**Obtaining the Distribution of Quiescent Periods Directly from Wave Power Spectral Densities:**

**The Basis for a Forward Planning Aid for**

**Maritime Operations**

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**Abstract.** There is a growing practical interest in the ability to increase the sea states at which marine operations can be safely undertaken by exploiting the quiescent periods that are well known to exist under a wide range of sea conditions. While the actual prediction of quiescent periods at sea for the control of operations is a deterministic process the long term planning of future maritime tasks that will rely on quiescent periods is a statistical process involving the estimation of the anticipated quiescence properties of the forecasted sea conditions in the geographical region of interest. It is in principle possible to obtain such data in tabular form either via large scale simulation or from field data. However such simulations are computationally intensive and libraries of appropriate field data are not common. Thus it is clearly attractive to develop techniques that exploit standard wave spectral models for the estimation of quiescence statistics directly from such spectra. The present study focuses upon such techniques to allow the production of computationally low cost quiescence estimation tools and compares their efficacy against simulations. Two significant properties emerge for a large class of wave spectral models that encompasses the ubiquitous Neumann and Pierson Moskowitz or Brettschnider forms. Firstly the probability of a given number of consecutive wave heights less than some critical value are predicted to be independent of the sea state. The only absolute dependence of wave height enters through the mean wave period, which is needed to convert the number of small waves into a quiescent time interval. Secondly the auto-correlation function of the wave envelope (and also of the wave profile) can be obtained analytically in terms of standard special functions. This considerably reduces the computational cost associated with quiescence estimation making desk top computer based planning tools a reality. In fact for a broad class of practically interesting wave spectra all that is required to compute in an operational planning tool is a rescaling from the number of runs in a quiescent period to its actual duration in time.

**1. Introduction.** There is a rapidly growing applications interest, (Giron\_Sierra, 2010, Duncan 20102, Belmont 2016) in the ability to exploit the quiescent periods (QPs) that are well known to exist in otherwise large seas. The ability to deterministically predict such QPs even a few tens of seconds into the future offers the prospect of allowing a marked increase in the sea states under which maritime operations can be safely performed.

To be of practical value two aspects of QPs are required. The first is knowledge of the statistical properties of QPs, i.e., knowledge of the probability of QPs under the anticipated or prevailing sea conditions. This is a vital operational forward planning requirement because clearly one cannot rely on exploiting QPs if they have a very low probability of occurrence and have durations that are to short to be useful. The second aspect is the deterministic side of QPs i.e., the ability to deterministically predict at sea in real time when they will actually occur and what their durations are. This is the process of Quiescent Period Prediction (QPP) and is required for the actual execution of a task rather than its planning. QPP is based upon the burgeoning new technology of Deterministic Sea Wave Prediction (DSWP) that has been of interest for some time, (Morris, 1992, Blondel 2008), and is close to becoming viable as an operational tool, (Belmont 2014).

Any vessel wishing to exploit QPP will of necessity be equipped with vessel motion sensors and clearly just prior to executing a mission it is a simple matter to continuously measure the statistics of QPs of the vessel (which is the actual operational requirement rather than merely the QPs of the wave system). However a major benefit stemming from the availability of QP information is its use in future mission planning which is clearly statistical in nature. Consequently it would be highly desirable to be able to estimate QPs statistics ahead of time for the typical sea characteristics for the regions and seasons of interest.

Apart from the direct empirical determination of the distribution function of QPPs at sea at the time of interest as described above there are three further possible approaches to estimating the statistics of QPPs. The first is a semi-analytical technique where mathematical models of the power spectral density of the sea type are employed in conjunction with analytic statistical models of the bivariate wave height to wave height probability distributions. In this case the bulk of the computations can be accomplished analytically producing a computational cheap and approach that is very convenient for users, This method is however limited to those wave spectral models that are amenable to analytic techniques involved (typically: Neumann, Pierson Moskowitz, Bretschnider etc) . The second approach is to use purely empirical models of the wave power spectral densities. In this case all of the necessary computations are performed numerically. This method is clearly broader in the scope of sea conditions it can deal with but it is far more computationally intense than the semi-analytic method and is less suitable for users. The third methodology is a large scale multi-directional sea simulation based numerical experiment which directly simulates the purely empirical data based at sea situation mentioned above. This is even more computational intensive than the second methodology described above. The present report deals with the semi-analytic approach which also provides the framework for the second numerically evaluated methodology, the latter being the subject of a future communication.

The methodology will be first to develop a statistical model for the chance that a run of *at least r consecutive waves is less than some critical threshold*. Secondly given that users are more interested in duration of quiescent periods that the number of quiescent waves the runs statistics are recast in terms of the duration of the quiescent periods. Given the ubiquity of wave spectral models the runs statistics will e developed from that perspective.

It transpires that the general background theory required has been extensively covered in what are in the main rather mature publications dating from the 1960’s to the 1980’s dealing with statistical wave theory. However the predominant practical motivation which drove such research was in the complementary problem, i.e., sequences of consecutive large rather than small waves. The present treatment will explore the most appropriate results from this literature, modifying and extending these where necessary, and applying the results obtained to the problem of estimating the properties of QPs based upon typical spectral wave models.

**2. Runs Theory for Sea Waves.** In statistical terms a *run* is a sequence of outcomes that all share some property, in this case waves either greater than or less than some critical value, . For entirely obvious engineering reasons the majority of the previous research interest has been focussed on sequences of consecutive large waves which are typically termed, *High Runs.* One very important exception to this is the work of Kimura, Kimura, 1980) who also considers the converse case of *Low Runs* which relates to QPs.

The simplest approach to modelling *runs* of sea waves was developed by Goda, (Goda 1970) who assumed that consecutive waves are statistically independent. As alluded to above Goda’s work concentrated upon what at the time was the most important case i.e., *High Runs.* However using Goda’s formalism a *Low Run* ,(i.e., a QP), of length, , can represented in the following manner: the *run* commences with a first wave having an elevation, less than, , which has a probability of, , and ends with a wave of wave height, , which has a probability, of, . Thus the probability, , of such a *Low Run* or QP, which is  waves long is simply given by:



Equation 1 is evaluated using the standard Rayleigh marginal probability distribution for wave heights.

Goda’s simple approach ignored the short term correlations that are well known to typically exist over up to three “half waves” ahead, (Wilson, 1972, Rye, 1974, Siefert, 1974). This prompted two more sophisticated approaches in the literature to describe the *runs* statistics of sea waves. One class of technique termed Wave Envelope methods was based directly upon Rice’s work, (Rice, 1944, 1958), on narrow band random signals. This area was reviewed in, Longuet-Higgins, 1984), and is essentially a signal processing technique that endeavours to model the properties of narrow band stochastic processes in terms of generalised amplitude modulation (i.e., envelopes). It relies on the simple property that any dynamical system whose frequency response function is localised around the frequency, , with a spread, , where, , is capable of exhibiting oscillatory behaviour at frequencies close to, , which is modulated by an envelope that fluctuates over timescales of the order of, . This envelope reflects the statistical dependence that affects *runs.* For sea waves the bandwidth parameter, , where the, , is the, , moment of the power density spectrum is typically used to measure, .

Such envelope behaviours are encapsulated in the theory of Analytic Signals, e.g., (Boashash, 1992), which uses the Hilbert Transform in the determination of the envelope functions of interest and has been applied to sea waves by Longeut Higgins, (Longeut Higgins, 1984).

The second alternative to Goda’s approach, (Goda 1970) developed by Kimura, (Kimura, 1980), employs a Markov Chain model which incorporated the appropriate conditional probability functions that depended upon the correlation coefficient, , between successive wave heights.

Using simulated wave data Kimura was able to demonstrate that incorporating such successive wave correlations significantly improved the ability to model *runs* over other approaches*.* As expected the findings agreed with those of Goda, (Goda, 1970), in the uncorrelated limit. Validation using actual sea data was subsequently provided by Battjes, (Battjes, 1984) Ref[16], who tested Kimura’s results using North Sea wave buoy data rather than simulations.

Comparing the wave envelope method with the Markov chain technique suggests that the former slightly under estimates the length of *runs*. Both Longuet Higgins, (Longuet-Higgins, 1984), and, Arhan, (Arhan, 1978), discuss the reasons why the Kimura approach would be expected to produced a better fit to experimental data. This discussion is important because the bivariate probability distribution used by Kimura, (Kimura, 1980), was actually that developed by Rice, (Rice, 1944, Rice, 1958), and hence the same narrow band restrictions apply.

**3. The Longuet-Higgins and the Kimura *Runs* Model for QPs.** As stated both Longuet-Higgins and Kimura used, Rice’s, (Rice, 1944, Rice, 1958), bivariate Rayleigh probability density function, , for consecutive wave height parameters, , and,  :

 

Equation 2 is the form Longuet-Higgins used in which the wave height parameters were mean to peak values and, , is the root mean square value of the instantaneous wave elevations (Kimura used peak to trough values with, , being the root mean square of that quantity). The parameter, , is the wave-wave correlation coefficient and the function, , in equation 2 is the modified Bessel function of order zero.

The phrase, “wave height parameters” for ,, and, , is used advisedly because as Longuet-Higgins, (Longuet-Higgins, 1984), points out in Kimura’s analysis ,, and, , are actually trough to crest values and not the wave heights (with the unstated assumption that they are the full width of the envelope of the wave peaks in the narrow bandwidth case). This is why the extra factors of 4 appear in, (Kimura, 1980), as opposed to the equation appearing in Rice’s original, (Rice, 1944, Rice, 1958), formulation. Equally Kimura’s, , parameter is actually equal to half of the standard wave-wave correlation parameter.

In Kimura’s case equation 2 was employed to derive the requisite coefficients of the transition probability matrix required for a Markov chain model. In contrast Longuet-Higgins used a simpler linked probability approach which exploited the fact that the wave-wave correlation was negligibly small beyond consecutive wave peaks, (Wilson, 1972, Rye, 1974, Siefert, 1974). The simplicity and flexibility offered by Longuet-Higgins simple linked probability approach makes it very straightforward to vary the assumptions and definitions associated with quiescent periods and mainly for this reason it will be used here despite its frequently discussed limitations.

***3.1 Crest verses Trough to Crest Measures:*** The issue of using trough to crest or wave height value is not simply a matter of rescaling by a factor of 2. This is because equation 2 actually only applies to wave heights and Kimura’s, (Kimura, 1980), implicit assumption that trough to crest height is double wave height is only justified for very narrow band spectra where the effects of higher frequencies do not introduce trough, crest asymmetries. This point has attracted significant attention in the literature, (Forristall, 1978, Haring, 1976, Longuet-Higgins, 1952, Longuet-Higgins, 1980, Cartwright 1956, Naess, 1985, Vinje, 1989), with several variants to equation 2 being proposed for describing trough to crest probability distributions. The key issue underpinning this literature debate is that results derived from equation 2 should be compared with experimental statistics derived solely from wave heights and not as is frequently the case against statistics derived from trough to height values.

***3.2 The Conditional Probabilities:***

The conditional probabilities required for estimating the probability of a run of small waves that constitute a quiescent period can be obtained from equation 2. The family of marginal and conditional probabilities that are relevant are then given by:

with obviously:











It can then be readily shown that:



and directly or via Bayes theorem that:



***3.3 Longuet-Higgins Approach:*** For the case of a run of large waves, which was the main topic of practical interest at the time, Longuet-Higgins, (Longuet-Higigns, 1984), started by assuming with absolute certainty that a wave occurred where, . The probability that the next wave also exhibited the property, , was then given by the conditional probability, . A run of , , such large waves was terminated by a small wave, the conditional probability of a large wave being followed by a small wave being, . Given the approximate independence of wave peaks beyond three half periods, (Wilson, 1972, Rye, 1974, Siefert, 1974), this allowed consecutive pair probabilities to be treated as statistically independent producing an expression for the overall probability, , of a run of , , large waves which after a little manipulation has the form:



Kimura’s Markov chain approach, (Kimura, 1980) Ref[7], employed the same assumptions and lead to the same result.

***3.3 Quiescent Periods:***

However the present authors propose that the assumptions underpinning a quiescent period are somewhat different to those employed by Longeut Higgins and by Kimura. Clearly a quiescent period of, , small wave peaks is bounded at each end by large waves. Thus the authors considered it necessary to first determine the probability of the starting wave being large, i.e., . In order to employ conditional probabilities (which require a certain first event) to estimate the probability of a large wave followed by a small wave then the conditional probability of the event comprising a large wave followed by a small wave is, . The remainder of the argument follows that adopted by Longuet-Higgins, (Longuet-Higgins, 1984), but in this case the probability, , of a quiescent period of , , small waves bounded at each end by a large wave has the form:



which is readily shown to have the form:



An outcome of simply starting with a small (or large) wave as in, (Longuet-Higgins, 1984), is that this does not exclude the possibility that the precursor to this wave was similarly small (or large) and so on back in time. It is thus vital that the quiescent period starts with a large wave whose probability of occurrence must be incorporated.

***3.3.1 Cumulative Quiescence:***

The practical quantity of interest to users wishing to exploit quiescent periods is not the probability of a given quiescent period of a specified length but the cumulative probability, , of all quiescent periods equal to and longer than some value minimum value, , i.e.:



which using equation 16 evaluates to:



***3.3.2 Quiescence Duration:***

Further to the issue of user requirements, applications will mainly require the time duration of quiescence rather than the number of consecutive quiescent wave peaks. The simplest approach to this, which is almost certainly adequate given the present aims, is to multiply the run length, , by an appropriate average period measure, such as, the mean zero up-crossing period, .

A somewhat more sophisticated approach, if deemed to be justified, is to compute a measure of the mean wave period, , for wave heights over the restricted range, , then to multiply this by the run length, , corresponding to the particular , , value of interest. Given that the authors have employed the Longuet-Higgins narrow band model in determining the runs length behaviour it was natural to use the joint wave height wave /wave period distribution developed by Longuet-Higgins from the same assumptions, (Longuet-Higgins, 1983).

The joint density function,  between wave height, , (zero to peak) and wave period,  , is then given by:



where the scaling constant, , is given by:  .

The mean wave period, , for wave heights up to, , derived from equation 20 is unbounded however the mean zero up-crossing period, , is bounded and given by:

 

Note that some care must be taken when using equation 21 because .

However as indicated above it is the authors view that for the purposes of the simple QPP operational planning tool, which is the main focus of this report, the above level of sophistication and computational cost involved is not justified. As stated the far simpler option is to merely multiply the minimum run length, , of interest whose probability is given by equation 19, by a measure of the mean wave period such as, , or,  .

***3.4 The Narrow Band Restriction and Quiescent Period Prediction:***

Both the envelope and the Markov chain approach use Rice’s (Rice, 1944, Rice, 1958), conditional probability distribution which is fundamentally a narrow band approximation, where it is required that strictly speaking the bandwidth parameter, , or at least, , is significantly less than unity. However as indicated previously Arhan, (Arhan, 1978), does present arguments to mitigate the effects of this restriction.

Fortunately quiescent period prediction (exploiting deterministic sea wave prediction to extend operational envelopes) is primarily of interest in high sea states where the wind speeds in themselves are not so excessive as to by themselves prohibit operations. Such conditions often represent large developing or well developed swell seas created by remote storm systems under which conditions, , is typically modest, (, for Pierson Moskowitz) if not actually strictly, .

**4. Using Spectral Parameters for Wave-Wave Correlation.**

***4.1 Wave by Wave Correlation Coefficient:***

By employing either simulations or actual wave data the wave correlation parameter, , defined as:



wan be determined, where, , are wave heights (mean to peak) of which, , is the mean.

can be obtained. This was shown to be related to the parameter, , in equation 2 by Uhlenbeck, (Uhlenbeck, 1943), and by Middleton, (Middleton, 1960):

 

in which, , and , are complete elliptic integrals of the first and second kind respectively.

The inconvenience of having to use equation 21 to obtain, , from, , can be avoided by recognising that unbiased estimates of the parameter,  , in equation 2 can obtained directly from the power density spectrum, . This is achieved by using a discrete form of the Weiner-Khinchin theorem.

A number of authors have discussed this approach, (Longuet\_higgins, 1984, Battjes, 1984, Arhan, 1978), from slightly different, but quantitatively equivalent, perspectives. Battjes, (Battjes, 1984), obtains,  , directly from the magnitude of the normalised autocorrelation function, , derived from the wave spectrum evaluated at a lag value given by the mean zero crossing period, . This exploits the fact that, .

Thus:



The present authors will follow , Battjes, (Battjes, 1984), noting that: , the parameters,and  being the zeroth and second moments of the energy spectrum respectively.

Longuet Higgins, (Longuet-Higgins, 1984), also derives, , via a normalised autocorrelation function which in this case is of the wave envelope. The lag value employed was,.

Specifically:



where:

 

and:



where, , is the centre frequency of the narrow band spectrum which physically is interpreted as the angular frequency at which the envelope of wave heights is modulated.

Now provided, , can be chosen such that, , the approaches of Battjes and Longuette Higgens become mathematically equivalent. In this regard an important practical point here is that for many commonly used wave spectral models the two mean wave period measures, , and,, are numerically very similar, e.g., for the Pierson Moskowitz spectrum, . So given that Rice’s narrow band model, (Rice, 1944, Rice, 1958), only approximately applies to wave spectra to within the anticipated accuracy of the approach it is probably equally legitimate to use either value for, , the lag value at which to evaluate the normalised envelope auto-correlation function in the estimation of,.

***4.2 Power Spectral Density Functions:***

This inverse Fourier approach to estimating, , is attractive because the statistical properties of sea waves are typically couched in terms of various classes of wave spectral model forms, e.g., Neumann, Pearson Moskowitz, Jonswap etc. It is very important to note at this point that while it is common parlance in oceanographic research to either implicitly or explicitly refer to all wave spectral models as wave power spectra this is not correct. Apart from the obvious factor of 2 difference between these two quantities the important point is that the power density spectrum, , is an even function of, , and many spectral models, for example one of the most commonly used forms, the Pierson Moskowitz, spectrum is an odd function of, .

It is the property of, , being an even function that forces the use of the one-sided trigonometric integrals in this context where in fact the true Fourier integral based definition of the power spectral density constituting the Weiner-Khinchin theorem spans, . What is well appreciated is that these commonly used spectral forms are with very limited exceptions curve fits and what is more the moments of such objects are in general only bounded up to a certain order.

Technically the solution to this problem is very simple and merely requires scaling spectral forms exhibiting odd, , symmetry by the function, , which evaluates as, , depending upon the sign of the angular frequency, .

Hence one can say that when treated appropriately it is possible to employ the “one sided form” of the Weiner Khinchin theorem to provide an unbiased estimator for, , that is given by:

,

where given the assumption that, , the function, , evaluated at a lag of,, can be either the auto-correlation function of the wave elevations or of the wave envelope.

As stated the authors will follow Battjes, Ref[16], and employ,, corresponding to the mean zero-crossing wave period.

Illustrative examples of,, are: for the Neumann spectrum,  , and for the Pierson Moskowitz form, , where in each case the parameter, , is a wind speed value at a given height above sea level.

**5. 0 A Significant General Property of Quiescence for a Range of Sea Types:**

Consider the following spectral form:



 This encompasses the power spectral density of a wide range of sea states including the commonly used Neumann, and Peirson Moskowitz/Bretschnieder, forms. In these spectral models the constants, , and, , are typically functions of wind-speed over the region where the waves are formed but are independent of angular frequency, . This will also be assumed to be the true in the general case of equation, 26. The only restriction on the parameters, , is that they are real and positive.

Given equation 26 the moments,, and, , in terms of angular frequency (radian/sec not Hz), are given by:

 

and:



 Using the change of variable:  , the mean zero up-crossing wave period, , can be written as:

Now recalling that:



and using the previous change of variables:



Substituting for, , gives:



Thus equation 32 means that, , is independent of the parameters,  , and, , and is hence independent of wind-speed and thus is independent of the absolute scale of the wave height. Now the bi-variate probability distribution, , given by equation 2, only depends upon: (i) , and, (ii) upon the ratio of wave height to root mean square wave height. Furthermore the absolute dependences on wave height associated with the integrals over, , and over the marginal probability density cancels because only the ratio of such integrals is used. Consequently there is no dependence of the probability of a run of, , small waves on the absolute level of the prevailing wave height. Only the functional “form” of the spectrum through the parameters, , and ,, affects this.

Now as stated in section 3.3.2 it is the time duration of the “runs of small waves” that are most likely to interest users, rather than the number of waves,  , or rather the cumulative number of runs of at least this length. Hence as stated previously the number of consecutive small waves in a run must be multiplied by the corresponding mean wave period, and this does in general vary with the absolute level of wave height.

Summarising, for the broad class of wave spectral object as defined by equation 24 the number of waves constituting a “run of small waves” (as opposed to its duration) only depends upon the “form” of the wave spectra and not upon the absolute wave height. However it must be born in mind that this finding only holds while the narrow band approximation holds legitimising the results of Rice.

This analysis has been undertaken using,  , however the same independence on absolute wave height can easily be shown by repeating the analysis if, , is used rather than,  .

**6.0 Semi-Analytic Evaluation of Quiescent Period Probabilities.**

Given that the well known parameterised spectral model forms are a commonly employed for both describing and forecasting sea conditions the ability to estimate the *runs* properties (in this case of QPs) of different categories of seas directly from such model spectra in the manner described is of considerable value in forward planning of maritime operations. This can be achieved for all power density spectra if the integrals involved in determining the *low runs* probability are evaluated numerically. A companion work will describes such a numerical approach and will present examples using power spectral densities obtained from both simulations and from actual sea trials data.

However for a set of interesting commonly met special cases encompassed by the form given in equation 26 , ( including the: Neumann, Peirson Moskowitz and Bretschnider), somewhat surprisingly it also is possible to obtain semi-analytic results which significantly reduces computational costs.

Such a semi-Analytic approach will be developed for the process described in sections 3 and 4 for spectra of the type, . This will be illustrated for the special cases of the Neumann and the Pierson and Moskovitz/ Bretscneider spectra. The early Neumann form is included because unlike the much more commonly cited Pierson Moskovitz /Bretscneider spectra it is an even function of frequency and thus is a true power density spectrum.

Given that the key parameter, , is derived from auto-correlation functions it is sensible to explore the literature on the analytic treatment of these w.r.t., wave spectral models. Expressions for the autocorrelation functions for both Neumann and the Pierson-Moskowitz wave spectra have been obtained by Latta and Balie, (Latta, 1969). Their method for the Neumann case involved an exponential operator technique applied via the Mellin transform and the Pierson and Moskovitz case was approached by setting up an associated differential equation. These methods were indirect, extremely intensive and very case specific. In contrast the frequency domain approach employed here very readily yields analytic expressions for the respective envelope auto-correlations in terms of standard special functions. Specifically the wave to wave correlation coefficient,, appearing in equation 2 can be evaluated analytically in terms of hypergeometric and MeijerG special functions which are standard function calls in symbolic language packages such as MAPLE and MATHEMATICA. This means that one layer of numeric integration in the runs evaluation process can be avoided making it realistic to produce computationally cheap user packages designed to provide advice on quiescence conditions over a wide range of sea types.

Given that the nomenclature of such special functions is rather cumbersome and these forms do not fall within most users mathematical experience the functional details are omitted and it is simply stated here that for integer values of, , and, , the required integrals of the type:

, and, , can be considered to be standard forms within the repertoire of typical symbolic language packages.

***6.1 Relevant Parameters of the Neumann, Pierson Moskowitz/Bretschneider Spectra:***

The Neumann wave spectral model, , which is a true power density spectrum, is given by:



where, , and, , in which, , is the gravitational constant, and, , the wind velocity 7.5m above the sea surface. The first and second moments of this spectrum are given respectively by:

  , and,  . The corresponding mean zero up-crossing period is given by: .

The Pierson Moskowitz form, which is not a true power spectral density function, is:



where, , and,  , where, g, is the gravitational constant and, , is the wind velocity 19.5m above the sea surface. The two moments, , and, , are given by:

, and, . The corresponding mean zero up-crossing period is given by: .

From the perspective of QP analysis the Bretschneider form is mathematically equivalent to Pierson Moskowitz, the fact that the constants, , and, , are chosen independently does not affect the results. Thus the value of the wave wave correlation parameter,  , for these two forms is the same.

**7.0 Illustrations**

The analysis described is applied in a comparison against results obtained from a linear sea wave simulation. The quantity of interest was the probability that under specific sea conditions (with a given mean wave period) what is the occurrence of quiescent periods of at least, , consecutive wave heights of values bounded above by, . Given the ubiquity of the Pierson Moskowitz/Brettschnieder spectral forms this case is used as an illustration of the approach, specifically using a wind speed of 19 at the standard reference height of 19.5above sea level. The phases were sampled from a uniform distribution over,, and the magnitudes of the components were obtained by randomly sampling from a Rayleigh distribution whose variance was provided by value of, , at each angular frequency value used. The latter is especially important, (Tucker, 1984), in order to obtain simulations with the correct runs properties rather than being merely “sea like”. In order to obtain confidence in the low probability events the simulations were run over an equivalent real time of 260,000 seconds using a sampling time-step of 1 second. The total number of zero up-crossings was approximately 28,000. The frequency band employed was, , and the deep water dispersion, , relationship was assumed.

Figure 1 compares the results of the present approach against the simulations. The parameter varied between each curve is the critical wave height value, . The results presented cover the range, . The minimum number of wave heights in the run, , was employed in figure 1 rather than the practically more interesting quiescence period duration because as shown in section 5 for two closely linked reasons:

1. For this class of wave spectra the cumulative probability of a value, , predicted by the analysis is independent of absolute wave height and as will be shown in figure 2 that to the anticipated level of accuracy this is also true of the simulation results.
2. The standard bandwidth width measure, , is also wind speed independent, (Tucker, 2001). Thus the extent to which the Moskowitz/Bretschnieder satisfies Rice’s, (Rice, 1944, 1958), narrow band requirement. Hence the validity of the results based upon this is assumption should be independent of wind-speed and thus of absolute wave height.

Hence figure 1 is a generic plot for the Pierson Moskowiz/Breteschieder case as a whole, for all wind speeds (and hence the absolute scale of wave heights). Conversion to a dependence on quiescence duration for a particular wind speed, , is simply a matter of multiplying the, , values by the appropriate values of, .

Cumulative probability of a value, , probability values were also derived from the simulations under conditions of interest to QPP. The results are presented in figure 2 for two cases, i.e.,  values of 3.6m and 6.7m. The results plotted are of the fractional change in the probability. Given that the standard deviation of these changes is less than 10 percent (well within the anticipated precision of the methodology) these results bear out the sea state independence for the Pierson Moskowitz form and it seems reasonable to assume that the same is likely to be true for the whole class of spectral forms given by equation 26.

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Figure 1. Plots of the natural logarithm of the probability of a given number of runs within a quiescent period. The parameter, h\_o, is equal to , where, , is the root mean square sea surface elevation. The data derived from the wave simulations described is denoted by discrete symbols while the predictions correspond to solid lines. 

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Figure 2. Scatter diagram of the fractional change in the probability of quiescence when the wind velocity increases from 19m/sec to 22m/sec, corresponding to M0 rising from 3.7m to 6.7m. The range of critical wave heights over which the data is determined is equivalent to that in figure 1.

**8.0 Discussion.** The results presented in figure 1 and figure 2 show a reasonable agreement between the simulations and the semi-analytic calculations using the present approach and demonstrate that under conditions where QPP might be employed the runs statistics are only weakly sensitive to absolute sea state. As might be anticipated the fits between theory and the simulations become increasingly worse as the number of peaks present in the quiescent period increases. The type of conditions where deterministic QPP might expect to be used (aircraft or small boat recovery and cargo transfers) would be typified by large well developed swell seas produced by remote storms with a dominant wave period of 10 to 12 sec. Typical prediction times of 1 minute would be useful, which in the above context is approximately 5 wave periods.

To set this in context the results in figure 1 indicated that quiescence probability of 5 wave periods can be reasonably well estimated provided the critical wave heights, , is given by, , i.e., . For comparison if traditional wave statistics were used to decide if an operation deemed safe for waves of height, , under prevailing conditions then one might expect that at least, . Thus under the conditions explored above deterministic QPP would be of considerable operational benefit and the present semi-analytic approach would provide a useful and reliable forward planning tool.

Of considerable practical importance is the fact that theoretical estimates tend to underestimate the probability of quiescence, the converse being clearly operationally dangerous.

Thus given that a goal of the present work was to provide the technical basis for a computationally low cost planning tool for applications using quiescent periods some success has been achieved. Of special relevance is regard are (i) the independence of the whole class of wave spectra of the type given in equation 26 and (ii) all that is required to obtain quiescent period durations is to scale the , , values by the appropriate wave period (which is sea state dependent).

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