UNIVERSITY OF SOUTHAMPTON

FACULTY OF BUSINESS, LAW AND ART

SOUTHAMPTON BUSINESS SCHOOL

Optimal production,scheduling,lot-sizing and power generation for a soft drink factory in absence of a power grid

by

Abubakar Yusuf Baba

A thesis submitted for the degree of

Doctor of Philosophy in Management Science

May 2019

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Abstract

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by Abubakar Yusuf Baba

Lot-sizing and scheduling has become an important aspect of manufacturing businesses nowadays. The effective and efficient management of scheduling and planning has the potential of reducing production cost which leads to increase in quality of products, which brings about competitive advantage. Lot-sizing and scheduling is a critical success factor especially in developing countries that lack technological infrastructure and resources available in developed countries. Due to the peculiar nature of the issue in Nigeria, the unit commitment problem (UCP) has to added to production planning to include energy considerations.

Inspired by the complexity and the challenges faced by the soft drink industry in Nigeria. The aim of this thesis is to develop mathematical models for production planning that minimises production and backorder cost, evaluate the models under various demand profiles and determine the power needs of the factory under various demand profiles. Secondly, the thesis examines the operational and performance of diesel generators in preparation for model development, develops models that help determine minimum investment and running costs of diesel generators which falls under the unit commitment problem.

The contributions of this thesis brings about development on a academic front by developing mathematical models that combine production planning and unit commitment with real world implications.

A case study of a Nigerian soft drink producing company provides motivation and provides practical importance for the contribution of this thesis in the real world.

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Declaration of Authorship

I, Abubakar Yusuf Baba , declare that this thesis titled, 'Optimal production, scheduling, lot-sizing and power generation for a soft drink factory in absence of a power grid' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed : **Abubakar Yusuf Baba**

Date : **May 2019**

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Chapter 1

Introduction

Chapter 1

Introduction

1.1 Motivation and context of the research

This study will conduct issue oriented academic research with an aim to achieve real-world impact. The case study used to develop this work concerns the development of a business case for a soft drink bottling factory in Nigeria. The development of an academic Ph.D study in conjunction with the development of a real business is a unique setting with the potential for providing both practical benefits and academic innovation.

Manufacturing has been the backbone of the Nigerian economy and other African countries. It has been providing employment and has been the source of livelihood especially for the population in the rural areas (Omorogbe et al., 2014). However in recent years, Nigeria has suffered massive decline in the manufacturing sector due to high dependence on the oil sector (Brian, 1989). Nigeria that was once a great exporter of food and livestock now heavily relies on imports (Osaghae, 1998). Attempts to revive the sector have failed due to the lack of technology and proper logistics (Adebambo and Toyin, 2011).

Manufacturing holds immense potential for enhancing and stabilizing the country. The increase in competition globally requires Nigerian manufactures to become more effective and efficient. Amongst the success factors necessary for achieving this are planning systems and a stable energy supply.

1.2 Context of the case study

The soft drink bottling factory under consideration is not yet built, but to be located in the Northern part of Nigeria on an available piece of land. The climate in this part of the country is arid, with a long dry season and short wet season. Demand for bottled (flavoured) soft drink is high due to the climate and the fact that bottled water is purified and safe to drink.

The land on which the factory is to be situated has the benefit of being able to offer a stable supply of source water via bore holes due to the favorable presence of underground water. The factory is expected to be able to provide bottles in various sizes and flavours to a school and, potentially, also to various shops in the area.

The case-based issue oriented research to be conducted has to help the developer assess the economic viability of the proposal, which includes assessing the day-to-day running costs of the facility, as well as developing insights into how to account for the impact of uncertainty.

Uncertainty arises first and foremost from not knowing the demand with certainty. While there is an agreement in principle with the school, the uptake from shops is unknown at present. This is in part determined by the competition from international soft drink companies. While the local production will offer the benefit of much lower transportation costs in comparison, and may create goodwill amongst the local population to support the local economy, the price per bottle and the availability of the products remain important considerations. The factory developer therefore seeks to better understand how to minimize production costs and maximize service levels (minimise back-orders) and this under various possible scenarios of demand uptake.

The lack of a reliable power supply network, however, requires the factory to generate its own source of power needed to run the production lines, as well as all supporting facilities. This will be done by means of diesel power generators. The additional investment cost in these diesel generators needs to be considered. The set of generators selected needs to be able to handle various power demand profiles while minimizing running costs of the generators. It is also to be investigated to which extend the (uncertainty on) diesel fuel price affects production costs.

Finally, the research should culminate into the development of a Net Present Value calculation to help assess this business opportunity, which may be needed to help attract investors to raise the necessary starting capital.

1.3 Research aim and academic approach

Considering the above stated expectations of the business developer, the aim of this research is *to provide an academic approach to help maximize the long-term profitability of the envisaged soft drink bottling factory, and create insight into the impact of uncertainty.*

To reach this aim, the research will primarily focus on the examination and suitable adaptation and application of OR/MS methods developed in two relevant areas, namely lot sizing and scheduling of production lines, and electricity generation in an island manufacturing setting, i.e. a setting without an external power supply. Both areas also need to be reviewed in the light of uncertainty about future demand.

There are several interesting issues to be addressed:

1.3.1 Production technology and production methods

The production problem encountered in the soft drink industry it has some features that are not present in the production line problems studied so far such as not having an electric grid conntected to the line.

Furthermore, the local context also matters. The availability (or lack thereof) of local natural and human resources and the accessibility (or lack thereof) of technology and supporting infrastructure has to match the envisaged factory set-up.

The acquisition of technologically advanced machinery, for example, may bring efficiencies into the process but one should also consider the cost and availability of sufficiently technically skilled personnel. Most importantly, this also concerns the level of technical support for proper maintenance and repair of such technology and the reliability of service provision.

The production set-up will therefore consist of adopting well-proven systems in the industry with modest levels of automation.

1.3.2 Power generation in absence of a reliable electricity/power grid

Related problems have been investigated in the literature on the so-called unit commitment problem in the context of so-called 'island manufacturing' (Mandelli et al., 2016). However, in the context of this study, the power generation is not through large power plants, providing electricity to a population of users, but has to come from a local source tailored to the factory needs.

The use of diesel generators has been selected by the business developer as these are commonly used by local manufacturing industry as well as by the more affluent local population. This technology can therefore be supported by good local services.

While the (combined) use of alternative power sources (solar power, wind energy) may reduce running costs, there are several drawbacks that make this a less attractive option. This includes the higher investment costs, the variable and less controllable output, and the lack of local expertise is servicing these technologies.

1.3.3 Production and power generation in the context of uncertainty

As mentioned earlier, not knowing final demand a priori is a major source of uncertainty for the business developer. This level of uncertainty is primarily related to the evolution of the demand in the long term.

While forecasting this evolution is difficult, the factory can expect, however, to have a good forecast of demand in the short term based on customers having placed their main orders at least one to two weeks in advance of expected delivery.

1.3.4 Net Present Value of the project

The application of previously developed methods is expected to lead to quantitative results to help establish the Net Present Value of the proposed project. This final phase of the project will thus require the application of project evaluation techniques from the area of corporate finance. Special attention may need to go to methods for dealing with (correlated) random inputs.

1.4 Research objectives

Having set out the main research aim and four areas of attention in the previous section, establishing a more detailed research objectives, structured around the following three areas are formed:

1.4.1 Production scheduling in a soft drink bottling factory

- Establishing the process logic of the envisaged factory through the use of literature review of the lot sizing and scheduling models of production lines, and soft drink bottling plants in particular;
- Develop a mathematical model for planning the production at minimum production and back-order costs. A model that is flexible enough to deal with the specific industrial issues. The issues are the design (best tank capabilities, number of production lines and power requirements). The mathematical model has to be extendable or even reformulated in order to deal with specific industrial issues of practical implementation.
- Using the mathematical model to determine the power needs of the factory under various demand profiles.

1.4.2 Power generation with diesel generators

- Examine the operational and performance characteristics of diesel generators in preparation of model development, which finds an optimal way to run a set of diesel generators to meet a given demand.

- Literature review of relevant areas, including unit commitment problems, in particular in the context of island manufacturing;
- Develop model(s) to help determine minimum investment and/or running costs of a set of diesel generators to meet a variety of power demand scenarios;
- To develop a mathematical model that combines optimises the investment, fuel consumption and lot sizing and production planning.

1.4.3 Net Present Value (NPV) of a bottling plant investment project

- Identify the main characteristics of the various cost and revenue components for the bottling plant factory project;
- Review the relevant area from corporate finance of how to determine the NPV of a project in the context of uncertainty (on revenues, costs, ...);
- Develop a model to calculate the NPV of the bottling factory proposal, making use of quantifiable results obtained in previous parts of the dissertation;
- Extract managerial insights about the profitability and risks of the project.

1.5 General structure of the thesis

The outline of the remainder of this transfer thesis is as follows.

Chapters 2, 4, and 6 are, respectively, dealing with the above identified problem areas of production, power provision, and overall project evaluation. These chapters serve to meet the research objectives reported in each of the corresponding sections in Section 1.4. For example, Chapter 4 will start with the development of the properties to be taken into account regarding the production of power by diesel generators, and then proceed with a review of literature on models and algorithms to solve similar power generation problems.

Chapter 3, 5, and 7 deal with the remaining objectives identified in Section 1.4 in the areas, respectively, of production, power provision, and NPV analysis of the overall project.

Chapter 10 summarizes the future work and conclusion.

Chapter 2

Production planning in a soft drink factory

Chapter 2

Production planning in a soft drink factory

2.1 Soft drink industry

The soft drink industry consists of companies that produce, package, sell and deliver beverages to their customers. Soft drinks are popular in many markets and demand has been increasing due to the low cost of soft drinks, and they have become repeat purchase items. South-America has the largest producers of soft drinks globally with over 800 plants, and a customer base consuming over 10 billion liters per year. The variety of soft drinks offered to consumers, the production scale and the advancement of current filling lines require production plants and facilities to adopt optimisation techniques that provide the plants with efficient and effective production schedules and lot sizes (Ferreira et al., 2008).

2.2 Modeling of lot sizing and scheduling problems

Production planning is an activity that selects the most optimal use of resources used in production in a way that satisfies the necessary requirements for a given period of time, known as the *planning horizon*. *Scheduling* refers to the process of allocating a number of available tasks to a number of available resources over time within the time horizon. *Lot sizing* decisions identify how and when a product should be produced, and the optimal level of production, so that relevant costs (e.g. set up and holding cost) are reduced to an optimal or desirable level (Jans and Degraeve, 2008; Karimi et al., 2003).

2.2.1 General problem characteristics

The following are common characteristics of production scheduling and lot sizing problem.

- **Planning horizon.** This is the overall length of time in which the production schedule has to be determined. Lot sizing and scheduling problems are typically modeled based on time buckets or time periods, where the sum of considered time buckets corresponds to the planning horizon. Models can be further categorized as small bucket or large bucket problems. In a small bucket problem the time period allows for the production of one product, while in a large bucket problem more than one product can be produced within one time period. If demand and other data do not change from period to period, the problem is static and the resulting scheduling solutions are typically simple and repetitive. This usually assumes an infinite planning horizon. When demand or the scheduling solutions may change from period to period, the problem is called dynamic. This is usually solved by assuming a finite horizon. In a rolling planning horizon approach, only the first time bucket scheduling solutions are implemented, and the model is re-optimized over the same time horizon, but shifted one time bucket into the future. In cases where there is uncertainty in data, a rolling horizon approach is possible but optimal solutions are not guaranteed (Karimi et al., 2003). The rolling horizon sometimes leads to additional costs, because schedules may have to be changed frequently (Kazan et al., 2000)

- **Number of levels.** Production can either be single or multi-level. In single level production, the production process is simple and only requires one action or stage. Multi-level production involves various stages, whereby the raw materials are going through numerous processes in various possible structures (serial, convergent or divergent, network, ...) (Karimi et al., 2003).

- **Number of Products.** The complexity of production scheduling and lot sizing typically increases as the number of products increases. Van Hoesel and Wagelmans (2001) illustrate algorithm/computational performance differences between single- and multi-level production settings.

- **Capacity constraints.** The production scheduling or lot sizing environment can also be categorized into *capacitated* or *un-capacitated* problems. A problem that has restriction on resources is know as a *capacitated* problem. Capacity constraints are important in most practical settings, and affect the complexity of the problem (Karimi et al., 2003).

- **Demand.** Demand is typically an input to the model. When the (dynamic or static) demand is known (as well as all other data), the problem is *deterministic*. When the demand is not known with certainty but governed by known probability distributions or stochastic information, the problem is *probabilistic*.

- **Set-up Cost/Time.** Set-up costs and time's model the costs or times incurred from 're-set' operations performed on a resource (machine, process), e.g. due to the changeover from one type of product to another. These set-ups typically introduce binary (0/1) variables in the mathematical model, which makes the problem more challenging. The change over cost from one product to another product incurs additional set-up cost and time. The set-up is *simple* when the set-up time and cost are independent from previous actions (within the same period or from previous periods). The set-up cost or time is incurred when e.g. changing production to a new type of product, but the set-up cost and time only depend on that new product type. The various types of *complex* set-up structures can be distinguished in the structure. *Sequence-dependent* set-up costs or times depend on the sequence of production, e.g. are dependent on which product type replaces which product type. If production can be continued using the settings of the previous period, then in order to continue in the current period, a *set-up carry-over* cost or time may be incurred. This may account for example that the machine has been stopped during the night, and needs e.g. a warm-up or minor re-calibration to restart the next day. There exist other varieties, such as e.g. larger set-up costs and time when changing between families of product types, and smaller set-up costs and times when changing product types within the same product family.

- **Backlogging cost.** These costs account for situations when there are shortages, i.e. when the product demand cannot be met on the due date. Such costs typically increase with the number of periods being late.

The characterization of the problem in this thesis is as follows:

- finite horizon, dynamic demand, 2-level production, 4*2 products, capacitated (flavor tanks, filling lines), set-up times and costs sequence-dependent.
- Flavor tanks: change-over = cleaning, preparation, usage by filling lines.
- Filling lines: change-over = cleaning & reset to next product type, usage.

- Deterministic dynamic demand. Justified see introduction ; assume known orders at least 2 weeks. Uncertainty on overall demand volume will be captured through examinations of different scenarios.

2.2.2 Formulation of the lot sizing and scheduling problem

As an illustration,the general formulation of the single-item uncapacitated lot sizing/scheduling problem as formulated in Karimi et al. (2003) is presented.

Assumptions

- The planning horizon is finite and has T periods.
- Demand of a period has to be satisfied at the end of the period.
- The unit production cost may depend on the period.
- A set-up cost is incurred when production occurs in the period.
- Shortages are not allowed.
- A holding cost is incurred for products in inventory at the end of a period.

Notations

- S_t set-up cost in period t
- Y_t 1 or 0 variable if the product is produced or not in period t
- C_t variable cost of production per unit in period t
- X_t amount of production in period t

- h_t inventory holding cost in period t
- I_t ending inventory at the end of period t
- $M_t = \sum_{t=1}^{T} dt$

The optimisation model is then:

$$
\begin{aligned}
\min Z = \quad & \textstyle\sum_{t=i}^{T} \Big(S_t Y_t + C_t X_t + h_t I_t \Big) & & \\
s.t. \quad & X_t + I_{t-1} - I_t = d_t & & (t = 1, ..., T), \\
& X_t \leq M_t Y_t & & (t = 1, ..., T) \\
& Y_t \in \{0, 1\} & & (t = 1, ..., T) \\
& X_t, I_t \geq 0 & & (t = 1, ..., T)
\end{aligned}
\tag{2.1}
$$

The objective function minimizes the sum of production, holding and set-up costs. The first set of constraints are mass-balance equations: the inventory at the start of a period plus the production in a period should cover for the demand in that period and the inventory at the end of the period. The third set of constraints express that it is not possible to produce in a period without incurring the set-up cost. The M_t represent large positive constants.

The above model formulation can be solved efficiently by means of dynamic programming (Wagner-Whitin algorithm) or shortest path calculations (Wagelmans et al., 1992; Aggarwal and Park, 1993; Federgruen and Tzur, 1991). The economic lot sizing problem with capacity constraints, limiting the level of production that can occur in each period, is NP-hard (Gallego and Shaw, 1997). An NP-hard problem is a problem that the algorithm for solving the problem can be translated into an algorithm for solving any non-deterministic polynomial time problem (NP-problem). Therefore, NP-hard means at least as hard as any NP-hard problem, though it can be harder.

Extensions of this basic model are often needed to better match the conditions existent in real industry problems, see e.g. Sambasivan and Yahaya (2005). In a multi-product setting, for example, decisions about schedules and lot sizes may need to occur within each period.

In the industry practice, the problem is often solved hierarchically. For example, first the production lot sizes are determined, and afterwards the production schedules. However, it is often of interest to integrate the two problems in the decision process. Production lot sizing and scheduling problems can be very difficult depending on the restrictions which have to be met and the combinatorial structure. The production manager needs to consider product demands, machine capacities, raw material availability, set-up times, among other aspects.

2.2.3 Types of lot sizing and scheduling problem

The literature on this type of problem is extensive. In this dissertation, the main focus on will be deterministic lot sizing. see section 2.3.1

The General Lot sizing and Scheduling Problem (GLSP) considers the scheduling and lot sizing of numerous products on one machine, subject to constraints. Demand (static or dynamic) over a finite planning horizon has to be met without backlogging, so that the total of set-up and inventory holding costs is minimized, (Fleischmann and Meyer, 1997; Meyr, 2000). The scheduling and lot sizing problem in general is classified as NP-hard (Chen and Thizy, 1990; Bitran and Yanasse, 1982).

There are five common variations of the lot-sizing problem, they are as follows: the capacitated lot sizing problem (CLSP), the economic lot sizing problem (ELSP), the discrete lot sizing problem (DLSP) and the proportional lot sizing problem (PLSP) (Karimi et al., 2003).

The economic lot sizing problem (ELSP) can be modeled as a single-level or a multi-level problem with stationary continuous demand, and infinite planning horizon. The multi-level economic lot sizing problem with capacity constraints is NP-hard (Gallego and Shaw, 1997).

The discrete lot sizing problem (DLSP) divides the periods in the capacitated lot sizing problem into smaller periods. The assumption of the discrete lot sizing problem is the all-or-nothing production, which means only a single product, can be produced in each period and the amount produced uses full capacity that is available. Based on this, the discrete lot sizing problem (DLSP) is known as a small bucket problem. The DLSP also introduces the connection between batching and lot sizing. Jordan and Drexl (1998) developed an algorithm that is used as an extension to solve the DLSP. Fisher et al. (2001) also considered the effect of inventory policies for DLSP and produced near-optimal results using the new algorithm.

The aim of the proportional lot sizing problem (PLSP) is to schedule the production of a second product using the remaining capacity, in cases where the capacity is not exhausted. The PLSP addresses the limitation of the CLSP. Under the assumption of the PLSP, the set-up state of the machine can be changed once in a period. Production can only take place when the machine has been set-up at the beginning or end of the period, which means that two products can be produced in a period (Drexl and Kimms, 1997).

2.3 Algorithms and Heuristics

2.3.1 Optimal algorithms

Most deterministic lot-sizing and scheduling problems can be formulated as mathematical programming problems, see e.g. the model (2.1). These can in principle be solved to optimality with the machinery of integer programming (Nemhauser and Wolsey, 1991). A benefit of formulating a problem as a mathematical programming problem is that (linear) programs can be solved with standard optimisation software. As most problems are NP-hard, the use of mathematical programming however may be restricted to problem instances of small sizes.

Certain lot-sizing and scheduling problems can also be solved with the technique of dynamic programming. This technique is in particular suited to solve deterministic dynamic (non-linear) problems, and can even be used when data is stochastic, see e.g. Puterman (2005) . Large instances of complex problems, however, remain difficult to solve to optimality due to the exponential growth of computational time and memory requirements (state space). Another practical drawback compared to integer programming is that a specific dynamic programming algorithm needs to be programmed by the researcher for each individual problem type.

2.3.2 Heuristics

2.3.2.1 Evolutionary Approaches

The evolutionary approaches like the ones adopting a 'Memetic Algorithms' (MA) were first introduced in the late 1980s (Moscato et al., 1989a,b). The introduction/conception of MAs as a group of meta-heuristics is crucially based on the hybridization of different algorithmic approaches. It is worth noting that most of the MAs approaches are population-based, which allows a set of cooperating and competing agents to engage in periodic individual improvements of problem solutions, on one hand, but at the same time interact sporadically (Vaid and Verma, 2014; Ma et al., 2017). Furthermore, in a study by Toledo et al. (2009), a multi-population hierarchical 'Genetic Algorithm' (GA) procedure is developed, which can be memetically adopted in solving the Synchronized and Integrated Two-Level Lot Sizing and Scheduling Problem (SITLSP).

2.3.2.2 The Decomposition and Relaxation Approach

Following from the discussion in the preceding sections, this section aims to describe the adopted mathematical model to represent the SITLSP and the solution approaches (Ferreira et al., 2009; Stadtler, 2011) . The adopted model follows the model in Toledo et al. (2006), and takes into account the synchrony between the production levels, and integrates the lot sizing and scheduling decisions (Toso et al., 2009; Copil et al., 2017). However, the current model differs from the model by Toledo et al. (2006) in several ways. This paper considered a simplification of the problem by assuming that each filling line has a dedicated tank. This follows that each tank can be filled with all the liquids needed by the associated machine. There are T macro-periods of the planning horizon. It is a 'big bucket' model; therefore, each of the macro-periods needs to be divided into a number of micro-periods (t), which provide the order at which the items will be produced. The total number of 't' is defined by the user, but should be set as the maximum number of set-ups in each macro-period. Furthermore, the size of the micro-period is defined by the model because it depends on the item's lot size, and is flexible.

At both levels, the total number of micro-periods is the same, and only one item (i.e. liquid flavor) can be produced in each micro-period.

2.3.2.3 Relax and fix strategies

The usual relax and fix strategy groups variables by periods (i.e. macro-periods), and only the integer variables are fixed at each iteration. Dillenberger et al. (1994) presents a model in which these criteria are used in the solution of a multi-machine multi-items lot sizing model. Hence, the heuristics iterations number is thus the number of periods. This heuristic has also been applied to a class of project scheduling problems, and in exploring various strategies to partition the set of binary variables (Escudero and Salmeron, 2005). The relax-and-fix heuristic can also be used in combination with meta-heuristics like the tabu search (Pedroso and Kubo, 2005; Ferreira et al., 2010; Kluabwang et al., 2012)

For example, Pedroso and Kubo (2005) presents a hybrid tabu search procedure in which the relax-and-fix heuristic is used either to initialize a solution or to complete partial solutions. To solve a big 'bucket' lot sizing problem with set-up and backlog costs, the hybrid approach is normally applied. At each iteration of the relax-and-fix heuristic, only variables of a given period that concern a single product are fixed. The strategy is commonly called relax-and-fix-one-product. Because it is hard to solve some mono-period, mono-machine multi-items lot sizing problems, this strategy helps to solve these smaller sub-models, which is the main advantage of the strategy.

Toledo (2005) proposed a relax-and-fix heuristic to solve the SITLSP model formulated in (Toledo et al., 2006), then the ones in Level 2 (i.e. from the last period to the first). There are other relax-and-fix strategies, which fix continuous variables

(Federgruen et al., 2007; Ferreira et al., 2009; Buschkühl et al., 2010). For example, Federgruen et al. (2007) classified the relax-and-fix heuristics as a particular case of progressive interval heuristics, and state that fixing continuous variables decreases the flexibility of the heuristic.

2.4 Conclusion

In this chapter, a general characterization of the modeling of lot sizing and scheduling problem is discussed. Additionally, integer programming formulation and standard IP software will be used because it turns out that the problem instance sizes in the thesis are small enough optimal solutions can be found in acceptable computational time using standard IP optimisation software .

Chapter 3

Production scheduling and lot sizing for a soft drink company in Nigeria

Chapter 3

Production scheduling and lot sizing for a soft drink company in Nigeria

3.1 Introduction

The problem addressed in this chapter is motivated by a real life case study that is found in bottling companies around the world. The production of soft drink involves two main stages with decisions about preparation, inventory and storage. In the first stage, the liquid is prepared and decisions about the amount of flavor to be stored in each available tanks are also made. While the bottling of each product demanded and scheduling of the different lines is determined in the second stage. In both stages the scheduling and the lot sizing problem have to be solved.

In this chapter we propose a mixed integer programming model that integrates the lot sizing and scheduling decisions that considers the synchronization between the stages of the liquid flavor preparation and liquid bottling of a Nigerian beverage production plant. This chapter will develop a Mixed integer linear programming (MILP) model that reflects the lot-sizing and scheduling problem of the bottling factory, which will form the basis for an extended Mixed integer linear programming (MILP) model presented in Chapter 6, where the power consumption needs are considered.

It is based on the GLSP (general lot sizing and scheduling problem) framework proposed in Fleischmann and Meyer (1997). The model adopts the model that was used by Ferreira et al. (2009), however there are major differences between the models. Initial and essential constraints were added to the model. Even though, the test bed for our work is small when compared to the industry case, the advantage of our work is that our AMPL programming enables us to consider the effect of different parameters on lot sizing and scheduling decisions of a soft-drink plants with sequence-dependent set-up costs and times. In addition,the time of program execution is less than one minute while for big test bed (industry case) they are not solved to optimality in a pre-specified amount of time and still difficult to solve.

3.2 Case study

The factory in this dissertation will be located in Kano, Nigeria. The following description of the system is derived based on visits to Nigerian bottling companies and existing literature. The data that has been presented in this chapter was collected from planned visits to factories that have similar characteristics to the factory referred to in this dissertation. The data in this chapter is used across the sections of this dissertation.

The production of soft drinks is conceptually presented in Figure 3.1. It involves two main phases: liquid flavor preparation, and bottling.

- Phase I (flavor preparation): Water from a source (large repository) is transferred to any of a number of tanks of possibly different sizes. The addition of one of a number of liquid concentrate allows for the preparation in a tank of a particular flavor. At any time, a tank cannot contain more than one type of flavoured water.
- Phase II (Bottling): The soft drinks are bottled in bottling or filling lines. The flavoured liquid from a single tank from Phase I can be pumped to

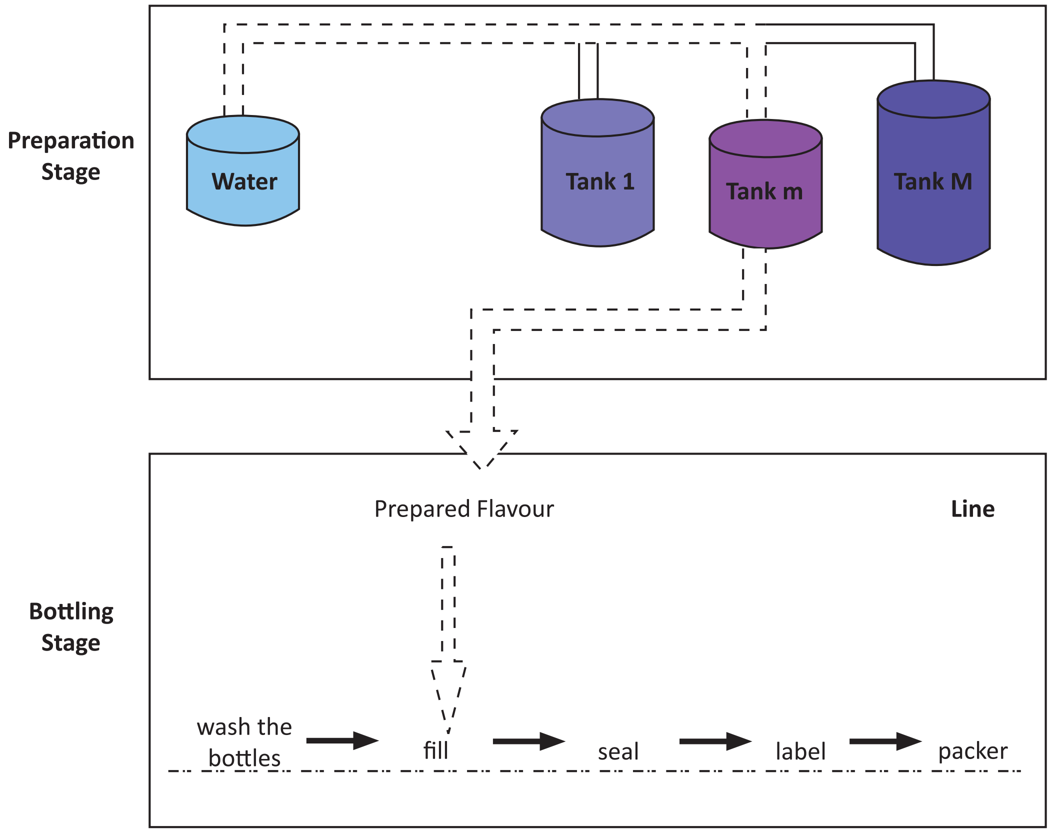

Figure 3.1: Conceptual model of the bottling process

one or multiple bottling lines simultaneously if they were bottling the same soft drinks, but a filling line can only receive liquid from one tank at any given time. A bottling line is made up of the conveyor belt, preceded by a sterilisation unit for the bottles to be filled. The filling lines then fill, seal, label and pack the soft drinks.

They can be packed in different kinds of packages. Packaged bottles can then be further order picked onto pallets or cages according to customer demand. These pallets can then be stored in inventory prior to distribution.

3.2.1 Sequence-dependent change-over costs and times

A tank has to be empty and cleaned before a new batch of flavoured liquid can be prepared. To assure uniformity, a minimum quantity of the liquid flavor has to be prepared. The tank propeller must always be completely covered in order for the flavor to be mixed properly.

Whenever a bottling line is started for production after having been unused, or after it has been used to produce a another kind of flavor, the filters and all of the channels and the water that passes through have to be sanitised, and machine adjustments have to be made. Whenever a line is stopped, there is likewise a loss of energy and time in order for the process to safely halt.

In both Phases, there are therefore set-up or change-over costs and times. These costs and times are also so-called 'sequence dependent', meaning that they depend on which flavor follows after another one. For example, if the soft drink of pineapple flavor has to be prepared next, cleaning time and adjustments will take longer if it was preceded by the production of orange flavor then when it would have been preceded by unflavoured water.

3.2.2 Synchronization

The production of the soft drinks has to account for requirements imposed by the synchronisation between Phase I and Phase II. A filling line has to be ready to accept the liquid flavor from a tank before production can begin. Likewise, if the required flavoured water is not ready in the tank, the filling line(s) have to wait for the tank preparation process to be completed.

Figures 3.2 and 3.3 illustrate, respectively, an in-feasible and a feasible production schedule for four kinds of soft drinks (berry, orange, pineapple and apple) which are produced from four kinds of flavors.

At the initial phase of the planning horizon, time has to be given to the tank to mix the flavors. In Figure 3.2, the filling line has already begun bottling soft drink 1, which is in-feasible. Therefore, the line has to be set in a way that it waits for the tanks to finish the preparation. The first change-over from flavor a to b and from type 1 to type 2 soft drink both have the same change-over time. However, the filling line is still ahead of the tank due to the continuation in the previous period. In Figure 3.2, the two phases of production are thus not synchronised, and the resulting schedule is in-feasible.

Figure 3.3 shows a feasible schedule. At the start of the day, the tank is prepared with berry flavor, during which the line has a waiting time. The first changeover on the tank involves cleaning the tank to remove the residue of the berry flavor and then prepare the tank with orange liquid flavor. In the meantime, the line is in change-over to prepare for filling with orange flavor. When the tank is empty, a third change-over is needed, during which the tank is refilled and prepares for the mango flavor. During this time, the line incurs a waiting time. The process continues until the line has completed the required lot size of the day for the orange soft drink. The line is then changed to accommodate mango soft drink, which uses the prepared flavor. In the meantime, the tank flow has to be stopped, and then a waiting time on the tank is incurred. When the line is ready, the process then continues filling soft drink. This process is completed just after the targeted completion time of the day.

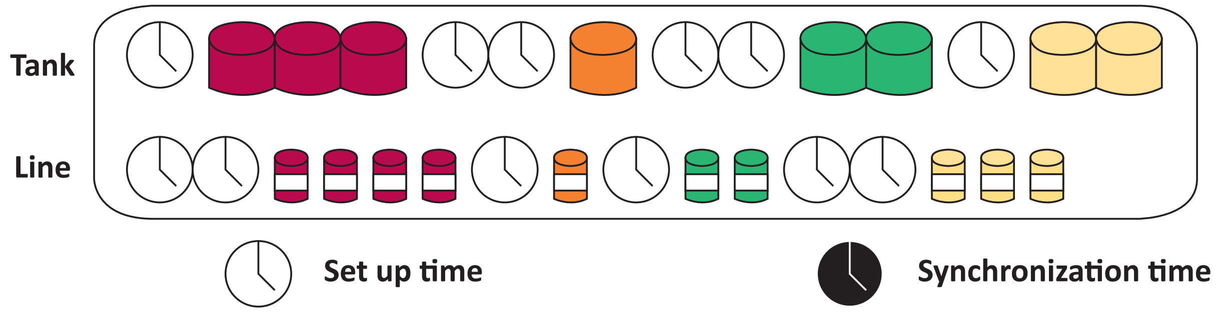

Figure 3.2: non synchronized schedule of the production process

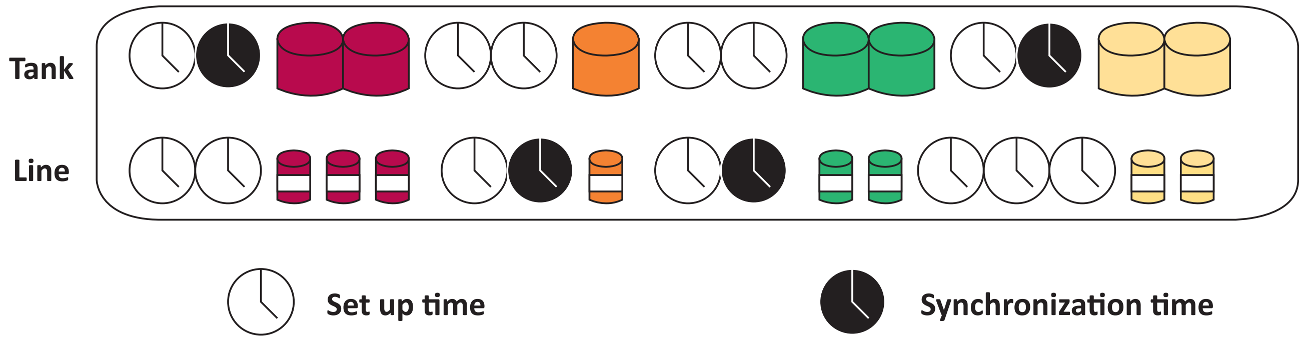

Figure 3.3: Synchronized schedule of the production process

3.3 The factory in Kano, Nigeria

Based on the available water source and expected demand volumes, the envisaged factory is rather small in scale in comparison to the average size of the industry, with two to three bottling lines for the production of up to four types of flavors.

The company still has to decide on the number and capacity of flavor tanks.

- Pumping the water from boreholes is a continuous process, where water is pumped from raw water tank and reaches the flavor tank. Filtration and other treatment is done during the travel.

- There are two kinds of tanks, the raw water tank,that has a capacity of 400,000 liters, and two types of flavor tank tanks that store treated water that is connected to each line with a capacity of 5000 or 10000 liters.
- Every line is able to produce 5,200 standardized size bottles per hour.

Table 3.1: Product details

Type	cost/bottle	Wholesale price	Retail price
p(1)	N42.00	N102.00	N120.00
p(2)	N145.00	N300.00	N335.00
a(1)	N42.00	N102.00	N120.00
a(2)	N145.00	N300.00	N335.00
o(1)	N42.00	N102.00	N120.00
o(2)	N145.00	N300.00	N335.00
b(1)	N42.00	N102.00	N120 .00
b(2)	N145.00	N300.00	N335.00

Table 3.2: Production Time

Type	Production time (hrs/case)
a(1)	.03075
a(2)	.02000
b(1)	.03075
b(2)	.02000
o(1)	.03075
o(2)	.02000
p(1)	.03075
p(2)	.02000

Table 3.1 provides the cost of producing all the products that are produced by the factory. The products are product p1, p2, a1, a2, o1, o2, b1 and b2. The information presented in table 3.1 is used in the NPV analysis that is carried out in chapter eight.

Table 3.2 provides the production time for all the products. The information provided in table 3.2 is used as data in the models that are developed in chapters three, six and seven. The numbers from this table were calculated based on the machine capacity that would be installed.

Table 3.3 and 3.4 provide changeover times from one flavour to another and the changeover time from one product to another product. The data provided in both tables is used in chapter three, six and seven.

Table 3.3: Liquid flavor changeover times

Flavor	a	b	o	p
a	2	3	3	3
b	3	2	3	3
o	3	3	2	3
p	3	3	3	2

Table 3.4: Type changeover time/hr

Type	a(1)	b(1)	o(1)	p(1)	a(2)	b(2)	o(2)	p(2)
a(1)	0	0	0	0	1	1	1	1
b(1)	0	0	0	0	1	1	1	1
o(1)	0	0	0	0	1	1	1	1
p(1)	0	0	0	0	1	1	1	1
a(2)	1	1	1	1	0	0	0	0
b(2)	1	1	1	1	0	0	0	0
o(2)	1	1	1	1	0	0	0	0
p(2)	1	1	1	1	0	0	0	0

Table 3.5: Bottling line details

Name	Qty	Acquisition Cost(Naira)	Power requirement
Water purification	1	N26,723,600.00	120 kWh
Bottle blower	2	N12,471,000.00	90 kWh
Rinse,filling & capping	2	N9,027,200.00	15 kWh
Labeling	2	N7,126,300.00	20 kWh
Shrink wrapper	2	N16,034,200.00	60 kWh

Table 3.5 provides power requirement data for the machines used in the factory. The data from table 3.5 is used in chapter seven models.The numbers from this table were collected from the producers of the machines that would be installed. Table 3.6 provides information on the salaries of those who run the factory, the information is used in chapter eight.

Table 3.6: HR Requirement

Name	Salary/Month
Unskilled personnel	N24,000.00
Semi-skilled personnel	N34,000.00
Skilled personnel	N45,000.00
Plant managers	N255,000.00
Sales Supervisor	N55,000.00
Packaging Supervisor	N50,000.00
Security Officers	N40,000.00
Admin & accounts manager	N80,000.00
Marketing Reps	N120,000.00

3.4 Model development

The mixed integer optimisation model presented in this section considers the synchronisation between the production stages and integrates the lot sizing and scheduling decisions. In this model,each line is refereed to as a machine m, each line has a dedicated tank and each tank can take the different kind of flavors. The planning horizon has been divided into macro-periods. In order to obtain the production schedule, each macro-period has been divided into a number of micro-periods.

The planning horizon is divided into T macro-periods. The macro periods consists of a fluctuating number of micro-periods that could have variable length. Each bottling line has a contrasting micro-period subdivision, i.e the number of micro-periods will differ from one bottling line to another. A single bottling line does not have to get micro-periods that are of the same duration.

The model size is defined by (J, M, F, T, N) representing:

- J the number of soft drink products (drink types)
- M the number of machines

- F the number of liquid flavors
- T the number of macro-period
- N the total number of micro-periods

Let (i, j, m, k, l, t, s) be the index set defined as:

- i, j ,$\in$ {1,...,J}
- $m \in$ {1, ...,M}
- k, l, $\in$ {1, ...,F}
- $t \in$ {1, ...,T}
- $s \in$ {1, ...,N}

The following sets and data are known.

Sets

- S_t time slots in period t;
- P_t First time slot in period t ;
- λ_j Bottling line that can produce drink type j;
- α_m Drink types that can be produced by bottling line m;
- β_m set of liquid flavor that can be produced on tank m;
- γ_{ml} set of drink types that can be produced on bottling line m and need liquid flavor l;

Data

- d_{jt} demand for drink type j in period t;
- E_j initial inventory for drink type j in period t;
- h_j inventory cost for one unit of drink type j;
- g_j backorder cost for one unit of drink type j;
- s^I_{kl} changeover cost from liquid flavor k to l;
- s^{IIk}_{ij} cost of change over from drink type i to j;
- b^I_{kl} changeover time from liquid flavor k to l;
- b^{II}_{ij} changeover time from drink type i to j;
- $a^{II}mj$ production time for one unit of drink type j on machine m;
- K^I_m total capacity of tank m, in liters of liquid m;
- K^{II}_{mt} total capacity time on machine m in period t;
- r_{jl} quantity of liquid flavor l necessary for the production of one unit of drink type j
- q^I_{lm} minimum production of liquid flavor l in tank m.
- I^+_{j0} initial inventory for drink type j;
- I^-_{j0} initial backorder for drink type j ;
- y^I_{ml0} 1 if tank m is initially set-up for liquid flavor l; 0 otherwise;
- y^{II}_{mj0} 1 if bottling line m is initially set-up for drink type j; 0 otherwise;

Variables

- I^{+}_{jt} inventory for drink type j at the end of period t;
- I^{-}_{jt} backorder for drink type j at the end of period t;
- x^{II}_{mjs} production quantity in bottling line m of drink type j in micro-period s;
- V^{II}_{ms} waiting time of machine m in micro-period s;
- y^{I}_{mls} $\{$ 1 if there is production in tank m of the liquid flavor l in micro-period s; 0 otherwise
- y^{II}_{mjs} $\{$ 1 if the bottling line m is set-up for drink type j in micro-period s; 0 otherwise;
- z^{I}_{mkls} $\{$ 1 if there is changeover in tank m from liquid flavor k to l in micro-period s; 0 otherwise;
- z^{II}_{mijs} $\{$ 1 if there is changeover in bottling line m from drink type i to j in micro-period s;0 otherwise

The two-stage multi-machine lot-scheduling model (P2SMM) is then:

$$\min \qquad Z = \sum_{i=1}^{J}\sum_{t=1}^{T}(h_j I_{jt}^{+} + g_j I_{jt}^{-}) + \sum_{s=1}^{N}\sum_{m=1}^{M}\sum_{k\in\beta_m}\sum_{l\in\beta_m} s_{kl}^{I} z_{mkls}^{I} + \sum_{s=1}^{N}\sum_{m=1}^{M}\sum_{i\in\alpha_m}\sum_{j\in\alpha_m} s_{ij}^{II} z_{mijs}^{I} \tag{3.1}$$

The objective function 3.1 minimizes the total sum of inventory, backorder and the changeover cost of machines and tanks. In the first stage (tank), the demand of flavor l is calculated based on the production variables that are available in the second stage (bottling). Which means that the demand for flavor l in the tanks m for a given micro-period s is $\sum_{j\in\gamma ml} r_{jl} K_{mjs}^{II}$

s.t

Stage 1 (Tank)

$$\sum_{j\in\gamma_m l} r_{jl} x_{mjs}^{II} \leq K_m^{I} y_{mls}^{I} \qquad m = 1,...,M; l \in \beta_m; s = 1,...,N \tag{3.2}$$

$$\sum_{j\in\gamma_m l} r_{jl} x_{mjs}^{II} \geq q_{lm}^{I} y_{mls}^{I} \qquad m = 1,...,M; l \in \beta_m, s = 1,...,N \tag{3.3}$$

$$\sum_{l\in\beta_m} y_{ml}^{I}(s-1) \geq \sum_{l\in\beta_m} y_{mls}^{I} \qquad m = 1,...,M; t = 1,...,T; s \in S_t - \{P_t\} \tag{3.4}$$

$$z_{mkls}^{I} \geq y_{mk}^{I}(s-1) + y_{mls}^{I} - 1 \qquad m = 1,...,M; k,l \in \beta_m; s = 2,...,N \tag{3.5}$$

$$z_{mkls}^{I} \geq \sum_{j\in\gamma_m k} y_{mj}^{II}(s-1) + y_{mls}^{I} - 1 \qquad m = 1,...,M; k,l \in \beta_m; t = 2,...T; s = P_t \tag{3.6}$$

$$\sum_{k\in\beta_m}\sum_{l\in\beta_m} z_{mkls}^{I} \leq 1 \qquad m = 1,...,M; t = 1,...T; s \in S_t \tag{3.7}$$

Hence, constraint 3.2 in addition to constraint 3.3 ensure that when flavor l is produced in tank m ($y_{mls}^{I} = 1$), the maximum and minimum tank capacity and size of micro-periods necessary to ensure homogeneity of the liquid will define the amount produced. Constraint 3.4 indicates the micro-periods where production does not take place at the end of macro-periods, which means that $\sum_{l \in \beta m} y^I_{mls}$ = 0 is possible. The change over between flavors is controlled by constraint 3.5. Although when the variable $y^I_{mk(s-1)}$ =0 during the last micro-period of a particular macro-period, constraint 3.5 does not account for the change over between the micro-periods. The set-up variables in the bottling stage show which flavor was prepared during the last micro-period of the each macro-period. Constraint 3.6 is therefore needed when there is changeover between macro-periods. Constraint 3.7 ensures that there is not more than one changeover in each tank during each micro-period.

Stage 2 (Bottling)

$$I^+_j(t-1) + I^-_{jt} + \sum_{m \in \gamma_j} \sum_{s \in S_t} K^{II}_{mjs} = I^+_{jt} + I^-_j(t-1) + d_{jt} \qquad j = 1, ..., J; t = 1, ..., T \tag{3.8}$$

$$\sum_{j \in \alpha_m} \sum_{s \in S_t} a^{II}_{mj} K^{II}_{mjs} + \sum_{i \in \alpha m} \sum_{j \in \alpha_m} \sum_{s \in S_t} b^{II}_{ij} z^{II}_{mijs} + \sum_{s \in S_t} V^{II}_{ms} \leq K^{II}_{mt} \qquad m = 1, ..., M; t = 1, ..., T \tag{3.9}$$

$$V^{II}_{ms} \geq \sum_{k \in \beta_m} \sum_{l \in \beta_m} b^I_{kl} z^I_{mkls} - \sum_{i \in \alpha_m} \sum_{j \in \alpha m} b^{II}_{kl} z^{II}_{mijs} \qquad m = 1, ..., M; s = 1, ..., N \tag{3.10}$$

$$K^{II}_{mjs} \leq \frac{K^{II}_{mt}}{y^{II}_{mjs}} \qquad m = 1, ..., M; \quad i, j \in \alpha_m; t = 1, ..., T; s = \in S_t \tag{3.11}$$

During the bottling stage, constraint 3.8 is responsible for balancing inventory in each macro-period. The production variables in each micro-period have to be defined in advance in order to obtain the total production of drink type j. The production variables associated with all machines have to also be calculated. Constraint 3.9 is the capacity of machine for given macro-periods. As discussed in the earlier section, the machines have to wait for the liquid to be prepared in the tanks. The continuous variables(v_{ms}^{II}) and constraint 3.10 calculate the waiting time for the machines at the beginning of each micro-period s. The difference between the changeover time of machine and tank is the waiting time. In scenarios where the changeover for different drink types i to j is greater than the changeover time for tank preparation (flavor k to l), the waiting variable is 0 and only the changeover time for the machine is considered in constraint 3.9.If the changeover times are equal V_{ms}^{II} is positive and all the waiting time is considered in constraint 3.10, synchronization is then established between the stages. Constraint 3.11 makes sure that production of drink types in micro period s only happens when the set up variable is 1.

$$\sum_{j \in \alpha_m} y^{II}_{mjs} = 1 \qquad m = 1, ..., M; s = 1, ..., N \tag{3.12}$$

$$z^{II}_{mkls} \geq y^{II}_{mi}(s-1) + y^{II}_{mjs} - 1 \quad m = 1, ..., M; \; i, j \in \alpha_m; s = 1, ...N \tag{3.13}$$

$$\sum_{i \in \alpha_m} \sum_{j \in \alpha_m} z^{II}_{mijs} \leq 1 \qquad m = 1, ..., M; s = 1, ..., N \tag{3.14}$$

$$I^+_{jt}, I^-_{jt} \geq 0 \qquad j = 1, ...J; t = 1, ..., T \tag{3.15}$$

$$z^I_{mkls}, z^{II}_{mijs}, v^{II}_{ms}, \mathcal{X}_{mjs} \geq 0 \qquad m = 1, ..., M; k, l \in \beta_m; \; i, j \in \alpha_m; t = 1, ..., T; s \in S_t \tag{3.16}$$

$$y^{II}_{mjs}, y^I_{mls} = 1 \text{ or } 0 \qquad m = 1, ..., M; k, l \in \beta_m; \; i, j \in \alpha_m; t = 1, ..., T; s \in S_t \tag{3.17}$$

Constraint 3.12 ensures that each machine m is set up for production of only one drink type at each micro-period. Constraint 3.13 counts the number of changeovers in each machine in each micro-period. Constraint 3.14 ensures that only one changeover at most per machine in each micro-period. Constraints 3.15,3.16 and 3.17 define the domain of variables. The changeover variables z^I_{mkls} and z^{II}_{mijs} have been set to continue. Constraint 3.5 and 3.13 in addition to the minimisation function only allows the variables to be 1 or 0 in a solution that is optimal.When defining the lot size and production scheduling, the factory requires that available inventory in the macro-period must be enough to cover future demand production in the next macro-period.

$$I^+_{jt} \geq d_{j(t+1)} \qquad j = 1, ..., J; t = 2, ..., T \tag{3.18}$$

Where $d_{j(t+1)}$ is the future forecasted demand for drink type j in period T + 1. Which is the first period of the next planning horizon.

$$\sum_{k \in \beta m} z^{I}_{mkl1} \geq y^{I}_{ml1} \quad m = 1, ..., M; l \in \beta m \tag{3.19}$$

$$\sum_{i \in \alpha m} z^{II}_{mij1} \geq y^{II}_{mj1} \quad m = 1, ..., M; j \in \alpha_m \tag{3.20}$$

$$I^{+}_{j0} = E_j \quad j = 1, ..., J \tag{3.21}$$

$$I^{+}_{jt} \geq F_j \quad j = 1, ..., J; t = T \tag{3.22}$$

$$I^{-}_{j0} = 0 \quad j = 1, ..., J \tag{3.23}$$

Constraints 3.19 and 3.20 are initial set up for the liquid flavor in the tanks and initial set up for the machines for first micro-period, respectively. Constraints 3.21, 3.22 and 3.23 refer to the initial inventory of products E_j, demand of the future period F_j and initial backorder for the products, respectively.

3.5 Computational tests

The P2SMM model was coded in the AMPL IDE modeling language (Version: 3.1.0.2015). The P2SMM model was solved using the optimization system CPLEX version 12.7.1.0. The runs were executed using an $Intel\ Core^{TM}$ i5-3320M 2.60GHz.An example considering a time horizon of two macro-periods, two periods for a production process with two flavors, four products, two tanks, and two lines. Each macro- period has six micro-periods of capacity. Suppose the following:

- flavor 1 produces only product 4

- flavor 2 produces product 1, 2 and 3
- Line 1 (machine 1) can bottle product 1, 2, 3 and 4
- Line 2 (machine 2) can bottle product 1, 2, 3 and 4
- Tank 1 can store flavor 1 and 2
- Tank 2 can store flavor 1 and 2

Three tests were generated based on above condition and other data in AMPL code:

- Test 1: capacity time of each line and each tank are 1000 unit and 1000 liters.
- Test 2: capacity time of each line and each tank are 1000 unit and 550 liters.
- Test 3: capacity time of each line and each tank are 700 unit and 1000 liters.

drink 1	drink 2	drink 3	drink 4
flavour 2	flavour 2	flavour 2	flavour 1

Figure 3.4: flavor mix and drinks

The computational test carried out in this chapter was done to investigate the design decision (the sizing of tanks in relation to line production rate) that was not investigated in earlier studies.

Figures 3.3, 3.4 and 3.5 show the results of an integration of production lot sizing and scheduling decisions of beverage plant with sequence-dependent set-up costs, times and synchronisation for the production in lines and the storage in tanks, which are compatible with each other throughout the time horizon.

3.5.1 Test 1

The changeovers in machine 1 are as follows from drink 3 to 1 in the micro-period 2, from drink 1 to 2 in the micro-period 5 ,from drink 2 to 3 in the micro-period 8, and from drink 3 to 1 in the micro-period 9. Machine 2 produces drink 4 only with no changeovers. In this test tank 1 is totally dedicated to producing drink 4. Tank 2 also avoided changeover of flavors, only flavor 1 is produced.

3.5.2 Test 2

There is no changeover for the drinks in the line and only drink 4 was produced. The changeovers in machine 2 are as follows from drink 3 to 1 in the micro-period 2, from drink 1 to 2 in the micro-period 3,from drink 2 to 4 in the micro-period 6,from drink 4 to 3 in the micro-period 9, from drink 3 to 1 in the micro-period 10 , from drink 1 to 2 in the micro-period 11.

3.5.3 Test 3

The changeovers in machine 1 are as follows from drink 3 to 1 in the micro-period 2, from drink 1 to 2 in the micro-period 3,from drink 2 to 4 in the micro-period 6. The changeovers in machine 2 are as follows from drink 4 to 3 in the micro-period 10 ,from drink 3 to 1 in the micro-period 11 ,from drink 1 to 2 in the micro-period 12.

Based on figure 3.6,3.7, 3.8,3.9, 3.10 and 3.11, the line production rate affects the solution time of algorithm. In practise there is little room to change the rate (technological constraint, safety). The size of tanks used in production also influences the production schedule and the cost of production.The test carried out show that bigger tanks are not always better.

Test1		Test2		Test3	
CPLEX 12.7.1.0: optimal integer solution		CPLEX 12.7.1.0: optimal integer solution		CPLEX 12.7.1.0: optimal integer solution	
74029 MIP simplex iterations		908780 MIP simplex iterations		2293903 MIP simplex iterations	
2583 branch-and-bound nodes		19086 branch-and-bound nodes		74750 branch-and-bound nodes	
CPU (Intel® Core™ i5-3320M 2.60GHz) time = 3 S		CPU (Intel® Core™ i5-3320M 2.60GHz) time = 16 S		CPU (Intel® Core™ i5-3320M 2.60GHz) time = 37 S	
Objective Function	**312.5**	**Objective Function**	**340.1**	**Objective Function**	**340.1**
Capacity time of the each period	**1000**	**Capacity time of the each period**	**1000**	**Capacity time of the each period**	**700**
Tank Capacity	**1000**	**Tank Capacity**	**550**	**Tank Capacity**	**1000**

Figure 3.5: Results of three tests

Based on figures 3.6, 3.7, 3.8, 3.9, 3.10 and 3.11 which are a pictorial representation of test1-test3, the line production rate affects the solution time of algorithm. In practise there is little room to change the rate (technological constraint, safety). The size of tanks used in production also influences the production schedule and the cost of production.The test carried out show that bigger tanks are not always better. Figure 3.2 summarises the results of three tests. The number of iteration for the Test 3 (2293903) is more than Test 1 and Test 2. Therefore, it takes more time to reach a solution (37s). The Objective function minimises total sum of product inventory, demand back-order, machine changeover and tank changeover costs. It increases with reducing capacity of tanks and lines. Objective function raises 8.8% with dropping 45% of tank capacity and 30% of line capacity.The conclusion raised from the three test that were carried out is that the size and number of tanks play a vital role in how production cost are incurred. The literature ignores the size of the tanks, due to the fact that electricity and other variables are not constrained like in the Nigerian context.

Period		1												2											
Slot			1		2		3		4		5		6		7		8		9		10		11		12
Machine 1	Product		3		1		1		1		2		2		2		3		1		1		1		
	Quantity		699		431		461		431		1162		3448		2719		746		431		431		597		
	Production time		21		13		14		13		70		207		163		22		13		13		18		0
	Consumption (lit)		166		125		134		125		337		1000		789		177		125		125		173		0
	Changover time in line			4						9						15		4							
	Waiting time	16		4		8		614		0		8		8		0		4		724		8		8	
Machine 1	Time sequence	16	21	8	13	8	14	614	13	9	70	8	207	8	163	15	22	8	13	724	13	8	18	8	0
Tank 1	Flavour		2		2		2		2		2		2		2		2		2		2		2		
Tank 1	Time sequence	16	21	8	13	8	14	614	13	9	70	8	207	8	163	15	22	8	13	724	13	8	18	8	0
Machine 2	Product		4		4		4		4		4				4		4		4		4		4		
	Quantity		3448		431		3448		3448		2148				1890		3448		3448		431		3448		0
	Production time		207		26		207		207		129		0		113		207		207		26		207		0
	Consumption (lit)		1000		125		1000		1000		623		0		548		1000		1000		125		1000		0
	Changover time in line																								
	Waiting time	193		8		8		8		8		0		208		8		8		8		8		0	
Machine 2	Time sequence	193	207	8	26	8	207	8	207	8	129	0	0	208	113	8	207	8	207	8	26	8	207	0	0
Tank 2	Flavour		1		1		1		1		1				1		1		1		1		1		
Tank 2	Time sequence	193	207	8	26	8	207	8	207	8	129	0	0	208	113	8	207	8	207	8	26	8	207	0	0

Figure 3.6: The two-stage multi-machine lot-scheduling model for Test1

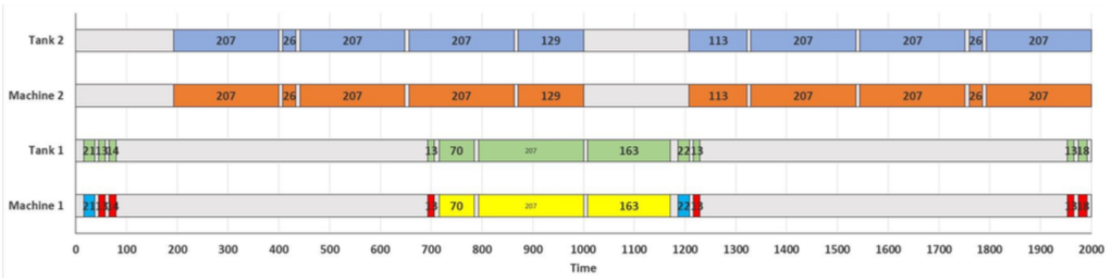

Figure 3.7: The two-stage multi-machine lot-scheduling model for Test1

Period		1												2											
Slot			1		2		3		4		5		6		7		8		9		10		11		12
Machine 1	Product		4		4		4		4		4		4		4		4		4		4		4		4
	Quantity		1897		1897		1545		1897		1897		1897		1050		1897		1897		237		1897		1897
	Time per unit		0.06		0.06		0.06		0.06		0.06		0.06		0.06		0.06		0.06		0.06		0.06		0.06
	Production time		114		114		93		114		114		114		63		114		114		14		114		114
	Consumption (lit)		550		550		448		550		550		550		304		550		550		69		550		550
	Changover time in line																								
	Waiting time	8		8		8		298		8		8		8		8		8		428		8		8	
Machine 1	Time sequence	8	114	8	114	8	93	298	114	8	114	8	114	8	63	8	114	8	114	428	14	8	114	8	114
Tank 1	Flavour		1		1		1		1		1		1		1		1		1		1		1		1
Tank 1	Time sequence	8	114	8	114	8	93	298	114	8	114	8	114	8	63	8	114	8	114	428	14	8	114	8	114
Machine 2	Product		3		1		2		2		2		4		4		4		3		1		2		2
	Quantity		699		1323		817		1897		1897		1897		1897		1897		746		1459		822		1897
	Time per unit		0.03		0.03		0.06		0.06		0.06		0.06		0.06		0.06		0.03		0.03		0.06		0.06
	Production time		21		40		49		114		114		114		114		114		22		44		49		114
	Consumption (lit)		166		384		237		550		550		550		550		550		177		423		239		550
	Changover time in line			4		9						30						30		4		9			
	Waiting time	486		4		0		8		8		0		480		8		0		4		0		8	
Machine 2	Time sequence	486	21	8	40	9	49	8	114	8	114	30	114	480	114	8	114	30	22	8	44	9	49	8	114
Tank 2	Flavour		2		2		2		2		2		1		1		1		2		2		2		2
Tank 2	Time sequence	486	21	8	40	9	49	8	114	8	114	30	114	480	114	8	114	30	22	8	44	9	49	8	114

Figure 3.8: The two-stage multi-machine lot-scheduling model for Test2

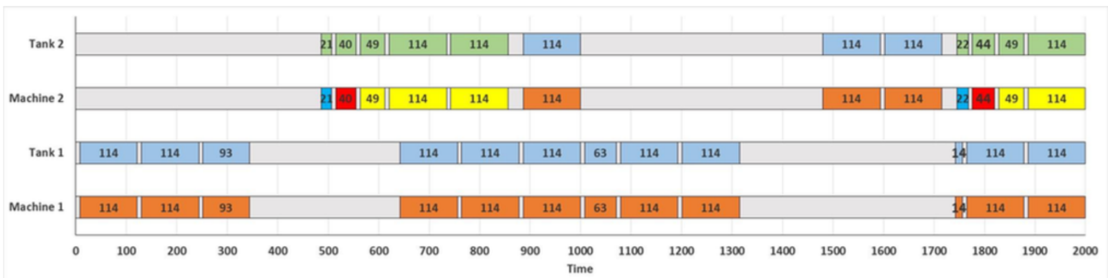

Figure 3.9: The two-stage multi-machine lot-scheduling model for Test2

Period		1												2											
Slot			1		2		3		4		5		6		7		8		9		10		11		12
Machine 1	Product		3		1		2		2		2		4		4		4								
	Quantity		699		1323		731		431		3448		3448		3448		3448								
	Time per unit		0.03		0.03		0.06		0.06		0.06		0.06		0.06		0.06								
	Production time		21		40		44		26		207		207		207		207		0		0		0		0
	Consumption (lit)		166		384		212		125		1000		1000		1000		1000		0		0		0		0
	Changeover time in line			4		9						30													
	Waiting time	93		4		0		8		8		0		8		8		270							
Machine 1	Time sequence	93	21	8	40	9	44	8	26	8	207	30	207	8	207	8	207	270	0	0	0	0	0	0	0
Tank 1	Flavour		2		2		2		2		2		1		1		1								
Tank 1	Time sequence	93	21	8	40	9	44	8	26	8	207	30	207	8	207	8	207	270	0	0	0	0	0	0	0
Machine 2	Product		4		4		4		4						4		4		4		3		1		2
	Quantity		431		2148		3448		3448						1890		3448		431		746		1459		2719
	Time per unit		0.06		0.06		0.06		0.06						0.06		0.06		0.06		0.03		0.03		0.06
	Production time		26		129		207		207		0		0		113		207		26		22		44		163
	Consumption (lit)		125		623		1000		1000		0		0		548		1000		125		177		423		789
	Changeover time in line																			30		4		9	
	Waiting time	16		99		8		8		0		0		8		8		8		0		4		54	
Machine 2	Time sequence	16	26	99	129	8	207	8	207	0	0	0	0	8	113	8	207	8	26	30	22	8	44	63	163
Tank 2	Flavour		1		1		1		1						1		1		1		2		2		2
Tank 2	Time sequence	16	26	99	129	8	207	8	207	0	0	0	0	8	113	8	207	8	26	30	22	8	44	63	163

Figure 3.10: The two-stage multi-machine lot-scheduling model for Test2

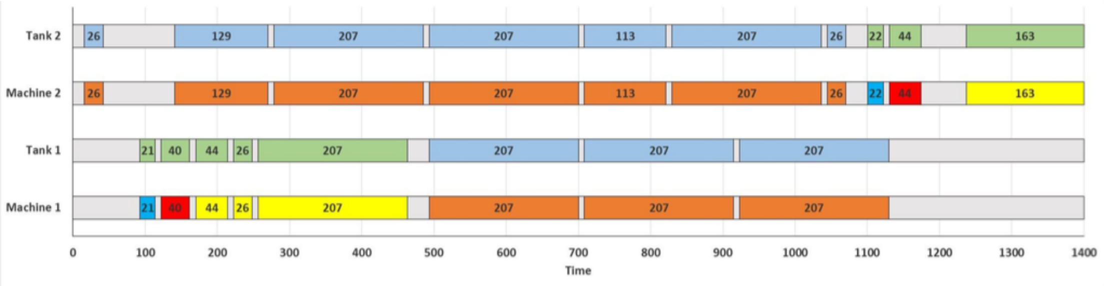

Figure 3.11: The two-stage multi-machine lot-scheduling model for Test3

3.6 Conclusion

Several studies have been carried out in the area of lot sizing and scheduling(reviewed in chapter 2). The formulations of the numerous models have been based on different assumptions to investigate and address different problems that face production plants globally. The various aspects of the lot sizing and scheduling problems such as the multiple levels of production, numerous time constraints,synchronisation constraints and many other constraints make the task of creating an effective and efficient model hard. The research carried out for this chapter has been tailored towards having an impact in a real factory that is currently in difficulty. The problems facing the bottling plant is special because it has to generate it's own electricity by using diesel generators. Therefore, the model had to take account of the special case of the production plant by modelling the problem in way that the impact of every variable is seen in the solution.

A practical and bespoke model has been put together to address the problems of the production plant using existing models that were found in the literature. The GLSP(Generalized lot sizing and scheduling problem) was used to formulate a model for application in a real-life situation. This chapter has been based on the production capacity for a small to medium production plant that has two filling lines, in that way this chapter has contributed to the field. This literature in this sections proves that based on existing models that have been studied, the lot-sizing problem is important and solving the large scale problem is not straightforward. The aim of this section was not to find solution procedure or create new algorithms, therefore AMPL a commercial software was used to help the factory understand the problem better. Future work could be focused on combining the unit commitment problem which deals with electricity and the GLSP (Generalized lot sizing and scheduling problem), which will address the electricity and production schedule concern.

Chapter 4

Manufacturing power with diesel generators

Chapter 4

Manufacturing power with diesel generators

4.1 Introduction

Due to the rise in standards of living and the growth in business developments, the demand for electricity in certain parts of the world has increased more rapidly than national grids are able to securely provide. In some remote areas, there may not be any provision at all. According to different sources, from 1.3 to 1.5 billion people worldwide lack access to grid-based electricity (Sipahutar et al.).

Nigeria is expected to be one of the most populous nations by 2050 (Nejad et al., 2012),(Tan and Muttaqi, 2014).Currently more than 40 percent of Nigerians do not have access to electricity, and those that have access to electricity often need to rely on unreliable and intermittent distribution (Dada, 2014). The penetration of national grids into remote areas is mostly restricted because of the costs. As estimated by ARE (2012), extending a grid can cost from 6,340 to 19,070 per kilometer for the least populated areas. Struggling with many difficulties such as insurgency, Nigeria does not currently have the resources to increase centralized power generation and provide access to electricity to remote areas.

As a consequence, production plants in Nigeria often need to rely on other alternatives to generate a reliable supply of power. This is typically accomplished by installing a set of company-owned diesel generators. The cost associated with generating power from such sources is often an important component of total operating expenses.

In this dissertation, we aim to develop Operational Research models to help decision-making on various aspects. The research, motivated by a case study of a soft drink bottling facility, is aspiring to develop decision models related to:

- Purchasing and operating a set of diesel generators to enable manufacturing in an environment with no access to an electricity grid;
- Investigate the impact of different loading profile assumptions, from constant, time-dependent but deterministic, to stochastic settings;
- Integrating the power supply decisions and the scheduling of the manufacturing as to maximise economic profit of the manufacturer.

Definition 4.1. When given a set of power generating units and a demand profile over time, the unit commitment problem (UCP) asks for decisions about when which power generating unit should be turned on, the power to be generated by each unit that is on, and when which units to turn off, while respecting various technical and economic constraints (Dalal and Mannor, 2015).

4.2 Case study

The research for this section is based on case study of the soft drink bottling plant in Kano, Nigeria. The need for a more effective production planning is highlighted by the first issue facing the plant which is the cost of generating power,due to the lack of electricity supply in Nigeria. The plant manager has to decide how the plant would generate power in order for production to be carried out. The factory has to determine what sort of generators would be used.

4.3 Island manufacturing with diesel generators

The electricity sector globally has been under review and is experiencing changes like deregulation, increase in consumer demand, renewable energy and increase in competition. These changes have promoted the need for optimisation in the electricity and power sector. Investing in energy operation, distribution, price setting and planning requires a step by step process in order to come up with an economically, operationally and environmentally sustainable power supply. This step by step process involves the unit commitment problem which plays a very important role in solving and planning power problems globally.

4.4 Diesel generator characteristics

A solution to a diesel generator UCP working for a particular manufacturing unit consists of identifying which generators from an available set need to work at which times and at which levels of loading as to meet the demand profile of the manufacturing unit. A solution need to be robust towards meeting this demand profile while also avoid that diesel generators are inappropriately loaded, which would otherwise lead to excessive maintenance costs or even break-down. The demand profile for electricity may be known in advance with reasonably certainty if the manufacturing schedule is known in advance. This makes the UCP in this context somewhat more manageable, and solutions to deterministic models may well be appropriate. If the diesel generators are to be used to also service machinery of which it is uncertain when they will need power (e.g. lifts), elements of randomness will however still be introduced into the problem.

Most generators run on diesel (8kVA TO 2.5mVA) or gas (240kVA to 1250kVA). Diesel generators come in different sizes or capacity, expressed in kW (kilo-Watt). The fuel consumption per hour will depend on the model and manufacturer, and then on the size of a diesel generator and the load at which it is being operated.

Table 4.1 illustrates a fuel consumption chart from a specific manufacturer of a certain type of diesel generators. It shows that, for example, a small diesel generator of 20 kW consumes 1.6 gallons per hour at full load, 1.3 gal/hr at 3/4 load, 0.9 gal/hr at half load, and 0.6 gal/hr at 1/4 load. A large generator of 2000 kW will consume, at these loads, respectively: 141.9, 103.5, 72.2, and 42.8 gal/hr. So there are clearly economies of scale in terms of fuel consumption. This data further shows that the relationship between diesel consumption and load factor is across all generator capacities fairly close to being linear.

It is not advisable to run a diesel generator at lower load levels than 25% because it leads to engine damage which reduces the reliability of the generator set, and neither is it advisable to run at higher than full capacity for long periods of time. Based on the site where the generator set is fixed, the size of the generator set has to be increased in order to attain maximum performance as altitude and temperature increases.

Limit: All generators have to be operated between an upper limit $P^{\max}$ and a lower limit $P^{\min}$ for all units .

$$P^{\min} \leq u^t \leq P^{\max} \tag{4.1}$$

Generator set sizing is also influenced by the type of duty application the generator will run for. There are three duties namely standby, prime and continuous. Prime and continuous duty applications are the most common especially in Africa.

The continuous is the diesel generator produces its maximum capacity i.e 100% constantly for an unlimited time, while prime loading occurs when peak power is produced to a load that changes over time for an unlimited amount of hours . When the load is variable the average load factor should be set around 80% and an overload of 10% is available for 1 hour in every 24 hours. Standby or emergency duty generators are limited to a usage of maximum 500 hours yearly with a load of 80% maximum in every 24 hours. Overloading standby generators is not permitted.

Table 4.1: Fuel consumption (gal/hr) as a function of size (kW) and load (%)

#	*kw*	25%	50%	75%	100%
1	20	0.6	0.9	1.3	1.6
2	30	1.3	1.8	2.4	2.9
3	40	1.6	2.3	3.2	4
4	60	1.8	2.9	3.8	4.8
5	75	2.4	3.4	4.6	6.1
6	100	2.6	4.1	5.8	7.4
7	125	3.1	5	7.1	9.1
8	135	3.3	5.4	7.6	9.8
9	150	3.6	5.9	8.4	10.9
10	175	4.1	6.8	9.7	12.7
11	200	4.7	7.7	11	14.4
12	230	5.3	8.8	12.5	16.6
13	250	5.7	9.5	13.6	18
14	300	6.8	11.3	16.1	21.5
15	350	7.9	13.1	18.7	25.1
16	400	8.9	14.9	21.3	28.6
17	500	11	18.5	26.4	35.7
18	600	13.2	22	31.5	42.8
19	750	16.3	27.4	39.3	53.4
20	1000	21.6	36.4	52.1	71.1
21	1250	26.9	45.3	65	88.8
22	1500	32.2	54.3	77.8	106.5
23	1750	37.5	63.2	90.7	124.2
24	2000	42.8	72.2	103.5	141.9
25	2250	48.1	81.1	116.4	159.6

The purchase price of diesel generators is a function of capacity and manufacturer. In March 2015, a new generator of 20 kW sells in the region of £7k; a generator of 200 kW around £20k; for 1250 kW you might need to pay over £100k; for 2000 kW this goes up to £300k to £400k; for 9900 kW this may well exceed £1.4 million.

More data needs to be collected to get an accurate picture, comparing ideally one brand and type of generator but for different output. The investigation carried out so far seems to indicate that the price tends to monotonically increasing with Power supply. If it would increase more than linear, then buying a set of smaller generators to reach a total output level would require a smaller investment then purchasing one large generator of a capacity equal to the same output. However, since smaller generators are less economical in diesel consumption, there would be a trade-off between investment costs and running costs. This trade-off will depend on the output needed over the life-time of the generators. If generators of different capacity have about the same life expectancy, and the required output is a given constant kW, then buying one large generator would most likely be better overall as it saves on running costs. If the relationships was (sub)linear, then clearly buying one larger generator would be best since then both investment costs and running costs are lower.

There is a healthy second-hand market of used generators. It may be that generators that run towards their end of their useful life may become less economical and consume more fuel per hour to provide a certain load factor, or that they may no longer provide their maximum output.

Diesel generators provide flexibility. A diesel generator can be shut down at any point in time at zero cost, and when shut down its running cost also reduces to zero. However, if a series of generators provide some total output, then since a generator does not perform reliably when turning its load below 25%, there will always be a sudden drop of output that is at least equal to 25% of the capacity of the generator being shut down. Diesel engines should only run at low loads for short periods of time in order to avoid maintenance, structural, and performance problems, and provided they are regularly brought up to their full load. Ideally, diesel engines should run at least at 60% to 75% of their capacity.

A diesel generator can be kicked into action at any point in time. However, if the generator is cold, there will be an initial period in which the generator will not perform optimally. This needs to be further investigated. It may be that the generator consumes more fuel per kW output during this initial period, and also that when turned to full load it may not generate an output equal to its capacity during this warm-up period. It seems that there is hence a set-up cost to be associated with starting up a (cold) generator, and possibly a reduced maximum output to be considered as well. when turning on the generator set, or when load its at its peak, unexpected load transients cause voltage disturbance when the generator is undersized.

4.5 Power environments

4.5.1 National grids

The literature on UCPs has traditionally been concerned with large-scale electricity provision through national grids. Since electricity itself cannot be stored, any power produced and not used at that time is wasted. The problem is hence to use the generators so that their supply can assuredly meet demand at any point in time while minimising the cost of overproduction. The costs to be minimised include not only production (fuel) cost, but also e.g. transition costs when starting-up or shutting-down a power generating unit (Labbi and Benattous, 2014).

Common constraints of large scale power generating units, such as a power plant based on steam produced from burning coal, are the starting-up and shutting-down constraints. These constraints imply that a power generating system cannot quickly react to sudden changes in demand. For this reason, power plants typically need to built up a spinning reserve. The spinning reserve is the sum of power generation available from generating units net of their current power generating produced. This reserve power available is to meet at short notice an unexpected increase in the required load or, or to alleviate the sudden failure of a generating unit (Fontes et al., 2012).

A national grid typically makes use of a combination of various types of generating units as to help overcome some of the above considerations. This includes gas or nuclear based power stations, wind farms, dams, and solar energy units. Most national grids also have standby diesel generating units to deal with peak demands. In addition, they may store the energy not needed by consumers by e.g. pumping water to higher levels, so that this energy can be converted back into electricity fast to help with peak demand periods.

The objective of a firm operating a power grid depends on the market situation. In a market where only a single utility company (usually government owned) provides the power, the objective function will be to minimize cost. In deregulated markets, there are multiple competing utility companies. The long-term objective function of individual providers is to maximize profit, and determining the optimal market share and price setting become important. In the short term, however, both regulated and deregulated markets have to use the same conditions and constraints when scheduling the generating units.

The complexity of the unit commitment problem depends on the size of the power system that is being considered. In the UCP, the main decisions are related to the starting up and shutting down of generators over a particular time horizon (Morales-Espana and Ramos, 2013). National grids, however, also consist of transmission lines and distribution facilities. Transmission lines deliver it throughout the country, and distribution facilities (transformers) provide it to end consumers. In these large scale networks, aspects of transmission losses and transmission facilities add an additional layer of complexity to the UCP problems, see e.g. Rahmat et al. (2013), Tumuluru et al. (2014), Nakawiro (2014), Coronado et al. (2012). Sophisticated grids may have to built in more sophisticated protection strategies that allows any major component to fail without overloading or customer shedding (Olsen et al., 2013).

The main factors of uncertainty in the UPC are related to the fuel price and the load forecasting (Rahmat et al., 2013). It is seldom possible to forecast exactly the power demand for the next day, or even for the next hour. Another important factor of uncertainty is the unexpected generator outage (Martinez and Anderson, 2015). The size and difficulty of the problem can increase when transmissions constraints are taken into account (Ostrowski and Wang, 2012).

In Nigeria, the national grid mostly relies on gas (39.8%), hydro (35.6%) and oil (24.8%) for power generation (Dada, 2014). The heavy weight of fossil fuels in the generation mix puts the country under pressure from the global community to reduce carbon emissions. The main challenge in managing a national grid is to match power supply with demand. Many national grids in developing countries struggle with large losses of energy in transmission and also lack real-time information on current loads and failures (Dada, 2014).

The integration of renewable energy generators into national grids can be effective to reduce costs and carbon emissions and to increase the overall capacity. However, wind and solar solutions also bring along a range of problems, including low inertia and variable fault levels (Li et al., 2014). More uncertainty is added to the generator network when it includes power units based on renewable resources, such as solar and wind generators. Their generation levels will heavily depend on not always easily predictable conditions such as the weather (Martinez and Anderson, 2015). This impacts the overall reliability of the system and calls for creating more reserves. Extra effort is needed to coordinate the operation of renewable energy generators with traditional ones.

The conventional approach to the UCP is to view the demand side as inflexible (Tumuluru et al., 2014). However, it is not the case in the present-day smart micro-grids that incorporate a two-way communication between the supply and demand sides. This communication can greatly improve power generation efficiency and minimise the costs (Chao, 2010). The factor of the demand side participation should be included when solving the UCP for smart grids.

4.5.2 Environments with intermittent or unreliable supply

Islands and remote rural areas mostly represent the environments where supply from a national grid may be intermittent or unreliable.

Islands often have a weak connection to the national grid because of the limited transmission capacity of submarine cables (Ji et al., 2011). Islands increasingly rely on renewable sources, such as tidal or geothermal energy, to cover local needs. These generators allow to mitigate for the fluctuations in external supply. However, the renewable energy is rather costly, which drives up the island energy prices. For example, in Corsica the energy price is 110 EUR/MWh, or in 2.5 times greater than in mainland France (Passarello and Notton, 2015).

The problems with grid supply in remote rural areas arise from financial and technical constraints. The small population density makes it more expensive to build new substations and transmission lines there. As the local energy consumption grows, the existing transformers and substations have to bear an increasing load. In addition, the grid equipment installed in rural areas is often obsolete and high-loss, which contributes to the problem. The result is low power supply voltage and frequent outages (Sun et al., 2012). Khoury et al. (2015) model the case of a remote residential area with a low energy profile and with intermittent electricity supply, complemented by a PV-battery back-up system. The system will store energy and can act as a back up during power outages.

4.5.3 Island manufacturing

Functional electricity grids are not always available due to a lack of infrastructure. In such cases where the grid is not reliable,other sources such as generators, solar panels and batteries are used to meet the energy demand. Designing a combination of such energy sources in the absence of the grid is known as island manufacturing. In island manufacturing, the manufacturing unit has no access to an outside source of electricity and has to rely on a set of dedicated generators. These sets are commonly referred to as remote area power supply (RAPS) systems (Tan and Muttaqi, 2014).

The majority of RAPS systems rely on generators powered by diesel or gas. However, there also is a growing number of hybrid systems that combine diesel with solar, wind, hydrogen or compressed air power (Hessami et al., 2011). The popularity of hybrid power systems is driven by growing fuel prices and environmental concerns (Nejad et al., 2012).

4.6 Basic formulations and algorithms

4.6.1 UCPs for national grid coverage

There exist various formulations of the UCP for national grids, see e.g. Wood and Wollenberg (1984); Swarup and Yamashiro (2002); and Fossati (2012).

Most models consider a time horizon split up in T time buckets or periods (typically of 1 hour duration), with a load demand D^t in period $t = 1, ..., T$. There is a set of N power generating units, each unit i is in state $x_{i,t} = 1$ if producing in t, $x_{i,t} = 0$ otherwise. The production level of i in t is $y_{i,t}$. The objective is to minimise total costs.

We consider here a basic formulation of the UCP, similar to Fernando and Carlos (2010):

$$\begin{array}{lll} \min & \sum_{i,t}\Big(c_{i,t}(y_{i,t}) + f_{i,t}(x_{i,t}) + s_{i,t}(x_{i,t} - x_{i,t-1})^{+}\Big) & \\ s.t. & \sum_i y_{i,t} \geq D_t & \forall t \\ & Y^{-}_{i,t}x_{i,t} \leq y_{i,t} \leq Y^{+}_{i,t}x_{i,t} & \forall i,t \\ & \sum_i Y^{+}_{i,t}x_{i,t} \geq D_t + R_t & \forall t \\ & x_{i,t+j} \geq x_{i,t+1} - x_{i,t} & j = 1, ..., \delta^u_i; \forall i,t \\ & x_{i,t+j} - x_{i,t+j-1} \leq x_{i,t} & j = 1, ..., \delta^d_i; \forall i,t \\ & y \geq 0, x \in \{0,1\} & \end{array} \tag{4.2}$$

using the following notation:

- $c_{i,t}$: variable production cost,
- $f_{i,t}$: fixed running cost,

- $s_{i,t}$: start-up cost,
- $Y^{-}_{i,t}, Y^{+}_{i,t}$: minimum and maximum production levels,
- R_t: spinning reserve,
- δ^u_i: minimum up-time,
- δ^d_i: minimum down-time.

The objective function minimises the variable and fixed production costs, as well as the start-up costs. The variable production costs are in general convex non-decreasing, but may be approximated by piecewise linear convex functions. The first constraint set is needed to meet the demand. The second constraint set specifies that, when up, a generating unit needs to operate between lower and upper bounds. The spinning reserves are expresses by the third set of constraints. The next two sets of constraints express, respectively, minimum up- and down-time whenever a generating unit is turned on or off, respectively.

Larger industrial models will also consider that the start-up costs may depend on whether it is a so-called a hot or cold start-up, and consider similar shut-down costs. In addition, these models will consider ramping constraints which limit the rate of change in production level between consecutive periods, and that production levels cannot exceed certain values when the unit is to be switched on or off. These ramping costs may also be explicitly included in the objective function.

4.6.2 Algorithms for the deterministic UCP

This section shows how the UCP can be very difficult, the very large numbers in 4.2 are the upper bounds of the enumerations that are required. The section shows the high dimentionality of the possible solution space.

Let there be N generating units available and assume that the load D_t in period t can be met by any combination of these units. Then the number of combinations to enumerate as to identify the least cost set of units to use would be given by (Wollenberg, 2014):

$$\sum_{i=1}^{N} C(N,i) = 2^N - 1, \tag{4.3}$$

where

$$C(N,i) = \frac{N!}{(N-i)!i!}. \tag{4.4}$$

Table 4.2 illustrates that when there are 24 periods in the time horizon, the size of this problem becomes quickly unmanageable.

Table 4.2: Number of possible combinations to enumerate.

N	$(2^N - 1)^{24}$
10	1.73×10^{72}
20	3.12×10^{144}
30	(very large number)

4.6.3 Algorithms for stochastic UCPs

A unit commitment problem, even if future demand cannot be perfectly forecasted, can still be solved as a deterministic problem (Bessa et al., 2014). This is accomplished by the incorporation of a sufficient reserve or margin as to meet unexpectedly high demands. However, if a peak demand does not occur, the reserve capacities, which may well amount up to 10 percent of actual power needs, will remain idle. While deterministic solutions to UCP are generally still considered reliable, they are not cost-effective (Safta et al., 2014).

4.6.4 Uncertainty modeling

The aim of this discussion is to provide an insight to key concepts of uncertainties and the core notion linked with uncertainty modelling. The primary idea here is to highlight the science of quantitative characterization, which allows the development of uncertainties in real and simulated world computations. Research conducted by Khatir et al (2013) highlight that the core notion linked with uncertainties is associated with the development of a modelling technique that allows classification of uncertainties in order to develop a viable and effective simulation mode. This is critically important concept that is associated with the development of a simulation process in various engineering problems. Gooi et al (2013) highlight that the core concept of uncertainty modelling is linked with developing a viable footprint for a cohesive analysis that would allow the development of a direct link between the actual environment and what can be simulated. This is a critically important concept in the area of power generation as the importance of effective simulation plays a virtually important role in this particular domain. Research conducted by Baldick (1995) highlights that he development of an effective and efficient modelling technique in the power systems domain. The core notion here is to simulate various outcomes and develop an effective and viable solution based on the simulation that is as close to the reality as it could be. The core element or one of the core elements within the simulation of power systems is linked with the concept of understanding the operation of electrical power systems. Baldick(1995) highlights that there are various techniques that can be used for the development of an effective simulation model, but the key driving force here is linked with the application usage of simulation process to understand the importance of short term impact of power systems, long term elements linked with transmission expansion and the planning required around that aspect and market analysis which is linked with consumption development of pricing and understanding the overall landscape of an effective simulation system. There are various aspects that can be used in order to understand the development of a viable modelling system, this may be linked with key elements of short circuit, transient stability and optimal dispatch of generating unit. As highlighted earlier the use of simulation in power systems is a very important concept, however, it needs to be highlighted that the core concepts here are linked with the use of unit commitment and how this is relevant in the current concept. The core notion of using unit commitment approach is about the development of finding or identifying the least cost dispatch a viable for generation of electric load. The main idea here is to provide a simulation model that cuts down costs and develops a direct link with the development of efficiency in a power system. This type of approach is often used in hydro power, nuclear and thermal power generation techniques. It needs to be further stated that the core concept here is linked with the development of simulation software that allows active development of how a plant is able to produce and effective transport this deliverable energy. Therefore, the concept of developing a simulation modelling technique is of utmost importance to the power generation units and hence unit-commitment plays a vitally important role in the development of an effective simulation model. As highlighted by Shevle and Fahd (1994), Unit commitment is one of the key application in power system operations, one that allows the development of optimal bidding strategies in an un-regulated power generation market. The concept of uncertainty and the development of unit commitment is important in the power generation fields; therefore it is important to highlight the relevance of these within an uncertainty modelling perspective. Uncertainty modelling is often divided into three key areas of developing a simulated approach, these are scenarios, uncertainty set and probabilistic constraints. Given p uncertain real parameters, an uncertainty set is the set of values that can possibly be taken by the p parameters considered (Vasant, 2016). The main focus in the scenario representation model is to develop as many situations of potential scenarios as possible, this allows the development of a two stage model one that moves down and populates various scenarios before simulating them. While effective, this types of approach is not always usable in a very large, dynamic power generation process. another important element that is important is the use of probabilistic forecasting. This is effectively based on optimisation of various algorithms and development of set of quantities in order to highlight and develop effective forecasting model. These approaches are all effective and often used based on the current requirements and the complexity of the power generation at hand. It needs to be state that uncertainty modelling when coupled with the use of UC approach allows the development of an effective modelling technique, however this needs to be further developed based on the bespoke set of requirements and the valid use case of the current requirements of the power generation industry.

The next step of the process is linked with the development of various algorithmic techniques that are feasible in the current context. One such approach from an algorithmic point of view is stochastic programming. Baldick (1995) states that the core notion of stochastic programming is linked with the development of a modelling optimization technique that involves core elements of uncertainties. The key notion here is to utilize the core aspects of effective modelling and couple this with the key notion of optimizing the model by combining scenarios, parameters and developing a direct link with the problem at hand. Another important method to move towards the development of an effective model for UC based problem is to utilize robust optimization. In contracts to the technique mentioned earlier, the core notion of using this approach is linked with incorporation of uncertainties without the information of underlying parameters. The core idea here in the case of UC is to minimize the worst case scenario outcome and optimize that core element to develop a cost effective solution to the problem using uncertainty modelling. This approach although much more focused, often has very limited results in terms of optimization of various scenarios. The core notion here is to provide a conservative set of results and then develop these into a viable solution for the UC problem at hand. Finally, it needs to be state that a dynamic approach to uncertainty modelling needs to be also developed. This approach is known as stochastic dynamic programming, which allows the development of an approach that is both effective and takes a multi-tiered development approach. This model is about the optimization of simulation and the idea is to include various variables liked with a power system from an uncertainty point of view. Overall, it is evident that the use of an effective UC modelling technique is in demand within the market. The core notion of having a viable uncertainty modelling technique is of critical importance one that cannot be ignored by organizations across the globe. The importance of this approach is significant and cannot be therefore ignored by organizations, while the overall forecasting methodologies for renewable energies has increased significantly the core notion of forecasting is still not very clear. This is why the importance of effectively combining both a viable uncertainty simulation technique along with the development of a viable business model to cope with the challenges of identifying core problems related to UC are of significant importance. This discussion just provides a brief overview of how uncertainty and UC can be utilized to answer and solve the important problem linked with power generation.

Due to the operating constraints on generating units and the forecasted load, it is rare that these very big numbers are reached. However, it still remains a challenge to optimize the unit commitment problem due to the huge dimensionality of the solution area. Historically, the most frequently used techniques in solving the unit commitment problem are as follows, see also Padhy (2004), Hobbs et al. (2001), and Wollenberg Bruce et al. (2014)

1. Priority Listing
2. Dynamic programming
3. Lagrange relaxation
4. Mixed Integer linear programming (MILP)

Advances in mixed integer programming seem to make it quickly establishing itself as the preferred technique (Labbi and Benattous, 2014). MILP solvers are advanced and they ensure optimality. The UCP model characteristics are well suited for MILP. Using MILP, requirements that are of high technicality such as spinning reserves are easily implemented (Koller and Hofmann, 2018) and (Delarue and D'haeseleer, 2007).

4.6.5 Priority Listing

To determine the commitment states of thermal units,new methods had to be introduced Kerr et al. (1966). Amongst these methods was the Priority Listing algorithm, where generating units were prioritized based on the average cost of production.Each hour units were committed one after the other based on their priority order till the constraints were satisfied. The minimum up and down times were checked during the listing process. After that, the shut-down process is applied to decommit some units to save cost Kerr et al. (1966). However, when applying priority listing the start-up cost and other constraints are ignored in the process. Therefore,the solutions that are obtained from this process are not optimal. Priority listing cannot be applied to hydro-units and fuel constrained units.

The priority listing method solutions can be obtained using exhaustive enumeration of all the units at various levels of load. The simplest solution to the unit commitment problem consists of creating a priority listing of units by start-up heuristics arranging the units by operating costs. Different versions of the priority listing method rank the units continuously, the ranking system is based on guidelines.

The priority listing method can be enhanced to become better through grouping of the generating units so that operating constraints can be met (Wollenberg Bruce et al., 2014). Baldwin et al. (1959) used a heuristic approach for unit commitment where all the units were shut down and started up in a strict priority order. The priority list was obtained based on the average full load cost of each unit Although solutions are obtained quickly under priority listing, a major drawback is that solutions are not optimal (Delarue and William, 2007). Priority listing method is also not capable of handling large-scale systems. Bawa and Kaur (2016) solved the unit commitment problem using an improved version of the priority listing method. They switched the generators ON and OFF to reduce the total operating cost of the units. The numerical results they provided showed that the improved version committed the most economic unit first. The improved version is much better than the existing Priority Listing methods.

4.6.6 Dynamic Programming

Dynamic programming was first applied to the unit commitment by Lowery (1966), and an improved version was introduced later by Ayoub and Patton (1971), They introduced Probability techniques to determine reserve. The Approach by Pang et al. (1981) selects nominal commitment that has been chosen to be good every hour. The selection is based on minimum hours,priority order needed to meet constraints. A set of units are chosen for optimisation, the units above are assumed to be off and the units below the set are assumed to be committed. To reduce the running time of the algorithm,Snyder et al. (1987) and Hobbs et al. (1988) used approaches to select the best states from all the states and applied economic dispatch. Dynamic Programming has many disadvantages with power systems that contain many generating units. This is due to the fact that the dynamic programming solution has to search over a few number of commitment states, which in turn decreases the combinations that have to be tested in each time period.

Dynamic programming is an optimisation technique that has many applications, It reduces the complexity by cutting the work down, therefore only hitting the sub-optimal combination rather than the optimal combination (Hobbs et al., 1988). The method begins from the optimality theorem which states that the optimal policy only contains optimal sub policies . However, when larger systems are involved, it becomes difficult to solve through dynamic programming. This method also creates the same priority list for use in the dynamic programming search. In its simplest form, dynamic programming for the unit commitment problem solves for all possible states at every interval. The states that are not optimal are rejected immediately, even when the problem size is small to medium, a high number of feasible solutions exists and this increases the time it will take to solve even for a very large computer. As a result, the techniques used are a simpler version of dynamic programming algorithm.

4.6.7 Lagrange Relaxation

In 1970, an observation was made that hard combinatorial optimization problems could be viewed as small or easier problems with relatively fewer set of constraints. By dualizing a problem, a Lagrangian problem is created which is easier to solve and has an optimal value whose optimal value is a lower bound for minimisation problems and vice versa for maximisation problems. Therefore, the Lagrangian problem can be used to replace linear programming relaxation because it provides bounds in branch and bound (Fisher, 1981).

Using Lagrangian approaches, the unit commitment problem is formulated as follows:

- Cost function based on a single unit.
- Set of constraints based on a single unit.
- Hourly generation and reserve constraints based on all units.

Lagrangian method is considered to be the fittest for large-scale environments, but the yielded solutions might not be technically feasible (Coronado et al., 2012).

4.6.8 Mixed integer linear programming

Mixed-integer programming (MIP) is a modified version of the standard integer programming that allows non-integer functions. Mixed-integer programming treats the objective function and the constraints as being continuous, while variables are treated as integers. The branch and bound method is one of the methods used in solving integer problems, it solves the problem by solving a simpler versions that are derived from the original problem. However, a number of activities have to be defined. They are as follows:

1. The problems on the branch and bound tree.
2. The method that will used in solving the problems on the tree.
3. The method that will be used to search.

By defining a corresponding problem,the branch and bound tree is considered to be fully defined when the top node of the tree and the method determining the branches of the tree Cohen and Sherkat (1987).

When compared to other methods, MILP has the following advantages:

- MILP can be optimized using commercial solvers.
- The equations are easy to write and adapt.
- MIP solvers are able to include start-up cost to the whole optimization.
- MILP allows the inclusion of spinning reserves in a detailed manner as part of the optimisation process, this enhances the optimisation and provides a unique optimal solution.

The unit commitment problem has been formulated using mixed integer programming, then the commitment schedule was solved using the regular standard integer programming algorithm Dillon et al. (1978). Another method that has been proposed in the literature changes the linear optimization problems into transshipment problems during the branch and bound search procedure Lidgate and Nor (1984). An algorithm was built by combining dynamic and mixed integer linear programming, the algorithm found feasible combinations of the units at particular points. The dynamic programming helped in identifying good scheduling routes for particular time domains.

An algorithm has been developed to improve the calculation of the MILP. it is know that adding the minimum up and downtime constraints to the model increases the calculation time. Therefore the algorithm relaxes the minimum up and downtime constraint initially and the feasibility of the solution is checked. If the solutions obtained are feasible,the minimum up and down constraints that were initially relaxed are included and new iteration is executed Wollenberg Bruce et al. (2014).

Compared to other methods and algorithms, MILP has been shown to have used linear cost functions although other algorithms are more accurate and precise. Presently, the branch and bound method has been only applied to small and medium sized problems using linear models Cohen and Sherkat (1987). For a large problem, the computation time using mixed integer linear programming is long.

4.6.9 Heuristics and alternative approaches

Data envelopment analysis (DEA) is one approach used when solving the unit commitment under deterministic environment. When using this non-parametric method, the input and output are provided initially. Most often, they are estimated from the previous operation experience of the power generation system.

This approach might result in overly conservative and expensive operating strategy (Bessa et al., 2014).

Another method used under the deterministic environment is the principal component analysis (PCA), where variables are kept to a minimal number. This method is good as a quick estimate but not as a comprehensive UCP solution as many important factors might be left outside the scope of this model (B. et al., 2013).

4.6.10 Stochastic programming

The formulation of stochastic problem and decisions are made without adequate or full information because the decisions are sometimes dependent on random events such as the future price of crude oil, weather conditions and so on. The first set of decisions to be made are known as the first-stage decisions and are represented by vector x, while at the second-stage the full information has been received and is based on a random vector ξ and corrective actions represented by y are made. The mathematical representation of the two-stage stochastic program is

$$\text{Minimize } C_x^T + E_\xi Q(x, \xi) \tag{4.5}$$

subject to

- $Ax = b$,
- $x \geq 0$,

In order to schedule the UC problem under uncertainty and the diesel generator constraints, adjustments have to made to the deterministic UC problem in the section above. Under the stochastic conditions, the reserve requirement for each period is determined endogenously which is done through Optimization. A cost is introduced into the objective function due to interruptions that might occur, also the spinning reserve is optimized through the trade-off the cost and benefits. Hence the objective function under the stochastic UC problem is as follows:

$$\text{Minimize} U_i^t, P_i^t [\sum_{t=1}^{T} \sum_{i=1}^{I} C_i(u_i^t, P_i^t) + S_i^t(u_i^t) + C_E(u_i^t, P_i^t] \tag{4.6}$$

The objective function above does not need system constraints because both the spinning reserve and optimal loads have been Optimized. However when dealing with stochastic UC problems, even the power needed to power low inertia loads such as pumps, fans , computers, e.t.c have to be considered. Additional operational constraints have to be added to generating units so that a feasible solution can be reached.

An alternative approach is the stochastic environment, where the solution of the unit commitment is not precise and equivalent to the load. A stochastic model uses a finite sample of load estimates to reduce the inherent uncertainty of this variable (Safta et al., 2014). To each estimate, a probability weight is assigned (Martinez and Anderson, 2015). Typically, these solutions are more flexible and less costly as compared to those developed in the deterministic environment.

4.6.11 Simulation and heuristic techniques

Evolutionary programming techniques for UCP include genetic algorithms, neural networks, fuzzy logic, artificial bee colony, particle swarm optimization, Monte-Carlo technique and simulated annealing (Rahmat et al., 2013). These approaches are also referred to as meta-heuristic or non-conventional methods (Coronado et al., 2012). They rely on probability scenarios to determine the possible future conditions of the power supply system operation. These methods are generally more flexible and practicable than traditional techniques as they yield a range of recommendations rather than a single solution. However, their common drawback is that they are very sensitive to the correct choice of architecture and manual parameter tuning (Dalal and Mannor, 2015). Moreover, scenario development and simulation me be time-inefficient.

Evolutionary programming methods are commonly used for stochastic environments, while numerical optimization methods are associated with deterministic environments (Martinez and Anderson, 2015). Evolution programming is found to be more effective to deal with uncertainty constraints (Rahmat et al., 2013). However, it might also present computational challenges.

Genetic algorithms (GA), stemming from the evolution theory, are a powerful tool for solving optimization problems (Labbi and Benattous, 2014). They generate a random set of stochastic variables through mutation and crossover, similarly to how it happens in living organisms. In UPC solution, genetic algorithms are often used in combination with other methods (Nakawiro, 2014). The major limitation of GA is that they have only been used for two-state models (Coronado et al., 2012).

The Monte-Carlo technique is often used to model the future demand, since its actual figures are inaccessible. Researchers favour this method due to the guaranteed convergence of the optimal solution value this method. However, it can also present a scalability challenge (Coronado et al., 2012).

One of the most recent approaches to UCP solution is the Markov Decision Process (MDP) framework, which has been extensively used in other research fields. This model considers the outcomes of a process to be partly random, partly the result of a decision makers actions .Dalal and Mannor (2015) applied it to develop a low-cost generation schedule. The result was a 27 reduction in operation cost as compared to the previously used simulated annealing model (Dalal and Mannor, 2015). The running time decreased from 2.5 hours to 2.5 minutes. However, this research model assumed that all generators were of the same type, with the same generation and start-up costs. Further research is needed to find out whether MDP will be as much advantageous in more diverse power grids, including solar and wind generators along with traditional ones.

In addition, there are a growing number of attempts to create reliable hybrid environments that combine the features of stochastic and deterministic ones (Restrepo and Galiana, 2011) ; (Tan and Shaaban, 2015). Such models use a deterministic approach for some variables and stochastic modelling for others. For example, the UC formulation by (Restrepo and Galiana, 2011) used the stochastic approach to model the day-ahead demand and combined it with the deterministic N-1 security criterion. Potentially, the hybrid environments can overcome the limitations of both methods. However, the field of their application is currently limited to small, stand-alone power generation capacities, mainly wind turbines.

4.7 Conclusion

This chapter starts with describing the current state of electricity in Nigeria where the factory will be based.Manufacturing plants in Nigeria rely on their own electricity sources for production. This chapter focused on the characteristics of diesel generators and the complexity of modelling the UCP. The chapter goes through the literature on power production such as grid and island manufacturing.This chapter will help in understanding the best way to deal with electricity issue. This chapter shows that a novel research can be carried out considering variations of the problem such as impact of loading profiles from deterministic to stochastic settings, combination of the different sources of electricity, energy storage and integrating of power decisions with manufacturing decisions. The knowledge gained from this chapter forms the foundations for the next chapters.

Chapter 5

Diesel generator models

Chapter 5

Diesel generator models

5.1 The optimal way to run a set of generators to meet a constant required output

In this first model, we consider a given set of possible generators. The problem is to establish which generators of this set to actually use and at which feasible load factor so as to provide a given total constant output level at total minimum running cost.

We assume the following data is available. Let $\mathcal{M}$ be the set of m available generators. For generator i in $\mathcal{M}$, the capacity is p_i (kW). Each generator consumes fuel per unit of time as a linear function of the load factor at which it is set to run, and the intersection at 'zero load' b_i (which is not part of the region of feasible loads) and the slope c_i of the curve are assumed known. Intersect and slope can be retrieved from available manufacturer specifications such as from the data given in Table **??**. The minimum load factor to run generator i is given by y_i^{min}. The total output level required is X (kW), and the price per gallon of diesel is g.

We introduce for every generator i a decision variable x_i that should be 1 if generator i is selected to run, and 0 otherwise, and a decision variable y_i (the load factor) that will determine at which load it should run.

The following MILP can then be formulated:

$$
\begin{array}{lll}
\min & \sum_{i=1}^{m} g(c_i y_i + b_i x_i) & \\
s.t. & \sum_{i=1}^{m} p_i y_i \geq X & \\
 & y_i \leq x_i & \forall i \in \mathcal{M} \\
 & y_i \geq y_i^{min} x_i & \forall i \in \mathcal{M} \\
 & x_i \in \{0,1\} & \forall i \in \mathcal{M} \\
 & y_i \geq 0 & \forall i \in \mathcal{M}
\end{array}
\tag{5.1}
$$

The objective function minimises the total cost of diesel consumed per unit of time over the set of generators being selected to run and at which load. If $x_i = 1$, then the second and third set of constraints specify that the load factor y_i has to be chosen within the range of y_i^{min} and 1. If $x_i = 0$, then the third set of constraints imposes the condition that then $y_i = 0$.

The first constraint specifies that the total output provided is to be at least X. Therefore, it is assumed that output generated in excess of the required output X is feasible; the excess output will be diverted to a dummy load and is assumed lost as waste. Therefore, if p_{min} is the capacity and y_{min} the minimal load factor of the smallest generator in $\mathcal{M}$, then feasible solutions for cases where $X < y_{min} p_{min}$ still exist, e.g. by letting this generator run at y_{min}. Also, if we consider, for example, only two generators in $calM$ with $p_1 = 200$ and $p_2 = 2000$ and $y_{min} = 0.25$, then feasible solutions for $200 < X < 0.25(2000) = 500$ exist as we can let generator 2 run at 25%. The chances of having non-zero excess are minimised when the generator set $\mathcal{M}$ contains a range of capacities; excess will only arise in an optimal solution by the impact of the minimum load factor constraints since the objective function will prefer any feasible solution in which excess is zero.

Property 1. (Infeasible region) No feasible solution exists when $X > \sum_{i=1}^{m} p_i$.

We exclude such instances from further consideration.

Property 2. (Price independence) An optimal solution is independent of the price of fuel.

This is obvious from the objective function.

Note that the model formulation only assumes a linear relationship for every generator between fuel consumption and load factor. It will identify the optimal running mode even if the set $\mathcal{M}$ includes generators of different type, manufacturer, and efficiency standards.

All generators, however, are known to run more efficiently i.e. consume less fuel per hour of kW provided, when running at higher load factors. (As illustrated by the data in Table 4.2 This means mathematically the following.

Definition 1. (Load efficiency) $\forall i \in \mathcal{M}$: $b_i > 0$.

Let $\mathcal{M}_k$ be a subset of $\mathcal{M}$ of identical generators and call it the class k of identical generators.

Property 3. (Identical generators property: when $y_k^{min} \leq 0.5$) If more than one generator of $\mathcal{M}_k$ is selected as part of an optimal solution and $y_k^{min} \leq 0.5$, then at most two of these identical generators will have to run at less than full load. If two generators need to run below full load, then their combined load will be larger than 1 and below $1 + y_k^{min}$.

Proof. Consider the subset of s generators of class k with load factors less than one. Let the load factors be $y_1, y_2, ..., y_s$. Each load factor must not be smaller than y_k^{min} and greater than 1. First consider that $s = 2$. The total output provided by these two generators is:

$$p_k(y_1 + y_2), \tag{5.2}$$

and the total fuel consumed is:

$$c_k(y_1 + y_2) + 2b_k. \tag{5.3}$$

and therefore if $y_1 + y_2 \leq 1$, we can make a re-assignment to make $y_1 = 1$ and $y_2 = 0$ and achieve the same output but we safe b_k on fuel consumption. (Since $b_k > 0$ due to load efficiency.) If $y_1 + y_2 \geq 1 + y_k^{min}$, we can make a re-assignment to make $y_1 = 1$ and keep $y_2 \geq y_{min}$ providing the same output and consuming the same amount of fuel. So in this case we do not need to have two generators running at less than full load (but may choose to do so.)

In the case that $1 < y_1 + y_2 < 1 + y_k^{min}$, however, it is impossible to make one of these generators have a load factor of 1 without decreasing the other's load factor below y_k^{min}. In any feasible solution in which both y_1 and y_2 receive non-zero values it must be that $2y_k^{min} \leq y_1 + y_2$, but if $y_k^{min} \leq 0.5$ then $2y_k^{min} < 1$ and this is then automatically satisfied in this case.

Now consider the case $s > 2$. The total output they provide is:

$$p_k \sum_{i=1}^{s} y_i, \tag{5.4}$$

and since any feasible re-assignment of load factors amongst this set will keep $\sum_{i=1}^{s} y_i$ constant, the total output will remain the same. The total fuel consumed is:

$$c_k \sum_{i=1}^{s} y_i + sb_k. \tag{5.5}$$

Let $\sum_{i=1}^{s} y_i = Y$, then it must be than $Y < s$. If Y is integer, we can make a re-assignment of load-factors to retain Y generators at full load and safe $(s - Y)b_k$ on fuel consumption. If Y is a fractional number, say $Y = r.f$, then if $.f \geq y_k^{min}$ we can make a re-assignment of load factors to have r generators running at full load and one generator running at load factor $.f$, saving $(s-r)b_k$ on fuel consumption. If $0 < .f < y_k^{min}$, we can re-assign load factors to have $(r-1)$ generators running at full load, saving $(s-(r-1))b_k$ on fuel consumption, and are left with two generators of which their total load is higher than 1 but smaller than $1 + y_k^{min}$. That case was considered above. □

If $y_k^{min} > 0.5$, then the property does no longer hold. For $y_k^{min} = 0.6$, the minimum number of generators running at less than full load may well be 3; for $y_k^{min} = 0.8$, 4; and for $y_k^{min} = 0.9$, 8.

Property 4. (Identical generators property: when $y_k^{min} > 0.5$) If more than one generator of $calM_k$ is selected as part of an optimal solution and $y_k^{min} > 0.5$, then at most ? of these identical generators will have to run at less than full load. If two generators need to run at below full load factors, then if $y_k^{min} \leq 0.5$ their combined load factor will be larger than 1 and below $1 + y_k^{min}$, and if $y_k^{min} > 0.5$ their combined load factor will be larger or equal to $2y_k^{min}$ and below $1 + y_k^{min}$.

Proof. Consider the subset of s generators of class k with load factors less than one. Let the load factors be $y_1, y_2, ..., y_s$. Each load factor must not be smaller than y_k^{min} and greater than 1. First consider that $s = 2$. The total output provided by these two generators is:

$$p_k(y_1 + y_2), \tag{5.6}$$

and the total fuel consumed is:

$$c_k(y_1 + y_2) + 2b_k, \tag{5.7}$$

and therefore if $y_1 + y_2 \leq 1$, we can make a re-assignment to make $y_1 = 1$ and $y_2 = 0$ and achieve the same output but we safe b_k on fuel consumption. (Since $b_k > 0$ due to load efficiency.) If $y_1 + y_2 \geq 1 + y_k^{min}$, we can make a re-assignment to make $y_1 = 1$ and keep $y_2 \geq y_{min}$ providing the same output and consuming the same amount of fuel. So in this case we do not need to have two generators running at less than full load.

In the case that $1 < y_1 + y_2 < 1 + y_k^{min}$, however, it is impossible to make one of these generators have a load factor of 1 without decreasing the other's load factor below y_k^{min}. In any feasible solution in which both y_1 and y_2 receive non-zero values it must be that $2y_k^{min} \leq y_1 + y_2$. Hence, if $y_k^{min} > 0.5$ then in this case it cannot be that $1 < y_1 + y_2 < 2y_k^{min}1$ since it is not feasible and therefore cannot be part of an optimal solution. However, if $y_k^{min} \leq 0.5$ then such a restriction does not apply to this case.

Now consider the case $s > 2$. The total output they provide is:

$$p_k \sum_{i=1}^{s} y_i, \tag{5.8}$$

and since any feasible re-assignment of load factors amongst this set will keep $\sum_{i=1}^{s} y_i$ constant, the total output will remain the same. The total fuel consumed is:

$$c_k \sum_{i=1}^{s} y_i + sb_k. \tag{5.9}$$

Let $\sum_{i=1}^{s} y_i = Y$, then it must be than $Y < s$. If Y is integer, we can make a re-assignment of load-factors to retain Y generators at full load and safe $(s - Y)b_k$ on fuel consumption. If Y is a fractional number, say $Y = r.f$, then if $.f \geq y_k^{min}$ we can make a re-assignment of load factors to have r generators running at full load and one generator running at load factor $.f$, saving $(s - r)b_k$ on fuel consumption.

If $0 < .f < y_k^{min}$, we can re-assign load factors to have $(r-1)$ generators running at full load, saving $(s-(r-1))b_k$ on fuel consumption, and are left with two generators of which their total load is higher than 1 but smaller than 1.25. That case was considered above. □

Definition 2. (Economies of scale) The set $\mathcal{M}$ is said to exhibit economies of scale if and only if, for any two generators i and j from $\mathcal{M}$ where $p_i > p_j$, the following relationship holds:

$$\frac{c_i y + b_i}{c_j y + b_j} < \frac{p_i}{p_j}, \tag{5.10}$$

for any value of $y \in (0.25, 1)$.

Economies of scale applies to the data given in Table 4.2. However, it may not hold in general when $\mathcal{M}$ consists of generators of various type and manufacturer.

Property 5. (On the case that $X = yp_i$) With economies of scale, it is never optimal to provide yp_i output with a set S of generators for which $\sum_{s \in S} p_s = p_i$ and each set at load y, if a generator i with p_i is in $\mathcal{M}$ for which $y \geq y_i^{min}$.

Proof. Assume it would be optimal, then it means:

$$\sum_{s \in S} (c_s y + b_s) < c_i y_i + b_i. \tag{5.11}$$

However, with economies of scale, each generator s in S has to respect per definition the property:

$$\frac{c_i y + b_i}{c_s y + b_s} < \frac{p_i}{p_s}, \tag{5.12}$$

Rearranging:

$$\frac{c_s y + b_s}{c_i y + b_i} > \frac{p_s}{p_i}, \tag{5.13}$$

and therefore taking the sum over all generators in S:

$$\frac{\sum_{s\in S}(c_s y + b_s)}{c_i + b_i} > \frac{\sum_{s\in S} p_s}{p_i}, \tag{5.14}$$

but since $\sum_{s\in S} p_s = p_i$, this reduces to:

$$\sum_{s\in S}(c_s y + b_s) > c_i y + b_i. \tag{5.15}$$

This contradicts the initial assumption. □

Corrolary 1. (On the case that $X = p_i$) With economies of scale, it is never optimal to provide p_i output with a set S of generators such that $\sum_{s\in S} p_s = p_i$ if a generator i with p_i is in $\mathcal{M}$.

Proof. This is a special case of Property 5 when $y = 1$. □

Proof. Assume it would be optimal, then it means:

$$\sum_{s\in S} c_s + b_s < c_i + b_i. \tag{5.16}$$

However, with economies of scale, each generator s in S has to respect per definition the property:

$$\frac{c_i y + b_i}{c_s y + b_s} < \frac{p_i}{p_s}, \tag{5.17}$$

For $y = 1$ it means:

$$\frac{c_s + b_s}{c_i + b_i} > \frac{p_s}{p_i}, \tag{5.18}$$

and therefore taking the sum over all generators in S:

$$\frac{\sum_{s\in S} c_s + b_s}{c_i + b_i} > \frac{\sum_{s\in S} p_s}{p_i}, \tag{5.19}$$

but since $\sum_{s \in S} p_s = p_i$, this reduces to:

$$\sum_{s \in S} c_s + b_s > c_i + b_i. \tag{5.20}$$

This contradicts the initial assumption. □

Corrolary 2. (On the case that $X = p_i$) With economies of scale, it is never optimal to provide p_i output with a set of s generators from $\mathcal{M}_k$ with $p_k = p_i/s$ if a generator i with p_i is in $\mathcal{M}$.

Proof. If $p_k = p_i/s$ then using s generators of class k provides an output of $\sum_{j=1}^{s} p_j = sp_k = p_i$, and hence Corrolary 1 applies. Assume it would be optimal, then it means:

$$s(c_k + b_k) < c_i + b_i. \tag{5.21}$$

However, with economies of scale, each generator j in class k has to respect per definition the property:

$$\frac{c_i y + b_i}{c_j y + b_j} < \frac{p_i}{p_j}, \tag{5.22}$$

where $c_j = c_k$, $b_j = b_k$, and $p_j = p_k$. For $y = 1$ it means:

$$\frac{c_j + b_j}{c_i + b_i} > \frac{p_j}{p_i}, \tag{5.23}$$

and therefore taking the sum over all s generators of this class:

$$\frac{s(c_k + b_k)}{c_i + b_i} > \frac{sp_k}{p_i}, \tag{5.24}$$

but since $sp_k = p_i$, this reduces to:

$$s(c_k + b_k) > c_i + b_i. \tag{5.25}$$

This contradicts the initial assumption. □

Property 5 requires that each generator in S operates at y. If load factors differ between options, then using the larger generator is not always best. For example, to provide 600 kW output, using one generator of 1000 kW at $y = 0.6$ is a solution with a gap of 2.29% compared to the solution of using a set of one 500 kW and five 20 kW generators all at full load (based on a model using the data of Table 4.2). At a price of $g = 3.68$ dollars per gallon, this is an extra cost of around 35,000 dollars per year.

Further work could look into identifying more properties; test alternative model with load factor re-defined by adjusting the objective function:

$$c_i(y_i' + 0.25) + b_i x_i = c_i y_i' + (b_i + 0.25c_i)x_i,$$

where $y' \in \{0, 1\}$. This eliminates the third set of constraints from the model, however, y' is no longer the load factor.

Chart 5.1 represent a larger model that was developed in which all generator classes were considered which ranges from .25(20) to 2250 are considered, an appropriate number is selected for the smaller and larger models.

5.2 Purchasing a set of generators to cover a constant output not known with certainty

We now consider the problem of purchasing a subset of a given set of potential generators and to use this subset of generators to provide a feasible and minimal

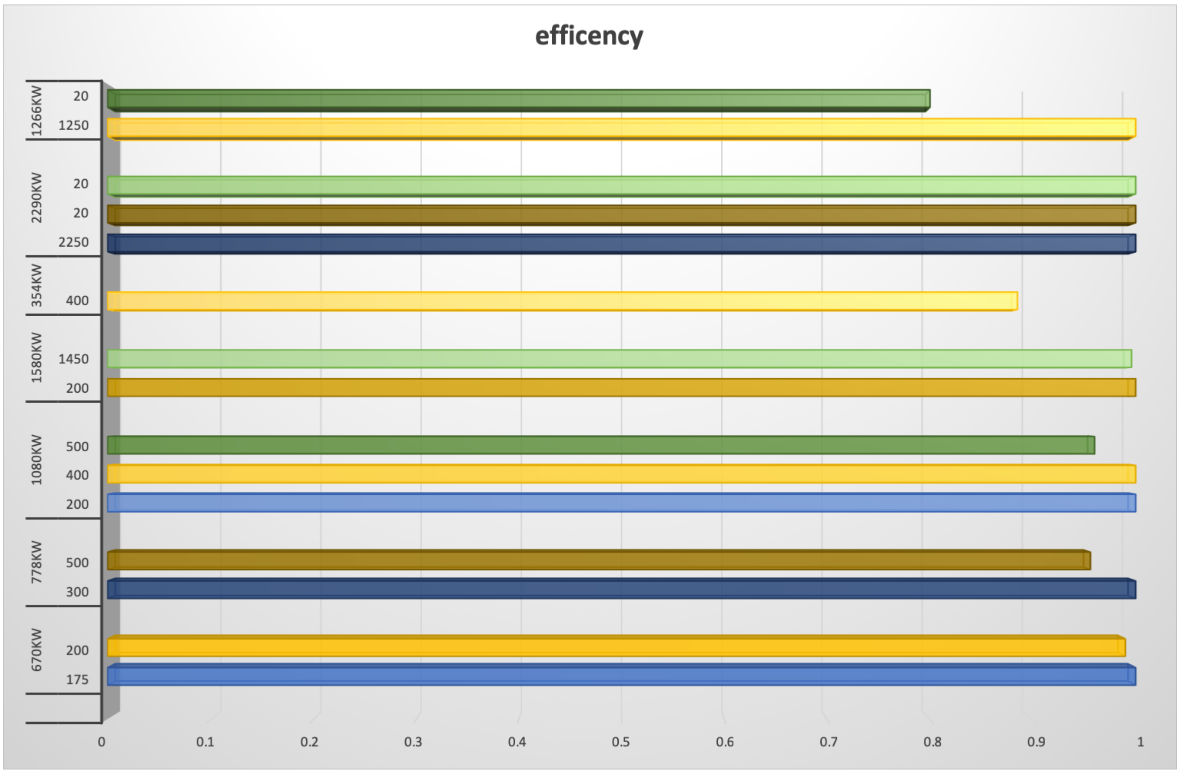

Figure 5.1: diesel generator selection model DIFFERENT GENERATOR/LOAD

total cost solution when the required output level is not known with certainty. In addition to total running costs, we will also consider total investment costs. Let f_i be the fixed investment cost of generator i per unit of time, derived with some appropriate accounting technique from the initial acquisition cost and the expected life-time of the generator.It is assumed that the factory will be getting a loan to buy the set of diesel generators. A percentage of investment cost will be paid by the factory to reimburse the bank for cost and interest over a period of time. Because of the trade-off between investment costs and running costs, the optimal decision does depend on the price of fuel g, which at the moment of purchasing the diesel generators is a parameter not known with certainty.

Let u indicate a state of nature, and let $\mathcal{U}$ represent the set of all possible values of u. We can formulate the following problem:

$$
\begin{aligned}
\min \textstyle\sum_{i=1}^{m} f_i x_i \quad & + \textstyle\sum_{i=1}^{m} g(u)(c_i y_i + b_i x_i) & \\
s.t. \quad & \textstyle\sum_{i=1}^{m} p_i y_i \geq X(u) & \forall u \in \mathcal{U} \\
& y_i \leq x_i & \forall i \in \mathcal{M} \\
& y_i \geq y_i^{min} x_i & \forall i \in \mathcal{M} \\
& x_i \in \{0, 1\} & \forall i \in \mathcal{M} \\
& y_i \geq 0 & \forall i \in \mathcal{M}
\end{aligned}
\tag{5.26}
$$

The above model has a random cost vector through $g(u)$ and a random right-hand side because of $X(u)$. It is clear that the cost of fuel g is determined by global market conditions, while X is governed by how much demand the particular manufacturer has. We thus treat these two parameters as being independent.

As the set of generators need to be able to handle all possible required output levels, we require a robust solution with respect to $X(u)$. We treat X as a random variable with discrete realisations. In particular, we define $\mathcal{N}$ as the set of n different possible output levels. Let $X_j \in \mathcal{N}$ be an output level of relative importance n_j, where n_j measures the relative fraction of time this output level needs to be used per unit of time. (It can also represent a probability measure.) For every generator i a decision variable y_{ij} (the load factor) determines at which load it should run in a scenario where total load required is X_j.

The following MILP can then be formulated:

$$
\begin{aligned}
\min \textstyle\sum_{i=1}^{m} f_i x_i \quad & + \textstyle\sum_{j=1}^{n} n_j \sum_{i=1}^{m} g(u)(c_i y_{ij} + b_i x_i) & \\
s.t. \quad & \textstyle\sum_{i=1}^{m} p_i y_{ij} \geq X_j & \forall j \in \mathcal{N} \\
& y_{ij} \leq x_i & \forall i \in \mathcal{M}, \forall j \in \mathcal{N} \\
& y_{ij} \geq y_i^{min} x_i & \forall i \in \mathcal{M}, \forall j \in \mathcal{N} \\
& x_i \in \{0, 1\} & \forall i \in \mathcal{M} \\
& y_{ij} \geq 0 & \forall i \in \mathcal{M}, \forall j \in \mathcal{N}
\end{aligned}
\tag{5.27}
$$

The above model has still the random cost $g(u)$. It seems most appropriate to handle this uncertainty through reformulating it as a chance constraint. Let $F(x, y, g)$ represent the objective function and $A(x, y) = b$ the set of constraints of the above model, then its reformulation is:

$$
\begin{aligned}
& \min v \\
s.t. \quad & Prob\{F(x, y, g) <= v\} >= \alpha \\
& A(x, y) = b
\end{aligned}
\tag{5.28}
$$

A stochastic program with a random cost vector can be further converted into a (non-linear) deterministic programming problem when assuming knowledge of the probability distribution from which the random vector is drawn.

5.3 Conclusion

In this chapter, a framework is proposed to classify models that will allow the incorporation of power generation and diesel generator decision making in production planning. The first model decides which generator/generators is/are the most feasible to use so that a given output is given at a minimum cost.However, this model considers a limited number of diesel generators. Therefore a larger model which considers generators ranging from .25(20kWh) to 2250kWh. Using the two models, extensive computational experiments were carried out to compare the different combinations and power requirements. The first contribution that comes from this chapter is that no known literature on properties related to optimal set of generators exists. Secondly, an important contribution which involves integrating power generation and production planning is envisaged. Finally, this chapter provides inputs that are used in chapter six.

Chapter 6

Model with energy usage considerations

Chapter 6

Model with energy usage considerations

6.1 Introduction

As stated in the previous chapter , efficient planning is the most important aspect for manufacturing plants. Ecological and sustainable business behavior is increasingly becoming more relevant globally. Entrepreneurs and businesses have to change the way they think due to the sensitivity about climate change and energy prices. When building mathematical models, this new perspective has to be considered in all areas, especially in the manufacturing process. Replacing or upgrading machines that are currently in use to Eco-friendly or more efficient ones is expensive and sometimes not possible in developing countries like Nigeria. Therefore strategies have to be developed that enhance already existing mathematical models so that power usage and other Eco-factors are considered when generating production schedules. If power usage and energy prices are ignored, they affect what the production schedule should be, what products should be produced, the arrangement what products are produced first and last, and entire profitability of the company. By making some changes to the algorithm, energy consumption can be optimized and overall cost minimised. It is therefore reasonable to take these additional factors into consideration when creating production plans.

6.2 Literature review

The problem presented here is based on the soft drink integrated production lot sizing and scheduling problem from (Ferreira et al., 2009). Contrary to Ferreira et al. (2009), we take into account the energy consumption of the system and include the energy cost within the total costs of the problem. Masmoudi et al. (2015) also include the energy consumption in a lot-sizing model. However, their model considers a linear production structure much simpler than the model in (Ferreira et al., 2009).

In our model, the production system is entirely supplied by a diesel generator. The diesel generator selection is a variable of the problem, and its investment cost is also included within the total costs. Therefore, the contribution of this chapter is the formulation of a model that optimise the decision of investment in a power generator, its fuel consumption,together with the lot sizing and scheduling of the system it is supplying.

6.3 Problem description

6.3.1 Energy consumption model

The energy cost here is the amount of fuel used by the diesel generator multiplied by the fuel cost. Diesel fuel consumption charts show that the relation between the fuel consumption of a diesel generator and the electricity produced by it can be well estimated by a linear function, i.e.

$$f = A \times p + B \tag{6.1}$$

Where f is the fuel consumption, in liter per hour, p is the power produced, in kW, and A and B are two constants whose values depend on the type of diesel generator. The value of B is generally non negligible, i.e. a diesel generator switched on and producing few or no electricity still consumes a significant amount of fuel.

Therefore, the total amount of fuel consumed by the generator during a specific period is related to the energy produced by it according to the following equation:

$$\begin{aligned} V = \int_T \quad & f = \int_{T_{on}} f + \int_{T_{off}} f = \int_{T_{on}} (A \times p + B) \\ V = \quad & A \times \int_{T_{on}} + \int_{T_{on}} B \\ V = \quad & A \times E + B \times D^{on} \end{aligned} \tag{6.2}$$

Where T is the time interval of the period, V is the total volume of fuel consumed during the period, in liter, E is the total amount of energy produced by the generator, in kWh, D^{on} is the switched-on duration of the generator during the period, in hours.

6.3.2 Problem formulation

The problem is formulated as follows:

Sets

- $t \in \{1, \ldots, T\}$ set of macro-periods;
- $s \in \{1, \ldots, N\}$ set of micro-periods;
- S_t set of micro-periods in macro-period t;
- P_t first micro-period in macro-period t;
- $i, j \in \{1, \ldots, J\}$ set of items;
- Ω_l set of items that can be produced with liquid flavor l;
- $k, l \in \{1, \ldots, F\}$ set of liquid flavors;
- $tk \in \{1, \ldots, TK\}$ set of tanks;
- $m, r \in \{1, \ldots, M\}$ set of machines/bottling lines;
- $k(m)$ tank connected to bottling line m;
- $dg \in \{1, \ldots, DG\}$ set of diesel generators;

Parameters

- $D_{j,t}$ demand for item j in macro-period t;
- ic_j inventory cost for one unit of item j ;
- bc_j backorder cost for one unit of item j;

- $cc_{k,l}$ changeover cost from liquid flavor k to l ;
- fc diesel fuel cost;
- gc_{dg} investment cost of diesel generator dg ;
- γ percentage of the investment cost that should be paid during the total time period of the problem;
- $P_{dg}^{\max}$ maximum power output of diesel generator dg;
- A_{dg} slope of the power to fuel consumption function of diesel generator dg;
- B_{dg} intercept of the power to fuel consumption function of diesel generator dg;
- C_{tk}^{max} maximum volume capacity of tank tk;
- C_{tk}^{min} minimum volume capacity of tank tk ;
- $v_{j,l}$ volume of liquid flavor l necessary for the production of one unit of item j;
- D duration of a micro-period;
- $Cttk_{k,l}$ changeover time, in tanks, from liquid flavour k to l ;
- $Ctm_{i,j}$ changeover time, in machines, from item $i$to j ;
- $p_{m,j}$ production time of item j in machine m ;
- ptk_{tk} power consumption of tank tk;
- pl_m power consumption of bottling line m ;
- $It_{j,0}$ initial inventory for item j;

- $B_{j,0}$ initial backorder for item j;

Variables

- $It_{j,t}$ inventory for item j at the end of period t;
- $B_{j,t}$ backorder for item j at the end of period t;
- Is_{dg} investment decision in generator dg;
- $s_{dg,s}$ indicates whether generator dg is switched on in micro-period s;
- $p_{dg,s}$ maximum power produced by generator dg in micro-period s;
- $E_{dg,s}$ energy produced by generator dg in micro-period s;
- $V_{dg,s}$ volume of fuel consumed by generator dg in micro-period s;
- $Dur^{on}_{dg,s}$ amount of time generator dg is switched on in micro-period s;
- p^T_s maximum power produced in micro-period s;
- E^T_s energy produced in micro-period s;
- $s_{tk,l,s}$ indicates whether there is production in tank tk of the liquid flavor l in micro-period s;
- $Ctk_{tk,k,l,s}$ indicates whether there is changeover in tank tk from liquid flavour k to l in micro-period s;
- $TD_{tk,s}$ amount of time tank tk is producing in micro-period s;
- $s_{m,j,s}$ indicates whether the machine m is set-up for item j in micro-period s;

- $Cm_{m,i,j,s}$ indicates whether there is changeover in machine m from item i to j in micro-period s;
- $q_{m,j,s}$ production quantity in machine m of item j in micro-period s;
- $W_{m,s}$ waiting time of machine m in micro-period s;
- $CT_{m,s}$ completion time of machine m in micro-period s
- $l_{m,\gamma,s}$ indicates whether the completion time of machine m is after the starting time of machine γ in micro-period s;

$$\min Z = \sum_{t=1}^{T}\sum_{j=1}^{J}(ic_j Iv_{j,t} + bc_j B_{j,t}) + \sum_{s=1}^{N}\sum_{tk=1}^{TK}\sum_{l=1}^{F}\sum_{k=1}^{F} cc_{k,l} Ctk_{tk,k,l,s} + (\sum_{s=1}^{N}\sum_{dg=1}^{DG} V_{dg,s}) fc + \gamma \sum_{dg=1}^{DG} gc_{dg} I \tag{6.3}$$

The objective function 6.3 minimises the total cost composed of the operation costs and the investment costs. The operation costs are the sum of inventory costs, back-order costs, changeover costs, and fuel cost. The investment cost refers to the cost of diesel generators.

$$\sum_{dg=1}^{DG} Is_{dg} = 1 \tag{6.4}$$

$$p_s^T = \sum_{dg=1}^{DG} p_{dg,s} \qquad s = 1, \ldots, N \tag{6.5}$$

$$E_s^T = \sum_{dg=1}^{DG} E_{d,s} \qquad s = 1, \ldots, N \tag{6.6}$$

$$p_{dg,s} \leq s_{dg,s} P_{dg}^{\max} \qquad dg = 1, \ldots, DG, s = 1, \ldots, N \tag{6.7}$$

$$E_{dg,s} \leq s_{dg,s} P_{dg}^{\max} D \qquad dg = 1, \ldots, DG, s = 1, \ldots, N \tag{6.8}$$

$$s_{dg,s} \leq Is_{dg} \qquad dg = 1, \ldots, DG, s = 1, \ldots, N \tag{6.9}$$

Constraint 6.4 ensures that only one diesel generator can be installed. Constraints 6.5 and 6.6 guarantee that the power and energy supplied to the system at any time are the ones delivered by the unique generator installed. A diesel generator can deliver power at a certain time only if it has been switched on at this time, which is translated by constraints 6.7 and 6.8. Naturally, a diesel generator can only be switched on if it has been installed 6.9.

$$V_{dg,s} = A_{dg} \times E_{dg,s} + B_{dg} \times D^{on}_{dg,s} \qquad dg = 1, \ldots, DG, s = 1, \ldots, N \tag{6.10}$$

$$\sum_{m/tk(m)=tk} \sum_{j \in \Omega_l} v_{j,l} q_{m,j,s} \leq s_{tk,l,s} C^{\max}_{tk} \qquad tk = 1, \ldots, TK, l = 1, \ldots, F, s = 1, \ldots, N \tag{6.11}$$

$$\sum_{m/tk(m)=tk} \sum_{j \in \Omega_l} v_{j,l} q_{m,j,s} \geq s_{tk,l,s} C^{\min}_{tk} \qquad tk = 1, \ldots, TK, l = 1, \ldots, F, s = 1, \ldots, N \tag{6.12}$$

$$\sum_{l=1}^{F} s_{tk,l,s-1} \geq \sum_{l=1}^{F} s_{tk,l,s} \qquad tk = 1, \ldots, TK, t = 1, \ldots, T, s \in S_t - P_t \tag{6.13}$$

$$Ctk_{tk,k,l,s} \geq s_{tk,k,s-1} + s_{tk,l,s} - 1, \qquad tk = 1, \ldots, TK, k, l = 1, \ldots, F, s = 1, \ldots, N \tag{6.14}$$

$$Ctk_{tk,k,l,s} \geq \sum_{j \in \Omega_l, m \in M_{tk}} s_{m,j,s-1} + s_{tk,l,s} - 1, tk = 1, \ldots, TK, k, l = 1, \ldots, F, t = 2, \ldots, T, s = P_t \tag{6.15}$$

Constraints 6.10 come from 6.2 and describe the relation between the fuel consumption of a diesel generator and the energy it produces during a specific period.Constraints 6.11 and 6.12 guarantee that, in case a certain liquid is produced in a tank, its volume will be between the minimum and maximum liquid capacity of the tank. As explained by authors in Ferreira et al. (2009), a minimum liquid quantity is necessary for liquid homogeneity. Constraints 6.13 enforce that the liquid production preferentially occurs at the beginning of each macro-period. Constraints 6.14 and 6.15 control the liquid flavor changeover.

$$\sum_{k=1}^{F}\sum_{l=1}^{F} Ctk_{tk,k,l,s} \leq 1, \qquad tk = 1,\ldots,TK, t = 1,\ldots,T, s \in S_t \tag{6.16}$$

$$Ivt_{j,t-1} + B_{j,t} + \sum_{m=1}^{M}\sum_{s\in S_t} q_{m,j,s} = Ivt_{j,t} + B_{j,t-1} + D_{j,t}, j = 1,\ldots,J, t = 1,\ldots,T \tag{6.17}$$

$$W_{m,s} \geq \sum_{k=1}^{F}\sum_{l=1}^{F} Ctk_{tk,k,l,s} cttk_{k,l} - \sum_{i=1}^{J}\sum_{j=1}^{J} C_{m,i,j,s} ctm_{i,j} \qquad m = 1,\ldots,M, s = 1,\ldots,N \tag{6.18}$$

$$p_{m,j} q_{m,j,s} \leq s_{m,j,s} D \qquad m = 1,\ldots,M, j = 1,\ldots,J, s = 1,\ldots,N \tag{6.19}$$

$$\sum_{j=1}^{J} s_{m,j,s} \leq 1 \qquad m = 1,\ldots,M, s = 1,\ldots,N \tag{6.20}$$

$$C_{m,i,j,s} \geq s_{m,i,s-1} + s_{m,j,s} - 1, \qquad m = 1,\ldots,M, i,j = 1,\ldots,J, s = 1,\ldots,N \tag{6.21}$$

$$\sum_{i=1}^{J}\sum_{j=1}^{J} C_{m,i,j,s} \leq 1, \qquad m = 1,\ldots,M, s = 1,\ldots,N \tag{6.22}$$

$$CT_{m,s} \geq \sum_{j=1}^{J} p_{m,j} q_{m,j,s} + W_{m,s}, \qquad m = 1,\ldots,M, s = 1,\ldots,N \tag{6.23}$$

$$CT_{m,s} - CT_{\gamma,s} + \sum_{j=1}^{J} p_{r,j} q_{\gamma,j,s} \leq l_{m,r,s} D, \qquad m,\gamma = 1,\ldots,M, m \neq \gamma, s = 1,\ldots,N \tag{6.24}$$

$$p_s^T \geq \sum_{l=1}^{F} s_{tk(m),l,s} ptk_{tk} + \sum_{j=1}^{J} s_{m,j,s} pl_m \quad + \sum_{\gamma/tk(\gamma) \neq tk(m)} (l_{m,\gamma,s} + l_{\gamma,m,s} - 1) ptk_{tk(\gamma)}$$
$$+ \sum_{r \neq m} (l_{m,r,s} + l_{r,m,s} - 1) pl_r \qquad m = 1, \ldots, M, s = 1, \ldots, N \tag{6.25}$$

Constraint 6.16 ensures there is not more than one liquid changeover at a time in each tank.Constraints 6.17 represent the inventory balancing constraints for each item in each macro-period. Following the method used by Ferreira et al. (2009) , constraints 6.18 ensure the synchronization between tanks and bottling lines. Constraints 6.19 guarantee that an item can be produced in a machine at a certain time only if this machine has been set-up for this item. Only one item at a time can be produced for each machine 6.20, and there cannot be more than one item changeover at a time in machine 6.21 and 6.22. Constraints 6.23 calculates the time when a machine finishes to produce within a micro-period. Following the method used by authors in Masmoudi et al. (2015), we introduce constraints 6.24 and 6.25. These equations help in identifying machines with overlapping producing schedules, and the resulting maximum power output necessary from the generator. Indeed, it can be showed that the value of $l_{m,\gamma,s} + l_{\gamma,m,s}$ that appears in 6.25, is always greater than 1 for $m \neq \gamma$, due to constraints 6.24. Moreover, its value is equal to 2 if, and only if, the two machines have overlapping producing schedules. Therefore, constraints 6.25 guarantee that the required power output during a given micro-period is greater than the power consumption of a specific machine, in addition to the power consumption of all the machine whose production schedule overlaps its own production schedule.

$$TD_{tk,s} \geq CT_{m,s} - CT_{r,s} + \sum_{j=1}^{J} p_{r,j} q_{r,j,s} \quad tk = 1, \ldots, TK, m, r/tk(m) = tk(r) = tk, s = 1, \ldots, N \tag{6.26}$$

par Constraints 6.26 control the production duration of a tank according to the production duration of all the machines connected to it.

$$E_s^T = \sum_{tk=1}^{TK} TD_{tk,s} ptk_{tk} + \sum_{m=1}^{M} \sum_{j=1}^{J} p_{m,j} q_{m,j,s} pl_m ds = 1, \ldots, N \tag{6.27}$$

$$\sum_{dg=1}^{DG} D_{dg,s}^{on} \geq CT_{m,s} - CT_{\gamma,s}, + \sum_{j=1}^{J} p_{\gamma,j} q_{r,j,s} m, \gamma = 1, \ldots, M, s = 1, \ldots, N \tag{6.28}$$

$$D_{dg,s}^{on} \leq s_{dg,s} D \qquad dg = 1, \ldots, DG, s = 1, \ldots, N \tag{6.29}$$

Constraints 6.26 control the producing duration of a tank according to the producing duration of all the machines connected to it. Constraints 6.27 computes the energy consumed, in each micro-period, as the sum of energy consumed by all tanks and bottling lines. Constraints 6.28 ensures that the generator must be switched on from the first moment a machine starts producing until the last moment a machine stops producing. Constraints 6.29 control that the switch-on duration of a generator in a specific micro-period is null if this generator has not been switched in this micro-period.

$$It_{j,t}, B_{j,t} \geq 0, j = 1, \ldots, J, t = 1, \ldots, T \quad p_{dg,s}, E_{dg,s}, V_{dg,s}, D^{on}_{dg,s} \geq 0, dg = 1, \ldots, DG, s = 1,$$
$$p^T_s, E^T_s \geq 0, s = 1, \ldots, N \quad TD_{tk,s}, Ctk_{tk,k,l,s} \geq 0$$
$$tk = 1, \ldots, TK, k, l = 1, \ldots, F, s = 1, \ldots, N \quad Cm_{m,i,j,s}, q_{m,j,s}, W_{m,s}, CT_{m,s} \geq 0$$
$$m = 1, \ldots, M, i, j = 1, \ldots, J, s = 1, \ldots, N \quad Is_{dg}, s_{dg,s} \in \{0, 1\}, dg = 1, \ldots, DG, s = 1, \ldots, N$$
$$s_{tk,l,s} \in \{0, 1\}, tk = 1, \ldots, TK, \quad l = 1, \ldots, F, s = 1, \ldots, N s_{m,j,s}, l_{m,\gamma,s} \in \{0, 1\},$$
$$m, \gamma = 1, \ldots, M, j = 1, \ldots, J, s = 1, \ldots, N \tag{6.30}$$

Constraints 6.30 define the variables domain. Even though variables $Cm_{m,i,j,s}$ and $Ctk_{tk,k,l,s}$ are continuous, constraints 6.14 and 6.21, together with the minimization objective of the problem, ensure that these variables will only take values 0 or 1 in an optimal solution of the problem, as noted by authors in Ferreira et al. (2009).

6.4 Case Study

The model has been tested using different case studies composed of eight different alternative set-ups for the soft-drink production system, and under four different demand scenarios.

6.4.1 Case study description

The eight system structures considered are described in table 6.1. The structure of system 2 is illustrated in fig 6.1. The four demand scenarios are composed of a pessimistic and an optimistic scenario, and each one of them is composed of a low price scenario and a high price scenario. In the pessimistic scenario, the relative demand split between items is supposed to be the same, regardless of the price scenario. The same applies to the optimistic scenario. Relative demand split is described in table 6.3, together with the expected sales depending on the price scenario. We consider four macro-periods of 1 week each, composed of seven micro-periods of 1 day. Eight different items can be produced, which correspond to four different liquid flavors sold in two different bottle sizes.

Table 6.1: Structure description of the soft-drink production systems

Alternative	Structure
1	tank1-line1 ,tank4-line2
2	tank1-line1, tank4-line2,tank4-line3
3	tank4-line1,tank5-line2
4	tank4-line1,tank5-line2,tank5-line3
5	tank1-line1,tank2-line2,tank3-line3
6	tank4-line1,tank2-line2,tank3-line3
7	tank4-line1,tank5-line2,tank3-line3
8	tank4-line1,tank5-line2,tank6-line3

The alternative description for each production system is a link "tank i-line j that, in this system, the tank i is connected to bottling line j.The tank can be connected to various bottling lines, but a bottling line can only be connected to one tank. Tanks one, two and Three have a capacity of 5,000 liters, while four,five and six have a capacity of 10,000 liters. There is no difference between the three bottling lines.

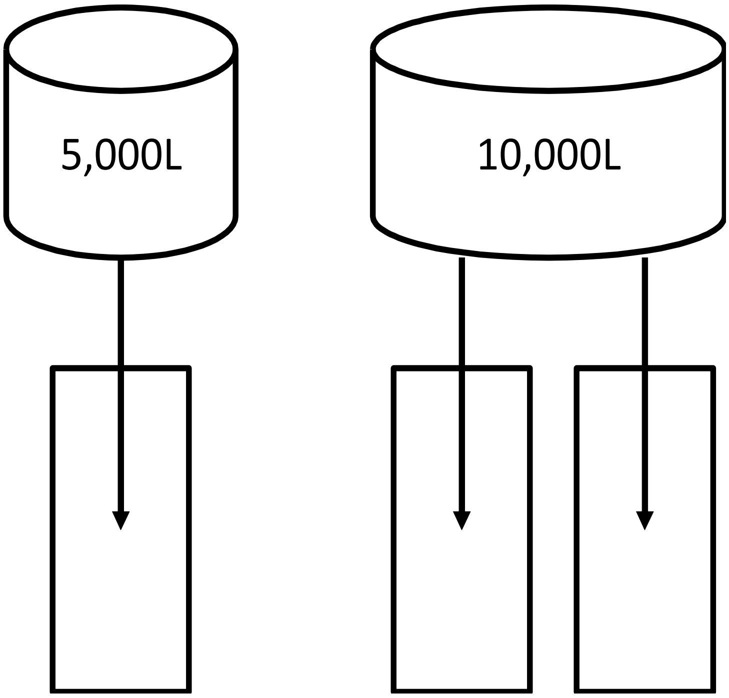

Figure 6.1: Alternative representation of the soft-drink production system 2.

6.4.2 Numerical results

The 32 alternatives have been executed using AMPL IDE modeling language(Version 3.1.0.2015) using CPLEX version 12.7.1.0 and the runs executed on Intel $core^{TM}$ i5-3320M 2.60 GHz. In order to accelerate the execution time, a time limit of 600 seconds has been fixed for each case study. On average, the integrality gap is around 10%. This large gap can be explained by the significant number of binary variables (1,600).Ferreira et al. (2009) provide some heuristic methods to significantly reduce the computation to find a good quality solution. However, this is out of the scope of this thesis.

The production schedule and power consumption of the case studies based on alternative 6, and alternative 7 respectively, considering the optimistic, high price, scenario, is illustrated in the figures below respectively. For the sake of clarity, only the first of the four weeks is represented. In the case study based on alternative 6, a diesel generator of 1000 kW has been installed, whereas in the case study based on alternative 7, a diesel generator of 350 kW has been installed. However, the energy consumed in both cases is roughly the same: 161,000 kWh in system 6, and 164,000 kWh in alternative 7. The diesel consumption is also similar: 47,400 liter in case 6, and 47,500 liter in case 7.

The fact that the energy is the same in both case is not really surprising. Indeed, the two cases assume the same demand scenario, there is no back-order in both cases, and the energy consumed should be roughly proportional to the amount of items produced.

However, the structure of the production system seems to have an impact on the optimal power consumption profile and generator investment decision. In alternative 7, there are two tanks of 10,000 liter instead of one, in alternative

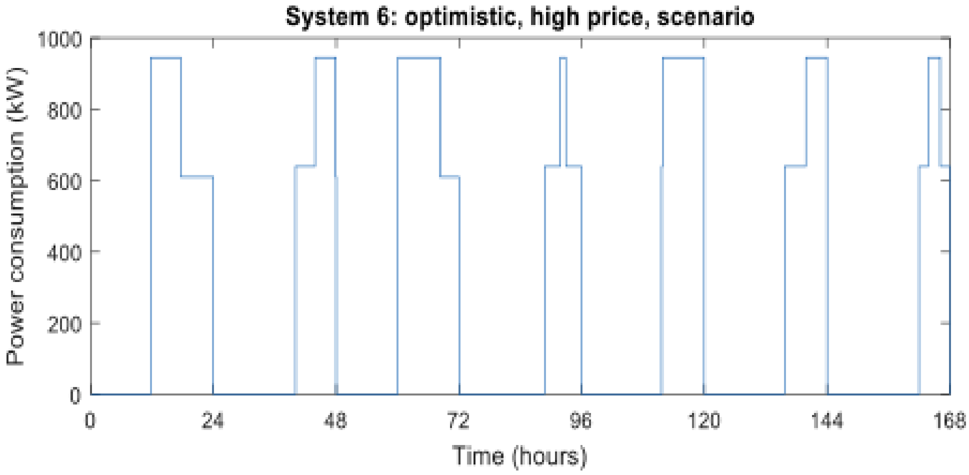

Figure 6.2: Optimistic high price scenario alternative 6

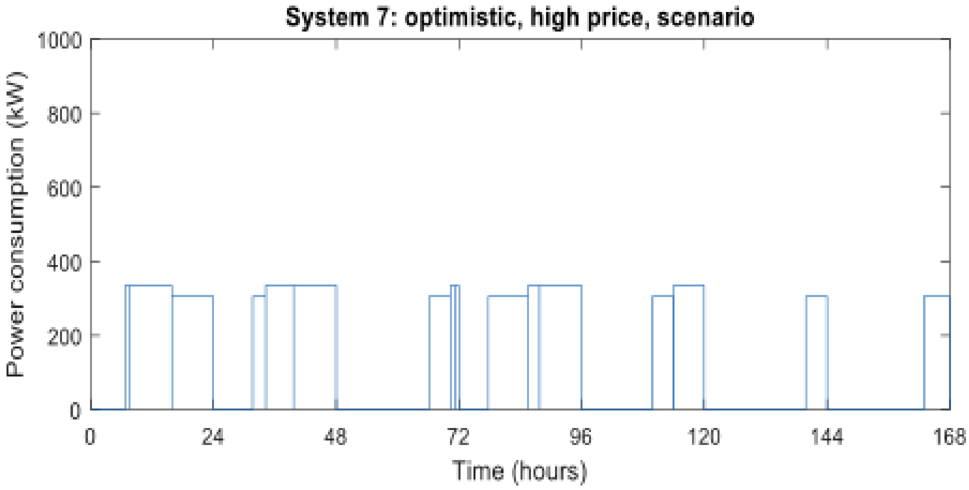

Figure 6.3: Optimistic high price scenario alternative 7

6. This larger tank capacity provides more flexibility to the alternative which can spread its energy consumption on lager time periods and lower its maximum power consumption. Therefore, the system is able to invest in a generator of a lower size while consuming the same amount of energy to produce the same quantity of items. The total costs are therefore lower in the alternative 7 than in the alternative 6: 63,000 GBP instead of 72,000 GBP.

Table 6.2: Cost of production for different alternatives under different scenarios

Alternatives	Optimistic(LP)	Optimistic(HP)	Pessimistic(LP)	Pessimistic(HP)
1	50,376	49,472	76,462	53,754
2	50,567	49,955	71,315	50,365
3	69,135	70,524	89,395	54,997
4	67,787	68,859	74,966	47,809
5	49,074	49,076	74,900	56,414
6	68,223	72,227	82,711	53,073
7	71,943	63,113	83,365	53,742
8	73,747	64,579	85,518	55,983

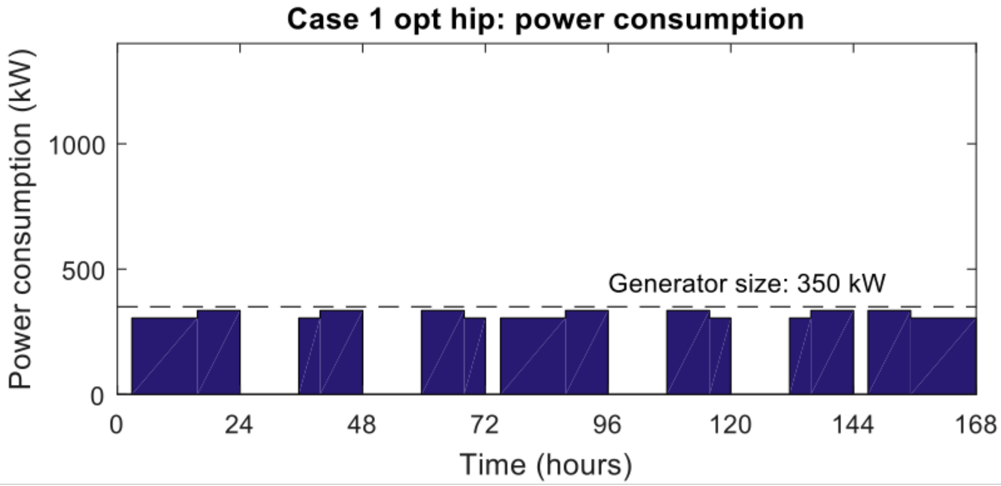

Figure 6.4: Alternative one optimistic high price

6.5 Power consumption figures for different Alternatives and Scenarios

The figures below present the diesel generator selection for the different alternatives, under different scenarios.Under alternative one, the optimistic high price scenario requires a 350kW generator, while the optimistic low price scenario requires a 400kW generator and the pessimistic low price scenario requires a 750kW generator. Although the demand under each scenario is different, each scenario

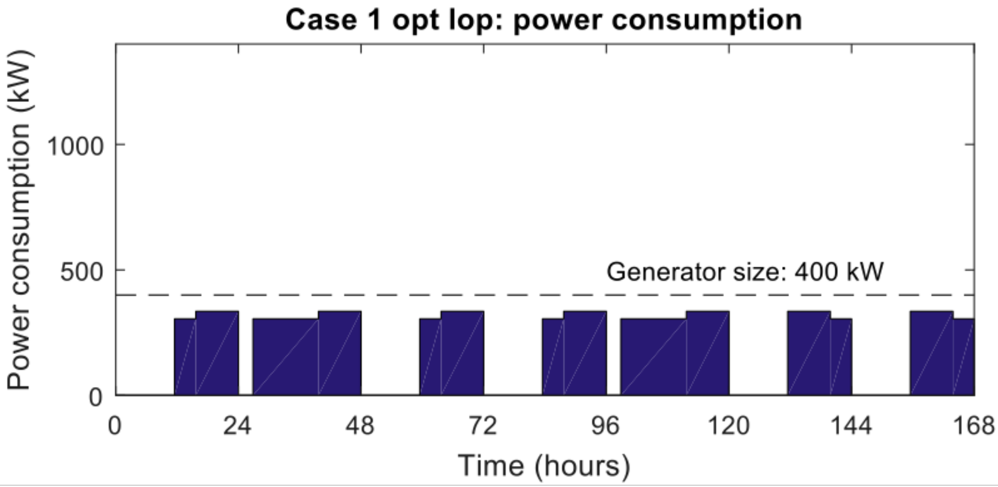

Figure 6.5: Alternative one optimistic low price

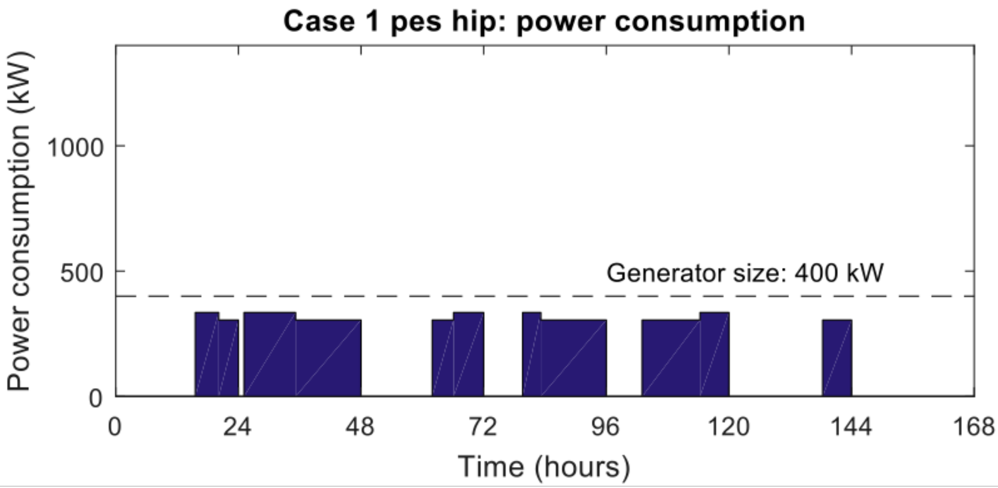

Figure 6.6: Alternative one pessimistic high price

has a chance of occurring. The investment cost of diesel generators is high and has to be decided early due to factory set-up and layout. The experiments show that a combination of generators of different sizes can considered. The experiments have highlighted that indeed diesel generator sizing and energy considerations are non negligible aspects of lot sizing problem in the context of developing countries.The difference in power requirements in alternative two is highlighted by figures 6.4, 6.5, 6.6, 6.8, 6.9 , 6.10 and 6.11.

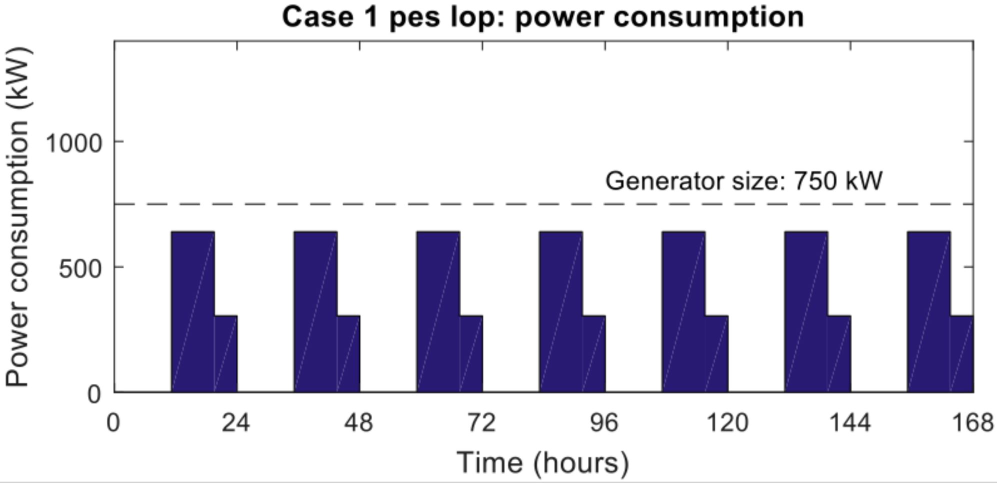

Figure 6.7: Alternative one pessimistic low price

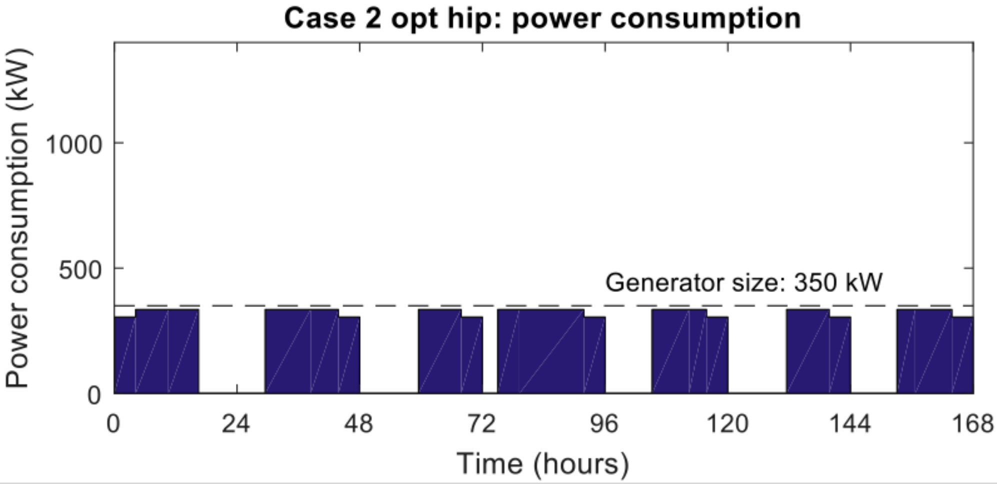

Figure 6.8: Alternative two optimistic high price

6.6 Conclusion

The contribution of this chapter is to model lot-sizing and planning scheduling with the consideration of diesel generators for developing countries without stable electricity. The lot-sizing problem model that considered diesel generators was proposed. The objective of the formulation was to minimise the total cost of production, diesel price and generators are a big component of production costs in developing countries. The model allows the soft-drink production factory to know

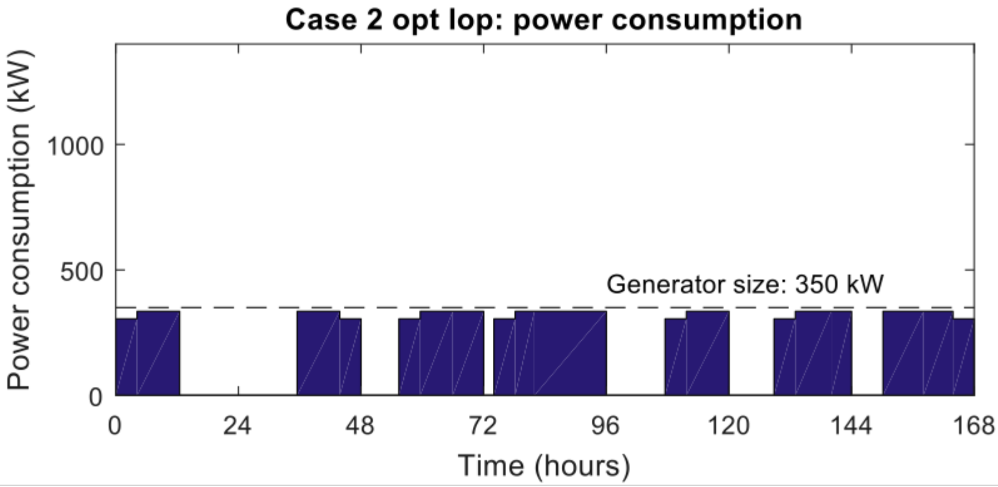

Figure 6.9: Alternative two optimistic low price

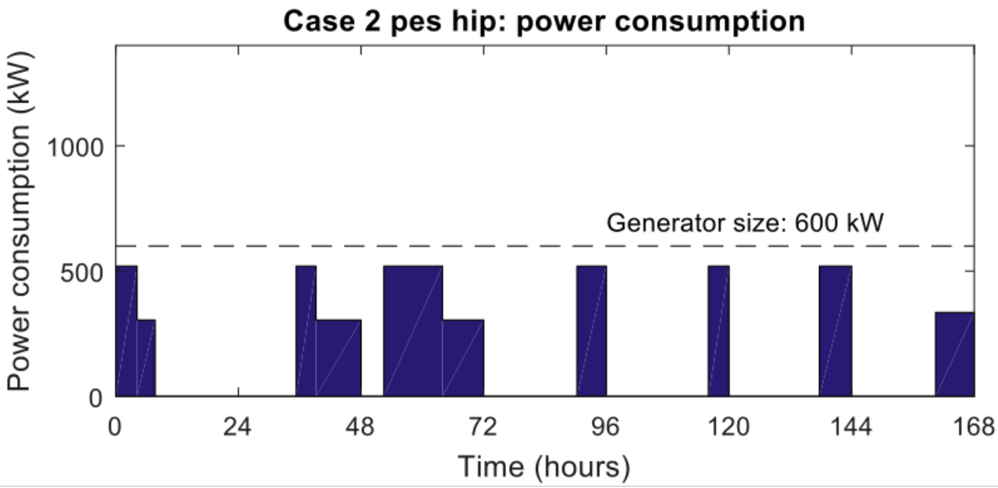

Figure 6.10: Alternative two pessimistic high price

in advance optimal quantities of diesel to order to cover given production periods. The experimental computations show that even a slight change in production will incur cost, therefore diesel generator consideration is not negligible. It is assumed that a factory falls into the category of one scenario at any given time.

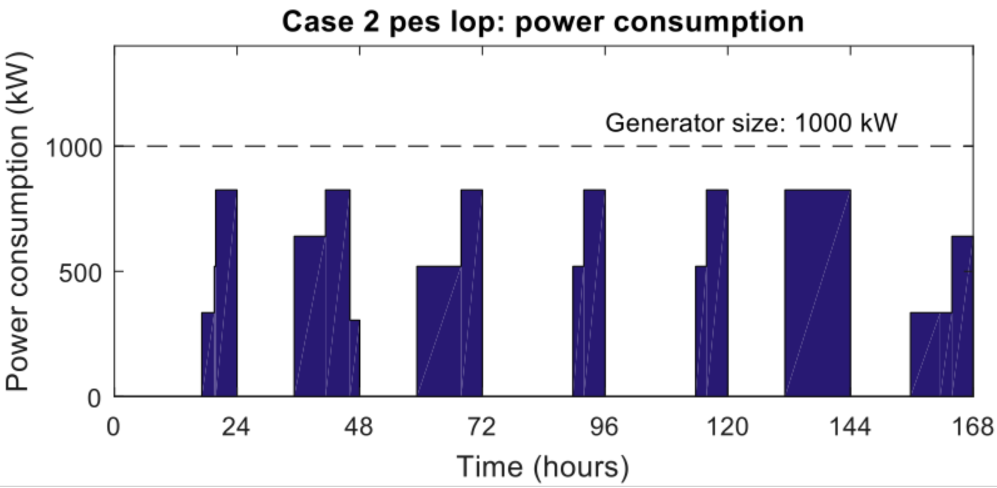

Figure 6.11: Alternative two pessimistic low price

Chapter 7

Manufacturing when system is connected to the grid,off-grid and energy not considered

Chapter 7

Manufacturing when system is connected to the grid,off-grid and energy not considered

7.1 Introduction

In this chapter we expand on the mixed integer programming model in the previous chapters that integrates the lot sizing and scheduling decisions that considers the synchronization between the stages of the liquid flavor preparation and liquid bottling of a Nigerian beverage production plant. This chapter applies the model to three different energy sources. The different energy sources considered in this chapter pronounce the impact of the thesis contribution and how important energy consideration is in production planning and scheduling.

7.2 Problem formulation

The problem is formulated as follows: **Sets**

- $t \in \{1, \ldots, T\}$ set of macro-periods;
- $s \in \{1, \ldots, N\}$ set of micro-periods;
- S_t set of micro-periods in macro-period t;
- P_t first micro-period in macro-period t;
- $i, j \in \{1, \ldots, J\}$ set of drinks;

- Ω_l set of drinks that can be produced with liquid flavor l;
- $k, l \in \{1, \ldots, F\}$ set of liquid flavors;
- $Tk \subset \{1, \ldots, TK\}$ set of tanks;
- $m, r \in \{1, \ldots, M\}$ set of bottling lines;
- $Bl \subset$ set of bottling lines;
- $g \in \{1, \ldots, G\}$ set of diesel generators;

Parameters

- $D_{j,t}$ demand for item j in macro-period t;
- c_j^+ inventory cost for one unit of item j ;
- c_j^- backorder cost for one unit of item j;
- $c_{k,l}^c$ changeover cost from liquid flavor k to l ;
- c^f diesel fuel cost;
- c_g^i investment cost of diesel generator dg ;
- c^p power cost;
- c^n energy cost;
- γ percentage of the investment cost that should be paid during the total time period of the problem;
- $P_g^{\max}$ maximum power output of diesel generator dg;
- a_g slope of the power to fuel consumption function of diesel generator dg;

- b_g intercept of the power to fuel consumption function of diesel generator dg;
- K_m^{max} maximum volume capacity of tank m;
- K_m^{min} minimum volume capacity of tank m ;
- $v_{j,l}$ volume of liquid flavor l necessary for the production of one unit of item j;
- D duration of a micro-period;
- $T_{k,l}^{liq}$ changeover time, in tanks, from liquid flavour k to l ;
- $T_{i,j}^{item}$ changeover time, in machines, from item $i$to j ;
- $T_{m,j}^{prod}$ production time of item j in machine m ;
- ptk_{tk} power consumption of tank tk;
- p_m^{cons} power consumption of bottling line m ;
- $I_{j,0}^+$ initial inventory for item j;
- $I_{j,0}^-$ initial backorder for item j;

Variables

- $X_{m,r}$ indicates whether the tank m is connected to the bottling line r;
- Y_m indicates whether the machine m is installed;
- $I_{j,t}^+$ inventory for item j at the end of period t ;
- $I_{j,t}^-$ backorder for item j at the end of period t ;

- u_g investment decision in generator g;
- $o_{g,s}$ indicates whether generator g is switched on in micro-period s ;
- $p^{gen}_{g,s}$ maximum power produced by generator g in micro-period s ;
- $E^{gen}_{g,s}$ energy produced by generator g in micro-period s;
- $V^{gen}_{g,s}$ volume of fuel consumed by generator g in micro-period s ;
- $D^{gen}_{g,s}$ amount of time generator g is switched on in micro-period s ;
- p^{Total}_{s} maximum power produced in micro-period s ;
- E^{Total}_{s} energy produced in micro-period s ;
- $y^{I}_{m,l,s}$ indicates whether there is production in tank m of the liquid flavor l in micro-period s ;
- $z^{I}_{m,k,l,s}$ indicates whether there is changeover in tank m from liquid flavor k to l in micro-period s ;
- $z^{IX}_{m,r,k,l,s}$ indicates whether there is changeover in tank m from liquid flavor k to l in micro-period s and tank m is connected to machine r ;
- $y^{II}_{m,j,s}$ indicates whether the machine m is setup for item j in micro-period s ;
- $y^{IIX}_{m,r,j,s}$ indicates whether the machine r is setup for item j in micro-period s and connected to tank m ;
- $z^{II}_{m,i,j,s}$ indicates whether there is changeover in machine m from item i to j in micro-period s ;
- $q_{m,r,j,s}$ production quantity in tank m and bottling line r of item j in micro-period s ;

- $W_{m,s}$ waiting time of machine m in micro-period s ;
- $C_{m,s}$ completion time of machine m in micro-period s ;
- $S_{m,s}$ starting time of machine m in micro-period s ;
- $D^{mch}_{m,s}$ amount of time machine m is producing in micro-period s ;
- D^{Total}_{s} amount of time the entire system is producing in micro-period s ;
- $f_{m,r,s}$ indicates whether the completion time of machine m is after the starting time of machine r in micro-period s ;

7.3 Problem with no energy consideration

$$Min \quad Z = \sum_{t=1}^{T}\sum_{j=1}^{J}(c_j^{+}I_{j,t}^{+} + c_j^{-}I_{j,t}^{-}) + \sum_{s=1}^{N}\sum_{m\in Tk}\sum_{l=1}^{F}\sum_{k=1}^{F} c^{c}_{k,l} z^{I}_{m,k,l,s} \tag{7.1}$$

The objective function 7.1 minimises the total cost with no energy consideration. These total costs are composed of the sum of inventory costs, backorder costs, and changeover costs.

$$\sum_{m\in Tk} X_{m,\gamma} = Y_\gamma, \qquad \gamma \in Bl \tag{7.2}$$

$$X_{m,\gamma} \le Y_m, \qquad m \in Tk, \gamma \in Bl \tag{7.3}$$

$$Y_m \le \sum_{\gamma \in Bl} X_{m,\gamma}, \qquad m \in Tk \tag{7.4}$$

$$y^{I}_{m,l,s} \le Y_m, \qquad m \in Tk, l = 1,\dots,F, s = 1,\dots,N \tag{7.5}$$

$$y^{II}_{m,j,s} \le Y_m, \qquad m \in Bl, j = 1,\dots,J, s = 1,\dots,N \tag{7.6}$$

$$\sum_{\gamma \in Bl} \sum_{j \in \Omega l} v_{j,l} q_{m,r,j,s} \leq y^{I}_{m,l,s} K^{ma}_{m}, \qquad m \in Tk, l = 1, \ldots, F, s = 1, \ldots, N \tag{7.7}$$

$$\sum_{\gamma \in Bl} \sum_{j \in \Omega l} v_{j,l} q_{m,r,j,s} \geq y^{I}_{m,l,s} K^{mi}_{m}, \qquad m \in Tk, l = 1, \ldots, F, s = 1, \ldots, N \tag{7.8}$$

$$q_{m,r,j,s} \leq \left(\frac{D}{T^{p}_{r,j}} \right) y^{II}_{r,j,s}, \qquad m \in Tk, \quad \gamma \in Bl, j = 1, \ldots, J, s = 1, \ldots, N \tag{7.9}$$

$$q_{m,r,j,s} \leq \left(\frac{D}{T^{p}_{r,j}} \right) X_{m,r}, \qquad m \in Tk, \quad \gamma \in Bl, j = 1, \ldots, J, s = 1, \ldots, N \tag{7.10}$$

Constraints 7.2 guarantees that a bottling line is installed if and only if there is a unique tank connected to this bottling line. Constraints 7.3 imposes that a tank is installed if at least one bottling line is connected to this tank. On the other hand, if a tank is installed, then at least one bottling line is connected to this tank 7.4. Neither a tank 7.5 nor a bottling line 7.6 can produce a drink if this machine is not installed. Constraints 7.7 and 7.8 guarantee that, in case a certain liquid is produced in a tank, its volume will be between the minimum and maximum liquid capacity of the tank. As explained by authors in Ferreira et al. (2009), a minimum liquid quantity is necessary for liquid homogeneity. Moreover, an item can be produced from a specific tank and bottling line only if this bottling line is setup 7.9, and if there is a connection between the tank and the bottling line 7.10.

$$m \in Tk, \quad t = 1, \ldots, T, \quad s \in S_t - P_t \tag{7.11}$$

$$z^{I}_{m,k,l,s} \geq y^{I}_{m,l,s-1} + y^{I}_{m,l,s} - 1, \quad m \in Tk, \quad k, l = 1, \ldots, F, s = 1, \ldots, N \tag{7.12}$$

$$z^{I}_{m,k,l,s} \geq \sum_{j \in \Omega_l,} \sum_{\gamma \in Bl} y^{IIX}_{m,\gamma,j,s-1} + y^{I}_{m,l,s} - 1, m \in Tk, k, l = 1, \ldots, F, t = 2, \ldots, T, s = P_t \tag{7.13}$$

$$y^{IIX}_{m,\gamma,j,s} \geq y^{II}_{m,j,s} + X_{m,r} - 1, \quad m \in Tk, \quad \gamma \in Bl, \quad j = 1, \ldots, J, s = 1, \ldots, N \tag{7.14}$$

$$\sum_{k=1}^{F}\sum_{l=1}^{F} z^{I}_{m,k,l,s} \leq 1, \quad m \in Tk, \quad t = 1,\ldots,T, s \in S_t \tag{7.15}$$

$$I^{+}_{j,t-1} + I^{-}_{j,t} + \sum_{m\in Tk}\sum_{\gamma\in Bl}\sum_{s\in S_t} q_{m,\gamma,j,s} = I^{+}_{j,t} + I^{-}_{j,t-1} + d_{j,t}, j = 1,\ldots,J,\ t = 1,\ldots,T \tag{7.16}$$

Constraints 7.11 enforce that the liquid production preferentially occurs at the beginning of each macro-period. Constraints 7.12, 7.13, and 7.14 control the liquid flavor changeover. Constraints 7.15 ensure there is not more than one liquid changeover at a time in each tank.Constraints 7.16 represent the inventory balancing constraints for each item in each macro-period.

$$\sum_{i=1}^{J}\sum_{j=1}^{J} z^{II}_{r,i,j,s} T^{it}_{i,j} r \in Bl, \quad s = 1, \dots, N \tag{7.17}$$

$$z^{IX}_{m,r,k,l,s} \geq z^{I}_{m,k,l,s} + X_{m,r} - 1, m \in Tk, r \in Bl, k, l = 1, \dots, F, s = 1, \dots, N \tag{7.18}$$

$$\sum_{j=1}^{J} y^{II}_{m,j,s} \leq 1, m \in Bl, s = 1, \dots, N \tag{7.19}$$

$$z^{II}_{m,i,j,s} \geq y^{II}_{m,j,s-1} + y^{II}_{m,j,s} - 1, m \in Bl, \quad i, j = 1, \dots, J, s = 1, \dots, N \tag{7.20}$$

$$\sum_{i=1}^{J}\sum_{j=1}^{J} z^{II}_{m,i,j,s} \leq 1, m \in Bl, \; s = 1, \dots, N \tag{7.21}$$

Following the method used by authors in Ferreira et al. (2009), constraints 7.17 and 7.18 ensure the synchronization between tanks and bottling lines. Only one item at a time can be produced for each machine 7.19, and there cannot be more than one item changeover at a time in machine 7.20,7.21.

$$S_{r,s} \geq W_{r,s}, r \in Bl, \; s = 1, \dots, N \tag{7.22}$$

$$C_{r,s} = \sum_{j=1}^{J} T^{prod}_{r,j} \left(\sum_{m \in Tk} q_{m,r,j,s} \right) + S_{r,s} r \in Bl, \; s = 1, \dots, N \tag{7.23}$$

$$C_{m,s} - C_{r,s} \geq (X_{m,r} - 1)\, D, m \in Tk, \quad r \in Bl, s = 1, \dots, N \tag{7.24}$$

$$S_{m,s} - S_{r,s} \leq (1 - X_{m,r})\, D, m \in Tk, \quad r \in Bl, s = 1, \dots, N \tag{7.25}$$

$$C_{m,s} \leq D, m = 1, \dots, M, \; s = 1, \dots, N \tag{7.26}$$

Constraints 7.22 and 7.23 calculates the time when bottling lines start and finish to produce within a micro-period. These starting and ending times can be calculated as well through constraints 7.24 and 7.25. These constraints ensure that the completion time of a tank is always after the completion time of any of the bottling line connected to this tank. On the other hand, the starting time of a tank is always before the starting time of any of the bottling line connected to this tank. Finally, any machine should have its completion time before the end of the micro-period 7.26.

7.4 Problem when the system is connected to the grid

$$Z^{ongrid} = Z + \sum_{s=1}^{N} c^p p_s^{Total} + \sum_{s=1}^{N} c^n E_s^{Total} \tag{7.27}$$

$$C_{m,s} - S_{r,s} \leq f_{m,r,s} D, m, r = 1, \dots, M, \quad m \neq r, s = 1, \dots, N \tag{7.28}$$

$$p_s^{Total} \geq \sum_{j=1}^{J} y_{r,j,s}^{II} p_r^{cons} + \sum_{m \neq r} \left(f_{m,r,s} + f_{r,m,s} - 1 \right) p_m^{cons} r \in Bl, \ s = 1, \dots, N \tag{7.29}$$

$$D_{m,s}^{mch} = C_{m,s} - S_{m,s}, m = 1, \dots, M, \ s = 1, \dots, N \tag{7.30}$$

$$E_s^{Total} = \sum_{m=1}^{M} D_{m,s}^{mch} p_m^{cons}, s = 1, \dots, N \tag{7.31}$$

The new objective function to minimize 7.27, and the constraints of the problems are 7.2 -7.31. From now on, the power and energy costs should be minimized together with the rest of the scheduling costs. We assume that the soft drink system is connected to the electrical grid. We assume a single energy cost (GBP per kWh) and a single power cost (GBP per kW) that should be paid for each micro-period.

Following the method used by authors in Masmoudi et al. (2015), we introduce constraints 7.28 and 7.29. These equations help identifying couple of machines with overlapping producing schedules, and the resulting maximum power output necessary from the generator. Indeed, it can be showed that the value of $f_{m,r,s} + f_{r,m,s} - 1$ that appears in 7.29, is always positive for $m \neq r$, due to constraints 7.28. Moreover, its value is equal to 1 if, and only if, the two machines have overlapping producing schedules. Therefore, constraints 7.29 guarantee that the required power output during a given micro-period is greater than the power consumption of a specific machine, in addition to the power consumption of all the machine whose producing schedule overlaps its own producing schedule.

Constraints 7.30 and 7.31 help compute the total energy consumption of the soft drink system based on the setup duration of each machine and its power consumption.

7.5 Problem when the system is off-grid

$$Z^{offgrid} = Z + \sum_{s=1}^{N} \sum_{g=1}^{G} c^{fuel} V_{g,s}^{gen} + \gamma \sum_{g=1}^{G} c_g^{ivst} u_g \tag{7.32}$$

$$p_s^{Total} = \sum_{g=1}^{G} p_{g,s}^{gen}, \ \ s = 1, \ldots, N \tag{7.33}$$

$$E_s^{Total} = \sum_{g=1}^{G} E_{g,s}^{gen}, s = 1, \ldots, N \tag{7.34}$$

$$p_{g,s}^{gen} \leq o_{g,s} p_g^{max}, g = 1, \ldots, G, \;\; s = 1, \ldots, N \tag{7.35}$$

$$p_{g,s}^{gen} \leq o_{g,s} p_g^{min}, g = 1, \ldots, G, \;\; s = 1, \ldots, N \tag{7.36}$$

The new objective function to minimize is the one in 7.32, and the constraints of the problems are 7.2-7.45. Here, we assume that the soft drink system is disconnected from the grid and connected to a set of generators. The costs to take into account are the fuel cost and the investment cost in a set of diesel generators.Constraints 7.33 and 7.34 guarantee that the power and energy supplied to the system at any time are the ones delivered by the unique generator installed. A diesel generator can deliver power/energy at a certain time only if it has been switched on at this time, which is translated by constraints 7.35 and 7.36.

$$E_{g,s}^{gen} \leq p_g^{max} D_{g,s}^{gen}, g = 1, \ldots, G, \;\; s = 1, \ldots, N \tag{7.37}$$

$$E_{g,s}^{gen} \geq p_g^{min} D_{g,s}^{gen}, g = 1, \ldots, G, \;\; s = 1, \ldots, N \tag{7.38}$$

$$o_{g,s} \leq u_g, g = 1, \ldots, G, \;\; s = 1, \ldots, N \tag{7.39}$$

$$V_{g,s}^{gen} = a_g E_{g,s}^{gen} + b_g D_{g,s}^{gen}, g = 1, \ldots, G, \;\; s = 1, \ldots, N \tag{7.40}$$

$$D_{g,s}^{gen} \leq o_{g,s} D g = 1, \ldots, G, \;\; s = 1, \ldots, N \tag{7.41}$$

$$D_s^{Total} - D_{g,s}^{gen} \leq (1 - o_{g,s})\, D g = 1, \ldots, G, \;\; s = 1, \ldots, N \tag{7.42}$$

$$D_s^{Total} \geq C_{m,s} - S_{r,s}, m, r = 1, \ldots, M, \;\; s = 1, \ldots, N \tag{7.43}$$

Constraints 7.37 and 7.38 are the energy equivalent constraints to 7.35 and 7.36. Naturally, a diesel generator can only be switched on if it has been installed 7.39. Constraints 7.40 come from Masmoudi et al. (2015) and describe the relation between the fuel consumption of a diesel generator and the energy it produces during a specific period. Constraints 7.41 and 7.42 ensure that the switch-on time of a generator is at least the producing time of the system if the generator is switched on, and zero otherwise. Finally, constraint 7.43 computes the producing time of the soft drink system as the duration between the moment the first machine starts and the moment the last machine stops.

7.6 Case Study

The three different manufacturing set-ups described above have been solved for 4 different demand scenarios .

7.6.1 Case study description

The available tanks and bottling lines to compose the structure are described in table 7.1.

The four demand scenarios are composed of a pessimistic and an optimistic scenario, and each one of them is composed of a low price scenario and a high price scenario. In the pessimistic scenario, the relative demand split between items is supposed to be the same, regardless of the price scenario. The same applies to the optimistic scenario. Relative demand split is described in table 7.2 and 7.3, together with the expected sales depending on the price scenario. We consider four macro-periods of one week each. Eight different items can be produced, which correspond to four different liquid flavors sold in two different bottle sizes.

Consistently with constraints 7.2, a bottling line can only be filled by one tank at a time. This is the same assumption as the one made by authors in Ferreira et al.

Table 7.1: Candidate structure components of the soft drink system

	Tanks and lines		
	Min volume(L)	Max volume(L)	Consumption (kW)
tk1	500	5000	120
tk2	500	5000	120
tk3	500	5000	120
tk4	1000	10000	150
tk5	1000	10000	150
bl1			185
bl2			185
bl3			185

[1]

Table 7.2: Relative demand split and potential sales

Item	Relative dmd split in the pessimistic scenario (%)	Relative dmd split in the optimistic scenario (%)
o1	30	9
b1	16	5
a1	13	4
p1	6.5	2
o2	15	36
b2	9	20
a2	7	16
p2	3.5	8

[2]

Table 7.3: Potential sales

Price scenario	Potential sales pessimistic scenario (l/week)	Potential sales in the optimistic scenario (l/week)
Low price	120,000	150,000
High price	80,000	135,000

(2009), and it helps avoiding conflicts between tanks. Indeed, allowing two tanks to fill the same bottling line would raise a few issues. The liquid flavor in the two tanks should be the same since they should both correspond to the liquid flavor used to produce the item in the bottling line. If the liquid flavors in the two tanks were not the same, the production in the bottling line should be split in two steps: one step for the item made of liquid flavor from one tank, and one step for the item made of liquid flavor from another tank. Due to the complication it brings to the filling process, the option of allowing more than one tank per bottling-filling line has been discarded.

For the off-grid(island manfacturing) alternatives, there are 49 candidate diesel generators of various sizes ranging from 150 kW to 2,250 kW. Cost of fuel is assumed to be equal to 1.2 GBP per liter. Technical data describing the link between the power output and the fuel consumption can be found in (Brighton, 2017). Investment cost data have been gathered from various seller websites. The parameter γ is the percentage of the investment cost that should be paid during the total time period of the problem. We assume here that the soft-drink system owner is contracting a loan to buy the set of diesel generators. γ represents the percentage of the generators investment costs the owner will have to reimburse to the bank (interests included) over the 28-days time period. In these case studies, it is assumed that $\gamma = 2\%$.

For the on-grid case studies, we assume a cost of electricity of 0.12 GBP per kWh and a cost of power of 0.10 GBP per kW per day.

The 3 optimisation problems described in section 7.2-7.4 are solved for each demand scenarios. In total, there are 12 case studies.

Table 7.4: Summary of results for no energy considered

	opt lp	opt hp	pes lp	pes hp
Structure installed	A	A	A	A
Generator Installed				
Max power(kW)				
Energy consumed (MWh)				
Fuel consumed (L)				
Backorder cost (£)	0	0	0	0
Inventory cost (£)	1093	323	240	19
Change cost (£)	1560	1520	1440	960
Fuel cost (£)				
Generator cost (£)				
Electricity cost (£)				
Power cost (£)				
Energy unit cost (£)				
Avg fuel consumed (£)				
Total cost (£)	2,653	1,843	1680	979

7.6.2 Numerical results

The 12 case studies have been executed using GAMS and an Intel Xeon computer, 144 GB RAM, 2.60 GHz. In order to accelerate the execution time, a higher time limit of 10800 seconds (3 hours) has been fixed for each alternative as compared to the time fixed in chapter six due to the higher number of alternatives . The integrality gap ranges from 10% to 50%. The large gap can be explained by the significant number of binary variables (7,000). Authors in Ferreira et al. (2009) provide some heuristic methods to significantly reduce the computation to find a good quality solution. However, this is out of the scope of this Thesis. A summary of the results can be found in tables 7.4, 7.5 and 7.6. Among the 12 alternatives, 2 different optimal system structures have been identified. These two structures A and B are depicted in Fig.

Table 7.5: Summary of results for off grid case(Island manufacturing)

	opt lp	opt hp	pes lp	pes hp
Structure installed	A	A	B	B
Generator Installed	1000kW	1000kW	1000kW	750kW
Max power(kW)	855	855	855	705
Energy consumed (MWh)	188	169	190	123
Fuel consumed (L)	57,152	51,675	55,880	36,725
Backorder cost (£)	0	0	0	0
Inventory cost (£)	1,151	441	3,139	1,612
Change cost (£)	2,080	1,880	1,760	1,180
Fuel cost (£)	68,582	62,010	67,057	44,070
Generator cost (£)	3,600	3,600	3,600	2,600
Electricity cost (£)				
Power cost (£)				
Energy unit cost (£)	0.36	0.37	0.35	0.36
Avg fuel consumed (£)	304	306	294	298
Total cost (£)	75,413	67,932	75,556	49,462

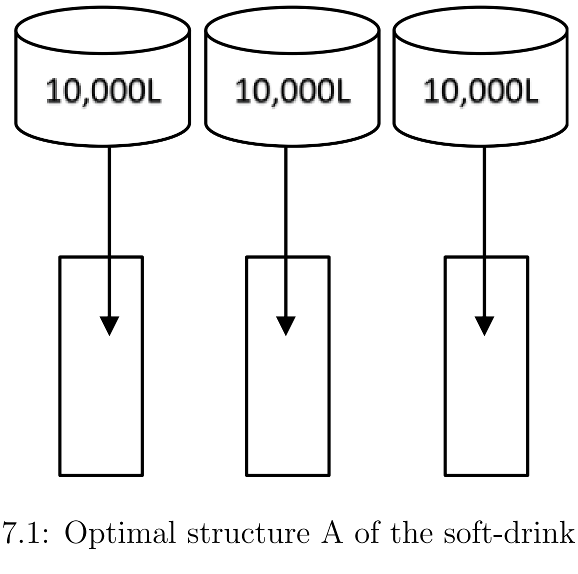

Figure 7.1: Optimal structure A of the soft-drink system

In all case studies, only the tanks with the biggest capacity (10,000L) have been installed, despite their minimum liquid volume requirement and their higher power consumption. It can be noticed that, when energy is discarded from the problem, the optimal soft-drink structure is always the structure A, in which the 3 big tanks

Table 7.6: Summary of results for off grid case(Island manufacturing)

	opt lp	opt hp	pes lp	pes hp
Structure installed	A	A	B	B
Generator Installed				
Max power(kW)	670	670	520	520
Energy consumed (MWh)	188	169	191	127
Fuel consumed (L)				
Backorder cost (£)	0	0	0	0
Inventory cost (£)	1,242	398	2,660	1,612
Change cost (£)	2,080	1,880	1,760	1,180
Fuel cost (£)	68,582	62,010	67,057	44,070
Generator cost (£)	3,600	3,600	3,600	2,600
Electricity cost (£)				
Power cost (£)				
Energy unit cost (£)	0.36	0.37	0.35	0.36
Avg fuel consumed (£)	304	306	294	298
Total cost (£)	75,413	67,932	75,556	49,462

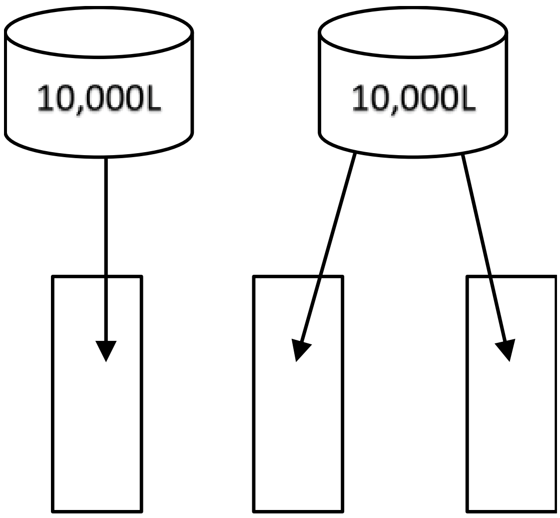

Figure 7.2: Optimal structure B of the soft-drink system

are installed and each of them is connected to a single bottling line. When energy issues are taken into account, however, this structure is optimal only for the optimistic scenario, in which there is more demand and for larger bottles. On the other hand, when there is less demand and for smaller bottles, the optimal structure is the one with only two tanks. This can be explained by the trade-off made between the higher consumption costs for the structure involving 3 tanks and the higher back-order costs there would be if there were less tanks installed. It is remarkable that the energy consumption is almost exactly the same for the on-grid and the off-grid cases. This is because the energy consumed by the soft-drink essentially depends on the machines installed, and not the production schedule. Since there is no back-order, exactly the same quantity of each item is produced, whether the system is on-grid or off-grid. And when each tank is connected to a single bottling line, the energy consumption will be the same in both cases. However, when one of the tank is connected to two bottling lines, the management of the production can be different and lead to slightly lower energy consumption when the system is supplied by a set of generators. On the contrary, the maximum power output is significantly higher when the system is off-grid than when the system is on-grid. This is due to the lumpiness of the generator investment decision. It should be noticed as well that, in the off-grid cases, it is always better to invest in a single large diesel generator instead of many small ones. This is because, even though the parameter a_g(the slope of the fuel-energy curve) does not depend on the size of the generator, the parameter b_g (the intercept of the fuel-energy curve) increases slower than the size of the generator (Brighton, 2017). Therefore, for a given maximum power need, a large single diesel generator will be more efficient, i.e. consumes less fuel per energy produced, than many small generators.

Finally, the total fuel costs in the off-grid cases are always 3 times higher than the total electricity costs in the on-grid cases. Moreover, the investment costs in diesel generators are higher than the costs of power.

These results seem to indicate that the connection to the grid is a better option than being off-grid and being supplied by a set of generators. However, these results should be assessed in the light of the assumptions made above. For instance, if the electricity cost when being connected to the grid was higher than 0.36 GPB per kWh, being off-grid would represent a better option. A higher cost of power would also make the off-grid a preferable option.

Moreover, the off-grid option offers an obvious reliability advantage. The owner has no control if there is a blackout in the electrical infrastructure, whereas an off-grid option offers more reliability as long as the owner has sufficient reserves of fuels.

Furthermore, as mentioned in the literature in chapter one and four more than half the population of Nigeria does not have access to the national electric grid. The reason for including an on-grid option in the analysis is to highlight to the government and stakeholders the economic importance of being connected to the grid. For example, under the off-grid option (opt lop) the total cost of production is £75,413 while the same (opt lop) under the on-grid option cost £26,954. Nigeria is currently facing an unemployment crisis and the government is diversifying the economy and promoting made in Nigeria products. Due to the high cost of electricity, Gaskiya textiles located in Kano, Nigeria had to close in 2005 where five thousand people lost their jobs. In the face of increasing competition , dozen of factories have been forced to close and thousands of people have lost their jobs (Muhammad, 2011).

7.7 Conclusion

In this chapter, a novel mixed integer programming formulation is presented for the two stage lot sizing and scheduling problem under three different cases. The cases are as follows; when the lot sizing and scheduling model is connected to the National grid at a certain price per kWh, when energy is not considered and when a diesel generator is incorporated to the lot sizing and scheduling problem. The model has been programmed in such a way that a large bucket problem is solved to optimality in seconds. The models have been built to handle several variations of parameters. The insights of this chapter have indicated the total energy consumption in kWh is almost exactly the same for the on-grid and off-grid case for a given demand scenario. However, the power consumption profile is differs significantly. When the system is on grid (connected to national grid), the power is very low with a wide spread production time. This is due to the fact that maximum power each day has a cost, therefore spreading the energy consumption among the time period to have a maximum power as low as possible. On the contrary, when the diesel generator is supplying the system, the production time is shorter and the power is higher. This is due to the fact that the ratio of fuel-energy is better when the working duration of the diesel generator is shorter (switch-on duration should be as low as possible).

Chapter 8

Conclusion & Future work

Chapter 8

Conclusion & Future work

8.1 Conclusion

This thesis approaches the production scheduling, lot-sizing and unit commitment problem in the soft drink production industry. The research conducted in this thesis is divided into two distinctive areas which are production scheduling, lot-sizing and the unit commitment problem. Chapters two and three are dedicated to production scheduling and lot-sizing. These chapters cover the literature review and the characterisation of the production scheduling and lot-sizing problem in relation to the thesis. A MILP that integrates the lot-sizing and scheduling decision and the synchronisation between the stages in the context of Nigeria is presented.The MILP model in chapter three forms the basis for the extended model presented in chapter six, where the power consumption needs are considered. The unit commitment on the other hand is presented in chapters four and five. Chapter four presents the characterisation of diesel generators and variations of power production. Chapter five presents mathematical models that find the optimal way to run a set of diesel generators to meet a given output and another mathematical model that makes the decision of which generators to purchase to cover a constant out put not known with certainty. The contribution of chapter five is that no known literature have been found on the properties related to the optimal set of generators and provides the input that will be used in the other chapters.Below are the contributions and summary of each chapter and potential future work.

8.2 Summary & Contributions

Chapter two reviews the literature and developments made in production planning and lot-sizing problems. The chapter provides a description and information of the soft drink production system that has been derived based on visits to Nigerian soft-drink production companies. The general problem characteristics that form a basis for this dissertation have been laid out in this chapter. The chapter updates the review with regards to modelling a soft drink production factory in the context of a developing country.

Chapter three proposed a mixed integer programming model that integrates the lot sizing and scheduling decisions which considers the synchronization between the stages of the liquid flavor preparation and liquid bottling of a Nigerian beverage production plant. This chapter will develop a MILP model that reflects the lot-sizing and scheduling problem of the bottling factory, which will form the basis for an extended MILP model presented in Chapter six, where the power consumption needs are considered. The various aspects of the lot sizing and scheduling problems such as the multiple levels of production, numerous time constraints,synchronization constraints and many other constraints make the task of creating an effective and efficient model hard. The research carried out in this chapter has been tailored towards having an impact in a real factory that is currently struggling. The problems facing the bottling plant is special because it has to generate its own electricity by using diesel generators.Therefore, the model had to take account of the special case of the production plant by modelling the problem in way that the impact of every variable is seen in the solution.

chapters four and five focused on developing Operational Research models to help decision- making on various aspects. Decision models related to purchasing and operating a set of diesel generators to enable manufacturing in environment with no access to an electricity grid; Investigating the impact of different loading profile assumptions,from constant,time-dependent but deterministic, to stochastic settings. Chapter 4 shows that a novel research can be carried out considering variations of the problem.Chapter five proposed a framework that classified models that will allow the incorporation of power generation and diesel generator decision making in production planning. The first model decides which generator/generators is/are the most feasible to use so that a given output is given at a minimum cost. Extensive computational experiments were carried out to compare the different combinations and power requirements. From this chapter, an important contribution which involves integrating power generation and production planning is envisaged.

Chapter six presented a problem based on the soft drink integrated production lot sizing and scheduling problem from (Ferreira et al., 2009), taking into account the energy consumption of the system and include the energy cost within the total costs of the problem. Masmoudi et al. (2015) also includes the energy consumption in a lot-sizing model. However, their model considers a linear production structure much simpler than the model in (Ferreira et al., 2009). In our model, the production system is entirely supplied by a diesel generator. The diesel generator selection is a variable of the problem, and its investment cost is also included within the total costs. Therefore, the contribution of this chapter is the formulation of a model that optimises the decision of investment in a power generator, its fuel consumption,together with the lot sizing and scheduling of the system it is supplying.

Chapter seven expands on the mixed integer programming model in the previous chapters that integrates the lot sizing and scheduling decisions that considers the synchronisation between the stages of the liquid flavor preparation and liquid bottling of a Nigerian beverage production plant. This chapter applies the model to three different energy sources. The different energy sources considered in this chapter pronounce the impact of the thesis contribution and how important energy consideration is in production planning and scheduling.

Chapter eight provides the thesis conclusion, summarises the thesis, highlights thesis contribution and future work.

8.3 Future work

8.3.1 Fuel purchasing strategies

The main aim of this section would focus on studying the value of purchasing diesel under fluctuating prices.

The running costs of a diesel generator are dominated by the cost of diesel. The price per liter of diesel varies over time and depends on the price of crude oil and other economic factors. Purchasing diesel in larger quantities may lead to discounts which reduce the price per liter. Another advantage of purchasing in bulk is that this can provide a safety stock of diesel because also in this market there can be disruptions in the supply chain. A final potential advantage of having the capacity to store diesel is that the company can build up speculative stocks. That is, purchase more diesel when anticipating a rise in diesel price in the future. The draw-back of purchasing in larger quantities is that the company needs investment in installing a larger storage tank, or more storage tanks.

This is a list of interesting ideas but they will likely remain ‘further research’ in the dissertation:

8.3.2 The optimal way to sequence a set of generators to provide for a dynamic output profile

In this section, a situation with a a given set of generators is considered but a required output profile that has a start time, runs for a period of time, and then shuts down. Such situations occurs in production environments where the plant does not operate 24 hours a day or seven days a week. If that the output profile can be represented by a piecewise constant function over time, a set of constant output profiles can be identified $\mathcal{N}$, each profile $X_j \in \mathcal{N}$ running for a time n_j. It is clear that then the corresponding model is similar to model (5.27) in which $f_i = 0$, $\forall i \in \mathcal{M}$, and that the price of fuel is then not determining the optimal use of the generators.

Using one large generator in order to be handle the peak loads may no longer run under an optimal load factor once the plant operates in normal continuous mode. An optimal strategy may in this case hence include the use of diesel generators that are only running in the phase of starting up large machines or equipment in the factory. Once the plant operates in normal continuous mode, these generators can be shut down. Such generators are said to run in prime operating mode (or even in standby mode if start-up occurs infrequently). A particular diesel generator that has, for example, a power output rating of 1000 kW when running in continuous mode, will typically have a prime power rating that is lower, say 850 kW, and it may be ran at 110% of this prime power rating for less than one hour in twelve, and for not more than 25 hours per year.

8.3.3 Purchasing a set of generators to provide for a dynamic output profile not known with certainty

Production plants may have some equipment or motors that can start up or shut down at random moment during plant operation. For example, pumps will typically automatically operate only when needed, or elevators in the factory will have their motor turn on only when in use.

8.3.4 The backup generator problem

Diesel engines may occasionally fail. To cover for those situations, having a back-up generator unit is indispensable when operating a diesel generator set, in particular in island mode applications.

It is usually not predictable when a generator will fail, and also the time needed to fix a failed generator may be hard to predict. It may well be the case that the emergency happens in a time when demand is high. If part of the demand load could be quickly reduced, then this may help keep the back-up generator unit smaller, but that may not be feasible.

Generators also need maintenance. However, this can typically be scheduled and may be possible to do when demand permits taking out a generator. In some situations this is not possible or costly and then the back-up unit may also be a solution.

To maximise reliability, it is assumed that a back-up generator unit may be called for at any given time during a variable demand load profile when the largest generator in the standard set fails and is then to provide coverage for a sufficiently long period of time in which 95% of the known repair times for all generators in the standard set fall.

The problem is hence to select from a range of possible generators one or set of back-up generators that can be used to help cover the unmet part of a required load profile at any given start time and for a set length of time.

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