

The Application of a Multi-reference Control Strategy to Noise Cancelling Headphones

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1 Active noise cancelling (ANC) headphones have seen significant commercial success
2 and a number of control strategies have been proposed, including feedforward, feed-
3 back and hybrid configurations, using both analogue and digital implementations.
4 Irrespective of the configuration or implementation approach, the strategies proposed
5 in the open-literature have focused on implementations where the control system for
6 each ear of the headphones operates independently. In this paper, a multi-reference
7 ANC strategy is proposed and investigated for noise cancelling headphones. As with
8 standard feedforward ANC headphones, the system utilises a single error microphone
9 and single reference microphone on each cup, however, in the proposed configuration
10 the left and right reference microphones are used to achieve control at both the
11 left and right ear cups. The performance of this controller design is compared to
12 a standard single reference feedforward controller implementation under a variety of
13 different sound field conditions. Although the proposed strategy requires an increased
14 computational demand, it is shown that there is a significant control advantage for
15 noise sources originating from the side of the user, whilst the performance for front
16 and rear sources is maintained.

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17 I. INTRODUCTION

18 Active noise control is an effective technology in application areas where it is not pos-
19 sible to achieve sufficient levels of noise control passively. This generally occurs where the
20 control of low frequency noise is important and where the size and weight restrictions on
21 passive treatments are limited. Although active noise control has been practically imple-
22 mented in applications including propellor induced noise in aircraft¹, engine and road noise
23 in the automotive sector²⁻⁴, ambient noise control in mobile phones⁵ and noise control in
24 the maritime environment⁶, the most commercially successful application has been in noise
25 cancelling headphones⁷⁻¹². As a result, a variety of control strategies for noise cancelling
26 headphones have been proposed and investigated in the open literature, and undoubtedly
27 many more have been developed, tested and realised without open publication.

28 Early investigations into ANC headphones were carried out in the 1950s and used simple
29 analogue feedback control strategies^{7,13}. Nevertheless, through careful tuning of the feed-
30 back gain, and the inclusion of phase lag compensation, these systems were able to achieve
31 significant levels of attenuation. For example, Meeker¹³ reported approximately 15 dB of at-
32 tenuation between 100 and 200 Hz. These early systems, however, were limited by the detail
33 of tunability achievable using simple analogue circuits and the heuristic compensator design
34 processes. Therefore, more recent analogue feedback ANC headphone systems have utilised
35 more advanced loop shaping methodologies and compensator realisations; for example, Bai
36 and Lee¹⁴ have utilised an H_∞ robust control design process to realise a feedback ANC

37 headphone system. The controller in this case is implemented using operational amplifiers
38 and attenuation of up to 15 dB between 200 and 800 Hz is reported.

39 To allow greater flexibility over the controller design and also to enable the controller
40 to adapt to changes in the acoustic environment, digital feedback controllers have been ex-
41 tensively investigated^{10,11,15,16}. For example, in¹¹ an Internal Model Control architecture
42 is employed and the control filter is adapted using the filtered-reference LMS (FxLMS)
43 algorithm¹⁷. The proposed adaptive feedback headphone system is evaluated using engine
44 noise samples dominated by tonal components and it is shown that the proposed method
45 achieves high levels of control of multiple tones. More recently, in¹⁶ the broadband per-
46 formance of a digital feedback controller is demonstrated and achieves comparable control
47 to the previous analogue designs. Although it might be expected that a digital controller
48 could outperform an analogue controller through the greater design flexibility, due to the
49 additional delays in the digital system the bandwidth becomes limited, as discussed by
50 Rafaely¹⁵. Digital ANC systems thus require careful design and selection of the full system
51 path including the converters, antialiasing and reconstruction filters and the sampling rates,
52 which inevitably brings a trade-off between computational demand and performance.

53 An alternative approach to the design of ANC headphones is the use of a feedforward
54 control strategy, in which a microphone external to the headphone ear cup is used to provide
55 a reference signal. Various implementations of feedforward ANC headphones have been
56 presented in the literature over an extended time period^{12,18-20}, but have generally used
57 some form of an FxLMS algorithm. The performance of these systems is essentially limited
58 by the coherence between the reference and error microphone signals and the time-advance

59 provided by the reference signal. As in digital feedback systems, this time-advance is strongly
60 influenced by the design of the digital system, as extensively investigated in¹², but is also
61 influenced by the passive characteristics of the headphones²¹. Nevertheless, well-designed
62 feedforward ANC headphones have been shown to be able to achieve attenuation between 5
63 and 25 dB between 200 and 2000 Hz¹² or broadband reductions of around 12 dB²⁰. However,
64 the performance of these single channel feedforward controllers has been shown to be strongly
65 dependent on the direction of arrival of the primary sound field compared to the orientation
66 of the reference and error microphones^{20,21}. This is because the time-advance provided by
67 the reference signal compared to the error signal depends on the direction of arrival; in
68 the extreme case, when the reference microphone is upstream of the error microphone the
69 time-advance is positive, whereas, when the reference microphone is downstream of the error
70 microphone the time-advance is negative and a non-causal controller would be required. To
71 overcome this limitation, Rafaely and Jones²¹ proposed a combined feedforward-feedback
72 ANC headphone system, in which a single channel feedforward system is complemented
73 by an analogue feedback controller, which performs largely independently of the primary
74 sound field. Although the hybrid control system proposed in²¹ required additional analogue
75 circuitry, it has also been demonstrated that the hybrid (feedforward-feedback) controller
76 can be implemented in a digital configuration²². However, there is the potential to reduce
77 the sensitivity of a feedforward controller to the direction of the incident sound field by using
78 multiple reference microphones, as suggested in²⁰.

79 Although the multichannel formulation of the FxLMS algorithm was presented in 1987²³,
80 the multi-reference stochastic version of this algorithm was not formalised and rigorously

81 analysed until 1997²⁴. However, the multi-reference algorithm was previously presented
82 and utilised in the context of automotive road noise control in 1994³. This first practical
83 demonstration of active road noise control used 6 reference signals in order to obtain both
84 sufficient multiple coherence and time advance between the reference and error signals and
85 reported a maximum attenuation of 7 dB³. Subsequently, multi-reference feedforward active
86 noise control has probably been most extensively utilised within the active road noise control
87 application, due to the complexity of the primary source and the resulting need for multiple
88 reference sensors in order to provide sufficient levels of multiple coherence between the
89 reference and error signals. For example, Oh *et al* present a comprehensive investigation
90 into the selection of accelerometer-based reference sensors for road noise control²⁵, Cheer
91 and Elliott investigate the use of interior microphones as reference sensors⁴ and Jung *et al*
92 utilise 8 reference sensors located around the four wheels to achieve a broadband reduction
93 of 4 dB up to 1 kHz²⁶.

94 Despite the application of multi-reference, multichannel FxLMS algorithms in practical
95 applications, there are a number of limitations in these control systems. Specifically, the
96 multichannel FxLMS algorithm may suffer from slow convergence due to the reference signals
97 being non-white and cross-correlated, and both dynamics and cross-coupling in the multi-
98 channel plant responses²⁷. Moreover, the application of the multichannel systems can be
99 limited due to the high computational requirements. The convergence of the multi-reference,
100 multichannel FxLMS can be improved by a preconditioning process, which whitens and
101 decorrelates the reference signals and compensates for the dynamics and cross-coupling in
102 the plant responses²⁷. This method is, however, not trivial to implement for practical mul-

103 tichannel systems, but a more practical variation has also been proposed²⁸. To overcome
104 the computational requirements, a variety of alternative control algorithm implementations
105 have been proposed in the literature, which include frequency domain implementations²⁹
106 and subband processing based methods^{30,31}.

107 In this paper a multi-reference feedforward control strategy is described and its applica-
108 tion to a noise cancelling headphone system is presented. Although the idea of using multiple
109 reference signals in an active noise control system is by no means novel, as discussed above,
110 it is presented here for the first time in the context of ANC headphones. This provides new
111 physical insight into the behaviour of ANC headphones and offers a potential improvement
112 over the single-reference strategies previously presented in the literature. In particular, it is
113 shown that the proposed strategy offers a significant control performance advantage for noise
114 sources incident from the sides of the user. Section II presents a description of the physical
115 noise cancelling headphone system and describes the single and multi-reference feedforward
116 control algorithms. Section III details the real-time implementation of the proposed strat-
117 egy on a prototype headphone system and presents the results of experimental testing and
118 Section IV draws conclusions.

119 **II. FEEDFORWARD ACTIVE NOISE CONTROL FOR HEADPHONE APPLICA-** 120 **TIONS**

121 This section will firstly describe the prototype noise cancelling headphones used in the
122 following study and then review the single-reference feedforward controller commonly em-

123 ployed in noise cancelling headphones before introducing the multi-reference feedforward
124 controller.

125 **A. System Description**

126 Figure 1 shows a schematic diagram of the ANC headphones considered in this study.
127 From this diagram it can be seen that each ear cup contains a loudspeaker, an error mi-
128 crophone and a reference microphone located on the outside of the ear cup. This setup
129 is consistent with previous feedforward ANC headphones presented in the literature. The
130 physical prototype has been realised using a customised pair of Beyerdynamic Custom One
131 Pro Plus headphones. The error microphones have been inserted into the ear cups and the
132 reference microphones have been integrated into the shell of the ear cup via a 3D printed in-
133 sert; the prototype headphones are shown in Figure 2. The reference and error microphones
134 were omnidirectional electrets with a ± 3 dB frequency response between 50 Hz and 16 kHz.
135 The control algorithms were implemented on a dSpace MicroLabBox, with a sample rate of
136 16 kHz.

139 In the first instance, the responses between the control loudspeakers and error micro-
140 phones were measured and the frequency and impulse responses, for the left and right ear
141 cup, are shown in Figure 3. From these responses it can be seen that the general character-
142 istics in the time and frequency domain are consistent for the two ear cups, although there
143 is a notable attenuation in the right ear response at frequencies above around 6.5 kHz. This
144 can be related to additional damping installed in the right ear cup introducing additional

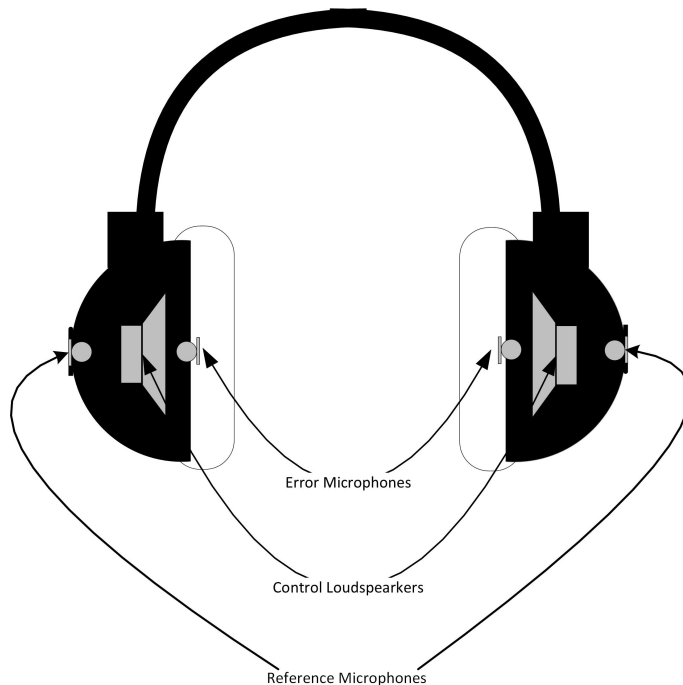


FIG. 1. Practical ANC headphone configuration showing the locations of the error microphone inside each ear cup, the reference microphones located on the outside of each ear cup and the control loudspeakers.

145 passive attenuation. That said, it can be seen from the impulse responses that the two
 146 responses are consistent and the initial time delay is 3 samples, or 0.2 ms, in both cases.

148 B. Single-Reference Control Algorithm

149 As detailed in the introduction, a variety of feedforward ANC headphone systems have
 150 been presented in the literature^{12,18–20} and these have all been based around using a single
 151 reference signal. This means that the two sides of the ANC headphones, as shown in Figure 1,
 152 operate independently. A block diagram of the single-reference FxLMS feedforward control



FIG. 2. Photo of the prototype headphones; note, although two microphones can be seen in the ear cup, only one has been used in the presented implementations.

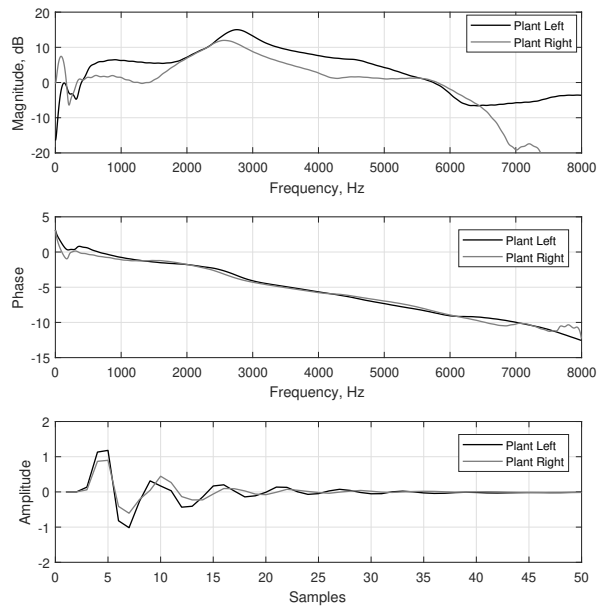


FIG. 3. The magnitude and phase of the frequency response and the impulse response of the plant.

153 algorithm is shown in Figure 4 for the left ear; an equivalent algorithm for the right ear can
 154 be obtained by exchanging the L subscripts for R subscripts.

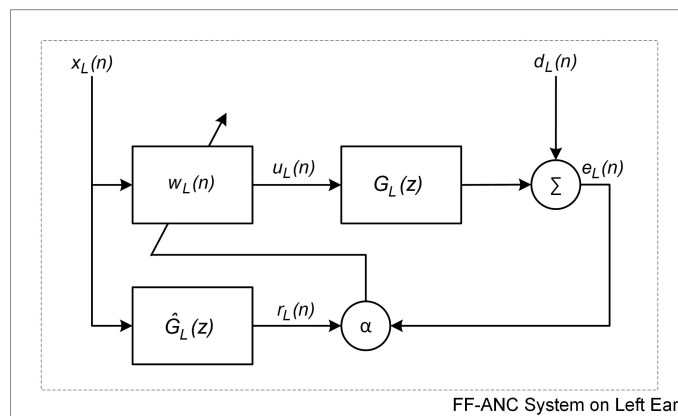


FIG. 4. Single-reference FxLMS feedforward control algorithm for the left ear. An equivalent control algorithm operates independently for the right ear.

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157 Initially, considering the single-reference implementation shown in Figure 4 the error
 158 signal at the left ear can be expressed as

$$e_L(n) = d_L(n) + \mathbf{g}_L^T \mathbf{u}_L(n), \quad (1)$$

159 where $d_L(n)$ is the disturbance signal at the left ear at the n -th sample, \mathbf{g}_L is the vector
 160 containing the impulse response of the plant and $\mathbf{u}_L(n)$ is the vector of current and previous
 161 samples of the control signal. The control signal is generated by filtering the reference signal,
 162 x_L in this case, with the control filter, \mathbf{w}_L , which can be expressed as

$$u_L(n) = \mathbf{w}_L^T(n) \mathbf{x}_L(n) \quad (2)$$

163 where \mathbf{w}_L is the vector of control filter coefficients, which has length I , and $\mathbf{x}_L(n)$ is the
 164 vector containing the current and $(I - 1)$ previous samples of the reference signal. The

165 control filter coefficients can then be calculated and adapted to minimise the error signal
 166 using the FxLMS algorithm, as in^{12,18–20}. In many practical applications it is beneficial to
 167 utilise the leaky version of the FxLMS algorithm due to its increased robustness and the
 168 vector of control filter coefficients in this case are updated as³²

$$\mathbf{w}_L(n+1) = (1 - \alpha\beta)\mathbf{w}_L(n) - \alpha\mathbf{r}_L(n)e_L(n), \quad (3)$$

169 where α is the convergence coefficient, β is the leakage parameter and $\mathbf{r}_L(n)$ is the vector of
 170 current and previous samples of the reference signal filtered by a model of the plant response,
 171 which is designated by the transfer function $\hat{G}(z)$ in Figure 4. In the following practical
 172 implementation, the normalised FxLMS algorithm is used, in which case the convergence
 173 gain is normalised by an estimate of the power of the filtered reference signals³³.

174 C. Multi-Reference Control Algorithm

175 Irrespective of the application, it is well understood that the performance of the FxLMS
 176 ANC algorithm will depend on both the coherence between the reference and error signals
 177 and the time advance provided by the reference signal over the error signal. For example,
 178 in the control of road noise in a car, it is common to position the reference signals as close
 179 as possible to the noise generating source or sources so as to maximise the available time-
 180 advance; however, this limits the coherence between the reference and error signals and thus
 181 multiple reference sensors are utilised to increase the multiple-coherence^{3,4,25,26}. It is clearly
 182 not practical in the ANC headphone application to position the reference microphones at
 183 a significant distance from the error microphones to maximise the time-advance, since they

184 must generally be integrated into the headphones. However, there is potential to utilise
 185 the two reference microphones shown in Figure 1 to control the signal at each of the error
 186 microphones, without any significant increase in hardware costs. The multiple-reference
 187 FxLMS algorithm is well established, as discussed in the introduction, but has not previously
 188 been investigated for the ANC headphone application and, therefore, will be described here
 189 for this application.

190 Figure 5 shows a block diagram of the multi-reference FxLMS algorithm for the left ear;
 191 an equivalent block diagram for the right ear can be obtained by exchanging the R and
 192 L subscripts. From Figure 5 it can be seen that the multi-reference controller is split into
 193 two parts: the upper part is consistent with the single-reference FxLMS algorithm shown in
 194 Figure 4 and described in the previous section, whilst the lower part shows a second FxLMS
 195 algorithm being used to update a second control filter, which operates on the reference signal
 196 measured by the reference microphone on the right ear cup to control the noise at the left
 197 ear error microphone. The control signal fed to the loudspeaker is thus given by

$$u_L(n) = u_{LL}(n) + u_{LR}(n), \quad (4)$$

198 where u_{LL} is the control signal generated to control the error signal at the left ear by filtering
 199 the reference signal from the left ear and u_{LR} is the control signal generated to control the
 200 error signal at the left ear by filtering the reference signal from the right ear. Equation 4
 201 can be expressed in terms of the vectors of control filter coefficients as

$$u_L(n) = \mathbf{w}_{LL}^T(n)\mathbf{x}_L + \mathbf{w}_{LR}^T(n)\mathbf{x}_R, \quad (5)$$

where \mathbf{w}_{LL} is the vector of control filter coefficients operating on the left reference signal to minimise the left error (or the ipsilateral control filter) and has length I_{LL} and \mathbf{w}_{LR} is the vector of control filter coefficients operating on the right reference signal to minimise the left error (or the contralateral control filter) and has length I_{LR} . The two vectors of control filter coefficients can be calculated using the leaky FxLMS algorithm, as given in equation 3 for the single-reference case. The update equations here are given as

$$\mathbf{w}_{LL}(n+1) = (1 - \alpha_{LL}\beta_{LL})\mathbf{w}_{LL}(n) - \alpha_{LL}\mathbf{r}_{LL}(n)e_L(n) \quad (6)$$

$$\mathbf{w}_{LR}(n+1) = (1 - \alpha_{LR}\beta_{LR})\mathbf{w}_{LR}(n) - \alpha_{LR}\mathbf{r}_{LR}(n)e_L(n), \quad (7)$$

202 where \mathbf{r}_{LL} and \mathbf{r}_{LR} are the reference signals from the left and right ear cups respectively,
 203 filtered by a model of the plant response between the left loudspeaker and left error micro-
 204 phone, which can be expressed by the transfer function $\hat{G}_L(z)$, and the subscripted conver-
 205 gence gains and leakage coefficients indicate that these can be set independently for the two
 206 paths. As in the previous section, the normalised version of the FxLMS algorithm has been
 208 employed.

209 It is possible to combine equations 6 and 7 and express the multi-reference FxLMS algo-
 210 rithm in its usual form as

$$\mathbf{w}_L(n+1) = (1 - \alpha\beta)\mathbf{w}_L(n) - \alpha\mathbf{r}_L(n)e_L(n), \quad (8)$$

211 where the $(2I \times 1)$ vector of filter coefficients is populated as

$$\mathbf{w}_L = [w_{LL_0}, w_{LR_0}, w_{LL_1}, w_{LR_1}, \dots, w_{LL_{I-1}}, w_{LR_{I-1}}]^T \quad (9)$$

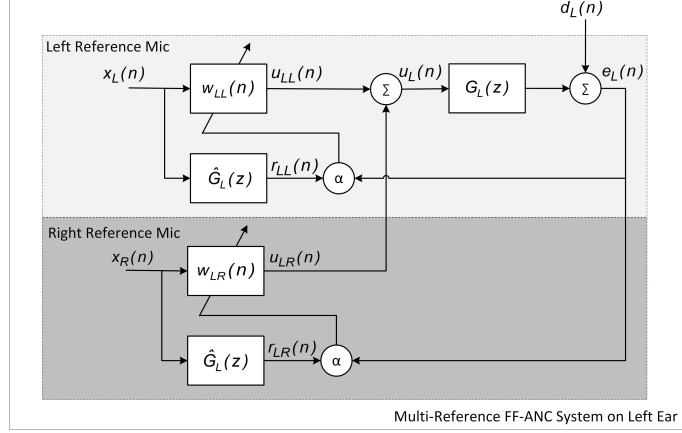


FIG. 5. Multi-reference filtered- x LMS feedforward control algorithm for the left ear. An equivalent control algorithm operates independently for the right ear.

where w_{LLi} and w_{LRi} are the i -th coefficients of the two control filters shown in Figure 5 and the $(2I \times 1)$ vector of filtered reference signals is populated as

$$\mathbf{r}_L(n) = [r_{LL}(n), r_{LR}(n), r_{LL}(n-1), r_{LR}(n-1), \dots, r_{LL}(n-I+1), r_{LR}(n-I+1)]^T \quad (10)$$

212 From equation 5, and the block diagram in Figure 5, it is evident that the multi-reference
 213 FxLMS algorithm provides the potential for the controller to benefit from the additional
 214 reference signal provided by the reference microphone mounted in the opposite ear cup. Since
 215 this additional reference signal is at a greater distance from the error microphone, depending
 216 on the direction of the incident unwanted sound field, it may provide an additional time-
 217 advance to the controller. However, it should also be noted that the multi-reference controller
 218 is potentially non-unique, due to correlation between the multiple reference signals. This
 219 non-uniqueness can potentially result in slow convergence properties and could ultimately

220 limit the expected advantages of the multi-reference controller. This can be overcome by
221 decorrelating the reference signals, as proposed in²⁷, but a more straightforward and often
222 more practical approach is to use a suitable level of leakage in the controller adaptation.
223 This aspects of the proposed multi-reference approach will be investigated experimentally
224 in the following section.

225 III. REAL-TIME IMPLEMENTATION AND PERFORMANCE COMPARISON

226 In this section, the performance of the multi-reference feedforward ANC headphone sys-
227 tem is compared to that of the typical single-reference control strategy. The algorithms
228 described in the previous section have been implemented on a dSpace MicroLabBox and
229 the performance of the prototype headphones described in Section II A have been tested
230 in real-time. The experimental setup is first described, including details of how each con-
231 troller is setup, and then the results of the experimental implementations are presented and
232 compared.

233 A. Experimental Setup

234 It has been shown in previous studies that the performance of ANC headphones utilising a
235 single-reference feedforward control strategy for each ear cup is dependent on the direction
236 of the incident sound field^{20,21}. The proposed improvement presented in this paper is to
237 utilise the two available reference signals to control the error signal at each ear and thus
238 reduce the dependency of the performance on the direction of incidence. To investigate this
239 dependency, the ANC headphone prototype has been mounted on a binaural dummy head

240 and this has been positioned in the large anechoic chamber at the Institute of Sound and
241 Vibration Research, as shown in Figure 6. The performance of the two control strategies
242 outlined in Sections II B and II C have then been measured when the incident, unwanted
243 sound field is generated by a single loudspeaker positioned in front of (at 0°), behind (at
244 180°) and to the right and left hand sides (at 90° and 270° respectively) of the user at a
245 distance of 1.3 m, as shown schematically in Figure 7. In each case, the primary loudspeaker
246 that generates the unwanted sound field is driven with pink noise.

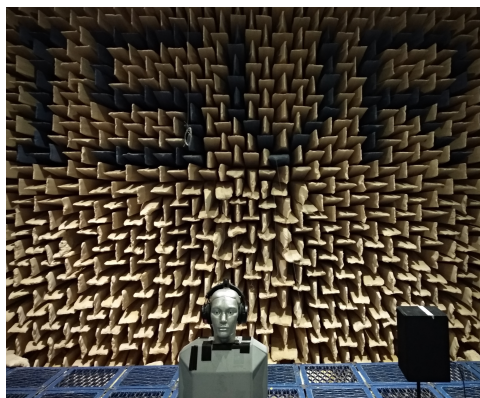


FIG. 6. Photograph of the dummy head with prototype headphones located in the large anechoic chamber at the ISVR, with the loudspeaker generating the primary sound field positioned to the left of the dummy head.

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250 As detailed in Section II, there are a number of parameters that must be set for each of
251 the two controllers. Specifically the lengths of the control filters, the convergence gains and
252 the leakage parameters. The lengths of the control filters in the two control algorithms have
253 been set such that a further increase in the filter length provides less than 1 dB improvement
254 in the broadband attenuation. This enables the upper limit on control performance to be as-

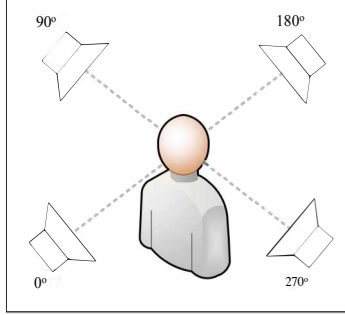


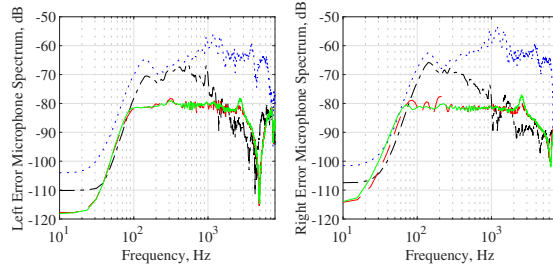
FIG. 7. Schematic of the experimental test configuration with primary sources at 0, 90, 180 and 270 degrees with respect to the user.

255 sessed in each case, whilst not unduly increasing the computational demand. Following this
 256 approach, it is worth highlighting that longer control filters are required for the contralateral
 257 terms, \mathbf{w}_{LR} and \mathbf{w}_{RL} , compared to the ipsilateral terms to achieve the maximum perfor-
 258 mance. This requirement can be related to the longer path length between the reference
 259 and error sensors in the contralateral cases. Ultimately, the single reference control filter
 260 length and the ipsilateral control filter length in the multi-reference controller have been set
 261 to 160 coefficients, whilst the contralateral control filter lengths in the multi-reference case
 262 have been set to 320. The convergence gains and leakage parameters in each of the two
 263 controllers have been set to provide the maximum convergence speed in each case. The con-
 264 vergence and leakage parameters for the single reference controller and the ipsilateral control
 265 filter have been set to 0.15 and 2×10^{-5} respectively. The convergence gain and leakage
 266 for the contralateral control filter have been set to 0.09 and 6×10^{-6} and this difference is
 267 largely related to the difference in the length of the contralateral control filter.

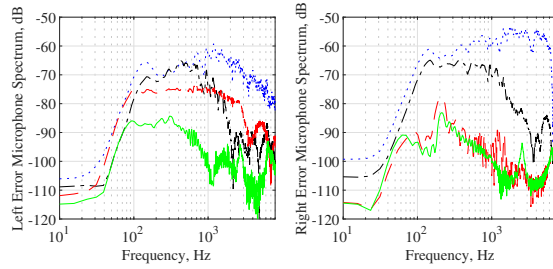
268 B. Control Performance

269 The performance of the two control strategies can be evaluated from the results presented
270 in Figure 8. The four subfigures in Figure 8 show the performance of the single and multi-
271 reference configurations for the sound field incident from 0° (a), 90° (b), 180° (c) and 270°
272 (d) of the dummy head. In each case, the power spectral density of the reference signal
273 is shown by the blue dotted line, along with the power spectral density of the error signal
274 without control (black dot-dashed line) and with control using the single (red dashed line)
275 and multi-reference (green solid line) configurations. Figure 8(a) shows the results for a
276 primary source incident from 0° , i.e. in front of the dummy head, and from these results it
277 can be seen that both the single and multi-reference configurations achieve the same level of
278 attenuation compared to the uncontrolled error signal. The broadband attenuation, between
279 0 and 8 kHz, in this case is 5 dB at both ears. A similar result is shown in Figure 8(c) for the
280 case when the primary source is located at 180° , which is behind the user. In this case, both
281 controllers achieve a broadband attenuation compared to the uncontrolled error of around
282 9 dB at both ears.

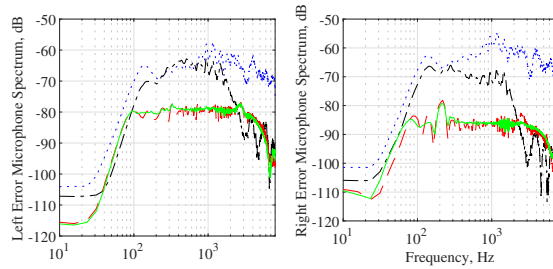
284 The performance of the two controllers begins to differ when the sound field is incident
285 from either side of the dummy head. Firstly, Figure 8(b) shows the performance at the two
286 ears when the primary field is generated by a loudspeaker positioned to the right of the
287 dummy head, at 90° to the normal. In this case it is clear that the attenuation provided by
288 the two control strategies is equal at the right ear, with a broadband attenuation of around
289 20 dB, but the multi-reference strategy provides a significant improvement at the left ear.



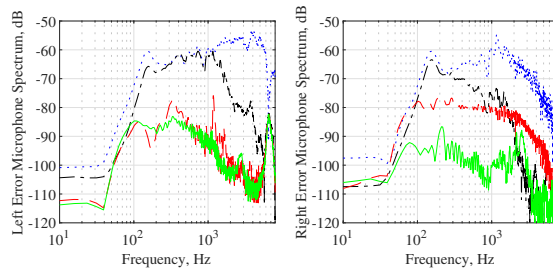
(a) Control performance for a 0° incident sound field.



(b) Control performance for a 90° incident sound field..



(c) Control performance for a 180° incident sound field..



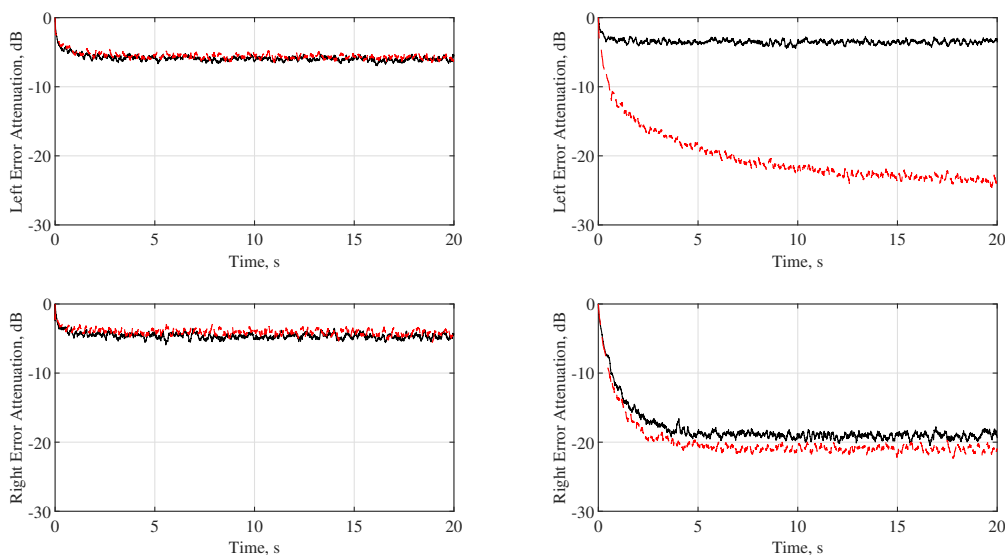
(d) Control performance for a 270° incident sound field..

FIG. 8. The power spectral density of the pressure measured at the reference (blue dotted) and error microphones (black dot-dashed) without control, and at the error microphone with single reference feedforward control (red dashed) and multi-reference feedforward control (green solid).

290 Specifically, the single-reference controller achieves a broadband attenuation of 4 dB, whilst
291 the multi-reference controller achieves 22 dB of attenuation. In this configuration, where
292 the primary source is located to the right-hand side of the dummy head, the increased
293 attenuation that is achieved by the multi-reference controller at the left ear is due to the
294 additional time-advance provided by the second reference microphone mounted on the right
295 ear cup. A similar performance advantage is provided by the multi-reference controller at
296 the right ear when the primary field is incident from the left of the dummy head and this
297 is shown by the results presented in Figure 8(d). In this case, the left reference microphone
298 provides an additional time-advance and the attenuation in this case is increased from 5 dB
299 with the single-reference controller to 20 dB with the multi-reference controller.

300 Although it is evident from the preceding results that the multi-reference controller of-
301 fers increased performance over the single reference controller after convergence, it is also
302 important to consider the convergence performance of the two controllers. In particular, it
303 is important to understand if correlation between the two reference signals limits the con-
304 vergence, as discussed in the introduction. Therefore, Figure 9 shows the convergence of the
305 single and multiple reference controllers for a primary source at 0° and at 90° . From Figure
306 9(a) it can be seen that for a primary source at 0° , the two algorithms reach the same level
307 of attenuation after convergence, as expected from the results presented in Figure 8, but
308 importantly, converge at the same rate. From Figure 9(b), which shows the results when
309 the primary source is located at 90° , it can be seen that the initial convergence of the two
310 algorithms is approximately equivalent, however, the multi-reference controller continues to
311 converge and reaches the higher level of attenuation expected from the results presented in

312 Figure 8. From the presented convergence plots, it is clear that the multi-reference con-
 313 troller does not achieve the additional control performance at the expense of limiting the
 314 convergence speed, thus further supporting the benefits of the proposed approach.



(a) Convergence for a 0° incident sound field. (b) Convergence for a 90° incident sound field..

FIG. 9. Convergence plots, showing the attenuation at the left and right error microphones for the single (black solid) and multi (red dashed) reference controllers.

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317 To provide further insight into the performance of the multi-reference controller compared
 318 to the typical single reference controller, Figure 10 shows the broadband attenuation achieved
 319 by the two controllers at the left and right ears for primary noise sources located at 30°
 320 intervals between 0 and 330° . From these results it can be seen that for the single reference
 321 controller, the performance at each of the two ears is limited for sources located on the
 322 opposite side of the head, whilst the multi-reference controller is able to achieve significant

323 levels of attenuation at both ears for primary sources located to both the left and right of
324 the user.

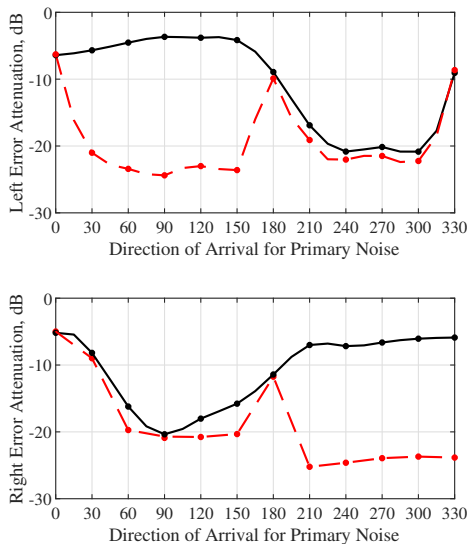


FIG. 10. Broadband attenuation for the single (black solid) and multi (red dashed) reference feedforward controllers plotted as a function of the angle of incidence of the primary source.

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327 As noted above, the increased attenuation achieved by the multi-reference controller is
328 due to the additional time-advance provided by the second reference microphone. This can
329 be investigated further via the group delay between, for example, the left error microphone
330 and the left and right reference microphones when a primary source is located to the right
331 of the dummy head at 90° ; this is plotted in Figure 11. From these results it can be seen
332 that the group delay between the right reference microphone and the left error microphone
333 is significantly greater than that provided by the left reference microphone. This means
334 that the right reference microphone provides a greater time-advance than the left reference

335 microphone and thus enables the significant increase in performance achieved by the multi-
 336 reference controller compared to the single reference controller. However, it is also worth
 337 noting that although the left reference microphone is geometrically further from the primary
 338 source, which is positioned to the right of the dummy head, than the left error microphone, it
 339 still provides a predominately positive group delay over frequency and, therefore, some time
 340 advance due to the passive isolation provided by the ear cup; this was previously investigated
 341 in²¹.

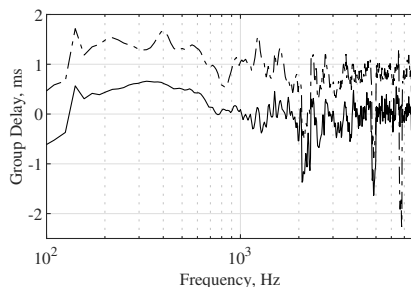


FIG. 11. Group delay between the left error microphone and the left (solid) and right (dot-dashed) reference microphones for a primary sound field generated to the right of the dummy head at 90° .

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 343

344 Finally, in practical applications it is unlikely that the primary source will be incident
 345 from only 1 direction and is more likely to be somewhat diffuse in nature. Therefore, Figure
 346 12 compares the performance of the single and multi-reference controllers when multiple
 347 primary sources surrounding the user are driven with uncorrelated pink noise. From these
 348 results it can be seen the multi-reference controller continues to outperform the single ref-
 349 erence controller under this more practical configuration and achieves an additional 9 dB
 350 attenuation.

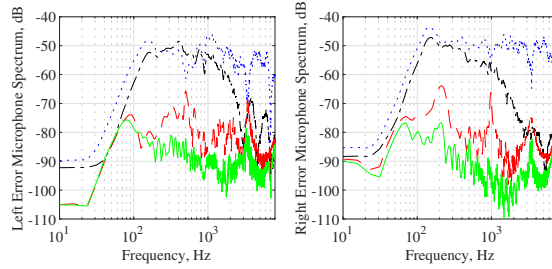


FIG. 12. The power spectral density of the pressure measured at the reference (blue dotted) and error microphones (black dot-dashed) without control, and at the error microphone with the single reference feedforward control (red dashed) and multi-reference feedforward control (green solid) for multiple primary noise sources distributed around the user.

352 IV. CONCLUSIONS

353 ANC headphones have seen significant commercial success and a variety of designs have
 354 been proposed and investigated in the open literature. These various implementations,
 355 whether using feedback, feedforward or hybrid strategies, have used independent controllers
 356 for each ear. This paper has investigated the potential of a multi-reference control strategy,
 357 where the signals from the reference microphones mounted on the exterior of each ear cup
 358 are both utilised by each of the individual ear controllers. Through experiments utilising
 359 a prototype ANC headphone system, it has been shown that this multi-reference control
 360 strategy reduces the sensitivity of the controller to the incidence direction of the unwanted,
 361 primary sound field. That is, for sounds incident from the left and right of the user, the
 362 investigated multi-reference controller is shown to achieve a broadband increase in attenua-
 363 tion of around 15 dB compared to the typical single-reference controller. This performance

364 increase has been related to the additional time-advance provided to the controller by the
365 second reference microphone signal, which is consistent with previous work in broader ap-
366 plications of ANC. Although the use of multiple reference signals in the ANC headphone
367 application does not significantly increase the hardware requirements, since the second ref-
368 erence microphone will already be in place, there is a modest increase in the computational
369 demand; although this is unlikely to be a limiting factor with modern processor capabilities.

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