# The value of public information in vertically differentiated markets*

Andrea Canidio[†] and Thomas Gall[‡]

**Abstract**

Generating public information about vertically differentiated products increases expected vertical differentiation and softens competition. We show that this will induce firms to overinvest (underinvest) in information generation, if the deadweight loss in the subsequent market equilibrium is high (low). Moreover, information generation by one firm has a positive externality on the other firm. It follows that coordination (e.g. via industry associations) increases information generation. When product qualities are endogenous, information generation may prevent quality degradation and thus have an additional social benefit.

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[†]IMT School for Advanced Studies, Lucca (Italy) and INSEAD, Fontainebleau (France); email: andrea.canidio@imtlucca.it.

[‡]Department of Economics, University of Southampton, Southampton SO17 1BJ (United Kingdom); email: T.Gall@soton.ac.uk.

# 1 Introduction

Do market equilibrium outcomes provide participants with efficient incentives to generate public information on the goods traded? This question is of considerable interest from both a regulatory and a researcher's point of view. We contribute to answering it by examining the case of a competitive market with vertically differentiated products, in which product qualities are uncertain but can be partially learned via public signals. These public signals can take the form of reviews by experts in the media (examples are *Consumer Reports* in the US or *Which?* in the UK), quality tests and certification by professional agencies (e.g., rating agencies for financial products, TÜV for industrial goods), industry competitions or trade shows. They are often commissioned by firms, which can invite experts to test their products or set up industry bodies for quality certification.

We use a canonical model of price competition with vertically differentiated products (as in Shaked and Sutton, 1982) augmented by the possibility that, before setting their prices, firms can generate costly signals correlated with the quality of their products, observed publicly by all consumers and all firms. We show that better information will have a twofold effect on the expected market outcome: an improvement in the allocation of heterogenous goods to agents is accompanied by an increase of equilibrium prices, because drawing a signal correlated with the quality of any product increases the expected quality distance between quality leader and follower. This will raise firms' profits and the deadweight loss in the pricing equilibrium. This observation has a number of intriguing implications.

First, in equilibrium firms may under- or overinvest in information generation relative to the social optimum. Which case will occur depends on the market shares of quality leader and follower. We show that under mild restrictions (log concavity, full support, and a sufficiently high minimum taste for quality) these market shares do not depend on the information generated by the firms, but only on the distribution of valuations in the population. When this distribution is such that the market share of the quality follower is large, there is a substantial deadweight loss in the pricing equilibrium. This loss strictly increases in the amount of information generated, because, when new information arrives, the expected quality distance between leader and follower grows, but their market share does not change. As a consequence, the social benefit of information generation (which is always positive) is smaller when the dead-weight loss in the pricing equilibrium is larger. Firms' private benefits of information generation are less sensitive to the deadweight loss and thus, when this deadweight loss is large, firms overinvest in information generation relative to the social optimum.

Conversely, when few consumers purchase from the quality follower, the deadweight loss is small and so is its sensitivity to new information. The social benefit of information generation then exceeds its private benefit and firms will underinvest in information generation relative to the social optimum. This underinvestment is most severe if the quality leader captures the entire market in the pricing equilibrium. In this case the quality leader's profit depends on the marginal valuation of its product, i.e. the willingness to pay of the least quality-sensitive consumer. The firm's private value of information generation is thus lower than its social value, because the former depends on the marginal consumer's valuation, whereas the latter depends on the average consumer's valuation.[1]

Second, information generation generates a positive externality among firms, because drawing *any* signal increases a firm's expected profits, including signals about the opponent's product. Hence, even when price competition in the product market is perfect, firms can soften competition by cooperating in generating unbiased, publicly available information about product quality, for example, by introducing a classification system or an industry-wide competition. Such coordination decreases aggregate surplus whenever the level of information generation in the Nash equilibrium already exceeds the social optimum, which provides a novel perspective on the regulation of industry cooperation.

Finally, we endogenize the initial quality levels by allowing firms to cost-

[1] Interestingly, in the underinvestment case, consumers may benefit from generating additional information. This paper focuses on firm behavior, see Terstiege and Wasser (2019) for optimal information generation from the consumers' perspective.

lessly degrade their quality before engaging in information generation. If firms do not expect any information generation, they will increase the expected quality distance in the product market by way of quality degradation. The possibility of generating public information mitigates this problem, because learning provides an alternative means to generate quality dispersion in the market. Hence, when quality degradation is a concern, encouraging firms to cooperate in information generation may be socially desirable, as it prevents quality degradation.

While there is an active literature on information generation in monopoly settings, considerably less is known about settings with competition.[2] For instance, Ottaviani and Prat (2001) find that a monopolist always benefits from generating and disclosing signals that are affiliated to the valuation of the buyer. A number of authors studied the incentives to generate public information in auctions (see, for example, the seminal paper by Milgrom and Weber, 1982). Closer to our model, Ganuza and Penalva (2010) study an auction, in which one side of the market (the firm) generates information related to the *other* side of the market (the buyers' valuations). They argue that the amount of information generated by the firm will fall short of the social optimum, because information increases the dispersion in buyers' evaluations and information rents. Roesler and Szentes (2017) examine a related issue, identifying the optimal information environment from the buyers' point of the view. We focus on the competitive effect generated by the presence of a second firm/seller, which affects the incentive to generate information even when the equilibrium outcome is a monopoly, because the identity of the monopolist may change with the arrival of new information.

The small literature that considers information generation in models with vertical differentiation tends to rely on very specific informational environments. For instance, Bouton and Kirchsteiger (2015) examine the role of reliable rankings of sellers and show that their presence can reduce consumers'

[2]There is, of course, the classical paradox that by construction fully revealing market equilibrium prices will not provide incentives for costly information generation (Grossman and Stiglitz, 1980), which has been resolved by allowing agents to take into account the effect of their actions on prices and beliefs of other agents (Milgrom, 1981; Verrecchia, 1982).

welfare. Bergemann and Välimäki (2000) consider a dynamic setting, in which information is generated through repeated purchases, and find that information generation increases firms' market power and may reduce social welfare. The main difference with respect to these two papers is that we consider a generic form of information generation (i.e., any unbiased signal correlated with the distribution of quality), examine incentives for over- or underinvestment in information, as well as the role of coordination in information generation.

Related to our work is also Board (2009), who introduces information disclosure into a model of vertical competition. He shows that firms with private information on the quality of their product can use the choice of information disclosure to increase vertical differentiation. We consider a similar environment, but our focus is on the incentives for generating new, public information. A related literature has studied information disclosure and information generation in the context of horizontal competition. In particular, Anderson and Renault (2000, 2009) show that information generation may decrease consumers' welfare. Levin, Peck, and Ye (2009) consider a Hotelling model and argue that when firms operate as a cartel they may disclose more information than under a duopoly. These results are clearly related to ours. In both vertical and horizontal competition, increasing distance between quality levels via information generation increases firms market power. However, the welfare consequence of an increase in distance are very different in the vertical and horizontal case. This is particularly evident when the quality choice is endogenous, because increasing vertical product differentiation by quality degradation is unambiguously harmful for welfare.

Finally, a number of papers have studied models in which firms generate private information, that is, signals that are informative relative to each consumer's idiosyncratic preferences (for example, Lewis and Sappington, 1994, Moscarini and Ottaviani, 2001, Johnson and Myatt, 2006). We instead study public information, which is about the quality of the goods sold in the market, where "quality" refers to the attributes of a product that are valued by all consumers. Hence, all consumers prefer higher quality to lower quality but may solve the tradeoff between quality and price differently.[3]

In the remainder of the paper we first present the model. Then in Section 3 we derive the equilibrium in the pricing game for given expected qualities. In Section 4 we solve the full game, in which firms can invest to generate information before setting prices. Section 5 adds a stage to the game, in which each firm can degrade its product at no cost. The last section concludes. All mathematical derivations missing from the text are in the appendix.

# 2 Model

Our starting point is the canonical model of a duopoly with vertically differentiated products (see Gabszewicz and Thisse, 1979, Shaked and Sutton, 1982, and Chapter 7 of Tirole, 1988's textbook). The market consists of 2 firms and a mass 1 of buyers. Each firm produces a good of quality $s_i \in [\underline{s}, \overline{s}]$ for $i \in \{1, 2\}$. A buyer's utility is given by

$$U = \begin{cases} \theta s_i - p_i & \text{if good } i \text{ is purchased} \\ 0 & \text{in case of no purchase,} \end{cases}$$

where $p_i$ is the price of the good produced and $\theta \in \mathbb{R}_+$ is an i.i.d. taste parameter with cumulative distribution function $F(x) = \text{pr}(\theta \leq x)$ that is continuous, differentiable, and has a continuous first derivative. We assume that $F(.)$ has a lower bound $\underline{\theta}$ (so that $F(x) = 0$ for all $x \leq \underline{\theta}$), and may or may not be bounded above. If an upper bound exists, we call it $\overline{\theta} > \underline{\theta}$, otherwise we write $\overline{\theta} = \infty$.

Each firm has zero marginal cost of production, so that profit is given by price times quantity sold.

[3]Because the taste for quality of each consumer is constant, but information generation affects the expected quality, in our setting new information always proportionally shifts the consumer's willingness to pay.

**Information and Learning**

We depart from the canonical model by assuming that the quality levels $s_i$ are unknown to both buyers and firms, who have common ex-ante beliefs $G_i(x) : [\underline{s}, \overline{s}] \to [0, 1]$ that reflect the probability that the "true" quality $s_i$ is below a level $x$.

Firms can generate new, public information on the quality levels by acquiring informative signals on each product's quality. For example, the products' technical specifications may be perfectly known to all agents, yielding symmetric information and the common priors. From the consumers' point of view, however, the consumption utility generated by the two products may depend on harder-to-measure attributes such as their aesthetic appeal, their ergonomics and ease of use, the presence of unexpected bugs or defects, both in absolute and relative terms. As a consequence, although the products' technical specifications are common knowledge, expert reviews and testing, industry events and competitions, objective rankings and classification systems provide informative signals on the products' expected qualities.

Firm $i$ can acquire information by paying a cost $k$ and drawing a signal $\sigma_i$, which is informative with respect to $s_i$ and may be informative with respect to $s_{-i}$ as well.[4] We allow for any possible correlation between any $\sigma_1$ and $\sigma_2$. Information generated is public: all market participants receive the signal and update their belief about quality. Information generation is thus best understood as submitting the product to a public quality review process. A good example of such public reviews are classification systems, such as the ones for wine.[5] Notably, and for future reference, classification systems are

[4]We abstract away from the choice of precision of the signal (as in the Bayesian persuasion literature, see in particular Gentzkow and Kamenica, 2016) as well as from the possibility of signal jamming. We will show below that both firms' expected profits increase in the precision of both signals. Hence, firms have no incentive to jam each other's signal, and, for given cost of drawing a signal, firms prefer the most precise signal available. Hence, our results carry over to such a case. However, if signals of different precision differ in their cost, firms will face a trade off. This trade off depends crucially on the details of the cost function, and we prefer to leave this extension for future research.

[5]E.g., for Bordeaux wines the wine classification of 1855, and, more importantly, its more recent and regularly updated offshoot Cru Bourgeois, and similar systems in Burgundy, Champagne, Douro, and other regions. While observable variables such as soil quality of the vineyard and the weather of the vintage determine expected qualities, the

often funded by industry bodies.[6]

Formally, given $G_i(x)$, call $q_i$ the initial expected quality of firm $i$'s product. Without loss of generality let $q_1 \geq q_2$. That is, before any additional information is generated firm 1's product has higher expected quality. Denote a specific realization of the vector of signals $\sigma$ by $\hat{\sigma}$. Let $G_i(x|\hat{\sigma})$ denote the posterior belief distribution conditional on $\hat{\sigma}$ and $\hat{q}_i$ the expected value given distribution $G_i(x|\hat{\sigma})$.

We adopt the convention that $\sigma_i = \emptyset$, if firm $i$ does not acquire information, so that $\hat{q}_i = q_i$ for $i \in \{1,2\}$ if $\sigma = (\emptyset, \emptyset)$. We also write $\sigma = (\emptyset, \sigma_i)$ when firm $i \in \{1,2\}$ acquires information, but not firm $-i$, and $\sigma = (\sigma_1, \sigma_2)$ when both firms acquire information. Note also that by iterating expectations $E[\hat{q}_i|\sigma] = q_i$ for any signal configuration $\sigma$. This means that ex ante, before any signals are drawn, the expected posterior quality is equal to the prior expected quality.

### Timing

To summarize, the timing of the game is as follows.

1. $G_i(x)$ for $i \in \{1,2\}$ is exogenously determined.[7]
2. Firms simultaneously decide whether to acquire information at cost $k$, yielding a vector of signals $\sigma$.
3. Realizations of signals $\hat{\sigma}$ are publicly revealed.
4. Firms announce prices simultaneously. Consumers decide if and from whom to buy and consume. Payoffs are realized.

### Solution Concept

To derive the outcome of the game described above we employ a subgame perfect Nash equilibrium of signal generation choices $\sigma_1$ and $\sigma_2$ and price choices $p_1$ and $p_2$ depending on the signals.

true quality of a fine wine often only realizes after years of storage.

[6]For example, associations of vintners.

[7]See Section 5 for the possibility that firms can affect expected quality $q_i$.

**Assumptions**

We conclude the description of the model by introducing some restrictions on the distribution of the taste parameter $\theta$. These restrictions guarantee the existence and uniqueness of a pure strategy Nash equilibrium in the pricing game (stage 4 in the timeline above), as we will show.

**Assumption 1** (Full Support)**.** *Call $f(\theta)$ the p.d.f. of $F(\theta)$. If at two $\theta', \theta'' \in \mathbb{R}$ we have $f(\theta') > 0$ and $f(\theta'') > 0$, then $\forall \theta''' \in [\theta', \theta'']$ then $f(\theta''') > 0$.*

If the distribution of the taste parameter has an upper bound, the above assumption implies that its p.d.f. is strictly positive for all $\theta \in (\underline{\theta}, \overline{\theta})$. If instead the distribution parameter has no upper bound, then the assumption implies that its p.d.f. is strictly positive for all $\theta > \underline{\theta}$.

**Assumption 2** (Log-concavity)**.** *The density $f(\theta)$ is log-concave.*

This assumption puts some useful structure on the distribution of $F(\theta)$, ensuring that both $F(\theta)$ and $1 - F(\theta)$ are log-concave (see Prékopa, 1973 and Bagnoli and Bergstrom, 2005). This, in turns, implies that $F(\theta)/f(\theta)$ increases, $(1 - F(\theta))/f(\theta)$ decreases, and $(1 - 2F(\theta))/f(\theta)$ also decreases, all facts that we will use extensively in our derivations. This assumption comes with only a very modest loss of generality, as log-concavity is satisfied by a host of widely used distributions.

Finally, we assume that the there is enough potential revenue in the left tail of the taste distribution, in the sense that $\underline{\theta}$ is sufficiently high and the taste distribution has enough mass at or near $\underline{\theta}$.

**Assumption 3** (Covered Market)**.** *Either $\underline{\theta} \cdot f(\underline{\theta}) > 1$, or*

$$\underline{\theta} \cdot m \geq \frac{\overline{s} - \underline{s}}{\overline{s}}, \tag{A3}$$

*where $m \equiv \min_{\theta \in [\underline{\theta}, \theta^*]} f(\theta)$ and $\theta^*$ is implicitly defined as $\theta^* = \frac{1-F(\theta^*)}{f(\theta^*)}$.*

Note that, because of log-concavity, $\frac{1-F(\theta)}{f(\theta)}$ is strictly decreasing and hence $\theta^*$ exists and is unique as long as $f(\underline{\theta})\underline{\theta} \leq 1$.

Condition (A3) guarantees that the least-quality sensitive consumer is quality sensitive enough so that in equilibrium all consumers prefer to purchase from one of the firms to not purchasing. as we will show.[8] Condition (A3) is a generalization of the standard *covered market* condition.[9] Indeed, any distribution that is bounded below can be scaled up by increasing $\underline{\theta}$. As a consequence of this rescaling $\theta^*$ decreases and $m$ (weakly) increases. If the new $\underline{\theta}$ is sufficiently large relative to the maximum possible dispersion in quality $\overline{s} - \underline{s}$, Condition (A3) will hold.

Note also that Assumption 3 implies that $f(\underline{\theta}) > 0$. Assumptions 1 to 3 therefore encompass a variety of commonly used distributions, such as uniform, truncated normal, exponential, Pareto.[10]

# 3 The Pricing Game

Consider a given realization $\hat{\sigma}$ of the signal vector $\sigma$. If this realization is such that $\hat{q}_1 = \hat{q}_2$, then the two firms compete á la Bertrand and set equilibrium prices $p_1 = p_2 = 0$. All consumers purchase from one of the two firms, and are indifferent between purchasing from firm 1 or 2. If instead $\hat{q}_1 \neq \hat{q}_2$, there is scope for price differentiation between firms. Since information generation may reverse the initial quality ranking of firms, we will refer to the quality leader by $L$ and the follower by $F$, so that $\hat{q}_L \equiv \max\{\hat{q}_1, \hat{q}_2\} > \hat{q}_F \equiv \min\{\hat{q}_1, \hat{q}_2\}$.

[8] To the best of our knowledge, (1)-(3) are the weakest conditions existing in the literature guaranteeing existence, uniqueness and full analytical characterization of the pricing equilibrium with covered market. Studies that examine the non covered-market case (Moorthy, 1988, Choi and Shin, 1992) or that do not impose ex-ante whether the market will be covered (Wauthy, 1996) restrict their attention to uniform taste distributions.

[9] For example, in Chapter 7 of Tirole (1988)'s textbook, the taste parameter is distributed uniformly with $\overline{\theta} - \underline{\theta} = 1$, and the model is solved assuming the covered market condition $\frac{|\hat{q}_1 - \hat{q}_2|}{\max\{\hat{q}_1, \hat{q}_2\}} \leq \underline{\theta}$. Condition (A3) is a generalization of this condition, because it applies to all possible distributions of the taste parameter, and to all possible quality levels (in Tirole, 1988 the quality levels are given exogenously.)

[10] See Bagnoli and Bergstrom (2005). Note also that the truncation of a lognormal distribution is also lognormal (Bagnoli and Bergstrom, 2005, Theorem 7). Hence, any distribution that is unbounded below or bounded below but with mass equal to zero at the lower bound will satisfy Assumptions 1 to 3 if appropriately truncated.

Denote a firm $i$'s posted price by $p_i$. The demand for goods can be characterized by two thresholds. The first threshold $X$ is given by the consumer who is indifferent between purchasing from either firm, if there is such a consumer, and by $\underline{\theta}$ ($\overline{\theta}$) if all consumers weakly prefer $L$ ($F$):

$$X \equiv \begin{cases} \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \in (\underline{\theta}, \overline{\theta}) \\ \underline{\theta} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \leq \underline{\theta} \\ \overline{\theta} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \geq \overline{\theta}. \end{cases}$$

The second threshold $Y$ is given by the consumer who is indifferent between the lower quality firm $F$ and not consuming, if there is such a consumer, and by $\underline{\theta}$ ($\overline{\theta}$) if all consumers weakly prefer $F$ (not to consume):

$$Y \equiv \begin{cases} \frac{p_F}{\hat{q}_F} & \text{if } \frac{p_F}{\hat{q}_F} \in (\underline{\theta}, \overline{\theta}) \\ \underline{\theta} & \text{if } \frac{p_F}{\hat{q}_F} \leq \underline{\theta} \\ \overline{\theta} & \text{if } \frac{p_F}{\hat{q}_F} \geq \overline{\theta}. \end{cases}$$

Noting that the quality leader can always out-price the follower these thresholds can be shown to have some useful properties.

**Lemma 1.** *In any pure strategy Nash equilibrium:*

- *A positive measure of consumers purchase from the quality leader:* $Y \leq X < \overline{\theta}$.
- *If not all consumers purchase from the quality leader (*$X > \underline{\theta}$*), then there is positive demand for the quality follower (*$X > Y \geq \underline{\theta}$*).*

Hence, in any pure strategy Nash equilibrium the demand for good $L$ is $1 - F(X)$ and the demand for good $F$ is $F(X) - F(Y)$.[11] Profits are given by:

$$\pi_L(p_L, p_F) = p_L \cdot (1 - F(X)) \text{ and } \pi_F(p_L, p_F) = p_F \cdot (F(X) - F(Y)).$$

[11]From now on we focus exclusively on pure strategy equilibria, even when we do not explicitly mention this.

Given this, we can derive the two best responses:

**Lemma 2.** *The quality leader's best response is*

$$p_L(p_F) = \max\left\{\frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F\right\},$$

*which is a continuous function. The quality follower's best response is*

$$p_F(p_L) = \begin{cases} [0, +\infty) & \text{if } p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F) \\ \min\left\{\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}\hat{q}_F\right\} & \text{otherwise.} \end{cases}$$

*which is a upper-hemicontinuous, compact valued, convex correspondence.*

A Nash equilibrium is a pair $p^*$ such that $p_i(p_{-i}(p_i^*)) = p_i^*$ for $i = L, F$. Depending on the distribution of the taste parameter $\theta$, this equilibrium can be a monopoly or a duopoly. The following proposition gives sharp conditions on the type distribution for either case.

**Proposition 1** (Market Equilibrium Outcome)**.**

*(i) If $1 \leq \underline{\theta} \cdot f(\underline{\theta})$, then in the unique pure strategy Nash equilibrium $X^* = Y^* = \underline{\theta}$, i.e. the quality leader supplies the entire market, and prices are $p_F^* = 0$ and $p_L^* = \underline{\theta}(\hat{q}_L - \hat{q}_F)$.*

*(ii) If instead $1 > \underline{\theta} \cdot f(\underline{\theta})$, then in the unique pure strategy Nash equilibrium $Y^* = \underline{\theta}$ and $X^* > \underline{\theta}$ where*

$$X^* = \frac{1 - 2F(X^*)}{f(X^*)} \tag{1}$$

*Equilibrium prices are $p_L^* = \frac{1-F(X^*)}{f(X^*)}(\hat{q}_L - \hat{q}_F)$ and $p_F = \frac{F(X^*)}{f(X^*)}(\hat{q}_L - \hat{q}_F)$.*

That is, $\underline{\theta} \cdot f(\underline{\theta})$ determines whether the market outcome will be a monopoly, with the quality leader supplying the whole market, or a duopoly, with the quality leader supplying consumers with $\theta \geq X^*$ and the quality follower supplying the remaining consumers. If $\underline{\theta} \cdot f(\underline{\theta}) \geq 1$ (monopoly) profits are given by

$$\pi_L = \underline{\theta}(\hat{q}_L - \hat{q}_F) \text{ and } \pi_F = 0,$$

if instead $\underline{\theta} \cdot f(\underline{\theta}) < 1$ (duopoly) profits are given by

$$\pi_L = \frac{(1 - F(X^*))^2}{f(X^*)}(\hat{q}_L - \hat{q}_F) \text{ and } \pi_F = \frac{(F(X^*))^2}{f(X^*)}(\hat{q}_L - \hat{q}_F).$$

Note also that the cutoff $X^*$, separating consumers buying from $L$ from those buying from $F$, cannot be greater than the median by construction. Hence, the quality leader's market share is greater than one half.

Perhaps surprisingly, in equilibrium the demand faced by leader and follower does *not* depend on the expected qualities $\hat{q}_L$ and $\hat{q}_F$. This is because both firms' best responses are proportional to the expected quality distance $\hat{q}_L - \hat{q}_F$, and therefore the equilibrium distribution of market shares is independent of that distance. Thus the demand faced by leader and follower depends only on the taste distribution $F(\theta)$, which determines the cutoff $X^*$. This fact will be very convenient because it implies that the signal realizations, and thus also the signal configurations, only affect the identity of quality leader and follower and their market prices, but not their demand.

To provide some illustration for Proposition 1 suppose that the taste distribution is uniform. Then the quality leader supplies the entire market if $\overline{\theta} \leq 2\underline{\theta}$, and there is a duopoly with $X^* = \frac{\underline{\theta}+\overline{\theta}}{3}$ otherwise. Which case will occur depends mainly on two intuitive effects. First, fixing either $\overline{\theta}$ or $\underline{\theta}$, as $\overline{\theta} - \underline{\theta}$ increases (and with it the variance of the distribution) the duopoly becomes more likely. This is because the quality leader will find it less profitable to attract the least quality sensitive consumers by lowering the price, thus leaving demand for the quality follower. By contrast, holding the range of the support $\overline{\theta} - \underline{\theta}$ constant, an equal increase in both $\overline{\theta}$ and $\underline{\theta}$ will make it more likely that the quality leader corners the market. This is because the incentive of the quality leader to sell to the least quality-sensitive consumer increases in her quality sensitivity.

# 4 Information generation

Equipped with the properties of the pricing equilibrium we are now in a position to examine the firms' choices of information generation. Depending on the properties of the type distribution, either the quality leader will corner the market (monopoly case) or both firms will supply some consumers. In the following we consider each case separately.

## 4.1 Case 1: Monopoly ($1 \leq \underline{\theta} f(\underline{\theta})$)

We start by considering the case $1 \leq \underline{\theta} f(\underline{\theta})$, in which the quality leader covers the entire market. Since all consumers consume the good that has higher expected quality, the pricing equilibrium is efficient.

**Social value of information generation.** Given the expected qualities $\hat{q}_1$ and $\hat{q}_2$ the social welfare is given by:[12]

$$S(\hat{q}_1, \hat{q}_2) = \max\{\hat{q}_1, \hat{q}_2\} E[\theta] \, .$$

Suppose that no firm acquires information; then $\hat{q}_i = q_i$ for $i = 1, 2$ and (by assumption) firm 1 is the quality leader. The ex ante expected social welfare is then $S(q_1, q_2) = E[\theta] \, q_1 = E[\theta] \, E[\hat{q}_1|\sigma]$, where the last equality follows from the law of iterated expectation and holds for any $\sigma$. The social benefit of acquiring information is therefore given by the difference between expected social welfare given a chosen signal configuration $\sigma$ and expected social welfare when no information is acquired:

$$\begin{aligned} E[S(\hat{q}_1, \hat{q}_2)|\sigma] - S(q_1, q_2) &= E[\theta] \, E[\max\{\hat{q}_1, \hat{q}_2\}|\sigma] - E[\theta] \, E[\hat{q}_1|\sigma] \\ &= E[\theta] \{ E[\hat{q}_1|\hat{q}_1 \geq \hat{q}_2, \sigma] \mathrm{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma\} + E[\hat{q}_2|\hat{q}_2 \geq \hat{q}_1, \sigma] \mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} \\ &\quad - E[\hat{q}_1|\hat{q}_1 \geq \hat{q}_2, \sigma] \mathrm{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma\} - E[\hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma] \mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\}\} \\ &= E[\theta] \, E[\hat{q}_2 - \hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma] \mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} \equiv E[\theta] \, \Delta(\sigma, q_1, q_2), \end{aligned}$$

where

$$\Delta(\sigma, q_1, q_2) \equiv \mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} E[\hat{q}_2 - \hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma], \qquad (2)$$

is the *expected quality gain.* It measures the value of information generated

[12] We assume throughout the paper that the investment in information generation $k$ by itself is not socially valuable.

by the signal configuration $\sigma$, i.e. the expected gain in quality in the event that the quality ranking changes and thus the good is supplied by a different firm, weighted by the probability that the quality ranking changes. If no signal is drawn, the quality ranking cannot change and $\Delta((\emptyset, \emptyset), q_1, q_2) = 0$. For any other configuration of $\sigma$ the expected quality gain and thus the social benefit of learning are determined by the characteristics of signals and of the quality distribution. For instance, as the priors $q_1$ and $q_2$ get closer, given a signal configuration $\sigma \neq (\emptyset, \emptyset)$, also the probability that the quality ranking reverses and the expected quality gain in the event of a reversal will increase.

To summarize: because the two signals are informative about the two qualities the law of iterated expectation applies and expected quality is independent from $\sigma$. In contrast, by a straightforward application of the Jensen's inequality, the expected maximum quality is a function of the signal configuration. More precisely, the above derivations show that, from a social point of view, information generation is beneficial if and only if there is a realization of the signal vector $\sigma$, such that the quality ranking reverses. If positive, the benefit of information generation increases in the probability of a ranking reversal and in the expected distance conditional on a ranking reversal.

For an intuition, suppose that a signal about the quality leader's quality is drawn. If the leader's quality is revealed to be better than expected, aggregate welfare will increase. If it is instead worse than expected, then aggregate welfare will decrease. The decrease in welfare is, however, bounded below by the quality of the ex-ante quality follower. This makes information generation about the quality leader socially beneficial in expectation. The case of the follower is analogous. This logic is remarkably general, extending for instance to the cases when each signal is informative of the other product as well, and when signals are correlated.

In our setting more information is better, in the sense that the expected quality gain is (weakly) monotone in the number of signals drawn:

$$\Delta((\emptyset, \emptyset), q_1, q_2) \leq \Delta((\emptyset, \sigma_i), q_1, q_2) \leq \Delta((\sigma_1, \sigma_2), q_1, q_2). \text{ for } i \in \{1, 2\} \quad (3)$$

This is because more information (in the form of two signals rather than one) increases the dispersion in the distribution of the posterior.[13] Note also that the value of information generation $\Delta(\sigma, q_1, q_2)$ depends only on the expected posterior quality distribution, but not on which firm draws a signal. Hence, if firms have access to the same signal technology, so that $\sigma_1$ and $\sigma_2$ induce the same posterior distribution given initial quality, then the social values of generating information on firm 1's and firm 2's products are identical.

The socially optimal investment in information generation then solves

$$\max_{\sigma} E[S(\hat{q}_1, \hat{q}_2)|\sigma] - \begin{cases} 2k \text{ if } \sigma = (\sigma_1, \sigma_2), \\ k \text{ if } \sigma = (\emptyset, \sigma_i) \text{ for } i \in \{1, 2\}, \\ 0 \text{ if } \;\sigma = (\emptyset, \emptyset). \end{cases}$$

Whether it is optimal to learn about only the quality leader, only the quality follower, or both will depend on the two expected qualities, on the two signals, and on the cost parameter $k$.

**Private value of information generation.** Given the outcome of the pricing game in Proposition 1 the firms' profits are

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = \begin{cases} \underline{\theta}|\hat{q}_i - \hat{q}_{-i}| & \text{if } \hat{q}_i > \hat{q}_{-i} \\ 0 & \text{otherwise,} \end{cases}$$

which increase in the distance between quality levels, strictly so for the quality leader. Computing the change in a firm's expected payoffs due to a change in the signal configuration yields the following statement (details are in the appendix).

**Proposition 2.** *Suppose there is a monopoly (i.e., $1 \leq \underline{\theta} f(\underline{\theta})$). Then for any two signal configurations $\sigma'$ and $\sigma''$ a firm $i$'s gain in payoffs from moving*

[13] To see this, suppose that two signals are drawn sequentially starting with $\sigma_1$. The derivation above implies that given $\hat{\sigma}_1$ drawing $\sigma_2$ is always weakly beneficial, strictly so if the ranking given $\hat{\sigma}_1$ changes for some realizations of $\sigma_2$. Hence, in expectation (i.e., before drawing $\sigma_1$) information generation by both firms generates weakly greater social welfare (excluding the signal cost) than by only one firm.

*from $\sigma'$ to $\sigma''$ is given by*

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] = \underline{\theta}\left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\right), \qquad (4)$$

*and is proportional to the gain in social welfare with a factor $\underline{\theta}/E[\theta] < 1$.*

Note that by (3) the benefit will be non-negative, if $\sigma''$ contains strictly more signals than under $\sigma'$.

Proposition 2 contains two important insights, summarised in two corollaries below. The first is that private returns to information generation are lower than the social returns, for any increase in the number of signals drawn and given any signal configuration. The reason is that the private returns are determined by the expected increase in the valuation of the least quality-sensitive consumer. Social benefits are instead determined by the expected increase in the average consumer's valuation. Note also that if the private benefit of information generation is lower than the social benefit, then information generation increases consumer surplus, because social welfare is the sum of profits and consumer surplus.

**Corollary 1.** *Suppose $1 \leq \underline{\theta} f(\underline{\theta})$, i.e. there is a monopoly. Then each firm's private benefit from generating information is strictly lower than its social benefit. Information generation strictly increases consumer surplus.*

The second insight from Proposition 2 is that any firm's benefit from information generation is independent of whether the firm is quality leader or follower. That means private returns from drawing a signal are *the same for all firms.* That is, the quality follower's benefit from drawing a signal on the leader's quality is the same as the leader's, and both are positive. Hence, there is a positive externality in information generation across firms.

**Corollary 2.** *There is a positive externality in information generation: Firm $i$'s private benefit from drawing a signal $\sigma_i$ is the same as the one of firm $j \neq i$.*

That is, the quality follower has a positive valuation for generating an informative public signal on the quality leader's product, despite the fact that the expected outcome is to confirm the leader's quality advantage. This is again because of the asymmetry of the firms' payoffs in expected quality (the quality follower already makes zero profits, but could corner the market with a positive probability). The fact that both firms have the exact same benefit from generating a signal on a product is driven by piece-wise linearity of profits in qualities.

**Subgame Perfect Nash Equilibrium** A firm's optimal choice of whether to acquire a signal, and thus the outcome of the two stage game, will depend on whether the expected increase in profits computed above outweighs the investment cost $k$. That is, the subgame perfect Nash equilibrium of the information generation cum pricing game will depend on the quality distribution $q_1$ and $q_2$, the signal technology, and the investment cost $k$. We list the pure strategy equilibria below:

- If $k > \underline{\theta}\left(\Delta(\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$ and $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ then there is an equilibrium in which only firm $i \in \{1, 2\}$ generates information. There are two equilibria (each corresponding to a different firm generating information), if these inequalities hold for both $i = 1$ and $i = 2$.

- if $k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$ and $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is a unique equilibrium in which both firms generate information.

- if $k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$, but $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \leq k$ for both $i = 1, 2$, then there are multiple equilibria: one in which no firm generates information, and one in which both firms generate information.

- Otherwise, there is no information generation in equilibrium.

Therefore multiple equilibria are possible in two cases. First, if it is beneficial for each firm to generate information individually but not jointly, then there could be two equilibria depending on which firm generates information.

Second, if the expected quality gain $\Delta(\sigma, q_1, q_2)$ increases in the number of signals drawn, then information generation choices will be strategic complements. An interesting case occurs when individual information generation is not profitable, but joint information generation is. This will be the case, for instance, when drawing one signal cannot perturb the posterior quality distribution sufficiently to reverse the quality ranking, but drawing two signals can (so that $\Delta((\emptyset, \sigma_1), q_1, q_2) = \Delta((\emptyset, \sigma_2), q_1, q_2) = 0$, but $\Delta((\sigma_1, \sigma_2), q_1, q_2) > 0$). If the cost $k$ is sufficiently small, this case would produce a familiar coordination failure: an equilibrium without information generation, Pareto dominated by another equilibrium, in which both firms acquire information. In what follows, if there are multiple equilibria that can be Pareto ranked, we will always focus on the Pareto-preferred one.

Which of the different cases will emerge depends not only on the signal structure, but also on the distance in ex-ante expected qualities $|q_1 - q_2|$. If, for given signals, this distance is sufficiently small, information generation by at least one firm is more likely in equilibrium. For intermediate $|q_1 - q_2|$, there may be multiple equilibria, in which either both firms generate information or neither does; the former equilibrium Pareto dominates the latter. If the distance is sufficiently large, neither firm will acquire any signal.

The characterization of the Nash Equilibrium and Proposition 2 imply the following proposition, derived in the appendix, stating that the equilibrium level of information generation is inefficiently low.

**Proposition 3.** *Suppose* $1 \leq \underline{\theta} f(\underline{\theta})$, *i.e. there is a monopoly. Then the number of signals drawn in equilibrium is lower than socially optimal, strictly so for some values of* $k$. *More precisely:*

*(i) Underinvestment: there are* $0 < \hat{k}_0 < \hat{k}_1$ *and* $0 < \hat{k}_2 < \hat{k}_3$ *such that if* $\hat{k}_0 < k < \hat{k}_1$ *or* $\hat{k}_2 < k < \hat{k}_3$ *the number of signal drawn in equilibrium is strictly lower than socially optimal. Thresholds* $\hat{k}_0$ *and* $\hat{k}_2$ *are increasing functions of* $\underline{\theta}$, $\hat{k}_1$ *and* $\hat{k}_3$ *increasing functions of* $E[\theta]$.

*(ii) Coordination Failure: There are* $0 < \hat{k}_4 < \hat{k}_5$ *such that if* $\hat{k}_4 < k < \hat{k}_5$ *there are multiple Nash equilibria, one with each firm drawing a signal,*

*but not the other. One of these equilibria is inefficient, because the firm with the less informative signal generates information.*

Hence, firms are more likely to draw fewer signals in equilibrium than efficient if the difference between private benefit (as measured by $\underline{\theta}$) and social benefit (as measured by $E(\theta)$) of information generation is large. We formalize this intuition in the following corollary.

**Corollary 3.** *Consider two distributions of the taste parameter $F(\theta)$ and $F'(\theta)$ with equal mean but different lower bounds $\underline{\theta} > \underline{\theta}'$. The set of $k$ for which there is an inefficient equilibrium under $F'(\theta)$ contains the set of $k$ for which there is an inefficient equilibrium under $F(\theta)$.*

For example, if $F'(\theta)$ is a mean preserving spread of $F(\theta)$ then the above corollary applies.

**Coordination in information generation.** When the firms can coordinate their information generation, they will choose a signal configuration that maximizes joint profits. By the previous derivations, the joint benefit of information generation by firm $i$ is:

$$2\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2),$$

and the joint benefit of information generation by both firms is

$$2\underline{\theta}\Delta((\sigma_1, \sigma_2), q_1, q_2).$$

Because information generation by one firm imposes a positive externality on the other firm, the firms' joint benefit from information generation is larger than each firm's individual benefit. Hence, firms may coordinate their choice of information generation via, for example, an industry body.[14] Therefore there are cost parameters $k$, for which no firm generates information in any equilibrium described above, but information generation by one or both firms

[14] As already noted, industry bodies are often responsible for creating and maintaining public information generation mechanisms such as classifications and competitions.

will occur when firms jointly decide on information generation and share its cost. Similarly, for some level of $k$ only one firm generates information in equilibrium, and both firms generate information when they can coordinate.

This increase in information generation will be socially beneficial, if the joint benefit of information generation is less than its social benefit, that is, when $2\underline{\theta} \leq E[\theta]$. In this case there is underinvestment in information generation, both with and without coordination, but the underinvestment will be less severe when firms can coordinate.[15] These observations yield the following corollary to our results above.

**Corollary 4.** *Firms that coordinate their choice of information generation generate more information than is generated by individual choices in the Nash equilibrium. If* $2\underline{\theta} \leq E[\theta]$*, coordination increases social welfare and consumer surplus.*

## 4.2 Case 2: Duopoly $(1 > \underline{\theta} f(\underline{\theta}))$

Turn now to the case of a duopoly, i.e., a consumer type distribution that satisfies $1 > \underline{\theta} f(\underline{\theta})$ and thus implies both firms will sell to some consumers, jointly covering the market.

**Social benefit of information generation.** In contrast to above case, now the pricing game has an inefficient outcome. In the pricing equilibrium, the social welfare is given by:

$$\begin{aligned} S(\hat{q}_1, \hat{q}_2) &= \hat{q}_L \int_{X^*}^{\overline{\theta}} \theta dF(\theta) + \hat{q}_F \int_{\underline{\theta}}^{X^*} \theta dF(\theta) \\ &= \max\{\hat{q}_1, \hat{q}_2\}(1 - F(X^*))E[\theta|\theta > X^*] + \min\{\hat{q}_1, \hat{q}_2\}F(X^*)E[\theta|\theta < X^*] \\ &= \max\{\hat{q}_1, \hat{q}_2\}E[\theta] - |\hat{q}_1 - \hat{q}_2|F(X^*)E[\theta|\theta < X^*]. \qquad (5) \end{aligned}$$

The first part of this expression is the first-best social welfare, resulting from all consumers consuming the high quality good. The second part is the

[15] When $2\underline{\theta} > E[\theta]$, firms coordination may lead to overinvestment in information generation, and may reduce social welfare. We discuss more in details this possibility in the next subsection.

deadweight loss generated by positive demand for the lower quality good.

Information generation therefore has two competing effects on social welfare. Similar to the monopoly case above, drawing a signal increases the expected highest quality in the market, which increases social welfare. In contrast to the monopoly case above, information generation also increases the expected quality distance, which in turn increases the deadweight loss, too. The strength of this second effect depends on the market share of the quality follower (i.e., on $F(X^*)$) and on the average taste for quality of the consumers purchasing the low-quality good (i.e., $E[\theta|\theta < X^*]$). Both quantities strictly increase in $X^*$, which is therefore a sufficient statistics for the deadweight loss in the pricing equilibrium.

The expected change in social welfare due to information generation is given by:

$$E[S(\hat{q}_1, \hat{q}_2)|\sigma] - S(q_1, q_2) = (E[\theta] - 2F(X^*)E[\theta|\theta < X^*])\Delta(\sigma, q_1, q_2) \geq 0. \quad (6)$$

The last inequality follows since by the definition of $X^*$ the majority of consumers purchase the high quality good ($F(X^*) < 1/2$), which implies:[16]

$$E[\theta] - 2F(X^*)E[\theta|\theta < X^*] > E[\theta] - E[\theta|\theta < X^*] > 0,$$

and the positive effect of information generation dominates. Therefore information generation always increases social welfare, strictly so when $\Delta(\sigma, q_1, q_2)$ is strictly positive.

**Private benefit of information generation.** To compare private and social returns, recall the firms' profits:

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = |\hat{q}_i - \hat{q}_{-i}| \begin{cases} \frac{(1-F(X^*))^2}{f(X^*)} & \text{if } \hat{q}_i \geq \hat{q}_{-i} \\ \frac{F(X^*)^2}{f(X^*)} & \text{if } \hat{q}_i \leq \hat{q}_{-i}. \end{cases} \quad (7)$$

[16]Recall also that in any Nash equilibrium of the pricing game the quality leader faces strictly positive demand and therefore $X^* < \overline{\theta}$.

Both linearly increase in the distance between quality levels, but because $F(X^*) < \frac{1}{2}$ this increase is steeper for the quality leader. Computing the change in a firm's expected payoffs due to a change in the signal configuration yields the following statement (the proof is analogous to the one of Proposition 2 and therefore omitted).

**Proposition 4.** *Suppose there is a duopoly (i.e., $1 > \underline{\theta} f(\underline{\theta})$). Then for any two signal configurations $\sigma'$ and $\sigma''$ a firm i's gain in payoffs from moving from $\sigma'$ to $\sigma''$ is given by*

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] = \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta(\sigma''; q_1, q_2) - \Delta(\sigma'; q_1, q_2)), \tag{8}$$

*and is proportional to the gain in social welfare with a factor*

$$\frac{\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)}{(E[\theta] - 2F(X^*)E[\theta|\theta < X^*])}.$$

As in the case above the benefit will be non-negative, if $\sigma''$ contains strictly more signals than $\sigma'$. Indeed, Proposition 4 is the duopoly version of Proposition 2 and differs mainly in that the factor of proportionality between private and social returns of information generation may be below or above unity, depending on $X^*$, i.e. firms' market shares. The following corollary states the implied condition for overinvestment.

**Corollary 5.** *Each firm's private benefit from generating information is strictly higher than its social benefit if, and only if:*

$$\frac{F(X^*)^2}{f(X^*)} + F(X^*)E[\theta|\theta < X^*] > \frac{1}{2}(E[\theta] - X^*). \tag{9}$$

Note that condition (9) reduces to $\underline{\theta} > E[\theta]$, when there is a monopoly (i.e., $1 \leq \underline{\theta} f(\underline{\theta})$ and thus $X^* = \underline{\theta}$), so that no qualifier is needed. Indeed, the LHS of (9) strictly increases in the threshold consumer $X^*$, while the RHS of (9) is strictly decreasing in $X^*$. Hence, if $X^*$ is sufficiently high (for example, close to the mean of the distribution of the taste parameter), then a firm's private benefit from acquiring a signal is higher than its social benefit. The reverse holds if $X^*$ is sufficiently low (for example, close to the marginal consumer $\underline{\theta}$). Whether firms have an excessive incentive to invest in information generation thus depends on the characteristics of the type distribution.

For intuition, recall that the deadweight loss in the pricing equilibrium is $|\hat{q}_1 - \hat{q}_2|F(X^*)E[\theta|\theta < X^*]$, and therefore strictly increases in $X^*$. Furthermore, $X^*$ determines the sensitivity of the deadweight loss to the arrival of new information that increases the expected distance between qualities. Therefore the higher is $X^*$, the further from efficiency is the pricing equilibrium for a given quality distribution, and the more likely it is that the private value of information generation exceeds its social value. This reflects the fact that a higher deadweight loss is associated to a higher market share of the lower quality good. This in turn means that less consumers will benefit from a reversal of the quality ranking due to new information.

For illustration we turn again to the example of a uniform distribution. The private benefit of information generation exceeds the social benefit if $\overline{\theta} > \underline{\theta}(2+\sqrt{3})$ and falls short of it if $\overline{\theta} < \underline{\theta}(2+\sqrt{3})$. Again, fixing either $\overline{\theta}$ or $\underline{\theta}$, as the range of the support $\overline{\theta} - \underline{\theta}$ (and thus the variance of the distribution) increases, so does the deadweight loss in the pricing equilibrium. As a consequence the social value of information generation decreases, eventually dropping below its private value. On the other hand, if both $\overline{\theta}$ and $\underline{\theta}$ increase by the same amount, holding constant the range of the support $\overline{\theta} - \underline{\theta}$, the deadweight loss decreases. As a consequence, the social value of information generation increases, eventually becoming greater than its private value.

The subgame perfect equilibria of the information generation and pricing game can be derived analogously to the analysis above. Using the characterization of the equilibrium (see appendix) and Proposition 4 yields the following analogue of Proposition 3.

**Proposition 5.** *Suppose $1 > \underline{\theta} f(\underline{\theta})$, i.e. there is a duopoly and that Condition (9) holds. Then the number of signals drawn in a Pareto-dominant Nash equilibrium is higher than socially optimal, strictly so for some values of $k$. More precisely:*

*(i) Overinvestment: there are* $0 < \hat{k}_0 < \hat{k}_1$ *and* $0 < \hat{k}_2 < \hat{k}_3$ *such that if* $\hat{k}_0 < k < \hat{k}_1$ *or* $\hat{k}_2 < k < \hat{k}_3$ *the number of signals drawn in a Pareto-dominant Nash equilibrium is strictly higher than socially optimal. Thresholds* $\hat{k}_0$ *and* $\hat{k}_2$ *are increasing functions of* $E[\theta]-2F(X^*)E[\theta|\theta < X^*]$*,* $\hat{k}_1$ *and* $\hat{k}_3$ *are increasing functions of* $X^* + 2\frac{F(X^*)^2}{f(X^*)}$*.*

*(ii) Coordination Failure: There are* $0 < \hat{k}_4 < \hat{k}_5$ *such that if* $\hat{k}_4 < k < \hat{k}_5$ *there are multiple Nash equilibria, one with each firm drawing a signal, but not the other. One of these equilibria is inefficient, because the firm with the less informative signal generates information.*

That is, under Condition (9) the market equilibrium level of information generation in a duopoly will never fall short of the social optimum and there may be over-investment in information generation. If instead Condition (9) fails, the private benefit from information generation falls short of the social benefit, and the result in Proposition 3 applies: the number of signals drawn in equilibrium is never higher than socially optimal and there may be under-investment in information generation. In any case a coordination failure may arise, where the "wrong" firm invests in information generation.

Note that, again, if Condition (9) holds in equilibrium firms are more likely to draw more signals than efficient when the difference between private benefit (as measured by $\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)$) and social benefit (as measured by $(E[\theta] - 2F(X^*)E[\theta|\theta < X^*])$) of information generation is large. By log concavity, for a given $E[\theta]$ this difference is strictly increasing in $X^*$, which implies the following corollary.

**Corollary 6.** *Consider two taste distributions,* $F(\theta)$ *and* $F'(\theta)$*, with equal mean but different* $X^{*\prime} > X^*$*. If Condition (9) holds at both* $F(\theta)$ *and* $F'(\theta)$*, then the set of* $k$ *for which there is an inefficient equilibrium under* $F'(\theta)$ *contains the set of* $k$ *for which there is an inefficient equilibrium under* $F(\theta)$*. If Condition (9) is violated at both* $F(\theta)$ *and* $F'(\theta)$*, then the set of* $k$ *for which there is an inefficient equilibrium under* $F'(\theta)$ *is contained in the set of* $k$ *for which there is an inefficient equilibrium under* $F(\theta)$*.*

Recall that the deadweight loss in the pricing equilibrium is also strictly increasing in $X^*$. The above corollary implies that, starting from no deadweight loss (so that (9) is violated), as the deadweight loss increases the set of $k$ for which there is under investment in information generation progressively shrink to zero. As the deadweight loss increases further, condition (9) holds and the set of $k$ for which there over investment in information generation expands. The relation between inefficiencies in the pricing stage and inefficiencies in the information-generation stage is therefore non-monotonic.

**Coordination in information generation.** As in the case of a monopoly, information generation by one firm imposes a positive externality on the other firm, because of (8). Hence, the firms' joint benefit of information generation exceeds each firm's private benefit. Allowing firms to coordinate in information generation can thus only increase the number of signals drawn, which will increase firms' joint profits. If condition (9) holds, the number of signals drawn in a Nash equilibrium is already higher than socially optimal, and thus coordination decreases social surplus. This observation implies the following corollary.

**Corollary 7.** *If (9) holds, then coordination in information generation decreases aggregate welfare and consumer surplus.*

If instead (9) does not hold, then similarly to the monopoly case examined above coordination in information generation may increase welfare .

# 5 Extension: endogenous quality

One possibility that we have so far ignored is that firms may intentionally degrade their products to increase the distance between their qualities. For example, Tirole (1988) shows that, if quality degradation is costless, the quality follower will always degrade its quality as much as possible in order to achieve maximum distance from the quality leader. Quality degradation is observed in some instances,[17] but is far from ubiquitous. In this section we argue that information generation may act as a strategic substitute to

[17]For example, several producers of electronic devices are known to intentionally reduce the performance and functionality of their products; e.g. the case of IBM printers.

quality degradation, and therefore explain why quality degradation is rarely observed. This also implies that, when quality is endogenous, information generation has an additional social benefit because it may prevent harmful quality degradation.

Denote by $q_i^0 \in [\underline{s}, \overline{s}]$ firm $i$'s initial quality with the convention that $q_1^0 > q_2^0$. Before the market opens, both firms simultaneously can decrease their expected quality at zero cost to any $q_i \in [\underline{s}, q_i^0]$.[18] Quality degradation is publicly observable. Recall that firms' profits increase in the distance between their expected quality levels. Hence, absent information generation, in the pure strategy Nash equilibrium of a quality degradation game the quality leader will maintain the initial quality $q_1 = q_1^0$, but the follower will degrade as much as possible to $q_2 = \underline{s}$.[19]

Turn now to a quality degradation and information generation game: after deciding whether to degrade its quality each firm can acquire information at a cost $k$. Introducing information generation may affect the choice quality degradation, because information generation provides an alternative means to increase the quality distance between firms. However, in contrast to degradation, information generation allows for upward revisions of the expected quality as well as downward revisions, increasing the expected highest quality and thus aggregate surplus.

Given that at least one firm generates information, the quality follower's profit can be written as:

$$E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma] = \left( X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)} \right) \Delta(\sigma, q_1, q_2) + \pi_2(q_1, q_2). \qquad (10)$$

[18] More precisely, firm $i$ can shift downward the quality distribution $F_i(s)$ to achieve any expected quality $q_i \in [\underline{s}, q_i^0]$. As discussed above the "consumption utility" generated by consuming a product $s_i$ is unknown, but the product's technical specifications are publicly known and determine the expectation of $s_i$. With this interpretation in mind, quality degradation can be achieved by designing a product with worse technical specifications.

[19] The pure-strategy Nash equilibrium in which the quality follower degrades always exists. A second pure-strategy Nash equilibrium, in which the quality leader fully degrades its quality, but the quality follower does not, exists for some $q_1^0, q_2^0$. In case both equilibria exist, they can be ranked in terms of efficiency, because the welfare loss is smaller when the quality follower degrades than when the quality leader degrades. For ease of exposition, we only discuss the Nash equilibrium in which the quality follower degrades.

Note that $\pi_2(q_1, q_2)$ decreases in $q_2$, but $\Delta(\sigma, q_1, q_2)$ increases in $q_2$, and strictly so if $\Delta(\sigma, q_1, q_2) > 0$. Hence, if $\Delta(\sigma, q_1, q_2) = 0$ and information generation has no value, the above result carries over and the quality follower is better off by degrading as much as possible to maximize the distance to the quality leader. When $\Delta(\sigma, q_1, q_2) > 0$, however, it is possible that $E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma]$ increases in $q_2$, and hence that there is no incentive to degrade quality. That is, quality degradation and information generation can be alternative ways to achieve vertical differentiation.

To provide a sufficient condition for this case to occur we note that

$$\pi_2(q_1, q_2) = \frac{F(X^*)^2}{f(X^*)}(q_1^0 - q_2),$$

is arbitrarily close to zero whenever $X^*$ is close to $\underline{\theta}$, because in this case the demand for the good sold by the quality follower is arbitrarily small. It is also arbitrarily close to zero if $q_1^0$ is close to $\underline{s}$, because the maximum distance that can be achieved between quality leader and follower is also arbitrarily small. In either of these cases, we have that

$$E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma] \approx \left( X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)} \right) \Delta(\sigma, q_1, q_2),$$

so that the quality follower's profit is strictly positive and strictly increases in $q_2$, if $\Delta(\sigma, q_1^0, q_2^0)$ is strictly positive, that is, if $q_1^0$ and $q_2^0$ are sufficiently close or if $\sigma$ is sufficiently informative (in the sense of dispersion in the posterior expected qualities). This observation implies the following lemma.

**Lemma 3.** *For any $X^*$, there are $q_1^0, q_2^0$ such that the quality follower will fully degrade quality if no information is generated, but neither firm will degrade quality if information is generated by at least one firm.*

The lemma states that information generation can prevent harmful quality degradation. Indeed, a sufficiently low cost $k$ will guarantee some information generation in equilibrium, which implies the following corollary.

**Corollary 8.** *There are $q_1^0, q_2^0$ and $k$ such that information generation and no quality degradation constitute a subgame perfect Nash equilibrium of the* *quality degradation and information generation game.*

Note that, if the case described in Lemma 3 and the following corollary do not apply, the quality follower may partially degrade, even though information is generated in equilibrium. For example, if the signals $\sigma_1$ and $\sigma_2$ are discrete, then the probability of a quality ranking reversal may be discontinuous in the amount of quality degradation by the quality follower. That is, this probability may be very small when the quality follower degrades by a small amount, but jump discontinuously if the amount of quality degradation passes a given threshold. If, at the same time, the benefit of increasing vertical distance is large, the quality follower may prefer to partially degrade quality. We will not consider this possibility here.

Turning to social welfare, assume the case described in Lemma 3, i.e. information generation prevents quality degradation. The social benefit is:

$$\begin{aligned} E[S(\hat{q}_1,\hat{q}_2)] - S(q_1^0,\underline{s}) &= E[S(\hat{q}_1,\hat{q}_2)] - S(q_1^0,q_2^0) + S(q_1^0,q_2^0) - S(q_1^0,\underline{s}) \\ &= (E[\theta] - 2F(X^*)E[\theta|\theta < X^*])\,\Delta((\emptyset,\sigma_i),q_1^0,q_2^0) + (q_2^0 - \underline{s})F(X^*)E[\theta|\theta < X^*]. \end{aligned} \tag{11}$$

The first term of this expression is the benefit of information generation given initial quality levels as in (6). The second term stems from (5) and is the benefit from preventing quality degradation. It increases in $q_2^0 - \underline{s}$ (the amount of quality degradation prevented by generating information), in the market share of the quality follower and in the average valuation of these consumers. The following proposition summarizes these observations.

**Proposition 6.** *There are $q_1^0, q_2^0$ and $k$ such that the social benefit of information generation with endogenous quality is strictly greater than the one with exogenous qualities.*

*Proof.* Immediate from Lemma 3, (11) and the following discussion. □

Since information generation can have an additional social benefit when quality is endogenous rather than exogenous, Proposition 5 may no longer apply. That is, when quality is exogenous, potentially the efficient number of signals is zero, but in equilibrium at least one firm generates information. Under endogenous quality choice the fact that a firm is expected to generate information prevents quality degradation. If the social benefit of preventing quality degradation (given by the second part of 11) is larger than the net social cost of an additional signal (given by the second part of 11 minus $k$), then one firm generating information is the socially optimal outcome with endogenous quality levels. Similarly, with exogenous quality levels there are situations in which the efficient number of signals is zero, which is also the equilibrium outcome. With endogenous quality levels, however, the absence of information generation leads to quality degradation and may, therefore, be inefficient. The following corollary summarizes these observations.

**Corollary 9.** *Suppose the case described in Lemma 3 holds. There are cases in which there is over-investment in information generation with exogenous quality, but the efficient level of information generation when quality levels are endogenous. Similarly, there are cases in which there is the efficient level of information generation with exogenous quality, but under-investment in information generation when quality levels are endogenous.*

# 6 Conclusion

We consider a standard duopoly with vertically differentiated products, and study firms' incentives to generate information. Our main result is that firms will under- or overinvest in information generation, depending on the inefficiencies in the pricing equilibrium. When, for a given quality distribution, the deadweight loss in the pricing equilibrium is low, firms will under-provide information. Conversely, when the deadweight loss is large in equilibrium, firms will over-provide information. We also show that information generation has a positive externality on the other firm's profit and thus firms benefit from coordinating their information generation activities. Finally, we introduce the possibility of quality degradation and show that quality degradation and information generation are substitutes for increasing vertical product differentiation. Therefore the possibility of information generation may reduce harmful quality degradation.

This last result implies that there are situations in which information generation should be discouraged if quality levels are exogenous—possibly via a tax—but information generation should be encouraged if quality levels are endogenous—possibly via a subsidy. This insight carries over to a more concrete policy application, i.e. the extent to which cooperation and coordination of competing firms will result in overinvestment in information generation. Our analysis shows that there are situations in which coordination in information generation should be prevented if quality levels are exogenous, but should be allowed or even encouraged if quality levels are endogenous. This, however, implies that the optimal policy may be time inconsistent, because the policymaker may want to revise the policy after quality levels are set; this is an intriguing question for future research.

Our analysis assumed a covered market: in equilibrium all consumers purchase some product. Removing our Assumption 3 would potentially allow for equilibria in which some consumers do not purchase at all. If firms' quality levels are sufficiently close, however, their profits are close to zero and the market is covered. The logic laid out above continues to apply: information generation by firms is privately valuable, because it increases expected vertical distance and profits. From the social point of view, information generation may cause some consumers to stop consuming, which generates an additional source of inefficiency relative to the case of a covered market considered above. If the initial quality distance between firms is large, so that not all consumers purchase, our results may no longer apply: e.g. firms' benefits from information generation may become negative. A thorough analysis of this case is deferred to future work.

# Appendix

## Proof of Lemma 1

Note first that $X = Y = \overline{\theta}$ quickly leads to a contradiction: if both prices are so high that no consumers purchase, then each one of the firms will earn strictly positive payoff by deviating to a small, but positive price, which will attract a positive measure of consumers because $F(\theta)$ is continuous and $\overline{\theta} > 0$.

Suppose that $X = \overline{\theta}$, i.e. the quality leader faces zero demand. Then, by the argument above, $Y < \overline{\theta}$ and the quality follower faces positive demand. This cannot be an equilibrium because the quality leader can set its price equal that of the quality follower, generate positive demand and earn positive profits.

Finally, suppose that $\underline{\theta} < X < \overline{\theta}$, i.e. the quality leader faces positive demand, but does not capture the entire market. Then $Y < X$. To see this suppose the contrary, i.e. $Y = X$. This cannot be an equilibrium because the quality follower will earn strictly positive payoff by setting a small, but positive price, which will attract a positive measure of consumers because $F(\theta)$ is continuous and $\overline{\theta} > 0$.

## Proof of Lemma 2

The best responses are:

$$\begin{aligned} p_L(p_F) &= \text{argmax}_{p_L} \{\pi_L(p_L, p_F)\} \\ p_F(p_L) &= \text{argmax}_{p_F} \{\pi_F(p_L, p_F)\}. \end{aligned}$$

Consider first the quality leader's problem. For given $p_F$ the leader can out-price the follower and set $p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$, so that $X = \underline{\theta}$. In this case the leader serves the entire market and its profit equals $p_L$. Hence, conditional on $X = \underline{\theta}$ the quality leader maximizes profits by setting $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$. If instead $p_L > \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$, the leader serves only a fraction of the total market, and $X > Y \geq \underline{\theta}$. Using the definition of $X$ the quality leader's problem becomes:

$$\max_{p_L \geq \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F} \left\{ p_L \left( 1 - F\left( \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \right) \right) \right\}.$$

The first derivative of the objective function is

$$1 - F(X) - \frac{p_L f(X)}{\hat{q}_L - \hat{q}_F}$$

or

$$\left( \frac{1 - F(X)}{f(X)} - \frac{p_L}{\hat{q}_L - \hat{q}_F} \right) f(X),$$

and equals zero at

$$p_L = \frac{1 - F(X)}{f(X)} (\hat{q}_L - \hat{q}_F), \tag{12}$$

which is unique due to log-concavity. Log-concavity also implies that the second derivative of the objective function is negative at $p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$. The quality leader's objective function therefore strictly increases for $p_L < \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$ and strictly decreases for $p_L > \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$.

We now turn to the quality follower $F$'s best response. Suppose the quality leader chooses a price $p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F)$. In this case for any $p_F$ the quality leader covers the entire market and the quality follower's profits are zero for any $p_L$. Then the quality follower's best response is

$$p_F(p_L) = [0, \infty) \text{ if } p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F).$$

If instead $p_L > \underline{\theta}(\hat{q}_L - \hat{q}_F)$, there are $p_L > 0$, such that both the demand faced by and the profit of the quality follower are positive. In this case the follower's profit function has a kink at price $p_F = \underline{\theta}\hat{q}_F$. Although the profit function is not differentiable at $p_F = \underline{\theta}\hat{q}_F$, it is well-behaved above and below, which allows us to characterize the follower's best response.

If $p_F \leq \underline{\theta}\hat{q}_F$, then all consumers purchase one of the goods and $Y = \underline{\theta}$, so that a change in $p_F$ only affects $X$. Conditional on $Y = \underline{\theta}$, the follower's profit function is

$$\max_{p_F \leq \underline{\theta}\hat{q}_F} \{p_F F(X)\}.$$

The objective function's first derivative is

$$\left( \frac{F(X)}{f(X)} - \frac{p_F}{\hat{q}_L - \hat{q}_F} \right) f(X).$$

The first derivative equals zero at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which is unique by log-concavity. Again, conditional on $Y = \underline{\theta}$, the follower's profit function is strictly concave at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which in turns imply that profits conditional on $Y = \underline{\theta}$ are first increasing then decreasing in $p_F$, reaching a maximum at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$.

If instead $p_F > \underline{\theta}\hat{q}_F$ some consumers will not purchase, so that a change in $p_F$ will affect both $X$ and $Y$. Conditional on $\underline{\theta} \le Y < X$, the follower's profit function is now

$$\max_{p_F \ge \underline{\theta}\hat{q}_F} \left\{ p_F(F(X) - F(Y)) \right\}.$$

The objective function's first derivative is

$$F(X) - F(Y) - p_F \left( \frac{f(X)}{\hat{q}_L - \hat{q}_F} + \frac{f(Y)}{\hat{q}_F} \right).$$

Now Condition (A3) becomes useful: it implies that the above expression is always negative, which implies that the quality follower always sets a price so that $Y = \underline{\theta}$ and the market is covered.

To see why, note that 12 implies that $X < \frac{1-F(X)}{f(X)}$ so that $X \le \theta^*$. Hence, by the definition of $m$ (see Assumption 3) $f(X) < m$ and $f(Y) < m$. Recall that the first order condition for the case $Y > \underline{\theta}$ is

$$F(X) - F(Y) - p_F \left( \frac{f(X)}{\hat{q}_L - \hat{q}_F} + \frac{f(Y)}{\hat{q}_F} \right).$$

Because $F(X) - F(Y) \le 1$, $p_F > \underline{\theta}\hat{q}_F$ (whenever $Y > \underline{\theta}$), $\frac{\hat{q}_F}{\hat{q}_L} \ge \frac{\underline{s}}{\overline{s}}$, and $f(X), f(Y) \ge m$, the above expression is always smaller than

$$1 - m\underline{\theta}\left(1 - \frac{\underline{s}}{\overline{s}}\right)^{-1},$$

which is negative under (A3). Hence the first order condition for the case $Y > \underline{\theta}$ is always negative, and the quality follower is always better off by setting $p_F$ such that $Y = \underline{\theta}$.

## Proof of Proposition 1

As a preliminary step, note that (A3) is equivalent to

$$\frac{1}{m\underline{\theta}} - 1 \leq \left(\frac{\overline{s}}{\overline{s} - \underline{s}}\right) - 1 \Leftrightarrow \left(\frac{1}{m\underline{\theta}} - 1\right)\left(\frac{\overline{s}}{\underline{s}} - 1\right) \leq 1. \tag{13}$$

Recall the quality leader's best reply as derive above:

$$p_L(p_F) = \max\left\{\frac{1 - F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F\right\}.$$

By log-concavity $\frac{1-F(X)}{f(X)}$ is decreasing and therefore is maximal for $x = \underline{\theta}$. Hence, whenever

$$\left(\underline{\theta} - \frac{1}{f(\underline{\theta})}\right)(\hat{q}_L - \hat{q}_F) + p_F \geq 0$$

the quality leader's captures the entire market. If $p_F > 0$, this cannot be an equilibrium because the quality follower should lower its price and earn positive profits. If instead $p_F = 0$ and $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F)$ then no firm can make a profitable deviation, and these prices constitute a Nash equilibrium. If $\underline{\theta} f(\underline{\theta}) \geq 1$, therefore, in equilibrium the leader captures the entire market.

Suppose instead $1 > \underline{\theta} f(\underline{\theta})$ from now on. The observations made in the text above imply that in this case the quality leader's best reply to $p_F = 0$ is $p_L = \frac{1-F(p_L/(\hat{q}_L - \hat{q}_F))}{f(p_L/(\hat{q}_L - \hat{q}_F))}(\hat{q}_L - \hat{q}_F)$. Hence, by Lemma 1 the Nash equilibrium will necessarily have $X > \underline{\theta}$ (implying $f(X) > 0$) and $p_F = \min\left\{\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}\hat{q}_F\right\} > 0$. Therefore there are two possible cases, depending on whether the quality follower's best response is a corner solution ($p_F = \underline{\theta}\hat{q}_F$) or an interior solution ($p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$).

Suppose first that the quality follower's best response has an interior solution:

$$\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F) \leq \underline{\theta}\hat{q}_F, \tag{14}$$

so that $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$. In this case, by definition of $X$, the equilibrium cutoff $X$ solves

$$X = \frac{1 - 2F(X)}{f(X)}. \tag{15}$$

This equation has a unique solution because, by log concavity, its RHS is decreasing in $X$ and we have assumed $1 > \underline{\theta} f(\underline{\theta})$. This constitutes a Nash equilibrium if indeed the solution $X$ of equation (15) satisfies condition (14). Condition (14) can be rewritten as

$$\frac{F(X)}{f(X)\underline{\theta}} \left( \frac{\hat{q}_L}{\hat{q}_F} - 1 \right) \leq 1.$$

Note that $\frac{\hat{q}_L}{\hat{q}_F}$ is at most $\frac{\overline{s}}{\underline{s}}$, and that by (15) $\frac{F(X)}{f(X)} = \frac{1}{2}\left(\frac{1}{f(X)} - X\right)$, which is at most $\frac{1}{2}\left(\frac{1}{m} - \underline{\theta}\right)$.[20] Therefore,

$$\frac{F(X)}{f(X)\underline{\theta}} \left( \frac{\hat{q}_L}{\hat{q}_F} - 1 \right) < \frac{1}{2} \left( \frac{1}{m\underline{\theta}} - 1 \right) \left( \frac{\overline{s}}{\underline{s}} - 1 \right) < 1,$$

where the last inequality follows by (13). Hence, (14) holds and thus $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$ and $p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, with $X$ defined implicitly by (15) is a Nash equilibrium.

To conclude the proof, we show that there is no equilibrium in which $1 > \underline{\theta} f(\underline{\theta})$, and hence the quality leader's best response has an interior solution:

$$p_L = \frac{1 - F(X)}{f(X)}(\hat{q}_L - \hat{q}_F),$$

but at the same time (14) is violated, and hence the quality follower's best response has a corner solution:

$$p_F = \underline{\theta}\hat{q}_F.$$

If such equilibrium exists, then by definition

$$X = \frac{1 - F(X)}{f(X)} - \underline{\theta} \left( \frac{\hat{q}_L}{\hat{q}_F} - 1 \right)^{-1} \tag{16}$$

This is consistent with a Nash equilibrium if indeed for this $X$ (14) is violated.

Note that by (16) $\frac{F(X)}{f(X)}$ is smaller than $\frac{1}{f(X)} - X$ which, in turn, is smaller

[20]Here we make use again of a fact established in the proof of Lemma 2 (see its last paragraph): that $f(X) \geq m$.

than $\frac{1}{m} - \underline{\theta}$. Also, $\frac{\hat{q}_L - \hat{q}_F}{\hat{q}_F}$ must be smaller than $\left(\frac{\overline{s}}{\underline{s}} - 1\right)$. It follows that

$$\frac{F(X)}{f(X)\underline{\theta}}\left(\frac{\hat{q}_L - \hat{q}_F}{\hat{q}_F}\right) \leq \left(\frac{1}{m\underline{\theta}} - 1\right)\left(\frac{\overline{s}}{\underline{s}} - 1\right) \leq 1,$$

where the last inequality follows by (13). Hence, (14) must hold and there cannot be a Nash equilibrium with $p_F = \underline{\theta}\hat{q}_F$.

## Proof of Proposition 2

Suppose $\sigma' = (\emptyset, \emptyset)$, that is, we are compare signal configuration $\sigma''$ to no information. Since firm 1 is the quality leader ex ante, if no information is generated then $\pi_1(q_1, q_2) = \underline{\theta}\,[q_1 - q_2]$ and $\pi_2(q_1, q_2) = 0$. Therefore firm $i$'s benefit of signal configuration $\sigma''$ relative to no information is:

$$E\big[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''\big] - \pi_i(q_i, q_{-i}) = E\big[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''\big] - \underline{\theta}\begin{cases} E\left[\hat{q}_1 - \hat{q}_2|\sigma''\right] & \text{if } i = 1 \\ 0 & \text{if } i = 2 \end{cases}$$

$$= \underline{\theta}\text{pr}\{\hat{q}_i \geq \hat{q}_{-i}|\sigma''\}E\big[\hat{q}_i - \hat{q}_{-i}|\hat{q}_i \geq \hat{q}_{-i}, \sigma''\big] -$$

$$\underline{\theta}\cdot\begin{cases} \text{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma''\}E[\hat{q}_1 - \hat{q}_2|\hat{q}_2 \geq \hat{q}_1, \sigma''] + \text{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma''\}E[\hat{q}_1 - \hat{q}_2|\hat{q}_1 \geq \hat{q}_2, \sigma''] & \text{if } i = 1 \\ 0 & \text{if } i = 2 \end{cases}$$

$$= \underline{\theta}\text{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma''\}E\big[\hat{q}_2 - \hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma''\big] = \underline{\theta}\Delta(\sigma'', q_1, q_2),$$

where, again, we used the fact that by the law of iterated expectations $\pi_1(q_1, q_2) = \underline{\theta}(q_1 - q_2) = \underline{\theta}E\left[\hat{q}_1 - \hat{q}_2|\sigma''\right]$.

The above derivation also implies that the benefit of going from no information to signal configuration $\sigma'$ is

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] - \pi_i(q_i, q_{-i}) = \underline{\theta}\Delta(\sigma', q_1, q_2).$$

We therefore have

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] =$$
$$(E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - \pi_i(q_i, q_{-i})) - (E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] - \pi_i(q_i, q_{-i})) = \underline{\theta}\left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\right).$$

## Proof of Proposition 3

We distinguish three cases:

1. It is socially optimal to generate no information, that is

$$2k > E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2) \text{ and } k > E[\theta]\Delta((\emptyset,\sigma_i),q_1,q_2) \quad i \in \{1,2\}.$$

By Proposition 2 each firm's best reply to the other firm not generating information is to not generate information either. Likewise, each firm $i$'s best reply to the other firm $-i$ generating information is not to generate information, if

$$k \geq \underline{\theta}\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_{-i}),q_1,q_2)\right),$$

which is always true, because

$$\underline{\theta}(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2))$$
$$\leq E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2) - \underline{\theta}\Delta((\emptyset,\sigma_i),q_1,q_2) \leq 2k - \underline{\theta}\Delta((\emptyset,\sigma_i),q_1,q_2) \leq k.$$

Hence, in the case when no information generation is socially optimal there is a unique Nash equilibrium, in which neither firm generates any information.

2. It is socially optimal for firm $i$ to generate information, but not firm $-i$, that is

$$E[\theta]\Delta((\emptyset,\sigma_{-i}),q_1,q_2) \leq E[\theta]\Delta((\emptyset,\sigma_i),q_1,q_2) \equiv \hat{k}_1 \text{ and}$$
$$\hat{k}_4 \equiv E[\theta](\Delta((\sigma_1,\sigma_2)q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)) < k < E[\theta]\Delta((\emptyset,\sigma_i),q_1,q_2). \tag{17}$$

The second inequality immediately implies

$$k > \underline{\theta}\left[\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right].$$

This means that if firm $i$ generates information, then firm $-i$'s best reply is to not generate information. Hence, there is no Nash equilibrium, in which both firms generate information.

Suppose that

$$k > \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \equiv \hat{k}_0. \tag{18}$$

Then neither firm finds it profitable to generate information if the other firm does not. Hence, in the unique Nash equilibrium there is no information generation.

If instead

$$\underline{\theta}\Delta((\emptyset, \sigma_{-i}), q_1, q_2) < k \leq \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2),$$

then there is a unique equilibrium in which firm $i$ generates information. Finally, if

$$k \leq \underline{\theta}\Delta((\emptyset, \sigma_{-i}), q_1, q_2) \equiv \hat{k}_5,$$

then there are multiple equilibria, in which each firm may generate information, while the other one does not. In one of these equilibria firm $-i$ generates information, but not firm $i$. This is inefficient, because by assumption $\Delta((\emptyset, \sigma_i), q_1, q_2) < \Delta((\emptyset, \sigma_{-i}), q_1, q_2)$, i.e. firm $i$'s signal generates more information (as measured by the dispersion of the posteriors) and higher social welfare than firm $-i$'s signal.

3. It is socially optimal for both firms to generate information, that is

$2k \leq E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2)$ and
$E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - 2k \geq E[\theta]\max\{\Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\sigma_1, \sigma_2), q_1, q_2)\} - k.$

That is, the net social benefit of drawing both signals is positive, and exceeds the net social benefit of drawing either individual signal. The above inequalities can be rewritten as

$$k \leq E[\theta]\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \right.$$
$$\left. \max\left\{\frac{1}{2}\Delta((\sigma_1, \sigma_2), q_1, q_2), \Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2)\right\}\right) \equiv \hat{k}_3.$$

A necessary condition for both firms to generate information in a Nash equilibrium (including the case of multiple equilibria) is

$$k < \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right),$$

for both firms $i = 1, 2$, or

$$k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \max\left\{\Delta((\emptyset, \sigma_1), q_1, q_2), \Delta((\emptyset, \sigma_2), q_1, q_2)\right\}\right) \equiv \hat{k}_2.$$

Therefore, if

$$k > \hat{k}_2 \text{ and } k \leq \hat{k}_3, \tag{19}$$

then the number of signals drawn in a Nash equilibrium is strictly less than in the social optimum. Otherwise, there will be a (possibly unique) Nash equilibrium that is efficient.

We therefore established that the number of signals drawn in equilibrium is always smaller than the socially optimal number of signals, strictly so if either both conditions (17) and (18) hold, or both conditions in (19) hold. Note also that, in both cases, the set of such $k$ for which fewer signals than optimal are drawn expands with $E[\theta] - \underline{\theta}$ and with the first difference of $\Delta(.)$.

We also established the possibility of a coordination failure: when the efficient number of signals is 1, either firm generating one signal may be a Nash equilibrium, and in particular only the firm with the less informative signal generating information may be an equilibrium, which is inefficient.

## Proof of Proposition 5

The pure strategy Nash equilibria of the information generation game for the case of a duopoly are similar to the ones derived for the case of a monopoly, modulo the different expression for the private benefit of information generation. We have:

- If $k > \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta(\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$ and $\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is an equilibrium in which only firm $i$ generates information.

- if $k \leq \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$ and $\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is a unique equilibrium in which both firms generate information.

- if $k \leq \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$, but $\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset, \sigma_i), q_1, q_2) \leq k$ for both $i = 1, 2$, then there are multiple equilibria: one in which no firm generates information, and one in which both firms generate information.

- Otherwise there is no information generation in equilibrium.

We follow the structure of the proof of Proposition 3 and consider different cases. For ease of notation let us define the social value of information generation as

$$S \cdot \Delta(\sigma, q_1, q_2) \equiv (E[\theta] - 2F(X^*)E[\theta|\theta < X^*])\,\Delta(\sigma, q_1, q_2),$$

and the private value of information generation as

$$P \cdot \Delta(\sigma, q_1, q_2) \equiv \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta(\sigma, q_1, q_2).$$

Condition (9) implies that $P > S$, so that the private benefit of information generation is higher than the social benefit. We distinguish three cases.

1. It is socially optimal to have no information generation, that is

$$k > S\Delta((\emptyset,\sigma_i),q_1,q_2) \text{ and } 2k > S\Delta((\sigma_1,\sigma_2),q_1,q_2).$$

At least one firm $i$ will invest if $k < P\Delta((\emptyset,\sigma_i),q_1,q_2)$, and thus the number of signals generated in equilibrium is higher than socially optimal if

$$\hat{k}_0 \equiv S\max\{\Delta((\emptyset,\sigma_i),q_1,q_2),\Delta((\sigma_1,\sigma_2),q_1,q_2)\} < k < P\Delta((\emptyset,\sigma_i),q_1,q_2) \equiv \hat{k}_1.$$

Otherwise, if the above condition does not hold, there may be an equilibrium, in which both firms invest, if

$$k \leq P\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right) \quad \forall i \in \{1,2\}.$$

In this case there are, however, multiple equilibria: one with both firms investing and one with neither firm investing.

2. It is socially optimal for firm $i$ to generate information but not firm $-i$, that is

$$S\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right) < k < S\Delta((\emptyset,\sigma_i),q_1,q_2) \text{ and } \Delta((\emptyset,\sigma_{-i}),q_1,q_2) < \Delta((\emptyset,\sigma_i),q_1,q_2).$$

Note that this case can only occur if $\Delta(.)$ has strictly decreasing differences in $\sigma$. Since $P > S$, at least one firm will invest in any Nash equilibrium, so the number of signals is at least the socially optimal one. For both firms to invest to be the unique Nash equilibrium it is necessary that

$$k < P\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right) \text{ and } k < P\Delta((\emptyset,\sigma_i),q_1,q_2),$$

Since $S < P$ the second condition holds. Hence, both firms will invest and there will be overinvestment if

$$\hat{k}_2 \equiv S\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right) < k$$
$$< P\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right) \equiv \hat{k}_3.$$

If the above condition is violated, but

$$\hat{k}_4 \equiv P\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right) < k < P\Delta((\emptyset,\sigma_{-i}),q_1,q_2) \equiv \hat{k}_5,$$

then in equilibrium only one firm invests. If firm $i$ invests then the equilibrium is efficient. If firm $-i$ invests, then the equilibrium is inefficient. In this last case, in equilibrium the information generated in equilibrium is *less* then the social optimum, because the firm with the least informative signal generates information in equilibrium.

3. It is socially optimal for both firms to generate information, that is

$2k < E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2)$ and
$E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2) - 2k > E[\theta]\max\{\Delta((\emptyset,\sigma_1),q_1,q_2),\Delta((\emptyset,\sigma_2),q_1,q_2)\} - k.$

A necessary and sufficient condition for a Nash equilibrium, in which both firms generate information, is:

$$k < P\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right),$$

for both firms $i = 1, 2$. Because $P > S$, there is always an equilibrium in which both firms generate information. Of course, there may also be another equilibrium, in which no firm generates information. But, as discussed in the text, when both equilibria are present the one in which both firms generate information Pareto dominates the other.

By restricting our attention to equilibria that are not Pareto dominated, we established that the number of signals drawn in equilibrium is always above the efficient one, strictly so in some cases. Also here, there is the possibility that the efficient number of signals is one, which is also the equilibrium one, but the "wrong" firm generates information in equilibrium.

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