven by positive feedback loops. The clearest examples of this from both empirical data and dynamic modelling practices can be seen within shallow lake systems (Carpenter et al. 2003; Carpenter 2005; Scheffer 2009). It is debated across the scientific community, whether the same principles of alternate stable states exist within coral reef systems, or whether they are single state systems, capable of dramatic phase shifts away from their single stable state (Knowlton 2004; Norström et al. 2009; Fung et al. 2011).

While it is difficult to determine whether real-world socio-ecosystems are capable of transitioning between alternative stable states, understanding the potential dynamics is of upmost importance affecting system management. Systems which transition between alternative stable states are difficult to reverse due to the nature of hysteresis.

## 7.3 Background

*Coral reef degradation*

Coral reefs provide goods and services to societies worldwide through seafood, resource consumption, recreational activity, tourism, culture and coastal protection (Pauly et al. 2005; Daily 1997). While they only occupy roughly 0.1-0.5% of the sea floor, they account for one third on the ocean's fish species (Moberg and Folke 1999), making them a key support mechanism for marine biodiversity.

Ecosystem degradation has been shown in coral reefs (Nyström et al. 2012), grasslands (Ravi et al. 2010) and shallow lakes (Scheffer 2009). Ecosystem degradation is the deterioration of the environment through depletion of resources, the destruction of ecosystems, habitat, extinction of wildlife and pollution (Johnson et al. 1997). When a coral reef is degraded, its ecosystem services are reduced (Nyström et al. 2012).

*"Coral reefs are major reservoirs of biodiversity and render goods and services that support the livelihoods of millions of people worldwide, but are being degraded rapidly."* (Fung et al. 2011).

Globally it has been near impossible to find a reef that has been and remains unaffected by human activity (Hughes et al. 2003, Halpern et al. 2008). As anthropogenic activity surrounding the reefs increase, knock on effects occur to the quality and quantity of the reef's good and services (Moberg and Folke 1999). Examples of coral reef net worth include coastal protection, valued between $820-1 000 000 per km coastline in Indonesia (Cesar 1996), waste assimilation services at $58 per hectare per year in the Galapagos (De Groot et al. 1992) and the financial value of tourism estimated at $8 900 000 000 in the 1990s over the Caribbean (Dixon et al. 1993). Overfishing, coral bleaching, runoff of nutrients and sediment from industrial activity, pollution, disease, dynamite fishing, cyanide fishing, use of coral as building materials and climate change have all been observed having adverse effects on the wellbeing of a reef (Wagner 1997; Jones and Hoegh-Guldberg 1999; Knowlton 2001; Hughes et al. 2003; Syvitski et al. 2005; Hoegh-Guldberg et al. 2007). These factors have all been shown to degrade the system causing decline in the species abundance, resilience and habitat structure.

*Hysteresis and feedback dynamics*

Hughes et al. (2010) express that while scientific effort surrounding coral reefs has concentrated on shifts into degraded states, less focus has been directed to the mechanisms (incl. feedback dynamics) which cause them and prevent or impair their recovery. The role of feedback loops as driving mechanisms within ecosystems should be a key subject of study regardless of whether coral reefs are capable of alternative stable states or not. Positive feedback loops not only drive transitions between states, but also retard the recovery of systems driven out of single stable states, both of which are costly to social and ecological spheres (Knowlton 2004). Studies of coral reefs which concentrate on the role of feedback dynamics have also seen use in decision support systems concerning coastal management programmes and practices (Chang et al. 2008).

The debate of alternative states vs. phase shifts has importance from a system recovery perspective and within the context of marine conservation biology (Knowlton 2004). Degradation of coral reefs generally lead them to becoming algal dominated; if alternative stable states exist within the system, then our ability to restore the reefs to coral dominance is hindered by the properties of hysteresis (lags in recovery) caused by ecological feedbacks which reinforce the algae dominant state (Nyström et al. 2012).

*"Reversal of environmental conditions to levels close to that which precipitated a degraded stable state will not lead to a recovery, these circumstances pose a particular challenge for managers"* (Fung et al. 2011).

Nyström et al. (2012) suggest that it may be a shift in driving ecological processes that lock a system into an alternative degraded state. In theory, a phase shift is easier to reverse (Beisner et al. 2003). However, even phase shifts can be difficult to reverse if they are driven by large, persistent environmental changes such as toxic chemical spills (Mumby 2009, Dudgeon et al. 2010).

It is much better to avoid undesirable state transitions as they can be difficult to reverse with increasing difficulty as the system degrades (Suding et al. 2004, Suding and Hobbs 2009)

*How can understanding feedback loops help us with hysteresis?*

In their study of feedbacks within degraded marine ecosystems, Nyström et al. (2012) introduce three concepts of feedbacks which we need to understand better in order to seek successful management of marine ecosystems:

1. The way human actions influence the strength and direction of feedbacks.
2. How different feedbacks interact.
3. The scales at which feedbacks operate.

Nyström et al. (2012) approach these concepts by investigating the extent to which critical feedbacks are represented within current management practices of degraded marine systems. Their work involves description and illustration of complex feedback synergies, real world examples and speculation on how to conduct feedback breaking through policy and management, but the analysis of the feedback mechanisms is taken no further. In this study, the work from Van de Leemput et al. (2016) is extended to show how LEEA, may help us to qualitatively and quantitatively gain a greater understanding of how feedback mechanisms hold influence within a coral reef model and in doing so, better understand points 1, 2 and potentially 3 of Nyström's introductory requirements for successful management practices.

While there are many different approaches to investigating and testing Nyström's three points, this study approaches them through the following: 1) is investigated by the manipulation of the major social variable within the model (fishing) to see how feedback mechanisms within the system are affected over its range of values. 2) is investigated by analysing how feedback loops are affected by the presence or absence of other loops. 3) is more difficult to investigate within this study as we only investigate one model where the feedbacks are set to act only within and between three main species of the reef.

Nyström et al. (2012) states that the purpose of studying these three points is to understand how and when we can influence these feedback mechanisms (What window of opportunity do we have?) in order to reduce the resilience of a degraded system. Expanding on this, better understanding of critical feedback processes may also allow us to increase and/or maintain the resilience of a preferred system state.

*Alternative stable states in coral reef systems*

Van de Leemput et al. (2016) investigate feedback loops within a reef system able to experience hysteresis (Scheffer et al. 2001) and therefore containing alternative stable states. This stems from an ongoing debate within the academic community as to whether or not coral reefs are capable of alternative stable states.

Coral reefs are capable of being dominated by either coral cover, or algae cover as the two compete for space on the reef. The debate is whether transitions between these two conditions occur as phase shifts within a system which has only one stable attractor, or as alternative stable states within a system that contains two (or more) stable attractors.

It is important to study and test alternative stable states numerically within system models as they can be difficult to test for in the real-world (Scheffer and Carpenter 2003). Mumby et al. (2013) discuss three reasons why alternative states can be difficult to test for within a real-world reef system: 1) In coral reef systems, the dynamics associated with corals occur slowly over year to decade timescales (Adjeroud et al. 2009; Halford et al. 2004). 2) Stable states are difficult to observe naturally as the reefs are frequently disturbed by stochastic perturbations i.e. cyclones (Hughes and Connell 1999). 3) Manipulation of reefs for the purpose of study would be largely prohibited, mainly due to the amount of anthropogenic impact already present across many reef systems (Petraitis and Dudgeon 2004). Some authors use the second point to argue against alternative attractors, finding that corals or macroalgal dominant states rarely exist in the real world (Bruno et al. 2009).

While a coral reef maintaining a non-coral dominant species may be indicative of a transition into an alternative state, the success of the new species could also be caused by the imprint of a catastrophic disturbance persisting within the system, changing and maintaining the ecological community without shifting it into an alternative state (Dudgeon et al. 2010). Alongside this, some reefs are naturally algal dominated: Vroom et al. (2006) explore 94 algal dominated reefs in the Central Pacific that are largely undisturbed by humans. Overall, the frequency of naturally algal dominated reefs worldwide is unclear (Fung et al. 2011).

Evidence for coral reefs experiencing multiple attractors is discussed within Scheffer and Carpenter (2003) and Mumby et al. (2013) with the use of field data, statistical models driven by field data and mechanistic ecological models. As Mumby et al. (2013) notes, one advantage of simulation models is that a system's ecology does not have to be simplified to achieve analytical tractability and alternative attractors can become emergent properties without conscious implementation.

Fung et al. (2011) found that in their model, critical transitions only occurred when parameters were set to the extremes of their empirically determined values. Fung et al. (2011) also discussed that even if reefs were capable of critical transitions, smooth phase shifts were much more likely within realistic model parameter ranges of reef systems.

Alternative studies of coral reef transitions focus on stressors that alone or in combination generate phase shifts or critical transitions between coral and algal dominance (Van de Leemput et al. 2016; Fung et al. 2011). Fung et al. (2011) discuss how it is usually difficult to separate the effects of each individual stressor in order to determine their impact on a phase shift.

These stressors often include anthropogenic factors which affect the reef, including overfishing, the aquarium trade, nutrification and sedimentation, as these external factors bring stresses to the reef that are often the cause of top-down changes from coral to macroalgae dominated systems (Norström et al. 2009). The anthropogenic factors are also stressors which policy and reef management practices could potentially have the most impact over, as they are largely driven by societal and economic pressures. In modelling practices, anthropogenic parameters are usually modelled indirectly through their effects on the ecological processes of the reef, i.e. Fung et al. (2011) model fishing as a decrease in herbivore biomass.

For their approach, Van de Leemput et al. (2016) construct a mechanistic model to explore a reef's ability to contain alternative stable states and the role of feedback mechanisms in the transitions between those states. Van de Leemput et al. (2016) focus on fishing as a controlling parameter for their simulation as this has a direct impact on the herbivore population, in turn creating knock-on effects to the death rate of the algal and coral communities.

## 7.4 Methodology

*Van de Leemput's Five Coral Reef Scenarios*

Van de Leemput et al. (2016) explore the potential for coral reef hysteresis through the manipulation of system structure and the change in fishing pressures upon the reef across a series of five scenarios. The five scenarios which they generate can be identified by the feedback mechanisms that are included within the model structure. Van de Leemput et al. (2016) show that at any level of fishing pressure, the first four scenarios of the system are only able to express monostability, having one stable state. However, under the same initial conditions as set in the first four scenarios, the feedback mechanisms within the fifth scenario when combined, are able to reinforce one another's dynamics, generating hysteresis and therefore bistability within the system.

The first scenario (scenario i) constitutes the base model which all scenarios stem from, portraying basic dynamic mechanisms known to operate within reef systems including competition for space between coral and macro-algae, predation of macro-algae by herbivores and fishing of herbivores as an anthropogenic pressure on the system. Van de Leemput et al. (2016) then present three new feedback mechanisms which when implemented individually to the base scenario, provide negligible driving pressures on the system. The three feedbacks are named 1) Herbivory-escape 2) Competition and 3) Coral-herbivore. When all three feedback mechanisms are included at once, their combined dynamics cause the system to experience hysteresis, meaning the system becomes bistable at certain levels of fishing.

Each scenario is defined through a set of equations determining which feedback mechanisms are present. Equations of scenario i, which included unoccupied space on the reef (S), reef covered by coral (C), reef covered by macroalgae (M) and herbivore abundance (H) take the following form:

$$S = 1 - C - M \tag{7.1}$$

$$\frac{dM}{dt} = (i_M + b_M M)S - gHM \tag{7.2}$$

$$\frac{dC}{dt} = (i_c + b_C C)S - d_C C \tag{7.3}$$

$$\frac{dH}{dt} = rH(1 - H) - fH \tag{7.4}$$

Despite Van de Leemput et al. (2016) coining the base model the 'No Feedbacks' scenario, there are eight feedback loops which are generated by these dynamics and are responsible for the behaviour that is expressed within the first model system (table 7.1).

| **Loop No.** | **Parameters and Sequance** | **Feedback type** |
|---|---|---|
| 1 | Cover by Coral → Coral Inflow → Cover by Coral | Positive |
| 2 | Cover by Coral → Coral Outflow → Cover by Coral | Negative |
| 3 | Cover by Macroalgae → Macroalgae Inflow → Cover by Macroalgae | Positive |
| 4 | Cover by Macroalgae → Macroalgae Outflow → Cover by Macroalgae | Negative |
| 5 | Herbivore Abundance → Herbivore Inflow → Herbivore Abundance | Positive |
| 6 | Herbivore Abundance → Herbivore Outflow → Herbivore Abundance | Negative |
| 7 | Cover by Coral → Available space on reef → Coral Inflow → Cover by Coral | Negative |
| 8 | Cover by Macroalgae → Available space on reef → Macroalgae Inflow → Cover by Macroalgae | Negative |

TABLE 7.1: Lists the eight loop structures within Van de Leemput et al. (2016) base scenario coral reef system

The additional feedback mechanisms introduced within scenarios ii-iv, named Herbivory-escape feedback, Competition feedback and Coral-herbivore feedback respectively were developed by Van de Leemput et al. (2016) based on 20 of the best documented positive feedbacks known to exist within coral reef systems having been qualitatively discussed in 60 papers (Van de Leemput et al. 2016 and references therein). Many of the dynamics of these 60 feedbacks were analysed within Mumby and Steneck (2008), Mumby (2009) and Nyström et al. (2012).

Feedback loop from scenario ii) Herbivory-escape

Feedback between macroalgae cover and herbivory rate - a scenario where consumption by herbivores saturates when algae are more abundant. This feedback does not form a structural loop, but generates a direct influence to the death rate of macroalgae). The herbivore-escape feedback affects the system when fishing levels are high as it decreases the herbivore population to the point where algae consumption saturates. At low fishing levels, when herbivore population is high, the feedback has little effect on the system.

Feedback loop from scenario iii) Competition (Interspecific)

The competition feedback generates a negative effect of macroalgae population on coral recruitment. The feedback causes a direct link to form between macroalgae and coral, where their interaction is no longer purely intraspecific from competing for space. Intraspecific competition is where: "species are indirectly affected by each other through space available" Van de Leemput et al. (2016). Interspecific competition: "proportion of macroalgae directly inhibits coral growth" Van de Leemput et al. (2016).

Feedback loop from scenario iv) Coral-herbivore

The coral-herbivore feedback introduces a direct reinforcing connection from coral cover to herbivore births, generating a feedback loop to coral through herbivore grazing of macroalgae. Corals promote herbivore growth through habitat and shelter. The loop structure connects coral cover to herbivore carrying capacity. By doing so it links the impact of herbivore population to the rest of the system as part of a feedback, instead of a simple linear input to macroalgae death.

The feedbacks present within scenario ii, iii and iv are manipulations of equations 7.2, 7.3 and 7.4 respectively and take the following dynamical form:

Herbivory-escape feedback:

$$\frac{dM}{dt} = (i_M + b_M M)S - \frac{gHM}{g\eta M + 1} \tag{7.5}$$

Competition feedback:

$$\frac{dC}{dt} = (i_c + b_C C)S(1 - \alpha M) - d_C C \tag{7.6}$$

Coral-herbivore feedback:

$$\frac{dH}{dt} = rH(1 - \frac{H}{(1-\sigma)+\sigma C}) - fH \tag{7.7}$$

Scenario v makes use of equations 7.1, 7.5, 7.6 and 7.7, the structures of which can be seen in table 7.2.

| **Scenario** | **Loop No.** | **Parameters and Sequance** | **Feedback type** |
|---|---|---|---|
| Herbivory-escape | 4+ | Cover by Macroalgae → Macroalgae Outflow → Cover by Macroalgae | Negative |
| Competition | 9 | Cover by Macroalgae → Coral Inflow → Cover by Coral → Unoccupied Space → Macroalgae Birth → Cover by Macroalgae | Positive |
| Coral-Herbivore | 10 | Cover by Coral → Herbivore Birth → Herbivore Abundance → Macroalgae Death → Cover by Macroalgae → Unoccupied Space → Coral Birth → Cover by Coral | Negative |
| All Feedbacks | 4+, 9 & 10 | N/A | Mixed |

Table 7.2: Lists the feedback loop structures within Van de Leemput et al. (2016) additional scenarios

The meanings and values associated with equations 7.1-7.7 can be seen in table 7.3.

Van de Leemput et al. (2016) show how the feedback mechanisms added to the base scenarios within scenarios ii-iv appear to generate little change in the ecosystem as the system always remains monostable across all levels of fishing. Van de Leemput et al. (2016) then use scenario v, which combines all previous feedback mechanisms together, to show how these feedback loops must not be dismissed as unimportant as their collective influence generates hysteresis within the system and therefore generate profound changes in state potential.

Symbols of the model equations and their corresponding values have all been replicated from Van de Leemput et al. (2016) in order to recreate their model scenarios. The values for each parameter to determine the stable and unstable system states were calculated by Van de Leemput et al. (2016) using parameter analyses in Matlab. The values for the base model ( scenario i), were based on two assumptions of the reef: 1) Coral growth rate is always exceeded by macroalgal growth rate. 2) With fishing set to zero, and therefore herbivore population being at maximum capacity, the macroalgae consumption rate will always exceed the natural death rate of the corals.

| Symbol | Name | Value |
|---|---|---|
| $C$ | Cover by Coral | 0-1 |
| $M$ | Cover by Macroalgae | 0-1 |
| $H$ | Herbivore Abundance | 0-1 |
| $S$ | Unoccupied space | 0-1 |
| $t$ | Time | 0-1 |
| $f$ | Fishing pressure | 0-1 |
| $b_C$ | Local expansion of existing adults of coral | 0.3 |
| $b_M$ | Local expansion of existing adults of macroalgae | 0.8 |
| $i_C$ | External import of propagules of coral | 0.05 |
| $i_M$ | External import of propagules of macroalgae | 0.05 |
| $d_C$ | Mortality of corals | 0.1 |
| $r$ | Growth rate of herbivores | 1 |
| $g$ | Grazing rate of herbivores | 1 |
| $\eta$ | Macroalgae handling time of herbivores | 1 |
| $\alpha$ | Proportion of macroalgae involved in direct inhibition of corals | 0.5 |
| $\sigma$ | Relationship strength between coral cover and herbivore carrying capacity | 0.6 |

Table 7.3: Adapted from Van de Leemput et al. (2016). Expresses the symbols, names and values attributed to construct each of the scenarios

Replication of results from Van de Leemput's 'All feedbacks' Scenario (scenario v), can be seen in figure 7.1. The parameter space where the system becomes capable of alternative stable states due to multiple feedbacks generating hysteresis occurs between 33-51% fishing.

Within each scenario, the level of fishing is able to be set anywhere between 0 and 1. 0 fishing indicates that no fishing takes place and therefore no fish are removed from the system via fishing within the catchment and 1 refers to 100% of the fish are caught and therefore removed from the catchment. It is through the fishing variable that the mono-bistability of scenario v, 'All feedbacks', is expressed. In order for a comparison of LEEA results across the different scenarios, one fishing level must be chosen.

For the analysis of Van de Leemput's model scenarios, fishing is set to a constant value of 0.51, or 51% fishing. This value was chosen from the parameter space as it is one of the final values within scenario v that lies within the bistable region, meaning that both the coral dominated and macroalgae dominated states could be compared against all other scenarios, while also making the system close to the point at which it transitions to a monostable, macroalgae dominated regime at 52% fishing. At 0.51 fishing, all monostable scenarios, i-iv are coral dominated. In total six scenarios are presented for feedback analysis and comparison. Of the six scenarios, five are coral

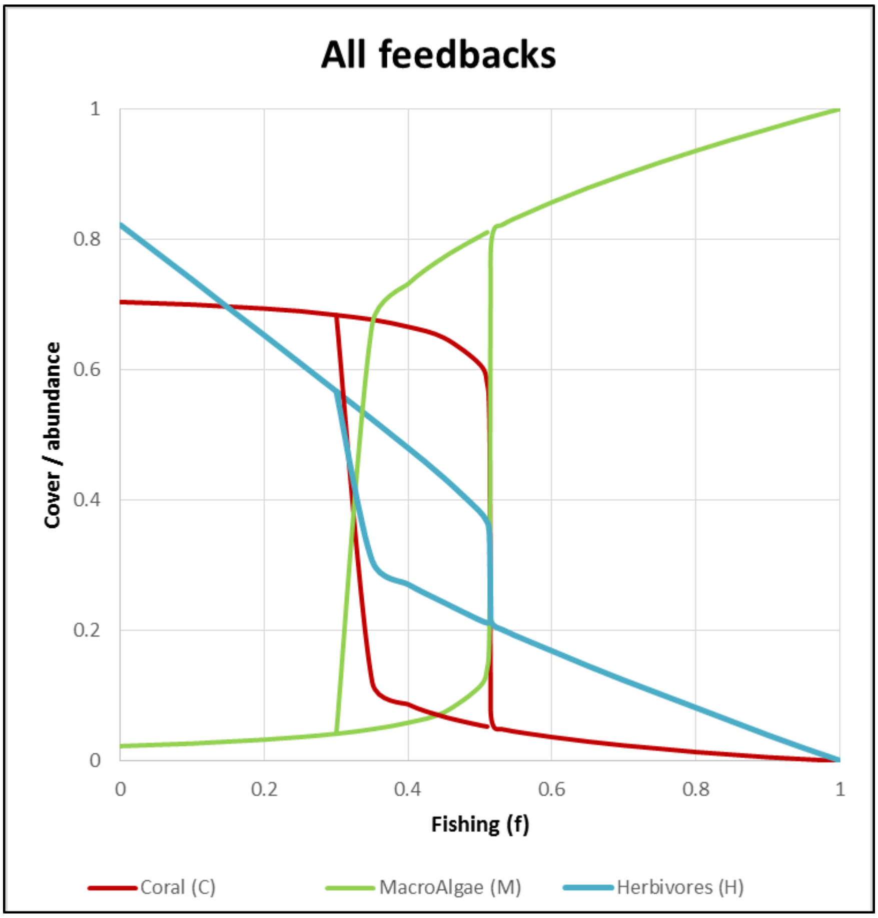



FIGURE 7.1: A replication of 'All feedbacks' scenario from Van de Leemput et al. (2016). The red line represents cover by coral. The green line represents cover by macroalgae. The blue line represents abundance of herbivores. The bistable zone occurs between 33% and 51% fishing.

dominated (scenarios i-v at fishing=51%) and one algae dominated (scenario v alternative state at fishing =51%).

*Extentions of the five scenarios using LEEA*

The work of Van de Leemput has been expanded using Loop Eigenvalue Elasticity Analysis (LEEA), a method developed to identify the influence which individual feedback loops hold over a system's behaviour (Kampmann and Oliva 2006; Kampmann 2012). LEEA is capable of identifying the most dominant loop mechanisms within a system. By analysing the loop dominance within Van de Leemput's coral reef system, the influence of each loop can be plotted relative to every other loop, effectively quantifying the significance of the mechanisms known to generate hysteresis within the system. LEEA is able to identify the impact which each feedback loop has on the system across its range of stable states.

Subjecting the five scenarios to structural loop analysis, a focus is drawn to the three feedback mechanisms of scenarios ii, iii and iv to compare the influence of these loops when analysed separately with respect to when they are comined in scenario v. The base scenario was analysed to act as a control for comparison across the other scenarios. When all feedback loops are built into one model they are shown to reinforce each other's behaviour. LEEA is used to evaluate how the reinforcing behaviours translate in terms of feedback influence to the system. e.g. Do the feedback loops reinforce each other equally, or cause one to become dominant?

Each of the 5 scenarios from Van de Leemput et al. (2016) have been built as a separate model and analysed under the same conditions, the only variation between scenarios is the structural addition of the new feedback loop and its associated variables. The structure of scenario v, which contains all eight original feedback loops and the three scenario extensions can be seen in figure 7.2. The additional connections formed by the new dynamics within equations 7.5, 7.6 and 7.7 are highlighted in red. The feedback loops created by these new dynamics are changed to dashed lines for scenario ii, dotted lines for scenario iii and bold lines for scenario iv.

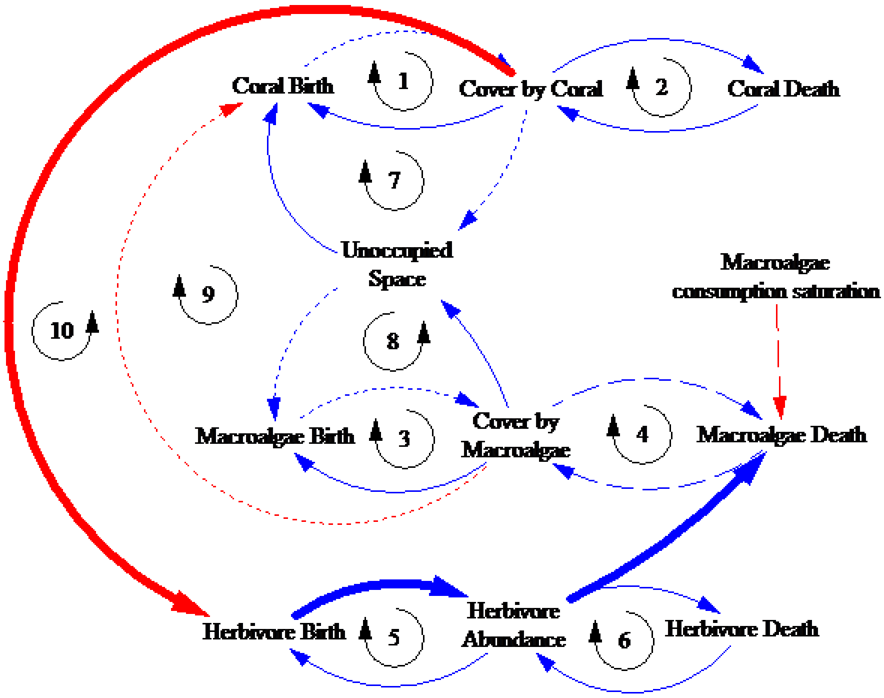
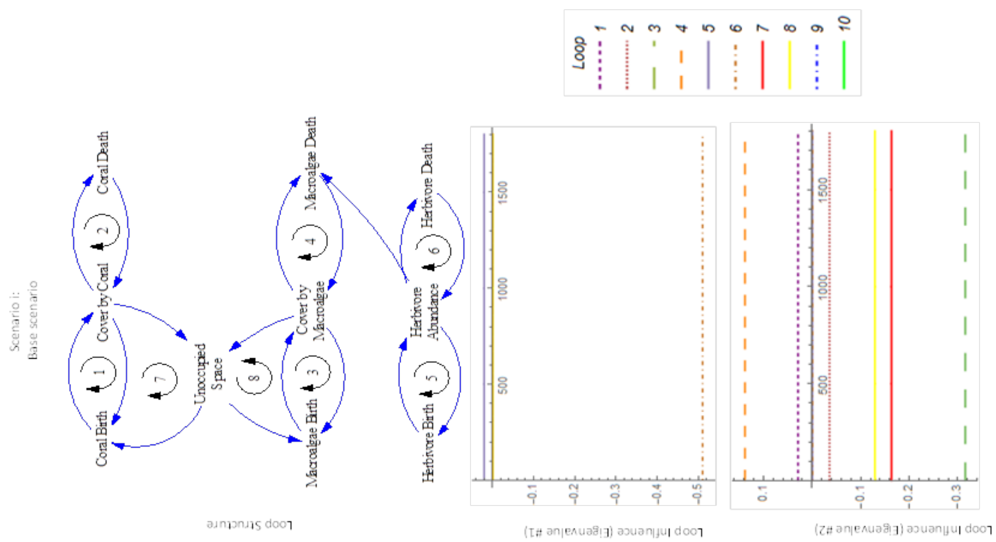



FIGURE 7.2: Cyclic diagram of Van de Leemput et al. (2016) scenario 'All feedbacks' containing all eight loops from the base scenario, labelled 1-8 and the three extensions which collectively form two new structural loops 9 & 10. Any interactions contained within the base scenario are represented with blue arrows and any interactions from the extension scenarios are represented with red arrows. Loops formed by the implementation of each scenario have their arrows adapted in the following manner: Scenario ii) Herbivory-escape - dashed arrows, Scenario iii) Competition - dotted arrows, Scenario iv) Coral-Herbivore - bold arrows. Interactions which are involved in more than one scenario are represented by the scenario which they first appear in.

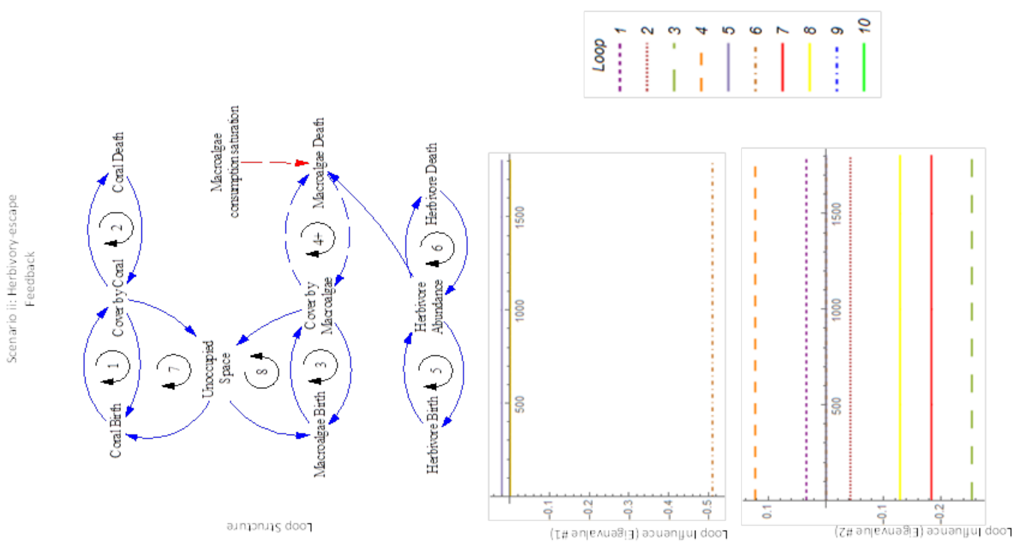
The scenarios have been subjected to model spin-up, allowing each simulation to move towards its stable state at 51% fishing before conducting LEEA. The systems have been plotted in stable state conditions in order to concentrate on the influence of loop structures which are maintaining the stable state, and exclude any movement towards these states which were deemed unnecessary for the comparison. From each scenario, a hierarchy of feedback loop influence can be calculated. The difference between these hierarchies can then be compared as different feedback mechanisms are added to the dynamics of the reef across scenarios i to v.

Each scenario has been modelled to 1800 time steps to show how the stability of the coral, macroalgae and herbivores remains constant throughout time. While the model was run for a much longer time (around 10000 time steps), 1800 proved sufficient as a check as almost all critical transitions occurred within this time period if they were going to occur at all. Results have been displayed in Figures 7.3a, 7.3b, 7.3c, 7.4a, 7.4b and 7.4c. Loop influence for Eigenvalues 1 and 2 have been displayed as these consistently show the highest contribution to system stability, by having higher absolute values than eigenvalue 3 while all eigenvalues remain negative throughout each simulation.

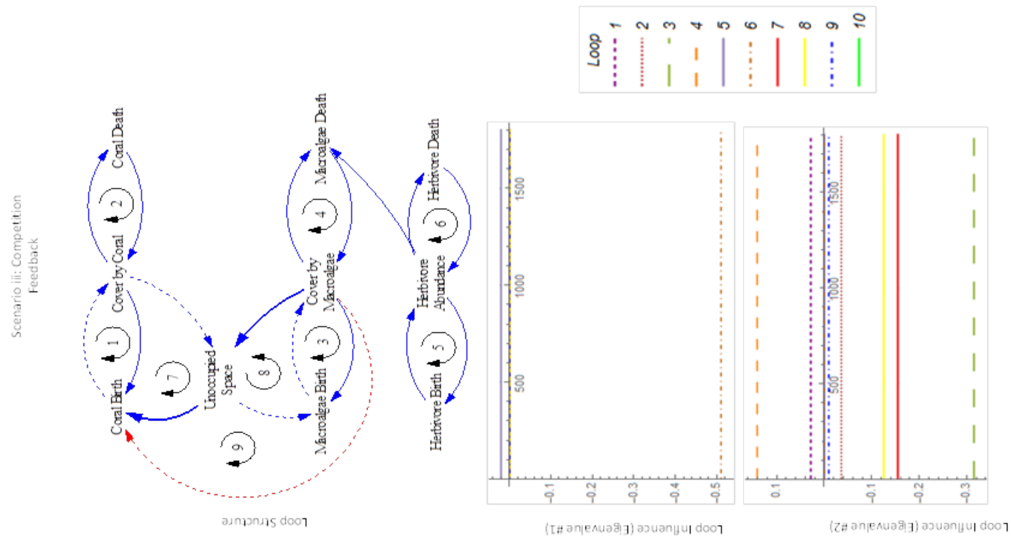
## 7.5 Results



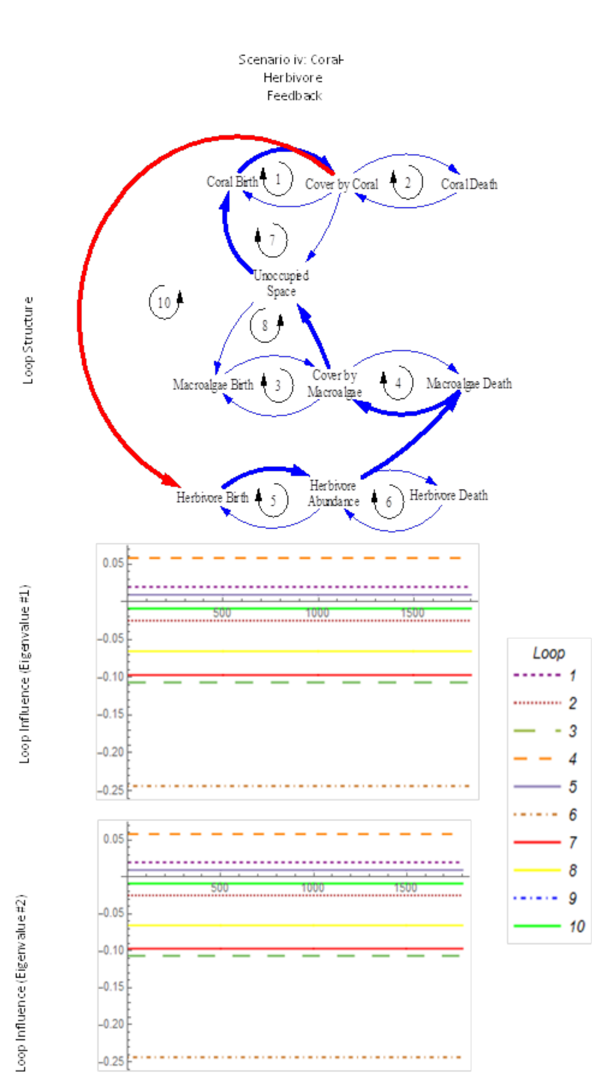

(A) Scenarios i, the base scenario showing system structures, LEEA Loop Influence (Eig 1), Loop Influence (Eig 2).



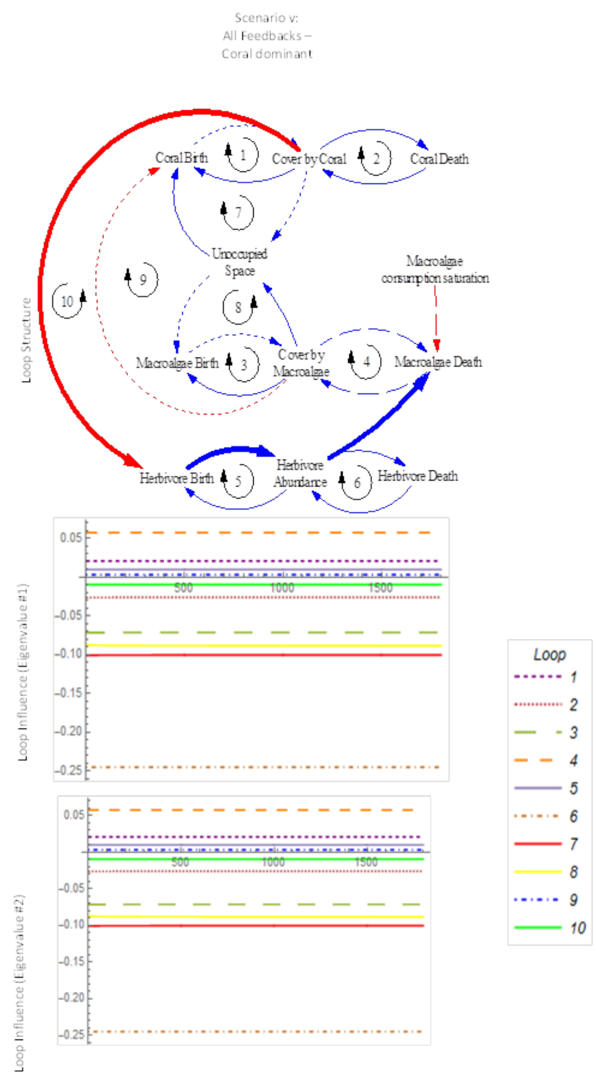

(B) Scenarios ii, Herbivory escape showing system structures, LEEA Loop Influence (Eig 1), Loop Influence (Eig 2).



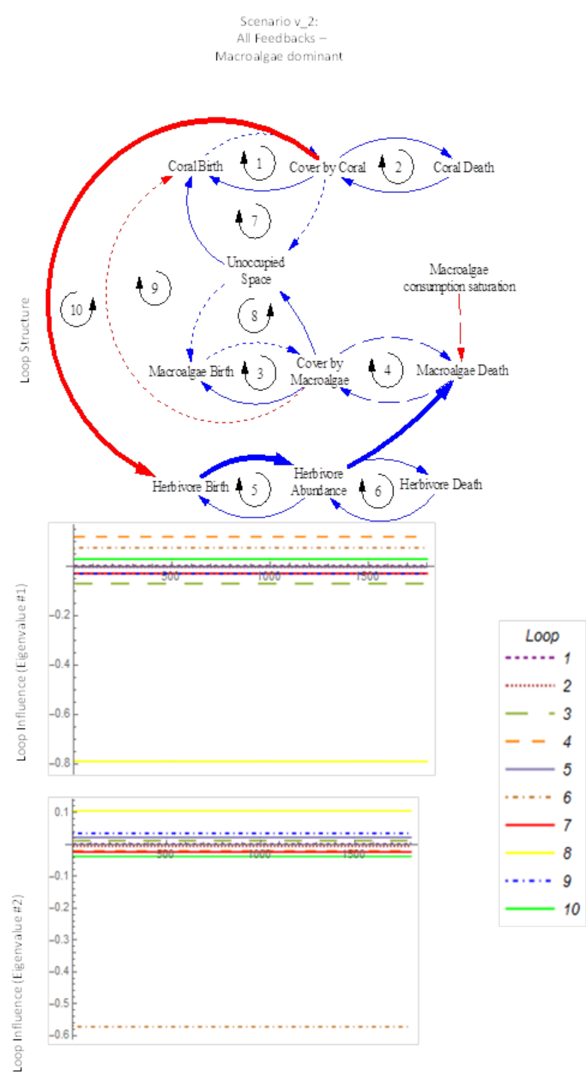

(C) Scenarios iii, Competition feedback showing system structures, LEEA Loop Influence (Eig 1), Loop Influence (Eig 2).




(A) Scenarios vi, Coral-herbivore feedback showing system structures, LEEA Loop Influence (Eig 1), Loop Influence (Eig 2).




(B) Scenarios v, All feedbacks: coral dominated showing system structures, LEEA Loop Influence (Eig 1), Loop Influence (Eig 2).




(C) Scenarios v, All feedbacks: algae dominated showing system structures, LEEA Loop Influence (Eig 1), Loop Influence (Eig 2).

The results are displayed in grid format with each column representing a separate model scenario and each row containing the model structure and loop influence for eigenvalues 1 and 2. Each of the first four scenarios has one column attributed to it, while scenario v, 'All feedbacks' has two separate states which are analysed separately.

LEEA is calculated where cover by coral, macroalgae and herbivore abundance are stable with respect to the initial conditions of the auxiliary variables. The stable states mean that LEEA plots as continuous horizontal lines as the hierarchy of feedback dominance does not shift when a system rests at equilibrium in this manner.

Structurally, scenario ii, 'herbivory-escape' feedback does not generate an extra feedback loop within the system, it only acts to reinforce one that is already present which is why the herbivory-escape feedback is not picked up as a separate structure which LEEA can identify. In scenario iii, 'Competition feedback', the loop structure that forms is represented by loop 9 (blue dotted line within the models) and in scenario iv, 'coral-herbivore', the new feedback mechanism is represented by loop 10 (solid bold line within the model). Scenario v, 'All feedbacks contains all the structures combined.

Between sceanrios i-iii, there is not much contrast between the loop influence values. All systems are dominated by loop 6, which provides stability to the system as its most dominant negative feedback loop. In terms of populations, in Herbivore-escape scenario ii, we can already start to see the effect of the 3rd positive feedback mechanism creating a decrease in herbivore abundance past the level of fishing which is occurring in the area. The stable level of herbivore abundance in the three previous models occurs at 0.49, which reflects that 51% of the fish are removed, leaving 49% to remain. However, the introduction of the Coral-herbivore loop acts to decrease herbivore levels even further.

From Eigenvalue 1: The presence of the coral-herbivore feedback loop (Loop 10, scenario iv) causes the influence of almost all feedback loops to increase while lowering the relative influence of loop 6 (herbivore death rate). The impact of fishing has been reduced relative to other feedback mechanisms.

Structurally, the difference between Coral-Herbivore feedback and All-feedback is the addition of one feedback loop (loop 9), despite it being the combination of three feedback mechanisms. The presence of loop 9 and 10 causes the loop hierarchy of loops 8, 7 and 3 to switch. However, the consequence of these changes is the difference between the absence and presence of hysteresis within the system.

## 7.6 Discussion

*Detecting the cause of hysteresis*

By analysing each scenario using structural loop analysis through LEEA, the contribution which each loop has over the system's behaviour could be compared to each other loop structure. Thus, whenever a feedback loop was added, generating a new scenario within the model, LEEA was able to show us exactly how much impact that loop was generating within the system.

Within each scenario, the individual feedback loops which were added specific to that scenario showed little to no impact to the system's output and stability. Across scenarios ii to iv, the lack of behavioural change within the system correlates to the low influence values detected through LEEA. When the addition of a feedback mechanism does not produce a high influence over the system, it may be assumed that the system's behaviour would not be affected by the presence of that loop. However, this study has shown this not to be the case as can be seen and explained through scenario V: 'All feedbacks'.

In the results of LEEA for scenario v, the additional feedback structures which are introduced to the system produce low levels of influence on the system's behaviour, inferring that these loops have little or no contribution to the system's output. However, we know from Van de Leemput et al. (2016) results that it is the addition of these loops which allow hysteresis within the system to be possible, so what has happened?

While the new feedback mechanisms have little influence over the system's behaviour in scenario v, they have profound effects on the other feedback loops present in the model. This can be seen by comparing the base Scenario i to Scenario v. The main trigger for the feedback influence hierarchy to change is the coral-herbivore feedback, producing the greatest change to the base scenario shown by the comparison with Scenario iv. Comparing across loop influence results for Eigenvalue 1, Scenarios i, ii and iii show only one dominant loop within the system (loop six: herbivore abundance - herbivore outflow) maintaining the system's stability. As soon as coral-herbivore feedback is introduced within scenario iv, all of the original eight feedback loops increase to higher levels of influence relative to their influence within the base scenario.

We know from Van de Leemput et al. (2016) that the coral-herbivore feedback alone is not enough to cause hysteresis under this set of conditions. However, it has shown to generate the greatest change in loop influence values across all loops in comparison to the herbivory escape and competition feedbacks. This change which the coral-herbivore feedback causes (increasing the influence of every feedback within the system) is due to the loop's structure forming a feedback which encompasses all three colonies of the system, linking Herbivore abundance (H) within a feedback structure to coral population (C) and macro-algae population (M), rather than simply being a linear driving pressure of macroalgae deaths.

Van de Leemput et al. (2016) identify the conditions to which the presence of hysteresis is satisfied, but the extension work conducted within this chapter identifies the interactions between feedback loops which produce hysteresis. LEEA has also allowed us to identify that the key loops required for hysteresis actually maintain little influence directly to the system's behaviour themselves.

While the system is capable of residing in either a coral-dominated or macroalgae-dominated state within its bistable region, the states are determined by the feedback loops that are dominating as shown in results scenarios 5 and 6. The stability of a coral dominated state (shown in scenario 5) is largely stabilised by loop 6 - the negative feedback loop associated with herbivore population and death rate with stabilising influences also arising from loops 7, 8 and 3 concerning the competition for space on the reef between corals and macroalgae. However, under the same environmental conditions, the macroalgae dominanted state, shown in scenario 6, is heavily dominated by loop 8 providing a stabilizing influence and loop 6 now having a much lower impact and contributing to system instability. How dominant loop 6 will be across these two regimes depends on the level of fishing within the system.

What does this imply in terms of model and feedback structure? The results from conducting LEEA on the coral reef study infer that a loop does not have to express high impact directly to the system to generate high impact on the system's behaviour. Feedback loops are capable of affecting a system's output by influencing and in this case reinforcing other loop structures.

This can be viewed as a downside of LEEA as the analysis is only able to show you a loop's impact on the system's behaviour, but not its impact on other loop structures. The additional loops to the base scenario appear to be hidden meta loops as their influence goes undetected in LEEA's output, but their presence causes change across all feedback structures within the system and are impactful enough to generate system bistability. The only way to see this is by removing the loop from the system and seeing how much this changes the output of both the system and of LEEA in a similar manner to Ford's Behavioural Approach (Ford 1999a).

This study shows us that influential loops over the system may not actually be picked up within LEEA if their presence affects the other loops, but have low impact as an individual structure. This has implications for the way that we may be able to influence these ecosystems and find leverage points (Meadows 2008). There may be multiple ways for us to influence the behaviour of a system, either targeting the most influential feedbacks, or targeting ones which the high impact feedbacks are being reinforced by.

*Error within a feedback loop*

The results from the coral reef chapter raise an interesting discussion about system error associated within feedback loops. The coral reef study is a perfect example of how feedback mechanics can reinforce each others dominance within the system and interact to form distinct system behaviours. With this in mind, the concept of how a single error might be able to propagate throughout a system model via its feedback loop interactions becomes an interesting topic for study.

## 7.7 Conclusion

This work set out to investigate feedback mechanisms within a coral reef model system, testing if a structural loop analysis known as Loop Eigenvalue Elastsicity Analysis (LEEA) can help to expand understanding of causal drivers of system mono vs bistability. LEEA was conducted on a generic model coral reef system in order to investigate its effectiveness when used on a socio-ecological system and its usefulness in regards to improving future models.

This study has shown that LEEA can be used as a valuable tool for further exploration and explanation of the potential causes of hysteresis within coral reef systems. Using LEEA, we have been able to identify the loop structures responsible for generating reinforcing behaviour within other loops of the system, despite providing limited behavioural changes directly to the system's output themselves. The results within this study have implications for future modelling practices concerning the role of feedback loops within socio-ecological systems and the way hysteresis may be generated within coral reef systems.

# Chapter 8

# Discussion

This thesis investigates whether a structural analysis technique known as Loop Eigenvalue Elasticity Analysis (LEEA) which was developed in the fields of business, industry and economics could be utilised within the field of socio-ecological systems (SES). Four main studies have been conducted assessing LEEA on grounds of its utility. Within this discussion LEEA is considered for its ability to provide serviceable information to systems whose dynamics are already well known in the field of SES and whether it has application to model systems outside the scope of this study. It is discussed whether LEEA has potential to be used to aid model scenario design and governmental policy implementation and whether it can be justified to introduce LEEA as part of a standard modelling practice.

The meta-analysis of LEEA and its application to policy as well as socio-ecology naturally invites multiple different avenues of discussion. To adhere to both the practical uses of LEEA which have been demonstrated within this thesis, as well as the speculative applications and future potential of the analysis, the discussion takes the following format:

- A summary of the studies within this thesis.
- A review of the research questions and motivations of the thesis.
- The implementation of LEEA as a standard analytical tool.
- Levarage points, Policy and LEEA.
- Exploring LEEA's limitations.
- LEEA in qualitative modelling.
- Future Work.
- Importance of accessibility - Terminology.

## 8.1 A summary of the studies within this thesis

### 8.1.1 Chapter 4: The PLUM model

The first study of this thesis applies LEEA to a small, but complex ecological system which simulates a shallow lake undergoing eutrophication. The study provides an introduction to loop analysis, acting both as a review and worked example of the methodology and interpretation. The model used is first developed into system dynamic form using dynamic equations from Carpenter (2005). The study presents LEEA so that its application, model context and utility of its outputs are accessible to a wide audience while focusing on a critical transition within an ecological system. The study highlights the benefits of using LEEA to gain information regarding dynamic changes which occur within the structures of a model system prior to a critical transition, when no behavioural change can be seen within the model's output.

### 8.1.2 Chapter 5: Investigating LEEA's limitations

LEEA has proven within the first study to have potential use in the context of ecological dynamics and concerns regarding SESs. One of LEEA's main limitations as discussed in Kampmann and Oliva (2006) is that the ability to run and interpret LEEA becomes increasingly difficult as model complexity and size increases. To explore this, the second study of this thesis takes the PLUM model from Chapter 4 and over a two stage iterative process, increases its complexity to investigate how the output of LEEA changes with model size while the dynamic behaviour of interest, a critical transition, remains the primary focus. The aim of this study was to inform on how small changes made to model structure and size could change the time and effort required to interpret analysis results. The study showed that LEEA still produced useful output when it was subjected to increasing model complexity, but it came at the cost of having to deal with a multitude of output which may not be so useful to the user. The dynamics introduced by simple additions to model structure could cause LEEA outputs to heavily differ between eigenvalues, and in some cases mask the critical transition dynamic, while making dominant feedback loops difficult to track consistently through the simulation. The chapter also discusses and provides an example of a model system where LEEA showed limited utility as its output was unrepresentative of the whole system.

### 8.1.3 Chapter 6: LEEA, DDWA and Sensitivity Analysis

With the successful application of LEEA to a shallow lake system's model and insight into its limitations, the third study analyses the outputs of LEEA itself. The relationship of model properties to LEEA output is assessed, conducting a form of sensitivity analysis on LEEA's output. With a focus on the loop influence hierarchy calculated during LEEA, it is asked to what extent the original values of the model have to be manipulated in order to affect the influence levels and dominant loops of the system. This is achieved by the execution and comparison of two alternative analysis techniques Dynamic Decomposition Weights Analysis (DDWA) and Sensitivity Analysis (SA) in order to identify highly influencial and sensitive variables within the model system. These variables were then manipulated in order to generate change within the outputs of LEEA. The techniques were chosen for their ability to identify key parameters regarding the system's sensitivity to change, using them to explore the connection of parameter dominance to dominance of entire loop structures. The study continues through a discussion on leverage points, considering LEEA's applicability to policy implementation if used alone or in conjunction with other techniques.

### 8.1.4 Chapter 7: Coral reef model

The application of LEEA to the shallow lake eutrophication model (PLUM model) demonstrated LEEA's utility and ability to provide serviceable information to an SES model. It was important to show that LEEA could be utilised by a wide range of socio-ecological model users, with the potential to be used across alternative SES model scenarios. To show this, the fourth main study of this thesis applied LEEA to an entirely different system; a coral reef where stability and coral growth are explored over a range of fishing levels. This study not only provided a second successful example of LEEA being used within an SES context, but it produced new insights into the mechanisms that the original study first analysed. The results showed how feedback loops may act collectively to achieve a dynamic behaviour and stresses the importance of understanding the interconnectvity and reinforcing behaviour of feedback structures.

## 8.2 A review of the research questions and motivations of the thesis

At the beginning of this thesis three problem areas were specified common to system dynamic research which were identified from Richardson (1996) and are still relevant today. The three areas consisted of 'Understanding model behaviour', 'Making models accessible' and 'Widening the base'. These concepts acted to form the initial direction with which the thesis was taken and helped to shape the motivations and constraints of the research.

The following section reviews the research questions and motivations as set out in the introduction of the thesis. Each research question is addressed individually, reflecting on whether the motivations behind each question remained relevant throughout the study. It is then considered whether each question was successfully addressed and if answers to specific questions were found within the case studies of the thesis.

*1) To what extent can the structure of a dynamic system model provide serviceable information about the behaviour of its output and therefore a real world system?*

This question was initially created to steer the examination of modelling techniques and analysis tools towards system structure. Prior to this thesis study and its literature review, it was known that feedback loops, causal chains, connection delays etc. played important roles as endogenous drivers of system behaviour, but to what extent we are able to quantify this was a key interest. The focus of the thesis has remained on system structure, with particular interest in feedback loops and their interconnected nature.

System structure alone can only provide limited information on a system's behaviour. With use of graph theory and structural identifiers, a model user is able to determine the number of feedback loops within a complex system and more importantly how many positive loops there are vs. negative loops. Graph theory can also be used to assess reachability between components, build causal loop diagrams and identify system components which are highly connected (West et al. (2001)). The identification of feedback loops and their polarity is important for determining potential sights of reinforcing or stabilising behaviour, but the identification alone give little insight into a system's behavioural drivers (Kampmann and Oliva 2008). The interconnected nature that feedback loops are often found in prevents causal drivers of system behaviour from being easily identified.

The proposed question progressed when LEEA was used to rank feedback loops for their dominance over a system behaviour. A key strength of LEEA is that it is capable of displaying loop dominance ranking through time alongside changes in system behaviour. Using LEEA, it was possible to identify changes in influence of feedback structures when there is no sign of system change in the model's output data.

Using structural analysis techniques, serviceable information (refering to analysis data that is both original and useful for making informed decisions) could be gained connecting system structure to model behaviour. The best demonstration of serviceable information being obtained from using LEEA can be found in chapters 4 and 7, in the lake and coral reef model studies respectively, where LEEA was originally used to test and evaluate the methodology, but also provided additional insight into the behaviours of the original models. In chapter 4, LEEA was used to show how loop dominance drives an increase in system instability prior to a critical transition. In chapter 7, LEEA was able to show how feedback loops are able to reinforce one another's behaviour to generate bistability in an otherwise mono-stable system, while generating little influence on the system directly.

*2) Complex models (high order systems, with numerous interactions) of dynamic systems can be computationally demanding, as well as time consuming for model creation and validation. Can an intermediate level of analysis be conducted alongside the running of a simulation, which would provide insight into the behaviour of the model, thus increasing model efficiency?*

All model analysis methods, are designed to increase the information that can be gained from a model system. The analysis tools compared within the literature review are all good examples of this, so whether analysis tools are able to increase model insight and efficiency is not the question here. The key term is an intermediate level of analysis, this refers to an analysis tool capable of bridging the gap between high levels of understanding on small simple models and low levels of understanding alongside simple acceptance of model output on much larger, high order models. With this in mind, can we count LEEA as an intermediate analysis tool and can its use increase model construction and simulation efficiency?

LEEA has been shown to contain limitations to its utility of large, complex models, notably on anything >13 stocks (Oliva 2016), the interpretation of its output begins to get difficult. The limitation of physically running LEEA on large models has largely been addressed with the creation of an updated online algorithm (Naumov & Oliva 2017). LEEA's ability to identify structural drivers of system behaviour and rank them through time could truly bridge the gap to our understanding of behavioural drivers in large system models. However, it is the outputs of LEEA itself that become increasingly numerous and harder to interpret with model size and complexity and may be the analysis' main downfall.

In terms of efficiency, having an initial grasp of behavioural drivers would allow the planning and construction of model scenarios to have a focus and LEEA could be used to identify structures most likely to induce change in the system when manipulated. Identifying dominant structures as well as structures generating little to no impact could heavily cut down, not only the planning time for scenario testing, but the number of scenarios run in total by specialising each scenario based on previous iterations and results of LEEA. A discussion on using LEEA to inform on policy leverage points and policy scenario testing can be found in chapter 6. Chapter 6 also looks into a user's ability to manually alter loop dominance through variable maniulation, which has implications for policy design and scenario testing where the goal is to induce change within a system.

*3) Can anything be generalised across system dynamic models which may aid the understanding of model capability when a new model is produced or an old one is reconstructed?*

Question three refers to the topic of model knowledge cross-over, allowing what has been learned from previous models to improve future models, or using knowledge from one model to inform on the output of another. It is important that what is learned from each modelling project does not become isolated and bound to that specific model, only to be forgotten about when building a new one.

In the context of feedback loop influence and its relationship to the dynamics which a system expresses, it is important to question whether the knowledge gained from implementing structural loop analysis on one model can be generalised across multiple models.

Using LEEA to investigate loop dominance hierarchy and change, in relation to a specific dynamic, (i.e. a critical transition) provides insight into how feedback loops are capable of causing that interaction. As an example, the relationship between feedback dominance and system behaviour in the PLUM model of chapter 4 may provide insight into the behaviours of other lake models, or at the very least inform a model user which dynamics need to be included in their lake model in order to simulate a transition. It should not however be assumed that a critical transition in one lake model is caused by the same structural properties. The PLUM model of chapter 4 shows a single positive feedback loop gaining influence and causing instability to build within the system prior to a critical transition. It should not be assumed that the build up to a critical transition will always be as easily identifiable, as shown within the model iterations of chapter 5, nor that it will always be the result of a change within a single dominant loop. It has already been shown in Chapter 7 that feedback loops are able to reinforce one another's behaviour, building on the capabilities of an individual feedback structure.

The role which feedback loops play on a specific system behaviour may change between model systems. In order to utilise LEEA to its fullest extent and truly appreciate the role which feedback loops play on system behaviour, feedback loop influence output in each system should be recorded. Collecting together the results of LEEA across multiple systems, would allow a catalogue to be constructed for cross comparison of loop influence over specific model behaviours. Model users could use this catalogue as a reference to find all the possible ways feedback loops have generated a specific behaviour (i.e. a critical transition) in previous models. In this way, it would be possible to form generalisations of feedback influences across an array of model systems.

## 8.3 The Implementation of LEEA as a Standard Analytical Tool

### 8.3.1 Where LEEA sits among current techniques

At the beginning of this study, LEEA was reviewed for its place among existing analytical techniques. While LEEA could provide useful information on a system model, there would be little to gain in adopting and utilizing its methodology if the information it provided could already be achieved within existing analysis techniques.

The theory on which LEEA is based largely stems from Graph Theory and Linear Stability Theory. Respectively, Graph Theory allows the user to gain a great deal of information about their system's structure, its connectivity and potential critical components, while Linear Stability Theory can provide insight into when a system is deemed to be stable or unstable and liable of transitioning into alternative regimes. When combined, the application of these techniques allow LEEA to identify critical structures of a system responsible for generating strength and stability or reinforcing weakness within a model system. While there are alternative techniques which can be used to identify critical structures in a system, or identify variables within a system which it is most susceptible to changes of, i.e. Ford's Behavioural Approach (Ford 1999b) and Pathway Participation Metrics (Mojtahedzadeh et al. 2004), none of them focus on the role of feedback loops within a system to the extent to which LEEA is able. The ability to identify a hierarchy of feedback loop dominance over system behaviour, show how it changes through time and how this reflects back on changes occurring within the system is valuable to SES research and analysis and appears to be unparalleled by any other current structural analysis technique.

While the outputs of LEEA explored within this thesis and the possible uses it could have on future modelling projects shows great promise for the technique being utilised on a broader scale in the SES field, it does not come without its limitations. Among these an inefficiency when used on models not driven by feedback loops and an increased level of complexity to the output when used on large models (13+ stock) appear to be the most prevalent. While these limitations can be largely addressed with further development of the technique and practice interpreting the outputs, they result in LEEA not being best suited to all modelling projects.

### 8.3.2 Implementing LEEA into Standard practices

The outputs of LEEA as a stand alone analysis have been shown to provide useful output to ecological dynamics, but have even greater potential when combined with the information gained through other analysis techniques, such as sensitivity analysis and its partnering analysis tool DDWA as shown in chapter 6. A user should not limit themselves to the output of a single technique, just because it works. Combining analysis results and not restricting oneself to the output and interpretation of a single one can lead to simulations and policies designed from a position of greater understanding over a system's dynamics and structural properties. The benefit of these techniques is that collectively they can be run within a relatively short space of time (a few days to a week), but provide valuable information when compared and contrasted that could save time and money down the line.

The following explores a step guide of how LEEA can be implemented within an ecosystem model, from early stages within the construction process to the application of LEEA to real world systems with the promotion of good practices throughout. The following order is taken from the author's own personal experience, through various projects, conversations and presentations of other modellers. The specific order will vary between users but the purpose behind each section remains constant between model projects. Of a 12 step modelling process, LEEA is applicable to: Step 5, concerning model construction; Steps 7, 8 and 9, regarding model behaviour testing and revisions; Step 11, concerning scenario testing and finally step 12 regarding policy design and implementation.

1) Model conceptualisation: Qualitative research with multiple organisations to generate a conceptual model.

System models simply start as an idea or a concept which translates to a mental model of the system. Every individual's mental model of the same system will be different (Carpenter et al. 2005) determined by their background knowledge and experiences. When first conceptualising a model it is important to have a focal point within a modelling team so that each individual's idea of what the model should contain and portray is represented in the concept model. At this early stage it can be important to discuss what the main aims, questions and motivations of the study are, in order to restrict the multitude of directions that a socio-ecological system model could be directed in.

2) Converting model into cyclic diagram.

A cyclic diagram is primarily a visual representation of the system's components and interactions. As interactions are added, the diagram will give an initial impression of the system's structure, allowing the user to gain a rough idea of the number of feedback loops and high connectivity nodes which the model may contain. At this stage in design, dynamic properties associated with stocks, i.e. differential equations of the system, can be used to aid the identification of system interactions, but they are not necessary to generate an initial diagram. In some cases specific equations are best left for a later stage in development so that focus stays on the model as a whole and does not become tied down to a specific dynamic.

3) Discussion of interactions.

Everyone involved in the design process of converting a real world system into a model will have different perceptions of which components interact together and where interactions should be held within a model. Disagreements may infer that the model is too basic, unable to represent multiple aspects of the system correctly. A priority must be that the model does not become unnecessarily complex, constraining the model's dynamics to specific questions and motivations of the study.

4) Conversion of cyclic diagram into system dynamic format.

Conversion of a cyclic diagram into a system dynamic model can be a relatively simple process as structurally the two model formats will remain the same. The conversion involves taking the variables and interactions of the cyclic diagram and converting them into a series of stocks, flows, auxillary variables, constants, sources and sinks. Finding dynamics to input into the system dynamic model can prove slightly challenging and can be generated as part of the modelling process using local knowledge of the specific interactions between components to build up the dynamic properties of a stock.

5) Data collection of inputs.

Collection of empirical data to be input into the model is important for both validation and accuracy of the model. Without quantifiable data, the outputs of the model and of the analysis will be unable to accurately reflect what is happening in the real world system. Data collection also provides an important base to be established for the value of each model component, which is important for designing and testing scenarios. Data collection is potentially one of the most time consuming parts of this process and is most susceptible to the time constraints and funding available to the project. Up to this point, LEEA has little use within a model project as it's strengths come from analysing the dynamics of a system. While an early use of LEEA using abstract parameter values may allow the user to identify which dynamic behaviours are associated with which feedback loops, LEEA requires the model to have empirical data input to it in order for the analysis to have any meaning in a real world context.

6) Running of model for output.

Running the model for output allows the users to reflect on the dynamics occurring within the model, how accurately they are reflecting empirical data and gain an initial output with which to base other scenarios.

7) Sensitivity analysis and Principle Component Analysis (PCA) for unsure parameters & identification of highly sensitive / unresponsive parts of model and strength of connections within the model using LEEA and DDWA.

Sensitivity analysis and PCA are useful analysis tools to help identify variables within the model to which the output is particularly sensitive (Cariboni et al. 2007) or to identify the principle components of a model output, the variables associated with the most variance within the output data (Abdi and Williams 2010; Wold et al. 1987). If the model is unable to express the dynamics required or cannot reflect the empirical data closely, then sensitivity analysis and PCA can help to identify parts of the model which may need expanding. At this stage, LEEA and DDWA could also be utilised to fill a similar niche within the modelling process identifying highly influencial vs. highly uninfluencial structures within the system.

8) Carry out LEEA, DDWA: analysis and discussion of results.

LEEA and DDWA can act as valuable analysis tools once a model and multiple scenarios have been established. The dominant, influencial structures which these techniques are designed to identify allow an additional comparison to be achieved to model dynamics and between simulations.

9) Discussions with organisations of preliminary results (may not always be possible).

Discussion of preliminary results ideally involves bringing back together all of the original organisations affiliated with the project to discuss the model output. Discussions may include whether the model reflects the aims and objectives during the construction of the cyclic diagram and whether the dynamics are represented properly. Using results from LEEA, DDWA and SA together, discussions may extend to structures and parameters within the model identified as primary drivers of system behaviour, vs. the sections of the system which each organisation feels they are able to easily manipulate. This part of the discussion is revisited in step 12.

10) Model revisions and back to step 6.

Model revisions occur as a result of steps 7 and 8. It is important for model construction and output to be an iterative process in order to achieve a structure which best reflects the real world system and to ensure the dynamic properties of the model are able to reflect the aims and objectives of the project. It is highly unlikely that the first generation of the model will be the best reflection of the project's requirements and opening the model reconstruction before further analysis is an important learning process in model development and understanding of the core system.

11) Scenario testing.

System dynamic models are not often used as predictive models. Their role is best suited for scenario and hypothetical policy testing, allowing the user to observe a multitude of potential futures given an initial set of conditions. Scenario testing can be conducted once a base model has been established and finalised in order to have a platform with which to edit system structure and parameter values to reflect the implementation of policy. Each scenario could be run and analysed using LEEA to investigate how dominant loop structures change with different initial conditions. In this step LEEA could be used as a reflexive and integrated tool as part of the testing process, meaning its analysis could be used to learn from and build new scenarios between iterations, rather than simply acting as an overarching analysis to prebuilt scenarios.

12) Discussion with original organisations to focus on leverage points and application to policy.

With the identification of key driving structures within the system, parameters and feedback mechanisms can be established as potential leverage points with which to focus policy and direct positive change or stability within the system. Discussion with the original corporations is once again a valuable part of the process as it is important to establish leverage points determined within the model by LEEA, DDWA and SA, vs. sections of the model which are the easiest (physically, economically and ethically) to manipulate. This information could then be used to start looking at alternative methods for manipulating dominant feedback structures if they are difficult to directly change in the real world. This may involve an attempt to completely remove the dynamics of a feedback loop as dicussed in chapter 6 or identifying an alternative set of feedback loops which are reinforcing a dominant loop's behaviour as discussed in chapter 7.

### 8.3.3 Model Design and Construction: Participatory Complexity Science

The process with which system models are constructed, validated and applied to understand their real world counterparts can heavily determine whether they are useful to aid policy development. The utility of a model can depend on how accurate the model is and how many dynamics of the real world it is able to simulate. Part of this thesis' discussion addresses where structural analysis techniques fit within the modelling development process. The following section explores where LEEA and DDWA fit into SES model design and development, starting with the importance of a structured and multi-disciplined approach to model design.

In order to effectively use LEEA and be able to apply the outputs of loop influence plots to achieve greater understanding of the target system and its controlling dynamics, the model which LEEA is applied to must be an accurate representation of the target system. This is because the techniques used within LEEA are purely analytical and so the outcome heavily relies on the dynamics, structure and initial conditions within the model. The output of LEEA is only as good as the model it is used on. To that end, it is important to ensure that each step taken in constructing a model system is designed to achieve the most accurate model ( in this case meaning low measurement error of parameters and structural fidelity) possible.

The first step to achieving this goal is through competent model design. When a model is designed by a single author, there is risk of bias. Models can be designed, sometimes unintentionally, capable of simulating a desired system behaviour, but not be representative of the real world system. This bias can be caused by the limitations of the user's knowledge and the desire for a model to verify a hypothesis or satisfy a client without the dynamics or structure of the model ever being questioned by another person. To avoid this, a technique known as participatory complexity science can be adopted, which is specifically designed to promote good practices during model conceptualisation and construction.

Dr. Alex Penn from the University of Surrey, England, describes Participatory Complexity Science as an approach to build models using the existing knowledge of local companies and authority figures when their combined knowledge and goals of the target system are brought together. The modelling technique refers to the construction of 'Fuzzy Cognitive Maps' (FCM) (Penn et al. 2017; Penn et al. 2013), a full description, methodology and advice regarding the technique can be found on their website (Cctool.herokuapp.com). The approach requires a great deal of of groundwork and interviews to bring different companies and research facilities together, making sure that everyone is on the same level in terms of a common goal. The model created is formed with nodes and connections with polarity, similar to the early stages of a system dynamic model: the network of a causal loop diagram, but the dynamics are purposely not discussed during early stages of development. Dynamics are left out of discussions at this early stage to avoid the group getting stuck on individual sections of the model, when the goal is to achieve an overall design and understanding of the system's structure. The model is analysed using graph theory to identify potential high influential levers within the system in the form of nodes with high connectivity. A variety of different agencies (i.e. local authority or industry) are then asked to rate the nodes in the model which they believe they have most vs. least control over. Often the aspects of the model that the different agencies believe are easy to control are different across companies.

Identifying which sections of a model different industries believe they have the most control over is important, both in terms of policy design, but also when interpreting the outputs of LEEA. LEEA can help us to understand where to focus our interest in controlling the system. However, there is a huge difference between the most influential component of a system (identified using LEEA) and the parts of the system that the community and local authorities actually have the power to change. Different agencies will have different perceptions of what they are capable of controlling within the system. A leverage point which is impossible to change for one company, may be

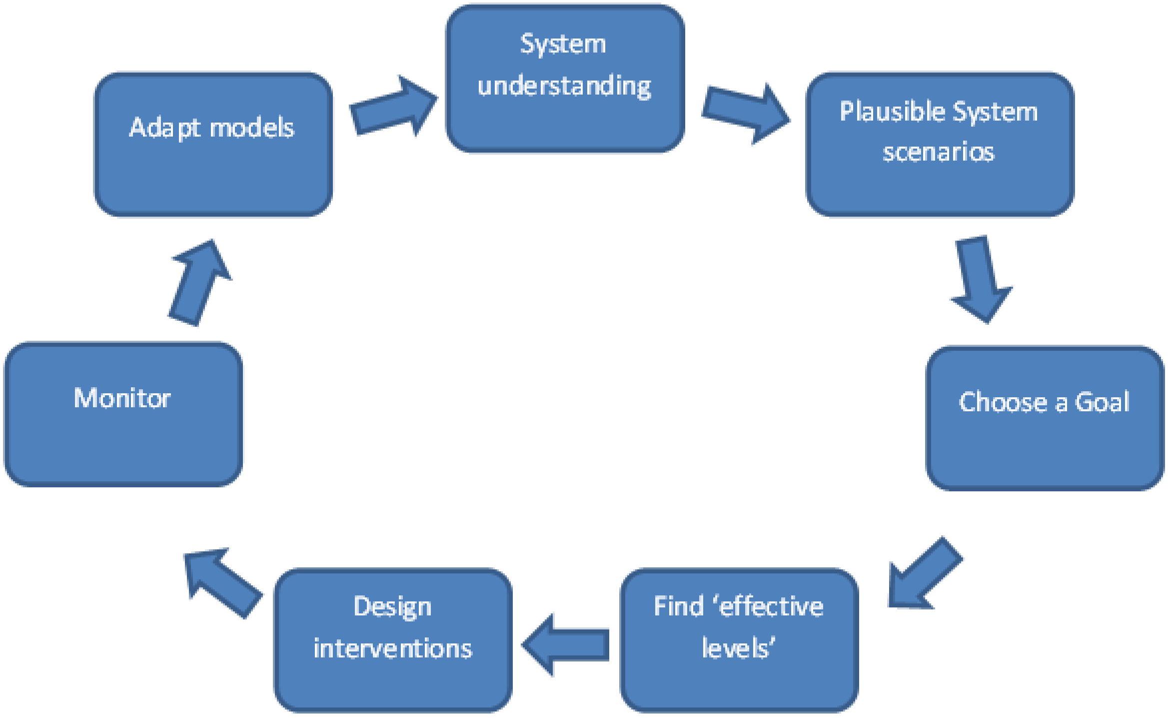



FIGURE 8.1: The process of participatory complexity science. Adapted from Dr Alexandra Penn, 2017 - guest lecturer at Geography and Environment, Southampton University. Originated from 'Steering Complex Human Systems' (Penn 2017).

easily manipulated by another, in which case the restriction lies within the knowledge and information flows between agencies.

The overall process of model design, under Participatory Complexity Science, can be seen visualised in figure 8.1.

The advantages of this approach stem from being able to draw from multiple areas of expertise over the target system. The main disadvantages are that assembling members from each company in the same space, at the same time can require a great deal of effort and discussions may easily go off track if not properly structured.

Participatory complexity science compliments the application of LEEA, because the early stages of design already format the model, as well as the mind-set of the participants in terms of the system's structural form. Structure and model dynamics being the main features of the model which LEEA analyses.

### 8.3.4 LEEA and model adaptability

As social norms and environmental habitats change through time, so too do the structural frameworks with which a system's behaviour is determined. In the real world this could equate to the introduction of a new policy, or a new species integrating itself into the local food web. Numerically this means an adaption of the local dynamics which are used to describe the system and in turn, a change to the feedback structure of the system. With this in mind, it is important to speculate on the utility of LEEA in the context of ever changing model structure.

In chapter 5, we see a simple model system of a shallow lake (PLUM) being adapted through multiple iterations as the complexity of the system represented within the model progressively increases. For this purpose, LEEA was used to compare changes in structural behaviours across model structures and therefore provide valuable insight as multiple stocks and feedback structures were added to the model.

However, in a scenario where a model's structure changed with time during model simulation, the current technique of executing LEEA could not keep up. Currently model structure and output is not automatically linked to LEEA calculation and output, requiring many manual steps and respective computational run time to gain analysis data. As the methodology and technique behind executing LEEA develops, connecting model simulation to an automated output of LEEA could bridge the gap between dynamic structural change and the speed at which feedback dominance information could be gained. While theoretically plausible, one would also have to compensate for the multitude of outputs that LEEA produces at each time step, as change within a model's structure (even small) could cause dramatic changes to the number of feedback loops, and dominant eigenvalues within the system.

### 8.3.5 Using LEEA for System Maintenance

Much of the discussion surrounding the use of LEEA and its potential uses in policy are based on the need to change a system in order to reverse or dampen an unwanted feedback mechanism which is dominating the current system. However, reversal and control of adverse system properties are not the only uses for recognising system drivers as LEEA could be used to identify feedback loops which are keeping a system in a desirable state. Not only this but due to LEEA's ability to analyse systems across a time series, it could indicate a beneficial feedback loop losing influence within a system and lead to the promotion of artificial intervention and reinforcement of that loop in order to maintain the system in a desirable state. An example of this from the field of socio-ecology is an agricultural field.

In order to promote the growth of a harvest and ensure the biggest yield possible from a field, farmers must keep the soil of their land maintained through ploughing, fertilizer and pesticides. While these practices keep the land clear and competition free for the crops, they keep the land out of its natural state of stability. That is to say that if the farming practices were to stop, the field would naturally diversify and become a grassy field full of natural plantation and wildlife over time. The maintenance of this unstable state can come at a cost to the nutrition in the land as constant turning of the soil and runoff can lead to the natural minerals in the soil being lost, eventually making the soil unsuitable for the desired crop, or requiring more and more nutrient supplements at the expense of the farmer.

By collecting data from the field on a regular basis and feeding that data into a live model of the system, LEEA could be used to identify changes which occur to the system's feedbacks. By conducting this analysis in parallel to the data collection on the farm, any changes which occur in the system's drivers could be observed and gained as instant feedback on the state and stability of the field. While this is an idealised situation in which to use LEEA, it could forewarn of unwanted system changes such as the build-up of a reinforcing feedback loop or the loss of influence from a favourable negative feedback.

## 8.4 Levarage points, Policy and LEEA

### 8.4.1 A review of ecological modelling and policy

The following section overviews a general application of ecological models and system dynamic modelling to policy implementation. Examples are used from the literature to explore good vs. bad practices in pragmatic model design, that is, models which have been designed for the purpose of policy implementation. The section following considers LEEA specifically, highlighting some of the limitations which the analysis has when used for ecological modelling and policy design in the real world, rather than idealistic scenarios.

Ecological models are used in conjunction with policy design, decision making and implementation because they allow the user to explore the consequences of alternative policies and management scenarios (Schmolke et al. 2010) without any disruption or economic requirement to the real world system. Policy makers are frequently asked to assess the social and economic impact which their strategies will have, particularly in regards to sustainable development (Boulanger and Bréchet 2005; Costanza 1992).

Good model practices are a necessity to link models to policy, but are reportedly lacking in many modern projects. These include a scrutinised iterative process during model design and involvement of decision makers from an early stage in the modelling process (Penn et al. 2013; Schmolke et al. 2010). On the reverse side of good modelling practice, some argue that science has frequently been used to push a pre-established political agenda, rather than designing policy based on the science.

*"science has become a commodity rather than a standard. Too many groups can go out and buy science, and then use science to reaffirm a political decision"* (Dietrich 1995).

Ecological models have long been used for building understanding of complex processes. Often models which have the most impact upon policy are the simplest ones. Toy models can have profound impacts, bringing together a diverse range of understanding on a subject matter, particularly cross-disciplinary between model user and policy maker, e.g. Carpenter et al. (1999a). However, policy geared towards specific targets or accounting for particular behaviours within the system, which require data-rich assessments demand much more in-depth, intricate models to be developed, e.g. Janssen et al. (2000).

Examples where ecological models have successfully been used to influence stakeholders include running fish stock simulations in the form of short computer games (Walters 1994). Understanding that social choice is also a vital part of accurate policy design is also becoming a key feature of many socio-ecological models (Janssen 1998).

An example where policy, guided by an ecological model could go wrong is discussed within Carpenter and Gunderson (2001) in terms of implementing fixed vs. adaptive policy. Their model involves a fish stock within a lake system, the level of fishing of which can be determined by the policy maker. As the fish level changes away from a desired state, the model infers a level of fishing which must be implemented in order to return the system to its optimal state. If a fixed policy is introduced, the policy ensures short term success in restoring the system, but in the long term causes the system's fish stock to oscillate wildly caused by non-linearities which change in intensity as the system evolves through time. Policies willing to adapt alongside the system prove more successful, using the model as a continuous point of reference, rather than a one-time solution to a goal.

Pindyck (2017) argues that what often differentiates between models which are somewhat useful to close to useless for policy implementation is the way a model user handles uncertainties. While the body of Pindyck's argument is based on Integrated Assessment Models of climate change, many of his points can be applied across all model scenarios. Namely, that pressure for models to be used for system forecasting and quantitative policy analysis often lead to parameter values that have no empirical or even theoretical grounding (Pindyck 2017). Pindyck also points out that many modeler's automatic response to uncertainty is to assign probability distributions to parameters and running Monte Carlo simulations, which he argues does not help matters.

Despite potential modelling drawbacks as explained above, the design of new policy can sometimes require precise predictions of future values within a model output, where an exact time and size of an event is needed. If this is the case, then LEEA may not be appropriate, or at least would have to be used alongside other methods. This is because the modelling platform which the analysis is often run on, system dynamic modelling, is often not used as a prediction tool. System Dynamics does have a use in policy design, but it performs best when used to teach the policy implications of uncertainty (Ford 2010). That is to say that instead of producing specific prediction values in its output, it can be used to run multiple scenarios side by side to view a range of potential outcomes i.e. the 'polluter pays principle' where the extent of environmental harm caused by an economic solution is unknown (Ford 2010).

An example where system dynamic modelling has been used with relative success was to explore renewable energy policy and energy dependence in Finland (Aslani et al. 2014). They used causal loop diagrams and system dynamic diagrams to explore encouragement packages, dependency, and energy demand, stating that one of the main advantages of using system dynamics was that their model could be easily adapted for other countries. Other examples where system dynamics has been used for policy making include urban health (Newell and Siri 2016) and High Nature value farming (Ribeiro et al. 2014). In both cases, they were able to suggest general implications of policy over their respective systems, but stayed away from specific value predictions, which may be seen as a requirement from other pragmatic models.

*Policy Briefs*

Policy Briefs are used to address particular issues and are used to outline the rationale behind specific policy choices. They are used to convey a concise summary of the problems and recommend options in order to deal with them. It is through policy briefs that computational modelling may first be recognised as a necessary step to fully understanding and solving a problem.

Reviewing a selection of policy briefs used within the ecological field, computer models are either not mentioned within the policy brief at all (CONCORD 2017; OECD 2001), or the briefs mention very specific models which are built to simulate a specific set of data (Defra 2018). System Dynamic modelling and loop analysis may not fit into a conventional structure of a policy brief because despite being able to handle specific data sets, they are more suited for scenario testing and policy exploration within a system, than predicting specific data values, which are usually required for policy briefs (Forrester 2009). However, loop analysis may still have a use as a thinking tool with which to explore a system and build a narrative of the issue and possible solutions. Using LEEA to identify system leverage points alongside basic concepts within system dynamics which emphasise system structure and variable interactions could benefit the storytelling of ecological issues; stories which are used to bridge the narrative gap between ecologists and policy makers. Concepts of dominant feedback loops could help to draw attention to causes of key problem areas within a system and set up a platform to discuss solutions, whilst maintaining the concise and clear rhetoric that are required by policy briefs.

### 8.4.2 Identifying System Leverage Points

One of the most important questions of investigating LEEA is how can it help us to improve our control over real world systems? The simplest way of assessing this is whether it is capable of improving our ability to identify control levers of a target system. Identifying system levers allows policy to target system drivers, achieving the most control over a target system's behaviour in order to achieve a desired change, while manipulating as few system components as possible (Meadows 1999).

In identifying system leverage points, we are interested in levers which, when manipulated, are most likely to generate the desired system behaviour with minimal change applied. Such levers are known as effective levers (Meadows 1999). The key difference between a lever and an effective lever being that structures which control a system's behaviour (a lever) can be found all over a system, but effective ones will depend on the goal that is sought.

LEEA does not tell us how to control a system, but is able to identify where we should be looking in order to control it. Through the identification of influential feedback structures, LEEA helps us to pinpoint structures of the system which can be used as effective leverage points. The next step in the process is to identify levers within these feedback structures which would allow us to best manipulate the feedback loops in order to fully influence the system's dynamic properties which can be achieved by using DDWA.

System levers may come in many forms, identified as a single variable, or in the form of an entire feedback loop which dominates a dynamic process. Individual variables may serve as effective system levers if dominant feedback processes are highly sensitive to relatively small changes in that variable's input, thus having the greatest impact over the system's dominant drivers with the least amount of change to the system's variables. Note that variables which feedback loops are most sensitive to might not be the ones directly connected to their structure as explored within chapter 6.

While effective levers are the best parts of a system to achieve desirable change, effective levers must also be addressed with an air of caution. As much as an effective lever can be manipulated in the interest of the user, they can also be a system's most vulnerable point and by their nature this could mean that the same lever is also the easiest part of a system to achieve undesirable change. If a lever can affect a system to an extent where you are able to change the current behaviour then there is potential for the lever to act against you as much as for.

### 8.4.3 The application of LEEA to policy design and implementation

LEEA has shown potential to provide a model user with greater insight over the controlling structural drivers of system behaviour. The ability to pinpoint the most dominant parts of a system at a critical moment in a time series, such as the build up to a critical transition, can aid the design of model scenario testing which focuses on the manipulation and outcome of dominant structures within the model.

Conducting scenario testing can, in part, help to improve a model from both the users understanding of the model's ability to perform dynamics and how these reflect back to the real world, but also as part of an iterative process of model design and restructuring of the model's base interactions. Scenario testing can also be an important part of policy design and implementation.

As mentioned throughout this work, most notably during the reviews on system dynamic models, the design of system dynamic models is best suited for testing the impact of different policies. Designing how a policy may change the structure of a model or impact the dynamics integral to parameter interactions and then viewing the change that occurs in the system can be a valuable experience for policy development. The main advantage being that a system dynamic model of a real world system can be experimented on for potential side effects before anything is attempted on a real world socio-ecosystem.

Using LEEA to help guide scenario testing for the purposes of policy design could streamline the testing process. Designing policy, which is based on the outputs of LEEA would help to address feedback structures responsible for unwanted behaviour. The benefit being that testing can take place with dominant feedbacks in mind, observing whether a specific policy change would dissolve or escalate the behaviour.

The nature of feedback loops within systems is that their interlinked nature can make the changes applied have unexpected consequences. Reducing the reinforcing behaviour of a dominating positive feedback loop in one part of the system, could cause other feedbacks to dominate in other areas of the system. LEEA could be used to identify any changes within the structural dominance within the system after each scenario. Conducting LEEA on each policy scenario not only allows for a change in model output to be compared and contrasted, but also any changes to structural dominance, which may not be so obvious from model output alone. These changes to feedback structural dominance could provide valuable insight into future issues or side effects of a policy originally designed at targeting a specific part of the system.

With LEEA having great potential to be incorporated into policy design and testing, the analysis is still in relative infancy and has yet to be used as an integral part of project design and implementation in a socio-ecological context. In order for LEEA to reach a greater potential where it can benefit policy design in such a manner as described above, it must become better known within the academic field, and its output must be portrayed in terms of what is useful for a policy design team.

Being able to show what the driving influential structures within a system model are may be interesting to a policy maker, but not useful without the context of how that information can be used to change the system as required. In light of this, showing what feedback loops are dominating a system may not prove useful without first conducting further research in how to interact and change those feedbacks in the model's real world counterparts. Local knowledge of the system and capabilities of local industries becomes necessary to link model theory to practical change. This is where DDWA, LEEA's sibling analysis comes in, providing the ability to identify high influencial parameters inside of feedback loops.

### 8.4.4 Acknowledging LEEA's limitations in the context of ecological modelling and policy making

From its most basic stand point, the purpose of a model is to help to bring assumption and biases on the target system together (Holling and Chambers 1973). So, in the first instance would LEEA help or hinder this process? Would a general understanding of how feedback loops interact and control a system aid or complicate the dialogue that must occur across disciplines and especially between ecologists, policy makers and local companies. As shown within this thesis, there is no doubt that understanding feedback loops within a system model through LEEA can greatly increase the understanding of how a target system works. However, is the language used in LEEA and the outputs it creates (eigenvalue plots, loop influence plots, loop influence point plots) transferable in the context of policy design and implementation?

Policy makers and local businesses may not have the time to learn how to interpret LEEA and what its outputs mean. With an overall understanding between parties that models can be complex and analysis outputs take time to interpret, it would be put upon the ecological modeller to convey the information from the analysis (in this case the outputs of LEEA) quickly and clearly. With that in mind, pointing at a series of lines from multiple line plots which LEEA produces may not be the best solution to convey important information regarding the system's feedback loops. Instead, conveying the information from LEEA's loop influence plots back into cyclic diagram form, which all parties may have been involved in building at early stages of the modelling process and where loop structures are easy to see, would greatly improve speed and understanding for delivering key information from the analysis.

In real policy and decision making contexts data for every variable in the system is often unobtainable, or limited at best, which presents a distinct limitation for using

LEEA. In situations where qualitative interactions are crucial or actors are strategic and adaptive, LEEA would not be able to account for the actions of these players, unless they were inherently described within the system's dynamics. This presents a common modelling problem of how to quantify concepts such as free choice or spontaneity within a model system.

LEEA fills an important niche as a model analysis tool which specialises in feedback loops, but it is limited by its requirement of the whole system model having associated dynamics and data. With all this in mind, LEEA's practical application to policy and decision making may best serve as an illustrative and critical thinking tool used best in conjunction with other modelling forms. It might be in the best interest of decision makers to apply their strategies to an appropriate mix of models, not all of which LEEA may be applicable to.

## 8.5 Exploring LEEA's limitations

The limitations affecting LEEA include limited application to low feedback models, practical limitations of the LEEA algorithms to system dynamic functions (i.e. STEP, IF, etc.), but increasing complexity and number of results with larger models is the most prevalent limitation restricting LEEA's utility across multiple model scales (Oliva 2016; Kampmann and Oliva 2006). To explore this limitation further and gain first hand understanding of interpreting results from increasingly complex models, PLUMGov and PLUMPlus were generated as extensions to the PLUM model within chapter 5.

The PLUMGov and PLUMPlus models explored how incorporating multiple endogenous dynamics into the PLUM model would affect the output gained from LEEA and our ability to recognise a critical transition dynamic from the original PLUM model. During analysis, it was observed how much of a difference if any, the additional features made to the output of PLUM. During LEEA analysis of PLUMGov and PLUMPlus, three features were expressed in the analysis output which had been identified as potential complications:

1) LEEA produces a lot of graphical data: Both Elasticity and Influence are generated as part of LEEA output and can both be used to infer structural drivers of system behaviour, but their output can differ greatly and must be interpreted separately as they do not mean the same thing. Throughout this thesis the main focus has been on interpreting loop influence values as these are generally considered to be the easiest to understand. Loop influence plots are also considered the most accessible to an outside reader and the outputs are not affected by eigenvalues which sit close to zero.

The PLUMPlus model showed how simple additions to model size and structure could make profound changes to a model's number of interactions, number of feedback loops and therefore analysis results. One of the most important discussions contained within the PLUMPlus study was interpretation of LEEA results became more challenging when multiple eigenvalues had to be interpreted simultaneously.

One feature of LEEA is that it outputs a multitude of data regarding a system's feedback loops corresponding to different eigenvalues. A large amount of analysis data to interpret can be seen as having both a positive and negative side. While more data to interpret and understand the system can be a good thing, the output of LEEA must be filtered in order to identify the most prominent drivers. This is largely achieved by identifying the eigenvalue expressing the greatest magnitude as this reflects the system's most dominant mode. However, when multiple eigenvalues express high values at the same time, their elasticity and influence values must be interpreted simultaneously. If the outputs from these multiple sources contradict each other, i.e. showing different loops dominating the system's behaviour at the same time, then interpretation and reflection back on the real world system can be difficult.

2) Our ability to interpret LEEA's outputs: Eigenvalues will contain complex numbers when the system is expressing oscillatory behaviour. This in turn affects the elasticity values and influence values output by LEEA, which also have complex values. Containing complex values means that there is both a real and imaginary part which must be interpreted in order to understand the feedback loop's impact on the system.

Alongside having to deal with complex numbers, loop plots of both elasticity and influence can be difficult to interpret purely based on the number of feedback loops in the system. Chapter 5 showed the PLUMPlus model which had a total of 17 feedback loops, each of which had their own trajectory to be interpreted for a given behaviour of the system and whose values could easily differ drastically between eigenvalues as well as between elasticity and influence plots of the same eigenvalue.

Outputs of loop elasticity and influnece plots can sometimes appear counterintuative. As an example, on loop influence plots, a negative feedback loop could display positive real values inferring that it is generating instability within a system or vice versa with a positive feedback loop displaying negative real values. While negative feedback loops are usually associated with stabilizing behaviour and positive feedbacks with destabilizing behaviour, the interaction and reinforcing behaviour held between feedbacks on top of properties internal to feedback structures, like a delay in an interaction, can cause feedbacks to express unexpected and sometimes opposite behaviours.

3) Artificial 'Ghost' loops: Ghost loops occur when the elasticity or influence value of a loop is reflected by another loop in the opposite polarity, effectively counteracting each other's influence. The presence of ghost loops can mean that a loop appearing

to express a high level of influence over the system is being countered by the dynamics within another loop. Sometimes this counter-action occurs across only a section of a time series and the two loops no longer mirror each other when the system changes behaviour. When a model contains artificial loops it can infer a problem with the dynamics and structure of the model, not reflecting the real world system correctly.

*Limitations in validating LEEA and its outputs*

As pointed out from the literature review alongside a discussion within chapter 5 of the PLUMPlus model, validation of LEEA and its outputs take the form of an internal and an external validation. The internal validation links to validation of the model LEEA is analysing, whilst external validation requires data collection and comparison to the real world. Data collection alongside model and output validation can, at the best of times, be difficult to achieve due to impracticalities or economic feasibility. Data validation is a universal issue across all models and a user would benefit to bear this in mind when thinking of using LEEA for a project.

## 8.6 LEEA in Qualitative Modelling

### 8.6.1 Early Model Design and consequences to LEEA: A Case Study

Qualitative research can play an important role in the early stages of model design and development, which ultimately determine the structural representation and dynamic properties of a system model. In turn these perceptions of the target system have great impacts on the outputs gained from loop analysis. The following section discusses a project conducted by the author in January of 2016 alongside a research group at the Harbour Branch Oceanographic Institute, at the University of Florida. The project investigated a prominent microbial species of freshwater lakes, Loxodes Rex (Hines et al. 2016), and its effects on surrounding freshwater lake systems using specialist knowledge of micro-biological systems, socio-ecology and local food webs.

Initial intentions of the project were to construct a system dynamic model of the local ecosystem and use LEEA to investigate the feedback loops which the newly discovered species Loxodes Rex could be influencing. Unfortunately due to limitations of available data and time constraints set for the project, a model accurate enough to reflect the target lake system could not be developed. Despite this, through the practices adopted by participatory complexity science, qualitative data and local knowledge of the system were enough to construct a structural representation of the target system. The cyclic diagrams produced proved to be an invaluable talking point with which to focus the remainder of the study, investigating methods of dispersal of Loxodes rex from local to regional scales, and could be used as a platform on which to build an effective system dynamic representation of the system in the future. Some outputs of this project can be seen below.

Loxodes Rex. Local vs Regional interactions

An organism's ability to populate a large region is controlled by properties of the local habitat alongside regional processes which facilitate dispersal (Havel and Shurin 2004). While studies of local habitat properties investigate controlling factors of population growth rates i.e. temperature, salinity, predator population and food availability (Finlay et al. 1986, Finlay and Berninger 1984), regional studies of dispersal tend to concentrate on the physical mechanisms by which the organisms spread (Wilkinson et al. 2012, Green and Figuerola 2005, Havel and Shurin 2004).

The bulk of the case study focused on network properties at regional scales which affect the species dispersal capabilities. To aid discussion, a cyclic diagram was constructed at the beginning of the project and subjected to an iterative process of design, discussion and redesign to visualise the complex nature of interactions and feedback processes which affect the local capability of Loxodes rex to populate its environment as shown in figure 8.2. This figure was used as a reference point throughout the project, particularly concerning the ecological contribution Loxodes rex makes towards its local ecosystem.

The project began by using system dynamics to investigate the freshwater bodies of Florida and building a structural representation of the drivers which affect the survival of Loxodes rex. The project then moved onto studying the ability for the Loxodes rex to successfully colonise a single water body and finally, investigating properties of a system which would allow the species to successfully spread across the entire State of Florida.

*Building a Cyclic Diagram and System Dynamic Model*

Constructing a cyclic diagram at the early stages of the project proved a valuable asset to the research group as it allowed discussion points to be made from a common level of understanding about the target system. This also kept the discussion centralised around the concepts of parameter interactions and vital vs. unnecessary components, which in turn made the transition from initial discussions to system dynamic model much smoother as the entire team working on the project could see how the model had been developed and justify its structure and components. Overall the cyclic diagram provided the group with a visual output of system interactions, consolidating all of their knowledge of the system into one simple diagram. It provided an excellent base and reference point to return to during any discussion within the project that could be easily adapted and adjusted as the project developed.

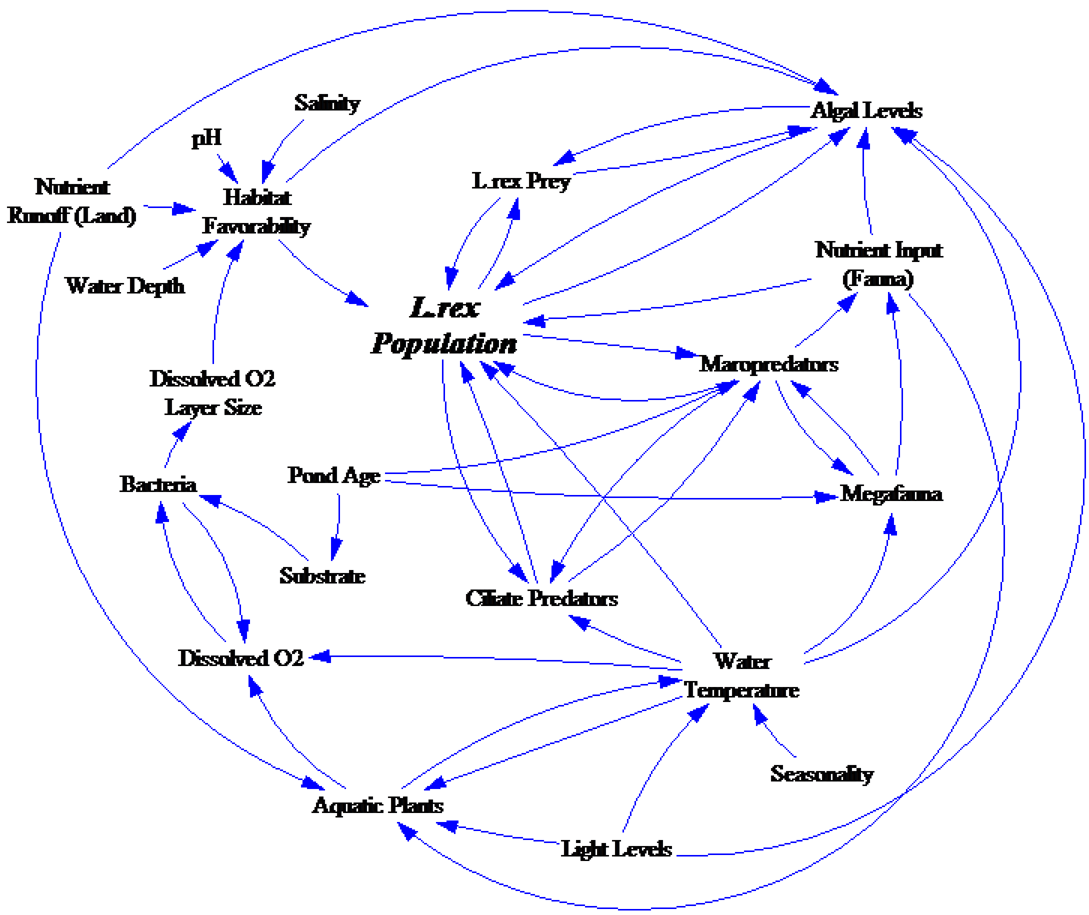



FIGURE 8.2: Cyclic diagram of factors affecting the success of a population of Loxodes rex

While limited data was available to begin running model scenarios specific enough to reflect the target system, the cyclic diagrams were converted into a system dynamics format (figure 8.3) in order to gain information regarding the structural properties of the system. This model had significance not only in investigating the system's feedback mechanisms present, but was designed to be able to conduct LEEA and sensitivity analysis on the system drivers pending more data at a later stage. LEEA was discussed as a tool with the potential to identify dominant structures of the ecosystem at critical times during the growth and colonization by Loxodes rex.

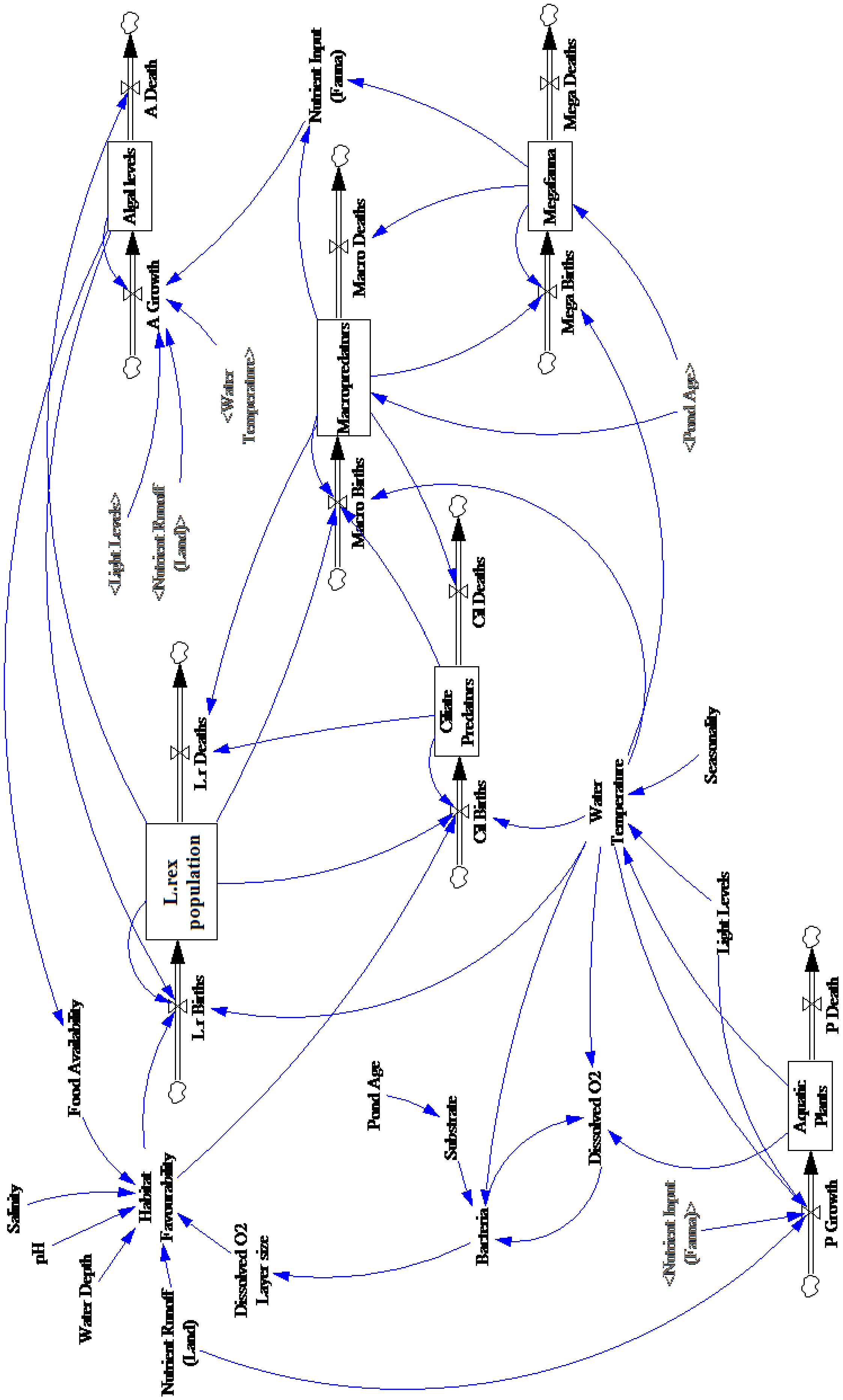



FIGURE 8.3: System dynamics diagram of the system drivers and interactions affecting Loxodes rex within its local freshwater environment.

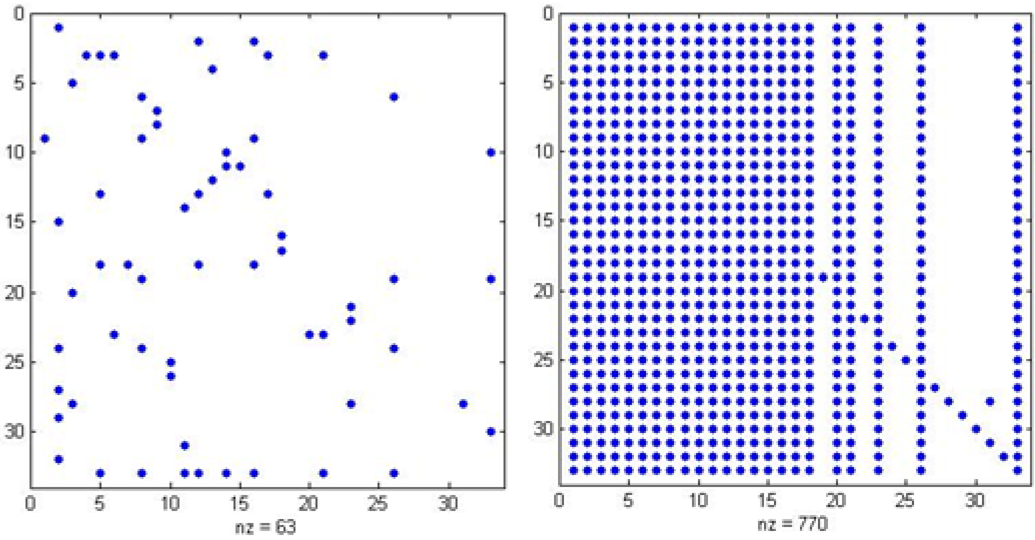



Figure 8.4: a) adjacency matrix of the variables within the Loxodes rex system dynamics model. 4 b) reachability matrix of the variables within the Loxodes rex system dynamics model. Axis represent the number of variables in the model and nz is the number of connections found within the structure..

Despite not being able to conduct LEEA and utilise its outputs to the fullest extent as shown in previous projects, methods used within graph theory were still adopted to help the team identify important features of the model system. Structural analysis was conducted on the system dynamics model, which found that there were 25 independent Feedback loops within the system. This analysis was undertaken using Oliva's structural analysis software (Oliva 2015) and the loops were identified using the shortest independent loop set (SILS) as described in Kampmann (2012) and Oliva (2004). Feedback loops act as key drivers in system behaviour, it is important to acknowledge their presence and influence when attempting to understand transitions in system states (Kampmann and Oliva 2006). The ability to identify the number of feedback loops within the target system and be able to identify them within the model proved a useful exercise as no member of the team was aware of the extent to which feedback loops existed in the system prior to the structural analysis results.

The adjacency matrix and reachability matrix can also be seen in figures 8.4, a & b respectively, where the x and y axis represent the number of variables in the model assigning each variable its own number and nz referring to the number of connections found within the structure. The adjacency matrix shows a record of which variable is able to directly reach another variable through a neighbouring connection. The adjacency matrix helps to get a sense of how connectivity is spread across a model structure. The reachability matrix is a representation of the pathways within a model structure. If it is possible to reach another variable through any given pathway along the model then the reachability matrix will show a connection between them. The reachability matrix is useful for investigating how easily a change in the system's behaviour may generate knock-on effects to the rest of the system.

Despite this section of the project stretching little beyond the capabilities of qualitative research, the team found that the system dynamics approach provided a new perspective for thinking about the systems main properties and interactions. Overall it was found that the initial methodology of systems thinking, visualising systems in terms of their interactions of components and being able to go through an iterative process of model design as a team benefitted the project greatly. Iterations of causal loop diagrams aided group discussion and gave the project a platform with which to find common ground as well as a focus with which to forward the project. The initial stages of this project lead to more confidence in the model structure and dynamics imposed on the interactions which would have the knock on effect of interpreting LEEA's results from a platform of confidence about the system model and understanding of the natural system when interpreting the output of influential loop structures.

### 8.6.2 Is There A Place For Qualitative Modelling Within System Dynamics?

As in the model construction of the Loxodes Rex. project, it can become apparent that there is not enough data or information in order to input quantitative data into every part of the system in order to run an accurate system dynamic model. Qualitative modelling in the field of system dynamics is a process of formulizing and analysing feedback loops without the process of simulating numerical output. Qualitative modelling of a system is otherwise known as soft Operational Research (OR) and occurs when quantitative modelling is either not possible or is undesirable (Coyle 1998). In OR, the structure of the model itself becomes a platform for discussion and learning.

A recurring question within the literature of system dynamics seems to be whether qualitative modelling has a place in the technique, or whether only quantitative methods leading to simulation hold a place in system dynamics. The phrase "When to map and when to model?" was coined by Richardson (1996) concerning the uses of qualitative vs quantitative system dynamic modelling.

It is generally accepted across the field of system dynamics, as discussed in Luna-Reyes and Andersen (2003) and first illustrated in Forrester (1994), that the amount of information available about a system decreases sharply as you go from a mental model (observations and experience) to a written data base, to a numerical data base. Starting from qualitative information and trying to provide quantitative data for every variable can be challenging.

System dynamic models are mathematical representations of problems and policy alternatives where often most of the information available to the modeller is not numerical in nature (Luna-Reyes and Andersen 2003).

While using quantitative data, there is the question of how much weight can be attributed to it: Who sourced the data? Was it from the client of the modelling process or the consultant? Might the collector of the data have had any bias towards the output? The validity of the simulation has to be questioned when there is so much uncertainty in the parameters of the model.

Data collected on a variable can differ depending on the individual doing the recording. For example, if a person were to record the temperature in a room, you could put 24C or 'hot vs cold'. Modellers do not always have the privilege of asking how they require data to be collected or have the ability to go back to record data themselves.

In the context of LEEA, the analysis cannot be run unless there is data with which to simulate the model. This would suggest that only a quantitative approach where differential equations and empirical data is available is important for LEEA. However, the outputs and interpretations of LEEA are so heavily reliant on a model's structure, that the qualitative stage of model design, interatively building a system's internal structure with a system's thinking approach (i.e. first representing the system with a causal loop diagram), should not be overlooked.

The following sections discuss qualitative modelling in the context of system dynamic modelling:

*For:*

In system dynamics, the importance of studying behavioural patterns has always been stressed, especially when the data is limited. But this is not wise when a client (for whom you are creating the model) believes that they are retrieving precise data from the model's output. *"Omitting structures or variables known to be important because numerical data are unavailable is actually less scientific and less accurate than using your best judgement to estimate their values"* (Sterman 2000).

Nuthmann (1994) coins the phrase 'plausible nonsense in our models' referring to the practice of quantifying a model without fully being able to validate the data going in and therefore producing output which seems correct, but in fact does not reflect the real world.

System dynamic models and causal loop diagrams are often capable of producing policy insights before any form of simulation is conducted because of how they map out system structure in a highly visual manner and are capable of showing large amounts of information in a single diagram. Coyle (1998) recounts of how a system about instability in Africa which was described over a series of 50 written pages was able to be represented by a causal loop diagram containing 57 variables and 200 links. While this diagram was far from simple, it did allow for all of the information about the system's structure to be visualised on a single page.

An example of qualitative data having practical uses is demonstrated in Coyle et al. (1999) who shows how new insights can be gained from a qualitative diagram of the treatment of psychogeriatric patients. Coyle shows how policy insights may be mapped onto an existing qualitative diagram. The diagram is capable of showing where information flows may be lacking or blocked from getting to the right people (in this case the medical staff of the facility) and this is purely determined by examining the system's structure, prior to or foregoing any simulation.

An example of the difficulties which arise when attempting to quantify data can often be seen when trying to model human choice or perception, a topic which is relevant to the field of soico-ecology. System dynamic models can be difficult to quantify more often than not when social components are required to be incorporated into the structure. For example tourist satisfaction at a resort may be represented on a scale of 1 to 5, but scales such as this can differ between each individual so it is difficult to quantify what 100% satisfaction is if compared to 50%. Likewise, when it comes to quantification of a variable with two potential outcomes (i.e. more likely to vote for party X or party Y) it is often tempting to enforce a multiplier from 0 to 1 which represents two extreme views of the choice, but this type of quantification runs into trouble when the value reaches 0.5 as it is uncertain what this would represent.

Coyle et al. (1999) points out that while you cannot infer dynamic behaviour simply from a complex diagram, being able to describe a system and visualise its structure has a use in itself. Coyle summarises the uses of influence diagrams (those which do not undergo simulation):

- They can condense multiple pages of description onto a single diagram.
- They are able to form a type of agenda and be useful at discussion meetings about the system.
- Identifying key structures in the system i.e. feedback loops can give insight into possible system behaviour.
- Studying the diagram may reveal the wider contexts of the modelling task.
- Finally they are the basis with which to start an informed quantified model.

Luna-Reyes and Andersen (2003) recount benefits of qualitative modelling which include: understanding of the modelling process, facilitate communication among modellers and clients and set up methodological framework to promote constructive discussion around the merits of qualitative versus quantitative modelling.

*Against:*

It is often accepted that in order for a system dynamic model to be applicable to guidance and policy, it must be possible to fully quantify (attribute equations and data to) its components.

Common forms in which qualitative data is collected include: interviews, oral history, focus groups, delphi groups (an extension of focus groups making use of survey and interview analysis), observation, participant observation and experimental approaches. An explanation of all of these may be found in Luna-Reyes and Andersen (2003). The main cost of collecting qualitative data in this manner is that it is time consuming and labour intensive, often having to go through an ethics check system as much of the data gathered can involve the public.

Despite the growing popularity of system dynamics and its growing use across multiple disciplines, (Lane 1999, Guo et al. 2001, Saleh et al. 2010) Forrester expresses how a lack of system dynamics in the education system and a lack of training at the individual level has created a rift between the practice of system dynamics (a quantitative approach) and the practice of System Thinking (a qualitative approach). Forrester (2007a) explains how System Thinking often tries to dumb down the concepts introduced in system dynamics, often through causal loop diagrams or management games which focus on decision making. However, the true power in system dynamics lies behind how policy design influences those decisions.

*" Making system dynamics simple is a losing game. System dynamics is not simple. The problems of complex feedback systems are not simple."* (Forrester 2007a)

While Forrester expresses how the origin of system dynamics focused on tackling major issues in the outside world, he criticises how much of the real world relevance has been lost to its use in academic papers (Forrester 2007b). Forrester (2007b) explains that system dynamics as a field has not taken the necessary steps that would give it a place in important issues including: trade deficits, the future of social security, immigration and political promises in the healthcare sector. Instead system dynamics has been misrepresented through a lack of proper training at both an individual and institutional level as well as a drive for academic paper publication rather than real world issues.

## 8.7 The future for LEEA - Steering Complex Systems

Given LEEA's strengths for identifying driving structures within a system and its ability to identify system leverage points, while also accounting for the limitations which a user of LEEA would face in terms of data collection and utility to a policy maker, it is important to consider its utility for the future. As it stands, LEEA's methodology is best suited as a key component of a 'steering complex systems' approach. As shown from the case study within chapter 6 of this thesis, LEEA is an extremely useful and informative method when used on its own, but the knowledge gained from its output becomes even more valuable when used alongside other techniques which together uncover and/or detect key and dominant parts of a system.

LEEA's role in future projects lies in the exploitation or mitigation of system feedbacks as effective levers. LEEA would provide a significant benefit alongside projects which prioritised system structure and component interaction as key focus points during model construction. LEEA's outputs would provide more utility to projects in which local authorities and local companies were involved to help differentiate between dominant system levers identified within LEEA and effective system levers which were physically, ethically and economically viable to manipulate. Finally, LEEA would benefit projects which used an array of analysis tools in order to gain the model knowledge out of their model. LEEA is built to provide a wealth of information regarding the feedback loops within a complex, dynamic system and in comparison to all other methodologies identified within this thesis, is almost unparalleled in its ability to do so, but its methodology has limited utility outside the scope of system feedbacks. Used in conjunction with other analysis techniques, LEEA has considerable potential for a future in 'steering complex systems', having great utility in its niche for identifying and analysing system feedback loops.

## 8.8 Future Work - What Next?

LEEA needs to be utilised on a wider range of model subjects in order to continually test its utility to a range of SES dynamics. The more dynamic behaviours and systems which LEEA is used to analyse, the greater idea we get of its applicability to all system dynamic models. In this thesis, LEEA has been shown to provide novel insight into key questions and concerns surrounding lake systems and coral reef ecosystems, but this does not mean that LEEA has a place in all studies of lake or coral reef models. LEEA will not be appropriate for all system dynamic models as demonstrated within Chapter 5 of this thesis, which explored the limitations of LEEA, but it will certainly have a greater application to critical transitions, or determination of bistability than that which is explored within this thesis.

The next logical step would be to use LEEA within a research site (such as a coral reef environmental centre or fishery research centre) where the model can be designed based on variables and interactions specific to that location where the data for the model is readily available. The analysis would be used to gain information of the system drivers which can then be compared and used alongside existing knowledge of the researchers and, testing its applicability to real world system in real time (i.e.

a week by week basis of data collection to model analysis output). The system dynamic model of the project can be designed with conducting structural loop analysis in mind, even utilising LEEA's ability to identify the extent of dominance within the system as part of the model testing and validation.

Improvements can be made to LEEA's automation. The most recent update to its online capabilities occurred in Sept 2017. It is important that the analysis and its development is live and is active, i.e. is still ongoing as this means that the utility of the technique is still improving. As mentioned previously within this discusson, the ability to gain analysis output simultaniously with model runs would greatly streamline the process of conducting LEEA and have many benefits with regards to instant feedback on changes to model structure made during scenario testing. This process would not come without its limitations, as the multitude of data LEEA outputs and the user's ability to select relevant data would still be an issue.

Design and creation of a common ground with which to explore and share the models and dynamic behaviours that LEEA has been used to analyse, could provide a database for the successes and failures of the technique across a range of disciplines. An online platform with which to search for specific dynamic behaviours or find other models of similar target systems where LEEA has already been used to great effect could greatly benefit the wider modelling community, as well as improve the accessibility and effectiveness of LEEA.

From personal observations and searches conducted by the author, there are many system dynamic models used within the SES community, but they are not always made readily available for further analysis. Sometimes, papers will display their system dynamic model, but not fully disclose the dynamic equations associated with that model. Often reproducing a model and its results is not possible as values used within the model are not stated or included within additional information sections. Often system dynamic models in SES appear to be used to generate an output and then not used any further which reduces the potential information gained from the model and prevents analysis like LEEA being easily run on the system. These types of information blocks need to be challenged alongside looking for new and alterative analysis techniques as one cannot be used to its full potential without the other.

## 8.9 Importance of Accessibility: A Review of LEEA Implementation and Subject Terminology

### 8.9.1 Improving LEEA Accessibility

With greater testing and implementation over a wider variety of systems, LEEA has potential to be a useful analysis tool across a wide range of socio-ecological projects and find itself as a mainstream analysis. However, for it to first be considered by the wider academic community and be justifiable as a tool for policy manipulation, its accessibility to academic, public and political audiences must be addressed.

Socio-ecology is by definition, a multidisciplinary field joining together social sciences, with environmental science, ideally capable of integrating all of the techniques used therein, capable of producing more accurate and realistic models. Model users within the SES field will approach LEEA with varying levels of knowledge, particularly surrounding their numerical and modelling background. For LEEA to be available to academics of multiple disciplines, it must have a standardised format from which to learn about and implement the technique. Implementing LEEA requires understanding of several concepts which are not all found within the same line of research e.g. knowledge surrounding feedback loops, eigenvalues, graphy theory and linear stability theory are all required in understanding LEEA, but their theory does not go hand in hand. A user must search multiple different areas/disciplines of research in order to gain full understanding of each step that LEEA undertakes. This is currently a great deal of extra work and effort required on the user's part and with little prior knowledge, the learning curve to understand Jacobian matrices, eigenvalues, system dynamics, Mathematica coding etc. can be steep. The current requirement to learn LEEA may be unwelcomed as much of the theory which the user comes across may be irrelevant in order to understand LEEA. A single place where the information required to learn and implement LEEA is consolidated and standardised would greatly improve LEEA's accessibility for a wide academic audience.

Naumov and Olivas online materials (Naumov & Oliva 2017) provide a great platform to implement LEEA, automating many of the required calculations: the Jacobian matrix, eigenvlaues, loop gains, loop elasticities and loop influence values. R. Oliva even provides a package to convert Vensim diagrams (a software for constructing system dynamic models) into a format mathematica can use.

The code can be prone to minor errors, which can be difficult to find the source of if the user is unfamiliar with the code. An error which occurred frequently was due to model variable names being the same as variables used to run the code, i.e. shortening a variable of rainfall to 'r' would overlap with a coding variable and cause the LEEA package to stop working.

The code currently contains each step of LEEA in a separate section which helps to translate the required steps of LEEA into the order in which the package carries out calculations. To improve accessibility, the introduction of each section and the notation within could be extended to better explain how the code processes each calculation. This would increase user understanding of code function and aid the identification of errors.

Finally a common, open access place in the form of website or forum could be initialised for users of the analysis to share findings, uses and errors of the analysis and associated software. Having a common place with which to share experience would benefit individual users seeking advice and feedback, as well as improving the overall analysis by identifying errors in the base code and providing extensions which individual users have built for specific projects.

### 8.9.2 Accessibility of system dynamic modelling to the public and policy makers

Making system dynamic models accessible is important, not just for outreach purposes, but also for the ability to communicate with businesses and industries which could be major players in a restoration or conservation project.

As it currently stands, system dynamics is accessible to a public audience as a visual method of modelling, which can be understood with some basic understanding of stocks and flows. Causal flow diagrams can also be used as a more simplified version of a system dynamic model so that dynamics do not need to be considered and a focus can be drawn to the components and interactions of the system.

However, system dynamic models are not trivial. Before any analysis is conducted, models naturally begin to get complicated as they grow in size; the ability to follow an individual chain becomes harder; the links between stocks become more complex and the ability to pick out feedback structures becomes more difficult.

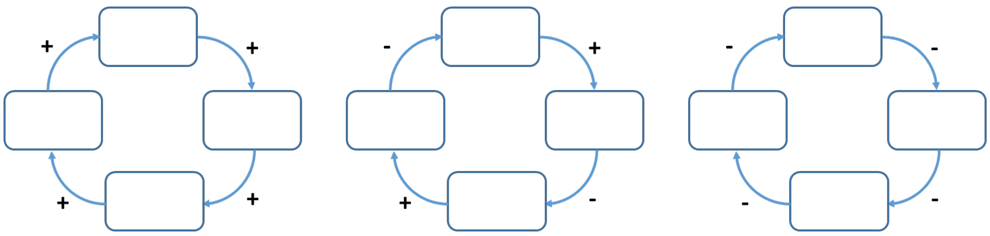
### 8.9.3 Discussion on the terms positive and negative feedback:

Positive and negative feedback have long been terms used to describe the two states which feedback loops can form. Understanding how they form, what they look like and what behaviours they are capable of generating is fundamental within the study of system dynamics and structural loop analysis. This section reflects on the terms positive and negative in terms of system dynamic feedback and explains how both are inappropriate and misleading and calls for an appropriate alternative to be introduced/ maintained within the field of system dynamics.

The very first time that we are introduced to the terms of positive and negative feedback is within the education system of primary and secondary school. From this environment we learn that positive feedback is something useful and encouraging, overall being a good thing, while negative feedback is seen as unhelpful and at times harsh, often viewed as something that would not be of any benefit. Unfortunately, in terms of system dynamics, these preconceptions can create a skewed and often false view of what positive and negative feedback loops mean for a system. Positive feedback in system dynamic terms often does not have positive consequences and in many systems where it generates runaway behaviour, it can often be detrimental. On the reverse side, negative feedbacks should not be prejudged as negative as they can often create beneficial stabilizing conditions within a system, or create limits which counteract positive feedback loops.

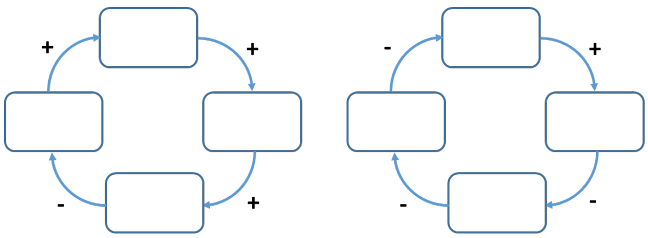
Phrases such as 'reinforcing' and 'runaway' or 'stabilizing' and 'opposed to change' are almost always found alongside the terms positive and negative to describe a feedback loop's effect on behaviour. The terms positive and negative give no indication as to what behaviour the structure will actually be having.

The terms positive and negative also fail to reflect the types of links that can be found in their associated loop structures (see figures 8.5 and 8.6). A positive feedback loop can be formed from links that all take a positive polarity, but may also be formed with links that all take a negative polarity. Positive feedback loops are also capable of containing links with both positive and negative polarity, provided that the number of negative links are even.



(A) Full positive Links (B) Even Negative Links (C) Full Negative Links

FIGURE 8.5: Positive Loop Structures



(A) Odd number of negative links (B) Odd number of negative links

FIGURE 8.6: Negative Loop Structures

These diagrams show different feedback loops which contain four variables and which category they fit into, but feedback loops can be formed from any number of variables, even from a variable containing a feedback to itself.

Positive and negative are even more complicated when trying to describe the connections or links between variables. It is often mistaken that a negative link means that variable A will decrease variable B, and that a positive link means that variable A will increase variable B. Instead a positive link means 'reflects the change' and a negative link means 'opposes the change'. To this end, even the symbols that are commonly used to visualise these structures '+' and '-' are misleading.

Overall the terms negative and positive when discussing system dynamic structures are counterintuitive to making the field accessible to a wider audience. The terms neither describe the system structures in terms of their internal links, the output they produce, nor their potential benefit/detriment to the system. New names for these terms could increase clarity and accessibility to public audiences and educational platforms. The terms 'reinforcing' and 'stabilizing' for feedback loops and 'reflects change' and 'opposes change' for causal links may not be appropriate for all scenarios, but are an example of four terms which give a better indication of the structure's capabilities.

# Chapter 9

# Conclusions

Loop Eigenvalue Elasticity Analysis (LEEA), a structural loop analysis tool for system dynamic models has been shown to have great potential for use in ecological and soico-ecological study. The principles on which it is based; identifying system structures as drivers of dynamic behaviour and attributing behavioural shifts to changes in feedback loop dominance are complimentary to evidence and theory surrounding many processes within socio-ecological systems. LEEA has been found to be an important tool to consider in a socio-ecological project, not for its ability to determine system stability, nor for its ability to attribute system behaviour to feedback loops as these are important features, but are already well known. The power gained from LEEA's utility lies in the ability to disentangle and prioritise feedback loops within a system from the 'black box' models on which our simulations rely. LEEA provides greater understanding and therefore control over our model systems.

LEEA's utility naturally matches and is able to compliment key dynamics which are hot topics within the conservation, sustainability and resilience communities. It excels at analysing and shedding light on non-linear processes including oscillations, system collapse, exponential growth and decay and has been shown within this thesis to have a practical use in studies of critical transitions, hysteresis and mono vs. bistability.

Despite its great potential, LEEA like many techniques of its kind does not come without practical limitations. A user's ability to interpret LEEA becomes increasingly difficult with model size and complexity. While this may not be such a problem for experienced users of the technique, one of the aims of this thesis is to introduce LEEA analysis and its way of thinking about complex system drivers to a new audience of socio-ecologists and therefore it is important to highlight this limitation to new users. The utility of LEEA hits a plateau when its output becomes just as complicated to interpret, if not more so than the complex model that is being analysed.

One main interest of the meta analysis of LEEA was to see if it could have a use in the design and implementation of environmental and conservation policy. To investigate this, LEEA was examined within the context of system leverage points, accessing whether the identification and quantification of dominant feedback loops could be justified as system leverage points and if so could they aid in the design of well informed, efficient and effective environmental policy.

For the purposes of gaining a greater understanding over a model system and its drivers, LEEA works well as a stand alone analysis and can provide the user with a wealth of information which is not attainable from simple data output or from other analysis techniques. However, when it comes to the identification of leverage points and its potential use in policy, it would be ill advised to use LEEA as the sole justification for policy design. LEEA's output is purely numerical based on the dynamics internal to the target system. LEEA does not therefore take into account concepts of a safe or just operating space and its output does not take into account the practicality, economic cost or ethics of manipulating dominant system structures unless these concepts are integrated into a model's structure and design.

LEEA has been shown within a lake system and coral reef system model. The emphasis within this thesis was exploring the breadth of LEEA's utility, but many more systems where LEEA is effective could be explored. This cannot be achieved within one thesis and needs to happen on a much wider scale across multiple different systems world wide. Implementing an online platform where LEEA results could be shared and discussed between model users would result in a common space with which to explore and understand dynamic behaviours and the interplay of feedback loops across a multitude of complex systems.

The next step would be to use LEEA within a socio-ecological project where the system's dynamics were not already well known. There would be nothing to compare the results of LEEA to and this would therefore give a new perspective on how useful LEEA was and the extent that its results could be relied upon and utilised as the main analysis tool of a project.

# Supplementary Information

## Supplementary Information 1: LEEA Methodology

The following calculations are required for LEEA analysis. A full breakdown of the formula required for LEEA analysis can be found in Kampmann (2012) with the original theory found within Forrester (1983) and Forrester (1982). A more detailed 10 step guide with model examples can be found in Güneralp (2006). Online material to help streamline the analysis and avoid manual calculations (i.e. of the Jacobian matrix and eigenvalues) can be found from Oliva (2015).

Calculation of loop eigenvalues and loop influence values is conducted via the following process:

1. The Jacobian Matrix of the dynamical system model.
2. Eigenvalues of the Jacobian matrix.
3. Loop Gains.
4. Loop Eigenvalue Elasticity & Loop Influence.

1) *The Jacobian Matrix* The original model to be analysed must first be converted into a series of partial differential equations (PDEs) and thus be cast as a dynamical system in order to calculate its Jacobian matrix. The model may already exist as a dynamical system (a set of differential equations) or as a system dynamic model (SDM), which has to be interpreted as a dynamical system. The Jacobian Matrix is an nxn square matrix and represents the links held between system stocks in their most compact form (Gonçalves 2009). Where each element of the matrix is equivalent to taking the partial derivative of one stock with respect to another system stock is a partial differential equation that represents how one stock affects another stock's derivative, when all other stocks are kept constant. The matrix therefore represents the links between two stocks (Gonçalves 2009). To calculate it, the differential equations for each stock of the system must be known. A Jacobian Matrix takes the generalised form:

$$J = \begin{vmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \cdots & \frac{\partial \dot{x}_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial \dot{x}_n}{\partial x_1} & \cdots & \frac{\partial \dot{x}_n}{\partial x_n} \end{vmatrix}. \tag{9.1}$$

Where $J$ is the Jacobian Matrix of the system and $\frac{\partial \dot{x}_1}{\partial x_n}$ represents the change in net rate of stock variable $x_1$ in response to an infinitesimal change in the value of stock variable $x_n$. (Saleh 2005). The calculation of a Jacobian matrix requires the system to be linearised. By calculating the entries of the matrix at any single point in time, all non-linear inputs can be taken as constants and the model can be linearised at that individual point in time (Kampmann 2012; Güneralp 2006; Forrester 1983). If the model is to be linearised at each time step, then the time steps or changes in the system between time steps, must be very small to assume linearity.

2) *Eigenvalues of the Jacobian* - Eigenvalues are calculated from a system's Jacobian matrix and therefore the number of eigenvalues of a system is equal to the number of stocks in that system. Eigenvalues ($\lambda$) are calculated for an $nxn$ matrix ($A$) by satisfying the equation: $det(A - \lambda I) = 0$, where $I$ is an $nxn$ identity matrix. From linear stability theory, eigenvalues can be used to determine if a system at a fixed point is stable or unstable (Glendinning 1994). Eigenvalues are capable of being real, or complex numbers with a real and imaginary part. If the real parts of all eigenvalues are negative, then the fixed point is stable. If the real parts of any of the eigenvalues positive, then the fixed point is unstable.

In linear systems and particularly in matters of system stability and exponential growth/decay, the dominant eigenvalue (i.e. the one with the greatest absolute value) should not always be taken as the one most important to understanding the system's behaviour. If the real part of any eigenvalue within a linear system holds a positive value, no matter how small its magnitude, it will eventually dominate the system's behaviour (Guneralp 2004; Emami-Naeini et al. 2002).

Changes in values to eigenvalues which sit at or close to zero are more likely to cause shifts in polarity than highly negative eigenvalues. When concerned with system stability, eigenvalues changing polarity is an extremely important phenomenon to be aware of, particularly when working with systems capable of experiencing critical transitions, regime shifts or system collapse. In the context of influential feedback structures, a shift in eigenvalue polarity often infers a change has occurred to which feedback loop is dominating within that eigenvalue. Changing from a dominant negative feedback to a dominant positive feedback can lead to a reinforcing or runaway behaviour, which eventually destabilizes the system. With regards to system stability, eigenvalues which sit at or close to zero and eigenvalues which hold large real positive magnitudes can be seen as more important than eigenvalues with larger negative magnitudes.

In determining which eigenvalues are important to focus on, it is also important that the user takes into account the changes that occur within an eigenvalue through time. The build-up to an eigenvalue gaining dominance or changing polarity within a system can be just as important for identifying feedback loops which are driving change than taking eigenvalues at isolated points in time.

3) *Loop Gains* - Loop gain is required to compare perturbations within the system's feedback loops against the system's eigenvalues. In order to calculate loop gain, the gain of each causal link within that loop must first be determined. A causal link refers to the link (edge) between two variables where a connection stems from a predecessor variable and leads to a successor variable. The strength of a link, or 'link gain' refers to the amount of influence a predecessor variable produces on a successor variable. The gain of a causal link ($a$) is the partial derivative of a successor variable $v_1$ with respect to a predecessor variable $v_2$:

$$a_{v_1 v_2} = \frac{\partial successor}{\partial predecessor} = \frac{\partial v_1}{\partial v_2} \tag{9.2}$$

The gain of a loop ($g$) is then calculated through the product of all causal link gains which join together to form that loop (Kampmann 2012):

$$g = a_{v_1 v_2}.a_{v_2 v_3}.a_{v_3 v_4}.\ldots a_{v_n v_1} = \frac{\partial v_1}{\partial v_2}.\frac{\partial v_2}{\partial v_3}.\frac{\partial v_3}{\partial v_4}.\ldots\frac{\partial v_n}{\partial v_1} \tag{9.3}$$

where $v_{1,2,3,4n}$ represent individual variables which link together to form a feedback loop.

4) *Loop Eigenvalue Elasticity* is a measure of the importance of a loop on the system's current behaviour and is defined as the partial derivative of the eigenvalue with respect to the gain of the loop, normalised to isolate the effect of changes in size (values) of the eigenvalues and link gains (Gonçalves 2009). It represents a relative change in a system eigenvalue ($\lambda$) with respect to a relative change in the gain of a feedback loop (Kampmann and Oliva 2008; Kampmann and Oliva 2006). One can attribute the importance of a feedback loop to the absolute value of its loop eigenvalue elasticity. Loop eigenvalue elasticity , is defined with respect to the loop's gain parameter, in the form:

$$\varepsilon = \frac{\partial \lambda}{\partial g}.\frac{g}{\lambda} \tag{9.4}$$

where $\lambda$ is an eigenvalue of the Jacobian matrixsystem chosen by the user and g the gain of a feedback loop as defined above. If the eigenvalue is a complex number:

$$\lambda = re^{i\theta} \tag{9.5}$$

where $r = |\lambda|$ is the absolute value of the eigenvalue and is known as the "natural frequency", $i$ is the imaginary value and $cos\theta$ is known as the "dampening ratio", then the loop eigenvalue elasticity will also be complex, holding real and imaginary parts:

$$R\{\varepsilon\} = \frac{dr}{dg}.\frac{g}{r} \tag{9.6}$$

$$Im\{\varepsilon\} = \frac{d\theta}{dg}.g \tag{9.7}$$

It is possible to measure the real and imaginary parts of elasticity separately as shown in Kampmann (2012):

$$\varepsilon_1 = \frac{dRe\{\lambda\}}{dg}.\frac{g}{|\lambda|} \tag{9.8}$$

$$\varepsilon_2 = \frac{dIm\{\lambda\}}{dg}.\frac{g}{|\lambda|} \tag{9.9}$$

These calculations are possible because the eigenvalues and loop gains of a system are inherently connected via the system's characteristic polynomial ($P(\lambda)$). The characteristic polynomial of a system takes the form:

$$P(\lambda) = |\lambda I - J| \tag{9.10}$$

Where $I$ is an identity matrix, $\lambda$ is the system eigenvalues and $J$ the system's Jacobian Matrix. Eigenvalues of a system are determined as the roots of the characteristic polynomial and as it turns out, the coefficients of the characteristic polynomial can be expressed in terms of the feedback loop gains. Kampmann (2012) provides an example and helpful breakdown of this within Theorem 2 of his article 'Feedback loop gains and system behaviour (1996)'. This direct link between the eigenvalues as roots of the characteristic polynomial and loop gains as the coefficients of the characteristic polynomial allows one to attribute the change in eigenvalues directly to changes in individual feedback loops by implicit differentiation of the equation; $P(\lambda, g_i) = 0$, i.e.

$$\frac{d\lambda}{dg_i} = \frac{\partial P(\lambda, g_i)}{\partial g_i}.(\frac{\partial P(\lambda, g_i)}{\partial \lambda})^{-1} \tag{9.11}$$

Where $g_i$ is the gain associated with one feedback loop, $i$. As noted by Kampmann (2012), this formulism can only be used and is valid if the change in the gain of a link can be attributed to the gain of one feedback loop independently of all others. This requirement is prevented by individual links potentially being part of multiple feedback loops at once. To combat this, the user must choose a particular loop set with which to describe the system (an Independent Loop Set (ILS) (Kampmann 2012) or Shortest Independent Loop Set (SILS) (Oliva 2004)) in order to give meaning to the relative importance of a loop within the context of the selected loop set.

A hierarchy of loop dominance over an eigenvalue can be established through the absolute values of loop elasticity. Loops with high absolute values of elasticity contribute the most to changes within an eigenvalue.

In Forrester's original work (Forrester 1983; Forrester 1982) it was suggested that partial differential equation for elasticity be determined by calculating the system's Jacobian matrix before and after a small change in loop gain (0.1%). The algorithm developed by Naumov & Oliva (2017) was built to maintain all calculations regarding Loop Elasticity in algebraic form, evaluating the exact derivatives at each point in time when the system is linearized. Only at the last minute are the values of the stocks introduced to obtain a numerical estimation of the analysis. Links to all the papers, models, and Mathematica code regarding the LEEA software toolset can be found in Supplementary information section 2.

*Loop Influence* Alternatively to Loop Elasticity, Loop Influence $\mu$, of a loop can be calculated and is defined thus:

$$\mu = \frac{\partial \lambda}{\partial g} . g \tag{9.12}$$

From loop influence it is possible to determine the type of contribution a loop is having over an eigenvalue. A positive value of loop influence indicates a loop generating instability within an eigenvalue, while negative values indicate the generation of stability (Kampmann and Oliva 2006). Similar to loop elasticity, the greater the absolute value of a loop's influence, the greater the contribution that loop makes to the system's current behaviour.

*ILS and SILS:*

Forming an ILS is a necessary part of LEEA as it is often the case that links that exist between variables will be part of multiple feedback loop structures. A change occurring within a link can therefore impact multiple feedbacks simultaneously. By forming an ILS, changes which occur within the links are given context within their loop set. An ILS is made up of a maximum number of loops which encompass the feedback complexity of the model, but whose links are linearly independent. Calculation of an ILS derives from concepts established within graph theory, involving the identification of shortest paths from the system's reachability matrix and finding a maximum number of independent feedback loops, within a cycle partition of the system, where the maximum independent loop number is equal to $E - V + 1$, where $E$ = edges (links) and $V$ = vertices within the cycle partition (Kampmann 2012). A full breakdown of this process, including pseudo-code for each step, can be found in Oliva (2004). The main issue was that the algorithm created to generate an ILS did not account for the complex structure which the loops could take within the set and perhaps more importantly, it did not consistently produce the same unique set. This meant that an ILS could change between users, even for the same model system. In order to make the loop sets consistent, the Shortest Independent Loop Set (SILS) was developed by Oliva (2004). The SILS is made up of the shortest loops considered for ILS by always making the smallest addition of edges to the loop set as possible.

## 9.1 Supplementary Information 2: LEEA Processing Details

The PLUM model and following results can all be reproduced using the tools and software packages found in the following links:

Naumov & Oliva (2017) Online LEEA toolset:

http://iops.tamu.edu/faculty/roliva/research/sd/sda/

PLUM forward and reverse Vensim Models via the Open Science Framework:

https://osf.io/h3g92/

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