Ionospheric Models for use in Radio Astronomy

by

Poppy L. Martin

Thesis for the degree of Doctor of Philosophy

October 2016
This thesis describes a variety of techniques for modelling the ionosphere of the Earth to improve low-frequency radio observations. The ionosphere of the Earth is a dynamic and inhomogeneous plasma which distorts low-frequency radio signals by causing absorption and scattering of the incoming radio wave and changes to the measured Faraday rotation and phase error. Within this thesis the phase errors, absorption and changes to the measured Faraday rotation caused by the ionosphere are investigated.

The assumption of a thin-layer model to correct for the phase errors is a method that is becoming more commonly used throughout radio astronomy. In this thesis we explore the errors arising from this assumption and the conditions under which the assumption can be applied to radio telescopes including the GMRT, VLA, LOFAR, MWA and SKA1-LOW.

Absorption can strongly affect faint low-frequency radio signals such as the 21 cm hydrogen line from the epoch of reionisation. It is therefore integral to studies of the epoch of reionisation to be able to accurately measure and model the amount of absorption caused by the ionosphere. In this thesis we use radio telescopes as multi-frequency riometers to study the amount of absorption that occurs in the lower ionosphere. We find that this method is not accurate enough to measure the absorption that occurs in a quiet, mid-latitude ionosphere. However we have designed and tested software that recreates electron density height profiles from high-latitude multi-frequency absorption measurements which is capable of recovering information about the lower ionosphere as well as information about the physical processes that affect the electron density within it. This method of measuring the electron density of the lower ionosphere is useful to ionospheric physicists as this region of the ionosphere is difficult to observe with existing methods.
Finally we investigate the structure of the turbulence within the ionosphere by observing changes to the measured Faraday rotation of pulsars. We first investigate whether information about the turbulent spectrum can be inferred from variations in the Faraday rotation of a simulated radio signal, finding that we can recover the power of the input turbulent spectrum. We then implement a pulsar observing mode at KAIRA that successfully observes full Stokes polarisation data at a high temporal resolution and develop a pipeline that converts the pulsar data into a format readable by pulsar analysis programs. We use the change in the rotation measures of the observed pulsars to investigate the turbulent spectrum above KAIRA.
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Acknowledgements

Thanks to my supervisor, Anna Scaife, for all her help and support throughout my PhD. Many thanks to the post docs and PhD students in Anna’s group, especially Justin, for all their advice. Thanks as well to Derek, Ian and Antti for helpful discussions, and Matt, Justin, Therese, Alex and Lemon for their proofreading. You all know that I appreciate the help that you’ve given me throughout my PhD. G&Ts are on me.

I would also like to thank those who have supported me outside of my academic work. With special thanks going to Matt, my parents, my brother and my new sister-in-law for their constant love and support. As well as to Hannah, Therese (and her constant supply of Netflix), Amy, Spesh, Adam, Andy, Lemon and Sam for always being there to distract me and provide hugs.
# Nomenclature

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<tr>
<td>$A$</td>
<td>absorption</td>
</tr>
<tr>
<td>$A_{\text{mod}}$</td>
<td>modelled absorption</td>
</tr>
<tr>
<td>$A_{\text{obs}}$</td>
<td>observed absorption</td>
</tr>
<tr>
<td>ACF</td>
<td>Autocorrelation Function</td>
</tr>
<tr>
<td>ADU</td>
<td>Analog Digital Units</td>
</tr>
<tr>
<td>AIPS</td>
<td>Astronomical Image Processing System</td>
</tr>
<tr>
<td>ALMA</td>
<td>Atacama Large Millimeter/submillimeter Array</td>
</tr>
<tr>
<td>ARIES</td>
<td>Advanced Rio-Imaging Experiment in Scandinavia</td>
</tr>
<tr>
<td>ATNF</td>
<td>Australia Telescope National Facility</td>
</tr>
<tr>
<td>$b$</td>
<td>baseline</td>
</tr>
<tr>
<td>$B$</td>
<td>bandpass</td>
</tr>
<tr>
<td>$</td>
<td>B</td>
</tr>
<tr>
<td>$B_0$</td>
<td>mean value of the magnetic field at the magnetic equator</td>
</tr>
<tr>
<td>$B_{ij}$</td>
<td>Bayes factor</td>
</tr>
<tr>
<td>$\chi$</td>
<td>polarisation angle</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light in a vacuum</td>
</tr>
<tr>
<td>CASA</td>
<td>Common Astronomy Software Applications package</td>
</tr>
<tr>
<td>COSPAR</td>
<td>Committee on Space Research</td>
</tr>
<tr>
<td>$D_t$</td>
<td>second order temporal structure function</td>
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<tr>
<td>$D_\theta$</td>
<td>second order spatial structure function</td>
</tr>
<tr>
<td>dec</td>
<td>declination</td>
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<tr>
<td>DM</td>
<td>Dispersion Measure</td>
</tr>
<tr>
<td>$DR$</td>
<td>dynamic range</td>
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<tr>
<td>$\epsilon_0$</td>
<td>permittivity of free space</td>
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<tr>
<td>$\epsilon_{\text{ion}}$</td>
<td>energy loss per ion formation</td>
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<td>$e$</td>
<td>electron charge</td>
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<td>electron</td>
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<td>energy</td>
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<td>$E_0$</td>
<td>characteristic energy of electron precipitation</td>
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<td>EISCAT</td>
<td>European Incoherent Scatter Facility</td>
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<td>ESR</td>
<td>EISCAT Svalbard radar</td>
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<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>GLONASS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GMRT</td>
<td>Giant Metrewave Radio Telescope</td>
</tr>
<tr>
<td>GMT</td>
<td>Greenwich Mean Time</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite Systems</td>
</tr>
<tr>
<td>GOES</td>
<td>Geostationary Operational Environmental Satellite</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck’s constant</td>
</tr>
<tr>
<td>HBA</td>
<td>High Band Antenna</td>
</tr>
<tr>
<td>HF</td>
<td>High Frequency</td>
</tr>
<tr>
<td>IGRF</td>
<td>International Geomagnetic Reference Field</td>
</tr>
<tr>
<td>IRI</td>
<td>International Reference Ionosphere</td>
</tr>
<tr>
<td>IRIS</td>
<td>Imaging Riometer for Ionospheric Studies</td>
</tr>
<tr>
<td>IMAGE</td>
<td>International Monitor for Auroral Geomagnetic Effects</td>
</tr>
<tr>
<td>$J_0$</td>
<td>flux of precipitating electrons</td>
</tr>
<tr>
<td>Jy</td>
<td>Jansky</td>
</tr>
<tr>
<td>$k$</td>
<td>conversion factor for kJy to ADU</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
</tr>
<tr>
<td>KAIRA</td>
<td>Kilpisjärvi Atmospheric Imaging Receiver Array</td>
</tr>
<tr>
<td>$\mathcal{L}$</td>
<td>log-likelihood</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wavelength</td>
</tr>
<tr>
<td>$\Lambda(s/R(E))$</td>
<td>rate of energy dissipation</td>
</tr>
<tr>
<td>$L(h)$</td>
<td>recombination rate</td>
</tr>
<tr>
<td>LBA</td>
<td>Low Band Antenna</td>
</tr>
<tr>
<td>LOFAR</td>
<td>LOw Frequency ARray</td>
</tr>
<tr>
<td>LOS</td>
<td>line of sight</td>
</tr>
<tr>
<td>LWA</td>
<td>Long Wavelength Array</td>
</tr>
<tr>
<td>$m_e$</td>
<td>mass of the electron</td>
</tr>
<tr>
<td>MCMC</td>
<td>Monte Carlo Markov Chain</td>
</tr>
<tr>
<td>MJD</td>
<td>Modified Julian Day</td>
</tr>
<tr>
<td>MSSS</td>
<td>Multifrequency Snapshot Sky Survey</td>
</tr>
<tr>
<td>MWA</td>
<td>Murchison Widefield Array</td>
</tr>
<tr>
<td>$\nu$</td>
<td>frequency</td>
</tr>
<tr>
<td>$\nu_{en}$</td>
<td>electron-neutral collision frequency</td>
</tr>
<tr>
<td>$\nu_p$</td>
<td>plasma frequency</td>
</tr>
<tr>
<td>$n$</td>
<td>refractive index</td>
</tr>
<tr>
<td>$N_e$</td>
<td>electron density</td>
</tr>
<tr>
<td>$N_{eq}$</td>
<td>quiet time electron density</td>
</tr>
<tr>
<td>$N_{mod}$</td>
<td>electron density height profile model</td>
</tr>
<tr>
<td>$N_{obs}$</td>
<td>observed electron density</td>
</tr>
<tr>
<td>NCP</td>
<td>North Celestial Pole</td>
</tr>
<tr>
<td>$\omega_g$</td>
<td>electron gyrofrequency</td>
</tr>
</tbody>
</table>
NOMENCLATURE

\( \phi \)  phase error
\( \phi_{\text{ion}} \)  ionospheric phase rotation
\( p \)  sample probability
\( P \)  power
\( P_0 \)  quiet day power
\( P_{\text{sky}} \)  true power
\( P_{\text{sys}} \)  total power received by the instrument
PRESTO  Pulsar Exploration and Search Toolkit
\( Q(h) \)  ionisation rate
\( Q_0(h) \)  quiet time ionisation rate
QDC  Quiet Day Curve
QDS  Quiet Day Surface
\( \rho(h) \)  mass density at altitude \( h \)
\( R_E \)  radius of the Earth
\( R(E) \)  effective range of electrons
RA  Right Ascension
RFI  Radio Frequency Interference
RM  Rotation Measure
\( \sigma_{\text{obs}} \)  uncertainty associated with observed values
\( s \)  atmospheric scattering depth
\( S \)  system equivalent flux density
SIC  Sodankyla Ion Chemistry
SIGPROC  Signal Processing and Communications
SKA  Square Kilometre Array
SMSE  Standardised Mean Square Error
SPAM  Source Peeling and Atmospheric Modeling
STEC  Slant Total Electron Content
\( \tau_g \)  geometric delay
\( T_e \)  electron temperature
\( T_{\text{instrument}} \)  noise due to receiver system
\( T_{\text{sky}} \)  sky noise
\( T_{\text{sys}} \)  total system temperature
TEC  Total Electron Content
TECU  TEC units
UDP  User Datagram Protocol
UHF  Ultra High Frequency
URSI  International Union of Radio Science
UT  Universal Time
VHF  Very High Frequency
VLA  Karl G. Jansky Very Large Array
VTEC  Vertical Total Electron Content
$z$    redshift
$Z$    TEC ratio derivative
$Z_i$  measured parameter
$Z_i^*$ estimated parameter
$Z_{\text{var}}$ variance of the measured parameter
Chapter 1

Introduction

The low-frequency radio sky is one of the least explored areas in astronomy. This is because the ionosphere of the Earth is a dynamic and inhomogeneous plasma which distorts low-frequency radio signals. Variation in the electron density of the ionosphere causes changes to the measured Faraday rotation and phase error, as well as causing absorption and scattering of the incoming radio wave. A strong understanding of the ionosphere is therefore paramount to low-frequency radio astronomy so that these effects can be corrected.

Low-frequency radio observations are integral for probing a range of astrophysical processes, from studying the highly-redshifted 21-cm line emission of neutral hydrogen from the epoch of reionisation ($z=6–20$) and the cosmic dawn phase ($z=20–50$), to exploring the diffuse, steep-spectrum synchrotron emission from clusters of galaxies and polarisation studies of cosmic magnetic fields. Low-frequency radio surveys are also essential in searching for highly-redshifted radio sources, previously undiscovered pulsars and cosmic radio transients.

The epoch of reionisation was one of the most significant events in the history of the intergalactic medium. Probing the structure of the 21-cm emission from neutral hydrogen gas in the intergalactic medium will help to reveal the astrophysics behind the formation of the Universe. The redshifted ($z > 6$) 21-cm emission should be visible as a faint and diffuse background below 200 MHz. The relationship between redshift and observing frequency is given by $\nu = 1420/(1+z)$ MHz (Bowman et al., 2013), therefore to observe the highest predicted redshift of the 21-cm emission line from the cosmic dawn, frequencies as low as 28 MHz are required. The majority of ionospheric effects on interferometric observations scale with $\nu^{-2}$ (Thompson et al., 2001) and are therefore worse at these low frequencies. At their worst, these ionospheric effects can have a deleterious effect on radio observations.
This thesis concentrates on modelling the electron density of the ionosphere, as well as presenting and analysing electron density models and methods of observing the ionosphere to further our understanding. In this thesis I aim to improve upon the current approaches used to calibrate the ionosphere in low-frequency radio observations, concentrating on the effects of phase delay, absorption and change in measured Faraday rotation.

In the remainder of this chapter I provide an overview of the ionosphere, describing its structure and temporal variation in Section 1.1, as well as describing a range of phenomena that affect its structure. In Section 1.2 I then discuss the techniques currently used to observe the ionosphere. Finally, in Section 1.3 I outline the structure of this thesis.

1.1 The ionosphere

The ionosphere is a partially ionised upper layer of the Earth’s atmosphere that ranges from altitudes of around 50 km to 1000 km (Hargreaves, 1992). It was first postulated in 1902 by Oliver Heaviside and Arthur E. Kenelly, before being confirmed by Edward Appleton when its effects on radio transmissions were observed in the early 1920s. The ionosphere affects radio observations and communications because it contains a sufficient number of temporally and spatially varying free electrons which influence the propagation of radio waves.

The ionosphere is dominated by atomic oxygen and nitrogen under very low pressure (Hunsucker and Hargreaves, 2002) but also contains free electrons and positive ions in equal numbers. This means that, as a medium, it is electrically neutral. The ionisation of the gas atoms and particles in the ionosphere is primarily caused by photo-ionisation from solar radiation at UV and X-ray wavelengths as well as by charged particles from the solar wind. The ionisation is therefore dependent on the geomagnetic latitude, time of day, season and the current level of solar activity, with ionisation primarily occurring during the day. The ionisation that occurs during the day is balanced out by recombination at nighttime.

During the daytime, ionisation of atoms (X or Y) and molecules (XY) in the ionosphere is mainly caused by solar radiation,

\[ h\nu + X \rightarrow X^+ + e^- \]

\[ h\nu + XY \rightarrow XY^+ + e^-, \]

where \( h \) is Planck’s constant and \( h\nu \) is a photon with energy and momentum dependent on its frequency, \( \nu \). \( X^+ \) and \( XY^+ \) are positive ions, with \( e^- \) being a free electron that is released when they were ionised. These free electrons can also form negative ions...
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\[(XY^-)\] by attachment,

\[XY^- + e^- \rightarrow XY^- . \quad (1.3)\]

The negative ions can then absorb photons to return to a neutral state via photodetachment,

\[XY^- + h\nu \rightarrow XY + e^- . \quad (1.4)\]

At night, the positive ions recombine,

\[X^+ + e^- \rightarrow X + h\nu \quad (1.5)\]

\[XY^+ + e^- \rightarrow X^* + Y^* , \quad (1.6)\]

where the second chemical equation is dissociative recombination which involves the splitting of a molecule into two excited atoms, \(X^*\) and \(Y^*\).

As well as this well-observed diurnal ionisation and recombination, and the seasonal and solar cycle variation of the electron content of the ionosphere, the ionosphere is also affected by a range of phenomena and disturbances. In Sections 1.1.1 and 1.1.2 I outline the large- and small-scale structure of the ionosphere and how it can be affected by these phenomena.

1.1.1 The large-scale distribution of electron density in the ionosphere

The ionosphere is divided into three layers based on the governing physical processes within that layer, named the D, E and F layers (see Figure 1.1). Edward Appleton originally used the letter ‘E’ in his diagrams to indicate that electromagnetic waves were reflected by the ionosphere. The D and F layers were subsequently discovered in 1924 by Appleton and arbitrarily labelled with the letters D and F, on the assumption that other layers of the ionosphere may be discovered in the future.

As mentioned earlier in this chapter, this thesis concentrates on three different ways in which low-frequency radio observations are affected by the ionosphere, these are phase delay, absorption and Faraday rotation. The phase delay and Faraday rotation (assuming that the magnetic field throughout the ionosphere is constant) are dependent on the column density of electrons in the ionosphere, known as the total electron content (TEC), and so are primarily affected by the layer of the ionosphere which has the highest density of electrons, the F layer. On the other hand, the absorption is primarily caused by the lower ionosphere (the D and E layers) due to the higher electron-neutral collision frequency and number of ionised gas molecules at these altitudes. In the rest of this section, I outline these layers of the ionosphere, as well as describing phenomena that can affect them.

The D layer of the ionosphere is located 50–100 km above the Earth’s surface. It has the highest concentration of gas molecules which are ionised during the day by solar
ionisation. At mid- and low-latitudes, and at high-latitudes during geomagnetically quiet conditions, solar ionisation and Lyman alpha radiation of NO are the main causes of ionisation (Browne et al., 1995). At high latitudes ionisation can also be caused by energetic electrons which precipitate into the D layer during geomagnetically active conditions. Solar flares can also produce a larger ionised population in this layer. Cosmic rays also cause ionisation at low altitudes below the D layer. The D layer almost entirely disappears at nighttime as recombination occurs, forming a neutral layer. During the daytime photodetachment also occurs.

Due to the high concentration of gas molecules in the D layer of the ionosphere, absorption of extraterrestrial radio waves primarily occurs 60–100 km above the surface of the Earth (Browne et al., 1995). The motion of the free electrons that are present in this layer is influenced by absorbing energy from propagating radio waves. This energy can then be lost to neutral atoms or molecules through collisions.

D layer chemistry is complex and difficult to model as it is made up of tens of neutral and ionised particle species with over 50 possible chemical reactions. The chemistry is affected by multiple processes including particle precipitation, changes in the level of
solar radiation, cosmic rays, heat conduction, adiabatic heating and cooling, and heat loss by radiation (Turunen, 1996). The chemistry of the lower ionosphere, including the D layer has been modelled by the Sodankylä Ion Chemistry (SIC) model (Enell et al., 2005). The D layer chemistry can also be affected by the Polar Mesospheric Summer Echo phenomenon which causes the appearance of ice, dust and aerosol particles which can become charged (Rapp and Lübken, 2004).

The E layer is located at altitudes of 90–150 km above the surface of the Earth. Ionisation of the E layer is again primarily caused by solar X-rays and ultraviolet radiation. This solar radiation causes ionisation of molecules, leading to the main ionised constituents of the E layer being $\text{N}_2^+$, $\text{O}_2^+$ and $\text{NO}^+$. Dissociative recombination causes the electron density in the E layer to decrease at night when the primary source of ionisation is no longer present. The D and E layers enable high frequency (HF) communication over long distances because they reflect lower frequency (<10 MHz) radio waves from within the HF band (3–30 MHz).

The F layer is located above the D and E layers, and can reflect radio waves of higher frequencies within the HF band. It is the primary contributor to the TEC of the ionosphere, as electron densities can exceed $10^{12}$ m$^{-3}$. It is the layer in which the peak electron density of the ionosphere occurs. An astronomical radio signal that is capable of penetrating the F layer should therefore reach radio telescopes on the surface of the Earth. The F layer extends upwards from an altitude of 150 km and is dominated by monatomic oxygen which is ionised by extreme ultraviolet radiation ($\lambda = 10–240$ nm). As altitude increases, the number of oxygen ions decreases and hydrogen and helium ions become more dominant.

During the daytime the F layer often splits into two separate layers, the $F_1$ layer and the $F_2$ layer. At night the electron density of the $F_2$ layer decreases due to to the recombination of molecular ions, whilst the $F_1$ layer completely disappears.

The plasmasphere is an ionised region of the Earth’s atmosphere that is toroidal in shape, as shown in Figure 1.2. It is located above the ionosphere at altitudes greater than 1000 km. The plasmasphere is responsible for 10–30% of the TEC along the line of sight at mid-latitudes and extends out to 3–7 Earth radii (Yizengaw et al., 2008). The primary constituent of the plasmasphere is hydrogen which outflows from the ionosphere into the plasmasphere. Above 2000 km the electron density contributes less than 5% to the integrated electron content and above 25000 km the contribution is negligible.

The TEC of the ionosphere is measured in TEC units (TECU), where one TECU is $10^{16}$ electrons m$^{-2}$ and the TEC along the line of sight (LOS) is given by

$$\text{TEC} = \int_{\text{LOS}} N_e(l) \, dl,$$

where $N_e$ is the free electron density along the path length $dl$. 


Bulk changes to the TEC of the ionosphere, such as the increase in ionisation at dawn, cause large-scale changes to the structure of the ionosphere. Phenomena such as a magnetospheric storm can also cause large-scale changes (Hargreaves, 1995). A magnetospheric storm can be caused by a large change in solar wind due to a coronal mass ejection. This can cause a geomagnetic storm on the Earth’s surface which makes the F$_2$ layer unstable and can even make it disappear completely. Enhanced X-rays from a solar flare can also cause a sudden ionospheric disturbance on the dayside of the Earth, often penetrating to the D layer and thus increasing absorption of incident radio waves.

Phenomena such as the winter anomaly, the semiannual anomaly and the equatorial anomaly can also affect the ionosphere on a large scale. The winter anomaly is a daytime phenomenon seen at mid-latitudes in the northern hemisphere. It causes the plasma density at the F-peak height to be greater in winter than in summer and has been seen in many observations, including Lee et al. (2011). It can also be seen in empirical models of the ionosphere, such as the International Reference Ionosphere (IRI), which is discussed in more detail in Section 1.1.1.1. The semiannual anomaly is where the electron density at the F-peak altitude is greater at equinox than at solstice (Lee et al., 2011). The equatorial anomaly is a trough in the ionisation of the F$_2$ layer within $\pm 20^\circ$ of the equator.

Travelling ionospheric disturbances are also common ionospheric phenomenon. A travelling ionospheric disturbance is a ripple or wave in the electron density that propagates horizontally and is often coupled to acoustic gravity waves. They can travel at between 300–700 kmh$^{-1}$ and cause variations in the TEC of 1–5% (Intema, 2009). A large-scale travelling ionospheric disturbance can have a wavelength of up to 1000 km, and is usually caused by auroral activity so is therefore generally found to move from the pole towards the equator. A medium-scale travelling ionospheric disturbance generally has
a wavelength of hundreds of kilometres (Otsuka et al., 2013) and is postulated to be caused by thunderstorms.

1.1.1.1 International Reference Ionosphere (IRI)

The International Reference Ionosphere (IRI) is an empirical model for the large-scale structure of the ionosphere that uses data from ground- and space-based instruments. It is developed by the Committee on Space Research (COSPAR) and the International Union of Radio Science (URSI) and it calculates monthly medians of ionospheric parameters using models for each region of the ionosphere (Bilitza and Reinisch 2008; Bilitza et al. 2011). The IRI provides information about electron density, electron temperature, ion temperature and ion composition, as well as other parameters that describe the plasma in the Earth’s ionosphere between the altitudes of 60–2000 km.

The IRI is regularly updated with both new models and new data for the ionosphere. It uses data from ionosondes, incoherent scatter radars, topside sounder satellites, GPS (Global Positioning System) and rocket observations. Further details of all of these methods of observing the ionosphere are given in Section 1.2. It also provides the user with choices over which models to use, allowing the user to alter the model according to the situation for which they are trying to obtain data. This model only provides data for a nonauroral ionosphere.

The IRI model is less reliable in certain areas and during certain time periods. For example, due to its primary use of GPS and ionosonde data, the F layer of the ionosphere is very well modelled by the IRI, however the D and E layers are not well observed using GPS or ionosonde data due to their lower electron densities. This means that the model of the D and E layers are based upon less data than the model of the F layer and so are less accurate. The model performs best in the northern hemisphere at mid-latitudes because many ionosondes are located at these mid-latitudes. In the same vein, there is also less ionosonde and GPS station coverage over oceans and other uninhabited areas, meaning that large areas have to be interpolated over (Bilitza et al., 2011).

1.1.2 The small-scale distribution of electron density in the ionosphere

At high latitudes, small-scale variations of the electron density in the ionosphere can be caused by substorms during active auroral periods, known as electron precipitation. This is where electrons with energies ranging from tens to hundreds of keV cause ionisation and changes in the ion chemistry of the high-latitude ionosphere. The precipitation is caused by substorms when electrons are injected from the Van Allen radiation belt (Wild et al., 2010). Electrons precipitate along the magnetic field lines in polar regions (at latitudes higher than 65°) and ionise the neutral particles in the ionosphere. Those with
energy greater than 30 keV ionise the neutral particles in the D layer of the ionosphere because they are capable of penetrating below altitudes of 90 km. This can cause an increase in polar cap absorption.

The shortest spatial scale of interest in the ionosphere is typically on the scale of a few km to tens of km. On these scales TEC variations are typically of the order of 0.1% (Intema et al., 2009). These small-scale inhomogeneities in the ionospheric electron densities cause random amplitude and phase shifts of radio waves as they propagate through the medium, this is known as scintillation.

Radio waves passing through the ionosphere show that it has the statistical behaviour of a turbulent medium (Thompson et al., 2001), obeying a spectral density power-law of

\[ P_N(q) \propto k^{-\alpha}, \]  

where \( k \) is the magnitude of the spatial frequency. The motion of a turbulent plasma is chaotic and thus not reproducible, even under controlled laboratory conditions. Instead the average properties can be predicted by a theory and thus reproduced. These small-scale ionospheric features can therefore be seen as stationary and scale-invariant (within a range of scales) and can be described by convenient mathematical functions such as power spectra or structure functions (Meyer and Watkins, 2011).

1.1.2.1 Kolmogorov model for turbulence

In a Kolmogorov turbulence model, large-scale turbulent motion transfers kinetic energy to smaller and smaller turbulent scale sizes until it dissipates. This is known as the turbulent energy cascade (Kolmogorov, 1941). According to the Kolmogorov model, fully-developed, isotropic turbulence (a turbulent flow that statistically appears unchanging in time and uniform in space) should have a steady rate of energy transfer from one scale to the next so that the total energy of the scale sizes does not increase or decrease with time.

Kolmogorov assumed that the characteristics of the turbulent eddies of scale size \( d \) are solely dependent on the scale size and the energy cascade rate, \( \epsilon_c \). The fraction of kinetic energy, \( dE \), contained in eddies with wave numbers \( k \) to \( k + dk \), where \( k = 2\pi/d \), is

\[ dE = E_k(k) \, dk, \]  

where \( E(k) \) is the energy spectrum function of the turbulence with units of m\(^3\)s\(^{-2}\). Pope (2000) shows that the only possible form of the energy spectrum function is

\[ E(k) = C\epsilon_c^{2/3}k^{-5/3}, \]
where $C$ is a unitless constant and the units of $\epsilon_c$ are $m^2 s^{-3}$. $E (k)$ is one-dimensional and is equal to the power spectrum averaged over all directions,

$$E (k) \, dk = P (k) \, dk^D.$$ (1.11)

This means that for a three-dimensional Kolmogorov power spectrum, the power-law in Equation 1.8 has an exponent of $\alpha = 11/3$, whilst for two dimensions $\alpha = 8/3$ and for one dimension $\alpha = 5/3$.

### 1.2 Methods of observing the ionosphere

Observations of the ionosphere provide a multitude of information about the small- and large-scale behaviour of the electron density, as well as information about temperature, ion mass and plasma velocity. Higher layers of the ionosphere, such as the F layer, can be measured directly using probes and satellites. At lower altitudes the ionosphere cannot be observed by satellites, however it is still too high in altitude to be observed with atmospheric balloons. Measurements of the D and E layer are therefore often made using radio wave propagation techniques.

In this section I give a brief overview of some of the current methods of observing the ionosphere, providing more detail about the methods if they are used in this thesis,

- **Incoherent scatter radar** - more detail is given in Section 1.2.1.

- **Coherent scatter radar** - observes non-thermal irregularities such as field-aligned plasma irregularities. Coherent scatter radar uses lower power levels and more compact antennas than incoherent scatter radars because the resulting echoes are stronger than that received by an incoherent scatter radar. The Super Dual Auroral Radar Network (SuperDARN; Chisham et al. 2007) is an example of a coherent scatter radar. It uses coherent backscatter of 8–20 MHz radio waves to observe the high- and mid-latitudes of the ionosphere.

- **Ionosonde** - more detail is given in Section 1.2.2.

- **Rockets** - capable of carrying out in situ measurements of the lower ionosphere. They carry scientific instruments that can measure a range of ionospheric properties.

- **GPS/GNSS** - more detail is given in Section 1.2.3.

- **Topside sounders** - As an ionosonde cannot measure above the $F_2$ peak, an ionospheric sounder is placed in orbit to operate from above and provide a measure of electron density from altitudes greater than the $F_2$ peak (Chapman and Warren, 1968).
1.2.1 Incoherent scatter radar

Incoherent scatter refers to the scattering of an electromagnetic wave by random fluctuations in a gas of particles (such as electrons). Radio waves transmitted by an antenna weakly scatter off free electrons in ionised media, for example the ionospheric plasma, and create an incoherent scatter return which is received by another antenna, as shown in Figure 1.3. Doppler shift caused by the thermal motion of the electrons results in a spread in frequency in the backscatter which can be detected by incoherent scatter radars (Gordon, 1958).

The electron density in the ionosphere is dependent on ion temperature, mass distribution and motion. This is due to the more massive and slower positive ions altering the distribution of the electrons. An incoherent scatter radar causes ionospheric electrons to start to oscillate due to the electric field of the wave transmitted by the radar. These oscillating electrons then radiate electromagnetic waves which have frequency that changes according to the movement of the electron. The electrons partially follow the motion of the much heavier ions. The radar observes a broad spectrum of the observed signal, with a shape that depends on variables, such as the temperature. Therefore this frequency spectrum also provides information about ion and electron temperatures, ion mass and plasma velocity as well as electron concentration.
In order to determine these characteristics, the autocorrelation function (ACF) is used. The ACF of the target is estimated from the ACF of the transmitted radio wave, which can be varied using the impulse response of the receiver and the transmission phase modulation technique. The transmitter radiates a modulated signal, the signal is then scattered back to the receiver, where it is sampled with a time delay corresponding to the altitude range of interest.

Incoherent scatter radars have the potential to continuously monitor the ionosphere with high time and spatial resolution, however this is expensive due to its power usage. Unlike the majority of methods of observing the ionosphere, they are capable of observing the D layer. The incoherent scatter technique is, however, difficult to apply when the nighttime D layer electron density is less than $10^9$ m$^{-3}$ (Hargreaves, 1992).

There are a variety of incoherent scatter radars located throughout the World, such as the European Incoherent Scatter Facility (EISCAT), Sondre Stromfjord, Millstone Hill, Arecibo, and Jicamarca radars. In this thesis I use one of these radars, EISCAT, and so I provide more detail on this radar below.

1.2.1.1 EISCAT

EISCAT (European Incoherent Scatter Facility) was built in 1981 and is a network of incoherent scatter radars distributed over four experimental sites which are used to investigate the Earth’s atmosphere and ionosphere (Folkestad et al., 1983). The EISCAT Svalbard radar (ESR) is located near Longyearbyen (Svalbard, Norway), whilst at Ramfjordmoen (near Tromsø, Norway) there is a transmitter and receiver for the EISCAT ultra high frequency (UHF) radar, as well as the mono-static very high frequency (VHF)
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Figure 1.5: The EISCAT VHF radar located at Ramfjordmoen (near Tromsø, Norway), longitude = 19.14° E, latitude = 69.58° N.

The EISCAT VHF radar. The remaining two EISCAT UHF receivers are located near Kiruna (Sweden) and Sodankylä (Finland).

The EISCAT UHF operates in the 931 MHz band with a peak transmitting power of more than 2 MW. They are fully steerable, parabolic dishes of 32 m. Figure 1.4 shows a picture of the EISCAT UHF dish at Ramfjordmoen. The EISCAT VHF (Baron, 1986) operates in the 224 MHz band and is a 120 × 40 m parabolic cylinder antenna which can be steered mechanically in the meridional plane from vertical to 60° north of zenith. Figure 1.5 shows a picture of the EISCAT VHF.

The EISCAT radars have a range of codes\(^1\) that enable them to be optimised for the relevant geophysical process being studied. For example, the codes used in this thesis are ‘manda,’ which is optimised to observe the D and E layers, and ‘beata,’ which is optimised to observe the F layer. They are capable of observing in the altitude range of 50–2500 km, and can have a temporal resolution of less than one second.

1.2.2 Ionosondes

Ionosondes, also known as radio sounders, transmit a range of frequencies and measure the range, amplitude and phase of the echoes that they receive from the ionosphere (Bibl, 1998). The main principle behind radio sounding and ionosondes is that radio signals reflect from the ionosphere when their frequency is less than the critical frequency.

As shown in Figure 1.1, the ionosphere can be split into multiple layers, each with different electron densities. This causes the refractive index of the ionosphere to vary with height, a feature which ionosondes rely on. As well as being proportional to the

\(^1\)see https://www.eiscat.se/about/experiments2/experiments for details on these observing codes.
electron concentration, the refractive index is also inversely proportional to the frequency of the transmitted wave. To recover the peak electron density of each layer, the ionosonde sweeps through a range of frequencies and finds the frequency at which the peak plasma frequency for that layer is exceeded. The primary difference between an ionosonde and a radar is that radars will generally only transmit at a single frequency.

Ionosondes are relatively cheap to build, and there are therefore a lot of them located all over the World with the majority located at mid-latitudes in the northern hemisphere, as shown in Figure 1.6. The major disadvantage of ionospheric sounding is that it cannot be used to probe altitudes above the peak in the F layer.

1.2.3 GNSS

GNSS (Global Navigation Satellite Systems) is a satellite system that is used to pinpoint a user’s geographic location. It is made up of the GPS (Global Positioning System;
USA), GLONASS (Global Navigation Satellite System; Russia) and Galileo (EU) navigation systems. It can also be used for ionospheric observations as it measures signal propagation delays between receivers and satellites by finding the difference in the delay at two frequencies (1575.42 MHz and 1227.60 MHz for GPS). The global coverage, continuous operation, high temporal resolution and near real-time data acquisition allow for TEC maps to be produced from GNSS data (Mannucci et al., 1999).

The Center for Orbit Determination in Europe (CODE) generates global ionosphere maps using data from approximately 200 GPS sites. These TEC maps are produced by using the dispersive delay to measure the STEC along the line of sight between each satellite and base-station. A thin-layer ionosphere at an altitude of 450 km is then assumed with a geographical resolution of $\Delta_{\text{lon}} = 5^\circ$ and $\Delta_{\text{lat}} = 2.5^\circ$ and a temporal resolution of 2 hours. Interpolation can then be carried out between these grid points. An example of a TEC map created by CODE is shown in Figure 1.7.

TEC maps are also created by the Royal Observatory of Belgium (ROB). Whilst these maps only use GPS data from within Europe, they are more finely gridded, with geographical resolution of $\Delta_{\text{lon}} = 0.5^\circ$ and $\Delta_{\text{lat}} = 0.5^\circ$ and temporal resolution of 15 minutes (Bergeot et al., 2014). An example of a TEC map created by ROB is shown in Figure 1.8.
1.2.4 Riometers

The variation in ionospheric electron density can also be measured using a riometer (Relative Ionospheric Opacity Meter, Honary et al. 2011). A riometer consists of a stationary antenna or group of antennas that measures the incident cosmic noise level at the Earth’s surface, and indirectly measures the ionospheric opacity by subtracting the cosmic noise from a predefined quiet day curve (Browne et al., 1995). The measured variation in the level of cosmic radio noise is caused by absorption which is proportional to the electron density in the lower ionosphere. The measurement of absorption by riometers is therefore a proxy for ionospheric density fluctuations.

The internationally reserved frequency for riometry is 38.2 MHz, so this frequency is often used. If the frequency that the riometry measurements are taken at is too low then the radio signal will be significantly deviated by the ionosphere, and at lower frequencies it will not be able to penetrate the F-layer peak. At low frequencies radio frequency interference (RFI) can also affect the absorption measurements. If the frequency is too high then the amount of absorption that occurs is small, so only major absorption events will be able to be measured as the riometer is not sensitive enough. The power received by the riometer is a function of geographic latitude, sidereal time, antenna beam size and pointing direction.
As shown in Figure 1.6, riometers are primarily located at high-latitudes. Riometers are complementary to the other methods of observing the ionosphere that have been discussed in this section because they observe a lower layer of the ionosphere. This extends the observable layer downwards. Unlike rockets and incoherent scatter radars which are limited to observing these low altitudes for short periods of time (5–20 minutes and a matter of hours, respectively) due to the nature of the method (in the case of rockets) or due to the cost of the method (in the case of incoherent scatter radars), riometers are a relatively cheap method to observe the lower ionosphere for an indefinite period of time.

Riometers are, however, limited in their ability to observe the ionosphere as they are sensitive to a very small range of altitudes and are only capable of measuring the amount of absorption that occurs in their field of view at one frequency. The electron density within the range of altitudes cannot be inferred from this single frequency measurement. Riometers are also limited by their inability to directly determine the altitude at which the radio wave absorption occurs.

1.3 Thesis structure

The following chapter introduces the techniques used in low frequency radio astronomy, outlining the use of interferometers and methods of calibration. The radio telescopes used within this thesis are also introduced. The main effects of the ionosphere on low-frequency radio observations are then discussed, with an emphasis on the three ionospheric effects on radio astronomy that will be covered in the future chapters of this thesis; phase delay, absorption and changes to the measured Faraday rotation.

Chapter 3 introduces how the ionosphere causes propagation delays that result in phase errors in the signals received by ground-based radio interferometers. In this chapter I discuss calibration techniques that assume that this can be corrected for by modelling the ionosphere as a thin layer. The error arising from this thin-layer assumption is investigated and the conditions under which the assumption can be applied to a variety of radio telescopes are explored. Parts of Chapter 3 have been published as Martin et al. (2016a).

In Chapters 4 and 5, a new method of using multi-frequency absorption data to infer an electron density height profile for the lower ionosphere is introduced and explored. Software, named IONONEST, that uses the nested sampling method has been developed to recover electron density height profiles from multi-frequency absorption data. In Chapter 4 IONONEST is tested by being used to fit electron density height profiles to simulated absorption data. In Chapter 5 IONONEST is applied to observed absorption data from the Kilpisjärvi Atmospheric Imaging Receiver Array (KAIRA), a radio telescope located...
in Finland than can be used as a multi-frequency riometer. The resulting electron density height profiles are then compared to electron density height profiles obtained using the co-located EISCAT VHF. Parts of Chapters 4 and 5 have been published as Martin et al. (2016b).

Chapters 6 and 7 concentrate on measuring the turbulence of the ionosphere which affects the radio signal by introducing additional phase errors and changing the Faraday rotation. To measure the turbulence, I observe the change in Faraday rotation of signals received from pulsars. Information about the turbulent spectrum of the ionosphere can then be inferred from these observations. This information can then be used to improve existing calibration techniques. In Chapter 6 simulations of turbulence screens are made to investigate whether information about the turbulent spectrum can be inferred from variations in the Faraday rotation of a simulated radio signal. In Chapter 7 I implement a pulsar observing mode at KAIRA. Methods of calibrating and obtaining rotation measures from the observed data are then applied. The techniques discussed in Chapter 6 of inferring information about the structure of the turbulent ionosphere are then applied to this pulsar data in Chapter 7.
Chapter 2

Radio astronomy

Karl Jansky detected the first astronomical radio waves from the Milky Way in the 1930s (Jansky, 1933), and since the realisation that astronomical sources emit radio waves, radio astronomy has contributed to many great astronomical discoveries. Pulsars, quasars, radio galaxies and the cosmic microwave background (CMB) were all first observed using radio telescopes.

In recent years, low-frequency radio astronomy has become important because the radio spectra of many sources are inverted due to synchrotron self-absorption or free-free absorption. High redshift radio galaxies and pulsars with very steep spectra can also only be observed at low frequencies (van Haarlem et al., 2013).

In this chapter I outline the techniques used in low-frequency radio astronomy and introduce the radio telescopes used in this thesis (Section 2.1). I also discuss the main effects of the ionosphere on low-frequency radio observations that are investigated in this thesis. Section 2.2 introduces phase delays, whilst Section 2.3 discusses absorption and finally Section 2.4 introduces Faraday rotation.

2.1 Interferometers

The best angular resolution, $\theta$, that is achievable by a single radio dish is limited by classical optical diffraction theory,

$$\theta \approx \frac{\lambda}{D}$$

where $\lambda$ is the wavelength and $D$ is the diameter of the telescope. Larger telescopes operating at higher frequencies therefore have better angular resolution. Due to the poor angular resolution of single radio antennas, interferometers were introduced to enable the study of the intensity, polarisation and frequency spectrum of radio sources (Thompson et al., 2001). An interferometer is a radio telescope made up of many small antennas or dishes.
Figure 2.1: A two element interferometer separated by a baseline, $b$, demonstrating how a wavefront from a source will arrive at a different time at each antenna. $\tau_g = (b/c) \sin \theta$ is the geometric delay, where $c$ is the speed of light in a vacuum.

The time delays needed to align the phases of the incoming radio waves at each antenna or dish can be applied by delaying the electrical signal from each detector. This can be done by adjusting cable lengths to delay the electrical signal. The VHF radar in Tromsø (introduced in Section 1.2.1.1) uses cables to adjust its pointing system. In the vertical direction, the radar is mechanically steered, however to adjust the horizontal pointing direction, the cables must be manually changed.

In the design of more recent interferometers, each antenna receives a signal from a source (as shown in Figure 2.1) which is then converted into a complex voltage vector representing the source signal and the receiver noise. Instead of adding extra cable to modify the pointing direction, once the signal has been digitised, a time delay, $\tau_g$, can be added. This is known as delay compensation and allows for the time delay to be changed very quickly and without signal degradation, which in turn allows for the pointing direction of the entire interferometer to be changed.

There are two types of interferometers, those that work by weighted addition of the input signals (known as beamformers), and those that work by multiplying the input signal (known as correlators). In Sections 2.1.1 and 2.1.2 I discuss these two types of interferometers as well as existing telescopes that use this technology.
2.1.1 Adding interferometers

Beamforming is the subsequent addition of the signals after they have been aligned using delay compensation (as discussed in Section 2.1). By creating copies of the signal received and applying different filters and delays to them using a collection of software pipelines, multi-frequency and multi-directional beams can be created. This is known as multiple beamforming, and a demonstration of the process using a two element interferometer is shown in Figure 2.2. Each beam formed by a beamformer receives signal from a certain area in the sky.

Aperture arrays are dense arrays of antennas which can be used with a receiver system to form a beam using the weighted addition of the input signals. This beam can be pointed by applying time delays to the received signal, allowing for the phases of the signals to be aligned. Each detector is capable of receiving incoming radio waves from a wide range of directions.

The use of aperture arrays has exploded in radio astronomy in recent years because an aperture array has a large field of view, can have multiple pointing directions (i.e., can observe multiple sources at once) and is cheaper than a radio dish.
In this thesis I use two beamforming telescopes, these are the Kilpisjärvi Atmospheric Imaging Receiver Array (KAIRA) and the Rawlings Array. I describe these two telescopes below.

2.1.1.1 The Kilpisjärvi Atmospheric Imaging Receiver Array (KAIRA)

KAIRA, the Kilpisjärvi Atmospheric Imaging Receiver Array is located in Kilpisjärvi, Finland (longitude = 20.76° E, latitude = 69.07° N) and is a project of the Sodankylä Geophysical Observatory (Vierinen et al., 2013). It is a radio telescope that uses the antenna system and signal processing of a station in the Low-Frequency Array (LOFAR; van Haarlem et al. 2013; introduced in Section 2.1.2.1) to form multiple beams on the sky (McKay-Bukowski et al., 2015). It was originally designed for radio astronomy but is primarily being used for prototyping work for EISCAT_3D (a new scatter radar capable of providing 3D monitoring of the atmosphere and ionosphere, currently being planned by the European Incoherent Scatter Scientific Association; Tjulin et al. 2013).

KAIRA consists of a High Band Antenna (HBA) array and a Low Band Antenna (LBA) array, shown in Figure 2.3. The LBA array is made up of 48 dual-polarisation, crossed inverted-V-dipole aerials that are scattered quasi-randomly in a circle of diameter 34 m. The HBA array is made up of 48 tiles which each contain 16 crossed-bowtie antenna elements. The KAIRA HBA array operates at 110–270 MHz, whilst the LBA array operates at 10–80 MHz. The layout of the LBA and the HBA arrays at KAIRA are shown in Figure 2.4.

The typical sampling clock frequency used at KAIRA is 200 MHz, providing a bandwidth of 100 MHz across. This received signal is then split by a polyphase filter bank into 512 sub-bands, each of width 0.195 MHz. When using 16-bit sampling the HBA and LBA
arrays can select up to 244 of these sub-bands, allowing up to 244 different pointing directions coupled with a frequency. These are known as beamlets. For 4- or 8-bit sampling, 976 or 488 beamlets can be used, though by reducing the sampling depth, the dynamic range worsens.

The power that KAIRA measures is dependent on the total system temperature, \( T_{\text{sys}} \). \( T_{\text{sys}} \) is the noise from the whole system and includes both the noise measured from the sky (\( T_{\text{sky}} \)) and noise due to the receiver system, \( T_{\text{instrument}} \).

\[
T_{\text{sys}} = T_{\text{sky}} + T_{\text{instrument}}.
\]  

(2.2)

The noise due to the receiver system is created by thermal noise in the electronic components that make up the array, caused by thermal agitation of free electrons, it typically has a noise spectrum that is flat with respect to frequency, known as ‘white noise’. The noise in each component is normally independent of noise from the other components, and the power due to the noise is given by (Thompson et al., 2001)

\[
P = k_B T \Delta \nu.
\]

(2.3)

where \( k_B \) is Boltzmann’s constant and \( \Delta \nu \) is the bandwidth. The total noise of all the components is the instrument noise, \( T_{\text{instrument}} \), and is calculated using the system equivalent flux density, \( S \), which has been measured for KAIRA by McKay-Bukowski.
et al. (2015). This allows the true power received from the sky, \( P_{\text{sky}} \), to be calculated as the instrumental noise can be calculated from the total power received by KAIRA, \( P_{\text{sys}} \), using

\[
P_{\text{sys}} = B (kS + P_{\text{sky}}),
\]

where \( B \) is the dimensionless bandpass (which has also been calculated by McKay-Bukowski et al. 2015) and \( k \) is a conversion factor which is needed to convert \( S \) from kJy to Analog Digital Units (ADU), assuming the gain is constant. \( k \) is required because \( P_{\text{sys}} \) and \( P_{\text{sky}} \) are both measured in ADU.

The primary science aims of KAIRA are to make bistatic measurements using multiple beams of the co-located EISCAT incoherent scatter radars at Ramfjordmoen, Norway, as well as to observe interplanetary and ionospheric scintillation, to observe radio emission from the Sun and to act as a riometer (further details on this use of KAIRA are given in Section 2.3.4.1).

### 2.1.1.2 The Rawlings array - UK608

The Rawlings array (Best and the LOFAR-UK Consortium, 2008) is a beamforming interferometer that is based in Chilbolton, UK, (longitude = 1.43° W, latitude = 51.14° N). It forms one of the international stations of the LOFAR array (which will be discussed in Section 2.1.2.1). Similarly to KAIRA, and like all LOFAR international stations, the Rawlings array comprises two antenna arrays: a HBA array made up of 96 tiles of 16 dual-polarised bowtie antennas operating at 120–240 MHz, and a LBA array made up of 96 dual-polarised antennas operating at 30–80 MHz.

The Rawlings array has a total of 244 useable imaging beams in 16-bit mode, each of which can be assigned a different frequency and a different pointing direction. When not used as part of the LOFAR array, it is typically used to search for pulsars and fast transients.
Figure 2.6: Two stations of a correlating interferometer separated by a baseline, \( b \), demonstrating how the signals from different stations are combined.

### 2.1.2 Correlating interferometers

Unlike a beamformer which receives a 1D signal from a certain area in the sky, a correlator outputs visibilities that may be used to create 2D images of the sky. The correlating interferometer is made up of antennas or stations that are pointed at an area of sky. As an example, Figure 2.6 shows two stations of a radio interferometer. The signals from the two stations are combined to measure a visibility on the baseline, labelled as \( b \) in Figure 2.6. The two spatially separated stations, \( p \) and \( q \), independently measure the voltage vectors, \( v_p \) and \( v_q \), which are then input into a correlator. This results in the correlations \( \left\langle v_{p1} v_{q1}^* \right\rangle \), \( \left\langle v_{p1} v_{q2}^* \right\rangle \), \( \left\langle v_{p2} v_{q1}^* \right\rangle \) and \( \left\langle v_{p2} v_{q2}^* \right\rangle \), where 1 and 2 refer to the different polarisation of each antenna feed. These correlations can then be written as a visibility matrix (Smirnov, 2011),

\[
\mathbf{V}_{pq} = 2 \begin{pmatrix} \left\langle v_{p1} v_{q1}^* \right\rangle & \left\langle v_{p1} v_{q2}^* \right\rangle \\ \left\langle v_{p2} v_{q1}^* \right\rangle & \left\langle v_{p2} v_{q2}^* \right\rangle \end{pmatrix}.
\]  

Each visibility is measured per baseline and per orientation with respect to the sky. The visibilities in the \( u-v \) plane (where \( u \) and \( v \) are the components of the spatial frequency) are the Fourier transform of the image on the sky. The correlations on baseline vectors therefore sample the visibilities at points in the \( u-v \) plane. The Earth’s rotation is therefore taken advantage of as it changes the orientations of the baselines with respect to the sky, allowing the \( u-v \) plane to be sampled more completely.

A measure of the errors on the image produced from these visibilities is given by the dynamic range. The dynamic range is the ratio of the peak brightness in the image and the root mean squared noise of a radio-quiet region (a region believed to be void of radio emission). The noise in the radio-quiet region provides a lower limit to the error in the brightness of non-radio-quiet regions. If the \( u-v \) plane is adequately covered and the data is well-calibrated, then the dynamic range can be assumed to be a good indicator...
of image reliability, providing a lower limit to the true, position-dependent errors in the image produced from the visibilities (Perley, 1999).

Perley (1999) defines an equation for calculating the dynamic range, $DR$ for $N$ antennas,

$$DR = \frac{N(N - 1)}{\sqrt{2\phi}} \approx \frac{N^2}{\sqrt{2\phi}},$$

where $\phi$ is a random phase error. Extending this to a case where there are many antennas with random phase errors on each baseline, then the dynamic range is divided by a factor of $\sqrt{N(N - 1)/2}$ (Perley, 1999),

$$DR = \frac{\sqrt{N(N - 1)}}{\phi} \approx \frac{N}{\phi}.$$  

(2.7)

The true visibilities will always be different to the visibilities observed using the interferometer because the interferometer is not a perfect instrument, and the incoming radio waves are affected by factors outside of the observer’s control, such as ionospheric effects, instrumental noise and radio frequency interference (RFI). Data that are affected by these factors can be excluded from the data set (‘flagged’) or the ionospheric or instrumental effect can be corrected for using mathematical models and self-calibration implemented in software packages such as AIPS (Astronomical Image Processing System), Miriad and CASA (the Common Astronomy Software Applications package).

Self-calibration is a technique pioneered by Cornwell and Wilkinson (1981) that allows radio astronomers to correct direction-independent effects such as errors associated with antennas. These errors include telescope pointing errors, gain-elevation dependence and changes in the relative delay caused by instrumental effects, atmospheric effects and imperfect knowledge of the position of the source and the position of the antenna. Much of the software available for calibration in radio astronomy, such as CASA, applies direction-independent self-calibration. In more recent years, self-calibration has advanced to also become capable of dealing with direction-dependent effects (effects that vary across the field of view).

In this thesis I discuss five correlating interferometers, these are

- **LOFAR** - Low-Frequency Array
- **VLA** - Karl G. Jansky Very Large Array
- **GMRT** - Giant Metrewave Radio Telescope
- **MWA** - Murchison Widefield Array
- **SKA** - Square Kilometre Array

I describe these five telescopes below.
2.1.2.1 LOFAR

The LOw-Frequency ARray (LOFAR; van Haarlem et al. 2013) is a phased-array radio interferometer which uses digital beam-forming in order to observe multiple sources at once, as well as being able to quickly repoint the array. The array is a pathfinder for the Square Kilometre Array (SKA; discussed in Section 2.1.2.5) and is made up of many aperture array stations with a core located in the northeast of the Netherlands (longitude = 6.87° E, latitude = 52.91° N). A further 18 stations are located in the Netherlands within 100 km of the core and several stations are located across Europe, in Germany, UK (UK608; see Section 2.1.1.2 for further details), France and Sweden, providing a current maximum baseline of 1158 km (van Haarlem et al., 2013). Stations are also being built in Poland and Ireland.

As discussed in reference to KAIRA (a telescope based on LOFAR technology; Section 2.1.1.1) and the Rawlings Array (part of LOFAR; Section 2.1.1.2), each LOFAR station is made up of low-cost antennas which comprise two arrays, the LBA array and the HBA array. The LBA observes between 10 and 90 MHz and the HBA observes between 110 and 250 MHz. Each of these individual LOFAR stations performs delay compensation and beamforming to combine the antenna signals into a station beam, then the signals from all the different stations are streamed to a central processing facility and correlated to produce visibilities.

The key science goals of LOFAR are to observe the high-redshifted 21-cm emission line from neutral hydrogen as a probe for studying the cosmic dawn and the epoch of reionisation, surveying the low-frequency sky and observing transients, pulsars and magnetic fields.

2.1.2.2 The Karl G. Jansky Very Large Array

The Karl G. Jansky Very Large Array (VLA; Perley et al. 2011) is a radio interferometer located in New Mexico, USA (longitude = 107.62° E, latitude = 34.08° N) that operates at 58 MHz–50 GHz. It is made up of 27 antennas, each of 25 m diameter that are positioned on three 21 km arms. The positions of the antennas can be changed, allowing the maximum baseline length to vary between 1 km and 36 km. The four possible configurations are A-array (maximum baseline of 36 km), B-array (maximum baseline of 10 km), C-array (maximum baseline of 3.6 km) and D-array (maximum baseline of 1 km).

The key science goals of the VLA are measuring the strength and topology of cosmic magnetic fields, observing objects that are shrouded by dust and are thus obscured at other wavelengths, observing transient sources, and tracking the evolution of objects in the Universe.
2.1.2.3 Giant Metrewave Radio Telescope

The Giant Metrewave Radio Telescope (GMRT; Swarup 1991) consists of 30 parabolic dishes that are 45 m in diameter. Fourteen of these dishes are placed randomly in a $1 \times 1$ km$^2$ area, with the other sixteen placed along three 14 km arms in a ‘Y’ shape, providing a maximum baseline of 25 km. The GMRT is located near Pune, India (longitude = 74.05° E, latitude = 19.09° N) and can operate in six frequency bands with central frequencies of 153, 233, 325, 610 and 1420 MHz.

Its primary science goals include searching for the highly redshifted 21 cm line from neutral hydrogen that is expected to have been emitted during galaxy formation in the early stages of the Universe, as well as studying and discovering pulsars and neutron stars, and observing galactic and extragalactic radio sources.

2.1.2.4 Murchison Widefield Array

The Murchison Widefield Array (MWA; Tingay et al. 2013) is a precursor to the SKA (Section 2.1.2.5). It is located 800 km north of Perth, in the mid-west of Western Australia (longitude = 116.67° E, latitude = 26.70° S), and operates at 80–300 MHz. The MWA is made up of 128 tiles with each tile consisting of 16 crossed dipoles. Fifty of these tiles make up a dense core which is 100 m in diameter, whilst 62 tiles are placed within a circle of diameter 1.5 km, and the remaining 16 are placed within a circle of diameter 3 km.

As well as being a precursor to the SKA, the primary science goals of the MWA are to detect fluctuations in the 21 cm neutral hydrogen line from the epoch of reionisation, to observe transients and to observe the heliosphere and the ionosphere.

2.1.2.5 The Square Kilometre Array

The Square Kilometre Array (SKA; Dewdney et al. 2013; Dewdney et al. 2016) was originally proposed by Wilkinson (1991) as a a telescope that has a collecting area of one square kilometre with the primary science goal of being able to detect neutral hydrogen at cosmological distances.

The SKA will have cores in Australia (located close to the MWA) and in South Africa. It will be split into two sub-arrays, SKA-low and SKA-mid, and there will be two stages of construction — SKA1 and SKA2. SKA-low will observe in the frequency range 50–350 MHz and will consist of a phased array of log-periodic, dual-polarised antennas. In the first phase of the SKA-low, SKA1-low, there will be approximately 131,000 of these antennas, with the majority located in a compact core of 1 km diameter. The remainder will be arranged in stations that are a few 10s of metres in diameter. In total there
will be approximately 512 stations that are distributed over an area of diameter 80 km. SKA-mid will be an array made up of dish antennas that observe in the frequency range of 0.35–14 GHz.

### 2.1.3 Effects of the ionosphere on radio astronomy

All ground-based radio telescopes have to look through the ionosphere in order to observe astronomical radio sources. The ionosphere affects radio observations in a variety of ways, including Faraday rotation, group delay, phase change and stability, frequency stability, absorption, and refraction (Thompson et al., 2001). Many of these effects can be mitigated by observing at higher frequencies as they scale with $\nu^{-2}$, however for science goals that require observations at low frequencies, the three main ionospheric effects are:

- Phase delays
- Absorption of radio waves
- Changes to the measured Faraday rotation

These three ionospheric effects are the main subjects of this thesis. In the following sections I elaborate on these effects.

### 2.2 Phase delays

Radio waves from astronomical sources, particularly at low frequencies, experience propagation delays due to the low-density plasma of the Earth’s ionosphere which results in phase errors in the visibilities. This effect is of concern in radio interferometry because it affects measurements of the phase differences between pairs of antennas that are used by radio synthesis telescopes to reconstruct images of radio emission from the sky.

Radio interferometry depends upon measurements of the phase difference between pairs of antennas separated by baseline vectors. If the lines of sight to a radio source from two antennas have different electron densities, the ionosphere will introduce a different phase on each antenna, causing an error in the phase measurement. For each antenna the propagation delay results in a phase rotation along the line of sight of

$$\phi_{\text{ion}} = -\frac{2\pi \nu}{c} \int (n - 1)dl,$$

(2.8)

where $\nu$ is the radio frequency, $c$ is the speed of light in a vacuum and the refractive index, $n$, can be written as,

$$n^2 = 1 - \frac{\nu_p^2}{\nu^2}.$$  

(2.9)
Figure 2.7: Taken from Lonsdale (2005). Low-frequency array calibration regimes for the ionosphere. $A$ is the longest baseline of the array, $V$ is the size of the field of view at ionospheric altitudes and $S$ is the scale size of the irregularities in the ionosphere.

The plasma frequency, $\nu_p$, is described as

$$\nu_p = \frac{e}{2\pi} \sqrt{\frac{N_e}{\epsilon_0 m_e}}, \quad (2.10)$$

where $e$ is the electron charge, $m_e$ is the electron mass and $\epsilon_0$ is the permittivity of free space. Typically $\nu_p$ is in the 1–10 MHz range and therefore if it is assumed that the observational frequencies are significantly larger than the plasma frequency ($\nu \gg \nu_p$), then $\phi_{\text{ion}}$ can be approximated as

$$\phi_{\text{ion}} \approx \frac{\pi}{c\nu} \int \nu_p^2 dl. \quad (2.11)$$

Therefore, by substituting in Equation 2.10 it can be shown that $\phi_{\text{ion}}$ is directly dependent on TEC,

$$\phi_{\text{ion}} \approx \frac{e^2}{4c\nu\pi\epsilon_0 m_e} \int N_e dl. \quad (2.12)$$

In low-frequency radio observations, there is typically substantial variation in the phase delay due to ionospheric structure across the field of view, as illustrated in regimes 3 and 4 from Lonsdale (2005), reproduced in Figure 2.7. Methods of calibration that only determine one phase correction across the field of view for each antenna are therefore inadequate. In Section 2.2.1, I describe a method of defining different phase corrections across the field of view by assuming that the ionosphere can be adequately modelled using a thin-layer assumption.
Figure 2.8: Typical profile of ionospheric electron density, showing layers of the ionosphere and some altitudes assumed in applications of the thin-layer model.

2.2.1 The thin-layer assumption

The dominant phase errors are assumed to originate from a limited height range in the ionosphere, near the altitude of maximum electron density in the F-layer. Because the ionospheric electron content is concentrated at a narrow range of altitudes (see Figure 2.8), it is typically modelled as a spherical shell of infinitesimal thickness, centred on the Earth. The assumed altitude of this thin layer above the Earth’s surface varies, ranging from 200 km to 500 km (Nava et al., 2007; Cohen and Röttgering, 2009; Intema et al., 2009; Intema, 2014b).

In the thin-layer model, the ionosphere is represented as a thin spherical shell surrounding the Earth at a fixed altitude, \( h \), above the mean Earth surface. The true distribution of ionospheric electron content with altitude shown in Figure 2.8 is neglected, and instead described only by the equivalent TEC along a vertical column (vertical TEC; VTEC) at each point on the shell. The slant TEC (STEC) along a line of sight can then be calculated based on the VTEC at the point at which the line of sight pierces the thin layer, as

\[
\text{STEC} = \frac{\text{VTEC}}{\cos \theta},
\]

where \( \theta \) is the angle of the line of sight to the thin layer at the pierce point.

The thin-layer model is not a perfect representation of the ionosphere, and under certain conditions — a telescope operating with widely-separated antennas at low frequencies, or observations with a highly active ionosphere — the error associated with this approximation may limit the dynamic range of the resulting radio image. Intema et al.
Figure 2.9: Illustration of $n_{\text{ant}} = 3$ antennas, part of a radio synthesis telescope, simultaneously observing $n_{\text{src}} = 2$ sources (in the far field). This results in $n_{\text{ant}} \times n_{\text{src}} = 6$ pierce points in the ionosphere (in the near field), which may be used for ionospheric calibration. This figure is taken from Martin et al. (2016a).

(2009) state that it is unclear under what conditions this occurs. It is possible to use more complex models to avoid this error. For example, Intema et al. (2011) model the ionosphere as three distinct layers at 100 km, 200 km, and 400 km.

Ionospheric calibration can correct for the substantial variation in the phase delay due to ionospheric structure across the field of view, as shown by regimes 3 and 4 in Figure 2.7 (Lonsdale, 2005), by using bright sources to form a model for the ionospheric structure. Corrections can then be applied for the ionospheric phase delay elsewhere in the field of view. In a typical radio observation at frequencies low enough to require ionospheric calibration, there are $n_{\text{ant}} \gtrsim 20$ antennas observing $n_{\text{src}} \gtrsim 10$ radio sources bright enough to be used for ionospheric calibration in a single field of view (van Weeren et al., 2009; Intema et al., 2011; Wykes et al., 2014), which leads to $n_{\text{ant}} \times n_{\text{src}} \gtrsim 200$ pierce points at which information can be obtained about the ionospheric electron content. Figure 2.9 illustrates this for three antennas observing two sources, resulting in six pierce points, whilst Figure 4 of Intema et al. (2009) provides an example of ionospheric pierce points used in a real observation.

Substituting Equation 1.7 into Equation 2.12, it can be seen that the ionospheric phase delay is (Intema et al., 2009)

$$
\phi \approx \frac{e^2}{4\pi\varepsilon_0 m_e c \nu} \text{STEC} \quad \text{(2.14)}
$$

$$
\approx 4840^\circ \times \left( \frac{\nu}{100 \text{ MHz}} \right)^{-1} \left( \frac{\text{STEC}}{\text{TECU}} \right), \quad \text{(2.15)}
$$
where STEC is the electron density along the line of sight. If the distribution of ionospheric electron content can be determined, the difference in STEC between two lines of sight can be calculated, and the resulting phase error is then subtracted from the measurements.

### 2.2.2 SPAM

SPAM (Source Peeling and Atmospheric Modeling; Intema et al. 2009; Intema 2014a) is a widely-used method for performing ionospheric calibration that uses a semi-automated set of data reduction scripts that include direction-dependent calibration and imaging. Direction-dependent ionospheric phase errors are measured by iteratively peeling bright sources within the field to determine the STEC along the line of sight between each source-antenna pair, relative to an arbitrary reference value. These errors are then modelled using a single- or multi-layer ionospheric phase model before the phase corrections are applied and the field is re-imaged. SPAM is capable of returning continuum images that are comparable to self-calibration.

In the original implementation of SPAM, each line of sight is assumed to pierce the ionosphere at a single point that is described by the thin-layer model. The STEC at each of these pierce points is then converted to an equivalent VTEC. Once a range of VTEC values have been calculated over the field of view, a power-law spectral density model can be fitted to these points, allowing the VTEC to be interpolated across the footprint of the field of view on the ionosphere. These VTEC values are converted back to STEC during the imaging process, allowing for the phase corrections for each point on the image to be calculated using Equation 2.15. This procedure corrects for horizontal variation in the ionospheric electron content and, when repeated for successive time intervals, it can also correct for temporal variation.

### 2.3 Absorption

Absorption is caused by collisions of electrons with ions or neutral particles. The incoming radio wave will affect the motion of any free electrons that are present and if these electrons subsequently collide with any of the neutral atoms or molecules in the ionosphere, then the energy is transferred to the heavier particle causing absorption of the radio signal. If the electron does not collide with any of the neutral atoms or molecules then the energy is re-radiated.

The electron density $N_e(h)$ as a function of altitude, $h$, is related to the radio wave absorption by (McKay-Bukowski et al., 2015)

$$A(\text{dB}) = 4.6 \times 10^{-5} \int \frac{N_e(h)\nu_{en}(h)}{(\omega \pm \omega_g)^2 + \nu_{en}(h)^2} dh,$$

(2.16)
where $\omega$ is the angular frequency of the radio wave, $\nu_{en}(h)$ is the electron-neutral collision frequency and $\omega_g$, is the electron gyrofrequency which is defined by

$$\omega_g = \frac{e|B|}{m_e c},$$

(2.17)

where $e$ is the charge on the electron, $m_e$ is the mass of the electron, $c$ is the speed of light in a vacuum and $|B|$ is the magnitude of the geomagnetic field. To calculate the electron gyrofrequency, the magnetic field is approximated as a dipole,

$$|B| = B_0 \left( \frac{R_E}{h + R_E} \right)^3 \sqrt{1 + 3\cos^2(\theta)}. \quad (2.18)$$

$B_0$ is the mean value of the magnetic field at the magnetic equator on the Earth’s surface, $R_E$ is the radius of the Earth and $\theta$ is the azimuth.

From Equation 2.16, it can therefore be seen that the absorption of radio waves is dependent on the electron-neutral collision frequency as a function of altitude as well as the electron density. Figure 2.10 shows typical values for the electron density and the electron-neutral collision frequency in the lower ionosphere.

The electron-neutral collision frequency is proportional to neutral density and therefore causes absorption to occur more at lower layers of the ionosphere, such as the D layer. It can be found using the number density of atmospheric constituents and the electron temperature as calculated from the NRL-MSISE-00 reference atmosphere (Picone et al.,
I consider five atmospheric constituents: atomic oxygen, hydrogen and helium, and molecular oxygen and nitrogen. The electron-neutral collision frequency can then be calculated using (Banks and Kockarts, 1973)

\[ \nu_{en} = \nu_{N_2} + \nu_{O_2} + \nu_O + \nu_H + \nu_{He}, \]  

where

\[ \nu_{N_2} = 2.33 \times 10^{-11} n_{N_2} (1 - 1.2 \times 10^{-4} T_e) T_e \]  
\[ \nu_{O_2} = 1.8 \times 10^{-10} n_{O_2} (1 + 3.6 \times 10^{-2} \sqrt{T_e}) \sqrt{T_e} \]  
\[ \nu_O = 8.2 \times 10^{-10} n_O \sqrt{T_e} \]  
\[ \nu_H = 4.5 \times 10^{-9} n_H (1 - 1.35 \times 10^{-4} T_e) \sqrt{T_e} \]  
\[ \nu_{He} = 4.6 \times 10^{-10} n_{He} \sqrt{T_e}. \]  

\[ n_X \] is the density of the atmospheric constituent \( (\text{cm}^{-3}) \) and \( T_e \) is the electron temperature \( (\text{K}) \).

Absorption is at a maximum in the D layer as the point at which \( \nu_{en}(h) \) reaches a peak is in this layer. Changes in pressure which affect the electron-neutral collision frequencies are small, therefore most deviations in absorption that occur are caused by changes in the electron density of the D layer.

### 2.3.1 Rhiometry

To observe the amount of absorption caused by the ionosphere a constant, stationary radio source is required. The cosmic radio noise is used for this purpose. As outlined in Section 1.2.4, a stationary antenna beam can then measure this cosmic radio noise as a function of the sidereal day, allowing comparisons to be made between sidereal days to recover the amount of absorption that occurs.

The cosmic noise absorption caused by the ionosphere, \( A \), can then be found using

\[ A(\text{dB}) = 10 \log \frac{P_0}{P}, \]  

where \( P_0 \) is the power that would be received if there was no absorption, and \( P \) is the actual power received by the radio telescope. \( P_0 \) can be given by the frequency dependent power in the quiet day curve, and using this and the measured power, \( P \), the absorption value for each time, frequency and pointing direction can be calculated. This absorption is dependent on the inverse square of the frequency.

The rhiometry technique of observing rapid variations in the level of absorption of cosmic radio noise as a proxy for ionospheric density fluctuations is limited by its inability to directly determine the altitude at which the radio wave absorption occurs. However
direct observations of the D layer are expensive and produce only a single profile, as
the D layer is observed primarily using rocket-borne instruments which produce only
one set of data per flight. Another method to observe the D layer of the ionosphere,
incoherent scatter radars, is detailed in Section 1.2.1, however this radar data is still
limited because it is less sensitive than the data obtained using rocket-borne instruments
and the incoherent scatter radars are both expensive to build and do not give a good
coverage of different latitudes as there are a limited number of them, mainly located
around the poles.

Riometers are primarily used to study ionospheric processes, including the precipita-
tion from the magnetosphere into the ionosphere of charged particles, caused by solar-
terrestrial interactions. These precipitation events cause a lot of energy transfer, with
electrons ranging from energies of tens to hundreds of keV penetrating into the iono-
sphere and mesosphere at high latitudes, affecting the ion chemistry and even being
linked to the destruction of ozone (Honary et al., 2005). Whilst riometers cannot di-
rectly monitor the precipitation, they can provide a proxy for the occurrence of this
electron precipitation, via observations of the increase in electron density in the D layer,
as riometers are most sensitive to ionospheric absorption occurring at altitudes between
50 and 110 km. At high-latitudes, this absorption is predominantly due to electron
precipitation (Hargreaves 1995; Kavanagh et al. 2002). Knowledge of these processes
informs us about space weather and enables us to communicate using radio communi-
cations in the 3–30 MHz band.

2.3.2 Quiet day curves

A riometer measures the amount of absorption that occurs in the ionosphere indirectly.
It is standard practice in riometry to define a quiet day curve (QDC) which represents the
power level that is received on a day when minimal absorption occurs and the ionosphere
is assumed to be transparent. The actual received signal power which varies depending
on the amount of absorption that is occurring in the ionosphere is subtracted from this
QDC. This QDC accounts for the received power level varying with sky temperature
in the pointing direction of the beam. The rotation of the Earth and the apparent
motion of the radio sky causes variation in the power received by the riometer. The
Sun also causes increases in the received power as well as ionisation in the D layer at an
altitude of about 80 km during daylight hours. This periodic variation of absorption in
the ionosphere is dependent on the Sun’s zenith angle and thus also contributes to the
characteristic shape of the QDC.

Quiet day curves are calculated in terms of sidereal time. Sidereal time is a method
of measuring time relative to the celestial sphere rather than relative to the Sun. The
sidereal second is shorter than the solar second because the Earth rotates faster relative
to stars than to the Sun by one day per year. By using sidereal time, the only difference
in location on the sky of stars is due to precession effects. QDCs need generating regularly throughout the year, as well as throughout the 11 year solar cycle, because the background solar ionisation varies according to time of year and solar cycle.

A variety of empirical methods of creating quiet day curves have been determined by different authors since the first riometer (Armstrong et al. 1977; Krishnaswamy et al. 1985; Drevin and Stoker 1990; Browne et al. 1995; Rodger et al. 2013). In addition to these empirical methods, QDCs can also be simulated (Grill, 2007). These simulated QDCs are inherently inaccurate because they are generally based on sky maps which contain inaccuracies due to sidelobe effects from the instrument that was used to create the sky map, different frequencies to that used for the riometer, and gaps in the observations that have been filled with interpolated data.

The majority of the empirical methods involve plotting the received signal according to sidereal time, and then either using visual estimation or a calculation using a percentage criterion to determine the QDC. Below I discuss the percentage criterion method, with particular emphasis on the methods detailed in Browne et al. (1995) and Rodger et al. (2013).

The percentage criterion method defines the QDC as the curve which separates a certain percentage of the data points in the distribution from the remainder (often a 10%, 90% split). The percentile values are often chosen qualitatively, or using visual estimates, often leading to arbitrary values being chosen. Armstrong et al. (1977) tried to reduce the arbitrariness with which this value was chosen by using a value corresponding to the inflection point on the high-signal side of the data distribution. This method is discussed and further advanced in Krishnaswamy et al. (1985).

Browne et al. (1995) discuss a method for generating a QDC that was developed for IRIS (Section 2.3.3). In this method, data are arranged into sidereal days, and 10 minute averages are calculated over each day. Each time interval corresponding to the same period on each day is then sorted in descending order of power, and the mean of the second and third highest values for each time interval is calculated. This mean is assumed to be a good estimate of quiet time absorption for that time interval in the QDC. Once a value for each time interval has been calculated, interpolation can be performed to obtain quiet values for each second of the sidereal day. Browne et al. (1995) find that this method requires minimal amounts of computation, yet gives curves that are accurate to 0.1 dB for IRIS.

An advanced variant of the method of Browne et al. (1995) is detailed in Rodger et al. (2013). Similar to Browne et al. (1995), contiguous data is smoothed with a median filter of at least 599 s prior to being binned in accordance with sidereal time. Each time bin is then sorted in descending order, and a mean of two of the values in that time bin is calculated. The choice of values depends on whether the observation period is a geomagnetically quiet time or whether it is a more active period. For geomagnetically quiet
periods, the fourth and fifth largest values are chosen for calculating the mean, whereas for more active periods the second and third largest values are chosen for calculating the mean.

Occasionally negative absorption is seen in the absorption curves created by riometers. This negative absorption can be explained to some extent by the random nature of the received signal, however if the negative absorption event is strong then it can be assumed that more power has been received than was expected. This can be due to solar activity (such as radio bursts), lightning and RFI (Grill, 2007).

### 2.3.3 Existing riometers

All existing riometers are made up of a single antenna or group of antennas that are connected to a receiving device that measures the cosmic background noise. The absorption is then calculated from these measurements as detailed in Section 2.3.1.

Wide-beam riometers have been used since the late 1950s. They are made up of an antenna which has a beam with an azimuthally symmetric radiation pattern (Honary et al., 2011) of an order of 100 km on the sky (Browne et al., 1995). This results in the absorption measured being averaged over a large area of sky. This means that wide-beam riometers have the disadvantage of being unable to observe structure smaller than about 100 km and have no imaging capabilities.

In the 1960s, riometers consisting of larger antennas were constructed, enabling smaller ionospheric structures to be observed. Using multiple antennas several narrow beams can be simultaneously formed, using a Butler matrix to allow for beamforming. If all the received signals are added together at the same phase then just one beam is formed, however if they are added together with phase differences then many beams are formed (Nielsen and Hagfors, 1997).

Nielsen and Greenwald (1978) compared the absorption detected with a wide-beam riometer with the absorption detected with a narrow-beam riometer. Figure 2.11 demonstrates the differences in absorption measured by the two riometers (located in Ramfjordmoen, Norway). Nielsen and Greenwald (1978) discuss how a wide-beam riometer should measure more absorption than a narrow-beam riometer assuming that the absorption is homogeneous in the beam. A wide-beam riometer consistently sees more absorption because contributions from the side have a longer path through the absorbing ionosphere. If the absorption measured by the wide-beam riometer is less than that measured by the narrow-beam riometer, then it indicates that the absorption feature in the D layer of the ionosphere was of a limited spatial extent.

Another type of riometer is a scanning riometer which can be used to observe variations in cosmic background noise that occur over a large range of azimuth angles. They
Figure 2.11: Comparison of absorption as measured by a narrow-beam riometer and a wide-beam riometer. The absorption measured by the wide-beam riometer should always be greater than the absorption measured by the narrow-beam riometer. If the absorption measured by the wide-beam riometer drops below the absorption measured by the narrow-beam riometer then it indicates an absorption feature that only partially fills the beam of the riometer. This can be seen at 20:43 UT, 20:46 UT and 20:57 UT. Taken from Nielsen and Greenwald (1978).

therefore have an advantage over wide-beam riometers because wide-beam riometers have difficulty observing such azimuth angles. A scanning riometer is similar to a narrow-beam riometer, however they use electronic phase shifters to scan one beam from north to south and another beam from east-west (Kikuchi et al., 1988). This means that they can only observe a cross-shaped area of sky.

Imaging riometers extend narrow-beam riometers, adding more antennas in a phased array system. They provide higher spatial resolution and, unlike wide- and narrow-beam and scanning riometers, allow for simultaneous observations of a large area of sky. They can create a wide beam by using a single antenna. An imaging riometer can provide more information on the spatial extent of electron precipitation than an incoherent scatter radar, which have a relatively narrow beam.

An imaging riometer known as the Imaging Riometer for Ionospheric Studies (IRIS) was developed in 1988 (Detrick and Rosenberg, 1990). Many of these imaging riometers are now located all over the World. An example is the IRIS imaging riometer located in Kilpisjärvi, Finland (Figure 2.12). This riometer comprises 64 antennas which form 49 beams using a 2D analogue Butler matrix. It is circularly polarised and operates at a frequency of 38.2 MHz.
Mills cross riometers rely on the cross-correlation of signals from two perpendicular fan beams (Mills, 1952), allowing for even higher spatial resolution than imaging riometers (Honary et al., 2011). In a Mills cross riometer, the antennas are distributed in a cross shape on the ground. Each arm of the cross forms several fan beams. When two perpendicular fan beams (a fan beam from each arm of the cross) are cross-correlated, a narrow pencil beam is then formed (Nielsen et al., 2004). As there are many combinations of fan beams from the two arms, many pencil beams are therefore formed on the sky. In comparison to an imaging riometer, a Mills cross riometer is capable of achieving higher spatial resolution with fewer antennas, however the antenna array has a reduced aperture and therefore the achievable noise performance is worse.

The Advanced Rio-Imaging Experiment in Scandinavia (ARIES, Honary et al. 2011) is an example of a Mills cross riometer which has two arms of 32 circularly polarised, crossed dipole antennas that have azimuthally symmetric radiation patterns. The antennas have a spacing of $\lambda/2$ between them. It is located in Ramfjordmoen, near Tromsø, and has a beam width of 6.56° at zenith, which overlaps with the field of view of the IRIS system.

### 2.3.4 Application of riometry to low-frequency radio telescopes

The majority of radio telescopes discussed in Sections 2.1.1 and 2.1.2 operate at similar frequencies to existing riometers, as well as having similar characteristics such as the ability to form a single beam like a narrow-beam riometer or multiple beams like an imaging riometer. Using a radio telescope as a multi-beam, multi-frequency riometer is advantageous as they have increased sensitivity due to the extra bandwidth.
Table 2.1: Pointing directions of KAIRA during the 2013 observing campaign and of the Rawlings array during all observations. The telescopes were pointed at zenith, the North Celestial Pole (NCP) and two of the A-Team sources, Cygnus A (3C405) and Cassiopeia A (3C461). Multiple frequencies are observed between the ranges displayed below.

<table>
<thead>
<tr>
<th>Pointing Direction</th>
<th>Elevation (degrees)</th>
<th>Azimuth (degrees)</th>
<th>RA (J2000) [h : m : s]</th>
<th>Dec (J2000) [° : ’ : ”]</th>
<th>Beamlets</th>
<th>Frequency range [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zenith</td>
<td>90</td>
<td>180</td>
<td>-</td>
<td>-</td>
<td>121</td>
<td>9.77–80.66</td>
</tr>
<tr>
<td>NCP</td>
<td>-</td>
<td>-</td>
<td>00:00:00</td>
<td>+90:00:00</td>
<td>39</td>
<td>24.41–77.73</td>
</tr>
<tr>
<td>3C405</td>
<td>-</td>
<td>-</td>
<td>19:59:28.36</td>
<td>+40:44:02.10</td>
<td>39</td>
<td>24.41–77.73</td>
</tr>
<tr>
<td>3C461</td>
<td>-</td>
<td>-</td>
<td>23:23:24.0</td>
<td>+58:49:54.0</td>
<td>39</td>
<td>24.41–77.73</td>
</tr>
</tbody>
</table>

In this section the use of aperture arrays to observe the absorption that occurs in the ionosphere has been investigated. The riometry technique has been implemented at both KAIRA and the Rawlings array.

### 2.3.4.1 KAIRA as a riometer

The multi-beam and multi-frequency capabilities of the KAIRA LBA mean that it is an ideal instrument to use as a riometer. If used with beams with multiple pointing directions and narrow fields-of-view, KAIRA can be used as an imaging riometer. In contrast to other riometers, KAIRA is capable of multi-frequency measurements which allows for the frequency dependence of absorption to be observed. The location of KAIRA is ideal for observing enhancements in electron density in the lower ionosphere caused by events such as electron precipitation inside the auroral oval.

My first KAIRA campaign was run in 2013 with riometry observations using KAIRA being made over a period of 5 weeks, from 10 October 2013 until 14 November 2013. Subsequent KAIRA observing campaigns were also run in 2015 and 2016. During the 2015 campaign, riometry observations were made between 01 March 2015 and 08 March 2015, and during the 2016 campaign observations were made from 15 March 2016 until 23 March 2016.

In the majority of the 2013 observations the pointing directions detailed in Table 2.1 were used. Whereas in the 2015 and 2016 observations and some of the 2013 observations, eight pointing directions were used with the KAIRA LBA array, with half the beamlets pointing at zenith and observing between 9.77 MHz and 80.66 MHz, with a frequency spacing of 0.5859 MHz. Twenty of the remaining beamlets were pointing at the north celestial pole and observing between 9.77 MHz and 76.56 MHz with frequency spacings of 3.5156 MHz, and the other 102 beamlets had static pointings above the EISCAT site, equally split between six pointing directions and observing between 9.77 MHz and
Table 2.2: KAIRA pointing directions, and corresponding altitude above the EISCAT VHF. Multiple frequencies are observed using KAIRA, between the ranges displayed below.

<table>
<thead>
<tr>
<th>Pointing Direction</th>
<th>Elevation (degrees)</th>
<th>Azimuth (degrees)</th>
<th>RA</th>
<th>Dec</th>
<th>Altitude above EISCAT VHF (km)</th>
<th>Beamlets</th>
<th>Frequency range [MHz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90 (zenith)</td>
<td>180</td>
<td>-</td>
<td>-</td>
<td>122</td>
<td>17</td>
<td>9.77–80.66</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>180</td>
<td>90</td>
<td>20</td>
<td>17</td>
<td>9.77–76.56</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>313.95</td>
<td>-</td>
<td>-</td>
<td>59.5</td>
<td>17</td>
<td>9.77–75.39</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>313.95</td>
<td>-</td>
<td>-</td>
<td>71.3</td>
<td>17</td>
<td>9.77–75.39</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>313.95</td>
<td>-</td>
<td>-</td>
<td>85.0</td>
<td>17</td>
<td>9.77–75.39</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>313.95</td>
<td>-</td>
<td>-</td>
<td>101.3</td>
<td>17</td>
<td>9.77–75.39</td>
</tr>
<tr>
<td>7</td>
<td>55</td>
<td>313.95</td>
<td>-</td>
<td>-</td>
<td>121.4</td>
<td>17</td>
<td>9.77–75.39</td>
</tr>
<tr>
<td>8</td>
<td>60</td>
<td>313.95</td>
<td>-</td>
<td>-</td>
<td>147.2</td>
<td>17</td>
<td>9.77–75.39</td>
</tr>
</tbody>
</table>

75.39 MHz, with frequency spacings of 4.1016 MHz. Table 2.2 lists the elevation angle of these pointing directions and corresponding altitude over the EISCAT VHF. In this chapter, I concentrate on the zenith pointings of the KAIRA telescope. The range of the full-width half maxima at zenith of the KAIRA LBA beams was between 8° at 80 MHz to 40° at 10 MHz. The average power for each beamlet and each polarisation were recorded with the KAIRA LBA array at a temporal resolution of 1 s.

I compare the amount of absorption that occurs on two days during the 5 week observing period in 2013. One of these days is chosen during a geomagnetically quiet time and the other is chosen during a geomagnetically active time. These days were chosen by looking at magnetograms from the magnetometer in Kilpisjärvi, Finland which is one of the 35 magnetometers that make up the International Monitor for Auroral Geomagnetic Effects (IMAGE) network based in Scandinavia (shown in Figure 2.13).

A magnetometer measures the components of the Earth’s magnetic field at ground level. The three components measured are \(X\) (representing the magnetic field strength in the direction of the north magnetic pole), \(Y\) (representing the magnetic field strength in the ‘magnetic east’ direction) and \(Z\) (representing the magnetic field strength in the local vertical direction). Data from magnetometers can be used to infer information about the relative strength of a magnetic storm in the magnetosphere. The relative strength of the storm is defined by the Kp index which is calculated by taking data from a variety of magnetometers, finding the difference between the maximum and minimum of the \(X\) component (known as the A index), and then averaging this index from all magnetometers as well as over a 3 hour interval.

It can be seen in the magnetograms displayed in Figure 2.14 that 14–15 October 2013 is geomagnetically active in comparison to 22–23 October 2013. The magnetograms for 22–23 October 2013 show a single peak in activity at 22:50 UT, whereas the magnetograms for 14–15 October 2013 show activity throughout the observation.
Unlike a standard riometer I used KAIRA to simultaneously observe with 244 beamlets, so that instead of a quiet day curve a quiet day surface (QDS) is created. To create the QDS the variation in power level is measured throughout the day at different frequencies for each beam pointing in Table 2.1. Observations to create the QDS for the geomagnetically active day of 14–15 October 2013 were made between 10 October 2013 and 19 October 2013. The QDS is shown in the top plot of Figure 2.15. The middle plot of Figure 2.15 shows the power received for the observations beginning at 12:30 UT on 14 October 2013 and ending at 11:30 UT on 15 October 2013 and the bottom plot shows the absorption for this time period, as calculated using Equation 2.25.

The geomagnetically quiet time that I compare Figure 2.15 to is 22–23 October 2013. To create the QDS (shown in the top plot of Figure 2.16) for this day, the observations made between 17 October 2013 and 27 October 2013 are used. The middle plot of Figure 2.16 shows the power received for the observations beginning at 18:00 UT on 22 October 2013 and ending at 12:00 UT on 23 October 2013 and the bottom plot shows the absorption for this time period, as calculated using Equation 2.25.
There are fewer absorption features visible in the observations made during the geomagnetically quiet time than during the geomagnetically active time. Only one strong absorption feature is seen in Figure 2.16, at 22:50 UT, corresponding to the peak in activity seen in Figure 2.14. On the other hand, strong absorption features are visible in Figure 2.15 from 17:20 UT until 09:30 UT. These absorption features do not appear to have such a clear correlation to the peaks in geomagnetic activity that are visible in Figure 2.14. Although the variation in geomagnetic activity appears to stop at 04:00 UT, absorption features are still seen in the multi-frequency riometry data from KAIRA.

For reference I also include absorption plots for the 2015 (Figure A.1) and 2016 (Figure A.3) observing campaign in the appendix, along with the corresponding magnetogram data (Figures A.2 and A.4 respectively). These figures are similar to Figure 2.15, as the observations both corresponded to geomagnetically active times and contain many absorption features.

It can be seen from Figures 2.15, 2.16, A.1 and A.3 that KAIRA suffers from both broadband and narrowband radio frequency interference (RFI). A moving window filter
Figure 2.15: TOP: The quiet day surface created for KAIRA (beam pointing at zenith) using observations from 10–19 October 2013. MIDDLE: The observed power at KAIRA for a beam pointing at zenith and observing between frequencies of 9.77 MHz and 80.66 MHz on 14–15 October 2013. BOTTOM: The absorption, $A$, as measured by KAIRA for a beam pointing at zenith. The absorption is found using Equation 2.25 with the values in the top plot as $P_0$ and the values in the middle plot as $P$. The vertical red lines in this absorption plot show peaks in absorption. RFI can be seen to dominate at all frequencies below 16.80 MHz. The vertical black lines indicate local sunset (15:39 UT) and local sunrise (05:58 UT).

is passed over the data in order to remove RFI. The RFI contaminated data are flagged, allowing for the removal of entire time periods or entire frequency bands if the time period or frequency is too heavily contaminated. Otherwise the data are filtered out using the average of the moving window filter. Below 20 MHz, RFI caused by shortwave radio communications dominates (McKay-Bukowski et al., 2015).

KAIRA also suffers from a certain type of RFI known as ‘Dragon’s teeth’ (see Figure 2.16). These manifest in the data as broadband RFI which is triangular in shape. The source of this RFI has yet to be determined.

The implementation of riometry at KAIRA enables constant observations of the absorption that occurs in the ionosphere. These observations can be used to monitor the geomagnetic activity, as shown by the comparisons to magnetometer data.
Figure 2.16: TOP: The quiet day surface created for KAIRA (beam pointing at zenith) using observations from 17–27 October 2013. MIDDLE: The observed power at KAIRA for a beam pointing at zenith and observing between frequencies of 9.77 MHz and 80.66 MHz on 22–23 October 2013. BOTTOM: The absorption, $A$, as measured by KAIRA for a beam pointing at zenith. The absorption is found using Equation 2.25 with the values in the top plot as $P_0$ and the values in the middle plot as $P$. The vertical red lines in this absorption plot show peaks in absorption. RFI can be seen to dominate at all frequencies below 16.80 MHz and ‘Dragons teeth’ RFI (described in the text) can be clearly seen in the middle and bottom plot at 20:04 UT, 22:49 UT, 01:21 UT, 06:59 UT and 09:55 UT. The vertical black line indicates local sunrise (06:18 UT).

### 2.3.4.2 Riometry with the Rawlings array

KAIRA uses the antenna system and signal processing of a LOFAR station. The method outlined above can therefore be used to measure absorption in multiple frequencies at LOFAR stations in stand-alone mode. One such LOFAR station is the Rawlings array (also known as UK608). In this section I describe its use as a riometer and present the resulting absorption measured using the Rawlings array.

As shown in Figure 1.6, the majority of riometers are generally located at higher latitudes (above 60°N) than radio interferometers. This means that by using the Rawlings array as a riometer I will be able to observe an area of the ionosphere not currently observed by other riometers, allowing me to potentially observe different ionospheric behaviour and to better understand the absorption that occurs at mid-latitudes.
Similarly to KAIRA, the Rawlings array can be used as a multi-frequency, multi-beam riometer. The Rawlings array is comparable to other riometers, with a similar spatial resolution to the ARIES system (beam width of 6.56°; Honary et al. 2011), because it has a beam width of 5.5° at a frequency of 30 MHz which corresponds to a spatial resolution of 8 km in the D layer of the ionosphere (Best and the LOFAR-UK Consortium, 2008).

As detailed in Section 2.1.2.1, LOFAR has a key science project that focuses on the epoch of reionisation and the cosmic dawn (van Haarlem et al., 2013). One of their main goals is to use the 21-cm neutral hydrogen emission line at high redshifts as a probe to study the cosmic dawn and the epoch of reionisation. This involves the use of the LOFAR LBA system to detect the global redshifted 21-cm signal, with the aim of placing stringent upper limits on it. For these observations, the absorption that occurs in the ionosphere is assumed to be a constant value of 0.06 dB at 40 MHz by Vedantham et al. (2014).

In December 2012, an observing proposal was submitted for the period-2 call for proposals for use of the Rawlings array in stand-alone mode. From this proposal, 262 hours throughout 2013 were allocated to riometry observations. The beam targets used are the same as those used by KAIRA in the 2013 observing campaign. These are displayed in Table 2.1. In this section I concentrate on the zenith pointing of the Rawlings array.

As the Rawlings array is an international station that is part of the LOFAR network, it can only be used in stand-alone mode when it is not being used for international observing as part of the full array. In general, the station is only put into stand-alone mode at weekends, when it is also used for millisecond radio transient surveys and for studying pulsars and the interstellar medium. This limits the ability of the Rawlings array to be used as a riometer because a riometer requires a regularly generated QDC created from multiple observations made within 10 days of the day that the user is interested in measuring the absorption on. My initial observations with the Rawlings array are spread over many months with weeks in between each observation due to the array being required for other uses. The dates and times of each observation are shown in Table 2.3. At the beginning of 2014, the array was dedicated to riometry from 02 January 2014 until 07 January 2014, providing data to create a QDS using the percentage criterion method of Browne et al. (1995).

The top plot in Figure 2.17 shows the QDS created for the Rawlings array using the data from 02 January 2014 until 07 January 2014, whilst the middle plot in Figure 2.17 shows the measured power between 10:00 UT on 04 January 2014 and 09:00 UT on 05 January 2014. The bottom plot in Figure 2.17 is the absorption between 10:00 UT on 04 January 2014 and 09:00 UT on 05 January 2014 as calculated using the data from the other two plots in Figure 2.17 and Equation 2.25.
Table 2.3: Start and end times of all riometry observations made using the Rawlings array.

<table>
<thead>
<tr>
<th>Start time</th>
<th>End time</th>
</tr>
</thead>
<tbody>
<tr>
<td>18/03/2013 15:57:56</td>
<td>19/03/2013 14:31:10</td>
</tr>
<tr>
<td>28/03/2013 15:16:56</td>
<td>29/03/2013 14:57:11</td>
</tr>
<tr>
<td>01/04/2013 09:13:51</td>
<td>02/04/2013 08:55:46</td>
</tr>
<tr>
<td>08/04/2013 11:29:21</td>
<td>09/04/2013 08:57:45</td>
</tr>
<tr>
<td>07/05/2013 09:36:48</td>
<td>08/05/2013 08:57:23</td>
</tr>
<tr>
<td>22/05/2013 09:05:05</td>
<td>23/05/2013 09:00:00</td>
</tr>
<tr>
<td>17/06/2013 13:34:51</td>
<td>18/06/2013 13:50:24</td>
</tr>
<tr>
<td>15/10/2013 09:03:55</td>
<td>16/10/2013 08:59:33</td>
</tr>
<tr>
<td>21/10/2013 09:02:55</td>
<td>22/10/2013 07:27:20</td>
</tr>
<tr>
<td>22/10/2013 15:52:15</td>
<td>23/10/2013 09:00:27</td>
</tr>
<tr>
<td>02/01/2014 09:14:33</td>
<td>03/01/2014 09:13:32</td>
</tr>
<tr>
<td>03/01/2014 09:16:11</td>
<td>04/01/2014 09:15:49</td>
</tr>
<tr>
<td>04/01/2014 09:25:32</td>
<td>05/01/2014 09:17:53</td>
</tr>
<tr>
<td>05/01/2014 09:20:37</td>
<td>06/01/2014 09:16:06</td>
</tr>
<tr>
<td>06/01/2014 09:18:53</td>
<td>07/01/2014 09:00:45</td>
</tr>
</tbody>
</table>

The vertical black lines in Figure 2.17 indicate local sunset at 16:13 UT on 04 January 2014 and local sunrise at 08:08 UT on 05 January 2014. Between sunrise and sunset, the frequencies below 30 MHz can be seen to be dominated by RFI.

No strong absorption features can be seen in the absorption plot in Figure 2.17. The lack of absorption features in this data could demonstrate that the lower ionosphere is stable and undisturbed at mid-latitudes, or the ionosphere could be especially quiet during the observing period. X-ray impact from solar flares should increase the electron density in the lower ionosphere and so I therefore investigate solar activity to see if anything about the activity of the ionosphere can be inferred during these observations.

Figure 2.18 shows the X-ray flux obtained from the Geostationary Operational Environmental Satellite-15 (GOES-15) for January 2014. It shows that solar activity was not particularly quiet during these observations with a C-class flare occurring during the observations, however it was reasonably constant. It should be noted that due to the fact that the Rawlings array can only be used in stand-alone mode when it is not being used for international observing as part of the full LOFAR array, fewer days of observations were used to make the QDS than were used to make the KAIRA QDS. This will have an adverse effect on absorption measurements as if there is no ‘quiet’ time during the period of observations, then there is no data to make a QDS from.

Vedantham et al. (2014) estimate that absorption at 40 MHz for a zenith pointing is 0.06 dB, however in Figure 2.17 I measure that the absorption occurring over the
Figure 2.17: TOP: The quiet day surface created for the Rawlings array (beam pointing at zenith) using observations from 02–07 January 2014. MIDDLE: The observed power at the Rawlings array for a beam pointing at zenith and observing between frequencies of 9.77 MHz and 80.66 MHz on 04–05 January 2014. BOTTOM: The absorption, $A$, as measured by the Rawlings array for a beam pointing at zenith. The absorption is found using Equation 2.25 with the values in the top plot as $P_0$ and the values in the middle plot as $P$. RFI can be seen to dominate during the daytime. The vertical black lines indicate local sunset (16:13 UT) and local sunrise (08:08 UT).

Figure 2.18: X-ray flux obtained from the Geostationary Operational Environmental Satellite-15 (GOES-15) for January 2014.
observation is not a constant value, but instead varies between -0.41 dB and 0.18 dB at 40 MHz, with a mean absorption of 0.01 dB. The mean absorption is lower than the value suggested by Vedantham et al. (2014). This is likely to be due to the observations at the Rawlings array being affected by RFI, and also due to there being little difference between the ionosphere on this sidereal day and the value given by the QDS. Browne et al. (1995) found that the method of QDC generation used to generate the QDS in Figure 2.17 only gives curves that are accurate to 0.1 dB for IRIS. Considering that the levels of absorption observed using the Rawlings array are smaller than this accuracy value, measurements of absorption using LOFAR stations in single-station mode are therefore unlikely to improve low-frequency observations that require high precision knowledge of the ionosphere.

Figure 2.17 shows that the effect of absorption at mid-latitudes on low-frequency radio observations is less than at high-latitudes. It also shows that the absorption is relatively constant; no strong absorption features are seen. The average absorption measured is lower overall than the value given by Vedantham et al. (2014) of 0.06 dB at 40 MHz, however it is within the error expected from the method of calculating the absorption that I have used.

### 2.3.5 Recreating electron density height profiles from absorption measurements

As shown by Equation 2.16, the absorption at altitude $h$ is proportional to the electron density. This relationship can be exploited to invert the absorption measurements into electron density height profiles.

Hargreaves and Friedrich (2003) investigated whether ground-based absorption measurements at a single frequency can be used to detail the electron density in the D layer, given that the relationship between radio absorption and electron density cannot be found precisely due to the spectrum of the ionising electrons not being constant. They took data from an incoherent scatter radar, EISCAT UHF (located in Ramfjordmoen, Norway), using Common Programme runs between 1985 and 1992 and found the median electron density values at 5 km intervals between 70 km and 100 km. This was then compared to the average absorption measurements, $A$, taken from four riometers located within 10 km of the EISCAT UHF. They then used the ratio $N_e(h)/A$ for the $N_e(h)$ values obtained from the EISCAT data to calculate if the auroral radio absorption could be interpreted to estimate the electron density. They concluded that an estimation of approximate electron density at altitudes between 80 km and 100 km could be obtained from measurements of radio absorption, however Hargreaves and Friedrich (2003) only used data from four riometers, all of which measured absorption at 27.6 MHz. If data are taken from a range of frequencies instead of just a single frequency, then the height profile of the electron density in the D layer can be recovered.
Kero et al. (2014) used KAIRA as a multi-frequency riometer, measuring absorption at a range of frequencies from 10–80 MHz. In their paper, they find that multi-frequency riometry measurements can provide realistic electron density height profiles when combined with an electron precipitation model detailed by Rees (1989). They fit the electron precipitation model to multi-frequency absorption measurements made using a riometer and compare the results to EISCAT electron density measurements. Figure 2.19 shows the comparison of the model fits to the EISCAT electron density measurements.

2.4 Faraday rotation

The final ionospheric effect investigated in the thesis is changes to the measured Faraday rotation of radio signals. Like the other effects investigated in this thesis, this is especially relevant to low-frequency observations, as Faraday rotation increases quadratically with wavelength.

Faraday rotation occurs when a linearly polarised radio wave propagates through the ionosphere or another magneto-ionic medium (Brentjens and de Bruyn, 2005). As the wave propagates, the intrinsic polarisation angle of the signal rotates,

\[ \chi (\lambda^2) = \chi_0 + \phi \lambda^2, \]  

where \( \lambda \) is the wavelength, \( \chi (\lambda^2) \) is the polarisation angle at \( \lambda^2 \), \( \chi_0 \) is the polarisation angle at \( \lambda = 0 \) and \( \phi \) is the Faraday depth. Assuming that the rotation of the polarisation angle is caused by a thermal electron density, \( N_e(l) \) with a vector magnetic field, \( B \), along the line of sight between the observer and the source, \( l \), then the rotation measure (RM)
Chapter 2 Radio astronomy

is proportional to the Faraday depth,

\[
\text{RM} = \frac{e^3}{8\pi\varepsilon_0 m_e^2 c^2} \int_0^D N_e (l) B (l) \cdot dl
\]  

(2.27)

where \( e \) is the charge of an electron, \( \varepsilon_0 \) is the permittivity of free space, \( m_e \) is the mass of an electron, \( c \) is the speed of an electromagnetic wave in a vacuum and the integral is over the path to the radio source. If the RM is positive then the magnetic field is considered to be pointing towards the observer.

For ground based observations of point sources, there are four primary contributions to the total measured Faraday rotation:

- the internal Faraday rotation of the source itself.
- the ionosphere.
- the Galactic interstellar medium.
- the inter-galactic medium (if the source is extra-Galactic).

Due to the form of Equation 2.27 the different components of the total Faraday rotation contribute additively. Studies of the RMs of astronomical sources can therefore be used to inform astronomers of the form of magnetic fields in the Galaxy (Manchester 1974; Han et al. 2006; Noutsos et al. 2008) as well as other galaxies, galaxy clusters and of a variety of Galactic objects, such as supernova remnants and HII regions (Sun et al., 2015)

The Stokes parameters, I (total intensity), Q and U (linear polarisation) and V (circular polarisation) are used to describe the polarisation properties of radio sources (Yan et al., 2011). The polarisation angle (which is caused to rotate due to Faraday rotation as the polarised radio signal propagates through the magneto-ionic medium) can be written in terms of the linearly polarised emission,

\[
\chi = 0.5 \tan^{-1} \left( \frac{U}{Q} \right).
\]

(2.28)

2.4.1 Using pulsars to observe changes in rotation measure

Pulsars are highly magnetised, rotating neutron stars. They are formed when the core of a massive star is compressed into a neutron star during a supernova but retains the majority of its angular momentum and so when it is compressed it starts rotating at a very high speed. The pulsar emits electromagnetic radiation at radio wavelengths along its magnetic axis (Manchester and Taylor, 1977). As the pulsar is spinning, the emitted radio signal appears to pulse when observed from Earth. Pulsars were first identified
by Hewish et al. (1968) at 81 MHz, and have been found to be brightest at low radio frequencies.

Faraday rotation occurring along the line of sight is relatively easy to observe in pulsars because they typically have a high degree of linear polarisation. Pulsars also have the unique advantage that the dispersion measure (DM) is also known,

\[ \text{DM} = \int_0^D N_e(l) \, dl. \]  \hspace{1cm} (2.29)

This allows for information about the magnetic field along the line of sight to be inferred as

\[ \frac{\text{RM}}{\text{DM}} = \frac{\int_0^D N_e(l) \, B(l) \, dl}{\int_0^D N_e(l) \, dl} = B(l). \] \hspace{1cm} (2.30)

### 2.4.2 Ionospheric Faraday rotation models

Over the past 15 years, many models have been constructed in order to correct for ionospheric Faraday rotation. Erickson et al. (2001) developed a model using four GPS receivers at the VLA (Kassim et al., 2007) in New Mexico. They used a thin-layer ionospheric model consisting of a VTEC with a gradient that was fitted to the TEC obtained from the GPS measurements from each receiver. This model is only of use to a radio interferometer that has GPS receivers located on site.

Afraimovich et al. (2008) developed a model and software to calculate the Faraday rotation and dispersion that occurs in the ionosphere based on the IRI (detailed in Section 1.1.1.1), the International Geomagnetic Reference Field (IGRF; Thebault et al., 2015; detailed in Section A.2) and TEC values obtained from GPS measurements.

Sotomayor-Beltran et al. (2013) developed a software package known as ionFR, which is similar to TECOR — a task available in AIPS (Greisen, 2003). Similarly to the thin-layer model, this software assumes a thin spherical shell around the Earth and then calculates the ionospheric Faraday depth at the point at which the line of sight pierces the shell. The ionospheric Faraday depth is therefore calculated from the VTEC, zenith angle and magnetic field at this piercing point. The issues of assuming a thin-layer model are discussed in Section 2.2.1.

The VTEC used by Sotomayor-Beltran et al. (2013) is obtained from CODE TEC maps (see Section 1.2.3) and is capable of calculating hourly values of ionospheric Faraday depth that allow for pulsar RMs to be calculated to very high precision (an absolute error of $\lesssim 0.1 \, \text{rad m}^{-2}$) when this method is used in the calibration of LOFAR data.
Chapter 3

Limits on the validity of the thin-layer model

For a ground-based radio interferometer observing at low frequencies, the ionosphere causes propagation delays and refraction of cosmic radio waves which result in phase errors in the received signal. These phase errors can be corrected using a calibration method that assumes a two-dimensional phase screen at a fixed altitude above the surface of the Earth, known as the thin-layer model. Here I investigate the validity of the thin-layer model and provide a simple equation with which users can check when this approximation can be applied to observations for varying time of day, zenith angle, interferometer latitude, baseline length, ionospheric electron content and observing frequency. Parts of this chapter have been published as Martin et al. (2016a).

Phase differences between pairs of antennas are used in radio synthesis astronomy to reconstruct images of radio emission from the sky. However, the ionosphere causes propagation delays and refraction of cosmic radio waves, which results in phase errors in the signals received by ground-based radio interferometers. Methods of calibration to correct for these errors are therefore vital to radio astronomers.

Telescopes operating at higher radio frequencies (>1 GHz) are generally able to correct for ionospheric effects using direction-independent, time-varying complex gains derived using self-calibration (described in Section 2.1.2), however arrays operating at lower frequencies such as the VLA (58–470 MHz; Section 2.1.2.2), GMRT (153–610 MHz; Section 2.1.2.3), MWA (80–300 MHz; Section 2.1.2.4) and LOFAR (30–240 MHz; Section 2.1.2.1) have found that the ionosphere can limit the achievable dynamic range of astronomical observations. Future low frequency arrays such as SKA1-LOW (50–350 MHz; Section 2.1.2.5) will also be affected by the ionosphere.
In Section 2.2.1, calibration techniques for correcting these ionospheric effects when using low-frequency telescopes are discussed. These techniques are based on the assumption that the ionosphere can be modelled as a thin layer, e.g. SPAM (outlined in Section 2.2.2). In this chapter, the validity of this thin-layer model is considered. Section 3.1 discusses the thin-layer model and the error that arises from it. Section 3.2 expands on this error and its consequent effect on the ionospheric phase calibration of a radio synthesis telescope, as well as investigating the conditions under which the thin-layer model approximation can be applied to observations without limiting the dynamic range of the resulting radio image.

### 3.1 The thin-layer model

The thin-layer model is an assumption used to compensate for the phase errors caused by propagation delays and refraction of cosmic radio waves via ionospheric calibration. A two-dimensional phase screen is assumed at a fixed altitude, $h$, above the mean surface of the Earth. This phase screen is created by developing a model of the spatial and temporal variation of the ionospheric electron content, calculating the resulting effect on the phases measured by the telescope, and applying direction-dependent phase corrections to the data to compensate (Intema et al. 2009; summarised in Section 2.2.1). The thin-layer model is implemented in data reduction in radio interferometric imaging through software such as SPAM (Section 2.2.2) but it is not a perfect representation of the ionosphere.

The quantity of interest for determining the ionospheric phase delay is the electron column density or TEC. The TEC along the line of sight (STEC) in the thin-layer model can be calculated using Equation 2.13. If the Earth is assumed to be flat, as shown in Figure 3.1, then the zenith angle, $\theta'$, of the line of sight from the observer is the same as the angle of the line of sight to the thin layer at the pierce point ($\theta$ in Equation 2.13). In this case, assuming that the horizontal variation of the TEC is neglected, the thin-layer model is perfectly accurate, and is independent of the value chosen for $h$.

For a more realistic, spherical Earth, as shown in Figure 3.2, the STEC calculated with the thin-layer model will differ from the true electron column density along the line of sight even if horizontal variation of the TEC is neglected. For the spherical Earth, the STEC is dependent on the assumed altitude, $h$, of the thin layer. The relationship between the angle of the line of sight to the normal of the thin layer at the pierce point, $\theta$, and the zenith angle of the line of sight of the observer, $\theta'$, can be derived using the law of sines,

$$
\sin \theta = \frac{R_E}{R_E + h} \sin \theta'
$$

(3.1)
Chapter 3 Limits on the validity of the thin-layer model

Figure 3.1: Illustration of vertical and slanted lines of sight through the ionosphere. The column density of electrons along a vertical line of sight (‘LOS 1’) is the VTEC. The column density of electrons along a slanted line of sight (‘LOS 2’) is the STEC and, assuming a thin-layer model, can be related to the VTEC by the angle $\theta$ at which the line of sight intersects the assumed thin layer (Equation 2.13). In this simplified case, ignoring the curvature of the Earth, $\theta$ is independent of the altitude $h$ of the thin layer, and is therefore equal to the zenith angle $\theta_0$ of the line of sight from the observer. This figure is taken from Martin et al. (2016a).

where $R_E = 6371$ km and is the mean radius of the Earth. This can then be substituted into Equation 2.13 to obtain the relationship between STEC and VTEC for the spherical Earth case,

$$\text{STEC} = \text{VTEC} \times \left(1 - \left(\frac{R_E}{R_E + h} \sin \theta_0\right)^2\right)^{-\frac{1}{2}}.$$  \hspace{1cm} (3.2)

To illustrate the imprecision of the thin-layer model due to the curvature of the Earth, the STEC calculated for a thin-layer ionosphere at an altitude of $h$ using Equation 3.2 (STEC$_{2D}$) can be compared with the STEC calculated for a three-dimensional ionosphere (STEC$_{3D}$).

I calculate STEC$_{3D}$ from a realistic altitude profile of electron density created by combining separate models for the ionosphere and for the plasmasphere. The plasmasphere is included in this profile because it typically contributes about 10% to the daytime TEC (Yizengaw et al., 2008). For the ionosphere up to an altitude of 2000 km, I use
Figure 3.2: Illustration of a slanted line of sight through the ionosphere for a spherical Earth. In the thin-layer model, the VTEC and STEC are related by the angle at the pierce point, $\theta$, as in Figure 3.1. However, unlike the flat-Earth case, the angle at the pierce point, $\theta$, is not equal to the zenith angle, $\theta'$, and depends on the assumed altitude $h$ of the thin layer. Since $h$ is only an approximation to the true distribution of the ionospheric electron content across a range of altitudes, this leads to inaccuracy in the thin-layer model. This figure is taken from Martin et al. (2016a).

an electron density profile taken from the IRI (see Section 1.1.1.1) for 12:00 UT on 17 February 2015. I assume a pierce point of the thin layer at 40° geomagnetic latitude and 0° geomagnetic longitude. The electron density profile obtained from the IRI model only extends up to altitudes of 2000 km, therefore the model of Gallagher et al. (1988) is used for the plasmasphere at higher altitudes. As the IRI model is a historical model based on contemporaneous measurements it is considered more accurate than the model of Gallagher et al. (1988) which is a parameterised fit to historical electron density variation for different latitude and time of day. The profile obtained from the model of Gallagher et al. (1988) is therefore renormalised to match the profile obtained from the IRI at the altitude where the profile transitions between the two models. The resulting profile is shown in Figure 3.3.

To calculate STEC_{3D} using the profile in Figure 3.3, the profile is divided into individual layers of 1 km thickness. I choose layers of 1 km thickness because the profile obtained from the IRI has step sizes of 1 km. Each of these layers is then approximated as a
distinct thin layer which can be summed to calculate $\text{STEC}_{3D}$,

$$\text{STEC}_{3D} = \sum_{i=60}^{30,000} \text{STEC}_i,$$

(3.3)

where $\text{STEC}_i$ is the STEC calculated using Equation 3.2 from the VTEC value for an individual layer at an altitude of $h = i$ km.

Figure 3.4 shows the comparison between $\text{STEC}_{2D}$ (as calculated assuming a thin-layer ionosphere using Equation 3.2) and $\text{STEC}_{3D}$ (as calculated assuming a three-dimensional ionosphere using Equation 3.3) for changing zenith angle. The thin-layer model significantly overestimates the STEC at large zenith angles for assumed thin-layer altitudes of 250 km and 350 km, but provides a somewhat closer approximation for an assumed thin-layer altitude of 450 km.
Figure 3.4: STEC\textsubscript{3D} calculated using Equation 3.3 for the electron density profile shown in Figure 2.8, and STEC\textsubscript{2D} obtained for the same profile using the thin-layer model in Equation 3.2, both for a range of zenith angles $\theta'$ as shown in Figure 3.2. At a zenith angle of zero, STEC\textsubscript{2D} = STEC\textsubscript{3D} = VTEC but the definitions of STEC\textsubscript{2D} for different thin-layer altitudes, $h$, generally diverge for lines of sight closer to the horizon. The accuracy of the thin-layer model depends on the assumed thin-layer altitude, with $h = 450$ km being approximately correct over the widest range of zenith angles.

### 3.2 Testing the thin-layer model

The use of the thin-layer model causes inaccuracy in the ionospheric phase calibration, as discussed in Section 2.2.1, through two different effects. Firstly, lines of sight that nominally pass through the same point on the assumed thin layer may sample different regions of the ionosphere above or below this altitude, which may have different electron content. Secondly, even if horizontal variation in the ionospheric electron content is neglected, the curvature of the Earth leads to inaccuracy in the derived STEC, as shown in Figure 3.4. In this section the magnitude of this latter error as a measure of the range of validity of the thin-layer model, and of the circumstances under which it is necessary to use a more complex model such as those in later implementations of SPAM (Intema et al., 2011), is investigated.

Two radio antennas, A and B, observing a source through the ionosphere as depicted in Figure 3.5 are considered. It is assumed that the ionosphere has been perfectly calibrated using the thin-layer model, and the VTEC is perfectly known at all points on the thin
layer. For simplicity, it is assumed that there is no local horizontal variation in electron content for each relevant layer of the ionosphere, which allows a direct comparison to be made between \( \text{STEC}_{2D} \) and \( \text{STEC}_{3D} \). The derived STEC along the line of sight from each antenna is therefore subject to an error that is associated with the thin-layer model,

\[
\Delta \text{STEC} = |\text{STEC}_{2D} - \text{STEC}_{3D}|. \tag{3.4}
\]

This error will lead to corresponding errors, \( \Delta \phi_A \) and \( \Delta \phi_B \), in the phase corrections on antenna A and antenna B. If the antennas are closely spaced, these errors will be almost identical, and the error in the phase correction on the baseline between these two antennas will be close to zero,

\[
\Delta \phi_{AB} = |\Delta \phi_A - \Delta \phi_B|. \tag{3.5}
\]

The larger the distance, \( b \), between the antennas, the greater the difference in \( \theta' \). This causes a difference in the phase corrections on antenna A and antenna B, \( \Delta \phi_A \) and \( \Delta \phi_B \), leading to a more significant error in the calibrated phase on this baseline, \( \Delta \phi_{AB} \).

In reality, it cannot be assumed that the VTEC is perfectly calibrated because the calibration process is also based on phase measurements along lines of sight through the ionosphere, as described in Section 2.2.2, and so is subject to the same discrepancy between the \( \text{STEC}_{3D} \) and the \( \text{STEC}_{2D} \) values inferred for a thin-layer ionosphere. It is the difference in this discrepancy between antennas due to differing zenith angles \( \theta' \) that is responsible for the phase error \( \Delta \phi_{AB} \), and this difference is not corrected by the calibration procedure. This is because a point on the ionosphere will be calibrated on the basis of nearby pierce points of lines of sight from multiple antennas, with differing zenith angles. Figure 2.9 shows how the fields of view of different antennas typically overlap when projected on the ionosphere, enabling calibration from nearby pierce points. The phase error resulting from this imperfect calibration is equivalent to the phase error for perfect VTEC calibration considered above, because in either case its magnitude is controlled by \( \Delta \theta' \).

The phase error, \( \Delta \phi \), for each antenna is related to \( \Delta \text{STEC} \) by Equation 2.15,

\[
\Delta \phi \approx 4840^\circ \times \left( \frac{\nu}{100 \text{ MHz}} \right)^{-1} \left( \frac{\Delta \text{STEC}}{\text{TECU}} \right). \tag{3.6}
\]

The difference in zenith angle between the two antennas as shown in Figure 3.5 is given by \( \Delta \theta' \) and can be approximated by

\[
\Delta \theta' \approx \frac{b}{R_E}. \tag{3.7}
\]
Figure 3.5: Illustration of two antennas (A and B), part of a radio synthesis telescope, observing a source through the ionosphere. Due to the curvature of the Earth, the zenith angle $\theta'$ of the source from each antenna, and the consequent pierce angle $\theta$ at the thin-layer ionosphere, will differ, depending on the baseline length $b$. The discrepancy $\Delta\text{STEC}$ due to the thin-layer model will therefore differ between the two lines of sight, leading to a phase error $\Delta\phi_{AB}$ across this baseline.

To find the phase error $\Delta\phi_{AB}$ on the baseline, the difference between $\Delta\text{STEC}$ for the two antennas needs to be found. This can be approximated for small values of $\Delta\theta'$ as

$$\Delta\text{STEC}_{AB} \approx \Delta\theta' \frac{d\Delta\text{STEC}}{d\theta'},$$  \hspace{1cm} (3.8)$$

where $\theta'$ is the zenith angle, and

To find an expression for how the difference between $\text{STEC}_{2D}$ and $\text{STEC}_{3D}$ varies with zenith angle, $\frac{d\Delta\text{STEC}}{d\theta'}$, the TEC ratio derivative, $Z$, is defined as

$$Z = \frac{d}{d\theta'} \frac{\text{STEC}_{2D}}{\text{STEC}_{3D}},$$  \hspace{1cm} (3.9)$$

which, by substituting in Equation 3.4, can be approximated as

$$Z \approx \frac{1}{\text{STEC}_{3D}} d\frac{\Delta\text{STEC}}{d\theta'}$$  \hspace{1cm} (3.10)$$
Figure 3.6: Diurnal variation of STEC and the thin-layer model STEC, and (lower panels) the derivative $Z$ of the ratio between the two. The STEC shows the expected increase during the day due to solar forcing, while $Z$ varies significantly, but its magnitude is minimised at all times by a thin-layer altitude of $h = 450$ km, except at large zenith angles. Plots are for the specified local times on 17 February 2014 (local winter) for an ionospheric pierce point at 40° geomagnetic latitude and 0° geomagnetic longitude.
Chapter 3 Limits on the validity of the thin-layer model

The STEC (ionospheric electron content) and the thin-layer model STEC are shown in Figure 3.7. The STEC is maximized around April 2014 and July 2014 (local spring and summer, respectively) while the difference between the two models shows little variation throughout the year. The STEC is almost zero at these times, but it increases significantly in the northern hemisphere during winter, especially at high latitudes. The variation in STEC with season depends on the latitude and is affected by phenomena such as the mid-latitude winter anomaly (seen in the northern hemisphere) and the semiannual anomaly, where the electron density at the F-layer peak height is greater at equinox than at solstice (Lee et al., 2011).

The STEC is maximized around April 2014 and July 2014 (local spring and summer, respectively), while the difference between the two models shows little variation throughout the year. The STEC is almost zero at these times, but it increases significantly in the northern hemisphere during winter, especially at high latitudes. The variation in STEC with season depends on the latitude and is affected by phenomena such as the mid-latitude winter anomaly (seen in the northern hemisphere) and the semiannual anomaly, where the electron density at the F-layer peak height is greater at equinox than at solstice (Lee et al., 2011).
Variation with latitude of STEC and the thin-layer model STEC, and the derivative $Z$ of the ratio between the two. The STEC is higher near the equator and lower near the poles. The magnitude of $Z$ is minimised at all latitudes by a thin-layer altitude of $h = 450$ km, except at large zenith angles. Plots are for 03:00 local time for an ionospheric pierce point at 0° geomagnetic longitude on 17 February 2015 (local winter).
Chapter 3 Limits on the validity of the thin-layer model

Figure 3.9: Range of possible values for the TEC ratio derivative $Z$, reflecting variation with time of day (sampled at one-hour intervals), season (sampled one day in each month), latitude (from $0^\circ$ to $90^\circ$), and solar cycle (i.e. over the past 11 years). Lines show the median value for each thin-layer altitude $h$, and shaded areas denote the $\pm 1\sigma$ range, encompassing 68% of all simulated values. Values of $Z$ from this plot for the appropriate zenith angle and thin-layer altitude may be taken as typical inputs to Equation 3.12.

provided that $\Delta\text{STEC} \ll \text{STEC}_{3D}$. By approximating the relationship between $\text{STEC}_{3D}$ and VTEC using Equation 2.13, $Z$ can then be written as

$$Z \approx \frac{\cos \theta'}{\text{VTEC}} \frac{d\Delta\text{STEC}}{d\theta'},$$

(3.11)

provided that the line of sight is not close to the horizon.

As the TEC ratio derivative $Z$ plays a key role in determining the phase error $\Delta\phi_{AB}$, I investigate its behaviour across a range of parameters. The ionosphere varies with time of day, season and latitude, therefore I consider the extent to which $Z$ is affected by these parameters. The same approach for determining $\text{STEC}_{2D}$ and $\text{STEC}_{3D}$ is used as in Section 3.1. Figures 3.6, 3.7 and 3.8 show the variation of $\text{STEC}_{2D}$, $\text{STEC}_{3D}$ and $Z$ with time of day, with season, and with latitude respectively.

The variation with time of day (Figure 3.6) shows that throughout the day the behaviour of $Z$ changes for the different thin-layer heights. This is expected as the ionosphere is known to vary drastically on a diurnal basis as seen in the STEC plots of Figure 3.6. The STEC significantly increases during the day, falling to a minima in the early morning. Whilst Figure 3.7 shows that STEC varies with season, $Z$ does not vary notably over
the year, regardless of the assumed height of the thin-layer ionosphere. From Figure 3.8 it can be seen that $Z$ does however vary with latitude. In all of the cases shown in Figures 3.6, 3.7 and 3.8, the strongest variation is with the zenith angle and the assumed altitude of the thin layer. A thin-layer altitude of $h = 450$ km minimises the magnitude of $Z$ for zenith angles of less than $55^\circ$.

The variation over all of these parameters is summarised in Figure 3.9, which provides a range of typical values for $Z$ for thin layers located at altitudes of 250, 350 and 450 km above the Earth’s surface with different latitude, time of year and time of day against zenith angle. The range shows the values of $Z$ lying within one standard deviation of the median value. The median and one standard deviation limit were calculated from data downloaded from the IRI for each hour on one day per month from the past 11 years (the length of the solar cycle). The latitude was also varied between $0^\circ$ and $90^\circ$.

The phase error associated with the thin-layer model on the baseline between two antennas can be found by substituting Equations 3.7 and 3.11 into Equation 3.8, and converting from STEC to phase with Equation 2.15. This phase error can be expressed by

$$
\Delta \phi_{AB} \approx 0.44^\circ \times \left( \frac{\nu}{100 \text{ MHz}} \right)^{-1} \left( \frac{b}{\text{km}} \right) \times \left( \frac{\text{VTEC}}{10 \text{ TECU}} \right) \left( \frac{Z}{0.001 \text{ deg}^{-1}} \right) \frac{1}{\cos \theta^\circ}.
$$

(3.12)

For comparison to the phase error that is calculated by this equation, Perley (1999) states that a radio image typically has a maximum dynamic range of 20,000:1 which corresponds to a phase error on a single baseline within an array of

$$
\phi \approx \frac{N}{DR}
$$

(3.13)

where $N$ is the number of antennas in the array and $DR$ is the dynamic range. Residual ionospheric phase errors will cause a systematic error in the phase measured on each antenna, however due to the largely random distribution of antennas in an array this can be treated as random error in a similar way to how the phase errors resulting from inaccuracy in baseline determination are treated by Perley (1999). If $\Delta \phi_{AB}$ exceeds the value given by Equation 3.13 then the use of the thin-layer model will degrade the dynamic range of an image.

In Table 3.1 I calculate the phase calibration precision required to achieve this dynamic range using Equation 3.13 for the radio synthesis telescopes mentioned at the beginning of this chapter (LOFAR, GMRT, VLA, MWA, and SKA1-LOW) and, as examples, calculate the typical values of the phase error for the longest baselines of these radio synthesis telescopes. For most of these arrays (the VLA, the GMRT, LOFAR, and SKA1-LOW),
Chapter 3 Limits on the validity of the thin-layer model

Figure 3.10: How dynamic range changes with baseline assuming the use of a thin-layer calibration model for a typical observation (assuming a zenith angle of $\theta'' = 45^\circ$) with LOFAR-HBA and SKA1-LOW. The dashed lines indicate a typical mid-latitude nighttime (4 TECU) VTEC and the solid lines indicate a typical mid-latitude daytime (10 TECU) VTEC (Verkhoglyadova et al., 2013).

the use of the thin-layer model will be a limiting factor at the listed frequencies assuming a typical mid-latitude daytime VTEC of 10 TECU (Verkhoglyadova et al., 2013). However for the more compact MWA, the use of the thin-layer model is not a limiting factor unless the ionosphere is extremely active and the VTEC is correspondingly high. If a typical mid-latitude nighttime VTEC of 4 TECU (Verkhoglyadova et al., 2013) is instead assumed, the use of the thin-layer model is not a limiting factor for the LOFAR HBA (Superterp) and the VLA in D-configuration as well as for the MWA.

Given the actual phase error for typical mid-latitude nighttime and daytime conditions shown in Table 3.1, I can calculate the limiting dynamic range for each telescope using Equation 3.13. The results of this are given in Table 3.2. These results will help users of the telescopes listed decide whether the thin-layer assumption is an adequate calibration technique to use depending on the accuracy required for their observations and depending on the VTEC of the ionosphere during their observations.

Figure 3.10 shows how the achievable dynamic range changes with baseline for SKA1-LOW and the LOFAR HBA for typical nighttime and daytime observations. The achievable dynamic range is always better during the typical nighttime observations. This is
Table 3.1: Typical phase error resulting from the thin-layer model for five radio synthesis telescopes: the GMRT, the MWA, LOFAR LBA and HBA (Dutch stations, core and Superterp), the VLA (A and D configurations) and SKA1-LOW. The table displays the frequency, latitude, maximum baseline and magnitude of the TEC ratio derivative, $Z$, assuming a zenith angle of $\theta = 45^\circ$ for each telescope. The lowest frequencies are considered for each telescope, providing a worst-case scenario for observers. The phase error is calculated for each telescope for typical mid-latitude daytime (10 TECU) and nighttime (4 TECU) VTEC ($\text{Verkhoglyadova et al., } 2013$), assuming a pointing azimuth of either due east or due west (i.e. fixed latitude) and a thin-layer altitude of 450 km. The VTEC can vary by a factor of 10 from the typical mid-latitude daytime VTEC and as the VTEC varies, it will cause a proportional change in the phase error. Assuming a dynamic range of 20,000:1, the limiting phase error is calculated for each telescope for comparison. If the phase error calculated for the telescope exceeds the limiting phase error, it is highlighted in red, whilst if it is less than the limiting phase error it is highlighted in green.

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Frequency</th>
<th>Baseline</th>
<th>$Z$</th>
<th>Actual phase error at 10 TECU</th>
<th>Actual phase error at 4 TECU</th>
<th>Max allowed phase error</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMRT</td>
<td>153.0</td>
<td>25.0</td>
<td>0.00020</td>
<td>0.80</td>
<td>2.01</td>
<td>30.00</td>
</tr>
<tr>
<td>LOFAR LBA (Superterp)</td>
<td>30.0</td>
<td>0.24</td>
<td>0.000015</td>
<td>0.03</td>
<td>0.43</td>
<td>10.07</td>
</tr>
<tr>
<td>LOFAR LBA (core)</td>
<td>30.0</td>
<td>3.5</td>
<td>0.00015</td>
<td>0.43</td>
<td>1.43</td>
<td>16.11</td>
</tr>
<tr>
<td>LOFAR HBA (Dutch stations)</td>
<td>110.0</td>
<td>121.0</td>
<td>0.00015</td>
<td>0.01</td>
<td>0.07</td>
<td>24.07</td>
</tr>
<tr>
<td>LOFAR HBA (Superterp)</td>
<td>110.0</td>
<td>2.04</td>
<td>0.000015</td>
<td>0.07</td>
<td>0.29</td>
<td>26.02</td>
</tr>
<tr>
<td>LOFAR HBA (core)</td>
<td>110.0</td>
<td>3.06</td>
<td>0.000015</td>
<td>0.07</td>
<td>0.37</td>
<td>28.05</td>
</tr>
<tr>
<td>MWA</td>
<td>80.0</td>
<td>4.06</td>
<td>0.000015</td>
<td>0.07</td>
<td>0.37</td>
<td>28.05</td>
</tr>
<tr>
<td>VLA (A configuration)</td>
<td>58.0</td>
<td>5.04</td>
<td>0.000010</td>
<td>0.07</td>
<td>0.37</td>
<td>28.05</td>
</tr>
<tr>
<td>VLA (D configuration)</td>
<td>58.0</td>
<td>5.04</td>
<td>0.000010</td>
<td>0.07</td>
<td>0.37</td>
<td>28.05</td>
</tr>
<tr>
<td>SKA1-LOW</td>
<td>50.0</td>
<td>5.04</td>
<td>0.000010</td>
<td>0.07</td>
<td>0.37</td>
<td>28.05</td>
</tr>
</tbody>
</table>
Table 3.2: The limiting dynamic range, given the actual phase error for typical mid-latitude nighttime (4 TECU) and daytime (10 TECU) conditions in Table 3.1.

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Limiting dynamic range at 4 TECU [°]</th>
<th>Limiting dynamic range at 10 TECU [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMRT</td>
<td>1:2,137</td>
<td>1:855</td>
</tr>
<tr>
<td>LOFAR LBA (Superterp)</td>
<td>1:11,642</td>
<td>1:4,657</td>
</tr>
<tr>
<td>LOFAR LBA (core)</td>
<td>1:3,193</td>
<td>1:1,277</td>
</tr>
<tr>
<td>LOFAR LBA (Dutch stations)</td>
<td>1:146</td>
<td>1:58</td>
</tr>
<tr>
<td>LOFAR HBA (Superterp)</td>
<td>1:42,687</td>
<td>1:17,075</td>
</tr>
<tr>
<td>LOFAR HBA (core)</td>
<td>1:11,709</td>
<td>1:4,683</td>
</tr>
<tr>
<td>LOFAR HBA (Dutch stations)</td>
<td>1:536</td>
<td>1:214</td>
</tr>
<tr>
<td>MWA</td>
<td>1:166,733</td>
<td>1:66,693</td>
</tr>
<tr>
<td>VLA (A configuration)</td>
<td>1:1,002</td>
<td>1:401</td>
</tr>
<tr>
<td>VLA (D configuration)</td>
<td>1:35,401</td>
<td>1:14,160</td>
</tr>
<tr>
<td>SKA1-LOW</td>
<td>1:18,341</td>
<td>1:7,336</td>
</tr>
</tbody>
</table>

because the VTEC is lower during typical nighttime conditions than during typical daytime conditions. These results should prompt observers to opt for nighttime observations over daytime observations.

### 3.3 Implications for epoch of reionisation experiments

The primary science goal of SKA1-LOW is to observe the highly-redshifted 21 cm neutral hydrogen line from the epoch of reionisation and earlier. To achieve this with SKA1-LOW, a dynamic range of $>10^6$ is required (Koopmans et al., 2015). In Figure 3.10, I show how the dynamic range changes with baseline for SKA1-LOW, assuming that the same number of antennas are used. As the same number of antennas are compressed within a smaller area (thus decreasing the baseline), dynamic range increases. It can be seen that a baseline of 1.19 km with a typical nighttime observation (a VTEC of 4 TECU) is capable of achieving the required dynamic range of $10^6$ when used in conjunction with the thin-layer approximation.

In reality the number of antennas would decrease as the maximum baseline became shorter as 512 stations, each being a few 10s of metres in diameter (Dewdney et al., 2016), cannot physically fit in an area of the smallest diameter shown in Figure 3.10, 0.25 km. I therefore include the dynamic range at the baselines for the LOFAR-HBA Dutch stations, core and Superterp, as shown in Table 3.1, for reference. It can be seen that as the number of antennas decreases with decreasing baseline, a smaller improvement in the dynamic range is seen.
Fortunately 512 stations can fit within a diameter of 1.19 km. It is therefore possible for an experiment to achieve a dynamic range of $10^6$, enabling detection of the epoch of reionisation signal, whilst using the thin-layer approximation if horizontal variation in the ionospheric electron content is ignored.

### 3.4 Conclusions

In this chapter I have numerically investigated the phase error that results from using the thin-layer model in ionospheric phase calibration for a radio synthesis telescope, independent of horizontal variation in the ionospheric electron content, using a realistic vertical electron density profile. I find that an assumed thin-layer altitude of $\sim 450$ km generally minimises this particular error. The final result allows observers to determine the circumstances under which the thin-layer model can reasonably be applied to radio observations: Equation 3.12, together with typical values for the TEC ratio derivative $Z$ from Figure 3.9, provides a value for the phase error resulting from the use of the thin-layer model. I find that this phase error limits the imaging fidelity for long-baseline arrays such as the VLA, the GMRT, and LOFAR LBA array and HBA array, as well as the future SKA1-LOW, as shown in Table 3.1.
Chapter 4

IONONEST

Obtaining high resolution electron density height profiles for the lower ionosphere as a well sampled function of time is difficult for most methods of ionospheric measurement. Here I present a new method of using multi-frequency riometry data for producing electron density height profiles of the lower ionosphere via inverse methods. To obtain these profiles I use the nested sampling technique, implemented through my code, IONONEST. Parts of this chapter have been published as (Martin et al., 2016b).

In this chapter a new method of using multi-frequency absorption data to infer an electron density height profile for the lower ionosphere using IONONEST is explored. IONONEST is a tool for finding the shape of the electron density height profile from multi-frequency absorption measurements. It can be used with any ionospheric model and has the ability to compare the quality of fit of those models. IONONEST uses the nested sampling method implemented through the publicly available MULTINEST software library (Feroz and Hobson 2008; Feroz et al. 2009). Telescopes such as KAIRA, the LOFAR LBA and the LWA (Long Wavelength Array; Ellingson et al. 2009) can use this method to recover electron density height profiles from multi-frequency absorption data. This will enable continuous observations of the lower ionosphere to be made at a range of latitudes, allowing models of the currently under observed D layer to be improved.

First I investigate the validity of three different electron density height profile models by fitting them to electron density height profiles obtained by an incoherent scatter radar, the EISCAT VHF radar, before introducing IONONEST. I test IONONEST by using it to fit electron density height profiles to simulated multi-frequency absorption data. Section 4.1 introduces the nested sampling method, as well as the electron density height profile models. Section 4.1 also investigates whether realistic electron density height profiles such as those observed by the EISCAT VHF can be recreated using
these models. Section 4.2 describes the implementation of IONONEST and how it uses the nested sampling method to recover electron density height profiles from absorption measurements, as well as detailing the application of IONONEST to simulated data sets to establish if IONONEST can return true electron density height profiles from absorption data.

4.1 Height profile determination

To create electron density height profiles from multi-frequency absorption data, a parameterised model that provides smooth electron density height profiles is required. A variety of lower ionosphere electron density height profile models exist, including models that describe the physical and chemical processes that create the lower ionosphere (outlined in Section 1.1.1) as well as models that simply describe the shape of the electron density height profile. In this section I introduce three electron density height profile models. By varying the parameters in the models I can simulate profiles which can then be compared to observed data using a method called nested sampling (Skilling, 2004). This nested sampling technique is based on Bayesian inference, which allows a set of parameters, \( \theta \), in a model, \( M \), given data, \( x \), to be estimated. It uses Bayes theorem, which states,

\[
p(\theta|x, M) = \frac{p(x|\theta, M)p(\theta|M)}{p(x|M)},
\]

(4.1)

where the posterior probability distribution is \( p(\theta|x, M) \equiv P(\theta) \), the likelihood is \( p(x|\theta, M) \equiv L(\theta) \) and the prior probability distribution is given by \( p(\theta|M) \equiv \Pi(\theta) \). \( p(x|M) \) is the Bayesian evidence, which is the factor that normalises the posterior over the prior volume and can be ignored in the case of parameter estimation. In the case of model selection the evidence is needed in order to determine which model performs best when using a common data set.

4.1.1 Nested sampling

Whilst most model fitting algorithms concentrate on parameter estimation, calculating the Bayesian evidence often as a by-product, the nested sampling technique calculates the Bayesian evidence directly. The nested sampling algorithm selects a range of points from the parameter space to create models, these models are then compared to the data and a likelihood value is returned. A weight is then assigned to the point in the parameter space with the lowest likelihood, before it is recorded and then discarded from the selection of points in the parameter space. A new point with a higher likelihood is then selected from the parameter space using a Monte Carlo method. This causes the sampler to converge on the likelihood peaks of the parameter distribution. Knowledge of the model parameters is defined by the priors before the model is fitted to the data.
A prior can be defined using the posterior of previous observations or as the available parameter space.

MULTINEST implements the nested sampling method, using Bayesian inference (e.g. Trotta 2008) to select a model using the evidence as the factor to normalise the posterior. This is done by transforming the multi-dimensional evidence integral into a one-dimensional integral in order to calculate the evidence (Feroz et al., 2009).

This method reduces the computational time required to fully explore the parameter space and is effective for posteriors which are multimodal. MULTINEST has recently been improved to also use importance nested sampling (Cameron and Pettitt, 2013) which uses all sampled points in the evidence calculation. To date MULTINEST has been used for a number of astronomical applications, including dispersion measure fitting (TEMPONEST; Lentati et al. 2014) and object detection (Feroz et al., 2008).

4.1.2 Electron density height profile models

The parameterised electron density height profile models of the lower ionosphere that have been chosen to be investigated in this chapter are:

- **Model 1**: A two parameter exponential model.
- **Model 2**: A polynomial model.
- **Model 3**: An electron precipitation model.

These models and the methods used to evaluate the strength of evidence for them are discussed below. For each model the parameters that are varied are stated along with the priors for the parameters. Information about the past applications of the models and any additional variable required is also given below.

4.1.2.1 Model 1: Two parameter exponential model

Model 1 is a simple two-parameter exponential model that was originally introduced as a parameterisation of the electron density profile of the lower ionosphere by Wait and Spies (1964). It is a model of the form,

$$N_e(h) = 1.43 \times 10^7 \exp[-0.15h'] \exp\left[(\beta - 0.15)(h - h')\right],$$  \hspace{1cm} (4.2)

where $h$ is the altitude above the Earth’s surface and is measured in km. The model parameters, $h'$ and $\beta$ are measured in km and km$^{-1}$ respectively. A larger value of $\beta$ causes the electron density to increase more rapidly with altitude and a larger value of
Figure 4.1: Taken from Cheng et al. (2006) (Figure 6). A comparison of nighttime electron density profiles derived by Cheng et al. (2006) using Equation 4.2 to profiles obtained from nighttime rocket experiments.

$h'$ increases the height of the profile. $\beta$ can vary between 0 and 1, whilst I allow $h'$ to vary from 60 km to 110 km.

This model has previously been successfully used and was found to be comparable to measurements made using VLF propagation and directly observed D layer profiles by many authors, for example, Sechrist (1974), Thomson (1993) and Cheng et al. (2006). Figure 4.1 shows a comparison of this model to profiles obtained by nighttime rocket experiments, demonstrating that although the model does not recreate small-scale variations in the electron density, it does recreate the overall variation in the lower ionosphere.

4.1.2.2 Model 2: Polynomial model

Parthasarathy et al. (1963), Stoker (1987) and more recently McKay-Bukowski et al. (2015) have all fitted a polynomial model to the electron density in the lower ionosphere, with Parthasarathy et al. (1963) using a cubic polynomial and McKay-Bukowski et al. (2015) assuming a quadratic polynomial. Here, I also investigate the possibility that the lower ionosphere can be fitted by a polynomial of the form

$$N_e(h) = \exp \left( a_0 + a_1 \left( \frac{h - h_{\text{ref}}}{50} \right) + \ldots + a_n \left( \frac{h - h_{\text{ref}}}{50} \right)^n \right),$$

where $a_i$ is a model parameter and $h$ is the altitude above the Earth’s surface. The parameters ($\theta = a_0, a_1, \ldots, a_n$) are allowed to vary over a wide range of standard uniform priors, where $-200 \leq \theta \leq 200$. The reference height, $h_{\text{ref}}$, is set equal to 80 km, which is the height at which $N_e(80) = \exp(a_0)$. 
4.1.2.3 Model 3: Electron precipitation model

Many authors, including Kero et al. (2014), have proposed that the electron density in the lower ionosphere can be accurately represented using an electron precipitation model as detailed by Rees (1989).

Electron precipitation, introduced in Section 1.1.2, causes ionisation in the ionosphere. The ionisation rate caused by electron precipitation, \( Q(h) \), is defined by

\[
Q(h) = \int_{0}^{\infty} \frac{E \Lambda \left( \frac{s}{R(E)} \right) \rho(h)}{R(E) \epsilon_{\text{ion}}} J_0 E^\gamma \exp \left( \frac{-E}{E_0} \right) dE, \tag{4.4}
\]

where \( \Lambda \left( \frac{s}{R(E)} \right) \) is a function describing the rate of energy dissipation through collisions with neutral particles and was obtained by Grün (1957) for air and by Barrett and Hays (1976) for \( \text{N}_2 \). \( \Lambda \left( \frac{s}{R(E)} \right) \) is a function of the fractional range \( \frac{s}{R(E)} \) where \( s \) is the atmospheric scattering depth (\( s = \int_{h}^{\infty} \rho(h) dh \)) and \( R(E) \) is the effective range of the electrons, as detailed in Rees (1989). The values for the energy dissipation function for monoenergetic electrons of isotropic angular dispersion up to 80° were obtained from Figure 4.2 which is taken from Rees (1963). The isotropic curve is used because Semeter and Kamalabadi (2005) found that the electrons with the highest energy tend to have a strictly isotropic distribution, and these high-energy electrons carry the majority of the energy flux. \( \rho(h) \) is the mass density and can be obtained from the NRL-MSISE-00 reference atmosphere (Hedin, 1991) and \( \epsilon_{\text{ion}} \) is the energy loss per ion formation. \( \gamma \) is the power law that determines the shape of the spectrum, \( E_0 \) [keV] is the characteristic energy of the electron precipitation, giving the energy of the precipitating electrons at the top of the ionosphere (del Pozo et al., 2002), and \( J_0 \) [m\(^{-2}\)s\(^{-1}\)sr\(^{-1}\)keV\(^{-1}\)] is the electron flux (the rate of flow of the precipitating electrons per unit area).

There are three unknowns in Equation 4.4, \( \gamma, E_0 \) and \( J_0 \). In Kero et al. (2014), an exponential flux distribution is assumed and so \( \gamma = 0 \) is set. I make the same assumption throughout this chapter. Priors for \( E_0 \) and \( J_0 \) were also obtained from Kero et al. (2014) where they are set to be \( 0 \leq \log(E_0) \leq 2 \) and \( 3 \leq \log(J_0) \leq 15 \).

It has been found via empirical methods that the value of \( \epsilon_{\text{ion}} \) is approximately the same for the major species in the upper atmosphere (\( \text{N}_2, \text{O}_2 \) and \( \text{O} \)). It is therefore common practice to set \( \epsilon_{\text{ion}} = 35 \) eV.

The effective range of the electrons is the distance from the source that an electron with an initial energy of \( E \) can travel before it is stopped. The parameter is derived experimentally, and can be approximately described by

\[
R(E) = 4.3 \times 10^{-7} + E^{1.67} \times 5.36 \times 10^{-6}, \tag{4.5}
\]

where \( R(E) \) is measured in g cm\(^{-2}\) and assuming that 0.2keV \( \leq E \leq 50\)keV.
Chapter 4 IONONEST

Figure 4.2: Taken from Rees (1963). The normalised energy dissipation distribution function for four different angular dispersions of incident electrons.

The electron precipitation model describes the electron density height profile as

\[ N_e(h) = N_{eq}(h) \sqrt{1 + \frac{Q(h)}{Q_0(h)}}, \]  

(4.6)

where \( N_{eq}(h) \) is the quiet day electron density profile, \( Q_0(h) \) is the quiet day ionisation rate and \( Q(h) \) is the ionisation rate on the day of observation.

Kero et al. (2014) calculate \( Q_0(h) \) and \( N_{eq}(h) \) using the Sodankylä Ion Chemistry (SIC; Enell et al. 2005) model, assuming a recombination rate, \( L(h) \), that does not vary throughout the observation. However \( Q_0(h) \) can also be calculated using \( N_{eq}(h) \) and the recombination rate, \( L(h) \). As access to the SIC model is restricted, I instead use approximate values for \( N_{eq}(h) \) and \( L(h) \) and calculate \( Q_0(h) \) using

\[ Q_0(h) = L(h)N_{eq}(h)^2. \]  

(4.7)

The values used for \( N_{eq}(h) \) are the values of electron density obtained from the IRI (Section 1.1.1.1) for the period of the EISCAT observations. The approximate values used for \( L(h) \) are obtained from empirical measurements made by Barabash et al. (2012) and are shown by the points in Figure 4.3. These values of \( L(h) \) only extend up to an altitude of 100 km, so I extrapolate them to higher altitudes. I do this by fitting a function using the Levenberg-Marquardt non-linear least squares algorithm to the
empirical measurements. Using this method, \( L(h) \) can be extended up to an altitude of 110 km. The best fit to the empirical measurements of \( L(h) \) is found to be

\[
L(h) = 0.02169h^2 - 0.4784h + 19.47. \tag{4.8}
\]

Figure 4.3 shows the functional form of \( L(h) \) as well as the empirical measurements of Barabash et al. (2012). It also shows an example plot for \( N_{eq}(h) \), as well as the resulting \( Q_0(h) \) profile.

Figure 4.3: Variation of \( L(h) \), \( N_{eq}(h) \) and \( Q_0(h) \) with height. As detailed in the text these profiles are obtained from a least squares fit to the empirical measurements of \( L(h) \) made by Barabash et al. (2012) and the IRI model.

4.1.2.4 Comparison of models

The evidence ratios (Bayes factors) of the model fits can be used to determine the optimal model using the Jeffreys scale (Jeffreys, 1961). The Jeffreys scale provides a formal way of evaluating which model is better supported by the data.

One model can be compared to another model using the Bayes factor,

\[
B_{ij} = \frac{p(x|M_i)}{p(x|M_j)} = \frac{\int p(\theta_i|M_i) p(x|\theta_i, M_i) d\theta_i}{\int p(\theta_j|M_j) p(x|\theta_j, M_j) d\theta_j}, \tag{4.9}
\]
Table 4.1: An empirical scale taken from Table 1 of Trotta (2008) for use in evaluating the strength of evidence when comparing two models, $M_i$ and $M_j$.

| $|\ln B_{ij}|$ | Odds | Probability | Strength of evidence |
|----------------|------|-------------|----------------------|
| $< 1.0$        | $< 3:1$ | $< 0.750$ | Inconclusive         |
| $1.0$          | $\sim 3:1$ | $0.750$ | Weak evidence        |
| $2.5$          | $\sim 12:1$ | $0.923$ | Moderate evidence    |
| $5.0$          | $\sim 150:1$ | $0.993$ | Strong evidence      |

where $x$ is the observed data and the two different models are $M_i$ and $M_j$ with parameters $\theta_i$ and $\theta_j$. This Bayes factor gives the change in relative probability of the two models. If the Bayes factor is greater than 1 then Model $i$ is favoured over Model $j$, and vice versa if $B_{ij}$ is less than 1. To evaluate the strength of the evidence when comparing the two models, I use a revised version of the Jeffreys scale as detailed in Table 1 of Trotta (2008). For reference, I include this table as Table 4.1.

The quality of the prediction can also be evaluated using the standardised mean squared error (Rasmussen and Williams, 2005). The standardised mean square error (SMSE) is calculated by computing the squared residual between the mean prediction and the target at each point, the mean of these values is then found (the mean squared error) and the value is then normalised by the variance of the data to output the SMSE,

$$\text{SMSE} = \frac{\sum_{i=1}^{n} (Z_i^* - Z_i)^2}{Z_{\text{var}}}, \quad (4.10)$$

where $Z_i^*$ is the estimated parameter, $Z_i$ is the measured parameter and $Z_{\text{var}}$ is the variance of the measured parameter. Smaller values obtained by the SMSE indicate a better goodness-of-fit between the estimated and measured parameters.

4.1.3 Validation of models

To validate the three models described in Section 4.1.2 I fit them directly to observed EISCAT electron density height profiles using the nested sampling method. This provides verification of whether the models are capable of producing realistic electron density height profiles. The strength of evidence is then assessed for each of the models using a modified Jeffreys’ scale (Trotta, 2008) as well as the SMSE.

4.1.3.1 EISCAT VHF observations

The EISCAT VHF observations that I use to fit the electron density height profile models to were made using a zenith pointing with a temporal resolution of 1 minute. The radar
Table 4.2: The order of the polynomial fit to the EISCAT height profile data with the fitted maximum likelihood polynomial coefficients, as well as the natural logarithm of the evidence for a 3σ prior volume (ln Z) and the difference between these (Δ ln Z). Figure 4.5 shows the resulting height profile for each polynomial order in comparison with the EISCAT height profile.

<table>
<thead>
<tr>
<th>Order</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>ln Z</th>
<th>Δ ln Z</th>
<th>SMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.66 ± 0.04</td>
<td>5.81 ± 0.09</td>
<td>–</td>
<td>–</td>
<td>-271.28 ± 0.07</td>
<td>–</td>
<td>20.73</td>
</tr>
<tr>
<td>2</td>
<td>8.90 ± 0.08</td>
<td>16.71 ± 0.51</td>
<td>17.19 ± 0.73</td>
<td>–</td>
<td>-13.04 ± 0.06</td>
<td>-258.24 ± 0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>3</td>
<td>8.97 ± 0.10</td>
<td>15.36 ± 1.24</td>
<td>-12.07 ± 4.33</td>
<td>-5.26 ± 4.36</td>
<td>-31.19 ± 0.15</td>
<td>18.15 ± 0.27</td>
<td></td>
</tr>
</tbody>
</table>

observing mode ‘manda’ (Section 1.2.1.1) was used for the observations as it is optimised for the measurement of plasma parameters in the D and E layers of the ionosphere, the heights at which absorption primarily occurs.

The observations began at 23:00 UT on 01 March 2015 and ended at 05:00 UT on 02 March 2015. Figure 4.4 shows the electron density height profile as measured by the EISCAT VHF.

![Electron density height profiles as measured by the EISCAT VHF between 23:00 UT on 01 March 2015 and 05:00 UT on 02 March 2015.](image)

**4.1.3.2 Establishing the order of the polynomial**

I begin by establishing the order of the polynomial needed for Model 2 to fit the electron density height profile data obtained by the EISCAT VHF. To do this, I vary the order of the polynomial to establish at what point the evidence reaches its maxima. This process is shown in Table 4.2, with the resulting height profiles for each polynomial order displayed in Figure 4.5. It can be seen from Figure 4.5 that both a second-order and a third-order polynomial fit the EISCAT height profile well, however the evidence for the models (Table 4.2) shows that a second-order polynomial creates the best-fitting profile. I therefore choose to use a second-order polynomial for Model 2.
Figure 4.5: The EISCAT height profile and the height profiles obtained by finding the maximum likelihood polynomial coefficients for varying polynomial order in Model 2. The polynomial coefficients for these height profiles are displayed in Table 4.2.

In Table 4.2 the SMSE has also been calculated for each polynomial fit. Whilst the calculation of the SMSE (Equation 4.10) will always show a better fit with increase in order of the polynomial, the Bayesian fitting integrates over all parameters in each model and so includes a penalty for complex models that include additional unnecessary parameters (Occam’s razor). A complex model should only return a larger posterior probability if it needs to be that complex to fit the data.

### 4.1.3.3 Model fits to EISCAT data

Using the nested sampling method, the models are fitted to the electron density height profiles observed by the EISCAT VHF. The log-likelihood that the electron density height profile data are described by the electron density height profile model is given by

\[
L = \sum_{i=1}^{n} \left( -\frac{1}{2} \ln \left( 2\pi \sigma_{\text{obs},i}^2 \right) - \frac{(N_{\text{obs},i} - N_{\text{mod},i})^2}{2\sigma_{\text{obs},i}^2} \right),
\]

where \( N_{\text{mod}} \) is the electron density height profile model, \( N_{\text{obs}} \) is the electron density measured by the EISCAT VHF, and \( \sigma_{\text{obs}} \) is the uncertainty associated with the measured electron density values.

Figure 4.6 shows the model fits to the observed electron density height profiles, whilst Figure 4.7 shows comparisons of the height of the peak electron density and the peak electron density output by the models to that measured by EISCAT between 80–110 km.
The top plots in Figure 4.6 show the EISCAT electron density height profiles for comparison to the model fits to the height profiles, shown in the middle plots. The bottom plots in Figure 4.6 show the ratios between the EISCAT electron density height profiles and the models fitted to the EISCAT electron density height profiles. I find that all three models return similar temporal variability in electron density to the EISCAT electron density height profiles.

It can clearly be seen from the model fits to the height profiles (Figure 4.6) that Model 2 is able to recreate the EISCAT electron density height profiles most effectively. The model is flexible enough to accommodate almost any profile shape and thus returns a better representation of the EISCAT data than the other models. It succeeds to some degree in returning the height of the peak electron density in the EISCAT data, as well as effectively returning the peak electron density, as shown in Figure 4.7b.

Model 1 returns increases in electron density that correspond to increases in the EISCAT data. The model does not, however, return the peaks in electron density at the same height as in the EISCAT data, nor does it return the true peak electron density when the ionosphere is particularly active. As Model 1 is an exponential model, the peak electron density will always be at the highest point in the altitude range, thus giving a constant peak electron density of 110 km in Figure 4.7a. Therefore the peak electron density is always overestimated to fit to the rest of the electron density profile and so Model 1 always overshoots at the top of the altitude range. In fact, as shown in Section 4.1.2.1, Cheng et al. (2006) only apply this model to data below 95 km, possibly for this reason. As well as being overestimated at higher altitudes in the height range, Model 1 also causes the electron density to be consistently overestimated at lower altitudes (80–82 km), but underestimated in the middle of the height range when the ionosphere is particularly active.

Whilst the height profiles fitted to the EISCAT data by Model 3 do return the same general shape as the EISCAT data, the electron density at the bottom of the height range (80–82 km) is consistently lower than that observed with EISCAT. Similarly to the Model 1 fit, the model does not return the peaks in electron density at the same height as in the EISCAT data, instead returning a constant peak electron density of 110 km (as demonstrated in Figure 4.7c). Figure 4.7c also shows that Model 3 overestimates the electron density at this peak height.

I evaluate the fits using the methods described in Section 4.1.2.4. I calculate the Bayes factor, finding that $B_{12} = 0.20$, $B_{13} = 0.34$ and $B_{23} = 3.41$. Therefore Model 3 is favoured over Model 1, however Model 2 is favoured over both Model 1 and Model 3. The value of $|\ln B_{21}|$ is found to be 2.25 and the value of $|\ln B_{23}|$ is found to be 1.17. Using the empirical scale of Trotta (2008) (reproduced in Table 4.1), this means that there is ‘weak evidence’ for Model 2 being favoured over Model 1 and Model 3.
Figure 4.6: TOP: EISCAT electron density height profiles. MIDDLE: Model fits to EISCAT data. BOTTOM: The ratio of the EISCAT height profiles and the model fitted to the EISCAT height profiles.
Figure 4.7: Comparison of the height of the peak electron density and the peak electron density output by the models to that measured by EISCAT between 80–110 km.
To compare the model fits to the EISCAT electron density height profiles using the SMSE, $Z_i$ is set to be the EISCAT electron density, $Z_{var}$ to be the variance in the EISCAT data and $Z_i^*$ to be the Model 1, 2 or 3 fit to the EISCAT electron density. For Model 1 the SMSE value was found to be 0.76, for Model 2 it was found to be 0.11 and for Model 3 it was found to be 0.53, again showing that Model 2 is the best fit to the EISCAT electron density height profiles as the SMSE value obtained is smaller than that found for Models 1 and 3. This indicates a better goodness-of-fit between the data estimated by Model 2 and the EISCAT data. The SMSE values also confirm that Model 3 is a better fit than Model 1, in agreement with the Bayes factors calculated above.

4.2 IONONEST

In order to recover electron density height profiles from absorption data, I implement the nested sampling method detailed in Section 4.1.1 by interfacing with MULTINEST. I have named this implementation IONONEST.

IONONEST is a tool that can find the shape of the electron density height profile from multi-frequency measurements of absorption. It assumes a model for the electron density height profile and varies the parameters in the model. The absorption profile for this modelled electron density height profile is then created using Equation 2.16 and compared to multi-frequency absorption data.

The inputs to IONONEST are an observed, or simulated, data set of multi-frequency absorption values, an electron density height profile model and priors, where the priors are the parameter space over which parameters in the electron density height profile models are allowed to vary. MULTINEST then selects values for the parameters from the priors and IONONEST creates an electron density height profile from which the associated absorption values at the observing frequencies are calculated. It then compares the calculated absorption values to the observed or simulated absorption values and returns a likelihood to MULTINEST. At any time, MULTINEST has 1000 live points, and once IONONEST returns the likelihood it has calculated to MULTINEST, MULTINEST drops the point with the lowest likelihood from the set of live points and selects new parameter values using a Markov Chain Monte Carlo method. This process terminates when MULTINEST finds the log-evidence to a precision of 0.1. A flow diagram of the inputs, outputs and processes for IONONEST is shown in Figure 4.8.

The noise on the KAIRA data is expected to be Gaussian and I assume uncorrelated noise between frequency channels. The log-likelihood that the absorption data are described
by the electron density height profile model is therefore given by

$$L = \sum_{i=1}^{n} \left( -\frac{1}{2} \ln \left( 2\pi \sigma_{\text{obs},i}^2 \right) - \frac{(A_{\text{obs},i} - A_{\text{mod},i})^2}{2\sigma_{\text{obs},i}^2} \right),$$

(4.12)

where $A_{\text{mod}}$ is the absorption due to the electron density height profile model, $A_{\text{obs}}$ is the observed absorption and $\sigma_{\text{obs}}$ is the uncertainty associated with the observed absorption values. For parameter estimation, the first term in this sum can be ignored.

To verify that IONEST is capable of returning accurate parameter estimates, I simulate absorption data and use IONEST to find fits to that simulated data.

### 4.2.1 Fit to simulated absorption data

Having shown in Section 4.1.3 that all three models provide reasonable fits to electron density height profiles derived from EISCAT data, I now run the IONEST code on absorption data sets simulated from generated height profiles. From this test it can be seen if the parameters input into the simulation can be returned by IONEST.

I choose to test IONEST on simulated electron density height profiles because this allows me to see any difference in result caused by IONEST, rather than caused by the model being an inaccurate fit to a real electron density height profile.

#### 4.2.1.1 Fit to a single simulated electron density height profile

Individual electron density height profiles are simulated using each of the models detailed in Section 4.1.2. These electron density profiles are simulated using the input parameters detailed in Table 4.3 and are shown in the top plots of Figure 4.9 (labelled with ‘Input’). Absorption values for frequencies in the range 17.38–56.05 MHz are calculated for the height profiles and white noise is added as well as the instrumental, frequency-dependent noise described in Section 2.1.1.1 at a similar level to that of a KAIRA observation.
Figure 4.9: TOP: Simulated model height profile (‘input’), with a height profile fitted to absorption data generated from this simulated height profile plotted over the top (‘output’). The shaded green region shows the error on the fit. BOTTOM: Simulated absorption data with error bars plotted, with the fit to the absorption data using the model as found by IONONEST plotted over the top.
Table 4.3: Values for the input parameters that created the simulated data and the parameters output for the best fit to the simulated data.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$h'$</td>
<td>66.351</td>
<td>64.474±3.453</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>0.293</td>
<td>0.276±0.034</td>
</tr>
<tr>
<td></td>
<td>$\log (k)$</td>
<td>-5.088</td>
<td>-5.083±0.027</td>
</tr>
<tr>
<td>2</td>
<td>$a_0$</td>
<td>7.977</td>
<td>7.934±0.964</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>12.858</td>
<td>13.359±5.513</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>-9.127</td>
<td>-11.389±9.418</td>
</tr>
<tr>
<td></td>
<td>$\log (k)$</td>
<td>-5.088</td>
<td>-5.087±0.030</td>
</tr>
<tr>
<td>3</td>
<td>$\log (J_0)$</td>
<td>1.989</td>
<td>1.883±0.366</td>
</tr>
<tr>
<td></td>
<td>$\log (E_0)$</td>
<td>4.556</td>
<td>4.915±0.376</td>
</tr>
<tr>
<td></td>
<td>$\log (k)$</td>
<td>-5.088</td>
<td>-5.082±0.017</td>
</tr>
</tbody>
</table>

Figure 4.10: Posterior distributions for each Model 1 parameter as output by iononest.

This is done by adding another parameter to fit with iononest, $k$, as introduced in Equation 2.4.

The simulated absorption data corresponding to the simulated electron density profiles are shown in the bottom plots of Figure 4.9. These simulated absorption profiles are then input into iononest. The priors are set to vary between the maximum and minimum
Figure 4.11: Posterior distributions for each Model 2 parameter as output by IONONEST.

parameter values obtained for each model in the fits to the EISCAT data shown in Figure 4.6 $\pm 3\sigma$. The prior for $k$ for the models is set to a standard uniform prior, $-5.5 \leq k \leq -4.5$. These priors are listed in Table 4.4. IONONEST returns the parameters to create the height profile once it fits an absorption profile to the simulated absorption data, as shown by the bottom plots of Figure 4.9. Figure 4.9 shows that IONONEST fits the simulated absorption data using Models 1, 2 and 3 within an error of $\pm 1\sigma$.

The parameters output by IONONEST and used to create these profiles are listed in Table 4.3 and labelled as ‘Output’ in Figure 4.9. From Table 4.3 it can be seen that IONONEST also returns these parameter values to within $1\sigma$ for all three models. The errors for the electron density height profiles (shown by the green shaded areas in Figure 4.9) are calculated using

$$
\sigma = \sqrt{\frac{\sum_{i=1}^{n} p_i (E_i - pF)^2}{\sum_{i=1}^{n} p_i}},
$$

(4.13)

where $p$ is the sample probability and $F$ is the electron density. The sample probability is an output from MULTINEST and is found by multiplying the sample prior mass by its
Figure 4.12: Posterior distributions for each Model 3 parameter as output by iONEST.

Table 4.4: Priors for the three models found using $\mu \pm 3\sigma$ as obtained from fitting the models directly to the EISCAT data.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$h'$</td>
<td>55.65</td>
<td>74.71</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>0.22</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>$\log (k)$</td>
<td>-5.5</td>
<td>-4.5</td>
</tr>
<tr>
<td>2</td>
<td>$a_0$</td>
<td>5.97</td>
<td>10.17</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>2.25</td>
<td>24.81</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>-28.36</td>
<td>5.42</td>
</tr>
<tr>
<td></td>
<td>$\log (k)$</td>
<td>-5.5</td>
<td>-4.5</td>
</tr>
<tr>
<td>3</td>
<td>$\log (J_0)$</td>
<td>0.27</td>
<td>3.25</td>
</tr>
<tr>
<td></td>
<td>$\log (E_0)$</td>
<td>4.28</td>
<td>5.55</td>
</tr>
<tr>
<td></td>
<td>$\log (k)$</td>
<td>-5.5</td>
<td>-4.5</td>
</tr>
</tbody>
</table>

In equation (4.14), the posterior distribution is calculated by taking the product of the likelihood and normalising it by the evidence. $p_F$ can then be calculated using

$$p_F = \frac{\sum_{i=1}^{n} p_i F_i}{\sum_{i=1}^{n} p_i}.$$  (4.14)
Figures 4.10, 4.11 and 4.12 show the 1D and 2D marginalised posteriors for the parameters as output by IONONSEST for the fits to the data shown in Figure 4.9. The colour of the 2D plots represents the marginalised density, with the contour lines representing the 1σ, 2σ, and 3σ levels. The dashed lines on the 1D plots represent the mean likelihood for each parameter, whilst the solid lines represent the marginalised likelihood. The vertical lines represent the ‘output’ values as shown in Table 4.3.

It is noted from Figures 4.10, 4.11 and 4.12 that the posterior probability distributions for these parameters are highly degenerate, especially for Models 2 and 3. This means that a large range of parameters are capable of producing absorption curves that fit the simulated absorption data which results in different electron density height profiles being reproduced by IONONSEST regardless of the close fit in absorption data. This is most likely to be the cause of the large errors in electron density at high altitudes seen in the top plot of Figure 4.9b. Model 2 has trouble reproducing the input profile above an altitude of 100 km as the uncertainties become large at altitudes greater than 100 km. The electron density height profiles returned are, however, similar to the simulated electron density height profiles, demonstrating that IONONSEST is capable of recovering electron density height profiles from simulated absorption data.

4.2.1.2 Fit to absorption data simulated from EISCAT height profiles

Having demonstrated that a single electron density height profile can be recovered from simulated absorption data, I now simulate absorption data for the height profiles obtained by fitting the models to the EISCAT data in Section 4.1.3, again adding noise at a similar level to that of a KAIRA observation. I then fit the three separate models to those absorption values in the same way as detailed in Section 4.2.1.1. The returned height profiles are then compared to those simulated from the EISCAT data.

The results of this test are shown in Figure 4.13. The height profiles obtained by fitting each model to the EISCAT VHF data are shown in the top plots of the figure, whilst the middle plots show the resulting electron density height profiles obtained by using IONONSEST to fit to the simulated absorption data. The bottom plots show the ratios between the fit to the EISCAT data and the fit to the simulated absorption data.

Figure 4.13a shows that when the electron density height profile of the ionosphere is assumed to have a two parameter exponential profile, the electron density height profile can be accurately recreated from the simulated absorption measurements. Some parameter variation is observed in $h'$ (Figure 4.14a), however this is expected due to the added noise. The temporal variation in $\beta$ is not recovered by IONONSEST. Nevertheless, the simulated electron density height profile can still be determined using Model 1.
Figure 4.13: (a) Model 1 (b) Model 2 (c) Model 3

(a) Model 1
(b) Model 2
(c) Model 3

Figure 4.13: TOP: Model fitted to measured EISCAT electron density height profile data. MIDDLE: Model fitted to simulated absorption data from the above height profile. BOTTOM: The ratio between the fit to EISCAT data and the fit to simulated absorption data.
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Figure 4.14: Variation in parameters input into and output by IONONEST for each model. The input parameters are those obtained by fitting each model to EISCAT data (the top plots in Figure 4.13) and the output parameters are the resulting parameters obtained by fitting each model to absorption data simulated using the input parameters (the middle plots in Figure 4.13).
Figure 4.13b shows the fit of the polynomial model to the absorption data generated from the Model 2 fit to EISCAT data. Again the fit to absorption data simulated from the fit to EISCAT data is similar to the fit to the EISCAT data, though occasionally the electron density at the higher and lower altitudes in the 80–110 km range are overestimated or underestimated. In general, in the middle of the 80–110 km altitude range, the correct electron density is recovered. This fits with what was observed in Figure 4.9b, where large errors are seen in the higher and lower altitudes of the electron density height profile.

Similarly to Model 1, when the electron density height profile of the ionosphere is assumed to be of the form of Model 3, the electron density height profile can be accurately recreated from the simulated absorption measurements. Figure 4.13c shows that the Model 3 simulated electron density height profile has been accurately recreated using IONONEST. Figure 4.14c shows that the parameter, $J_0$, is very closely recreated. The temporal variation in $E_0$ is, however, not so well recreated.

Model 2 is a more complicated model than Models 1 and 3, which means that there are many more combinations of the parameters capable of producing the same or similar absorption curves, causing the posterior distributions to be degenerate (Figure 4.11). However, the combinations of different parameters do still succeed in recreating a similar electron density height profile.

Only the output parameters of $h'$ (Model 1), $a_0$ (Model 2) and $J_0$ (Model 3) are recovered effectively by the models, as shown in Figure 4.13, however the electron density is successfully reproduced regardless. All three models succeed at returning the simulated electron density height profiles, with Models 1 and 3 recreating the simulated data set more accurately than Model 2.

In order to investigate the quality of fit, each model is fitted to the three simulated sets of absorption data. As in Section 4.1.3, the quality of fit is investigated using the SMSE (Equation 4.10). In this case, $Z_i$ is the absorption data simulated from the model fits to the EISCAT electron density profiles, $Z_{var}$ is the variance in this simulated absorption data and $Z_i^*$ is the model fit to the simulated absorption data. The SMSE values obtained for the fits to the absorption data simulated using Model 1 and Model 3 electron density height profiles show that Model 1 and Model 3 provide the best fits to the data respectively. However, for the fit to the absorption data created using Model 2 electron density height profiles, it is found that Model 1 has a lower SMSE value than Model 2. The SMSE values obtained for fitting Models 1 and 2 to absorption profiles simulated from Model 2 electron density height profiles are 0.007 and 0.16 respectively. This demonstrates that whilst Model 2 is the best model for accurately fitting a realistic electron density height profile, as shown in Section 4.1.3.3, Model 1 is a better fit to absorption data.
Chapter 4 IONONEST

4.3 Conclusions

IONONEST, a new technique for obtaining electron density height profiles for the lower ionosphere from multi-frequency absorption measurements, has been introduced. IONONEST enables the posterior probability distribution of parameters that describe an electron density height profile to be found by comparing measured or simulated absorption data to absorption data calculated from a parameterised electron density height profile model. IONONEST has the benefit over many other inverse methods of returning uncertainties on these returned parameters, as well as sampling efficiently from multi-modal or degenerate posterior distributions, something that standard Markov Chain Monte Carlo methods, such as the Metropolis-Hastings algorithm, struggle with. IONONEST also returns Bayesian evidence values for electron density height profile models, giving IONONEST the ability to compare different electron density height profile models.

I have demonstrated that IONONEST can return accurate parameters for ionospheric electron density height profiles from multi-frequency absorption data calculated using simulated ionospheric electron density height profiles. The output parameters of $h'$ (Model 1), $a_0$ (Model 2) and $J_0$ (Model 3) are recovered effectively by the models. The ability to recover the magnitude of the electron flux, $J_0$, is especially beneficial because it provides useful information for satellites as high fluxes of energetic electrons can cause penetration of the electrons into spacecraft components, resulting in deep-dielectric charging (a buildup of charges). Once the built up charge becomes too large, it can cause discharge or arcing which results in anomalous behaviour of the satellite, or a temporary or even permanent loss of functionality.

In this chapter I have considered three models for ionospheric electron density height profiles, a two parameter exponential model (Model 1), a polynomial model (Model 2) and an electron precipitation model (Model 3). By comparing Bayes factors and SMSE values, I have found that Model 2 is capable of recreating more realistic electron density height profiles, whilst Model 1 and Model 3 consistently overestimate the electron density at the top of the modelled height range.

Model 2 is a more accurate model of a realistic electron density profile as it is capable of reproducing an electron density height profile similar to the observed EISCAT data. Unlike Models 1 and 3, this polynomial model does not find that the peak electron density is always at 110 km. It recovers the peaks at lower heights and is therefore able to recover realistic electron density height profiles far more accurately. It should be noted, however, that when Model 2 is used with IONONEST it has trouble reproducing the input electron density height profile from simulated absorption data above an altitude of 100 km as the uncertainties become large at altitudes greater than 100 km (see Figure 4.9b).
The approximations used in this chapter for Model 3 are inherently inaccurate as I do not have access to an ion chemistry model such as the SIC model. This means that I cannot use this model in conjunction with real multi-frequency absorption data. However, the approximations used here have enabled me to investigate the use of this model with IONONEST and test whether IONONEST is capable of returning information about electron precipitation. I have demonstrated that this model can be used with IONONEST to find electron density height profiles, as well as returning the temporal variability of the input parameter $J_0$. Users with access to ion chemistry models such as the SIC model will therefore be able to apply IONONEST to infer information about electron precipitation using values obtained from the ion chemistry model for $N_{eq}(h)$ and $L(h)$. 
Chapter 5

Application of IONONEST to multi-frequency riometry data

I apply IONONEST to new data from the KAIRA instrument used as a multi-frequency riometer and consider two electron density models. I compare the recovered height profiles from the KAIRA data with those from incoherent scatter radar using data from the EISCAT instrument and find that there is good agreement between the two techniques, allowing for instrumental differences. Parts of this chapter have been published as (Martin et al., 2016b).

In Chapter 4 I have outlined IONONEST and demonstrated that it is capable of returning electron density height profiles from simulated absorption data. I now apply IONONEST to absorption data obtained by using the KAIRA instrument (detailed in Section 2.1.1.1) as a multi-frequency riometer over two observing campaigns, in October–November 2013 and March 2015. The generation of a QDS for each of these observing campaigns is detailed in Section 2.3.4.1.

The electron density height profiles recovered from the KAIRA absorption data using IONONEST can be compared to electron density height profiles measured by the incoherent scatter radar, the EISCAT VHF (detailed in Section 1.2.1.1) which is located just 85 km away from KAIRA. Due to its ability to observe with many pointing directions and its proximity to the EISCAT site, KAIRA is capable of obliquely observing above the EISCAT VHF.

In Chapter 4.2 I investigated the capability of three models to recreate electron density height profiles from simulated absorption data. I found that all three models were capable of recreating electron density height profiles from simulated absorption data within the expected error. Model 2 is a polynomial model and is more flexible than
Models 1 and 3 and therefore can fit to more realistic electron density height profiles such as those observed using the EISCAT VHF. Model 1 is a simple two parameter exponential model that is incapable of returning a peak electron density at a height lower than the top of the height range being investigated. Model 3 is very dependent on a range of parameters that are ideally found using an ion chemistry model. It is therefore inherently inaccurate without access to an ion chemistry model such as the SIC model. As I was unable to gain access to an ion chemistry model, I do not feel that Model 3 is accurate enough to inform us of the electron density height profiles using the approximations discussed in the previous chapter. I therefore do not use Model 3 in this chapter, instead concentrating on Models 1 and 2.

The absorption measurements made using KAIRA are compared to the electron density height profiles measured with the EISCAT VHF instrument in Section 5.1. Section 5.2 details the application of IONONEST to KAIRA absorption data and compares the resulting electron density height profiles to those obtained using the EISCAT VHF.

5.1 Comparison of KAIRA absorption data to EISCAT VHF data

In this section I compare absorption plots obtained from using KAIRA as a multi-frequency riometer during the observing campaigns in October–November 2013 and March 2015 to the corresponding observations taken using the EISCAT VHF.

The LBA array at KAIRA was used for absorption measurements in order to exploit the multi-frequency dependence of the absorption signal. In the observations, eight pointing directions were used, as described in Section 2.3.4.1. In this section, I concentrate on the zenith pointings of the KAIRA telescope. The range of the full-width half maxima at zenith of the KAIRA LBA beams used ranged between 8° at 80 MHz to 40° at 10 MHz. The average power for each beamlet and each polarisation were recorded with the KAIRA LBA array at a temporal resolution of 1 s.

Two separate EISCAT VHF observations were made during the 2013 observing campaign at 18:00–22:00 UT on 22 October 2013 and 02:00–07:00 UT on 25 October 2013. Observations were also made with the EISCAT VHF during the 2015 observing campaign, beginning at 23:00 UT on 01 March 2015 and ending at 05:00 UT on 02 March 2015. The observing mode used during these observations is outlined in Section 4.1.3.1.

Figure 5.1 shows the observations made using the EISCAT VHF at 18:00–22:00 UT on 22 October 2013 and the corresponding absorption measured by KAIRA. The observations were made during a geomagnetically quiet period and there are therefore
Figure 5.1: TOP: Electron density height profiles as measured by the EISCAT VHF between 18:00–22:00 UT on 22 October 2013. The white sections on the plot indicate that the electron density is less than $10^9 \text{ m}^{-3}$ which means that the threshold of detectability for the EISCAT VHF has not been met and so the value for electron density at that altitude has been returned as a zero. BOTTOM: The absorption, $A$, as measured by KAIRA for a beam pointing at zenith for the same time period.

few features of interest in the EISCAT VHF electron density height profiles to compare to the absorption measurements made using KAIRA. The observations made at 02:00–07:00 UT on 25 October 2013 have even fewer features of interest.

Figure 5.2 shows a comparison of the electron density measured by the EISCAT VHF with the absorption measured by KAIRA between 23:00 UT on 01 March 2015 and 05:00 UT on 02 March 2015. Unlike Figure 5.1, these observations were not made during a geomagnetically quiet period, and therefore peaks in electron density can be seen throughout the EISCAT VHF data, and many absorption features can be seen in the KAIRA data. The absorption features measured by KAIRA often correspond to peaks in electron density in the EISCAT VHF height profiles. The peaks in absorption at 00:10 UT, 01:00 UT, 02:00–03:00 UT and 03:30–05:00 UT all correspond to peaks in electron density measured by the EISCAT VHF.

The peak in the KAIRA absorption data at 03:30–05:00 UT is weaker than the peak in absorption data at 02:00–03:00 UT. This could be due to physical differences between the two instruments, for example,

- EISCAT and KAIRA are located 85 km apart and therefore are not observing the same volume of ionosphere.
Chapter 5 Application of IONONEST to multi-frequency riometry data

Figure 5.2: TOP: Electron density height profiles as measured by the EISCAT VHF between 23:00 UT on 01 March 2015 and 05:00 UT on 02 March 2015. The white sections on the plot indicate that the electron density is less than $10^9$ m$^{-3}$ which means that the threshold of detectability for the EISCAT VHF has not been met and so the value for electron density at that altitude has been returned as a zero. BOTTOM: The absorption, $A$, as measured by KAIRA for a beam pointing at zenith for the same time period.

- The EISCAT VHF beam is also very narrow, with a beam size of $0.6^\circ \times 1.7^\circ$ whereas KAIRA has a beam size of $10^\circ - 20^\circ$ (depending on frequency). This means that events that affect the electron density of the D layer such as thin filaments due to electron precipitation may fill the EISCAT beam and appear as bright absorption events causing a peak in the electron density. In the KAIRA observations, however, the bright absorption event caused by the thin filament will be averaged out over the large beam and therefore only the averaged electron density over the whole beam is output in the electron density height profiles, as demonstrated by Figure 2.11.

- KAIRA is also more sensitive to absorption of radio waves caused by increases in electron density at lower altitudes. Whilst there is a strong peak in electron density at 03:30–05:00 UT, the peak is at a higher altitude and therefore does not produce as strong a peak in absorption as is expected.

As there are few features of merit in the EISCAT VHF or KAIRA data from 2013 for be compared when IONONEST returns height profiles from the KAIRA absorption data, I begin by using IONONEST on the KAIRA data obtained in 2015.
5.2 Application of IONONEST to KAIRA data – 2015

I now apply IONONEST to the absorption profiles obtained from the KAIRA pointings at zenith taken during the 2015 observing campaign, as well as the KAIRA pointings intersecting with the EISCAT VHF beam at points A and B in Figure 5.3. The electron density height profiles output by IONONEST are then compared to the profiles as measured by EISCAT.

5.2.1 Pointings above KAIRA (zenith)

First the absorption profiles obtained from the KAIRA zenith pointing taken between 23:00 UT on 01 March 2015 and 05:00 UT on 02 March 2015 are input into IONONEST. Figures 5.4 and 5.5 show comparisons between height profiles obtained using the EISCAT VHF (the top plot in each figure) and resulting height profiles obtained by IONONEST for Model 1 and Model 2 respectively (the middle plot in each figure). The bottom plots show the ratio between these height profiles and the EISCAT VHF height profile.

Both Model 1 and Model 2 show some agreement with the EISCAT VHF electron density profiles, generally recreating the shape of the electron density variation with time. Similarly to Figure 4.6, the ratio in the bottom plot of Figure 5.4 shows overestimation by Model 1 of the electron density at the top and bottom of the height range. It also shows an underestimation of the electron density by Model 1 in the middle of the height range. Figure 5.6a shows that the magnitude of the peak electron density is reasonably well recreated by Model 1, however due to the nature of the model, the height of the peak electron density is always at the maximum of the height range being modelled. On the other hand, Figures 5.5 and 5.6b show that when lower heights of the peak electron density are seen in the EISCAT data, lower heights of the peak electron density are also seen in the electron density height profiles created by Model 2. However the magnitude of the peak electron density for Model 2 (as seen in Figure 5.6b) is smaller than that measured by the EISCAT VHF.

The electron density height profiles returned by Model 1 and Model 2 show an underestimation of electron density in comparison to the measurements made by the EISCAT VHF at the beginning and end of the observation (Figure 5.6). In Chapter 4 it was shown that IONONEST is capable of returning simulated height profiles using these models and so we consider that these are real features that differ from the EISCAT data because of physical differences between the two instruments, as discussed in Section 5.1.

Figure 5.6 shows that the electron density height profiles obtained by Model 1 show a similar peak electron density to the EISCAT data, whilst Model 2 shows a consistently lower peak electron density. However, in Figure 4.7a, I demonstrated that a Model 1 fit to the EISCAT data overestimated the peak electron density. Bearing in mind the
Figure 5.3: The EISCAT VHF and KAIRA pointings. The KAIRA pointings are detailed in Table 2.2. The lowest intersection with the EISCAT beam is at 59.5 km and the highest is at 147.2 km. Only two of these pointings (elevations of 45° and 50°, labelled as A and B) intersect with the EISCAT VHF beam within the 80–110 km height range above the Earth’s surface. I concentrate on these two pointings as the absorption is highest at these heights. I cannot fit below 80 km because the data from the NRL-MSISE-00 reference atmosphere is inaccurate below this height. Access to an ion chemistry model would enable extension down to lower heights.
Figure 5.4: These plots show a comparison between the height profiles generated using Model 1 fitted to the KAIRA absorption data for the zenith pointing and the height profiles obtained from EISCAT. TOP: The EISCAT height profiles for 23:00 UT on 01 March 2015 until 05:00 UT on 02 March 2015. MIDDLE: The Model 1 height profiles fitted to the KAIRA absorption data for the same time period and pointing. BOTTOM: The ratio between the EISCAT data and the Model 1 fit to KAIRA absorption data.

smaller size of the EISCAT beam, it could be assumed that if Model 1 was a more realistic model then it should also underestimate the data.

Using the quality of fit assessment outlined in Section 4.1.2.4, I can evaluate the quality of the prediction using the SMSE (Equation 4.10). When calculating the SMSE, $Z_i^*$ is the estimated parameter, which is the absorption fit to the KAIRA data, whilst $Z_i$ is the measured parameter, which is the absorption data observed by KAIRA. $Z_{\text{var}}$ is the variance of the measured parameter, and so in this case it is the variance of the absorption data observed by KAIRA.

The SMSE values obtained for Model 1 and Model 2 are 0.21 and 0.35 respectively. Whilst the lower value for the Model 1 estimated absorption fit to the data implies that it is an improved fit when compared to Model 2, reinforcing what can be seen in the ratio plots in Figure 5.6, both models are reasonable fits to the data.
Figure 5.5: These plots show a comparison between the height profiles generated using Model 2 fitted to the KAIRA absorption data for the zenith pointing and the height profiles obtained from EISCAT. TOP: The EISCAT height profiles for 23:00 UT on 01 March 2015 until 05:00 UT on 02 March 2015. MIDDLE: The Model 2 height profiles fitted to the KAIRA absorption data for the same time period and pointing. BOTTOM: The ratio between the EISCAT data and the Model 2 fit to KAIRA absorption data.

I also evaluate the fits by calculating the Bayes factor, finding that $B_{12} = 1.087$. This again shows that Model 1 is favoured over Model 2, however the value of $|\ln B_{12}|$ is found to be 0.10. According to the empirical scale given in Table 1 of Trotta (2008) this means that the evidence is ‘inconclusive’ for Model 1 being favoured over Model 2.

### 5.2.2 Pointings above EISCAT

To enable the two instruments to observe the same volume of ionosphere, albeit with KAIRA’s beam pointing obliquely, the KAIRA absorption data from the pointings labelled A and B in Figure 5.3 can be used.

In order to recover the electron density height profile, a homogenous electron density profile over the EISCAT and KAIRA sites is assumed. This allows the assumption of
Figure 5.6: The height and magnitude of the peak electron density as output by IONONSEST using Models 1 and 2 compared to the height and magnitude of the peak electron density as measured by the EISCAT VHF.

an electron density profile whilst taking into account increased absorption due to the longer path length. The electron density height profile is mapped to the path and the values along the path length are interpolated. The absorption occurring along this path is then calculated in the same way as in Sections 4.2 and 5.2.
For the pointing elevation of 45°, the KAIRA beam crosses the EISCAT beam at 85 km above the EISCAT site, which is 120 km along the KAIRA beam, whilst the KAIRA beam at a pointing elevation of 50° crosses the EISCAT beam at 101 km above the EISCAT site, which is at 132 km along the KAIRA beam. Figures 5.7 and 5.8 show the electron density height profiles calculated by IONONEST using Model 1 and Model 2 for the 45° and the 50° pointings respectively. The top plots in Figures 5.7 and 5.8 show the EISCAT VHF electron density height profiles for comparison.

Figure 5.7: These plots show a comparison between the height profiles generated using both models fitted to the KAIRA absorption data for the 45° pointing over EISCAT and the height profiles obtained from EISCAT. The purple lines indicate the points where the two telescope beams intersect. TOP: The EISCAT height profiles for 23:00 UT on 01 March 2015 until 05:00 UT on 02 March 2015. MIDDLE: The Model 1 height profiles fitted to the KAIRA absorption data for the same time period. BOTTOM: The Model 2 height profiles fitted to the KAIRA absorption data for the same time period.

The purple lines marked on the plots in Figures 5.7 and 5.8 indicate the points where the two telescope beams intersect. Figure 5.9 shows comparisons of the EISCAT electron density at these intersection points, with the electron densities as calculated from the KAIRA absorption data by IONONEST for the two models. For comparison, the same plots are also presented for the corresponding ionospheric heights for the zenith pointing.
Figure 5.8: These plots show a comparison between the height profiles generated using both models fitted to the KAIRA absorption data for the 50° pointing over EISCAT and the height profiles obtained from EISCAT. The purple lines indicate the points where the two telescope beams intersect. TOP: The EISCAT height profiles for 23:00 UT on 01 March 2015 until 05:00 UT on 02 March 2015. MIDDLE: The Model 1 height profiles fitted to the KAIRA absorption data for the same time period. BOTTOM: The Model 2 height profiles fitted to the KAIRA absorption data for the same time period.

The electron densities corresponding to the intersections of the KAIRA and EISCAT beams, as created by iONOnest for the oblique KAIRA beams are generally closer to the electron densities measured by the EISCAT VHF. However, at certain times the electron densities calculated using iononest from the KAIRA beam pointing at zenith appear to be as close, if not closer to the EISCAT VHF electron densities.

These electron densities are far more similar to the EISCAT data than those obtained by the KAIRA beam pointing at zenith seen in Figures 5.4 and 5.5. The fit to pointing B appears better than that of pointing A when compared to the EISCAT data, though it still fails to recreate the peak in electron density at 04:00–05:00 UT. However a peak in electron density is seen at 02:00–03:00 UT in the KAIRA data as well as the EISCAT data, although the EISCAT peak is stronger than the KAIRA peak. This could again be caused by the mismatch in beamsizes. Between 04:00–05:00 UT the absorption measured
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Figure 5.9: The electron density at the points (A and B) at which the KAIRA beam intersect with the EISCAT beam. At an elevation of 45°, the KAIRA beam intersects with the EISCAT beam at point A, which is 85 km above the EISCAT site and 120 km along the KAIRA beam. At an elevation of 50°, the KAIRA beam intersects with the EISCAT beam at point B, which is 101 km above the EISCAT site and 132 km along the KAIRA beam.

for each frequency with KAIRA is more prominent in the high-frequency (and thus smaller) beams, implying that the absorption feature is not a broad, all-sky structure.

The difference between the EISCAT and KAIRA electron densities could also be caused by density enhancements not contributing to the absorption with the same effectiveness at heights of 100 km because, although the electron density becomes high, the neutral density is relatively low and hence the electron-neutral collision frequency is lower at this altitude (as shown in Figure 2.10). The majority of the electron density at 04:00–05:00 UT is located at a higher altitude, therefore it contributes less to the absorption measurements made by KAIRA, which could lead to an underestimation in the electron density.

To investigate the true causes of the peaks in electron density between 02:00–03:00 UT and 04:00–05:00 UT all-sky observations corresponding to the EISCAT and KAIRA
observations could be used, allowing for comparison with auroral activity. There is no
increase in AE index at 04:00–05:00 UT, implying that the peak in electron density
could be due to an increase in localised auroral activity which would be seen by all-sky
observations. Unfortunately these could not be obtained because although there are
all-sky cameras located at the EISCAT site in Ramfjordmoen, there was cloud cover on
the day these observations were taken.

The quality of fit is again assessed using the SMSE (Equation 4.10), setting $Z_i$ to be
the KAIRA absorption data for the two pointing directions, $Z_{var}$ to be the variance of
the KAIRA absorption data for each pointing direction and $Z_i^*$ to be the Model 1 or
Model 2 fit to the KAIRA absorption data. The average SMSE across the frequency
range for Model 1 is found to be 0.11 for pointing A and 0.13 for B, whilst for Model 2
it is 0.13 for A and 0.15 for B, showing that both models are good fits to the KAIRA
absorption data.

5.3 Application of iononest to KAIRA data – 2013

As in Section 5.2, I now apply iononest to the absorption profiles obtained from the
KAIRA zenith pointing taken between 18:00–22:00 UT on 22 October during the 2013
observing campaign and compare the electron density height profiles output by ionon-
est to those measured by EISCAT.

Figures 5.10 and 5.11 show comparisons between the EISCAT VHF and the resulting
height profiles obtained by iononest for Models 1 and 2 respectively. Unlike the ionon-
est fit to the 2015 data (shown in Figures 5.4 and 5.5), the iononest fit to the 2013
data does not generally appear to recreate the shape of the electron density variation
with time. Figure 5.12 reinforces how poorly the electron density height profiles pro-
duced by iononest appear to match the EISCAT data. The maximum electron density
within the 80–110 km altitude range rarely matches in magnitude or altitude between
the EISCAT and KAIRA electron density height profiles.

It should, however, be noted that the EISCAT VHF data is noisy, and the altitudes
at which the electron density reaches its maxima within the height range are often
due to this noise. The limiting value of electron density for the EISCAT VHF occurs
when the level of random error in an estimate of a plasma parameter becomes too
high and the threshold of detectability is reached because the measurement becomes
meaningless. This generally occurs at altitudes below 80 km. Hargreaves (1992) state
that the incoherent scatter method is difficult to apply when the nighttime D layer
electron density is less than $10^9 \text{ m}^{-3}$. If the threshold of detectability is not met then
the value for electron density at that altitude is returned as a zero. When no electron
density has been measured by the EISCAT VHF, it is plotted as white in Figure 5.1.
It can also be seen from Figure 5.1 that the absorption measured by KAIRA does not vary significantly over the observation. It maximises at approximately 0.3 dB. As discussed in Section 2.3.2, the QDS method used here is only accurate to within 0.1 dB which means that the inaccuracy in the KAIRA absorption measurements could also be the cause of the discrepancy between the electron density height profiles returned by IONONEST and the electron density height profiles measured by the EISCAT VHF.

I assess the quality of fit using the SMSE using Equation 4.10 in the same way as in Section 5.2.1. The SMSE values obtained for Model 1 and Model 2 are found to be 1.13 and 1.00 respectively. This shows that whilst neither model provides a good fit to the absorption data, Model 2 provides a better fit than Model 1.

Figure 5.10: A comparison between the height profiles generated using Model 1 fitted to the KAIRA absorption data for the zenith pointing and the height profiles obtained from EISCAT. TOP: The EISCAT height profiles for 18:00–22:00 UT on 22 October 2013. MIDDLE: The Model 1 height profiles fitted to the KAIRA absorption data for the same time period and pointing. BOTTOM: The ratio between the EISCAT data and the Model 1 fit to KAIRA absorption data.
Figure 5.11: A comparison between the height profiles generated using Model 2 fitted to the KAIRA absorption data for the zenith pointing and the height profiles obtained from EISCAT. **TOP:** The EISCAT height profiles for 18:00–22:00 UT on 22 October. **MIDDLE:** The Model 2 height profiles fitted to the KAIRA absorption data for the same time period and pointing. **BOTTOM:** The ratio between the EISCAT data and the Model 2 fit to KAIRA absorption data.
Figure 5.12: The height of the peak electron density and the peak electron density as output by IONONEST using the models compared to the height of the peak electron density and the peak electron density measured by EISCAT.
5.4 Conclusions

I have found that information can be inferred about the electron density of the lower ionosphere by measuring the absorption that occurs. Multi-frequency absorption measurements made by KAIRA have been compared to electron density height profiles from the EISCAT VHF, demonstrating that during peaks in electron density, peaks in absorption are also observed.

I have used IONONEST to fit to KAIRA multi-frequency absorption data and have compared the returned electron density height profiles to data from the EISCAT VHF to qualitatively verify the fits. I have investigated the use of IONONEST during quiet periods of geomagnetic activity (during the 2013 observations), as well as during active periods (the 2015 observations). I have demonstrated that IONONEST is capable of returning realistic electron density height profiles that are comparable to those obtained by the EISCAT VHF during geomagnetically active periods, bearing in mind the differences between the two instruments. The electron density height profiles returned using IONONEST for the 2015 KAIRA pointings over the EISCAT VHF site showed similar temporal variability and magnitude as the measured EISCAT VHF electron densities at those heights.

On the other hand, I have found that the electron density height profiles returned by IONONEST during geomagnetically quiet periods fail to show the same temporal variability and magnitude as the EISCAT VHF electron densities observed over the same period. This is due to the inaccuracies in the absorption measurements made by KAIRA, as well as the inability of the EISCAT VHF to observe at low altitudes when the ionospheric electron density at that altitude is lower than $10^9 \text{ m}^{-3}$.

Unlike EISCAT, KAIRA is capable of providing continuous measurements of absorption which can then be converted to continuous electron density height profiles. Whilst EISCAT has the advantage of being able to measure electron densities at higher altitudes because KAIRA is only sensitive to increases in electron density at altitudes where the electron-neutral collision frequency is also sufficiently high, KAIRA has the potential to return electron density height profiles that extend down to lower altitudes. This can be achieved by using IONONEST in conjunction with an ion chemistry model that extends to lower altitudes than the NRL-MSISE-00 reference atmosphere which is used in this chapter to calculate the electron-neutral collision frequency.
Chapter 6

Simulations of a turbulent ionosphere

The ionosphere is a turbulent medium that affects all ground-based, low-frequency radio telescopes. The statistical behaviour of radio waves passing through the ionosphere can be used to probe the properties of the turbulent medium. Here I present a method of obtaining the statistical properties of a turbulent medium by simulating two- and three-dimensional turbulence screens and investigating their structure functions.

A turbulent medium is a fluid (such as a plasma) that does not have a laminar (smooth) flow, but instead has chaotic, irregular, eddying motions with a spectrum of eddy sizes. As the motion of the turbulent plasma is chaotic it is not reproducible and so only the average properties can be predicted from theory and reproduced.

The ionosphere is a medium where turbulence is fully developed (Thompson et al., 2001). Whilst it is known that a turbulent plasma will introduce additional phase errors to the observed radio wave which depend on time, frequency, and position, as well as introducing time- and position-dependent changes to the observed rotation measure (RM) of the radio signal, there is no generally accepted theory of ionospheric plasma turbulence. Fluctuations in the phase errors and RMs are caused by density fluctuations in the plasma due to this turbulence. This provides the observed phase errors and RMs with a certain statistical behaviour in time and position, with calibration software such as SPAM (Section 2.2.2; Intema et al. 2009) implementing thin-layer turbulence models to account for these fluctuations.

In this chapter, the effect a turbulent ionosphere has on radio observations and the statistical properties of a turbulent field are discussed in Section 6.1. Section 6.2 discusses methods for simulating turbulence screens as well as a tool for analysing the
statistical properties of turbulence to investigate its structure. One of these turbulence simulation methods is then developed and implemented to simulate two-dimensional and three-dimensional turbulence screens (Section 6.3). The statistical properties of these simulated turbulence screens are then investigated prior to running a simulation in Section 6.3.3 to check that the statistical properties of turbulence can be returned from polarisation data obtained by observing pulsars with KAIRA, an investigation that is carried out in Chapter 7.

6.1 Turbulence in the ionosphere

A good understanding of the effect that the ionosphere has on the polarisation observed by ground based low-frequency radio telescopes is paramount for studies of interstellar and intergalactic polarised sources. Observing polarised sources does not only return the RM of the source, it also returns the contribution to the RM of the polarised source from all regions of magnetionic plasma along the line of sight, allowing e.g. for the mapping of Galactic magnetic fields and probing of HI and HII regions (Manchester 1974; Han et al. 2006; Noutsos et al. 2008). Studies of the Galactic magnetic field can point to the presence of star-forming regions as well as information about the deflection caused by the Galactic magnetic field of ultra-high-energy cosmic rays.

As in the case of ionospheric phase errors (discussed in Chapter 3), the change in RM of radio sources is dependent on the total electron density along the line of sight from the observer to the source. Most research on ionospheric processes deals with electron density features down to the scale of a hundred kilometres, often relying on information from GPS satellites which fails to show the true turbulence of the ionosphere. The smallest scale required for calibration is a few kilometres (van der Tol, 2009). In this project, I aim to pioneer a method for observing the turbulence of the ionosphere by using the change in RM that occurs spatially in the ionosphere.

In order to observe the turbulence of the ionosphere, and thus to establish the turbulent power spectrum, I use the change in RM of pulsars due to the ionosphere. A pulsar is a highly-magnetised, rapidly-rotating neutron star that produces highly polarised emission (Section 2.4.1; Stappers et al. 2011). This makes them useful for investigation of variation in RM. The change in RM of these pulsars can be used to find the second order spatial structure function,

\[ D_\theta(\delta\theta) = \frac{1}{N} \sum_i \left[ RM(\theta) - RM(\theta + \delta\theta) \right]^2 \]  

(6.1)

where \( N \) is the number of pulsars in each bin, \( \theta \) is the position of a pulsar and \( \theta + \delta\theta \) is the position of the pulsar that it is being compared to, with \( \delta\theta \) being the distance between the two pulsars.
For fully developed turbulence it is stated by the Taylor hypothesis (Taylor, 1938) that the spatial average of the velocity and the time average are equivalent. This means that the time variation will follow the same power law as the spatial variation (Wright, 1996). The spatial structure function can be related to the temporal structure function, $D_t$, by assuming the frozen screen hypothesis (Taylor, 1938). This hypothesis assumes that the turbulent eddies remain fixed during the time that it takes for the turbulent medium to move across the baseline, $b$. The temporal and spatial structure functions are therefore related by $D_t(\delta t) = D_\theta(\theta = \nu_s \delta t)$, where $\nu_s$ is the velocity of the turbulent medium (Thompson et al., 2001). The temporal structure function can be written as

$$D_t(\delta t) = \frac{1}{N} \sum_i \left[ RM(t) - RM(t + \delta t) \right]^2_i. \quad (6.2)$$

If ionospheric phenomena such as travelling ionospheric disturbances are not taken into account, the ionospheric medium appears to have the statistical behaviour of a turbulent layer (Intema et al., 2009). This means that when plotted, the second order spatial structure function should have a power law of the form

$$D_\theta(\delta \theta) \propto \delta \theta^\gamma, \quad (6.3)$$

where $\gamma = \alpha - 2$ and $\alpha$ is the power in Equation 1.8. Many authors (Thompson et al. 2001; Intema et al. 2009; van der Tol 2009) have assumed that the mid-latitude ionosphere can be adequately described by the Kolmogorov model for turbulence (introduced in Section 1.1.2.1; Kolmogorov 1941). The Kolmogorov model for turbulence is self-similar (Lane et al., 1992), meaning that it will look similar at whatever scale that it is viewed.

### 6.1.1 Structure functions

It can be assumed that the differences in electron density in the ionosphere are driven by fully developed turbulence, therefore the likely difference in electron density between two points in the ionosphere is a function of the distance between the points, as in Equation 6.1. This is known as the structure function.

As Kolmogorov turbulence is self-similar, the structure function can be described by a power law. The theory of Kolmogorov turbulence predicts that its structure function will have a power law with a gradient of $5/3$. The power law of Kolmogorov turbulence is only true for a subrange of wavenumbers (Steiner et al., 2009). If the spectrum was purely Kolmogorov then the ‘inner scale’ should be zero ($l_0 = 0$) and the ‘outer scale’ should be infinity ($L_0 = \infty$). However in reality, the energy-producing scales and dissipation scales (small and large wavenumbers respectively) cause the structure function to be dominated by small- and large-scale irregularities at the inner and outer scales of spatial frequency. In the case of simulations of Kolmogorov turbulence screens,
the computational grid size provides the ‘inner scale’ causing a break in the power law, whereas in the case of true ionospheric observations, the ‘inner scale’ is thought to be of the order of the ion gyroradius (Booker 1979; Vedantham and Koopmans 2015). The ‘outer scale’ of true ionospheric conditions is stated by Vedantham and Koopmans (2015) to be the spatial wavenumber which corresponds to the energy injection scale, and can be 10s–100s of kilometres.

To create a structure function of the electron density of the ionosphere, the difference in VTEC between two points is found and plotted against the distance between those two points on the sky. Stil et al. (2011) plot structure functions which compare the variation of RMs across the sky. In this work, the same binning method as Stil et al. (2011) is used. The size of bins used by Stil et al. (2011) are 0.1°, however if a bin contains less than 20 source pairs then the size is expanded until it contains this minimum number of source pairs. As I am using kilometres on the sky instead of degrees, I use the equivalent of 0.1° on the sky at an assumed ionosphere altitude of 450 km, which is 12 km.

To find the power law of the structure functions in this chapter, the Levenberg-Marquardt non-linear least squares algorithm is used. This algorithm uses the data as binned above, with the standard deviations of the data in each bin being used as weights by the least-squares solver.

To account for the inner and outer scales discussed above, a broken power law is implemented using the Levenberg-Marquardt non-linear least squares algorithm by fitting power laws to the data incrementally (i.e., the first power law starts by fitting just the initial 3 values). The optimal value for the power law is found and this is output along with the covariance matrix. If the variance of the parameter estimate in the covariance matrix is a minima for the data then I choose this index as the best linear fit to the data. If the data are of the form of a broken power law then once the first subset of data has had a power law fitted to it, the data points that were used to fit the power law are removed from the data set and the same process is followed again to find the index of the next section of the broken power law.

### 6.2 Turbulence screen simulation methods

To investigate what information can be found about a turbulent ionosphere, I simulate turbulence screens with known statistics. Many different methods of simulating turbulence screens have been developed. These methods include,

- **The Fast Fourier Transform (FFT) method.** A FFT algorithm converts a signal from its original domain (such as time or space) to the frequency domain (known as computing the discrete Fourier transform). It is considered to be ‘fast’ because the algorithm reduces the complexity of the computation of the
Fourier transform from $O(n^2)$ to $O(n \log n)$, where $n$ is the data size. Using this algorithm to simulate a turbulence screen is computationally expensive because the simulation depends on the outer scale of the turbulence, $L_0$, rather than the size of the turbulence screen, $D$, so when performing simulations with $D << L_0$, a large number of samples is required. McGlamery (1976) simulated turbulence over 16 times the area required, which significantly improves the approximated phase structure, however requires substantially higher computational cost. A FFT based simulation is computationally intensive and when generating volumes with $> 10^9$ elements, aliasing makes the use of an FFT based simulation inefficient (McGlamery, 1976).

- **The Zernike polynomial method.** The use of Zernike polynomials was investigated by Roddier (1990). This method reproduces the Kolmogorov turbulence accurately, however it is not computationally superior to the FFT method described above. This method is used to create turbulence screens in SPAM (discussed in Section 2.2.2; Intema et al. 2009).

- **The addition of subharmonics method.** The addition of subharmonics methods is outlined by Lane et al. (1992) and is a simple technique where additional random frequencies are generated and their effects are added to the sampled frequencies. The addition of these subharmonics creates a ringing phenomenon caused by the irregular spacing of the samples at the edge of the phase screen. Similarly to the FFT method, the effect of this can be removed by generating a phase screen that is larger than that required and only using the central section.

- **The random mid-point displacement algorithm.** I detail this method in Section 6.2.1 below. It is advantageous over the other methods due to its speed (Lane et al., 1992).

In this study, I use the random mid-point displacement algorithm as a method for simulating turbulence screens due to its speed. This method for simulating turbulence screens is discussed below.

### 6.2.1 The random mid-point displacement algorithm

The random mid-point displacement algorithm is a method of simulating a turbulence screen that was developed by Fournier et al. (1982) and further built upon by Lane et al. (1992). The process of simulating a turbulence screen using this method begins by a few points of the surface being selected and defined, then the random mid-point displacement algorithm selects smaller and smaller regions and defines them, allowing the fractal surface to be refined.
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Figure 6.1: The diagram on the left shows how values in a $3 \times 3$ two-dimensional grid are interpolated from the original corner values (labelled by $A$, $B$, $C$ and $D$). The face value (labelled as $f_1$) is calculated using Equation 6.4 and the four edge values (labelled as $e_1$) are calculated using Equation 6.5 and the appropriate neighbouring corner values. The diagram on the right shows how values in a $5 \times 5$ two-dimensional grid are interpolated from the values in the $3 \times 3$ two-dimensional grid. The edge values (labelled as $e_2$) are again calculated using Equation 6.5 and the appropriate neighbouring values and the face values (labelled as $f_2$ and $f_2^*$) are calculated using Equation 6.4 and either the four values diagonally adjacent or the four values horizontally and vertically adjacent, as shown in the diagram.

The implementation of the two-dimensional random mid-point displacement algorithm developed for this thesis begins with four defined corner points, $A$, $B$, $C$ and $D$ with values given by a random number selected from a Gaussian distribution. A new point is then defined between the four points, with a value assigned to it of $f_1$,

$$f_1 = \frac{A + B + C + D}{4} + \epsilon,$$  \hspace{1cm} (6.4)

where $\epsilon$ is a random number selected from a Gaussian distribution and scaled by the structure function.

Figure 6.1 shows the position of the four corner points with respect to $f_1$. The variance of $\epsilon$ is dependent on the size of the grid and the structure function and reduces as the algorithm continues and the sampling of the screen becomes finer. The points on the edges of the grid, $e_1$, are found using a one-dimensional random mid-point displacement algorithm which can also be used to generate a Brownian motion spectrum. The position of the new point on an edge is therefore given by

$$e_1 = \frac{A + B}{2} + \epsilon,$$  \hspace{1cm} (6.5)

where $\epsilon$ is again dependent on the size of the grid and is scaled by the structure function. Once each point in the screen is populated, the screen is increased in size. Between each
Figure 6.2: The generation of a two-dimensional turbulence screen using the random mid-point displacement algorithm as shown in Figure 6.1. I begin with a $3 \times 3$ grid, and progress to a $2049 \times 2049$ grid.

row a new row is inserted and between each column a new column is inserted. This new array is then split into smaller $3 \times 3$ squares and the centre points and outer edge points of the newly inserted rows and columns are populated in the same way as above. The inner edges of each of the $3 \times 3$ squares in the screen (the points listed as $f_2^*$ in Figure 6.1) are then calculated using the four vertically and horizontally neighbouring values.

The implementation of this algorithm is shown in Figure 6.2, demonstrating the description above and what is shown in Figure 6.1. Initially a $3 \times 3$ grid is generated and from this grid a $5 \times 5$ grid is then generated, the same method is then implemented until a $2049 \times 2049$ grid is produced. The value of $\gamma$ used to create the example turbulence in Figure 6.2 was set to $\gamma = 2/3$.

The random mid-point displacement algorithm can also be extended to the three-dimensional case. Nikolic et al. (2007) used a three-dimensional generalisation of the
Figure 6.3: This diagram shows how values in a $3 \times 3 \times 3$ three-dimensional grid are interpolated from the original corner values (labelled by $A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$). The centre value (labelled as $c_1$) is calculated using Equation 6.6, the six face values (labelled as $f_1$) are calculated using Equation 6.7 and the appropriate neighbouring values, finally the twelve edge values (labelled as $e_1$) are calculated using Equation 6.8 and the appropriate neighbouring values.

random mid-point displacement algorithm in order to calculate the structure function of phase errors caused by variations in water vapour in the troposphere for the Atacama Large Millimeter/submillimeter Array (ALMA; Brown et al. 2004). Nikolic et al. (2007) directly simulates the three-dimensional volume, assuming an atmospheric volume that is frozen is moving across the array. The three-dimensional volume simulated is a statistical realisation of a Kolmogorov field with the structure function given by Equation 6.3 (Nikolic et al., 2008).

Similarly to Nikolic et al. (2007), I extend the method of Lane et al. (1992) to a three-dimensional volume. I begin with a $3 \times 3 \times 3$ cube with the eight corner values ($A$, $B$, $C$, $D$, $E$, $F$, $G$ and $H$) populated by random numbers (in $xyz$ coordinates, these corners are described by 000, 002, 020, 200, 022, 202, 220, 222, as shown in Figure 6.3). From
these eight corner values, the central point of the cube \((xyz = 111)\) can be found using
\[
c_1 = \frac{A + B + C + D + E + F + G + H}{8} + \epsilon. \tag{6.6}
\]
The central value of each face can then be found using the four corners bounding that face and the central point. For example,
\[
f_1 = \frac{A + C + E + G + c_1}{5} + \epsilon. \tag{6.7}
\]
This means that for the central face value with \(xyz\) coordinates of 011, the points used to calculate the value are 000, 020, 002, 022 and 111. Finally the edge values can be found using the bounding two corners and the adjacent face values,
\[
e_1 = \frac{A + E + f_1 + f_1}{4} + \epsilon. \tag{6.8}
\]
For example, for the edge at \(xyz = 021\), the points at 020, 022, 011 and 121 are used to calculate the value.

A pictorial demonstration of how the centre, face and edge values of a 3 \(\times\) 3 \(\times\) 3 three-dimensional grid are calculated is shown in Figure 6.3. As in the two-dimensional algorithm, once the grid is fully populated, slices are inserted between each value of \(x\), \(y\) and \(z\), padding out the array from a 3 \(\times\) 3 \(\times\) 3 grid to a 5 \(\times\) 5 \(\times\) 5 grid, this is then split into smaller 3 \(\times\) 3 \(\times\) 3 grids, each of which is iterated over in the same way as described above.

### 6.3 Implementation of turbulence screen simulation methods

Spatial variations in ionospheric electron density, \(N_e\), can be modelled as a Gaussian random field with a Kolmogorov turbulence power spectrum. In this section I simulate a series of screens with turbulent spatial variations in ionospheric electron density. I begin by simulating a two-dimensional turbulence screen prior to simulating a three-dimensional turbulence cube which is then collapsed into a two-dimensional turbulence screen.

#### 6.3.1 Two-dimensional turbulence screen

I first assume a two-dimensional turbulence screen located at an altitude of 450 km, simulated using the mid-point displacement method detailed in Section 6.2.1 with dimensions 2049 km \(\times\) 2049 km. This simulated turbulence screen is shown in Figure 6.4.
To initially test the simulation, I select 1000 random points from the turbulence screen with coordinates \((x,y)\) and create a structure function from these points by calculating the difference in VTEC between each point and plotting this against the distance between the points on the sky. To create the structure function plot shown in Figure 6.5, the binning method detailed in Section 6.1.1 is used. There are two distinct regions to the plot as expected from the discussion in Section 6.1.1. To find the gradients of each of these sections, a broken power law is fitted to the structure function. Figure 6.5 shows this broken power law fitted to the data using the Levenberg-Marquardt non-linear least squares algorithm. The broken power law has two gradients, \(\gamma_0\) (the purple line) and \(\gamma_1\) (the red line). The power laws of this structure function are therefore given by \(\gamma_0 = 1.42 \pm 0.004\) and \(\gamma_1 = 2.42 \pm 0.49\). \(\gamma_0\) is the relevant value, whilst \(\gamma_1\) is caused by the scale effects discussed in Section 6.1.1.

The power law, \(\gamma_0\), found for the structure function in Figure 6.5 is lower than the value of \(\gamma\) that the turbulence screen was simulated with, \(\gamma = 5/3 = 1.67\). However, a single iteration of a turbulence screen will not necessarily return the correct statistics. By simulating multiple turbulence screens the structure function created for all of these screens should return the expected power law. To show this 100 two-dimensional turbulence screens were simulated using the same method as that used to create Figure 6.4. The structure function is then calculated for 1000 random points selected from each of these turbulence screens. The resulting value of the power law, \(\gamma_0\), is then calculated for each of these structure functions and plotted as a histogram in Figure 6.6. The most frequently found value of \(\gamma_0\) is close to the theoretical value of 5/3, however there is
Figure 6.5: A power law fitted to the structure function of VTEC as calculated by selecting random points in the turbulence screen shown in Figure 6.4. The power laws are $\gamma_0 = 1.42 \pm 0.004$ (the purple line) and $\gamma_1 = 2.42 \pm 0.49$ (the red line).

Figure 6.6: The power laws fitted to the structure functions for electron density of 1000 points selected at random from 100 simulated turbulence screens using the Levenberg-Marquardt non-linear least squares algorithm. The mean value of the power laws was found to be $\gamma_0 = 1.60 \pm 0.14$.

considerable spread over a range of values of $\gamma_0$. The mean value of these power laws was found to be $\gamma_0 = 1.60 \pm 0.14$.

### 6.3.2 Three-dimensional turbulence screen

The same investigation is now conducted for the three-dimensional case. Similarly to Nikolic et al. (2007), I use a three-dimensional generalisation of the mid-point displacement algorithm extended from Lane et al. (1992), as detailed in Section 6.2.1. I create
Figure 6.7: Five slices of the \(1025 \times 1025 \times 1025\) three-dimensional turbulence cube generated using a three-dimensional generalisation of the mid-point displacement algorithm.
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Figure 6.8: A thin-layer turbulence screen created by summing each slice of the three-dimensional turbulence cube (slices of which are shown in Figure 6.7) together.

I create a thin-layer screen from this turbulence cube by summing each slice together. Figure 6.8 shows the thin-layer screen created from the three-dimensional turbulence cube. As in the two-dimensional case, the thin-layer screen is placed at an altitude of 450 km and 1000 points are selected from the screen at random. The structure function for these points is then plotted in Figure 6.9. When a broken power law is fitted to the data, the power laws are given by \( \gamma_0 = 1.78 \pm 0.02 \) (the purple line) and \( \gamma_1 = 2.15 \pm 0.07 \) (the red line).

The gradient of the power law for the points selected at random from this thin-layer screen is higher than expected, however it is consistent with the values of \( \gamma_0 \) obtained for the simulation of 100 turbulence screens shown in Figure 6.6. Unfortunately, due to the computationally intensive nature of generating a three-dimensional turbulence cube, I cannot repeat the simulation hundreds of times and so it is not viable to recreate a histogram similar to that shown in Figure 6.6. As the two-dimensional and flattened three-dimensional turbulence screens return similar statistics, I elect to use the two-dimensional turbulence screen for further work because the three-dimensional turbulence screen is too computationally intensive.
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Figure 6.9: The structure function of RM for a thin-layer ionosphere (shown in Figure 6.8) created by summing every layer in a simulated three-dimensional turbulence cube. The power laws are $\gamma_0 = 1.78 \pm 0.02$ (the purple line) and $\gamma_1 = 2.15 \pm 0.07$ (the red line).

6.3.3 Simulation of KAIRA observations

In Sections 6.3.1 and 6.3.2 it has been demonstrated that the correct turbulence statistics of a simulated turbulent ionosphere can be returned. In this section I create a simulation of the intended pulsar observations with KAIRA to verify that pulsar observations can return useful information about the statistics of a turbulent ionosphere.

I simulate 8 pulsars over a period of 18 hours (shown in Figure 6.10 and detailed in Table 6.1) and put their RMs (as obtained from literature) through a two-dimensional simulated turbulent electron density screen with assumed uniform magnetic field. The simulated turbulence screen changes the RM of each pulsar, and using these variations in RM as well as the distance on the sky between the pulsars, the structure function (Equation 6.1) can be calculated. The gradient of the structure function should be consistent with the expected Kolmogorov index.

As well as assuming a uniform magnetic field, an ionosphere that exhibits no temporal fluctuations during the simulation is also assumed. This assumption is Taylor’s frozen turbulence hypothesis and it is used here to simplify the model because the time scales involved in the development of turbulence are much longer than the time taken for a turbulent volume element to pass across the field of view of a telescope, therefore for a short time scale I can assume no temporal fluctuations in the simulation. To estimate the magnetic field for these simulations, I use the IGRF-12 as discussed in Section A.2. The results are shown in Table 6.2. A two-dimensional thin-layer ionosphere at an altitude of 450 km is also assumed. This is an oversimplifying assumption that I find to be a limitation in phase calibration for interferometers with long baselines in Chapter 3. I am however using an aperture array rather than a long baseline interferometer for this...
Figure 6.10: Pulsar tracks through the sky visible from the KAIRA HBA array (located at latitude = 69.07° N, longitude = 20.76° E) on 03–04 March 2015, plotted on an altitude/azimuth polar graph. The dashing of the lines indicate when the telescope is observing the pulsar.

Figure 6.11 shows a modelled two-dimensional turbulence screen as previously shown in Figure 6.4, with the points at which the pulsars pierce the thin-layer ionosphere throughout the observation plotted with dashed lines. The dash of the lines indicate when the pulsar is observed, i.e., a dash of the line indicates that the pulsar is being observed for 2 minutes and when there is no line, this indicates that the pulsar is not being observed during that period.

Once the RMs have been calculated for the pulsars, I can find the differences in change in RM between each pair of pulsars and plot this against the distance between the two
### Chapter 6: Simulations of a turbulent ionosphere

Table 6.1: A list of the pulsars observed using the KAIRA HBA. The RMs taken from Noutsos et al. (2015) are obtained from LOFAR using RM synthesis with the assumed RM due to the ionosphere (calculated using ionfr; Sotomayor-Beltran et al. 2013) subtracted.

<table>
<thead>
<tr>
<th>Pulsar</th>
<th>RA (UTC) [h:m:s]</th>
<th>Dec (UTC) [d:m:s]</th>
<th>Period [s]</th>
<th>Pulsation Frequency [Hz]</th>
<th>DM [cm⁻³pc⁻¹]</th>
<th>RM [rad m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>J0332+5434</td>
<td>03:32:59.368</td>
<td>+54:34:43.57</td>
<td>0.7145</td>
<td>1.3995</td>
<td>26.76</td>
<td>-63.7</td>
</tr>
<tr>
<td>J0814+7429</td>
<td>08:14:59.50</td>
<td>+74:29:05.70</td>
<td>1.2922</td>
<td>0.7738</td>
<td>5.73</td>
<td>-14.00</td>
</tr>
<tr>
<td>J1509+5531</td>
<td>15:09:25.629</td>
<td>+55:31:32.394</td>
<td>0.7397</td>
<td>1.3519</td>
<td>19.61</td>
<td>1.28</td>
</tr>
<tr>
<td>J2022+5154</td>
<td>20:22:49.873</td>
<td>+51:54:50.233</td>
<td>0.5292</td>
<td>1.8897</td>
<td>22.65</td>
<td>-6.5</td>
</tr>
<tr>
<td>J2113+4644</td>
<td>21:13:24.307</td>
<td>+46:44:08.70</td>
<td>1.0147</td>
<td>0.9855</td>
<td>141.26</td>
<td>-224</td>
</tr>
<tr>
<td>J2219+7429</td>
<td>22:19:48.139</td>
<td>+47:54:53.93</td>
<td>0.5385</td>
<td>1.8571</td>
<td>43.49</td>
<td>-35.93</td>
</tr>
<tr>
<td>J2225+6535</td>
<td>22:25:52.721</td>
<td>+65:35:35.58</td>
<td>0.6825</td>
<td>1.4651</td>
<td>36.08</td>
<td>-22.99</td>
</tr>
</tbody>
</table>

*Manchester (1972), †Noutsos et al. (2015).*
Table 6.2: Magnetic field model results obtained for the KAIRA HBA array (located at latitude = 69.07° N, longitude = 20.76° E) on 03–04 March 2015 using IGRF-12, assuming an ionospheric altitude of 450 km above the surface of the Earth.

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Declination [deg]</td>
<td>9.06</td>
<td>0.54</td>
</tr>
<tr>
<td>Inclination [deg]</td>
<td>77.97</td>
<td>0.22</td>
</tr>
<tr>
<td>Horizontal Intensity [nT]</td>
<td>11,120</td>
<td>133</td>
</tr>
<tr>
<td>North Component [nT]</td>
<td>10,981</td>
<td>138</td>
</tr>
<tr>
<td>East Component [nT]</td>
<td>1,751</td>
<td>89</td>
</tr>
<tr>
<td>Vertical Intensity [nT]</td>
<td>52,182</td>
<td>1650</td>
</tr>
<tr>
<td>Total Intensity [nT]</td>
<td>53,353</td>
<td>152</td>
</tr>
</tbody>
</table>

Figure 6.11: Pulsar tracks over a thin-layer ionosphere, modelled as a two-dimensional turbulence screen with an exponent of $\gamma = 5/3$ (also shown in Figure 6.4). The dashing of the lines indicate when the telescope is observing the pulsar. The centre of the turbulence screen is located at KAIRA’s zenith and the turbulence screen is located at an altitude of 450 km.
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Figure 6.12: The structure function of RMs as seen by the pulsars observed by KAIRA assuming a frozen thin-layer ionosphere at an altitude of 450 km created by simulating a two-dimensional turbulence screen. A broken power law is fitted to the structure function, the purple line has a gradient of $\gamma_0 = 1.79 \pm 0.17$ and the red line has a gradient of $\gamma_1 = 3.78 \pm 0.20$.

pulsars. To do this, I use a moving window algorithm. Each RM is compared to the RMs observed in the 20 minutes prior to the observation of the current RM as well as all those RMs observed in the 20 minutes after. The structure function is plotted in the same way as for Figure 6.5, and the resultant structure function over the entire observation period using the moving window algorithm and the Stil et al. (2011) binning method, assuming a frozen (not varying with time) ionosphere, is shown in Figure 6.12. Fitting a broken power law to this structure function results in gradients of $\gamma_0 = 1.79 \pm 0.17$ (the purple line) and $\gamma_1 = 3.78 \pm 0.20$ (the red line).

There is more variation in the structure function in Figure 6.12 than is seen in the previous structure functions in this chapter (Figures 6.5 and 6.9). This could be due to there being fewer values in each bin, thus the error is larger on each bin. The sampling is also not at random as it was in the creation of the structure functions in Figures 6.5 and 6.9. Instead it follows the tracks of the pulsars around the simulated turbulence screen as shown in Figure 6.11 which means that the turbulence screen is not being fully sampled.

It should, however, be noted that the value of $\gamma_0$ obtained for the structure function in Figure 6.12 is consistent to within $2\sigma$ with the expected variation seen in Figure 6.6. I therefore conclude that this method of observing the statistics of a turbulence screen is capable of returning the correct result.

For this simulation, I have assumed an ionosphere that does not vary with time (a frozen ionosphere). This is something that cannot be assumed for any long period of time. I therefore test whether I can recover information about the structure of the turbulent ionosphere from short time periods of pulsar observations, assuming the same type of
Figure 6.13: The powers obtained from fitting a broken power law to the structure functions of each time section using a least-squares algorithm. The red line indicates the mean of all of these values, finding an average value of $1.37 \pm 0.88$.

observation as used to generate the structure function in Figure 6.12 – one pulsar is observed for 2 minutes, then the next pulsar is observed for 2 minutes, until all 8 pulsars have been observed once (with the pulsar J1509+5531 being observed three times), the loop then goes back to the beginning and the same pulsars are observed again in the same order. For each pulsar, the RMs observed in the time period $\pm 20$ minutes of the pulsar observation can therefore be used to create the structure function. In Figure 6.13, the gradients obtained for each structure function are plotted against time.

As discussed in Section 6.3.1, the statistics of a single turbulence screen will not return the exact statistics expected for Kolmogorov turbulence, as these statistics are an average. This is why, when a limited selection of points are used to analyse a single turbulence screen in Figure 6.13, the power is not consistently at $5/3$. The values of $\gamma_0$ in Figure 6.13 do however have a mean of $1.37 \pm 0.88$ which is again consistent with the expected value.

### 6.4 Conclusions

In this chapter I have researched methods of simulating ionospheric turbulence, before running simulations using the random mid-point displacement algorithm to create ionospheric turbulence screens. The turbulence statistics of these screens were investigated to demonstrate that information can be obtained about a turbulent ionosphere. It was found that a single simulation of a turbulent ionosphere does not always create a structure function that returns the expected turbulence statistics precisely, however repeated
Simulations of a turbulent ionosphere demonstrate the expected variation in structure function.

As well as a two-dimensional turbulence screen, a three-dimensional turbulence screen was also developed. A comparison of the turbulence statistics obtained for the two- and three-dimensional screens shows that a two-dimensional turbulence screen returns the same statistics as a flattened three-dimensional screen, therefore as a two-dimensional turbulence screen requires less computational power to generate, I assume a two-dimensional turbulence screen for the pulsar simulation in Section 6.3.3.

In Section 6.3.3 I investigated the ability of observations of the RM of 8 pulsars to return information about the turbulent ionosphere. It was found that throughout the time period investigated, the power law was not consistently at $\gamma = 5/3$, however the average value of all of the simulated observations was found to be $1.37 \pm 0.88$, which is consistent with the simulated turbulence. This shows that this method of observing pulsars does return meaningful information about the turbulence statistics of the ionosphere.
Chapter 7

Constraining turbulence using pulsar observations

The change in rotation measure caused by the ionosphere can be used to probe the turbulence statistics of the plasma. A pulsar observing mode is therefore implemented at the KAIRA instrument and a method is developed to process the data. The calibrated and analysed data are then used to investigate the statistics of the high-latitude turbulent ionosphere.

If the turbulent ionosphere can be described by pure Kolmogorov turbulence then the power-law relation between the structural function of the phase rotation of radio waves or the structural function of change in measured Faraday rotation should be expected to be $\gamma = 5/3$. This power-law relation has been investigated in detail for mid-latitudes. Cohen and Röttgering (2009) measured $\gamma$ using the VLA (Section 2.1.2.2) at 74 MHz and found a typical nighttime value of $\gamma = 1.0$ and a typical daytime value of $\gamma = 1.38$ for scales of 10–100 km, whilst Intema et al. (2009) describe a power-law relation that has a wide distribution and peaks at $\gamma = 1.5$ and van der Tol (2009) found a value of $\gamma = 1.60$.

In this chapter I investigate the statistics of the high-latitude turbulent ionosphere by measuring the change in Faraday rotation of observed pulsars using KAIRA (Section 2.1.1.1). The KAIRA HBA array is well suited to studies of RM because it has both a wide relative bandwidth and a high spectral resolution (McKay-Bukowski et al., 2015). Pulsars are used because they have a high degree of linear polarisation which makes the Faraday rotation easier to observe.

I begin by discussing the implementation of a pulsar observing mode at KAIRA in Section 7.1, before describing the calibration and analysis of the pulsar data observed
using KAIRA. In Section 7.2 a range of methods of obtaining RMs from the observed pulsar data are considered and RMs are then calculated using the chosen method. As outlined in Chapter 6, these RMs are then used to investigate the turbulence of the high-latitude ionosphere in Section 7.3.

7.1 Observing pulsars with KAIRA

During a series of observing campaigns in November 2013, March 2015 and March 2016, a selection of pulsars were observed using KAIRA. Prior to November 2013, KAIRA had no pulsar observing mode and so one of the main goals of this campaign was to design and implement a pulsar observing mode that was capable of observing the full polarisation of pulsars with a high enough temporal resolution to resolve their pulses. A number of pulsars were observed during this campaign to establish which pulsars should be used to probe the ionosphere. The implementation of this pulsar observing mode is described in Section 7.1.1.

Having implemented the pulsar observing mode at KAIRA and developed a method to process the data, two further observing campaigns were carried out in March 2015 and March 2016. During both of these campaigns, eight pulsars were observed in succession for periods of 2 minutes per pulsar, for 18 hours in March 2015 and for 20 hours in March 2016.

7.1.1 Implementation of a pulsar observing mode at KAIRA

Unlike standard LOFAR stations, KAIRA had no software for outputting Stokes I, Q, U and V (Section 2.4) at high temporal resolution. Instead software had to be developed for this purpose. This was done using a UDP (User Datagram Protocol) packet recorder which records beamlet data from the four outputs (known as lanes) at KAIRA. Each lane outputs data from 61 subbands. The UDP packets are written into a buffer as a 16-bit, complex number for each of the orthogonal, linear feeds of the antenna, $x$ and $y$. From the buffer, the real and imaginary parts are found for $x$ and $y$, these are then used to form averaged Stokes parameters. These Stokes parameters (IQUV) can be calculated using (Hamaker and Bregman, 1996)

\[
I = \langle A_x^2 \rangle + \langle A_y^2 \rangle \\
Q = \langle A_x^2 \rangle - \langle A_y^2 \rangle \\
U = 2A_xA_y \cos \delta_{xy} \\
V = -2A_xA_y \sin \delta_{xy}
\]

(7.1)
where $A_x$ and $A_y$ are time-varying complex amplitudes and $\delta_{xy}$ is the phase difference between the two oscillating components. These equations can also be written as

$$\begin{align*}
I &= \sum ((\Re x \times \Re x + \Im x \times \Im x) + (\Re y \times \Re y + \Im y \times \Im y)) \\
Q &= \sum ((\Re x \times \Re x + \Im x \times \Im x) - (\Re y \times \Re y + \Im y \times \Im y)) \\
U &= 2\sum (\Re x \times \Re y + \Im y \times \Im x) \\
V &= -2\sum (\Im x \times \Re y - \Re x \times \Im y),
\end{align*}$$

(7.2)

where $\Re$ indicates the real part of $x$ or $y$ and $\Im$ indicates the imaginary part. KAIRA is capable of sampling data with a sampling time of $8.192 \times 10^{-5}$ s (McKay-Bukowski et al., 2015), however an averaging factor of 100 is used to reduce the quantity of data output. This results in a sampling time of $8.192 \times 10^{-3}$ s. These data are then output to files where they can be analysed as detailed in the following section.

### 7.1.2 Adopting LOFAR standards

The Stokes IQUV data output by KAIRA are in a raw format and are therefore not readable by any of the existing software available for pulsar data analysis. For initial tests, software for folding and dedispersing the KAIRA data was developed. This software uses the rotation frequency and the dispersion measure from the ATNF pulsar catalogue (as listed in Table 6.1). Pulsar folding forms pulsar profiles by summing up multiple consecutive, individual pulses (Stappers et al., 2011) as a single (unfolded) pulse cannot be distinguished from the instrument noise. This method relies on the periodicity of the pulsar and an exact measurement of the rotation frequency of the pulsar. Once the individual pulses have been summed, the data also need to be dedispersed.

As described in Section 2.4.1, the signal of a pulsar is dispersed as it propagates through a plasma because the speed of this propagation is dependent on the frequency of the radio wave — low-frequency waves travel slower than high frequency waves. Figure 7.1 shows the output of this software after folding and then after dedispersion for the pulsar J0332+5434.

In order to utilise already existing software for pulsar data, the raw KAIRA data were converted into sigproc-style filterbanks format (Lorimer, 2011), which is readable by PRESTO (the pulsar exploration and search toolkit). PRESTO is a set of search and analysis software for pulsar data that was developed by Ransom (2001).

A SIGPROC-style filterbanks file is made up of a header followed by data in a little-endian format consisting of 4 bits per data point. The header of the file contains information about the data within the file. An example of the information provided in the header for the pulsar J0332+5434 is shown in Table 7.1.
The raw data from KAIRA were converted into the format required for SIGPROC style filterbanks files by taking all the values in a channel, calculating the maximum ($I_{\text{max}}$) and minimum ($I_{\text{min}}$) of the data in that channel and normalising each value ($I_i$) so that they are between 0 and 15 and can be written as 4-bit values,

$$I_i = \frac{15.0 \times (I_i - I_{\text{min}})}{I_{\text{max}} - I_{\text{min}}}. \quad (7.3)$$

Once this has been done for two consecutive data points, the two 4-bit values are packed as unsigned 4-bit integers into a single byte. This byte is then written to the file. This process is continued for each time sample and channel, until all data for one polarisation is written to the file. The same process is then followed for the other three polarisations, writing each set of data to separate SIGPROC-style filterbanks files. A python module, KAIRA2SIGPROC, was written to perform this conversion as standard practice.
7.1.3 Calibration

Frequency-dependent effects caused by conversion to the frequency domain are corrected at the correlator, however other frequency-dependent effects are caused by the physical structure of the individual receiving elements, and cause the bandpass to peak near the resonance frequency of the dipole. These frequency-dependent effects can be corrected post-correlation, using a calibrator source.

A bandpass for the telescope can be created by observing a calibrator source and comparing the pre-determined spectral energy distribution model of the calibrator to the observed power. The best sources to use for calibration of radio instruments are selected from the 3C catalogue (Edge et al., 1959). They are chosen because they have well-understood spectral energy distributions across the full LOFAR bandpass, as well as having high flux densities, dominating the visibility function, and being compact in comparison to the angular resolution of the instrument. Cygnus A and Cassiopeia A, whilst being some of the brightest radio sources in the sky, are therefore not suitable candidates because they are not compact compared to the angular resolution of the instrument and thus have extremely complex morphologies which would make calibration difficult. Scaife and Heald (2012) fit polynomials to six selected 3C sources using a Monte Carlo Markov chain (MCMC) method to create a spectral calibration model.
The polynomials are of the form,
\[
\log S_{\text{src}} = \log A_0 + A_1 \log \nu + A_2 \log^2 \nu + \ldots \tag{7.4}
\]

The 3C source chosen to calibrate KAIRA’s bandpass is 3C295. For 3C295, the spectral model is fitted with a fourth order polynomial with \( A_0 = 97.763 \pm 2.787 \), \( A_1 = -0.582 \pm 0.045 \), \( A_2 = -0.298 \pm 0.085 \), \( A_3 = 0.583 \pm 0.116 \) and \( A_4 = -0.363 \pm 0.137 \).

To correct the bandpass, a radio-quiet point on the sky close to 3C295 is found by consulting the Haslam 408 MHz map (Haslam et al. 1982; Remazeilles et al. 2015). The radio-quiet point selected is located at right ascension of 137.29° and declination of +75.66° (J2000). Using the variation in power with frequency observed with 3C295 and the blank sky, the System Equivalent Flux Density (\( S \)) of the telescope can be calculated. It is assumed that the radio-quiet point is equivalent to the power output by the system, therefore the true power of the source is
\[
A_{\text{src}} = A_{\text{obs}} - A_{\text{sys}} \tag{7.5}
\]

where \( A_{\text{src}} \) is the true power of the source without any system effects, \( A_{\text{obs}} \) is the measured power of the source and \( A_{\text{sys}} \) is the power due to system effects (the power measured by observing the patch of blank sky). The system equivalent flux density, \( S \), can then be calculated using \( A_{\text{obs}} \) and \( A_{\text{sys}} \) and the spectral calibration model described in Equation 7.4, \( S_{\text{src}} \).
\[
S_{\text{src}} = S \frac{A_{\text{src}}}{A_{\text{sys}}}
= S \frac{A_{\text{obs}} - A_{\text{sys}}}{A_{\text{sys}}}
= S \left( \frac{A_{\text{obs}}}{A_{\text{sys}}} - 1 \right). \tag{7.6}
\]

The system equivalent flux density corrects for the bandpass and converts the observed data from ADU to Janskys. Figure 7.2 shows the stages of the system equivalent flux density creation. The top plot shows the true flux of 3C295 across the KAIRA bandpass as calculated by Equation 7.4 (Scaife and Heald, 2012), the middle plot shows the observed power received from the blank sky patch and 3C295, and the bottom plot shows the system equivalent flux density as calculated using Equation 7.6. This system equivalent flux density is then applied to the pulsar data to correct for the frequency-dependent response of the instrument.
Figure 7.2: The stages of the system equivalent flux density creation, where the top plot shows the true flux of 3C295 as calculated by Equation 7.4 (Scaife and Heald, 2012), the middle plot shows the observed power received from the blank sky patch and 3C295, and the bottom plot shows the system equivalent flux density as calculated using Equation 7.6.
Chapter 7 Constraining turbulence using pulsar observations

7.1.4 Removal of RFI

As well as automatically clipping all strong, transient, DM=0 signals in the data, PRESTO can also be used to remove RFI from the data in the SIGPROC-style filterbanks files using the task RFIFIND (Ransom, 2001). RFIFIND removes the RFI by searching over both the time and frequency domains with integration times of a few seconds, looking for both strong narrow-band RFI (such as that seen in Figure 7.1) and broad-band RFI of a short duration. RFIFIND splits each frequency channel into short time intervals prior to analysing it for RFI. RFI is identified using a statistical clipping method in both the time domain and the frequency domain. If the RFI is greater than or less than ±6σ in the time domain then RFIFIND flags it and then masks the values, however if it is greater than or less than ±10σ then the time interval is rejected. In the frequency domain the RFI has to be greater or less than ±4σ for that section of the frequency channel to be rejected. The percentage of the observation that is masked by RFIFIND in the KAIRA data is normally less than 5%. Figure 7.3 shows an example of the output produced by RFIFIND. The coloured areas represent RFI, where red is periodic RFI and blue and green are caused by time-domain statistical issues.
Figure 7.4: PREPOLD plots of Stokes IQUV data for the pulsar J0332+5434.
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7.1.5 Outputting pulsar data from PREPFOLD

Having converted the raw KAIRA data into a SIGPROC-style filterbanks format that is readable by pulsar analysis programs such as PREPFOLD, as well as correcting for frequency-dependent effects and RFI, the Stokes Q and U pulse profiles can be obtained.

First each polarisation for each time period for each pulsar is run through PREPFOLD, returning plots as shown in Figure 7.4 as well as data (such as the best dispersion measure) that can be used by the other PREPFOLD commands. The commands used with PREPFOLD include specifying the number of subbands (244), the number of bins each pulse profile is split into (256), the number of integrated pulse profiles in the observation (20), the mask output by RFIFIND and a pulsar parameter file containing data from the ATNF pulsar catalogue\(^1\) (Manchester et al., 2005). The pulsar parameter file contains the pulsar name, right ascension (J2000), declination (J2000), barycentric rotation frequency (Hz), first time derivative of barycentric rotation frequency (s\(^{-2}\)), epoch of period (MJD) and dispersion measure (cm\(^{-3}\)pc).

The pulsar data are then dedispersed using the best dispersion measure found by PREPFOLD, then the best profile from the Stokes I data is selected. The plot in the top

\(^1\)http://www.atnf.csiro.au/research/pulsar/psrcat/
left of Figure 7.5 shows an example of a best profile. The bins in the best profile that correspond to the pulsar are found by calculating the mean and standard deviation of the profile and selecting any bins that are greater than the mean plus $3\sigma$. Figure 7.6 demonstrates this process. If no bins exceed the value of the mean plus $3\sigma$ then it is assumed that the data is too noisy to find the pulse and the time period for that pulsar is therefore discarded. The majority of the data for the pulsars J2022+5154, J2113+4644 and J2225+6535 are discarded in this way as their pulses have lower signal-to-noise than the other observed pulsars.

Once the bins corresponding to the pulsar have been found, all 4 polarisations are output as three-dimensional arrays with axes of the subbands, the number of bins in each profile and the number of integrated pulse profiles in the observation. The mean values along the integrated pulse profiles in the observation axis are found, returning a two-dimensional array and then the bins corresponding to the pulse, shown in Figure 7.6, are selected for each subband for the Q and U data. The mean background (the off-pulse part of the pulsar profile) for the Q and U data is then calculated for each subband and subtracted from the on-pulse bins, before the Q and U values are written to a file.

### 7.2 Methods of obtaining rotation measures

Once the Stokes Q and U data have been obtained from the pulsar observations, they can be used to find the RM of the pulsar. A common method of obtaining RMs from polarisation data is polarisation angle fitting. The premise of this method is that it determines a best fit to the constant linear gradient using the polarisation angle, $\chi$, and
the square of the wavelength, $\lambda^2$ to find the RM,

$$\text{RM} = \frac{d\chi (\lambda^2)}{d\lambda^2}. \quad (7.7)$$

This method is limited by its inability to detect a rotation of $n\pi$ radians between different observing wavelengths. This means that to use this method to obtain correct results, the frequency resolution must be high enough to resolve this $n\pi$ ambiguity (Brentjens and de Bruyn, 2005). This method is also not valid if the radio source has multiple Faraday components (a composite Faraday structure) or is affected by Faraday depolarisation (a decline in fractional polarisation with wavelength; Farnes et al. 2014) as this affects the linear relationship, causing the method to be unreliable in returning the correct linear gradient (Farnsworth et al. 2011; O’Sullivan et al. 2012).

Due to the limited nature of the polarisation angle fitting method, many methods have subsequently been developed to find the RM when the observed data do not have a high enough frequency resolution. Sun et al. (2015) compare some of these algorithms for determining the RM. The algorithms that they consider are

- **RM synthesis** – a method that minimises the effects of the $n\pi$ ambiguity and enables the user to recover emission at multiple Faraday depths along the line of sight (Brentjens and de Bruyn 2005; Heald 2009) causing polarised flux at different values of $\phi$. This method was first proposed by Burn (1966).

- **Wavelet decomposition** – a method suggested by Frick et al. (2010) that involves decomposing the Faraday spectrum into wavelet coefficients by transforming the polarised intensity.

- **Compressive sampling** – a method that assumes a sparse sample can represent the Faraday spectrum in a set of analysis functions. The Faraday spectrum can then be reconstructed by minimisation.

- **QU-fitting** – a method where models are fitted to the observed $Q (\lambda^2)$ and $U (\lambda^2)$ data. This method is detailed in Farnsworth et al. (2011) and O’Sullivan et al. (2012).

Sun et al. (2015) used the above algorithms on simulated data sets which comprised models with sources that have one Faraday thin component, two Faraday thin components and one Faraday thick component. They found that whilst all methods work well with one Faraday thin component and none of the methods work well for Faraday thick components, QU-fitting performs best with two Faraday thin components. This is because most polarisation studies find the RM by using Equation 2.26 and fitting for the RM, however this method is only valid when there is only one uniform Faraday screen in the foreground. I therefore opt to use QU-fitting to recover the RMs, with the details of how the method is implemented discussed below.
7.2.1 QU-fitting

The QU-fitting method used here relies on a Metropolis-Hastings algorithm. This algorithm is an MCMC method which selects random points from a probability distribution to approximate the distribution. It is used if the probability distribution is multi-dimensional and cannot be directly sampled. A point, $x_t$, from within the probability distribution is selected, a new point, $x'$, is then chosen with a probability density for the new point given by the previous point, $Q(x'|x_t)$. $Q(x'|x_t)$ is often assumed to be a Gaussian distribution, meaning that points closer to $x_t$ are more likely to be sampled next. An acceptance ratio for this newly selected point is then calculated. This ratio depends on the posterior probability ratio between $x_t$ and $x'$. If the acceptance ratio is greater than 1 then $x_{t+1}$ is set to be $x'$, otherwise $x_{t+1}$ is $x_t$ (i.e., the selected point is rejected). As in this case the location of the peak of the probability distribution is completely unknown a method known as ‘burn-in’ is used. This is where the initial point for the MCMC is selected arbitrarily, and the chain is then run for many iterations with a larger step size than is used in the normal Metropolis-Hastings algorithm. This allows the algorithm to get closer to the peak of the distribution before it begins the sampling of the probability distribution. The ‘burn-in’ values are forgotten before the proper sampling of the probability distribution is begun.

To implement this QU-fitting method, a prior is first set stipulating the range that the RM should be within. As shown in Table 6.1, the RM for most of the observed pulsars is within the range $-50 \text{ rad m}^{-2} \leq \text{RM} \leq 50 \text{ rad m}^{-2}$, therefore for those pulsars this was chosen as a flat prior. For J0332+5434, one of the pulsars in Table 6.1 with a RM outside of this range, the prior is chosen as $-100 \text{ rad m}^{-2} \leq \text{RM} \leq 100 \text{ rad m}^{-2}$. For the other pulsar in Table 6.1 with a RM outside of this range (J2113+4644) the prior is chosen as $-300 \text{ rad m}^{-2} \leq \text{RM} \leq 300 \text{ rad m}^{-2}$.

The weighted mean is then subtracted from each value of Q and U. The weighted mean is calculated using the equation

$$
\bar{x} = \frac{\sum_{i=1}^{244} w_i x_i}{\sum_{i=1}^{244} w_i}
$$

(7.8)

where $x$ is set to Q or U. As the location of the peak in the probability distribution is unknown, the ‘burn-in’ for the Metropolis-Hastings QU-fitting method is first undertaken, allowing the algorithm to start closer to the peak of the distribution when the Metropolis-Hastings QU-fitting method is run again for the final sampling of the probability distribution. Figure 7.7 shows the Q and U polarisation data after the weighted means have been subtracted for the pulsar J0814+7429. The fit to these data as obtained by the Metropolis-Hastings QU-fitting method is plotted over the top.

Figure 7.8 shows the Faraday spectrum for the time sample of the pulsar J0814+7429 shown in Figure 7.7. It can be seen that the Faraday spectrum does not just consist
of a component at the RM of the pulsar, but instead there is a large peak at RM= 0. This is caused by frequency-independent instrumental effects (Brentjens and de Bruyn 2005; Farnes et al. 2013) which can be removed (or at least diminished) by full polarisation calibration. The QU-fitting method used here, however, disregards anything with RM= 0, whilst returning the same RM as is obtained through the RM-synthesis method illustrated in Figure 7.8. Therefore these frequency-independent instrumental effects can be safely ignored.

### 7.2.2 Conditioning of KAIRA data for QU-fitting

The RMs for each bin are calculated using the Metropolis-Hastings QU-fitting method. The median value and standard deviation of these RMs across the bins is then calculated and if the RM is greater or less than \( x_{med} \pm \sigma \) then the RM is discarded. The median is used here instead of the mean because it is more robust against outliers and skew. The median value of the remaining RMs across the bins is then calculated and output for each time sample and for each pulsar. For each pulsar, the variation from the median of the RMs across the entire period of the observation is calculated,

\[
\delta RM = RM - \bar{RM}. \tag{7.9}
\]
Figure 7.8: The plot on the left shows peaks in the Faraday depth, where the Faraday depth is equivalent to the RM, for the pulsar J0814+7429. The peak at RM = 0 is caused by frequency-independent instrumental effects. The plot on the right shows the variation of the Q and U values for the pulsar J0814+7429 averaged over the 2 minute time sample. The value of RM is the same as that obtained through QU-fitting

This is then plotted in Figures 7.9 and 7.10, which show the variation in RM due to the ionosphere for the pulsar observations made on 03–04 March 2015 and 17–18 March 2016 respectively. Only four pulsars are included in each figure. This is because the majority of the data for the pulsars J2022+5154, J2113+4644 and J2225+6535 are discarded as their pulses have low signal-to-noise and so cannot be discerned from the background noise.

Figures 7.9 and 7.10 also show the TEC above the KAIRA site as obtained from CODE, for comparison. It can be seen that the increase in TEC at dawn on 03–04 March 2015 corresponds to a change in δRM. A similar increase in TEC is also seen on 17–18 March 2016 at dawn, however it does not appear to correspond to a change in δRM. The TEC on 17–18 March 2016 is significantly lower than the TEC on 03–04 March 2015.
Figure 7.9: TOP: The variation from the median of the RM\(^s\) for each pulsar and their variation with time as observed using KAIRA on 03–04 March 2015. The solid vertical black line indicates local sunrise (05:49 UT). The dashed vertical lines on each plot indicate the separation between nighttime (20:00–04:00 UT), dawn (04:00–08:00 UT) and daytime (08:00–14:00 UT). BOTTOM: TEC data above the KAIRA site obtained from CODE (Section 1.2.3).
Figure 7.10: TOP: The variation from the median of the RMs for each pulsar and their variation with time as observed using KAIRA on 17–18 March 2016. The solid vertical black lines indicate local sunset (16:44 UT) and local sunrise (04:44 UT). BOTTOM: TEC data above the KAIRA site obtained from CODE (Section 1.2.3).
7.3 KAIRA data and the structure function

Once the RMs obtained by KAIRA have been corrected for their line of sights through the ionosphere using the thin-layer approximation, $\delta$RM can be plotted against the distance between the two pulsars as detailed in Section 6.1.1, forming a structure function. The structure functions for the entire 18 hours of the observation on 03–04 March 2015 and the 20 hours of the observation on 17–18 March 2016, assuming a temporally frozen ionosphere are shown in Figure 7.11. To find the power law of these structure functions, the Levenberg-Marquardt non-linear least squares algorithm is used. The structure

![Graph of structure function for KAIRA data](image)

Figure 7.11: The structure functions for RMs for the pulsars observed by KAIRA over the entire observation. The lines fitted to the data by the Levenberg-Marquardt non-linear least squares algorithm have gradients of $1.39 \pm 0.86$ (purple) and $-20.44 \pm 29.27$ (red) for the observations on 03–04 March 2015. The lines fitted to the data for the observations on 17–18 March 2016 have gradients of $1.62 \pm 0.20$ (purple) and $-0.45 \pm 38.70$ (red).
function obtained using this method results in gradients of $\gamma_0 = 1.39 \pm 0.86$ (purple) and $\gamma_1 = -20.44 \pm 29.27$ (red) for the observations on 03–04 March 2015. For 17–18 March 2016, the structure function has gradients of $\gamma_0 = 1.62 \pm 0.20$ (purple) and $\gamma_1 = -0.45 \pm 38.70$ (red).

The gradients obtained here for the observations on 03–04 March 2015 and 17–18 March 2016 are lower than the gradients expected if the turbulence in the ionosphere is assumed to be Kolmogorov. However, in Section 6.3.1 a single simulation of a two-dimensional Kolmogorov turbulence screen was also found to produce a structure function with a gradient that was lower than that expected for Kolmogorov turbulent structure. This is because a single iteration of a turbulence screen will not necessarily return the correct statistics.

Multiple turbulence screens were simulated in Section 6.3.1 and the structure function was found for each of these individual turbulence screens, finding that the mean gradient is $\gamma_0 = 1.60 \pm 0.14$. The gradient found for the observations on the 17–18 March 2016 is therefore within one standard deviation of a Kolmogorov turbulent spectrum, whilst the gradient found for the observations on the 03–04 March 2015 are within two standard deviations. Therefore, as one turbulence screen is not guaranteed to return the correct statistics, to obtain the true turbulence statistics of the ionosphere, the ionosphere needs to be sampled many times assuming the same conditions each time, in a similar way to how the 100 turbulence screens and structure functions were created in Section 6.3.1.

As previously discussed in Section 6.3.3 there is quite a lot of variation in the structure functions in Figure 7.11. This is again likely to be due to there being fewer values in each bin, causing the errors to be larger. The sampling is also worse than Figure 6.12 as there are fewer pulsar observations than were simulated due to the signal to noise not being high enough in the pulsars J2022+5154, J2113+4644 and J2225+6535.

In this experiment I have made a number of assumptions that could lead to errors in the results. These include assuming a frozen ionosphere and assuming that the ionosphere can be adequately modelled by a thin-layer approximation at an altitude of 450 km. The assumption of a frozen ionosphere is incorrect, and this is investigated below. The assumption that the ionosphere can be adequately modelled by a thin-layer approximation at a fixed height is also flawed. In reality, the altitude of the thin-layer should vary with time, as it is dependent on the electron density profile of the ionosphere which is dependent on factors such as time of day, time of year and solar activity.

As in Section 6.3, the change in structure of the ionosphere throughout the observing period can be calculated by using a moving window algorithm. This moving window algorithm allows the structure to be calculated by comparing the change in RM between the pulsars observed within 20 minutes of each pulsar. By looking at the variation of the gradient of the structure function with time, a frozen ionosphere is no longer assumed, however it is still assumed that the ionosphere can be modelled as a thin layer at a
height of 450 km. This enables us to see how the structure of the ionosphere changes with time. Figure 7.12 shows the variation of the structure function with time. The average power obtained from fitting lines to the 03–04 March 2015 structure functions is $\gamma_0 = 0.34 \pm 1.49$, and is $\gamma_0 = 0.67 \pm 1.34$ for the 17–18 March 2016 structure functions. As there are fewer valid RM values in each time period than were simulated in Section 6.3.3, there is more variation in $\gamma_0$ because there are fewer points in each structure
function. This causes the large errors on the average values of $\gamma_0$. The errors on these values are too large to be able to infer anything about the temporal variation of the turbulent structure of the ionosphere. To obtain information about the temporal variation of the structure of the ionosphere more observations of pulsars with high signal to noise pulses are required so that there are more values in each bin in the structure function.

7.4 Conclusions

A pulsar observing mode has been implemented at KAIRA, capable of observing full Stokes data at a high temporal resolution. A pipeline for converting the data output from KAIRA into a format readable by pulsar analysing programs such as PRESTO has been written, implementing PRESTO tools such as RFIFIND and PREPFOLD in order to output Q and U data that can be used to investigate changes in RM due to the ionosphere.

Structure functions of the ionosphere above KAIRA have been created using the data obtained on 03–04 March 2015 and 17–18 March 2016. The powers obtained by fitting a broken power law to these structure functions are consistent with the expected variation in $\gamma$ seen in Section 6.3.1, implying that the ionosphere above KAIRA has the statistics expected for Kolmogorov turbulence.

The assumption that the ionosphere above KAIRA should follow Kolmogorov turbulence comes from the fact that the mid-latitude ionosphere has been found to have a power law similar to that expected for Kolmogorov turbulence. Further investigation is required to establish whether the high-latitude ionosphere has the same turbulence statistics as the mid-latitude ionosphere. Suggestions for further work to verify this are detailed in Section 8.3.
Chapter 8

Conclusions and future work

Having introduced the requirements in low-frequency radio astronomy for a good understanding of the ionosphere, this thesis has discussed ionospheric modelling techniques. The aim of these techniques is to reduce the effects of the ionosphere on low-frequency radio observations. Three of the main ionospheric effects on low-frequency radio observations have been investigated — phase delays, absorption and Faraday rotation. Whilst these effects can all be mitigated by observing at higher frequencies, many of the key science goals of telescopes such as LOFAR, the MWA and SKA-LOW require the ability to observe at low frequencies. These key science goals include studies of the high-redshifted 21-cm emission line from neutral hydrogen as a probe for investigating the cosmic dawn and the epoch of reionisation, as well as discovering and observing pulsars which are brightest at low frequencies.

In Chapter 3 the phase error that results from using the thin-layer assumption in ionospheric phase calibration for a radio synthesis telescope was numerically investigated. In Chapters 4 and 5 a new method of using multi-frequency absorption data to infer an electron density height profile for the lower ionosphere using iononest was introduced and explored. Chapters 6 and 7 concentrated on measuring the turbulence of the ionosphere using the change in Faraday rotation of pulsars, allowing information to be inferred about the structure of the ionosphere. In this chapter I present the conclusions for each of the ionospheric effects discussed in this thesis, as well as suggesting ideas and directions for future work and research.

8.1 The thin-layer assumption

In Chapter 3 the phase error that results from using the thin-layer assumption in ionospheric phase calibration for a radio synthesis telescope was numerically investigated. This assumption is regularly used throughout radio astronomy, and I found that whilst
an assumed altitude of $\sim 450$ km will generally minimise this error, there are circumstances under which the thin-layer assumption becomes the limiting factor for imaging fidelity. I have presented the phase error as calculated for typical mid-latitude nighttime and daytime VTEC for a range of long-baseline arrays (GMRT, LOFAR, MWA, JVLA and SKA-LOW) when assuming a thin layer at an altitude of 450 km and have concluded that the MWA is the only array for which the phase error introduced by this assumption does not limit the imaging fidelity.

The thin-layer assumption has been extended by Intema et al. (2009) into a number of thin layers that make up a multi-layer ionospheric model. This has been applied in more recent versions of SPAM. The work in Chapter 3 could therefore be continued to encompass the multi-layer ionospheric model and to calculate how many and at which altitudes these layers should be placed in order to reduce the phase error caused by the thin-layer assumption for long-baseline arrays.

### 8.2 Absorption measurements

In Section 2.3.4 the use of aperture arrays to observe the absorption of radio signals that occurs in the ionosphere was investigated. Two aperture arrays were used: KAIRA, located at a high latitude and the Rawlings array, located at a middle latitude. A QDS was created for each array for each observing campaign, however the QDS does not represent a perfectly quiet ionosphere, therefore for a stable or quiet ionosphere the absorption cannot be found accurately.

As the Rawlings array is part of the international LOFAR array, it was difficult to obtain enough time to create an accurate QDS. The evidence is therefore inconclusive as to whether telescopes at a mid-latitude are capable of returning meaningful absorption measurements. On the other hand, the measurements obtained by KAIRA demonstrate that constant observations of absorption are possible using this technique.

IONONEST, a new technique for obtaining electron density height profiles for the lower ionosphere from multi-frequency absorption measurements was introduced in Chapters 4 and 5. Testing in Chapter 4 demonstrated that it was capable of returning accurate parameters for ionospheric electron density height profiles from realistic, simulated multi-frequency absorption data using three parameterised electron density height profile models.

It was found that all three electron density height profile models were capable of simulating realistic electron density height profiles, with the polynomial model (Model 2) able to return height profiles that were most similar to those observed using the EISCAT VHF radar. When absorption data were simulated from electron density height profiles created using these models, it was found that IONONEST was more capable of
reproducing the electron density height profiles simulated by the two parameter exponential model and the electron precipitation model (Models 1 and 3). This is because these models are simpler and therefore have less degenerate posterior distributions.

The use of Model 3, a model that considers the effect of electron precipitation on a quiet electron density height profile, found that information about the electron flux of the incident electron precipitation can be inferred. This makes IONONEST, coupled with Model 3 and an ion chemistry model, a useful tool for observing ionospheric processes at high-latitudes. It also provides useful information for satellites as high fluxes of energetic electrons can cause penetration of the electrons into spacecraft components which results in buildup of charges, known as deep-dielectric charging. Once the built up charge becomes too large, it can cause discharge or arcing which results in anomalous behaviour of the satellite, or a temporary or permanent loss of functionality.

As well as the useful information about electron flux that can be found using Model 3, information about the electron density can be deduced through the use of IONONEST with all three models. This is a useful tool for ionospheric physicists who are interested in observing ionospheric processes.

In Chapter 4, just three electron density height profile models were compared, however there are many more within the literature that could be implemented in IONONEST. Furthermore, these data provide a powerful resource for developing D layer models further. Monitoring observations also raises the possibility of probing D layer changes on short time scales, for example during solar events.

Chapter 5 has demonstrated that information can be inferred about the lower ionosphere by measuring the absorption that occurs. Comparisons were first made between absorption measurements and electron density height profiles obtained by the EISCAT VHF, which reflected that absorption was generally higher during peaks in the electron density measured by the EISCAT VHF.

IONONEST was then used to return electron density height profiles by fitting absorption curves to the observed absorption data. It was found that whilst IONONEST is capable of returning realistic electron density height profiles that are comparable to those obtained by the EISCAT VHF during geomagnetically active periods, it is not capable of returning realistic electron density height profiles during geomagnetically quiet periods due to inaccuracies in the QDS method. The electron density height profiles returned by IONONEST were compared to measurements by the EISCAT VHF, finding good agreement in the observations taken at the geomagnetically active period. The comparison between the KAIRA and the EISCAT VHF height profiles during the geomagnetically quiet period shows less agreement, failing to show the same temporal variability and magnitude as the EISCAT VHF electron densities observed over the same period. This could be due to the inaccuracies in the absorption measurements made by KAIRA, as
well as the inability of the EISCAT VHF to observe at low altitudes when the ionospheric electron density at that altitude is below the sensitivity limit of the EISCAT VHF.

Having shown that iononest is capable of recreating electron density height profiles if the absorption measurements measured by the KAIRA instrument are significantly large enough, this method could now be extended to similar facilities such as LOFAR and the LWA during periods of high absorption, as well as future radio telescopes operating at low frequencies such as SKA-LOW. This could enable continuous measurements of D layer electron density height profiles over Europe, Australia and South Africa, allowing for observations to be made simultaneously at a range of mid-latitudes. Models such as the IRI could use these continuous measurements to improve their models of the currently under-observed D layer.

Measurements of absorption data were also made with KAIRA during an observing campaign in March 2016, with three separate corresponding measurements of electron density made by the EISCAT VHF. These were at 13:00 UT–19:00 UT on 16 March 2016, 23:00–05:00 UT on 16–17 March 2016 and 13:00 UT–19:00 UT on 17 March 2016. Due to the use of NRL-MSISE-00 to obtain information such as the electron-neutral collision frequency, I cannot use iononest to convert these data into electron density height profiles yet, as NRL-MSISE-00 currently only has data up to February 2016.

8.3 Constraining turbulence using pulsar observations

Simulations of ionospheric turbulence were presented in Chapter 6. These simulations have demonstrated that the turbulence statistics of a turbulent ionosphere can be returned. Whilst a single simulation of a turbulence screen does not create a structure function that returns the expected turbulence statistics precisely, repeated simulations of a turbulence screen do enable the creation of a structure function that returns the expected turbulence statistics precisely.

In Chapter 7 a pulsar observing mode was successfully implemented at KAIRA, consisting of software that successfully observed full Stokes polarisation data at a high temporal resolution. A python module, kaira2sigproc, was also developed that converts the raw pulsar data output by KAIRA into a format readable by pulsar analysis programs such as prepfold.

From pulsar observations during observing campaigns at KAIRA in March 2015 and March 2016, Q and U data were obtained from which RM values were calculated. The structure functions that were subsequently created using these data were found to have gradients that match that expected, assuming Kolmogorov turbulence, based on the turbulence simulations that were run in Chapter 6. This fits with observations of the
mid-latitude ionosphere which has also generally been found to have similar statistics to that expected for Kolmogorov turbulence. Further investigation is, however, required. Repeats of the observations carried out in Chapter 7 need to occur, allowing data covering different periods of solar activity and different times of year to be compared. Other ionospheric events that occur at high-latitudes but not at mid-latitudes should also be considered.

To improve the quality of the structure functions, more pulsars with high signal to noise pulses should also be observed. Measurements of the ionosphere from other instruments can also be incorporated into the KAIRA structure functions to add extra data points. This would validate that the change in RMs due to the ionosphere as measured by KAIRA are accurate.

The EISCAT UHF can be operated using the observing code ‘beata’ (Section 1.2.1.1). This enables the electron density in the ionosphere up to F layer altitudes to be measured. An experiment using the EISCAT UHF in this mode was run on 03 March 2015 at 23:19 UT until 05:00 UT 04 March 2015, corresponding to KAIRA observations of pulsars. The EISCAT UHF was in a mode which allows it to track a celestial source and was used to track the pulsar J1509+5531, measuring ionospheric electron density height profiles along the line of sight to the source. Figure 8.1 shows the electron density height profiles obtained by the EISCAT UHF. Due to time constraints, this data was not included in the structure function plots in Chapter 7.

As well as EISCAT UHF measurements from 23:19 UT on 03 March 2015 until 05:00 UT on 04 March 2015, ionosonde data can also be obtained from the UK Solar System Data Centre\(^1\) for the ionosonde located at Tromsø (geodetic latitude= 69.586°, geodetic

\(^{1}\)http://www.ukssdc.ac.uk
Figure 8.2: The VTEC as obtained by the ionosonde located at Tromsø. The time resolution is 15 minutes and there are no data available between 22:15 UT on 03 March 2015 and 03:30 UT on 04 March 2015. The vertical black line indicates local sunrise.

longitude=19.227°. Whilst ionosonde data is not available from 19:00 UT on 03 March 2015 until 03:30 UT on 04 March 2015, data can be obtained corresponding to the remainder of the observations, as shown in Figure 8.2.

8.4 Concluding remarks

In summary this thesis has discussed and investigated new modelling techniques that can reduce the ionospheric effects of phase delays, absorption and changes in Faraday rotation on low-frequency radio observations.

The numerical result from investigations into the error that arises from using the thin-layer assumption in ionospheric phase calibration is of use for all astronomers using radio interferometers with long baselines. It is especially useful for the design of the SKA-LOW and future radio interferometers.

The development of the riometry technique to measure absorption using radio telescopes at multiple frequencies is of use to radio astronomers using high-latitude low-frequency radio telescopes, such as KAIRA. The development of IONEST is of use to ionospheric physicists as it allows for information about the electron density of the currently under-observed D layer to be inferred. IONEST can be used with any electron density model of the lower ionosphere and allows for model comparison. It is also capable of returning information about processes occurring in the ionosphere such as electron precipitation.

Finally, a pulsar observing mode has been implemented at KAIRA, as well as a pipeline for converting the data output by KAIRA into a standard pulsar format. These pulsar
observations were then used to investigate whether Kolmogorov turbulence is an adequate approximation for the turbulence in the high-latitude ionosphere. My results are consistent with Kolmogorov turbulence, however further work is required to verify this. Thin-layer turbulence models are implemented in calibration software such as SPAM, therefore a good understanding of the turbulent ionosphere above radio telescopes is paramount for further development of calibration software.
Appendix A

A.1 Riometry with KAIRA

Figure A.1: TOP: The quiet day surface created for KAIRA (beam pointing at zenith) using observations from 01–08 March 2015. MIDDLE: The observed power at KAIRA for a beam pointing at zenith and observing between frequencies 9.77 MHz and 80.66 MHz on 01–02 March 2015. BOTTOM: The absorption, $A$, as measured by KAIRA for a beam pointing at zenith. The absorption is found using Equation 2.25 with the values in the top plot as $P_0$ and the values in the middle plot as $P$. The vertical red lines in this absorption plot show peaks in absorption. RFI can be seen to dominate at all frequencies below 16.80 MHz and ‘Dragons teeth’ RFI can be clearly seen in the middle and bottom plot at 00:58 UT, 03:26 UT, 06:00 UT and 08:40 UT. The vertical black lines indicate local sunset (15:02 UT) and local sunrise (05:45 UT).
Figure A.2: Magnetograms for the magnetometer in Kilpisjärvi, Finland for 01–02 March 2015.

Figure A.3: TOP: The quiet day surface created for KAIRA (beam pointing at zenith) using observations from 15–23 March 2016. MIDDLE: The observed power at KAIRA for a beam pointing at zenith and observing between frequencies of 9.77 MHz and 80.66 MHz on 16–17 March 2016. BOTTOM: The absorption, $A$, as measured by KAIRA for a beam pointing at zenith. The absorption is found using Equation 2.25 with the values in the top plot as $P_0$ and the values in the middle plot as $P$. The vertical red lines in this absorption plot show absorption, peaking between 06:00 UT and 07:00 UT. RFI can be seen to dominate at all frequencies below 16.80 MHz. The vertical black lines indicate local sunset (16:44 UT) and local sunrise (04:48 UT).
A.2 The International Geomagnetic Reference Field (IGRF)

The magnetic field of the Earth near the surface is typically 45,000 nT (horizontal with a magnitude of 30,000 nT near the equator and vertical with a magnitude of 60,000 nT near the poles) and is mainly caused by the electric currents in the Earth’s fluid core.

The IGRF is a mathematical description of the Earth’s main magnetic field. It is made up of a series of mathematical models and uses data from satellites. It varies with standard annual variation (secular variation) and is a reasonable approximation to the magnetic field near to and above the surface of the Earth. The model begins in 1900 and is updated every 5 years, it then interpolates between these 5 year points by linear interpolation. If the data required is after the most recent update of the model, then a linear model is used for forward extrapolation.

The IGRF is limited in many ways,

- It only represents the longer wavelengths of the magnetic field, ignoring the contribution from from magnetised rocks in the crust of the Earth (which typically contribute 200–300 nT with short wavelengths to the magnetic field).
- It purposefully does not try to model certain contributions to the observed field, such as contributions from man-made and natural sources (such as electric currents in the ionosphere and magnetosphere). The contribution from the ionosphere and magnetosphere can be as large as 1000 nT during magnetic storms.
- The uncertainties of the model are difficult to estimate, and so order-of-magnitude approximations to the errors are given.
• The geomagnetic field does not vary linearly with time due to secular variation, however this approximation is used and is generally valid until 2000. After 2000, the use of linear interpolation occasionally results in increased errors.

In this thesis, we use IGRF-12 (Thebault et al., 2015) which is valid for 1900–2020.
References


REFERENCES


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