ISSN 0140 3818
SSSU 6/81

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April 1981
Ship Science Report No. 6/81

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ISSN 0140 3818

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CONTENTS

1. Introduction

This report contains the information for inputting ship data into the lateral ship motions program. The program enables the calculation of roll, sway, and yaw motions in regular or irregular waves, accounting for the appendages of the hull that are not usually catered for in other programs.

The program does not requires any estimation of roll damping, as all the damping terms are calculated from the appendage information.

As with the vertical motions program (1), much time has been saved, computationally, by producing for all the sections, the added mass and damping coefficients at a set of pre-determined frequency parameters. These parameters contain the section draught and frequency of encounter. After the matrix of values have been calculated, the program uses an interpolation subroutine to calculate the values needed in the specified calculation.

The method that is used for calculating the added mass and damping coefficients is the so called Frank Close Fit Method (2). This replaces a segment of a hull section with a pulsating source at the middle of the section.

The program allows the choice of one of two methods for calculating the forcing functions. The first is the so called total solution, which uses the pressures at the seven offset points per section to produce the roll and yaw moments and the sway force. This requires substantial computer time, since for each frequency for each section there is a total of 34 interpolations to be calculated. The second method is the so called long wave approximation which replaces some of the calculations of moments and forces by simple additions of the added mass and damping. The method is outlined in Appendix B. This method requires but 8 interpolations.

The results for the two methods give agreement to within 5% of each other, in general.

Two other options are programmed for the motion prediction of the ship. The ship can be force rolled in beam seas by either its fins or rudder. The results are presented only in regular waves over the range of frequencies specified, and then over a three octave band at $^1/_3$rd octave band intervals. The centre frequency is the roll natural frequency. The wave angle must be specified as 90°.

Although it may appear at first sight that only irregular sea results are calculated, it is possible to compute regular sea responses by specifying NSEA as zero.

To arrive at the response curve in irregular seas, it is necessary to know the significant roll response in that seaway in order to calculate the viscous roll damping. The program has a predetermined starting value, which is used as an initial estimate of significant roll. After the whole spectrum of wave frequencies response values have been calculated, a test is performed as to the accuracy vis-a-vis the initial guess. If the result is not close enough the calculations are re-computed until satisfactory agreement is reached. At this point the response is printed out. It is thus possible for different responses for the ship in the same waves, for different sea spectra.

A similar iterative scheme is used for regular waves but the iteration scheme is centred on the value of roll response and roll natural frequency.

2. Input Description

CARD (1) FREE FORMAT

IFIN	0 - Usual Type Calculations
	1 - Forced Roll, with fins
	2 - Force Roll, with rudder
ALG	Angle of fins or rudder in degrees.

CARD (2) 2 INTEGER (2I3)

ID — Wave Spectra Type

- 0 Canadian Spectra (Station India)
- 1 Pierson - Moskowitz
- 2 Jonswap
- 3 Two-parameter ITTC

IF — Wave Spreading

- 0 No spreading
- N Even number, power of cosine

CARD (3) 6 REALS (6F10.4)

EL	Length between perpendiculars (ft)
HCG	Height of CG above waterplane (ft)
GMIN	Metacentric Height (ft)
RNF	Roll natural frequency (rad / sec)
RRG	Roll radius of gyration ÷ beam
YRG	Yaw radius of gyration ÷ EL

NB. 1. If metacentric height is not specified on input, the programme will use a value computed from offset data. This GM is not corrected for free internal surfaces.

2. Either RNF or RRG must be specified, with the other input as 0.0, if RRG is input, note that it is assumed that the hydrodynamic moment of inertia is lumped with the ship moment of inertia. For small warship forms RRG lies in the range 0.35 to 0.4.

3. If YRG is unknown, use 0.25 as an input.

CARD (4) 1 INTEGER (I3)

NSP — Number of speeds (MAX 10)

CARD (5) 3 REALS (3F10.4)

WANGI	Initial wave angle (degs)
WANGA	Final Wave angle (degs)
DWANG	Increment (degs)

NB. If wave spreading is specified and only required on 180° then put WANGI = 180°, and DWANG as the incremental value to $\pm 90^\circ$

CARD (6) NSP REALS (8FIO.4)

VV(I) Ship speeds (knots) 8 to a card

CARD (7) 1 INTEGER 2 REALS (I5, 2F10.5)

NFR Number of wave frequencies at which responses are to be calculated
FR1 Lowest wave frequency (rad / sec)
FR2 Highest wave frequency (rad / sec)

The range of frequencies is divided into equal intervals of frequency.

CARD (8) 1 INTEGER (I3)

IFT Control
0 Ship has no tanks or fins
1 Fins only
2 Tanks only

CARD (9) 2 INTEGERS (2I3)

NSEA Number of seaways for which motions are to be computed (MAX 10)
NPOS Number of positions at which swaying motion in irregular seas are to be computed (MAX 10)

NB. Cards (10),(11) are ignored if either NSEA or NPOS = 0

CARD (10) 3 REALS (3F10.4)

HSW(I) Significant wave height of seaway I (ft)
TSW(I) Mean period of seaway I (sec)

NB. 1. If ID < 1, then TSW(I) all are zero
2. If ID > 1, then TSW(I) is the energy averaged period of seaway.

CARD (11) 2 REALS (2F10.4)
Calculation of motion at a point.

XPOS(I) x-coordinate of position I (Station aft or FP)
ZPOS(I) z-coordinate of position I (ft above CG)

CARD (12) 1 INTEGER (I3)

NST Number of stations for which offsets are input. For each station one each of cards (13), (14) and (15) are required.

NB. 1. The program uses 21 equally spaced stations. Stations numbered 1 - 21, 1 is the forward perpendicular 21 is the aft perpendicular.
2. Offsets must be given for station 2 - 20 offsets for station 1 are only given if the ship has a bulbous bow. Station 21 is given if a transom stern.
3. Thus there are values of NST of
19 no bulb, no cruiser stern
20 bulb or cruiser stern
21 bulb and cruiser stern

CARD (13) 1 REAL 1 INTEGER (F10.3, I3)

XA(I) Station number NB. AP is 21 FP is 1
IEDDY(I) Control integer for eddy making calculation.

O if the station is unlikely to produce eddies as the ship rolls.

1 for V or U shapes station such as those normally far forward.

2 for station triangular at the keel, such as aft stations of cruiser stern hulls.

3 for full, almost rectangular section.

NB. if a bilge keel spans I, set IEDDY(I) to be zero as these calculations are accounted for later.

CARD (14) 8 REALS (8F10.4)

YA(I,J) J = 1, 8 horizontal offsets of station I (ft).

CARD (15) 8 REALS (8F10.4)

ZA(I,J) J = 1, 8 vertical offsets of station I (ft)

NB. 1. Exactly 8 offset points must be specified for each station.

2. The first point is at the intersection of the centreline with the station contour while the eighth is at the intersection of load waterline with the station contour (fig. 1).

3. The vertical offsets are input as heights above the hull baseline (waterline zero)

4. For stations spanned by bilge keels, one of the offsets must be exactly the same as the bilge keel offset called for on card (18).

5. On very beamy stations, such as occur with transom sterns, do not use more than 4 points on the slightly sloping horizontal surface, as computational inaccuracies may otherwise result.

CARD (16) 1 INTEGER (I3)

NBKP Number of bilge keel parts (max of 5) if O, then card (17) is ignored, for NBKP > O one card (17) is required for each pair of bilge keels.

CARD (17) 2 INTEGERS (2I3)

NFBK(I) First station spanned by bilge keel I
NSBK(I) Number of stations spanned by bilge keel I

CARD (18) 4 REALS (4F10.4)

YBK(J)	horizontal bilge keel offset at station J (ft)
ZBK(J)	vertical bilge keel offset at station J(ft)
BKK(J)	bilge keel breadth at station J (ft)
ELBK(J)	bilge keel length at station (J) (ft)

See figure 2 for clarification.

CARD (19) 3 REALS (3F10.4)

XSK	X-coordinate of aftermost point where skeg meets hull (station aft of FP)
BSK	skeg breadth (ft)
ELSK	skeg length (ft)

See figure 3 for clarification

CARD (20) 1 INTEGER 9 REALS (I3 9F8.3).

NFP Number of fin pairs (MAX of 4)
FCK1
FCK2
FCK3
FCB1
FCB2
FCB3
FSA1
FSA2
FSA3

CARD (21) 1 INTEGER (I3)

NSH Number of propeller shafts (MAX of 2)

CARD (22) 8 REALS (8F8.3)

X(I)	X-coordinate of foil I (stations aft of FP)
Y(I)	Y-coordinate of foil I (ft)
Z(I)	Z-coordinate of foil I (ft)
B(I)	Span of Foil I (ft)
CR(I)	Root chord of foil I (ft)
CE(I)	Tip chord of foil I (ft)
CLA(I)	Lift curve slope of foil I (rad^{-1})
GAM(I)	Dihedral angle of foil I (deg)

NB. 1. Antirolling fins and propeller shaft bracket are regarded as foils. Thus there are NFP + NSH cards.

2. Coordinates are input for the port foil.

3. Figure 4 gives details of x, y, z.

4. For NSH = 2, the data card for the outboard bracket must precede the card for the inboard bracket.

5. If CLA(I) is unknown, the programme calculates a value.

CARD (23) 7 REALS (7F8.3)

X(NFS) X-coordinates of rudder stock (station aft of FP)
Y(NFS) Y-coordinates of rudder stock (ft)
Z(NFS) Z-coordinate of rudder stock (ft)
B(FNS) Rudder span (ft)
CR(NFS) Rudder Root chord (ft)
CE(NFS) Rudder Tip chord (ft)
CLA(FNS) Rudder lift curve slope (rad^{-1})

(a) If Y(FNS) = 0.0, the ship is assumed to have a single rudder, while Y(FNS) > 0, indicate twin rudders each with the specified span and chord.

(b) The same convention for x, y, z apply as for the previous card.

CARD (24) 9 REALS (9F8.3)

RCK1Y
RCK2Y
RCK3Y
RCB1Y
RCB2Y
RCB3Y
RSA1
RSA2
RSA3

CARD (25) 6 REALS (6F8.3)

RCK1R
RCK2R
RCK3R
RCB1R
RCB2R
RCB3R

CARD (26) 2 REALS (2F10.3)

BETNOM Normal fin angle (deg)

NB. 1. This is used for fin-fin and fin-bilge keel interference.

2. Not required if NFP = 0.

FLINC

CARD (27) 10 REALS (10F8.4)

TL Longitudinal length of anti-rolling tank (ft)
W Width of tank connection duct (ft)
W1 Bottom width of tank vertical leg (ft)
Y Average fluid depth in tank vertical leg (ft)
ALFA Inclination of outside wall of tank vertical leg (deg)
H Height of tank connecting duct (ft)
R Distance from (C.G.) to bottom of tank connecting duct (ft)
XT Tank x-coordinate (ft)
RHO Tank fluid density (Slug / ft^3)

VS Tank valve resistance coefficient

NB. 1. Figure 5 explains the input

2. XT is measured from C.G. XT is positive when tanks longitudinal centre is forward of C.G.

3. Tank valve resistance coefficient may be assigned any value between zero and infinite (see ref (1) fig. 8).

<u>CARD (28)</u> 1 INTEGER I3

INDC Type of càlculation used

0 total solution using pressures generated by sources

1 long wave approximation

<u>CARD (29)</u> 1 INTEGER I3

IFILE file manipulation for added mass, damping coefficients and pressures.

-1 write results to channel 7 and print values
0 write results to channel 7 and no printout
1 read results from channel 7 and print values
2 read results from channel 7 and no printout
3 read results from multi-coefficient file and print values, channel 7
4 read results from multi-coefficient file and no printout, channel 7

<u>CARD (30)</u> 1 INTEGER I3

NEXT control integer

> 0 implies more ships

≤ 0 finish programme

Appendix A Equations of Motion

The underlying theory of the program is standard for strip theory

1. Ship response is a linear function of wave excitation.

2. Ship length is much greater than either beam or draught.

3. All viscous effects other than roll damping are negligible.

4. The hull does not develop appreciable planing lift.

The equations given below are written with respect to stability axes fixed in the ship. This is different from the usual translating earth axes. The use of stability axes makes the coefficients from the equations of motion much simpler. The axis system is illustrated in figure 1. The origin of the axis system is at the centre of gravity. The x-axis is directed horizontally forward, the z-axis is vertically upward and the y-axis to port.

The coupled equations of sway, roll and yaw are familiar, see reference (2).

Sway $$(A_{22} + m)\ddot{x}_2 + B_{22}\dot{x}_2 + A_{24}\ddot{x}_4 + B_{24}\dot{x}_4 + A_{26}\dot{x}_6 + (B_{26} + mU)\dot{x}_6 = F_2 \qquad \text{(A1)}$$

Roll $$A_{24}\ddot{x}_2 + B_{24}\dot{x}_2 + (A_{44} + I_4)\ddot{x}_4 + B_{44}\dot{x}_4 + C_{44}x_4 + A_{46}\ddot{x}_6 + B_{46}\dot{x}_6 = F_4 \qquad \text{(A2)}$$

Yaw $$A_{62}\ddot{x}_2 + B_{62}\dot{x}_2 + A_{64}\ddot{x}_4 + B_{64}\dot{x}_4 + (A_{66} + I_6)\ddot{x}_6 + B_{66}\dot{x}_6 = F_6 \qquad \text{(A3)}$$

where x_2 is sway, x_4 is roll, and x_6 is yaw. A_{jk} and B_{jk} are the added mass and damping coefficient, C_{44} is the roll restoring force. F_j are the exciting forces and moments.

$$A_{ij} = A_{ij}^{H} + A_{ij}^{F} \; ; \quad B_{ij} = B_{ij}^{H} + B_{ij}^{F} + B_{ij}^{C}$$

$$F_i = F_i^{H} + F_i^{F} + F_i^{C} \qquad \text{(A4)}$$

Are the general form of the coefficients; where the superscript H denotes hull terms derived from strip theory, F denotes contributions due to appendages; C denotes hull circulatory terms.

For the B_{44} term there is the additional viscous roll damping term, which takes into account bilge keels, skeg, rudder and other appendages.

To obtain the equations and coefficients used in A1, A2, A3 the following transformation is required to transform those of reference (2)

$$\dot{x}_2 \rightarrow \dot{x}_2 + Ux_6 = \dot{x}_2 - \frac{U\ddot{x}_6}{\omega^2}$$

$$\dot{x}_4 \rightarrow \dot{x}_4$$

$$\dot{x}_6 \rightarrow \dot{x}_6$$

Thus the hull added mass and damping coefficients are

$$A_{22}^{H} = \int_L a_{22}\, dx$$

$$B_{22}^{H} = \int_L b_{22}\, dx$$

$$A_{24}^{H} = \int_L b_{24}\, dx$$

$$B_{24}^{H} = \int_L b_{24}\, dx$$

$$A_{26}^{H} = \int_L a_{22}\, x\, dx$$

$$B_{26}^{H} = \int_L b_{22}\, x\, dx$$

$$A_{44}^{H} = \int_L a_{44}\, dx$$

$$B_{44}^{H} = \int_L b_{44}\, dx$$

$$A_{46}^{H} = \int_L a_{24}\, x\, dx$$

$$B_{46}^{H} = \int_L b_{24}\, x\, dx$$

$$A_{62}^{H} = A_{26}^{H} - UB_{22}^{H}/\omega^2$$

$$B_{62}^{H} = B_{26}^{H} + U\, A_{22}^{H}$$

$$A_{64}^{H} = A_{46}^{H} - UB_{24}^{H}/\omega^2$$

$$B_{64}^{H} = B_{46}^{H} + UA_{24}^{H}$$

$$A_{66}^{H} = \int_L a_{22}\, x^2\, dx$$

$$B_{66}^{H} = \int_L b_{22}\, x^2\, dx$$

The above integrations are over the length of the ship. The two dimensional added mass (a_{jk}) and wave making damping (b_{jk}) are computed for each section.

a_{22}, b_{22} are from sway motions

a_{44}, b_{44} are from roll motions

a_{24}, b_{24} are from cross-coupling between sway and roll

$C_{44} = mg\,\overline{GM}$. $\overline{GM}$ is the metacentric height.

Forcing Functions

$$F_j^H = \rho a \int_L (f_j + h_j)\, dx \qquad j = 2, 4$$

$$F_6^H = \rho a \int_L \{(f_2 + h_2)\, x + \frac{Uh_2}{i\omega}\}dx$$

where a is the amplitude of the incident wave and integration is over the hull length. f_j and h_j are the sectional incident and diffraction forces.

$$f_j(x) = -g \exp(-ik_w x\cos\beta) \int_{c(x)} n_j \exp(ik_w y\sin\beta + k_w z')ds \qquad \text{(A5)}$$

$$h_j(x) = \omega_w \exp(-ik_w x\cos\beta) \int_{c(x)} \phi_j \; (in_3 - n_2\sin\beta) \exp(ik_w y\sin\beta + k_w z')ds \qquad \text{(A6)}$$

f_j is also commonly referred to as the Froude-Krilov force. A physical interpretation of f_j and h_j is that f_j results directly from the action of the incident waves on the hull, while h_j represents a convection for ship-wave diffraction. The integrations are performed over the submerged hull section C(x). n_2 and n_3 are the y and z components of the unit outward normal to the hull at (x, y, z)

$$n_4 = y\, n_3 - z\, n_2$$

$$z' = z + h_{CG}$$

ϕ_2, and ϕ_4 are the two-dimensional section potentials for sway and roll motions. ω_w is the wave frequency, k_w is the wave number. h_{CG} is the height of the CG above the waterline. β is the wave heading, O is following seas, 180 head seas. The frequency of encounter is given by

$$\omega = \omega_w - k_w\, U \cos\beta$$

Appendix B Long Wave Approximation

The forcing function used in Appendix A do not, because of their mathematical complexity, give much insight into the physical properties of the equations. However when the wavelengths are long compared with the ship beam, fairly simple approximations can be derived for the forcing function.

In equation A5 the complex potential is given by

$$I = \int_{c(x)} n_j \exp(k_w z')(\cos p + i \sin p)\, ds \tag{B1}$$

where $p = k_x y \sin\beta$

From symmetry consideration the n_j cosp term integrates to zero for j = 2, 4 because the normal vectors are of the same sign but opposite magnitudes.

Thus equation (B1) gives

$$I = i \int_{c(x)} n_j \exp(k_w z') \sin p\, ds \tag{B2}$$

when the wavelength is long relative to beam k_w is small thus

$$\sin p \sim p \quad ; \quad \exp(k_w z') \sim 1$$

Thus $$I = i\, k_w \sin\beta \int_{c(x)} n_j\, y\, ds \tag{B3}$$

For j = 2, since $n_2 = \frac{dz}{ds}$

$$I = ik_w \sin\beta \int_{c(x)} y\, dz = ik_w \sin\beta\, A(x) \tag{B4}$$

where A(x) is the sectional area.

For $j = 4$

$$I = ik_w \sin\beta \int_{c(x)} (yn_3 - zn_2)\, y\, ds \qquad \text{(B5)}$$

To evaluate (B5) consider an element of hull area dA, located at (x, y, z). The ship rolls through a small angle x_2, the hydrostatic force acting on dA increases by

$$df = -\rho g\, x_2\, y\, dA$$

df acts normal to the hull, with y and z components

$$df_y = -\, n_2\, df$$

$$df_z = -\, n_3\, df$$

Thus the rolling moment exerted by df is

$$dK = y df_z - z df_y = (-yn_3 + zn_2)\, df$$

$$= -\rho g x_2 y\, (-yn_3 + zn_2)\, dA$$

$$= -\rho g x_2 y (-yn_3 + zn_2)\, ds\, dx$$

Integration over the hull gives the total roll-restoring moment

$$K = -\, \rho g x_2 \int_L Mx\, dx$$

Which evaluates (B5).
Where M_x is the sectional contribution to the roll-restoring moment

$$M_x = \int_{c(x)} y(-yn_3 + zn_2)\, ds$$

Thus equation B5 is

$$I = -ik_w \sin\beta \; M_x \qquad \text{(B6)}$$

Thus the long wave approximations to the forcing functions are

$$f_2 = -ik_w \, g \sin\beta . \; A(x) \exp(-ik_w x \cos\beta)$$

$$f_4 = ik_w \, g \sin\beta \; M(x) \exp(-ik_w \, x \cos\beta)$$

The sectional diffraction forces h_j when expanded become

$$I = \int_{c(x)} \phi_j \exp(k_w z')(in_3 - n_2 \sin\beta)(\cos p + i \sin p) \, ds$$
$$= \int_{c(x)} \phi_j \exp(k_w z')(i(n_3 \cos p - n_2 \sin\beta \sin p)$$
$$- n_3 \sin p - n_2 \sin\beta \cos p) \, ds \qquad \text{(B7)}$$

from symmetry properties

$$\phi_j \text{ (port)} = - \phi_j \text{ (stbd)}$$
$$n_2 \text{ (port)} = - n_2 \text{ (stbd)}$$
$$\sin p \text{ (port)} = - \sin p \text{ (stbd)}$$

(B7) becomes

$$I = - \int_{c(x)} \phi_j \exp(k_w z')(n_3 \sin p + n_2 \sin\beta \cos p) \, ds \qquad \text{(B8)}$$

Now for long waves $\cos p \sim 1$.

$$\therefore I = - \int_{c(x)} \phi_j (n_3 k_w y \sin\beta + n_2 \sin\beta) \, ds$$

$$\therefore I = - \sin\beta \int_{c(x)} \phi_j \, n_2 \, ds \qquad \text{(B9)}$$

since k_w is small for long waves.

But the definition of

$$\int_{c(x)} \phi_j \; n_2 \; ds = \frac{i}{\rho\omega} \; \{\omega^2 \; a_{2j} - i\omega \; b_{2j}\} \qquad \text{(B10)}$$

$$\therefore \; h_j = - \frac{\omega_w}{\rho} \sin\beta \; (b_{2j} + i\omega \; a_{2j}) \; \exp\; (-ik_w x \sin\beta) \qquad \text{(B11)}$$

In the particular case of beam seas $\beta = 90^\circ$ the long wave approximations give for sway, if roll, yaw coupling to sway is neglected,

$$x_2 = i \; a$$

i.e. The sway equals the wave amplitude and leads the wave phase by 90°.

If the more general decay of wave force with depth is used i.e. in equation (B7) replace $\exp(k_w z')$ as $\exp(-k_w \; C_M T)$, where T is the section draught, C_M is the midship section area coefficient

$$\therefore \; x_2 = i \; a \; \exp\; (-k_w \; C_M T)$$

If now roll is looked at, without yaw coupling, using the asymptotic value of x_2

$$\left[- \omega^2 (A_{44} + I_{44}) + i \; \omega \; B_{44} + C_{44}\right] \; x_4$$

$$= F_{I4} + H_4 + F_4^F - i \; a \; (- \omega^2 \; A_{24} + i \; \omega \; B_{24})$$

$$= a\left[ik_w \; mg \; \overline{GM} - \omega(B_{24}{}^H + i \; \omega \; A_{24}{}^H)\right.$$

$$\left. - \; \omega(B_{24}{}^F + i \; \omega \; A_{24}{}^H) + \omega(B_{24} + i \; \omega \; A_{24})\right]$$

$$= i \; k_w \; mg \; \overline{GM} \; a \; = \; i \; C_{44} \; a \; k_w$$

Thus as $\omega \to 0$ $\; x_4 \to i \; ak_w$

Thus the roll amplitude equals the wave slope at roll leads wave phase by 90°.

Similarly for equation (A3)

$$x_6 = \frac{(\omega^2 A_{64} - i\,\omega\, B_{64})}{-\omega^2 (A_{66} + I_{66}) + i\,\omega\, B_{60}} \quad x_4 \tag{B12}$$

Since the denomination of (B12) is always larger than the numerator, x_4 is $\sim ak_w$,

x_6 is very small for large wave lengths.

All these approximations are true for beam seas only. Thus the classical roll decoupling is verified, but there is no reason to suppose that the same can be said for other wave headings, since the frequency of encounter is the wave frequency.

Appendix C Viscous Roll Damping

The viscous roll damping coefficient may be expressed as follows :

$$B_{44}^{v} = B_{BK} + B_E + B_H + B_F .$$

where B_{BK}, B_E and B_H are the contributions from bilge keels, eddy-making resistance of the hull, and hull skin friction. B_F represents the viscous effect of appendages other than bilge keels, at zero speed.

Viscous roll damping is non-linear with respect to roll angle, and to overcome this problem, the amount of work done during one roll cycle is equated to the torque.

$$x_4 = \bar{r} \sin \omega t, \qquad \bar{r} \text{ is the roll amplitude.}$$

$B\dot{x}_4$ represents the torque about the CG.

$$\therefore 4 \int_o^{r} B \ \dot{x}_4 \ dx_4 = \pi \ \omega B \ ^2 \qquad \text{(C1)}$$

If the energy dissipated is E

$$\therefore B = \frac{E}{\pi \ \omega \ r^2} \qquad \text{(C2)}$$

Bilge Keels.

Kato (4) gives the following formula for the energy dissipated during one roll cycle

$$E = 4\rho \ \ell \ b_k \ r_1 \ r \ \left[\frac{r_1 r}{T}\right]^2 C_o \ C_a \ C_k \ C_n \ \ B \ \ F^{-\alpha} \qquad \text{(C3)}$$

where ℓ is the bilge keel length, b_k is the bilge keel breadth, r_1 is the distance from the centre of the bilge keel to the CG and T is period. C_o, C_a, C_k, C_n, B and F are coefficients depending on ship form and Reynolds number.

The coefficient F depends upon r_1, r, T, b_r, and Γ the angle between the waterline, CG and bilge keel root.

$$F = \frac{3.13\ r\ r_1}{T\ \sqrt{gb_k}} \quad \Gamma^{1.7}$$

The index α is also a function of r, b_k, and Γ ;

$$\alpha = 0.6 - 2.03 \exp(-25z_2)$$

$$\text{where } z_2 = \frac{b_k}{r_1\ \Gamma^{0.75}}$$

The coefficient (B) depends upon the length of girth from bilge keel root to waterline S, beam B, height of CG above keel KG, draught d, and rise of floor F_r :

$$B \quad z \cos\gamma + \frac{S}{2b_k r_1} \ \{q + p_o - (p_o - p_1)\ f(\lambda)\}$$

where γ is the angle made by the plane of the bilge keel with the straight line passing through the CG and the bilge keel root.

$$q = \left[\frac{B}{2} \tan\left(\frac{\pi}{4} - \frac{\varepsilon}{2}\right) + F_r - KG\right] \sin\left(\frac{\pi}{4} + \frac{\varepsilon}{2}\right)$$

$$\varepsilon = \tan^{-1}\left(\frac{2F_r}{B}\right)$$

$$p_o = KG - \frac{d}{3} - \frac{2F_r}{3}$$

$$p_1 = 0.88 \left\{KG - d - 0.54\left(\frac{B}{2} - (d - F_r) \tan\left(\frac{\pi}{4} + \frac{\varepsilon}{2}\right)\right)\right\}$$

$$f(\lambda) = \frac{1.34 \sin\left(\frac{\pi\lambda}{3.6}\right)}{1 + 0.162 \sin\left(\frac{\pi}{1.8}(\lambda - 0.9)\right)}$$

$$\lambda = \frac{R}{d - \frac{F_r}{B}(B - 2R)}$$

C_n is the normal pressure coefficient for a rectangular plate moving with a uniform velocity in the direction perpendicular to its plane :

$$C_n = \begin{cases} 1.98 \exp(-11\, b_k/\ell) & b_k/\ell < 0.048 \\ 1.17 & b_k/\ell \geq 0.048 \end{cases}$$

The coefficient C_K depends upon ship form and in particular upon R, the bilge radius

$$C_k = 1 + 3.5\, e^{-9k}$$

$$k = \frac{R(1 + F_r/B)^2}{\sqrt{\frac{B.KG}{2}}}$$

where ν is the kinematic viscosity.

$$C_a = \begin{cases} 1 & \text{when } R_N \geq 10^3 \\ 1.95 - 0.25 \log R_N + 0.2 \sin\{\frac{\pi}{0.54}(\log R_N - 2.1)\} & \text{when } R_N < 10^3 \end{cases}$$

C_o depends upon Γ and also scales the whole equation

$$C_o = 14.1 + 37.3\, \Gamma^3$$

Hull Friction

Consider an element of hull surface ds. The skin friction is drag force $\underset{\sim}{dF}$ acting on ds as a result of the rolling velocity $\dot{x}_4$ is

$$\underset{\sim}{dF} = \frac{1}{2} \rho\, r_2 (y\, n_2 + z n_3)\, \dot{x}_4 \, |\dot{x}_4|\, C_{D_F}\, ds\, (n_3 \underset{\sim}{j} - n_2 \underset{\sim}{k})$$

where C_{D_F} is the skin friction drag coefficient, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are unit vectors along the y and z axes.

$r_2 = \sqrt{y^2 + z^2}$ is the distance from the CG.

dF exerts a torque about the rolling axis given by

$$dT = -\frac{1}{2}\rho\ r_2\ (y\ n_2 + z\ n_3)^2\ \dot{x}_4\ |\dot{x}_4|\ C_{D_F}\ ds$$

The energy dissipated by dT during one roll cycle is

$$dE = \frac{4}{3}\rho\ r_2\ (y\ n_2 + z\ n_3)^2\ \omega^2\ r_1^{\ 3}\ C_{D_F}\ ds$$

$$\therefore\quad B_H = \frac{4}{3\pi}\rho\ \omega\ r_1\ C_{D_F}\ \int_L dx \int_{c(x)} r_2\ (y\ n_2 + z\ n_3)^2\ d\ell$$

If forward speed U is non-zero, the Schoenherr rule based on smooth turbulent flow is used to evaluate C_{D_F}

$$C_{D_F} = 0.004 + \left(3.46 \log\left(\frac{UL}{\nu}\right) - 5.6\right)^{-2}$$

If $U = 0$, C_{D_F} is evaluated by the following method

$$C_{D_F} = 1.328\ R_N^{\ -0.5} + 0.014\ R_N^{\ -0.114}$$

$$R_N = \frac{3.22}{T_o}\ (F\ x_4)^2$$

$$F = \frac{1}{\pi}\ \{(0.887 + 0.145C_B)(1.7T + BC_B) + 2(KG - T)\}$$

C_B is the block coefficient, KG the CG height above the keel. B the ship beam, T the draught. R_N is the Reynolds Number based on the average roll velocity, and distance from the CG.

By regarding the rudders and fins as oscillating flat plates, at zero forward speed, the method above gives

$$B_F = \frac{4}{3\pi}\rho\ \ \omega\ x_4\ \ \sum\ \ (x^2 + y^2)^{3/2} SC_n$$

where the summation is over all foil elements. C_n is the normal force coefficient for a flat plate inclined at a large angle to the flow. Hoerner gives 1.17 for C_n when the angle is greater than 40°.

Eddy-Making Roll Damping

Tanaka (5) results are presented for the coefficient for eddy-making drag coefficients.

Consider first V or U shaped sections. Tanaka obtained the following empirical equation

$$C = T_1 \frac{B}{KG} T_2 \left(\alpha, \frac{R_e}{d}\right) \exp\left(- u \frac{R_e}{d}\right)$$

where α is the angle of inclination of the ship side at the waterline. R_e is the effective radius at the keel, u is a function of x_4, T_1 and T_2 are tabulated functions. Quadratics have been fitted to the data to give

$$u = 14.1 - 46.7\, x_4 + 61.7\, x_4^2$$

$$R_e = \frac{B}{2} \left| 4.12 - 3.69 \frac{KG}{B} + 0.823 \left(\frac{KG}{B}\right)^2 \right| \qquad \frac{KG}{B} < 2.1$$

$$= 0 \qquad \frac{KG}{B} > 2.1$$

For very full almost rectangular sections, the same equations are used with r as the distance from the CG to the bilge, R_e the bilge radius with $T_e = 1$.

For triangular sections, found aft in cruiser sterns C is a function of B/KG

Then $C = 0.438 - 0.449\,(B/KG) + 0.236\,(B/KG)^2$

Thus to determine the eddy making damping B_E, the sections used in the strip theory must be classified.

Appendix D Lift Surface Contributions

When U > O, hull appendages such as rudder, skeg and propeller shaft brackets act as lifting surfaces to generate damping and exciting forces. Their contribution to roll damping is significant. Of lesser importance are the added mass terms of the appendages which are indpendent of speed.

The results are summarized as

$$A_{22}^{F} = \sum a\ p\ \sin^2\Gamma$$

$$B_{22}^{F} = \frac{1}{2}\rho\ U \sum s\ C_{L\alpha}\ C(K)\ \sin^2\Gamma$$

$$A_{24}^{F} = -\sum a\ p\ \sin\ \ (y\ \cos\ \ + z\sin\ )$$

$$B_{24}^{F} = -\frac{1}{2}\rho\ U \sum sC_{L\alpha}\ C(K)\ \sin\Gamma(y\ \cos\Gamma + z\sin\Gamma)$$

$$A_{26}^{F} = \sum a\ p\ s\ \sin^2\Gamma = A_{62}^{F}$$

$$B_{26}^{F} = \frac{1}{2}\rho\ U \sum s\ C_{L\alpha}\ C(K)\ (s - C/4)\ \sin^2\Gamma$$

$$A_{44}^{F} = \sum a\ p\ (y\ \cos\Gamma + z\ \sin\Gamma)^2$$

$$B_{44}^{F} = \frac{1}{2}\rho\ U \sum s\ C_{L\alpha}\ C(K)\ (y\ \cos\Gamma + z\ \sin\Gamma)^2$$

$$A_{46}^{F} = -\sum a\ p\ s\ \sin\Gamma\ (y\ \cos\Gamma + z\ \sin\Gamma)$$

$$B_{46}^{F} = -\frac{1}{2}\rho\ U \sum s\ C_{L\alpha}\ C(K)\ \sin\Gamma\ (s - C/4)\ (y\ \cos\Gamma + z\ \sin\Gamma)$$

$$B_{62}^{F} = \frac{1}{2}\rho\ U \sum x\ s\ C_{L\alpha}\ C(K)\ \sin^2\Gamma$$

$$B_{64}^{F} = -\frac{1}{2}\rho\ U \sum x\ s\ C_{L\alpha}\ C(K)\ \sin\Gamma\ (y\ \cos\Gamma + z\ \sin\Gamma)$$

$$A_{66}^{F} = \sum a\ p\ s^2\ \sin^2\Gamma$$

$$B_{66}^{F} = \frac{1}{2}\rho\ U \sum s\ C_{L\alpha}\ C(K)\ (s - C/4)\ x\ \sin^2\Gamma$$

The terms are summed over all elements, Γ is the dihedral angle Γ as illustrated in figure . ap is the added mass of a foil being accelerated perpendicular to its surface. For large aspect ratios, such as A brackets

$$ap = \pi \rho \; b \; (C/2)^2$$

Otherwise use empirical formulae.

The bilge keels can be regarded as very low aspect ratio foils, so an additional term to B_{44} is obtained,

$$B_{BK} = \pi \; \rho \; U \; b_k^{\;2} \; r^2$$

b_k is the bilge keel length, r is the distance from bilge keel to the CG.

The existing forces are modified as follows

$$F_2^F = \sum f_2^F \sin\Gamma$$

$$F_4^F = - \sum f_2^F \; (y \cos\Gamma + z \sin\Gamma)$$

$$F_6^F = \sum f_2^F \; s \; \sin\Gamma$$

where f_2^F is the sway exciting force acting on the individual foil element.

$$f_2^F = - \omega_w \; (\sin\Gamma \; \sin\beta_s + i \cos\Gamma) \; (\tfrac{1}{2} \rho \; U \; s \; C_{L\alpha} \; G(K) + i \; \omega \; ap)$$

$$\exp \; (- k_w (h + i \; (x \cos\beta_s - y \sin\beta)))$$

The functions G(k) is a modified form of Jones gust function, k is the reduced frequency. See references (6), (7).

Appendix E Hull Circulatory Effects

Let the damping be given by

$$B_C = \frac{\pi\rho}{2} UT^2 \qquad \text{where T is the draught}$$

Then $B_{22}{}^C = B_C$

$$B_{26}{}^C = B_C\, x_p = B_{62}{}^C$$

$$B_{66}{}^C = B_C\, \left(\frac{1}{2} C_p L\right)^2 + U \int_L a_{22}\, x\, dx$$

where x_p is the x-coordinate of the centre of area of the hull underwater profile, C_p is the prismatic coefficient.

The effect of the hull swaying and yawing is estimated by assuming that the hull is a wing of length L and span varying with sectional draught. Thus the results for forcing moments in sway and yaw are :

$$f_2{}^C = \omega_w\, T_x\, \sin\beta\, \exp(-k_x(i\, x \cos\beta + T_x/2))$$

$$F_2{}^C = -\frac{B_C}{S_p} \int_L f_2{}^C\, dx$$

$$F_6{}^C = -\frac{B_C}{S_p} \int_L f_2{}^C\, x\, dx$$

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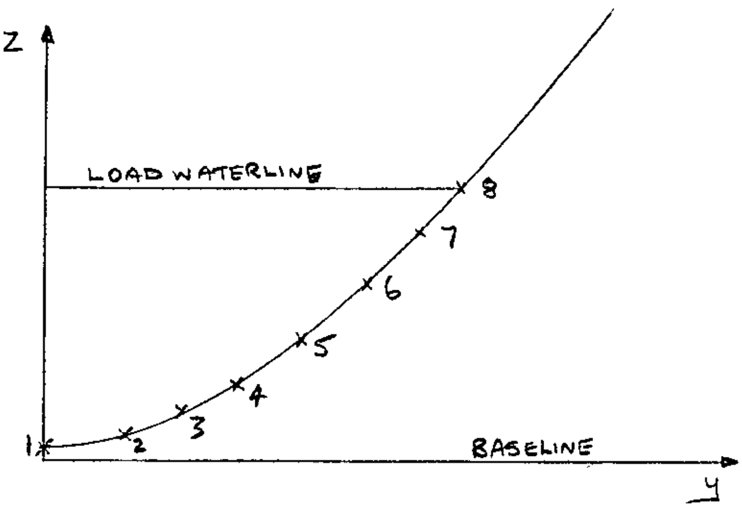

Fig 1 Station Offsets

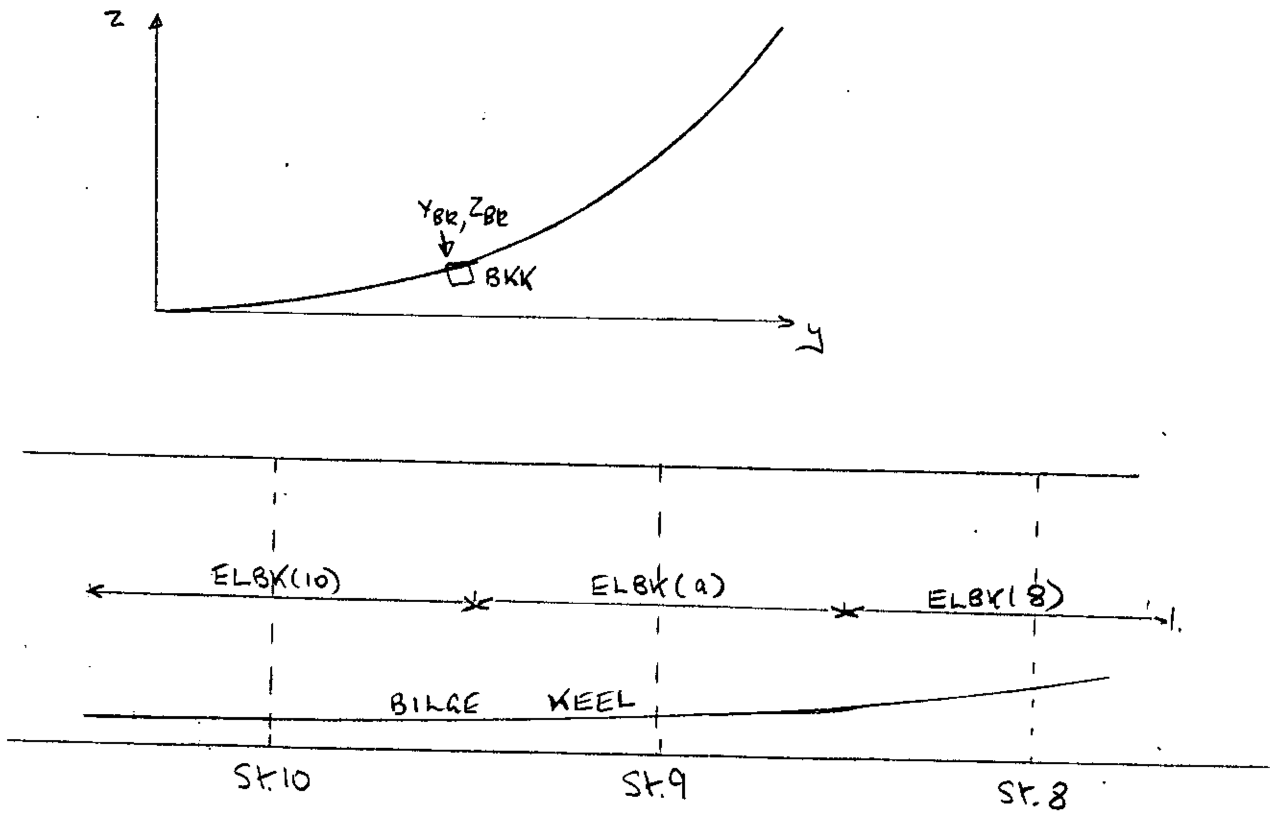

Fig 2 Bilge Keel Inputs

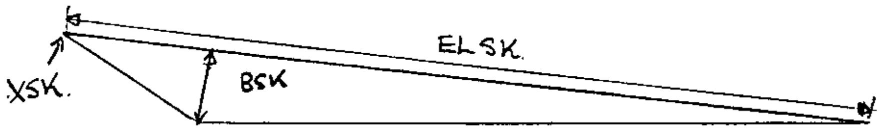

Fig 3 Skeg Inputs

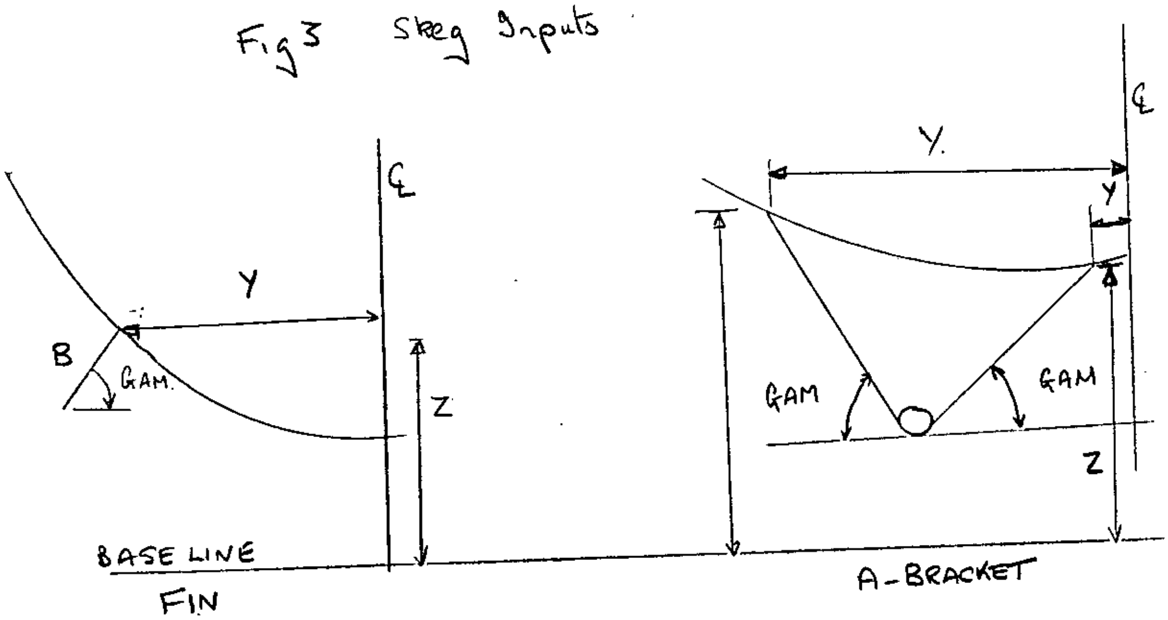

Fig 4 Foil Inputs

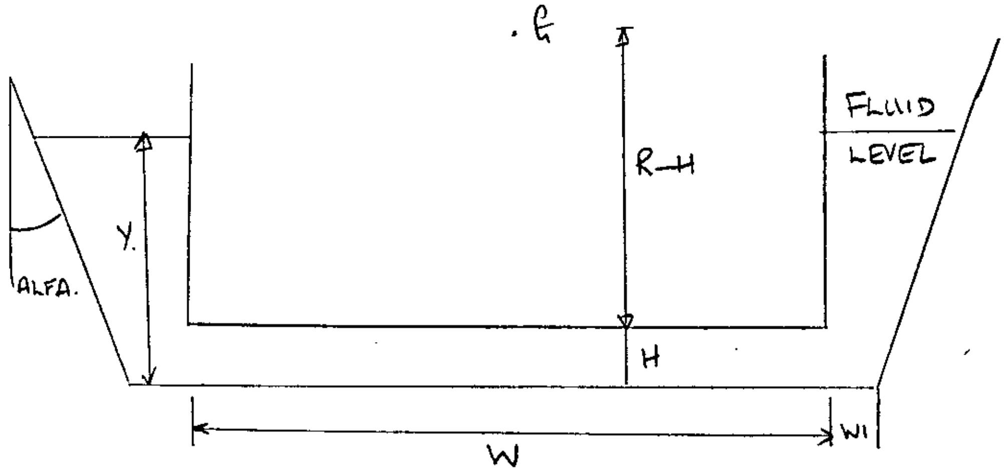

Fig 5 Tank Inputs.