Tests of Asset Pricing with Time-Varying Factor Loads*

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Abstract

This paper proposes an empirical asset pricing test based on the homogeneity of the factor risk premia across risky assets. Factor loadings are considered to be dynamic and estimated from data at higher frequencies. The factor risk premia are obtained as estimates from time series regressions applied to each risky asset. We propose Swamy-type tests robust to the presence of generated regressors and dependence between the pricing errors to assess the homogeneity of the factor risk premia and the zero intercept hypothesis. An application to US industry portfolios shows overwhelming evidence rejecting the CAPM, and the three and five factor models developed by Fama and French (1993, 2015). In particular, we reject the null hypotheses of a zero intercept, homogeneous factor risk premia across risky assets and the joint test involving both hypotheses.

Keywords: Asset pricing, realized measures, slope homogeneity tests, stochastic discount factor, two-pass regressions.

JEL classification codes: E22, G11, G12, C12.

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1 Introduction

Early asset pricing models exploit the covariance between the return on a financial asset and the market portfolio. In this category we find the CAPM of Sharpe (1964) and Lintner (1965), based on the mean-variance efficiency of the market portfolio, Black (1972)'s version of the CAPM, characterized by the zero-beta portfolio return, the cross-sectional CAPM of Fama and MacBeth (1973) or the intertemporal CAPM of Merton (1973). Extensions of these models incorporate other risk factors beyond the market portfolio with power to explain the cross-section of asset returns. Seminal contributions in this literature are Ferson, Kandel, and Stambaugh (1987), Fama and French (1993, 2015) and Carhart (1997) specifications, among many others. All of these models can be regarded as special cases of a unifying framework developed in Harrison and Kreps (1979), which states that, under no arbitrage, there exists a single stochastic discount factor (SDF) for pricing all risky assets in a market. Asset pricing theories based on the existence of a common stochastic discount factor are exploited in Cochrane (1996), Lettau and Ludvigson (2001), Kogan and Papanikolaou (2013) and Penaranda and Sentana (2015), among many others.

The asset pricing theories developed by Sharpe (1964), Lintner (1965), Fama and MacBeth (1973), Ross (1976) and subsequent literature have been extensively tested in the finance literature by the so-called 'traditional methodologies.' These methodologies are characterized by, given a particular data-generating process for the returns, testing the restrictions imposed by the corresponding asset pricing model as parametric constraints on the return-generating process. It has been standard in the empirical asset pricing literature to validate these asset pricing specifications, characterized by tradable common factors such as portfolio returns, by applying simple hypothesis tests for the statistical significance of the intercept of asset pricing equations, see e.g., early work by Sharpe (1964) and Lintner (1965) on the CAPM, Ross (1976) for multifactor asset pricing models, and more recent contributions such as Fama and French (1993, 2015). For asset pricing models characterized by non-tradable factors or by the absence of a risk-free asset the testable restrictions imposed by the model are different. In Black (1972)'s zero-beta portfolio CAPM, the correct specification of the asset pricing model implies a specific form of the unrestricted intercept that depends on the expected return on the zero-beta portfolio. More generally, Fama and MacBeth (1973) propose joint tests for the correct specification of the asset pricing model. The null hypothesis in these tests not only considers the statistical significance of the intercept but also tests the equality between the factors' risk premia and the expected value of the pricing factors using estimates of the latter two quantities. Ferson, Kandel, and Stambaugh (1987) propose tests that focus on assessing the contribution of non-market risk factors in addition to a market portfolio return for pricing a cross-section of time-varying expected returns.

Recent empirical asset pricing tests have also been developed based on the more general SDF theory. These studies have been focused on testing the pricing restrictions in terms of the SDF model rather than on restrictions imposed by specific asset pricing models. Both the traditional and SDF methodologies assume the existence of a common factor risk premia for pricing the entire cross-section of risky assets that precludes the presence of arbitrage opportunities. These factor risk premia entail a discount rate associated to each pricing factor common across risky assets. This argument is well understood in the theoretical asset pricing literature, however, to the best of our knowledge, the empirical asset pricing literature has not developed tests to statistically assess the homogeneity of the factor risk premia for the cross-section of risky assets.

The main contribution of this paper is to propose the use of statistical tests to assess the homogeneity of the factor risk premia across risky assets and with it the correct specification of asset pricing models. In contrast to the related asset pricing literature, we do not impose the factor risk premia to be common across risky assets, instead, these parameters are estimated from time series regressions between the excess returns on risky assets and the corresponding risk factor loadings. By doing so, we allow for the possibility of heterogeneity in the factor risk premia across assets and develop a test to statistically assess this condition. We focus on a static set of factor risk premia, but the proposed methodology can also be extended to dynamic factor risk premia as discussed in Lettau and Ludvigson (2001), and more recently, Nagel and Singleton (2010) and Gagliardini, Ossola, and Scaillet (2016). To make the model tractable, the factor loadings that measure the quantity of risk born by each asset need to be dynamic, otherwise the factor risk premia and the intercepts of the corresponding asset pricing regression equations cannot be identified.

The presence of dynamics in the factor loadings of asset pricing equations is consistent with the recent literature in financial economics that suggests that market betas may vary with conditioning variables, see Hansen and Richard (1987), Jagannathan and Wang (1996) and Wang (2003). We are interested in obtaining low frequency (quarterly) estimates of the factor loadings from high frequency (daily) data on excess returns on the risky assets and common factors. In particular, we exploit realized measures recently developed in statistics by Andersen, Bollerslev, Diebold, and Ebens (2001), Andersen, Bollerslev, Diebold, and Labys (2001), Andersen, Bollerslev, Diebold, and Labys (2003) and Barndorff-Nielsen and Shephard (2004), among many others. The application of this strategy to obtain realized betas has also been proposed in Andersen, Bollerslev, Diebold, and Wu (2006), among a few others. The asset-specific estimates of the factor risk premia obtained from time series regressions are compared by means of a statistical test with a pooled estimate of the factor risk premia obtained from panel data methods that is common across assets by construction.

The homogeneity hypothesis for the factor risk premia constitutes the basis for our test of correct specification of the asset pricing equation. We exploit the econometric literature on panel data and adapt slope homogeneity tests developed by Ando and Bai (2015) to an asset pricing context. These tests are very well suited for testing the restrictions imposed by our asset pricing framework under the presence of cross-sectional dependence among the pricing errors. In particular, we test for the absence of intercept and homogeneity of slopes of N asset pricing regression equations. We contribute to the econometric literature on tests of homogeneity for the parameters of regression models by deriving the asymptotic distribution of Ando and Bai (2015)'s test under the presence of generated regressors, in our case characterized by time series of estimated factor loadings. For comparison purposes, we also present the analogue slope homogeneity test under cross-sectional independence similar in spirit to the Swamy type tests introduced in Pesaran and Yamagata (2008), and derive its distribution under the presence of generated regressors.

In an online appendix, we present a comprehensive Monte-Carlo study with the performance of these tests in finite samples. The results show adequate performance of both proposed tests under the presence of generated regressors, with correct empirical size and high power against selected alternatives.

We apply the tests to a comprehensive empirical application to U.S. quarterly data on industry portfolio returns spanning the period July 1963 to December 2014. We illustrate our testing procedure for three popular asset pricing models developed in the literature, namely, the CAPM model of Sharpe (1964) and Lintner (1965), and the three and five factor models of Fama and French (1993, 2015). The conclusions of our empirical analysis reject the null hypothesis of correct specification of these models for different hypotheses and test statistics. In particular, we reject the null hypotheses of a zero intercept, homogeneous factor risk premia across risky assets and the joint test involving both hypotheses. These results are robust to the evaluation period and choice of test statistic and provide ample evidence of misspecification of different asset pricing factor models that use tradable assets as common factors when the factor loadings are dynamic. Our results are in line with the findings for the four-factor model obtained in Gagliardini, Ossola, and Scaillet (2016). These authors reject the suitability of this model for a large cross-section of assets and also for industry portfolios in a conditional asset pricing factor setting.

The rest of the paper is organized as follows. Section 2 introduces the asset pricing framework, derives the testable asset pricing equations and outlines the test. Section 3 deploys the econometric methodology and is divided in three stages: first, estimation of the dynamic factor loadings; second, time series regressions to obtain asset-specific factor risk premia; and third, we propose our asset pricing test. This is a joint test based on the homogeneity of the factor risk premia and the null intercept hypothesis. We discuss different versions of this test under the presence of cross-correlation between the pricing errors following Ando and Bai (2015) and under the assumption of absence of such correlation following Pesaran and Yamagata (2008). In Section 4, we apply this testing methodology to assess the Sharpe (1964) CAPM and Fama and French (1993, 2015) asset pricing models with quarterly data of U.S. industry portfolio returns spanning the period July 1963 to December 2014. Section 5 concludes the paper. A separate online appendix presents different extensions of the results. Appendix A collects the mathematical proofs of the results presented in the paper. Appendix B contains an extension of our tests for exact asset pricing factor models. In Appendix C we develop a detailed Monte-Carlo exercise showing the finite-sample performance of the tests.

2 Asset pricing tests

This section is divided into two blocks. First, we provide a brief review of linear asset pricing models. In the second block we introduce the null hypothesis of homogeneity of the factor risk premia for asset pricing tests.

2.1 A review of linear asset pricing factor models

Traditional asset pricing linear factor models propose a return-generating process for the excess returns, $r^e_{i,t+1}$. A typical example is a K-factor model

$$r^e_{i,t+1} = \alpha_i + \beta_i^\top f_{t+1} + e_{i,t+1}, \tag{2.1}$$

where f_{t+1} is the realized value of a K-vector of systematic risk factors at time $t+1$. These common factors have unit variance and are uncorrelated across factors, i.e., $\mathrm{Cov}(f_j, f_k) = 0$ for $j, k = 1, \ldots, K$, α_i is the intercept of the regression model and $\beta_i = \mathrm{Cov}[r^e_{i,t+1}, f_{t+1}]$ are the factor loadings of the excess returns on the common factors. The random variable $e_{i,t+1}$ is the idiosyncratic risk of the asset that satisfies $\mathrm{E}[e_{i,t+1}|f_{t+1}] = 0$. We hereafter use $\mathrm{E}[\cdot]$, $\mathrm{Cov}[\cdot,\cdot]$ and $\mathrm{Var}[\cdot]$ to denote the expected value, covariance and variance of random variables and $\mathrm{E}_t[\cdot]$, $\mathrm{Cov}_t[\cdot,\cdot]$ and $\mathrm{Var}_t[\cdot]$ to denote the corresponding conditional statistical moments.

No-arbitrage arguments (Ross (1976), Al-Najjar (1998)) show that for tradable common factors the corresponding beta asset pricing model is

$$\mathrm{E}[r^e_{i,t+1}] = \beta_i^\top \lambda, \text{ for } i = 1, \ldots, N, \tag{2.2}$$

with λ a vector of factor risk premia that captures the price of risk corresponding to the pricing factors f_{t+1}, and β_i the factor loadings that capture the quantity of risk corresponding to each risky asset. The asset pricing model (2.2) entails the testable restrictions

$$\alpha_i = 0 \text{ for } i = 1, \ldots, N,$$

and

$$\lambda = \mathrm{E}[f_{t+1}].$$

The main feature of this asset pricing theory is the existence of a common parameter λ that prices the cross-section of risky assets. This assumption along with the restriction of a zero excess return for those risky assets uncorrelated with the common factors entail the absence of arbitrage opportunities in asset pricing models characterized by tradable common factors. Most of the empirical asset pricing literature focuses on evaluating the intercept restriction, namely, $\alpha_i = 0$ for all $i = 1, \ldots, N$, in the time series regression (2.1), and assumes that the factor risk premia λ is homogeneous across risky assets, see early work by Sharpe (1964) and Lintner (1965) on the CAPM, Ross (1976) for multifactor asset pricing models, and Fama and French (1993, 2015) on univariate versions of this restriction. A seminal contribution extending the restriction to a multivariate setting is Gibbons, Ross, and Shanken (1989). These authors propose a Wald type test for the joint null intercept hypothesis. A recent contribution by Pesaran and Yamagata (2018) propose Wald type tests robust to the presence of correlation in the pricing errors.

Alternative asset pricing tests for the null intercept hypothesis use information on the cross-section of risky assets. These tests are based on the so-called two-pass regressions and focus on testing the significance of the intercept of a cross-sectional regression between the expected excess return on the risky asset and the corresponding estimated beta factor loadings. A seminal example is Fama and MacBeth (1973) and, more recently, for conditional linear factor pricing models, see Gagliardini, Ossola, and Scaillet (2016). The assumption that the factor risk premia λ are common across risky assets implies that the discount rates associated to each pricing factor are the same across assets. This argument is well understood in the theoretical asset pricing literature, however, to the best of our knowledge, the empirical asset pricing literature has not developed tests to statistically assess this condition. For example, two-pass regressions are by construction not able to test this condition as this testing methodology assumes a common price of risk parameter across risky assets.

The main contribution of this paper is to propose the use of statistical tests to assess the joint hypothesis of a null intercept and homogeneity of the factor risk premia for a cross-section of asset pricing equations. By doing so, we provide a statistical method to test for general forms of misspecification in asset pricing equations that are reflected in heterogeneity across factor risk premia.

The factor model (2.1) entails the representation

$$r_{i,t+1}^{e} - \mathrm{E}[r_{i,t+1}^{e}] = \beta_i^{\top} \widetilde{f}_{t+1} + e_{i,t+1},\ i = 1, \ldots, N, \tag{2.3}$$

with $\widetilde{f}_{t+1} = f_{t+1} - \mathrm{E}[f_{t+1}]$ being the demeaned common factors, and recalling that $\mathrm{E}[e_{i,t+1}] = 0$. Under the correct specification of the beta asset pricing model (2.2), expression (2.3) can be written as

$$r_{i,t+1}^{e} = \beta_i^{\top} \lambda + \beta_i^{\top} \widetilde{f}_{t+1} + e_{i,t+1},\ i = 1, \ldots, N. \tag{2.4}$$

Equation (2.4) presents the time series counterpart of the cross-sectional two-pass regression models introduced in Fama and MacBeth (1973) for asset pricing. In this paper, to test the beta asset pricing model we assume a generalized form of (2.2) given by

$$\mathrm{E}[r_{i,t+1}^{e}] = \alpha_i + \beta_i^{\top} \lambda_i. \tag{2.5}$$

In particular, we extend standard asset pricing tests based on the null intercept hypothesis, see Gibbons, Ross, and Shanken (1989). For each pricing factor $k = 1, \ldots, K$, we add the testable restriction $\lambda_{1k} = \lambda_{2k} = \ldots = \lambda_{Nk} = \lambda_k$, to the standard null intercept condition. More formally, our null hypothesis to assess the correct specification of the asset pricing model is

$$H_0^{\alpha,\lambda} : \alpha_1 = \ldots = \alpha_N = 0 \text{ and } \lambda_1 = \ldots = \lambda_N = \lambda, \tag{2.6}$$

for λ a vector of parameters, against the alternative $H_1^{\alpha,\lambda} : \alpha_i \neq 0$ or $\lambda_i \neq \lambda_j$, for some $i, j \in 1, \ldots, N$. For notational convenience, we define $\eta_i = (\alpha_i, \lambda_i^{\top})^{\top}$ and $\eta_0 = (0, \lambda^{\top})^{\top}$, such that the null hypothesis can then be simply written as $H_0^{\alpha,\lambda} : \eta_i = \eta_0$, for all $i = 1, \ldots, N$, against $H_1^{\alpha,\lambda} : \eta_i \neq \eta_j$ for some $i, j \in 1, \ldots, N$. For asset pricing models characterized by non-tradable factors the intercept of the asset pricing regression model can be different from zero and the null hypothesis of interest only concerns the homogeneity of the factor risk premia.

For comparison purposes, we define the test for the joint null intercept hypothesis as

$$H_0^{\alpha} : \alpha_i = 0, \tag{2.7}$$

for all $i = 1, \ldots, N$, against the alternative $H_1^{\alpha} : \alpha_i \neq 0$, for some $i, j \in 1, \ldots, N$. We also introduce a hypothesis test for the homogeneity of the factor risk premia as

$$H_0^{\lambda} : \lambda_i = \lambda, \tag{2.8}$$

for all $i = 1, \ldots, N$, against the alternative $H_1^{\lambda} : \lambda_i \neq \lambda_j$, for some $i, j \in 1, \ldots, N$.

2.2 Asset pricing equation for a known factor structure

Under this generalized expression for the expected excess returns on risky assets using a known factor error structure model the relevant time series regression equation is the following:

$$\begin{aligned} r_{i,t+1}^e &= \alpha_i + \beta_i^\top \lambda_i + u_{i,t+1}, \\ u_{i,t+1} &= \beta_i^\top \widetilde{f}_{t+1} + e_{i,t+1}. \end{aligned} \tag{2.9}$$

The error term $u_{i,t+1}$ has zero mean but exhibits cross-sectional correlation for $i = 1, \ldots, N$, due to the presence of the demeaned common factors $\widetilde{f}_{t+1}$ in the error term.

Note that α_i and λ_i cannot be identified from the regression model (2.9). This identification issue can be solved by replacing the static factor loadings β_i by the dynamic quantity $\beta_{it} = \mathrm{Cov}_t[r_{i,t+1}^e, f_{t+1}]$. This is possible if we assume the common factors to be constructed from tradable assets, see Fama and French (1993) as a seminal example. In this case the dynamic factor loadings β_{it} are unbiased estimators of β_i. More formally, by the law of total covariance, it follows that $\beta_i = \mathrm{E}[\mathrm{Cov}_t(r_{i,t+1}^e, f_{t+1})] + \mathrm{Cov}(\mathrm{E}_t[r_{i,t+1}^e], \mathrm{E}_t[f_{t+1}])$, with $\mathrm{E}_t[\cdot]$ denoting a conditional expectation with respect to the information set available up to time t. For tradable common factors characterized by financial returns on investment portfolios the efficient market hypothesis implies the absence of return predictability and entails the condition $\mathrm{E}_t[f_{t+1}] = c$, with c a constant, such that $\beta_i = \mathrm{E}[\beta_{it}]$. The regression equation (2.9) is replaced by

$$
\begin{aligned}
r^e_{i,t+1} &= \alpha_i + \widehat{\beta}_{it}^\top \lambda_i + \nu_{i,t+1}, \qquad (2.10) \\
\nu_{i,t+1} &= \beta_i^\top \widetilde{f}_{t+1} + \varepsilon_{i,t+1},
\end{aligned}
$$

with $\widehat{\beta}_{it}$ a consistent estimate of β_{it} for $i = 1, \ldots, N$. The sequence of estimates of the factor loadings is interpreted as a set of generated regressors in our empirical asset pricing specification (2.10). Note that the error term u_{it} in (2.9) is replaced by ν_{it} in (2.10). This error term reflects the estimation of β_{it} that introduce additional uncertainty into the model. Moreover, our regression model (2.10) will be estimated using the proposed interactive fixed-effects model of Bai (2009) and Song (2013), and thus it allows for the factor structure in the error term to be correlated with the covariates, i.e., $\widehat{\beta}_{it}^\top$.

2.3 Asset pricing equation for an unknown factor structure

Testing for $H_0^{\alpha,\lambda}$ rather than H_0^α offers important information for building an appropriate empirical asset pricing model. In fact, the definition of a strict factor structure can, in some cases, be sufficiently stringent that it is unlikely that any large asset market has a usefully small number of factors (Chamberlain and Rothschild (1983)). Thus, even if a strict $K-$factor structure existed, K might be so large that a more useful model would be an approximate factor model with fewer factors. In econometric terms this approximate factor model structure entails the omission of relevant common factors in the pricing equation. In this context, H_0^α may lack power for detecting the omission of relevant factors for pricing the cross-section of risky assets if the exposure of the excess risky returns on the omitted factors is weak. A similar observation has been noted by Pesaran and Yamagata (2018) for motivating the presence of cross-correlation in asset pricing equations.

We can augment model (2.1) to consider the following factor model as the correct specification

$$
r^e_{i,t+1} = \alpha_i + \beta_i^\top f_{t+1} + \phi_i^\top h_{t+1} + e_{t+1}, \qquad (2.11)
$$

with h_{t+1} a set of additional common factors omitted from the model and ϕ_i the corresponding factor loadings. Model (2.11) is flexible and allows for common factors omitted from (2.1). Under specification (2.11) and $\mathrm{E}[e_{i,t+1}|f_{t+1}, h_{t+1}] = 0$, the representation in (2.3) becomes

$$r^e_{i,t+1} - \mathrm{E}[r^e_{i,t+1}] = \beta_i^\top \widetilde{f}_{t+1} + \phi_i^\top \widetilde{h}_{t+1} + e_{i,t+1},\ i = 1, \ldots, N, \tag{2.12}$$

with $\widetilde{h}_{t+1} = h_{t+1} - \mathrm{E}[h_{t+1}]$ being the demeaned unobserved common factors, and

$$r^e_{i,t+1} = \alpha_i + \beta_i^\top \lambda_i + \beta_i^{*\top} \widetilde{g}_{t+1} + e_{i,t+1},\ i = 1, \ldots, N, \tag{2.13}$$

where $\widetilde{g}_{t+1} = (\widetilde{f}^\top_{t+1}, h^\top_{t+1})^\top$ and $\beta_i^{*\top} = (\beta_i^\top, \phi_i^\top)$.

Under this generalized expression for the excess returns on risky assets, the relevant time series regression equation is

$$\begin{aligned} r^e_{i,t+1} &= \alpha_i + \hat{\beta}_i^\top \lambda_i + \nu_{i,t+1}, \\ \nu_{i,t+1} &= \beta_i^{*\top} \widetilde{g}_{t+1} + \varepsilon_{i,t+1}. \end{aligned} \tag{2.14}$$

The error term $\nu_{i,t+1}$ has zero mean but exhibits cross-sectional correlation for $i = 1, \ldots, N$ due to the presence of the demeaned common factors $\widetilde{g}_{t+1}$ in the error term. Note that the presence of unknown common factors entails the same empirical representation as the model with a known factor structure, as long as the cross-sectional dependence is properly treated when modeling the error term. In what follows, we employ the regression equation (2.14) in the empirical implementation of our asset pricing tests.

3 Testing Procedures

This section describes the testing procedures used in the paper, which are divided in three stages. First, we describe the computation of the factor loadings. Second, we describe the estimator for the parameters of interest in the construction of the test statistics. We then formalize the null hypothesis, the test statistics, and the corresponding limiting distributions of the tests.

3.1 First Stage: Realized Covariance Measures

Although nonstandard the assumption that β_{it} is dynamic is consistent with recent literature in financial economics. This literature suggests that factor loadings may vary with conditioning variables, see Hansen and Richard (1987), Jagannathan and Wang (1996) and Wang (2003). The first step is the construction of the estimates of the factor loadings β_{it}. This has been a difficult task in the empirical asset pricing literature. Our paper overcomes this problem by using data at higher frequencies to obtain accurate measures of the factor loadings at lower frequencies. We follow Andersen, Bollerslev, Diebold, and Wu (2006) and explicitly allow for continuous evolution of β_{it} over time. The theoretical background for this assumption is the theory of quadratic variation and covariation. The following paragraphs provide an introduction to this theory and define the realized covariance measure that we will use to estimate the dynamics of β_{it}.

Let Δ denote the sampling frequency and $m = 1/\Delta$ be the number of sample observations per period t. We denote the intra-period continuously compounded returns from time $t+h\Delta$ to $t+(h+1)\Delta$ by $R_{t+(h+1)\Delta} = p_{t+(h+1)\Delta} - p_{t+h\Delta}$ with $h = 0, \ldots, m-1$ and $m\Delta = 1$. The corresponding inter-period return is defined as $R_{t+1} = \sum_{h=0}^{m-1} R_{t+(h+1)\Delta}$. For our purposes, it is helpful to consider this return process as a multidimensional process formed by the excess returns r^e_{t+1} on the N individual risky assets and K factor returns f_t. We can do this under the assumption that the common pricing factors f_t are returns on traded assets. Let $R_{t+1} = (r^e_{1,t+1}, \ldots, r^e_{N,t+1}, f_{1,t+1}, \ldots, f_{K,t+1})$; under these conditions, the $(N+K) \times (N+K)$ realized covariance matrix at time $t+1$ is

$$\widehat{\Omega}_{t+1} = \sum_{h=0}^{m-1} R_{t+(h+1)\Delta} R^{\top}_{t+(h+1)\Delta}. \tag{3.1}$$

This matrix is positive definite provided $N + K < m$. Moreover, this covariance measure can be defined over l periods as

$$\widehat{\Omega}^{(l)}_{t+l} = \sum_{j=1}^{l} \widehat{\Omega}_{t+j}.$$

The assumption of no-arbitrage in financial markets also entails a logarithmic $(N+K) \times 1$ price process, p_τ, with $\tau \in [0, T]$, that is in the class of semi-martingales. Then, it has the representation

$$p_\tau = p_0 + A_\tau + M_\tau, \tag{3.2}$$

where A_τ is a predictable drift component of finite variation, and M_τ is a local martingale, and such that $A_0 = 0$ and $M_0 = 0$. A typical example of process within this class is a multivariate continuous-time stochastic volatility diffusion process,

$$dp_\tau = \mu_\tau d\tau + \Omega_\tau dW_\tau, \tag{3.3}$$

where W_τ denotes a standard $N+K$-dimensional Brownian motion, and both the process for the $(N+K)\times(N+K)$ positive definite diffusion matrix, Ω_τ, and the $(N+K)$-dimensional instantaneous drift, μ_τ, are strictly stationary and jointly independent of the W_τ process. The cumulative return process at time $t+1$ associated to (3.3) is $R_{t+1} = p_{t+1} - p_t$. Then, for any partition Π_m of the interval $[t, t+1]$ defined as $\Pi_m = \{t = \tau_0 < \tau_1 < \ldots < \tau_m = t+1\}$, the quadratic covariation (QC) of the return process from time t to $t+1$ is defined as

$$\mathrm{QC}_{t+1} = \mathrm{plim}_{||\Pi_m|| \to 0} \sum_{h=0}^{M-1} R_{\tau_{h+1}} R_{\tau_{h+1}}^\top \text{ as } m \to \infty, \tag{3.4}$$

with plim denoting the limit in probability, and $||\Pi_m|| = \max_{h=0,\ldots,m-1} (\tau_{h+1} - \tau_h)$. This process measures the realized sample path variation of the squared return process. For the process (3.3), it follows that

$$\mathrm{QC}_{t+1} = \int_t^{t+1} \Omega_\tau d\tau,$$

with $\int_t^{t+1} \Omega_\tau d\tau$ the integrated diffusion matrix at time $t+1$.

The main result derived and extended in Andersen, Bollerslev, Diebold, and Ebens (2001), Andersen, Bollerslev, Diebold, and Labys (2001), Andersen, Bollerslev, Diebold, and Labys (2003) and Barndorff-Nielsen and Shephard (2004), is that the realized covariance matrix converges in probability to the quadratic variation measure as the sampling frequency m increases. Mathematically,

$$\widehat{\Omega}_{t+1} \xrightarrow{p} \mathrm{QC}_{t+1}, \text{ as } m \to \infty. \tag{3.5}$$

It is important to note that the process QC_{t+1} is different from the conditional return covariance matrix $\mathrm{Cov}_t[R_{t+1}, R_{t+1}]$. Nevertheless, Andersen, Bollerslev, Diebold, and Labys (2001) show that if the price process is square integrable and satisfies some further regularity conditions on the predictable drift component A_τ in (3.2), it follows that $\mathrm{Cov}_t[R_{t+1}, R_{t+1}] = \mathrm{E}_t[\mathrm{QC}_{t+1}]$. In our asset pricing setting outlined above, we are interested in the submatrix of $\mathrm{Cov}_t[R_{t+1}, R_{t+1}]$ determined by the upper-right block matrix of dimension $N \times K$. The elements of this matrix correspond to the dynamic quantities $\mathrm{Cov}_t[r^e_{i,t+1}, f_{j,t+1}]$ with $i = 1, \ldots, N$ and $j = 1, \ldots, K$, that are identified above as the dynamic factor loadings β_{it}. Hence, we have the identity $\beta_{it} = \mathrm{E}_t[\mathrm{QC}^{ur}_{i,t+1}]$, with $\mathrm{QC}^{ur}_{i,t+1}$ denoting the upper-right block-matrix of QC_{t+1} with the quantities describing the quadratic variation between excess returns and common factors. These vectors can be consistently estimated by the vector $\widehat{\Omega}^{ur}_{i,t+1} = (\omega_{(i,N+1),t+1}, \ldots, \omega_{(i,N+K),t+1})$ obtained from realized estimators computed with data at higher frequencies. Most importantly, note that for a grid sufficiently dense ($m \to \infty$), expression (3.5) shows that we can assume for each time t that

$$\beta_{it} = \mathrm{E}_t[\widehat{\Omega}^{ur}_{i,t+1}]. \tag{3.6}$$

This quantity is unobserved so we need to approximate it by a forecast of β_{it} obtained from a time series model. For simplicity, we propose the following autoregressive process for each element $\omega_{(i,j),t+1}$ of the vector $\widehat{\Omega}^{ur}_{i,t+1}$ with $i = 1, \ldots, N$ and $j = N+1, \ldots, N+K$[1]

$$\omega_{(i,j),t+1} = \delta_{ij,0} + \delta_{ij,1}\omega_{(i,j),t} + v_{ij,t+1}, \tag{3.7}$$

with $\omega_{(i,j),t+1}$ converging to the quadratic covariation between $r^e_{i,t+1}$ and each of the pricing factors

[1]The autoregressive strategy in (3.7) can be improved by constructing estimators of the conditional covariance matrix $\mathrm{Cov}_t[R_{t+1}, R_{t+1}]$ that preserve the symmetric positive definiteness property of covariance matrices, see Noureldin, Shephard, and Sheppard (2012), Golosnoy, Gribisch, and Liesenfeld (2012) and Jin and Maheu (2013). To do this, one has to assume that $\widehat{\Omega}_{t+1}$ conditional on the information available up to time t follows a $N+K$-dimensional central Wishart distribution $W(\nu, S_t/\nu)$ where $\nu > N+K-1$ denotes the degrees of freedom and S_t/ν is a positive definite symmetric scale matrix of order $N \times K$. This assumption defines a conditional autoregressive Wishart model such that $\mathrm{E}_t[\widehat{\Omega}_{t+1}] = S_t$.

$f_{j,t+1}$ for $m \to \infty$. Furthermore, the autoregressive process implies that $\mathrm{E}_t[\omega_{(i,j),t+1}] = \delta_{ij,0} + \delta_{ij,1}\omega_{(i,j),t}$. By construction, see (3.6), $\mathrm{E}_t[\omega_{(i,j),t+1}] = \beta_{ij,t}$ under the assumption that model (3.7) provides a correct specification of the dynamics of each element of the vector $\widehat{\Omega}^{ur}_{i,t+1}$. In this context, a consistent estimator of the time series $\beta_{ij,t}$ is

$$\widehat{\beta}_{ij,t} = \widehat{\mathrm{E}}_t[\omega_{(i,j),t+1}] = \widehat{\delta}_{ij,0} + \widehat{\delta}_{ij,1}\omega_{(i,j),t}, \tag{3.8}$$

with $(\widehat{\delta}_{ij,0}, \widehat{\delta}_{ij,1})$ the OLS parameter estimators of the time series regression equation (3.7).

3.2 Second stage: Estimation of factor risk premia

The observable estimated factor loadings obtained from (3.7) allow us to obtain estimates of the factor risk premia from time series regressions between the excess returns on the risky assets and the estimated dynamic factor loadings. For ease of presentation, we reproduce the notation for the relevant quantities. Let $\eta_i = (\alpha_i, \lambda_i^\top)^\top$ denote the model parameters for $i = 1, \dots, N$, $\widehat{X}_{it} = (1, \widehat{\beta}_{it}^\top)$ with $\widehat{\beta}_{it}$ defined in (3.8), and $X_{it} = (1, \beta_{it}^\top)$ with $\beta_{it} = \mathrm{E}_t[\mathrm{QC}^{ur}_{i,t+1}]$.

We propose the following time series regression equation that mimicks (2.10) for testing the above hypotheses:

$$r^e_{i,t+1} = \widehat{X}_{it}\eta_i + \nu_{i,t+1}, \tag{3.9}$$

with

$$\nu_{i,t+1} = \beta_i^{*\top}\widetilde{g}_{t+1} + \varepsilon_{i,t+1}$$

being the pricing error of the asset pricing equation. The regression model (3.9) exhibits cross-correlation with the rest of asset pricing equations indexed by $i = 1, \dots, N$ due to the presence of observed and unobserved common factors in the pricing errors $\nu_{i,t+1}$. We assume R factors that will be treated as unobserved components and that will determine a factor model. The presence of these factors generates cross-sectional dependence in the test statistic. The error term $\varepsilon_{i,t+1}$ satisfies Assumption C in Ando and Bai (2015), namely, $\mathrm{E}[\varepsilon_{it}] = 0$, $\mathrm{E}[|\varepsilon_{it}|^8] < C < \infty$ for all i and t, ε_{it} and ε_{js} are independent for $i \neq j$ and $t \neq s$. The error term is also independent of the regressors $\widehat{X}_{it}$, parameters η_i and factors $\widetilde{g}_t$.

The quantities η_i, β_i^* and $\widetilde{g}_{t+1}$, for $i = 1, \ldots, N$ and $t = 1, \ldots, T$ are estimated from the following minimization problem:

$$l(\eta_i, \beta_i^*, \widetilde{g}_{t+1}) = \sum_{i=1}^{N} \sum_{t=1}^{T} (r_{i,t+1}^e - \widehat{X}_{it}\eta_i - \beta_i^{*\top} \widetilde{g}_{t+1})^2, \tag{3.10}$$

subject to the normalization $\frac{G^\top G}{T} = I_R$ and $\frac{\beta^{*\top}\beta^*}{N}$ being diagonal, with $G = (\widetilde{g}_1, \ldots, \widetilde{g}_T)^\top$ a $T \times R$ matrix and $\beta^* = (\beta_1^*, \ldots, \beta_N^*)^\top$ a $N \times R$ matrix. For convenience of implementation, we adopt the iterative principal component approach initially proposed by Bai (2009) and extended by Song (2013). This approach decomposes the original estimation problem into two steps: the estimation of the individual coefficients given common factors, and the estimation of the common factors given individual coefficients. Following these authors, we maintain the assumption that the number of factors, i.e., R, is known. The extension to an unknown number of factors under heterogeneous regression coefficients is cumbersome (see Song (2013)) and beyond the scope of this paper.

Bai (2009) and Song (2013) propose a tractable solution to the estimation problem by concentrating out the factor loadings from the objective function (3.10). More specifically, these authors assume that the factor loadings β_i^* satisfy a relationship of the form $\beta^* = (G^\top G)^{-1} G^\top (r_i^e - \widehat{X}_i \widehat{\eta}_i)$, with r_i^e the $T \times 1$ vector representation of the excess returns in (3.9). Then, replacing this expression into (3.10), the minimization problem is equivalent to

$$\max_G \operatorname{tr} \left[G^\top \left(\frac{1}{NT} \sum_{i=1}^{N} (r_i^e - \widehat{X}_i \widehat{\eta}_i)(r_i^e - \widehat{X}_i \widehat{\eta}_i)^\top \right) G \right] \tag{3.11}$$

subject to $\frac{G^\top G}{T} = I_R$, with tr denoting the trace of the matrix. Therefore, the estimator $(\{\widehat{\eta}_i\}_{i=1}^N, \widehat{G})$ with $\widehat{\eta}_i = (\widehat{\alpha}_i, \widehat{\lambda}_i^\top)^\top$ should simultaneously solve a system of nonlinear equations

$$\widehat{\eta}_i = (\widehat{X}_i^\top M_{\widehat{G}} \widehat{X}_i)^{-1} \widehat{X}_i^\top M_{\widehat{G}} r_i^e, \tag{3.12}$$

and

$$\left[\frac{1}{NT} \sum_{i=1}^{N} (r_i^e - \widehat{X}_i \widehat{\eta}_i)(r_i^e - \widehat{X}_i \widehat{\eta}_i)^\top \right] \widehat{G} = \widehat{G} \widehat{V}_{NT}, \tag{3.13}$$

where $\widehat{V}_{NT}$ is a diagonal matrix of R largest eigenvalues corresponding to $\widehat{G}$, and $M_{\widehat{G}} = I_R -$

$\widehat{G}(\widehat{G}^\top\widehat{G})^{-1}\widehat{G}^\top$. The actual estimation procedure can be implemented by iterating each of the two steps in (3.12) and (3.13) until convergence. The unknown factor loadings are obtained as $\widehat{\beta}^* = (\widehat{G}^\top\widehat{G})^{-1}\widehat{G}^\top(r_i^e - \widehat{X}_i\widehat{\eta}_i)$.

The presence of generated regressors $\widehat{X}_{it}$ in (3.9) replacing X_{it} implies that the variance of the vector of parameter estimators $\widehat{\eta}_\cdot = (\widehat{\eta}_1, \ldots, \widehat{\eta}_N)^\top$ needs to be corrected. In what follows, we establish several asymptotic results when m and N, T go to infinity sequentially. The sequential asymptotics is defined as m diverging to infinity first, and then N, T.[2] For a detailed discussion on sequential and simultaneous asymptotics for panel data, see Phillips and Moon (1999, 2000). In particular, we adopt the following notation: $(m, N, T)_{seq} \to \infty$ means that first $m \to \infty$ and then $(N, T) \to \infty$.

We notice that the generated regressors in this paper have two layers of potential estimation errors. First, as equation (3.1) shows, we compute the realized covariance matrix with high-frequency data. Second, as shown in equation (3.7), we employ an autoregressive framework to model the correlation in the elements of that variance. Therefore, the sequential asymptotics helps with the first issue, since by assuming that m diverges first, we obtain consistent estimates for the realized covariance, which are used in the autoregressive fitting, and only have to account for the estimation error of the autoregressive model. We also notice that the sequential asymptotics assumption is plausible in our application since we use daily data to estimate quarterly observations.[3]

First, we derive the variance of the statistic $\sqrt{T}(\widehat{\eta}_\cdot - \eta_\cdot)$, with $\eta_\cdot = (\eta_1, \ldots, \eta_N)^\top$ the true parameters of the model, as this quantity plays a fundamental role for deriving the asset pricing tests. Let S be a $N(K+1) \times N(K+1)$ block-diagonal matrix with elements $S_{ii} = [X_i^\top M_G X_i]/T$, and let L be a $N(K+1) \times N(K+1)$ matrix with elements $L_{ij} = a_{ij}(X_i^\top M_G X_j)/T$ with $a_{ij} = (\beta_i^*)^\top (G^\top G/N)^{-1}\beta_j^*$. Also, let H be a block-diagonal matrix with elements H_i defined as $T \times K$ matrices with columns given by the vectors $Z_{ij}\left(\frac{Z_{ij}^\top Z_{ij}}{T}\right)^{-1}\frac{Z_{ij}^\top v_{ij}}{\sqrt{T}}$ for $i = 1, \ldots, N$ and $j = 1, \ldots, K$; $Z_{ij} = (1, \mathrm{QC}_{(i,j)})$ with $\mathrm{QC}_{(i,j)}$ the (i, j) element of the quadratic covariation matrix QC^{ur} defined

[2] In the definition of the simultaneous asymptotics, m, N and T tend to infinity at the same time.

[3] As noted by an anonymous referee we usually face a bias when using a plugging based on in-fill asymptotics, see for instance Li and Xiu (2016). In our paper, by employing the sequential asymptotic framework we are able to use the consistent estimates of the realized covariances into the autoregressive model, and hence, reduce the estimation error from the first step to only the error produced by using forecasts of the autoregressive model (3.7) without having to rely on a bias-corrected estimator for the second step.

in (3.4) and containing the covariance between the excess returns r_t^e and the factors f_t.

Lemma 3.1. *Under assumptions A-D and GR1-GR3 in the Online Appendix, the asymptotic variance of the quantity* $\sqrt{T}(\widehat{\eta}. - \eta.)$ *is*

$$\left[\left(S - \frac{1}{N}L^\top\right)\right]^{-1} W \left[\left(S - \frac{1}{N}L\right)\right]^{-1}, \tag{3.14}$$

as $(m, N, T)_{seq} \to \infty$*, with* W *a* $N(K+1) \times N(K+1)$ *block-diagonal matrix given by*

$$W = \lim_{T\to\infty} Var\left[\left(\lambda \odot \frac{M_G X}{T}\right)^\top H\right] + \lim_{T\to\infty}\left(\frac{X^\top M_G X}{T}\right) Var(\varepsilon). \tag{3.15}$$

The following lemma proposes a consistent estimator for the asymptotic variance of $\sqrt{T}(\widehat{\eta}. - \eta.)$. To do this, we need a set of assumptions to be satisfied by the vector of unobservable regressors β_{it} and the error term v of the regression equation (3.7). These high level assumptions can be found before Theorem 2 in Pagan (1984). We also need some set of assumptions for the error term ν_{it} in (3.9) limiting the amount of cross-sectional dependence on the pricing errors. These assumptions can be found in Assumptions A to D in Song (2013).

Lemma 3.2. *Under assumptions in Lemma 3.1 and assumptions E–F in the Online Appendix, a consistent estimator of the variance of* $\sqrt{T}(\widehat{\eta}. - \eta.)$ *is*

$$\left[\left(\widehat{S} - \frac{1}{N}\widehat{L}^\top\right)\right]^{-1} \widehat{W} \left[\left(\widehat{S} - \frac{1}{N}\widehat{L}\right)\right]^{-1}, \tag{3.16}$$

where $\widehat{S}$ *be a* $N(K+1)\times N(K+1)$ *block-diagonal matrix with elements* $\widehat{S}_{ii} = (\widehat{X}_i^\top M_{\widehat{G}} \widehat{X}_i)/T$*,* $\widehat{L}$ *be a* $N(K+1)\times N(K+1)$ *matrix with elements* $\widehat{L}_{ij} = \widehat{a}_{ij}(\widehat{X}_i^\top M_{\widehat{G}} \widehat{X}_j)/T$ *and* $\widehat{a}_{ij} = (\hat{\beta}_i^*)^\top (\hat{G}^\top \hat{G}/N)^{-1} \hat{\beta}_j^*$*, and* $\widehat{W}$ *a block-diagonal matrix with elements*

$$\widehat{W}_i = \left(\widehat{\lambda}_i \odot \frac{M_{\widehat{G}}\widehat{X}_i}{T}\right)^\top T^{-1} diag\left(\widehat{H}_i^\top \widehat{H}_i\right)\left(\widehat{\lambda}_i \odot \frac{M_{\widehat{G}}\widehat{X}_i}{T}\right) + \left(\frac{\widehat{X}_i^\top M_{\widehat{G}}\widehat{X}_i}{T}\right)\widehat{\sigma}^2, \tag{3.17}$$

with $\widehat{H}_i$ *defined as* $T \times K$ *matrices with columns given by the vectors* $\widehat{Z}_{ij}\left(\frac{\widehat{Z}_{ij}^\top \widehat{Z}_{ij}}{T}\right)^{-1} \frac{\widehat{Z}_{ij}^\top \widehat{v}_{ij}}{\sqrt{T}}$ *for* $i = 1, \ldots, N$ *and* $j = 1, \ldots, K$*, where* $\widehat{Z}_{ij} = (1, \omega_{(i,j)})$ *with* $\omega_{(i,j)}$ *the elements of the realized* *covariance matrix* $\widehat{\Omega}^{ur}$ *defined in* (3.1) *and* $\widehat{v}_{ij}$ *are the residuals of the regression model* (3.7). *The quantity* $\widehat{\sigma}^2$ *is defined as* $\widehat{\sigma}^2 = \frac{1}{NT-N(K+1)-(N+T)R}\sum_{t=1}^{T}\sum_{i=1}^{N}(r^e_{i,t+1} - \widehat{X}_{it}\widehat{\eta}_i - \widehat{\beta}_i^{*\top}\widehat{g}_{t+1})^2$.

In order to complete the estimation of the model parameters, we introduce a panel data estimator for the factor risk premia λ. In contrast to the above estimators of λ that are idiosyncratic to each risky asset our panel data estimator is common across the N risky assets. More formally,

$$\widehat{\lambda} = \frac{1}{N}\sum_{i=1}^{N}\widehat{\lambda}_i. \tag{3.18}$$

This estimator is an average of the estimators of the factor risk premia obtained from each individual asset pricing regression equation. Our strategy in the next section to test the null hypotheses $H_0^{\alpha,\lambda}$, H_0^{α} and H_0^{λ} is to compare the estimates $\widehat{\eta}_i$ obtained from each individual asset pricing regression equation with the vector $\widehat{\eta} = (0, \widehat{\lambda}^\top)^\top$.

3.3 Third Stage: Homogeneity Tests

Now we consider asset pricing tests with cross-sectional dependence. There are several testing procedures for slope homogeneity available in the literature, see, for example, Pesaran, Smith, and Im (1996), Phillips and Sul (2003), Pesaran and Yamagata (2008), Blomquist and Westerlund (2013), Su and Chen (2013). Ando and Bai (2015) extend these tests by accommodating the presence of cross-sectional correlation between the error terms of the different regression models indexed by $i = 1, \dots, N$. However, to the best of our knowledge there is no available test of slope homogeneity in panel data models that accounts for generated regressors. In the following, we adapt existing tests, namely Pesaran and Yamagata (2008) and Ando and Bai (2015)'s test to our asset pricing context characterized by the presence of estimated factor loadings. We also extend these tests to be suitable for testing the null hypothesis $H_0^{\alpha,\lambda}$ that includes testing for the statistical significance of the intercept of the asset pricing models. We should note that the test in Pesaran and Yamagata (2008) corresponds to exact asset pricing factor models and Ando and Bai (2015) to approximate asset pricing factor models. We discuss first Ando and Bai (2015)'s test as this is more relevant empirically due to the presence of cross-sectional correlation between the pricing errors.

We consider the following test statistic:

$$\widehat{\Gamma}_{\alpha,\lambda} = \frac{T(\widehat{\eta}_{\cdot} - \widehat{\eta}\iota_N)^\top \left(\widehat{S} - \frac{1}{N}\widehat{L}^\top\right)\widehat{W}^{-1}\left(\widehat{S} - \frac{1}{N}\widehat{L}\right)(\widehat{\eta}_{\cdot} - \widehat{\eta}\iota_N) - [(N-1)K+N]}{\sqrt{2[(N-1)K+N]}}, \tag{3.19}$$

with ι_N denoting a vector of ones of dimension N and $(N-1)K+N$ denoting the number of restrictions under the null hypothesis $H_0^{\alpha,\lambda}$.

Proposition 3.1. *Under the null hypothesis $H_0^{\alpha,\lambda}$, and assumptions in Lemma 3.1 and 3.2,*

$$\widehat{\Gamma}_{\alpha,\lambda} \xrightarrow{d} N(0,1),$$

as $(m,N,T)_{seq} \to \infty$, with $\frac{\sqrt{T}}{N} \to 0$.

The above method can be also used to test the joint null hypothesis $H_0^{\alpha} : \alpha_i = 0$ for $i = 1, \ldots, N$. In particular, we can apply the results in Lemma 3.2 to the intercept parameter estimator only. This test statistic extends Gibbons, Ross, and Shanken (1989)'s Wald type test by considering the effect of using estimated factor loadings on the variance-covariance matrices W_α and accommodating the presence of cross-sectional dependence in the pricing errors. The corresponding test statistic is

$$\widehat{\Gamma}_{\alpha} = \frac{T\widehat{\alpha}^\top \widehat{B}_\alpha \widehat{\alpha} - N}{\sqrt{2N}}, \tag{3.20}$$

with $\widehat{B}_\alpha$ the submatrix of $\left(\widehat{S} - \frac{1}{N}\widehat{L}^\top\right)\widehat{W}^{-1}\left(\widehat{S} - \frac{1}{N}\widehat{L}\right)$ only containing the intercept parameter elements. Note that the joint null hypothesis H_0^{α} entertains N restrictions.[4] This test statistic will be used in the empirical application as a valid Wald type test to assess the null hypothesis of no intercept under the presence of generated regressors.

[4]Gagliardini et al. (2016) also develop a test that extends Gibbons, Ross, and Shanken (1989)'s Wald type test. Their test accommodates estimated factor loadings within an approximate linear factor structure for large N and T. Our approach differs from that in Gagliardini et al. (2016). We work on an unconditional linear factor model, use high-frequency data to obtain consistent estimates of the factor loadings (assuming that $m \to \infty$), and our tests do not only consider the hypothesis of null intercept but also consider the homogeneity of the slope parameters that characterize the price of risk for the cross-section of risky assets.

Proposition 3.2. *Under the null hypothesis H_0^{α}, and assumptions in Lemma 3.1 and 3.2,*

$$\widehat{\Gamma}_{\alpha} \xrightarrow{d} N(0,1),$$

as $(m, N, T)_{seq} \to \infty$, *with* $\frac{\sqrt{T}}{N} \to 0$.

As discussed earlier, we also consider tests for the null hypothesis H_0^{λ}, that is, slope parameters only. The corresponding test statistic is

$$\widehat{\Gamma}_{\lambda} = \frac{T(\widehat{\lambda}_{\cdot} - \widehat{\lambda}\iota_N)^{\top}\widehat{B}_{\lambda}(\widehat{\lambda}_{\cdot} - \widehat{\lambda}\iota_N)^{\top} - (N-1)K}{\sqrt{2(N-1)K}}, \tag{3.21}$$

with $\widehat{B}_{\lambda}$ the submatrix of $\left(\widehat{S} - \frac{1}{N}\widehat{L}^{\top}\right)\widehat{W}^{-1}\left(\widehat{S} - \frac{1}{N}\widehat{L}\right)$ only containing the slope parameter elements. Note that the joint null hypothesis H_0^{λ} entertains $(N-1)K$ restrictions. This test statistic will be used in the empirical application as a valid Wald type test to assess the null hypothesis of slope homogeneity under the presence of generated regressors.

Proposition 3.3. *Under the null hypothesis H_0^{λ}, and assumptions in Lemma 3.1 and 3.2,*

$$\widehat{\Gamma}_{\lambda} \xrightarrow{d} N(0,1),$$

as $(m, N, T)_{seq} \to \infty$, *with* $\frac{\sqrt{T}}{N} \to 0$.

Propositions 3.1, 3.2, and 3.3 formalize the limiting distributions of the asset pricing tests with cross-sectional dependence. In the Online Appendix B we also consider the simpler case of asset pricing tests without cross-sectional dependence. In this scenario we assume no cross-sectional dependence among the pricing errors ν_{it} in (3.9). Although this is not possible in our model set-up due to the presence of correlation in the pricing errors induced by the observed common factors $\widetilde{f}_{t+1}$ in (2.10) or the set of observed and unobserved common factors $\widetilde{g}_{t+1}$ in (2.14), we present for completeness the extension of the Swamy type tests derived in Pesaran and Yamagata (2008) to entertaining generated regressors. In turn, these tests, together with the standard Swamy-type tests, serve to illustrate the contribution of our tests for achieving correct size and power.

We study the finite sample performance of these tests with Monte Carlo simulations. The results are provided in the Online Appendix C. The test results show that: (i) the tests are able to separate heterogeneity arising from the intercepts (i.e., α_i) and the slopes (i.e., λ_i); (ii) in the presence of unobserved factors, Ando and Bai (2015) type tests are the only ones that have correct empirical size; (iii) the presence of generated regressors requires a variance correction that is achieved with our proposed test. We also note in this detailed simulation exercise that there are no significant differences in the finite-sample properties of the tests for the null intercept hypothesis between Ando and Bai (2015) and our correction for generated regressors. Nevertheless, there are significant differences, however, with the Gibbons, Ross, and Shanken (1989) test that is oversized when the factor loadings are estimated. Overall, our proposed tests, $\widehat{\Gamma}_\alpha$, $\widehat{\Gamma}_\lambda$ and $\widehat{\Gamma}_{\alpha,\lambda}$, have the best performance in terms of correct empirical size and power for detection of departures from the different null hypotheses.

4 Empirical Application

In this section we apply the asset pricing tests developed above to explain the excess returns on 47 U.S. industry portfolios maintained in Kenneth French's data library spanning the period July 1963 to December 2014.[5] The return on the risk-free asset is proxied by daily returns on the U.S. three-month Treasury bill also available from this website. Our aim is to test the suitability of the beta asset pricing model for different specifications of the pricing factors. The empirical strategy for the practical implementation of the above tests is as follows.

First, for each individual industry portfolio we compute the realized covariance measures $\hat{\beta}_{it}$ proxying the dynamic quantities β_{it}. These observed measures are unbiased estimators of the actual unobserved conditional covariances between the excess returns on the risky portfolios and the pricing factors. In a second stage, we regress the excess industry portfolio returns on the

[5] The dataset provided in Kenneth French's website comprises the returns on 49 industry portfolios at daily frequencies, however, reliable information at daily frequency is not available for the full sample period for Healthcare (available from July 1969) and Computer Software (available from July 1965), hence, these industries are removed from the empirical study. In a similar vein, Fama and French (2008) suggest to exclude from the empirical datasets the firms (and thus the industry porfolios) with a Standard Industrial Classification Codes between 6000 and 6999 (banking, insurance, real estate and trading sectors).

proxies $\hat{\beta}_{it}$ and obtain the parameter estimates of the vector (α_i, λ_i) for $i = 1, \ldots, N$, and estimates of the unobserved factors $\widetilde{g}_t$ for a predetermined number of factors R. Using these estimates we can compute the panel data estimator of the slope coefficients λ in (3.18) and the different test statistics. In particular, we compute $\widehat{\Gamma}_{\alpha,\lambda}$, $\widehat{\Gamma}_\alpha$ and $\widehat{\Gamma}_\lambda$ corresponding to the corrected version of Ando and Bai (2015)'s test; $\widehat{\Gamma}^{AB}_{\alpha,\lambda}$, $\widehat{\Gamma}^{AB}_\alpha$ and $\widehat{\Gamma}^{AB}_\lambda$ are the uncorrected versions of the tests. Finally, for comparison, we also compute the corrected versions (for generated regressors) of the Pesaran and Yamagata (2008) tests denoted by $\widetilde{\Gamma}_{\alpha,\lambda}$, $\widetilde{\Gamma}_\alpha$ and $\widetilde{\Gamma}_\lambda$, together with the uncorrected tests $\widetilde{\Gamma}^{PY}_{\alpha,\lambda}$, $\widetilde{\Gamma}^{PY}_\alpha$ and $\widetilde{\Gamma}^{PY}_\lambda$. Note that $\widehat{\Gamma}_\alpha$ coincides with $\widehat{\Gamma}^{AB}_\alpha$, and $\widetilde{\Gamma}_\alpha$ with $\widetilde{\Gamma}^{PY}_\alpha$, and thus only the former is presented in each case. For the approximate factor models we present test results for $R = 2, 5$, unobserved factors.

4.1 Models and Risk Factors

The asset pricing tests developed in this paper are applied to different asset pricing models proposed in the literature for describing the cross-section of risky assets. We concentrate on models defined by observable common factors given by financial returns on tradable portfolios. In particular, we study the suitability of the CAPM proposed by Sharpe (1964), the Fama and French (1993) three-factor model and the Fama and French (2015) five-factor model. Gagliardini, Ossola, and Scaillet (2016) propose a four-factor model for testing their conditional approach to asset pricing. These authors use a very similar evaluation period as well.

Our specifications comprise the CAPM one-factor model defined by the market excess return *MKT* (i.e, $K = 1$); the three-factor model (3F) with pricing factors given by *MKT*, *SMB*, and *HML* (i.e, $K = 3$); and the five-factor model (5F) defined by the pricing factors *MKT*, *SMB*, *HML*, *RMW* and *CMA* (i.e, $K = 5$). In particular, *MKT* stands for a value-weighted average market portfolio return; *SMB* denotes a small-minus-big portfolio constructed as the difference between the returns on diversified portfolios of small and large asset size; *HML* is high-minus-low portfolio constructed as the difference between the returns on diversified portfolios of high and small book-to-market equity; *RMW* defines a robust-minus-weak profitability portfolio constructed as the difference between the returns on diversified portfolios of stocks with robust and weak profitability.

The factor CMA stands for a conservative-minus-aggressive portfolio constructed as the difference between the returns on diversified portfolios of stocks of low (conservative) and high (aggressive) investment firms. Details of the definition of the portfolio returns that determine the different factors can be found in Fama and French (2015).[6]

Our testing regression equation is (3.9) that we reproduce here for convenience:

$$
\begin{aligned}
r^e_{i,t+1} &= \alpha_i + \widehat{\beta}_{it}^{\top}\lambda_i + \nu_{i,t+1}, \\
\nu_{i,t+1} &= \beta_i^{*\top}\widetilde{g}_{t+1} + \varepsilon_{i,t+1}.
\end{aligned}
$$

The pricing factors are standardized to have unit variance such that the estimates of the dynamic factor loadings, $\widehat{\beta}_{it}$, can be computed as the conditional covariances $Cov_t(r^e_{i,t+1}, f_{t+1})$ using the autoregressive model (3.7). The observed quantities $\omega_{(i,j),t}$ used for (3.7) are constructed from expression (3.1) with the vector R_{t+1} comprised by the 47 daily industry returns $r_{i,t+1}$ for $i = 1, \ldots, N = 47$, and K common pricing factors given by the daily returns on the factor portfolios introduced above. The risk-free return is proxied by the daily time series of the 3-month U.S. Treasury bill. This analysis leads to a time series of quarterly realized dynamic betas obtained from $m = 22 \times 3$ daily observations.

We study the suitability of the asset pricing models for quarterly returns evaluated over the period July 1963 to December 2014, and then over blocks of twenty years of observations that cover different turmoil periods in financial markets.

4.2 Tests Results

We first focus on tests for the null hypothesis $H_0^{\alpha} : \alpha_i = 0$ for all assets in the cross-section of returns. This is the test usually implemented in the empirical asset pricing literature to assess the correct specification of an asset pricing equation. This test can be implemented as a t-test for time series asset pricing equations, see Fama and French (1993, 2015) type of models, or as a joint F-test to assess the suitability of an asset pricing equation for the whole market. The latter is done

[6] A detailed explanation of the formation of the industry portfolios can be found at `http://mba.turc.dartmouth.edu/pages/faculty/ken.french.`

using Gibbons, Ross, and Shanken (1989) Wald type test. We extend these methods in two ways. First, we correct the variance of the statistic due to the fact that we use generated regressors, and second, we accommodate the presence of dependence between the pricing errors of the different asset pricing equations.

Table 1 reports the value of the test statistics and corresponding p-values for the versions of the tests that correct for the presence of generated regressors. In particular, we consider $\widehat{\Gamma}_\alpha$ computed for $R = 2, 5$, with R the number of unobserved factors, and $\widetilde{\Gamma}_\alpha$ that denotes the correction of the test statistic by Pesaran and Yamagata (2008). In order to obtain a clear insight over the performance of the asset pricing equations across different evaluation periods, we divide our sample in a full period covering $1963 - 2014$, and four additional subsamples covering twenty years of data each. The results for all tests and periods present overwhelming evidence rejecting the suitability of any of the asset pricing factor models under study.

We extend this analysis to assess the slope homogeneity hypothesis. In this case our interest is in the hypothesis $H_0^\lambda : \lambda_i = 0$ for all assets in the cross-section of returns. Test statistics and p-values are reported in Table 2. We also compute the uncorrected versions of the tests given in this case by $\widehat{\Gamma}_\lambda^{AB}$ and $\widetilde{\Gamma}_\lambda^{PY}$. As for the previous test, the results show overwhelming evidence rejecting the null hypothesis of slope homogeneity. These results suggest that the factor risk premia cannot be considered to be the same across asset pricing equations. The effect of the generated regressors on the tests can be observed in the differences in the value of the test statistics, nevertheless, in most cases, the magnitude of the test statistics leads to clear rejections of the null hypothesis. These results are robust to the sample period, choice of asset pricing model and the number of unobserved factors R, with $R = 2, 5$.

[Insert Tables 1 - 3 about here]

To complete the analysis, we also evaluate the joint hypothesis test $H_0^{\alpha,\lambda}$ in Table 3. The results of the tests in this case offer more discussion. For the full sample period, we observe that whereas Ando and Bai (2015)'s based tests do reject the null hypothesis, the tests based on the *i.i.d.* assumption provide statistical evidence not to reject for the three asset pricing specifications. Similar findings are observed for the period $1993 - 2014$. This result suggests that careful consideration about the possibility of cross-sectional dependence in the pricing errors is needed as the results are in stark contrast. The overall results suggest the rejection of the joint null hypothesis across periods and test statistics, however, it is worth noting that the corrected versions of Ando and Bai (2015)'s test for both $R = 2, 5$ and the three asset pricing equations do not reject the null hypothesis of correct specification of the asset pricing equation over the period $1973 - 1993$.

In summary, our empirical results for a cross-section of 47 industry portfolios excluding Health and Computer Software sectors are similar to other recent empirical studies testing the validity of empirical asset pricing models. Thus, Ang, Liu, and Schwarz (2017) using two-pass asset pricing regression models reject the hypothesis that the cross-sectional risk premia are equal to the mean factor portfolio returns using a dataset of portfolios. Our results are also similar to the empirical findings in Gagliardini, Ossola, and Scaillet (2016) that reject the empirical validity of different factor models to price the cross-section of industry portfolios using a novel test in a conditional asset pricing setting, and also the standard Gibbons, Ross, and Shanken (1989) F-statistic. The overall rejection of popular asset pricing factor models seems to be robust to the testing method, evaluation period and specific choice of investment portfolios.

5 Conclusion

This paper shows that the correct specification of an asset pricing equation not only involves a zero intercept but also homogeneity of the slope coefficients that characterize the factor risk premia associated to the common pricing factors. The latter condition is very important as the rejection of the null hypothesis due to heterogeneity on the factor risk premia raises concerns about the existence of a common set of pricing factors, and more importantly, about the possibility of arbitrage opportunities. To overcome this, we have developed novel statistical tests based on Swamy type tests for exact and approximate linear factor models for asset pricing. In particular, we proposed versions of Ando and Bai (2015) and Pesaran and Yamagata (2008) tests that accommodate the presence of generated regressors. These generated regressors arise in our model setup due to the choice of dynamic factor loadings in the different asset pricing specifications that are estimated by high-frequency realized covariance measures. Our methodology can be also interpreted as an extension of the popular Gibbons, Ross, and Shanken (1989) Wald type test to correct for the presence of generated regressors in two-pass regression models, see for example Fama and MacBeth (1973) type of framework.

An empirical application to the cross-section of U.S. industry portfolios rejects the validity of different linear factor models such as the CAPM and Fama and French (1993, 2015) three- and five-factor models. These results are robust to the evaluation period and choice of test statistic. These empirical findings are in line with recent literature, see Gagliardini, Ossola, and Scaillet (2016), that rejects a four-factor model using a conditional asset pricing factor approach.

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Table 1: Empirical application. Tests for intercepts

	$\widehat{\Gamma}_\alpha(R=2)$	$\widehat{\Gamma}_\alpha(R=5)$	$\widetilde{\Gamma}_\alpha$	$\widehat{\Gamma}_\alpha(R=2)$	$\widehat{\Gamma}_\alpha(R=5)$	$\widetilde{\Gamma}_\alpha$	$\widehat{\Gamma}_\alpha(R=2)$	$\widehat{\Gamma}_\alpha(R=5)$	$\widetilde{\Gamma}_\alpha$
	$K=1$			$K=3$			$K=5$		
	Full sample 1963-2014								
Stat.	36.367	14.328	5.708	50.188	25.164	7.764	190.047	488.650	152.022
p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	1963-1983								
Stat.	27.623	54.832	9.075	222.601	174.693	22.358	492.101	655.485	47.731
p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	1973-1993								
Stat.	17.378	50.780	4.930	274.051	169.400	30.904	460.803	450.162	56.467
p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	1983-2003								
Stat.	104.547	22.955	25.944	231.052	62.514	42.336	228.462	119.128	76.986
p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	1993-2014								
Stat.	19.987	19.232	4.919	62.034	53.512	19.655	43.514	30.487	14.108
p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table 2: Empirical application. Tests for slopes

K		$\widehat{\Gamma}_\lambda(R=2)$	$\widehat{\Gamma}_\lambda^{AB}(R=2)$	$\widehat{\Gamma}_\lambda(R=5)$	$\widehat{\Gamma}_\lambda^{AB}(R=5)$	$\widetilde{\Gamma}_\lambda$	$\widetilde{\Gamma}_\lambda^{PY}$
		Full sample 1963-2014					
1	Stat.	32.577	33.773	20.563	22.004	5.754	5.927
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3	Stat.	31.402	35.516	18.916	21.698	3.472	4.180
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.007)	(0.000)
5	Stat.	21.352	26.473	26.988	31.236	5.503	6.922
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
		1963-1983					
1	Stat.	134.705	137.401	334.660	339.822	28.711	29.196
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3	Stat.	140.738	144.236	261.319	265.673	19.753	20.727
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.007)	(0.000)
5	Stat.	89.994	95.712	276.311	282.667	45.942	50.770
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
		1973-1993					
1	Stat.	16.004	16.149	59.233	59.5469	2.594	2.614
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)	(0.004)
3	Stat.	126.750	130.951	76.760	79.637	8.780	11.215
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.007)	(0.000)
5	Stat.	178.459	184.742	186.177	189.971	13.763	16.959
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
		1983-2003					
1	Stat.	104.466	108.082	33.775	34.27969	20.960	21.398
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3	Stat.	122.206	128.296	41.991	45.610	25.059	27.970
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.007)	(0.000)
5	Stat.	91.953	98.901	59.942	66.454	28.602	32.103
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
		1993-2014					
1	Stat.	12.369	12.934	13.246	13.872	5.176	5.508
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
3	Stat.	38.251	47.042	27.000	30.540	8.941	10.413
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.007)	(0.000)
5	Stat.	23.770	1.271	16.886	21.839	2.185	3.950
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.014)	(0.000)

Table 3: Empirical application. Tests for intercepts and slopes

K		$\widehat{\Gamma}_{\alpha,\lambda}(R=2)$	$\widehat{\Gamma}^{AB}_{\alpha,\lambda}(R=2)$	$\widehat{\Gamma}_{\alpha,\lambda}(R=5)$	$\widehat{\Gamma}^{AB}_{\alpha,\lambda}(R=5)$	$\widetilde{\Gamma}_{\alpha,\lambda}$	$\widetilde{\Gamma}^{PY}_{\alpha,\lambda}$
		Full sample 1963-2014					
1	Stat.	8.778	9.619	2.338	3.352	-2.372	-2.251
	p-val.	(0.000)	(0.000)	(0.010)	(0.000)	(0.991)	(0.988)
3	Stat.	5.468	9.021	2.583	4.986	-3.167	-2.555
	p-val.	(0.000)	(0.000)	(0.005)	(0.000)	(0.992)	(0.947)
5	Stat.	4.798	9.465	5.988	9.859	0.356	1.649
	p-val.	(0.000)	(0.000)	(0.000)	(0.000)	(0.308)	(0.050)
		1963-1983					
1	Stat.	1.288	3.184	7.585	11.216	70.813	71.154
	p-val.	(0.099)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
3	Stat.	-0.564	2.458	3.847	7.607	83.237	84.079
	p-val.	(0.714)	(0.007)	(0.000)	(0.000)	(0.000)	(0.000)
5	Stat.	-1.031	4.179	5.957	11.748	44.387	48.786
	p-val.	(0.849)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
		1973-1993					
1	Stat.	0.592	0.694	2.397	2.618	-3.492	-3.478
	p-val.	(0.277)	(0.244)	(0.008)	(0.004)	(1.000)	(1.000)
3	Stat.	-1.458	2.170	-1.258	1.228	-2.109	-0.006
	p-val.	(0.928)	(0.015)	(0.896)	(0.110)	(0.983)	(0.503)
5	Stat.	-1.255	4.470	-1.631	1.826	8.572	11.484
	p-val.	(0.895)	(0.000)	(0.949)	(0.034)	(0.000)	(0.000)
		1983-2003					
1	Stat.	-0.310	2.233	-0.308	0.047	11.771	12.080
	p-val.	(0.622)	(0.013)	(0.621)	(0.481)	(0.000)	(0.000)
3	Stat.	2.314	7.575	-3.236	-0.110	17.622	20.136
	p-val.	(0.010)	(0.000)	(0.999)	(0.544)	(0.007)	(0.000)
5	Stat.	2.434	8.765	-3.958	1.976	35.406	38.596
	p-val.	(0.007)	(0.000)	(1.000)	(0.024)	(0.000)	(0.000)
		1993-2014					
1	Stat.	3.644	4.041	-1.660	-1.220	-2.328	-2.094
	p-val.	(0.000)	(0.000)	(0.952)	(0.889)	(0.990)	(0.982)
3	Stat.	5.962	13.555	0.517	2.712	-0.118	1.153
	p-val.	(0.000)	(0.000)	(0.303)	(0.003)	(0.547)	(0.124)
5	Stat.	5.524	12.358	2.536	7.049	-3.079	-1.472
	p-val.	(0.000)	(0.000)	(0.006)	(0.000)	(0.999)	(0.929)