

Problems with Higgspllosion

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A recent calculation of the multi-Higgs boson production in scalar theories with spontaneous symmetry breaking has demonstrated the fast growth of the cross section with the Higgs multiplicity at sufficiently large energies, called “Higgspllosion.” It was argued that Higgspllosion solves the Higgs hierarchy and fine-tuning problems. In our paper we argue that: (a) the formula for Higgspllosion has a limited applicability and inconsistent with unitarity of the Standard Model; (b) that the contribution from Higgspllosion to the imaginary part of the Higgs boson propagator cannot be re-summed in order to furnish a solution of the Higgs hierarchy and fine-tuning problems.

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I. THE AMPLITUDE BEHAVIOR WITH THE LARGE SCALAR MULTIPLICITY

One of the flaring questions for the modern elementary particle physics is the question about the energy scale of new physics. All current experiments are in excellent agreement with the Standard Model (SM). Moreover, the Higgs mass $m_H \simeq 125$ GeV means that all the couplings of the theory are small above the electroweak scale, and perturbative calculations in non-Abelian QFT, which is the core of the SM, should provide a consistent approach. Most of the coupling constants of the theory become smaller with increasing energy. The only two couplings which grow with the energy scale are the $U(1)$ hypercharge coupling constant and the Higgs self coupling λ . However, the scale of new physics related to this coupling evolution with the energy—the Landau pole—is proportional to $\exp(1/\lambda)$ and significantly exceeds the Planck scale. Therefore, it is normally assumed that SM can be trusted as a perturbative QFT at all energies that can, even hypothetically, be probed in collisions. The only scale that may appear in the SM framework is the one associated with the metastability of

the electroweak (EW) vacuum, but this scale, even if present, is very large $\sim 10^{10}$ GeV.

At the same time, it has long been known that theories of self-interacting scalars (which also include the Higgs boson of the SM) have problems with the application of perturbation theory at high energies. The first observations of subtleties in the scalar multiparticle production demonstrated that at the tree level, owing to the large number of contributing diagrams, the n -particle amplitudes have factorial dependence on the number of particles [1–5]

$$A_n^{\text{tree}}(0) = n! \left(\frac{\lambda}{8}\right)^{\frac{n-1}{2}}. \quad (1)$$

This factorial growth of the amplitude indicates the breakdown of the usual perturbative calculations for $n \gtrsim \lambda^{-1}$. It was found [6–8] that the corresponding $1 \rightarrow n$ cross section can be written in exponential form

$$\sigma(E, n) \propto \exp\left(\frac{1}{\lambda} F(\lambda n, \varepsilon)\right), \quad (2)$$

where $\varepsilon \equiv (E - nm_H)/nm_H$ is the average kinetic energy of the final-state Higgs particles. The function $F(\lambda n, \varepsilon)$ was obtained by following a specific semiclassical approach [8] valid in the limit

$$\lambda \rightarrow 0, \quad n \rightarrow \infty, \quad \text{with fixed } \lambda n, \varepsilon. \quad (3)$$

Moreover, there is a conjecture [7], that to exponential precision the result does not depend on the details of the initial state, given that the initial number of particles is

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small and therefore, without loss of generality, one can focus on calculation of $1 \rightarrow n$ process, even though the initial particle is off shell. For small $\lambda n \ll 1$ and small energies of the final particles $\varepsilon \ll 1$ the exponent of the cross section is [6–8]¹

$$F(\lambda n, \varepsilon) = \lambda n \ln \frac{\lambda n}{16} - \lambda n + \frac{3}{2} \left(\ln \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \lambda n \varepsilon + 2B\lambda^2 n^2 + O(\lambda^3 n^3) + O(\lambda^2 n^2 \varepsilon) + O(\lambda n \varepsilon^2), \quad (4)$$

where

$$B = \frac{\sqrt{3}}{8\pi}.$$

As $\lambda n \rightarrow 0$, $F(\lambda n, \varepsilon) \rightarrow -\infty$ and the cross section equation (2) is exponentially suppressed, while in the opposite regime for large λn the cross section grows exponentially, thereby contradicting the unitarity of the theory, at least at the level of perturbation theory.

The equation (4) for $F(\lambda n, \varepsilon)$ is valid for $\lambda n \ll 1$, $\varepsilon \ll 1$. The logarithmic and lowest order terms correspond to tree level contributions, the term of the order $O(\lambda^2 n^2)$ is the first radiative correction. Note, that in the range of the validity of Eq. (4) the function $F(\lambda n, \varepsilon)$ is negative. At tree level (for $\lambda n \ll 1$) the energy dependence for arbitrary energies ε was found in [11,12] and again leads to an exponentially suppressed result. However, the problem of finding the expression for arbitrary large λn and ε is still open.

Recently authors of [9,10] have extended the thin-wall approximation of [13] and have found the cross section for the opposite, $\lambda n \gg 1$ limit:

$$F(\lambda n, \varepsilon) = \lambda n \left(\ln \frac{\lambda n}{4} + 0.85\sqrt{\lambda n} - 1 + \frac{3}{2} \left(\ln \frac{\varepsilon}{3\pi} + 1 \right) - \frac{25}{12} \varepsilon \right). \quad (5)$$

An important feature of this solution is the increase of $F(\lambda n, \varepsilon)$ at sufficiently large λn for a fixed value of ε . This result was then used to argue that at large multiplicities [or, equivalently, large energies $E \sim n(\varepsilon + m_H)$] the $1 \rightarrow n$ width grows exponentially. One should note that the thin-wall semiclassical solution, leading to Eq. (5) exists only in the $\lambda\phi^4$ theory with spontaneous symmetry breaking in $3 + 1$ dimensions.

We would like to stress, however, that nonvanishing ε is required for the result of Eq. (5) to be positive, since at zero ε the logarithmic term is infinitely negative which gives zero cross section at the threshold. At the same time the

contribution $0.85\sqrt{\lambda n}$ in Eq. (5) was obtained at the kinematical threshold, that is for $\varepsilon \rightarrow 0$. This is a subtle point. One should also note that the full result of Eq. (5) is obtained from a combination of the large λn contribution with the tree level result, which has the factorized form

$$F(\lambda n, \varepsilon) = \lambda n(f_0(\lambda n) + f(\varepsilon)).$$

This form is valid at tree level and at one loop [cf. Eq. (4)]. We would now like to point out that higher order quantum corrections are expected to contain terms which depend *both* on ε and λn , e.g., terms like $O(\lambda^2 n^2 \varepsilon)$ in Eq. (4). Such terms could play an important role. We argue here that without the knowledge of these terms it is not possible to determine the validity region of the result, Eq. (5), with respect to the value of ε . We discuss this in detail in the next section. Such mixed terms may prevent the exponential growth of the cross section. The exponential growth of the $1 \rightarrow n$ width was suggested to be by itself a solution to the hierarchy problem in [14] where authors conclude that such exponential growth of the self-energy leads, after resummation, to exponential suppression of the scalar propagators at high energies.

In this paper we review in detail the validity and consequences of such fast-growing amplitudes in the context of unitary, local and Lorentz invariant quantum field theory.

II. UNITARITY AND 1PI RESUMMATION

It has been known for many years [2] that exponentially growing amplitudes lead to a violation of unitarity. In [14] the authors have proposed a mechanism to recover unitarity through the effect of the off-shell $1 \rightarrow n$ amplitude on the resummed scalar Feynman propagator. The authors suggested that if the two-point function falls off faster with energy than the amputated $1 \rightarrow n$ matrix element, unitarity can be restored via the so-called Higgespersion mechanism.

However, this argument requires a propagator which falls off faster than the amputated $1 \rightarrow n$ matrix element. In other words, we require the two-point function to be decaying exponentially with energy. This is a peculiar form of the two-point function that is known to cause problems with unitarity [15]. However, it has been proposed [14] that this form appears in a theory with exploding amplitudes.

The problem we see here is the following. An exponentially decreasing propagator has been obtained in [14] because the authors have used the perturbation theory to sum up single-particle irreducible (1PI) Green's functions, which is a valid procedure only for a convergent geometric series. Namely, it has been claimed that the exact two-point function $\Delta_F(p^2)$ can be obtained from the 1PI Green's function $\Sigma(p^2)$ via

¹We quote here the result for the theory with spontaneous symmetry breaking, which was used in the recent calculations [9,10].

$$\begin{aligned}\Delta_F(p^2) &= \int d^4x e^{ip \cdot x} \{ \theta(x_0) \langle 0 | \phi(x) \phi(0) | 0 \rangle \\ &\quad + \theta(-x_0) \langle 0 | \phi(0) \phi(x) | 0 \rangle \} \\ &= \frac{i}{p^2 - m_0^2 - \Sigma(p^2)}\end{aligned}\quad (6)$$

where m_0 is the bare mass of the theory. However, if Σ is exponentially growing with p^2 , at sufficiently large p^2 this series is no longer convergent. In this case, one may not use the resummed form of the above expression. Since resummation is not valid, instead of exponentially falling with p^2 , Δ_F will uncontrollably grow with p^2 . This leads to unitarity violation of the Higgsploding theory, assuming that Eq. (5) is valid for large λn values and nonvanishing ε . Under this assumption, one may ask whether the aforementioned problem is related to the application of the perturbation theory where it is not valid. It is illustrative to examine the functional form of the two-point function using nonperturbative “language” of dispersion relations. In this procedure we closely follow [16]. Consider the momentum-space Feynman propagator

$$\begin{aligned}\Delta_F(p^2) &= \int d^4x e^{ip \cdot x} \{ \theta(x_0) \langle 0 | \phi(x) \phi(0) | 0 \rangle \\ &\quad + \theta(-x_0) \langle 0 | \phi(0) \phi(x) | 0 \rangle \}\end{aligned}\quad (7)$$

where we anticipate that the Δ_F is Lorentz invariant and hence only a function of p^2 .

Using the integral representation of the θ -function,

$$e^{ip_0 \cdot x_0} \theta(\pm x_0) = \frac{1}{2\pi i} \int dp'_0 e^{ip'_0 \cdot x_0} \frac{1}{p'_0 - p_0 \mp i\varepsilon} \quad (8)$$

one has

$$\begin{aligned}\Delta_F(p^2) &= \int d^4x \int \frac{dp'_0}{2\pi i} e^{-i\vec{p} \cdot \vec{x} + ip'_0 \cdot x_0} \\ &\quad \times \left\{ \frac{\langle 0 | \phi(x) \phi(0) | 0 \rangle}{p'_0 - p_0 + i\varepsilon} - \frac{\langle 0 | \phi(0) \phi(x) | 0 \rangle}{p'_0 - p_0 - i\varepsilon} \right\}.\end{aligned}\quad (9)$$

Setting the variable of integration $\vec{x} \rightarrow -\vec{x}$ in the second term and using translation invariance of the vacuum,

$$\begin{aligned}\Delta_F(p^2) &= \int d^4x \int \frac{dp'_0}{2\pi i} \langle 0 | \phi(x) \phi(0) | 0 \rangle \\ &\quad \times \left\{ \frac{e^{ip'_0 \cdot x_0 - i\vec{p} \cdot \vec{x}}}{p'_0 - p_0 + i\varepsilon} - \frac{e^{ip'_0 \cdot x_0 + i\vec{p} \cdot \vec{x}}}{p'_0 - p_0 - i\varepsilon} \right\}.\end{aligned}\quad (10)$$

Now we insert a complete set of (*in* or *out*) states. In the language of [14], this corresponds to a kinematically unique one-particle state, plus a continuum of multiparticle states. We let σ_n denote all the internal quantum numbers of

an n -particle state, including its phase space. Assuming that $|\langle 0 | \phi(0) | n, \sigma_n \rangle|^2$ is Lorentz invariant,

$$\begin{aligned}\Delta_F(p^2) &= \int d^4x \int \frac{dp'_0}{2\pi i} \sum_{n, \sigma} |\langle 0 | \phi(0) | n, \sigma_n \rangle|^2 (p_n^2) e^{-ip_n \cdot x} \\ &\quad \times \left\{ \frac{e^{ip'_0 \cdot x_0 - i\vec{p} \cdot \vec{x}}}{p'_0 - p_0 + i\varepsilon} - \frac{e^{ip'_0 \cdot x_0 + i\vec{p} \cdot \vec{x}}}{p'_0 - p_0 - i\varepsilon} \right\}\end{aligned}\quad (11)$$

where p_n is the total four-momentum of the n -particle state.

In the case of [14], complications will arise due to the divergence of this integrand. To see how difficulties appear, let us consider the scenario where $\sum_{n, \sigma} |\langle 0 | \phi(0) | n, \sigma_n \rangle|^2$ is a polynomial of order N in p^2 .

Exchanging $p'_0 \rightarrow -p'_0$ and $x_\mu \rightarrow -x_\mu$ in the second term gives

$$\begin{aligned}\Delta_F(p^2) &= \int d^4x \int \frac{dp'_0}{2\pi i} \sum_{n, \sigma} e^{i(p'_0 - p_{n,0}) \cdot x_0 - i(\vec{p} - \vec{p}_n) \cdot \vec{x}} \\ &\quad \times |\langle 0 | \phi(0) | n, \sigma_n \rangle|^2 (p_n^2) \\ &\quad \times \left\{ \frac{1}{p'_0 - p_0 + i\varepsilon} + \frac{1}{p'_0 + p_0 + i\varepsilon} \right\}.\end{aligned}\quad (12)$$

Combining both terms in the curly bracket and making the $i\varepsilon$ prescription implicit,

$$\begin{aligned}\Delta_F(p^2) &= \int d^4x \int \frac{dp_0}{2\pi i} \sum_{n, \sigma} e^{i(p'_0 - p_{n,0}) \cdot x_0 - i(\vec{p} - \vec{p}_n) \cdot \vec{x}} \\ &\quad \times |\langle 0 | \phi(0) | n, \sigma_n \rangle|^2 (p_n^2) \frac{1}{(p_0^2 - p_0^2)}.\end{aligned}\quad (13)$$

At this point, one might be tempted to swap the order of integration and perform the x -integral. However, the remaining integrand would be an order $N - 1$ polynomial in p^2 . This integrand is not convergent at $p'_0 = \pm\infty$, so the straightforward change of the integration order is not valid here. Before we can swap the order of integration, we must perform N subtractions of the form

$$\frac{1}{p_0^2 - p_0^2} = \frac{1}{p_0^2} + \frac{p_0^2}{p_0^2(p_0^2 - p_0^2)}.\quad (14)$$

In this way, Eq. (13) may be written as $(p_0^2)^N$ times a convergent integral, plus an order $N - 1$ polynomial in p_0^2 where the coefficients are functions of $\Delta_F(0)$. For example, the first term in Eq. (14) simply gives $\Delta_F(0)$.

The contribution from the convergent integral is

$$\begin{aligned}(p_0^2)^N \int dp_0 \int \left\{ (2\pi)^3 \sum_{n, \sigma} |\langle 0 | \phi(0) | n, \sigma_n \rangle|^2 (p^2) \delta^{(4)}(p' - p_n) \right\} \\ \times \frac{-i}{(p_0'^2 - p_0^2)(p_0^2)^N}\end{aligned}\quad (15)$$

where $p^\mu \equiv (p'_0, \vec{p})$ and we recognize the term in the curly brackets as the Kallen-Lehmann spectral function $\rho(p'^2)$. Given that \vec{p} is fixed, one may change the variable of integration from p'^2_0 to p'^2 , giving

$$\Delta_F(p^2) = \Delta_F(0) + p^2 \Delta_F^{(1)}(0) + (p^2)^2 \Delta_F^{(2)}(0) + \dots + (p^2)^N \int dp'^2 \rho(p'^2) \frac{-i}{(p'^2)^N (p'^2 - p^2)}. \quad (16)$$

From this form it is evident that if $\rho(p^2)$ is an order- N polynomial in p^2 , knowledge of $\rho(p^2)$ only defines the two-point function up to some order- N polynomial. The functional form of $\Delta_F(p^2)$ is allowed to change dramatically without any change in the amputated $1 \rightarrow n$ matrix element.

In the case of [14], the situation is even more extreme. In this case, the spectral function $\rho(p^2)$ is the sum of terms proportional to the multiparticle rate

$$\mathcal{R}(p^2) \equiv \frac{1}{2M_h^2} \sum_n \int d\Pi_n |\mathcal{M}(1 \rightarrow n)|^2 \quad (17)$$

where M_h is the Higgs mass, Π_n is the n -particle phase space element and $\mathcal{M}(1 \rightarrow n)$ is the matrix element for $1 \rightarrow n$ Higgs decay. If one assumes that $\mathcal{R}(p^2)$ is exponentially growing in p^2 , all predictive power for Δ_F from $\rho(p^2)$ is lost, due to the infinite number of subtractions required for a convergent integral in Eq. (13). Although one may know the exact form of $\rho(p^2)$, one may add an arbitrary analytic function to both the left-hand side and right-hand side of Eq. (16) such that the Feynman propagator is allowed to change its functional form wildly without having any apparent effect on the multiparticle rate $\mathcal{R}(p^2)$.

This feature is just a statement that for an order- N polynomial $g(z)$ with a branch cut $\Delta g(z)$ along the real axis beginning at z_0 , one can integrate $\Delta g(z)$ via contour integration. In order to discard the contribution from the $|z| \rightarrow \infty$ curve, one performs N subtractions such that

$$g(z) = (z-y)^N \frac{1}{\pi} \int_{z_0}^{\infty} \frac{\Delta g(x)}{(x-z)(x-y)^N} dx - (z-y)^N \frac{d^{N-1}}{dy^{N-1}} \left(\frac{g(y)}{(y-z)} \right) \quad (18)$$

where the latter term is the residue at the $x = y$ pole [17]. The price one pays for convergence is the addition of an order- N ‘‘polynomial of integration’’ which must be fixed by extra conditions of the theory.

Returning to the Higgspersion scenario, we would like to stress that given that $\Delta_F(p^2)$ may include an arbitrary analytic function of p^2 there is no reason why it should fall

off exponentially with p^2 in the high-energy limit. In fact, Eq. (16) suggests precisely the opposite—that the two-point function should grow uncontrollably in this limit. The discrepancy between the amplitude growth with p^2 we observe and the exponential fall proposed in [14] arises because the latter was calculated using perturbation theory. Namely, the 1PI Green’s function $\Sigma(p^2)$ was summed into a geometric series in order to put Σ into the denominator of $\Delta_F(p^2)$. However, if Σ grows exponentially with p^2 , at sufficiently large p^2 this series is no longer convergent and one must instead use the form

$$\Delta_F(p^2) = \frac{i}{p^2 - m_0^2} \sum_{n=1}^{\infty} \left(-i\Sigma(p^2) \frac{i}{p^2 - m_0^2} \right)^n \quad (19)$$

where m_0 is the bare mass of the theory. In this form Δ_F will uncontrollably grow with p^2 , in agreement with Eq. (16). In this way, the Higgspersion mechanism only compounds the unitarity violation in the Higgsplosion theory.

III. CONCLUSIONS

We have explored the Higgsplosion effect and related Higgspersion mechanism behind it in detail and have found its limitation and problems.

In particular, assuming the correctness of the Eq. (5) for $F(\lambda n, \varepsilon)$ derived for $1 \rightarrow n$ process in [9,10] beyond the thin-wall approximation, we have found that the amplitude for $1 \rightarrow n$ process increases exponentially rather than decreases at sufficiently high energies as stated in [14]. We have found this effect and the respective discrepancy because one cannot use the resummation of the self-energy insertion when that self-energy grows exponentially. Since the respective series is divergent for sufficiently large momentum one can not resum it into a correction of the propagator. Previously [14] it was argued that such a correction will play a crucial role in ‘‘shutting off’’ the propagator at sufficiently large energies and solving hierarchy problem. In light of our finding we would like to state that such a resummation is not possible and that, assuming Eq. (5) is correct, the $1 \rightarrow n$ amplitude will grow exponentially thereby violating unitarity. It may be also interesting to note another nonperturbative resummation of a class of diagrams with multiple intermediate particle lines using 2PI formalism, which does not give suppression of Higgs boson propagator at high energies [18].

The fact that Eq. (5) implies unitarity violation leads us to conclude that this equation is likely not generic enough and that additional higher order cross terms of $O(\lambda^2 n^2 \varepsilon)$ form in Eq. (4) are expected to play an important role on restoration of unitarity. Indeed, unitarity should be restored, since it was present in the theory in the first place from the hermiticity of the Hamiltonian. If some theory has a real

unitarity problem (which is, however, not the case of the SM framework we discuss here) one of the natural solutions could be a composite nature of the Higgs boson which at certain characteristic energy scales would cure nonunitary growth via the respective form factor and the related new physics sector.

In the case of the Standard Model we conclude that the $1 \rightarrow n$ multiscalar final state amplitude should be consistent with unitarity, but that in any case if it exponentially grows it can not be resummed. Such behavior is not consistent with unitarity and does not provide a solution to the hierarchy problem. We believe that the correct evaluation of $1 \rightarrow n$ amplitude for multiscalar final states above the threshold requires an extension of Eq. (5) and remains still an open and very nontrivial problem.

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