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This report is based on a thesis submitted by the author as part of the requirement for the degree of B.Sc. (Hons) in the University of Southampton.

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CONTENTS

LIST OF SYMBOLS AND ABBREVIATIONS

a	keel depth
a_R	Rudder depth
a^*	Depth at keel trailing edge
$A(x)$	$\oint \phi_1 \, dy$
B	Beam (maximum)
CE	Centre of effort of sails
CLR	Centre of lateral resistance of hull
C_{L_1}	Coefficient of sideforce, constant ½
C_{L_π}	Coefficient of sideforce, constant π
C_{L_T}	Total Sideforce coefficient (constant π)
δ_R	Rudder deflection angle
δ^1	Slenderness parameter
δ	Lead distance
$F(x)$	$\oint \phi_o \, dy$
$F(x,y)=Z$	Function of Body Shape
I_Z'	Moment of inertia about vertical axis through C of G (normalised)
L	Hull sideforce
$L(x)$	Sideforce per unit length at position x along the yacht
ℓ	Waterline length of yacht
$m(x)$	Added mass / unit length at position x
m_T	Added mass at hull body end or effective trailing edge
$m'(x)$	Source strength in a line source at position x
M	Asymmetric moment
M'	$\int_o^\ell m(x) \, dx$
M'_x	$\int_o^\ell x \, m(x) \, dx$
M'_{x^2}	$\int_o^\ell x^2 \, m(x) \, dx$
n	outward normal from a 3-D body.
N	Total hull moment
N_x	Fore and aft component of the unit normally drawn outwards from the body in 3-D.
P_o	Elemental pressure due to ϕ_o
P_1	Elemental pressure due to ϕ_1
q	Resultant velocity in Bernoulli's equation

r	Hull body radius
r^*	Hull body radius at keel trailing edge position
$r(t)$	Yaw angular velocity as a function of time
Re( )	Real part of ( ).
$R=R(x)$	Radius of body of revolution at x, for a line source
t	time
U	Craft Velocity, or free stream velocity
v	Outward velocity of fluid from a line source
V_T	Total lateral velocity of a yacht at a particular x position
x	Lengthwise coordinate along the yacht
$\bar{x}$	Normalised variable CLR position
$\bar{x}_1$	Normalised hull upright CLR position
y	Measurement of depth in frame of reference
Y	Sway force, effectively L
Z	Measurement of beam in frame of reference

α	Leeway angle
$\bar{\varepsilon}$	$(^r/_{r^*})$ for positions aft of keel trailing edge
θ	Heel angle
λ	Beam / Draught ratio
$\bar{\mu}$	Proportionally constant for variable θ, α
ρ	Density of liquid, i.e. water
σ	Sectional area coefficient
σ_1'	Non-dimensional stability roots of yaw / sway equations of motion
ϕ'	Velocity potential for unit crossflow velocity
ϕ_o	Streamwise disturbance due to the hull body (ex fins at $\alpha = 0^o$)
ϕ_1	Crossflow potential due to angle of leeway, α.
Φ	Total Velocity potential
ϕ_{OZZ}	$\frac{\partial^2 \phi o}{\partial Z^2}$, ϕ_o differentiated twice with respect to Z.

1. BALANCE

The quest for speed of a sailing yacht necessitates a finely tuned yacht which has inherently good balance. This balance requires that a yacht when closehauled will maintain a straight course with little helm deviation from the centreline of the vessel. Rudder actions are kept to a minimum, since it is detrimental from a drag point of view and it leads to decreased speed through the water. The balance of a yacht is achieved by a reduction of the turning moments to a minimum (under a no-helm condition), and hence the centres of the aerodynamic and hydrodynamic forces need to be designed purposefully into a yacht.

There are basically two types of situation to be investigated in a balance consideration, namely steady state forces and transient forces that are encountered by a yacht. The response of a sailing vessel to transient forces such as wind gusts and instantaneous directional disturbances is known as directional stability. Both steady stability and directional stability requires the minimum helm condition.

The diagram below illustrates the basic force and moment mechanism for a sailing yacht as it affects balance.

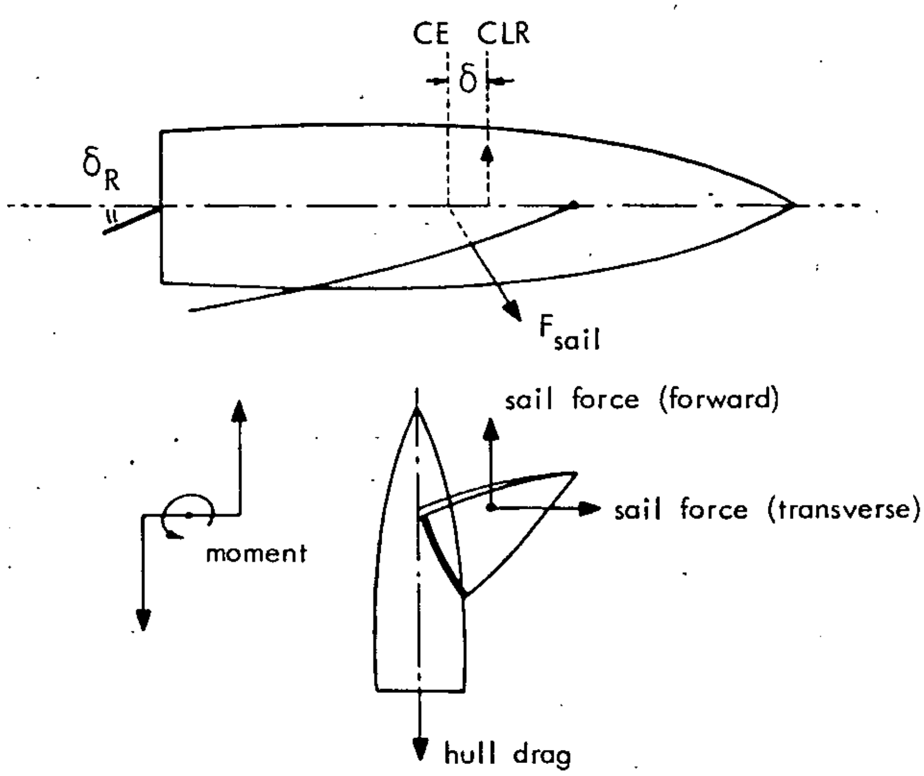


FIG. 1 BALANCE MECHANISM FOR A YACHT.

The centre of effort (C.E.) of the sail is that point on the hull centreline that intersects with the projected line of the resultant sail force. The centre of lateral resistance (CLR) can be defined as that point on the hull through which a single force acting would produce the same effect on the hull as all the water forces.

Since there exists a distance δ between the centre of lateral resistance of the hull (CLR) and CE in the above diagram the rudder needs to be deflected to an angle δ_R to bring the CLR aft to coincide with CE for equilibrium, and as already stated this angle, δ_R, should be a minimum to reduce drag.

The conventional approach to designing balance into a yacht is via the geometric centres of area of the sails and the yacht underwater profile centre of area, and involves a consideration of various area ratios of the parameters keel, rudder and sail areas. For the purposes of reasonable balance a yacht designer can implement a "lead" which is a certain distance as a percentage of the hull length, of the sail C.A. forward of the geometric C.A. of the hull, and this lead is determined by comparison of balance in successful yachts of comparable type. In designing balance based on centres of area the lead is necessary because the centres of area of underwater hull profile and sail plan are far apart from the actual hydrodynamic and aerodynamic centres of force on the sail and on the hull. There is thus a need for tuning of the yacht on the water by various adjustments, which is a result of the largely empirical approach to designing balance into yachts using centres of area. This simple basis to balance design does not always produce consistent results due to the lack of correlation between geometric centres and the aero-hydrodynamic centres that occur in reality.

This report considers the use of slenderbody theory in finding the hydrodynamic CLR position which is, in fact, not constant but varying with respect to heel and leeway angles. Graphs which define the force distribution on a yacht and hence the resultant CLR position are utilised.

The graphs used for yacht form force distribution may also be used effectively to define in a quantitative form the handling ability of a yacht by evaluating some "stability derivatives" which may be explained later in Section 3 under Directional Stability.

The report is specifically laid out in the following manner. Section 2 contains the method for determining CLR position and this is coupled with the graphs in the Appendices. Subsequently, Section 3 is a short review on Directional Stability, and the "Stability Derivatives" may be determined from the method found in the Appendices, again using the set of graphs.

Section 4 concludes the main body of the report. The Appendices contain, firstly the graphs used in CLR and derivative calculations, and secondly the relevant slenderbody theory used in the report. The third and final part of the Appendices is concerned with the mathematics of stability derivatives calculation and stability conditions.

2.0 HYDRODYNAMIC CLR

The CLR is, in fact, subject to variation due to heel angles and leeway angles. It may be shown to conform with good accuracy to a simple equation describing CLR position as due to a constant upright position and an increment for CLR position due to heel and leeway angles.

Let us assume the total yawing moment, N is made up of (i) a moment due to sideforce acting at a fixed centre x_1 ; and (ii) a moment proportional to heel angle(θ)due to asymetry of the heeled hull form : M

$$N = x_1 L + M$$

x_1 = upright CLR position
L = Sideforce on hull
M = Additional Asymetric Moment

$$\text{CLR Position, } x = \frac{\text{MOMENT}}{\text{SIDEFORCE}}$$

$$x = x_1 + \frac{M}{L}$$

Now $M \propto \theta$, heel angle

$L \propto \alpha$, leeway angle

Hence $x = x_1 + \mu\frac{\theta}{\alpha}$ where μ = constant

Normalise by dividing by the waterline length : $\bar{x} = \frac{x}{L}$, etc.

$$\bar{x} = \bar{x}_1 + \bar{\mu} \quad \frac{\theta}{\alpha} \qquad (1)$$

A typical plot of CLR position is shown in Fig.2(a)and the equation shows a surprisingly good agreement with experiments on a yacht as carried out by Ref. (5). Complete agreement would in fact require all the lines on Fig2(a) to meet at $\theta = 0^o$.

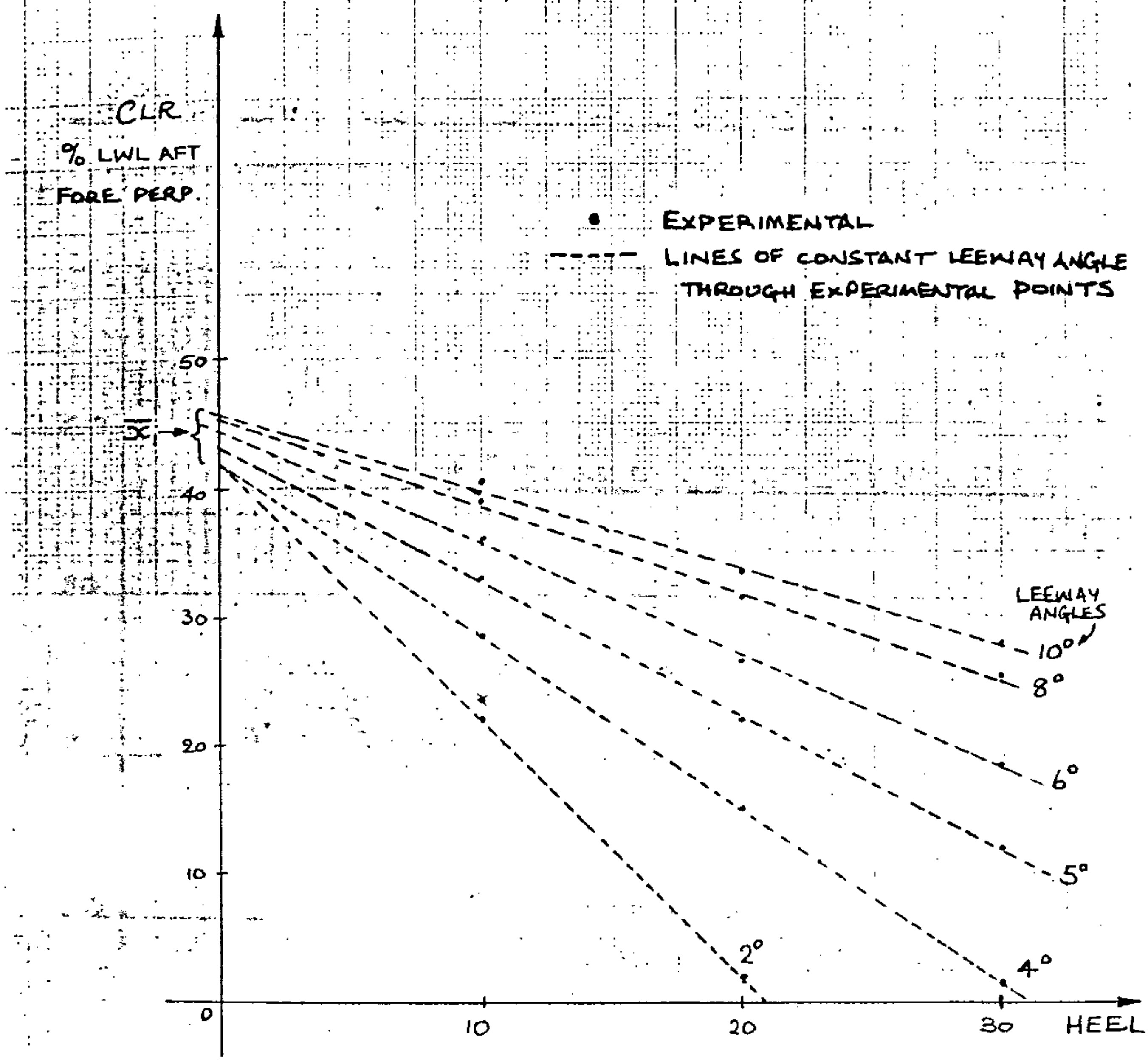
If the constant parameters in the equation, namely $\bar{x}_1$ and $\bar{\mu}$ are known then the CLR position can be analysed for variable sailing conditions.

The aim is to derive a simple quadrature method for computing $\bar{x}$ and $\bar{\mu}$ from the hull geometry based on the use of slender body theory.

FIG. 2 (A)

REFERENCE (5) MEASURED CLR DATA

CLR POSITION vs. HEEL ANGLE

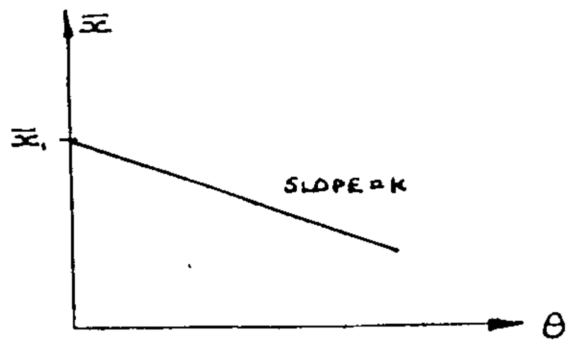


$\bar{x}_1$ is the intercept on the line: Heel Angle = 0°

μ is found from the slope of the plotted lines

$$\bar{x} = \bar{x}_1 + \frac{\mu\theta}{\alpha}$$

Let $\bar{x} = \bar{x}_1 + k\theta$



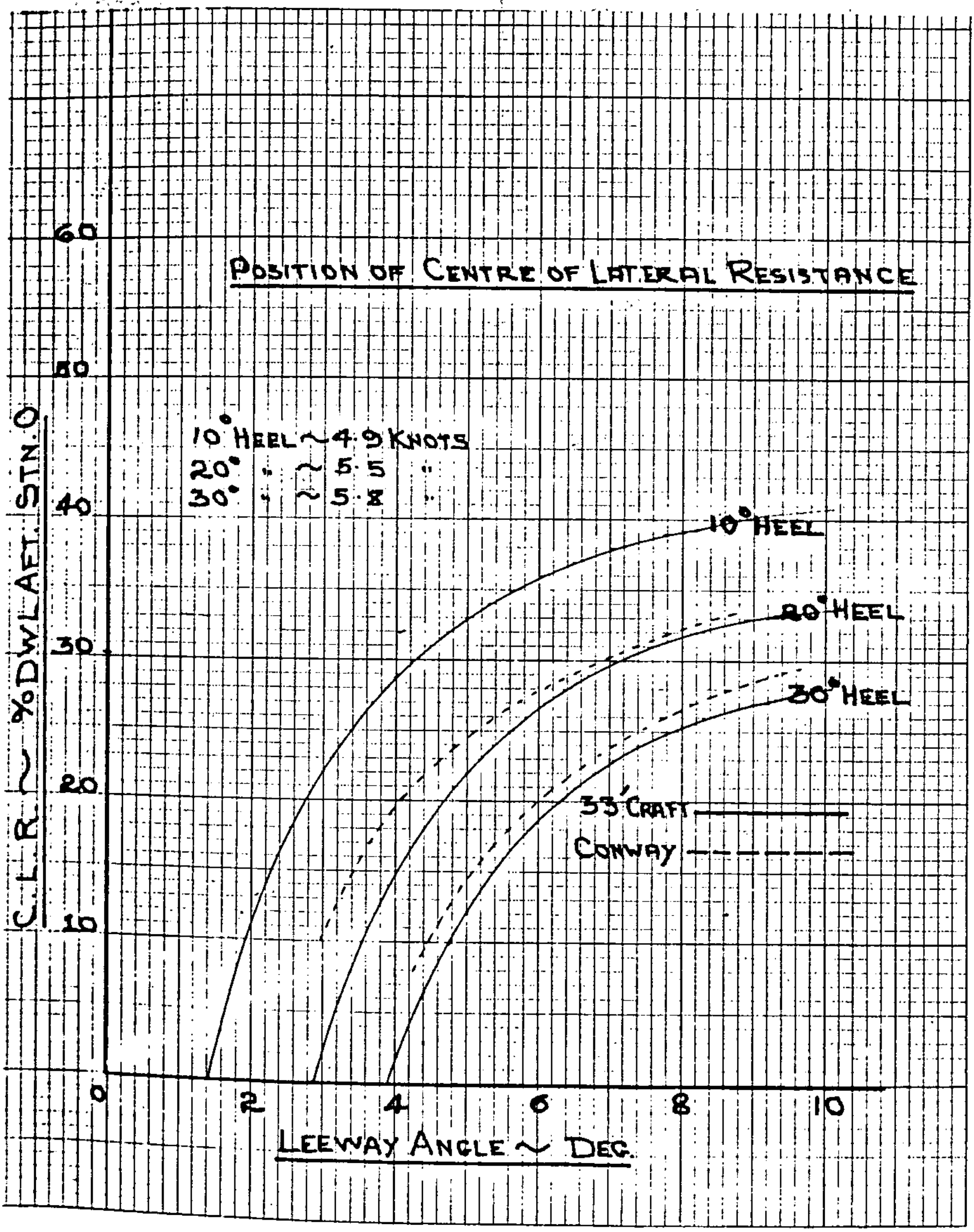
K is NEGATIVE

$\mu = K\alpha$

FIG. 14 CLR POSITIONS

FIG. 2 (B)

REFERENCE (5) MEASURED CLR DATA



2.1 THE USE OF SLENDERBODY THEORY

Slenderbody theory is a simplified theory which, in this report, allows the resolution of forces and moments on a yacht. It is a practical 2-D theory since 3-D theory is too complex. In order to use slenderbody theory in a simple manner the free surface (water surface) is assumed flat and for the necessary boundary condition that there is no flow of fluid through this free surface a system of images is used. This effectively produces cancellation of any fluid velocity components perpendicular to the free surface, and therefore the theory is more accurate for lower Froude numbers (a velocity parameter) where the water surface in reality becomes increasingly flat.

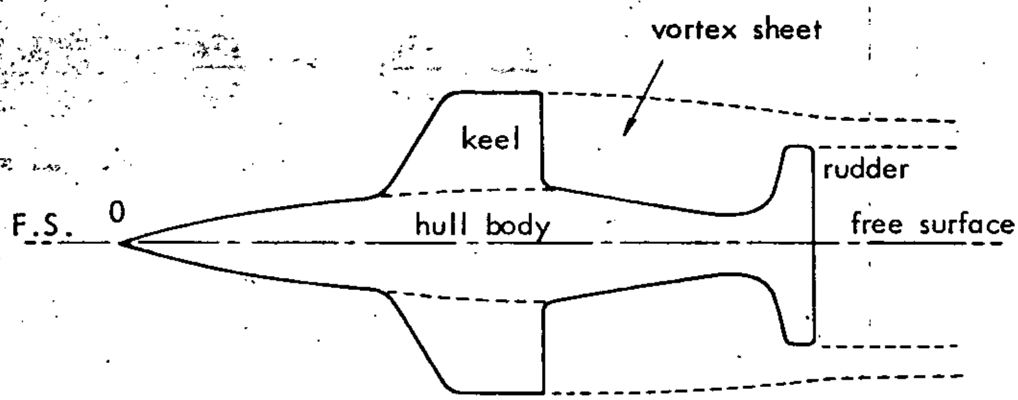


FIG. 3 IMAGE SYSTEM FOR A YACHT

The procedure for using slenderbody theory is based on reducing three dimensional equations to a two dimensional form whilst systematically retaining all terms of similar magnitudes that may not be neglected. The slender form is compared with a slenderbody of revolution (a "cigar" shape with circular cross sections) and hence the orders of magnitude of the various relevant terms are compared.

Initially the frame of reference must be clarified

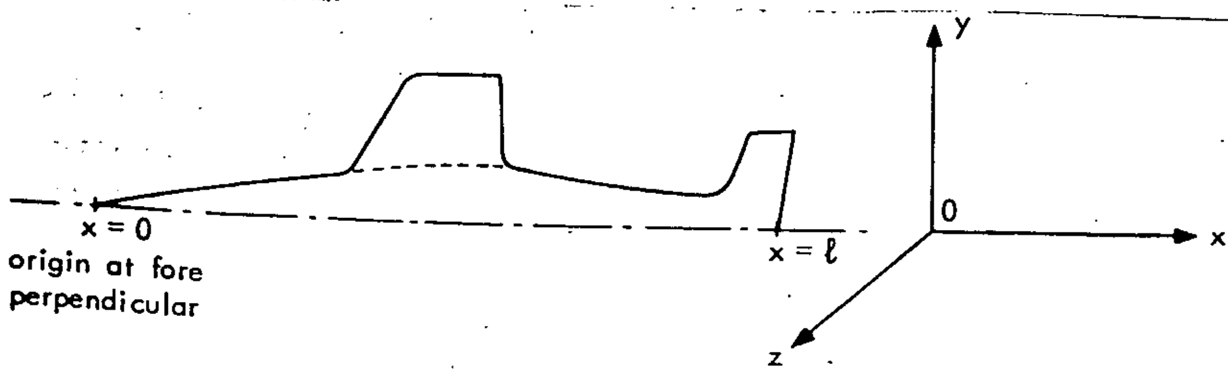


FIG. 4 FRAME OF REFERENCE.

Typically, draught is measured by y-values, beam by z-values and length by x-values, the origin being at the bow.

The basic equation for velocity potential is given by

$$\Phi = Ux + U\alpha z + \phi_o + \phi_1 \tag{2}$$

where α is the leeway or yaw angle

ϕ_o is the streamwise disturbance due to the body (ex fins and at $\alpha = 0$), and

ϕ_1 is the cross flow term due to yaw.

The exact three dimensional equations are reduced to a set of two dimensional equations by neglecting all but the largest terms in the full equations. It is found that :

$$\frac{\partial^2\phi}{\partial x^2} << \left(\frac{\partial^2\phi}{\partial y^2}, \frac{\partial^2\phi}{\partial z^2}\right)$$

so that Laplace's equation becomes :

$$\phi_{oyy} + \phi_{ozz} = 0 \tag{3}$$

$$\phi_{1yy} + \phi_{1zz} = 0 \tag{4}$$

These results are the essence of the two dimensional approach and neglect the interactions due to the shape upstream of the section under consideration.

The slender approximation also leads to the following simplified forms of Bernoulli's equations after discarding relatively small terms :-

$$P_o = -\tfrac{1}{2}\rho\{2U\phi_{ox} + \phi_{oy}^{\ 2} + \phi_{oz}^{\ 2}\} . \tag{5}$$

$$P_1 = -\rho\{U\phi_{1x} + \phi_{oy}\phi_{1y} + \phi_{oz}(U\alpha + \phi_{1z})\} \tag{6}$$

Thus an appraisal of crosswise force distribution and also the yawing moments due to asymmetry can be made by utilising these pressures at elemental positions about the cross-sectional contours and integrating to produce lift force values.

For the case of the crossforce distribution due to an angle of leeway of a yacht, the lift force at each section of a yacht can be deduced from the expressions :

$L(x) = \oint p_1$ dy which yields the result (see appendices)

$$L(x) = \rho U \frac{\partial}{\partial x} \oint \phi_1 \, dy \qquad (7)$$

This is a very useful result. It gives the lift force at position x as being proportional (for uniform velocity) to the rate of change of ϕ_1 integrating around the section contour, along the x axis. Thus, it is essentially a two-dimensional deduced force and a direct result of slender simplification. The equation has been used for a variety of slender forms. Originally used by Munk in the 1920's for airships it is now more important for the study of submarines, ships, yachts, aircraft and rockets, but has also been adapted to the problem of fish propulsion (such as anguillar-form) as demonstrated by Wu (Ref. (1)) and Lighthill (Ref. (2)). There is also an application to planar foils and delta shaped wings.

The contour integral $\oint \phi_1$ dy is directly related to the added mass of the 2-D section for sway motion and it can be shown that the added mass m is :

$$m(x) = \rho \{ \oint \phi_1 \, dy - \text{AREA} \} \qquad \text{AREA = X-Sectional Area}$$

In a similar manner to finding equation (7) an asymmetric moment due to longitudinal flow can be given as :

$$M = \rho U \int_0^{\ell} F(x) \, dx \qquad (8)$$

$$F(x) = \oint \phi_0 \, dy \qquad (9)$$

It is worth noting various points here before proceeding to actual evaluation of the constants within the CLR equation. Firstly, the total force on that part of a yacht ahead of position x is equal to the value of

$$L(x) = \rho U \oint \phi_1 \, dy \text{ of the section actually at x.}$$

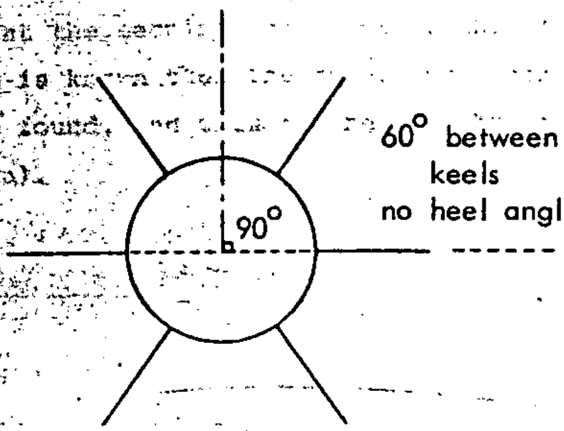
Secondly, due to the convection of the keel trailing vorticity, $\oint \phi_1$ dy in the wake of the keel trailing edge is less than at the trailing edge and hence the hull experiences a negative lift on the "afterbody".

Thirdly, it must be said that strictly speaking $\oint \phi_1$ dy should be found for the actual sections of the yacht but it may be sufficiently accurate to evaluate the integral for a number of approximate sections chosen to be representative of yacht forms and having a range of parameters (such as beam / draught ratios) to cover the practical cases.

Finally, the calculation of lateral forces may be for more than one fin and hence calculations can be performed for bilge keel forms. For the simple case of a circular section with two keels m(x) is given by :

$$m(x) = \pi \rho \left\{ 2^{\frac{1}{3}} a^2 \left[1 + \left(\frac{r}{a} \right)^6 \right]^{\frac{2}{3}} - r^2 \right\} .$$

where r = hull body radius
a = keel depth



A short study on these cross-sections demonstrates the relative ineffectiveness of bilge keels as lift producing surfaces and the drag is also increased due to induced and friction drag increases.

Now it remains to use slenderbody theory, effectively equations (7), (8) and (9), in the determination of the constants $\bar{x}_1$ and $\bar{\mu}$ in the CLR equation (1).

2.2 EVALUATION OF THE CONSTANT $\bar{x}_1$

In this report the slenderbody theory reduces to evaluation of added mass coefficients for sway motion of the two-dimensional shapes that comprise the sections of the yacht. Two-dimensional added mass is the added mass (in this case for translational motion) which is associated with unit thickness of a two-dimensional shape, and it is the portion of liquid around a section which is "entrained", or moves with the section. The added mass of each section may be related to the crossforce or lateral force at the section, and thus if the distribution of all these lateral forces is known then the resultant force centre, or centre or pressure, may be found, and this is the upright yacht CLR position (x_1 in the formula).

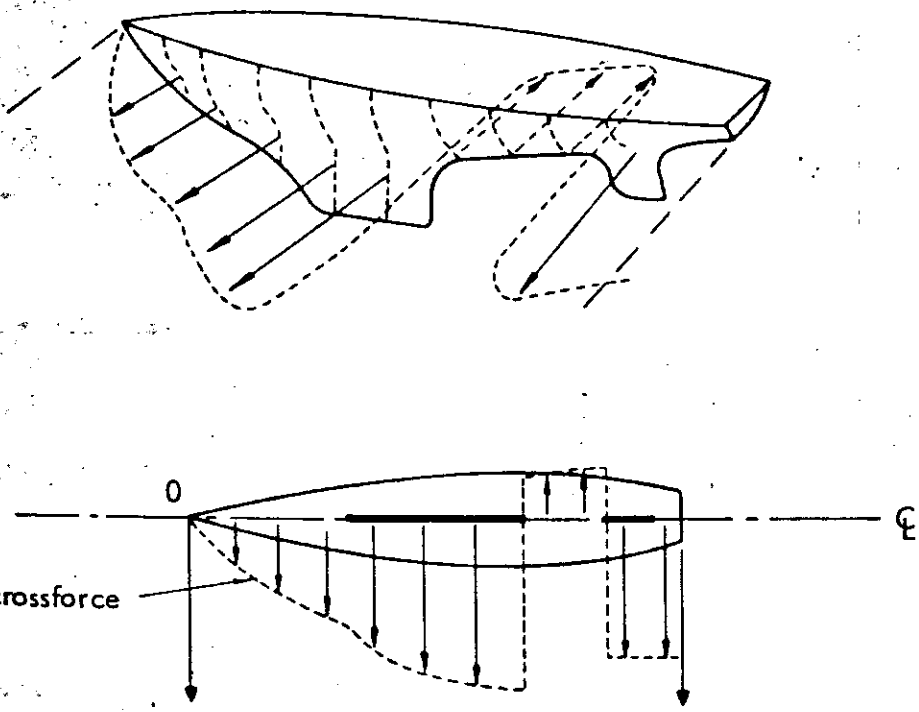


FIG. 5 . DISTRIBUTION OF CROSS FORCES.

The diagram above illustrates the kind of force distribution which may be found by slenderbody analysis.

The constant $\bar{x}_1$, or the upright CLR position is found from the expression :-

$$\bar{x}_1 = \frac{\text{MOMENT}}{\text{SIDEFORCE}} = \frac{\int_0^{\ell} x \frac{dL}{dx}\,dx}{L} = \ell - \int_0^{\ell} \frac{m(x)\,dx}{M_T} \qquad (10)$$

where ℓ = Waterline length

$m(x)$ = Added mass at position x

M_T = Added mass value at body end (perhaps including the rudder).

Hence the added mass value at each section of a yacht is needed and these values may be integrated along the yacht's length by simple means such as Simpson's rules integration. In order to find the added mass $m(x)$ for a section the corresponding value of lift coefficient is read off one of the crossforce graphs, and then converted to added mass using the equations below :

(iii) (Using a semicircular hull section approximation),

$$\frac{m}{\rho} = \oint \phi' \,dy - \text{AREA}$$

$$C_{L\pi} = \frac{L}{\pi \rho U^2 \alpha a^2}$$

$$L = \rho U^2 \alpha \oint \phi' \,dy.$$

m = added mass

ρ = density

$\oint \phi'$ = integral of velocity potential around the contour

AREA = cross sectional area

a = keel depth

These equations give : $C_{L\pi} = \frac{1}{\pi a^2} \{\frac{m}{\rho} + \text{AREA}\}$ AREA = πr^2

$$m = \pi\rho\{C_{L\pi} a^2 - r^2\} \qquad (11)$$

This equation (11) may be used to define m(x) at any position x, but the sections of a yacht can be subdivided into specific types, which require different treatment. These different sections are bow sections, keeled sections, afterbody sections (with downwash of vortex sheet effects) and also sections with rudders at the body end. Different input parameters are necessary to read off lift coefficient values from graphs for these four section types. The graphs mentioned above have been computed utilising equations (7) and conformal mapping techniques and an explanation of them follows.

2.21 ADDED MASS FROM GRAPHS

The magnitudes of the crossforces at particular cross sections along the x axis of the yacht may be found from various graphs computed from one of the following three models :

(i) Ahead of the fin trailing edge the cross forces are calculated in the case of the upright yacht for Lewis sections using conformal mapping techniques. The Lewis sections are upright symmetrical mathematical representations of real yacht-like sections and they are defined with respect to certain shape parameters, which are deduced from the actual sections of the yacht in question. The parameters required at each section are beam-draught ratios, sectional area coefficients and hull body depth to keel depth ratios, (See example Lewis Section Graph, Fig. 7.)

(ii) In order to calculate cross forces aft of the keel trailing edge it is necessary for mathematical reasons to assume a semicircular shape to the hull body. The lateral forces are calculated by a computer and displayed graphically (See Fig. 8.) for sections of semicircular hull bodies (becoming full circles using the image system) and fins, but also for the combination of circles and a vortex sheet shed from the keel trailing edge. A vortex sheet is a distribution of vortices shed from the trailing edge of the solid keel, and this moves aft along the yacht's centreline in the hydrodynamic model within this report.

(iii) The results of cross force coefficients are also given graphically for heeled circular sections with either keels or a vortex sheet outboard. The circle and fins model is a model where the keel part of the sections are made up of a line distribution of vortices which provide a solid boundary that the fluid must flow over. The following diagram illustrates this idea.

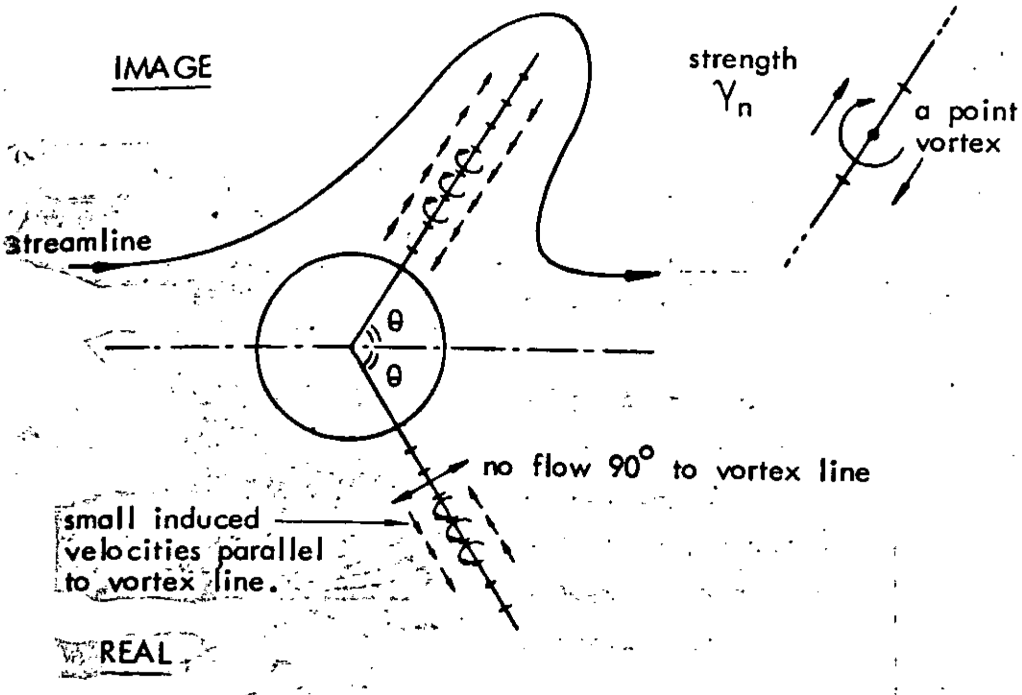
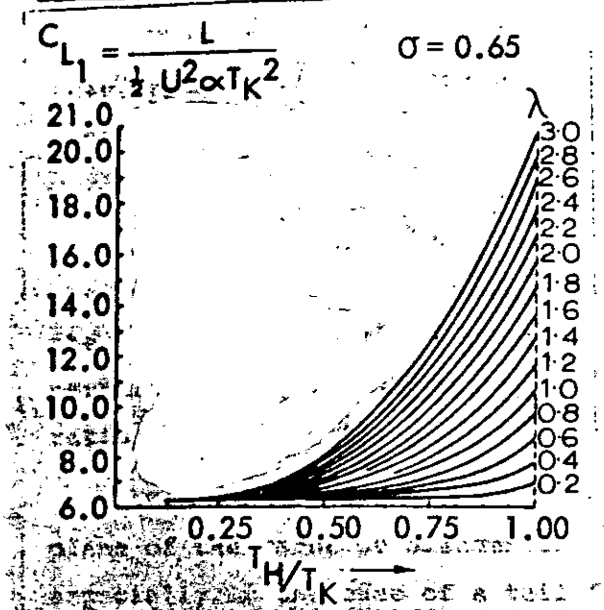


FIG. 6 — LINE VORTEX DISTRIBUTION TO MODEL FIN SEGMENTS.

The cross flow force coefficients displayed graphically are given for any section forward or aft of one keel trailing edge and hence the distribution of forces acting transversely at all positions along the yacht is determined. The essence of the graphical results is that they allow a simple resolution of the upright CLR position (x_1 in the formula defined earlier), and this resolution can be made by simple quadrature, such as Simpson's Rule, rather than a full-scale hydrodynamic CLR computation.

TYPICAL CROSS FORCE GRAPHS

Lewis Sections Graph - Fig. 7.



C_{L_1} = Coefficient of Side Force

σ = Sect. Area Coefficient for Hull Body = $\left(\frac{\text{AREA}}{\text{BEAM x DRAUGHT}}\right)$

$$C_{L_1} = \frac{L}{\frac{1}{2}\rho U^2 \alpha a^{*2}}$$

ρ = Density of Fluid

U = Speed of Craft

α = Angle of Leeway

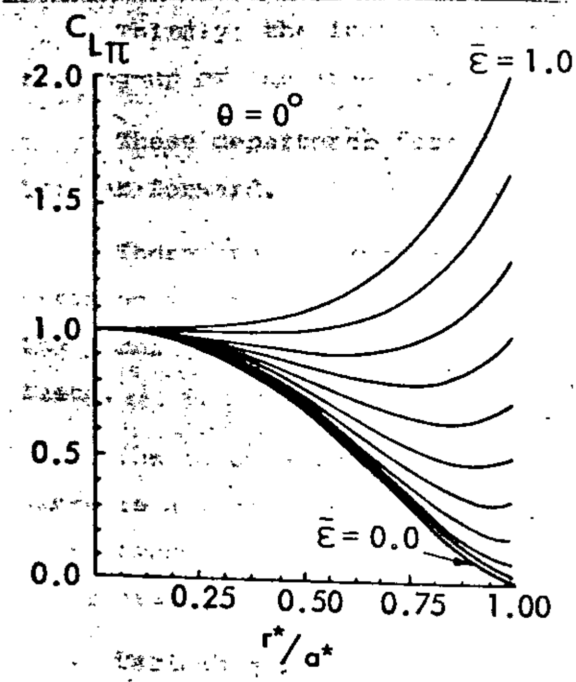
a^* = Keel depth

r^* = Hull body depth

λ = Beam / Draught of Hull

Different graphs for varying σ values

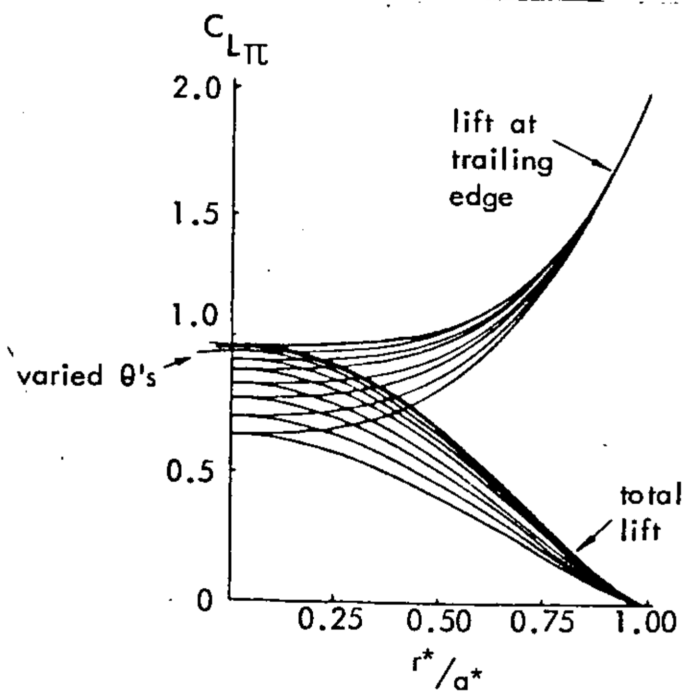
Circular Sections $C_{L\pi}$ Graph - Fig. 8.



Different graphs for different θ, heel angles.

$\bar{\varepsilon}$ = (r / r^*) for positions aft of keel trailing edge.

Trailing Edge Lift and Total Lift for Various Heel Angles - Fig. 8a.



2.22 CALCULATING $\bar{x}_1$

There are various inaccuracies for the slenderbody theory used in this report which can induce possible departure from good theoretical prediction.

The first possible inaccuracy is the fact that for highly swept leading edges of the kind specifically designed for Concorde, vorticity shed into the stream from the leading edges results in a contribution to the lift proportional to the square of the angle of incidence, and a certain similarity may be expected for planar foils of small aspect ratio penetrating the free surface.

Secondly, the assumption that the vortex sheet moves along the centre-plane of the yacht or slenderform leads to inaccuracies. In fact, especially in the case of a tail fin or skeg rudder system, the downwash experienced by the afterbody is less than predicted by the theory.

Thirdly, the instability of the trailing vortex sheet encourages a rolling-up of the sheet which also decreases the downwash on the after-body.

These departures from the theory result in a predicted CLR position too far forward.

Therefore an experimental correction factor is necessary in defining $\bar{x}_1$ to place the value of $\bar{x}_1$ with an accuracy of 1 - 2% of the yacht's length. It is planned to obtain these factors from model tests.

The calculation of $\bar{x}_1$ in practice takes about fifteen minutes and hence is a useful design tool for a yacht designer, and may well improve consistency in the design of a yacht which has no close comparison with other similar and successfully balanced yachts.

Various example calculations have been completed by reading off the graphs as follows :

2.23 EXAMPLE CALCULATION OF $\overline{X}_1$

Below is a table demonstrating the input parameters needed to lift off $C_{L\pi}$ values from the graphs, for the various section types :

SECTION TYPE	r/a	$\bar{\varepsilon}$	a_R/a^*	m
Bow	1.0	Use $\bar{\varepsilon} = 1.0$	-	$\pi\rho r^2$
Keeled	<1.0	Use $\bar{\varepsilon} = 1.0$	-	$\pi\rho\{C_{L\pi}a^2 - r^2\}$
Afterbody	Use r*/a*	$\bar{\varepsilon} < 1.0$	-	$\pi\rho\{C_{L\pi}a^{*2} - r^2\}$
Rudder	Use r*/a*	$\bar{\varepsilon} = 0.0$	0	$\pi\rho\{C_{L\pi}a^{*2} - r^2\}$

r = body radius for circular section of same area as actual section
r/a = body radius/keel draught (assuming semicircular section)
r*/a* = (r/a) at the trailing edge of the keel (T.E.)
$\bar{\varepsilon}$ = body radius of section/body radius at T.E. section = (r/r*)

It may be noted that for the bow and keeled sections the coefficient used is $\bar{\varepsilon} = 1.0$, but in fact the radius of the body at these sections is not necessarily equal to the body radius of the keel trailing edge section. The hull sectional area is decreasing aft of the keel trailing edge, and therefore $\bar{\varepsilon} < 1.0$ for these sections, and if the rudder is placed at the body end then the $\bar{\varepsilon} = 0.0$ curve is used here.

For the afterbody and rudder sections the geometry of the trailing edge section is important and the table thus shows that the ratio (r/a) used is (r*/a*) and the added mass is defined with respect to a*.

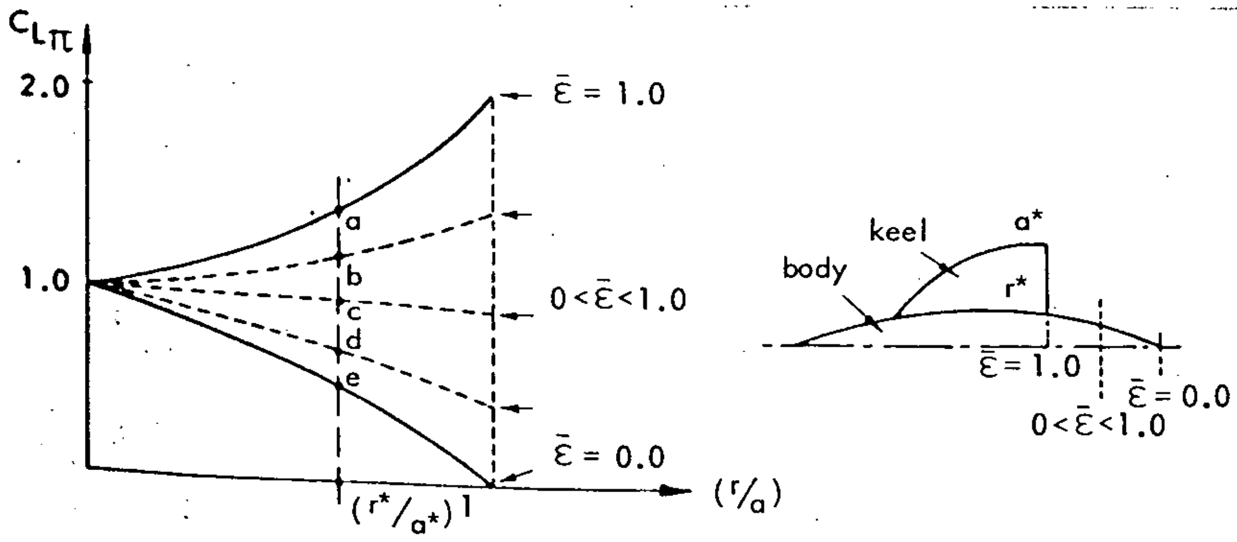


FIG. 9 EXAMPLE $C_{L\pi}$ READOFF.

The graph above shows a typical case for $(r^*/a^*)'$. The small letters (a to e) show the points corresponding to various afterbody sections for this value of (r^*/a^*) according to the values of $\bar{\varepsilon}$.

Finally, the rudder section at the hull body end is considered, and the important parameter to use in the graph (CH25) from Ref. (3) is : (Rudder depth/T.E. keel draught) or (a_R/a^*).

Results are given from this graph for $0 < r^*/a^* < 1.0$ and for $a_R/a^* = 0, 0.25, 0.50, 0.75, 1.0$. The dotted line on this graph shows the limit at which the rudder depth is greater than the depth of the vortex sheet at the rudder section and the added mass then depends only on the rudder itself to the neglect of the traling vortex sheet from upstream.

Tables for results have been prepared which can be used to calculate CLR position (x_1) and they are based on using a Simpson's integration of added mass values in accordance with equations (10) and (11).

SECTION	x	AREA	r	a	a^2	r^2	(r/a)	C_{LR}	$C_{LR}a^2 - r^2$	SM	SM.(1)
0		0.00	0.00	0.00	–	0.0000	0.00	2.00	0.00	1	0.00
1		0.70	0.67	(0.870)	–	0.4489	0.800	2.00	0.4489	4	1.7956
2		2.20	1.18	(1.26)	–	1.3924	1.000	2.00	1.3924	2	2.785
3		3.87	1.57	(1.78)	–	2.4649	1.000	2.00	2.4649	4	9.8596
4		5.25	1.83	3.10	9.61	3.3489	0.590	1.120	6.2611	2	12.5222
5		6.15	1.98	4.62	21.344	3.9204	0.429	1.032	18.107	4	72.428
6		6.00	1.95	4.64	21.5296	3.8025	0.420	1.029	18.352	2	36.703
7		4.95	1.77	4.64	21.5296	3.1329	0.381	1.020	18.827	4	75.3092
			r	a^*	a^{*2}	r^2	r/a^*	C_{LR}	$C_{LR}a^{*2} - r^2$		
8		3.20	1.43		21.5296	2.045	0.808	0.941	18.214	2	36.4288
9		1.10	0.85		21.5296	0.7056	0.475	0.858	17.767	4	71.0672
10		0.00	0.00		21.5296	0.00	0.000	0.815	16.841	1	16.841

$\Sigma 1 =$ 335.139

r^*	1.77
a^*	4.64
(r^*/a^*)	0.381
TOTAL C_{LT}	0.815
δx	0.1 ← LET $l = 1.0$
$\Sigma 1 \times \delta x/3$	11.1713 (A)
m_T	16.841

$x_1 = x_T - (A)/m_T$

$\bar{x}_1 = 1 - (A)/(m_T \cdot x_T)$ ($x_T = 1$)

$\bar{x}_1$	33.7%

"ANTIOPE" CLR POSITION

(UPRIGHT)

STATION	x	(HALF) AREA	T	a	a^2	T^2	(T/a)	C_{uR}	$C_{uR}a^2 \cdot T^2$	SM	SM.(10)
0		0.00	0	0		0	0	0	0	1	0
1		11.92	2.755	(4.6)		7.589	1.00	2.00	7.589	4	30.36
2		38.38	4.943	(6.6)		24.433	1.00	2.00	24.433	2	48.87
3		67.88	6.574	(7.85)		43.214	1.00	2.00	43.214	4	172.86
4		95.76	7.808	(8.70)		60.963	1.00	2.00	60.963	2	121.93
5		121.21	8.784	21.75	473.06	77.165	0.404	1.03	410.09	4	1640.4
6		135.25	9.279	24.00	576.0	86.103	0.387	1.025	504.3	2	1008.6
7		137.58	9.359	24.00	576.0	87.586	0.390	1.025	502.8	4	2011.2
8		123.13	8.854	24.00	576.0	78.387	0.369	1.020	509.13	2	1018.26
							$\bar{E}$ =				
			T	a^*	a^{*2}	T^2	T/a^*	C_{uR}	$C_{uR}a^{*2}T^2$		
9		92.93	7.692	24.00	576.00	59.161	0.869	0.969	498.98	4	1995.9
10		52.73	5.794	24.00	576.00	33.569	0.654	0.874	469.86	2	939.72
11		16.57	3.248	24.00	576.00	10.549	0.367	0.796	447.95	4	1791.8
				a_R =			a_R/a^* =				
12		0.00	0.00	18.25	576.0	0.00	0.760	0.853	491.33	1	491.33
								↑ (FROM: CH 25)		$\Sigma 1$ =	11271.22

T^*	8.854	
a^*	24.00	
(T/a^*)	0.369	
TOTAL C_{LT}	0.853	FOR SIMPLICITY
δx	$\ell/12 = 1/12$	LET $\ell = 1.0$
$\Sigma 1 \times \delta x/3$	$\Sigma 1/36 = 313.09$	(FOR SIMPSON'S RULE) (A)
m_T	491.33	

$x_1 = x_T - (A)/m_T$

$\bar{x}_1 = 1 - (A)/(m_T \cdot x_T)$ $(x_T = \ell)$

$\bar{x}_1$	36.3%

"WESTERLY 33."

CLR POSITION, UPRIGHT METHOD (2)

2.24 COMPARISONS OF X_1 DETERMINATION

(1) ANTIOPE, a 5.5m yacht Ref. (4)

(2) CRUISING YACHT Ref. (5)

(1) ANTIOPE

The comparisons of CLR determination for "Antiope" are possible between the body of revolution model with a keel, with the slender wing profile (no hull-keel effects) and with the keel profile alone using lifting line theory. Letcher, Ref. (7), has carried out a comparison and the predicted CLR varies a good deal.

METHOD	$\bar{x}_1$
Lifting line theory on wing	0.43
Profile of slender wing	0.38
Slenderbody theory (*)	0.337

(fractional position $\bar{x}_1 = x_1/\ell$)

The experimental results can be found in the table for reduced data (from the original tests on the full scale "Antiope") in Ref. (6). In particular for near zero heel angles and angles of leeway between 2.5° and 3°, the CLR position is given as :

α	SPEED	θ	C_L	C_M	CLR($\bar{x}_1$)
2.67	2.0	0	4.47	1.72	0.385
2.69	3.0	1	4.32	1.57	0.363
2.70	4.0	1	4.56	1.66	0.364
2.74	5.0	3	4.71	1.66	0.352
2.79	6.0	5	4.80	1.83	0.381

$\bar{x}_1 = 0.369$

Hence the CLR position is found to be about 3% of the length of the yacht different for the method used in this report(*) which gives the result $\bar{x}_1 = 0.337$.

(2) CRUISING YACHT (with skeg, keel T.E. sweepback, and a rudder at the hull body end)

The experimental value is obtained by projecting the results for different heel angles and leeway angles using equation (1) until the no-heel angle result is given (see Fig. 2.)

PREDICTED $\bar{x}_1$	EXPERIMENTAL $\bar{x}_1$
0.363	0.42

2.3 EVALUATION OF THE CONSTANT $\bar{\mu}$

The constant $\bar{\mu}$ may be found from tank tests on a yacht and thus from the gradient of the lines in Fig. 2. However, it is probably more useful to define $\bar{\mu}$ from theory and the geometry of the yacht just as $\bar{x}_1$ was derived from the yacht's shape. Thus, one of the aims of the report is to examine the yaw moments due to the canoe hull body asymmetry, when heeled, by a source element method.

If a number of parent forms are investigated and their yawing moments determined then the requisite values of the proportionality constant μ may be determined in the equation defined earlier for CLR position. This may be carried out for any yacht whose parent form yacht yawing moment coefficient is known, and hence provides part of the simple design computation procedure.

ASYMMETRIC YAWING MOMENTS

There are various assumptions in the calculation of the asymmetric moment due to an angle of heel in the computer source modelling program. These assumptions neglect the fundamental factors that firstly displacement must stay nearly constant in the heeled condition (but may increase due to a component of vertically down sail force), and secondly the trim will alter. Within the program the displacement increases since the heel is presumed to act about the intersection of the waterplane and yacht centreline in the upright mode, and equally there is no account of the change in trim associated with heeling.

The yawing moment is due essentially to the differences between the radii of curvature of the waterplane surfaces on either side of the 'centreline'.

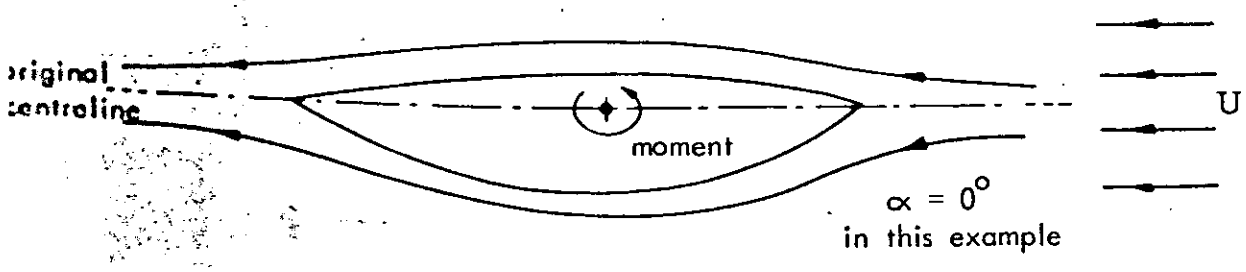


FIG. 10 ASYMMETRIC YAWING MOMENT DUE TO HEEL.

By utlising the fundamentals of slenderbody theory as it affects a body ex-fins at zero incidence a model of the contours of the asymmetric shape is made up by source modelling.

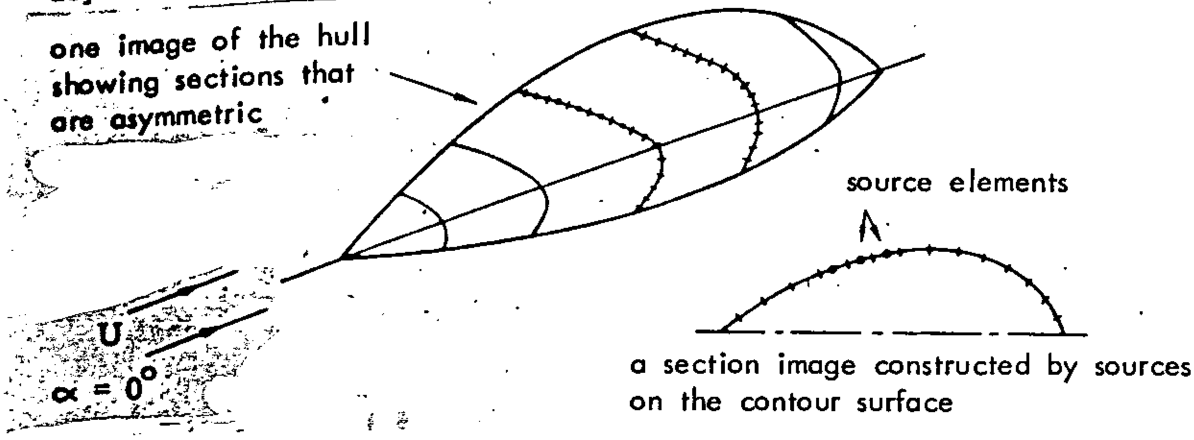


FIG. 11 SOURCE MODELLING OF ASYMMETRIC HULL SURFACE.

On finding the induced normal velocity components at the centres of straight line elements around each section contour, these are added by superposition to derive values for the boundary condition given by equation (12).

$$\frac{\partial \phi}{\partial n} = + UN_x \qquad (12)$$

where N_x is the fore and aft component of the unit normal drawn outwards from the body in three dimensions. Equally by superposing all the elements of velocity potential due to the other sources and itself at one particular source, then repeating this process for all the sources, a distribution of velocity potential around the section contour is deduced which yields the asymmetric moment M as defined in equations (8) and (9).

As a check on the program an ellipsoid of revolution has been source modelled and at an angle of attack α, the resulting moment, M is close to that predicted from slenderbody theory in equation (13) :

$$M = -\frac{8}{3}\pi\rho U^2 \alpha\ a\ b^2 \qquad (13)$$

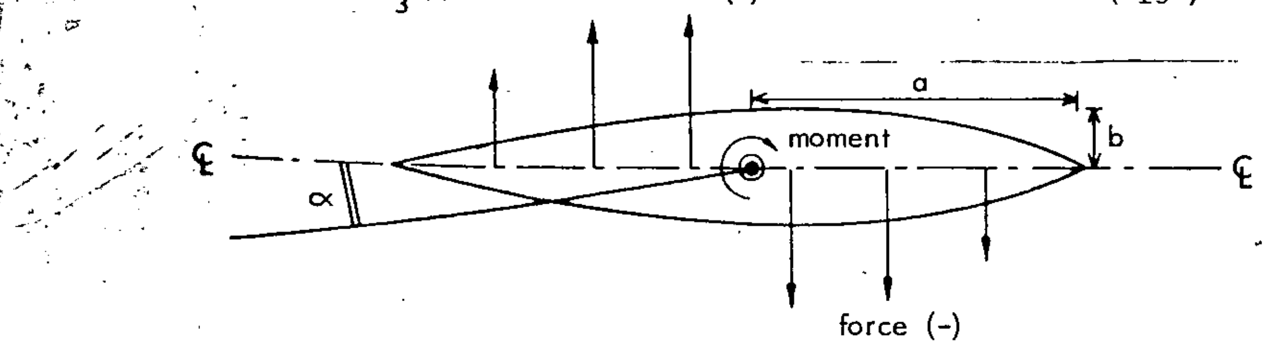


FIG. 12 MOMENT ON BODY OF REVOLUTION. (plan view)

2.31 μ CALCULATION

$\bar{\mu}$ is the coefficient in the CLR position equation (0.2):

$$\bar{\mu} = \frac{M\alpha}{L\theta} \qquad \begin{array}{l} M = \text{Moment from Asymmetry} \\ L = \text{Lift force} \end{array}$$

Hence to find $\bar{\mu}$, M must be determined:

Now, velocity potential, $\phi = \left[L^2T^{-1}\right]$

since $\frac{\partial\phi}{\partial x}$ = velocity = $\left[LT^{-1}\right]$.

From equations (8) and (9) :

$$M = \rho U \int_L F(x)\,dx \text{ and } F(x) = \oint \phi_o\,dy.$$

Let $\phi \propto UB$; B = beam of yacht

$F(x) \propto UB^2$

$M \propto U^2B^2\ell.$

Let a moment coefficient be :

$$C_M = \left\{\frac{M}{\pi\rho U^2 B^2 \ell\theta}\right\} \qquad (14)$$

The purpose of defining a moment coefficient C_M is that it may be used to determine coefficient of $\bar{\mu}$ for yachts. This may be done by assuming a calculated C_M to be constant for all yacht forms that are derived by stretching a given parent form. The source modelling programme can be used to calculate M for the parent hull, then a C_M can be defined which may be used for other yachts to find the coefficients $\bar{\mu}$. This may be done by substituting into the $\bar{\mu}$ formula various values :

$$C_M = \frac{M}{\pi\rho U^2 B^2 \ell\theta}$$

$$\bar{\mu} = \frac{\alpha M}{\theta L} = \left(\frac{B^2\ell}{a_*^2}\right) \frac{C_M}{C_{L_T}} \qquad (15)$$

2.4

A typical form for the CLR equation for varied heel and leeway angles and the one calculated to fit experimental data from the Wolfson Marine Craft Unit, (Ref. 5) is :

$$\bar{x} = 0.42 - 0.052 \left(\frac{\theta}{\alpha}\right)$$

Such an equation as this for a particular yacht is clearly a neat and simple way of displaying the variable CLR position, and as such will help the designer of yachts in the balance design stages.

3. DIRECTIONAL STABILITY

Directional stability is a vital design parameter for yachts since modern yachts have a high sail area to displacement ratio, and expecially off the wind yachts need to be directionally stable with as much sail up as is feasible under the circumstances. There are three main considerations for yacht directional stability :

1. There is enough rudder force and moment to balance the moments developed by the sails and hull to allow the yacht to be kept on course.

2. STATIC directional stability requires that if the boat, when initially balanced and on course, is given a small deviation from this course, it will then return due to an induced moment that is not from helm action.

3. DYNAMIC directional stability is concerned with whether oscillations about the desired course will increase or decrease. If the yacht's oscillatory deviations increase then it is dynamically unstable.

Static directional stability is of primary importance and since the acceleration induced inertia forces are complex and on the wind often of smaller magnitude, dynamic stability will not be analysed in detail here. It is important to analyse the directional stability when there is a windshift or change in windstrength or otherwise which will change the apparent wind angle on the yacht. A yacht should inherently have directional stability and this will obviously aid the installation of modern self-steering gears in the case of cruising yachts. It will also help to reduce radical helm deviations and reduce the distance travelled through the water and drag due to exaggerated rudder action.

3.1 STATIC DIRECTIONAL STABILITY

For this kind of stability the helmsman should not need to alter the helm if a deviation in apparent wind direction is encountered. The mechanism which acts when the apparent wind direction is changed is as follows :

The force on the sails changes such that the lateral component of forces on the sail and the hull are changed. The yacht also heels further (if the increase of lateral force is positive) and this will produce an asymmetric hull moment. Another effect is that the forward component of force on the sail will, when coupled with the drag force, produce a turning moment which increases as heel increases.

The balance of forces between the keel and rudder is altered as can be seen from the following argument :

Let the angle of leeway $= \alpha$

Angle of flow at the rudder $= \alpha_R = \alpha + \Omega$.

If the angle of leeway becomes $\alpha + \delta\alpha$ then the fractional increases are :

KEEL : $\frac{\delta\alpha}{\alpha}$

RUDDER : $\frac{\delta\alpha}{\alpha + \Omega}$

Hence there is a redistribution of forces, and a moment is created which increases weather helm or increases lee helm depending on the sign of Ω. If Ω is positive then the fractional increases in force (proportional to angle of attack) will alter the balance such that the keel force being greater than the rudder force will pull the bow round to weather .

BROACHING

The running yacht is subject to considerable instability downwind due to the interplay of forces and turning arms whereas on the wind there is some cancellation of factors. As the yacht is heeled over in a broach, the CLR moves forward while the CE of the sails moves aft. As the weather helm increases, more pressure is applied to the rudder. The yacht slows down as it round up and the apparent wind causes the sails to be over-trimmed. This, in turn, further increases the heel which finally stalls the rudder and renders it largely ineffective, and as a result the yacht rounds up out of control. Design of the underwater hull and steering ability must be carefully planned to aid as much as is feasibly possible the yacht's stability but, in fact, steering off the wind can be achieved largely by sail steering (of the spinnaker and mainsail) and is thus a contributory tool to broach prevention.

3.2 STABILITY DERIVATIVES

The main application of the theory in this report is to steady motion, but theory for unbounded fluid is directly applicable to unsteady motion such as that associated with dynamic stability or directional stability. The graphs for cross flow forces may be used to determine the degree of directional stability of a yacht in a mathematical form on knowing the magnitudes of various "stability derivatives". They describe quantatively the tendency of a yacht to return to its original direction after being given an instantaneous disturbance which changes its direction, and are thus useful to compare the handling ability of different yachts.

The significant hydrodynamic forces affecting a yacht are primarily the sway force and yaw moment. If the way a yacht handles can be related at least in part to a mathematical relationship then this must aid the design of directional balance into yachts. It is hardly surprising that a realistic method is not used due to the amount of tank testing that has to be made and this is clearly not feasible financially in the majority of cases. But, by using the graphs for $C_{L\pi}$, a comparable mathematical method for determining the stability roots can be used.

CRITERION OF STABILITY

It can be shown that the stability derivatives are related relatively simply to the added mass values. A stability derivative is basically a rate of change of force or moment with respect to a horizontal movement in the free surface plane. The horizontal movement may be in the form of yaw or way in this consideration. For example, on letting the sway force = Y, yaw moment = N:

$Y_r = \frac{\partial Y}{\partial r}$ = Rate of change of the sway force with r where r = yaw angular velocity

$r = \frac{\partial \psi}{\partial t}$ (ψ = angle of yaw).

The roots and stability conditions in the yaw equation are important:

$$A\ddot{r} + B\dot{r} + Cr = 0 \qquad (16)$$

If the primes denote a non-dimensional form then the yaw angular velocity can be found from the roots of the yaw equation above to be :

$$* \; r'(t) = \mathrm{Re}\,\{\Omega_1 e^{\sigma'_1 t'} + \Omega_2 e^{\sigma'_2 t'}\} \qquad (17)$$

Here σ_1' and σ_2' are the roots of the yaw equation and they provide a comparison between yachts for a criterion of stability.

$$\sigma_i' = \frac{-B + (-1)^{i-1}\sqrt{B^2 - 4AC}}{2A} \quad (i = 1,2) \qquad (18)$$

As may be seen in the Appendices the coefficients A, B and C may be shown to be related to the yaw angular velocity r and the sway velocity v by :

$$A = (Y_{\dot{v}}' - m')(N_{\dot{r}}' - I_z') - N_{\dot{v}}'Y_{\dot{r}}' > 0$$

$$B = (Y_{\dot{v}}' - m')N_r' + Y_v'(N_{\dot{r}}' - I_z') - N_{\dot{v}}'(Y_r' - m') + N_v'Y_{\dot{r}}'$$

$$C = N_r'Y_v' - N_v'(Y_r' - m').$$

It is also shown that for stability C > 0.

Therefore there are two stability criteria:

(a) the magnitude of the roots σ_1, σ_2

(b) the magnitude of the coefficient C in the yaw equation, as defined by the stability derivatives (in non-dimensional form). The magnitude of C (where C > 0) will indicate the degree of stability of a vessel.

COURSE KEEPING ABILITY

The course keeping ability of a yacht is really dependent upon several factors but four of these factors are :

1. $\mathrm{Re}\{\sigma_1, \sigma_2\}$
2. $|N_r'|$
3. $|N_{\dot{r}}'|$
4. Value of C.

The real part of the roots σ_1 and σ_2 must not be too large otherwise turning ability may be hampered, and this characteristic may also be the result of having the other three main parameters, as affecting course keeping characteristics, too large.

According to Milgram, Ref. (7) Re σ_1' is not as important as $|N_r'|$ and $|N_{\dot{r}}'|$ since these two derivatives have become markedly less with the modern designs due to the shorter keel and higher sail area-displacement ratios for the racing yacht. As $|N_r|$ and $|N_{\dot{r}}'|$ decrease it becomes easier to turn a yacht since for unit ($r = \partial\psi/\partial t$) and ($\dot{r} = \partial^2\psi/\partial t^2$) where ψ is the yaw angle, there is less yawing moment created.

The condition C > O is shown in Appendix 2 to be equivalent to

$$m_T \; |x_T| > \{ \frac{mM' + M_x' m_T}{(M' + m)} \} \qquad (19)$$

If either m_T or x_T are increased then the stability criterion is improved in the sense that (19) becomes more unequal and increasingly stable.

m_T may be increased by trimming the yacht further aft or perhaps by altering the aft sectional shapes for constant area but for increasing lift producing surfaces, and here the CH graphs for the Lewis sections may give guidance. Alternatively, the rudder may be moved further aft or a skeg introduced. These could be investigated with reference to the graphs for the vortex trailing sheet and the rudder section in this report. Moving the rudder aft will increase $N_{\dot{r}}'$ and this has various effects. If contributions Y_r', $Y_{\dot{r}}'$ and N_r' from the aeroforces on the sails are large compared to contributions due to the water surface than increasing $N_{\dot{r}}'$ is a stabilising influence. It also adds to the turning moment and there is less yaw due to a wave.

If a condition for (19) is made such that

$$M_x' + x_{cg} \, m = O \text{ and } x_{cg} = O$$

(i.e., the frame of reference origin is at the centre of gravity), then (19) reduces to

$$m_T \; |x_T| \; > \; \{\frac{mM'}{M' + m}\} \qquad (19b)$$

Hence another effect on the stability criterion is that if the centre of gravity is moved forward then the stability is increased.

It may be noted that the sign of C is not dependent on U, the forward velocity and the U is only important here if wave effects are significant.

The procedure for determining the stability derivatives may be found in Appendix 2. It must be noted that the existence of the graphs are the reason that the stability derivatives may be found simply, and wherever the lateral added mass of a section is necessary the graphs will provide this numerically.

MAGNITUDES OF STABILITY DERIVATIVES

The graphs within this report have not, as yet, been used to determine the stability derivatives but below there are two main references from which the expected orders of magnitude of the various stability derivatives and roots may be obtained, Ref. (7) and Ref. (8)

Gerritsma has a particularly good set of experimental results and shows the stability roots in 3 categories :

(i) Coupled Sway - Yaw equations of motion

(ii) Sway - Roll - Yaw equations

(iii) Sway - Roll - Yaw plus aerodynamic forces

Including first roll, then aerodynamic forces increasingly destabilises the yachts investigated, i.e. the Reel part of the roots σ_1, σ_2 become less negative :

Dimensionless stability roots of the coupled sway-roll-yaw equations of motion including aerodynamic forces

Ship		S_1	S_2	S_3	S_4	S_5
half ton yacht	F_n=0.243	-2.50 ±2.98 i		-1.54 ± 5.55 i		-0.02
" " "	F_n=0.486	-2.27 ±1.35 i		-1.72 ± 3.22 i		0.32
Columbia	F_n=0.168	-1.61 ±0.36 i		-1.13 ± 7.38 i		-0.02
"	F_n=0.251	-1.56 ±0.35 i		-1.19 ± 4.94 i		-0.01
"	F_n=0.335	-1.60 ±0.48 i		-1.61 ± 3.37 i		0.04
Valiant	F_n=0.163	-0.38	-3.08	-1.78 ± 8.19 i		0.04
"	F_n=0.244	-0.37	-3.01	-1.91 ± 5.08 i		0.09
"	F_n=0.325	-0.74	-3.97	-1.76 ± 3.59 i		0.16

Milgram has calculated whole range of stability derivatives which will be shown here for four 12m yachts :

CALCULATED STABILITY DERIVATIVES AND ROOTS FOR 12m YACHTS

	COLUMBIA	INTREPID (1967)	INTREPID (1970)	VALIANT
Y_v	-0.116	-0.116	-0.116	-0.116
$Y_{\dot{v}}$	-0.0554	-0.049	-0.0389	-0.0366
Y_r	0.0391	0.0340	0.0276	0.0204
$Y_{\dot{r}}$	0.0047	0.0034	0.0021	-0.0001
N_v	-0.0163	-0.0150	-0.0113	-0.0162
$N_{\dot{v}}$	0.0047	0.0034	0.0021	-0.0001
N_r	-0.0090	-0.0073	0.0057	-0.0049
σ_1, σ_2	-2.604 ±0.857i	-2.533 ±0.841i	-2.531 ±0.72i	-2.370 ±0.602i

CONCLUSIONS ON STABILITY DERIVATIVES

In hydrodynamics, the stability derivatives, the roots of the sway and yaw equations and the margin of stability given by the coefficient $C > 0$ are very useful for both a description and for a comparison between yachts. In reality, there are very few ways of describing the motion of yachts and their comparisons in as simple mathematical layout as for the stability derivative method.

Milgram (Ref (7)) has determined the stability derivatives for various yachts using a lifting surface method but not taking into account the body thickness or the way a streamline convects aft to the body end due to the body thickness inducement. This report offers a straightforward method of determining the derivatives taking into account body thickness and the geometry of the trailing vortex sheet.

There are three factors neglected in the appraisal of stability derivatives using the graphs here, namely flow separation in the afterbody, the effects of aerodynamic forces on the sails and the cross coupling of sway and yaw with roll, the latter being large effects since the CE of the sails moves transversely with rolling. Experimental results and the coupled equations of motion may be found in Ref (8).

Experience and utilisation of derivatives will allow the correct magnitudes for various required characteristics to be known and hence the method becomes an effective tool for design.

4. CONCLUSIONS

This report has demonstrated how hydrodynamic theory may be applied to yacht design where rules of thumb are mostly in use at present. The need for consistency and a need for wider knowledge of balance and stability will lead the design of yachts in a more scientific direction; thus a hydrodynamic basis for design will, in the future, become more apparent.

In this report slenderbody theory has been used to prepare a numerical method to find the centre of pressure positions and the stability derivatives for yacht forms. In order to bring the CLR position within 1-2% of actual experimentally measured CLR positions, the programme of research in the Ship Science Department of Southampton University for 1978 / 79 will definitely involve a series of tests and corresponding CLR calculations on varying yacht forms. Firstly, this will allow an accurate correction method for the final part of the CLR slenderbody calculations, to achieve the accuracy potential which is inherent in a hydrodynamic appraisal. Secondly, these tests will define more asymmetric moment coefficients $\bar{\mu}$ in the CLR equation:

$$\bar{x} = \bar{x}_1 + \bar{\mu}\,\frac{\theta}{\lambda}$$

and thus, with a suitable range of yachts tested in the tank, a designer can simply transfer the $\bar{\mu}$ value of a parent yacht to his own design.

Calculations are quick and simple, and perhaps take only slightly longer than the method of balancing the yacht's shape to find the centroid of area. In the hydrodynamic method we have a sound basis for which a "lead" distance is not necessary.

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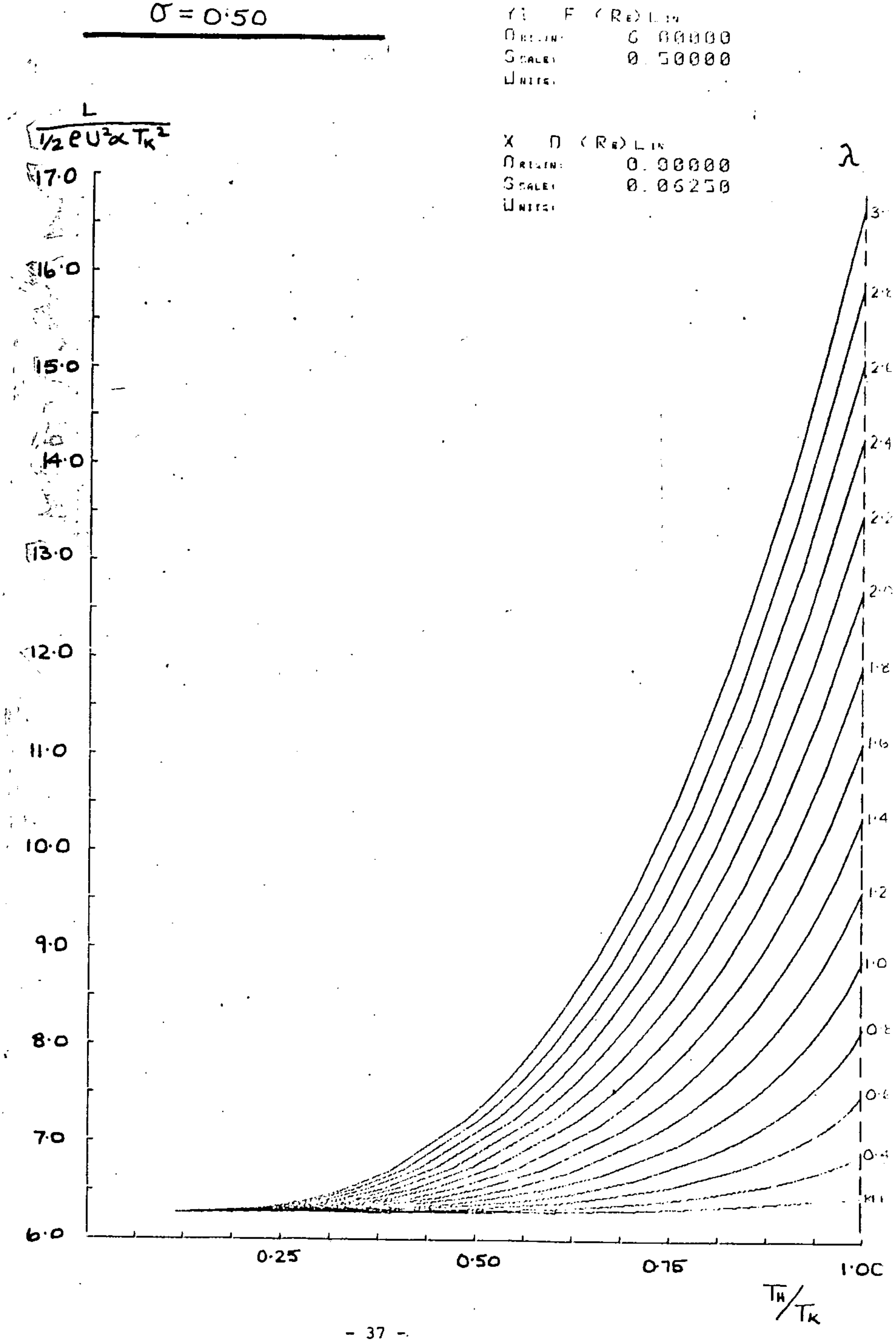
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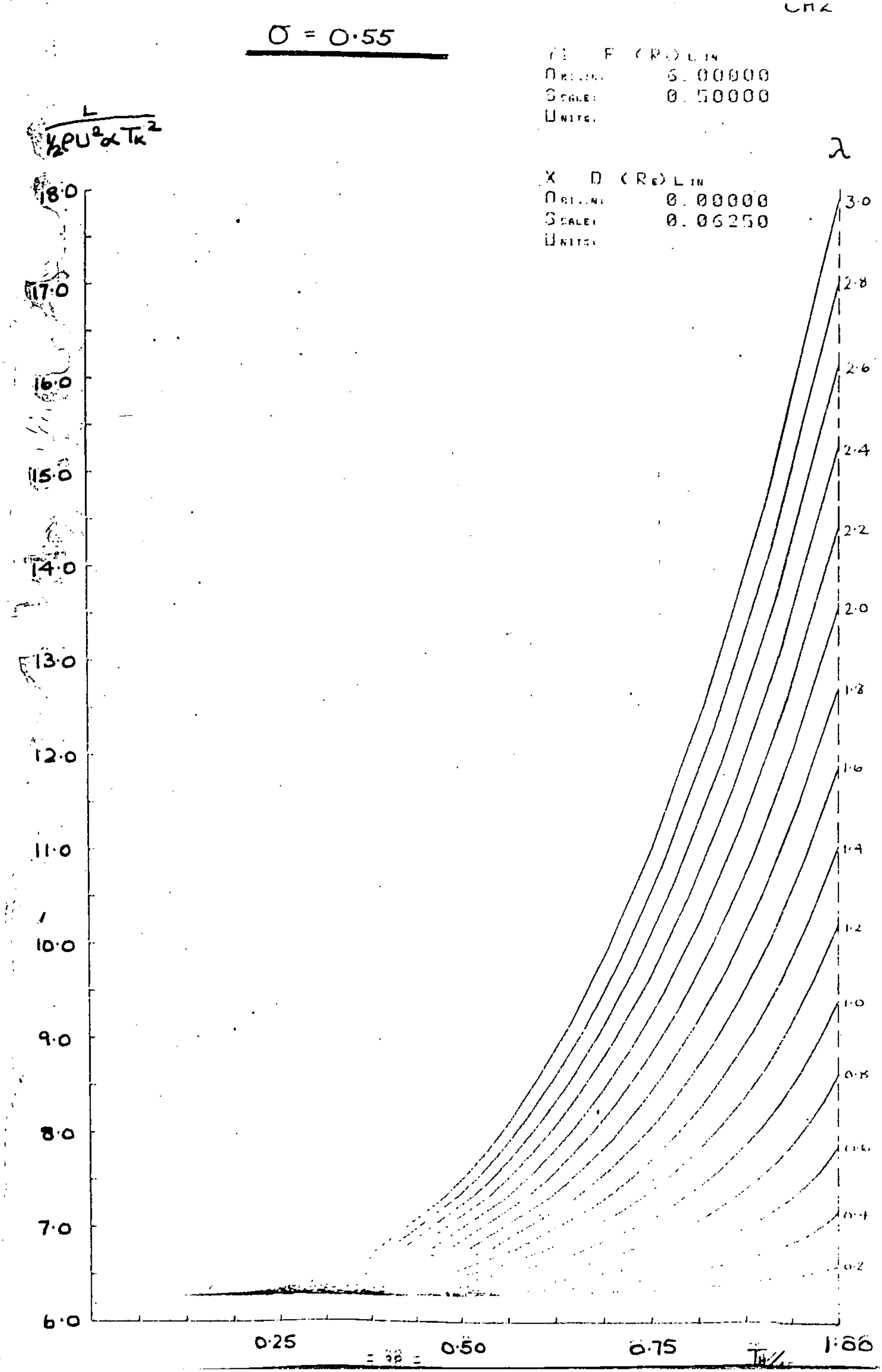
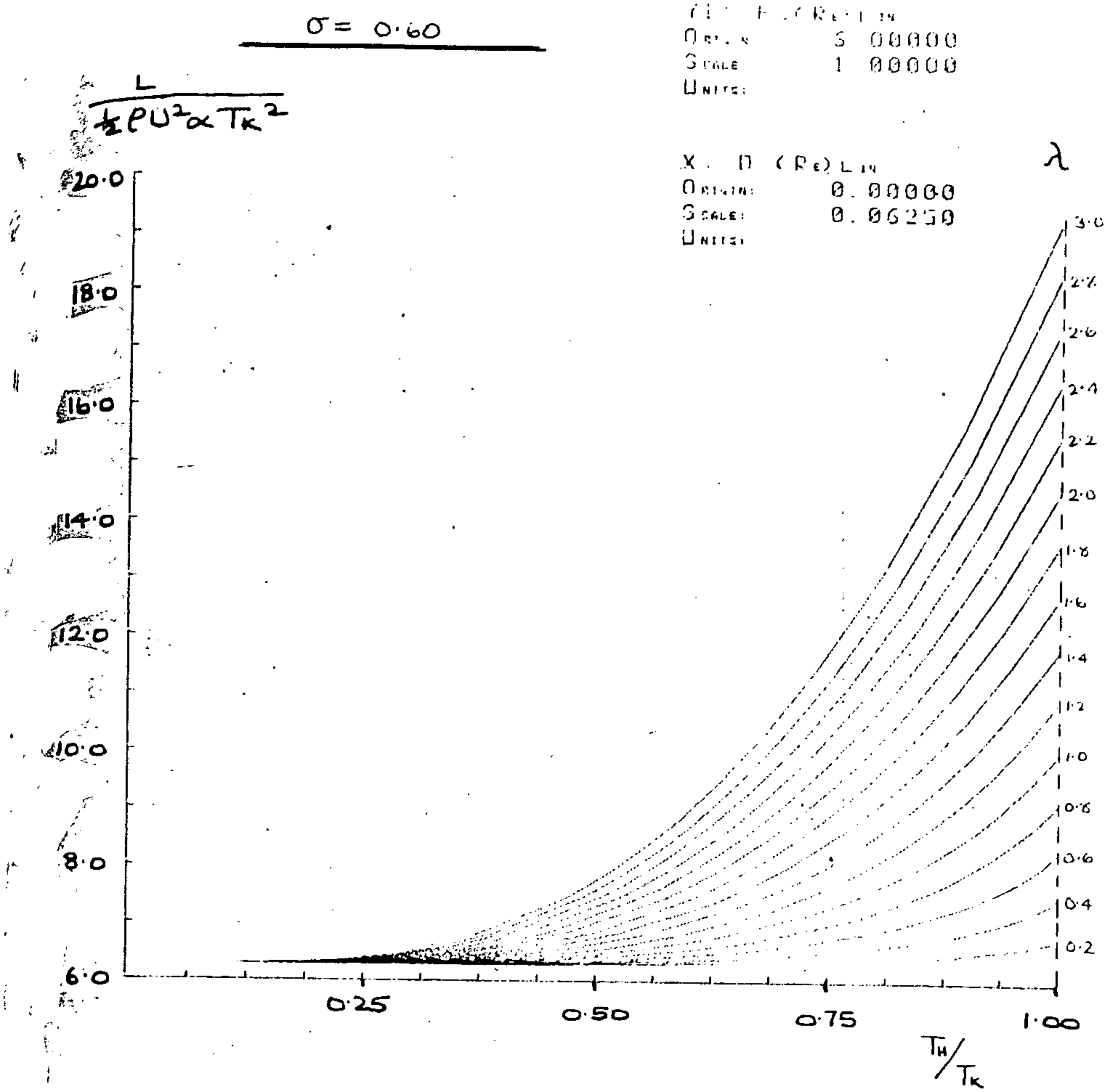
Scale: 0.50000
Units:
X: D (Re) Lin
Origin: 0.00000
Scale: 0.06250
Units:
L / ½ρU²αT_K²
17·0
16·0
15·0
14·0
13·0
12·0
11·0
10·0
9·0
8·0
7·0
6·0
0·25
0·50
0·75
1·00
T_H/T_K
λ
2·4
1·4
1·2
1·0


CH2
σ = 0·55
Y F (R6) LIN
ORIGIN 6.00000
SCALE 0.50000
UNITS
X D (R6) LIN
ORIGIN 0.00000
SCALE 0.06250
UNITS
L / ½ρU²αTk²
λ
18·0
17·0
16·0
15·0
14·0
13·0
12·0
11·0
10·0
9·0
8·0
7·0
6·0
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0·25
0·50
0·75
1·00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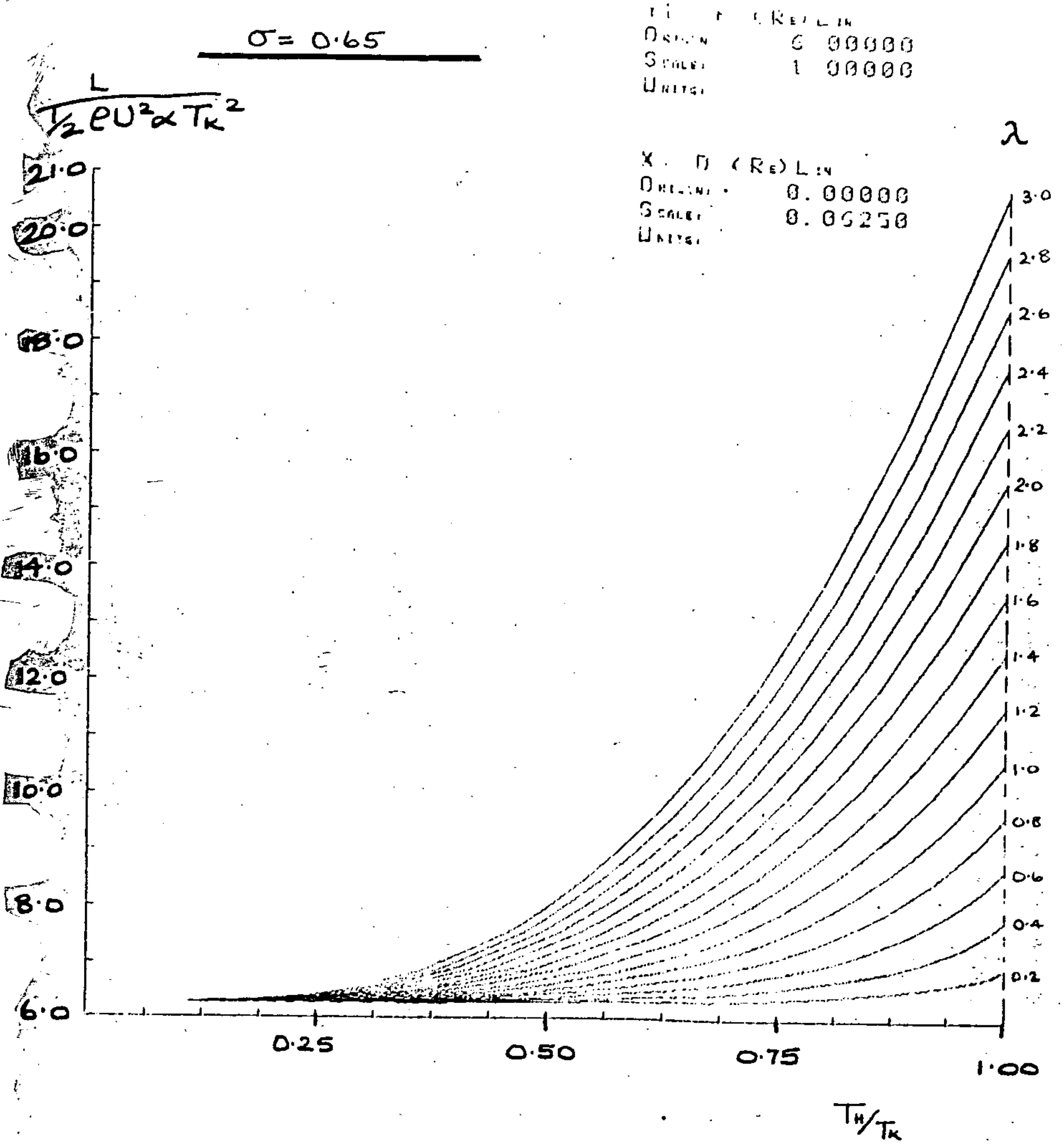
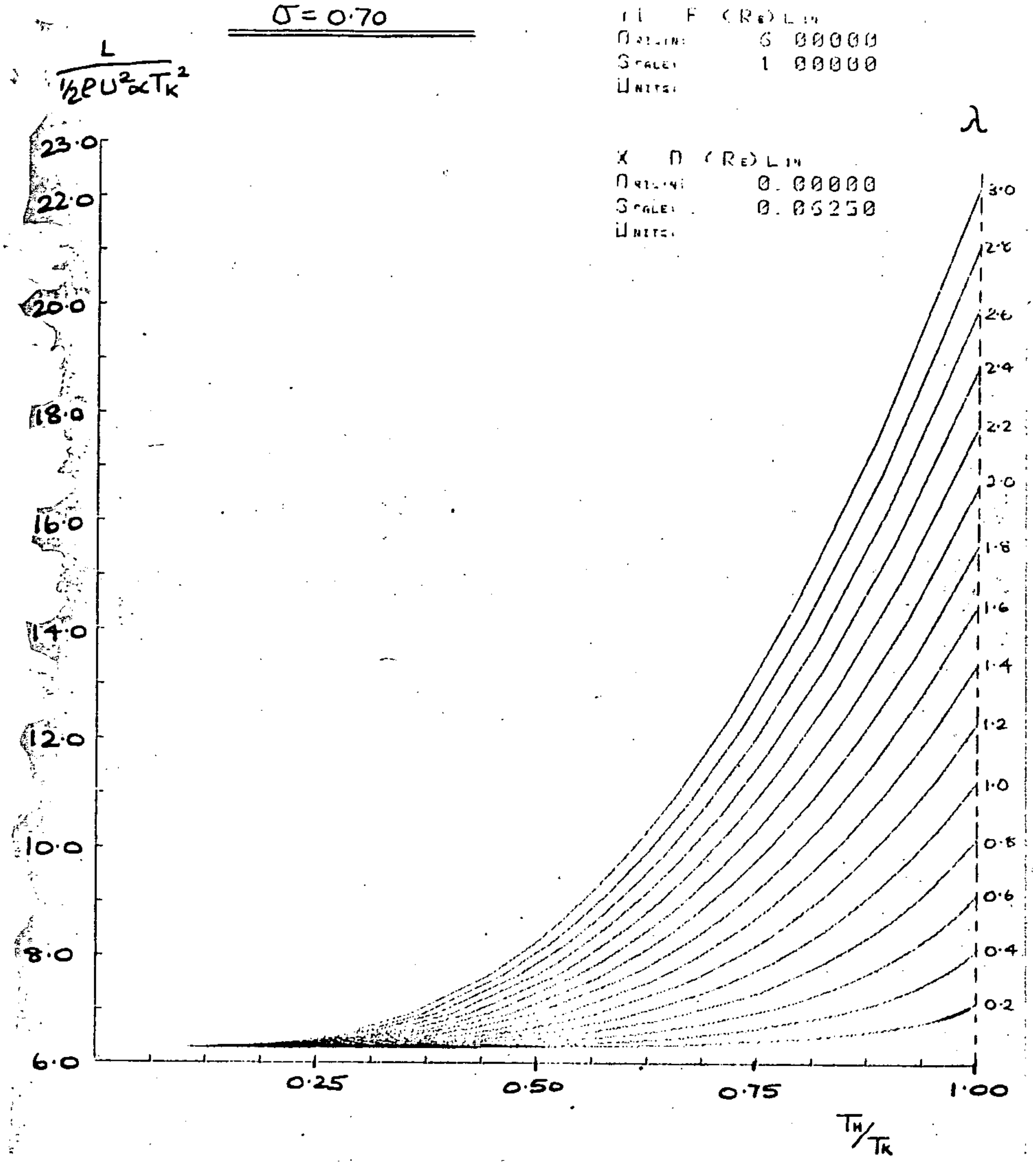
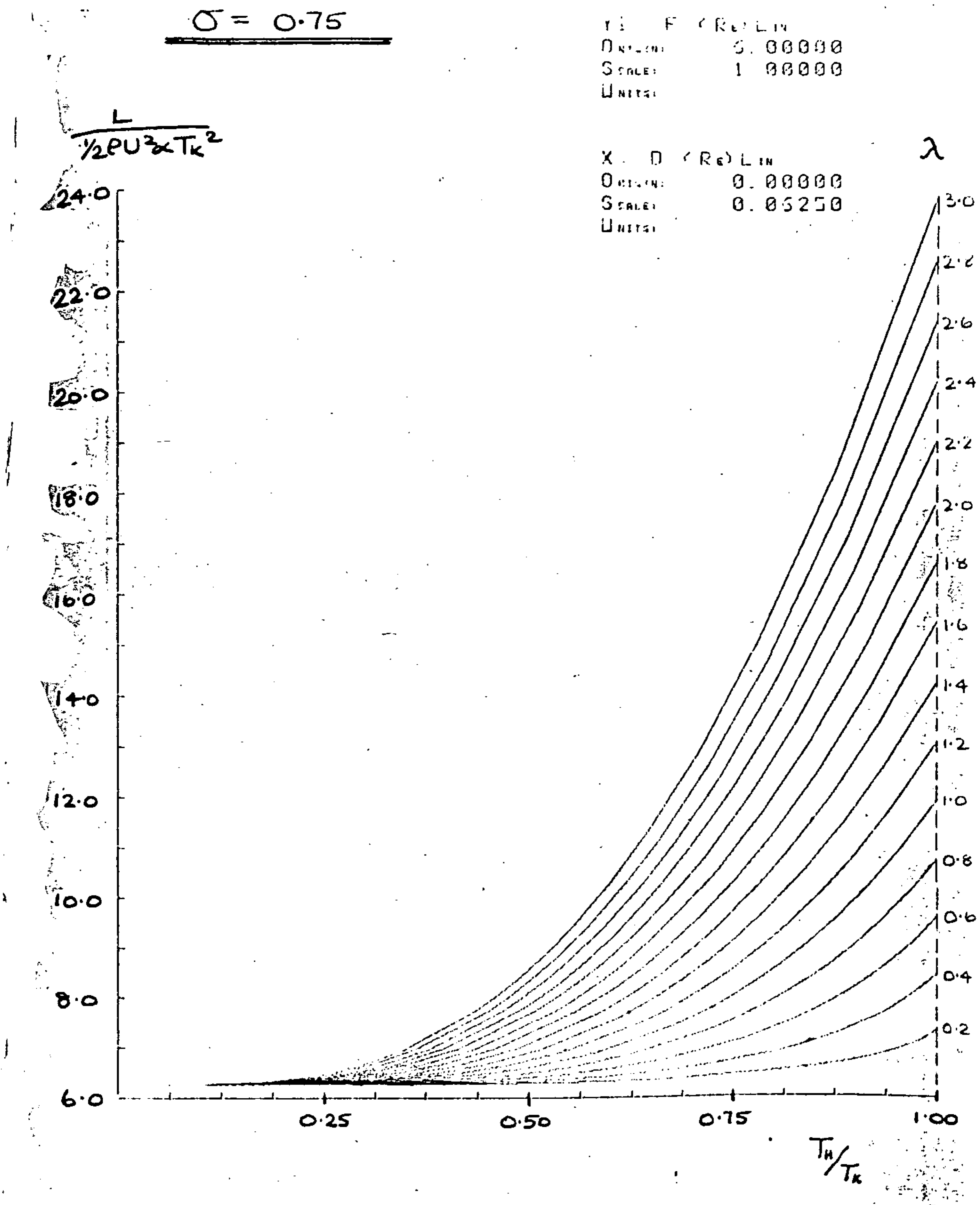
L / ½ρU²α T_K²
ORIGIN 5.00000
SCALE 1.00000
UNITS:
X . D (RE) LIN
ORIGIN: 0.00000
SCALE: 0.06250
UNITS:
λ
20·0
18·0
16·0
14·0
12·0
10·0
8·0
6·0
0·25
0·50
0·75
1·00
T_H/T_K
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2


σ = 0·65
L / ½ρU²αTK²
Origin 0.00000
Scale 1.00000
Units
λ
X · D (Re) L in
Origin 0.00000
Scale 0.06250
Units
21·0
20·0
18·0
16·0
14·0
12·0
10·0
8·0
6·0
0·25
0·50
0·75
1·00
TH/TK
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2



σ = 0·70
L / ½ρU²αT_K²
λ
23·0
22·0
20·0
18·0
16·0
14·0
12·0
10·0
8·0
6·0
0·25
0·50
0·75
1·00
T_H/T_K
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2
ORIGIN: 6.00000
SCALE: 1.00000
UNITS:
ORIGIN: 0.00000
SCALE: 0.06250
UNITS: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Y: F (Re) LIN
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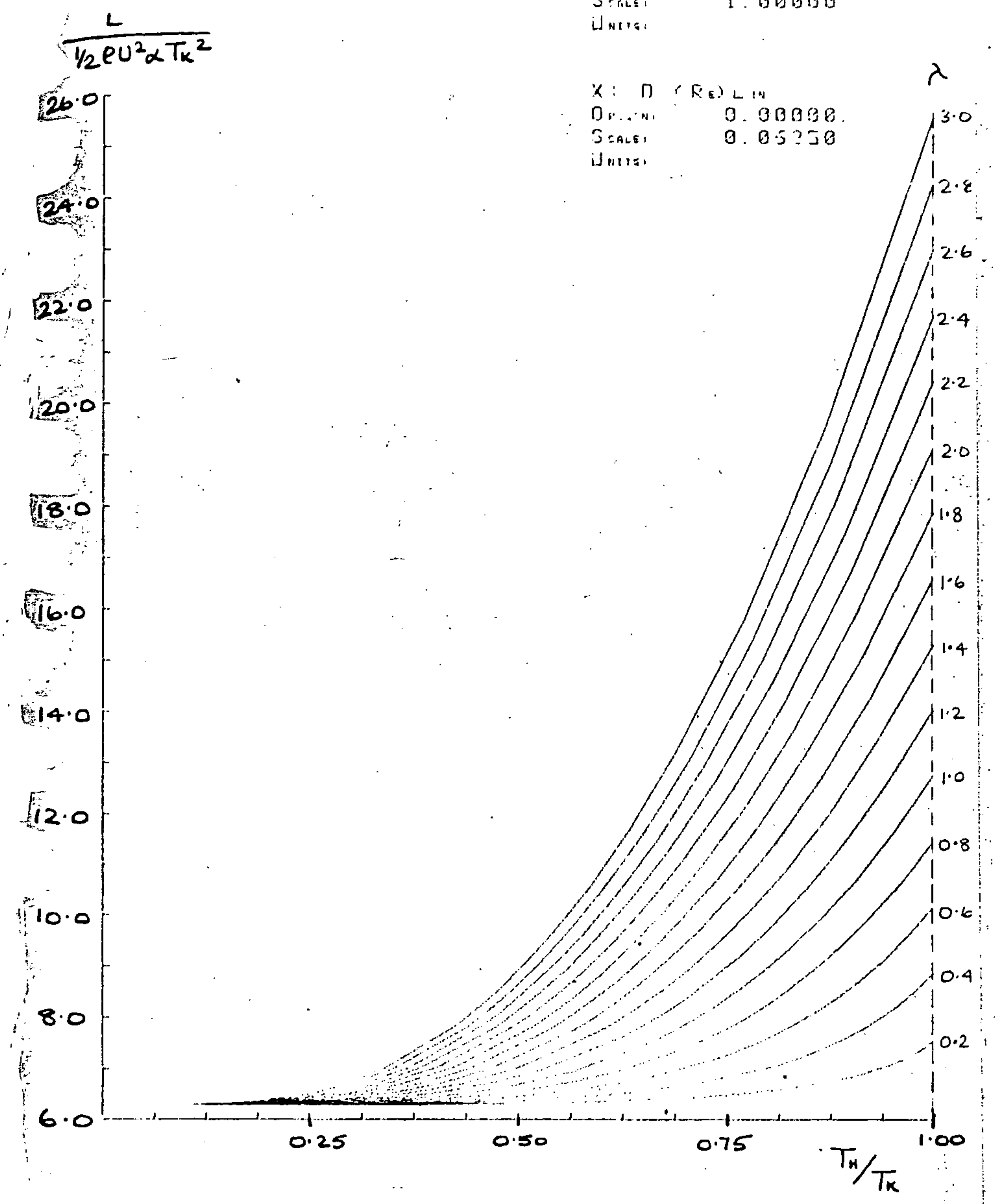
SCALE: 1.00000
UNITS:
X: D (Re) LIN
ORIGIN: 0.00000
SCALE: 0.05250
UNITS:
L / ½ρU²αTk²
λ
24·0
22·0
20·0
18·0
16·0
14·0
12·0
10·0
8·0
6·0
0·25
0·50
0·75
1·00
TH/TK
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2

σ = 0·80



6.00000
1.00000
0.00000
L / ½ρU²αTk²
26·0
24·0
22·0
20·0
18·0
16·0
14·0
12·0
10·0
8·0
6·0
0·25
0·50
0·75
1·00
TH/TK
λ
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2

START OF OUTPUT

[112, 014]

16:34:25-JAN-78

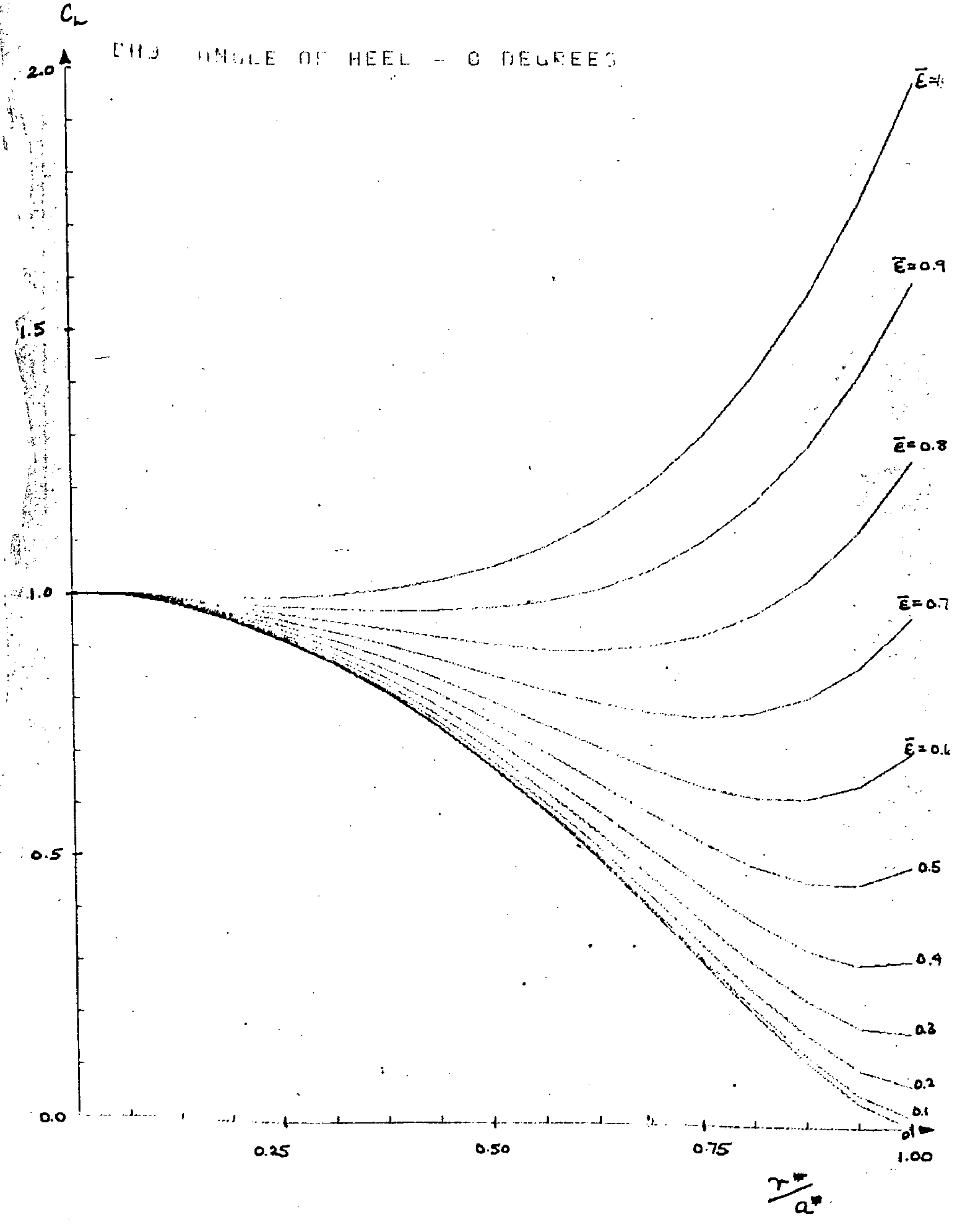
SCALES

Y1: F (RE) LIN
ORIGIN: 0.00000
SCALE: 0.10000
UNITS:

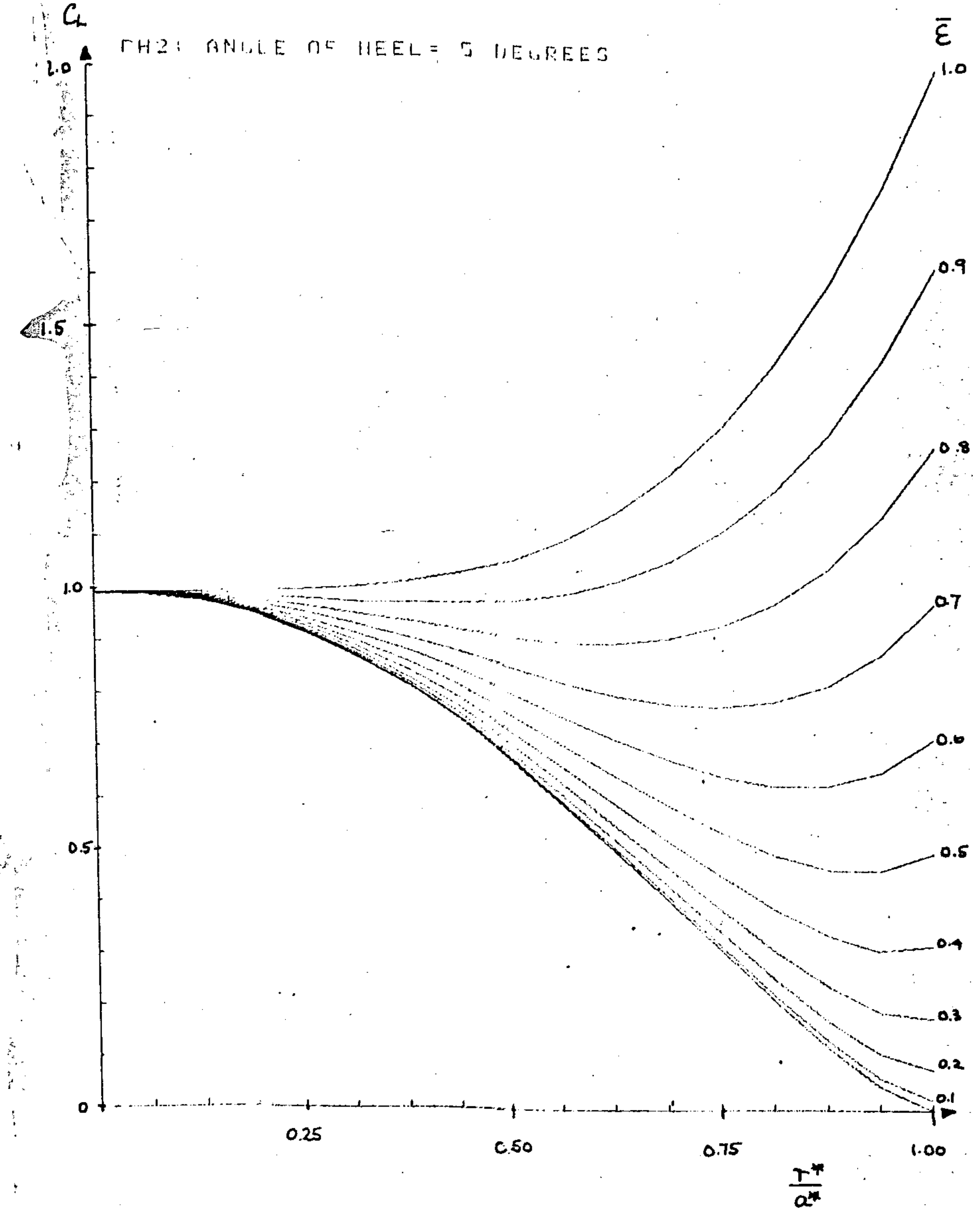
X: D (RE) LIN
ORIGIN: 0.00000
SCALE: 0.06250
UNITS:

$$C_L = \frac{L}{\pi \rho U^2 \alpha a^{*2}}$$

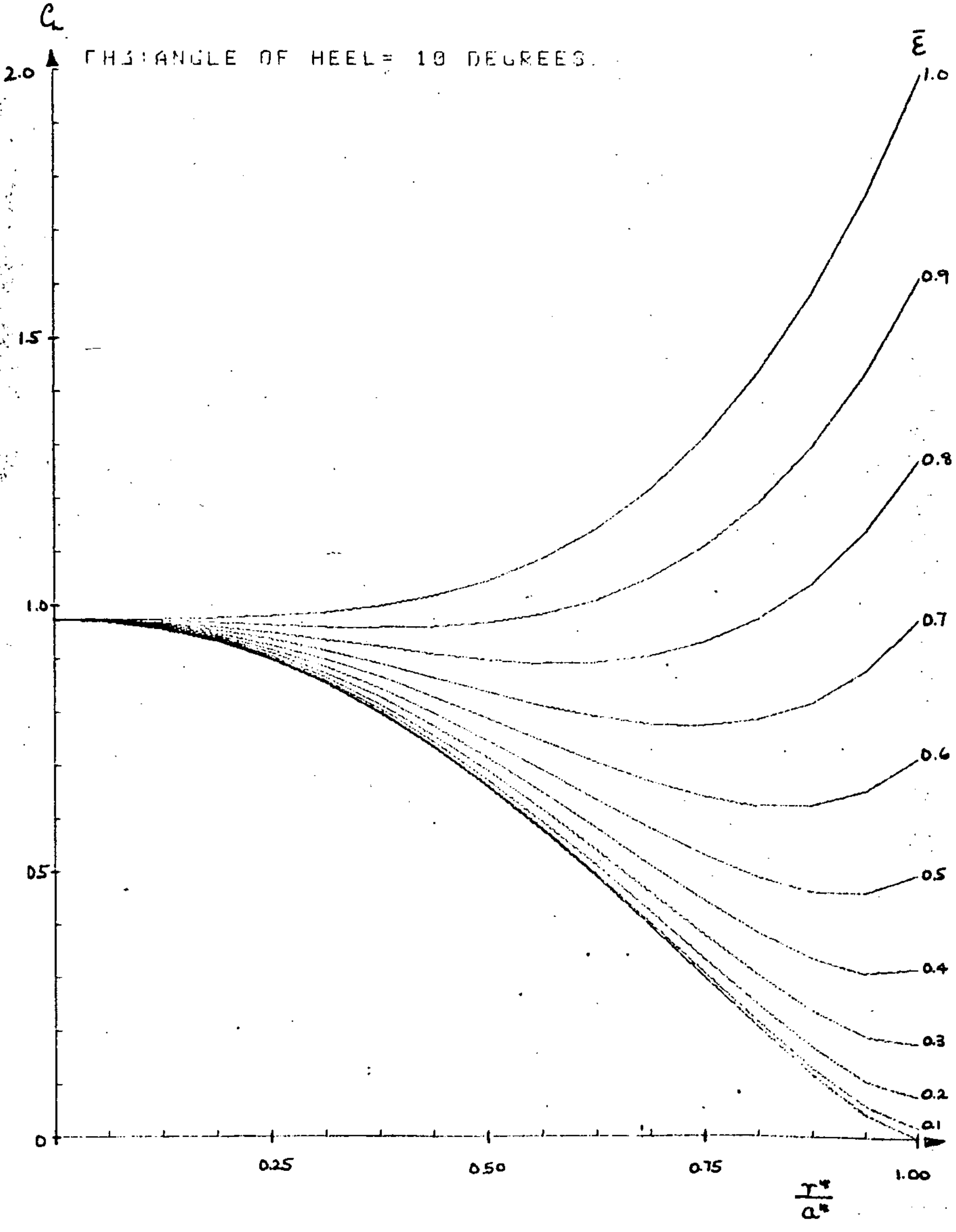
NON-DIMENSIONAL LIFT FORCES FROM THE TRAILING EDGE TO THE BODY END, TAKING INTO ACCOUNT VORTEX SHEET CONVECTION.



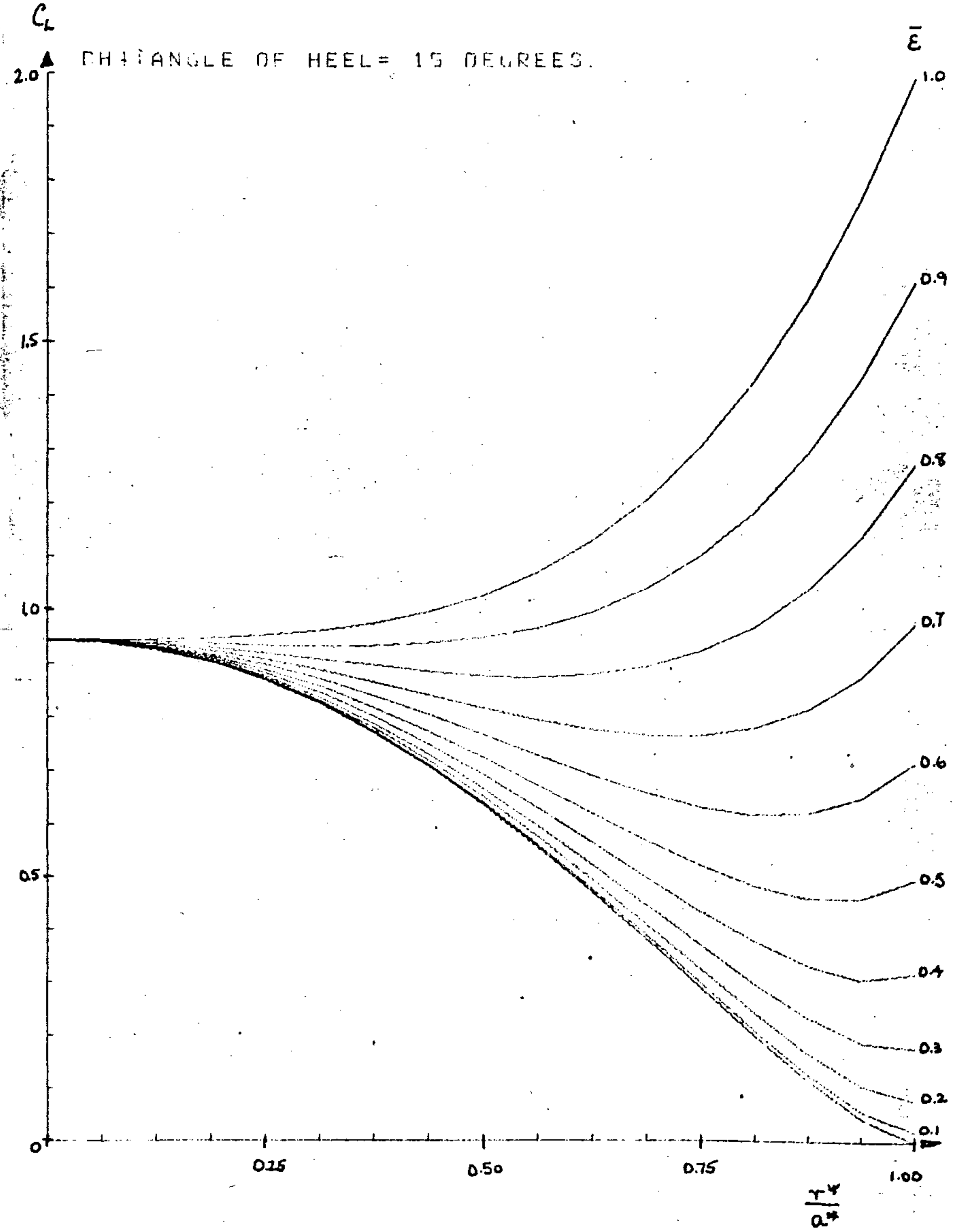
C_L
CH3 ANGLE OF HEEL - 0 DEGREES
2.0
1.5
1.0
0.5
0.0
$\bar{\varepsilon}=1$
$\bar{\varepsilon}=0.9$
$\bar{\varepsilon}=0.8$
$\bar{\varepsilon}=0.7$
$\bar{\varepsilon}=0.6$
0.5
0.4
0.3
0.2
0.1
0
0.25
0.50
0.75
1.00
r^*/a^*



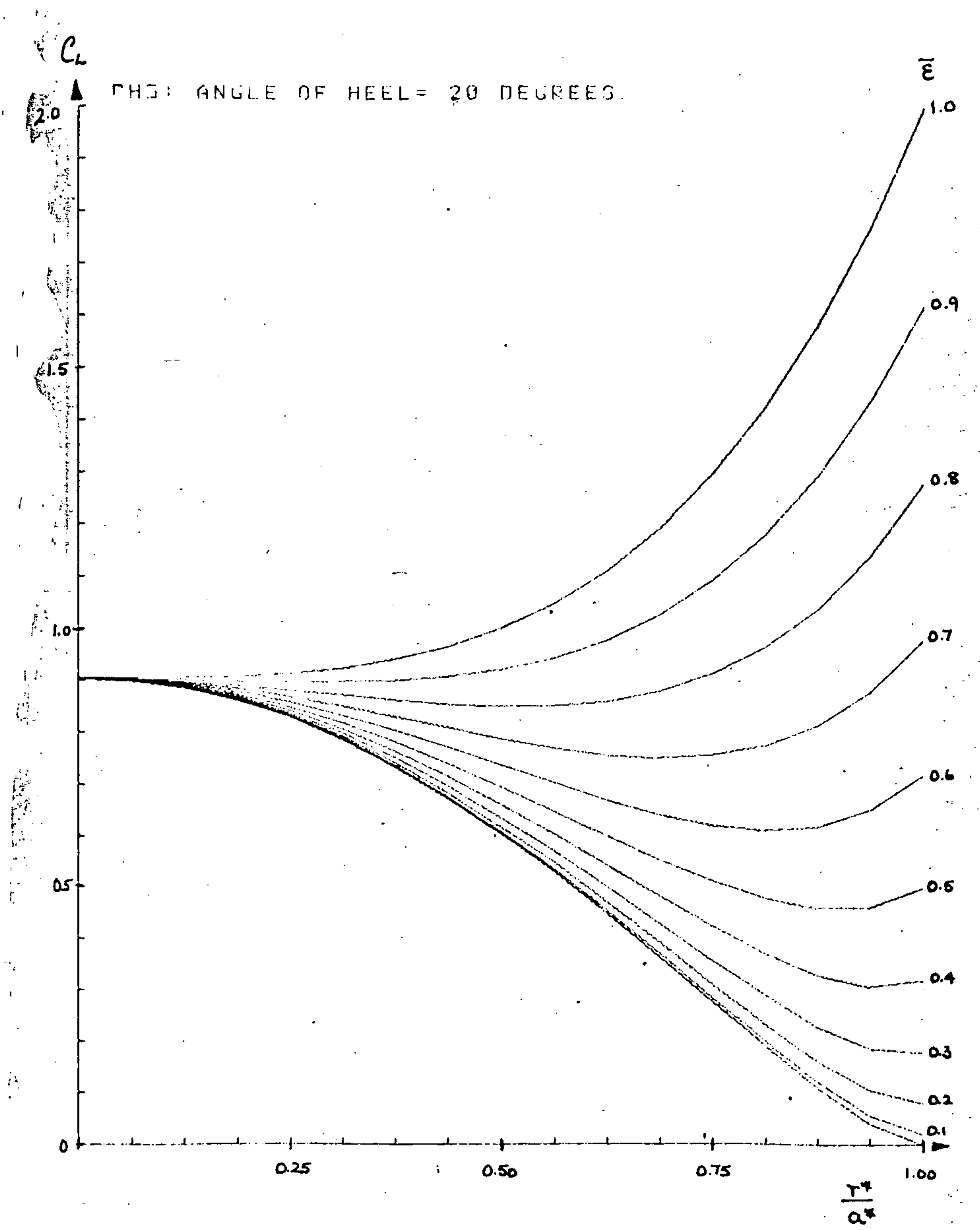
C_L
CH2: ANGLE OF HEEL= 5 DEGREES
$\bar{\varepsilon}$
2.0
1.5
1.0
0.5
0
0.25
0.50
0.75
1.00
$\frac{r^*}{a^*}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1



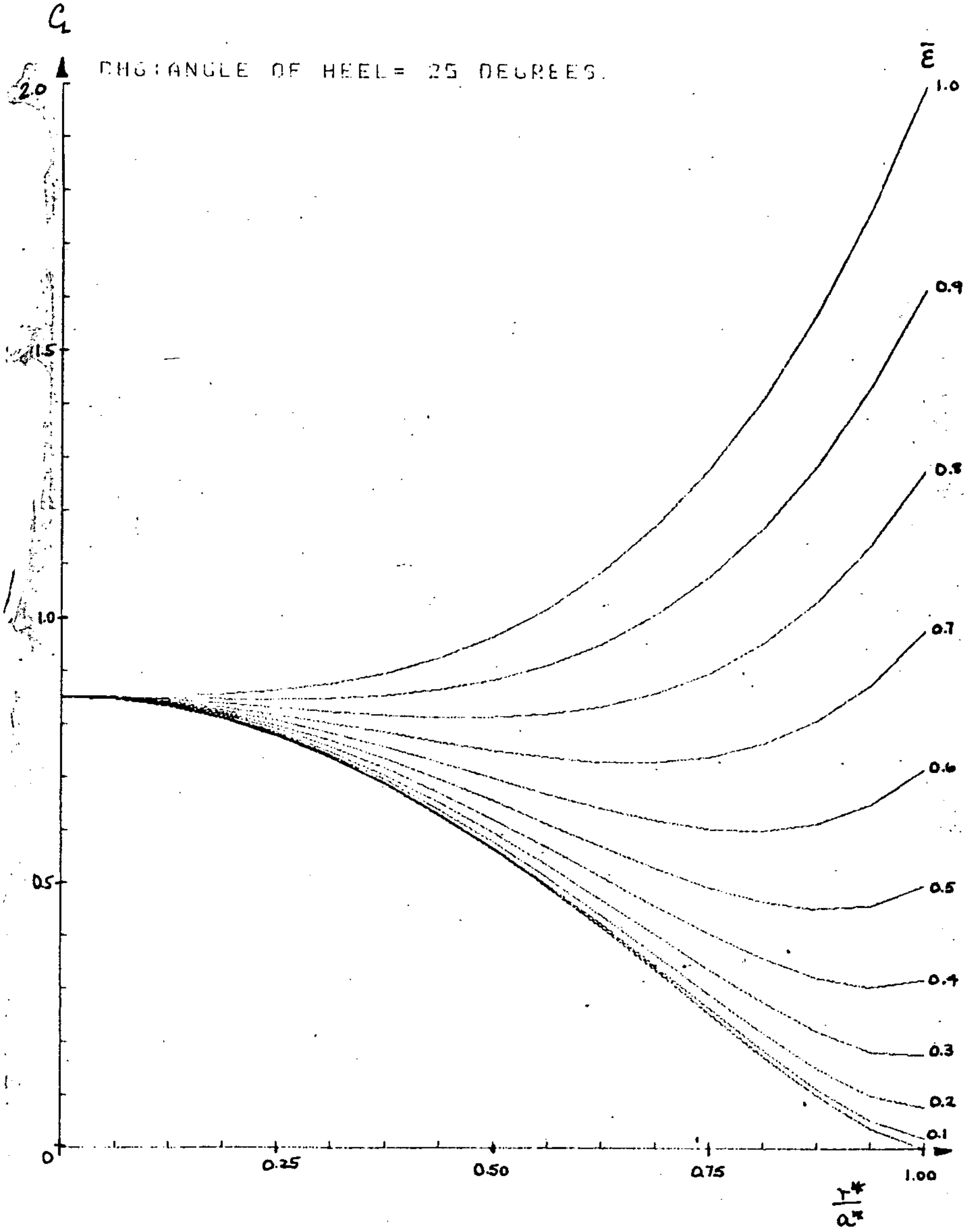
C_L
FH3:ANGLE OF HEEL= 10 DEGREES.
$\bar{\varepsilon}$
2.0
1.5
1.0
0.5
0
0.25
0.50
0.75
1.00
$\frac{r^*}{a^*}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1



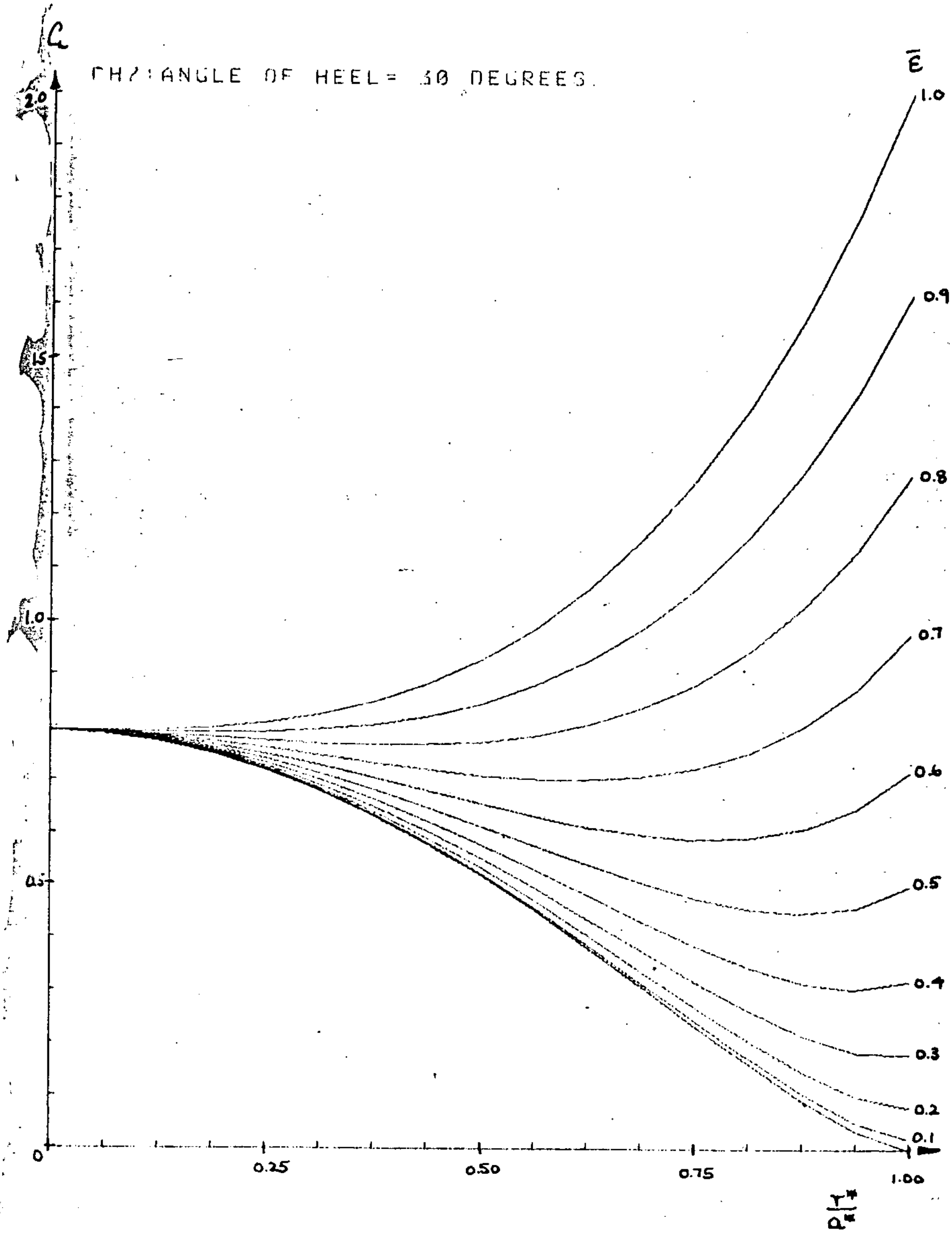
C_L
CHI ANGLE OF HEEL= 15 DEGREES.
$\bar{\varepsilon}$
2.0
1.5
1.0
0.5
0
0.25
0.50
0.75
1.00
$\frac{r^*}{a^*}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

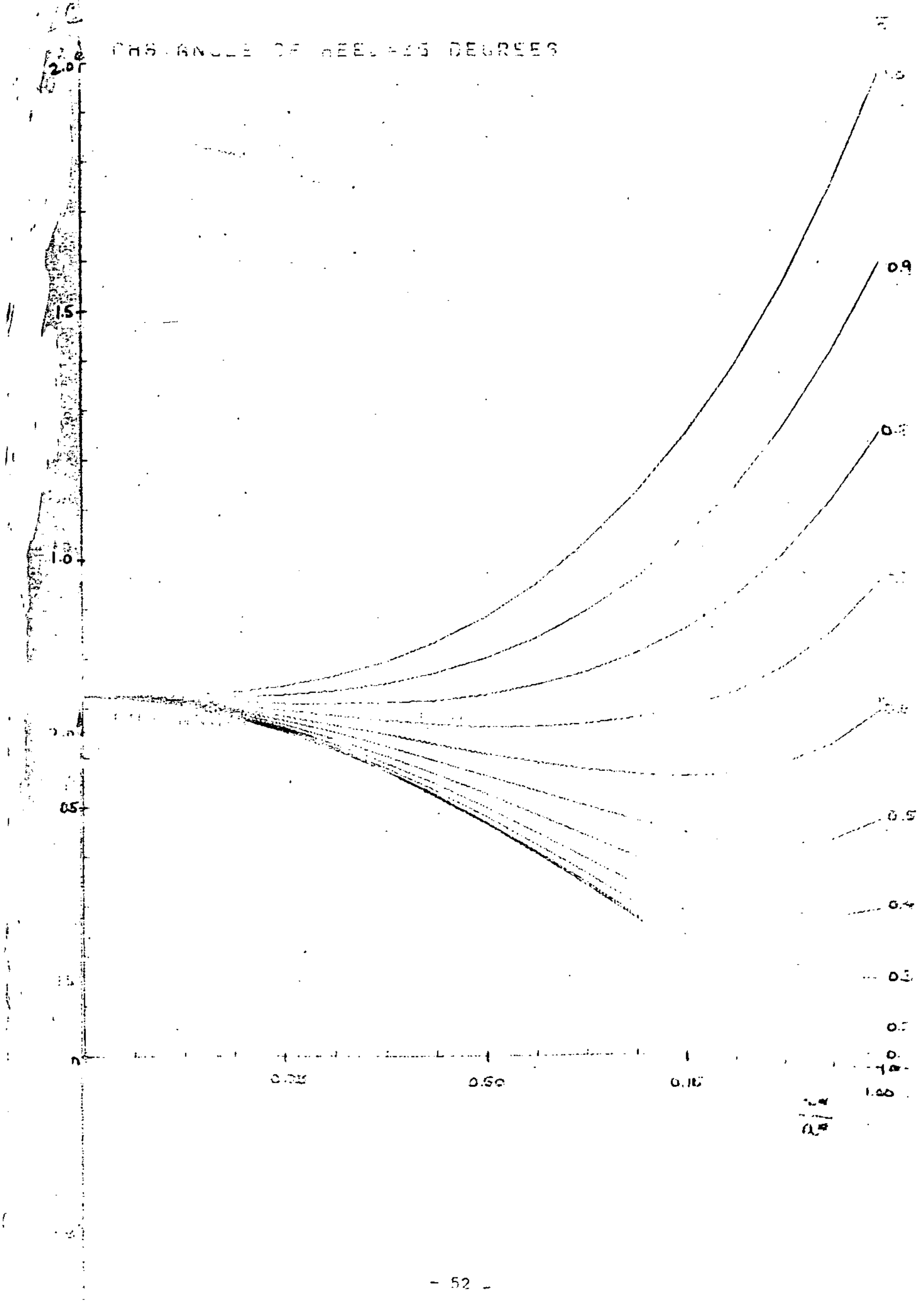


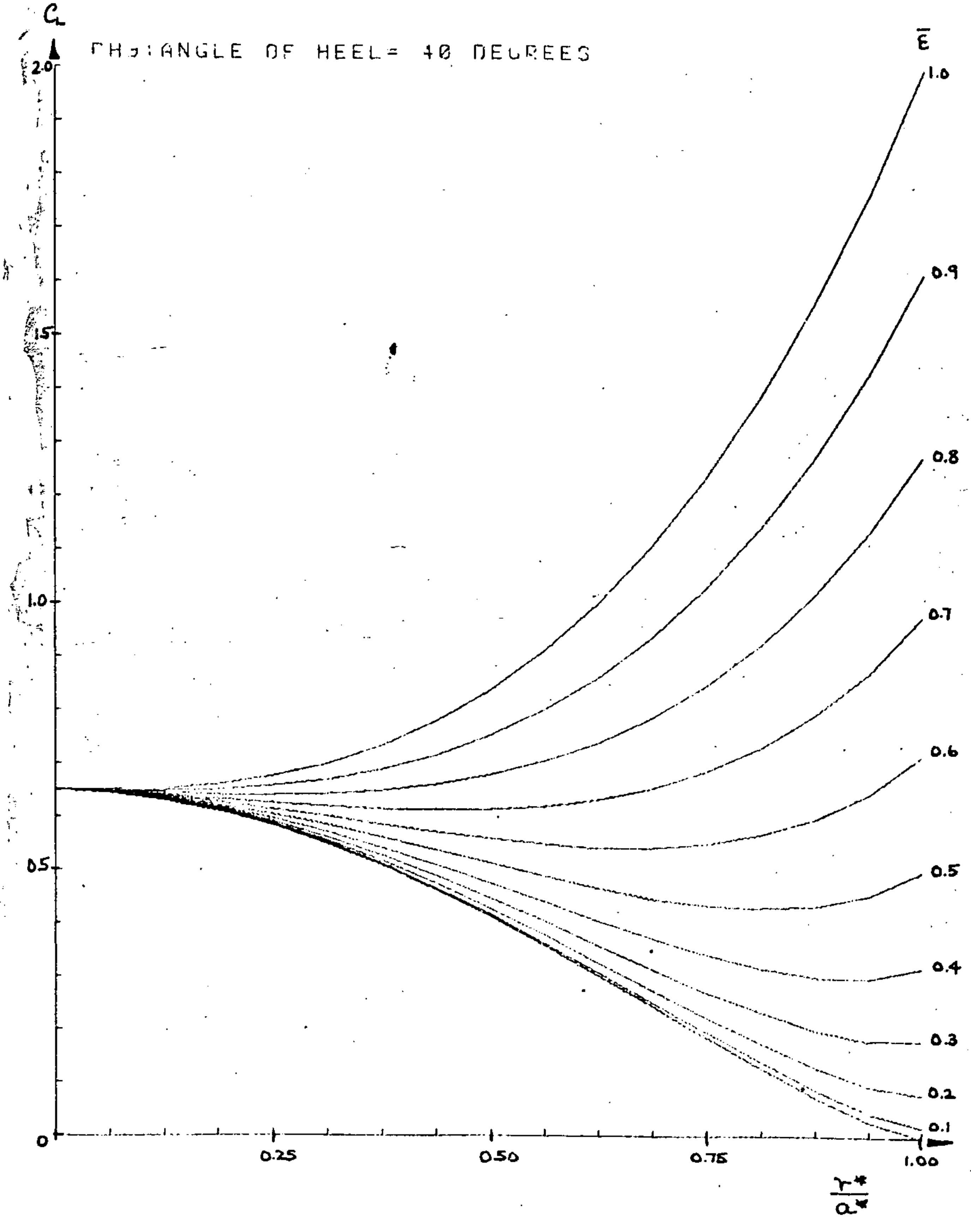
C_L
PHG: ANGLE OF HEEL= 20 DEGREES.
$\bar{\varepsilon}$
2.0
1.5
1.0
0.5
0
0.25
0.50
0.75
1.00
$\frac{r^*}{a^*}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1



C_L
CHG:ANGLE OF HEEL= 25 DEGREES.
2.0
1.5
1.0
0.5
0
$\bar{\varepsilon}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.25
0.50
0.75
1.00
$\frac{r^*}{a^*}$

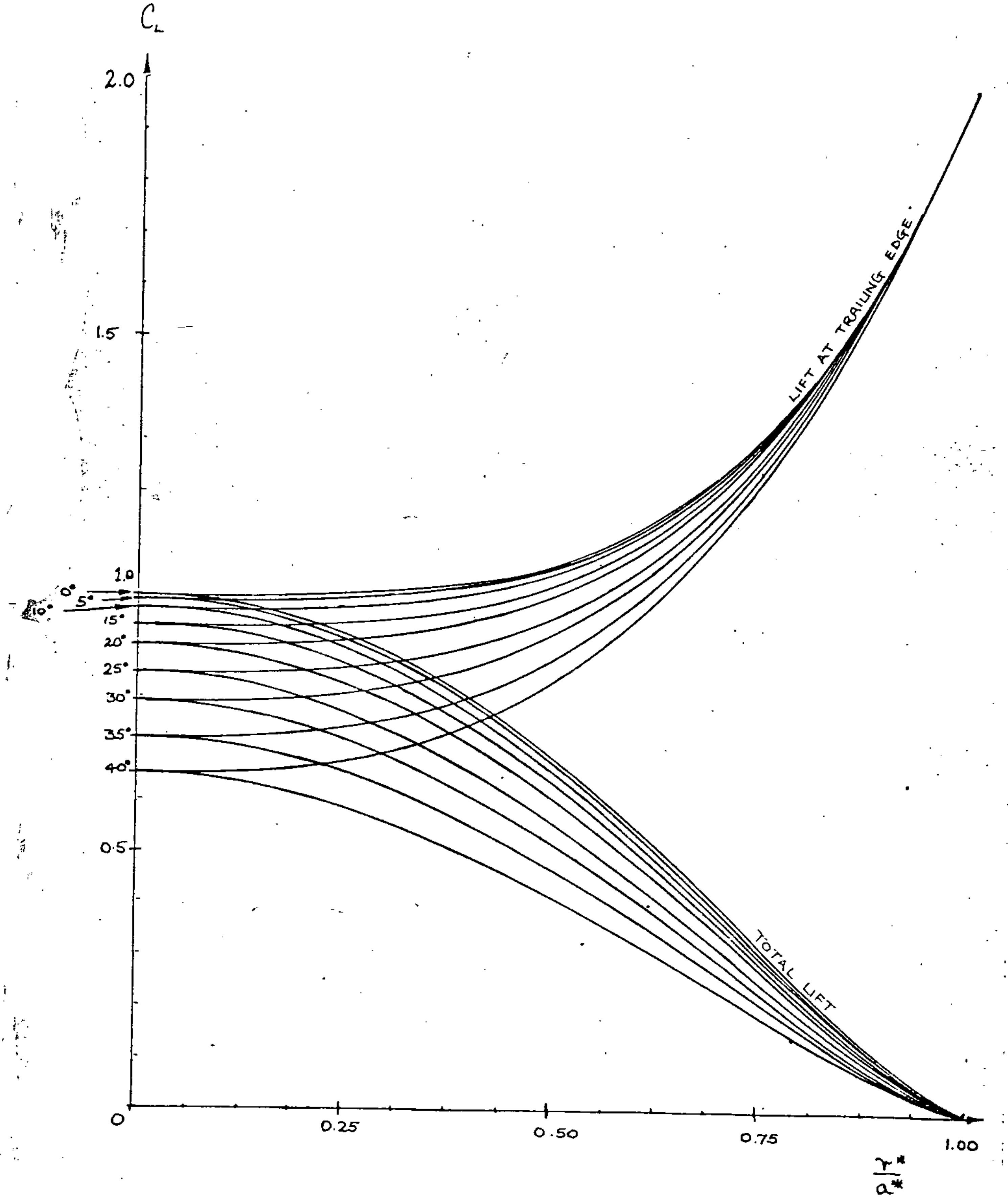

CH7: ANGLE OF HEEL= 30 DEGREES.
C
2.0
1.5
1.0
0.5
0
0.25
0.50
0.75
1.00
r*/a*
Ē
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1


2.0
1.5
1.0
0.5
0.9
0.5
0.4
0.50
1.00

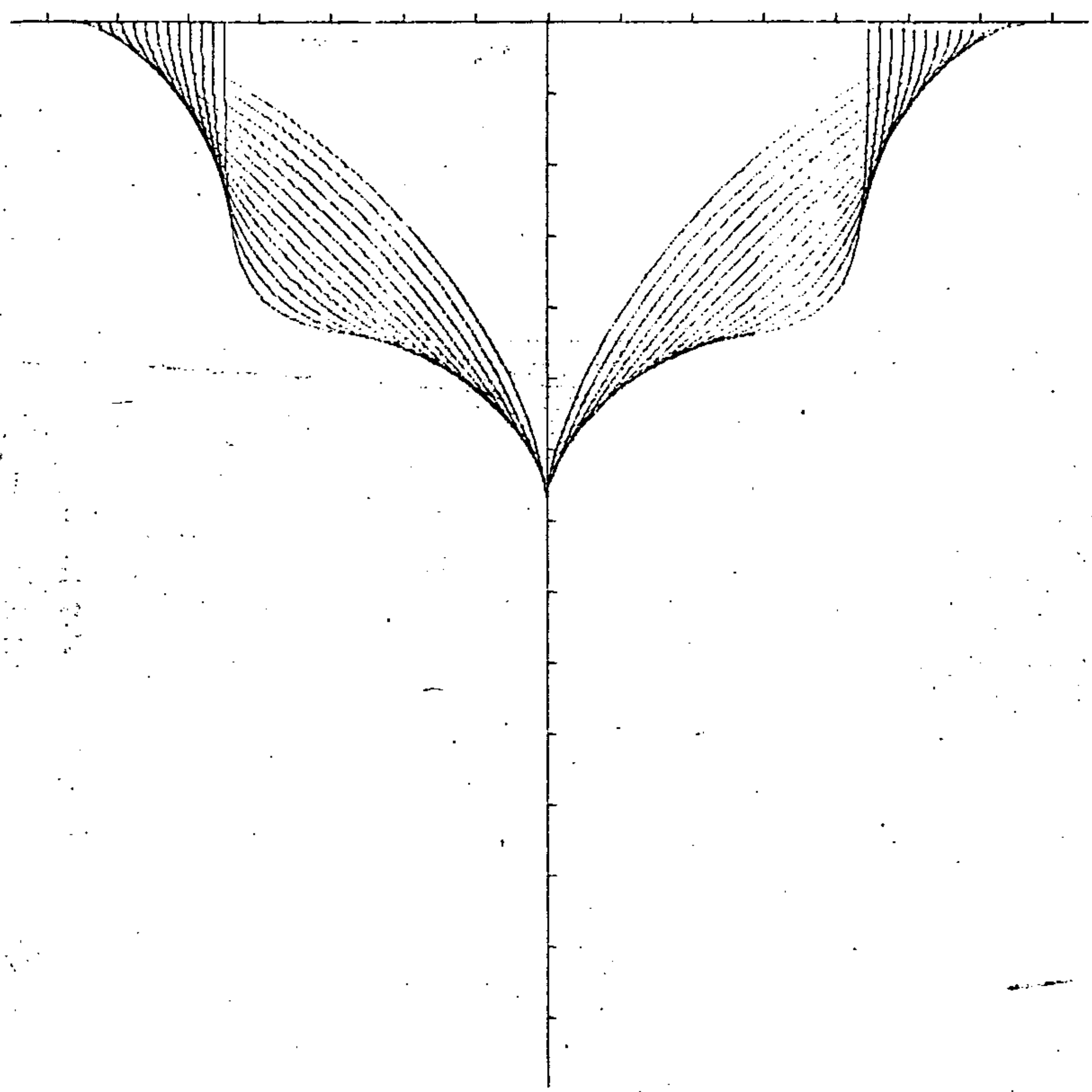


C_L
CH3:ANGLE OF HEEL= 40 DEGREES
$\bar{E}$
2.0
1.5
1.0
0.5
0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.25
0.50
0.75
1.00
$\frac{r^*}{a^*}$

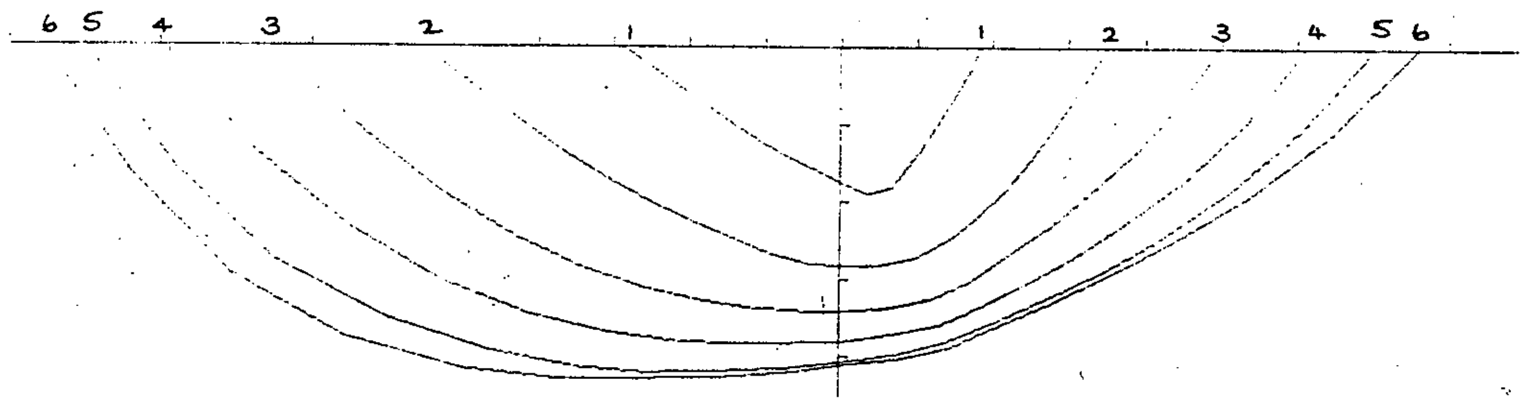
TRAILING EDGE LIFT AND TOTAL LIFT FOR VARIOUS HEEL ANGLES.



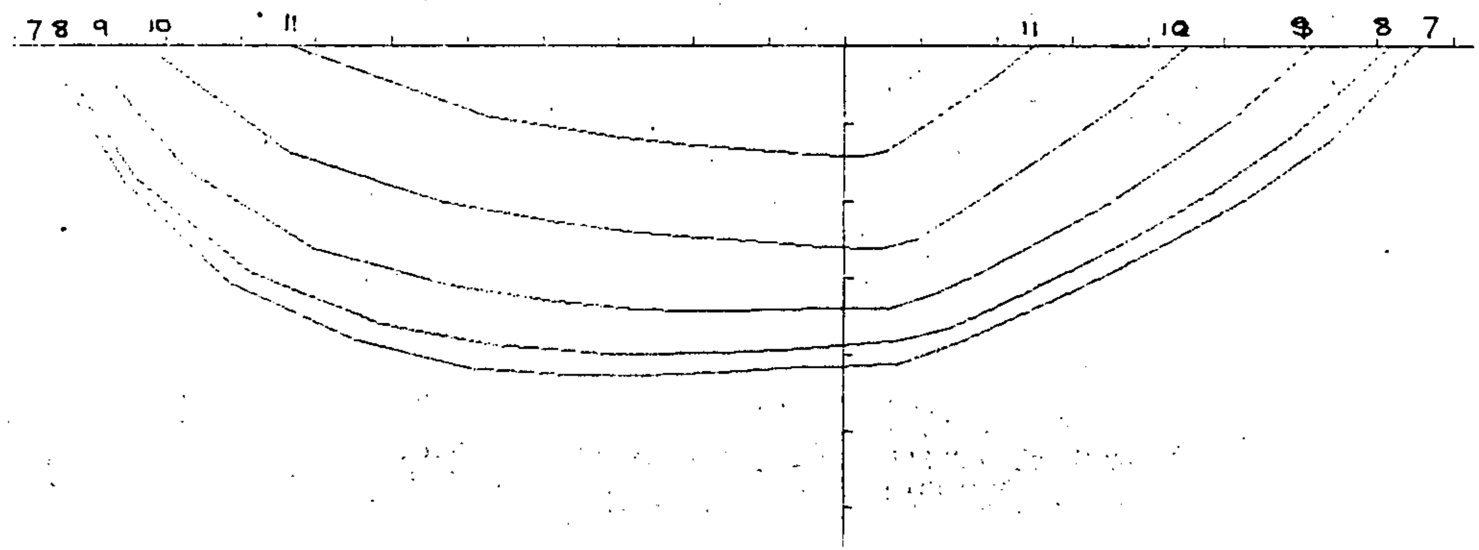
LEWIS SECTIONS : $\lambda = B/T = 2.0$ ($0.3 < A/BT < 0.95$).

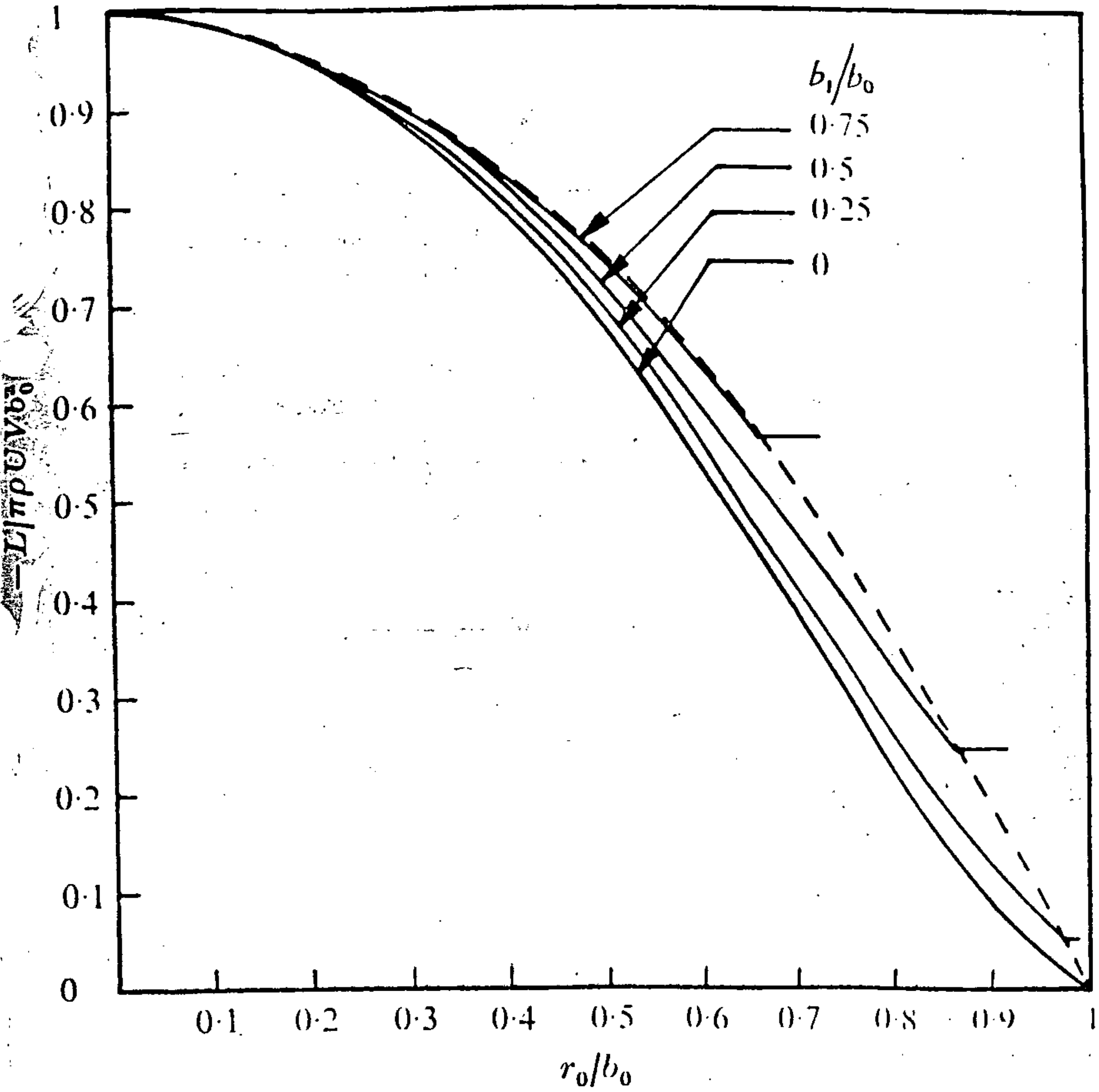


FORESECTIONS FOR WESTERLY AT HEELANGLE 15°



AFT SECTIONS FOR WESTERLY AT HEEL ANGLE 15°.





$L/\pi\rho UVb_0^2$
r_0/b_0
b_1/b_0
0·75
0·5
0·25
0
1
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1

$r_0 = r^*$
$b_0 = a^*$
$b_1 = a_R$

APPENDIX 1: SLENDERBODY THEORY

The slenderbody theory contained in this report is based on Newman and Wu's theory for fish-like forms (see Ref (13)) with some details filled in by Wellicome, J.F., of Southampton University.

This slenderbody theory is derived for a calm free surface, for steady yaw angle and is divided into two sections after the general results for boundary conditions and Bernoulli's slenderised equations are deduced:

1. CROSSFLOW LIFT FORCES
2. ASYMMETRIC YAWING MOMENTS.

Let there be a coordinate system.

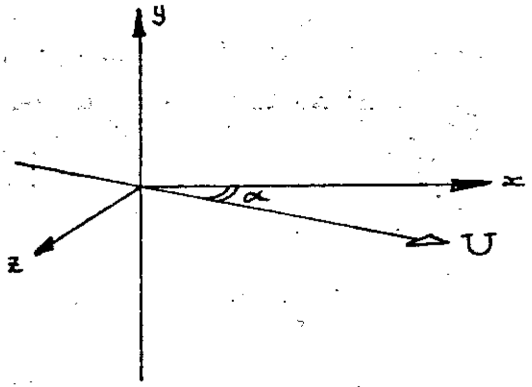


FIG. 30 SLENDERBODY COORDINATE FRAME

The body is symmetric about the x, z plane.

$$\Phi = Ux + U\alpha z + \phi_o + \phi_1 \qquad (1.1)$$

α = yaw angle

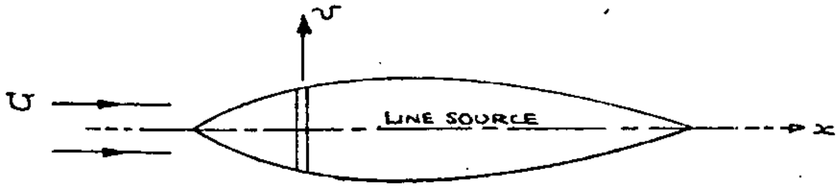
ϕ_o = stream disturbance due to body (ex fins) at $\alpha = 0^o$

ϕ_1 = cross flow term due to yaw

Order of magnitude of ϕ_o, ϕ_1 and derivatives are based on velocities around a slender body of revolution:

$$r = \delta' R(x) \qquad [\delta' \text{ is a small parameter}]$$

Represent body of revolution by a line source distribution along ℄:
If v is a small transverse flow at a strip, the body of revolution can be considered as a two-dimensional line source:



$v = \frac{m'(x)}{2\pi r} = U\frac{dr'}{dx}$ (1.2) for flow tangential to body.

$$\phi_o = \frac{m'(x)}{2\pi} \ell n\ r. \tag{1.3}$$

$$\frac{m'(x)}{2\pi} = U\delta'^2 R(x)R'(x) = U\delta'^2 RR' \tag{1.4}$$

Combining these and using derivatives with respect to x and r, and together with an equation for $_1$:

$$\phi_1 = U\alpha\left|r' + \frac{\delta'^2 R^2}{r'}\right|\cos\theta \tag{1.5}$$

a table of the orders of magnitudes of each derivative may be assembled:

	DERIVATIVES	MAGNITUDE	ORDER OF MAGNITUDE
	ϕ_o	$U\delta^2 RR'\ell n(\delta R)$	$O(U\delta^2 \ell n\ \delta)$
	ϕ_{ox}	$U\delta^2 \ell n\ \delta (RR')' + O(\delta^2)$	$O(U\delta^2 \ell n\ \delta)$
	ϕ_{or}	$U\delta R'$	$O(U\delta)$
	ϕ_{oxx}	$U\delta^2 \ell n\delta (RR')''$	$O(U\delta^2 \ell n\delta)$
	ϕ_{orr}	$-UR'/R$	$O(U)$
	ϕ_1	$2U\delta\alpha R\cos\theta$	$O(U\delta\alpha)$
	ϕ_{1x}	$2U\delta\alpha R'\cos\theta$	$O(U\delta\alpha)$
:on $\lvert r = \delta R\rvert$	ϕ_{1r}	$U\alpha(1 - (\delta^2 R^2/r^2)\cos\theta$	$O(U\alpha)$
	ϕ_{1xx}	$2U\delta\alpha(R'' + (\frac{R'}{R})^2\cos\theta$	$O(U\delta\alpha)$
	ϕ_{1rr}	$2U\alpha\frac{\delta^2 R^2}{r^3}\cos\theta$	$O(U\alpha/\delta)$

The assumption will be that for an arbitrary slenderbody ϕ_o, ϕ_1 will be of the same order in δ', α on a body: $z = \delta' Y(x, y)$ as indicated by the body of revolution case.

EQUATIONS OF CONTINUITY

$$\phi_{oxx} + \phi_{oyy} + \phi_{ozz} = 0$$

$$O(U\delta^2 \ln \delta) + O(U) + O(U) = 0$$

$$\phi_{oyy} + \phi_{ozz} = 0 \qquad (1.6)$$

$$\phi_{1xx} + \phi_{1yy} + \phi_{1zz} = 0$$

$$O(U\delta\alpha) + O(\frac{U\alpha}{\delta}) + O(\frac{U\alpha}{\delta}) = 0$$

$$\phi_{1yy} + \phi_{1zz} = 0 \qquad (1.7)$$

If $\alpha = O(\delta)$ then terms retained in both equations are $O(U)$ since $\ln\delta \approx O(1)$ terms neglected in both equations are $O(U\delta^2)$.

GENERAL BODY BOUNDARY CONDITIONS

Body shape: $z = F(x, y)$.

For a small vector $\{\delta x, \delta y, \delta z\}$ lying in the body surface

$$\delta z = F_x \delta x + F_y \delta y$$

For a small vector $\{\delta x, \delta y, \delta z\}$ lying along a streamline:

$$\delta x = (U + \phi_{ox} + \phi_{1x}).\delta t$$

$$\delta y = (\phi_{oy} + \phi_{1y}).\delta t \qquad (1.8)$$

$$\delta z = (U\alpha + \phi_{oz} + \phi_{1z}).\delta t$$

Hence for stream flow following the body shape:

$$U + \phi_{oz} + \phi_{1z} = F_x(U + \phi_{ox} + \phi_{1x}) + F_y(\phi_{oy} + \phi_{1y})$$

For a slenderbody: $F_x = O(\delta)$

$F_y = O(1)$.

$$U\alpha + \phi_{oz} + \phi_{1z} = F_x U + F_x\phi_{ox} + F_x\phi_{1x} + F_y\phi_{oy} + F_y\phi_{1y}$$

$$O(U\alpha) + O(U\delta) + O(U\alpha) = O(U\delta) + O(U\delta^3 \ln \delta) + O(U\delta^2\alpha) + O(U\delta) + O(U\alpha).$$

Retaining only terms: $O(U\delta)$ or $O(U\alpha)$ of comparable magnitude:

$$U\alpha + \phi_{oz} + \phi_{1z} = F_x U + F_y\phi_{oy} + F_y\phi_{1y} \quad \text{on body.}$$

This equation implies two equations:

$$\phi_{oz} = F_x U + F_y\phi_{oy} \qquad (\alpha = 0) \qquad (1.9a)$$

$$U\alpha + \phi_{1z} = F_y\phi_{1y} \qquad \text{(Cross flow)} \qquad (1.9b)$$

Both equations are to be satisfied on the body and fin surface.

BERNOULLI'S EQUATIONS

In steady motion ignoring the buoyancy we have:

$$P + \tfrac{1}{2}\rho q^2 = \tfrac{1}{2}\rho U^2.$$

Substitution gives:

$$P = \tfrac{1}{2}\rho\{U^2 - (U + \phi_{ox} + \phi_{1x})^2 - (\phi_{oy} + \phi_{1y})^2 - (U\alpha + \phi_{oz} + \phi_{1z})^2\}$$

$$P = -\tfrac{1}{2}\rho\{2U(\phi_{ox} + \phi_{1x}) + \phi_{ox}^2 + 2\phi_{ox}\phi_{1x} + \phi_{1x}^2 + \phi_{oy}^2 + 2\phi_{oy}\phi_{1y}$$

$$+ \phi_{1y}^2 + U^2\alpha^2 + 2U\alpha(\phi_{oz} + \phi_{1z}) + \phi_{oz}^2 + 2\phi_{oz}\phi_{1z} + \phi_{1z}^2\}$$

The largest terms are $O(\delta^2, \delta\alpha, \alpha^2)$ as seen below:

$$P = -\tfrac{1}{2}\rho\{2U(O(U\delta^2\ln \delta) + O(U^2\delta\alpha) + O(U^2\delta^4(\ln \delta)^2) + O(U^2\alpha\delta^3\ln\delta)$$

$$+ O(U^2\delta^2\alpha^2) + O(U^2\delta^2) + O(U^2\delta\alpha) + O(U^2\alpha^2) + O(U^2\alpha^2) + O(U^2\delta\alpha)$$

$$+ O(U^2\alpha^2) + O(U^2\delta^2) + O(U^2\delta\alpha) + O(U^2\alpha^2)\}.$$

Neglecting other terms gives:

$$\Rightarrow \quad P = -\tfrac{1}{2}\rho\{2U\phi_{ox} + \phi_{oy}^2 + \phi_{oz}^2\} - \tfrac{1}{2}\rho\{2U\phi_{1x} + 2\phi_{oy}\phi_{1y} + \phi_{1y}^2 + U^2\alpha^2 + 2U\alpha\phi_{oz} + 2U\alpha\phi_{1z} + 2\phi_{oz}\phi_{1z} + \phi_{1z}^2\}.$$

Let $P = P_o + P_1$ $\quad$ P_o = Dynamic pressure at $\alpha = 0$

P_1 = Dynamic pressure due to yaw α.

$$P_o = -\tfrac{1}{2}\rho\{2U\phi_{ox} + \phi_{oy}^2 + \phi_{oz}^2\} \tag{1.10}$$

$$P_1 = -\tfrac{1}{2}\rho\{2U\phi_{1x} + 2\phi_{oy}\phi_{1y} + 2\phi_{oz}\phi_{1z} + \phi_{1y}^2 + (U\alpha + \phi_{1z})^2 + 2U\alpha\phi_{oz}\}$$

or $$P_1 = -\rho\{U\phi_{1x} + \phi_{oy}\phi_{1y} + \phi_{oz}(U\alpha + \phi_{1z}) + \tfrac{1}{2}\phi_{1y}^2 + \tfrac{1}{2}(U\alpha + \phi_{1z})^2\}.$$

Note that the last terms are $O(U^2\alpha^2)$ compared to $O(U^2\delta\alpha)$ for the earlier terms in P_1. If $\alpha \ll \delta$ so that the lateral displacement of each station from track is small compared to body beam then the equation above may be further reduced to the form:

$$P_1 = -\rho\{U\phi_{1x} + \phi_{oy}\phi_{1y} + \phi_{oz}(U\alpha + \phi_{1z})\}. \tag{1.11}$$

Clearly P_o does not contribute to cross forces and moments on the body in the upright condition since there is then symmetry of the sections port and starboard. P_o is only important when the section becomes asymmetric.

The assumption that α is small compared to δ does not affect the derivation of equations (1.6, 1.7, 1.9).

CONVECTION OF TRAILING VORTICITY

The assumption ($\alpha. << \delta$) implies that the trailing vortex sheet lies close to the centreplane z = 0 on which $\phi_{oz} = 0$. Thus on the sheet we have:

$$P_1 = -\rho\{U\phi_{1x} + \phi_{oy}\phi_{1y}\}$$

The values of ϕ_1 at different but adjacent points on opposite sides of the sheet will be different but pressure must be continuous since any pressure difference will result in an infinite transverse acceleration on the sheet (which has zero mass):

Writing the difference in ϕ_1 across the sheet as ϕ_1':

$$P' = 0 = -\rho\{U\phi_x' + \phi_{oy}\phi_y'\}$$

or $$U\phi_x' + \phi_{oy}\phi_y' = 0 \quad \text{on the sheet.}$$

Now: $$\delta\phi' = \phi_x'\delta x + \phi_y'\delta y = 0$$

$$\delta\phi' = 0 \qquad \frac{\delta y}{\delta x} = -\frac{\phi_x'}{\phi_y'} = \frac{\phi_{oy}}{U} \tag{1.12}$$

But this implies that ϕ' = constant along streamlines in the flow produced at zero yaw, and that the trailing vortex lines lie along these lines.

1. CROSSFLOW FORCES

The crossflow per unit length of body, L(x) is given by:

$$L(x) = \oint P_1 dy = -\rho\oint\{U\phi_{1x} + \phi_{oy}\phi_{1y} + \phi_{oz}(U\alpha + \phi_{1z})\}dy$$

Using (1.9b):

$$L(x) = -\rho\oint\{U\phi_{1x} + \phi_{oy}\phi_{1y} + \phi_{oz\ 1y}F_y\}dy \tag{1.13}$$

Define $A(x) = \oint \phi_1 \, dy$.

Consider 2 contours C_1 and C_2 spaced δx apart.

Let $\phi_1 = \phi_1$ at A

$\phi_1 = \phi_1'$ at B for same y value as A.

In moving from A to B there is thus a lateral movement on the surface $z = F(x, y)$ of distance:

$$\delta z = F_x \delta x \qquad (y = \text{constant})$$

Then:
$$\phi_1' = \phi_1 + \{\phi_{1x} + \phi_{1z} F_x\} dx$$

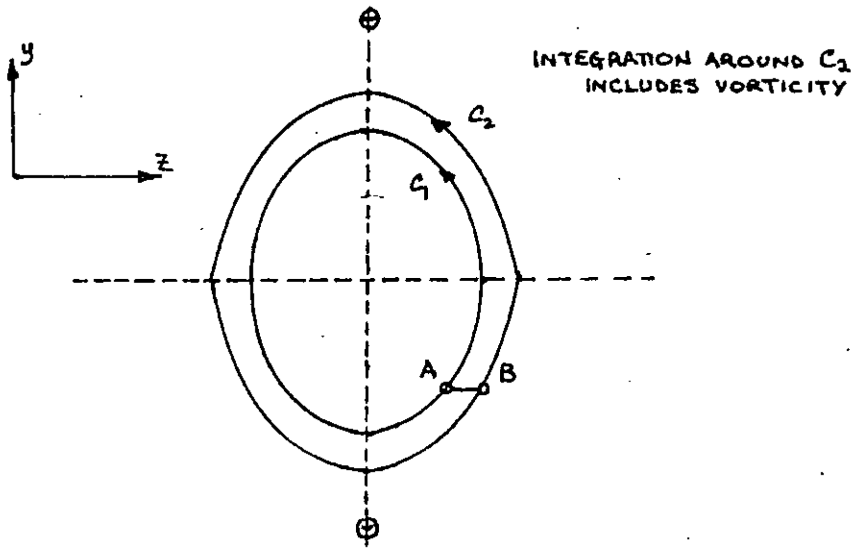


FIG. 32 NEIGHBOURING SECTIONS

At C_2:
$$A(x) + A'(x)\delta x = \oint \phi_1' \, dy$$

giving:

$$A'(x) = \oint \{\phi_{1x} + \phi_{1z} F_x\} dy.$$

Using (1.9a) $\quad UF_x = \phi_{oz} - \phi_{oy} F_y$ on body.

$$UA'(x) = \oint \{U\phi_{1x} + \phi_{oz}\phi_{1z} - \phi_{1z}\phi_{oy}F_y\} dy \qquad (1.14)$$

Adding (1.13) to (1.14):

$$UA'(x) + \frac{L(x)}{\rho} = \oint \{\phi_{oz}\phi_{1z} - \phi_{oy}\phi_{1y} - (\phi_{oz}\phi_{1y} + \phi_{1z}\phi_{oy})F_y\} dy \qquad (1.15)$$

By applying Gauss' theorem it is possible to show that the RHS of (1.15) is zero.

By deriving the Gauss theorem in two dimensions:

$$\oint_{c_1} (H dy + G dz) = - \iint_{s_2} \left(\frac{\partial H}{\partial z} - \frac{\partial G}{\partial y}\right) dy dz$$

where curve c_2 encloses c_1 and s_1 is plane inside c_1, s_2 is plane exterior of c_1, and nocing that at fixed x: $\delta z = F_y \delta y$ on $z = F(x, y)$, this gives:

$$\oint (H + GF_y) dy = - \iint_{s_2} \left(\frac{\partial H}{\partial z} - \frac{\partial G}{\partial y}\right) dy dz \tag{1.16}$$

The velocity components in (1.15) will, in fact, vanish sufficiently rapidly to apply (1.16) so that (1.15) becomes

$$UA'(x) + \frac{L(x)}{\rho} = - \iint_{s_2} \left\{ \frac{\partial}{\partial z}(\phi_{oz}\phi_{1z} - \phi_{oy}\phi_{1y}) + \frac{\partial}{\partial y}(\phi_{oz}\phi_{1y} + \phi_{1z}\phi_{oy}) \right\} dy dz$$

This is expanded as:

$$UA'(x) + \frac{L(x)}{\rho} = - \iint_{s_2} \{ \phi_{oz}\phi_{1zz} + \phi_{ozz}\phi_{1z} - \phi_{oy}\phi_{1yz} - \phi_{oyz}\phi_{1y}$$

$$+ \phi_{oz}\phi_{1yy} + \phi_{ozy}\phi_{1y} + \phi_{1yz}\phi_{oy} + \phi_{1z}\phi_{oyy} \} dy dz$$

$$= - \iint_{s_2} \{ \phi_{oz}(\phi_{1zz} + \phi_{1yy}) + \phi_{1z}(\phi_{oyy} + \phi_{ozz}) \} dy dz$$

$= 0$ since slenderised Laplace $\nabla^2 \phi = 0$.

$$UA'(x) + \frac{L(x)}{\rho} = 0$$

$$UA'(x) = U \frac{d}{dx} \oint \phi_1 dy$$

$$L(x) = -\rho U \frac{d}{dx} \oint \phi_1 \, dy. \tag{1.17}$$

Hence the equation giving the cross force at a particular section is derived. Now the asymmetric sections problem will be considered.

2. ASYMMETRIC YAWING MOMENTS

CLOSED CONTOUR CROSS FORCE INTEGRATION

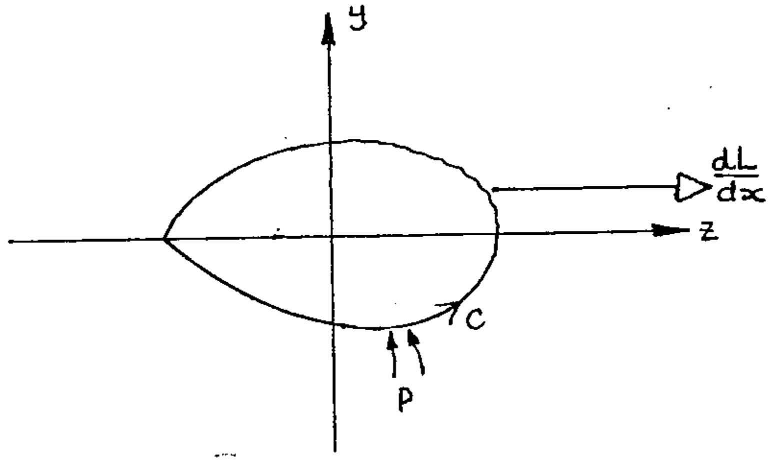


FIG. 33 CONTOUR

Local cross force on the body is given by:

$$\frac{dL}{dx} = + \oint P_o dy$$

Substituting for P: $$\frac{dL}{dx} = -\rho\oint\{ U\phi_{ox} + \tfrac{1}{2}\phi_{oy}^{2} + \tfrac{1}{2}\phi_{oz}^{2}\}dy$$

Define $$F(x) = \oint\phi_o \, dy \qquad (1.18)$$

$$\delta\phi = \phi_x\partial x + \phi_z\partial z \quad \text{at fixed } y.$$

$$\frac{\delta\phi}{\partial x} = \phi_x + \phi_z z_x^{*}$$

Four two contours extending over the similar range of y:

$$F'(x) = \oint\{\phi_x + \phi_z z_x\}dy.$$

---

* : Assume subscript o.

But on the body $-UZ_x = \phi_y Z_y - \phi_z$.

$$\rho UF'(x) = \rho\oint\{U\phi_x + \phi_z\phi_y Z_y - \phi_z^2\}dz$$

$$\frac{dL}{dx} + \rho UF'(x) = \rho\oint\{\phi_y\phi_z Z_y - \tfrac{1}{2}(\phi_z^2 - \phi_y^2)\}dy$$

$$= \rho\oint\{\phi_y\phi_z dy - \tfrac{1}{2}(\phi_z^2 - \phi_y^2)dy\}.$$

Again Gauss' theorem can show RHS = 0:

$$\frac{dL}{dx} = -\rho UF'(x).$$

YAWING MOMENT

$$M = \int_L x\frac{dL}{dx}.dx = \rho U\int_L xF'(x)dx$$

$$M = -\rho U\{xF(x)|_L - \int_L F(x)dx\}.$$

At ends of the body there is no contour about which to integrate, i.e., $F(x) = 0$.

This yields the result:

$$M = \rho U\int_L F(x)dx \qquad (1.19)$$

$$F(x) = \oint\phi_o dy. \qquad (1.18)$$

EVALUATION OF THE BOUNDARY CONDITION

The physical conditions on the boundary of the contour must be related by appropriate mathematical statements. A kinematic boundary condition is appropriate on any boundary surface with a specified geometry and position. In the case of a body moving with a prescribed velocity U through the fluid the velocity of the surface is non zero and the physically relevant boundary condition is that the normal component (V.n) of the fluid velocity must be equal to the normal velocity (U.n) of the boundary surface itself. In other words, no fluid can flow through the boundary surface:

$$\frac{\partial\phi}{\partial n} = U.n.$$

In particular one can derive a slender boundary condition in two dimensions as follows:

By relating orders of magnitude taking into account slenderness the following equations result :

$$\phi_{yy} + \phi_{zz} = 0 \rightarrow \quad \text{Solution as } \phi = \phi(y, z) + G(x)$$

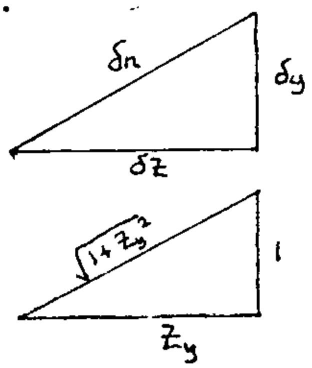
$$\phi_z = -UZ_x + \phi_y Z_y \quad \text{on } z = Z(x, y).$$

At constant x: $\delta z - Z_y \delta y = 0$

$$\{\delta z, \delta y\}. \{1, -Z_y\} = 0$$

$\{1, -Z_y\}$ is thus normal to the contour.

$$\delta z = \frac{\delta n}{\sqrt{1 + Z_y^2}}$$



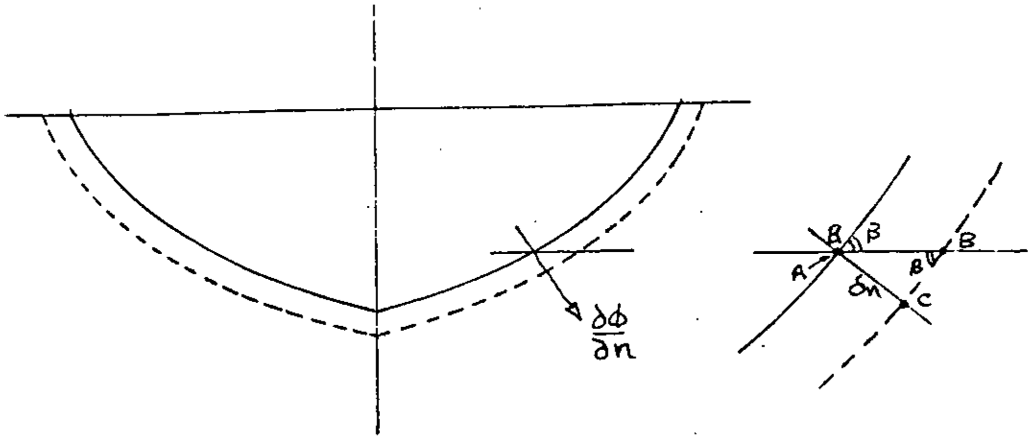
$$\delta y = \frac{-\delta n Z_y}{\sqrt{1 + Z_y^2}}$$

$$\delta\phi = \phi_y \delta y + \phi_z \delta z$$

$$= (\phi_z - \phi_y Z_y) \frac{\delta n}{\sqrt{1 + Z_y^2}}$$

$$\frac{\delta\phi}{\partial n} = (\phi_z - \phi_y Z_y) \cdot \frac{1}{\sqrt{1 + Z_y^2}}$$

$$\phi_z = -UZ_x + \phi_y Z_y \quad : \quad \frac{\partial\phi}{\partial n} = -\frac{UZ_x}{\sqrt{1 + Z_y^2}}$$



$$AC = \frac{z_x \delta x}{\sqrt{1 + z_y^2}} \; ; \qquad AC = N_x \delta x$$

$$\frac{\partial\phi}{\partial n} = +UN_x \qquad (1.20)$$

Thus the derivation of relevant equations for upright sections that are symmetric, and for the heeled asymmetric sections, have been deduced. Now the actual computer modelling system to determine the asymmetric yaw moments on a heeled hull will be reviewed in Appendix 2.

APPENDIX 2:

The coordinate system is fixed with respect to the yacht and the three motions of surge, sway and yaw are considered such that the system is restricted to the free surface plane. The body is assumed rigid and the image system is used, Ref. (14).

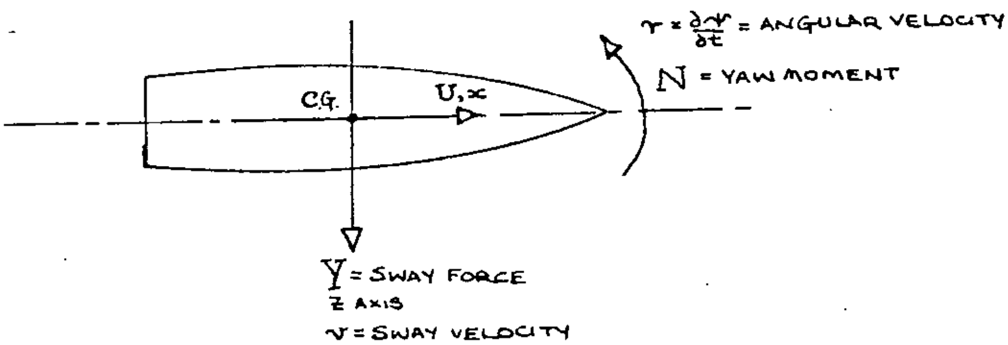


FIG. 16 DERIVATIVES COORDINATE SYSTEM

$$\text{Let } M' = \int_{\ell} m(x)\, dx \tag{9.1}$$

$$M'_x = - \int_{\ell} m(x) x\, dx \tag{9.2}$$

$$M_{x^2} = \int_{\ell} m(x) x^2 dx \tag{9.3}$$

The total lateral velocity is given as:

$$V_T(x, t) = v(t) - xr(t).$$

Local force acting on the hull is:

$$Y(x) = -(\frac{\partial}{\partial t} - U \frac{\partial}{\partial x}) \left[V_T m(x)\right]$$

$$= -(\frac{\partial}{\partial t} - U \frac{\partial}{\partial x}) \left[(v - xr) m(x)\right].$$

$$Y(x) = -(\dot{v} - x\dot{r}) m(x) + Uv \frac{\partial}{\partial x} m(x) - Ur \frac{\partial}{\partial x} \left[x m(x)\right].$$

The total side force is thus given as:

$$Y = -\dot{v}M' - rM'_x - Uvm(x_T) + Urm(x_T).x_T \tag{9.4}$$

where x_T = distance from origin to the effective trailing edge.

Multiplying by -x gives:

$$N = -\dot{v}M_x{}' - \dot{r}M'_{x^2} + Uv\left[M' + x_T m_T\right] + Ur\left[M'_x - x_T{}^2 m_T\right] \tag{9.5}$$

On differentiating Y and N with respect to r, ṙ, v and $\dot{v}$ the following table is produced.

TABLE OF STABILITY DERIVATIVES (9.6)

	DERIVATIVES		NORMALISED	GRAPHICAL USE
Y_v	$-Um_T$	$Y_v{}'$	$Y_v/\frac{1}{2}\rho UL^2$	$-2\pi(a*/\ell)^2 C_{LT}$
Y_r	$-Ux_t m_T$	$Y_r{}'$	$Y_r/\frac{1}{2}\rho\ell^3 U$	$-2\ x_T(a*^2/\ell^3).C_{LT}$
$Y_{\dot{v}}$	$-M'$	$Y_{\dot{v}}{}'$	$Y_{\dot{v}}/\frac{1}{2}\rho\ell^3$	$-M'/\frac{1}{2}\rho\ell^3$
$Y_{\dot{r}}$	$+M_x{}'$	$Y_{\dot{r}}{}'$	$Y_{\dot{r}}/\frac{1}{2}\rho\ell^4$	$M'_x/\frac{1}{2}\rho\ell^4$
N_v	$-U\left[M' + x_T m_T\right]$	$N_v{}'$	$N_v/\frac{1}{2}\rho\ell^3 U$	$-2M'/\rho\ell^3 + Y_r{}'$
N_r	$U\left[M'_x - x_T{}^2 m_T\right]$	$N_r{}'$	$N_r/\frac{1}{2}\rho\ell^4 U$	$2M'_x/\rho\ell^4 + (x_T/\ell)Y_r{}'$
$N_{\dot{r}}$	M'_x	$N_{\dot{v}}{}'$	$N_{\dot{v}}/\frac{1}{2}\rho\ell^4$	$M'_x/\frac{1}{2}\rho\ell^4$
$N_{\dot{r}}$	$-M'_{x^2}$	$N_{\dot{r}}{}'$	$N_{\dot{r}}/\frac{1}{2}\rho\ell^5$	$-M'_{x^2}/\frac{1}{2}\rho\ell^5$
m'	$2\nabla/\ell^3$			
v'	v/U			
$\dot{v}'$	$\dot{v}\ell/U^2$			
r'	$r\ell/U$			
$\dot{r}'$	$\dot{r}\ell^2/U^2$			
$I_z{}'$	$I_z/\rho\ell^5$			

$$C_{LT} = C_{L\pi}\big|_{x=x_T}$$

Some of the above expressions are deduced from the relation between the added mass and $C_{L\pi}$.

From Appendix 6 (3):

$$L = \rho U^2 \alpha \left[\frac{m}{\rho} + \text{AREA}\right]$$

$$C_{L\pi} = \frac{1}{\pi a^{*2}}\left[\frac{m}{\rho} + \text{AREA}\right]$$

$$m = \rho\{C_{L\pi}\pi a^{*2} - \text{AREA}\}.$$

Since $C_{L\pi}$ aft of the trailing edge is defined with respect to a^*

$$m_A = m_{AFT} = \pi\rho\{C_{L\pi}a^{*2} - r^2\} \quad (9.7)$$

$$m_F = \pi\rho\{C_{L\pi}a^2 - r^2\} \quad (9.8)$$

where a becomes the keel depth at sections forward of T.E.

Thus $m_T = m(x_T) = \pi\rho\{C_{LT}a^{*2}\}$. (9.9)

For example,

$$M' = \pi\rho\{\int_{-x_b}^{x_{TE}} (C_{L\pi}a^2 - r^2)dx + \int_{x_{TE}}^{x_T} (C_{L\pi}a^{*2} - r^2)dx\}$$

M', M'_x and M'_{x^2} can be found simply by numerical integration perhaps using a trapezium integration or Simpson's integration.

Thus almost all the stability derivatives considered in the water surface plane are found on using the graphs. It remains only to determine the centre of gravity, m' and I_z'. The origin of movement and the origin of the frame of reference defined earlier is the centre of gravity.

PROCEDURE

1. Three characteristics M', M'_x; and M'_{x^2} are calculated:

$$M' = \int_0^{\ell} m(x)dx = \text{Total Lateral Added Mass}$$

$$M'_x = \int_0^{\ell} m(x).x.dx = \text{Cross Coupled Added Mass between Sway and Yaw (for symmetry about } x = 0\text{, it is zero)}$$

$$M'_{x^2} = \int_o^{\ell} m(\ddot{x}).x^2.dx = \text{Added Moment of Inertia for yaw acceleration.}$$

These may be found by Simpson's Integration.

2. Y_v', Y_r', $Y_{\dot{v}}'$, $Y_{\dot{r}}'$, N_v' $N_{\dot{v}}'$, N_r', $N_{\dot{r}}'$ can all now be found simply from table 9.6.
(Here $C_{LT} = C_{L\pi}$ at $x = x_T$, where x_T = distance from the coordinate system origin to the effective trailing edge).

3. m' is $2\nabla/\ell^3$: ∇ = volume of displacement
ρ = yacht's length

4. $I_z' = I_z/\frac{1}{2}\rho\ell^5$: This is the moment of inertia about a vertical axis through the centre of gravity.

5. Solve the equations (9.14).

6. The resulting values of C, σ_1 and σ_2 are important in the stability consideration.

Now follows the derivation of stability conditions and roots. This derivation is included in the main body of the report to keep the stability section compact and in one location only.

DERIVATION OF STABILITY CONDITION AND ROOTS

The three equations for surge, sway and yaw may be defined (Ref. ( )) as:

$$\dot{u}(X_{\dot{u}} - \rho\nabla) + uX_u = 0 \quad \text{(SURGE)} \qquad (9.12a)$$

$$\dot{v}(Y_{\dot{v}} - \rho\nabla) + r(Y_r - \rho\nabla u_o) + vY_v + \dot{r}Y_{\dot{r}} = 0 \quad \text{(SWAY)} \qquad (9.12b)$$

$$\dot{v}N_{\dot{v}} + vN_v + rN_r + \dot{r}(N_{\dot{r}} - I_z') = 0 \quad \text{(YAW)} \qquad (9.12c)$$

u is forward or surge velocity
X = forward force

Write in normalised form: (but presume primed values ')

$$v\left[D(Y_{\dot{v}} - m) + Y_v\right] + r\left[DY_{\dot{r}} + (Y_r - m)\right] = 0$$
$$v\left[DN_{\dot{v}} + N_v\right] + r\left[N_r + D(N_{\dot{r}} - I_z)\right] = 0 \qquad (9.13)$$

$$\frac{\{D(Y_{\dot{v}} - m) + Y_v\}}{\{DY_{\dot{r}} + (Y_r - m)\}} - \frac{\{DN_{\dot{v}} + N_v\}}{N_r + D(N_{\dot{r}} - I_z)} = 0$$

This becomes a quadratic equation:

$$D^2\left[(Y_{\dot{v}} - m)(N_{\dot{r}} - I_z) - N_{\dot{v}}Y_{\dot{r}}\right] + D\left[(Y_{\dot{v}} - m) + Y_v(N_{\dot{r}} - I_z) - N_{\dot{v}}(Y_r - m) + N_rY_{\dot{r}}\right]$$

$$+ \left[Y_vN_r - N_v(Y_r - m)\right] = 0$$

$$AD^2 + BD + C = 0$$

$$A = (Y_{\dot{v}} - m)(N_{\dot{r}} - I_z) - N_{\dot{v}}Y_{\dot{v}} \qquad (9.14a)$$

$$B = (Y_{\dot{v}} - m)N_r + Y_v(N_{\dot{r}} - I_z) - N_{\dot{v}}(Y_r - m) + N_vY_{\dot{r}} \qquad (9.14b)$$

$$C = Y_vN_r - N_v(Y_r - m) \qquad (9.14c)$$

By considering the relative magnitude of all these terms it may be shown that

$\frac{B}{A} > 0$; $\frac{C}{A} > 0$ for STABILITY (Roots MUST be BOTH NEGATIVE)

and this implies

$$C > 0$$

or

$$\frac{Y_v}{N_v} > \frac{(Y_r - m)}{N_r} \qquad (9.15)$$

A, B and C are the coefficients of the equation:

$$A\ddot{r} + B\dot{r} + Cr = 0 \qquad (9.16)$$

and the roots of the equation give a solution of the form:

$$r'(t') = \text{Re}\ \{P_1'e^{\sigma_1't'} + P_2'e^{\sigma_2't'}\} \qquad (9.17)$$

where $$\sigma_i' = \frac{-B + (-1)^{i-1}\sqrt{B^2 - 4AC}}{2A} \qquad (9.18)$$

$i = 1, 2$

where $$\sigma_i' = (\frac{\sigma_i \ell}{U}).$$

STABILITY CONDITIONS

1. Real part of σ_1, σ_2 must be positive.

2. $C > 0$.

EVALUATION OF C IN TERMS OF ADDED MASS

$$C = (-Um_T)(U(M_x' - x_T^2 m_T)) + \left[U(M' + x_T m_T)(-Ux_T m_T - m')\right]$$

$$= U^2\{-M_x' m_T + x_T^2 m_T^2 - M' x_T . m_T - x_T^2 m_T^2 - m'(M' + x_T m_T)\}$$

$$= -U^2\{M_x' m_T + M' x_T m_T + m'(M' + x_T m_T)\}$$

$$= -U^2\{m'M' + m_T x_T(M' + m') + M_x' m_T\} > 0 \quad \text{for stability}$$

$$mM' + M_T x_T(M' + m') + M_x' m_T < 0$$

$$m_T |x_T| > \left\{\frac{m'M' + M_x' m_T}{(M' + m')}\right\} \qquad (9.19)$$

This is the stability condition in terms of added mass calculations.