Copyright © and Moral Rights for this thesis and, where applicable, any accompanying data are retained by the author and/or other copyright owners. A copy can be downloaded for personal non-commercial research or study, without prior permission or charge. This thesis and the accompanying data cannot be reproduced or quoted extensively from without first obtaining permission in writing from the copyright holder/s. The content of the thesis and accompanying research data (where applicable) must not be changed in any way or sold commercially in any format or medium without the formal permission of the copyright holder/s.

When referring to this thesis and any accompanying data, full bibliographic details must be given, e.g.

Thesis: Author (Year of Submission) “Full thesis title”, University of Southampton, name of the University Faculty or School or Department, PhD Thesis, pagination.

Data: Author (Year) Title. URI [dataset]
Declaration of Authorship

I, Rebecca Louise French, declare that this thesis titled, “Exploiting disorder for snapshot spectral imaging with complex media and compressive sensing” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed: 

______________________________________________

Date: 

______________________________________________
List of Publications


International Conference Oral and Poster Contributions


Abstract
Faculty of Engineering and Physical Sciences
Doctor of Philosophy
Exploiting disorder for snapshot spectral imaging with complex media and compressive sensing
by Rebecca Louise FRENCH

Seeing beyond the capability of the human eye is an appealing concept, as it allows us to explore new physical phenomena. In particular, extracting wavelength information from a scene, from which we would ordinarily observe a combination of three colours, is advantageous for identifying chemical signatures of materials. Spectral imaging devices allow us to access hidden wavelength information, for example, for environmental sensing and to determine the composition of stars. It is increasingly more popular to produce compact spectral imagers that can be easily transported, however, many current approaches are limited by the dependence of device footprint on spectral resolution. In this thesis, the study of snapshot spectral imaging systems, which acquire spatial and spectral information in a single measurement, are investigated. Rather than using traditional dispersive optics, the disorder of complex media is exploited for wavelength characterisation.

The first of three approaches employs a 1.7 µm multiple scattering layer of gallium phosphide nanowires in combination with a lenslet array to allow simultaneous acquisition of spatial and spectral information. A spectral resolution on order of 4 nm is achieved with a throughput of 18%. Tikhonov regularisation (TR) and Compressive Sensing (CS), or more specifically $l_1$-minimisation, are used to recover spectral information from 64 independent spatial positions in the image. CS is used to reduce data collection at the acquisition stage for efficient processing.

Utilising a bundle of multimode fibres, a second approach is demonstrated by characterising the wavelength dependent speckle patterns produced by up to 3000 independent cores. The spectral resolution of the multicore multimode fibre is shown to be directly dependent on its length, with sub-nanometre spectral resolution achievable. Spectral information is obtained from only 16 pixels in each speckle pattern by employing CS for data acquisition below the Nyquist-Shannon limit. The angle dependence of speckle patterns is also probed to increase the aperture of the system.

Finally, snapshot spectral imaging using a multiple scattering medium and a multispectral transmission matrix is presented. Using a phase retrieval technique and speckle pattern simulations, prVAMP, phase, amplitude and wavelength information are simultaneously reconstructed from one speckle pattern for up to 2 spectral components.
Acknowledgements

“Life need not be easy, provided only that it is not empty.”

Lise Meitner, Nuclear Physicist

During the 4 years of this PhD journey, I have been very privileged to have had a
great deal of support and encouragement from a long list of people.

Firstly, to my supervisors, Otto and Sylvain, I am forever grateful for the opportunity
to work with you both. Thank you for all of your guidance and support.

This PhD would not have been possible without the financial support of the De-
fence Science & Technology Laboratory (DSTL), for which I am very grateful. A
special thank you to my technical partner, Chris Howle, for all of your suggestions,
guidance, and proofreading over the last 4 years.

Thank you to my examiners Riccardo Sapienza and Vasilis Apostolopoulos for tak-
ing the time to read through my thesis, and for making many helpful suggestions to
improve this body of work.

To my Southampton colleagues, Dan, Leo, Simon, Nic, Matt, Bigeng, Wei, Christoph,
Theo, Marilena and Angela, as well as my “office husband” Roman, thank you for
your friendship inside and outside of Office 3073.

To my colleagues in France, Mickael, Hugo, Thomas, Baptiste, Jonathan, Tom, Saroch,
Teng-Fei, Benneth, Hilton, Antoine and Fernando, thank you for welcoming me into
your group every time I came to visit Paris. I will miss the science discussions as
well as Thursday night beers! Special thanks go to Mickael, Jonathan and Fernando
for your help with multispectral imaging. Thank you also to Laurent Daudet for
introducing me to the world of compressive sensing.

To my office mates, Tom, Olly, Elena, Johanna, Elena and Woody, thank you for
brightening up my days in the physics building. Sam and Azaria, managing UCAS
days wasn’t easy, but I’m glad we had each other’s friendship to make it through!
Sanja and Tamsin, thank you for your words of advise and for always lending a
shoulder to cry on.

To the astronomy group, thank you for adopting me as one of your own. To my
friends Georgios, Rob, Katy, Miika, Amy, Andy, James and Fred, Friday evenings at
the pub made PhD life that much easier. To Mat, the world would be a much
darker place without you. Thank you for your friendship and eternal optimism.

A core part of my support network has been thanks to the Women’s Physics Net-
work at Southampton. To Elena, thank you for encouraging (forcing!) me to become
part of the first WPN committee. Without your mentoring and friendship, I would
never have had the courage to take advantage of all of the opportunities the last 4
years have had to offer. To Zoë, chairing the WPN committee with you was one
of the highlights of my time in Southampton. I couldn’t imagine a more kind and thoughtful person to have shared the responsibility with. To Sadie, thank you for your words of encouragement (#YouGotThis) and for our chats before (and during!) every WPN meeting. To my WPN girls, Nina, Charlotte, Magui, Florence, Anna and Angela, and our number one supporter, Pearl, I am so glad that I get to call you my friends.

To my girls at home, thank you for always listening when times are tough, in person or over a message. You have been the best friends I could ever have asked for.

Chris, thank you for being a best friend and for making me smile, even in the most stressful of times. I owe a lot to you for all of your help and support, and I’m so grateful you’ve been able to come on this journey with me. Our travels in Chile are some of my fondest memories from the last few years. I’m looking forward to many more adventures to come.

There are two people in my life who didn’t get to see me finish this journey. Nana and Granny, thank you for all of your kindness and encouragement. I love and miss you both. To Grandpa and Grandad, thank you for all your love and support over the years. I hope I’ve made you proud.

Finally, to Mom, Dad, Anna and Maya, thank you for sharing your grammar expertise and for reminding me of the important things in life. Most of all, thank you for your unwavering love and support.
# Contents

Declaration of Authorship iii

Abstract vii

Acknowledgements ix

1 Introduction 1
   1.1 Spectral imaging ........................................... 2
      1.1.1 Commercial spectrometers .................................. 2
      1.1.2 Scanning spectral imaging systems ....................... 3
      1.1.3 Snapshot spectral imaging .................................. 3
      1.1.4 Dispersion elements in spectral imaging systems .......... 5
   1.2 Complex media in imaging and sensing ....................... 5
   1.3 Reducing data collection and storage in imaging ............ 8
   1.4 Thesis outline .................................................. 9

2 The behaviour of light in complex media 11
   2.1 Multiple scattering ........................................... 12
      2.1.1 Scattering mean free path .................................. 12
      2.1.2 Transport mean free path ................................... 13
      2.1.3 Number of supported modes ................................ 14
      2.1.4 Spectral correlation width ................................ 14
   2.2 Multimode fibres ............................................... 18
      2.2.1 Number of supported modes ................................ 19
      2.2.2 Frequency dependence ...................................... 20
   2.3 Comparison between multiple scattering and modal interference ... 21

3 Computational techniques 23
   3.1 The linear problem ............................................ 23
      3.1.1 Input signal ............................................... 24
      3.1.2 Output signal ............................................. 24
      3.1.3 The transmission matrix .................................. 25
   3.2 Sampling rates ................................................ 27
      3.2.1 Nyquist-Shannon sampling limit ............................. 27
      3.2.2 Oversampling ............................................... 27
      3.2.3 Undersampling ................................................ 28
3.3 Transmission matrix inversion for information recovery ............................................. 28
3.3.1 Method for comparison of computational techniques .............................................. 28
3.3.2 Conjugate transpose ............................................................................................. 30
3.3.3 Moore-Penrose pseudoinverse .............................................................................. 31
3.3.4 Tikhonov Regularisation ....................................................................................... 32
3.3.5 Summary ............................................................................................................... 33
3.4 Compressive sensing ............................................................................................... 33
3.4.1 Compressive sensing in imaging and sensing ......................................................... 34
3.4.2 L_1-norm ............................................................................................................... 36
3.4.3 Summary ............................................................................................................... 38

4 Hyperspectral imaging using multiple scattering media ............................................. 41
4.1 Introduction .............................................................................................................. 41
4.2 Method .................................................................................................................... 43
4.2.1 Experimental set-up .............................................................................................. 43
4.2.2 Multiple scattering nanowires ............................................................................. 44
4.2.3 Calibrating the transmission matrix ...................................................................... 46
4.2.4 Sampling rates ...................................................................................................... 47
4.2.5 Computational techniques ................................................................................... 49
4.3 Characterisation ....................................................................................................... 50
4.3.1 Nyquist-Shannon sampling limit ........................................................................... 50
4.3.2 Noise ....................................................................................................................... 50
4.3.3 Number of wavelengths ....................................................................................... 52
4.4 Spectral imaging ........................................................................................................ 52
4.4.1 Multiplexing spatial and spectral information ....................................................... 52
4.5 Compressive sensing limits .................................................................................... 55
4.5.1 Recovering spectral information from a monochrome image .................................. 55
4.5.2 Recovering spatial information from a monochrome image .................................... 56
4.5.3 Reconstructing multispectral and spatial information ........................................... 56
4.6 Discussion ................................................................................................................ 58
4.7 Summary ................................................................................................................... 61

5 Snapshot spectral imaging with multimode fibres ..................................................... 63
5.1 Introduction .............................................................................................................. 63
5.2 Method .................................................................................................................... 65
5.2.1 Experimental method .......................................................................................... 65
5.2.2 Multicore multimode fibres (MCMMFs) .............................................................. 66
5.2.3 Spectral correlation width .................................................................................... 66
5.2.4 Algorithm for spatial and spectral imaging ........................................................... 68
   Building the spectral intensity transmission matrix .................................................. 68
   DBSCAN ..................................................................................................................... 68
5.2.5 Computational techniques ................................................................................... 70
5.3 Characterisation of fibre imaging spectrometer ....................................................... 70
5.3.1 Nyquist-Shannon sampling limit ........................................ 71
5.3.2 Number of wavelengths .................................................... 73
5.3.3 Noise ........................................................................... 74
5.4 Hyperspectral reconstruction .................................................. 74
5.5 Compressive sensing limit ...................................................... 77
5.6 Angle dependence ............................................................... 79
  5.6.1 Experimental set-up ......................................................... 80
  5.6.2 Angle correlation ............................................................. 80
5.7 Discussion ........................................................................... 81
5.8 Summary ............................................................................. 83

6 Multispectral imaging with a complex medium ......................... 85
  6.1 Introduction ....................................................................... 85
  6.2 Uncovering phase and amplitude information .......................... 87
    6.2.1 The monochromatic transmission matrix ......................... 87
    6.2.2 The multispectral transmission matrix .............................. 88
  6.3 The multispectral imaging problem ......................................... 89
  6.4 Phase retrieval ................................................................. 90
    6.4.1 Phase retrieval techniques for the transmission matrix ...... 91
    6.4.2 The prVAMP algorithm ............................................... 92
  6.5 Simulation of a multispectral complex system ......................... 93
  6.6 Recovering a monochromatic object from a multispectral speckle pattern 95
  6.7 Global reconstruction of spatial and spectral information .......... 99
  6.8 Outlook for multispectral imaging .......................................... 102
  6.9 Summary ........................................................................... 102

7 Conclusions ........................................................................... 105
  7.1 Summary ........................................................................... 105
    7.1.1 Computational techniques ............................................. 105
    7.1.2 Multiple scattering in spectral imaging systems ................ 106
    7.1.3 Multimode fibre spectral imaging .................................... 108
    7.1.4 The multispectral transmission matrix for spectral imaging .. 109
  7.2 Outlook and future work ...................................................... 110
    7.2.1 Wavefront sensing with multiple scattering media ............ 110
    7.2.2 Spectral imaging in the world of Artificial Intelligence ........ 111

A Building a spectral intensity transmission matrix ....................... 113

B Reconstruction of spectral information via inversion techniques .......... 117

C Performing the l_1-norm with the spectral intensity transmission matrix .. 119

D Simulating a multispectral transmission matrix ............................. 121
# List of Figures

1.1 Traditional spectrometer designs. ........................................... 2  
1.2 Scanning and snapshot spectral imaging data cubes. ............... 3  
1.3 Integral Field Spectrometers. ........................................... 4  
1.4 Complex media in spectroscopy. ......................................... 6  
1.5 Traditional imaging versus imaging with a complex ‘lens’. ........ 8  
2.1 Multiple scattering in complex media. .................................. 14  
2.2 Spectral dependence of multiple scattering materials. ............. 17  
2.3 A comparison between a single mode fibre and a multimode fibre. 19  
2.4 Spectral dependence of multimode fibres. ............................ 20  
3.1 Schematic of a linear system. ............................................. 24  
3.2 Building a transmission matrix. ......................................... 26  
3.3 Using the transmission matrix. .......................................... 29  
3.4 Intensity transmission matrix reconstruction with the conjugate transpose. ......................................................... 30  
3.5 Intensity transmission matrix reconstruction using pseudo inversion. 31  
3.6 Intensity transmission matrix reconstruction using TR. ............ 33  
3.7 Unit circles representing norms in different real spaces. ........... 36  
3.8 Intensity transmission matrix reconstruction using CS. ............. 38  
4.1 Experimental set-up for nanowire spectral imaging system. ....... 43  
4.2 Camera images depicting speckle pattern grids. ....................... 44  
4.3 Characterising the MSM. ................................................... 45  
4.4 Measuring the spectral intensity transmission matrix. .............. 46  
4.5 Schematic of the nanowire spectral imaging system. ............... 48  
4.6 Reconstructing spectral information above the Nyquist-Shannon sampling limit. ......................................................... 49  
4.7 Characterising the spectral imaging system for TR and CS. ....... 51  
4.8 Speckle-based spectral imaging. ......................................... 53  
4.9 Using TR and CS to reconstruct spatial information. ............... 54  
4.10 Reconstructing spectral information above and below the Nyquist-Shannon sampling limit. ............................................. 55  
4.11 Reconstructing spatial information from an image containing one wavelength. ......................................................... 56  
4.12 Camera images depicting snapshot measurements. ............... 57
4.13 Spectral reconstruction of composite image containing 3 wavelengths. 58
4.14 Spatial reconstructions when sampling above (Y/X=14.2) and below (Y/X=0.8) the Nyquist-Shannon limit. 59

5.1 Fibre imaging spectrometer experimental set-up. 65
5.2 Schematic of fibre imaging spectrometer. 66
5.3 Spectral correlation width of the spectral imaging system. 67
5.4 Dependence of fibre length on spectral correlation bandwidth. 68
5.5 Schematic of STM construction. 69
5.6 Characterisation of fibre imaging spectrometer. 71
5.7 Spectral imaging system robustness to noise. 73
5.8 Schematic of hyperspectral imaging experiment. 74
5.9 Spectral reconstruction from composite image containing 16 wavelength components. 75
5.10 Spatial reconstruction of 16 letters. 76
5.11 Reconstructing information below the Nyquist-Shannon sampling limit. 77
5.12 Reconstructions of monochromatic object using two different sampling rates. 78
5.13 Experimental set-up to probe angle dependence of MCMMF. 79
5.14 Angle dependence of speckle patterns. 80
5.15 Dependence of angle correlation width variation, $\delta \theta$, with incident angle of light, $\theta$. 81

6.1 Measuring a transmission matrix and reconstructing spatial information. 87
6.2 Schematic of multispectral imaging through a complex medium. 93
6.3 Measuring the MSTM. 95
6.4 Reconstructing a multispectral object containing 2 wavelength components. 96
6.5 Reconstructing a multispectral object containing 3 spectral components. 97
6.6 Schematic of MSTM for a global multispectral recovery. 98
6.7 Schematic of the global multispectral system. 99
6.8 Reconstruction of a multispectral object containing 2 spectral components using a global approach. 100
6.9 Reconstruction of a multispectral object containing 3 wavelength components. 101

7.1 Shack-Hartmann wavefront sensor 111
7.2 Speckle spectrometer neural network 112
# List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCD</td>
<td>Charge-Coupled Device</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal Oxide Semiconductor</td>
</tr>
<tr>
<td>CS</td>
<td>Compressive Sensing</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous Wave</td>
</tr>
<tr>
<td>DMD</td>
<td>Digital Micromirror Device</td>
</tr>
<tr>
<td>ECDL</td>
<td>External Cavity Diode Laser</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>MCMMF</td>
<td>Multi Core Multi Mode Fibre</td>
</tr>
<tr>
<td>MSTM</td>
<td>Multi Spectral Transmission Matrix</td>
</tr>
<tr>
<td>SLM</td>
<td>Spatial Light Modulator</td>
</tr>
<tr>
<td>STM</td>
<td>Spectral intensity Transmission Matrix</td>
</tr>
</tbody>
</table>
For my family
Chapter 1

Introduction

“If we assume we’ve arrived: we stop searching, we stop developing.”

Dame Jocelyn Bell Burnell, Astrophysicist

Over the last century, we have seen the rapid development and growth of the technological world. The age of space exploration, the development of the internet and the World Wide Web, and the invention of personal computers are some of the many advancements that have dramatically changed the way we live. While many breakthroughs have been made, our demands are now moving towards inexpensive, compact technologies that combine many instruments into one portable device. Out of all of these technologies, possibly the most pronounced redesigns in recent years has been that of cameras. From film-based acquisition to the development of “digital” charge-coupled devices (CCDs) and complementary metal-oxide semiconductor (CMOS) cameras in the second half of the 20th Century, taking photographs has never been more easy. In fact, with so many of us having access to cameras on our phones, Oxford Dictionaries named “selfie” the “Word of the Year” in 2013, in reference to the popularity of people taking photos of themselves.\(^1\) It is almost impossible to imagine a world without this technology at our fingertips.

With this fascination in camera technology also comes the substantial investment in many forms of imaging and sensing. It is increasingly more desirable to take pictures of subjects beyond the capability of the human eye. While colour cameras are readily available to us, these only allow us to see within broad colour bands in the visible regime, mimicking our vision. However, numerous schemes rely on the measurement of wavelengths of light that we can’t detect with our eyes, for instance, environmental sensing and threat detection. Many materials, from solids to gases, emit different spectral signatures, which can be used to identify the chemicals they are made from. The cameras designed to detect these signatures are called spectral imaging systems. They combine the use of spectroscopy and imaging techniques to uncover hidden wavelength information in scenes, with many relying on the integration of computational techniques to improve the efficiency of image acquisitions.

In this thesis, I will present three different spectral imaging techniques that incorporate novel wavelength characterisation components to obtain both spatial and spectral information in one measurement. Computational techniques will be used to minimise data collection at the acquisition stage, to avoid computational bottlenecks for fast processing of spectral images.

1.1 Spectral imaging

1.1.1 Commercial spectrometers

Spectrometers are used for a multitude of detection purposes, from measuring the composition of stars and planetary atmospheres, to chemical identification of unknown substances. Spectroscopy systems commonly consist of a spectral dispersion element, such as a prism or diffraction grating, and a detector, as shown in Figure 1.1. Spectrometers function by capturing light from an object of interest through a narrow slit before spectrally separating the light into narrow wavelength bands. As only spectral information is decoded, the information obtained does not give any indication of the spatial features of the probed subject.

In order to capture both spatial and spectral information simultaneously, a spectral imaging system can be used. A wavelength characterisation component in combination with a 2D imaging array can be employed to capture a 3D spatiotemporal data cube. There are various classifications of spectral imaging that determine the resolution of spectral information obtained, including multispectral and hyperspectral imaging. Multispectral imaging relates to broadly-spaced wavelength bands, and hyperspectral imaging performs high spectral resolution measurements in narrow spectral bands. As an additional dimension must be obtained during spectral imaging measurements, it is necessary to separate the measurement up, either by collecting multiple 2D slices of the information sequentially, or pre-calibrating the
1.1. Spectral imaging

Scanning and snapshot spectral imaging data cubes. (a) Scanning system measures 2D slices of a data cube in each measurement corresponding to one spatial and one spectral dimension \((x, \lambda)\). (b) Snapshot systems measure 3D of information within a single acquisition.

...system before measuring the 3D information simultaneously. This leads to two distinct forms of spectral imaging approaches: scanning systems and snapshot systems.

1.1.2 Scanning spectral imaging systems

Some of the most established scanning systems are based on ‘pushbroom’ and ‘whiskbroom’ techniques \([1]\). Pushbroom systems are comprised of a slit to capture a 2D slice of the scene in one spatial dimension and one spectral dimension. The system is then scanned across the remaining spatial dimension to build up a 3D image, as shown in Figure 1.2(a). Whiskbroom systems also rely on spatially scanning a scene, but instead a 1D slice is measured in the spectral domain and an image is built up by point scanning over 2 spatial dimensions. Due to the time-sequential measurements performed by both of these systems, they rely on stabilisation platforms in order to prevent blurring of images during acquisition. However, pushbroom and whiskbroom spectral imaging system are commonly used in remote sensing on satellites and aircrafts, where mobile systems can be employed.

1.1.3 Snapshot spectral imaging

The first examples of snapshot spectral imaging were shown in the first half of the 20\(^{th}\) Century, however, the development of these systems has seen rapid acceleration in recent years \([2]\). Snapshot systems measure both spatial and spectral data in a single measurement. These systems enable measurements with a high degree of accuracy due to the stability gained with decreasing measurement acquisition times when collecting a full 3D datacube of spatial and spectral, as illustrated in
Figure 1.3: Integral Field Spectrometers (IFS). (a) Slicing mirror IFS collects light from a source before ‘slicing’ the image using mirrors angled in different directions. The different segments of the image are projected onto a diffraction grating (DG). (b) Fibre bundle IFS spatially filters incoming light. DG is used to obtain spectrum from each fibre. (c) Lenslet array separates out incoming light and DG uncovers spectrum for each portion of filtered light.

Figure 1.2(b). However, snapshot imaging systems rely on the calibration of large amounts of data prior to image collection.

The earliest snapshot techniques, developed for use in astronomy, are frequently used to carry out fast measurement of celestial objects. These systems, more commonly known as Integral Field Spectrometers, are comprised of a spatial filtering component, such as slicing mirrors, lenslet arrays, or fibre bundles, and a wavelength characterisation element, such as a grating or a prism, as illustrated in Figure 1.3 [2]. The original mirror-based systems rely on mechanically sophisticated arrangements, which are difficult to calibrate and expensive to build, however, large amounts of spectral data can be acquired using these methods [3]. Fibre bundles were later introduced to efficiently capture light, before coupling to a dispersive spectrometer [4]. Difficulty coupling into small diameter fibres (single mode) and modal noise produced by large diameter fibres (multimode) leads to some difficulties with alignment and detection, however, the high throughput of the fibres makes them desirable for detection of low intensity objects. The design of lenslet array snapshot spectral imagers allows many lenses to spatially filter incoming light, before light is projected on to a diffraction grating [5, 6, 7]. As with the other approaches, a spectrum can be obtained for each spatially filtered portion of light. These approaches are used at many world-leading telescope facilities including at the William Herschel Telescope and the Very Large Telescope.

More recently, new spectral imaging approaches have been developed, with many building on the foundations laid out by the astronomy community, such as Image Mapping Spectroscopy [8]. A novel technique known as Coded Aperture Snapshot Spectral Imaging (CASSI) was designed to take advantage of a new computational approach to reduce data collection at the acquisition stage, where an estimated data cube is measured from the binary projections acquired [9, 10]. CASSI allows high spectral resolution using a wide aperture, due to the multiplexing of spectral information across all spatial positions on the detector, unlike traditional spectroscopy
which requires a restricted aperture. CASSI is leading the way for spectral imaging with reduced data acquisition [11, 12, 13, 14, 15].

While snapshot spectral imaging approaches have many advantages over scanning techniques, there are still some tradeoffs to consider. Experimentally, snapshot systems require advanced experimental arrangements that utilise many new technologies. As well as that, significant amounts of data are needed to determine all spatial and spectral components within a single measurement, which is computationally demanding to process. However, the fast measurement times of snapshot spectral approaches are a key advantage over scanning systems, meaning that time-dependent measurements can be easily acquired.

### 1.1.4 Dispersion elements in spectral imaging systems

The majority of the spectral imaging approaches described thus far rely on dispersive media to spectrally characterise input light. However, the spectral resolution of dispersion-based systems is directly dependent on the footprint of the device. Therefore, a larger distance between the dispersion element and the detector results in a higher spectral resolution, and vice versa. However, if a high resolution is desired, the device may not be easily portable. It is therefore beneficial to determine a method of spectroscopy that removes device size dependence on spectral resolution.

Many techniques have been suggested to remove the size dependence of dispersive media with spectral resolution, using thin-film filters to non-periodic diffractive optics. Filters permit narrowband light to be transmitted, while blocking other wavelength regions from detection using particular absorption or reflection properties of the filter materials [16]. This procedure allows 2D images to be measured in single wavelength bands, however, it requires sequential scanning across all desired spectral components by changing the filter using a filter wheel [17]. As an alternative, diffractive optics utilise the surface scatterers of precisely etched structures to produce wavelength dependent intensity patterns [18, 19, 20, 21]. These components allow high transmission of light, meaning they are advantageous in spectroscopy and spectral imaging. However, diffractive elements require simulations to be performed at the production stage in order to meticulously design and fabricate the structures. A more desirable approach would be to make use of natural materials that don’t require expensive and time-consuming fabrication processes. One avenue to explore is the use of complex media, exploiting their natural disorder in order to achieve high spectral resolution, compact and cost-effective solutions for spectral imaging.

### 1.2 Complex media in imaging and sensing

In imaging and sensing, the goal is to achieve image acquisition that performs as well as, or better than, the human eye. In order to build new imaging systems, it is
crucial to understand the behaviour of light, for instance, for biomedical imaging. When looking through biological tissue, however, it is almost impossible to transmit visible light through to see underneath the skin. Multiple scattering limits the depth light can penetrate into tissue before it becomes too disperse. It has, therefore, been the goal of scientists to minimise the effects of scattering in imaging.

In pursuit of imaging through multiple scattering materials, techniques have been developed such as optical coherence tomography (OCT), and have utilised wavelengths of light that are not attenuated by the medium, such as X-rays. However, for many applications, it is still desirable to image using visible light. As well as being easy to use with inexpensive detector production, light in the visible regime is also more safe for human exposure than other regions of the electromagnetic spectrum. However, in many applications, multiple scattering media appear to scramble light that interacts with them, resulting in a seemingly random interference pattern, also known as a ‘speckle’ pattern. Rather than focusing our attempts on minimising its effects, new ideas have been developed to understand how we can utilise multiple scattering. While multiple scattering media significantly distort the original signal, the information is preserved. Furthermore, using modern experimental equipment and computational techniques, this information can be retrieved.

It is well known that intensity patterns, or ‘speckle’ patterns, produced by light travelling through a multiple scattering medium are wavelength dependent [22, 23, 24]. That is, the arrangement of ‘speckles’ within one of these patterns changes as the input wavelength of the light is varied. These unique speckle patterns behave as the input wavelength’s fingerprint. Provided that the input position of light remains constant, we can characterise wavelength-dependent speckle patterns in order to identify an arbitrary wavelength after light has travelled through the multiple scattering medium at a later stage, as illustrated in Figure 1.4, and in References [25] and [26].
Recently it was shown that a compact spectrometer could be developed using the complex wavelength dependent speckle patterns of a multimode fibre [27, 28, 29, 30]. Demonstrated initially by Redding et al., the spectrometer used a spectral intensity transmission matrix composed of wavelength dependent speckle patterns to characterise different sources of light after travelling through the medium, with a resolution of up to 1 pm for a narrow bandwidth, and 1 nm for a broadband acquisition [27, 31, 32, 33]. Furthermore, it was illustrated that this procedure could be extended to a disordered photonic chip, showing promise for multiple scattering media in spectral characterisation [34]. Other recent examples of speckle-based spectrometers have utilized the memory-effect or principal-component analysis of speckle patterns to achieve novel spectrometer designs [35, 36]. As the spectral resolution of these spectrometers depends only on the scrambling strength of the material being used, and not the spacing between the detector and grating as in commercial spectrometers, the system can be completely scaled down in size. This technique has paved the way for compact, high resolution, speckle-based spectrometers.

Over the last decade, there have been many exciting developments involving light control in multiple scattering media [37, 38]. Many of these breakthroughs have been made possible by the introduction of spatial light modulators (SLMs) to control light. Iterative approaches utilising SLMs to focus monochromatic light through complex media have paved the way for the use of multiple scattering media as lenses [39]. Further to this result, the methodical approach of mapping input degrees of freedom to an output intensity image, in order to measure the monochromatic transmission matrix of a multiple scattering medium, further propelled the use of multiple scattering media in imaging and sensing [40, 41]. Gaining access to the phase and amplitude of light after travelling through an multiple scattering medium using a transmission matrix has led to controlled focusing and imaging through complex media [42, 43, 44, 45]. The realisation that spatial manipulation of light can be achieved has opened up new windows of opportunity for light control across many other complex media, including multimode fibres [46, 47, 48, 49, 50].

With the control of monochromatic light in complex media, the natural extension is to achieve full spectral and spatial control of complex media. The multispectral transmission matrix (MSTM) was built based on the protocol for the monochromatic case, using the knowledge that different wavelengths of light do not interact [51, 24, 23]. The MSTM was measured to allow selective control of many spectral components for focusing [52]. Spatiotemporal control of an ultrashort broadband pulse was later achieved by individually applying the MSTM to each spectral component of an input light source [53, 54].

The concept of using speckle to characterise wavelength in novel spectroscopy devices has also been extended to spectral imaging. It was shown that spectral imaging could be carried out by exploiting the imaging response of a multiple scattering component, also known as the point spread function (PSF) [55]. The spatial and spectral multiplexing of light using the PSF of the system allow spectral imaging of
images over broad wavelength ranges with large calibrated spectral bands \cite{56}.

With this new pursuit into making opaque multiple scattering media ‘transparent’, new techniques to harness the properties of these materials for technological advancement have been realised. Despite the initial appearance of complex media, we now know that disorder can be exploited for applications in imaging to biocompatible lasing \cite{57, 58, 59, 60}. In this thesis the properties of complex media are harnessed for snapshot spectral imaging, utilising the wavelength sensitivity of speckle patterns.

1.3 Reducing data collection and storage in imaging

A problem faced in snapshot spectral imaging is the amount of information to be acquired within a single measurement, in order to resolve a signal. Processing a 3D data cube in one measurement is challenging, due to the significant computational power required. Approaches to minimising computational bottlenecks include either limiting the working range of the spectral imaging device, or using a scanning spectral imaging device instead. However, one avenue to be explored is the use of computational algorithms for streamlined data acquisition and processing to avoid the restrictions imposed by the alternative solutions.

The use of random media in controlling information has followed our understanding that disorder provides new opportunities for highly multi-mode data processing \cite{38, 61, 62}. In fact, it has been shown that multiple scattering media can make computational processing faster and more efficient \cite{61}. Large volumes of information can be stored within speckle patterns, as many complex media have the ability to mix all input degrees of freedom. Therefore, in theory, all information about an input signal can be contained within a small region of a speckle pattern, as illustrated in Figure 1.5(b). This principle was exploited by Liutkus et al. to build
1.4 Thesis outline

The ultimate goal of the work presented in this thesis is to produce new methods of portable snapshot spectral imaging that can be utilised for a broad range of applications. Building on the field of research into Integral Field Spectrometers, two spectral imaging approaches are presented: one using a lenslet array, and the other using a fibre bundle. As an alternative to dispersive optics, complex media are utilised to remove the spectral dependence of the system on the device footprint, in order to produce more compact devices. The use of CS is explored in combination with speckle pattern processing for optimised collection and storage of data. Lastly, an approach using the MSTM of a multiple scattering medium for multispectral imaging will be discussed. If both spatial and spectral information can be mapped onto a single speckle pattern, it avoids the need for spatial filtering. This transmission matrix approach can then be used to uncover the behaviour of information travelling through multiple scattering media. By exploiting the properties of complex media, full spatial and spectral acquisition in a single measurement can be achieved. The organisation of this thesis is outlined below.

Chapter 2 introduces the the underlying concepts of this thesis, with an overview of the behaviour of light transport in complex media. This chapter delves into the frequency dependence of complex media and describes how the properties can be harnessed for spectral sensing.

In Chapter 3, an overview of the role of the linear equation in imaging is given. The concept of spectral intensity transmission matrices of complex systems is described, as well as the mathematical approaches that can be used to recover information in conjunction with a transmission matrix. A modern algorithm to allow sampling of information below the traditional sampling threshold is introduced, and a comparison is carried out between the different computational approaches.

Chapter 4 presents a new technique for snapshot spectral imaging using a multiple scattering medium as a spectral characterisation component. Spatial filtering
of incoming light is achieved using a microlens array, in order to multiplex spatial and spectral information. An inversion technique and a compressive computational technique are employed in combination with a spectral intensity transmission matrix (STM) to achieve reconstruction of spectral objects. The sampling rate required for efficient recovery is probed, and the benefits of minimising data collection at the acquisition stage are presented. The results showcased in this chapter are published in References [68, 69, 70].

Following the work by Redding et al. on speckle spectrometers using multimode fibres, Chapter 5 extends this concept to a fibre bundle for high transmission spectral imaging [27]. Using a clustering algorithm, independent speckle patterns produced by up to 3000 fibre cores are identified and a spectral intensity transmission matrix is built. A computational technique is used to minimise the amount of data required for measurement and storage of spectral images. In order to characterise the system further, the angle dependence of the multimode fibres is investigated to effectively increase the aperture of the spectral imaging device. The results presented in this chapter are published in Reference [71].

Chapter 6 explores the reconstruction of spatial and spectral information from a single speckle pattern for snapshot spectral imaging without prior spatial filtering. This recovery problem deviates from a linear system, and therefore a novel approach is required to allow the recovery of phase, amplitude and spectral information in a single snapshot measurement. Using a simulation of the MSTM approach laid out in [52], and an advanced computational technique, known as ‘phase retrieval’, multispectral images are reconstructed.

A summary of the work presented in this thesis can be found in Chapter 7. The outlook of spectral imaging using complex media is discussed, and future work, which builds on these techniques, is proposed.
Chapter 2

The behaviour of light in complex media

“Humans are allergic to change. They love to say, “We’ve always done it this way.” I try to fight that.”

Grace Murray Hopper, Computer Scientist & US Navy Rear Admiral

The incorporation of light into technology has proven fundamental in shaping the world today. Light is used in many devices, from the displays in our smartphones to the transportation of the internet across the globe. When we can predict and understand the behaviour of light, it can be utilised. However, when light behaves in seemingly unpredictable ways, we endeavour to find ways to eliminate it. This could not be more true than in reference to the behaviour of light in complex media.

In many interactions, a finely structured intensity pattern is produced, known as a ‘speckle pattern’. These outwardly random patterns were thought to be the remnants of lost information that were considered to be irretrievable. However, it has been determined that speckle patterns preserve information about the signal prior to transmission through complex media. In fact, the original input signal can be recovered if either the behaviour of the medium is characterised prior to the signal transmission, or a powerful computational technique is used.

The science behind optically simple components, such as lenses and mirrors, has been well understood for hundreds of years. However, it was towards the end of the 20th Century that it was first hinted that complex interactions could be deciphered. Freund carried out pioneering work into the field of multiple scattering, in which it was determined that speckle patterns could be used in high precision optical instruments, rather than be seen as a nuisance [72]. In research investigating the manipulation of ultrasonic waves, time reversal symmetry was exploited to focus ultrasound waves through disordered media [73]. Later, this was extended to electromagnetic waves, moving towards increasing transmission through complex materials using wavefront shaping techniques, and converting opaque materials into lenses for precision focusing [74, 39, 75, 42, 43]. This realisation that complex media
can be understood has opened up a new realm of possibilities for future technologies.

In this chapter, I will discuss the concepts behind two types of complex media: multiple scattering materials and multimode fibres. The scrambling properties of each of these structures are introduced, and their use in imaging and sensing is presented.

### 2.1 Multiple scattering

Much of the world around us is visible thanks to light scattering effects. The sky appears blue throughout the day due to Rayleigh scattering of light by particles in the atmosphere, and rainbows are formed when light from the sun is scattered by water droplets. Many of these phenomena are apparent due to single scattering, although there are other media which scatter light many more times than this. When light is reflected by, or travels through materials such as white paint, milk, and clouds in the sky, it makes them appear opaque. This is caused by multiple scattering.

For many areas of research it is valuable to be able to see through multiple scattering media. From biomedical imaging through tissue, to monitoring the skies for defence purposes, much work has been done to find order in the disorder of multiple scattering media. In pursuit of optimisation of light transmission in multiple scattering media, some interesting properties of these media have been realised. In this section, the characteristics of multiple scattering media will be introduced, and the ways in which these properties can be utilised are presented.

#### 2.1.1 Scattering mean free path

An important factor to consider in scattering regimes is the scattering mean free path $l_s$. This defines the average distance that light can travel before interacting with a scatterer. This parameter is dependent on the density of the scatterers $\rho$, and their scattering cross-section $\sigma_s$:

$$ l_s = \frac{1}{\rho \sigma_s} $$

There are three different types of light transport regimes in scattering media: ballistic, single scattering, and multiple scattering. Ballistic transport occurs when light propagates through the medium without being scattered. The amount of ballistic light decreases exponentially as the scattering mean free path of a material is reduced. The relationship follows the Beer-Lambert law, $I(r) = I(0) \exp \left( -\frac{|r|}{l_s} \right)$, where $I(r)$ is the specific intensity at position $r$, which describes extinction of light travelling in a scattering medium. Even for a strongly scattering medium, a small number of photons are able to travel through the material in a straight line without any deviation from their original course. This principle is used in medical imaging in order to observe tissue and bone, for instance, in optical coherence tomography (OCT).
2.1. Multiple scattering

When \( l_s \approx L_{MSM} \), where \( L_{MSM} \) is the thickness of the medium, single scattering occurs. Light has the opportunity to interact with one scatterer, and so the original path of the input becomes partially distorted. However, the signal is traceable. In the regime where \( l_s < L_{MSM} \), multiple scattering occurs. The probability of light interacting with many scatterers is increased, and the original propagation direction is lost. Due to the difference in path length of the light travelling through the multiple scattering medium, a relative phase shift is introduced. The interference of many coherent light paths within the multiple scattering medium produces a seemingly random speckle pattern, as shown in Figure 2.1(c). These speckle patterns contain information about the original signal, however, the light is scrambled. It will be demonstrated later in this thesis that information can be recovered from speckle patterns.

2.1.2 Transport mean free path

While \( l_s \) describes the scattering ability of a material, it does not account for anisotropic light transport. The transport mean free path \( l^* \) represents the average distance light travels before its direction is completely randomised. \( l^* \) is given by:

\[
  l^* = \frac{l_s}{1 - \langle \cos \theta \rangle},
\]

where \( \theta \) represents the mean of the scattering angle. If \( \langle \cos(\theta) \rangle \) is 0, then the medium is considered to be isotropic. On the other hand, as \( \langle \cos(\theta) \rangle \to 1 \) and \( l^* \to \infty \), the transport is in the forward direction, and therefore the direction of propagation of the light is not fully randomised. \( l^* \) can be used to determine many other light transport parameters for a particular complex medium. The diffusive nature of light travelling through a multiple scattering medium can be described by the diffusion constant, which is dependent on \( l^* \). It is denoted as:

\[
  D = \frac{1}{3} \nu_E l^*,
\]

and is related to the energy velocity \( \nu_E \). The energy transport velocity describes the transfer of energy within scattering media, and takes into account light trapping caused by internal resonances of individual scatterers [76]. Due to the nature of many multiple scattering media, only some of the light entering the material is transmitted, while the rest is reflected. The transmission of light \( T \) through a multiple scattering medium is therefore related to the scattering properties of the medium:

\[
  T \approx \frac{l^*}{L_{MSM}}.
\]

Intuitively, for a medium with a large \( L_{MSM} \), the transmission of the light will
be proportionally small. The relationship in Equation 2.4 for light transport is analogous to Ohm’s law for electronic conduction. However, for experimental comparison $T$ requires a correction factor, due to internal reflections at the boundaries of the scattering layer \[77, 78\].

2.1.3 Number of supported modes

The number of degrees of freedom that a system has can be described by the number of transverse modes it can support. The number of transverse modes defines the number of diffraction limited points in free space within the illumination area on the input of the multiple scattering medium $A$. The number of modes supported in an multiple scattering medium is given by:

$$N \sim \frac{2\pi A}{\lambda^2},$$  \hspace{1cm} (2.5)

where $\lambda$ is the input wavelength of light. The factor of 2 corresponds to independent and orthogonal polarisation contributions. Light coupled to these modes experiences a retardation that changes its phase, while the light remains coherent. This coherence leads to complex interference patterns at the output, earlier introduced as speckle patterns.

2.1.4 Spectral correlation width

The interference of light after travelling through a multiple scattering medium encompasses many short and long range correlations within the resulting speckle pattern. It is essential to understand the behaviour of these correlations so that relationships can be derived between deterministic speckle patterns and the input light.
conditions. One of the most important short range correlations for this thesis is spectral correlation. That is, the correlation between speckle patterns produced by different wavelengths. Here I discuss the origin of these correlations, following the derivation in References [79], [22] and [80]. Consider light propagating through a multiple scattering medium, following the diffusion equation:

\[
\frac{\partial n(r, t)}{\partial t} = D \nabla^2 n(r, t),
\]

where the solution obtained in three dimensions is of the form:

\[
n(r, t) = \frac{1}{(4\pi D t)^{3/2}} e^{-t r^2 / 4 D t}.
\]

This solution represents the probability density of a particle being at position \( r \) at time \( t \).

Similar to the diffusion of particles, multiple scattering of light can be shown to reduce to a diffusion equation for the intensity. The local intensity can therefore be determined using the relationship \( I(r, t) = \frac{v_E}{4\pi} u(r, t) \), where \( v_E \) is the energy transport velocity, and \( u(r, t) \) is the energy density, which is analogous to the probability density \( n(r, t) \) given by Equation 2.7. The intensity distribution is therefore given by:

\[
I(r, t) = \frac{v_E}{4\pi} \frac{1}{(4\pi D t)^{3/2}} e^{-r^2 / 4 D t}.
\]

This equation describes the intensity resulting from a point source at \( r=0 \) and \( t=0 \).

In order to determine the diffuse intensity of the system, a more general description intensity propagator \( H(r, t) \), can be defined, describing the propagation of intensity in the medium. In the microscopic picture, \( H(r, t) \) begins with a scattering event and ending outside the medium. \( H(r, t) \) is equal to the integral over a time-dependent operator that describes the diffuse propagation through the medium, and the Green’s functions of the system described by \( I(r, t) \), which leads to the approximation:

\[
H(r, t) \simeq \frac{1}{t^{-3/2}} e^{-r^2 / 4 t} e^{-t / \tau_a},
\]

where \( \tau_a \) is the absorption length of the medium [79, 22, 80]. Equation 2.9 is the general equation for the intensity propagator with a position \( r \).

Up to now, I have considered a medium of infinite thickness, however, it is of interest for this thesis to consider a slab of finite thickness \( L_{MSM} \). The propagator for slab geometry can be obtained by applying boundary conditions to the diffusion equation at the interfaces of the slab, which are described below. The position vector \( r \) can be expanded further into two components. Inside a slab of width \( L_{MSM} \), the propagator must be considered at depths \( z \) in relation to the surfaces of the medium, as well as at a transpose position, denoted \( r_\perp \). The intensity propagator then becomes \( H(z_{in}, z_{out}, r_\perp, t) \), where \( z_{in} \) denotes the propagation of light travelling into
the multiple scattering medium, and $z_{\text{out}}$ denotes the propagation out of the multiple scattering medium. The resulting $H$ propagator is given by:

$$H(z_{\text{in}}, z_{\text{out}}, r, t) \simeq \frac{1}{t} e^{-\frac{L}{\tau_0}} e^{-\frac{r^2}{4Dt}} \times \sum_{n=1}^{\infty} e^{-\frac{\sigma^2 v^2 Dt}{L_{\text{MSM}}}} 4 \sin \left[ \pi n (z_{\text{in}} + \tau_0) / L_{\text{MSM}} \right] \sin \left[ \pi n (z_{\text{out}} + \tau_0) / L_{\text{MSM}} \right], \quad (2.10)$$

where the thickness of the medium corresponds to $L_{\text{MSM}} = L_{\text{slab}} + 2\tau_0$, where $L_{\text{slab}}$ is the physical thickness of the slab and $\tau_0$ is the extrapolation length beyond the surfaces of the multiple scattering medium, derived from the Milne problem, where $\tau_0 \simeq \frac{2\nu}{3} [81, 82]$. For the discussion of spectral correlations in multiple scattered light, it is convenient to transform $H(z_{\text{in}}, z_{\text{out}}, r, t)$ to momentum $p$ and frequency $\omega$ space. The Fourier transform of the intensity propagator $H(z_{\text{in}}, z_{\text{out}}, p, \omega)$ indicates the profile of the outgoing light after travelling through a multiple scattering medium. A full derivation can be found, for example, in de Boer et al., however, here I focus on the final result [79, 22]. The diffusion propagator of the slab is therefore given by:

$$H(z_{\text{in}}, z_{\text{out}}, p, \omega) \simeq \frac{\sinh [(z_\min + \tau_0)Q]}{\sinh [L_{\text{MSM}}(z_\max - \tau_0)Q]} \frac{\sinh [(L_{\text{MSM}} - z_\max - \tau_0)Q]}{\sinh [LQ]}, \quad (2.11)$$

where

$$Q = (p^2 + \frac{1}{D\tau_a} - \frac{i\omega}{D})^{1/2}, \quad (2.12)$$

and $z_\min$ is the minimum distance between $z_{\text{in}}$ and $z_{\text{out}}$, and $z_\max$ is the maximum distance between $z_{\text{in}}$ and $z_{\text{out}}$. It can be seen that the formation of speckle patterns has a dependence on the diffusive properties of the multiple scattering medium and the thickness of the multiple scattering medium, as well as the frequency and the incoming angle of the incident beam. The latter two dependencies are classified as short range correlations, in other words $C^{(1)}$ correlations, which define the granular nature of speckle patterns.

The $C^{(1)}$ correlation defines a width within which a small range of input frequencies and angles appears to produce similar interference between light paths, leading to a speckle pattern that is not completely de-correlated. For a shift in incident angle, the speckle patterns produced by the multiple scattering medium are shifted by an proportional angle [83]. This spatial dependence of the speckle pattern is known as the ‘memory effect’. The memory effect has been exploited for imaging light reflected from multiple scattering medium surfaces, for imaging around corners [84, 85, 86].

It is of particular interest to study the spectral dependence of light propagating through multiple scattering media. The unique light paths taken by light of different frequencies produce speckle patterns that behave like wavelength ‘fingerprints’, as
2.1. Multiple scattering

\( \lambda_1 \)

\( \lambda_2 \)

<table>
<thead>
<tr>
<th>Intensity (norm.)</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

**Figure 2.2:** Spectral dependence of multiple scattering materials. (a) Two speckle patterns produced by different wavelengths of monochromatic light, \( \lambda_1 \) and \( \lambda_2 \). (b) Spectral correlation function for a multiple scattering medium with \( D=14 \text{ m}^2/\text{s} \) and \( L_{\text{MSM}}=1.7 \text{ pm} \). The full width at half maximum of the correlation curve gives the spectral correlation width \( \delta \omega \) of the multiple scattering medium.

demonstrated in Figure 2.2(a). This behaviour can be seen by observing the correlation function given by:

\[
C(\Delta \omega) = \frac{\langle I(\omega)I(\omega + \Delta \omega) \rangle}{\langle I(\omega) \rangle \langle I(\omega + \Delta \omega) \rangle} - 1, \quad (2.13)
\]

where \( I(\omega) \) is a frequency-dependent speckle pattern in intensity, and \( I(\omega + \Delta \omega) \) is a speckle pattern produced by a frequency shifted by \( \Delta \omega \). The first investigations into the frequency-dependent correlation of speckle patterns was carried out by Feng et al. [23], and the spectral properties have since been utilised in novel spectrometer designs [34].

Following the derivation of de Boer et al. [79, 22], and maintaining the contributions when incoming amplitude and complex conjugate exit the multiple scattering medium with the same angle and wavelength, the correlation function can be written as:

\[
C(\Delta \omega) = \frac{2L_{\text{MSM}}^2 \Delta \omega}{\cosh[L_{\text{MSM}}(\frac{2\Delta \omega}{D})^{\frac{1}{2}}] - \cos[L_{\text{MSM}}(\frac{2\Delta \omega}{D})^{\frac{1}{2}}]}, \quad (2.14)
\]

where the variables are defined above. As shown in Figure 2.2(b), the correlation curve can be used to describe the behaviour of speckle patterns as the input wavelength of light is changed. Furthermore, the width of the correlation curve [87], and hence the spectral correlation width of the multiple scattering medium, \( \delta \omega \), is found
to be:
\[ \delta \omega = \frac{1.46D}{L_{MSM}^2}. \] (2.15)

Therefore, a short interaction length and a large thickness will result in greater sensitivity to changes in input wavelength and position of the light. This implies that the spectral dependence of a material can be tailored by selecting a multiple scattering medium with the appropriate characteristics.

The spectral correlation width is related to the light confinement time within the multiple scattering medium, known as the Thouless time \( \tau_D \), by \( \delta \omega \sim \frac{1}{\tau_D} \). The number of spectral degrees of freedom the speckle pattern has is given by \( N_\lambda = \frac{\delta \omega}{4\pi} \), and this determines the number of speckle grains within the speckle patterns. Furthermore, the contrast \( C \) of the speckle pattern, defining the ratio between the bright and dark speckle grains, is given by \( C = \frac{1}{\sqrt{N_\lambda}} \). The speckle contrast can, therefore, be measured to determine the diffusion properties of the multiple scattering medium [88]. A linear polariser can also be used to increase the contrast by a factor of \( 1/\sqrt{2} \) [51].

The scientific results presented in this thesis are heavily based on the spectral dependence of speckle patterns produced by complex media. The wavelength dependence will be exploited, as in [25, 35, 29, 36, 68], to characterise spectral properties of multispectral objects.

## 2.2 Multimode fibres

Optical fibres are used across the world in fibre optic communication networks and in sensing devices, due to their high transmission over large distances. The acceleration of internet speeds and ease of access to internet services are largely as a result of the use of fibres to transport large amounts of data around the globe. Optical fibres are made from glass that is drawn out to a width similar to that of a human hair, with lengths on the order of \( 10^1-10^7 \) m. Two types of multimode fibre can be manufactured, known as step-index and graded-index fibres. Step-index fibres have a uniform core refractive index, while graded-index fibres cores have a gradually varying refractive index with the highest refractive index in the centre of the core. In this thesis, step-index fibres are studied and utilised.

Fibres are able to transport data across large distances with low losses, thanks to highly efficient data transfer within fibre cores. The large difference in refractive index between the fibre core, \( \eta_{core} \), and the fibre cladding, \( \eta_{cladding} \), where \( \eta_{core} > \eta_{cladding} \), causes total internal reflection of light inside the structure, coercing the light to travel along its length.

The core diameter of a fibre dictates how many modes the structure can support, and, hence, the amount of information it can carry. There are two distinct categories of fibre: single mode and multimode. As suggested in the name, single mode fibres can only support one light path within the fibre, and have fibre core diameters of
around 10 µm. Multimode fibres tend to have larger core diameters, on the order of >50 µm, and can support many modes. Single mode fibres are used to transport light over substantial distances, while multimode fibres are used for high power transmission over short distances. Figure 2.3 illustrates the difference between both classes of fibre. The single mode fibre shows only one mode, or light path, within the structure, which allows only a narrow wavelength range to be transmitted through the fibre. On the other hand, multimode fibres allow broadband transmission of light along many optical paths. However, with many propagating modes comes the increased chance of interference between every optical path. Each propagating mode has a different phase velocity due to differences in incident angle of the light. This leads to large phase delays between each optical path. If the phase delays are sufficiently large, this modal interference leads to the production of a highly structured intensity pattern, similar in appearance to speckle patterns produced by multiple scattering media, provided that the input light is coherent. The speckle patterns produced by multimode fibres are also known as modal noise.

### 2.2.1 Number of supported modes

The number of propagation modes supported by a multimode fibre is approximately equal to the number of speckle grains contained in the speckle pattern at the output. The number of modes in a fibre is given by [89]:

\[
N_m \approx \frac{4}{\pi^2} V^2 = \frac{4}{\pi^2} \left( \frac{\pi D \times (NA)}{\lambda} \right)^2, \tag{2.16}
\]

where \( V \) is the V-number of a step-index fibre, \( D \) is the diameter of a fibre core, \( \lambda \) is the propagating wavelength of light, and NA is the numerical aperture of a fibre.
Chapter 2. The behaviour of light in complex media

surrounded by air, which is given by:

\[ NA = \sqrt{\eta_{\text{core}}^2 - \eta_{\text{cladding}}^2} \]  

(2.17)

This relationship infers that, if light only couples to a small number of modes, the observed output speckle pattern will contain a proportional number of speckle grains. Intuitively, increasing the fibre diameter results in a greater number of supported modes, and potentially a larger number of speckle grains in the resulting speckle pattern. By exploiting the full NA of the fibre, an optimal amount of information can be transmitted along the structure, as the maximum number of modes can be coupled to. However, the transported information becomes mixed.

2.2.2 Frequency dependence

As is the case with multiple scattering media, multimode fibres also produce frequency-dependent speckle patterns, as demonstrated in Figure 2.4. However, due to the fundamental difference in light propagation in multiple scattering media, it is crucial to understand the impulse response of multimode fibres as the input spectral properties of light are changed.

For a step-index multimode fibre the field distribution is given by:

\[ E(r, t) = \sum_{m} \alpha_m \psi_m(r)e^{i(\beta_m(\omega)L - 2\pi\omega t)}, \]  

(2.18)

where \( \alpha_m \) and \( \psi_m \) are the amplitude and phase coefficients, respectively, \( \beta_m \) represents the propagation modes of the fibre, \( L \) is the length of the fibre, \( \omega \) is the frequency of the incident light, and \( I = |E|^2 \) \cite{51}. As in Section 2.1.4, the spectral correlation function for a multimode fibre is of the form in Equation 2.13. As the spectral correlation function for a multimode fibre has been shown to be dependent on the amount of dispersion within the fibre, it is important to establish the difference in propagation time between the shortest and longest propagation paths \cite{51}. The
shortest propagation time occurs for the fundamental mode $\beta_1$, with $t_{\text{min}} = \frac{L_{\text{core}}}{c}$, where $c$ is the speed of light in a vacuum. The longest propagation time is found when light is reflected from the core-cladding boundary at the critical angle $\phi_c$ for $\beta_c$, leading to $t_{\text{max}} = \frac{L_{\text{core}}}{c \phi_c}$. The time difference is therefore approximated by:

$$\Delta t = t_{\text{max}} - t_{\text{min}} \approx \frac{L(NA)^2}{2c \phi_{\text{core}}}. \tag{2.19}$$

As the spectral correlation function is proportional to the inverse of $\Delta t$, the width of the correlation function, known as the spectral correlation width is given by:

$$\delta \omega \sim \frac{1}{\Delta t} \sim \frac{1}{L(NA)^2}. \tag{2.20}$$

The spectral correlation width denotes the frequency shift required of the incident light to significantly change the phase delays inside the fibre, resulting in a change in the output speckle pattern. The relationship between $\delta \omega$ and the physical fibre properties has been used to develop high resolution multimode fibre spectrometers [27, 31, 28]. From Equation 2.20, the spectral sensitivity can be adjusted by increasing the length of the fibre. This feature of multimode fibres has been exploited to achieve picometre spectral resolution by using a 20 m-length of fibre [32].

### 2.3 Comparison between multiple scattering and modal interference

While the physics behind multiple scattering and modal interference is significantly different, some parallels can be drawn between both multiple scattering media and multimode fibres. One of the most obvious similarities is the formation of speckle patterns at the output of both structures. Speckle patterns in multiple scattering media and multimode fibres are formed by interference between light paths within the structure. Phase delays are introduced through scattering events by many particles in multiple scattering media, or between the different modes in multimode fibres. Moreover, any change in the incident light field has the power to manipulate the speckle pattern structure at the output.

In both multiple scattering media and multimode fibres, the scrambling strength can be determined. At first glance, the properties of each medium look entirely different, but the mixing of modes within both media can be related back to the probability of light interfering within them. For multiple scattering media, the scattering strength depends heavily on the diffusive nature of the materials. A small diffusion constant $D$ and large thickness $L_{\text{MSM}}$ leads to an increased chance of interference between each random walk within the medium. Hence, speckle sensitivity and scattering strength can be easily tuned by changing either of these properties. On the
other hand, multimode fibres cannot be characterised based on a particular scrambling constant, such as mean free path. However, similarities can be drawn between multiple scattering medium thickness and the length of multimode fibres. The $NA$ of a multimode fibre also has a bearing on the number of modes propagating within the fibre. Therefore, an increased number of modes within a longer fibre length leads to a greater probability of interference between light paths within the fibre. Hence, the sensitivity of the output speckle pattern can be tailored to the desired application. In both media, the spectral correlation width is strongly affected by the scrambling strength of the structures. This finding will play a role in Chapters 4 and 5.

Another analogy between multiple scattering media and multimode fibres is the dependence of the number of modes each medium can support on the square of the input wavelength. The number of modes coupled to in a multiple scattering medium is also related to the incident illumination area of the material, while those in a multimode fibre are linked to the diameter of the fibre core. This relationship seems intuitive, due to the increased probability of coupling to the complex medium’s modes, but given the striking differences between multiple scattering media and multimode fibres it is interesting to observe. Therefore, this shows that multiple scattering media and multimode fibres may be used almost interchangeably under certain circumstances.

A key difference between multimode fibres and multiple scattering media is the allowed transmission through each medium. The transmission through multiple scattering media is highly dependent on the scattering strength of the material. The shorter the mean free path of the material, the lower the transmission through the material. This means that, in order to achieve a high speckle sensitivity to its input conditions, transmission must be sacrificed. However, while it is difficult to quantify the transmission through multimode fibres in general, due to the defects which vary from fibre to fibre, it is obvious that multimode fibre are designed to transmit a high proportion of light. Fibres are used to efficiently transport light across the globe, or to deliver light in keyhole surgery in a biomedical setting. Hence, a high fibre transmission is desirable.

The complete mixing of all input and output degrees of freedom in speckle patterns is of great interest for sensing applications. It has been shown that these properties can be utilised for faster and more efficient processing of data [62]. These characteristics can be harnessed for new imaging and sensing technologies, allowing for fast computational imaging using compact and cost-effective components.

In the next chapter, the recovery of information from speckle patterns is explored using several computational techniques. The restriction of a minimum measurement sampling threshold is introduced, as well as ways in which this limit can be overcome. Algorithms that can be used to extract information are highlighted, from inversion techniques to modern compressive methods, to enable the reconstruction of spectral and angular input information after travelling through complex media.
Chapter 3

Computational techniques

“Science and everyday life cannot and should not be separated.”

Rosalind Franklin, Chemist & Crystallographer

3.1 The linear problem

In many fields of study, one of the most well-known and, arguably, most probed schemes is the linear system. Linear relationships can be observed throughout physics, from classical mechanics to astrophysics. A linear equation is mathematically defined by the relationship between a variable (or a set of variables) \( x \), and coefficients (or a set of coefficients) \( a \) and \( b \):

\[
ax + b = 0,
\]

(3.1)

and any system that follows this equation can be described as linear.

Another more common form the linear equation can take is given by:

\[
y = Ax + c,
\]

(3.2)

where \( x \) is an input value or set of values, and the output \( y \) is determined by a known operation matrix \( A \) and a constant value \( c \). In graphical terms, \( A \) is a measure of the gradient of a straight line, and \( c \) is the \( y \)-intercept of the plot. This equation is used to approximate many physical systems. If the gradient \( A \) and intercept \( c \) are known, they can be used to determine \( y \) given \( x \), or vice versa.

The research presented in this thesis can be condensed into a generalised view of the linear equation, and so it is beneficial to realise how this applies to an optical system. In this section, I will discuss how the incorporation of complex media into an imaging system can be defined by the equation above, as in Figure 3.1, as well as how complex media, and their disordered nature, can aid imaging systems rather than hinder them.
3.1.1 Input signal

The majority of imaging systems are used to acquire both spatial and spectral information. In other words, they are used to measure different wavelengths of light reflected or emitted from objects, which are either detected with high resolution to identify chemical signatures or grouped together into different colour bands. The input signal explored in this thesis is therefore made up of multidimensional information, with two spatial and one spectral dimension, and so it is valuable to understand how to recover this information during detection. In this thesis, the detection and recovery of spectral and spatial information with the help of a complex medium is investigated.

3.1.2 Output signal

Light detected by an imaging system is captured using either photographic film or an electronic detector, such as a charge coupled device (CCD) or complementary metal-oxide semiconductor (CMOS). In digital photography, in order to collect colour features, a colour filter array can be used to establish the RGB values of the incident light. However, this concept only allows for broad wavelength regions to be grouped and represented as one colour, and therefore fine spectral features are
3.1. The linear problem

lost. In order to encode more detailed wavelength information in an image, another method must be determined.

In Chapter 2, the spectral and spatial dependence of the speckle patterns produced by light travelling through complex media were both introduced. Hence, when using complex materials in imaging devices, the output information $y$ detected by the device takes the form of speckle patterns, which can be measured by monochrome cameras. Speckle patterns behave like unique fingerprints, which can be characterised during a calibration procedure. By incorporating deterministic speckle information into a camera image, information can be extracted about the wavelength components in the original object, with a higher spectral resolution than determined using a colour camera. In this thesis, this concept is used to develop new spectral imaging modalities.

3.1.3 The transmission matrix

To understand how input light transforms from spatial and spectral information into a speckle pattern, a calibration needs to be performed. Transmission matrices describe the linear propagation of light through an optical medium [90]. Light transmission within any medium can be described using Maxwell’s equations, and therefore all input light travelling through an multiple scattering medium can be linearly mapped onto its output. If the input and output properties of light travelling through a scattering object are known, we can begin to understand how the light travels through the system. By mapping input points from a source to their corresponding outputs after travelling through a medium, the transmission matrix can be defined. The matrix is essentially a set of continuity conditions for light travelling from one medium to another. Any object or medium that light can pass through, either totally or partially, has a unique transmission matrix determining the path taken through the medium. For example, the transmission matrix of a thin lens is taken to be $\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$, where $f$ is the focal length of the thin lens.

Some objects do not immediately behave as ideal components in an optical system, such as linear multiple scattering media. Although light can be transmitted through these objects, all of the information appears to be lost when the light is, seemingly, randomly scattered. This scattered light may initially appear as a nuisance, however, the information transmitted through a multiple scattering medium is preserved. The original signal is scrambled by a linear operator: the transmission matrix of the system. If the transmission matrices of multiple scattering media can be measured, speckle patterns can be utilised.

The transmission matrix of complex media has been measured by controlling input light to establish its resulting output speckle pattern. The information it stores may contain data on the spectral properties of the light, or even the how the phase and amplitude of the light is manipulated. If this matrix describes precisely how light travels through a particular material, a signal can be sent through the complex
medium and the scattered light can be rearranged upon exiting the medium \[40, 27\]. This has been achieved by sending known wavelength information, or by modulating the input phase or amplitude of light, and observing how the speckle pattern changes \[27, 42, 39\].

Figure 3.2 shows regions from speckle patterns, each corresponding to different input information, which are subsequently stored in a transmission matrix \(A\). The number of camera pixels in the area of interest (AOI) selected from \(y\) are denoted \(Y\), and the number of input channels are denoted \(X\). As all of the input information in the system is mixed and contained within the output, a sub-region of the speckle pattern can be selected and used to recover all of the input signal. The transmission matrix can be used to, in effect, unscramble the speckle pattern captured by the camera to uncover the original information. Once the matrix has been measured, the medium can behave as a lens for focusing or imaging, or even a method of characterising different wavelengths of light \[42, 55, 27, 91, 92, 39\]. However, it is important to note that the linearity in the former examples of imaging and focusing comes from accessing the complex field from the output camera image. In Chapter 6, the extraction of spatial information from an intensity image with a quadratic non-linear dependence on the input information will be studied.

In the next section, the amount of information from the output signal needed to be stored in the transmission matrix will be discussed. A classical signal processing threshold will be applied to the linear system, for full recovery of an input signal. An alternative outlook to signal processing will also be introduced, in order to limit data collection at the measurement stage.
3.2 Sampling rates

When measuring a transmission matrix, it is important to consider the amount of information required to be stored in order to fully reconstruct an input signal. If the matrix is large, a proportional amount of storage space and computational power is also needed to utilise the stored information. It is therefore desirable to determine the minimum amount of information to be stored in the matrix, to minimise computational bottlenecks.

In this section, the idea of a minimum sampling rate will be discussed, as well as the conditions required to sample a signal above and below this threshold. The effects of the sampling rate on the transmission matrix are also presented.

3.2.1 Nyquist-Shannon sampling limit

In 1949, Claude E. Shannon published work on a sampling theorem that would go on to form an integral part of signal processing. While others had suggested the result years beforehand, Shannon derived the theorem in the context of communications systems, which introduced the idea of a sampling limit to a mainstream audience. The theorem states that, in order for a continuous signal to be fully recovered, it must be sampled at a rate of at least twice the highest frequency in the signal; in other words, at half the size of the smallest feature in the signal. Applying this concept to a transmission matrix, when acquiring a signal corresponding to an object of size X, from Y measurements, the Nyquist-Shannon limit is reached when X and Y are equal, and therefore when the matrix is square. The matrix can also be elongated in the column dimension, where each column corresponds to the amount of information stored after each calibration measurement. Although measuring and storing less information at the acquisition stage is, strictly speaking, forbidden by the Nyquist-Shannon sampling theorem, it is possible to sample below this threshold and fully recover a signal, as will be discussed later in this chapter.

3.2.2 Oversampling

In signal processing, one way to classify the regime in which a signal is measured is to compare it with the Nyquist-Shannon sampling limit. When a signal is sampled at a rate greater than the Nyquist-Shannon limit, it is referred to as an oversampled signal. This implies that more information is collected than is required by the sampling theorem. There are many reasons why oversampling is beneficial for acquiring a signal, one of which is the advantage of reducing the amount of noise in the reconstruction. As the measurement size increases, the signal-to-noise ratio increases proportional to the square root of the measurement size. However, a consequence of oversampling is the large amount of data needed to be stored in order to overcome large amounts of noise. This leads to high computational costs, making it undesirable to oversample unless it is necessary.
3.2.3 Undersampling

If oversampling demands significant computational resources, then it would seem advantageous to undersample a signal, that is, to sample below the Nyquist-Shannon limit. In this ideal case, the amount of information stored in the transmission matrix would be less than the size of the signal to be reconstructed. To put this into context, there would be more unknowns to determine than there would be measurements. As discussed earlier in this section, undersampling is not permitted by the Nyquist-Shannon theorem. However, if sufficient prior information is known about the system or assumptions can be made, modern computational techniques can be used to exploit this regime. One of these techniques is known as Compressive Sensing, which is explored in a later section.

In the context of sampling rates, the most straightforward option would be to calibrate a square transmission matrix, and therefore sample at the Nyquist-Shannon limit. However, in many cases this is not possible. Some systems do not have access to a sufficient amount of information to sample at the Nyquist-Shannon sampling rate, and others require additional information in order to gain a greater signal-to-noise ratio. Therefore, it is desirable to determine computational techniques that are suited to both oversampling and undersampling. In the following sections, different computational techniques are presented and the sampling regimes in which they can be used are shown.

3.3 Transmission matrix inversion for information recovery

Once the transmission matrix is calibrated it is beneficial to use it to gain access to a hidden input, as in Figure 3.3. The unknown input $x$ can be recovered by computing $x = A^{-1}y$, where $A^{-1}$ corresponds to the inverse of the transmission matrix. Among the computational techniques that can be used to solve the linear system are pseudoinversion, the conjugate transpose, and Tikhonov regularisation [93]. In this section, I will discuss the classical methods which can be applied to the transmission matrix to extract information from a seemingly random output speckle pattern, in order to uncover the input signal. The computational techniques are all based around direct inversion of the transmission matrix, and therefore all rely on the matrix being sampled above the Nyquist-Shannon limit. Prior to that, I will describe an experiment used to probe each method, to determine the reconstruction ability of each one.

3.3.1 Method for comparison of computational techniques

To test the reconstruction quality of inversion methods using a transmission matrix approach, an experiment was devised to extract information from speckle patterns,
3.3. Transmission matrix inversion for information recovery

\[
A^{-1} \ast y = x
\]

**Figure 3.3:** Using the transmission matrix to recover spectral information. An example of a spectral intensity transmission matrix containing information about the spectral response of multimode fibre, with \( X \) spectral channels and \( Y \) camera pixels from each resulting speckle pattern. The inverse of the transmission matrix is used to uncover spectral information from a speckle pattern output \( y \).

Built as in Figure 3.2. Wavelength-dependent speckle patterns, produced by a multimode fibre, were measured on a monochrome camera for 67 spectral channels, similar to the work by Redding et al. [27]. To probe the efficiency of the spectral reconstructions, the information was compiled into a transmission matrix, where each spectral channel \( X \) in the matrix contained speckle intensity information for 225 pixels from the camera image \( Y \). A ratio of \( Y/X \geq 1 \) was chosen to guarantee a sampling rate that satisfied the Nyquist-Shannon limit. The known spectral input information was mapped on to the output camera, and the resulting speckle information was extracted. The computational experiment consisted of measuring one speckle pattern dataset, which was used to compile the transmission matrix and was also used as an ‘output’ signal. Varying amounts of independent and identically distributed (i.i.d.) noise were added to the output signal, to probe the accuracy of reconstructed signals in a noisy environment. The known input index associated with each spectral channel was compared with the ‘reconstructed’ index, in order to observe the fidelity of the transmission matrix system.

To test the ability of each computational technique to reconstruct information from the speckle pattern output, 0%, 10% and 40% noise were added to the output signal. When no noise was added, a perfect reconstruction of the original signal was expected using each computational technique. However, with random noise added to the output, the robustness of each method to noise was unknown.

With the aim of determining the computational technique most suited to reconstructing information from speckle patterns in noisy environments, the remainder of this section explores the different inversion techniques that could be employed. An overview of the mathematical background of each technique is given, and the ease of implementing the method in MATLAB is reported.
3.3.2 Conjugate transpose

The conjugate transpose of a matrix, is one approach to determining the inverse of a non-square matrix. The conjugate transpose effectively reflects the matrix entries across the matrix diagonal, so that all row and column entries are interchanged. If the matrix is complex, the conjugate of each complex number replaces the original entry. The transpose is denoted by $A^*$ and can be implemented easily in MATLAB:

$$A^* = \begin{pmatrix}
    a & b & c & d \\
    e & f & g & h \\
    i & j & k & l \\
    m & o & p & q
\end{pmatrix}^* = \begin{pmatrix}
    a^* & e^* & i^* & m^* \\
    b^* & f^* & j^* & o^* \\
    c^* & g^* & k^* & p^* \\
    d^* & h^* & l^* & q^*
\end{pmatrix}.$$

It was shown that the conjugate transpose could be used to invert a transmission matrix measured in phase and amplitude to focus light and to image through a multiple scattering material [42, 40]. This method retained the phase and amplitude of the matrix entries but reversed the direction of propagation. However, it was determined by the group that phase conjugation could only be used for reconstruction if a large amount of information was used to compile the transmission matrix or if a series of averages were taken after many iterations.

In Figure 3.4, the reconstructions of spectral signals, containing one wavelength component, with varying amounts of noise added to the output signal are shown. The expected reconstruction of any technique able to reconstruct a significant amount of information, is a background corresponding to zero reconstruction amplitude with a strong diagonal where the input index is equal to the output. The output was measured in intensity, which effectively reduced the conjugate transpose to the transpose, due to the matrix containing real values rather than complex. The accuracy of all three reconstructions was found to be limited, with no obvious dependence on the amount of noise added to the output. This suggests that, for a large amount of noise, the transpose conjugate could give some insight on the information contained within speckle patterns. However, a computational technique needs
3.3 Transmission matrix inversion for information recovery

Figure 3.5: Transmission matrix reconstruction using pseudo inversion. The known input indices were compared with the reconstructed, output indices, when 0%, 10% and 40% noise was added to the output speckle information. The scale bar denotes the reconstruction ability of the pseudoinverse.

to be utilised that performs well under many noise conditions.

3.3.3 Moore-Penrose pseudoinverse

For a matrix $A$ which is not considered to be ‘invertible’, for example, a matrix that contains more speckle information, $Y$, than calibrated channels, $X$, the generalised inverse can be employed to determine an approximation of the matrix inverse. The generalised inverse of $A$, $G$, exists if the following is true:

$$AGA = A.$$

(3.3)

A commonly used form of the generalised inverse is the Moore-Penrose pseudoinverse, which can be easily computed using many programming languages. The pseudoinverse $A^\dagger$ is given by:

$$A^\dagger = (A^*A)^{-1}A^*$$

(3.4)

and $A^*$ is the conjugate transpose of the matrix $A$.

The pseudoinverse can also be determined by factorising $A$ into three matrices using a singular value decomposition, as used in the MATLAB \texttt{pinv} function. If $A$ can be decomposed into matrices $U$, $\Sigma$ and $V$, as in

$$A = U\Sigma V^*,$$

(3.5)

where $\Sigma$ is a diagonal matrix and $U$ and $V$ are unitary matrices, then the inverse $A^{-1}$ can be found using the following equation:

$$A^\dagger = V\Sigma U^*.$$  

(3.6)
Figure 3.5 shows the results of a computational experiment to observe how the pseudoinverse performs in noisy environments. As expected, when 0% noise is added, the method is able to reconstruct all of the information contained within the speckle pattern. As progressively more noise is added, the fidelity of the reconstruction deteriorates until the information can no longer be extracted from the speckle pattern.

The pseudoinverse was shown to work in the ideal case when there is no experimental noise in the measurement, and with only a small amount of noise contained in the measurement signal. Due to the many small values contained within the transmission matrix, which are close to zero when the matrix is inverted, the small eigenvalues become amplified and overtake the signal. Therefore, in an experimental scenario, it is not always practical to use the pseudoinverse. However, in low noise environments, the pseudoinverse is desirable due to the ease of computation and the efficiency of the reconstruction.

3.3.4 Tikhonov Regularisation

A method of inversion called Tikhonov regularisation (TR) can be used to minimise the small intensity values contained within the transmission matrix, which become amplified during the inversion process [93]. This method is similar to pseudoinversion, although it enables the system to compensate for varying amounts of noise. A noise parameter, $\sigma I$, can be selected to help identify solutions with smaller eigenvalues, that is, the signal among the noise, where $\sigma$ corresponds to a percentage of noise in the matrix and $I$ is the identity matrix. The inverse of the matrix can therefore be determined using the following equation:

$$A^{Tik} = (A^* A + \sigma I)^{-1} A^*,$$

where $\sigma$ is a noise factor which can be tuned to minimise small noise values that become amplified during inversion, and $A^*$ is the conjugate transpose.

As shown in Figure 3.6, Tikhonov regularisation performs well in noiseless and noisy environments. When 0%, 10% and 40% noise were added to the output, $\sigma$-values of 0%, 3% and 33% of the maximum value in the transmission matrix were used, respectively. The ideal $\sigma$-value was found by cross-correlating the known input signal with the reconstructed signal for many values of $\sigma$, and selecting the value corresponding to the largest correlation coefficient. The optimum result for the noiseless case was determined when TR was reduced to the pseudoinverse, illustrating that the pseudoinverse is well-suited to environments with no noise. While the fidelity of TR appears similar to that of the pseudoinverse, its performance is marginally better. Therefore, TR is expected to be robust to larger amounts of noise.
3.3.5 Summary

In this section, three inversion techniques were introduced to solve a system, consisting of a multimode fibre and a monochrome camera, with oversampled wavelength-dependent speckle information contained in a transmission matrix. A computational experiment was devised to probe the fidelity of the reconstructions by each technique. The conjugate transpose was also used to reconstruct information from a speckle pattern output signal. While the original input information could be determined using this approach, the signal-to-noise ratio was poor. The accuracy of the spectral reconstruction was also found to be independent of the measurement environment. The pseudoinverse was shown to perform well under noiseless conditions, and was able to extract information from output speckle patterns when up to 40% noise was added. However, the signal-to-noise ratio of the reconstruction was found to decrease significantly as increasing amounts of noise were added. TR was used to compensate for the varying degrees of noise added to the output signal. A noise value $\sigma$ was adjusted to maximise the signal-to-noise ratio of the reconstructions of spectral information. The results were similar to those of the pseudoinverse, with a high fidelity of the extracted information, while overcoming large amounts of noise.

In the next section, the realm of undersampling is explored and a computational technique is introduced that permits sampling below the Nyquist-Shannon limit. As presented here, the computational technique is probed under different noise conditions to determine if its performance is comparable with that of TR.

3.4 Compressive sensing

Classical inversion techniques offer methods that are easy to implement in many programming languages. However, when attempting to minimise the amount of data stored within a transmission matrix, or performing a measurement that does not have access to sufficient information required by the Nyquist-Shannon sampling
Chapter 3. Computational techniques

Theorem, undersampling is required. In this case, a different branch of signal processing must be employed to recover the original signal.

A promising area of computational techniques to investigate is the field of compressive sensing (CS). In many forms of imaging, a high quality image is acquired before being compressed for storage, for example, in the form of a JPEG file. It seems superfluous to measure an image with a high spatial resolution before sacrificing most of the information during the storage process. A more desirable approach would be to compress the amount of information collected at the measurement stage, while also scaling down the size, and hence the cost, of the detectors needed to access the information. Over the last decade, CS has been exploited to undersample information at the measurement stage, while still enabling accurate reconstructions of input signals. CS works on the basis that, if it is assumed that we have a sparse input signal, information about the signal can be obtained using a sampling rate below the Nyquist-Shannon limit. In effect, the number of camera pixels required to resolve a signal can be significantly reduced.

In this section, an overview of CS in computational imaging is given. The properties of CS are discussed, and a computational approach using the transmission matrix to resolve undersampled information is proposed.

3.4.1 Compressive sensing in imaging and sensing

At wavelengths longer than visible light (for example, infrared), spectral sensors become more bulky and impractical to use. Not only do they require large detectors to record the spectral information, they also demand a proportional amount of storage space to record the information. The human race are collecting and storing more information now than during any other period in the history of humankind. It was estimated that approximately 63 zettabytes (10^{21} bytes) of information was created in 2016 alone, and this is projected to increase significantly year on year [94]. In fact, scientists are battling to develop new storage options that can keep up with growing demands. By only recording a small sample of the signal, CS provides a means of developing a more compact device for this region of light by dramatically reducing the data collection at the acquisition stage using a smaller detection region.

In 2006, Candès, Romberg, Tao and Donoho introduced the concept of sampling signals below the Nyquist-Shannon limit, based on the idea of exploiting sparsity within the signal [65, 67]. It was proposed that the l_0-norm could be utilised to reconstruct a signal using undersampled information at the measurement stage, provided the regime was known in which the signal contained a small number of non-zero values. It was also established that the l_1-norm would be a more efficient computational approach, which could be exploited to compress the sampling rates required...
to fully reconstruct a signal, where the $l_p$-norm is given by

$$
\|x\|_p := \left( \sum_{i=1}^{n} |x_i|^p \right)^{1/p}.
$$

(3.8)

In the next section, the $l_1$-norm will be discussed in detail. This step towards compressive measurements in signal processing has opened a new window of opportunity for portable technologies and fast computational processing.

Imaging devices for visible light, such as complementary metal oxide semiconductor (CMOS) and charge-coupled devices (CCD) cameras, have made a big impact on the world today. It has become cheap and easy to acquire high spatial resolution technologies for smartphones, computers, and security devices. However, it is more costly to build devices designed for other regions of the electromagnetic spectrum, such as infrared or ultraviolet. A breakthrough in imaging and sensing came about when CS was exploited to reduce the size of the detectors required. The single pixel camera was introduced to acquire information about a signal in much fewer measurements than a regular camera [95]. As suggested by the name, the single pixel camera used one photodiode (equivalent to one camera pixel), rather than $10^6$ pixels found in commercial cameras, to detect spatial information. This new concept proposed using a digital micromirror device (DMD) to project a large number of binary patterns, although well below the Nyquist-Shannon limit, onto a single optical sensor. The inner product of the photodiode intensity and each corresponding DMD pattern was determined, and an image was reconstructed. CS, in the form of $l_1$-minimisation, was used to compress the number of measurements at the acquisition stage, in order to accelerate the rate at which the input signal could be recovered. There have been many advances using this technique in recent years, including 3D imaging and the use of structured illumination to mimic the behaviour of the human eye [96, 97, 98]. However, the operational speed of single pixel imaging is ultimately limited by the large number of binary patterns to be scanned through, with the result that it can take several minutes to acquire an image. Despite this, the technique is extremely beneficial for cost-effective detection of light beyond the visible region.

It is important for many applications to acquire spectral information using an imaging system. Spectral imagers are used to obtain both spatial and spectral information to aid with the detection of chemical signatures. Traditionally, these devices have involved lab-based equipment, but there is growing desire to move towards compact portable applications. Coded aperture snapshot spectral imaging (CASSI) employs CS to reduce the number of measurements required by the Nyquist-Shannon theorem for detection of multispectral images. CASSI uses either a single-disperser or a dual-disperser approach to spectrally filter light, in combination with a coded mask, for spectral imaging in only a few measurements [9, 10]. The CASSI concept has since been extended to three dimensional imaging and to video imaging, making it a highly adaptable approach [99, 30, 100].
In 2014, it was suggested that a transmission matrix of a complex medium, measured in phase and amplitude, could be utilised in a CS device \cite{62}. As a multiple scattering material can completely scramble an input signal, every speckle grain on the camera should contain some information about the original image before travelling through the scattering medium. This implies that only a small region of the resulting output signal needs to be detected to reconstruct the original input. Hence, due to the mixing the input and output degrees of freedom, speckle patterns produced by complex media aid CS. If the nature of speckle patterns can be harnessed for compressive imaging and sensing, compact devices can be built that require limited computational processing power and storage space.

Over the last decade, CS has opened up promising opportunities for imaging and sensing technologies. The potential to scale down detectors used to capture light enables current devices to be manipulated for compact and inexpensive imaging across wavelength regions beyond visible light. CS can be harnessed to limit the amount of calibration data required for spectral imaging systems, and the indication that speckle patterns can be exploited to allow compressive measurements will be investigated in subsequent chapters of this thesis. Next, the mathematical concepts behind a CS technique are discussed, and the performance of the CS method used in the remainder of this thesis is probed.

### 3.4.2 L\(_1\)-norm

The most well-known and accessible CS method is \(\ell_1\)-minimisation. The \(\ell_1\)-norm attempts to find a solution that results in the fewest non-zero values in the solution. This sparsity condition can be explained by observing the unit circles constructed for three different norms in Figure 3.7. The intersection of a function and the unit circles are shown for the \(\ell_0\), \(\ell_1\) and \(\ell_2\)-norms, represented by a red circle. The \(\ell_0\) and \(\ell_1\)-norms, used in CS for sparse recovery, provide a solution that is likely to
occur on the axes of the parameter spaces, where the parameter is generally taken to be 0. On the other hand, the $l_2$-norm gives solutions evenly distributed along the circumference of its unit circle. This gives an intuition as to why the $l_0$ and $l_1$ can be used almost interchangeably, as well as why the sparsity condition can be enforced when using these techniques to undersample information. Due to the fact that the $l_1$-norm is convex, implementing the $l_1$-norm is easier to compute, making it a desirable approach for reconstructing a signal.

Assuming the system is linear, as described in Section 3.1, the $l_1$-norm can be used to solve the problem by performing the following:

$$\text{minimize } \|Ax - y\|_1$$
$$\text{subject to } x \geq 0$$

(3.9)

where the $l_1$-norm is taken to be

$$\|x\|_1 := \sum_{i=1}^{n} |x_i|,$$

(3.10)

and $A$ is the measured transmission matrix, $y$ is the speckle information measured by the camera, $x$ is the unknown spatial and spectral input, and $n$ is the signal dimension. This technique is able to reconstruct an input signal as it attempts to solve the system as in Equation 3.9 for an $x$ with few non-zero terms, and therefore enables a sparse $x$ to be reconstructed when sampling below the Nyquist-Shannon limit. This mathematical approach allows measurements of $y, Y$, which are much smaller than the size of the input $x, X$, provided that the sparsity condition is enforced.

In this and the following chapters, Equation 3.9 will be solved using a MATLAB convex optimisation extension called “CVX” [101]. CVX performs an iterative process to determine the optimal vector solution of $x$, using a 2-dimensional matrix $A$ and vector $y$.

Figure 3.8 shows the outcome of the experiment described in Section 3.3.1, to probe the the robustness of CS to recovering information in the presence of noise. The sampling rate employed was the same as was used with the inversion techniques. When CS was used to extract spectral information from a speckle pattern in a noiseless environment (noise=0%), the technique was found to reconstruct the signal with a high degree of accuracy. Due to the nature of $l_1$-minimisation, the reconstruction noise was reduced, as the technique aimed to find solution with the smallest number of non-zero values possible. A similar result was seen when a small amount of noise was added to each monochromatic speckle pattern output image (10% added noise). While fluctuating background noise was present, the signal was fully reconstructed as in the noiseless case. This promising result suggests that, unlike the inversion techniques presented previously, this iterative approach can adapt to its environment.
When significant amounts of noise were added to the output signal (40% of the maximum values contained in the transmission matrix), a noticeable degradation of the reconstructed spectral signal was observed. While the signal-to-noise ratio of the reconstruction was relatively large, the accuracy of the recovery was diminished, with the algorithm failing to extract the correct information from a small percentage of the monochromatic speckle patterns. However, promising reconstructions were seen at lower noise levels, making CS an encouraging method to extract information from speckle patterns.

It should be noted that the reconstructions shown in Figure 3.8 were determined with a sampling rate equivalent to that used in Section 3.3; therefore, the undersampling of CS was not explored in this computational experiment. In the remainder of this thesis, the realm of sampling below the Nyquist-Shannon limit will be investigated. Sparsity in spectral signals will be exploited to enable undersampling, and combined spatial and spectral sparsity will be used to enable multispectral imaging through a complex medium.

### 3.4.3 Summary

In this section, a modern signal processing technique was introduced as a method of sampling below the Nyquist-Shannon limit [66, 67]. This breakthrough, by Candès, Romberg, Tao and Donoho, enables signals to be fully recovered using a relatively small amount of information. It was reported that the $l_1$-norm could be used to reconstruct an input signal $x$ in fewer measurements than permitted by the Nyquist-Shannon theorem, if a measurement matrix $A$ and output signal are known, and provided that the signal fulfils a sparsity condition. This CS technique was used to recover spectral signals under noiseless and noisy conditions to determine the method’s robustness to noise. CS was shown to reconstruct signals to a high degree of accuracy in environments containing up to 40% added noise. These results were comparable with those in Section 3.3, as demonstrated by TR. However, the signal-to-noise ratio achievable using CS was superior.
This chapter has demonstrated that information can be extracted from speckle patterns using a pre-calibrated transmission matrix. However, it should be noted that only one wavelength was recovered in each measurement. In the next chapter, TR and CS are used to reconstruct spectral information containing one or more wavelength components.

In an age where the world is collecting more data than can be stored, it is essential that new ways are found to limit measurement sizes at the acquisition stage while preserving the original information. In the next chapter, the performance of CS will be compared with that of TR when oversampling information. CS will be employed in regimes of undersampling to investigate the required sparsity to recover input spectral and spatial information from the linear system. To examine the versatility of CS, the fidelity of the reconstructions will also be investigated, in order to explore the potential for sampling the information contained within speckle patterns above and below the Nyquist-Shannon limit in a spectral imaging device.
Chapter 4

Hyperspectral imaging using multiple scattering media

“Don’t let anyone rob you of your imagination, your creativity, or your curiosity.”

Mae Jemison, Physician, Engineer, & NASA Astronaut

4.1 Introduction

Over the last decade, we have seen many great technological advancements. With smartphones and powerful computational equipment readily available, it is possible to harness the properties of these devices for a multitude of applications. The growth of present-day research fields, such as ‘nanotechnology’, reflects a change in the demands of consumers towards compact and efficient products. The task for the science community now is to develop new structures, devices, and processes, to pre-empt the next generation of technologies, in the push towards more portable designs.

One approach taken in the quest for fast and efficient imaging, which avoids adding further hardware, is to employ computational techniques to extract masked information in images. The increased spatial resolution of modern cameras makes it possible for additional pixel-space to be utilised, to encode a further dimension of information within a monochrome image. In particular, the incorporation of spectral data into a spatial image is of interest, as it enables fast measurements of multispectral data. Advances towards handheld spectral imaging devices have been demonstrated using diffractive or refractive elements to spectrally encode information [102, 9, 8]. High-resolution compact imaging spectrometers are highly desirable, and while impressive results have been achieved using dispersive optics, both the spectral range and spectral resolution scale linearly with the device footprint, which can be a drawback for fast, portable applications. An alternative solution is to distribute spectral information equally across an image, to exploit the full dynamic range of the system. One such example, removing the relationship between
device size and spectral resolution, demonstrated a diffractive element which could be characterised and used to detect multispectral images [21]. However, it is more desirable to utilise materials that are readily available, for cost effective solutions.

In recent years, the use of multiple scattering media in imaging and sensing applications has seen a rapid growth. The emergence of wavefront shaping techniques to image and focus through seemingly opaque materials has verified that multiple scattering can provide powerful tools for manipulating information in the spatial domain [42, 40, 39]. As well as spatially controlling light, the wavelength-dependent properties of speckle patterns have been harnessed to develop compact, high-resolution spectrometers [27, 28, 32, 35, 36, 103, 34]. It has also been shown that multiple scattering aids the compression of sampled data, due to the effective mixing of input and output degrees of freedom, inferring that only a small amount of output information is needed for recovery [62, 96].

Complex media differ from diffractive elements in the way spectral characterisation is achieved. Dispersive gratings and other etched diffractive structures, rely on surface scatterers to dictate the spectral dispersion of light incident on the medium. On the other hand, complex multiple scattering media depend on the scattering strength of volume scatterers within the medium. As shown in Chapter 2, the spectral correlation width of multiple scattering media scales with the diffusion constant and inversely with the material thickness. Both of these parameters can be altered to fine tune the spectral response of the material. In contrast, for diffractive elements, surface scatterers can only be altered with surface depth. Furthermore, these elements produce linear, deterministic dispersion, rather than the seemingly random speckle patterns produced by multiple scattering media.

In this chapter, I present a method for compact snapshot spectral imaging that combines multiple scattering and compressive sensing, as published in [68]. In other words, a detection device that requires only a single shot to extract a full hyperspectral dataset, using an undersampled calibration datacube. Wavelength-dependent speckle patterns are captured over a wide range of input spatial positions, before being stored in a spectral intensity transmission matrix. The transmission matrix is used to detect spectral information at each speckle pattern position, and the recovered information can be viewed in different spectral channels to observe spatial information. Two computational techniques, TR and CS, are observed to determine their reconstruction fidelity when faced with environmental noise, decreased speckle contrast, and as the sampling rate of pixel information is decreased. CS is tested for viability as a processing technique for sparse and broadband signals. This method is a step towards inexpensive, portable, spectral imaging, in which the employment of CS minimises the collection of data at the measurement stage. The device can be easily tailored to suit the detection spectral resolution and range, as well as the spatial resolution, suggesting that it is an extremely versatile snapshot imaging concept.
4.2 Method

4.2.1 Experimental set-up

A white light supercontinuum laser (Fianium, SC-400-2) was spectrally filtered using a 600 lines/mm reflection diffraction grating mounted on a rotation stage, and a 10 µm single mode fibre was used to select a portion of the separated light, as shown in Figure 4.1(a). This resulted in a tunable calibration source for the spectral imaging system, with a spectral resolution of 0.5 nm, and a linewidth of 1 nm, which could scan a large spectral range in the visible and near-infrared. Light from the single mode fibre was collimated to act as an illumination source for spectral imaging. A spectrometer was used to verify the output wavelength (Ocean Optics, USB-4000) from the fibre. Light incident on the system was spatially separated to create individual point sources, which were focused on to a multiple scattering medium producing a diffraction limited spot of 10 µm, to obtain spatial information about the input signal. This was done using a microlens array with a pitch of 300 µm, which produced a grid of around $N = 100$ input point sources. The resulting grid sources of light travelled through the multiple scattering medium, creating a grid of independent speckle patterns, as shown in Figure 4.2(a). The speckle patterns were imaged by a 1:1 imaging system, consisting of two 3 cm lenses with a numerical aperture of 0.5, on to a 2-bit, 5 MPixel monochrome CMOS (AVT Guppy) array with a pixel size of 2.4 µm. Each speckle grain within each of the speckle patterns...
corresponded to 2-3 camera pixels, in order to maximise the amount of spectral information encoded. Figure 4.2(b) shows an example of the camera image produced after a letter-shaped aperture (see Figure 4.1(c)) was placed in front of the microlens array. The aperture was used to aid the demonstration of spatial recovery, as shown later in this chapter. The speckle contrast was increased by a factor of $\frac{1}{\sqrt{2}}$ using a linear polariser, as discussed in Chapter 2, in order to maximise the reconstruction of many spectral features.

### 4.2.2 Multiple scattering nanowires

A layer of strongly scattering gallium phosphide nanowires, of thickness 1.7 µm, was used to provide strong, uniform scattering, with a comparably high transmission, $T$, of approximately 18 %. A Scanning Electron Microscopy image of the cross-section through the nanowire mat is shown in Figure 4.1(b). The nanowires were grown using metal-organic vapour phase epitaxy, which is described in detail by Muskens et al. [104, 105]. The samples were not fabricated during the time span of this PhD thesis. This method allowed fabrication of uniform layers of nanowires over a large wafer.

Multiple scattering in the nanowire mat resulted in a transport mean free path of $\ell^* = 300$ nm. The nanowires had a spectral correlation width, shown in Figure 4.3(b), proportional to the inverse Thouless time $\tau_D^{-1} = D/L^2 = 4.8$ THz (corresponding to a bandwidth of 13.9 nm at 650 nm wavelength), where $D = 14 \pm 1$ m$^2$/s was the measured diffusion constant of light inside the nanowire mat [106]. Around 100 independent transmission channels were contained within the illumination area of the nanowire mat [106], which allowed the encoding of spectral information but was also small enough to provide sufficient experimental stability.

An advantage of using thin layers of nanowires is the small diffuse spreading, i.e. a small number of transmission channels are excited as light travels through the material. This means that completely independent speckle patterns can be achieved.
when illuminating neighbouring lenses in the microlens array, which allows a higher spatial resolution to be achieved in the spectral imaging system.

Unique speckle patterns were produced when the input wavelength of light was changed by more than an amount known as the spectral correlation bandwidth, $\delta \lambda$, as illustrated in Figure 4.3(a). The correlation bandwidth, given by Equation 2.15, was measured by cross-correlating a series of speckle patterns, each measured after changing the wavelength by a small increment, $\Delta \lambda$.

Figure 4.3(b) shows the correlation function for a thin layer of GaP nanowires. The correlation curve follows the expected shape of the function presented in Figure 2.2, where the known $D$ and $L$ of the GaP nanowires were used to plot the theoretical approximation. The full-width half-maximum of the spectral correlation function reveals $\delta \lambda$ of the material. For a 1.7-µm layer of GaP nanowires $\delta \lambda = 13.9$ nm, as illustrated in Figure 4.3(b). This implies that the spectral resolution of the imaging spectrometer based on this multiple scattering medium will be on the order of a few nanometres. A small difference between the experimentally-determined correlation width and the theoretically-defined width of approximately 6-7 nm is found, due to measurement errors as well as the discrepancies between the formed speckle patterns and the theoretical prediction. The residual correlation in the spectral correlation function is due, in part, to the changes in the Gaussian distribution of each wavelength dependent speckle pattern.
Chapter 4. Hyperspectral imaging using multiple scattering media

4.2.3 Calibrating the transmission matrix

The main element of the hyperspectral imaging device is the data cube containing information about the spatial and spectral fingerprints produced by the multiple scattering medium. This data cube is known as the spectral intensity transmission matrix (STM) of the scattering material. The STM describes how input light of different wavelengths and input positions maps to speckle patterns displayed on the camera. Once the STM has been calibrated, it can be used to recover arbitrary input signals which are detected by the spectral imaging system [25, 26, 107]. A transmission matrix approach can be used to store the different spectral fingerprints for a desired frequency range in order to characterize arbitrary wavelengths.

After obtaining wavelength-dependent speckle intensity patterns from the experimental set-up, the input spatial and spectral information were mapped on to each output speckle pattern, to calibrate the wavelength-dependent STM. For every speckle position and wavelength in the desired range, a small number of camera pixels were selected and stored within a transmission matrix, $A(Y, X, N)$, where $Y$ represented the camera pixels selected and contained the corresponding intensities, $X$ was the wavelength of the input light, and $N$ was the speckle pattern position. In the demonstrations presented in this chapter, $X$ was calibrated between 610 nm and 670 nm, with each spectral channel separated by $\Delta \lambda$. $N$ was defined by the number of lenslets in the microlens array.

A consideration to make when calibrating the STM was the way in which the speckle pattern positions could be determined. The grid of point sources produced by the STM allowed the coordinates to be easily determined on a 2D regular square grid. Rather than having to measure all 100 speckle patterns individually, which would have been time consuming, row and column values for each centre coordinate were found. The speckle pattern positions were then found at the permutation...
of every row and column value. This resulted in the measurement of only 19 speckle patterns, with 20 coordinate measurements taken in total. By exploiting the design of the microlens array, the time taken to find all speckle position coordinates was greatly decreased. Therefore, the measurement time for each calibration of the STM is significantly reduced.

As illustrated in Figure 4.4, the 2D speckle patterns were reshaped into column vectors to create a wavelength-dependent layer of the STM for each speckle pattern position. The STM was then used to reconstruct full spatial and spectral information for a given image, where each speckle pattern position became one “pixel” in the reconstruction, as shown in Figure 4.5. The spatial resolution of the system could be adjusted by tailoring the microlens array pitch, provided that the resulting independent speckle patterns did not interact. The measured STM remained stable for periods of up to an hour, and was mainly limited by the optical set-up. Once the transmission matrix was measured, an arbitrary input signal could be reconstructed by solving the linear problem, \( y = Ax + c \) for every speckle pattern position, N, where \( y \) was the camera image, \( x \) was the input signal and \( c \) was noise.

Figure 4.5(a) shows a schematic of the spectral imaging device design in which the STM is measured. A reconstruction of the letter ‘H’ for a single-spectral input is shown in Figure 4.5(b). The individual speckle pattern positions of the microlens array were converted to a bitmap image (grayscale) representing the spatial information at the chosen wavelength of 661 nm. Each ‘pixel’ in the reconstructed image has a retrieval amplitude corresponding to the intensity of the chosen spectral component of the input light. One speckle pattern position was selected, and TR was used in combination with the STM, to identify the wavelength information at that coordinate.

### 4.2.4 Sampling rates

The number of pixels, \( Y \), selected from the camera image and used to build the STM defines the nature of the retrieval process used to reconstruct the original signal, as discussed in Chapter 3. If a classical inversion technique were employed, the output signal would need to be oversampled at a rate greater than the limit imposed by the Nyquist-Shannon sampling theorem, in order to recover the input. However, if the signal to be reconstructed is sparse, this approach collects a lot of information which is ultimately redundant. The aim of this thesis is to find a technique that permits undersampling well below the limit defined by the Nyquist-Shannon theorem for sparse signals, as well as oversampling for the recovery of more dense spectra. As defined in Chapter 3, undersampling and oversampling are defined by the ratio between the \( Y \) pixels selected from each speckle pattern, and the \( X \) wavelength increments in the calibrated STM: for undersampled signals, \( Y/X < 1 \), and for oversampled signals, \( Y/X > 1 \), where \( Y/X = 1 \) represents the Nyquist-Shannon sampling limit.
Chapter 4. Hyperspectral imaging using multiple scattering media

Transmission matrix measurement

N sources (produced using a microlens array) Multiple scattering medium

Array of speckle patterns

Input laser source

(a) Measuring the transmission matrix

Transmission matrix measurement

Figure 4.5: Schematic of the nanowire spectral imaging system. (a) The transmission matrix measurement carried out by recording speckle patterns produced by different wavelengths of light at each speckle pattern position. Each layer shows the relationship between the speckle patterns and wavelength, and each layer corresponds to the spatial position of each speckle pattern. (b) The transmission matrix is used to reconstruct spatial and spectral information about an arbitrary input object. Here, the letter ‘H’ is reconstructed in one spectral channel, and the corresponding spectral information is also recovered using TR.
In this chapter, two computational approaches are used: TR and CS. A comparison between the performance of the techniques with spectral imaging data will be shown in the case of oversampling $Y$. As a large amount of data needs to be stored in the STM, the realm of undersampling will be explored, and a CS will be tested to investigate its ability to reconstruct spectral information from a snapshot spectral image.

### 4.2.5 Computational techniques

Once the STM has been calibrated, it can be used to recover some unknown input signal. As discussed in Chapter 3, the retrieval of spectral information using the STM can be treated as a linear problem $y = Ax + c$, where $x$ is the input signal, $y$ is the resulting output signal, and $A$ is the STM. Knowledge of the STM allows us to solve the system to find the original input, and hence to determine the spatial and spectral information of the original input signal. Several mathematical inversion techniques can be used to do this. Here, we study the inversion method of TR, to explore its ability suppress small divergences in the singular values which we see as noise in the reconstruction [40, 93]. However, in order to reconstruct an input signal, TR requires that the camera image is oversampled, that is $Y/X > 1$, for both calibration and detection.

Another approach that can be used to recover any input signal in the spectral imaging system is to employ CS in our computational reconstruction; more specifically, using $l_1$-minimization, as laid out in Chapter 3. Candès, Romberg, Tao, and Donoho established that a signal can be completely reconstructed in a number of measurements less than the Nyquist-Shannon limit when you exploit its sparsity [65, 67]. In the CS reconstructions shown in this thesis, a package for specifying and solving convex programs called CVX is used [101].

Figures 4.6(a) and (b) show a comparison between the known wavelength sent into the spectral imaging system and the reconstructed wavelength information, for one speckle pattern position when oversampling during calibration and detection.
Over a wavelength range of 610 nm to 670 nm, both TR and CS can accurately reconstruct spectral information after it has been detected by the spectral imaging system. Furthermore, CS is able to minimize reconstruction noise, resulting in a much cleaner recovery than TR. The width of the diagonal on both graphs shows the correlation between speckles corresponding to neighbouring wavelengths. The reconstruction amplitudes are slightly higher towards 670 nm due to an increase in available laser intensity and correspondingly better signal to noise ratio. There is also an increase in the retrieval amplitude at 610 nm and 670 nm, as these wavelengths have no neighbouring calibrated spectral channels in the STM. Therefore, an increase in the correlation for these two spectral channels is observed.

In the next section, the chosen computational techniques are tested for fidelity as the sampling rate is decreased to below the Nyquist-Shannon limit. The task of reconstructing more than one wavelength in a single measurement is probed, and the ability of the system to reconstruct spectral information in noisy environments is investigated.

4.3 Characterisation

4.3.1 Nyquist-Shannon sampling limit

Spectral imaging systems rely on significant amounts of data being collected and stored for calibration and detection. In order to avoid oversampling the camera information, TR and CS should be tested for fidelity as the sampling rate, $Y/X$, is decreased. Figures 4.7(a) shows the results of a computational experiment in which two test datasets, measured consecutively using the same set-up, were employed. One dataset was used to compile the STM, and the other was used as the ‘output’ dataset from which input spectral signal could be recovered. As the sampling rate is reduced, moving from the oversampled ($Y/X>1$) to the undersampled ($Y/X<1$) regime, the fidelity of the results produced by the TR method is seen to decrease. In comparison, CS performs consistently well even when sampling below the Nyquist-Shannon sampling limit. This outcome is very promising for the spectral imaging system, as it means the amount of data collected at the acquisition stage can be reduced.

4.3.2 Noise

To investigate the spectral imaging system’s robustness to noise in the measurement environment, an experiment was carried out using one dataset to build an STM, as well as to be used as an ‘output’ dataset. After the STM was calibrated, and increasing amount of random independent and identically distributed (i.i.d.) noise was
4.3. Characterisation

![Characterisation Diagram]

**Figure 4.7:** Characterising the spectral imaging system for TR (red crosses) and CS (blue circles). (a) A spectral intensity transmission matrix with dimensions Y by X corresponding to the camera pixels and the number of calibrated wavelengths, respectively, is shown. The relationship between the reconstruction ability and Y/X is plotted with Y/X > 1 denoting sampling above the Nyquist-Shannon limit, and Y/X < 1 sampling below the limit. (b) Fidelity of the spectral reconstruction is tested as random i.i.d. noise is added to the ‘output’ speckle patterns. (c) The number of wavelengths contained within the input spectral signal is increased and the reconstruction ability is observed when sampling above the Nyquist-Shannon limit, and (d) below the Nyquist-Shannon limit.
added to the ‘output’ dataset. Figure 4.7(b) compares the fidelity of the measurements for TR and CS. Despite TR being optimised to perform in a noisy environment, CS outperforms TR in terms of reconstruction ability. CS can withstand up to 20% noise (correlation > 0.5) whereas TR fails to reconstruct spectral information above 5% noise. Hence, CS is a versatile technique that can adapt to noise levels.

4.3.3 Number of wavelengths

To investigate the ability of our system to reconstruct a full spectrum, rather than just individual wavelengths, CS and TR were both used to recover an increasing number of wavelengths. The experiment was carried out as in 4.3.1, with one dataset used to build the STM and another to act as the ‘output’ spectral signal. Speckle patterns corresponding to different wavelengths, with uniform spectral separation on the order of the spectral correlation bandwidth, were computationally added to increase the density of the spectral signal. Figures 4.7(c) and (d) show the correlation between the two experimental datasets when oversampling and undersampling, respectively. Using a large number of measurements (Y/X=7) results in the reconstruction of up to 10 wavelengths using CS (correlation > 0.5). From Figure 4.7(c), it appears that TR outperforms CS above 14 wavelengths when over-sampling information from the speckle pattern, however this is due to correlation with the noise reconstructed by TR.

When oversampling and undersampling, CS visibly outperforms TR. The promise for CS in overcoming speckle contrast reduction as spectra become more complex is also greater than for TR in both sampling regimes. However, while promising results have been achieved when undersampling using CS, the sparsity condition which is tied to the benefit of undersampling implies that the method is less advantageous for a large number of wavelengths.

Promising prospects for the use of CS to process spectral and spatial information in the speckle-based system have been demonstrated in these characterisation experiments. It can be seen that CS is not only robust to noise, but also allows speckle information to be undersampled for sparse spectral reconstructions. Even when oversampling, CS permits a much smaller sampling rate than the optimised TR inversion technique. This outcome leads to faster computational times in noisy environments, as well as removing the requirement of a large storage capacity.

4.4 Spectral imaging

4.4.1 Multiplexing spatial and spectral information

Up to now in this thesis, the results presented have involved reconstructing a spectral signal, with no spatial information taken into account. In effect, I have demonstrated a speckle-based spectrometer using a multiple scattering medium. In this
section, the spectrometer concept is extended to multiple input point sources, as described in Section 4.2, for spectral imaging.

In this approach, light incident in the spectral imaging system is spatially separated, to capture spatial information for an arbitrary signal. Each STM layer, corresponding to individual speckle pattern positions, is applied in parallel across the resulting image to extract spectral information at each position. A spectral image is then built from the spectral reconstructions. From the reconstruction, each spectral channel can be observed to see a monochromatic recovery.

As before, TR and CS were compared when reconstructing a spatial and spectral image using experimentally measured spectral objects. The input objects studied were letter aperture masks of 1.5 mm x 1.5 mm in size, and an example of the resulting output is seen in Figure 4.2(b). Light of one wavelength was used to illuminate the aperture to produce a speckle image on the camera. To create a multispectral image, speckle images corresponding to different wavelengths were computationally superimposed, as can be seen in Figure 4.8(a) and (b). The STM was then applied to the resulting output speckle image and the reconstruction of spatial and spectral information was observed. To the human eye, the resulting monochromatic image

Figure 4.8: Speckle-based spectral imaging. (a) A schematic of the input which consists of four letters, each of a different wavelength, combined to produce a composite image. (b) Camera image showing composite image from (a). Two speckle pattern positions, Pixel A and Pixel B, are highlighted in the image. Spectral reconstructions using TR and CS for: (c) Pixel A, with wavelength components corresponding to the letters C, H and J. (d) Pixel B, with wavelength components corresponding to the letters E and H.
does not appear to be made up of different letters or wavelengths of light.

Using both TR and CS, spectral information at each speckle pattern position in the image was reconstructed at a sampling rate of Y/X=25, as shown in Figure 4.8(c) and (d). From the snapshot image in (b), two speckle pattern positions were observed and spectral information was extracted. In Figure 4.8, Pixel A contains spectral information about letters C, H and J, and Pixel B reveals wavelength information about E and H. While TR recovers the correct spectral peaks, there is a significant amount of noise in the reconstruction, due to small non-zero values in the STM which are amplified after inversion. CS, on the other hand, produces no fluctuating background noise, allowing for a good signal to noise ratio.

While both methods were able to reconstruct the spatial information within each spectral channel, CS minimised any reconstruction noise, as can be seen in Figure 4.9. Despite being optimised for the recovery by adjusting a noise parameter, TR still produced a lot of reconstruction noise. With decreasing contrast, as wavelength components are increased, it is expected that the fidelity of the TR reconstructions would decrease more rapidly than for CS, as studied in Figure 4.7(c) and (d).

As noted in Chapter 3, CS assumes that any input signal it reconstructs is sparse in some domain. In this case, the spectra are assumed to be sparse, hence, CS determines the most probable wavelength peaks before minimising all other values. This means that, instead of the input signal being degraded by computational noise, as seen with TR, a clearly defined spectral and spatial reconstruction is observed. Furthermore, CS allows the recovery of a more complex spectrum than TR, when oversampling, as it is able to suppress background reconstruction noise.
4.5 Compressive sensing limits

4.5.1 Recovering spectral information from a monochrome image

Due to its ability to reconstruct well-defined spectral signals, CS will be used throughout the remainder of this chapter to probe the spectral imaging system. This section will explore undersampling and oversampling of camera information in the spectral imaging system when using CS.

A comparison between a section of a STM built when oversampling (Y/X = 14.2) and when undersampling (Y/X = 0.8) is shown in Figure 4.10(a). The first STM clearly contains more intricate detail about how the speckle pattern changes with wavelength than the second STM. Therefore, a more accurate reconstruction would be expected for Y/X=14.2. Figure 4.10(b) shows a spectral reconstruction, measured for the speckle pattern position highlighted in Figure 4.11 (orange box), for four different sampling rates, Y/X=0.3, 0.8, 1.2, 14.2. The correct wavelength is identified, with varying degrees of background noise fluctuations in the reconstruction which do not correlate with Y/X. Spectral reconstruction is seen for Y/X=0.3, despite sampling at a rate well below the Nyquist-Shannon limit, albeit a noisy recovery with a broad peak. The spectral reconstructions produced by the other sampling rates used (Y/X=0.8, 1.2, 14.2) are accurate, with recovery noise independent of sampling rate. As the number of pixels collected for calibration and detection can be reduced from 441 pixels (Y/X=14.2) to 25 pixels (Y/X=0.8) without a large trade-off in reconstruction fidelity, it is desirable to exploit CS for undersampling of data. This result...
Chapter 4. Hyperspectral imaging using multiple scattering media

4.5.2 Recovering spatial information from a monochrome image

The corresponding spatial reconstructions are shown in Figure 4.11, as well as a speckle pattern in the camera image with the corresponding area of interest (AOI) selected from each. As expected, when sampling above the Nyquist-Shannon limit, the letter recoveries have a high fidelity. Undersampling leads to the decrease in fidelity of the letter reconstructions. However, the reconstruction corresponding to Y/X=0.8 is still comparable in quality to the oversampled reconstructions. This provides evidence that there is enough information about the original input signal stored with a small area of the speckle pattern that a sparse signal can be reconstructed. Hence, the amount of data required to be collected for calibration and detection can be significantly reduced without a great compromise with the signal-to-noise.

4.5.3 Reconstructing multispectral and spatial information

Up to now, the viability of undersampling and oversampling using CS has been shown when reconstructing an input signal containing one input wavelength. A more realistic input to probe is an input signal made up of multiple wavelengths. This requires the detection and recovery of more complex spectral information within one measurement. It is experimentally challenging to generate a hyperspectral image, due to the difficulty of directing all wavelength components along the same path, and therefore, a hyperspectral image was simulated for this demonstration.

To simulate a spectral signal with spatial details, the three images shown in Figure 4.12 were computationally superimposed to produce a conjugate image, shown
in Figure 4.13. Using the pre-calibrated STM, as measured in Section 4.2.3, spectral reconstruction was achieved with $Y/X=14.2$ and $Y/X=0.8$, respectively. The width of the reconstructed spectral peaks is approximately 4 nm, suggesting that this is the true resolution of our spectrometer, despite the measured spectral correlation width of $\delta \lambda = 13.9$ nm for our MSM. This finding is also presented in speckle-based spectrometer techniques [36, 103].

Figure 4.13(a) and (b) each show a spectral reconstruction from two different speckle pattern positions in the conjugate camera image, comparing two different sampling rates. In both graphs the known input wavelengths, $\lambda_{in}$, are illustrated. As expected, the input spectral information is recovered when sampling at a rate of $Y/X=14.2$. When undersampling ($Y/X=0.8$), the wavelength information is determined to a lesser degree of accuracy, with the recovery of artificial neighbouring peaks and large noise contributions. Therefore, as the sparsity of the input signal decreases, i.e. the amount of spectral information increases, it is more difficult to exploit CS to undersample pixel information and achieve accurate reconstructions.

Figure 4.14(a) shows an example of the region of pixels selected when oversampling ($Y/X=14.2$) and undersampling ($Y/X=0.8$), for one speckle pattern in a camera image. The AOIs were used to build both the transmission matrix and to reconstruct the input signal from, for each sampling case. When sampling above the Nyquist-Shannon limit, the letters were fully recovered with minimal reconstruction.
noise. Furthermore, when working with a small number of camera pixels in the undersampled regime, spatial information was still reconstructed, however, there was some noise in the reconstruction (see Figure 4.14(b)). The reconstruction accuracy for sparse signals was greater when undersampling, due to the trade-off defined by CS. Nevertheless, it is promising that spatial and spectral information can be extracted simultaneously using such a small amount of information.

When undersampling pixel information, despite the recoveries containing reconstruction artefacts, spectral and spatial information can be retrieved from the output speckle information. This provides evidence that speckle-based spectral imaging devices could be combined with CS for fast and efficient processing of hyperspectral datasets. Reducing the amount of data stored within the STM, as well as using a smaller region of information from the camera image, leads to the possibility of calibrating for a larger spectral range, or accounting for other parameters such as polarisation effects on speckle patterns. As the optical footprint of the spectral imaging system is independent of spectral resolution, there is potential for this technique to be used for compact hyperspectral imaging.

4.6 Discussion

Spectral imaging has seen a rapid movement towards the development of snapshot systems in recent years. While scanning-based approaches have allowed access to
4.6. Discussion

![Image of spatial reconstructions](image)

**Figure 4.14**: Spatial reconstructions when sampling above (Y/X=14.2) and below (Y/X=0.8) the Nyquist-Shannon limit. (a) A cropped image showing one speckle pattern from the composite image in Figure 4.12. The squares represent the area selected from the speckle pattern in calibration and detection, with the largest and smallest squares denoting oversampling and undersampling, respectively. (b) The corresponding letter reconstructions from the composite image in Figure 4.12, for three different spatial channels when undersampling and oversampling, respectively.

Vast amounts of data for environmental analysis and security purposes, they can be time-consuming to use, as well as unstable. In many instances, scanning techniques, such as pushbroom and whiskbroom devices, will remain desirable due to the large calibration datacubes required by non-scanning approaches. However, snapshot technologies have opened up a new window of opportunity for fast acquisition of signals, and also offer more stable real-time measurements. The introduction of Integral Field Spectrometers in astronomy has enabled acquisition of images of large areas of the sky, while simultaneously detecting spectral information from objects to determine their chemical composition. Many approaches use dispersion or filters to access spectral details, however this makes for large devices, which scale with resolution, or provide narrow wavelength ranges [5, 6, 7]. In this chapter, a new technique for portable snapshot spectral imaging which utilises the spectral-dependence of multiple scattering media was realised. The hyperspectral
imaging device presented here eliminates the need for a prism or a grating, and uses a computational technique called compressive sensing, to limit data collection at the acquisition stage.

The multiple scattering material chosen for the demonstration of the speckle-based spectral imaging device was a thin layer of gallium phosphide nanowires. The 1.6 µm-layer of nanowires offered a good trade-off between transmission of light through the material and spectral resolution. Due to low diffuse spreading within the medium, there is also the potential to increase the spatial resolution of the imaging system, by altering the pitch of the microlens array used, while maintaining low cross-talk between neighbouring speckle pattern positions. For the GaP nanowires the minimum spacing achievable is on the order of a few microns \[106\].

One of the drawbacks of using snapshot spectral imaging systems is the vast amount of data to be stored during calibration. To minimise the effects of handling large quantities of information, and to increase the efficiency of the device at the processing end, CS was employed. CS permits the undersampling of data, in this case the number of pixels selected from the camera image during calibration and detection, to avoid redundancy in the stored information. However, CS requires that the reconstructed signal is sparse in some domain. In this chapter, the spectral information was assumed to be sparse. It was shown that sparse spectral information could be reconstructed accurately with a sampling rate of \(Y/X = 0.8\), as all calibrated wavelength information was contained within a small area of the resulting speckle patterns. For more dense spectral information, the same CS technique could be used when oversampling pixel information. Furthermore, despite not being able to reduce the data collection below the Nyquist-Shannon sampling limit for broadband signals, it was shown that CS outperforms an inversion technique, TR, at sampling rates close to this threshold in the oversampled case.

While advantageous for specific applications, this scattering-based technique is clearly limited in spectral resolution. In this chapter, a spectral resolution of approximately 4 nm was observed for a gallium phosphide nanowire layer with a spectral correlation width of 13.9 nm. To achieve a higher spectral resolution, the thickness of the nanowire mat would need to be increased to gain additional scattering strength. However, with this increased resolution comes a trade-off in transmission. In removing the relationship between the spectral imaging device footprint size and spectral resolution, it has been replaced with another compromise. Another limitation of using gallium phosphide nanowire mats is that the bandgap of GaP limits the calibrated wavelength range as the input signal approaches 580 nm. This obstacle could be overcome by utilising a material that allows transmission across the desired wavelength range, such as a layer of white paint. We have not exploited the near-infrared (NIR) in this demonstration, however, GaP nanowires transmit and multiple scatter light within this range \[106\]. With the appropriate calibration source, this wavelength range could be exploited.

The snapshot imaging system described here takes advantage of a transmission
matrix approach to efficiently measure and recover wavelength and spatial information from an arbitrary input signal from one measurement. A challenge which still remains is, as the number of discrete spectral components within the input spectrum at each speckle pattern position increases, the recovery will breakdown due to the decrease in contrast. However, there is still a window of opportunity for this method to be viable, with the system demonstrated in this chapter allowing successful reconstruction of up to 10 wavelengths. This has potential applications in the detection of discrete, sparse spectral signals. For example, for an input signal consisting of a discrete number of narrowband sources (i.e. lasers or LEDs) the system could be calibrated for only these wavelengths, therefore allowing effective analysis of only these contributions. Applications include spectral imaging of narrowband light sources like LEDs in machine vision, or fingerprinting techniques like Raman spectroscopy. For the recovery of broadband signals, using a material with a larger spectral correlation bandwidth than the nanowire mat used here would overcome the complication of contrast, due to the increase in spectral width of each independent wavelength channel in the system.

A limitation of the system is the throughput required to overcome the device’s signal-to-noise ratio for reconstruction. In the current configuration, the microlens array was selected to provide sufficient spatial resolution while maximising the throughput at each speckle pattern position. An array with a smaller pitch would distribute the equivalent intensity over a larger number of speckle pattern positions, which would result in more difficulty in detecting spectral information. In the next chapter, an alternative approach to speckle-based spectral imaging systems is investigated which allows a greater transmission.

The proposed spectral imaging system is flexible in its design: the number of speckle pattern positions and the speckle patterns spacing can be adjusted by replacing the microlens array; the spectral resolution can be tailored by changing the multiple scattering medium; the calibration wavelength step sizes and the wavelength range can be tuned using alternative calibration sources. While this technique has been demonstrated using a multiple scattering layer of gallium phosphide nanowires, this could be replaced with a layer of white paint, or even a sugar cube.

4.7 Summary

In this chapter, I have shown that a spectral imaging system can be created using a microlens array to capture spatial information, a multiple scattering material to identify wavelength components, and a monochrome camera. A 1.7 µm layer of gallium phosphide nanowires is used to achieve a spectral resolution of 4 nm. The nanowire mat permits low diffuse spreading of light and is therefore suited for producing independent speckle spatial channels with small spacing between neighbouring channels. The nanowire mat offers a good trade-off between transmission and spectral
resolution. The technique described here can be easily tailored to suit the desired application, from increasing the spectral and spatial resolution to tuning the working bandwidth.

A spectral intensity transmission matrix (STM) was built to enable simultaneous measurements of both spatial and spectral information. I compared the recovery of an input signal using two computational techniques: CS and TR. It was found that CS outperforms TR in the oversampling regime, as it is robust to environmental noise and can extract spectral information at a much lower speckle contrast. CS can also reconstruct a sparse signal when undersampling pixel information during calibration and detection. This leads to the conclusion that multiple scattering aids the compression of data to prevent the collection of redundant information. This is a promising low-cost compact spectral imaging technique for the recovery of sparse and complex signals.

Due to the independence between the device footprint and spectral resolution, as observed in common dispersion-based techniques, this approach proves to be advantageous for portable applications. Combining compact hardware with a system that can sample below the Nyquist-Shannon limit results in a fast and efficient portable spectral imaging system.
Chapter 5

Snapshot spectral imaging with multimode fibres

“Don’t be afraid of hard work. Nothing worthwhile comes easily. Don’t let others discourage you or tell you that you can’t do it.”

Gertrude B. Elion, Nobel Prize-winning Biochemist & Pharmacologist

5.1 Introduction

Multispectral and hyperspectral imaging are vital for applications ranging from environmental sensing and threat detection, to astronomy and agriculture. There is a high demand for compact and efficient spectral imaging devices that are compatible with turbulent environments. As discussed in Chapter 1, many current approaches scan over spatial or spectral dimensions, such as in pushbroom and whiskbroom spectral imaging systems [1]. However, despite the many advantages gained with these well-known techniques, the devices require long acquisition times making them prone to instabilities. In recent years, snapshot spectral imaging, in which spatial and spectral information are obtained in one measurement, has seen a growth in interest. Commonly found in astronomy, Integral Field Spectrometers aim to acquire full hyperspectral data cubes within single snapshot images, as presented in Chapter 1, with many based on lenslet arrays, similar to the system described in Chapter 4, and fibre bundles [2, 6, 5, 7, 4, 3]. These approaches require the collection and storage of large amounts of calibration data, which allows fast acquisition of spectral images. However, despite advances in computational power, it is desirable to limit the amount of data collected to prevent computational bottlenecks.

As demonstrated in Chapter 4, CS has opened up a new window of opportunity for undersampling information while fully recovering signals. Classically, in order for meaningful data to be extracted, a signal needs to be sampled at or above the Nyquist-Shannon sampling limit. CS techniques allow redundant information to be thrown away for the reconstruction of sparse signals. Modern techniques, such
as CASSI and the single pixel camera, are moving towards minimizing data collection at the acquisition stage by using computational methods to sample below the Nyquist-Shannon limit [10, 9, 95]. Recently, it has been shown that multiple scattering of light works in harmony with CS, as information from all inputs can be accessed from a small number of speckle pattern measurements, due to mixing of input and output degrees of freedom [62, 68]. Hence, the amount of data measured and stored at the acquisition stage can be minimised, to well below the traditional sampling threshold.

Another drawback of using traditional spectral imaging systems is the use of bulky dispersive elements, where the device footprint scales with spectral resolution and range. Alternative techniques using filters have been employed which could be scaled more easily for portable applications [16, 108]. However, in many cases, these are limited to narrow spectral ranges with a restricted spectral resolution and require scanning over the spectral domain. As presented in Chapter 4, the concept of speckle spectrometers using multiple scattering elements as a thin wavelength characterisation component has been recently introduced [26, 25, 35, 36, 68]. The replacement of a diffraction grating or a prism with a compact element in a spectral imaging system is highly desirable, as the spectral range and resolution is independent of the size of the device. Complex systems have the advantage of allowing transmission over broad wavelength ranges with a spectral resolution that depends mainly on the scrambling strength of the medium. Although, the throughput of light through the material was found to be relatively low. It is, therefore, beneficial to explore the performance of other complex media to achieve a greater transmission of light.

In recent years, a new category of spectrometers has been proposed, which exploits the properties of optical fibres [27, 29, 34, 107, 28]. Multimode fibre spectrometers, which harness the wavelength-dependent properties of speckle patterns, have the upper hand when it comes to throughput and spectral resolution. Fibre technology receives a lot of interest for imaging and spectroscopy, owing to their use in applications such as remote sensing and endoscopy [109, 110]. Recent advances in fibre-based spectrometers have seen spectral resolution of picometers in the near-infrared and nanometers in the visible region [32, 28]. By taking advantage of off-the-shelf fibre technology, the potential cost of these speckle-based spectrometers can be significantly lower than traditional spectroscopy devices.

In this chapter, a multicore multimode fibre (MCMMF) is used, in combination with a monochrome camera, to demonstrate high resolution and high throughput spectral imaging. This extension of the multimode fibre spectrometer work done by Redding et al. utilises over 3000 individual multimode fibre cores to perform snapshot spectral imaging using a transmission matrix approach [27]. The spectrometer makes use of the wavelength-dependent speckle patterns produced by each fibre core of a 30 cm-length fibre bundle. As in Chapter 4, CS is employed, in combination with a clustering algorithm, to measure a STM of each core within the MCMMF.
Sparse and broadband signals are reconstructed using undersampled and oversampled speckle information, to demonstrate efficient acquisition of spatial and spectral information in one measurement for snapshot spectral imaging.

5.2 Method

5.2.1 Experimental method

The device was constructed and calibrated as shown in Figure 5.1. The spectral imaging system is similar in design to the multiple scattering medium system in Chapter 4. However, the microlens array and multiple scattering medium were replaced by a 308.5 mm-length of multicore multimode fibre (MCMMF). Spectrally-filtered, collimated light, with power on the order of a few milliwatts (mW), was imaged on to the facet of a multicore multimode fibre (MCMMF) (Edmund Optics, Fiber optic image conduit, 40-643). The resulting beam spot size on the MCMMMF facet extended across the total diameter of the fibre. The end of the MCMMF was imaged onto a monochrome camera using a relay lens, consisting of two achromatic doublet lenses, each with focal length of 3 cm. A 12-bit, 5 MPixel monochrome CMOS camera array (AVT Guppy) of 2.2 µm x 2.2 µm pixel size was used. The system was aligned such that individual speckle grains were imaged on to approximately one camera pixel, in order to maximise the information stored within the
image. In practice, on average, one speckle grain was observed over 4 neighbouring camera pixels.

5.2.2 Multicore multimode fibres (MCMMFs)

The MCMMF used here, with a diameter of 3 mm, contained approximately 3000 individual fibre cores, with each core producing a wavelength-dependent speckle pattern. The packing density of the MCMMF was exploited to develop a spectral imaging system with a much higher spatial resolution than achievable with the multiple scattering medium, as well as a high throughput. Each fibre core had a diameter, $D$, of 50 µm, with transmission over a wavelength range of 400-750 nm. For a MCMMF with $D = 50$ µm, $NA = 0.06$, and propagating wavelength $\lambda = 670$ nm, $N_m$ was calculated to be approximately 85 modes, corresponding to the number of speckle grains imaged on the camera for each core. For portable devices, although the MCMMF was a rigid glass rod, it could be coiled into a more compact footprint after the application of heat.

5.2.3 Spectral correlation width

The spectral correlation width of the MCMMF was determined after observing a change in the speckle patterns produced, as the input wavelength of light was changed. A series of images of the MCMMF facet were measured as the wavelength of light was decreased in small increments. An AOI was selected from a speckle pattern produced by one fibre core. For each wavelength-dependent image, the AOI was cross-correlated with that of another in the series, produced by the same fibre core. This result was averaged over approximately 1000 fibre cores, and the standard deviation was determined.
In order to probe the spectral correlation bandwidth of the MCMMF, a Continuous Wave External Cavity Diode Laser (ECDL), with a central wavelength of 780 nm over a spectral range of 5 nm (1 MHz linewidth), was used [111]. The spectral correlation between the patterns is presented in Figure 5.3(a), and shows a full-width at half-maximum of 1.4 nm. For calibration and detection over a large bandwidth, an additional light source was required. A spectrally tunable source was obtained by filtering light from a supercontinuum laser (Fianium, SC-400-2) using a 600 lines/mm reflection diffraction grating and a 10 µm single-mode optical fibre, resulting in a output spectral bandwidth of 1 nm. The input wavelength was confirmed by a commercial spectrometer (Ocean Optics, USB4000). The spectral correlation width of the MCMMF device in combination with the calibration source was found to be 2.1 nm (see Figure 5.3(b)). Due to the increase in the spectral bandwidth of the calibration source in relation to the spectral correlation bandwidth of the MCMMF, the speckle contrast in each spectral channel was reduced.

It was shown by Redding et al. that the spectral resolution of a multimode fibre spectrometer is dependent on the length of the fibre, L [27]. To confirm this result, the spectral correlation functions of two further MCMMF lengths were determined. The results are shown in Figures 5.4(a) and (b) for L = 25.4 mm and L = 152.4 mm, respectively. Figure 5.4(c) demonstrates a linear relationship between the spectral correlation widths and 1/L, within the error of the experiment, implying that high spectral resolutions can be achieved with long MCMMFs. This result is due to a greater probability of modal dephasing within a longer fibre propagation length.
5.2.4 Algorithm for spatial and spectral imaging

Building the spectral intensity transmission matrix

To calibrate the STM for the MCMMF, a similar protocol to that presented in Chapter 4 was used. The calibration source was scanned across a broad range of wavelengths (654-696 nm) and the resulting speckle information was recorded by a monochrome CMOS camera. An AOI from every spectrally-dependent speckle pattern, produced by each fibre core in the MCMMF, was selected and unravelled into a column vector. The resulting vector was stored within a matrix location corresponding to the spectral and spatial information the pixel information contained. Therefore, every layer of the STM has the ability to reconstruct spectral information contained within each fibre core. An example of a STM is shown in Figure 5.5.

DBSCAN

The STM calibration requires that the locations of all fibre coordinates in the image are known. Rather than following a time-consuming approach of manually measuring each speckle pattern position, a Density-Based Spatial Clustering of Applications with Noise (DBSCAN) algorithm is employed [112, 113]. DBSCAN enables the identification of over-densities in an image. In the case of the MCMMF device, the clusters correspond to the speckle patterns produced by every fibre core, i.e. a region of high intensity speckle grains, separated by areas of low intensity between each core. Once the speckle pattern positions are located, the corresponding centre coordinates can be determined.

DBSCAN requires the following initial parameters: the maximum distance between individual speckles for them to be considered part of the same speckle pattern, incorporated into a parameter \( \varepsilon \) or \( \epsilon \), and the number of speckles contained within one speckle pattern for it to be considered a cluster, \( \text{MinPts} \). Unlike other
well-known clustering algorithms, DBSCAN does not require prior knowledge of the total number of unique ‘clusters’, or speckle patterns. This allows the algorithm to identify over a thousand speckle patterns produced by the MCMMF, as well as store the centre coordinate information of each fibre core, within a couple of seconds. Occasionally when implementing DBSCAN, one speckle pattern produced by a single fibre core can be identified as containing two or more clusters. To minimise the detection of false clusters, the standard deviation of the distance between consecutive cluster centroids can be determined, and a threshold enforced to ensure that only one coordinate is retained for every speckle pattern. DBSCAN can sometimes also mistake multiple fibre cores for a single core, if the packing of the fibres isn’t uniform across the bundle. This leads to a small number of defects in the reconstructed images where the STM has not been calibrated.

For the purposes of this demonstration, parameters of $\text{eps} = 3$ and $\text{MinPts} = 13$ were used for the MCMMF spectral imaging system. On average, approximately 1400 individual speckle patterns were detected, out of the possible 3000, due to the non-uniform illumination used during calibration. After thresholding the image to remove non-zero values between central fibre cores, the speckle patterns produced by fibres at the edge of the bundle appeared less pronounced. To process the detected signals more efficiently, only a selected area of fibre cores, which contained approximately 800 speckle patterns coordinates, were calibrated. The resulting coordinates were used to identify the locations where speckle information should be collected to be stored in the STM, as well as for post-calibration detection. DBSCAN significantly reduces the time taken to gather speckle pattern coordinate information from the MCMMF, making it a crucial component in the spectral imaging system.
5.2.5 Computational techniques

Following the calibration of the STM, a computational technique needed to be established to aid the recovery of spatial and spectral information. As in Chapter 4, the spectral imaging system is described by a linear equation: \( y = Ax + c \), where \( y \) is the intensity image on the camera, \( x \) is the input object, \( c \) is the inherent noise in the system, and \( A \) is the STM. Once \( A \) was calibrated, and after observing \( y \) on the camera, \( x \) could be determined. As before, a computational technique was required that could compensate for the noise, \( c \). Given the promising results presented in Chapter 4 using \( l_1 \)-minimisation, shown in Equation 3.9, the same technique was employed with the MCMMF.

As previously discussed, in order for information to be obtained about a signal, a sufficient sampling rate must be employed, known as the Nyquist-Shannon sampling theorem. Ordinarily, inversion techniques, which are easy to implement, are utilised to recover information. However, when working with vast quantities of data, inversion processes can have high computational demands. If the signal is sparse, or contains few non-zero terms, a lot of redundant information is collected to meet the requirements of the sampling theorem. To minimise pressure on computing systems, this sparsity can be taken advantage of, in order to lower data collection during acquisition, using the CS approach presented in Chapter 3 and 4.

In the case of the spectral imaging system, the detected spectra measured at each fibre core were assumed to be sparse, when undersampling was desired. As the spectral information was mixed throughout each speckle pattern, every area of the speckle pattern contained sufficient information about the input spectral signal. Therefore, for recovery of spectral information, only a few speckle grains, i.e. a small number of camera pixels, were needed from each speckle pattern. Hence, when undersampling, the STM could be calibrated with fewer camera pixels than calibrated spectral bands.

In subsequent sections of this chapter, the use of CS in undersampling and oversampling is explored. The aim is to find a universal technique which can resolve both sparse and dense spectra when using different sampling rates.

5.3 Characterisation of fibre imaging spectrometer

To make use of the STM and CS in spectral imaging, the system needs to be characterised, so that its strengths and limitations can be probed. An aspect of the device to take into consideration is the sampling rate at which spectral information can be retrieved. In this section, the sampling rate will be explored when retrieving a sparse spectrum and a dense spectrum. Another area which is investigated is the number of wavelengths the system can recover in one measurement before the speckle contrast becomes insufficient to extract meaningful information. As well as limitations
5.3. Characterisation of fibre imaging spectrometer

Figure 5.6: Characterisation of fibre imaging spectrometer. (a) A speckle pattern produced by a single fibre core. Three AOIs are selected, corresponding to $Y/X=10.3$, 1.1 and 0.84, from the largest square (green) to the smallest (blue), respectively. (b) A layer of a STM, illustrating $Y$, the number of pixels in the AOI, and $X$, the number of spectral channels. (c) Correlation between a known spectrum, containing $N_l=1$ and 10 wavelengths, and a reconstructed spectrum using CS, as the sampling rate $Y/X$ is varied. The shaded lines correspond to the sampling rates in (a) and (d), and the dashed line denotes the Nyquist-Shannon sampling limit at $Y/X=1$. (d) Correlation between a known spectrum and a reconstructed spectrum as the number of wavelengths, $N_\lambda$, contained within the spectrum increase (relative sparsity, $N_\lambda/X$, decreases), for the sampling rates indicated in (a).

due to the device design, environmental noise also needs to be taken into consideration. The device is probed to determine its robustness to noise, in order to mimic conditions faced in “real world” applications.

5.3.1 Nyquist-Shannon sampling limit

To determine the sampling rate at which speckle information could be converted into spectral information, a test was carried out to reconstruct both a sparse and dense spectrum at varying sampling rates above and below the Nyquist-Shannon limit. Figure 5.6(a) shows a speckle pattern, corresponding to one wavelength, produced by one fibre core. A STM was built by selecting an AOI, $Y$, and by defining
a number of spectral channels, $X$, as shown in Figure 5.6(b). Three AOIs were chosen to demonstrate the amount of speckle information which needs to be stored and detected for different sampling rates. For 43 calibrated spectral bands, the largest square (green) selected in Figure 5.6(a) corresponds to an area used for oversampling pixels for detection and calibration, at $Y/X=10.1$. In order to sample at a rate approximately equivalent to the Nyquist-Shannon limit, $Y/X$ was chosen to be 1.1. Finally, the smallest square in Figure 5.6(a) corresponds to an undersampled speckle pattern, with a rate of $Y/X=0.84$. Each of the sampling rates selected were probed in all characterisation measurements.

The sampling rate required to recover spectral information was investigated by observing the reconstruction quality against the ratio between the selected AOI, $Y$, and the number of independent spectral channels, $X$, where $X=43$. The fidelity of the measurements was confirmed by performing a cross-correlation between the input test spectrum and the reconstructed spectrum. The STM was built using a single dataset, and the test spectrum was compiled using a second dataset which was obtained consecutively using the same set-up. Figure 5.6(c) illustrates the results of the experiment, using a sparse ($N_\lambda=1$, $N_\lambda/X = 2.3\%$) and dense ($N_\lambda=10$, $N_\lambda/X = 23\%$) test spectrum, where $N_\lambda$ represents the number of nonzero wavelength components, and $N_\lambda/X$ is the relative sparsity. The green, red, and blue lines correspond to the sampling rates depicted in (a), and the dashed line represents the Nyquist-Shannon limit at $Y/X=1$.

When sampling at a rate greater than the Nyquist-Shannon limit, the fidelity of the reconstructed spectra was found to be high for both $N_\lambda=1$ and $N_\lambda=10$. The reconstruction quality deteriorated below this threshold, however, a good amount of spectral information could still be obtained, especially for the sparse test spectrum. With only 9 camera pixels stored within the smallest STM, it could be seen that sparse spectral information was reconstructed. However, a noticeable degradation in fidelity of the dense reconstructed spectrum was seen when undersampling speckle information. In fact, the lowest sampling rate that provided a satisfactory reconstruction quality was obtained for $Y/X=0.45$, using a correlation $>0.5$.

It is important to note that the calibration source used in this experiment had a bandwidth approximately 1.5 times that of the spectral correlation width of the MCMMF. This implies that $N_\lambda$ should be scaled by 1.5 to taken into account the reduced contrast of the speckle patterns decoded using the STM. Also, while the Nyquist-Shannon limit refers to the mathematical ratio of $Y$ to $X$, there are correlations induced by the speckle grains themselves. Each speckle grain corresponds to approximately 4 camera pixels, meaning that there is a reduction in the information stored in the STM. Hence, the Nyquist-Shannon sampling limit is effectively shifted to a higher sampling rate, $Y/X$, for the spectral imaging device, which accounts for the observed decrease in fidelity for the dense spectrum above $Y/X=1$, observed in Figure 5.6(c).
5.3. Characterisation of fibre imaging spectrometer

5.3.2 Number of wavelengths

For a spectrometer to be viable in a practical sense it needs to be able to simultaneously recover multiple wavelengths. The fidelity of spectral reconstructions, decreasing in sparsity, was probed for the three sampling rates shown in Figure 5.6(a). As before, the STM was calibrated using one dataset, while a second ‘output’ dataset was used to construct the test spectra. The input spectra and reconstructed spectra were cross-correlated to determine the reconstruction quality, where a correlation >0.5 was considered to represent a satisfactory reconstruction.

Figure 5.6(d) shows the reconstruction quality of the spectral imaging system for three different sampling rates. Using sampling rates of $Y/X=0.84$ and $Y/X=1.1$, the STMs were capable of successfully reconstructing spectra of containing up to 35 and 25 wavelength components, respectively. Although as the relative sparsity was reduced (> 0.6), the fidelity of the reconstructions significantly decreased.

When oversampling at a rate of $Y/X=10.1$, high reconstruction quality was found when $N_\lambda$ was $\leq 30$ (correlation > 0.9). The spectral imaging system was also able to resolve up to 43 independent nonzero spectral components in a single measurement, i.e. the STM could recover spectra with a relative sparsity of 1.

The shaded area in Figure 5.6(d), and following figures, represents the standard deviation of the reconstruction performance over many fibre cores. While there is some variation between cores, the resulting reconstruction fidelity follows a similar trend over the whole bundle.

From this experiment, it can be concluded that CS can be used to under-sample sparse spectral information using the spectral imaging system. This realisation allows efficient processing of speckle patterns for fast acquisition which could help overcome computational bottlenecks caused by the large quantities of data stored in the STM for snapshot imaging. For broadband spectra, the same technique can be used when sampling above the Nyquist-Shannon limit. Hence, the spectral imaging system is highly adaptable, due to its use of CS.
Chapter 5. Snapshot spectral imaging with multimode fibres

Figure 5.8: Schematic of hyperspectral imaging experiment. A letter-shaped aperture is used to create objects for spectral imaging. Each letter is observed on the monochrome camera, after being illuminated with light of one wavelength. Each image, corresponding to a different letter and wavelength, are computationally superimposed to create a conjugate image. The resulting monochrome camera image containing 16 spectral objects is shown.

5.3.3 Noise

Another challenge faced in portable applications is the noise which is inherent in the detection region. As it is difficult to tune a computational technique using a noise parameter during real-time measurements, it is beneficial to use a computational method which can overcome noisy environments. To probe the system’s robustness to noise, an experiment was constructed to recover a single spectral component \( N_\lambda = 1 \), as the amount of noise was increased. Random independent, identically distributed (i.i.d.) noise was added in increasing amounts, from 0% to 50% of the average intensity, to an ‘output’ camera image. Unlike before, the same dataset was used to build the STM as used for the test speckle pattern. Therefore, the amount of noise applied to the output image could be controlled exactly.

Figure 5.7 shows the reconstruction fidelities for the three sampling rates used in Figure 5.6. While the amount of information captured from the output images differs vastly, there is no noticeable dependence between the reconstruction ability and the different sampling rates. A correlation > 0.5 is observed for up to 40% added noise. This result confirms that the spectral imaging system is robust to noise. CS accounts for unknown amounts of noise in detection scenes, making it a versatile computational technique for portable spectral imaging.

5.4 Hyperspectral reconstruction

After characterisation of the system, the ability of the device to reconstruct hyperspectral information was probed. To demonstrate spectral imaging, a composite image was created using 16 letter-shaped objects, with each located at a different optical wavelength, as illustrated in Figure 5.8. The resulting output camera image
5.4. Hyperspectral reconstruction

is also shown. While there are small variations in intensity over the image, due to the weights of the different objects, it is not possible to distinguish spatial or spectral information by eye.

To recover the wavelength components and spatial information, the STM was applied to every calibrated fibre core position originally detected by DBSCAN. The STM was constructed for wavelengths in the range of 654 nm to 697 nm, in increments of 0.4 nm. CS was used, with a sampling rate of $Y/X=4$, to determine the corresponding spectral information at each fibre core, and to obtain spatial information from every monochromatic spectral channel. Figure 5.9 shows two spectral signatures extracted from two different speckle patterns in the output camera image, denoted by the red and orange circles. The central wavelengths in the reconstructed spectra agreed well with the known input wavelengths, measured using a commercial spectrometer (red dashed lines). Figure 5.9(a) shows the accurate recovery of a dense spectrum, which contains 9 spectral components. The smaller reconstruction peaks within the spectrum are, to an extent, due to a partially illuminated fibre core with a low intensity within certain spectral channels (e.g. letter S at 665.9 nm). Other small peaks correspond to reconstruction noise. As the linewidth of the laser was greater than the spectral band separation in the STM, reconstruction was found in three consecutive, independent wavelength channels. A similar result was seen for sparse spectra, as in Figure 5.9(c). Good reconstruction quality was achieved when extracting 3 wavelength components from the speckle pattern produced by one fibre core. The sparsity of the spectra of the two selected cores in Figure 5.9 was 12% and 4%, respectively, when taking into account the linewidth of the input source. While the spectral correlation bandwidth of the calibrated system was measured to be 2.1 nm, the reconstructed spectra were resolved with considerably higher spectral resolution.

To obtain spectral images, spatial intensities at a selected wavelength were assembled using the known fibre coordinates, and a kernel convolution was performed.
Chapter 5. Snapshot spectral imaging with multimode fibres

\[ \lambda_{\text{in}} = 654.7 \text{ nm} \]
\[ \lambda_{\text{in}} = 656.9 \text{ nm} \]
\[ \lambda_{\text{in}} = 657.8 \text{ nm} \]
\[ \lambda_{\text{in}} = 658.7 \text{ nm} \]
\[ \lambda_{\text{in}} = 660.9 \text{ nm} \]
\[ \lambda_{\text{in}} = 661.9 \text{ nm} \]
\[ \lambda_{\text{in}} = 663.8 \text{ nm} \]
\[ \lambda_{\text{in}} = 665.9 \text{ nm} \]
\[ \lambda_{\text{in}} = 667.9 \text{ nm} \]
\[ \lambda_{\text{in}} = 670 \text{ nm} \]
\[ \lambda_{\text{in}} = 672 \text{ nm} \]
\[ \lambda_{\text{in}} = 674.2 \text{ nm} \]
\[ \lambda_{\text{in}} = 675.9 \text{ nm} \]
\[ \lambda_{\text{in}} = 677.8 \text{ nm} \]
\[ \lambda_{\text{in}} = 680.1 \text{ nm} \]
\[ \lambda_{\text{in}} = 681.5 \text{ nm} \]
\[ \lambda_{\text{in}} = 683.7 \text{ nm} \]
\[ \lambda_{\text{in}} = 685.8 \text{ nm} \]
\[ \lambda_{\text{in}} = 687.7 \text{ nm} \]
\[ \lambda_{\text{in}} = 689.7 \text{ nm} \]

Figure 5.10: Spatial reconstruction of 16 letters from Figure 5.8, each detected in an independent spectral channel.

to plot the points with the equivalent size to the fibre core diameters. Figure 5.10 shows the spatial reconstruction of 16 letter-shaped objects extracted from the camera image in Figure 5.9(b), with very little apparent cross-talk between spectral channels. It is noticeable that some spatial coordinates are missing from the reconstruction, due to fibre cores remaining undetected by the DBSCAN algorithm. Therefore, a small amount of spatial information is lost from the reconstructions. Further to this observation, the gaps seen in letters such as ‘P’ and ‘R’ are features inherent in the stencil-type letter aperture used. However, it can be seen from the fibre spectral imaging device that spectral images containing >1000 spatial channels can be recovered to a high degree of accuracy in one measurement.

In many of the spectral channels, small amounts of noise can be seen distributed evenly over the spatial reconstruction. These fluctuations account for the small noise values seen in the reconstructed spectra in Figure 5.9. However, CS minimises many of the noise components that would otherwise be recovered using an inversion technique, due to the mathematics of using the $l_1$-norm to reconstruct a signal. The $l_1$-norm attempts to find a solution with the fewest non-zero values, thereby decreasing the fluctuations caused by environmental noise.

It is important to note that the spectral imaging system was used to characterise arbitrary speckle information within the order hours after the STM calibration was performed. This procedure was followed due to the potential decorrelation of speckle patterns caused by environmental influences, such as temperature changes, which may have rendered the calibration unusable. When observing the stability of the system over a period of hours, a small change in the speckle patterns was noted. However, CS was able to compensate for the discrepancies between the calibrated speckle information and the measured signals. Over longer time scales on the order of days, it is unlikely that the calibration would remain valid, leading to regular STM measurements.

In Chapter 4, it was shown that a limited number of wavelengths could be simultaneously extracted from each speckle pattern. The fibre spectral imaging system developed here displays many advantages over the nanowire device, mainly the huge amount of reconstructed data attainable in one snapshot image. The greater speckle contrast as well as the behaviour of the fibre modes permits a high transmission of
5.5 Compressive sensing limit

Thus far in this chapter, spectral images have been reconstructed using a sampling rate of \( Y/X = 4 \). However, in circumstances where sparse recovery is required, it is desirable to find a sampling rate lower than the Nyquist-Shannon threshold. In this section, oversampling and undersampling of information are explored when reconstructing spatial information across different spectral channels.

To explore sampling rates in terms of pixel information from a camera image, it is beneficial to visualise the amount of data required for calibration and detection. Figure 5.11(a) shows a speckle pattern produced by one fibre core, corresponding to a wavelength of 674.17 nm. Four AOIs are selected, labelled A to D, where each corresponds to a sampling rate of \( Y/X = 0.14, 0.32, 0.90, 2.03 \), respectively. The highest sampling rate, corresponding to D, samples above the Nyquist-Shannon limit. Despite the fact that this AOI represents oversampling of speckle information, the number of selected pixels is relatively small compared to the total size of the speckle light combined with well-defined spectral ‘fingerprints’. CS performs well when oversampling information, to minimise noise values and to accurately determine spectral signals. This proves that CS is an advantageous computational technique for the recovery of broadband signals as well as sparse signals.
Chapter 5. Snapshot spectral imaging with multimode fibres

Figure 5.12: Reconstructions of monochromatic object from Figure 5.11 in neighbouring spectral channels for two different sampling rates: Y/X=4 and Y/X=0.32.

pattern. B, C and D correspond to undersampling speckle information. In Figure 5.11(b), the output camera image, produced by a monochromatic letter-shaped object, is shown with coordinates overlaid corresponding to those determined by DBSCAN. As discussed earlier in the chapter, DBSCAN occasionally ‘misses’ a fibre core, leading to gaps in the collected coordinates and, hence, in the reconstruction.

The STM was applied to the image in Figure 5.11(b) at each fibre position. Figure 5.11(c) shows that Reconstructions B, C and D shared a great likeness with the input object. Good imaging performance was obtained when the system was sampled at a rate above the Nyquist-Shannon limit, as well as below. A noticeable degradation was seen for Reconstruction A, where Y/X=0.14, although it is still possible to resolve the shape of the letter above the noise levels by eye.

A concern with spectral imaging techniques is that false identification of an object may occur in other spectral bands. As seen in the previous section in Figure 5.9 and Figure 5.10, despite using an efficient computational technique, CS still reconstructs some small noise values. Therefore, the system is at risk of losing the correct wavelength reconstruction within the noise located at neighbouring spectral channels. Figure 5.12 shows reconstructions of the object in Figure 5.11 across many spectral bands, using two different sampling rates. Figure 5.12(a) corresponds to a sampling rate of Y/X=4. As the central wavelength of the letter-shaped object was 674.17 nm, with a linewidth of approximately 1 nm, spatial recovery is observed across two spectral bands which lie on either side of the central wavelength. Small amounts of fluctuating noise are seen in neighbouring bands, however, it is clear that the spectral objects are only detected in the correct spectral channels. When sampling at a rate well below the Nyquist-Shannon limit, where Y/X=0.32, as shown in Figure 5.12(b), a larger amount of noise is seen in neighbouring channels. However, the signal is an order of magnitude lower than the object detected in bands corresponding to 674.2 nm and 673.8 nm.
The spectral imaging device demonstrated here is suited to employing a compressive sensing approach in order to undersample information from speckle patterns. Redundant information can be thrown away to ease demands on computational processing, with satisfactory reconstructions achieved when $Y/X=0.32$ for sparse spectral signals. The spatial reconstructions obtained using CS are accurately recovered in the correct spectral bands, showing that not only can the spectral imaging system obtain spatial information from a camera image, but it only detects the spatial information when spectral signatures are extracted from every speckle pattern.

5.6 Angle dependence

A drawback of using fibre-based spectrometers is the dependence of incident angle on the speckle patterns, produced by the interference between fibre modes. If the calibration has only been carried out for a fixed set of input parameters, the spectral signal may be impossible to decode if the incident light does not fall within calibrated range.

As monochromatic light has the ability to interfere with itself, a change in the incident wavefront of the light will result in a vastly different speckle pattern. This occurs for light which enters the MCMFF with different incident angles. Even if the STM calibration were to take into account the different angles of light entering the system, the interactions between light paths would be too complex to immediately predict the resulting speckle pattern using CS. A more advanced approach, such as phase retrieval, is needed to decipher the speckle information [114].

An interesting phenomenon studied in complex media is the so-called ‘memory effect’. This effect describes the short range correlations present in speckle pattern measurements, which can be observed as the properties of the incident wavefront change - the speckle pattern appears to ‘remember’ its initial form [83]. If the range...
Figure 5.14: Angle dependence of speckle patterns. Camera images of speckle patterns produced by MCMMF cores, with varying incident angles. The corresponding correlation functions are shown for input angles of: $\theta = 0^\circ$, $\theta = 2^\circ$, $\theta = 4^\circ$. The angle correlation width are $\delta\theta = 1.15^\circ$, $1.06^\circ$, and $0.99^\circ$, respectively.

in which this correlation holds is measured, it can help to define the angular region in which the STM calibration, measured for one angle of incidence, will hold.

5.6.1 Experimental set-up

Figure 5.13 shows the experimental set-up used to investigate the angle dependence of speckle patterns produced by the MCMMF. A monochromatic light source, of wavelength 670 nm, was mounted on a rotation stage to control the angle of incidence, $\theta$, of the beam on the MCMMF facet. Light from the MCMMF was imaged onto a monochrome CCD array (AVT Stingray-II F-033B) of 9.9 $\mu$m x 9.9 $\mu$m pixel size, using a 3 cm and 20 cm lens. The size of individual speckle grains was scaled to correspond to approximately one camera pixel in order to optimize the information content of the image.

5.6.2 Angle correlation

Examples of the speckle patterns produced by the MCMMF are shown in Figure 5.14, for angles of incidence of $\theta = 0^\circ$, $2^\circ$, $4^\circ$. For an incident angle of $0^\circ$, the number of modes, $N_m$, is approximately equal to 1-2 modes, indicating that light can only couple to a small number of modes at small incident angles. Hence, only a small number
of speckle grains can be seen at the output. Furthermore, for light coupled into the MCMMF with a large incident angle, a greater number of modes are excited, as deduced from the number of speckle grains produced by each core. The corresponding angle correlation functions for each angle of incidence, are shown beneath the camera images, and their corresponding correlation widths are found to be $\theta = 1.15^\circ$, $1.06^\circ$, $0.99^\circ$, respectively.

Another illustration of this angle dependence is shown in Figure 5.15, where the angle correlation width of the MCMMF is plotted against different angles of incidence. The trend seen in this figure leads to the conclusion that the angular width of the speckles depends strongly on the incident angle. Therefore, in sacrificing spectral resolution, through reducing the number of speckle grains within each speckle pattern, the numerical aperture of the device can be increased by exploiting the short range correlations.

5.7 Discussion

Dispersion-based instruments, such as prisms and diffraction gratings, have long been relied upon for spectroscopic systems. However, there is a growing demand for portable spectroscopy devices. Due to the nature of dispersive techniques, spectral resolution and range scale with the footprint of the device. Therefore, it is difficult to produce a compact, high-resolution sensing system for which broadband detection can be carried out. Thin-film filters offer a solution to many problems faced by traditional spectrometers, however, they offer limited spectral range in snapshot detection. Wavelength scanning methods can be employed in combination with filters.
to increase spectral range, although this unstable approach is not practical ‘in the field’.

The introduction of speckle spectroscopy using multimode fibres has marked a new approach to sensing systems. Multimode fibres are desirable in many imaging systems as they have a high throughput and are able to transmit a broad range of wavelengths. The sensitivity of the speckle patterns produced by multimode fibres can be harnessed to produce an extensive range of sensors, to detect everything from temperature changes to spectral information [115, 116, 117, 118]. However, if the input conditions of light are not well known, they can cause decorrelation of the calibrated data.

The spectral imaging system described in this chapter was found to be stable throughout the duration of the measurements, on the order of a few hours. However, the STM was required to be re-calibrated before each measurement session, due to decorrelation of the wavelength-dependent speckle patterns. It is thought that temperature fluctuations accelerated the degradation of the calibration. Environmental sensitivity derived from temperature changes has been shown to be easily compensated for by observing shifts in wavelength identification, implying that additional data does not need to be collected to account for temperature variations post-calibration [32]. Unlike previous multimode fibre spectroscopy demonstrations, the MCMMF is inflexible in its nature, and so the speckle patterns are relatively insensitive to accidental bending of the bundle.

A limitation of the MCMMF device is its restricted angular range. By taking angle dependence of the incident light into consideration during calibration, it is thought that the aperture of the device could be increased. Although, the interaction of light of different incident angles within the fibres needs to be deciphered, with the help of computationally demanding algorithms, in order to extract information about wavelength and angle from each speckle pattern. With the current approach, a numerical aperture of 0.017 is achieved, which permits incoherent imaging with a small field of view.

As with many snapshot spectral imaging devices, the technique presented in this chapter requires the calibration and detection of vast amounts of data. Using CS, the amount of speckle information can be minimised during the measurement stage, further reducing computational bottlenecks. In order to take advantage of this approach, the system is required to be sparse in some domain, therefore only sparse spectral data can be recovered. However, CS can still be employed to accurately reconstruct broadband spectral signals when sampling above the Nyquist-Shannon limit. Despite needing to oversample speckle information, it was shown in Figure 5.11 that only a relatively small amount of speckle information is required for calibration and detection. It is possible that by taking the global sparsity of the system into account, or in other words, both the spectral and spatial sparsity, the reconstruction could be improved.
The spectral imaging system presented in this chapter was able to achieve sub-nanometer spectral resolution, as shown in Figure 5.9, across 10^3 independent spatial channels. Furthermore, the spectral resolution can be tailored by increasing or decreasing the fibre length, as demonstrated in Figure 5.4. For broadband applications, where a high spectral resolution isn’t critical, a shorter MCMMF with lower spectral resolution could be used, as in Figure 5.4. This alteration would provide a greater spectral range, due to the increase in speckle contrast within each wavelength increment. This flexibility implies that fibre-based spectral imaging with CS is a highly versatile approach, allowing the device specifications to be tailored to a desired application.

5.8 Summary

Throughout this chapter, I have explored the use of a multicore multimode fibre as a wavelength characterisation component in a snapshot spectral imaging system. Fibres are often used in imaging and sensing devices due to their high throughput and broad spectral range. Here, wavelength-dependent speckle patterns, produced by the interference of fibre modes, were characterised for over 3000 individual fibre cores. Unlike the multiple scattering system presented in Chapter 4, the MCMMF spectral imaging system achieves sub-nanometre resolution using a 30 cm-length of fibre. It was shown that the MCMMF spectral correlation width, and hence spectral resolution, scaled with fibre length. Therefore, the spectral resolution of the spectral imaging system could be tailored easily by selecting a MCMMF with the desired properties.

Due to the vast amounts of data collected during the calibration of the STM, a compressive sensing technique, CS, was employed, in combination with a clustering algorithm, to sample below the classical Nyquist-Shannon limit. L_1-minimisation was used to recover both sparse and broadband spectral signals when under- and oversampling, respectively. DBSCAN allowed the identification of up to 3000 speckle patterns, produced by individual fibre cores, for fast calibration and detection of spatial information. It was found that not only could CS permit the sampling of signals below the classical threshold, but it was also robust to large amounts of noise. Spectra with a relative sparsity of approximately 50% were reconstructed using undersampled speckle information, while up to 43 wavelengths (of a possible 43) could be simultaneously detected when oversampling the speckle signal.

Hyperspectral imaging was demonstrated using a composite spectral image, which consisted of 16 monochromatic letter-shaped objects. Each object was reconstructed from one camera image by applying the STM to each speckle pattern coordinate defined by DBSCAN. Spectral information was extracted from each fibre core, and spatial information was revealed in every spectral channel. While this information was extracted using a sampling rate of Y/X=4, the system was probed to determine the minimum amount of information required for sparse spectral image recovery. It
was found that good reconstruction of a spatial and spectral signal could be reconstructed with a rate of $Y/X=0.32$. However, using a STM calibrated by selecting 16 camera pixels for each fibre core ($Y/X=0.14$), a remarkable amount of hyperspectral information could be recovered.

Speckle-based spectral systems have a slight limitation in that speckle patterns produced by complex media are dependent on the input angle of light entering the system. Many demonstrations have utilised single mode fibres to couple light into multimode fibres to better control the incident wavefront [27]. As it is impractical to couple a single mode fibre bundle to the MCMMF in order to restrict the incident field, a study was carried out to determine the strength of the dependence of angle on the resulting speckle information in the MCMMF system. Using a 30 cm MCMMF, the fibre was found to have an angle correlation width of approximately 1°, and a numerical aperture of 0.017, when the incident angle was 3.5°. Due to the complex nature of monochromatic light interactions inside the fibre, it was not possible to calibrate the STM to recover angle information from the output speckle information. However, it is thought that, with access to additional information, a phase retrieval method could be employed to uncover the original angle of the incident light, thereby increasing the aperture of the spectral imaging system. The angle dependence could also be exploited for wavefront sensing akin to light field imaging.

The technique described here is adaptable for many applications. Sparsity in spectral information can be exploited to limit data collection at the measurement stage, while the same technique can be used to recover broadband information. The length of the MCMMF can be altered to tailor to the required specifications. This approach combines a low-cost, compact multimode fibre bundle with fast computational techniques for snapshot spectral imaging, as an alternative to unstable scanning approaches.
Chapter 6

Multispectral imaging with a complex medium

“I was taught that the way of progress was neither swift nor easy.”

Marie Skłodowska Curie, Nobel Prize in Physics & Chemistry

6.1 Introduction

Disorder caused by multiple scattering and modal interference has long been seen as a nuisance in many experimental fields. From medical imaging to coherent light experiments, scientists have strived to reduce the scattering of light. Over the course of this thesis, I have discussed how the properties of scrambled light can be harnessed for spectral imaging and compressive measurements to improve the efficiency of computational reconstructions. In Chapter 4 and Chapter 5, the intensities of wavelength-dependent speckle patterns were recorded and stored in a STM, requiring experimentally simple set-ups to extract spectral information. However, in order to decipher spatial information, the input wavefront was required to be spatially filtered, using either a microlens array or a bundle of fibres. This means that the spatial resolution of these systems is ultimately limited by the diffuse spreading of the multiple scattering medium or the packing efficiency of the fibre bundle. Light transmission is also decreased in the fibre bundle due to increased absorption within the cladding between fibres.

If multispectral object information can be reconstructed from one speckle pattern, rather than reducing one speckle to a single pixel in an reconstructed image, the spatial resolution and dynamic range can be improved upon. Since the measurement of the monochromatic transmission matrix in phase and amplitude, as discussed Chapter 1, new imaging techniques have been demonstrated to utilise the seemingly random scrambling of light to extract spatial information. With the realisation that both spatial and spectral information are preserved in complex systems, we can begin to explore the possibilities of extracting multispectral images from a single speckle pattern for snapshot multispectral imaging.
With easy access to wavefront shaping devices such as spatial light modulators (SLMs) and digital micromirror devices (DMDs), controlling the path of light through complex media has become almost trivial. Wavefront modulation and monochromatic transmission matrix approaches have been able to uncover phase and amplitude information stored within intensity images, allowing the reconstruction of images or focusing of light after undergoing transmission through complex materials [42, 40, 75]. Another window of opportunity for light control was demonstrated through the measurement of the multispectral transmission matrix (MSTM) [52]. This matrix enabled spatiotemporal focusing of a pulsed laser through a multiple scattering layer [53]. Furthermore, the broadband transmission matrix was also measured for broadband focusing, with potential to extend to nonlinear imaging for biomedical applications [119].

Many of the experimental approaches described above rely on interferometry techniques to uncover the phase information for calibration of transmission matrices and, hence, recovery of spatial information. Figure 6.1(a) shows a reference arm created using a 50:50 beam splitter to redirect an un-modulated portion of the incoming beam. An additional experimental arm leaves these methods open to instabilities during and after measurement of the transmission matrix, and consequently for recovery of images or focusing. However, in recent years, computational techniques, such as phase retrieval, have been utilised to build transmission matrices and to recover information from monochromatic speckle patterns, without the additional hardware demands required by interferometric designs. Figure 6.1(b) shows a more experimentally simplistic approach, using only a few components. Phase retrieval has been used across many fields of science, including astronomy, x-ray diffraction, and ptchography, and more recently it has been extended to optical systems to see through opaque media. Phase retrieval uses mathematical techniques to recover phase information that is lost during the detection of the intensity image on the camera, without the need for a reference field, provided that prior information is known about the measurement.

In this chapter, I will explore the use of phase retrieval techniques to recover phase, amplitude and spectral information from a speckle pattern in a single measurement. A computational MSTM approach will be used to calibrate the system, before applying phase retrieval to uncover an arbitrary multispectral object. To my knowledge, this has not been achieved before, due to the difficulty of the extracting complex information from an incoherent sum of intensities. The limitations of using this technique are discussed, and outlook for the future of this method is reported.
6.2 Uncovering phase and amplitude information

6.2.1 The monochromatic transmission matrix

In order to transmit an image through a multiple scattering medium, the transmission matrix must be measured in the spatial domain. In other words, phase and amplitude information must be recovered. Experimentally, this is achievable using a spatial light modulator (SLM), which can control the input wavefront of the monochromatic light. Popoff et al. showed that the light travelling from a phase-only SLM could be mapped onto the transmitted signal recorded on a CCD camera, after travelling through a multiple scattering medium, in order to build a transmission matrix of the system \[41\].

In the monochromatic case, the system condenses to a linear equation of the form:

\[ y = |Ax + c|, \tag{6.1} \]

where \( y \in \mathbb{R}^{N_{CCD}} \) is the square root of the camera intensity image, \( A \in \mathbb{C}^{N_{CCD} \times N_{SLM}} \) is the transmission matrix, \( x \in \mathbb{C}^{N_{SLM}} \) is the input phase object from the SLM, \( c \) is noise, \( N_{CCD} \) is the number of CCD pixels, and \( N_{SLM} \) is the number of SLM pixels. The reference field needed to extract phase information can be measured either using an external arm, or, in the case of Popoff et al., it can be measured by leaving an unmodulated part of the incident wavefront to propagate with the modulated field \([41]\).

In order to extract complex information from the intensity image, a “four-phase” method can be implemented to uncover phase and amplitude. This approach requires that an input SLM phase mask is projected through the system as the relative phase is shifted by \( \varphi = 0, \pi/2, \pi, 3\pi/2 \). The intensity images from the camera, \( I^\varphi \), are
recorded before performing the following operation to access the complex information:

\[ T = \frac{(I_0 - I_\pi)}{4} + i\frac{(I_3^{\pi} - I_\pi^{\pi})}{4} \]  

(6.2)

where \( T \) is the effective transmission matrix, containing the complex amplitude of both the field from the measured output reference and the matrix describing how the input light maps to the output field [41]. Although, as mentioned in Section 6.1, the phase of the complex field can also be recovered using a phase retrieval method, removing the need for an unstable reference component or phase-stepping procedure in the measurement.

Once the matrix is measured, it can be used to focus monochromatic light through the calibrated medium, or to recover monochromatic images transmitted through the scattering layer. However, no spectral information is contained within the transmission matrix, and so additional experimental measurements are required to extract wavelength features from speckle patterns.

### 6.2.2 The multispectral transmission matrix

The multispectral transmission matrix was first measured to allow independent control of multiple wavelengths for multispectral focusing of light through multiple scattering media [52, 53]. It was also used to enable spatiotemporal modulation, to counteract the effects of temporal broadening of broadband pulsed light [119]. The added benefit of spectral control is desirable for imaging purposes, however, it is more challenging to extract multispectral spatial information from a single speckle intensity pattern.

The multispectral transmission matrix (MSTM) follows a similar protocol to that of the monochromatic transmission matrix. The MSTM contains multiple monochromatic transmission matrix layers with each corresponding to a different wavelength. Each layer is measured using continuous wave (CW) light, before wavefront control is carried out using pulsed light. As with the STMs in Chapters 4 and 5, the spectral correlation width of the material has to be considered when measuring the MSTM. In order to maximise the amount of spectral information stored within the MSTM, while ensuring each monochromatic layer is completely independent from all others, the spectral correlation width of the complex medium acts as the minimum wavelength spacing between each calibrated matrix layer. This means that the potential spectral resolution of the MSTM is inherently lower than that of the STMs studied earlier in this thesis.

Further to the measurement of the MSTM, it was also shown that the broadband transmission matrix could be measured using a pulsed laser, rather than a CW laser, during the calibration procedure [119]. The same protocol as in Section 6.2.1, using a co-propagative reference beam, was used to uncover the phase information lost after light propagation through a multiple scattering medium. This effectively treated the broadband transmission matrix as a monochromatic transmission matrix with a
6.3. The multispectral imaging problem

Throughout this thesis, I have explored complex systems, using the linear equation to understand how light travels from an input to an output. Unlike the real spectral intensity matrices described in Chapters 4 and 5, phase and amplitude information extracted from speckle patterns also needs to be considered. In this case, complex information needs to be recovered from an intensity image. While this has been demonstrated in the monochromatic regime, the multispectral problem is more challenging as information cannot simply be recovered using an inversion operation.

As seen in previous chapters, when light travels through multiple scattering materials, each wavelength contained within the incident beam will produce its own characteristic speckle pattern, due to the fact that different frequencies of light do not interfere. This means that the problem can be treated as the summation of many monochromatic complex speckle patterns. The multispectral problem can be represented by:

\[ I = \sum_{\nu} |A_{\nu}x_{\nu}|^2 = |A_1x_1|^2 + |A_2x_2|^2 + \cdots + |A_{\lambda}x_{\lambda}|^2 \] (6.3)

where \( I \in \mathbb{R}^{N_{CCD}} \) is a monochrome intensity image, \( A_{\nu} \in \mathbb{C}^{N_{CCD} \times N_{SLM} \times N_{spectral}} \) is a multispectral transmission matrix (MSTM), \( x_{\nu} \in \mathbb{C}^{N_{SLM} \times N_{spectral}} \) is one wavelength component of a multispectral input object, and \( N_{spectral} \) is the number of spectral channels.

As can be seen from Equation 6.3, extracting phase and amplitude from the sum of squares is a seemingly impossible task, as the system is no longer linear. As discussed in Section 6.2.1, when reconstructing a monochromatic image, the complex information can be extracted using either a phase retrieval algorithm or via a “four phase” technique, if the transmission matrix is known. However, the added complexity of the multispectral problem makes it difficult to adapt to existing algorithms. Furthermore, not only does complex multispectral information need to be reconstructed from an intensity image, but a large enough sampling rate needs to be
used in order to meet the criteria of many recovery techniques. It may be possible to use modern compressive phase retrieval techniques, provided that the problem can be formulated in such a way that satisfies the format of these algorithms. However, many current approaches are not designed for use with an approximately Gaussian transmission matrix measured using a phase-only SLM, such as the MSTM.

One recently developed transmission matrix-based phase retrieval algorithm, prVAMP, focuses on improving computational processing efficiencies, rather than compressing measurements, by employing a significantly faster algorithm. In the remainder of this chapter, prVAMP will be introduced before it is utilised to extract spectral and phase information from an intensity pattern to enable the reconstruction of a multispectral object. In the next section, I will discuss various methods of phase retrieval that have been developed recently to recover monochromatic spatial information from an intensity image.

6.4 Phase retrieval

Phase retrieval has been used for decades to recover phase from intensity images, using prior information, in order to observe spatial information or correct for spatial aberrations. These techniques have been implemented due to the fact the modern cameras cannot record the phase of an incoming wave. Phase recovery has been exploited by many fields, from astronomy and optical imaging to crystallography and microscopy. One of the earliest methods, which utilised the Fast Fourier Transform to recover phase from an intensity measurement, was the Gerchberg-Saxton (GS) method. Light waves have an interesting property, in that their far-field is the Fourier Transform of their near-field. Therefore, given some known prior information, the information stored in the optical far-field can be used to uncover monochromatic spatial information using computational techniques [121].

Phase retrieval solves the linear problem given by

\[ y = |Ax + c|, \]

where, in the case of the multispectral problem, the camera image \( y \) and the MSTM \( A \) are known. Reconstructions of monochromatic images, as well as the calibration of transmission matrices, have been demonstrated in recent years, using AMP (Approximate Message Passing)-based algorithms, such as prSAMP and prVAMP [122, 123]. However, the non-linear problem of multispectral imaging through complex media has not yet been solved. Therefore, in order to utilise these phase retrieval techniques, the problem laid out in Equation 6.3 must be linearised.

In this section, I will discuss modern phase retrieval techniques that have the potential to recover multispectral information from speckle intensity patterns. Each of the phase retrieval algorithms have been used previously to reconstruct monochromatic spatial information from an intensity pattern using a transmission matrix.
6.4. Phase retrieval

6.4.1 Phase retrieval techniques for the transmission matrix

The measurement of the transmission matrix of a multiple scattering medium has been seen as a breakthrough in the progression of wavefront shaping techniques in complex media. Exploiting the time-symmetry of light travelling through complex media has many applications, from adaptable focusing techniques to imaging approaches for biomedical imaging. However, a drawback of many current methods is the requirement of a reference arm or un-modulated region of the input light to access phase information. An alternative approach, developed in recent years, is the use of phase retrieval methods to extract phase information from a speckle intensity image, before storing the data in a transmission matrix.

The first demonstration using a phase retrieval method to measure a monochromatic transmission matrix was shown by Drémeau et al. using the prVBEM algorithm [124, 125]. This computational approach, based on a variational Bayesian technique, allowed a digital micromirror device (DMD) to be used to build a transmission matrix using intensity measurements and binary modulation, rather than using a more expensive phase or amplitude modulated SLM as in other transmission matrix measurements. The complex-valued transmission matrix estimate was determined using real-valued inputs and outputs, and eliminated the reference arm, normally a requisite in the measurement of transmission matrices. This opened the field of complex media research to the possibility of using a two-step approach to build a transmission matrix as well as to use it for focusing and imaging.

Later, in 2016, a double phase retrieval method was suggested by Rajaei et al. for dual estimation of a transmission matrix and reconstruction of spatial information from an intensity image [126]. Based on a group of algorithms named Approximate Message Passing ‘AMP’, or generalised-AMP ‘GAMP’, the algorithm developed to perform double phase retrieval was named Phase Retrieval Swept Approximate Message Passing, or prSAMP [122, 127, 128]. AMP algorithms were originally built to solve linear systems, and were designed to be utilised for compressive measurements. Due to the versatility of the algorithm, it was shown to be possible to extend this to solve a wide variety of systems [127, 128]. Similar to the work of Drémeau et al., it was demonstrated that prSAMP could estimate a complex transmission matrix using binary inputs from a DMD, however, the performance of prSAMP was superior to prVBEM due to the greater variety of transmission matrix classes that could be computed and used with prSAMP [124, 125, 126, 122]. Furthermore, the calibrated transmission matrix was shown to perform compressive imaging, in order to reconstruct spatial information from an intensity image without the use of a reference arm. The prSAMP approach allows high resolution reconstructions of monochromatic signals after transmission through a complex medium. Thus, prSAMP is attractive to consider for multispectral imaging, due to the compressive sampling rates permitted by the algorithm. However, despite the robustness of the measurements performed by prSAMP, the phase retrieval method was shown to be much slower than other AMP algorithms.
Chapter 6. Multispectral imaging with a complex medium

In late 2017, a new phase retrieval algorithm was introduced to combine the performance of prSAMP with the speed of other AMP approaches. Phase Retrieval Vector AMP (prVAMP) was shown to allow double phase retrieval to build a transmission matrix and recover monochromatic images from speckle pattern intensity images [123]. Conversely to the approaches in References [126] and [125], the measurements were carried out using both amplitude-only and phase-only SLMs. The algorithm showed significant improvements over other phase retrieval approaches in terms of speed, outperforming both prSAMP and prVBEM. However, compressive imaging was not demonstrated. Nevertheless, it was suggested that the prior initialisation information of the algorithm could be altered to encourage the efficient reconstruction of sparse signals using undersampled measurements - a property beneficial to solving the multispectral problem [123].

6.4.2 The prVAMP algorithm

The prVAMP algorithm is laid on the foundations of an algorithmic approach known as Generalised Approximate Message Passing (GAMP) [128]. Based on a process called linear mixing, GAMP determines a set of inputs from known outputs through a series of transformations [127]. An unknown input vector $x$ undergoes a linear transform to a vector $z$ via an operation matrix $A$. A vector $y$ is subsequently produced from $z$ and is determined through an observable channel, such as a camera image. In effect, GAMP recovers $x$ from $z$ using known values of $A$ and $y$, and this constitutes the basis of all GAMP-based algorithms.

The structure of the GAMP algorithms is formed around sum-product message passing propagation based on Bayesian networks [128]. The algorithm works by passing real values along nodes, in a process called ‘messaging’. Each ‘message’ is updated iteratively until the algorithm converges on a solution. The underlying distributions injected into GAMP via this technique are Gaussian, similar to the distribution of transmission matrices of multiple scattering media. The programs used to perform GAMP recovery can be found in a repository created by Rangan et al. [128, 129].

The approach used in prVAMP extended the GAMP concept to solve the phase retrieval problem found when imaging through complex media [123]. prVAMP allowed a monochromatic transmission matrix to be constructed from a series of intensity measurements. Using the approximation of the transmission matrix, arbitrary spatial information could be reconstructed from a speckle pattern. A SLM was used in the measurement process to project binary amplitude or phase patterns on to a multiple scattering medium. However, unlike the monochromatic transmission matrix measurement described above, prVAMP enabled phase information to be extracted from speckle patterns without the need for a reference arm or four-phase technique. Avoiding the need for a reference arm for the measurement increased the stability of the system. The added advantage of the efficiency contributed by
6.5 Simulation of a multispectral complex system

prVAMP, as well as its versatility and robustness to noise, makes it a promising algorithm to employ in the multispectral problem.

While prVBEM, prSAMP and prVAMP have displayed promising results when shaping or imaging monochromatic light after travelling through a complex medium, none have been extended to the non-linear multispectral regime [124, 125, 126, 122, 123]. Furthermore, while exploiting CS is advantageous in spectral imaging, as in prSAMP, it is equally desirable to utilise an efficient algorithm to work towards real-time acquisition of spatial and spectral information from an intensity image. In the next section, I will extend prVAMP to reconstruct sparse multispectral objects, using a pre-calibrated MSTM. Two approaches are used: a method using each layer of the MSTM to reconstruct one monochromatic component from a multispectral speckle pattern at a time, and a global approach to reconstruct multispectral information simultaneously.

In the following section, I will describe how simulations can be carried out to understand how multispectral phase and amplitude information can be extracted from a single speckle pattern, in order to build a multispectral transmission matrix.

The following computational experiments were carried out with my supervisors, as well as Jonathan Dong and Mickael Mounaix from Laboratoire Kastler Brossel. M.M. provided the speckle pattern simulation code for the MSTM, and J.D. helped with the research into each phase retrieval algorithm. Fernando Soldevilla also contributed towards the discussion. I combined the MSTM simulation with the phase retrieval algorithm, and worked towards implementing them for multispectral image reconstruction. Thanks also go to Christopher Metzler and co-authors for providing access to their prVAMP code, which is used later in this chapter [120].

6.5 Simulation of a multispectral complex system

To computationally generate a speckle pattern, and to build a MSTM, geometric optics can be employed. As in previous chapters, when constructing the MSTM, the
problem can be treated linearly, as in Equation 3.1, where $y$ is the intensity image on the camera, $A$ is the MSTM, and $x$ is a multispectral input object. It is therefore straightforward to map an known input onto an accessible output in order to calibrate the MSTM. This section describes how $A$, $x$ and $y$ can be computationally simulated to probe the reconstruction of multispectral objects.

A Gaussian beam was computationally initiated that was dependent on the desired speckle grain size at the output, as well as the amount of pixel information stored from the CCD. A SLM phase object propagating through the complex medium was created after interacting with the input beam. A speckle pattern in phase and amplitude, with Gaussian correlation, was then generated by performing a Fast Fourier Transform (FFT) on the envelope. The resulting speckle pattern was stored in a column of the MSTM. This was performed for every SLM pixel number and every spectral channel desired, in order to build up a MSTM of dimensions $N_{\text{CCD}} \times N_{\text{SLM}} \times N_{\text{spectral}}$. To ensure that there was no correlation between each spectral layer of the MSTM, a Lorentzian smoothing process was employed along the spectral dimension, consisting of a Lorentzian scaling factor and a convolution process. This technique confirmed that each spectrally-dependent layer of the MSTM was independent from all others.

SLM phase masks were created using a binary mask, $x_{\text{binary}}$, for each spectral component, before converting them to phase objects, $x$:

$$x = e^{2\pi i (x_{\text{binary}})},$$  

which were used as the multispectral objects to be reconstructed. The SLM mask was propagated through the complex medium, and acquired its wavelength properties by applying the appropriate spectral layers of the MSTM to the mask, as depicted in Figure 6.3. A fraction of i.i.d. noise was added to each output spectral component. Next, every spectrally dependent speckle pattern layer was summed, and the modulus squared of the speckle pattern was taken as the intensity image on the CCD. This resulted in a deterministic speckle pattern which was later used to extract phase and amplitude information from. Further information on the MSTM simulation can be found in Appendix D.

Once the speckle simulations were carried out, a method to reconstruct phase information from an intensity image was employed. The use of the four-phase technique was not possible, due to the complexity imposed by the Equation 6.3. While phase retrieval approaches had been utilised in previous imaging demonstrations involving linear systems, no known technique had been used to recover information from a non-linear system.

In the next section, I will demonstrate the extraction of monochromatic information from a speckle pattern containing a multispectral signal. Using a simulated MSTM in combination with the prVAMP algorithm, I work towards multispectral imaging through a multiple scattering material. While demonstrations shown here
6.6 Recovering a monochromatic object from a multispectral speckle pattern

In order to reconstruct a multispectral object after propagation through a complex medium using current computational approaches, Equation 6.3 must be written as a linear system. The first approach presented here used individual spectral components of a pre-calibrated MSTM, as depicted in Figure 6.2. The monochromatic matrix layer was applied to the multispectral intensity output, and was used to reconstruct a monochromatic component of the original object. When no spatial information was detected at the wavelength selected, only noise was reconstructed. In doing so, the interpretation of the non-linear problem effectively treated all other spectral contributions as noise elements in the intensity image.

Beginning with a spatially and spectrally sparse object, a computational experiment was carried out to reconstruct multispectral information. The MSTM was calibrated with $N_{SLM}=256$ and $N_{spectral}=20$. Three different CCD pixel areas of $N_{CCD}=16384, 18225, 19600$, corresponding to $140 \times 140, 135 \times 135$, and $128 \times 128$ pixels, respectively, were used to alter the sampling rate. These values of $N_{CCD}$ were based on simulations, using a double phase retrieval method renders the use of an experimental reference arm redundant, potentially leading to a more stable approach.
Chapter 6. Multispectral imaging with a complex medium

Figure 6.4: Reconstructing a multispectral object containing 2 wavelength components at $N_{\text{spectral index}}=1$ and 15 when $N_{\text{CCD}}=16384$, $N_{\text{CCD}}=18225$, $N_{\text{CCD}}=19600$, with $N_{\text{SLM}}=256$ and $N_{\text{spectral}}=20$.

chosen as they corresponded to a narrow range of sampling rates within which multispectral reconstruction was both unsuccessful and achievable. Figure 6.4 shows the reconstruction of a multispectral object, consisting of 2 spectral components. Two SLM masks, each corresponding to a spectral index number of 1 and 15, were simultaneously propagated through the complex system, with each monochromatic component made up of a flat phase background (unmodulated) and a small phase shifted region of pixels to simulate the multispectral object.

Each layer of the MSTM was applied individually to the subsequent speckle pattern that was produced by the multispectral object, as described in Figure 6.2. In every iteration, the reconstruction process determined if spatial information could be recovered for one wavelength channel. As shown in Figure 6.4, when $N_{\text{CCD}}=19600$, both spectral components of the object were successfully reconstructed at the correct wavelength channel. However, the phase of the object was reversed in sign, due to the approximation of the solution given by prVAMP and the overall uncertainty on the global phase of the object. It is thought that the phase retrieval priors could be optimised for future recoveries. Reconstruction was achieved for $N_{\text{CCD}}=18225$, although the retrieved spectral components contained a significant amount of noise. The recovery of the multispectral object was not possible for $N_{\text{CCD}}=16384$. This demonstrated that a sufficient amount of output information from the camera was needed for a good reconstruction. In fact, a sampling ratio $N_{\text{CCD}}/N_{\text{SLM}}=10^2$ (equivalent to $Y/X$ in previous chapters) was required to observe a reconstruction of the object above the noise.

To observe how well prVAMP could reconstruct multispectral objects with decreasing spectral sparsity, the same computational experiment was carried out but
with an additional wavelength component incorporated into the object. Wavelength channels $N_{\text{spectral}}$ index=1, 15, 20 were contained within the multispectral object. The additional wavelength component consisted of a flat phase mask with a few phase-shifted pixels, which did not overlap with the other spectral elements. The MSTM was calibrated with $N_{\text{CCD}}=22500$, $N_{\text{SLM}}=256$, and $N_{\text{spectral}}=20$.

Figure 6.5 shows the reconstruction at each wavelength channel. Recovery of phase information at $N_{\text{spectral}}$ index=1, shown in Figure 6.5(a), is achieved. However, there is also the presence of a reconstruction at $N_{\text{spectral}}$ index=20. This demonstrates that the recovery begins to break down as the spectral sparsity is decreased. It appears that prVAMP is unable to distinguish between spectral components, but it is still able to reconstruct the correct phase information. In Figure 6.5(b), it can be seen that the recovery of spatial information fails. However, successful recovery of information in $N_{\text{spectral}}$ index=20 was achieved, as in Figure 6.5(c), albeit with a partial reconstruction of the information in $N_{\text{spectral}}$ index=1 as well.
Chapter 6. Multispectral imaging with a complex medium

A 2-D matrix is formed by arranging each monochromatic layer of the MSTM along the diagonal. The dimensions of the new matrix are $N_{\text{SLM}} \times N_{\text{spectral}}$ by $N_{\text{CCD}} \times N_{\text{spectral}}$.

It can be seen from Figure 6.5 that the recovery of more than 2 spectral components using prVAMP is not viable when treating each reconstruction as a monochromatic input with a substantial amount of noise. The system requires that a correlation between the correct wavelength channels in the MSTM and the multispectral object are found. The chosen phase retrieval algorithm would need to compete with vast amounts of noise to recover any meaningful information. In contrast to the transmission matrix approaches in Chapters 4 and 5, the spectral signatures produced by light travelling through complex media are affected by spatial degrees of freedom as well. This makes it more difficult for the algorithm to converge on the correct solution when reconstructing phase, amplitude and spectral information. The contrast of the relevant speckle information decreases significantly as more spectral components are added, i.e. as the ‘noise’ contributions increase.

Solving the multispectral problem by addressing individual elements of the incoherent sum of spectrally-dependent speckle patterns is a straightforward way of recovering information. However, this is an inefficient process, as the algorithm has to iterate through every spectral layer of the MSTM. A more attractive approach would be to recover multispectral objects using a global approach that considers the full MSTM in a single measurement. Therefore, prVAMP would have access to all the spectral layers of the MSTM during the reconstruction process, as well as enable the reconstruction to take place in one measurement. In the next section, I present an approach which allows every layer of the MSTM to be used simultaneously to
6.7 Global reconstruction of spatial and spectral information

Treating the multispectral problem as many incoherent monochromatic systems with added noise is not an efficient way to reconstruct information. It is more desirable if the system can resolve every spectral component simultaneously. To solve the problem in one global measurement, the 3D MSTM must be translated into a 2D matrix.

One way to do this is to place every spectral layer of the MSTM along the diagonal of a large matrix of dimensions $N_{CCD} \times N_{spectral}$ by $N_{SLM} \times N_{spectral}$, as illustrated in Figure 6.6. Relating this back to the linear equation, the full MSTM is represented by $A$, $x$ is the input multispectral vector of length $N_{SLM} \times N_{spectral}$, and $y$ is an output vector from the camera image of dimension $N_{CCD}$. However, as $y$ does not have access to the spectral dimension of the multispectral object, it must be compensated for in another way. In this second approach, $y$ will take on the dimension of $N_{CCD} \times N_{spectral}$ by forming a large vector containing $N_{spectral}$ copies of the CCD pixels, as illustrated in Figure 6.7. Therefore, the problem can be solved using prVAMP in one measurement.

Evidently, processing such a large amount of information in one measurement requires a proportional amount of computational power. For this reason, the demonstration shown here uses $N_{spectral} \leq 5$. As discussed above, prVAMP is inherently not

![Figure 6.7: Schematic of the global multispectral system. The output CCD image, corresponding to $y$, is repeated within a vector of length $N_{CCD} \times N_{spectral}$. The problem can be treated as linear if the 3D MSTM, as in Figure 6.6, is substituted for $A$. $x$ is a multispectral object of length $N_{SLM} \times N_{spectral}$.](image-url)

reconstruct a multispectral object. The limitations are discussed, as well as hopes for the future of the reconstruction of spectral and spatial information in a snapshot measurement.
Chapter 6. Multispectral imaging with a complex medium

Figure 6.8: Reconstruction of a multispectral object containing 2 spectral components using a global approach ($N_{\text{spectral}}=4$). Input SLM phase masks, corresponding to $N_{\text{spectral}}\text{index}=1$ and 2, respectively, are reconstructed using prVAMP after travelling through a complex medium, where $N_{\text{SLM}}=16$ and $N_{\text{CCD}}=27225$.

designed for compressive measurements. Therefore, there is no opportunity to undersample CCD information at the measurement stage. However, there is potential to alter the prior conditions to exploit the sparsity of input signals for compressive phase retrieval.

As in Section 6.6, to demonstrate the viability of multispectral imaging through a multiple scattering medium, a sparse multispectral object, containing 2 spectral components, was reconstructed. However, unlike the previous section, the system had the advantage of having access to all of the wavelength-dependent layers of the MSTM, rather than just one layer. The diagonal MSTM was employed with prVAMP, with $N_{\text{spectral}}=4$, $N_{\text{SLM}}=256$, and $N_{\text{CCD}}=27225$. Figure 6.8 shows the reconstructed objects in their individual wavelength channels, $N_{\text{spectral}}\text{index}=1$ and 2. In one measurement, the spatial and spectral information was reconstructed from a single intensity pattern. An improvement to the previous demonstration was the identification of the correct phase shift of each pixel (no reverse in the sign). It should be noted that the reconstruction was only possible if the input spectral information was sparse. Reconstruction of 2 spectral components was not possible for a MSTM calibrated with $N_{\text{spectral}}=2$. This was found to be independent of the sparsity prior during initialisation of prVAMP.

To probe the global approach and its ability to recover more than 2 spectral channels from a multispectral object, an additional wavelength component was added to the input. The MSTM was calibrated with $N_{\text{spectral}}=5$, $N_{\text{SLM}}=256$, and $N_{\text{CCD}}=27889$. 
Figure 6.9: Reconstruction of a multispectral object containing 3 wavelength components. Using a global approach ($N_{\text{spectral}}=5$). Input SLM phase masks, corresponding to $N_{\text{spectral}}\text{index}=1, 2$ and 5, are reconstructed after travelling through a complex medium, where $N_{\text{SLM}}=16$ and $N_{\text{CCD}}=27889$.

Figure 6.9 shows the reconstructed object at 3 different spectral channels. The phase and amplitude of the multispectral object was reconstructed, however, the object was found in every calibrated spectral channel. While prVAMP showed promising results for the recovery of 2 spectral components, it was clear that above this threshold the technique could not decipher hidden spectral information in the intensity image. Interestingly, for $N_{\text{spectral}}\text{index}=2$, the phase shift of the reconstructed object was reversed. Again, this was due to the approximation of the relative phase shift of the multispectral object as determined by prVAMP.

The results presented in this section show a great improvement on the reconstruction quality when using a global approach over a monochromatic method. For a spectrally sparse object, the correct phase and spectral properties were determined within a single measurement using prVAMP. By employing a diagonal MSTM, the process became much more efficient as there was no need to iterate over every calibrated wavelength channel. However, this method required that a substantial amount of information was measured and stored in the MSTM. This led to difficulties in computational processing, with the size of the MSTM being limited by the memory of the computer. As prVAMP was built for computational speed over the ability to compress measurements, a CS approach could not be utilised. Other algorithms that are designed with CS in mind, such as prSAMP, may lead to more optimal reconstructions in future investigations. Further to this drawback, prVAMP could not identify more than 2 spectral components from an intensity image. It is hoped that by exploiting the CS regime that a sufficient amount of information can be collected in order to recover phase, amplitude and spectral information in one measurement.
6.8 Outlook for multispectral imaging

While the techniques presented in Chapters 4 and 5 showcase a promising new approach to hyperspectral imaging, spatial filtering of the incoming signal led to a proportionally reduced signal intensity within each speckle pattern. The advantage of using one speckle as in this MSTM approach is the increased speckle pattern signal, as there is no necessity for a spatial filtering component, which absorbs or reflects some of the input light. Therefore, a larger portion of the input energy can be directed into one spatial channel.

While the MSTM has only been measured experimentally using a reference arm, there is the potential for one to be measured without the requirement of a reference. As prVAMP, along with many other phase retrieval techniques, was designed to build a transmission matrix as well as recover information from an intensity pattern, the process removes the necessity to have an unstable reference. However, the addition of a spectral dependence on the speckle behaviour adds further complication to this measurement. Although, as transmission matrix methods are integrated into portable technologies, the elimination of an experimental element in the multispectral recovery system is highly desirable.

There is still a long way to go in terms of creating a speckle-based multispectral imaging device that acquires spatial and spectral information from one speckle pattern. In this chapter, I showed that a phase retrieval algorithm could be used to recover a multispectral object containing up to 2 spectral components. However, for a spectral imaging device, this level of performance is not suitable, as snapshot spectral imaging techniques require the collection of many discrete wavelength components within one measurement. As the prVAMP algorithm used here depends mainly on the sampling rate being well above the Nyquist-Shannon threshold, it is hoped that the algorithm initialisation conditions can be adjusted to allow compressive measurements. In the global approach, prVAMP was unable to recover any information about the multispectral object which contained the same number of wavelength components as the calibrated $N_{\text{spectral}}$ layers in the MSTM. However, only a small amount of information could be stored in the diagonal matrix, due to the impractical amount of computational resources needed. A reduction in the necessary CCD camera data would allow the MSTM to be calibrated for a greater $N_{\text{spectral}}$, as the computational demands would be decreased. Thus, a greater reconstruction efficiency for multispectral objects could be acquired.

6.9 Summary

In this chapter, I presented an approach to reconstruct both spatial and spectral information from a single speckle pattern. The technique utilised the work of Metzler et al., Andreoli et al., and Mounaix et al., combining a phase retrieval technique, prVAMP, and the MSTM, to allow the recovery of a multispectral object, such as
layers of white paint [123, 53, 52]. No approach had previously allowed the recon-
struction of a multispectral object from a speckle pattern in intensity using a MSTM,
due to the non-linear nature of the system. Extracting spectral and phase informa-
tion from an incoherent sum of intensities proved to be a difficult task, especially as
the problem had to be moulded into a linear problem.

Two approaches demonstrated in this chapter aimed to reduce the multispectral
problem to a linear system. The first treated the system as a monochromatic imaging
system, with the additional spectral components treated as noise. This method was
conceptually simple, as prVAMP could be implemented with one layer of the MSTM
and the raw output intensity pattern. It was shown that 2 spectral components could
be reconstructed simultaneously from a speckle pattern. However, the system was
rapidly overcome by noise, meaning that a multispectral object with more than 2
spectral components could not be recovered. As every layer of the MSTM had to be
processed individually, the recovery was slow. The reconstruction quality was also
restricted, as prVAMP did not have access to the full MSTM.

The second method implemented a global approach that utilised the full MSTM
for a snapshot recovery of a multispectral object. Each layer of the MSTM was placed
along the diagonal of a larger matrix, before prVAMP was employed. The global ap-
proach returned a faster recovery, with a greater reconstruction quality when recon-
structing a multispectral object containing 2 spectral components. However, when
the multispectral object contained 3 or more wavelength components, the system
could not resolve any spectral information. Nevertheless, phase and amplitude in-
formation could be reconstructed, thus, the full object was observed in every wave-
length channel. The global approach was limited by the amount of information that
could be stored and processed in a single measurement. As new phase retrieval tech-
niques are developed in the coming years it is hoped that an efficient CS approach
can be utilised to maximise the amount of information stored in the MSTM.

Despite only being able to reconstruct the spatial and spectral information from
an object containing 2 wavelengths, this approach has demonstrated some promis-
ing results. By collecting information from a multispectral object in one speckle
pattern, the signal intensity is maximised, unlike the spatial filtering techniques in
previous chapters. With improved computational techniques, the possibility of de-
veloping a spectral imaging system using a multiple scattering material as a char-
acterisation component may become more probable. Hence, only a small amount
of experimental equipment would be required, with powerful algorithms removing
the need for these unstable elements. As mentioned, the potential spatial resolution
of the system could be much greater than the devices presented in Chapters 4 and
5, however, the spectral resolution is heavily restricted by the spectral correlation of
the complex material used. While the throughput of a multiple scattering medium is
limited by the scattering strength of the material, it is possible that this method could
be extended to multimode fibres. It is hoped that the proposed device presented here
can be used for applications in high spatial resolution multispectral imaging.
Chapter 7

Conclusions

7.1 Summary

In this thesis, I have shown that complex media can be exploited for spectral imaging systems. With the aim of removing the dependence of spectral resolution on the required footprints of traditional, dispersive, wavelength characterisation elements, there is great scope for utilising complex media for the development of portable devices. Complex media, including multiple scattering media and multimode fibres, are easily accessible and do not require intricate fabrication processes, making them desirable for cost effective solutions in spectral imaging. Due to the large volume of information acquired in snapshot imaging systems, a method to decrease the required amount of information was introduced. Compressive Sensing (CS) allowed speckle pattern information to be sampled at rates much lower than the traditional Nyquist-Shannon limit, suggesting that not only is CS an ideal computational technique to use to improve processing efficiencies, but also that complex media are ideally suited to compressive techniques due to the complete mixing of input and output degrees of freedom.

There are many approaches that can be used to achieve snapshot spectral imaging, including harnessing fibre bundles and lenslet arrays to spatially filter incoming light. Both of these approaches were used here in combination with natural disorder found in multiple scattering media and multimode fibres to perform spectral imaging. Finally, a technique to remove the requirement of a spatial filtering system was introduced, with the aim of uncovering both spatial and spectral information from a single speckle pattern.

In this chapter, the outcomes of this thesis are detailed, from the computational techniques used, to the three spectral imaging techniques designed throughout the duration of this Ph.D.

7.1.1 Computational techniques

An important aspect of snapshot spectral imaging systems is the computational techniques that can be used to analyse and process output signals. In Chapter 3, it was
highlighted that many spectral imaging systems, and hence the algorithms used to uncover the spatiotemporal information, follow the form of a linear equation. The three inversion methods investigated for spectral imaging devices with complex media included the transpose conjugate, pseudoinversion and Tikhonov regularisation (TR). In the scenarios encountered by spectral imaging devices, the computational techniques were required to be able to reconstruct input signals with high precision and also be robust to noise. TR, with a noise factor to account for changes in environment, was shown to be the most reliable and versatile for resolving spectral information contained within speckle patterns.

To overcome the data acquisition bottlenecks encountered by many snapshot systems, with the vast amount of information acquired, a modern computational technique was probed. In particular, CS was shown to allow sampling of information with rates well below those permitted by the traditional Nyquist-Shannon theorem, using $l_1$-minimisation. It was determined that the spectral sparsity of signals measured by speckle-based spectral imaging systems could be exploited to undersample output speckle information. Utilising this approach gave potential to optimised reconstruction algorithms for efficient processing of output signals.

In order to utilise the presented techniques, the information stored in wavelength dependent speckle patterns had to be characterised. As in demonstrations by Popoff et al. and Redding et al., it was determined that a transmission matrix approach could be used to store the wavelength information \[27, 42\]. The transmission matrix contained $X$ spectral channels, and $Y$ pixel values from each output speckle pattern. The sampling rate used was determined using the ratio of $Y/X$. The classical sampling threshold held when $Y/X \geq 1$, and below this threshold was considered to be undersampling.

It was determined that CS and TR were two potentially viable computational techniques to extract information from speckle patterns. In subsequent chapters, one or both of these methods were implemented.

### 7.1.2 Multiple scattering in spectral imaging systems

Following on from the computational techniques described in Chapter 3, TR and CS were utilised in recovering spectral information from speckle patterns produced by a multiple scattering layer of GaP nanowires. The nanowire mat was used due to its relatively high transmission (18 %) and impressive scattering strength ($D = 14 \pm 1 \text{ m}^2/\text{s}$). As shown in Chapter 2, the spectral correlation bandwidth of multiple scattering media was found to be directly related to the diffusion constant of the medium. Therefore, a greater sensitivity to input wavelength could be achieved for high spectral resolution while maintaining a high throughput. For a 1.7 µm layer of GaP nanowires, the spectral correlation width was found to be approximately 14 nm. The system could find use in the field of multispectral imaging, in which wide calibrated spectral channels are used.
In order to acquire spatial information in a single shot, a lenslet array was used, as in many Integral Field Spectrometer approaches for telescope acquisition [5, 6, 7]. When light interacted with the array, many different focal spots were formed on the surface of the nanowire mat, before each produced an independent speckle pattern. A 3D STM was measured containing spectral information for many independent speckle pattern positions on the camera screen. TR and CS were used to reconstruct spectral images and their reconstruction fidelity was compared.

In the face of noise, CS was shown to be robust to up to 20%, while TR could not withstand more than 5% noise. As expected CS performed well under many sampling conditions, from oversampling to undersampling, while TR was unable to reconstruct spectral information from a speckle pattern when approaching the Nyquist-Shannon threshold and below, thus showing that CS could be exploited to greatly undersample speckle information at the acquisition stage. Again, CS outperformed TR when reconstructing multiple wavelengths simultaneously, and it was shown that up to 12 spectral components could be recovered in one measurement using CS at a high sampling rate. When sampling below the Nyquist-Shannon limit, while TR could not recover any information, CS was able to recover between 6-8 wavelength components. The decrease in simultaneous spectral acquisition for CS was due to the sparsity constraints of the $l_1$-norm.

Further to this characterisation study, combined spectral and spatial acquisition was achieved using a STM to extract speckle information from a monochromatic CMOS camera. Both techniques were able to reconstruct spatial information, however, the reconstruction ability of CS was visibly superior to that of TR. The corresponding spectral information obtained from each ‘pixel’, or speckle pattern position, in the output signal was determined using TR and CS. A comparison between the two techniques demonstrated that the reconstruction efficiency of CS was closer to the original input, due to the minimisation of background reconstruction noise in the resulting spectral recovery.

Following on from these findings, CS was probed to determine its performance under many different sampling conditions when reconstructing both sparse and dense spectral objects. The experimental sampling rate ranged from oversampling to undersampling, with one sampled at approximately the Nyquist-Shannon limit. It was shown that there was no apparent difference between reconstructions of a sparse spectral signal using sampling rates of $Y/X=14.2, 1.2, 0.8$. However, the reconstruction efficiency decreased significantly for $Y/X=0.3$. Similarly, for the recovery of spatial information of a spectrally-sparse object, using sampling rates of $Y/X=14.2, 1.2, 0.8$, a good reconstruction was achieved, although the fidelity of the letter reconstructions decreased with sampling rate. This result showed that CS could be harnessed to minimise the amount of information required for storage in the STM ($Y/X\leq1$) for detection of sparse signals. However, when reconstructing multiple spectral components, a marked difference between oversampled and undersampled reconstructions was observed. Due to the decrease in sparsity of the
input signal, the performance of CS deteriorated. However, for non-sparse signals, CS showed good reconstruction efficiency.

CS was shown to be a versatile computational technique for the recovery of information from speckle patterns. It confirmed our understanding that complex media could be used to perform efficient and fast acquisition of signals in imaging and sensing. The multiple scattering medium system was ultimately limited by the throughput of the medium as well as the speckle contrast produced by the medium. Nevertheless, this technique could have applications in high intensity detection where only a small number of spectral bands are detected.

7.1.3 Multimode fibre spectral imaging

In order to design a spectral imaging system with a high throughput, a 30 cm multicore multimode fibre (MCMMF) with approximately 3000 cores was used. The fibres were found to have a spectral correlation bandwidth of 1.4 nm, suggesting an achievable sub-nanometre spectral resolution. This work extended on the demonstration by Redding et al. using a multimode fibre as a high resolution spectrometer. Similar to the approach presented in Chapter 4, the MCMMF produced wavelength dependent speckle patterns with a strong spectral dependence on the length and the numerical aperture of the fibre. Each fibre core produced an independent speckle pattern, which was characterised for a range of wavelengths. A clustering algorithm, DBSCAN, was utilised to discover the coordinates of each speckle pattern position for efficient measurement of the STM. Once the STM was calibrated, the reconstruction efficiency of the system was probed.

As in Chapter 4, the system was characterised based on both the number of wavelengths reconstructed simultaneously and the sampling rate required to successfully recover information. Utilising CS demonstrated that a high reconstruction fidelity could be achieved when undersampling sparse signals, and could also be employed when oversampling to recover more dense spectral objects. It was also shown that 100% of all calibrated wavelengths (43 in total) could be recovered in a single snapshot measurements using CS, demonstrating that CS is a versatile technique that can be employed for a multitude of applications.

In a demonstration of combined spatial and spectral recovery, 16 letters, each of a different wavelength, were successfully detected and the corresponding spectral information was extracted from each spatial channel. Both sparse and dense spectra were detected in one measurement, showing that snapshot spectral imaging is viable for a range of detection purposes. Moreover, it was shown that a sampling rate of as low as Y/X=0.14 could be used to recover one spectral component. This corresponded to selecting 16 camera pixels from each speckle pattern.

The angle dependence of speckle patterns produced by multimode fibres, is well known and is often seen as a restriction for speckle-based spectral detectors. To overcome this, in Chapter 5 the angle dependence of the resulting output speckle
patterns from the multicore multimode fibre was probed to determine if the aperture of the device could be extended. It was demonstrated that, within a narrow angular range on the order of $1^\circ$, the STM calibration could continue to be utilised. Therefore, the angle dependent ‘memory effect’ could be exploited to increase the viable aperture of the fibre spectral imaging system. However, to extend beyond this range would require the complexity of interfering angle dependent speckles to be deciphered using advanced computational techniques.

The high transmission and spectral resolution of the MCMMF spectral imaging system is desirable and fulfils the criteria not satisfied by current approaches. The ease of access to MCMMFs makes this system an attractive solution for low cost, compact spectral imaging systems.

7.1.4 The multispectral transmission matrix for spectral imaging

The final stage of this thesis involved the challenge of extracting spectral and spatial information from a single speckle pattern produced by a multiple scattering medium. This work entailed the simulation of speckle patterns with a dependence on the input wavelength and phase of the object. Information about the phase, amplitude and wavelength of light travelling through the multiple scattering medium was calibrated and stored within a MSTM. The recovery of arbitrary objects was carried out using a phase retrieval algorithm and the information contained within the MSTM.

The algorithm used to recover 2D information within a single shot was prVAMP. Based on Bayesian techniques and using the Gerchberg-Saxton algorithm to produce an initial ‘guess’, the iterative phase retrieval algorithm was used to recover up to 2 spectral components simultaneously using two different approaches.

Due to the nonlinearity of the multispectral problem, the system had to be linearised in order to utilise prVAMP. One approach treated the problem as monochromatic with large amounts of added noise. Effectively, this problem was solved using a scanning approach, with spectral information obtained over many post-processing measurements. As the number of spectral components added to the reconstruction was increased, the fidelity decreased significantly. For an object containing 3 or more spectral components, no spatial or spectral information could be recovered. In a second approach, the MSTM was diagonalised within one 2D matrix and used to reconstruct every wavelength component in one measurement. While only 2 spectral contributions could be correctly identified in a single shot, for an object containing 3 or more wavelengths the phase and amplitude information could still be recovered. It was hoped that, by tuning the prior conditions of the prVAMP algorithm, the complexity of input objects could be increased by exploiting sparsity in a known domain and utilising CS techniques. If this approach were able to reconstruct more wavelength and spatial information simultaneously, it would be a highly desirable technique, as phase, amplitude and wavelength information could be extracted from
an intensity image within one measurement. However, more development is needed to enable us to move towards a phase retrieval technique tailored to resolving the multispectral problem.

### 7.2 Outlook and future work

With the advancement of compact, powerful computational equipment, there has been a rapid increase in the interest in snapshot spectral imaging for portable technologies. While snapshot imagers require a significant amount of computational power to simultaneously measure three dimensions of information, these techniques are now viable in commercial systems. Current approaches based on gratings that spectrally disperse light are well-understood and straightforward to use, however, as explained in this thesis, the spectral resolution of these devices are limited by the size of the system. Complex media offer a solution to this dilemma, as they produce wavelength dependent speckle patterns that can be harnessed for spectroscopy and spectral imaging. While this knowledge has been exploited for less than a decade, it has gained rapid interest in the sensing community [130].

Complex media have many advantages over traditional dispersive media as a high resolution of up to picometre resolution can be achieved without compromising on the size of the spectral characterisation system. Disordered structures, from multiple scattering media to multimode fibres, are easily accessible and cheap to acquire, meaning that they aren’t reliant on complicated fabrication processes. The parameters of these media can also be easily adjusted in order to produce the desired characteristics, such as spectral resolution and throughput. However, there are still many aspects of speckle production that are not well understood, including how to simultaneously extract multiple dimensions of information from a single speckle pattern and the effects of environmental variation on calibrated systems. Nevertheless, much progress has been made in understanding these systems and they may offer a logical solution for compact spectral imaging and spectroscopy devices.

#### 7.2.1 Wavefront sensing with multiple scattering media

An area of investigation that could expand on the multiple scattering medium spectral imaging system in Chapter 4 utilises the angle dependence of speckle patterns to perform wavefront sensing. Shack-Hartmann wavefront sensors are commonly found in adaptive optics systems, and employ the use of lenslet arrays to determine the local tilt of incident wavefronts. As shown in Figure 7.1, the lenslet array focuses light down onto a detector, such as a CMOS or CCD, and the system is calibrated for light incident to the normal of the array plane. A shift or tilt of the wavefront is detected if the focal spot on the camera is displaced by an amount. The phase shift of the wavefront can then be calculated based on the displacement of the focal spot on the detector. It is thought that this technique could be combined with a thin multiple
7.2. Outlook and future work

![Figure 7.1: An example of a Shack-Hartmann wavefront sensor. A lenslet array is used to focus light onto a detector. (a) A uniform wavefront produces a uniform grid of focal spots on the detector. (b) A non-uniform wavefront displaces the grid of focal spots.](image)

scattering medium in order to classify spectral information as well as the shape of incoming wavefronts.

In Chapter 4, the angle dependence of speckle patterns was not probed, however, it is known that the memory effect observed in Chapter 5 can also be exploited in multiple scattering media [51, 131]. The memory effect of multiple scattering media has been utilised in imaging and sensing to ‘see’ around corners [85, 84]. However, beyond this region the speckle pattern significantly decorrelates as the incident angle is changed. Therefore, there will be an apparent shift on the camera screen, as in the Shack-Hartmann wavefront sensor. If this change of speckle pattern arrangement and speckle pattern displacement can be characterised, incident wavefronts and spectral details can be recovered.

### 7.2.2 Spectral imaging in the world of Artificial Intelligence

Throughout this thesis, I have explored the use of wavelength dependent intensity patterns to characterise spectral information. 3D spatial and spectral information was measured and stored in a STM to recover spectral images within one measurement. However, utilising a transmission matrix approach requires on the order of seconds or minutes to resolve input information. When measuring in real-time, it is desirable to recover information instantly.

One of the most exciting developments in recent years has been the introduction of machine learning and neural networks in experimental physics. These computational techniques can be harnessed for many applications as they have the potential to observe and detect patterns that are not immediately obvious to the human brain. Neural networks have been utilised in combination with complex media to ‘learn’ about the behaviour of these systems [132, 133, 134]. Similar to the measurement of transmission matrices, neural networks rely on the feeding of speckle pattern information, in order to build an understanding of observed changes and fluctuations in speckles produced by complex media. Once neural networks are trained, the
time scale on which they can identify and understand an output signal is potentially much faster than transmission matrices.

It is thought that a neural network could be trained using wavelength dependent speckle pattern information to allow fast acquisition of spectral components. As in the STM measurement, a neural network could be developed using a large speckle pattern data set corresponding to many wavelengths, or measured over a large time scale to account for any instability of the medium, as illustrated in Figure 7.2. However, neural networks can take on the order of days to ‘learn’ enough about their field of interest, depending on the complexity of the network, and so a long calibration time may be required for the network to be functional.

This incorporation of complex media into neural network approaches could lead to a better understanding of the behaviour of light in disordered materials. Integrating computational techniques into physical systems may help us advance much further into developing novel imaging and sensing devices exploiting complex media.
Appendix A

Building a spectral intensity transmission matrix

The results demonstrated in Chapters 3, 4 and 5 would not have been possible without the extensive post-processing code developed over the course of this Ph.D. In the Appendices, I summarise the code used to build the spectral intensity transmission matrix, perform the inversion techniques from Chapter 3, and initiate the CS method using $l_1$-minimisation.

Appendix A contains a simplified version of the MATLAB script required to build a spectral intensity transmission matrix (STM).

In order to build a STM, a series of data must be collected as described in Chapters 4 and 5. The image files labelled here, each corresponding to different wavelength dependent speckle patterns, were labelled with an index number starting from ‘000’. The initialisation procedure in the STM code simply opens the files depending on their location, name and index number. The parameters `firstindex` and `lastindex` correspond to the first and last index number of the files, so that a specified range of speckle images can be called into the script. The second portion of the initialisation normalises each image to the highest intensity value within the speckle pattern. At this stage, it is important to confirm that no saturated pixels are contained within the image. Finally, once each file has been imported, the files (TIFF format) are closed to minimise the amount of computer memory required.

In order to measure a STM, the known coordinates of the speckle pattern centroids must be known. An area of interest (AOI) must be chosen that corresponds to the pixel area selected from the camera image. The coordinates are then screened along with the chosen AOI to confirm that the AOI region lies within the image and does not travel outside the boundary of the image. Once this has been performed, the coordinates are used to select pixel AOIs from the camera image at each speckle pattern position, before storing them in a matrix column. Hence, the length of a column in the STM is equal to the AOI. This process is performed for every wavelength dependent speckle pattern image, in order to build a full STM.
for index = firstindex:lastindex
  % Call in files containing speckle patterns
  jindex = index - firstindex + 1;
  numindex = num2str(index);
  if (index<100)
    numindex = ['0' num2str(index)];
  end
  if (index<10)
    numindex = ['0' numindex];
  end
  fileloc = [filenamebase numindex ext];
  filename = [dir1 fileloc];
  image = dlmread(filename);

  % Normalise to highest value
  myRange = getrangefromclass(image(1));
  newMax = myRange(2);
  newMin = myRange(1);
  imagenorm = (image - min(image(:))) *(newMax - ...
              newMin)/(max(image(:)) - min(image(:))) + newMin;

  fclose('all');
  % Call in coordinates of speckle patterns
  for coord = 1:coord_num
    XX = round(coordinate(coord,1));
    YY = round(coordinate(coord,2));
    % Ensure coordinates + selected AOI fall within image
    if XX>aoi && YY>aoi
      % Coordinate for one speckle
      xy = [XX, YY];

      % Define the x and y centre coordinates of the speckle
      aoix = xy(1);
      aoiy = xy(2);

      % Select AOIs from speckle patterns in image (imagenorm)
      image(1:(2*aoi+1),1:(2*aoi+1),jindex) = ...
        imagenorm(aoiy-aoi:aoiy+aoi,aoix-aoi:aoix+aoi);
  }
Appendix A. Building a spectral intensity transmission matrix

```matlab
39     tmatrix(:, jindex, coord) = ...
        reshape(image(1:(2*aoi+1),1:(2*aoi+1),jindex), ...
                 colsizex^2, 1);
40
41     end
42     end
43     end
```
Appendix B

Reconstruction of spectral information via inversion techniques

In Chapters 3 and 4 inversion techniques are carried out on the STM to enable the reconstruction of information from speckle patterns. These techniques were pseudoinversion, transpose conjugation and Tikhonov regularisation (TR).

Performing an inversion on a 3D STM requires each 2D layer to be treated separately. In this code, the STM is is separated along the axis corresponding to speckle pattern position in order to treat each wavelength dependent speckle pattern independently. The pseudoinverse is determined using MATLAB’s \texttt{pinv} function which computes the Moore-Penrose pseudoinverse of each 2D matrix, as in line 7. The transpose conjugate is determined using MATLAB’s inbuilt transpose function, denoted by ‘ \texttt{'} \texttt{'} in line 11. The computation determines either the complex conjugate of the matrix if it contains any non-real elements, or the transpose if the matrix only contains real values.

The inversion technique used in Chapter 4, TR, is computed in several stages between lines 15 and 27. First the maximum values in the STM are determined by performing a singular value decomposition (SVD) using an inbuilt MATLAB function. A percentage of the maximum values is used as the sigma value, or noise tuning parameter, in the TR matrix calculation. Finally, TR is computed as in Equation 3.7 for each layer of the STM, before being stored as slices in the 3D matrix \texttt{tik}. Rather than being computed in one line of code, the calculation is broken down into several stages to avoid a computational error.

Once the inverse of the STM is computed, it can be applied to an ‘output’ speckle pattern image \texttt{y} that has been unwrapped and converted into a matrix as in Appendix A. The output matrix is 2D, corresponding to an AOI (equivalent to that selected to build the STM) and the number of calibrated speckle pattern positions. The resulting computation between the inverse of the STM and the output gives an insight into the original input signal \texttt{x}. 
Appendix B. Reconstruction of spectral information via inversion techniques

```matlab
for iteration=1:coord
    % Process every layer corresponding to different speckle ... pattern
    % separately
    tmatrix_=tmatrix(:,:,iteration);

    % Determine pseudoinverse
    T_=pinv(tmatrix_);
    T(:, :, iteration)=T_;%

    % Determine transpose
    tmatrixtransp=tmatrix_';
    transp(:,:,iteration)=tmatrixtransp;

    % Singular value decomposition to determine maximum values ...
    in STM
    [U, MATSVD2, V] = svd(tmatrix_);
    MATSVD=1./MATSVD2;
    max_value(:, iteration)=max(diag(MATSVD2));

    % Sigma value for Tikhonov regularisation
    sigma=0.01*max_value(:,iteration);

    % Calculate Tikhonov inverse
    L1=tmatrixtransp * tmatrix_;
    L2=(L1+sigma * eye(nrow));
    L3=inv(L2);
    tik_=L3*tmatrixtransp;
    tik(:,:,iteration)=tik_;%
end
```
Appendix C

Performing the $l_1$-norm with the spectral intensity transmission matrix

In Chapters 3-5, a compressive sensing technique known as $l_1$-minimisation, denoted CS, is used to reconstruct information using a STM and a set of known output speckle patterns. Unlike the inversion techniques, the STM is not processed separately before applying to the output speckle pattern image. In contrast, the original input is calculated through an iterative process using a convex optimisation package called ‘CVX’ [101].

The computation treats each speckle pattern position separately, as in the inversion case, before determining the snapshot reconstruction. The CVX package performs the process laid out in Equation 3.9 using the initiating line `cvx_begin` and the closing line of `cvx_end`. The ‘output’ $\mathbf{y}$ is a speckle pattern output image that is unwrapped using the same procedure as for the STM measurement. Once CVX has performed the computation, the reconstructed input signal can be extracted. Finally, the reconstruction $\mathbf{z}$ is compiled into a large 3D matrix $\mathbf{x}_{cvx}$, corresponding to the full recovery of spectral information at each speckle pattern position.

```
1 % Determine the size of the STM
2 A=tmatrix;
3 [d1, d2, d3] = size(A);
4
5 for iteration = 1:d3
6   % Resolve for each speckle pattern position separately
7   A = tmatrix(:,:,iteration);
8
9   % Build output in the same way as STM
10   % Extract selected speckle AOI from output image
```
% Resolve system for each speckle pattern position separately
b = output(:,iteration);

% Perform convex optimisation (Compressive Sensing) for unknown ...
input z

    cvx_begin quiet
    variable z(d2,1)
    minimize(norm(A*z - b,1))
    subject to
       z >= 0
    
    cvx_end

% Reconstructed input
z;

x_cvx(:,iteration) = z;

end
Appendix D

Simulating a multispectral transmission matrix

In Chapter 6 a multispectral transmission matrix (MSTM) was simulated to enable testing of the prVAMP algorithm for reconstruction of multispectral images [123]. The protocol used to simulate the incident laser beam travelling through a multiple scattering medium before forming a speckle at the output is outlined, and a MSTM is built from the resulting speckle information. The speckle simulation used in this work was provided by Mickael Mounaix.

A Gaussian beam is generated in the following form:

$$b = \exp \left( \frac{-2(\sqrt{X^2 + Y^2})^2}{(1/T_{grain})^2} \right), \quad (D.1)$$

where $X$ and $Y$ are $x$ and $y$-coordinates of a spatial light modulator (SLM) and $T_{grain}$ denotes a parameter based on the speckle grain size on the camera.

A SLM phase mask is next created by taking the exponential of a randomly generated binary matrix of size $b$ containing complex factors of $2\pi$. In order to simulate the Gaussian beam interacting with the SLM mask, the product of the two matrices is taken: $g = b \ast SLM$.

Next, a Fourier Transformation to the frequency space is performed to generate a speckle pattern on the camera. A region of speckle information is selected from the camera and stored as a column in a MSTM. This process is performed to build a matrix of dimensions $N_{CCD} \times N_{SLM} \times N_{spectral}$, as described in Chapter 6.

To ensure that each spectral layer of the MSTM is independent of all others, a Lorentzian smoothing process is used. The smoothing process uses a scaling factor along the spectral dimension that is dependent on the spectral correlation width of the multiple scattering medium being used. The scaling factor is given by

$$Z = \frac{\left(\frac{\nu_0}{2\pi}\right)}{z^2 + \left(\frac{\nu_0}{2}\right)^2}, \quad (D.2)$$

where \( \omega_0 \) corresponds to the inverse of the spectral correlation width, and \( z \) is a linear vector equal in length to the MSTM. The scaling factor is applied to the MSTM to create \( N_{\text{spectral}} \) independent layers within the matrix.

Once the MSTM is calibrated, it can be used to simulate a multispectral SLM phase object travelling through a multiple scattering medium, in order to create an output speckle pattern for reconstruction testing with prVAMP. After the speckle pattern is generated using the first steps of the approach described above, a percentage of noise is added to it to create a realistic output speckle pattern. Chapter 6 shows the results of two experiments using the MSTM to reconstruct multispectral images after travelling through multiple scattering media.
Bibliography


