Optical Anapoles

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Preface. The anapole, a non-radiating charge-current configuration, was recently observed in a variety of artificial materials and nanostructures. We provide a brief overview of this rapidly developing field and discuss implications for spectroscopy, energy materials, electromagnetics, as well as quantum and nonlinear optics.

Introduction

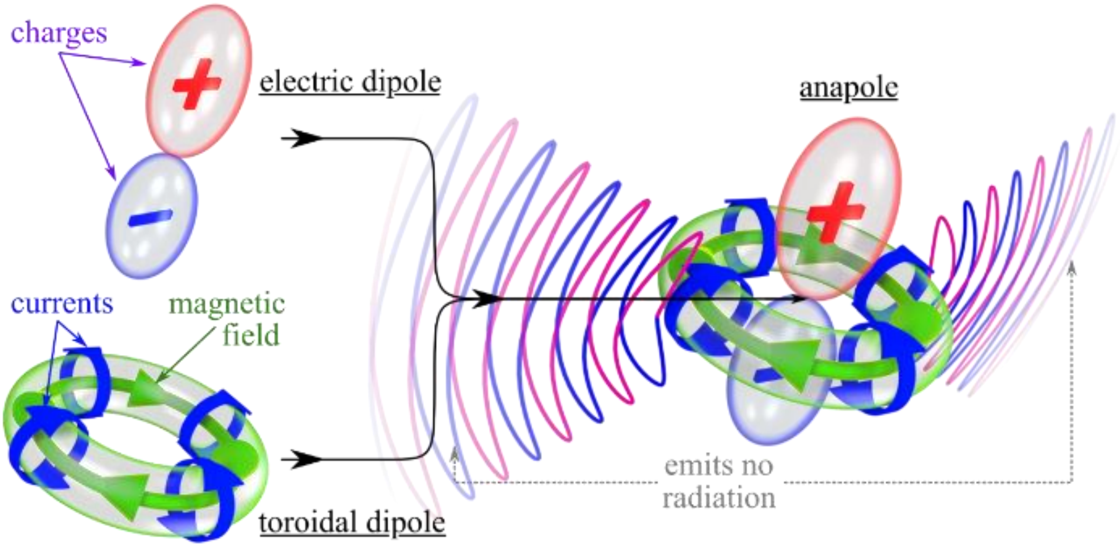

Figure 1. Structure of a dynamic anapole. Anapole is a balanced superposition of electric and toroidal dipoles. Electric dipole corresponds to a pair of opposite charges. Toroidal dipole corresponds to a poloidal current on a torus. Anapole emerges when the fields radiated by the electric and toroidal dipoles cancel each other out.

Toroidal electrodynamics, a new chapter of electromagnetic research is currently attracting considerable and growing attention [1, 2, 3, 4]. It includes the study of toroidal multipoles and anapoles (see Fig. 1). The recent surge of interest in toroidal multipoles is driven by the emerging understanding that alongside the well-known electric and magnetic multipoles they are necessary for a complete characterization of the electromagnetic properties of matter [2]. Indeed, while electromagnetic fields in free-space can be fully characterized with transverse electric (TE) and transverse magnetic (TM) multipoles [5], the characterization of current density requires three multipole series, the electric, magnetic and toroidal multipoles [6, 7] (see Fig. 2). The distinctive role of toroidal multipoles is particularly apparent in the optical properties of matter containing large molecules or structural elements of toroidal symmetry and of size comparable to the electromagnetic wavelength. Dynamic toroidal response of metamaterials had been the subject of intense discussions since 2007 [8, 9], but the first unambiguous experimental demonstration of dominant toroidal response in matter was recorded in a microwave metamaterial in 2010 [1] (see Fig. 3). Subsequently, dynamic toroidal response has been observed in metallic [10, 11, 12, 13, 14, 15], plasmonic [16, 17, 18, 19, 20, 21, 22] and dielectric metamaterials [23, 24] at frequencies ranging from microwave through to terahertz and up to near-infrared/visible parts of the spectrum (see Fig. 3). The analysis of transmission, reflection [12] and polarization phenomena [13] in complex molecular systems and metamaterials is incomplete without account of the dynamic toroidal response. Toroidal resonances could play a role in nano-lasers [19], sensors [25] and data storage devices [3, 26]. We shall also note that static toroidal dipoles, also known as 'static anapoles' introduced by Ya. B. Zeldovich in the context of parity violation in nuclear physics [27], have been observed in magnetism [26] and could be the only allowed electromagnetic form-factor for dark matter candidate particles [28].

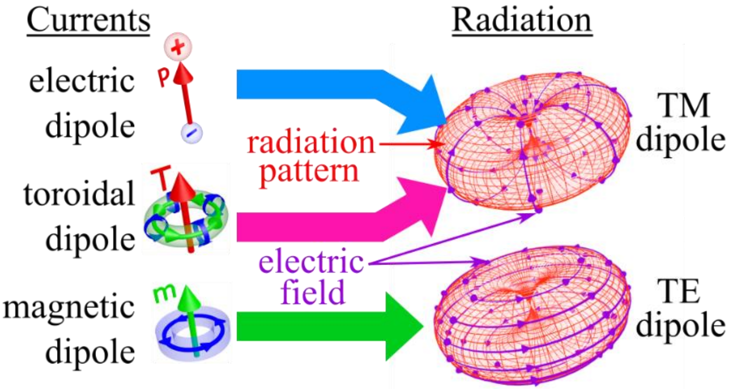

Figure 2. **Radiation patterns of the electric (p), magnetic (m) and toroidal (T) dipoles.** Oscillating electric, magnetic and toroidal dipoles have identical energy radiation patterns (emitted power per solid angle; red shells on the right). However, the electric and magnetic dipoles are distinguishable by the polarization state of the emitted light (TE – transverse electric; TM – transverse magnetic). Purple arrow lines on the surfaces of the shells show the direction of electric field oscillations. The electric and toroidal dipoles are indistinguishable by their far-field emission.

An electric dipole (a pair of oscillating charges) together with a toroidal dipole (oscillating poloidal current on a torus (see Fig. 1)) can form non-radiating charge current configuration, known as 'dynamic anapole' [29, 30, 31, 32]. The anapole state emerges at a particular frequency of oscillations when the fields radiated by the co-located electric and toroidal dipoles cancel each other through destructive interference. Crucially, electric and toroidal dipoles have identical radiation patterns (see Fig. 2), thus the net emission of an anapole is zero. The dynamic anapole, a non-radiating energy "reservoir", has inspired a broad search for anapole excitations in matter (see Fig. 3). Perfect anapoles do not emit or absorb light and therefore cannot be detected by far-field observations. Anapole excitations can only be detected if they are (weakly) coupled to electromagnetic modes interacting with free-space radiation or if they are not perfectly balanced, i.e. the electric dipole emission does not precisely cancel out the toroidal dipole radiation. A slightly off-balance anapole will create a narrow peak in the scattering spectrum. Electromagnetic anapoles were first detected as narrow transmission peaks in the spectra of a microwave metamaterial in 2013 [30]. Since then, a number of alternative nanostructures supporting anapoles were discussed theoretically [31, 33, 34]. Anapole excitations were observed in dielectric nanoparticles [35, 36, 37, 38, 39] and in metallic [40] as well as plasmonic metamaterials [41].

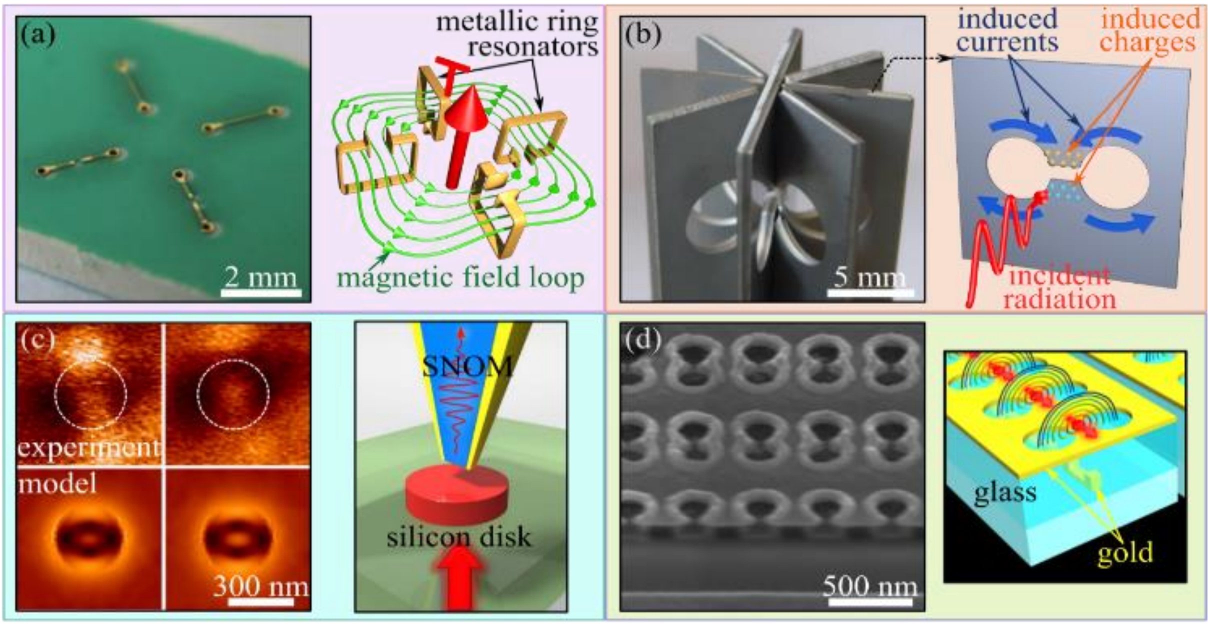

Figure 3. **Pioneering observations of dynamic toroidal and anapole excitations. (a)** 2010. Observation of toroidal resonance. A photograph of the unit cell of microwave metamaterial exhibiting resonant toroidal dipole response, and a schematic of the unit cell supporting toroidal dipole excitation. Closed loops of magnetic field lines are characteristic of the toroidal response[1]. (**b**) 2013. Observation of anapole excitation. A photograph of the unit cell, with 8-fold rotational symmetry, of microwave metamaterial supporting anapole mode of excitation[30]. The schematic shows a fragment of the structure with a dumbbell cut. The light wave polarized across the dumbbell gap induces electric (due to charges) and toroidal (due to currents) response in the structure. The emission of electric and toroidal excitations interferes destructively at the resonant frequency. **(c)** 2015. Observation of anapole mode in a silicon nano-disk[35]. The colour maps show the electric field distribution in the disk, as mapped experimentally and modelled numerically, indicating the excitation of an anapole mode. The field was mapped using scanning near-field optical microscopy, as shown in the schematic. **(d)** 2018: Plasmonic metamaterial supporting anapole mode of excitation[41]. Scanning electron microscope image shows a cross-section of the nanostructure. It consists of a dumbbell-perforated section of the gold film with an additional gold split ring resonator below it. The schematic shows the unit cell of the metamaterial, as well as the sketch of the resonant mode, simultaneously supporting electric and toroidal dipoles.

Detection of anapoles

Recent detection of anapole modes in a diverse range of structures is a significant achievement in the field of toroidal electrodynamics that illustrates the independent physical nature of electric and toroidal dipoles despite them having identical far-field radiation pattern (see recent discussion of this subject in refs. [42, 43]). Indeed, although their far-field emission patterns are identical, the electric and toroidal dipoles correspond to entirely different charge and current distributions (see Fig. 1,2). Furthermore, the oscillating vector potential emitted by electric and toroidal dipoles differs in a way that is irremovable through gauge transforms [29].

The independent physical significance of toroidal and electric dipoles also manifests itself in relativistic electrodynamics. Although fields emitted by electric and toroidal dipoles, when in inertial motion, are identical, linear acceleration of oscillating electric and toroidal dipoles changes the polarization properties of the respective radiated fields in a different way: the

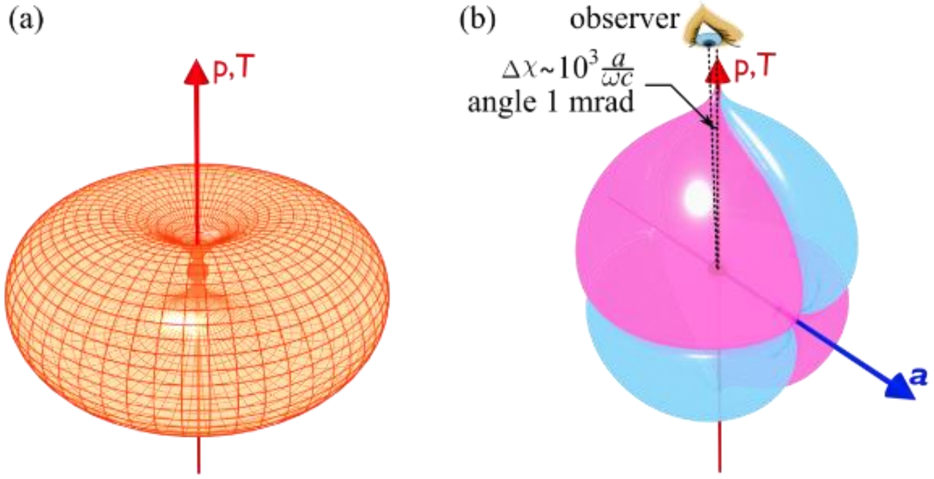

Figure 4. Radiation of accelerated electric (p) and toroidal (T) dipoles. (a) The radiation patterns of non-accelerating electric (**p**) and toroidal (**T**) dipoles, oscillating at frequency ω, are identical. However, emissions of accelerated oscillating electric and toroidal dipoles differ. Pannel **(b)** shows the logarithmic plot of the difference between the ellipticity of emission of electric (χ_{p}) and toroidal (χ_{T}) dipoles accelerated with acceleration **a**. Pink and blue colours corresponds to positive and negative ellipticity difference $\Delta\chi = \chi_{\mathrm{T}} - \chi_{\mathrm{p}}$ (c is the speed of light in the vacuum). Strongest effect due to ellipticity difference will be observable at small angle relative to dipole axis, where radiated power is still significant and ellipticity difference is large.

absolute value of ellipticity of the toroidal dipole radiation becomes greater than that of electric dipole [44]. The difference in ellipticities $\Delta\chi$ diverges along the dipole axis (see Fig. 4). Therefore, an ideal anapole, that is well-balanced to emit no radiation when at rest, will emit light and interact with light, when accelerated.

The effect described above requires extremely high accelerations, and is challenging to observe. However, there is a more accessible way of revealing the difference between the electric and toroidal dipoles. It exploits the difference in coupling of electric and toroidal dipoles to electromagnetic fields in ambient media, and can thus be seen as a form of solvatochromism, a phenomenon of changing the colour of a chemical substance depending on the host-solvent [45]. Indeed, the power (P) emitted by a point-like electric dipole (**p**), and a point-like toroidal dipole (**T**), oscillating with angular frequency ω, depends on the ambient refractive index (n) in a different way [46, 47, 48]:

$$\text{electric dipole:} \quad P_{\mathrm{p}} = \frac{\mu_0 \omega^4}{12\pi c} \cdot n \cdot |\mathbf{p}|^2$$

$$\text{toroidal dipole:} \quad P_{\mathrm{T}} = \frac{\mu_0 \omega^6}{12\pi c^3} \cdot n^5 \cdot |\mathbf{T}|^2$$

where μ_0 is the vacuum permeability and c is the speed of light in vacuum. While the power emitted by electric dipole scales as linearly proportional to the refractive index n, emission of toroidal dipole scales as n^5. Such a difference can be detected by measuring the decay rates of atoms, molecules or artificial metamaterials, exhibiting toroidal and electric dipole resonances, in ambient environments, e.g. solvents, with different indices of refraction [46]. Similarly, contributions from electric and toroidal dipoles can be distinguished by measuring absorption of artificial metamaterials immersed in liquids with different refractive index [49].

Future perspective

The study of anapoles promises some intriguing discoveries. It has been shown that dynamic anapole modes are supported in artificial metamaterials. Could the dynamic anapole be present in organic matter that is often built from molecules with elements of toroidal symmetry such as benzene rings? Indeed, some fullerenes support static anapoles [50]. Moreover, static anapoles have been recently theoretically identified in some cyclic molecules [51], diatomic molecules [52] and chiral molecules [53]. Interactions between toroidal currents allegedly break reciprocity, which could have implications for energy and information transfer at the molecular level and for the dynamics of chemical and biochemical processes [54]. As anapoles are energy reservoirs with a long lifetime, they could be of considerable importance as qubits for quantum technologies [55]. High quality anapole-related resonances can be used in enhancing nonlinear electromagnetic properties of materials [38, 56, 57, 58] and in sensor applications [34, 41]. Matter with high density of anapoles could be an exotic energy storage material from which energy bursts could be released by sudden changes to ambient conditions. Spectroscopy of anapoles presents considerable challenges due to weak coupling to free-space electromagnetic waves, as explained above. However, the use of structured light, most notably space-time non-separable pulses with toroidal topology may help, as they are better suited to drive toroidal excitations than transverse pulses [59]. Alternatively, toroidal and electric dipole constituents of an anapole mode could be engaged with electron beam excitations [60].

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