**Supplementary material**

The supplementary material provided here offers additional details of the derivations on the equivalences of helical axes and SARA/SCoRE. The material is structured into 3 sections, where section S1 demonstrates the derivation of the vector of angular velocity as a function of and S2 shows that the position of an instantaneous helical axis (IHA) is identical to those of the finite helical axis (FHA) or SARA, and S3 verifies that the mean finite helical axis (MFHA) and the SARA axis are mathematically equivalent.

**S1**

Here we derive for a representation as function of and. Using equation (6) for and equation (20) for , i.e.

one obtains

and further with the relationships, see equation (3),

Since with also holds and together with this yields finally

As expected, does not depend on. The final result is the well-known relationship between the vector of angular velocity, the rotation axis unit vector, and the angle of rotation. Here, we have demonstrated that the notation of the unique parameterization of the shortest geodesic path in is consistent with this definition.

**S2**

In order to determine the position of the IHA-axis as a function of and, we use equation (19)

and insert in this equation the terms for , equation (11), equation (23), and , equation (21), into equation (19) and obtain:

which yields

Thus finally we have, again with

For the unique point on the axis with minimal norm, i.e. using one obtains. This is the analytical proof that the instantaneous helical axis (IHA) coincides with the FHA- or the SARA-axis.

**S3**

In order to show the mathematical equivalence between the MFHA approach and SARA we start from the equation for the MFHA (27):

In which we replace now the terms with the results from (16), i.e.

and according to (17), considering that holds:

this yields:

Setting or, since leads to:

Further, with and, the relationship can be written as:

This equation (S33) above is identical to equation (8) in (Ehrig et al., 2006) and was presented in there as the closed form solution of the SCoRE approach for the determination of the centre of rotation of a ball joint, demonstrating the mathematical equivalence between the MFHA approach and SARA.

It is important to note that this derivation does not depend on how exactly the axes were determined and which specific method was used (FHA, IHA, etc.); the only requirements are a representation of each axis as the unique norm minimal point on the axis together with the unit axis vector.

In summary, with the formulae (S31) and (S33) differ only in that (S31) defines the centre in terms of rotations and translations whereas (S33) describes the centre as a function of and. It therefore follows that the derivation of equivalence also holds for the mean instantaneous helical axis.

**References**

R.M. Ehrig, W.R. Taylor, G.N. Duda, and M.O. Heller (2006). A survey of formal methods for determining the centre of rotation of ball joints. J Biomech 39: 2798-809.