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DEPARTMENT OF SHIP SCIENCE

FACULTY OF ENGINEERING AND APPLIED SCIENCE

THE APPARENT SHAPE OF TOWING TANKS

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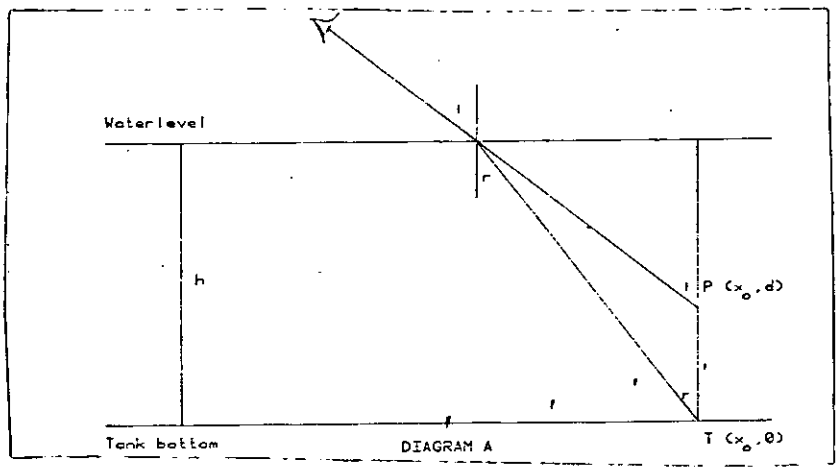
THE APPARENT SHAPE OF TOWING TANKS

At the University of Southampton the Department of Ship Science has a small towing tank which is used mainly for undergraduate project work. The immediate comment by all students when they first see the tank is, 'Is the tank bottom really curved?'. It is soon shown that this is an optical illusion by walking along the tank whereupon the curve moves with them. This technical note is written to help to predict the apparent shapes of towing tanks.

Consider the following diagram of a longitudinal section of a tank.



If the observer is situated at E, above the waterlevel, a height e above the tank bottom, with water of depth h, is looking at a point T, a distance x_0 ahead of the observer; then T appears to be at a shallower depth P, directly above T.

The phenomenon is brought about by the light being refracted between the two media. Snell's law allows the calculation that the depth P appears to be from the following formulae.

$$\frac{\sin i}{\sin r} = \mu \qquad (1)$$

Where i is the angle of incidence of the light at some distance x ahead of the observer, r is the angle of refraction of the light, μ is the relative refractive index of the two media. For water and air the refractive index is 1.33, it is in fact the ratio of the velocities of light in the two media.

If the observer looks directly below him at the tank bottom, it appears to be at a depth of $^1/1.33$ of the real depth, i.e. $^3/4$ depth. This is a useful check for the limiting value of the following theory.

Making use of Snell's Law and

$$\sin r = \frac{x_o - x}{\sqrt{h^2+(x_o-x)^2}} \qquad (2)$$

$$\sin i = \frac{x}{\sqrt{x^2+(e-h)^2}} \qquad (3)$$

∴ Equation (1) can be rewritten with equations (2) and (3) substituted as follows

$$x^4(\mu^2-1)-2x_o(\mu^2-1)x^3+x^2(x_o^2(\mu^2-1)-h^2+\mu^2(e-h)^2)-2x_o(e-h)^2\mu^2x$$

$$+\mu^2(e-h)^2x_o^2 = 0 \qquad (4)$$

Thus for known values of μ, e, h, and x_o it is possible to find the depth co-ordinate of P, i.e.

$$d = h - \frac{(e-h)(x_o-x)}{x}$$

Equation 4 is most easily solved using Newtons method to find a root of a polynomial. Although equation 4 could be solved by Ferrari or Tartalgia, general methods of solution for quartics. If this theory is now extended, to a three dimensional view of the bottom of the towing tank, as in diagram B, then it is possible to

calculate the apparent shape of the opposite bottom edge of the towing tank.

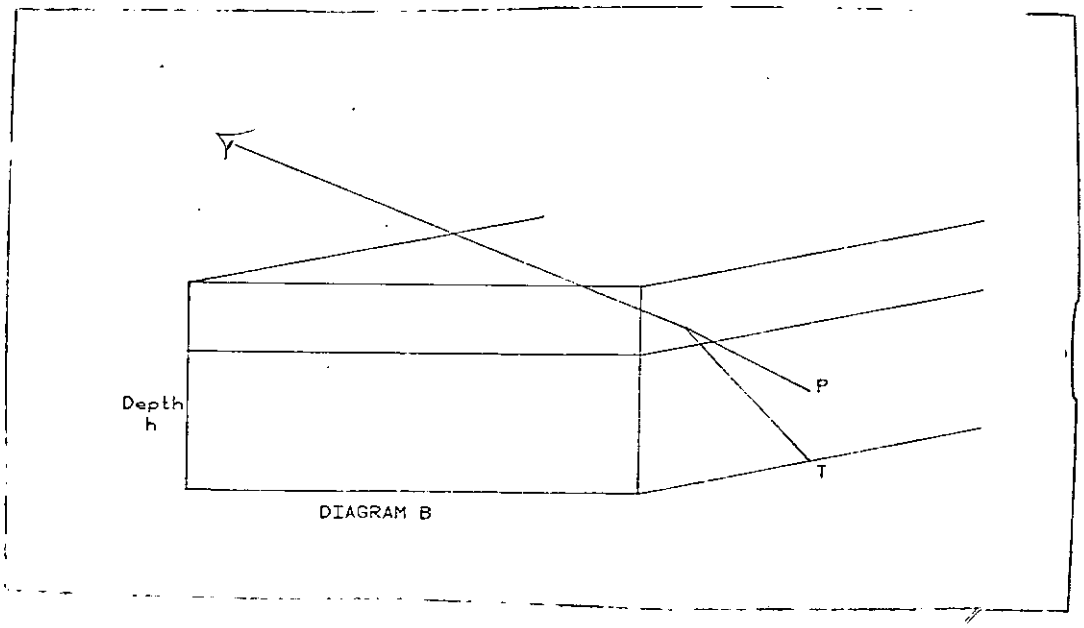


DIAGRAM B

This method of solution has been applied to eight towing tanks in England, for which the depth and width has been found from published data.

In each of the eight figures, except figure 4, there are two curves. The upper curve presents the apparent shape of the bottom edge of the tank directly below the observer as a function of tank length. The lower curve represents the apparent shape of the opposite tank bottom edge as seen by the observer. In all cases, except figure 4, the observer is 1m above the water level.

In figure 4, because of the width of this tank, an observer 1 metre above the water level cannot see the opposite bottom edge, because of total internal refraction of the light. In this case, three curves are presented. The upper curve is for an observer 3m above the waterlevel, looking at the edge directly below the observer. The middle and lower curves are for observers 3m, and 2m above the water level looking at the opposite bottom edge.

In all seven cases the theory predicts the sharply curved shape of the edge directly below the observer. The theory also predicts that narrow towing tanks appear, when viewing the opposite bottom edge to be highly curved, whereas wide tanks appear to be almost flat, if a little shallow!

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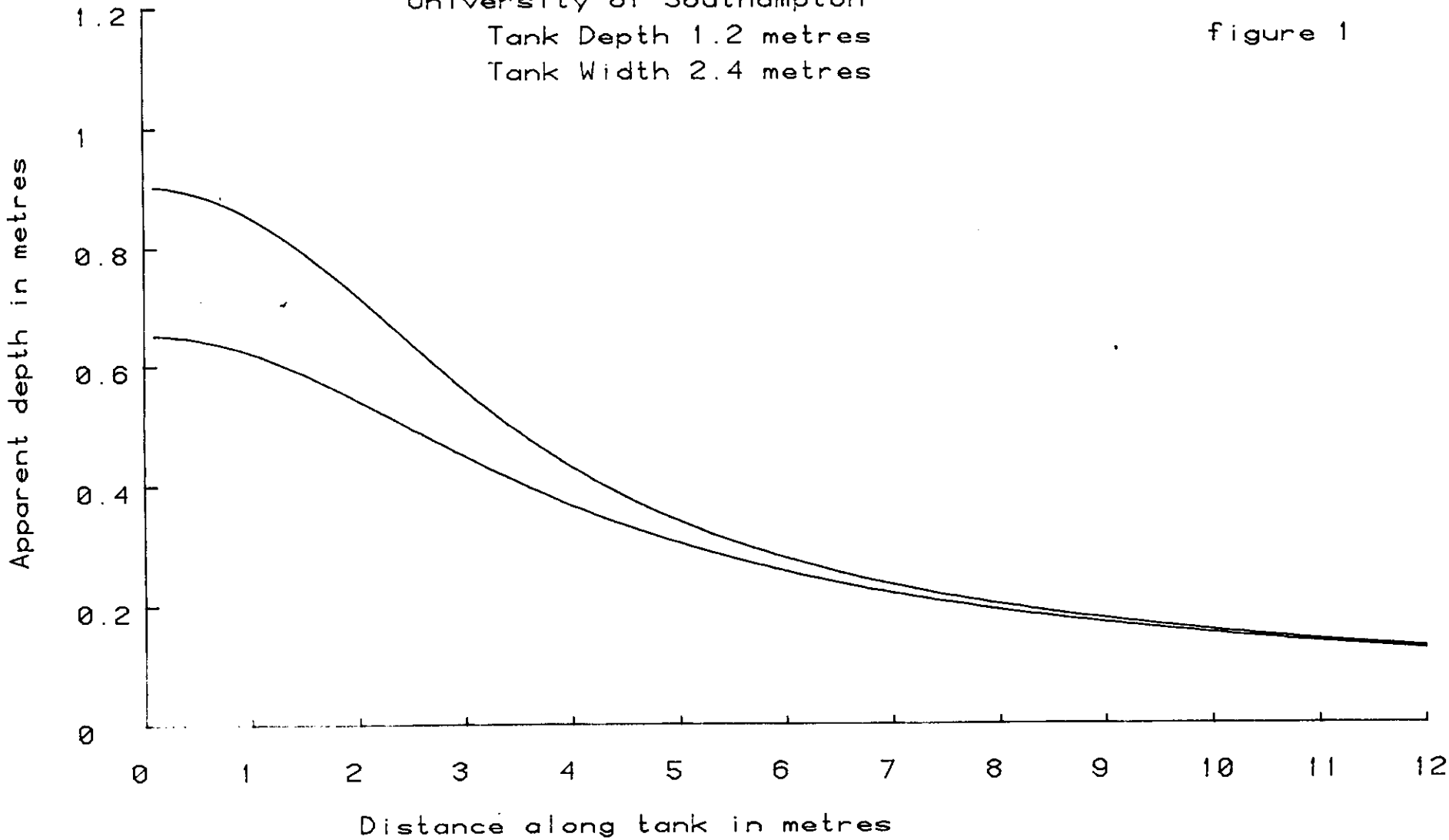
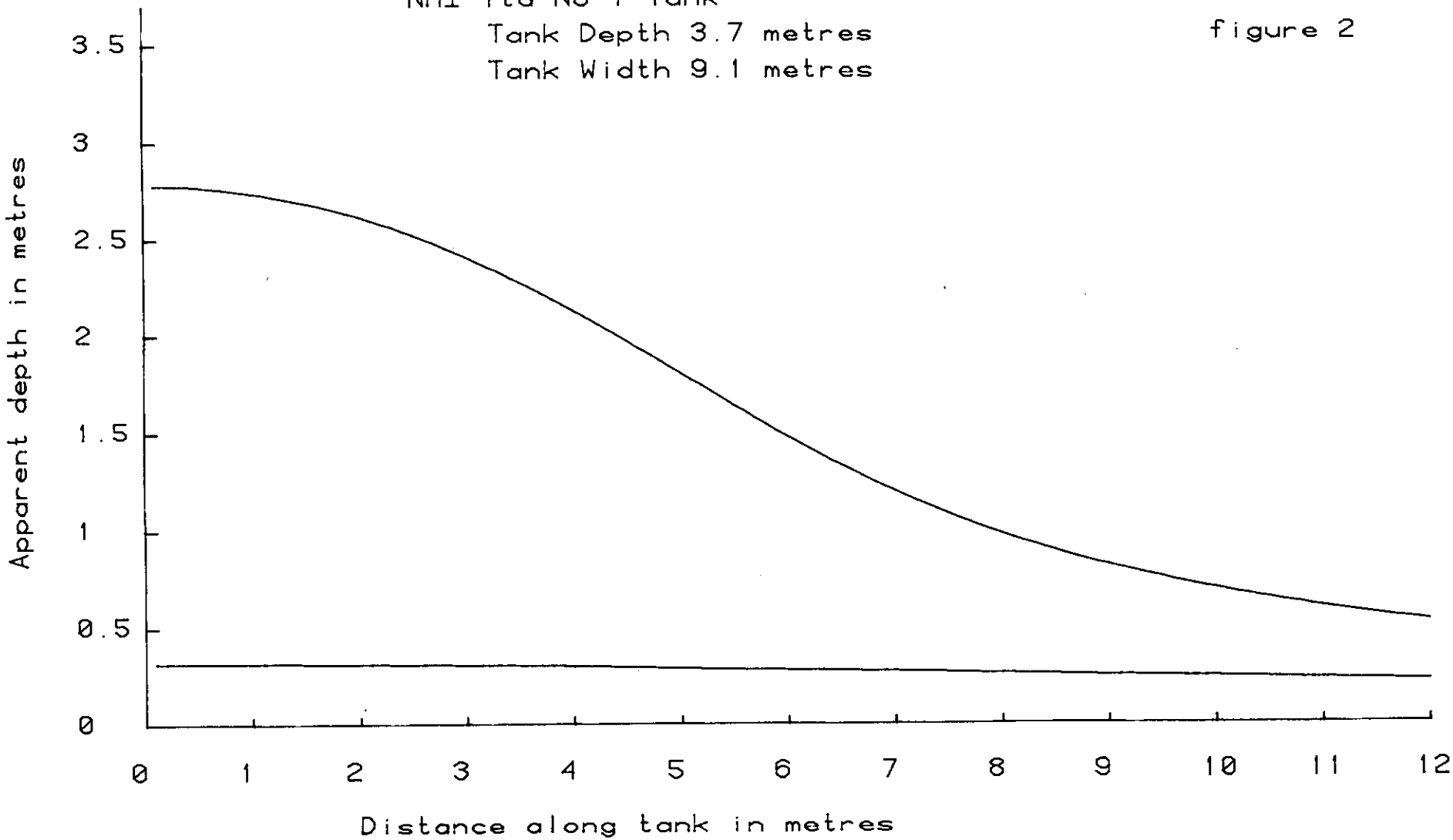
Tank Depth 1.2 metres
Tank Width 2.4 metres

figure 1



NMI ltd No 1 Tank
Tank Depth 3.7 metres
Tank Width 9.1 metres

figure 2



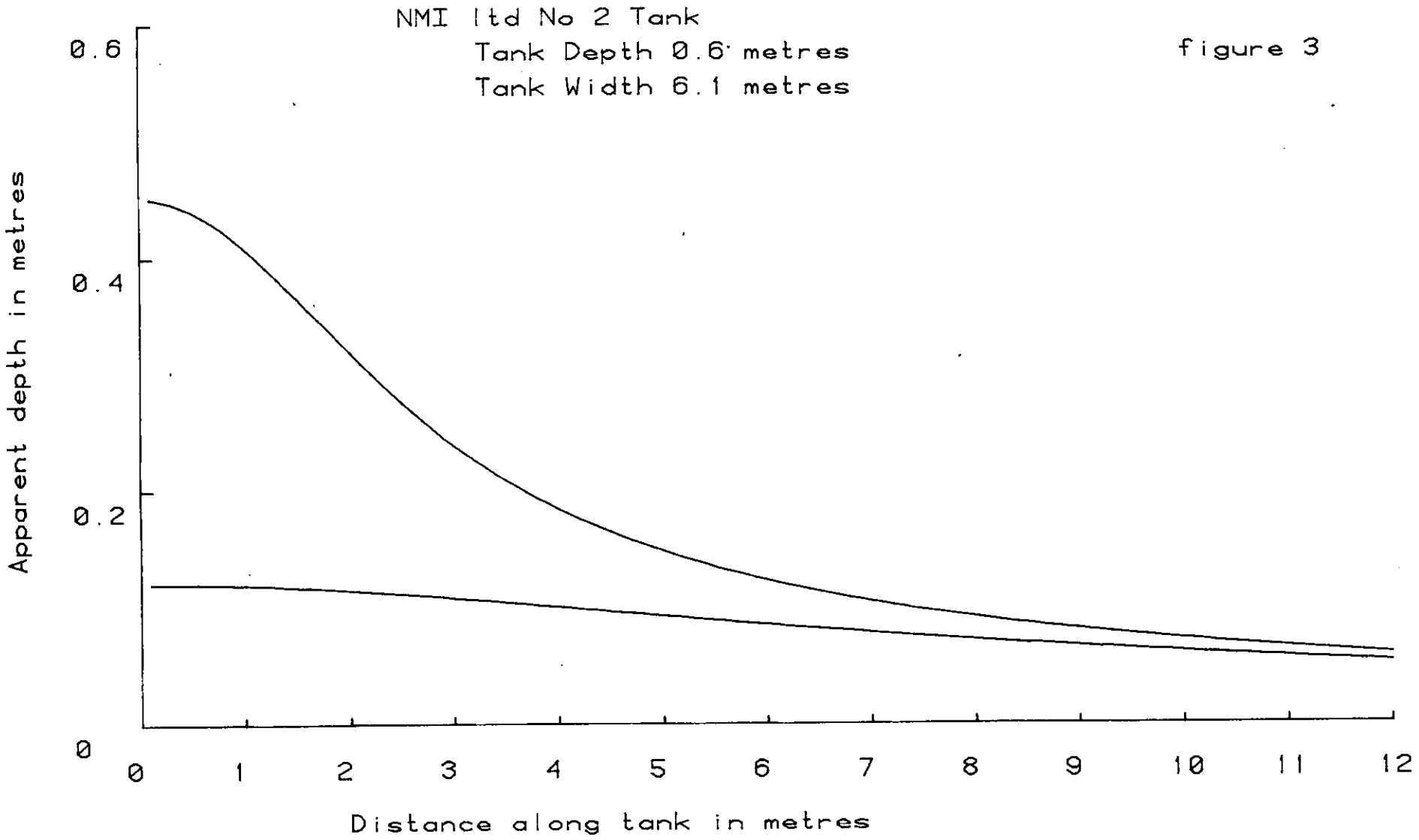
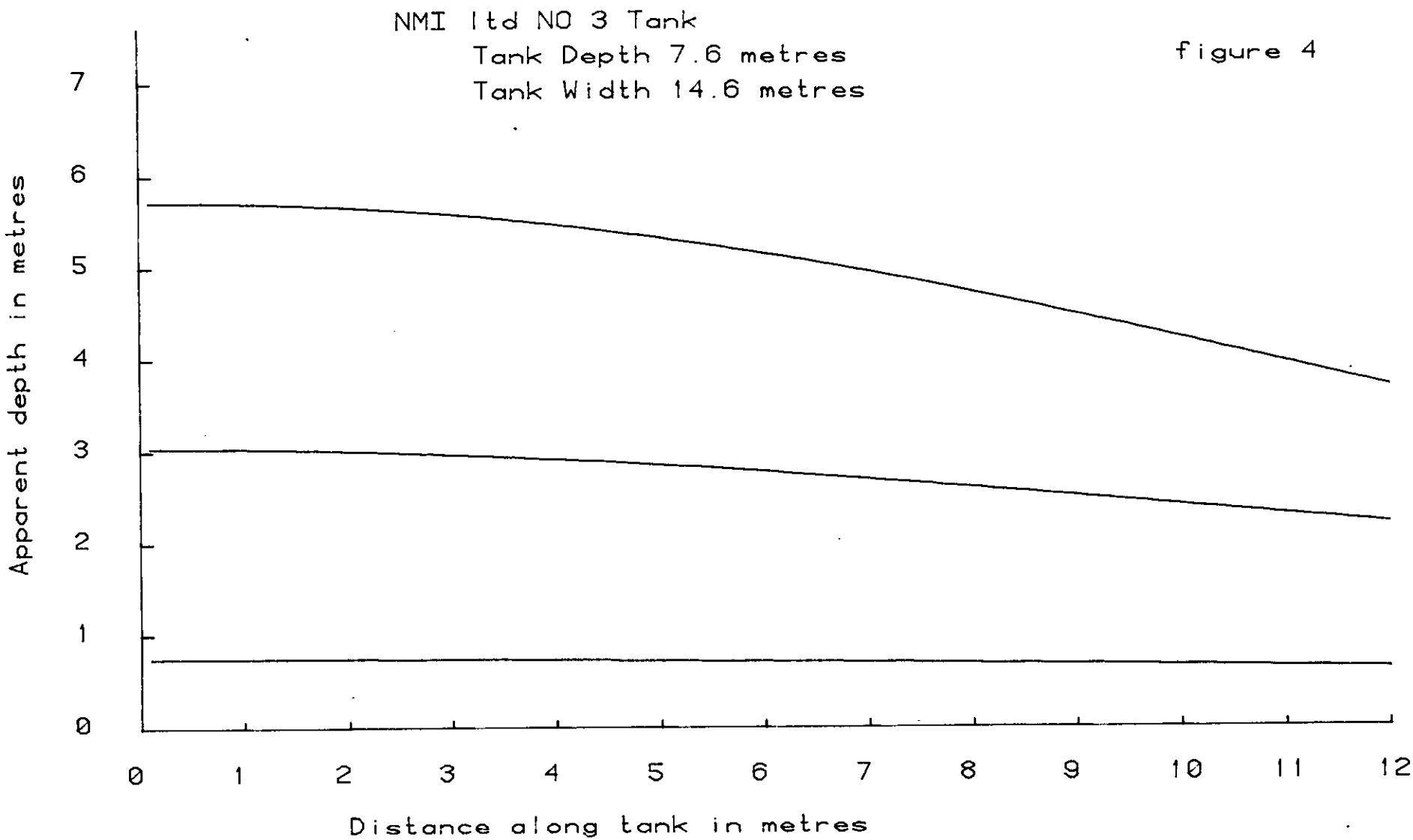

NMI ltd No 2 Tank
Tank Depth 0.6 metres
Tank Width 6.1 metres
0.6
0.4
0.2
0
Apparent depth in metres
0
1
2
3
4
5
6
7
8
9
10
11
12
Distance along tank in metres

figure 3

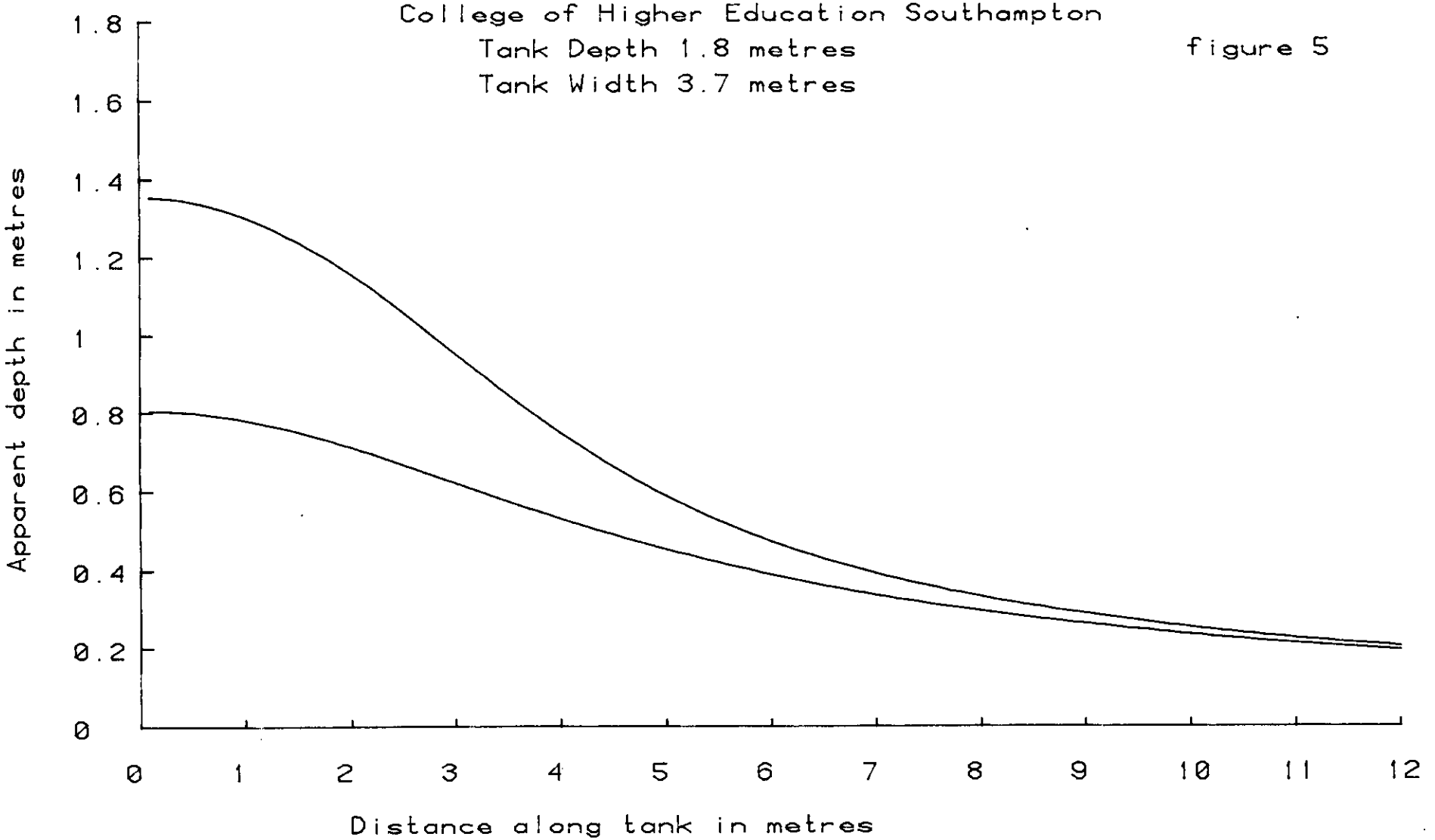

NMI ltd NO 3 Tank
Tank Depth 7.6 metres
Tank Width 14.6 metres
figure 4
Apparent depth in metres
0
1
2
3
4
5
6
7
0
1
2
3
4
5
6
7
8
9
10
11
12
Distance along tank in metres

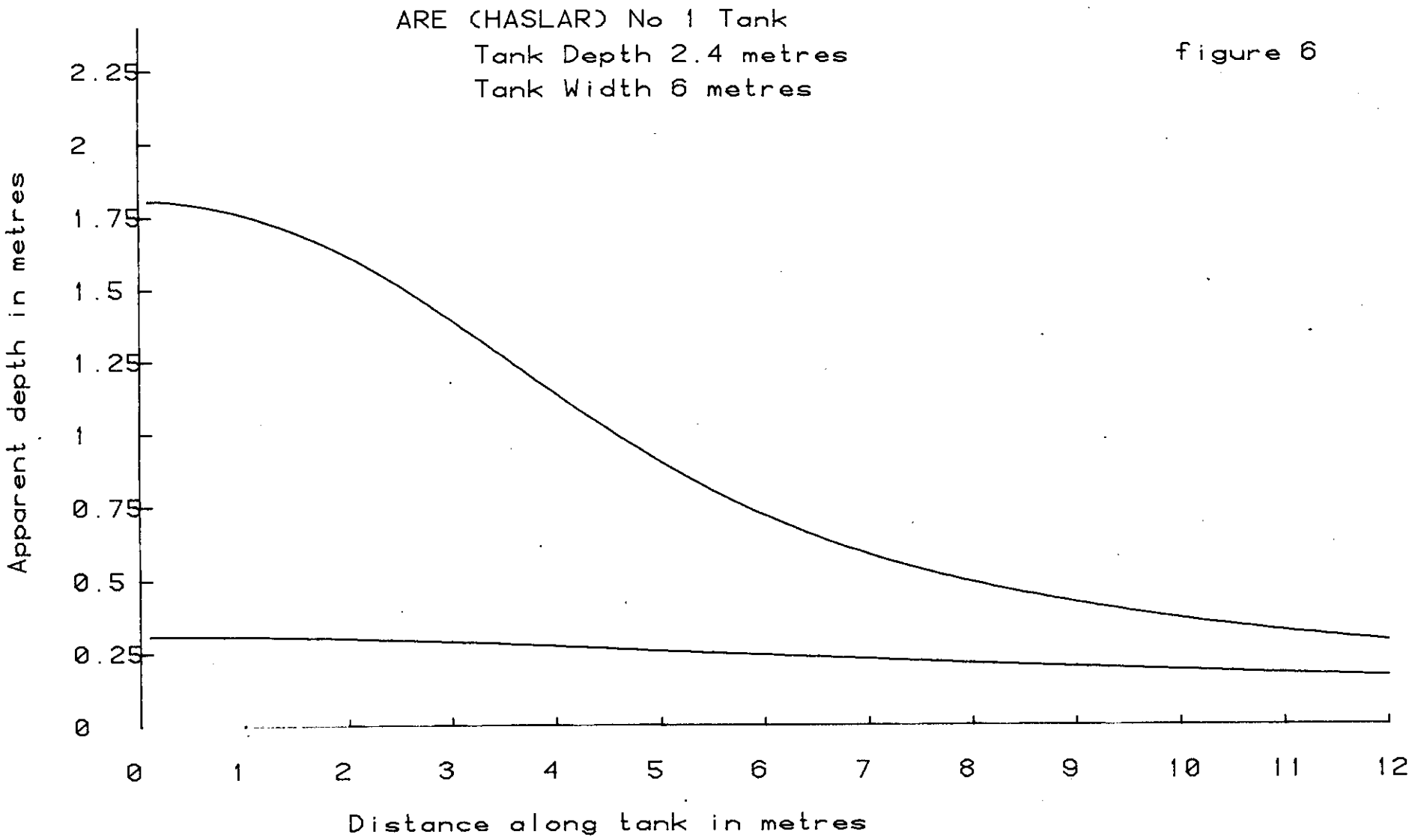
College of Higher Education Southampton

Tank Depth 1.8 metres

Tank Width 3.7 metres

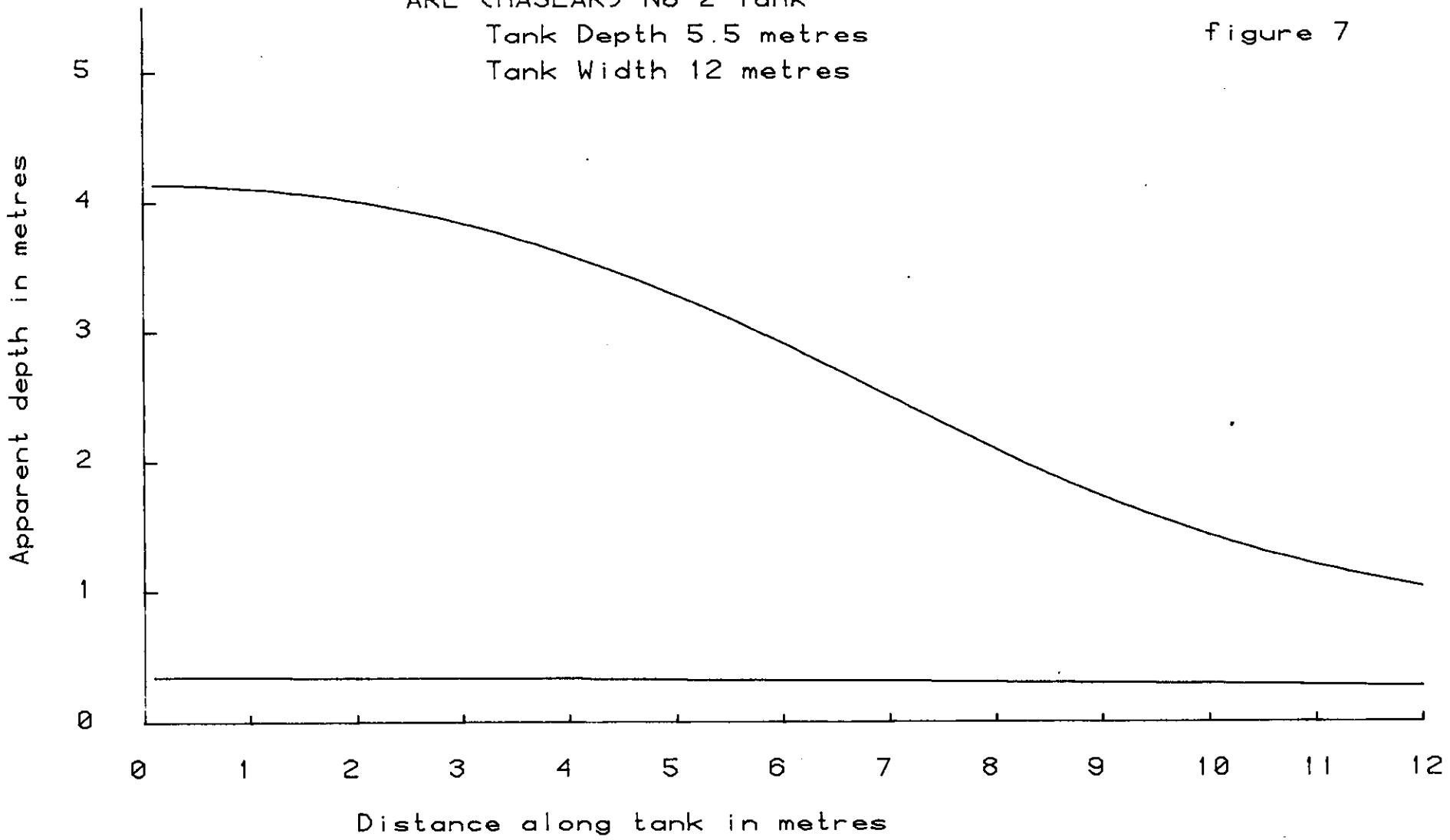
figure 5

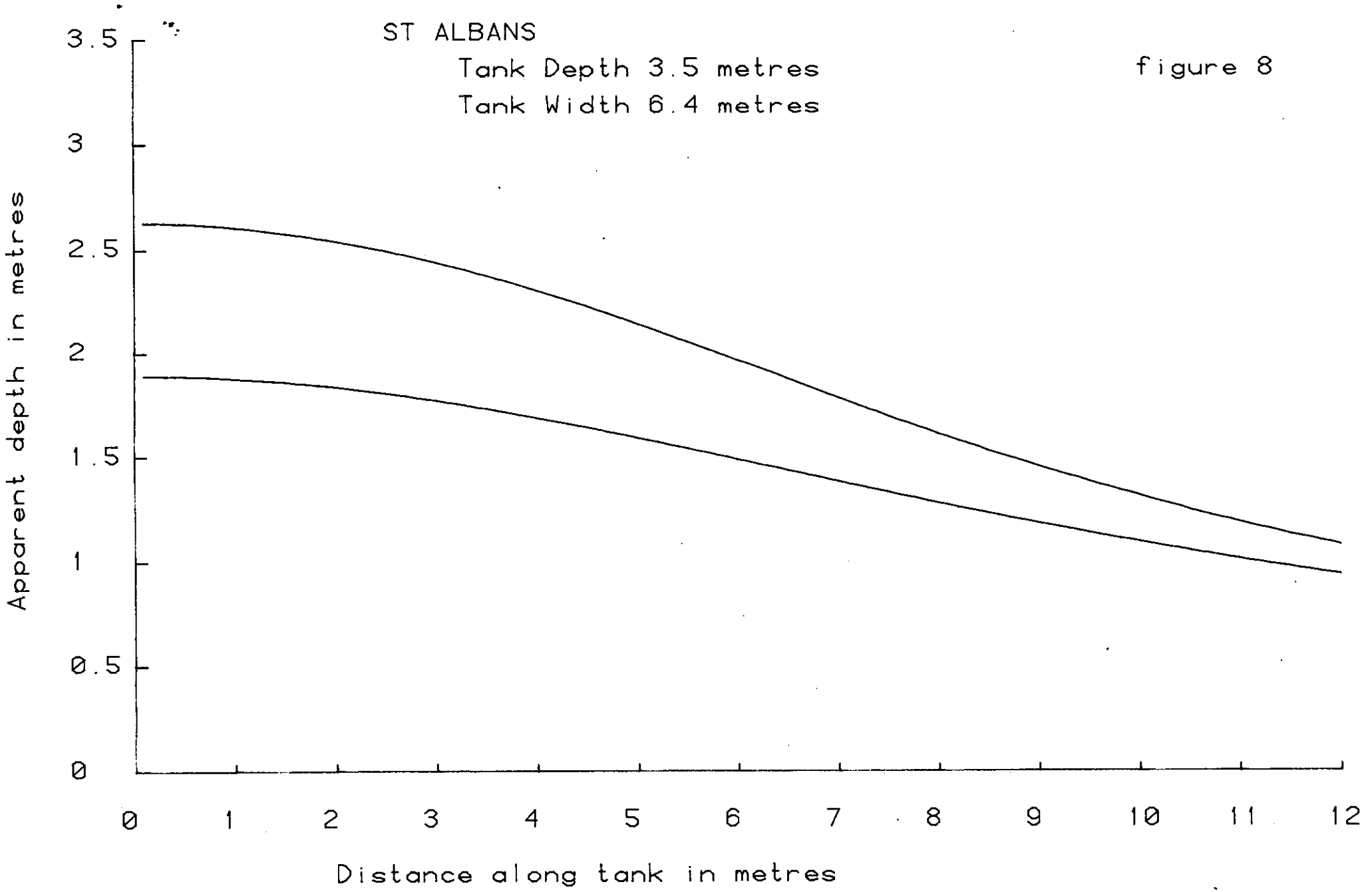



ARE (HASLAR) No 1 Tank
Tank Depth 2.4 metres
Tank Width 6 metres
figure 6
Apparent depth in metres
2.25
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
0
1
2
3
4
5
6
7
8
9
10
11
12
Distance along tank in metres

ARE (HASLAR) No 2 Tank
Tank Depth 5.5 metres
Tank Width 12 metres

figure 7




ST ALBANS
Tank Depth 3.5 metres
Tank Width 6.4 metres
figure 8
Apparent depth in metres
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
7
8
9
10
11
12
Distance along tank in metres

Reduce PPW

A & B.

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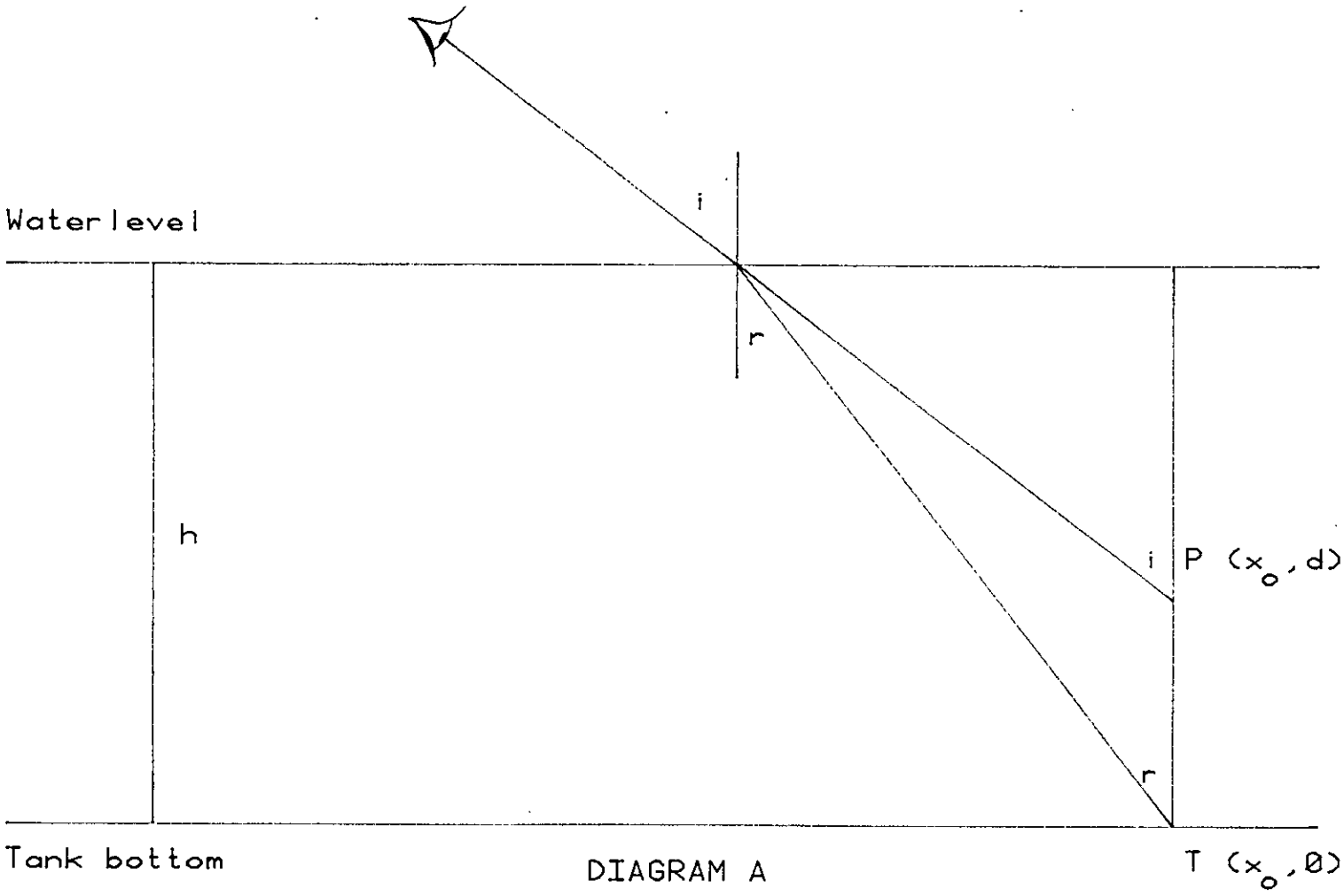

DIAGRAM A

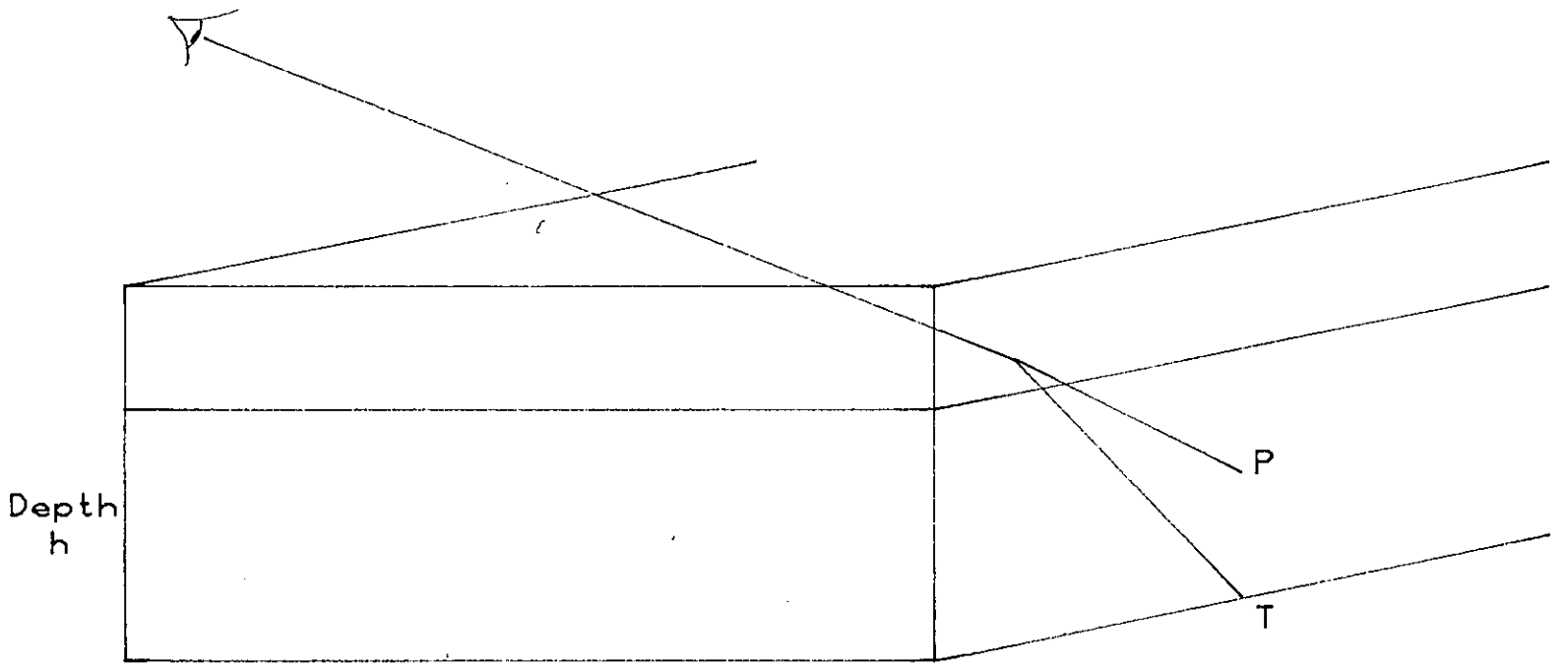

DIAGRAM B