Differential-Detection Aided Large-Scale Generalized Spatial Modulation Is Capable of Operating in High-Mobility Millimeter-Wave Channels

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Abstract—A large-scale differential-detection aided generalized spatial modulation (GSM) system is proposed, which relies on a novel Gram-Schmidt basis set and an adaptive low-complexity detector, and is evidently suitable for high-mobility millimeter-wave (mmWave) channels. We consider non-stationary time-varying mmWave channels and assume that the beam-angles remain relatively fixed, while the channel coefficients vary rapidly. In this scenario, it is a challenging task to find the accurate estimates of channel coefficients for digital beamforming, which becomes an even more severe problem, as the numbers of subarrays and subcarriers increase. Our analog-beamforming-aided nonsquare differentially-detected scheme achieves a higher transmission rate than the conventional coherent multiple-input multiple-output schemes because the pilot overhead and the complex-valued feedback are eliminated. Our simulation results following the IEEE 802.11ad specifications show that the performance of our proposed nonsquare differential GSM improved upon increasing the number of subarrays, where the maximum transmission rate of 16 [bps/Hz] was considered.

Index Terms—MIMO, OFDM, millimeter-wave communications, index modulation, spatial multiplexing, spatial modulation, differential modulation, differential spatial modulation, differential space-time block codes.

I. INTRODUCTION

Given its ample bandwidth, millimeter-wave (mmWave) communications have the promise of a multi-Gbps throughput for supporting compelling wireless applications [1–4]. A large number of antenna elements are used to form a directed beam, in beamforming (BF), for mitigating the high propagation loss of mmWave carriers. Since the full-digital BF circuit is power-thirsty, the associated analog phase shifters are typically combined with digital precoding to simplify both the transmitter and the receiver architectures [4–6]. Indoor mmWave communications in the unlicensed 57–66 GHz band have been developed under the IEEE 802.11ad specifications for supporting up to 7 Gbps throughput [7, 8], with the aid of orthogonal frequency division multiplexing (OFDM). Since 2017, the IEEE 802.11ay protocol set has been discussed, which relies on multiple-input multiple-output (MIMO) technologies [9] and aims for supporting a 100 Gbps throughput through eight independent data streams [10].

At the time of writing, index modulation has gathered tremendous attention [11–14], and it has also been applied to mmWave communications [15–30], because its unique on/off switching structure simplifies both the transmitter and the receiver implementation. Generalized spatial modulation (GSM) [31] activating more than one transmit subarrays has been used for indoor mmWave communications in order to obtain a sufficiently high BF gain [25–30]. Although having a low number of activated subarrays is inefficient in terms of BF, the GSM family is capable of achieving similar performance trends to the classic spatial multiplexing based MIMO family, such as Bell Laboratories layered space-time (BLAST) that activates all the transmit subarrays [13], despite having a low detection complexity.

In general, having a strong and stable line-of-sight (LoS) path is indispensable for high-rate mmWave communications. In LoS-dominant, large-scale and high-mobility scenarios the channel state information (CSI) has to be updated quite often and its increased feedback rate requires substantial computational resources [32, 33]. In contrast to LoS-dominant scenarios, the channel coefficients may change rapidly [34–36] in non-LoS (NLoS) scenarios, while the corresponding direction angle is relatively fixed [37]. For example, when a few people walk at a fast pace in a room, the channel’s coherence time is reduced compared to the slow-paced scenario [34]. Since the variance of path loss in NLoS scenarios is higher than that of LoS scenarios [35, 36], the estimation of channel coefficients inevitably becomes frequent and time-consuming [37], which erodes the high rate of MIMO-aided mmWave communications.

To circumvent the channel-estimation overhead in sub-GHz MIMO wireless communications, the differential space-time coding (DSTC) philosophy has been proposed [38–41]. In particular, the sparse-matrix-based DSTC family [42–49]...
achieves a high reliability, despite its lower complexity than that of the classic dense-matrix-based scheme [38–40]. The sparse DSTC scheme is referred to as differential spatial modulation (DSM) [42], since it is a differential counterpart of coherent SM [50]. The DSM codeword has a sparse structure, hence it is capable of reducing the encoding and decoding complexities [13].

The DSM scheme that is capable of dispensing with the channel estimation overhead may be deemed attractive for high-rate high-mobility mmWave communications. However, the major impediment of the conventional DSM family is its performance loss in high-rate scenarios [13], which is imposed by having to satisfy the unitary constraint. The encoding and the decoding complexities inevitably escalate as the number of transmit antennas increases, even though the sparse structure may be exploited for reducing the complexity. This has been an open issue since 2000, and it was solved in [51, 52] by invoking a simple square-to-nonsquare matrix mapping, which achieved a high throughput of $R = 12$ [bps/Hz] for $M = 1024$ transmit antennas. This high-rate capability was also verified by other authors in [53, 54] based on [51].

Instead of the high-complexity square-matrix-based DSM, the low-complexity nonsquare-matrix-aided DSM scheme of [51–54] may be the best solution for high-rate mmWave communications. Nevertheless, the conventional nonsquare scheme of [51–54] requires a carefully designed forgetting factor relying on time-consuming optimization in a specific channel environment. Thus, it is a challenge for the conventional scheme to communicate over time-varying mmWave channels. Furthermore, the nonsquare-matrix-based scheme exhibits severe error floors in high-Doppler high-mobility scenarios [51, 52], which implies that the nonsquare concept is less attractive for realistic mobile wireless communications.

Against this background, we propose a high-rate nonsquare differential scheme for static and high-mobility mmWave communications, where a special form of the proposed scheme may be deemed equivalent to the differential counterpart of GSM.1 In high-mobility scenarios, we consider non-stationary and time-varying mmWave MIMO-OFDM channels, where it is a challenging task to accurately track the channel coefficients. The straightforward application of the conventional nonsquare concept [51, 52] does not achieve a convincing performance gain in this scenario, due to the rapidly fluctuating channel coefficients and the fixed forgetting factor. To the best of our knowledge, the square and nonsquare DSTC family has not been applied to MIMO-aided mmWave communications because of its modest performance eroded by having to satisfy the unitary constraint and owing to the lack of channel coefficients for digital BF.2 The major contributions of this paper are summarized as follows:

- We propose a novel basis set relying on the Gram–Schmidt process [56], which converts the conventional square and sparse DSM constellation into differential GSM. This proposed scheme activates an arbitrary number of transmit subarrays and it is capable of striking a BF gain vs. detection complexity trade-off.
- We propose two adaptive forgetting factors for the proposed nonsquare DSTC, where both time-invariant and time-varying scenarios are considered. The proposed forgetting factor allows the detector to adaptively track the rapidly-fluctuating channel coefficients, hence improving the system’s reliability.
- We demonstrate that the performance of our scheme monotonically improves upon increasing the number of subarrays and approaches that of its coherent counterpart having perfectly accurate channel coefficients, while the channel estimation overhead of the coherent MIMO escalates with the system’s scale. When considering high-mobility scenarios, the proposed differential GSM scheme achieves a similar performance trend to that of its nonsquare counterpart that activating all the transmit subarrays, whilst reducing the detection complexity.

Furthermore, since the differential family has not been applied in MIMO-aided mmWave communications, we detail the main advantages of the proposed differential system as follows:

- **Reduced feedbacks.** Our system only requires real-valued signal-to-noise ratio (SNR) feedback for adjusting the beam angle, but does not require full CSI feedbacks. The CSI feedback contains a large number of complex-valued channel coefficients. Its frame length increases with the number of subarrays and subcarriers, as defined by the IEEE 802.11 specifications [57, p. 746], which reduces the effective throughput.
- **No channel estimation overhead.** Our scheme eliminates the pilot subcarriers, the interpolation-aided channel tracking, and the fixed-length channel estimation sequences, which increases the effective throughput. In the IEEE 802.11ad specifications, 16 pilot subcarriers are reserved for the entire set of 355 subcarriers [58], which represent about 5% of subcarriers. Here, the frequency domain channel transfer function coefficients are directly given for the pilot subcarriers, while for the data subcarriers they are estimated by interpolation [59–61]. Additionally, the IEEE 802.11ad frames include the fixed-length channel estimation field, whose duration is twice as long as the 512-subcarrier OFDM symbol duration [58].
- **Adaptive operation.** Our scheme subsumes the conventional DSTC family. In the IEEE 802.11ad specifications, single-input single-output (SISO) π/2-differential binary phase-shift keying (DBPSK) is used before setting up the BF link [58], because DBPSK exhibits a high robustness against unexpected channel conditions. Our system can readily switch between the proposed and the conventional DPSK arrangement by simply changing the corresponding codebook.
- **High BF gain.** The conventional square-matrix-based single-active DSM scheme only achieves a limited analog BF (ABF) gain, because merely a single transmit subarray is activated [25]. By contrast, the proposed scheme activates multiple subarrays at the same time and thus

1Note that the conception of differential GSM was first heralded in [43].
2Note that the first analog-circuit-based mmWave broadcasting system in the 1980s adopted differential PSK [55].
it achieves a substantial ABF gain that is comparable to that of the conventional spatial multiplexing scheme such as BLAST.

The remainder of this paper is organized as follows. Section II defines the channel model considered in this paper. Section III reviews the conventional square- and nonsquare-matrix-based DSTC family, while Section IV proposes our adaptive nonsquare differential GSM scheme. Section V provides our performance comparisons between the conventional and proposed schemes. Section VI concludes this paper.

We use the following notations in this paper. Italicized symbols represent scalar values, bold symbols represent vectors/matrices, and calligraphic symbols represent tensors. $j = \sqrt{-1}$ denotes the imaginary number, while $j$ denotes an index. $(\cdot)^T$ denotes the transpose of a matrix, while $(\cdot)^H$ denotes the Hermitian transpose of a matrix. Furthermore, $\mathcal{C}N(\mu, \sigma^2)$ denotes the complex normal distribution of a random variable having a mean of $\mu$ and a variance of $\sigma^2$. Finally, $\mathbb{C}^{A \times B \times C}$ represents a set of $(A \times B \times C)$-sized complex-valued tensors, $I_m$ represents the $(m \times m)$ identity matrix, $\text{bdiag}[\cdot]$ constructs a block diagonal of its arguments, $\text{diag}[\cdot]$ extracts the diagonal element of a matrix, and $\text{circuit}[\cdot]$ constructs a circulant matrix from a vector.

II. TENSOR-BASED TIME-IN Variant AND TIME-VARYING MMWave Channel Model

In this paper, we consider the popular clustered mmWave channel model of [37, 62], where we have $N_{sc}$ subcarriers, $M_T$ transmit antenna (TA) elements, $N_R$ receive antenna (RA) elements, and the $L$ multipath components. The TA elements are equally separated by half the wavelength, and are grouped into $M$ subarrays. Thus, each transmit subarray has $U_T = M_T/M$ TA elements. Similarly, the receiver has $N$ subarrays and each subarray has $U_R = N_R/N$ RA elements. Here, the direction angle is fixed and perfectly known at both the transmitter and the receiver, while the channel coefficients vary based on the autoregressive (AR) model designed for mmWave channels [63].

A. Tensor-Based Modeling

In this paper, we use a tensor-based formalism. A tensor is a multi-dimensional array [64]. We represent $L$ number of $N \times M$ channel matrices as a three-dimensional (3D) tensor $\mathcal{H}^{BF} \in \mathbb{C}^{N \times M \times L}$ as follows: [37, 62]

$$\mathcal{H}^{BF}_{i,j} = \frac{1}{\sqrt{L}} \mathbf{W}_T^H \mathbf{a}_R(l) \mathbf{a}_T(i,l) \mathbf{W}_T \mathbf{a}_T(j,l) \mathbf{W}_T \in \mathbb{C}^{N \times M},$$

(1)

where $\mathcal{H}^{BF}_{i,j} \in \mathbb{C}^{M \times L}$ ($n = 1, \cdots, N$), $\mathcal{H}^{m}_{i,m} \in \mathbb{C}^{N \times L}$ ($m = 1, \cdots, M$), and $\mathcal{H}^{BF}_{i,j} \in \mathbb{C}^{N \times M}$ ($l = 1, \cdots, L$) denote the two-dimensional (2D) sections of the 3D tensor $\mathcal{H}^{BF}$. This notation is also detailed in [64]. Additionally, the integer $i$ denotes the transmission index for $i \geq 1$. In Eq. (1), $\mathbf{a}_T(i,l) \in \mathbb{C}$ is generated for each $n$ and $m$. We use the following vector notation for simplifying the ABF weights:

$$\mathbf{a}_k^K(\phi) = [e^{j(k-1)\cos \phi} \cdots e^{j(k-K-1)\cos \phi}]^T \in \mathbb{C}^K,$$

(2)

which relies on a uniform linear array. In Eq. (1), $\mathbf{a}_T(l) = a_{nT}^N(\phi_T(l))/\sqrt{N_T}$ and $\mathbf{a}_R(l) = a_{nR}^N(\phi_R(l))/\sqrt{N_R}$ denote the transmit and receive steering vectors associated with the $l$-th multipath component, where $\phi_T(l)$ and $\phi_R(l)$ represent the corresponding angle-of-departure (AoD) and angle-of-arrival (AoA), respectively. The antenna weight matrices $\mathbf{W}_T$ and $\mathbf{W}_R$ are constructed by [65]

$$\mathbf{W}_T = \frac{1}{\sqrt{U_T}} \text{bdiag}[\mathbf{a}_1^{U_T}(\phi_T(1)), \mathbf{a}_U^{U_T}(\phi_T(2)), \cdots, \mathbf{a}_U^{U_T}(\phi_T(M))],$$

and

$$\mathbf{W}_R = \frac{1}{\sqrt{U_R}} \text{bdiag}[\mathbf{a}_1^{U_R}(\phi_R(1)), \mathbf{a}_U^{U_R}(\phi_R(2)), \cdots, \mathbf{a}_U^{U_R}(\phi_R(N))].$$

Note that $\text{bdiag}[\cdot]$ represents the block diagonalization.

Next, we introduce a four-dimensional (4D) tensor $\mathcal{H}_{Time} \in \mathbb{C}^{N \times M \times N_{sc} \times N_{sc}}$ that represents the time-domain impulse responses. The 2D sections of $\mathcal{H}_{Time}$ are circularly symmetric matrices of the form [66]

$$\mathcal{H}_{Time_{n,m:}} = \text{circuit}[\begin{bmatrix} \mathcal{H}_{nm:}^{BF}, & 0, \cdots, 0 \end{bmatrix} \in \mathbb{C}^{N_{sc} \times N_{sc}},$$

(3)

Here, function $\text{circuit}[\cdot]$ generates a circulant matrix from an $N_{sc}$-length vector.

Then, we represent the channel coefficients in the frequency-domain as a 3D tensor $\mathcal{H}_{Freq} \in \mathbb{C}^{N \times M \times N_{sc}}$ as follows:

$$\mathcal{H}_{Freq}^{nm:} = \text{diag}[\mathbf{W}_{N_{sc}}^{BF_{n,m:}} \mathbf{H}_{N_{sc}}^{H}],$$

where $\mathbf{W}_{N_{sc}} \in \mathbb{C}^{N_{sc} \times N_{sc}}$ denotes the $N_{sc} \times N_{sc}$ discrete Fourier transform (DFT) matrix, and $\text{diag}[\cdot]$ represents the diagonal elements of $\mathbf{W}_{N_{sc}}^{BF_{n,m:}} \mathbf{H}_{N_{sc}}^{H}$.

Finally, the received symbol block at the transmission index $i$ and the subcarrier index $j$ ($j = 1, \cdots, N_{sc}$) is given by

$$\mathbf{Y}(i,j) = \mathbf{H}(i,j) \mathbf{S}(i,j) + \mathbf{V}(i,j) \in \mathbb{C}^{N \times T},$$

(5)

where we have $\mathbf{H}(i,j) = \mathcal{H}_{Freq}^{nm_{ij}}$. Fig. 1 shows the relation between $\mathcal{H}_{Freq}^{nm_{ij}}$ and $\mathcal{H}_{j}^{Freq}$, where we have $N = 3$.

Note that the circulant matrix can be diagonalized by DFT matrices [66].
$M = 4$. As shown in Fig. 1, $H_{\text{freq}}$ extracts the $N_{\text{sc}}$-length vector associated with a specific index pair of $(n, m)$. The channel matrix $H(i, j) = H_{\text{freq}}^{n,m}$ is generated from the 2D section of $H_{\text{freq}}$ associated with subcarrier index $j$. In Eq. (5), $S(i, j) \in \mathbb{C}^{M \times T}$ represents the space-time codeword, and $V(i, j) \in \mathbb{C}^{N \times T}$ denotes the additive white Gaussian noise (AWGN) that obeys a zero-mean complex-valued Gaussian distribution of $CN(0, \sigma^2_v)$. According to Eq. (5), the SNR is defined by $\gamma = E_{i,j} (||S(i, j)||_F^2) / (T \cdot \sigma_v^2) = 1/\sigma_v^2$.

### B. Time-Varying Channel Coefficients

Based on the existing mmWave model of [63], we represent the time-varying channel coefficient $\lambda(i, l) \in \mathbb{C}$ in Eq. (1) as follows:

$$\lambda(i, l) = \begin{cases} CN(0, 1) & (i = 1) \\ r\lambda(i-1, l) + \sqrt{T-r^2}CN(0, 1) & (i > 1) \end{cases},$$  

(6)

which is modeled by a first-order AR process [67]. Here, the time-domain (TD) correlation function is defined by

$$\rho(\tau) = E_i[\lambda^*(i, l)\lambda(i+\tau, l)] / E_i[\lambda^*(i, l)\lambda(i, l)] = r^\tau,$$

(7)

where $\tau \in \mathbb{Z}$ is a discrete time-lag. The process of determining an appropriate $r$ of Eq. (6) is described as follows. First, the coherence time $T_c$ is defined by the duration, where the TD correlation function is above 0.5 as follows: [68]

$$T_c = 0.423 \cdot \frac{v_c}{v} \cdot \frac{1}{f_c}.$$  

(8)

Here, $v_c$ is the speed of light, $v > 0$ is the speed of receiver\(^4\), and $f_c$ is the carrier frequency. In this paper, we assume that the carrier frequency is $f_c = 60$ GHz, the DFT size is 512, the OFDM sample rate is 2640 MHz, and the normalized guard interval is 1/4, which obey the IEEE 802.11ad specifications [58]. Thus, the OFDM symbol duration is calculated by $T_s = 512 / (2640 \cdot 10^9) \cdot (4 + 1) / 4$ [s] = 242.4 [ns], which includes the guard interval. Secondly, the number of OFDM symbols generated within the coherence time $T_c$ is then calculated by $N_{\text{OFDM}} = \lceil T_c / T_s \rceil$. Since the TD correlation function $\rho(\tau) = r^\tau$ of Eq. (7) has to be above 0.5 between the time indices $i = 1$ and $i = N_{\text{OFDM}}$, we arrive at the following new constraint

$$r^{N_{\text{OFDM}}} = 0.5 \iff r = \exp \left( \frac{\log 0.5}{N_{\text{OFDM}}} \right) = \exp \left( -\frac{0.6931472}{T_c/(242.4 \cdot 10^{-9})} \right).$$  

(9)

According to Eq. (9), we find that the AR coefficient $r$ depends on the coherence time $T_c$, while directly depends on the mobile speed $v$. These relationships are exemplified in Table I.

\begin{table}[h]
\centering
\caption{Time-varying channel parameters based on the IEEE 802.11ad specifications.}
\begin{tabular}{|c|c|c|}
\hline
Speed $v$ [km/h] & Coherence time $T_c$ [ms] & AR coefficient $r$ \\
\hline
1 & 7.608732584039999 & 0.999977915640254 \\
5 & 1.521746516800000 & 0.999989579590400 \\
10 & 0.760873258404000 & 0.999979156419148 \\
100 & 0.076087325840400 & 0.99787922524068 \\
\hline
\end{tabular}
\end{table}

### III. Conventional Square and Nonsquare DSTC

The conventional square-matrix-based DSTC scheme of [38–41] generates a unitary matrix, multiplies it by the previous unitary matrix, and then transmits the multiplied matrix. Since the receiver knows that the received matrix is generated based on the previous matrix, it can detect the transmitted matrix without channel coefficients, where the previous matrix acts as if it was a pilot symbol block. By contrast, the conventional nonsquare-matrix-based DSTC scheme maps the unitary matrix onto a nonsquare matrix [52], and estimates the original unitary matrix from its mapped counterpart. Later in this section, we will omit the subcarrier index $j$ for simplicity of notation.

#### A. Square and Nonsquare Differential Encoding

The $i$th unitary matrix $\hat{S}(i) \in \mathbb{C}^{M \times M}$ is generated by differential encoding [38] as follows:

$$\hat{S}(i) = \begin{cases} \hat{I}_M & (i \leq M/T) \\ \hat{S}(i-1)\hat{E}_1 & (i > M/T) \end{cases},$$  

(10)

where a data matrix $X(i) \in \mathbb{C}^{M \times M}$ is associated with the $B$-length input bit sequence and satisfies $X(i)X(i)^H = \hat{I}_M$.\(^5\)

Then, the $i$th space-time matrix of Eq. (5) is generated by $\hat{S}(i) = \hat{S}(i)\hat{E}_1 \in \mathbb{C}^{M \times T}$, where a basis $\hat{E}_1 \in \mathbb{C}^{M \times T}$ converts an $M \times T$ matrix into an $M \times T$ matrix. Here, the basis set $\{\hat{E}_1, \cdots, \hat{E}_{M/T}\}$ is calculated by the DFT matrix as follows: [52]

$$\hat{I} = [\hat{E}_1, \cdots, \hat{E}_{M/T}] = \text{diag} \left[ W_{N_0}, \cdots, W_{N_0} \right] \in \mathbb{C}^{M \times M},$$

(11)

where $N_0$ denotes the number of nonzero elements in each basis column. For the $M = T$ and $N_0 = 1$ scenario, $\hat{I} = \hat{E}_1$ is equal to $\hat{I}_M$, which is identical with the classic differential encoding. In Eq. (11), the DFT matrix is defined by

$$W_M = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \omega & \omega^2 & \cdots & \omega^{M-1} \\ \omega^2 & \omega^4 & \cdots & \omega^{2(M-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^{M-1} & \omega^{2(M-1)} & \cdots & \omega^{(M-1)(M-1)} \end{bmatrix},$$

\(^5\)Based on the previous studies [45, 48, 69], this differential encoding can be extended to support a scaled semi-unitary matrix, i.e., $X(i)X(i)^H = \beta \hat{I}_M$, where $\beta$ is a scalar. In our simulations, we have observed that this extension causes severe error propagations due to the recursive construction of Eq. (13). As will be shown in Section IV-A, the resultant constellation following $X(i)X(i)^H = \hat{I}_M$ is similar to the star-QAM signaling.
where we have $\omega = \exp(-2\pi j/M)$, and $j$ denotes the imaginary number, i.e., $j^2 = -1$.

As a reference symbol, during $1 \leq i \leq M/T$, $\hat{I}$ is transmitted over $M$ symbol duration, both for the square scenario of $M = T$ and nonsquare case of $M > T$. Then, for the $M/T + 1 \leq i \leq W/T$ transmission indices, $W/T - M/T$ data symbols $\mathbf{S}(i) \in \mathbb{C}^{M \times T}$ are transmitted, where $W$ denotes the frame length. The ideal transmission rate is calculated by $R = B/T$, and the effective transmission rate is $R_{\text{eff}} = (1 - M/W) \cdot R$. The conventional nonsquare scheme of [51] transmits reference symbols $\hat{I}$ every 100$M$ blocks, i.e., we have $W = 100M$. In our simulations, the channel coefficients are randomized every $W = 1000M$ blocks. Thus, the rate loss is kept below 0.1%, and the periodic reference symbol insertions are nearly negligible.

Let us consider for example the family of nonsquare Alamouti codes [38,70] having $(M,T,R) = (2,1,2)$. At the $i = M/T + 1$ transmission index, a BPSK-aided square space-time codeword $\hat{\mathbf{S}}(i) = \hat{\mathbf{S}}(i-1)\mathbf{X}(i) = \mathbf{I}_M\mathbf{X}(i) = \mathbf{X}(i)$ is given by [70]

$$
\hat{\mathbf{S}}(i) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}.
$$

Then, the DFT basis set having $M = 2$ and $N_b = 2$ is given by

$$
\mathbf{E}_1, \mathbf{E}_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & -1 \end{bmatrix},
$$

which corresponds to the first and second columns of the DFT matrix. Finally, a nonsquare codeword is calculated by

$$
\mathbf{S}(i) = \hat{\mathbf{S}}(i)\mathbf{E}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.
$$

Further examples are found in [52].

### B. Noncoherent Detection

The $i$th data matrix $\mathbf{X}(i)$ is estimated by minimizing the following criterion [52]

$$
\hat{\mathbf{X}}(i) = \arg \min_{\mathbf{X}} \| \mathbf{Y}(i) - \hat{\mathbf{Y}}(i-1)\mathbf{X} \|_F^2, \quad (12)
$$

which is known as the maximum likelihood (ML) detector. Here, we have the difference equation of

$$
\hat{\mathbf{Y}}(i) = \begin{cases} 
\sum_{k=1}^{M/T} \mathbf{Y}(k)\mathbf{E}_k^H \\
\mathbf{Y}(i)\mathbf{E}^{(1-\alpha)} + \hat{\mathbf{Y}}(i-1)\mathbf{E}^{(\alpha)} 
\end{cases} \quad (i = M/T) \\
\mathbf{Y}(i)\mathbf{E}^{(1-\alpha)} + \hat{\mathbf{Y}}(i-1)\mathbf{E}^{(\alpha)} \quad (i > M/T).
$$

Ideally, $\hat{\mathbf{Y}}(i)$ approaches $\mathbf{H}(i)\hat{\mathbf{S}}(i) \in \mathbb{C}^{M \times M}$, which is estimated from the mapped symbols $\mathbf{H}(i)\hat{\mathbf{S}}(i)\mathbf{E}_1 \in \mathbb{C}^{M \times T}$.

In Eq. (13), the constant matrices are defined by $\mathbf{E}^{(1-\alpha)} = (1 - \alpha)\mathbf{E}_1^H$ and $\mathbf{E}^{(\alpha)} = \mathbf{I}_M - \mathbf{E}_1\mathbf{E}^{(1-\alpha)}$. Here, $\alpha$ is a forgetting factor that determines a combination ratio between the newly received symbol $\mathbf{Y}(i)$ and the previous symbols $\mathbf{Y}(i-1), \cdots, \mathbf{Y}(1)$. This single factor $\alpha$ is designed for minimizing a specific squared error [51,52], which will be detailed in Section III-C. Note that for the $M = T$ and $\alpha = 0$ scenario, Eq. (12) becomes equivalent to the conventional square-matrix-based detector, i.e.,

$$
\hat{\mathbf{X}}(i) = \arg \min_{\mathbf{X}} \| \mathbf{Y}(i) - \mathbf{Y}(i-1)\mathbf{X} \|_F^2.
$$

When relying on a complex-valued basis set, the ML detection complexity of Eq. (12) per receive subarray is given by [52]

$$
2^{RT+1} \big[ 2 \cdot \max(N_b, N_c) + 1 \big] + 4M/T(N_b + N_c) + 4N_b, \quad (14)
$$

where $N_c$ denotes the number of nonzero elements in each column of $\hat{\mathbf{S}}(i)$. Then, the number of activated subarrays is calculated by $P = \max(N_b, N_c)$ [52], which is equal to the number of nonzero elements in each column of $\hat{\mathbf{S}}(i)\mathbf{E}_1$. Eq. (14) is lower-bounded by $\Omega(2^{RT} \cdot \max(N_b, N_c)) = \Omega(2^{RT} \cdot P).$

### C. Forgetting Factor Design

The conventional detector of Eq. (12) requires a carefully-designed $\alpha \in \mathbb{R}$ ($0 < \alpha < 1$), which determines the overall system performance. In the previous studies [51,52], the forgetting factor $\alpha$ is designed for minimizing the following squared error:

$$
\alpha_c = \arg \min_{\alpha \in (0,1)} J_{\text{MSE}}, \quad (15)
$$

where $J_{\text{MSE}}$ is defined as [51]

$$
J_{\text{MSE}} = \frac{T}{W - M} \frac{1}{NM} \sum_{i=M/T+1}^{W/T} E \left[ \| \mathbf{Y}(i) - \mathbf{H}(i)\hat{\mathbf{S}}(i) \|_F^2 \right].
$$

Later, we will represent the forgetting factor designed for quasi-static channels by $\alpha_c$. Here, it is a time-consuming task to obtain $\alpha_c$, because Eq. (16) should be averaged over random $\mathbf{H}(i)$, $\mathbf{V}(i)$, and $\mathbf{X}(i)$ matrices, and the forgetting factor $\alpha_c$ has to be searched on a finely grained discrete set such as $\alpha = 0.01, 0.02, \cdots, 0.99$. In case we assume quasi-static Rayleigh fading channels, Eq. (15) can be simplified, and the forgetting factor $\alpha_c$ can be obtained by solving [51]

$$
2(K - 1)\alpha_c^{2K+2} + \alpha_c^{2K+1} - (2K + 1)\alpha_c^{2K} - K\alpha_c^3 + (K + 2)\alpha_c^2 + (K - 1)\alpha_c - K + 1 = 0, \quad (17)
$$

where we have $K = W/M$. As seen in Eq. (17), the forgetting factor $\alpha_c$ is dependent both on the number of transmit subarrays $M$ and on the frame length $W$. Since this simple criterion cannot be applied to time-varying channels, we have to rely on $J_{\text{MSE}}$ of Eq. (16), which will be addressed in Section IV.

### IV. PROPOSED ADAPTIVE DIFFERENTIAL GSM

The conventional nonsquare-matrix-based scheme of [52] relies on the DFT basis and on the static forgetting factor, as introduced in Sections III-A and III-C, which result in performance loss, especially for time-varying mmWave channels. Thus, in this section we propose a novel basis set and a novel forgetting factor design. Later, the transmission and subcarrier indices $i$ and $j$ are omitted, if they are not necessary.

$\Omega(\cdot)$ denotes the Donald Knuth’s big Omega notation [71].
A. Gram-Schmidt Process Aided Basis Design

The DFT basis imposes the additional constraint of \([M/N_0] = M/N_0\). For example, if we consider the \(M = 16\) case, \(N_0\) is limited to 1, 2, 4, 8 and 16. Furthermore, the DFT basis is unable to support some DSTC schemes, such as the algebraic DSM (ADSM) [48] having \(O > 1\) independent PSK symbols. This problem is encountered when the DFT projection is a non-injective function. Specifically, let us consider the \((M, T) = (4, 1)\) and \(N_0 = 2\) case. Here, the DFT basis is calculated by setting \(E_1 = [1 0 0 0]^T/\sqrt{2}\). Through the projection of \(S\) to \(SE_1\), only the first and second columns of \(S\) are extracted, while the third and fourth columns are discarded. Since the discarded columns include data-carrying symbols, this setup results in severe performance loss. Therefore, multiplying the basis \(E_1\) has to be an injecting mapping.

We propose a basis set relying on the Gram-Schmidt process (GSP) [56], which imposes no limitation on the number of nonzero elements \(N_0\). As seen in Eq. (14), the ML detection complexity can be improved by reducing \(N_0\) and \(N_c\). In mmWave communications, the number of activated subarrays \(P = \max(N_b, N_c)\) determines the BF gain. Here, we have a trade-off between the ML complexity and the BF gain, which can be controlled by using a flexible GSP-aided basis.

When assuming perfect CSI at the receiver, the pairwise-error probability (PEP) between the two arbitrary symbols \(\tilde{S}(f)E_1\) and \(\tilde{S}(g)E_1\) for \(1 \leq f < g \leq 2^B\) is upper bounded by [56, 72]

\[
\text{PEP}(f \rightarrow g) \leq \frac{1}{d(f, g)^N} \left( \frac{1}{\sigma^2} \right)^{-m'N},
\]

where \(m'\) represents the minimum rank of the Hermitian matrix \(C(f, g) = (\tilde{S}(f) - \tilde{S}(g))^T E_1 E_1^H (\tilde{S}(f) - \tilde{S}(g))^H \in \mathbb{C}^{M \times T}\). In Eq. (18), the diversity order is given by \(D = m'N\) and the coding gain is given by the Euclidean distance of \(d(f, g) = \prod_{m=1}^{m'} \mu_m \in \mathbb{R}\), where \(\mu_m\) represents the \(m\)th eigenvalue of \(C(f, g)\).

Although Eq. (18) is derived for coherent detection scenarios, this upper bound is also valid for designing differential codewords [44, 73]. Thus, we design the basis \(E_1 \in \mathbb{C}^{M \times T}\) so as to maximize the minimum Euclidean distance (MED) of

\[
\text{MED} = \min_{1 \leq f < g \leq 2^B} d(f, g).
\]

Here, \(E_1\) has \(N_0\) nonzero elements in each column, and satisfies the constraint of \(E_1^H E_1 = I_T\), which implies \(\|E_1\|_F^2 = \text{trace}(E_1^H E_1) = T\). Some of the GSP basis sets designed are exemplified in Appendix A.

After designing the first basis \(E_1\), the other bases \(E_2, \ldots, E_{M/T}\) are generated by the GSP, where the dense DFT bases of Section III-A are exploited. The conventional DFT basis set having \(N_b = M\) will be denoted by \(\{E_1, \ldots, E_{M/T}\}\) in this section. Based on the first basis \(E_1\) designed, the next basis \(E_k\), whose index is increased from \(k = 2\) to \(M/T\), is expressed by

\[
E_k = \left( I_M - \sum_{k' = 1}^{k-1} E_{k'} E_{k'}^H \right) E_k,
\]

and then normalized by \(E_k := \sqrt{T} \cdot E_k/\|E_k\|_F\). This normalization is required to satisfy the norm constraint of \(\|E_k\|_F^2 = T\). For example, the GSP basis set for the ADSM scheme having \((M, T, O, L) = (4, 1, 2, 16)\) and \(N_0 = 2\) is given by

\[
\tilde{E} = \begin{bmatrix}
0.00 & 0.56e^{0.00\pi} & 0.44e^{1.98\pi} & 0.70e^{1.98\pi} \\
0.76e^{0.95\pi} & 0.40e^{1.26\pi} & 0.23e^{1.31\pi} & 0.46e^{0.26\pi} \\
0.00 & 0.56e^{1.00\pi} & 0.82e^{0.01\pi} & 0.09e^{1.20\pi} \\
0.65e^{1.39\pi} & 0.47e^{0.70\pi} & 0.27e^{0.75\pi} & 0.54e^{1.70\pi}
\end{bmatrix},
\]

which is a unitary matrix. A part of the nonsquare ADSM codewords \(SE_1\) result in

\[
\begin{bmatrix}
0.65e^{1.64\pi} \\
0.00 \\
0.76e^{0.95\pi} \\
0.00
\end{bmatrix},
\begin{bmatrix}
0.00 \\
0.76e^{1.08\pi} \\
0.00 \\
0.00
\end{bmatrix},
\begin{bmatrix}
0.76e^{1.08\pi} \\
0.00 \\
0.00 \\
0.65e^{1.32\pi}
\end{bmatrix},
\begin{bmatrix}
0.00 \\
0.76e^{0.95\pi} \\
0.00 \\
0.00
\end{bmatrix},
\]

all of which contain \(P = \max(N_b, N_c) = \max(2, 1) = 2\) nonzero elements in each column. This setup is equivalent to the differential counterpart of the conventional GSM [31]. It is worth noting that all the TAs are selected with an equal probability due to the permutation matrix of the ADSM scheme, without relying on the sophisticated algorithm proposed in [74].

Furthermore, Fig. 2 shows illustrative examples for the nonsquare ADSM symbols having \((M, T, O, L) = (16, 1, 2, 4)\), where three growing MEDs were considered for characterizing the MED-maximization design process. Here, we used the GSP basis having \(N_0 = 12\) and generated 1000 \(M\) number of random codewords. The best GSP basis achieving \(\text{MED} = 0.877168\) is given in Appendix A. Since a fraction of \(1 - N_b/M = 1 - 12/16 = 25\%\) of the symbols are zero, we omitted them for clear illustration. As shown in Fig. 2, upon growing the MED, the nonsquare ADSM symbols converged on a spiral pattern, and the cardinality of these symbols remained finite for all the cases under the multiplication-based differential encoding. Most of the conventional differential MIMO schemes generate infinite and near-continuous constellation, which imposes additional complexity on the transmitter [45, 46]. Our GSP basis is capable of avoiding this infinite-cardinality problem.

B. Adaptive Forgetting Factor Design

Since the evaluation of the conventional design criterion in Eq. (15) is a high-complexity time-consuming task for the time-varying scenarios, we propose a low-complexity alternative. First, the \(J_{MSE}\) definition of Eq. (16) is transformed into

\[
J_{MSE} = \frac{T}{W-M} \frac{1}{NT} \sum_{i=M/T+1}^{W/T} \mathbb{E}\left[ \|Y(i)E_1 - H(i)\tilde{S}(i)E_1\|^2_F \right],
\]

(21)
because the average of the Frobenius norm in Eq. (16) is equivalent to
\[
E \left[ \left\| \hat{Y}(i) - H(i)\hat{S}(i) \right\|_F^2 \right] = \frac{M}{T} E \left[ \left\| \hat{Y}(i) - H(i)\hat{S}(i) \right\|_F^2 \right].
\]

Here, we have the relationships of \( Y(i) = H(i)\hat{S}(i)E_1 + V(i) \) and \( \hat{Y}(i) = (1 - \alpha)Y(i)E_1^H + \hat{Y}(i - 1)\hat{X}(i)(I_M - (1 - \alpha)E_1E_1^H) \).
Thus, the Frobenius norm of Eq. (21) can be further simplified to
\[
\hat{Y}(i)E_1 - H(i)\hat{S}(i)E_1 = \hat{Y}(i)E_1 - V(i) + \alpha(\hat{Y}(i) - \hat{Y}(i - 1)\hat{X}(i)E_1) + V(i).
\]

Later, we represent \( Y(i) - \hat{Y}(i - 1)\hat{X}(i)E_1 \) as \( D(i) \) for simplicity. Then, \( J_{\text{MSE}} = f(\alpha) \) can be transformed into
\[
f(\alpha) = \frac{1}{W - M} E \left[ \sum_{i=M/T+1}^{W/T} \left\| D(i) \right\|_F^2 \right].
\]

The derivatives of \( f(\alpha) \) with respect to \( \alpha \) are calculated by
\[
f'(\alpha) = \frac{1}{W - M} E \left[ 2\alpha \sum_{i=M/T+1}^{W/T} \left\| D(i) \right\|_F^2 \right] - 2\sum_{i=M/T+1}^{W/T} \text{Re} \left[ \text{tr} \left( V(i)H D(i) \right) \right]
\]
and
\[
f''(\alpha) = \frac{1}{W - M} E \left[ 2 \sum_{i=M/T+1}^{W/T} \left\| D(i) \right\|_F^2 \right].
\]

Since we have \( f''(\alpha) \geq 0 \), solving \( f'(\alpha) = 0 \) yields the best forgetting factor in terms of Eq. (21) as follows:
\[
\alpha = \frac{\sum_{i=M/T+1}^{W/T} \text{Re} \left[ \text{tr} \left( V(i)H D(i) \right) \right]}{\sum_{i=M/T+1}^{W/T} \left\| D(i) \right\|_F^2}.
\]

However, the AWGN term \( V(i) \) appeared in Eq. (24) is not available at the receiver, as defined in Eq. (5). Then, we transform \( \text{Re} \left[ \text{tr} \left( V(i)H D(i) \right) \right] \) into
\[
\text{Re} \left[ \text{tr} \left( V(i)H D(i) \right) \right] = \text{Re} \left[ \text{tr} \left( V(i)H \hat{S}(i)E_1 \right) \right] - \text{Re} \left[ \text{tr} \left( V(i)H \hat{Y}(i - 1)\hat{X}(i)E_1 \right) \right] + \text{Re} \left[ \text{tr} \left( V(i)H V(i) \right) \right].
\]

Furthermore, we use the assumption of \( \hat{Y}(i - 1)\hat{X}(i) \approx H(i - 1)\hat{S}(i - 1)\hat{X}(i) \approx H(i - 1)\hat{S}(i) \approx H(i)\hat{S}(i) \) as follows:
\[
\text{Re} \left[ \text{tr} \left( V(i)H D(i) \right) \right] = \text{Re} \left[ \text{tr} \left( V(i)H \hat{S}(i)E_1 \right) \right] - \text{Re} \left[ \text{tr} \left( V(i)H \hat{Y}(i - 1)\hat{X}(i)E_1 \right) \right] + \text{Re} \left[ \text{tr} \left( V(i)H V(i) \right) \right].
\]

Here, the mean of Eq. (25) converges to \( E \left[ \text{Re} \left[ \text{tr} \left( V(i)H V(i) \right) \right] \right] = N \cdot T \cdot \sigma^2 \).
Thus, Eq. (24) is approximated by
\[
\alpha_p = \frac{\left( \frac{W}{T} - \frac{M}{T} \right) \cdot N \cdot T \cdot \sigma^2}{E \left[ \sum_{i=M/T+1}^{W/T} \left\| D(i) \right\|_F^2 \right]}.
\]

Based on Eq. (26), we propose a pair of low-complexity adaptive schemes for the time-invariant and time-varying channels, respectively. Furthermore, we introduce an adaptive forgetting factor \( \alpha_q(i) \), and \( \hat{Y}(i) \) of Eq. (13) for \( i > M/T \) is redefined as follows:
\[
\hat{Y}(i) = (1 - \alpha_q(i)) D(i)E_1^H + \hat{Y}(i - 1)\hat{X}(i).
\]

1) Time-Invariant Channels: In quasi-static channels, the adaptive forgetting factor \( \alpha_q(i) \) is calculated from Eq. (26) as follows:
\[
\alpha_q(i) = \frac{(i - \frac{M}{T}) \cdot N \cdot T \cdot \sigma^2}{\sum_{i'=M/T+1}^{W/T} \| D(i') \|_F^2}.
\]

Eq. (28) contains the received symbols spanning from the \( (M/T + 1) \)-st block to the \( i \)-th block, which may overflow.
when the transmission index is high. Thus, Eq. (28) is rewritten by

\[
\frac{1}{\alpha_q(i)} = \left(1 - \frac{1}{i - M/T}\right) \frac{1}{\alpha_q(i-1)} + \frac{1}{i - M/T} \cdot \frac{\|D(i)\|^2_F}{N \cdot T \cdot \sigma_v^2},
\]

(29)

which only relies on the current \(D(i)\) and on the previous forgetting factor \(\alpha_q(i-1)\). This reduces the space complexity\(^8\) as compared to Eq. (26). When the transmission index \(i\) is sufficiently high, \(1/(i-M/T)\) approaches 0, and \(\alpha_q(i)\) converges to a finite value, i.e., \(\alpha_q(i) = \alpha_q(i-1)\).

2) Time-Varying Channels: In time-varying channels, we only consider the recent two blocks rather than all the \(i - M/T\) blocks. Specifically, the adaptive forgetting factor \(\alpha_v(i)\) is given by

\[
\alpha_v(i) = \frac{N \cdot T \cdot \sigma_v^2}{\|D(i)\|^2_F},
\]

(30)

Since we have \(D(i) = Y(i) - \hat{Y}(i-1)\hat{X}(i)E_1\), Eq. (30) only relies on the \(i\)-th and \((i-1)\)-st received blocks, which is different from \(\alpha_v(i)\) of Eq. (29).

The forgetting factor derived may violate the domain of definition \(0 < \alpha_v(i) < 1\) due to the approximation assumed in Eq. (25). Thus, we use its clipped counterpart of \(\min\{\max\{\alpha_v(i), 0.01\}, 0.99\}\) in our simulations. Note that the forgetting factors \(\alpha_q(i)\) and \(\alpha_v(i)\) proposed in Eqs. (29) and (30) are also applicable to general channel models such as time-invariant and time-varying Rayleigh, Rician, and Gamma-Gamma fading.

Fig. 3 exemplifies the static and adaptive forgetting factors \(\alpha_c, \alpha_q(i)\) and \(\alpha_v(i)\), where the mobile speed was \(v = 1, 5\) [km/h] and the SNR was set to \(\gamma = 20\) and 40 [dB]. As shown in Fig. 3, the adaptive forgetting factor \(\alpha_v(i)\) fluctuated against the block index \(i\), while \(\alpha_q(i)\) converged to a near-constant value. Ideally, \(\alpha_q(i)\) has to converge to \(\alpha_c\) that minimizes \(J_{\text{MSE}}\) of Eq. (16). For the SNR of \(\gamma = 20\) [dB] case, \(\alpha_q(i)\) converged to \(\alpha_c\), while we observed a gap of about 0.08 at \(i = 400\) for the SNR \(\gamma = 40\) [dB] case. This slight gap was imposed by the approximation of Eq. (25). It is worth noting that the proposed forgetting factors decreased as the SNR increased, while the conventional criterion of Eq. (17) only considered high SNRs and it was unable to support the SNR shift.

V. PERFORMANCE COMPARISONS

In this section, we compare the proposed scheme both to the conventional differential orthogonal space-time coding (DOSTC) [38, 70] and to the differential unitary coding (DUC) [41] as well as to the conventional coherent BLAST [9] and GSM [31, 75] schemes that have perfect estimates of CSI (PCSI) in terms of their MED, bit error ratio (BER), and \(J_{\text{MSE}}\). Later, the abbreviation of the square-matrix-based schemes starts with “S-”, while that of the nonsquare-matrix-based schemes starts with “N-”. For the \(M \cdot R \leq 32\) case, we did our best to simulate the conventional S-DOSTC schemes, although its computation time is so large that it is difficult to reproduce the same results in a typical computing environment.\(^9\) Since achieving a high throughput is crucial for mmWave communications, we only focus on the high-rate scenarios, where the transmission rate is increased up to \(R = 16\) [bps/Hz].

As in the IEEE 802.11ad specifications [58], the number of data subcarriers was \(N_{\text{sc}} = 339\).\(^{10}\) The number of TA and RA elements was set to four times the number of subarrays, i.e., we have \(M_T = M_R = 4M = 4N\), which were used for ABF. We considered both stationary and non-stationary scenarios, where the mobile speed was increased from \(v = 0\) to \(v = 100\) [km/h]. The number of multipath components was \(L = M\), where the largest setup considered was \(M = 16\).\(^{11}\) We basically used the GSP basis for the N-ADSM scheme, and the DFT basis for the N-DOSTC and the N-DUC schemes. The GSP bases used in our simulations are provided in Appendix A, and the DUC constellation factors are provided in Appendix B. Additionally, the number of nonzero elements in each codeword column is denoted by \(P\).

A. Small-Scale Scenarios

First, we conducted MED and BER comparisons for small-scale scenarios, where the numbers of transmit and receive subarrays were smaller than \(M = N \leq 4\).

Fig. 4 shows the MED comparisons between the conventional square schemes and the proposed nonsquare counterparts, where the basis \(E_1\) mapped both the DOSTC [38] and \(\text{PCSI}\) in terms of their MED, bit error ratio (BER), and \(J_{\text{MSE}}\). Later, the abbreviation of the square-matrix-based schemes starts with “S-”, while that of the nonsquare-matrix-based schemes starts with “N-”. For the \(M \cdot R \leq 32\) case, we did our best to simulate the conventional S-DOSTC schemes, although its computation time is so large that it is difficult to reproduce the same results in a typical computing environment.\(^9\) Since achieving a high throughput is crucial for mmWave communications, we only focus on the high-rate scenarios, where the transmission rate is increased up to \(R = 16\) [bps/Hz].

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\(^8\)It is worth noting that for the \(M \cdot R > 32\) case the number of codewords exceeds 2\(^{12}\) and 2\(^{12}\)-length array of \(M \times M\) complex matrices may overflow.

\(^9\)According to the survey of [4, Table X], some channel models assume that the number of multipath components obeys the Poisson distribution having the mean of 1.8, 1.9, 9, 10 and 18.
The transmission rate was $R = 8$ [bps/Hz] and the mobile speed was set to $v = 0$ [km/h]. Additionally, the normalized frame length $W/M$ was varied from $10^1$ to $10^4$, which corresponded to the reference insertion ratio of 10% and 0.01%, respectively. Here, we considered the low-rate scenario of $(M, N, R) = (2, 2, 2)$ in order to obtain the precise average of BER for the normalized frame length of $W/M = 10^4$% scenario, i.e., the pilot insertion ratio of 0.01%, which required $2\cdot10^4\cdot10^7$-block Monte-Carlo simulations. As shown in Fig. 5, although the conventional forgetting factor $\alpha_c$ was found by exhaustive search at each point, the proposed forgetting factor $\alpha_v(i)$ dispensing with previous optimization achieved the best BER. Furthermore, the achievable BER remained unchanged upon decreasing the reference insertion ratio. This implies that the performance of the proposed adaptive nonsquare scheme is not dependent on the frame length and on the reference insertion ratio, obeying the same trend as the conventional square DSTC scheme. Based on this observation, we use the pilot insertion ratio of 0.1% for all the simulation setups.

Fig. 5 shows our BER comparisons between the proposed N-DUC and the conventional S-DOSTC schemes. Here, we considered the $(M, N) = (2, 2)$ and $R = 8$ [bps/Hz] scenario. The mobile speed was set to $v = 0$ [km/h]. As shown in Fig. 6, our proposed scheme outperformed the coherent SM and BLAST schemes that have PCSI for SNR $\gamma \geq 25$ [dB]. The achievable diversity order of the proposed scheme having $T = 1$ is calculated as $\sqrt{N\cdot T} = 2$. Thus, the conventional S-DOSTC scheme having $T = 2$ outperformed the proposed scheme for SNR $\gamma \geq 50$ [dB]. By contrast, in Fig. 6, the proposed scheme exhibited an error floor formation for the $v = 5$ [km/h] case, which is a typical walking speed [77]. This implies that the proposed concept is not effective for small-scale scenarios, and it is recommended to use the conventional square schemes instead of the nonsquare schemes.

Similar to Fig. 6, in Fig. 7, we compared the proposed N-
DUC and the conventional S-DOSTC schemes, where we had \( M = 4 \) transmit subarrays and \( N = 4 \) receive subarrays. The transmission rate was \( R = 12 \) [bps/Hz], and the mobile speed was set to \( v = 0 \) [km/h]. As shown in Fig. 7, the N-DUC scheme having the \( P = 4 \) DFT basis and the ADSM scheme having the \( P = 2 \) GSP basis achieved a similar trend to the coherent SM and BLAST schemes. As in Fig. 6, the proposed forgetting factor \( \alpha_q(i) \) achieved some modest gains against the conventional forgetting factor \( \alpha_c \), while \( \alpha_v(i) \) does not require any high-complexity exhaustive search. Note that the ADSM scheme having the DFT basis exhibited severe performance loss, as anticipated in Section IV-A.

Overall, in the small-scale, high-rate and time-varying scenarios, our proposed scheme exhibited error floors, as shown in Figs. 6 and 7. This performance loss was caused by the recursive construction of Eq. (13). Specifically, when the number of receive subarrays is small, \( Y(i) \) in Eq. (13) contains less information concerning the channel coefficients, and \( \widetilde{Y}(i) \) becomes different from \( \mathbf{H}(i)\mathbf{S}(i) \).

### B. Large-Scale Scenarios

Secondly, we conducted \( J_{\text{MSE}} \) and BER comparisons for large-scale scenarios, where the numbers of transmit and receive subarrays were increased up to \( M = N = 16 \).\(^{12}\) Here, the number of antenna elements was as large as \( 4 \cdot M = 48 \), which required about 30 [cm] of space. Note that the MED comparison of large-scale scenarios was omitted, because the achievable gains became explicit in Fig. 4.

In Fig. 8, we compared \( J_{\text{MSE}} \) of Eq. (16), which has a dominant effect on the achievable BER performance. Here, we considered the conventional static forgetting factor \( \alpha_c \) of Eq. (15), the proposed adaptive forgetting factors \( \alpha_q(i) \) of Eq. (29), and \( \alpha_v(i) \) of Eq. (30). The conventional static forgetting factor \( \alpha_c \) was designed for minimizing \( J_{\text{MSE}} \) for each simulation parameter set, which was a time-consuming task as described in Section III-C. The numbers of transmit and receive subarrays were \( M = N = 4, 8, \) and 16, which were the same as those used in Figs. 7, 9, and 11. As shown in Fig. 8, the static forgetting factor \( \alpha_c \) achieved the lowest \( J_{\text{MSE}} \) at each point, but it required high-complexity optimization. Here, the proposed forgetting factor \( \alpha_v(i) \) achieved the same \( J_{\text{MSE}} \) as \( \alpha_c \), although \( \alpha_v(i) \) dose not require any previous optimization. Since the adaptive forgetting factor \( \alpha_q(i) \) was only designed for quasi-static channels, i.e., for \( v = 0 \) [km/h], it exhibited severe losses for \( v > 0 \) scenarios. It is worth noting that \( J_{\text{MSE}} \) significantly improved, as the numbers of transmit and receive subarrays increased. For the \( M = N = 16 \) case, \( J_{\text{MSE}} \) was below 10.0 between velocities of \( v = 0 \) and 60 [km/h]. This implies that our proposed scheme is especially beneficial for large-scale scenarios, while the pilot estimation overhead becomes a heavy burden for the conventional coherent MIMO schemes.

Fig. 9 shows our BER comparisons between the proposed N-DUC and the conventional coherent schemes, where the transmission rate was \( R = 4, 8 \) and 12 [bps/Hz]. The BER curves of the differential star-QAM scheme were plotted for reference. The conventional S-DOSTC scheme could only be considered in Fig. 9(a) due to the complexity issue. As shown in Fig. 9, for the \( v = 0 \) [km/h] case, the proposed N-DUC scheme achieved similar BER to the coherent SM and BLAST schemes. As the SNR increased, the performance gap between the conventional coherent scheme and the proposed scheme decreased. This is because the error \( J_{\text{MSE}} \) composed of the noise term \( V(i) \) improves upon increasing the SNR. A comprehensive analysis is provided in [52]. For the \( R \geq 8 \) [bps/Hz] and \( v = 5 \) [km/h] scenarios, the proposed scheme exhibited error floors, while the floors were improved by the
proposed adaptive forgetting factor $\alpha_v(i)$.

In order to investigate the error-floor-mitigation effect of $\alpha_v(i)$, as observed in Fig. 9, we varied the mobile speed from $v = 0$ to 100 [km/h] in Fig. 10. Here, the BER at SNR = 30 [dB] was calculated for each scheme. As shown in Fig. 10, the proposed forgetting factor $\alpha_v(i)$ outperformed the conventional $\alpha_c$ across the entire region. Thus, the proposed $\alpha_v(i)$ is beneficial for high-mobility scenarios.

According to Fig. 8, the error $J_{MSE}$ can be mitigated by increasing the numbers of transmit and receive subarrays. Then, we considered the $M = N = 16$ setup in Fig. 11, where the mobile speed was increased from $v = 0$ to 40 [km/h]. The transmission rate was $R = 16$ [bps/Hz] for the BPSK-aided BLAST scheme, $R = 12$ [bps/Hz] for the BPSK-aided GSM scheme, and $R = 11$ [bps/Hz] for both the N-DUC and N-ADSM schemes, where those different rates were compared vs. $E_b/N_0$ instead of SNR. In Fig. 11, we additionally considered the channel estimation errors of $\mathcal{CN}(0, 1/SNR)$ [78]. As shown in Fig. 11, for the $v = 0$ [km/h] and $E_b/N_0 \geq 0$ [dB] case, the proposed schemes outperformed the coherent BLAST and GSM schemes that have PCSI. Here, the performance of the N-ADSM scheme approached that of the N-DUC scheme, while the detection complexity was reduced by using the $P = 12$ GSP basis. Additionally, for the $v = 40$ [km/h] case, the proposed schemes outperformed the coherent schemes below $E_b/N_0 \leq 2$ [dB], provided that the channel estimation errors existed. This implies that our proposed scheme is capable achieving performance gains below $v = 40$ [km/h], while eliminating the channel estimation overhead. Thus, the proposed scheme is especially beneficial.
for large-scale high-mobility scenarios.

VI. CONCLUSIONS
In this paper, we proposed the novel GSP-aided basis set, which imposes no limitation on the number of nonzero elements $N_b$ and achieves a high coding gain over the conventional DFT basis set. Then, we also proposed a pair of adaptive forgetting factors for the general nonsquare DSTC scheme, that support the general time-invariant and time-varying channels, respectively. The proposed differential mmWave system only relies on ABF both at transmitter and receiver, and does not require the complex-valued channel coefficient feedback, while its frame length becomes longer as the numbers of subarrays and subcarriers increase. In our simulations, the proposed nonsquare scheme achieved similar performances to the classic BLAST and GSM schemes having PCSI. Specifically, the proposed nonsquare scheme achieved similar performances to the conventional static forgetting factor that required time-consuming exhaustive search.

The key feature of the proposed scheme is its high reliability in large-scale scenarios. The BER performance improves upon increasing the number of subarrays, although the channel estimation overhead escalates with the system’s scale. Furthermore, our proposed scheme also performed better than the ideal coherent MIMO schemes for $v \leq 40$ [km/h] scenarios. Based on these observations, we conclude that the proposed large-scale differential GSM system is capable of reliable operation in high-mobility mmWave channels.

APPENDIX A
THE GSP BASIS SETS FOR ADSM
For the $(M, O, L) = (4, 2, 16)$ and $N_b = 2$ case, we used the following basis:

$$E_1 = [0, -0.753584 + 0.112447i, 0, -0.218068 - 0.609525i]^H,$$

which achieved the MED of 0.0638601. Additionally, for the $(M, O, L) = (16, 2, 4)$ and $N_b = 12$ case, we used the following basis:

$$E_2 = [0.145493 + 0.200765i, 0.0319892 - 0.0912603i],
-0.10222 + 0.0536548i, 0.102279 + 0.0610215i],
0.259023 + 0.410747i, -0.220858 + 0.0084931i],
-0.0521544 - 0.115707i, 0.115496 - 0.0806238i],
0.295856 - 0.044537i, 0.377234 + 0.373951i],
-0.0332786 - 0.309139i, 0, -0.0848214 + 0.0480732i, 0, 0]^H,$$

which achieved the MED of 0.877168.

APPENDIX B
THE DESIGNED CONSTELLATION FACTORS FOR DUC
The DUC codewords are designed so as to maximize the diversity product [41]. In this Appendix, we use the same mathematical symbols as [41], such as the number of TAs $M$, the number of codewords $L$, the constellation design factors $[u_1 \cdots u_M]$, and the diversity product $\xi$.

In Fig. 6, we used the factors of $[u_1 u_2] = [1 75]$ for the $(M, L) = (2, 256)$ case, which achieved the diversity product of $\xi = 0.3143625$.

In Fig. 7, we used $[1 575 1059 1921]$ for the $(M, L) = (4, 4096)$ case, which achieved $\xi = 0.5672944$.

In Fig. 9, we considered $M = 8$ and used $[1 5 5 5 5 5 6 7]$ for $L = 16$, $[1 84 87 88 91 91 97]$ for $L = 256$, and $[1 16722 17014 20852 23231 23781 24192 29994]$ for $L = 65536$, each of which achieved $\xi = 0.7168117, 0.5197687, \text{and } 0.2244139$, respectively.

In Fig. 11, we used $[1 446 516 555 609 640 644 712 718 723 724 724 772 787 805]$ for the $(M, L) = (16, 2048)$ case, which achieved $\xi = 0.5681189$.

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