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COMPUTING SUBJECTIVE MOTION

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With computer programs becoming more available to calculate the transfer functions of a ships' motion to regular waves, questions are naturally asked as to the relative worth of one vessel vis a vis another. Various parameters are used to assess this relative worth, for example root mean square heave in a specified seaway, relative bow motion, bow wetness and more recently subjective motion {1}.

This parameter gives some measure for the "comfortableness" of the vessel, since it is a number that involves the r.m.s. absolute acceleration as well as the frequency of acceleration. This note is thus written in order to shed some light on the calculation of the subjective motion, when the spectrum is truncated.

In evaluating ship responses, various moments of the ships transfer function are used. The n^{th} moment being given by :

$$m_n = \int_0^\infty \omega^n \ \phi(\omega) \ T^2(\omega) d\omega \qquad (1)$$

where ω is the radian wave frequency

(ω) is the sea spectrum function

$T(\omega)$ is the ships transfer function

for more details see {2}.

The transfer function can be any one of the rotational or translational modes of the ship, or combinations of any of these modes, and their derivatives. In the case of subjective motion, $T(\omega)$ is usually the absolute heave response at a particular station along the ships length, including the coupling from pitch.

The problem with a theoretical analysis of such an expression as equation (1) is that $T(\omega)$, the transfer function is not known à priori, until the ship details are given. Thus to overcome this problem, it is proposed that a unit transfer function be used, i.e. $T(\omega) = 1.0$ for all ω. Thus the ship heaves in phase with the regular waves for all frequencies. It is in effect the motion of a cork.

The subjective motion parameter is defined in {3} by

$$S.M. = m_4^{0.715} \left\{ 3.087 + 1.392 \left[\log_e \left(\frac{1}{2\pi} \sqrt{\frac{m_6}{m_4}} \right) \right]^2 \right\}$$

It is thus necessary to calculate the fourth and sixth moments of the spectrum. The problem is often posed with the Pierson-Moskowitz spectrum. namely

$$\phi(\omega) = \frac{A}{\omega^5} e^{-B/\omega^4}$$

To solve the subjective motion problem, it is necessary to evaluate

$$m_n = \int_0^\infty \omega^n \phi(\omega) d\omega \qquad n = 4, 6 \qquad (2)$$

In {4} a similar problem was posed, here the authors investigated the effect of truncation of spectra on the spectral width parameter, and the correlation of this cut-off point with experimental data.

In that paper m_n was evaluated using n = 0, 2, 4. The method being to evaluate equation (2) with the upper limit replaced by a multiple of the frequency corresponding to the maximum value of the spectral energy. Thus equation (2) is replaced by

$$m_n = \int_0^{p\omega_m} \omega^n \phi(\omega) d\omega \qquad (3)$$

where $\omega_m = (0.8B)^{\frac{1}{4}}$ is the frequency corresponding to that of maximum of a Pierson-Moskowitz spectrum.

With this definition

$$m_o = \frac{A}{4B} \exp\left\{ - \frac{5}{4p^4} \right\}$$

$$m_2 = \sqrt{\frac{\pi}{B}} \frac{A}{4} \operatorname{erfc}\left\{ \sqrt{\frac{5}{4p^4}} \right\}$$

where erfc(x) is the complementary error function

$$\mathrm{erfc}(x) = \sqrt{\frac{2}{\pi}} \int_x^\infty e^{-x^2}\, dx \qquad \mathrm{erfc}(0.0) = 1.0$$

$$\mathrm{erfc}(\infty) = 0.0$$

$$m_4 = \frac{A}{4} E_1 \left(\frac{5}{4p^4}\right)$$

E_1 is the exponential integral {5}.

Thus as $p \to \infty$ $\quad m_o \to \frac{A}{4B} \quad m_2 \to \frac{A}{4}\sqrt{\frac{\pi}{B}}$ and $m_4 \to \infty$

In the analysis of the effect of truncation points on the spectral width, they concluded that for all but the severest sea states, a value of $p = 6$ was appropriate.

Using this approach to solve for m_6

$$m_6 = \int_o^{p\omega_m} \omega^6 \frac{A}{\omega^5} e^{-B/\omega^4}\, d\omega$$

let $x^2 = B/\omega^4$

$$\therefore \quad m_6 = \int_\Gamma^\infty \frac{A\sqrt{B}}{2} \frac{e^{-x^2}}{x^2}\, dx$$

$$\therefore \quad m_6 = \frac{A\sqrt{B}}{2} \left\{ \frac{e^{-\Gamma^2}}{\Gamma} - \sqrt{\pi}\ \mathrm{erfc}(\Gamma) \right\}$$

where $\Gamma^2 = 5/4p^4$

Thus as $p \to \infty$ $m_6 \to \infty$. Thus as with m_4, m_6 is an unbounded integral.

In the evaluation of the subjective motion parameter the ratio m_6/m_4 is used.

$$\frac{m_6}{m_4} \to \frac{A\sqrt{B}}{2} \frac{e^{-\Gamma^2}}{\Gamma} \frac{4}{AE_1(\Gamma^2)} = \frac{2\sqrt{B}\, e^{-\Gamma^2}}{\Gamma\, E_1(\Gamma^2)}$$

$\Gamma \to 0 \quad p \to \infty, \quad E_1(\Gamma^2) \to -0.8004 + 4\log p.$

$$\therefore \quad \frac{m_6}{m_4} \to \frac{2\sqrt{B}}{5} \frac{4p^4}{\{-0.8004 + 4\log p\}} \to \infty \quad \text{as } p \to \infty$$

Thus the <u>subjective motion parameter is unbounded.</u>

Thus the subjective motion parameter depends upon the cut off frequency.

To assess this effect of truncation of spectrum two typical spectra have been investigated, numerically.

A. ITTC Spectrum

For this spectrum, in metric units,
$A = 0.7795$, $B = 3.11 / h^2$ where h is the significant waveheight

The calculations were performed for significant waveheights 1 m to 9 m in 2 metre intervals. The results of these calculations are plotted in figure 1.

It can be seen that for a value of $p = 6$, the peak of all the curves has been passed, for all except a value of 1 metre.

To give some idea of the frequencies involved, the modal frequency ω_m is

$$\omega_m = (0.8 \times 3.11 / h^2)^4$$

So for

$h = 10$ m $\quad \omega_m = 0.397$ rad / sec
$\therefore$ truncation point is 2.38 rads / sec.
or a wavelength of 11 metres.

$h = 2$ m $\quad \omega_m = 0.888$ rad / sec.
$\therefore$ truncation point is 5.33 rad / sec.
or a wavelength of 2.17 metres.

B. ISSC Two Parameter Spectrum

$$A = 173\, h^2 / T_1^{\,4} \qquad B = 691 / T_1^{\,4}$$

where $T_1 = 1.086\, \bar{T}$, $\bar{T}$ is the average wave period.

Figures 2, 3, 4 show the calculations for subjective motion for T_1 = 6, 8, 10 seconds, respectively.

Again it can be seen that a value of p = 6 gives a good indication of the cut off frequency.

Conclusions

Although the above calculations in no way tries to emulate the subjective motions of a real ship, they do attempt to show the effect of cut off frequency. They are probably an overestimate of the real calculations.

For the case considered, that of a cork motion, a cut off frequency of six times the frequency corresponding to the peak of the spectrum is judged to be acceptable, because beyond this point the waves are very short and contain little energy.

For a ship the truncation point is probably a lot lower.

References

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(3) Brown, D.K. and Marshall P.D. 'Small Warships in the Royal Navy and the Fishery Protection Task'. Paper 4 Symposium on Small Fast Warships and Security Vessels, March 1978, London.

(4) Price W.G. and Bishop R.E.D. 'On Truncation of Spectra' Department of Mechanical Engineering, U.C.L.

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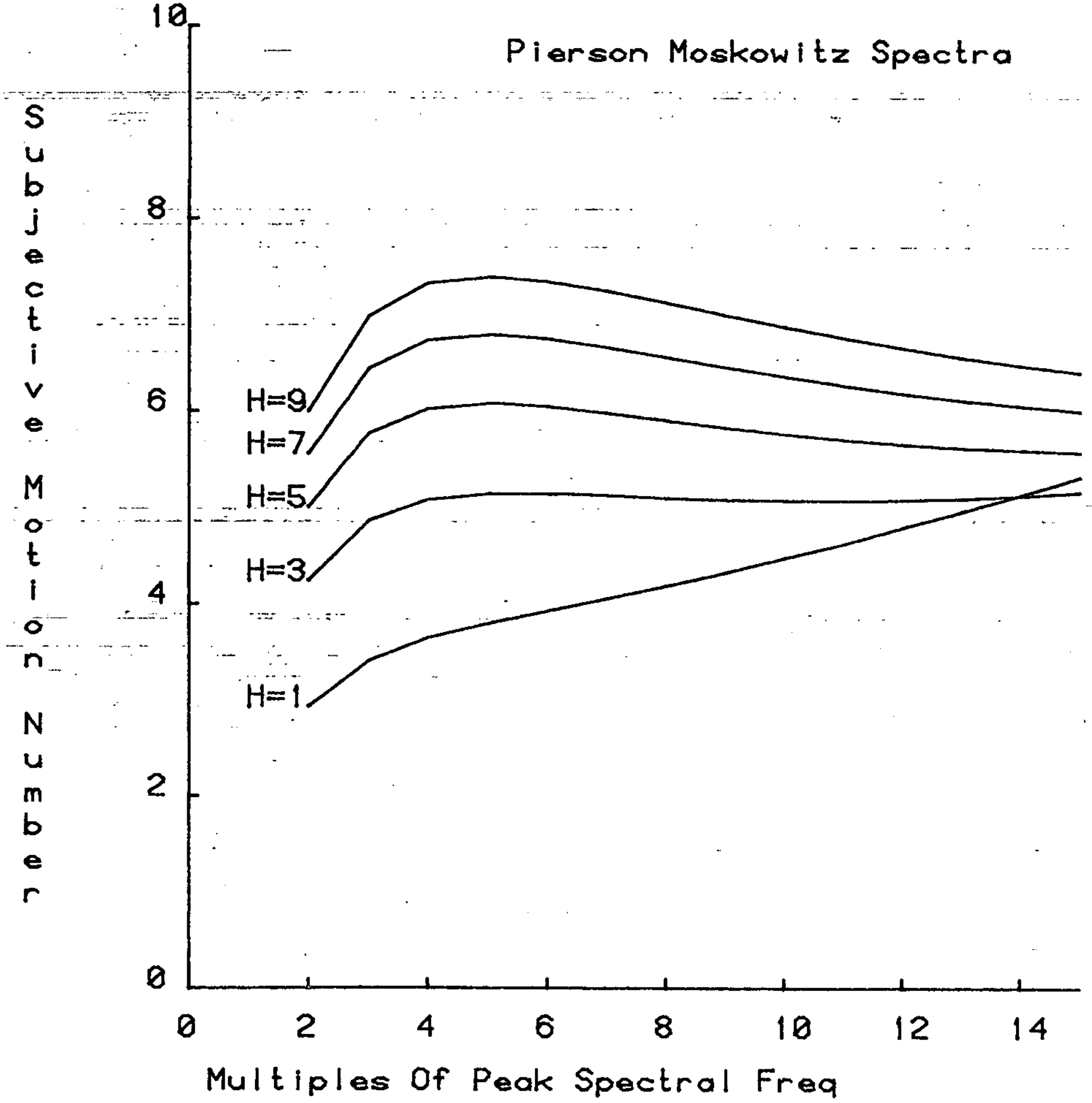
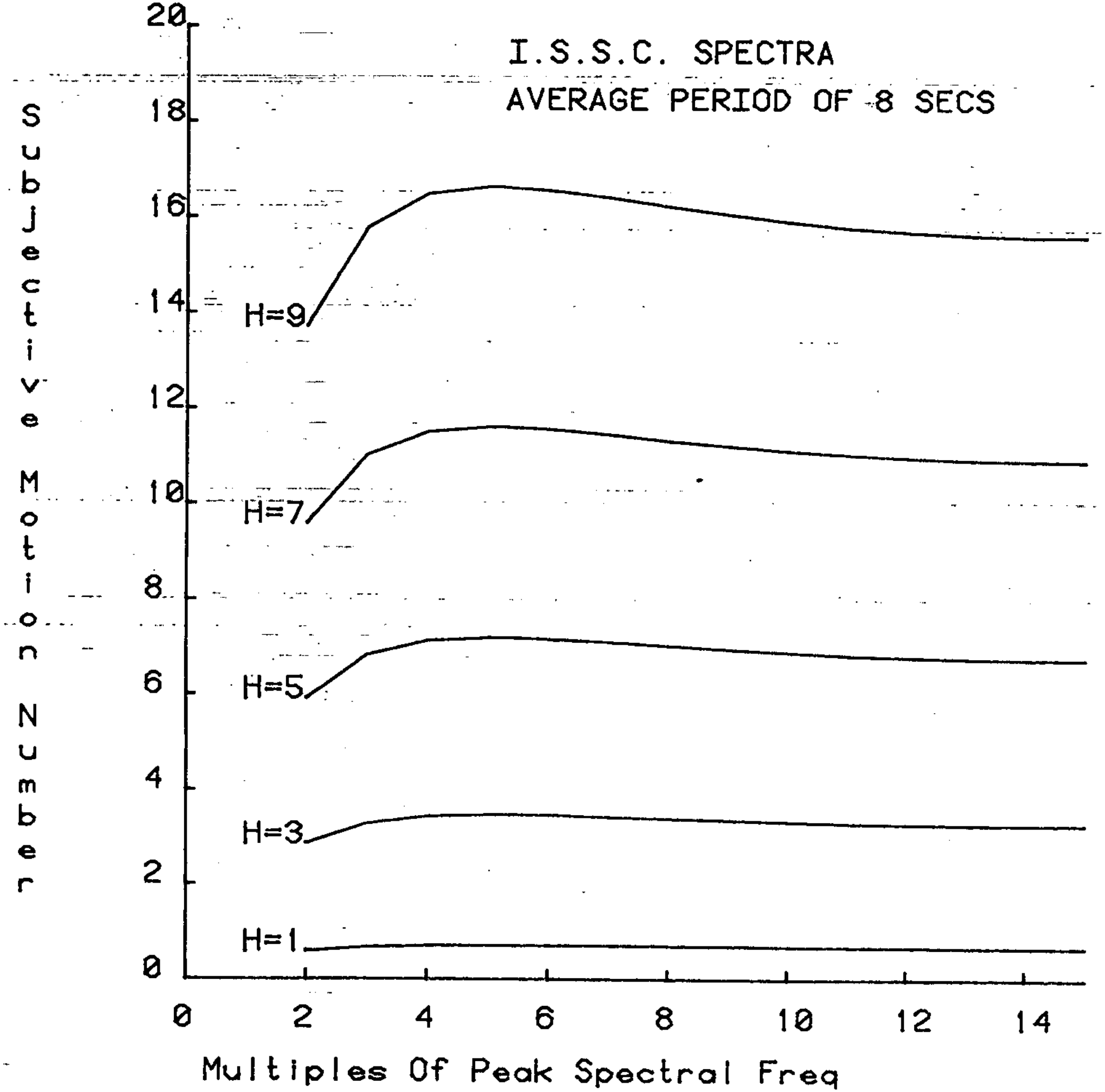
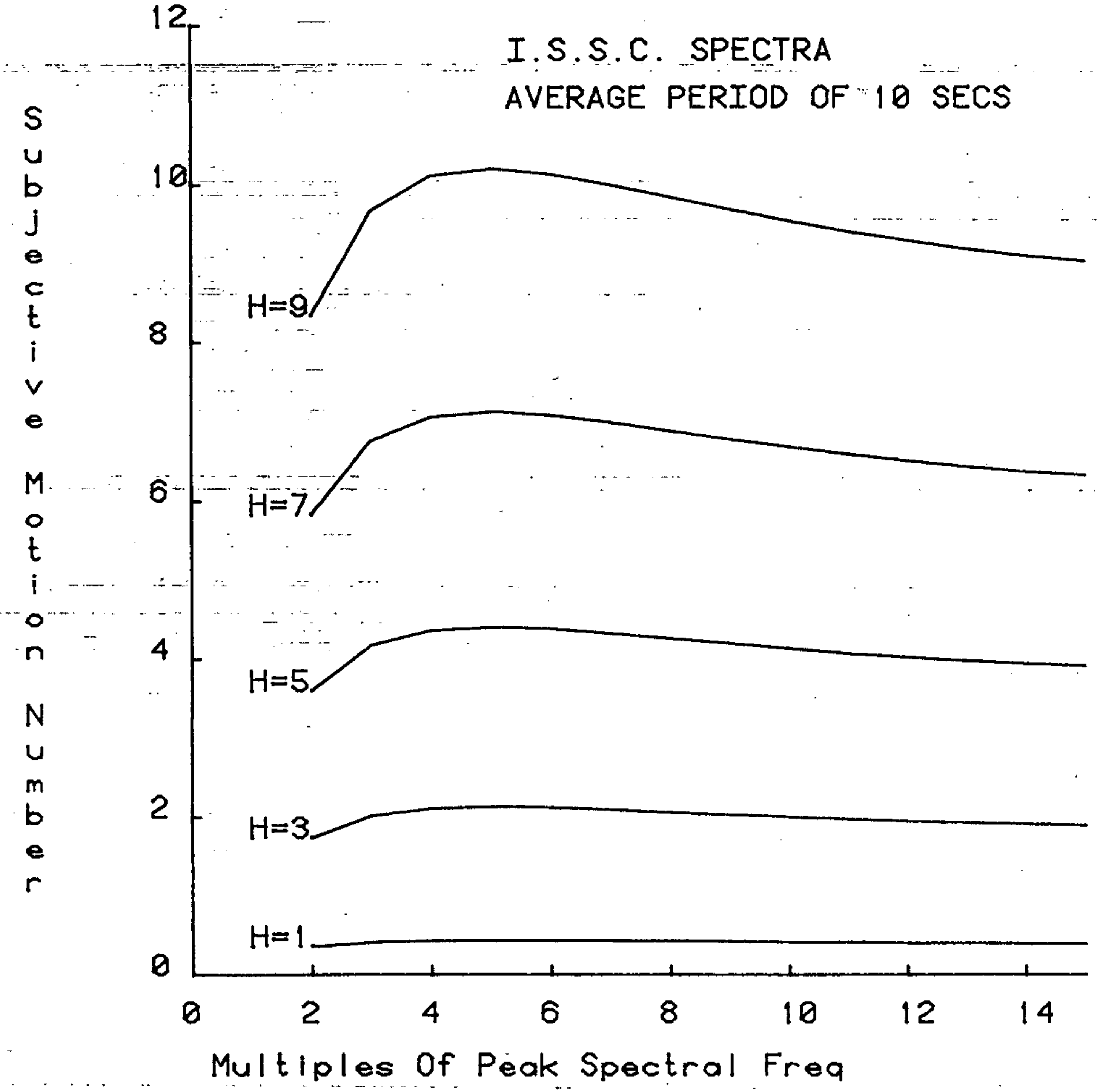
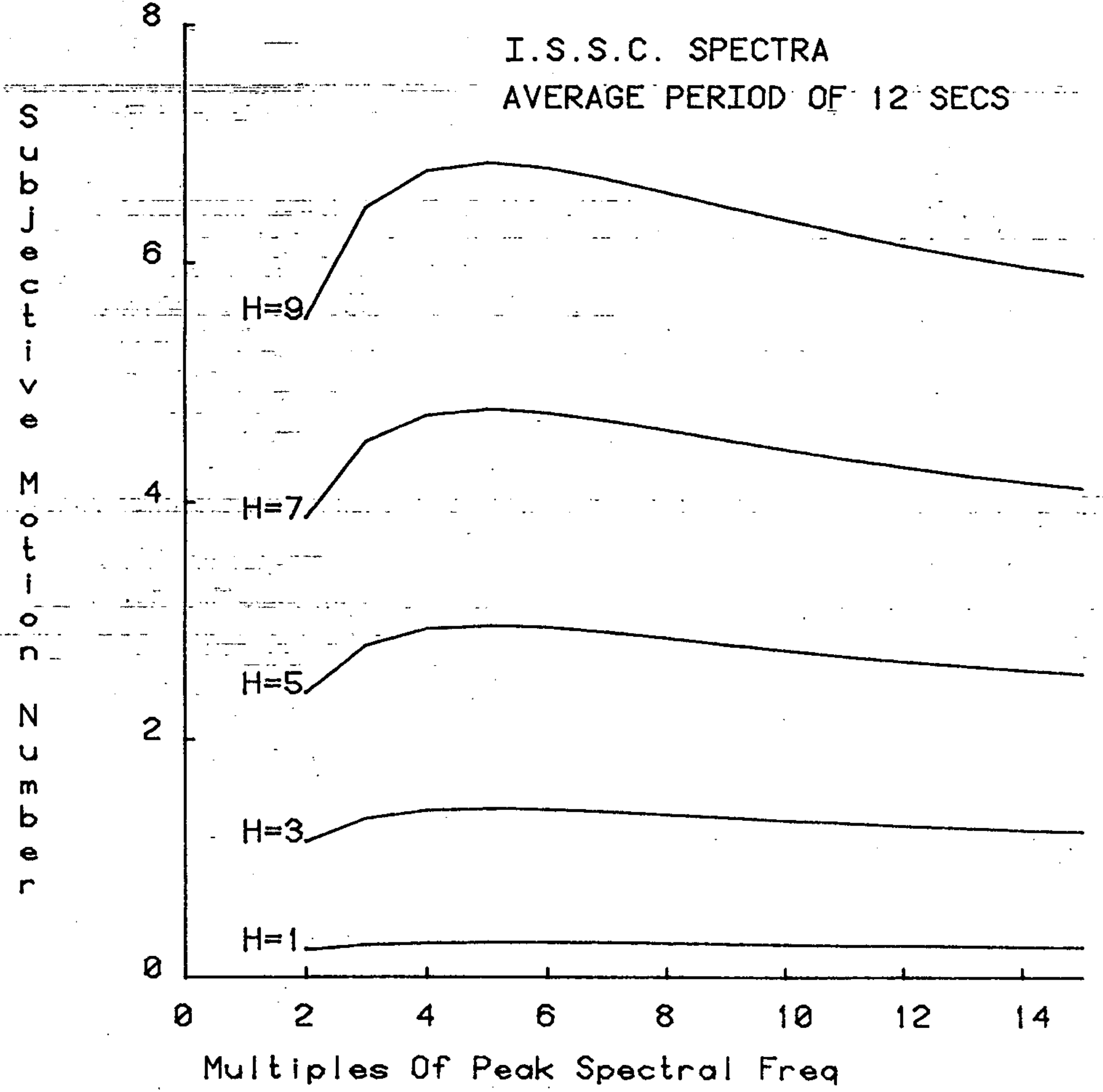

Pierson Moskowitz Spectra
Subjective Motion Number
10
8
6
4
2
0
H=9
H=7
H=5
H=3
H=1
0
2
4
6
8
10
12
14
Multiples Of Peak Spectral Freq


I.S.S.C. SPECTRA
AVERAGE PERIOD OF 8 SECS
Subjective Motion Number
20
18
16
14
12
10
8
6
4
2
0
H=9
H=7
H=5
H=3
H=1
0
2
4
6
8
10
12
14
Multiples Of Peak Spectral Freq


I.S.S.C. SPECTRA
AVERAGE PERIOD OF 10 SECS
Subjective Motion Number
Multiples Of Peak Spectral Freq
H=9
H=7
H=5
H=3
H=1
0
2
4
6
8
10
12
14


I.S.S.C. SPECTRA
AVERAGE PERIOD OF 12 SECS
Subjective Motion Number
8
6
4
2
0
H=9
H=7
H=5
H=3
H=1
0 2 4 6 8 10 12 14
Multiples Of Peak Spectral Freq