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Planning and Analysing Competing En-route Charging Stations for Electric Vehicles: A Game-theoretic Approach

by

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Abstract

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En-route charging stations are required to ensure the adoption of Electric Vehicles. However, careful planning is necessary due to high cost in infrastructure and potential queues, and literature on charging station competition is scarce. To address this and similar problems, this thesis proposes a versatile game-theoretic model for investor competition, where competing firm investors aim to maximise individual net profit by choosing locations, capacities, prices and the speed of service at their firms. On the other hand, self-interested customers aim to minimise the expected cost of acquiring the service firms sell. This includes a cost to access each firm, the fee for the service and an expected cost due to congestion at the firm. In addition, extraneous competition outside the investor system is considered as an option for customers. The solution combines analytical and algorithmic techniques to obtain subgame-perfect equilibria, and enables to assess both qualitative and quantitative aspects of firm competition. The model is applied to building charging stations for Electric Vehicles, and it is shown theoretically and empirically that equilibrium charging prices deviate upward of the marginal charging cost due to the inability to satisfy charging demand immediately, even with vast improvements in charging technology. Further results show that private investors will prefer to compete on the same route, because stations on longer routes have to set lower prices at their stations and this consists a significant disadvantage. Moreover, the more drivers are willing to pay in order to save time from their journey, the more investors will increase their profits at the expense of drivers. The inclusion of price choice and extraneous competition reinforces the existence of pure strategy Nash equilibria in capacity choice, and SPE solutions are highly efficient compared to optimal firm allocations when it comes to system-wide social welfare. Last, this thesis examines subsidies to stations as incentives to expanding rapid charging stations. Results show that subsidising the purchase of charging units for stations can have a significant beneficial effect for both EV drivers and station investors. In contrast, subsidies on the energy price for stations could provide incentive to investors to reduce capacities and increase prices. Finally, it is shown how the proposed model can be used to calculate the monetary gain or loss for drivers and investors due to subsidies, and to determine optimal subsidy levels according to certain requirements.

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Declaration of Authorship

I, Efstathios Zavvos, declare that the thesis entitled *Planning and Analysing Competing En-route Charging Stations for Electric Vehicles: A Game-theoretic Approach* and the work presented in the thesis are both my own, and have been generated by me as the result of my own original research. I confirm that:

- this work was done wholly or mainly while in candidature for a research degree at this University;
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Nomenclature

Abbreviations

EV Electric Vehicle

NE Nash equilibrium(a)

PoA Price of Anarchy

SLCOP Station Location, Capacity, charger Output and charging Price decision

SPE(s) Subgame-Perfect Equilibrium(a)

UK United Kingdom

Mathematical Symbols

$*$ Used in the power of a symbol to denote that it comes from or is an equilibrium.

$\mathbb{E}[r^k(c)|s]$ The expected utility for investor k of playing in profile c, given customers choose firms in s.

$\mathbb{E}[r_j^k(c)|s]$ The expected utility for a firm owned by investor k in location j, in pure strategy profile c.

$\mathbb{E}[u_i(\cdot)|s]$ The expected utility for i given that customers play in mixed strategy s (implicit).

$\mathbb{E}[u_i^j(x)|s_{-i}]$ The expected utility for i of choosing firm j where x other customers go to j in mixed strategy s.

μ The number of locations.

Θ The upper capacity limit for the investors' location, capacity and speed of service game.

A_i The set of actions available to customer i.

b_j The cost to build one server at firm j, when each investor is bound to a particular location.

b_j^k The cost to a server in the firm in location j owned by investor k.

C The set of pure strategy profiles in the location, capacity, and speed of service game.

c	A pure strategy profile (a choice of capacities for each location and the type server, by each investor).
C_k^0	The set of capacity choices available to investor k.
c_j	The capacity at station j, when each investor is bound to a particular location.
c_j^k	The capacity (number of servers) at the firm in location j owned by investor k.
c_L^k	A pure strategy (a choice of server type, and of a capacity for each location).
f	A pure strategy profile in prices (a choice of one price for each location by each investor).
f_j	The fee a customer pays at firm j, when each investor is bound to a particular location.
f_j^k	The fee at the firm in location j owned by investor k.
F_k	The set of actions available to investor k in the investors' price game.
F_k^0	The set of prices available to investor k.
f_L^k	A pure strategy in prices, by investor k (a choice of one price for each location).
f_m	The fee for using the outside option.
G	The set of server type options available to investors.
h	The cost of offering the service when it is the same for all firms.
h_j	The cost of offering the service at firm j, when each investor is bound to a particular location.
h_j^k	The cost of offering the service by a firm in location j owned by investor k.
I	The set of firm investors.
i	Indicates a customer.
j	Used to indicate a firm in the customers' model, and a location in the investors' model.
k	Used to indicate an investor.
L	The set of locations available to investors.
m	The number of choices available to each customer.
N	The set of customers.
n	The number of customers.
o_j	An one-time building cost for firm j, when each investor is bound to a particular location.
o_j^k	An one-time building cost for the firm in location j owned by investor k.

R	The time needed to serve one customer when it is the same in all firms.
$r^k(\cdot)$	The utility for investor k (implicit definition).
R_j	The service time at firm j, when each investor is bound to a particular location.
R_j^k	The service time at the firm in location j, owned by investor k.
$r_j^k(c)$	The utility for a firm owned by investor k in location j, in pure strategy profile c.
S	The set of mixed strategy profiles.
s	A mixed strategy profile (a vector of the choices of mixed strategy by all customers).
S_i	The set of mixed strategies available to customer i.
s_i	The mixed strategy of customer i.
s_i^j	The probability that customer i will choose firm j.
s_{-i}	The mixed strategy of customers other than i.
s_{-i}^j	The probability that customers other than i will go to firm j.
t_j	The cost to access firm j.
t_m	The access cost for the outside option.
$u_i(\cdot)$	The utility for customer i (implicit definition).
$u_i^j(x)$	The utility for customer i of choosing firm j, given x other customers are also going there.
$u_i^m(x)$	The utility for customer i of choosing the outside option, when x other customers make the same choice.
v_d	The value of time for obtaining the service from a firm.
v_m	The value of time for the outside option.
w	Gross profits normalisation parameter.
X	The set of subgame-perfect equilibria.
z	The number of investors.

Chapter 1

Introduction

Electric vehicles (EVs) benefit from zero direct emissions. Of course, the electricity they use comes at the cost of emissions in power generation for the grid. For modern EVs, however, these indirect emissions are already less per EV than the emissions of a conventional fossil-fueled vehicle (Holdway et al., 2010). Energy generation from renewable sources, like wind or sunlight, is becoming increasingly popular and energy production technology is constantly refined. At the same time, alternative energy sources like solar panels are already widely available at a consumer level. Consequently, the EV market is likely to benefit increasingly in the coming years as power generation becomes greener.

For these reasons, the popularisation of EVs has become a long-term goal for some of the more economically developed countries' governments. In the UK, a study was commissioned by the Committee on Climate Change regarding how the de-carbonisation of the vehicle fleet could be achieved. The Committee's target is for both pure and hybrid EVs to have a 16% market share by 2020, a 60% share by 2030 and finally for zero emission EVs to have 100% market share by 2040, in order for the total de-carbonisation of the fleet to be achieved by 2050. That study concluded that the EVs' limitations in range together with long charging times are the main reasons why they are not yet popular. At the moment the study was carried out, the committee deemed the charging infrastructure as sufficient for the country's EV fleet. However, due to the drivers' need to drive longer distances, it is believed that investing in more infrastructure can help break the barrier of the initial adoption of EVs. As a result, developing an extensive, well-planned network of rapid charging stations would be the best course of action in order to boost confidence in EVs (Element Energy et al., 2013). Furthermore, despite the fact that rapid charging is believed to deteriorate battery lifetime more quickly and puts more strain on the power grid, only rapid charging can effectively extend the range of EVs (Botsford and Szczepanek, 2009).

The aforementioned concerns about the small range and slow charging times of EVs are well founded. Top of the line luxury EVs, such as the Tesla model S, are capable of performing trips of up to 200-260 miles on a single charge, but most affordable EVs like the Nissan Leaf only have a range of 75-120 miles[1]. Furthermore, depending on battery capacity, EVs can take up to 4-12 hours to recharge in slow charging stations, while charging at rapid charging stations can be as fast as 30 minutes for an 80% charge[2]. Moreover, other studies are in agreement with the need for expanding en-route charging networks, as providing more charging infrastructure is also believed to be more cost efficient than investing in bigger batteries (Morrow et al., 2008).

In the UK, there currently exists a network of rapid charging stations[3] but the vast majority are slow park 'n charge stations (3-7kW output). Most rapid charging stations (30kW output or more) are focused at the highway arteries of the North-South route, and are typically of small capacity[4]. They mostly feature up to three charging units, which means only very few vehicles can recharge at a station every half an hour -which is the time needed to fill the battery to a sufficient level with a typical 50kW DC charger. This situation can lead to long queues if the traffic flow toward stations increases, which in turn can be detrimental to total travel time to reach a destination.

Thus, expanding this en-route rapid charging network is very important to popularising EVs and to increasing their effective trip range. However, arbitrary expansion could prove catastrophic to some private investors. At the same time, maintaining reasonable driver convenience by minimising queuing times is going to be challenging as EVs become increasingly popular and the charging market is privatised. As a result, such an undertaking necessitates planning the locations and capacities of charging stations, which in turn is a problem contingent on both the convenience of EV drivers and the cost for charging station owners.

Having established the necessity for planning en-route rapid charging stations, by taking into account the general constraints of EV driver convenience and investor profit, in Section 1.1 the research problem this thesis will discuss will be defined in more detail.

1.1 Research Problem

An expansion of the rapid charging station network can be achieved by increasing the capacity of existing stations, by building new ones or by both. Consequently, questions arise concerning the locations stations should be built at, their capacity and charging prices as well as what type of charging units they should use. Furthermore, questions arise on whether existing stations should be expanded, retained or even abandoned. In addition, considering private investors includes answering substantive questions such as what incentives can be given to private investors, what forms of subsidies can be effective and how much should stations be subsidised by, and how much drivers are going to pay for recharging. Addressing these questions is going to be the basis of this work. This problem will be referred to as the "Station Location, Capacity, charger

[1]According to the U.S. Department of Energy database: https://www.fueleconomy.gov
[2]According to ChargingPoint: http://thechargingpoint.azurewebsites.net/
[3]See https://www.nextgreencar.com/electric-cars/charging-points/
[4]Number of charging units

Output and charging Price decision problem", in short SLCOP from now on. As was identified in the Introduction earlier, this problem consists of two main aspects; that of driver convenience and that of profit maximisation for station owners.

Let us now briefly examine the SLCOP problem from the EV drivers' point of view. It is reasonable to consider that drivers are *selfish* in the sense that they generally wish to spend the minimum amount of time traveling. At the same time, drivers also wish to refuel paying the least amount of money possible. However, drivers usually are not myopically biased toward achieving the very minimum travel time or the very minimum refueling price, but it is reasonable to assess that a trade-off between travel time and monetary costs is involved when deciding their journey. That is especially expected to be so in the case of EV drivers, who in addition to travel time and refuelling cost also have to take into account potential queues at charging stations and waiting time for recharging. Drivers can obtain such information either through a navigation device that communicates station information, or they can learn it empirically through using the road network. Then it is evident that as long as an EV driver needs to recharge, *station choice depends on expected travel times for the routes that lead to the desired destination, the potential choices of other EV drivers, station locations and capacities*[5]*, recharging fees and charging unit specifications*.

With regard to charging stations, owners are expected to behave *selfishly* in the sense that their actions are motivated by individual profit. When the owner is a private investor, it is reasonable to consider that they would not want to sacrifice any portion of their profit, so as another could enlarge theirs. Additionally, whereas the state may open stations that are not economically viable for a variety of reasons, it makes no sense for private investors to do so. A high-level viewpoint on charging station profit reveals that profit depends on the number of vehicles that recharge at a station, the charging fee at the station, and the cost for the station to recharge each EV. Since a station's profit is reliant on EV traffic flow through the station, in conjunction with the EV driver behaviour analysed above, then station profit additionally depends on the location, capacity and the power output of charging units at the station. Thus *expected profit depends on the potential decisions of EV drivers, the location, capacity and charging unit output at the station, the charging fee at the station, as well as the locations, capacities, outputs and fees of other competing stations*. Finally, each station has a building cost which is particular to that station's location, capacity and the type of charging units. As a direct consequence, the problem of station location and capacity decision is, in the end, a problem of net profit maximisation from the station owners' point of view (Jia et al., 2012; Sadeghi-Barzani et al., 2014).

Another observation at this point is that the routing problem for the drivers is actually in conflict with station owners' interests. This happens because reducing travel costs (in time and money) for the drivers, is also contingent on reducing the length of potential queues and charging prices at the stations. As a result, in order for drivers to be accommodated, owners have to invest in increasing their stations' capacities to a level that guarantees desirable queuing times. Investors could expand their stations arbitrarily or by trial and error seeing how driver flows are influenced,

[5] i.e. the number of charging units at stations

but this would be a lengthy and costly procedure. Instead, it would be best to suggest a solution to investors that takes all the aforementioned factors into account, and which investors have incentive to follow.

To summarise, an EV driver's station choice partly depends on the capacities and prices at stations, because a lower queuing time combined with low charging cost will make the station more desirable. At the same time a station investor's business plan is then dependent on drivers' choices, as profit is directly influenced by them. Thus, questions arise about the best course of action for station investors, so that profit is guaranteed and drivers' needs are taken into account. To this end, our research problem is to find a solution for determining station locations, capacities, charging unit power outputs and prices, so that the station owners' profits are maximised, while at the same time travel costs (time cost and monetary cost) for EV drivers are minimal.

The SLCOP problem was defined here explicitly, however it can be similar to various firm competition scenarios where investors can set locations, the number of servers and the speed of service, as well as prices. That is true as long as customers choose which firm to obtain the service from, based on some cost expectancy that depends on customer congestion at firms. The SLCOP problem is, for example similar to network pricing problems such as the one presented in Hayrapetyan et al. (2007), albeit more complex. An example that shows many similarities is selling cloud services especially in a Platforms as a Service (PaaS) context. There, providers offer developing tools to customers who build services and deploy them through the provider's cloud. It becomes apparent that the performance of these services may depend on the number of other services the provider hosts, and customers may be faced with similar issues when choosing a provider for their service. Furthermore, Software as a Service (SaaS) cloud providers offer different software packages to customers who use them through the cloud, and the performance of these packages may again depend on the number of customers who use them. Choosing an Internet provider can also be formulated similarly, where customers may experience reduced connection speed and latency due to congestion at peak hours, and providers can set their service's price, as well as make various decisions on infrastructure. A final example can be investing in coffee shops, where the investor can choose better infrastructure and/or more experienced staff to improve service, and the customer may have to wait in line to get served during peak hours (e.g. when going to work).

1.2 Research Objectives

In the context of EV charging, the research problem is motivated by the hypothesis that if we optimise competing charging station locations, capacities and prices by including consideration for how drivers choose routes, then the welfare of drivers should improve. At the same time, it should also result in an economically sustainable station network with predictable profit for investors. The goal is to achieve the best of both worlds, as maximising profit or minimising travel costs individually can have severe consequences for investors and drivers.

As we will see in Chapter 2, the vast majority of literature related to EV charging station allocation optimisation deals with state monopolies and offers little insight into the practical and theoretical aspects of charging station competition. Although the public sector does indeed operate a large number of charging points, even today a large portion of the charging station market is private, especially when it comes to rapid charging. With the energy market becoming more and more competitive, it is unreasonable to consider that there can exist a public monopoly in EV charging, in developed free markets. Thus an important motive for and objective of this work is to delve more deeply into charging station competition, and analyse quantitative and qualitative characteristics and idiosyncrasies that may emerge.

A further objective of this work will be to examine how incentives to private investors can improve the quality of the road network for EV drivers. It is not uncommon in many market sectors, such as the energy sector, for governments to subsidise goods and services, either to improve their quality or to guarantee their delivery. Thus it would be interesting to know what forms of subsidies can help drivers and investors, and to develop a methodology for evaluating those subsidies.

The above notions, in conjunction with the problem in Section 1.1, lead to the research question of *determining the optimal locations, capacities, charging unit power outputs and prices of charging stations*, so that competing station investors can maximise their profit at the same time the drivers minimise their travel costs, while still operating within their selfish interests. The objectives, therefore, consist not only of finding a final solution to the research problem, but also of analysing the complex behaviour of the customer-investor system in the process of designing a solution.

1.3 Research Requirements

In order to solve the SLCOP and similar problems, and to achieve the objectives set previously, the behaviour of customers and firm investors must be considered. This means that any solution must include the economic and behavioural aspects of customers and investors, as well as some more specific, derived requirements which emerge from the research problem in Section 1.1. The requirements for solving the category of problems similar to the SLCOP are the following:

1. Customers are self-interested and choose the firm which minimises their expected cost of acquiring the service. This includes a cost to access the firm, the fee for being served, as well as the expected queuing (or congestion) cost at the firm.

2. We need to consider expected queuing at firms, which depends on the number of customers arriving at each firm, and the number and type of servers at the firm.

3. Expected queuing cost thus indicates that a customer's choice of firm depends on other customers' potential choices.

4. Firm investors are self-interested and their choices are motivated by maximising long-term expected profit.

5. A firm's long-term expected profit depends on the potential decisions of customers, the number and type of servers, the fee for the service and, finally the firm's building cost.

6. The building cost of a firm depends on its location, and the number and type of servers.

7. Firm investors can choose different locations, numbers and types of servers, and prices at their firms.

A characteristic of the model, which as we will see later on, is that it is computationally complex when it comes to investor choices. It is not unusual to make simplifying assumptions in complex models, and it will be done so. It will be challenging, however, to make simplifications that will not obstruct the research objectives that were set in Section 1.2 and that will not prevent from reaching a useful solution.

1.4 Research Contributions

To address the problem identified in Section 1.1, this thesis proposes a general model for firm competition that utilises game theory. A three-stage extensive-form game is defined, each stage of which addresses a component of the SLCOP problem as it was defined in Section 1.1. In this game, customers and firm investors behave selfishly and their desires are modelled using utility theory. A non-coordination scenario for the customers' choice of firm is considered to be represented by mixed strategy Nash equilibria. Furthermore, the Nash equilibrium of the investors' decisions is also considered, and solutions are obtained by combining the mixed strategy Nash equilibrium for the customers and the pure strategy Nash equilibria for investors through the concept of Subgame-Perfect Equilibrium (SPE).

The idea behind this is that a SPE will provide a solution that represents the best customers and investors can do while operating within their selfish interests. Consequently investors have incentive to follow a SPE solution. This process may result in more than one subgame-perfect equilibria. Choosing a final solution out of those can be done according to the requirements of the planner (government, private trust). For example, one can choose that equilibrium which maximises the customers' welfare. The model is evaluated, in the context of the SLCOP problem, using mostly duopoly examples which are straightforward to manipulate and to interpret behaviourally. However, the model can be used in a larger scale and also to represent other firm competition scenarios where customer and investor behaviour can be abstracted similarly.

This approach advances the state of the art in firm competition and charging station allocation optimisation in the following ways:

1. This work expands upon current literature in firm competition to highlight interesting properties theoretically and empirically. Toward this, a game-theoretic model that models both customer and firm investor behaviour is utilised. The model is the first to combine aspects that include stochastic queuing times, prices, the number of servers and the speed of each server, locations, building and operational costs, investor competition and extraneous competition. In the model, investors can own several firms that sell the same product or service, and make several decisions about their firms. Customers, choose which firm to obtain the service from based on the expected cost of acquiring the service, which in turn depends on customer congestion at firms. The model is solved through a combination of theoretical and algorithmic techniques to obtain subgame-perfect equilibria in investor and customer choices. The model is novel in that it combines several aspects from network pricing games, Stackelberg games and spatial competition. It is shown that subgame-perfect equilibria are highly efficient solutions for the social welfare of firms and customers, compared to optimal firm allocations, showing worst-case efficiency of 93% within reasonable competition constraints.

2. The model is applied in representing a more abstract form of the electric vehicle charging station location, capacity, charging unit power output and charging price decision problem. This models the interdependency between EV driver and station investor choices, that exists through prices and queues at stations. Current literature in charging station allocation generally disregards competition and the market dynamics of the SLCOP problem, with many models considering static traffic flows from conventional vehicle data, or disregarding queues and/or prices. It is shown in this thesis that the more EV drivers are willing to pay in order to save time, the more station investors will take advantage of them and increase profit at the drivers' expense. Furthermore, it is also shown that the more EV drivers are inclined toward using their EVs, the more station investors will take advantage of them.

3. Existing literature that considers charging prices generally sets them at marginal cost, under the framework of a state monopoly. Literature on firm competition shows that service prices may deviate from the marginal charging cost due to goods differentiation. This work confirms this finding theoretically and empirically through product heterogeneity that is induced by different costs to access firms. This implicitly means that investors would rather compete on the same than different locations. However, this work also finds that in the SLCOP and similar problems prices will be higher from the marginal cost due to the fact that demand may not be satisfied at once. For example, in the case of the SLCOP problem immediate demand satisfaction requires charging units with extremely high power output.

4. This thesis shows empirically that subsidising the building costs for charging units at stations has a significant beneficial effect for drivers and stations. On the other hand, subsidising the price of electricity for stations can cause private investors to decrease capacity and increase prices. Furthermore, subsidising investors to build on disadvantageous routes

can have a harmful effect to other investors. Last, it is shown that with the proposed model it is possible to determine the monetary gain due to the subsidy for stations and drivers, and to calculate optimal subsidies according to a variety of criteria.

1.5 Thesis Outline

The remainder of this thesis is organised as follows:

- Chapter 2 examines literature related to charging station allocation optimisation, charging station and more general firm competition, and electric vehicle driver behaviour. In addition, an overview relevant game-theoretic concepts is provided.
- Chapter 3 presents the formal model for this work. Firm investor and customer choices are modelled as normal-form games. These are combined into a three-stage extensive form game where investors first choose locations, capacities and the speed of service for their firms, then investors choose prices at their firms, and last customers choose among firms stochastically based on these. The game is solved by backward induction in order to obtain subgame-perfect equilibria. The Chapter also presents a theoretical evaluation of the model, and performance metrics are defined.
- Chapter 4 evaluates the customers' model and the investors' choice of capacity empirically, in the context of EV charging station capacity competition.
- Chapter 5 extends the empirical analysis to pricing competition, in the context of the SLCOP problem, to confirm the behaviour shown in the theoretical analysis of Chapter 3, and to extract new qualitative properties of investor competition. Furthermore, the possibility that only an uncertain portion of demand may be satisfied is evaluated, and it is shown that the model is robust against reasonable variations in important parameters. Moreover subgame-perfect equilibria are found to be highly efficient for system-wide utility compared to optimal charging station allocations.
- Chapter 6 presents an evaluation of the location and speed of service choices investors make, defines metrics for evaluating the efficiency of subsidies to investors, and the model is used to examine the effectiveness of subsidies toward the purchase of charging units, and toward the cost of electricity for private EV charging stations.
- Chapter 7 presents an overview of the conclusions that have been reached throughout the thesis and proposes future improvements.

Chapter 2

Background

In this chapter we will discuss current literature that is relevant to the SLCOP problem which we analysed in the previous chapter. In addition, this chapter provides some background in game theory that is necessary to understand and discuss this work. More specifically, Section 2.1 provides an overview of solutions for charging station monopolies and Section 2.2 discusses additional monopolies with consideration for power requirements. Section 2.3 provides background in game theory concepts relevant to this work. Next, relevant competitive market models are presented in Section 2.4, while EV driver behaviour is discussed in Section 2.5. Last, Section 2.6 provides an overview of some alternatives to rapid charging, and Section 2.7 concludes this chapter with a short summary.

2.1 Monopolistic Models

Literature relevant to the general charging station allocation optimisation problem mainly focuses on allocating stations in *monopolistic markets*. While there are vast amounts of papers that delve into charging station monopolies, these will not all be presented as they do not directly relate to the research problem this thesis examines. The papers that will be reviewed in this section, therefore, are characteristic of the assumptions made in monopolies. Allocation is done according to various criteria, but generally few papers address some form of the economic aspect of expanding charging station networks, and even fewer consider some form of profit or queues.

One work that examines the profit of stations is presented by Jia et al. (2012), who introduce a model that optimises the number, locations and capacities of charging stations in order to minimise the overall investment and operational cost. In their model, they use real-world conventional vehicle data to determine how much time vehicles remain stationary and they define the "vehicle hours" unit, which is the product of the number of vehicles and their corresponding time of stay at one place. It is assumed that the larger the vehicle hours at one place, the larger the demand for charging will be in that place. They abstract the road network into a graph and vehicle hours from neighbouring places are aggregated into the corresponding node of the graph to determine charging demand. Furthermore, they consider the cost of a station's construction and operation, by taking into account the aforementioned demand, and also consider the cost of charging for drivers. They define a mixed-integer quadratic programming problem with an objective function that aims to minimise the integrated cost of investment and operation. Driver behavioural patterns are taken into account for generating vehicle routes (traffic flows) utilising a shortest path algorithm. Finally, although it is assumed that vehicle hours is an indicative measure of charge demand, it is not clear what the unit exactly represents given that real-world vehicle data are utilised and that drivers in reality may spend time in one place for a variety of reasons (e.g. traffic).

Their approach to economic modeling though is consistent with the approach of Sadeghi-Barzani et al. (2014), who emphasise that rapid charging is critical to the success of EVs. They consider the development and electrification cost of stations, which they formulate similarly to the previous approach. Additionally, the cost of power losses in the grid due to EVs charging is also taken into account. A mixed-integer non-linear problem is formulated, with the aim of minimising the aforementioned costs, which the authors solve with the help of a genetic algorithm. Both these papers presented above focus purely on the economic aspect of the station allocation problem for a monopoly. Although these model a problem quite different from the SLCOP (Section 1.1), some economic aspects presented such as a linear cost with respect to stations' capacities, and the costs of power losses are useful to take into account given the requirements that were set in Section 1.3.

In addition to station development cost, Hess et al. (2012) consider the routing problem for the EVs. In their model, vehicles travel until they detect a low battery state. Meanwhile, they receive advertisements of stations via ad-hoc or cellular networks, and they divert to the closest stations when needed. The economic aspect is also modeled, by considering finite resources, building cost and capacity as well, but this does not include a profit model. Genetic programming is used to obtain a set of feasible solutions for station locations and capacities, by minimising the stations' development cost as well as average trip time for drivers. By doing so, the aspect of driver happiness is also addressed. Although parameters such as vehicle routes, travel time and capacities are taken into account, drivers still divert to the closest station without considering queues. Furthermore, initial route planning does not consider the fact that vehicles will need to recharge, thus vehicle routes depend solely on the destination. An additional approach that examines the routing problem is presented by Worley et al. (2012), who propose a model for finding an optimal set of routes and station locations for commercial goods delivery vehicles. In this model, vehicles originating from one depot (which is always a charging station) must satisfy all the delivery demands without travelling more than their range without recharging. In their model they define driver convenience in terms of cost in utility and they also consider the cost of recharging and the investment cost. They find an optimal solution to their problem via an objective function, that minimises the sum of travel costs, recharging costs and costs of locating charging stations. This model, however, refers to a version of the problem that is more similar to the travelling salesman problem, rather than a generalised, real-world transportation network with a large volume of vehicles. As a result, queuing is not present as a concept here either and instead focus is shifted toward the reduction of total delivery costs for the courier company, which also owns the stations. Finally, the goal of the drivers is to go through pre-determined delivery way-points and consequently routes are largely pre-determined by the deliveries they have to fulfill, something which results in station locations adjusting to vehicle routes, rather than route selection adjusting as well.

Lam et al. (2014) introduce population coverage, in addition to considering routing. Their project's aim is for the stations to achieve maximum population coverage, while at the same time routes are served more efficiently. They formulate the problem on the basis of charge demand at the nodes of their undirected graph and additionally include traffic conditions and queues. They formulate a mixed-integer linear programming problem and also propose a greedy algorithm as an alternative. The optimisation is based on station coverage and the convenience of drivers. However, because the driver convenience aspect is examined on a minimum distance travelled basis, queues do not influence the decision of drivers, but rather only influence the location and capacity for the stations. Similarly, Wang et al. (2010a) propose a model with the aim of maximising population coverage, in which they account for factors such as the distribution of charging demand, municipal planning, the power grid and energy consumption. Their algorithm is based on charging demand priority and the usage of existing gas stations. If demand in an area is high and there is a gas station nearby, that station is utilised. However, there is no concept of driver convenience present, apart from serving an area's population adequately. Additionally, in these two last examples no economy is taken into account.

To include both some form of economy and driver convenience, Wang and Wang (2010b) propose a solution which includes with the dual objectives of minimising locating cost and maximising population coverage. The model takes into account only the traffic flows of the shortest paths between cities for intercity journeys and assumes the capacity of stations is unlimited. Furthermore, it is assumed that the cost of locating stations is the same for every station. Their algorithm balances location cost against population coverage, so again traffic flows do not adapt to the positions of stations, but only the stations adapt to the given flows. In this approach, considering only the shortest path routes is restrictive in the sense that it discards the concept of potential combinations of non-shortest path choices that may, however, result in lower travel time. The observation on static traffic flows is characteristic in the work of Hiwatari et al. (2011), in which a road traffic simulator is presented with the goal of analysing the layout of charging infrastructure. Their goal is to reduce the number of EVs running out of power. Traffic data is used from house surveys in Japan and trips are generated according to this data. Their model takes into account many temporal variables such as consumption rate and charge, as well as station capacities. The idea behind their approach is that stations should be located in locations where many EVs run out of power. The model consists of the EV layer and the station layer. Initially, an arbitrary number of stations is chosen and are spread uniformly over the area under examination. The locations of the stations are then transferred to the EV layer, the simulations are carried out and the locations where EVs have run out of power are transferred to the station layer. Finally, the stations are rearranged according to the locations where EVs ran out of power and their capacity is optimised. In the station layer, stations are free-floating positively charged particles and EVs static negatively charged particles. The stations are rearranged autonomously this way due to the attraction force between stations and EVs and the repulsion force between stations. They show that the percentage of EVs running out of power is reduced successively and significantly as the process is iterated.

All these approaches consider monopolistic markets but this thesis argues that monopoly in EV charging is not realistic in developed free markets and that it is of interest to examine charging station competition. Moreover, while some economic aspects are included, these are not enough to capture the interactions between private station investors. Finally, we saw that some works consider EV driver convenience, but this is largely defined as the ability of stations to capture incoming static traffic, and not traffic that dynamically changes due to the presence of stations.

2.2 Power Distribution Models

In the previous section we reviewed literature that includes some form of the economic aspect or the driver convenience aspect of the charging station allocation problem. In addition to these, other researchers design monopolistic models from an engineering perspective. Such is the work of Ge et al. (2011), whose aim is to allocate stations and capacities in such a way that the users' cost of power losses on the way to the station is minimised. They calculate charge demand nodes from real-world traffic data and choose the best location for a station within a given zone. In their model, they utilise variables such as capacity, travel time and mileage. The location of a station is optimised so that the sum of the distances of a station from all demand nodes within the zone, is minimal. A solution even more focused on the engineering aspect of building infrastructure is proposed by Yao et al. (2014), who propose a model that considers static, fixed traffic flows. The aim of their project is to minimise annual investment cost and energy losses, while maximising annual traffic captured by the charging stations. In their model, they consider slow charging stations as the main means of charging and rapid charging stations as a complementary means. The model is complex and takes into account variables like capacity and investment cost, as well as ones related to power distribution systems. Additionally, it considers the amount of traffic flow captured by the stations as a measure of convenience for the drivers, but does not consider queuing. The authors then define three sub-problems for optimisation which are cost minimisation, energy loss minimisation and the maximisation of traffic captured. The model is solved by an evolutionary algorithm that seeks non-dominated solutions, but no details are provided on the genetic algorithm itself, or the decision process.

Additionally, Liu et al. (2013) consider multiple factors such as power losses in the network, building and operational costs, avoiding wasting resources (power) and also technical variables for the power distribution system such as transformer capacitor limits and voltage limits at power buses. They optimise investment cost for the charging stations and the viability of the power distribution system. As such, the model neither examines the EV routing problem, nor considers the stations' profit. An extension of this approach by Wang et al. (2013) adds more technical information on power distribution systems, and additionally introduces the concept of driver convenience into this model by aiming to maximise the traffic flow captured by the stations. Still, traffic flow is static as in Yao et al. (2014), thus vehicle routes do not change due to the presence of stations. They construct a robust multi-objective planning algorithm that converges fast and provides Pareto-optimal solutions, with convergence occurring when all routes' traffic flows are served sufficiently. Due to the focus on power distribution, these two attempts do not consider station capacity constraints, or queuing.

Although the power delivery problem for charging stations is interesting and significant, it is complex in its own right. The SLCOP problem is significantly complex already as we will see in later chapters. As a result, incorporating even more parameters that relate to power delivery would over-complicate the model which would make interpreting results confusing, and would divert attention to matters that are irrelevant to the points this thesis is trying to make.

2.3 Background in Game Theory

This section provides some necessary background information in game theory, with respect to the work presented in this thesis. The goal of this discussion is to familiarise the reader with concepts utilised in this work, and to discuss concepts in Game Theory that might be relevant. This background will be provided in the form of more intuitive and informal definitions[1]. The provided definitions follow closely the intuitive definitions given by Shoham and Leyton-Brown (2009), Fudenberg and Tirole (1991) and Nisan et al. (2007).

As was discussed in Section 1.4, the model that will be presented in this thesis is entirely based on game theory. For the purpose of this work, *noncooperative game theory* is utilised, which models mathematically the beliefs, preferences and possible actions of self-interested individuals with the goal of studying the interactions between them. We will call these individuals players from now on. A *self-interested* player is not necessarily one who strives to undermine others, but is rather a player who has a clear picture of desirable states of the world (interests) and strives to bring about these interests. In order to model the players' preferences over the available actions, *utility theory* is used. This theory makes several assumptions with regard to individuals. Among these, it is assumed that people act as if they behaved according to a utility function that ranks their preferences, and they try to maximise the expected values of this utility function. A further important assumption utility theory makes is that individuals are *rational*, that is there exists no circularity among an individual's preferences. If an individual prefers

[1] Formal definitions will be presented in later chapters throughout the model, where appropriate.

action A over action B and action B over action C, then it must be that the individual prefers action A over action C.

Under these assumptions, it is straightforward for a player to act optimally even under uncertainty, as long as the outcomes of actions and the likelihood of those outcomes are known to the player. However, taking an optimal action becomes more complicated when there are several utility maximising players whose actions affect the other players' utilities. Such is the case in this work, where several customers and firm investors strive to minimise service costs and maximise profit respectively. In order to model these strategic interactions between players, several games in *normal form* will be utilised. A normal form game is a representation of every player's utility for every possible state of the world, and it is assumed that the state of the world depends solely on the actions of the finite set of participating players. The goal of using such games is to determine Nash equilibrium (NE) states that may exist. In a Nash equilibrium state, every player's chosen action is a best response to the other players' actions thus no player has incentive to deviate unilaterally from the equilibrium strategy. We will utilise *pure strategy Nash equilibria* in which players myopically choose and play the single action that maximises their utility given the other players' actions. *Mixed strategy Nash equilibria* will also be used, where players choose over available actions with the probability distribution that maximises their expected utility given the other players' mixed strategies. For example, the customers' choice of firms is represented with a normal form game where we seek mixed strategy NE in firm choices. The firm investors' capacity choice is modelled with another normal form game where pure strategy NE in capacities are sought.

Normal-form games operate under the assumption that players all make their choices at the same time. But these are only static representations of otherwise interactive real-world situations. In this work, players are first separated into two groups; EV drivers and station investors. It is reasonable to assume that EV drivers make their decisions after investors have announced their choices (or rather after stations are at an operational state). Then again, station investors do not decide simultaneously on all matters, but first consider some locations and investment levels (capacities), then consider prices and so on. All these different levels of decision-making are thus considered as independent normal-form games[2]. To represent this dynamic interaction between different decision levels, a game in *extensive form* is used, which consists of independent normal-form games, or subgames, that are played one after the other. At each stage of this extensive-form game, players are able to observe the initialisation and events that transpire in previous stages, that is they have *perfect information*.

This existence of perfect information enables the utilisation of the *subgame perfect equilibrium* (SPE) concept which is a generalisation of Nash equilibrium, and which suggests that the equilibrium outcome of a sequential extensive-form game consists, in fact, of the individual Nash equilibria of its subgames. Speaking about the individual Nash equilibria of the subgames, there are a few additional concepts in Game Theory that might be relevant.

[2]Within each game players still decide synchronously.

2.3.1 Potential games

The concept of potential games is worth discussing in the context of this work. Potential games are games in which a global function, called the potential function, is used to express the incentive of all players to alter strategies. In the context of this work, the customers' firm choice game (Section 3.1) is indeed a potential game, and specifically a congestion game. In more detail, congestion games are a class of games where the utility of using a resource decreases as the number of players who choose the same resource increases (Shoham and Leyton-Brown, 2009). This is similar to the setting for EV drivers presented in Section 1.1, where the more the EVs that travel to the same station, the higher queuing time may be. A major advantage potential games offer is that pure strategy Nash equilibria, which are guaranteed to exist in potential games, are actually local optima of the potential function and are therefore relatively straightforward to locate. However, when it comes to customer choices mixed strategy Nash equilibria are of more interest for the purpose of this thesis, as in the SLCOP and similar problems there is no apparent customer coordination mechanism which would justify customers playing pure strategies. Additionally, the existence of many pure NE is problematic as to their interpretation. These issues are discussed in more detail in Sections 2.5 and 3.1.1. Regarding the firms' choice of capacities, locations and speed of service (Section 3.3), it is shown in Section 3.5.2 that there are cases where pure strategy Nash equilibria do not exist, and therefore it cannot be a potential game. As for the pure Nash equilibrium in firms' prices (Section 3.2), the possibility of a potential game was explored, but the prices game was not found to belong to any of several known forms of potential games. Furthermore, the analysis in Section 3.5.1, shows that there is a unique equilibrium in prices which is found analytically in a straightforward way, therefore a potential game would not contribute in terms of ease to locate the solution.

2.3.2 Network pricing games

Network pricing games are a category of games that are more relevant to this work. An advanced version of a network pricing game is presented by Hayrapetyan et al. (2007), where a set of network managers compete for users who want to use a network, by selecting prices for their service. This is similar to the SLCOP problem and Internet-based services competition identified in 1.1. The authors find that a single pure strategy equilibrium in prices exists, and is efficient for system-wide social welfare. To measure social welfare, they assume that the quality of service for the user depends on congestion at the network, and social welfare is defined as the sum of providers' profits minus the cost due to congestion for users. Although this network pricing game is closely related to the work in this thesis, the model that is proposed in this thesis goes above and beyond in many aspects. First, this work presents a more complex firm model where investors can also choose locations, service rates (capacity), and the speed of service for their firms in addition to prices. Secondly, Hayrapetyan et al. (2007) do not consider an explicit customer model, but pose assumptions on customers instead. These are that customers will choose the firm which minimises the sum of service latency and purchase price, and that demand decreases with disutility. Furthermore, they assume deterministic concave demand which is suitable to model services with a comparable alternative that users will switch to if the price of the service is too high. This thesis takes these a step further, with a full customer model where customers choose firms stochastically based on the expected cost to acquire the service. This includes prices and expected congestion at firms, and customers have an alternative option outside the firm system allowing for uncertain demand satisfaction based on expected congestion at the alternative option. This results in subgame-perfect equilibria in firm locations, capacities, service speeds, prices and customer firm choices which this thesis shows to be very highly efficient for system-wide social welfare compared to optimal firm allocations.

2.3.3 Mechanism design

This thesis involves evaluating subsidies to investors as incentives for improving the service for the customers. One branch of game theory that could be used to address this is mechanism design. In games of mechanism design, there is a set of players one of whom is the principal player. Other players report their type, that is their preferences or information on the product they sell, to the principal player. The principal has already decided on a mechanism that maps types to outcomes, and the players receive the outcome according to that mechanism (Shoham and Leyton-Brown, 2009). Mechanism design offers an interesting framework for subsidies, in particular because it allows to examine truthfulness in reporting to the principal, and because it works by starting from the desired outcome. A work that examines subsidies using mechanism design is of Sorana (2000), where the authors present a mechanism for subsidising telecommunications providers to serve high-cost areas with affordable rates. They show that in many cases auctioning mechanisms are more efficient than traditional subsidy schemes, however they also show that auction mechanisms are particularly vulnerable to collusion among bidders in certain settings.

A similar mechanism could be assumed here to guarantee certain qualities of the subsidies, such as that the subsidies end up to the firms who need them the most, or are distributed so that customer costs are minimised. However, there are two issues regarding mechanism design in relation to this work. First, this thesis is not involved with efficient subsidy distribution so as to design mechanisms for subsidy allocation, but is rather more involved with qualitative characteristics of the subsidies themselves. For example is a particular type of subsidy useful or not? What happens when we subsidise only one investor? These are questions that are straightforward to examine using the model presented in this thesis, and mechanism design would over-complicate the interpretation of some aspects. For instance, investors in this thesis are assumed to have unlimited budget, so it is difficult to design and interpret a mechanism for just distribution. Secondly, although this thesis will also include determining the performance of subsidies, applying mechanism design is problematic in that a game of mechanism design requires private information. However, decisions in this work are made in stages, and while within each stage private information exists, there needs to exist perfect information among stages; that is, information is propagated from one stage to the next in order to obtain subgame-perfect equilibria. These issues highlight that mechanism design is not directly compatible with the methodology followed here, and designing mechanisms for subsidy distribution would be a vast, complex undertaking in its own right should the constraints set in this thesis be maintained.

2.4 Competitive Models and Firm Competition

In Sections 2.1 and 2.2 we reviewed work that captures some form of the aspects of the SLCOP problem we defined in Section 1.1, albeit for charging station monopolies. Here, examples that explicitly include some form of firm competition, which are more closely related to the work in this thesis, will be presented.

In order to better understand firm competition, it is instructive to first discuss some classical models of firm competition, such as Cournot competition and Bertrand competition. In short, Cournot competition assumes that two competing firms sell a homogeneous product to customers at fixed prices. The firms decide their production output simultaneously, knowing the prices they and the opponent will set for given outputs. Cournot competition is not particularly interesting for the purpose of this work, as we will see later in Section 3.5.1 that the product firms sell in this work is, in fact, heterogeneous and in addition firms should compete in prices as well. Of more interest is Bertrand competition, that addresses pricing competition. In the Bertrand model, it is assumed that two firms are able to satisfy all demand for a homogeneous good immediately, and customers are assumed to buy the good from the firm that offers the lowest price. Interestingly, the Nash equilibrium in prices in Bertrand competition is for both firms to offer the good at marginal cost (Mas-Colell et al., 1995; Singh and Vives, 1984). Let us now move on to more advanced forms of competition. Toward this, Section 2.4.1 presents some approaches to charging station competition, Section 2.4.2 presents archetypical spatial competition, and Section 2.4.3 discusses sequential competition which is of more interest for this thesis.

2.4.1 Charging station competition

In the context of charging stations, Escudero-Garzás and Seco-Granados (2012) analyse a charging station oligopoly based on Bertrand competition. In this game-theoretic model, two competing station owners with set locations decide on their prices given the other owner's strategy. The model operates under the fundamental assumption behind Bertrand models; that demand is satisfied immediately. As such, station capacity is defined in terms of total electric current the stations can provide and queues are not a component of this model. However, they relax Bertrand's assumption on goods' homogeneity, based on the idea that a given amount of energy at a given price may hold different value for drivers who are further away, or who do not want to recharge. They show empirically that this relaxation causes prices to deviate upwards from the Bertrand result (marginal cost), sometimes significantly. In contrast, this thesis argues that due to power grid limitations and/or high investment cost, peak charging demand may not be satisfied at once. This is not only because of queues but also because recharging an EV still takes some measurable time even without queues. Furthermore, this thesis confirms that goods heterogeneity causes prices to deviate from marginal cost, and enhances this by showing theoretically that another important reason why equilibrium prices will be higher than marginal cost, is the fact that demand cannot be satisfied at once in the SLCOP and similar problems.

Competitive pricing is also addressed in Gerding et al. (2013), where authors present a solution for competing charging station pricing in a two-sided market. Via this market, EV drivers can make advance reservations in charging stations. Drivers are buyers who arrive at the market dynamically over time. They report their preferences for time slots and charging locations, with the goal of reserving a time slot for charging their EV at an available station. Charging stations that participate in the market report their availability and charging costs. Both drivers and stations are rational, profit-maximising entities. The authors apply online mechanism design, in order to develop a mechanism for pricing in which drivers have no incentive to misreport their preference or delay their reservations to exploit the mechanism. Furthermore, they explore a number of payment mechanisms on the charging station side and evaluate their mechanism in two realistic scenarios; en route charging and park 'n charge. Through the concept of reservations, the authors manage to eliminate the problems relevant to queuing. Consequently the drivers' routing choices are primarily focused on shorter travel time, which includes charging time. Using principled equilibrium analysis, they show that in the case of en route charging their proposed Reverse Vickrey learning and Posted Price learning mechanisms achieve 90-95% efficiency of optimal, while at the same time the Reverse Vickrey learning mechanism achieves a stable deficit in the region of 18% in buyer welfare compared to optimal welfare. This work is significantly different from the work presented here for a variety of reasons. First, in the above paper it is assumed that stations exist in predetermined locations and feature predetermined capacities. In contrast, in this work stations compete in locations, capacities and charging unit output in addition to prices. Second, the above work considers dynamic arrivals at stations where drivers have a reservation and thus there are no queues. This thesis focuses on peak EV traffic that arrives at stations at once, which results in oversaturated queues.

2.4.2 Spatial competition

This thesis further involves location competition among firms. Important classical models in this field include Hotelling (1929)'s spatial competition model of homogeneous firms, the correction on Nash equilibria in Hotelling competition by Osborne and Pitchik (1987) and spatial competition with heterogeneous firms proposed by Vogel (2008). Spatial competition is involved with the location choice of firms in a given uniform area, like a marketplace, over which customers are evenly spread. Firms can locate anywhere within that space and, depending on the position and product of firms, those that are closer to each other will compete more intensely for customers, thus owners may choose to place their firm away from or close to competition. Although the firms' location choice in the model presented in this thesis is a form of spatial competition, it is not a typical example. The concept of location in this thesis is more abstract and refers to different access costs incurred to customers in order to reach firms. This may as well be translated as spatial competition within the same uniform space, or may refer to firm allocation in non-uniform space. This thesis will be more involved with the latter. For example, this work is more interested in allocating charging stations across different routes, rather than fine-tuning locations within the same route. This abstraction helps reduce complexity, because spatial competition within the same uniform space would necessitate introducing too many new parameters in certain problems such as the SLCOP (such as distance from power substations), which would make the model more complex and difficult to evaluate. Furthermore, different costs for *accessing* different firms, that is goods heterogeneity, is not used in typical spatial competition and in addition typical models do not consider queuing.

2.4.3 Sequential competition

More advanced price competition with queues, has been proposed by Sattinger (2002) who followed a game-theoretic approach to model competing firms' pricing decisions when customers have to wait to be served. This may resemble our research problem more at first glance. However, it is fundamentally different in that (1) customer flows to firms are deterministic, (2) service rate is always greater than the arrival rate, (3) each firm has only one server. In contrast, in our case customer flows to firms are stochastic, we are especially interested in the situation where queues are oversaturated, and firms can serve multiple customers synchronously. Indeed, the model presented in this thesis resembles a Stackelberg game where players compete by moving sequentially. This is similar to the situation described in Section 2.3, where different groups of players decide sequentially over different matters. However, the problem negotiated in this thesis is much more complex and features several sequential stages, where decisions within each stage are made simultaneously. To address a sequence of actions in the context of firm competition, the Stackelberg leadership model has been proposed in which one firm acts as a leader and moves first, and the remaining firms follow in sequence, and the problem is solved using subgame-perfect equilibria (von Stackelberg, 2010). Although today the term 'Stackelberg game' is used generically to denote various sequential models, traditional Stackelberg competition is different than the work presented in this thesis. Stackelberg competition refers to competition in production output, and assumes there is a known price function which depends on the firms' production outputs, that is it is a refinement of Cournot competition. In contrast, firms in this thesis compete on several levels, including service rate, service speed, locations and prices. Furthermore, firms are assumed here to move simultaneously for certain decisions; the sequence of actions separates decisions regarding service rate and infrastructural decisions from price decisions, and from customer decisions.

Further sequential competition more similar to the work in this thesis includes the work of Deneckere and Peck (1995), who propose a model for firm competition in which firms first decide production capacities and prices simultaneously, and then customers choose firms based on these. It is possible that an uncertain portion of customers may not be served in the end (rationing), if the number of customers arriving at the firm is larger than the production capacity of the firm. Deneckere and Peck (1995) make several interesting deductions on firm competition. They identify discontinuous jumps in demand, as the main reason why equilibria may not exist in models of many competing firms. They note further that the possibility of demand being only partially satisfied helps eliminate discontinuities in demand. In addition, they prove that when demand is uncertain a pure strategy equilibrium in investment levels and prices exists, if the number of competing firms is sufficiently large. This indicates that a stochastic model for customer choices may reinforce the existence of any pure strategy Nash equilibria in the work presented here, and the possibility of demand not being satisfied completely can also be taken into account to reinforce existence. Furthermore, they find that under their setting prices will be higher than the marginal cost for firms. However, there also exist major differences with the model this thesis proposes. First, in this thesis prices are decided in a separate stage than capacities. This allows for theoretical analysis on the prices equilibrium, which shows that when customers have to wait to get served, prices are indeed higher than the marginal cost as Deneckere and Peck (1995) find, but they converge to the marginal cost as the service speed improves. Second, the problem this thesis examines is focused in situations where customers queue up at firms to receive the service, and thus incur an expected cost due to congestion at the firm (e.g. a time cost in waiting at charging stations, or reduced quality when using online services). Customers then select firms based on the expected cost of acquiring the service. Therefore excess customers are not rationed as in Deneckere and Peck (1995). Instead, an alternative option is given to customers outside the competing firm system and rationing is determined by the expected cost of choosing the outside option and the choices of firms, which allows for uncertain demand satisfaction.

2.5 Customer Behaviour

In the context of the SLCOP, so far we have reviewed literature that may include some aspect of convenience for EV drivers, but this was carried out in an indirect way, since queues are largely disregarded and traffic flows to stations are generic in most models. In this section, work that addresses the EV routing problem more explicitly, with consideration on how stations may affect EV drivers' choices will be examined. EV drivers

In the context of EV driver route selection, congestion games are utilised by Malandrino et al. (2015), who present a solution for addressing EV drivers' assistance in route choice when in need of recharging. In that work, it is assumed that a central navigation service collects data regarding vehicles' positions, speed and heading, as well as data like the occupancy at battery switching stations and their expected times to serve the driver. Drivers can use an intelligent transportation system (ITS) to request route advice from the central navigation service when they are low on battery or when they want to recharge. This approach is based on the assumption that the central service has all available information from all vehicles on the road, which guarantees that there is no better alternative to the proposed route, thus the driver has incentive to follow the proposed pure strategy Nash equilibrium.

EV driver coordination can be achieved by using an ITS, but in general it is more realistic to assume that drivers follow a stochastic strategy, as is the case in other work. For example de Weerdt et al. (2013, 2016) introduce an intention-aware routing system (IARS) for electric vehicles. In their model, drivers determine their routes so that the expected journey time is minimised. They do so by taking into account other drivers' intentions, which are used to compute predicted queuing times at charging stations. The authors stress that taking into account these intentions is important, because electric vehicles that have to recharge en-route may encounter significant queuing times if many other vehicles choose the same station. The IARS is simulated with real-world data on charging station locations, travel times, road networks and journeys, and it is shown that the routing algorithm achieves over 80% improvement in waiting times at charging stations, and more than 50% reduction in journey times.

In the context of charging station allocation a stochastic approach to driver decisions is implemented by Xiong et al. (2015) and Xiong et al. (2017). The authors study the charging station placement problem with the goal of minimising charging cost, by taking into consideration the EV drivers' strategic behaviour, the impact of drivers' choices on traffic conditions and the service quality of charging stations. They formulate an optimisation problem with the government acting as a market regulator in order to minimise social cost. In their game-theoretic driver model, drivers play mixed strategies meaning that they choose over the set of available routes and recharging options with a probability distribution. They calculate the Nash equilibrium for the drivers, for the sets of parameters to be explored (station capacities, locations) and then they choose the parameter set for stations which minimises the social cost for drivers. Social cost for the drivers considers time costs such as queuing time and traffic congestion. Finally, they use a brute force approach to solving the problem and also develop a heuristic to approximate the optimal solution more quickly. However, two integral assumptions in this model are that (1) there exists a monopoly in charging stations and (2) the fee that drivers pay for recharging is fixed. In contrast, this thesis proposes a model where several self-interested investors are competing with each other, which affects both the optimal prices and capacities, as well as the choice of charging units and locations.

Last, Anshelevich and Ukkusuri (2009), emphasise that in both transportation and communication networks there exist selfish flows. In their approach, every agent sending a flow over the network desires to get it to its destination as soon as possible. The drivers play pure strategies and the authors consider the concept of dynamic flow or flow over time. They examine the Nash equilibria in both time-dependent and non time-dependent network routing problems. Finally, they use the price of anarchy concept, which measures efficiency loss due to the selfish behaviour of a system's agents, to measure the quality of the Nash equilibrium solutions, where they exist, in comparison with the best possible solution. While the concept of pure strategies for drivers is not as interesting for modelling drivers in this thesis, the concept of the price of anarchy, on the other hand can be utilised to measure the efficiency of solutions for investors and customers.

2.6 Alternatives to Rapid Charging

A popular alternative to rapid charging in research, that has also been utilised in small scale in the real world is battery switching. Works such as Wang et al. (2011), Jamian et al. (2014) and several more, examine the concept of stations that, instead of recharging EVs directly, switch the battery of the EV with a pre-charged one. The station then charges the empty battery after the vehicle leaves. The claimed advantages of this approach are that this concept can save space at the stations which they could use to install large, complex charging systems that improve charging efficiency and extend battery lifetimes. The most important advantage, however, is that it can dramatically reduce the time a vehicle needs to stay at a station. These approaches are less complex than traditional station allocation problems, something that allows for better economic modeling and optimising net income. However, Avci et al. (2014) argue that battery switching is an environmentally unfriendly concept, in addition to the fact that it introduces high cost and risk for investors. This is because investors need to purchase different types of batteries in large numbers, in addition to building charging infrastructure. Furthermore, it is arguable that when switching demand is high this necessitates large numbers of pre-charged batteries, something which defeats the concept of slowly charging empty batteries at the stations' leisure with lower cost. Finally, this approach has been applied to a small extent in the real-world (e.g. Better Place), and has already failed for the reasons stated above, so it is not straightforward at all how battery switching can be applied in large scale[3].

Li et al. (2010) propose a different concept, that of Nomadic Portable Charging stations (PCSs). This idea relies on an operational centre, which sends trucks that drop PCSs in areas where charging demand is high. In order to determine the high-demand areas, real-time traffic and charge demand are evaluated by the centre. A reward function is defined for each vehicle in a one-dimensional line, which is then expanded to two dimensions. At each potential high-demand set of coordinates, the reward functions of all the vehicles are aggregated, which results in ranking each set of coordinates according to charging demand. PCSs are then sent to the areas where demand is high so that EVs can charge off them. The authors study this model in a single highway scenario and show that the capacity of the PCSs is an important factor for queuing time. This research proposes an interesting concept, but does not produce results that justify considering portable charging stations. It is argued, that the cost of rapid-charging infrastructure is already significant, even without considering PCSs. Furthermore, deployment of PCSs that

[3] Some more information on this can be found in:
https://www.nytimes.com/2013/06/02/automobiles/fallout-from-failure-of-battery-swap-plan.html
https://www.fastcompany.com/3028159/a-broken-place-better-place
https://www.theguardian.com/environment/2013/mar/05/better-place-wrong-electric-car-startup

multiple EVs can charge from requires parking spaces at regular intervals in highways, space that could be used to expand a proper rapid charging station. Finally, the use of PCSs induces further power losses that make charging even more expensive, because it necessitates charging PCSs that in turn charge EVs.

Finally, Chen et al. (2013) propose in their paper a solution strictly for slow charging stations. Here, household survey data is used to determine vehicle parking locations and the durations of all trips away from home. In order to identify optimal charging station locations they utilise parking demand, interpreted from that data. The goal of this approach is to identify station locations which are within walking distance of the drivers' ultimate destination. Their model takes into account budget constraints, but assumes infinite capacity, thus queues are also not present here. Consequently, the authors form a mixed-integer optimisation problem in which the objective function aims to reduce the total access cost as a function of walking distance between the station and the actual destination. Although slow charging stations are undoubtedly useful for situations such as recharging while at work or while shopping, as it was argued in Chapter 1 slow charging cannot extend the effective range of EVs, and cannot resolve range problems the way en-route rapid charging can.

2.7 Summary

In this chapter we have reviewed several existing approaches in solving variants of the SLCOP problem we defined in Section 1.1. The approaches we presented, generally offer different viewpoints on the typical EV rapid charging problem, with a few considering alternative solutions rather than rapid charging. As it is apparent, some researchers consider the economic aspect of the SLCOP problem, while others focus on power distribution or population coverage. Although a large charging station market is a strong future possibility, very few approaches attempt to put all the necessary pieces together in order to study charging station investor competition, even at an abstract level.

In regards to economic modelling, of interest are the approaches of Jia et al. (2012); Sadeghi-Barzani et al. (2014), who formulate the cost of building charging stations as linear with respect to capacity. This formulation is in accordance with our requirements 6 and 7, that we set in Section 1.3, which require for the building cost and profit to depend on capacity. In general, however, the current literature cannot address our research problem as it was set in Section 1.1. This happens because current literature mostly takes into account static, given traffic flows, which are derived from real-world conventional vehicle data. In itself using real-world data is not restrictive for our purpose. What is restrictive is the fact that these flows do not change when the station network's layout changes. According to the point of view this thesis presents, flows towards charging stations change because of expected queues at stations, something which is reflected in requirements 1,2,3 and 6. In consequence, we cannot apply the solutions that consider static flows or disregard queues to our research problem.

Whereas some papers do consider the routing problem for the EVs, interdependency between EV drivers' and station owners' decisions cannot be derived from those models. In Hess et al. (2012), although routing is done with the goal of minimising travel time, queuing times are not taken into account when vehicles decide their routes, nor when they choose where to charge en route. Similarly, Worley et al. (2012) also do not consider queuing because it is not a significant factor in the goods delivery problem they examine. Their vehicles' routes are also predetermined to a large extent by the deliveries they have to make. Additionally, Lam et al. (2014) consider routing with minimum distance travelled being the measure of driver happiness, a concept which is de facto incompatible with queuing times. Queues are addressed in de Weerdt et al. (2016) where authors examine the general EV routing problem under uncertainty. In addition they are addressed by Xiong et al. (2015) and Xiong et al. (2017) who also consider stochastic behaviour for EV drivers, to accommodate queuing times and traffic congestion. However, drivers are indifferent to prices and there is no competitive aspect for the stations. Nevertheless, the idea of a stochastic model for EV driver route choice is in accordance with requirements 1, 2, 3 and 6 in Section 1.3.

Finally, economic sustainability in the EV charging problem is in general disregarded. Examples that examine competitive pricing are scarce, because the overwhelming majority of literature assumes monopoly in EV charging. One such example of competing stations was the work of Gerding et al. (2013), where they present a competitive market for EV drivers and station owners. However, in their approach they devise a reservation-based model that eliminates the concept of queuing which this thesis perceives as important. A further competitive pricing example was shown by Escudero-Garzás and Seco-Granados (2012), but that model assumed immediate satisfaction of demand which is unreasonable to consider at the current level of technology. In contrast, it is shown in this thesis that the fact that demand cannot be immediately satisfied has a significant effect on charging prices. Hayrapetyan et al. (2007) provide a solution for equilibrium prices in a network pricing game, and although similar to the SLCOP, that problem is simpler and only involves pricing. Nevertheless, it will be useful to build upon it as it poses several assumptions such as customer homogeneity and an alternative substitute service for customers, which are in line with the SLCOP problem as perceived in this thesis.

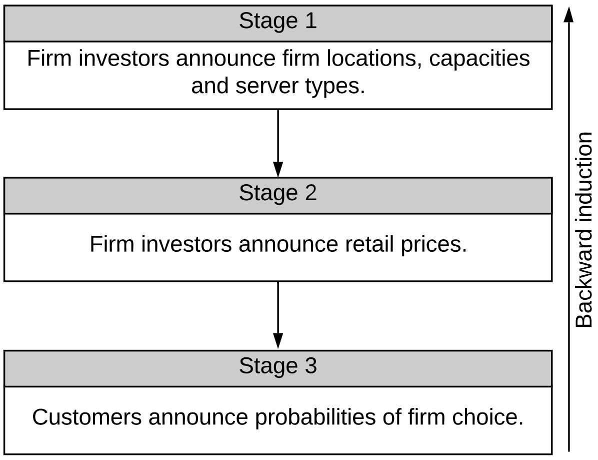
Chapter 3

Model for Firm Competition

This chapter will present and discuss the model this thesis proposes, for addressing the SLCOP (Section 1.1) and similar problems. Before presenting the model, it is necessary to first reflect on how customers and firm investors may interact in the real world. First of all, a customer who is planning on acquiring a service will consider firms which are known to provide the desired service. Secondly, a course of action where investors announce that a firm is open, and then decide on the number of servers watching how many customers ask for the service, or one in which service prices are determined before the magnitude of investment is decided would be rather unusual. A more reasonable course of action for the investor is to first build, or rather establish a firm of a certain non-zero capacity to provide the service, then decide the fee for providing the service, and then accept customers into the firm. This reasoning can be distilled into a *sequence* of actions performed by investors and customers. More specifically, the sequence that will be considered here is that firm investors first decide on their firms' locations, number of servers and speed of service. Then, they decide on prices, and finally customers choose stochastically among firms based on these. To denote the number of servers in each firm, the term 'capacity' will be used from now on. Furthermore, the 'speed of service' is used in the context where each investor can choose among different types of servers that can influence the speed of delivery of the service to the customer, once the customer is first in line.

To model this sequence of actions by investors and customers, a three-stage extensive-form game is used, which is suitable for modelling temporal or sequential actions performed by individuals. At every stage of the extensive-form game, players are able to observe the outcomes of the previous stages and the initialisation. The stages of this extensive form game can be seen in Figure 3.1. Each of the stages is itself defined as normal-form game, that is a game in which all players decide simultaneously. Subgame-perfect equilibrium solutions to the extensive form game are found by backward induction, hence in what follows the stages are presented in the order they are solved. Most formal and intuitive definitions in this chapter follow closely those given by Shoham and Leyton-Brown (2009) and Nisan et al. (2007). Any other source will be stated clearly where appropriate.

Figure 3.1: The three stages of the extensive-form game.



The rest of this chapter is structured as follows. The customers' model for choosing firms is presented in Section 3.1, the investors' price choice model follows in Section 3.2, and the investors' choice of locations, capacities and the speed of service is explained in Section 3.3. More information on the extensive form game, the solution concept and algorithm used to solve the extensive-form game is presented in Section 3.4. Last, Section 3.5 discusses some general theoretical findings, in particular with relation to equilibrium prices and capacities at firms, and Section 3.6 sets metrics for measuring the efficiency of the model.

3.1 Stage 3: Customers Choose Firms

In order to model how customers behave, some context needs to be defined first. It is assumed for the purpose of this thesis that all customers need to buy the same service from firms that offer it. Each firm offers only the particular service under examination, and congestion at firms with many customers means a customer may experience delay in obtaining the service due to having to wait in a queue. Alternatively, the quality of the service itself may degrade due to congestion (e.g. Internet). Nevertheless, the customer cannot switch firms, that is the customer is committed upon choosing a firm, to receive the service from that firm only. Furthermore, customers may have an alternative option, that is an alternative service that can satisfy the customers' needs which is provided by a provider extraneous to the firm investor system under examination. For simplicity, it will be assumed that all customers are alike, that is the cost for different customers to access the same firm is equal, and customers value their time identically.

A very complex customer model would be more difficult to integrate into the already complex competition for the investors that will be presented later on, and this is mainly for two reasons. Firstly, it would increase computational complexity significantly. Secondly, it would introduce too many parameters that would make game-theoretic analysis very difficult and fragile, raising questions on where the valuations for these parameters come from. For these reasons, it is common to make such assumptions in game-theoretic models and this is true even for simpler firm competition models than the one this thesis presents. For example, Escudero-Garzás and Seco-Granados (2012) make similar assumptions on EV driver homogeneity. Hayrapetyan et al. (2007) also assume homogeneous customers in network pricing games, which are simpler competitive models than the model in this thesis, where customers' routes have the same start and destination points. More general firm competition models as in the work of Hotelling (1929), Osborne and Pitchik (1987) and Sattinger (2002) in addition assume homogeneous firms. Some other models, for example Xiong et al. (2017), consider more complex customer behaviour, but in turn simplify by considering firm monopoly, set prices and set locations for firms. It would be prudent to reduce complexity here, rather than to simplify firm competition, as a very realistic customer model would add little to the points this thesis attempts to make. With this context in mind, Section 3.1.1 presents a normal-form game as model for customer choices, Section 3.1.2 discusses Nash equilibrium with focus on the particular model presented, Section 3.1.3 defines customer utility and expected utility, and Section 3.1.4 discusses the outside option for customers. Last, the solution to the customers' equilibrium and boundary conditions are explained in 3.1.5.

3.1.1 Normal-form n-player customers' game

Normal form games are fundamental to strategic interaction in game theory, and are ideal to help pursue the aims of this work straightforwardly. In the case where the states of the world depend solely on the players' combined actions, as is the case here, a game in normal form amounts to a representation of every player's utility for every state of the world.

In this game, a finite number of customers simultaneously choose among several firms. This models a particular time of day when demand for the desired service is high, and congestion at firms can occur. Firms can be at different locations, with different capacities (i.e. number of servers) and prices, and different firms can have different types of servers, affecting the customers' utility. Because this model does not include consideration for traffic on the way to the firm, customers that are indifferent to the modelled service are irrelevant.

Definition 3.1 (Customers' game)**.** More formally, this game is defined as a tuple $\langle N, A, u(\cdot)\rangle$, where N is the finite set of n *customers* (players) ($n \in \mathbb{N}$, $n > 1$). Let $A_i = \{1, \dots, m\}$ be the finite set of m *actions* (firm choices) available to customer i. Then, $A = A_1 \times \dots \times A_n$ is the set of action profiles. Each vector $a = \langle a_1, \dots, a_n\rangle$ of the Cartesian product A is an action profile which contains all the actions a_i played by each customer i in that action profile. Last, $u(\cdot) = \langle u_1(\cdot), \dots, u_n(\cdot)\rangle$ is itself a n-tuple of customer *utilities*, where $u_i(\cdot) : A \mapsto \mathbb{R}$ is a real-valued utility (or payoff) function for customer i.

To promote the discussion on how the customers' behaviour can be evaluated in game-theoretic terms, it is instructive to define some concepts with regard to the choice of firm customers may make.

Definition 3.2 (Pure strategy)**.** We call a strategy of selecting a single action and playing it a pure strategy; a choice of pure strategy for each player is called a pure strategy profile. Consequently, a pure strategy profile in our case is essentially an action profile $a \in A$ as it was defined in Definition 3.1.

The concept of Nash equilibrium (NE) intuitively proposes that in some games there exist strategy profiles, in which no player has incentive to deviate from the chosen strategy. In the case examined here, if customers play only pure strategies, and depending on the specific setting of the game, there may exist some *symmetric* pure strategy profiles which are Nash equilibria, in which all customers choose the same firm. Such a situation can arise, for example, when one firm is very close to customers and the utility for customer i of choosing that firm is better than choosing any other firm, even if everyone else went to the same firm as i. Furthermore, there may exist some pure strategy profiles which are *asymmetric* Nash equilibria, in which customers choose different firms. Depending on the parameters of the game such as number of firms, number of customers, and so on, there may be many symmetric or asymmetric pure strategy equilibria.

Therefore, the potential existence of many pure strategy NE and especially asymmetric ones, raises the question of how the observer determines which equilibrium customers will reach. More importantly, how can customers actually reach a particular pure strategy NE? The answer is that in order for customers to reach a pure strategy Nash equilibrium, they must know all relevant parameters at each firm, including the amount of congestion. However, in order to determine congestion at firms, it must be known what the choices of other customers are. In normal form games players are assumed to choose simultaneously, or at least to not be able to observe the other players' choices. Such a representation was chosen because it reflects a realistic situation for customers, and is in accordance with the problem definition that was provided in Section 1.1. According to that, customers may not have a clear picture[1] on what the utility for choosing a firm may be, because of potential queues/congestion. In order for customers to reach a pure strategy NE in this setting where congestion is uncertain, there must exist some form of indirect coordination. This could be done through using an intelligent central service that collects information on customers and calculates optimal choices, as was the case in Malandrino et al. (2015), but it is arguable whether this is feasible and whether players have incentive to follow the suggestions of the service[2]. To address these issues, employing mixed strategies for customer choices provides a more suitable representation of the research problem in Section 1.1, given also that a customer may not necessarily make the same choice of firm each time the service is needed.

Definition 3.3 (Mixed strategy)**.** We call the strategy of randomising over the set of available actions according to a probability distribution a mixed strategy; a choice of mixed strategy by

[1]While it may be acceptable in simple games with few players (like the prisoner's dilemma) that a player can reason deterministically given known utilities, this is not straightforward at all when the player must decide *and* calculate the utility in a game with many players.

[2]For example Malandrino et al. (2015) assume de facto that drivers will not perceive as better an alternative choice than the one proposed, so drivers will follow the suggestion.

all players is called a mixed strategy profile. Let $\Pi(A_i)$ be the set of all probability distributions over all firm options in A_i. Then the set of mixed strategies for customer i is $S_i = \Pi(A_i)$. $S = S_1 \times \ldots \times S_n$ is called the set of mixed strategy profiles. Each vector $s = \langle s_1, \ldots, s_n \rangle \in S$ is a mixed strategy profile which contains the mixed strategies $s_i = \{s_i^1, \ldots, s_i^m\}$ played by each player in that mixed strategy profile. $s_i^j \in [0, 1]$ denotes the probability that customer i chooses firm $j \in A_i$. At the same time, all the probabilities over firm choices in player i's mixed strategy s_i must add up to 1. That is: $\forall i : \sum_{j \in A_i} s_i^j = 1$.

Utilising mixed strategies is based on the idea that customers may be able to gain some information through experience. Moreover, while an infallible central service that collects data from all customers may not be feasible, it is not unrealistic to consider that a device or other service (e.g. TomTom or Google) may be able to provide information on firms. This information can include capacities, locations, service times and prices. Such a device may also be able to provide information on *expected congestion* at firms, or propose firm choices based on queue expectancy that may not necessarily be optimal for the customer. Utilising such information, customers may then be able to make stochastic decisions on where to receive the desired service from, randomising their choices each time based on the *expected utility* for choosing a firm. Before discussingthe utility and expected utility for choosing a firm, however, it is instructive to first discuss about mixed strategy Nash equilibria in this game.

3.1.2 Mixed strategy Nash equilibrium

Mixed strategy Nash equilibria, like the pure strategy equilibria that were analysed earlier, may also be symmetric or asymmetric. In symmetric mixed NE, customers choose from available firms with the same probability distribution, whereas in asymmetric mixed NE they may choose with different distributions. Calculating all asymmetric mixed equilibria, however, is problematic with respect to their use. This is so because the intended use is to calculate customer flows to firms that can later be utilised to find equilibria in firm investor choices. The potential existence of many equilibria in firm investor choices for just one mixed strategy equilibrium of the customers raises the question of which mixed NE will be reached by the customers. Using asymmetric mixed NE may therefore result in over complicating the model computationally and will make extracting theoretical deductions on firm competition difficult. Of more use for the purpose of this thesis are symmetric mixed NE, which offer closed form solutions that can be calculated in a straightforward way, and which can be approximated in polynomial time if need be (Daskalakis and Papadimitriou, 2007). Therefore, to guarantee a symmetric mixed NE and extract theoretical properties in a general setting that can encompass various problems, the assumption that all customers are identical was made to make this game symmetric for customers.

It is worth noting at this point that this will not affect the qualitative properties of firm competition and the analysis this thesis will make. Furthermore, although symmetry is assumed to promote theoretical and qualitative analysis, it is not a necessity when applying the model quantitatively on specific problems. For example, one could utilise the concept of *anonymous games* to still guarantee a symmetric mixed NE for customers in a problem-specific application. This would allow for some customer heterogeneity in a way that customers still only care about how many go where, rather than who goes where (Daskalakis and Papadimitriou, 2007). In the case of EV drivers, this could mean that drivers can have different starting and destination points, but still need to recharge by the same amount and have the same type of vehicle to guarantee a symmetric equilibrium. Alternatively, one could consider true customer heterogeneity in all aspects, and determine asymmetric mixed NE for more applied use of the model.

As was mentioned, customer i will play a mixed strategy s_i in mixed strategy profile s. Given that i is uncertain about what other customers might do, the choice of mixed strategy s_i will be reliant on the expected utility for i of playing in mixed strategy profile s. If $u_i(\cdot)$ is the utility for i of playing in action profile $a \in A$, then the expected utility for i of playing in s, given that customers will choose firms with the probability distributions in s is $\mathbb{E}[u_i(\cdot)|s]$.

Definition 3.4 (Mixed strategy Nash equilibrium)**.** In a mixed strategy profile $s = (s_i, s_{-i})$, let s_i be a strategy of customer i and s_{-i} be a strategy of all customers except for i. A strategy profile $s^* = \left(s_i^*, s_{-i}^*\right) \in S$ is a *Nash equilibrium*, if no unilateral deviation in strategy by any single customer is profitable for that customer. Let $s' = \left(s_i', s_{-i}^*\right)$ be a mixed strategy profile, in which all other customers except for i play the same strategy as in s^*, but customer i plays a different strategy s_i' than the one he/she plays in s^*. Then s^* is a Nash equilibrium if:

$$\forall i, s_i \in S_i : \mathbb{E}[u_i(\cdot)|s^*] \geq \mathbb{E}[u_i(\cdot)|\left(s_i', s_{-i}^*\right)]$$

Having established the existence of a symmetric mixed strategy Nash equilibrium for the customers' game, in the next section the utility and expected utility of firm choice, as well as the expected utility of playing in a mixed strategy will be discussed and defined more explicitly.

3.1.3 Utility and expected utility of firm choice

It is now time to introduce the utility function of a customer. Given that customers choose firms simultaneously, it can be deduced that customers do not know at which place of a firm's queue they will arrive, or alternatively how much congestion they will encounter. Therefore, it is reasonable to assume that a customer can arrive at any place in the queue with the same probability. Under this assumption, customer i that chooses firm j will experience an average delay due to congestion:

$$Q(x) = \frac{\sum_{\kappa=0}^{x} \left\lfloor \frac{\kappa}{c_j} \right\rfloor}{x+1} R_j \tag{3.1}$$

where x is the number of other customers that choose firm j, $c_j \in \mathbb{N}^+$ is the number of servers (capacity) at firm j and $R_j \in \mathbb{R}^+$ is the time it takes to serve a single customer at that firm. However, this does not provide a closed form solution that we can later use in investor competition. The floor function in Equation (3.1) can be also written as $\lfloor \frac{\kappa}{c_j} \rfloor = \frac{\kappa}{c_j} - \{\frac{\kappa}{c_j}\}$, where $\{\frac{\kappa}{c_j}\}$

is the fractional part of $\frac{\kappa}{c_j}$. Queuing time can then be approximated by discarding the fractional part $\{\frac{\kappa}{c_j}\}$ which results in:

$$Q(x) = \frac{x}{2c_j} R_j \tag{3.2}$$

This produces a slight overestimate in queuing time, especially when capacity at firms is small. It is a slight deviation from true queuing time, but it will retain the customers' behaviour when choosing firms.

As was mentioned in Section 1.1, customers should be able to make trade-offs between monetary costs and other costs. To achieve this it will be useful, if possible, to consider all costs in the same scale. This work examines the particular case where other costs are paid in time, and to that end the *value of time* parameter is utilised. The value of time represents how much money an individual is willing to pay in order to save a given amount of time. Multiplying time costs with this value thus gives a monetary evaluation of the timme costs for the customer. Government agencies that study, for example, transportation problems determine values of time for different types of individuals and modes of transport. Calculating the value of time is a complicated issue in econometrics and takes into account multiple factors and sources of data (Department for Transport, 2015), thus is beyond the scope of this thesis. If non-monetary costs represent some other quantity, this can represent the value of that quantity for the customer.

In addition to the fee for the service $f_j \in \mathbb{R}^+$ to be paid at firm j, the customer may also incur a cost $t_j \in \mathbb{R}^+$ to *access* the firm. This can, for example, represent travel time to reach the firm, or access costs such as a membership fee, line installation costs for a new Internet line and so on, or even combinations of such congestion-independent costs. Moreover, the customer will also incur cost $Q(x)$ due to congestion at the firm. This, as defined in Equation (3.2), depends on how many other customers chose the same firm. Finally, given identical customers, a further cost R_j may be incurred when customers queue up at firms to receive a service, which is the time customer i needs to get served once first in line. Assuming all these costs, apart from f_j, are time costs, they need to be multiplied by a value of time $v_d \in \mathbb{R}^+$. Given these, the utility for customer i of choosing firm j, given that x other customers choose the same firm is defined as:

$$u_i^j(x) = -v_d(t_j + \frac{x}{2c_j} R_j + R_j) - f_j \tag{3.3}$$

Note that the goal of customers is to minimise costs. For convenience, all costs have been defined as negative in order to define a utility function which should now be maximised.

In this game it has been assumed that identical customers make stochastic decisions and this results in the existence of a symmetric mixed strategy equilibrium. To determine this equilibrium, it is necessary to determine the *expected utility of choosing a firm*, given that customer i is uncertain what other customers might do. This expected utility is reliant on the potential unordered combinations of customers' actions, and specifically on the potential combinations of

$n-1$ customers choosing the same firm j as customer i with probability s_{-i}^{j}, or not choosing that firm with probability $1-s_{-i}^{j}$. From equation (3.3), the utility for customer i of choosing firm j given the other players' actions is already known. After trivial binomial transformations, the expected utility for player i of choosing firm j, given that the other customers will play mixed strategy s_{-i} is:

$$\mathbb{E}[u_i^j\left(x\right)|s_{-i}] = -v_d\left(t_j + \frac{s_{-i}^j\left(n-1\right)}{2c_j}R_j + R_j\right) - f_j \tag{3.4}$$

Now that utility and expected utility of choosing a firm have been defined, it is time to define the expected utility for customer i of playing in mixed strategy profile s.

Definition 3.5 (Expected utility of mixed strategy)**.** In order to determine the expected utility for customer i of playing mixed strategy s_i in mixed strategy profile s, the probability of reaching each outcome given the strategy profile must be calculated. Then, the expected utility is the weighed arithmetic mean of the payoffs of all outcomes, where each outcome is weighed by its probability. Given the customers' game that has been defined, the expected utility $\mathbb{E}[u_i\left(x\right)|s]$ for player i, of playing in mixed strategy profile $s=\langle s_1,\ldots,s_n\rangle$ is:

$$\mathbb{E}[u_i\left(x\right)|s] = \sum_{j=1}^{m} s_i^j \mathbb{E}[u_i^j\left(x\right)|s_{-i}] \tag{3.5}$$

where $\mathbb{E}[u_i^j\left(x\right)|s_{-i}]$ is the expected utility for choosing a firm in equation (3.4).

3.1.4 Introducing an outside option for customers

So far it has been assumed that customers have no other option than to buy the service from one of the firms that offer it. While this will allow for investor competition, it will not put competition into perspective with reality. This may result in investors choosing arbitrarily high prices and low capacities when the number of investors is small enough to allow for big profit margins. That is investors may compete by undercutting prices to expand their customer base rather than by offering better service. To control this behaviour and to promote a selection of more realistic prices and capacities by investors, an outside option is introduced for the customers. This outside option can be considered as an alternative means of acquiring the service, or an alternative service altogether. The customers' model remains conceptually the same as already explained so far, only now customers can also opt to use the outside option. This enables to examine scenarios in which an uncertain portion of demand may be satisfied, and according to Deneckere and Peck (1995) the inclusion of this possibility is expected to enhance the existence of pure strategy equilibria in capacities for investors.

The game's definition in Section 3.1.1 remains unchanged. The outside option is considered as the m^{th} action available to customers in their action set A_i. The utility for customer i of using the outside option, when x other customers also make the same choice is defined as:

$$u_i^m(x) = -v_m \cdot t_m - x \cdot D - f_m \tag{3.6}$$

where $v_m \in \mathbb{R}^+$ is the value of time for using the outside option, $t_m \in \mathbb{R}^+$ is the cost to access the outside option, and $f_m \in \mathbb{R}$ is the monetary cost of using the outside option. Note that the value of time, depending on what the outside option exactly represents, may not necessarily be the same as the value of time for the service firms offer. For example a train passenger would value time differently than a driver, and a passenger in an EV would also value time differently than the person driving it (Department for Transport, 2015). As before, the value of time can instead represent the value of any other quantity costs may represent.

The parameter D is used to express the customer's disappointment at not using the desired service. The logic is that customers expect services offered by firms to be usable, and would rather not substitute the service with an alternative. Therefore, a further cost in disappointment is incurred upon substitution of the service by the outside option. This disappointment becomes even greater with an increasing number of other customers who also have to use the outside option. Therefore, if we treat D as a parameter to calibrate the model, we can select a value which will yield a desired or reasonable satisfaction of demand by the firms, depending on the situation modelled. This will be explained in more detail with a more specific example in Section 5.2.1. The expected utility for customer i of using the outside option given that an uncertain number of other customers will make the same choice is calculated using the same logic as for choosing a firm in equation (3.4). Given that the outside option is the m^{th} option available to customer i, this makes the expected utility for customer i of choosing action j:

$$\mathbb{E}[u_i^j(x)\,|s_{-i}] = \begin{cases} -v_d\left(t_j + \dfrac{s_{-i}^j(n-1)}{2c_j}R_j + R_j\right) - f_j & ,j \leq m-1 \\ -v_m t_m - s_{-i}^m(n-1)\,D - f_m & ,j = m \end{cases} \tag{3.7}$$

3.1.5 Equilibrium solution and boundary conditions

The goal of this approach is to find a closed-form solution for the symmetric mixed Nash equilibrium in customers' game. According to Definition 3.3 in Section 3.1.2, in mixed Nash equilibrium customers must have no incentive to deviate from the chosen mixed strategy. Intuitively, in order for this to be true, it must be that for customer i, choosing an action j yields the same expected utility as choosing any other action. In addition, because the equilibrium is symmetric, not only all customers other than i choose action j with the same probability s_{-i}^j, but customer i also chooses that action with the same probability as the other customers; that is $s_i^j = s_{-i}^j$. Consequently, in order to determine the mixed strategy Nash equilibrium s^*, the mixed strategy s_i^* which customer i will employ in equilibrium must be calculated. This amounts to solving the following $m \times m$ system of linear equations:

$$\begin{aligned} \mathbb{E}[u_i^1(x)\,|s_{-i}^*] &= \mathbb{E}[u_i^2(x)\,|s_{-i}^*] \\ &\dots \\ \mathbb{E}[u_i^{m-1}(x)\,|s_{-i}^*] &= \mathbb{E}[u_i^m(x)\,|s_{-i}^*] \\ s_{-i}^{*j} + \ldots + s_{-i}^{*m} &= 1 \end{aligned} \tag{3.8}$$

Of course, the solution is subject to boundary conditions for the probabilities. In order to obtain a more clear picture on what these boundary conditions mean for the customer, it is instructive to solve a simple instantiation of the customers' game with two firms and no outside option; firm 1 and firm 2. How the customers' equilibrium is found is shown analytically in Appendix A.1. Assuming service time R_j is the same at both firms for simplicity, the customers' equilibrium probabilities of firm choice in this case are:

$$\begin{aligned} s_i^{1*} &= \frac{c_1 v_d R(n-1) + 2c_1c_2v_d(t_2-t_1) + 2c_1c_2(f_2-f_1)}{v_d(n-1)(c_1+c_2)R} \\ s_i^{2*} &= \frac{c_2 v_d R(n-1) + 2c_1c_2v_d(t_1-t_2) + 2c_1c_2(f_1-f_2)}{v_d(n-1)(c_1+c_2)R} \\ & \qquad\qquad\qquad\qquad v_d, c_1, c_2 > 0 \quad n > 1 \end{aligned} \tag{3.9}$$

Let us now explore the boundaries for these probabilities. First, in order to have $s_i^{1*} < 0$ since the denominator of s_i^{1*} in equation (3.9) is always positive, it must be:

$$\begin{aligned} s_i^{1*} &< 0 \Leftrightarrow \\ c_1 v_d R(n-1) + 2c_1c_2v_d(t_2-t_1) + 2c_1c_2(f_2-f_1) &< 0 \Leftrightarrow \\ v_d\frac{(n-1)}{2c_2}R + v_dt_2 - v_dt_1 + f_2 - f_1 &< 0 \Leftrightarrow \\ -v_dt_1 - f_1 &< -v_dt_2 - v_d\frac{(n-1)}{2c_2}R - f_2 \Leftrightarrow \\ -v_dt_1 - v_dR - f_1 &< -v_dt_2 - v_d\frac{(n-1)}{2c_2}R - v_dR - f_2 \Leftrightarrow \\ -v_d(t_1+R) - f_1 &< -v_d\left(t_2 + \frac{n-1}{2c_2}R + R\right) - f_2 \Leftrightarrow \\ \textit{Replacing from eq. } (3.3) \quad u_i^1(0) &< u_i^2(n-1). \end{aligned}$$

This means that $s_i^{1*} < 0$ when the utility for customer i of going to firm 1 with no congestion, is smaller than the utility of going to firm 2, even if all $n-1$ other customers went to 2 as well. In that case, i will simply play the pure strategy of going to firm 2. For larger numbers of firms, the result is that the utility for going to the firm with negative probability is smaller than the utility of visiting all other firms together, even if $n-1$ customers also visited all the other firms. For example for three firms where firm 1 yields a negative probability, it is $u_i^1(0) <$

$u_i^2(n-1) + u_i^3(n-1)$. In that case, the firm whose probability is negative, is conceptually out of the competition and there is no way a customer would choose it. The probability of choosing such a firm should then be set to zero and the equilibrium must be recalculated. Now when a probability is greater than 1, this simply means that the probability for some other action or actions is negative. More information on how this is actually handled can be found in Section 3.4.2.

3.2 Stage 2: Investors Choose Prices

This section will introduce the crucial aspect of firm investor pricing competition, as the second stage of the extensive form game (seen in Figure 3.1 at the start of this chapter). In order to delve into the model, some context needs to be defined first. This stage assumes that investors have already played the first stage of the extensive form game. Furthermore, each investor may own multiple firms that offer the same service, but can own at most one firm at each available location. It is further assumed that peak congestion at firms can occur a given number of times per day and that when peak traffic arrives at the firms, queues are empty. Last, in the previous section j was used to denote one of the options available to customers. To avoid more confusing notation, j will be used here to denote a firm an investor may own in a particular location. Given these, the formal model for the investors' price choices follows in Section 3.2.1, the utility and expected utility for firms and investors is defined in Section 3.2.2, and the pure strategy Nash equilibrium in prices is discussed in Section 3.2.3.

3.2.1 Normal-form z-player investors' price game

The firm investors' price choice is modelled as a normal-form subgame, in which a finite number of firm investors compete with each other in order to maximise their individual net profit by simultaneously selecting prices for their firms. They do so given that out of n customers, an uncertain portion will visit each firm, and given that investors have already chosen locations, capacities and the speed of service at their firms.

Definition 3.6. This subgame is defined as a tuple $\langle I, F, r(\cdot)\rangle$, where I is the finite set of z *firm investors*. Let $L = \{l_1, \cdots, l_\mu\}$ be the finite set of locations available to investors and $F_k^0 = (-\infty, +\infty)$ be the infinite set of price options available to investor k. Then, $F_k = (l_1 \times F_k^0) \times (l_2 \times F_k^0) \times \cdots \times (l_\mu \times F_k^0)$ is the set of *actions*[3] available to investor k. $F = F_1 \times \ldots \times F_z$ is the set of pure strategy profiles and $f = \langle f_L^1, \ldots, f_L^z\rangle \in F$ is a pure strategy profile. Thus pure strategy f_L^k contains the prices f_j^k investor k chose for each location $j \in L$ in pure strategy profile f. Finally, $r(\cdot) = \langle r^1(\cdot), \ldots, r^z(\cdot)\rangle$ is the z-tuple of *utilities* for the investors.

It can be argued at this stage that maybe mixed strategies should be considered, because investors in reality may adjust their firms' prices each day. However, these will not offer much insight

[3]Investor k will choose from an infinite amount of real-valued price options for each of the firms k owns.

with regard to firm competition. In addition, whereas adjusting prices may be a reality, doing so randomly does not make much sense for the following two reasons. The fact that investment levels have already been decided in the first stage of the extensive form game indicates that an investor has a clear picture of the costs that need to be overcome. Moreover, investors would be able to observe competitors' prices in the previous day easily, given that the number of direct competitors is rather unlikely to be large enough to render observation difficult or partial.

Having defined the game for investor pricing competition, the next section discusses firm and investor utility and expected utility.

3.2.2 Firm and investor utility and expected utility

A firm's utility is introduced as the net profit of the firm. That is the normalised earnings minus the costs. If x customers go to firm j that investor k owns, the utility for firm j is defined as:

$$r_j^k(f) = x(f_j^k - h_j^k) \cdot w - b_j^k c_j^k - o_j^k \tag{3.10}$$

where h_j^k is the cost for the firm to serve each customer, b_j^k is the cost of one server, and c_j^k (capacity) is the number of servers at the firm. In order to prevent investors from arbitrarily building firms, the parameter o_j^k is utilised as an one-time building cost for firm j. The parameter w is used to normalise earnings for a given time frame (e.g. 1 year), assuming peak congestion can occur a given number of times in a day and that the firm's customer base consists of peak traffic and some traffic throughout the rest of the day. Note that further costs can be integrated into firm utility. For example, if there are other daily costs, these have to be subtracted from utility and weighed by another normalisation parameter that reflects the correct time-frame. For example if the model is to be run for a time-frame of 1 year, where each day peak traffic occurs 6 times, then $w = 365 * 6$, and the weight for daily costs should be 365. Maintenance costs can be considered this way, or they can be integrated directly into the cost of building a server b_j^k for the whole year or time-frame under examination.

Given that n customers are going to select firms in mixed strategy profile s, the expected traffic flow toward firm j of investor k then is $s_i^{jk} n$ which is, in fact, a function of the prices of all m options available to customers. Then the expected utility for firm j of setting price f_j^k in pure strategy profile f, given that customers are going to choose in mixed strategy s, is:

$$\mathbb{E}[r_j^k(f)|s] = \begin{cases} s_i^{jk}(f) \cdot n \cdot (f_j^k - h_j^k)w - b_j^k c_j^k - o_j^k & , c_j^k > 0 \\ 0 & , c_j^k = 0 \end{cases} \tag{3.11}$$

Note that when $c_j^k = 0$, $s_i^{jk}(f)$ is not defined as was seen in Equation (3.9). In that case, expected utility is explicitly set to 0 to reflect a state where the firm is not open therefore there is no profit or loss. Then, the expected utility for investor k of playing in pure strategy profile

f is the sum of the expected utilities of all potential firms investor k can own across available locations.

$$\mathbb{E}[r^k(f)|s] = \sum_{j \in L} \mathbb{E}[r_j^k(f)|s] \tag{3.12}$$

3.2.3 Pure strategy Nash equilibrium in prices

In this game, as was explained in Section 3.2.1, pure strategies will be considered. Furthermore, pure strategy Nash equilibria will be useful in finding deterministic solutions investors can actually use[4] and understand, and thus have incentive to follow.

Definition 3.7 (Pure strategy NE in prices)**.** In a pure strategy profile $f = (f_L^k, f_L^{-k})$, let f_L^k be a strategy of investor k and f_L^{-k} be a strategy of all players except for k. A strategy profile $f^* = (f_L^{k*}, f_L^{-k*}) \in F$ is a *Nash equilibrium*, if no unilateral deviation in strategy by any single player is profitable for that player. Let $f' = (f_L^{k\prime}, f_L^{-k*})$ be a pure strategy profile, in which all other investors except for k play the same strategy as in f^*, but player k plays a different strategy $f_L^{k\prime}$ than the one played in f^*. Then f^* is a Nash equilibrium if:

$$\forall k, f_L^k \in F_k : \; \mathbb{E}[r^k((f_L^{k*}, f_L^{-k*}))|s] \geq \mathbb{E}[r^k((f_L^{k\prime}, f_L^{-k*}))|s]$$

In pricing competition as it has been defined here, it is expected that pure strategy Nash equilibria do exist. Whereas the set of an investor's price options been defined as $(-\infty, +\infty)$, it is not difficult to imagine that there can exist two numbers P_1, P_2 far enough apart so that (P_1, P_2) includes all realistic price options investors might consider[5]. In such a case where the action set is infinitely large but compact, Glicksberg (1952) notes that pure strategy equilibria are guaranteed to exist if additionally the utility function is continuous. Conceptually, negative prices would mean that an investor would pay customers to visit the firm. Such a strategy, considering the bigger picture of the extensive form game, is always dominated by the strategy of not opening the firm at all in the first place, regardless of how much the investor pays customers.

Now let us discuss how the pure strategy NE in prices is found. Utilising the concept of backward induction to solve the three stages shown in Figure 3.1, the first step toward finding equilibrium prices is to determine the equilibrium probabilities customers choose firms with. According to Definition 3.7 that was given previously, investors should not have incentive to deviate from the equilibrium strategy. More intuitively, this means that in equilibrium each investor's price choices are a best response to the other investors' best responses; that is each investor's price choices provide the maximum utility given the other investors' prices (Fudenberg and Tirole, 1991). This translates to solving the following $z\mu \times z\mu$ linear system:

$$\forall k \in I : \; \forall j \in L : \; \frac{\partial \mathbb{E}[r^k(f^*)|s^*]}{\partial f_j^{k*}} = 0 \tag{3.13}$$

[4]For example the model can be rerun every day to adjust prices taking new information into account.

[5]i.e. price options that are not infinitely far away from the equilibrium price

3.3 Stage 1: Investors Choose Location, Capacity and Service Speed

This section explains the first stage of the extensive-form game in Figure 3.1. This models the situation in which several investors are called to decide simultaneously on their firms' locations, capacities and speed of service. By speed of service, it is meant that the investor makes a choice other than the number of servers (capacity), which will influence the time needed to deliver the service to the customer. In order to consider these choices, it is necessary to determine how an investor would reason about building a firm. First of all, an investor would consider building one or multiple firms at locations that have potential for economic profit. Given a set of locations for consideration (or other options that influence the cost to access the firm), the investor would reason on building a number of firms in the available locations, choosing locations, capacities and the speed of service in an effort to maximise net profit.

At the same time, these three choices of location, capacity and speed of service are considered in the same stage, as these are related to each other more directly in problems such as the SLCOP that was explained in Section 1.1. The assumptions on investors are the same as in Section 3.2, with the addition of the assumption that an investor will choose the same speed of service across all the owned firms. This is done to keep the problem more tractable computationally, and is not expected to affect the findings of this work. Last, the term 'location' can obviously refer to the actual physical location of a firm, which will determine the cost for customers to access the firm and the cost for the investor to establish the firm at that location. However, conceptually it can be used to represent other decisions related to the access cost to the firm or the cost of building a firm, when the customer is indifferent to where the firm is physically located. For example, when choosing an Internet provider the customer rarely is interested on where the providers' physical headquarters are. More interesting information may include the cost to obtain an Internet line, and perhaps the distance to the providers' multiplexers (DSLAMs). When offering online applications, a choice of host by the firm may be an important factor that may influence the customer's experience, as well as the cost of providing the application for the firm, and so on.

With these in mind, the game where investors choose locations, capacities and the speed of service is defined in Section 3.3.1. Utility, expected utility and Nash equilibrium are then discussed in Section 3.3.2.

3.3.1 Normal-form z-player firm investors' location, capacity and speed of service game

As in the price choice game (Section 3.2.1), a normal-form game is utilised to model the strategic interaction among a finite number of investors who simultaneously choose locations, capacities and the speed of service for their firms. Each investor can have at most one firm in each location

Definition 3.8 (Investors' location, capacity and speed of service game)**.** This sub-game is defined as a tuple $\langle I, C, r(\cdot)\rangle$, where I is the set of z investors. Let $C_k^0 = [0, \Theta] \subsetneq \mathbb{N}$ be the finite set of capacity choices available to investor k and G be the finite set of options that can influence the speed of delivering the service under consideration. If $L = \{l_1, \cdots, l_\mu\}$ is the finite set of locations available to investors, then $C_k = ((l_1 \times C_k^0) \times (l_2 \times C_k^0) \times \cdots \times (l_\mu \times C_k^0)) \times G$ is the set of *actions*[6] available to investor k. Then, $C = C_1 \times \ldots \times C_z$ is the set of pure strategy profiles and each vector $c = \langle c_L^1, \ldots, c_L^z\rangle \in C$ is a pure strategy profile. So pure strategy c_L^k contains the capacities c_j^k investor k chose for each location $j \in L$, and the speed of service choice g^k in pure strategy profile c. Finally, $r(\cdot) = \langle r^1(\cdot), \ldots, r^z(\cdot)\rangle$ is the z-tuple of investor *utilities*.

In this game, pure strategies will be assumed. The potential existence of many symmetric or asymmetric pure strategy NE in this game is something that will offer better insight into firm competition. In the customers' game, in Section 3.1, it was argued that customers have difficulty reasoning about pure strategies because although they may have information on firms, they cannot have absolute and accurate information on congestion at firms. Consequently, they may randomise their choices each day. It can be argued that the situation for investors is not much different in the sense that station investors also cannot observe the other players' actions, since they play simultaneously. However, in reality an investor's behaviour is expected to show long-term commitment, especially when it comes to deciding capacities which in many cases may dictate the magnitude of the investment to a large extent. Moreover, investors should have a clear picture on how much they are willing to invest given some expectation for traffic. Because of these reasons, pure strategies are a good representation of investor behaviour that can be used to extract more concrete results on firm competition.

Having established that firm investors play pure strategies, it is time to discuss the utility and expected utility of firms and investors in the next section.

3.3.2 Utility, expected utility and Nash equilibrium

A firm's utility and expected utility are the same as in Section 3.2.2, only now conceptually they are considered functions of pure strategy profile c instead of f. Therefore, the expected utility of firm j investor k owns is:

$$\mathbb{E}[r_j^k(c)|s] = \begin{cases} s_i^{jk}(c) \cdot n \cdot (f_j^k - h_j^k)w - b_j^k c_j^k - o_j^k & , c_j^k > 0 \\ 0 & , c_j^k = 0 \end{cases} \tag{3.14}$$

where the cost of serving a customer h_j^k and the cost of adding a server b_j^k may or may not depend on the chosen location, and the cost of establishing the firm o_j^k is directly associated

[6] the investor will choose a capacity in C_k^0 for each of the μ locations and a speed of service to use across all his/her firms.

with the chosen location. The expected utility of investor k then is:

$$\mathbb{E}[r^k(c)|s] = \sum_{j\in L} \mathbb{E}[r^k_j(c)|s] \tag{3.15}$$

The pure NE in locations, capacities, and speed of service is defined similarly as in Definition 3.7 for prices, only now investors play in pure strategy profile c, and utilities are considered functions of c. Pure strategy Nash equilibria do not always exist in finite non-cooperative games. It is worth noting at this point that nonexistence has not been encountered in comprehensive explorations of the full model. Specific cases of non-existence in reduced versions of the model will be discussed in Section 3.5.2. The pure strategy NE in locations, capacities and speed of service is solved with an algorithmic approach, and this together with a discussion on the solution concepts used is discussed in the following section.

3.4 Solving the Model

This section explains how the three normal form games presented in this chapter are combined into an extensive form game, and how solutions to this extensive form game are found. Toward this, Section 3.4.1 discusses the extensive form game and the concept of subgame-perfect equilibrium and Section 3.4.2 presents the solution concept and algorithm for solving the extensive form game.

3.4.1 Subgame-perfect equilibrium (SPE)

Extensive form games are finite representations that, in contrast to normal form games, do not mandate players to be acting simultaneously. As was mentioned in the beginning of this chapter, the model presented here considers three stages; first, investors decide on their firms' locations, capacities and speed of service. Then investors decide on prices at their firms. Last, customers choose among firms. These three stages were formalised into three separate normal-form games in which players within the same game act simultaneously. Now, these games will be considered as subgames of a higher level extensive-form game in which the firm investors' location, capacity and speed of service game (Section 3.3) takes place first, the investors' price choice game (Section 3.2) takes place second given the outcome of the previous game, and the customers' firm choice game (Section 3.1) takes place last, given the outcomes of the previous two. It is also assumed that players at each stage are able to observe the initialisation of the first stage.

Because customers in the third stage are able to observe the outcomes of the first and second stages, investors in the second stage are able to observe the outcome of the first stage, and at all stages players can observe the initialisation of the first stage, players in this extensive form game have *perfect information*. This enables using the concept of subgame-perfect equilibrium (SPE) to obtain a solution. This makes solving the model more efficient computationally, since rather than computing all the possible permutations of strategies by investors and customers, choices in previous stages are taken for granted. The concept of SPE intuitively proposes that when players have perfect information, the Nash equilibria of the extensive form game consist of the individual Nash equilibria of its subgames. More specifically, if investors choose locations, capacities and speed of service in pure strategy NE c^* and choose prices in NE f^* given c^*, and customers choose firms with a mixed strategy NE s^* given c^* and f^*, then this combination of c^* and f^* investor choices, and s^* customer choices is a subgame-perfect equilibrium of the extensive form game. In the next section, the solution concept for solving SPE in this game, as well the algorithm used are explained.

3.4.2 Solution concept and algorithm

A common solution concept used in solving subgame-perfect equilibria is backward induction. Backward induction follows a bottom-to-top approach in solving the model, that is in this particular instance the customers' game is solved first. The result is then assumed to solve the investors' NE in price choices, and last both these results are assumed in solving the investors' NE in locations, capacities and speed of service. This concept is implemented in Algorithm 1 seen on the next page, which combines analytical and algorithmic techniques and will now be explained.

As discussed in Section 3.1.5, calculating the symmetric mixed NE for the customers is straightforward. This is done analytically (line 8, Algorithm 1) by obtaining a closed-form solution for equilibrium probabilities. This requires solving a $m \times m$ system of linear equations, which is easy to solve symbolically[7] for probabilities.

Having obtained the closed-form solution for customers' probabilities the next step is to find the pure strategy NE in prices (line 9, Algorithm 1). To do this, the customers' probabilities are replaced in firms' expected utilities (equation (3.11)), and then the system in equation (3.13) is solved again symbolically. The pure NE in prices, although linear with respect to firms' prices, is more difficult to compute symbolically. That is because it is non-linear with respect to capacities. More on the complexity of the NE in prices will follow in Section 5.1.1. It is worth noting at this point that the investor's expected utility of equation (3.12) is now governed by the same conditions as the customers' equilibrium is, due to substitution. This means that it is not defined for a capacity of zero (i.e. not opening a firm in that location), and thus the firm's utility for zero capacity has to be explicitly set to 0. In the case where all firms have zero capacity and there is no outside option for customers, the utility for investors is explicitly set to $-\infty$, to ensure that there is at least one firm open to serve the customers.

Next, the pure strategy NE in locations, capacities and speed of service is solved and subgame-perfect equilibria are obtained. Pure strategy NE are generally complex to compute exhaustively.

[7] i.e. by not replacing any parameters in the customer's utility and using a symbolic solver such as Matlab's solve() function.

Algorithm 1 Algorithm for obtaining SPE of the extensive-form game. An Iterated best response (IBR) is employed in solving the investors' location, capacity and speed of service game.

1: **procedure** FIND SPEs
2: $K \leftarrow$ Repeats threshold
3: $X \leftarrow \{\}$ ▷ Set of all SPEs found
4: $I \leftarrow$ set of z investors
5: $\Theta \leftarrow$ set of μ locations
6: $G \leftarrow$ set of charger type options
7: $T \leftarrow$ capacity limit
8: $s^* \leftarrow$ solve system (3.8) symbolically
9: $f^* \leftarrow$ solve system (3.13) symbolically given s^*
10: **for** *threshold*$= 1 \rightarrow K$ **do**
11: $currentCapacityState \leftarrow$ *randomise* $\{c_1^1, \ldots, c_\mu^1, \ldots, c_1^z, \ldots, c_\mu^z\}$
12: $currentSpeedState \leftarrow$ *randomise* $\{g^1, \ldots, g^k\}$
13: $O \leftarrow$ *shuffle*(I) ▷ Randomise investor order
14: **repeat**
15: *previousCapacityState* $\leftarrow$ *currentCapacityState*
16: *previousChargerState* $\leftarrow$ *currentChargerState*
17: **for** $k = 1 \rightarrow z$ **do**
18: $player \leftarrow O(k)$
19: *currentCapacityState, currentChargerState* $\leftarrow$

$$\underset{c_L^k \in C_k}{\arg\max}\, \mathbb{E}[r^k(c_L^k, c_L^{-k}) | s^*, f^*]$$

20: **end for**
21: **until** *previousCapacityState*=*currentCapacityState* **and** *previousChargerState*=*currentChargerState* ▷ SPE found
22: $c^* \leftarrow$ *(currentCapacityState, currentChargerState)*
23: $X \leftarrow X \cup \{\{s^*, f^*, c^*\}\}$
24: **end for**
25: **end procedure**

A classic method of locating pure NE is using utility matrices, and checking for every strategy profile whether any investor can deviate. This is a simple procedure that, however, can be computationally intensive. In this particular case, we have z investors each of whom has $\Theta + 1$ capacity options for each of the μ available locations. In addition there are ψ service speed options available to each investor. Meaning each investor's utility function is a $(\Theta + 1)^{\mu z} \psi^z$ table. Going through each of the strategy profiles and checking whether any investor can deviate requires $O(z^2 (\Theta+1)^{\mu z} \psi^z)$ time. While this approach may be adequate for very small instances of the investors' game, a better approach for larger problems is to utilise an Iterated Best Response (IBR) algorithm. Instead of calculating all the utilities, investors are initialised in a random state and take turns playing their best strategy given the other players' strategies. This way, in each iteration of the algorithm it is only necessary to calculate the utilities of the current player for a given state the other players are in, and this reduces computational time significantly.

The IBR algorithm (lines 11-23, Algorithm 1) will now be explained. First, a random capacity for each location, and a random speed of service are initialised for each investor (lines 11 and 12). Then, investors participate in rounds deciding their capacities and speed of service one after the other in a random order. After the playing order has been randomised (line 13), rounds are repeated (line 14). A round consists of each investor in the playing order looking at the other investors' capacities, speed of service, and prices, and choosing the capacity and speed of service combination that maximises the expected utility (lines 17-20). This leads to a new pure strategy profile for the next investor to take into account. Locating a pure strategy equilibrium in locations, capacities and speed of service (lines 21,22) is based on the idea that a new round begins with the pure strategy profile that resulted from the previous round. Then, if a full round passes without change in that pure strategy profile, no investor has incentive to deviate and thus the pure strategy profile investors started the round with is a pure strategy NE.

Note that in the maximisation process for the investor (line 19), prices are actually calculated numerically first. Then, these prices are used to calculate the customers' probabilities numerically. If the probabilities for some firms are negative, this falls under the conditions explained in Section 3.1.5. There are two ways to handle this situation, which have been found to be equivalent. One way is to remove the firms with negative probabilities (they are dominated by the other firms) from the customers' equilibrium system resulting in a reduced system. Then, the reduced system must be recalculated, as must be the new equilibrium in prices. This need not be done every time; all reduced permutations of the customers' and prices equilibria can be calculated symbolically before starting the IBR[8]. The second method is to fetch the numerical probabilities, set those which are negative to zero, and normalise each probability by dividing with the sum of the new probabilities.

Reduced versions of the problem in which locations and the speed of service for each investor are set can be further reduced by employing hill-climbing or simulated annealing algorithms. However, this is problematic for multiple locations and service speeds, because these introduce local maxima in investor utilities[9]. This could be overcome by running multiple hill-climbers in parallel for each investor, but it is doubtful whether this can provide any benefits given that the IBR is quite efficient even for large problems.

Last, it must be noted that the IBR algorithm can locate only one SPE with a given instantiation and playing order. To locate all possible equilibria, the IBR is repeated several times (line 10), and each time the resulting SPE is added to the set of SPEs if not present (line 23). Determining the number of repetitions needed is discussed in detail in Section 5.1.1.

[8] e.g. if there are 3 locations with 2 investors, that is 6 potential firms. Then solve symbolically also for 5, 4, 3, and 2 firms

[9] e.g. how do you transition from a state where the investor has a firm in one location, to a state where the investor switches location?

3.5 Analysis and Observations

This section will expand on some interesting theoretical aspects of firm competition as it has been defined in this chapter. Specifically, Section 3.5.1 will analyse theoretically the equilibrium in prices, and Section 3.5.2 will provide insights into the existence of pure strategy equilibria at stage 1 (see Section 3.3) of the extensive form game.

3.5.1 Prices equilibrium analysis

Having defined the pure NE in prices and a way to solve it in the Section 3.2.3, there are some noteworthy findings the analytical solution to equilibrium prices reveals. For demonstration purposes, simple notation as in Section 3.1 will be used. This assumes each investor has only 1 firm, therefore the terms 'Firm j' and 'Investor j' are equivalent.

Theorem 3.9. *Let* $(\{Firm\,1, Firm\,2\}, \{F_1, F_2\}, \{r^1(f), r^2(f)\})$ *be a two-investor instance of the investors' price game that was defined in Section 3.2.1, where investors have already chosen locations, capacities and the speed of service. Without loss of generality, it is assumed for simplicity that each investor has only one firm, that the cost of serving customers is the same for both firms ($h_1 = h_2 = h$), and that the speed of service is the same in both firms ($R_1 = R_2 = R$). Last, it is assumed that there is no outside option for customers. Then this game has a unique Nash equilibrium in prices $f^* = (f_1^*, f_2^*)$:*

$$\begin{aligned} f_1^* &= h - \frac{1}{3}v_d(t_1 - t_2) + Rv_d(n-1)\frac{2c_1 + c_2}{6c_1c_2} \\ f_2^* &= h - \frac{1}{3}v_d(t_2 - t_1) + Rv_d(n-1)\frac{c_1 + 2c_2}{6c_1c_2} \\ & \qquad v_d, c_1, c_2 > 0 \quad n > 1 \end{aligned} \tag{3.16}$$

in which charging prices will deviate from the marginal charging cost[10] h *due to the inability to satisfy charging demand immediately, and due to goods differentiation different firm access costs impose.*

It is noted at this point that immediate demand satisfaction is not necessarily related with the capacity of firms. In services which take some time to deliver, such as charging an EV, making a coffee, or downloading an application, it is understandable that demand may be de facto impossible to satisfy immediately, even if firms have enough servers to alleviate congestion. Additionally, this service time may be significant in some services. For example, it would take a charger with an output of about $2000kW$ to recharge a small EV such as the Nissan Leaf with a $24kW$ battery in time that is comparable with refuelling a conventional vehicle, and it would take even higher power output for charging times to be accepted as almost immediate.

[10] i.e. Bertrand equilibrium

Proof. For proving Theorem 3.9, the two-firm example that was solved in Section 3.1.5 is useful. The next step is to substitute the probabilities of equation (3.9) into investor expected utilities given by equation (3.12), and solve the price equilibrium from equation (3.13). This part is relatively long and is presented in Appendix A.2, but results in the equilibrium prices shown in equation (3.14).

It is evident from equation (3.14) that prices in equilibrium deviate upward of the marginal cost h because of the $Rv_d(n-1)\frac{2c_1+c_2}{6c_1c_2}$ and $Rv_d(n-1)\frac{c_1+2c_2}{6c_1c_2}$ terms. There also is some fluctuation in prices due to product differentiation different firm access costs impose, making the product of firms heterogeneous through the $vd(t_1-t_2)$ and $vd(t_2-t_1)$ terms. A firm with a lower access cost has an advantage in the ability to ask a higher fee, while a firm with a higher access cost has to reduce price to remain competitive. It must be noted here that this fluctuation may, in theory, result in price lower than h for a very disadvantaged firm (i.e. investor tries to minimise losses). However, in the greater extensive form game this strategy is always strictly dominated by not opening the firm at all in that location if it is doomed to constant loss, as we will see later in Section 5.1. Finally, from equation (3.16) it is straightforward to deduce that:

$$\lim_{R\to 0} f_1^* = h - \frac{1}{3}vd(t_1-t_2)$$
$$\lim_{R\to 0} f_2^* = h - \frac{1}{3}vd(t_2-t_1)$$

This means that in the case where firms have the same access cost and thus sell a homogeneous product (i.e. $t_1 = t_2$), equilibrium prices converge asymptotically to the marginal cost h with an increasing speed of service (decreasing service time R), something which is in line with the outcome of Bertrand competition. □

3.5.2 On the existence of pure NE in capacities

Let us consider a reduced version of the model in which two investors can own only up to one firm each, and locations, the speed of service and prices are set. In addition, there is no outside option for customers. Investors will first choose capacities, and then customers will choose firms. As discussed in Section 3.4.2, firm expected utility in Equation 3.11 is not defined for capacities of 0 due to substituting probability when solving with backward induction. A capacity of 0 is used to conceptually represent the situation where the investor chooses to not open a firm, and expected utility for that location is explicitly set to 0. In addition, a state where no firms are open is not desirable when there is no outside option for customers. Therefore, in the special case where one investor plays a capacity $c_j = 0$ while the other investor has also chosen a zero capacity, the utility for firm j is set to -∞. This introduces an inconsistency in utility which raises the following interesting situation.

Even in the case where all parameters for investors are the same (i.e. the problem is symmetric for investors), investors may play different capacities because capacities are discrete. Let us now consider the same setting, where in addition the fee f_j and the cost of providing the service h_j are the same for all firms. Because all customers must and will buy the service, there is a constant amount of total gross profit to be made by the firms, which is the buying power of customers:

$$p = (f_j - h_j)nw \tag{3.17}$$

Then, the expected utility for firm j is:

$$\mathbb{E}[r^k(c)|s] = s_i^j p - b_j c_j - o_j \tag{3.18}$$

Taking into account the probabilities, a small utility matrix for this scenario is shown in Table 3.1 below. For simplicity, the cost o_j is omitted. The left utility in each cell belongs to Firm 1, and the right utility in the cell belongs to Firm 2.

Table 3.1: 3x3 Matrix: Investors' capacity game.

		Firm 2		
	c_j	0	1	2
	0	$-\infty, -\infty$	$0, p - b_2$	$0, p - 2b_2$
Firm 1	1	$p - b_1, 0$	$1/2p - b_1, 1/2p - b_2$	$1/3p - b_1, 2/3p - 2b_2$
	2	$p - 2b_1, 0$	$2/3p - 2b_1, 1/3p - b_2$	$1/2p - 2b_1, 1/2p - 2b_2$

Now imagine that building cost is so high that firm utility is always negative. Let us say that the total profit that can be made is $p = 3$, while the building cost for a charging unit is $b_1 = b_2 = 4$. Then, the utility matrix becomes the matrix in Table 3.2. In this situation, if Firm 1 plays a capacity of 0, then Firm 2's best strategy is to play 1 despite incurring losses. If Firm 1 plays any other strategy, Firm 2's best strategy is to remain closed. The situation is the same for Firm 1, which leads to the existence of two asymmetric Nash equilibria $(c_1 = 0, c_2 = 1)$ and $(c_1 = 1, c_2 = 0)$. If both investors were allowed to not open, the equilibrium in this case would be $(c_1 = 0, c_2 = 0)$. So this equilibrium is artificially imposed by the assumption that customers have to recharge, and the investors do not have real incentive to open a firm leading to this irrational behaviour that a firm might be open even though it may be recording net loss. Let us now define this situation more generally.

Table 3.2: 3x3 Matrix: Investors' capacity game with $p = 3$ and $b_1 = b_2 = 4$. Maxima are indicated in bold numbers

		Firm 2		
	c_j	0	1	2
	0	$-\infty, -\infty$	**0**, **-1**	**0**, -5
Firm 1	1	**-1**, **0**	$-2.5, -2.5$	$-3, -6$
	2	-5, **0**	$-6, -3$	$-6.5, -6.5$

Theorem 3.10. *When each of m investors can own only up to one firm, there is no outside option for customers, and the problem of choosing capacities at firms is symmetric with respect to prices, service times and access costs, then investor j has real incentive to open a firm in strategy profile c only when:*

$$b_j < \frac{f_j - h_j}{c_j + \sum\limits_{-j \in I} c_{-j}} nw - \frac{o_j}{c_j} \tag{3.19}$$

where $-j$ indicates an investor other than j.

Proof. When the problem is symmetric for investors with respect to the costs that customers are interested in, that is when the access cost t_j, service fee f_j and service time R_j are the same across all firms, then it is straightforward to deduce from equation (3.4) that the customers' equilibrium in equation (3.8) reduces to:

$$\begin{gathered} \frac{s_i^1}{c_1} = \frac{s_i^2}{c_2} = \ldots = \frac{s_i^m}{c_m} \\ s_i^1 + s_i^2 + \ldots + s_i^m = 1 \end{gathered} \tag{3.20}$$

In turn, solving the system of (3.20) is fairly simple and reveals that for m firms, the probability of going to firm j is:

$$s_i^j = \frac{c_j}{c_1 + \ldots + c_m} = \frac{c_j}{c_j + \sum\limits_{-j \in I} c_{-j}} \tag{3.21}$$

Then in order for the investor of firm j to have real incentive to open the firm in strategy profile c (i.e. positive expected net profit) it must be:

$$\begin{aligned} & \mathbb{E}[r^k(c)|s] > 0 \Leftrightarrow \\ \textit{replacing from } (3.14) \textit{ and } (3.15) \quad & 0 < s_i^j (f_j - h_j) nw - b_j c_j - o_j \Leftrightarrow \\ \textit{from } (3.21) \quad & b_j c_j < \frac{c_j}{c_j + \sum\limits_{-j \in I} c_{-j}} (f_j - h_j) nw - o_j \Leftrightarrow \\ & b_j < \frac{f_j - h_j}{c_j + \sum\limits_{-j \in I} c_{-j}} nw - \frac{o_j}{c_j} \end{aligned}$$

which is the outcome in equation 3.19. □

This behaviour is not really an issue, as the situation where no firm is open is of little significance when customers do not have an outside option, and represents a rather unrealistic situation. In addition, the inclusion of the outside option relaxes the assumption that demand must be satisfied.

Moving on to examine what happens when firms' utilities are not always negative, a glance back at equation (3.21) shows that the probability of going to firm j increases at a reducing rate for a unilateral increase in capacity by that firm, and at the same time that probability has an upper bound of 1. This means that there is an upper bound in expected gross profit for each firm which, to pick up from the previous examples, would be p (equation (3.17). Looking at Equation (3.18), $-b_j c_j$ decreases linearly with an increasing capacity. Therefore, when expected gross profit $s_i^j p$ is added, this will introduce an upper bound in firm utility. As an investor will consider selecting capacity from 0 to some larger number Θ, utility will increase from 0 to a positive peak for some capacity, after which it will start decreasing. This is a good indication that equilibria might exist as investors will always have a maximum in expected utility, given the other investor's strategies. Let us assume now, still keeping in mind identical parameters for investors, that the total profit that can be made is $p = 15$ and the building cost for both investors to build each server is $b_1 = b_2 = 2$. A utility matrix for this case is shown in Table 3.3 below (again ignoring o_j).

Table 3.3: 4x4 Matrix: Investors' capacity game with $p = 15$ and $b_1 = b_2 = 2$. Maxima are indicated in bold.

		Firm 2			
	c_j	0	1	2	3
Firm 1	0	$-\infty, -\infty$	$0, \mathbf{13}$	$0, 1$	$0, 9$
	1	$\mathbf{13}, 0$	$5.5, 5.5$	$3, \mathbf{6}$	$1.75, 5.25$
	2	$11, 0$	$\mathbf{6}, 3$	$\mathbf{3.5}, \mathbf{3.5}$	$\mathbf{2}, 3$
	3	$9, 0$	$5.25, 1.75$	$3, \mathbf{2}$	$1.5, 1.5$

Here, if Firm 1 plays 0, Firm 2's best strategy is to play 1, if Firm 1 plays 1, Firm 2's action that maximises utility is playing 2. For a strategy $c_1 = 2$, Firm 2's best strategy is to play 2, and if Firm 1 plays 3, Firm 1's best action is playing 2. The situation is mirrored for Firm 1, and this indicates a pure strategy Nash equilibrium $(c_1 = 2, c_2 = 2)$. While in symmetric scenarios equilibria have been always found to exist in this model, there can be situations in which Nash equilibria do not exist. Non-existence in the presented model is attributed to severe differences between parameters for the investors. If, for example, the building cost is disproportionately low for one investor, the other investor may end up getting negative utilities for most capacity choices. This forces the maxima of that player toward the edge of the matrix, and the two investors' maxima can never coincide in the same strategy profile. Such a situation is displayed in the next table, Table 3.4, which shows the utilities when total possible profit is $p = 15$, building cost for Firm 1 is $b_1 = 2$ but for Firm 2 it is $b_2 = 7$.

Notice how the maxima of Firm 2 have now clustered at the left of the matrix, while the situation for Firm 1 is unchanged utility-wise, compared to the previous example of Table 3.3. This, of course, does not mean that capacity choice does not have an impact on the other player. If we follow the line of a strategy of Firm 1, Firm 1's utilities do change as Firm 2 increases capacity, as this changes the probability of customers choosing Firm 2. It means, however, that building cost for one player does not affect the utility of the other player directly, but it does so through a different choice in capacity Firm 2 will make. This is expected, as a disadvantaged investor with higher building costs cannot do anything to counter that in this form of the game where prices

Table 3.4: 5x5 Matrix: Investors' capacity game with $p = 15, b_1 = 2, b_2 = 7$. Maxima are indicated in bold.

		Firm 2				
	c_j	0	1	2	3	4
Firm 1	0	$-\infty, -\infty$	$0, \mathbf{8}$	$0, 1$	$0, -6$	$0, -13$
	1	$\mathbf{13}, 0$	$5.5, \mathbf{0.5}$	$3, -4$	$1.75, -9.75$	$\mathbf{1}, -16$
	2	$11, \mathbf{0}$	$\mathbf{6}, -2$	$\mathbf{3.5}, -6.5$	$\mathbf{2}, -12$	$\mathbf{1}, -18$
	3	$9, \mathbf{0}$	$5.25, -3.25$	$3, -8$	$1.5, -13.5$	$0.43, -19.4$
	4	$7, \mathbf{0}$	$4, -4$	$2, -9$	$0.57, -14.6$	$-0.5, -20.5$

are set, such as reduce price to attract more customers. Therefore, customers will select firms with the same probabilities for a particular capacity combination, regardless of the building costs to which customers are indifferent. For similar reasons, equilibria may also not exist when firm prices are vastly different. However, in that case customers do reason about prices, so Firm 1's utility is expected to be affected when the price of Firm 2 changes with a given combination of capacities. Let us now define equilibrium existence with regard to building costs more generally.

Theorem 3.11. *When investor j can own only up to one firm, there is no outside option for customers, and the problem of choosing capacities at firms is symmetric with respect to prices, service times and access costs, then in order for strategy profile c^* to be a pure strategy Nash equilibrium it must be that:*

$$\forall j \in I :$$
$$\frac{(f_j - h_j)nw \sum\limits_{-j \in I} c^*_{-j}}{\left(c^*_j + \sum\limits_{-j \in I} c^*_{-j}\right)^2 + c^*_j + \sum\limits_{-j \in I} c^*_{-j}} \leq b_j \leq \frac{(f_j - h_j)nw \sum\limits_{-j \in I} c^*_{-j}}{\left(c^*_j + \sum\limits_{-j \in I} c^*_{-j}\right)^2 - \left(c^*_j + \sum\limits_{-j \in I} c^*_{-j}\right)}$$
$$\textit{and}$$
$$f_j > \frac{b_j \sum\limits_{-j \in I} c^*_{-j}}{nw} + h_j,\ c_j, c^*_{-j} > 0,\ n > 1 \tag{3.22}$$

where $-j$ indicates an investor/firm other than j.

Proof. Let us assume, for now, that the capacities domain is continuous. From equation (3.18), if we replace s^j_i from equation (3.21), we have that expected utility for firm j in strategy profile c is:

$$\mathbb{E}[r^k(c)|s] = \frac{c_j p}{c_j + \sum\limits_{-j \in I} c_{-j}} - b_j c_j - o_j = \left(\frac{p}{c_j + \sum\limits_{-j \in I} c_{-j}} - b_j\right) c_j - o_j \tag{3.23}$$

In Appendix A.3 it is proven that firm j's expected utility in equation 3.23 has one critical point in $(0, \Theta)$ provided that $f_j > \frac{b_j \sum_{-j \in I} c^*_{-j}}{nw} + h_j$, and is concave down in $(0, \Theta)$.

According to Definition 3.7 where we defined pure NE in prices, extending it to capacities means that when investors are in pure strategy NE c^*, the utility for investor j is at least as good as the utility if j alone deviated from c^* and played a different strategy. It has already been shown that the utility for investor j in this case is concave down and can have only one critical point in $(0, \Theta)$ for any given capacity combination by the other investors. Remembering that capacity in our problem is actually a natural number, if c_j is j's current capacity and it is not a maximum, then a deviation by either $+1$ or -1 must yield better utility given utility is concave down with only one critical point (i.e. we can hill-climb utility). Alternatively, when investors are in equilibrium c^*, then a deviation from c^*_j by $+\alpha \in \{-1, 1\}$ must yield at most the utility that equilibrium does:

$$\forall j \in I: \quad \mathbb{E}[r^k((c^*_j, c^*_{-j}))|s] \geq \mathbb{E}[r^k((c^*_j + \alpha, c^*_{-j})|s] \Leftrightarrow$$

$$b_j \alpha \geq p \frac{\alpha \sum_{-j \in I} c^*_{-j}}{\left(c^*_j + \sum_{-j \in I} c^*_{-j}\right)^2 + \alpha \left(c^*_j + \sum_{-j \in I} c^*_{-j}\right)} \qquad \textit{Full expansion in Appendix A.4.}$$

When $\alpha = 1$ it must be:

$$b_j \geq p \frac{\sum_{-j \in I} c^*_{-j}}{\left(c^*_j + \sum_{-j \in I} c^*_{-j}\right)^2 + c^*_j + \sum_{-j \in I} c^*_{-j}}$$

and when $\alpha = -1$ it must be:

$$b_j \leq p \frac{\sum_{-j \in I} c^*_{-j}}{\left(c^*_j + \sum_{-j \in I} c^*_{-j}\right)^2 - \left(c^*_j + \sum_{-j \in I} c^*_{-j}\right)}$$

Finally, in order to guarantee equilibrium, utility in c^* must be at least as good as in a deviation $a = -1$, *and* a deviation $a = 1$, thus we need to consider these two conditions together. In addition, it must be taken into account that a maximum in firm utility exists in $(0, \Theta)$ only when the price is not very low. Hence replacing p from equation (3.17), it must be that:

$$\forall j \in I :$$
$$\frac{(f_j - h_j)nw \sum\limits_{-j \in I} c^*_{-j}}{\left(c^*_j + \sum\limits_{-j \in I} c^*_{-j}\right)^2 + c^*_j + \sum\limits_{-j \in I} c^*_{-j}} \leq b_j \leq \frac{(f_j - h_j)nw \sum\limits_{-j \in I} c^*_{-j}}{\left(c^*_j + \sum\limits_{-j \in I} c^*_{-j}\right)^2 - \left(c^*_j + \sum\limits_{-j \in I} c^*_{-j}\right)}$$
$$\textit{and}$$
$$f_j > \frac{b_j \sum\limits_{-j \in I} c^*_{-j}}{nw} + h_j,\ c^*_j, c^*_{-j} > 0,\ n > 1 \tag{3.24}$$

□

3.6 Welfare and Efficiency Metrics

When demand must absolutely be satisfied and customers do not have any other choice than to purchase the product/service, it is self-evident that it is not possible to measure efficiency for firms, because the optimal strategy would be to set capacity at 1 and an infinite price (or at some artificial price limit). However, the inclusion of an outside option for customers makes it now possible to measure the efficiency of SPEs for firm investors. In order to measure this efficiency, or rather inefficiency, of subgame-perfect equilibria the concept of the Price of Anarchy (PoA) will be used. The PoA concept intuitively proposes that the selfish behaviour of players, steers them away from making decisions that are optimal for the community. The inefficiency of equilibria, therefore, can be measured by comparing the worst-case social welfare in equilibrium, with the maximum social welfare that can be obtained (Anshelevich and Ukkusuri, 2009).

Definition 3.12 (Price of Anarchy for investors)**.** Let $X = \{\{c^*, f^*, s^*\}_1, \ldots, \{c^*, f^*, s^*\}_\lambda\}$ be the set of all λ subgame-perfect equilibria in investor capacities and prices, and customer choices. Then, the worst-case social welfare for investors is defined as the sum of investor utilities in subgame-perfect equilibrium ρ in which the sum of utilities is minimum:

$$\textit{Welfare}_{SPE} = \min_{\{c^*,f^*,s^*\}_\rho \in X} \sum_{j \in I} \mathbb{E}[r^j(c^*, f^*)|s^*]$$

It is assumed that the maximum social welfare that investors can obtain is the social welfare when a central agency dictates the players' strategies so that the sum of utilities is optimised (centralised optimum). Therefore, the maximum social welfare that can be obtained is defined as the combination of capacities and prices which maximises the sum of utilities of the investors, given customers will play in mixed strategy Nash equilibrium s^*:

$$\textit{Welfare}_{max} = \max_{c \in C, f \in F} \sum_{j \in I} \mathbb{E}[r^j(c,f)|s^*]$$

Then, the price of anarchy for investors is defined as the ratio of the maximum social welfare, over the worst-case social welfare in SPE:

$$\textit{PoA} = \frac{\max_{c \in C, f \in F} \sum_{j \in I} \mathbb{E}[r^j(c,f)|s^*]}{\min_{\{c^*,f^*,s^*\}_\rho \in X} \sum_{j \in I} \mathbb{E}[r^j(c^*,f^*)|s^*]} \tag{3.25}$$

This gives us a measure of efficiency for investors, but does not clarify the picture for customers. A similar concept could be applied for customers, however it is not clear how centralised customer coordination and a comparison against it could be interpreted meaningfully, given it may be unrealistic in many applications and given this thesis does not attempt to compare routing policies for customers. A more meaningful measure of efficiency is to measure system-wide efficiency *given that customers will play in mixed strategy NE*.

Definition 3.13 (System-wide SPE efficiency)**.** Let $X = \{\{c^*, f^*, s^*\}_1, \ldots, \{c^*, f^*, s^*\}_\lambda\}$ be the set of all λ subgame-perfect equilibria in investor capacities and prices, and customer choices. System-wide social welfare is defined as the sum of the utilities of all players, and the worst-case system-wide social welfare is defined as the sum of utilities in the SPE where the sum of utilities is minimum:

$$\textit{Social Welfare}_{SPE} = \min_{\{c^*,f^*,s^*\}_\rho \in X} \left(\sum_{j \in I} \mathbb{E}[r^j(c^*,f^*)|s^*] + nw\mathbb{E}[u_i(x)|s^*, f^*, c^*] \right)$$

Notice that the utility for all customers is actually the expected utility of the mixed strategy for customer i (Equation (3.5)), multiplied by the number of customers n and the profit scaling factor w. Now the maximum social welfare is defined as the the maximum sum of utilities of all players across capacity and price strategies:

$$\textit{Social Welfare}_{max} = \max_{c \in C, f \in F} \left(\sum_{j \in I} \mathbb{E}[r^j(c,f)|s^*] + nw\mathbb{E}[u_i(x)|s^*, f, c] \right)$$

Finally, the ratio we are interested in is the Social Welfare Ratio (SWR):

$$\textit{SWR} = \frac{\max_{c \in C, f \in F} \left(\sum_{j \in I} \mathbb{E}[r^j(c,f)|s^*] + nw\mathbb{E}[u_i(x)|s^*, f, c] \right)}{\min_{\{c^*,f^*,s^*\}_\rho \in X} \left(\sum_{j \in I} \mathbb{E}[r^j(c^*,f^*)|s^*] + nw\mathbb{E}[u_i(x)|s^*, f^*, c^*] \right)}$$

This will provide a useful measure of efficiency given that the goal of the model is to examine strategies for investors. SWR follows a similar logic as the Price of Anarchy, and social welfare in Hayrapetyan et al. (2007). However, because in this model both investors and customers make decisions, SWR here considers that customers will always play in mixed NE, and compares against the firm allocation which maximises social welfare, which is a meaningful metric. For the true system-wide Price of Anarchy, customers should follow a centralised optimal routing policy in finding *Social Welfare*$_{max}$, but this has little value given this thesis does not compare routing policies for customers.

Chapter 4

Basic EV Driver and Station Investor Behaviour

With this chapter, the application of the model for firm competition that Chapter 3 proposed will begin on the SLCOP problem (Section 1.1). Applying the model will generally be carried out in steps of increasing complexity, so as to promote empirical discussion on the various aspects of charging station investor behaviour, as well as EV driver behaviour. Hence, this chapter will mostly involve a duopoly example, that is a two-station setting where each charging station belongs to a different investor, and where EV charging demand must be satisfied, that is there is no outside option for EV drivers. Furthermore, it is considered that prices, locations and the speed of service are set, that is investors only decide on capacities for their stations, and then drivers choose stations. With these in mind, Section 4.1 discusses the model in relation to the SLCOP problem, Section 4.2 evaluates the EV drivers' behaviour when choosing stations, and Section 4.3 presents an empirical evaluation of charging station investor behaviour when choosing capacities.

4.1 Context and Parameters of the SLCOP

In order to determine and model how EV drivers behave, some context regarding EV usage needs to be defined. First of all, it is assumed for the purpose of this thesis that drivers drive pure electric vehicles, that is vehicles that are solely powered by electricity. Whereas hybrid EVs which combine internal combustion engines with electric motors exist today, these are generally indifferent to en-route charging. Hybrid EVs have small batteries and are ideal for using the electric motor for short intra-city trips, like going to work, where speed is lower and distances are relatively short. However, hybrid EVs generally have to rely on the internal combustion engine for higher speeds and travelling longer distances. This means that hybrid EV drivers can use the electric motor to complete short trips, or longer trips in combination with the combustion motor, without being constrained by the available battery charge as pure EV drivers are. Hybrid

EVs additionally regenerate charge from using the combustion engine, and after completing the trip they can replenish battery charge at leisure and low cost in slow park 'n charge stations. Thus, it is unreasonable to consider that a hybrid EV driver would regularly stop to recharge at en-route rapid charging stations at higher cost and with the additional potential of having to wait in a queue. As regards pure EV drivers now, a realistic context for using en-route charging stations extensively is for inter-city trips, as for shorter trips drivers can again recharge slowly while at work or at home during the night. Thus in order to analyse the effects of potential queues it will be considered in this work that EV drivers go on trips that they cannot complete even with a full battery.

Furthermore, it will be assumed that all EV drivers drive the same type of pure EV, that they recharge the battery for its full capacity in charging stations, and that they always have enough charge to reach the chosen station. Last, it will be assumed that all EV drivers perform the same trip, that is they have the same starting and destination points, as network users do in Hayrapetyan et al. (2007). These assumptions guarantee that the game of choosing stations is symmetric for drivers, and thus a symmetric mixed NE, which can help promote analysis, exists. However, any asymmetric mixed NE can be considered in the model, but considering those would further complicate the model and would add little to the findings of this thesis.

In relation to parameters in the customers' firm choice game (Section 3.1) in the SLCOP domain (Section 1.1), the access cost t_j is used to represent the travel time needed to reach the destination, if station j is chosen to recharge at. The speed of service R_j is the time station j needs to recharge one EV driver, the fee f_j will be the fee for recharging at station j, and c_j is the station's capacity (number of charging units). The value of time v_d is the value of time for driving an EV.

With regard to charging stations, as stated earlier a two-station example will be considered, where h_j now represents the cost of electricity for station j to recharge an EV, b_j is the cost of installing one charging unit, and o_j is an one-time building cost for station j.

A more detailed explanation of parameter choices will now follow. First, it is assumed that drivers drive the Nissan Leaf EV with a $24kW$ battery configuration. The charging efficiency of charging units is set to 85% which is consistent with the chosen model and rapid DC charging units (Channegowda et al., 2016). This results in an energy requirement of $E = 28.24kW$ for a unit to fully recharge the battery. Charging unit power output is set to $50kW$ which is a common output for rapid DC chargers (Channegowda et al., 2016). For simplicity, it is assumed that the output of charging units is linear over time, and this makes the charging time required to fully charge the Leaf's battery roughly 33 minutes and 40 seconds. In the model, however, time will be represented in half-hours which makes charging time $R = 1.1294$ half-hours. For a more realistic setting that will be useful in later chapters, it is assumed that EV drivers perform the trip from Central Southampton to Central London. This trip's usual length is 80 miles, which at an average speed of 60mph is travelled in $10/3$ half-hours and which is a realistic value without traffic. The value of time given the trip length and mode of transport has been determined to be $v_d = 12.56$ *£/half-hour* from the tables that the UK Department for Transport provides (Department for Transport, 2015).

The building cost for each charging unit is set to $b_1 = b_2 = \pounds 36000$. Rapid charging unit installation costs can vary, but these values were realistic to consider for $50kW$ rapid DC charging units at the time of the experiments, including a cycle of yearly maintenance (U.S. Dept. of Energy, 2015). One-time building cost for each station is set to $o_1 = o_2 = \pounds 30000$. The magnitude of this cost is not particularly important, but the parameter is introduced to prevent investors from building arbitrarily on all locations later on. With regard to the price station investors buy electricity at, this is set to $\pounds 0.1/kWh$. Electricity prices fluctuate from day to day, but this was a realistic commercial price at the time of the experiments. Then, given an energy requirement of $E = 28.24kW$ to recharge each EV, the cost of recharging each EV for station investors is $h_1 = h_2 = \pounds 2.8235$. The number of drivers is set to $n = 30$.

In order to normalise profits in proportion to the cost of building, the parameter w is utilised. Normalisation is necessary, because when utility for investors is always negative for positive capacities, then the maximum strategy for each investor would be to not open the station. In order to normalise profits, it will be assumed that the extensive form game represents a situation where a peak traffic of n drivers arrives at stations at once. This peak traffic can occur three times a day and encounters an empty queue. The station's daily income consists of the profit from recharging EVs during these three peak traffic occurrences, plus the income from the rest of the day when traffic is more scarce. Income throughout the rest of the day is assumed to be equal to the income in peak hours and the game is played with a horizon of 1 year, that is 365 days, for profit. This makes profit normalisation $w = 365 * 6 * 1 = 2190$.

These settings will represent reference settings, but many will be varied in the course of the experiments. It will be stated clearly where appropriate which parameters deviate from these reference settings.

4.2 Evaluation of the EV Drivers' Equilibrium

This section presents an empirical evaluation of the EV drivers' model that was shown in Section 3.1. One goal is of course to determine whether the model behaves within reason. A second goal for the evaluation is to determine potential weaknesses that will have to be addressed.

For the purpose of the experiments, it is assumed that EV drivers are called to select between two existing charging stations unless otherwise stated. The parameters that are of interest for studying the mixed NE include station capacities and prices, route travel times, charging times, the number of drivers playing the EV drivers' game, the value of time for drivers, and last the number of stations. Each experiment will involve varying a parameter while keeping other parameters constant.

Given the reference settings outlined in Section 4.1, first the EV drivers' equilibrium will be evaluated with relation to station capacities and charging unit power output in Section 4.2.1. Next, an evaluation with regard to charging fees at stations will be presented in Section 4.2.2. Last, sensitivity to other parameters will be discussed in Section 4.2.3.

4.2.1 Station capacities and charger output

The first experiment that will be discussed is varying the two stations' capacities. For this setting, it is assumed that both stations are on the same route ($t_1 = t_2$) and charge the same fee for recharging EVs ($f_1 = f_2$). The number of drivers who play the station choice game is $n = 32$. Looking back at Equation (3.21), this means that the equilibrium probability of driver i going to station 1 in this case is $s_i^{1*} = \frac{c_1}{c_1 + c_2}$ and for going to station 2 is $s_i^{2*} = \frac{c_2}{c_1 + c_2}$. The point that should be taken from this observation is that regardless of how high capacities are, there will always be some probability of going to a station no matter how high the capacity of the other station. Of course, if one station is closed ($c_j = 0$) a driver would play a pure strategy of going to the other station. This behaviour is induced by the approximation that was done in expected queuing time in Equation (3.2) which leaves some residual average expected queuing time no matter how high capacity is.

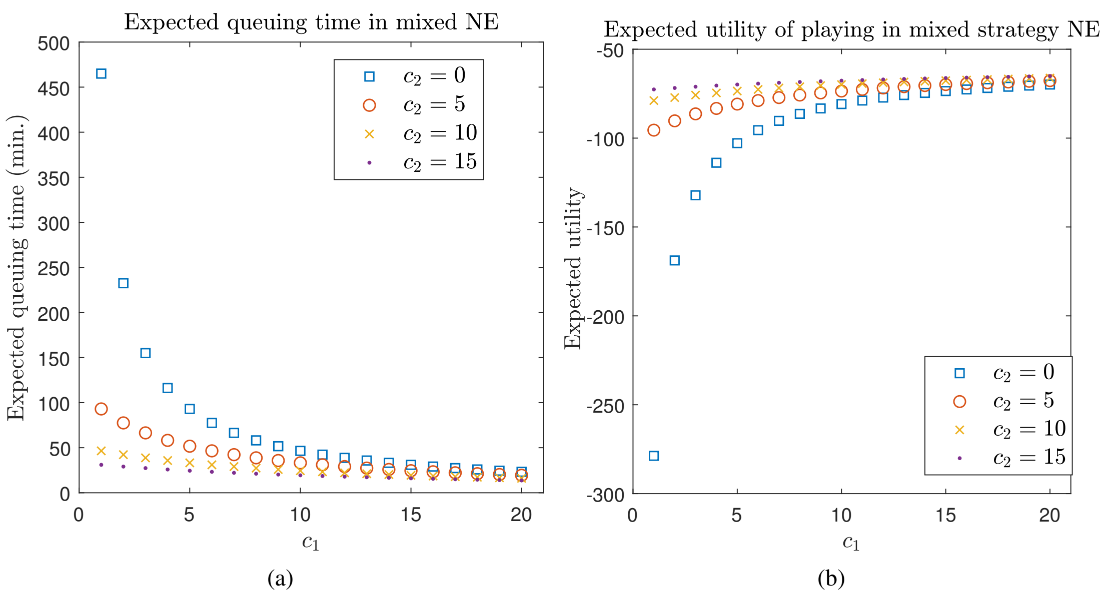


Figure 4.1: Expected queuing time in mixed strategy NE (left) and expected utility of playing in mixed strategy NE (right) for varying station capacities.

Figure 4.1a shows the expected queuing time in NE, that is the sum of average expected queuing times for each station weighed by the respective probability of station choice. The aforementioned residue can be observed, as expected queuing time never really reaches zero even when $c_1 + c_2 > n = 32$.

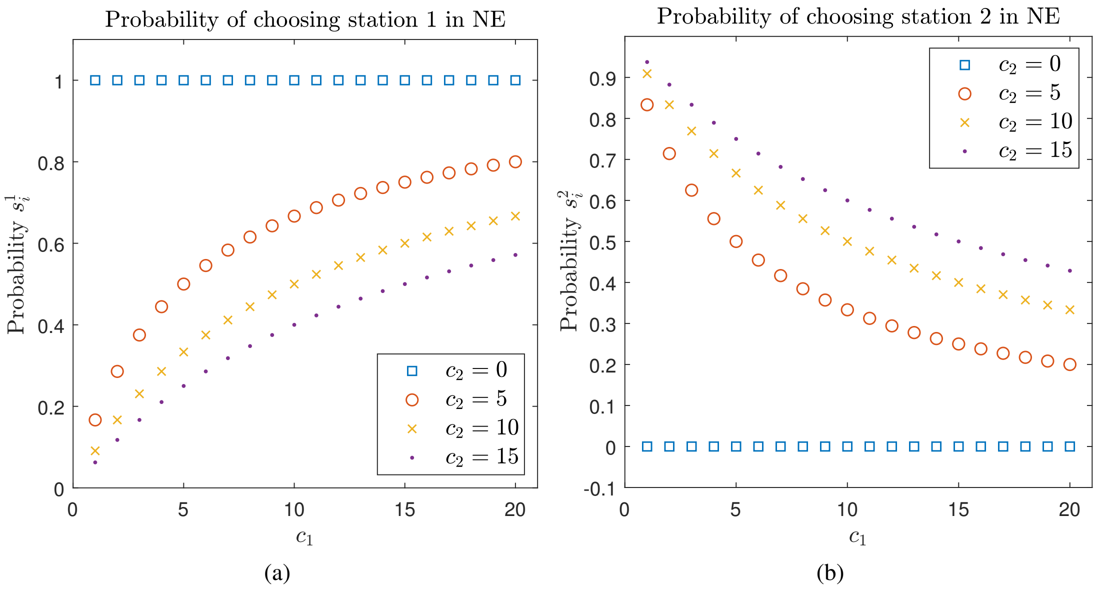


Figure 4.2: Probabilities of choosing station 1 (left) and station 2 (right) with varying station capacities.

This is an issue when reporting queuing time because the number is not entirely accurate, but in general the resulting behaviour is not problematic for the following reasons. First, conceptually the idea that some probability exists of selecting a station no matter the other stations' capacities is not necessarily inconsistent with how drivers act in reality. Second, despite the discrepancy behaviour is maintained. That is when stations have the same capacity there is the same probability of choosing each (0.5), and probabilities are proportionally correct for different capacities, when total capacity is less than the number of drivers. Last, we will see this in later chapters but it is very unlikely that investors will build so much capacity given realistic station parameters.

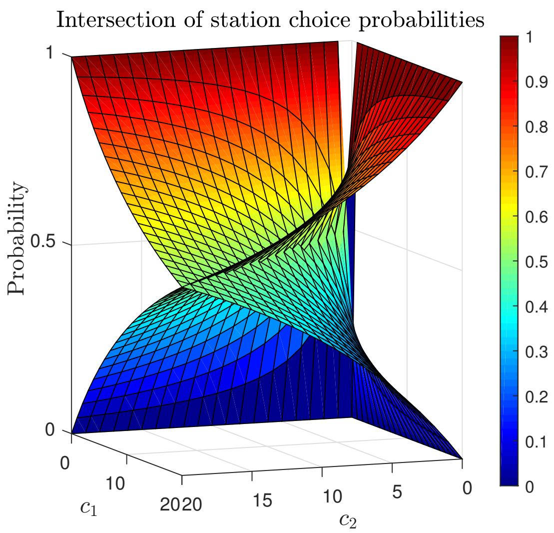


Figure 4.3: Intersection of probabilities of choosing stations in NE for varying station capacities

Moving on to the expected utility of playing in mixed strategy NE, which can be found in Figure 4.1b we can observe that expected utility increases in a manner identical to how expected queuing time decreases with increasing capacities. This is expected as station capacities influence the driver's expected utility through queuing time. With regard to the probabilities of station choice, if we overstep and differentiate them as if they were continuous, the rate of change $s_i^{1*}(c_1)' = \frac{c_2}{(c_1+c_2)^2}$ indicates the probability of choosing station 1 given the capacity of station 2 (c_2) should show a declining rate of increase with an increasing capacity c_1, something which is shown in Figure 4.2a. In the same figure it is also noted that the probability of choosing station 1 increases with an increasing capacity c_1 and decreases with an increasing capacity c_2 which is reasonable behaviour. The opposite should be true for station 2 and this is indeed observed in Figure 4.2b. In addition, in Figure 4.3 that shows the intersection of the aforementioned probabilities it can be observed that when stations have the same capacity drivers will select each station with the same probability (0.5).

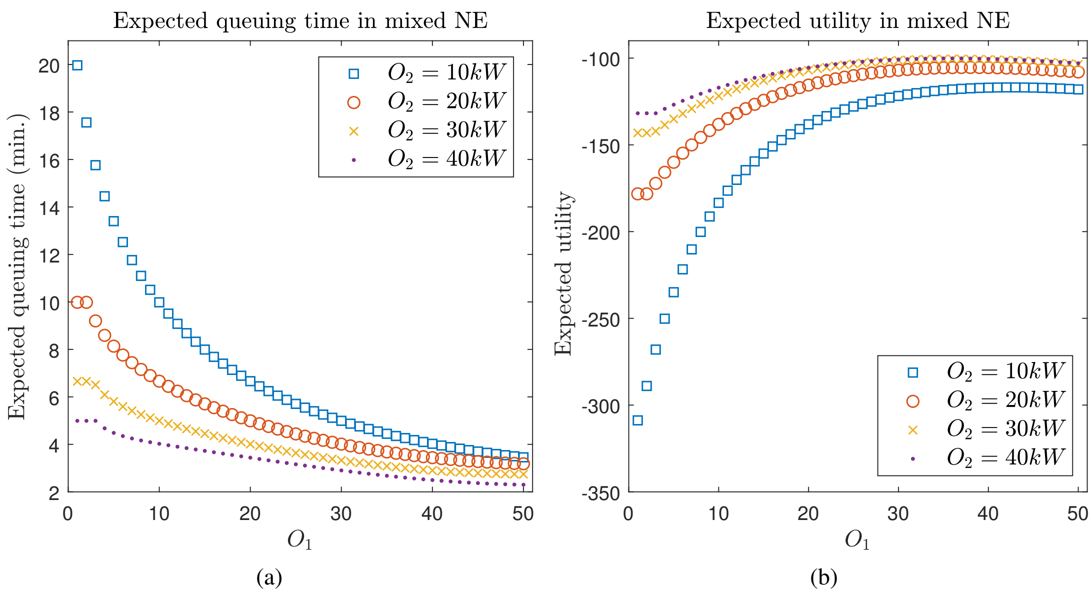


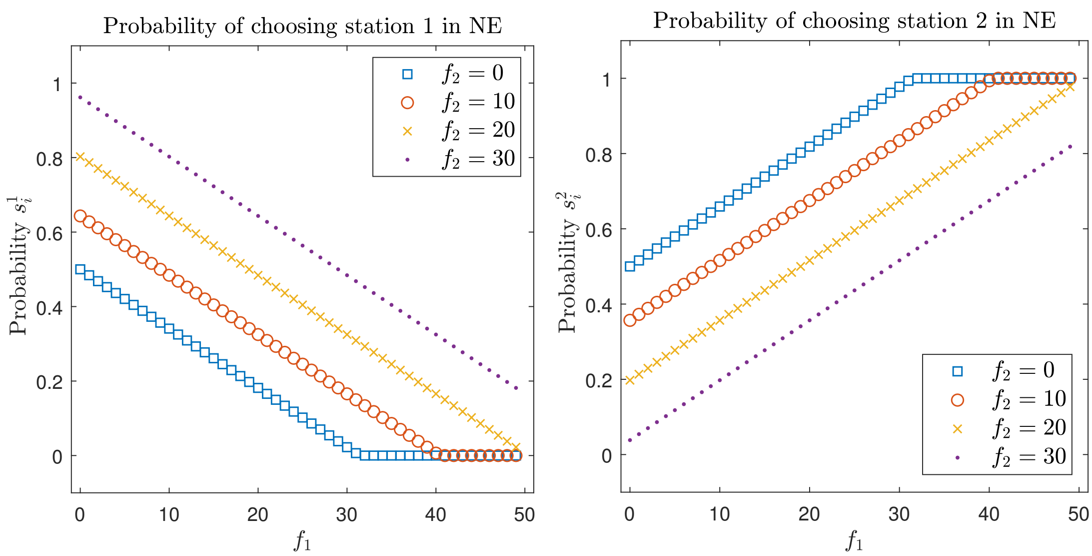
Figure 4.4: Expected queuing time in mixed strategy NE (left) and expected utility of playing in mixed strategy NE (right) for varying charger outputs.

By setting capacities at $c_1 = c_2 = 7$ and varying the charging units' power output, similar EV driver behaviour is obtained. The charging units' output O_j influences charging time R_j at station j. More specifically, charging time is defined as the ratio of the energy needed to recharge an EV over the charging output at station j, times two because the time measure is half-hours. That is $R_j = 2E/O_j$. Taking into account different charging speeds at each station changes the equilibrium probabilities of (3.9) slightly to:

$$s_i^{1*} = \frac{c_1 v_d R_2(n-1) + 2c_1c_2v_d(R_2 - R_1) + 2c_1c_2v_d(t_2 - t_1) + 2c_1c_2(f_2 - f_1)}{v_d(n-1)(c_1R_2 + c_2R_1)}$$
$$s_i^{2*} = \frac{c_2 v_d R_1(n-1) + 2c_1c_2v_d(R_1 - R_2) + 2c_1c_2v_d(t_1 - t_2) + 2c_1c_2(f_1 - f_2)}{v_d(n-1)(c_1R_2 + c_2R_1)}$$
$$v_d, c_1, c_2 > 0 \quad n > 1$$

The expected queuing time (Figure 4.4a) and expected utility for driver i (Figure 4.4b) show similar characteristics to those for varying capacities, and this is expected as charging time R_j influences the driver's utility for choosing a station in a very similar way.

4.2.2 Fees at charging stations



(a) (b)

Figure 4.5: Probabilities of choosing station 1 (left) and station 2 (right) in mixed strategy NE for varying prices.

Having explored station capacities and charging times, it is now time to explore the other crucial parameter of the EV drivers' station choice and that is station prices. In order to do so, capacity in both stations is set at $c_1 = c_2 = 7$ while the number of drivers is set to $n = 30$ and the fees for both stations are varied. By differentiating the probability of going to each station (Equation (3.9)) with respect to that station's fee, it is straightforward to deduce that the rates at which the probabilities change are:

$$s_i^{1*}(f_1)' = -\frac{2c_1c_2}{v_d(n-1)(c_1+c_2)R}$$
$$s_i^{2*}(f_2)' = -\frac{2c_1c_2}{v_d(n-1)(c_1+c_2)R}$$

From these, it is expected that the probability of going to a station will decrease linearly with an increasing fee in that station and this behaviour can be observed in Figure 4.5a for

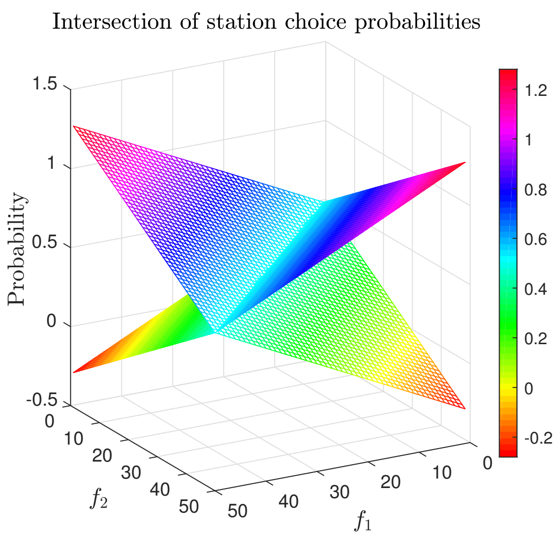


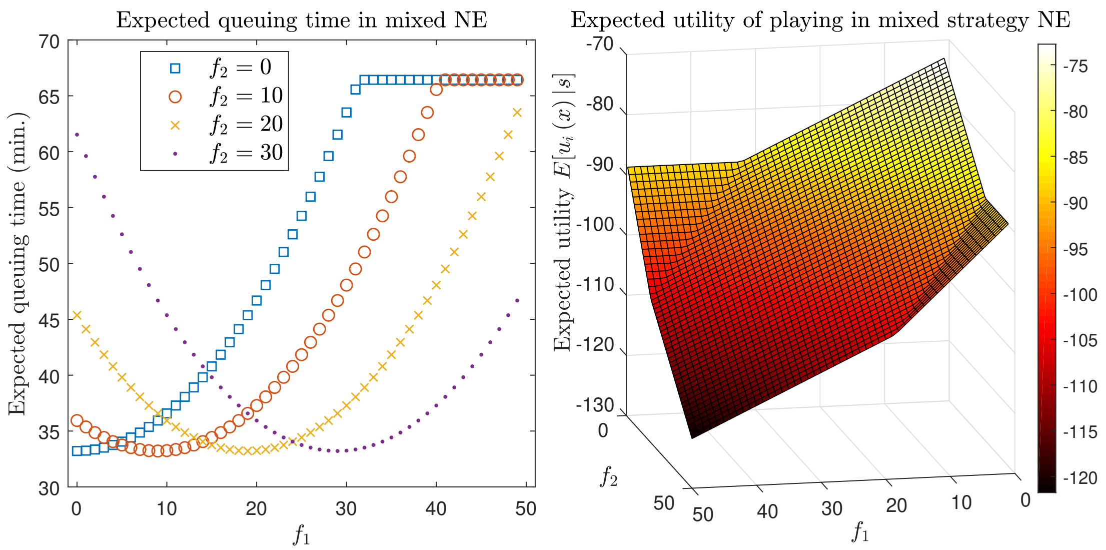
Figure 4.6: Intersection of non-bounded probabilities of choosing stations in NE for varying station fees

station 1 and Figure 4.5b for station 2. Another difference here is that with varying fees it is possible for the probabilities of station choice to go out of bounds. This behaviour can be observed in Figure 4.6 and is corrected by applying boundary conditions in the manner and logic that was explained in Section 3.1.5. Overall, the behaviour of drivers is reasonable in that they select stations with the same probability when they offer the same charging price, while a unilateral increase in price from a station results in a reduced probability of going to that station.

With regard to queuing time, we can see in Figure 4.7a that queuing time remains constant with an increasing price for both stations, when stations offer the same price, which is reasonable since in that case drivers select either station with the same probability. If one station unilaterally increases price, this will cause drivers to choose the other station with a higher probability, and this probability increases the higher the price increase. This results in the behaviour of Figure 4.7a which resembles a second degree polynomial with respect to a station's price given a constant price of the other station. This is reasonable behaviour given that the average expected queuing time driver i will experience for choosing station 1 is $s^1_{-i}(n-1)R/2c_1$ and for choosing station 2 is $s^2_{-i}(n-1)R/2c_2$. Then, the average expected queuing time in mixed strategy NE is $s^{1*}_i(s^{1*}_{-i}(n-1)R/2c_1) + s^{2*}_i(s^{2*}_{-i}(n-1)R/2c_2)$. In the symmetric mixed strategy NE driver i chooses station j with the same probability as other drivers, that is $s^{j*}_i = s^{j*}_{-i}$ which results in average expected queuing time for playing in mixed strategy NE being:

$$(s^{1*}_i)^2(n-1)R/2c_1 + (s^{2*}_i)^2(n-1)R/2c_2$$

Both probabilities are linear functions with respect to a station's price thus the result is, of course, a second degree polynomial given the strategy of the other station.



(a) (b)

Figure 4.7: Expected queuing time in mixed strategy NE (left) and expected utility of playing in mixed strategy NE (right) for varying station prices.

As regards driver i's expected utility for playing in mixed strategy NE, this decreases linearly with increasing prices. This might seem odd at first given that expected queuing follows a second degree polynomial as was discussed above. However, the driver's expected utility in mixed NE also contains the expected price term $s_i^{1*} f_1 + s_i^{2*} f_2$. Expected price is also a second degree polynomial with respect to each station's price, only this time concave down. This results in the drivers' utility being always linear with respect to a station's price, given the other station's price (this is proven in Appendix A.5), which produces the flat plate seen in Figure 4.7b. Notice that in both expected utility and expected queuing time there are two flat 'wings' in areas when a station has a very high price and the other station has a very low price. This is a result of the probability going to the station with the high price reaching 0 as seen in Figure 4.5.

4.2.3 Other parameters

Having explored the main parameters that stations can determine to influence driver decisions, other parameters will be discussed more generally. Regarding travel times, these are explored by setting $n = 30$ drivers, capacities $c_1 = c_2 = 7$ and charging fees at $f_1 = f_2 = £10$. It is assumed that one station exists in each of two routes and the travel times for those routes are varied. Results are very similar in this experiment to those that were discussed for varying prices.

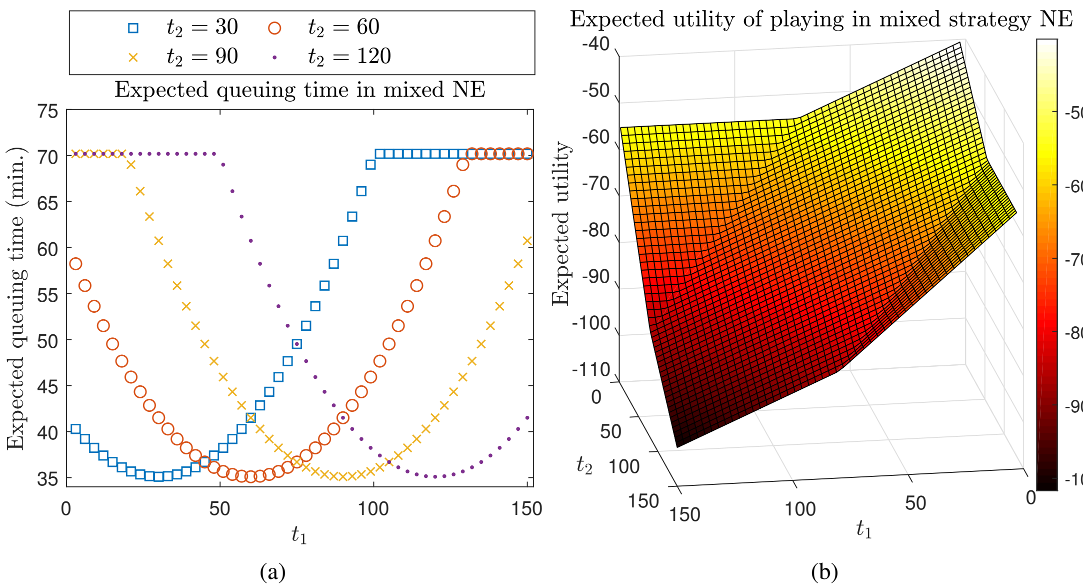
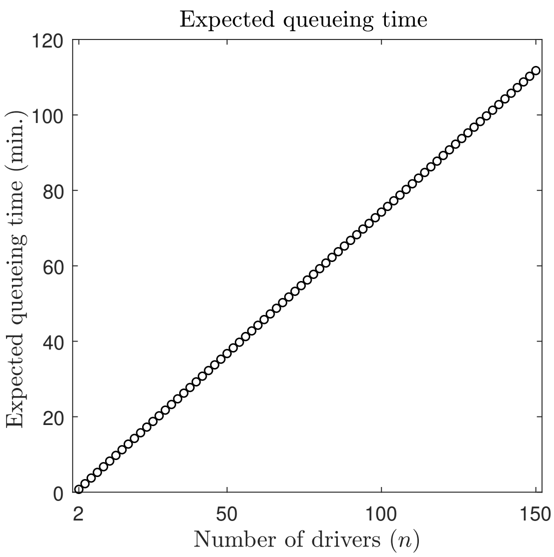


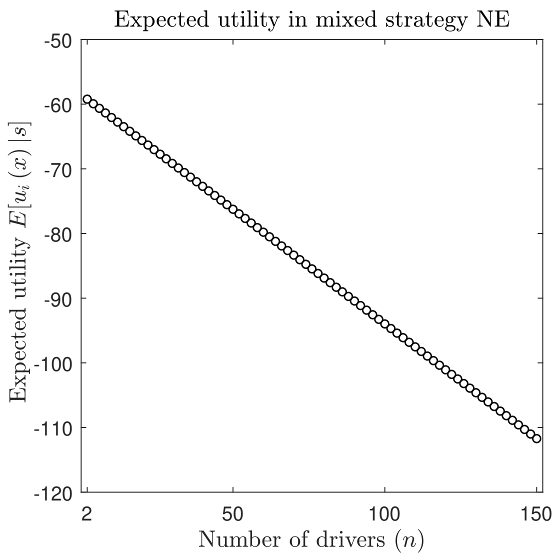
Figure 4.8: Expected queuing time in mixed strategy NE (left) and expected utility of playing in mixed strategy NE (right) for route travel times.

Expected queuing time shown in Figure 4.8a is again a second degree polynomial to the station's route travel time, and so is expected travel time $s_i^{1*} t_1 + s_i^{2*} t_2$. Therefore, drivers show similar behaviour to that for varying prices that were discussed in the previous section. This results in a linear decrease in expected utility (Figure 4.8b) with an increasing travel time for one station, given the the travel time of the other station. Again, there is a change in the slope of expected utility when travel times are such that all drivers go to one station.

To explore the number of drivers n, station capacities are set to $c_1 = c_2 = 7$ as in the previous experiment, while the number of drivers is varied. Given that the probabilities of station choice do not change, expected queuing time increases linearly with an increasing number of drivers (Figure 4.9a) while the expected utility of playing in mixed strategy NE decreases in a similar manner due to an increase in queuing times (Figure 4.9b).



(a)

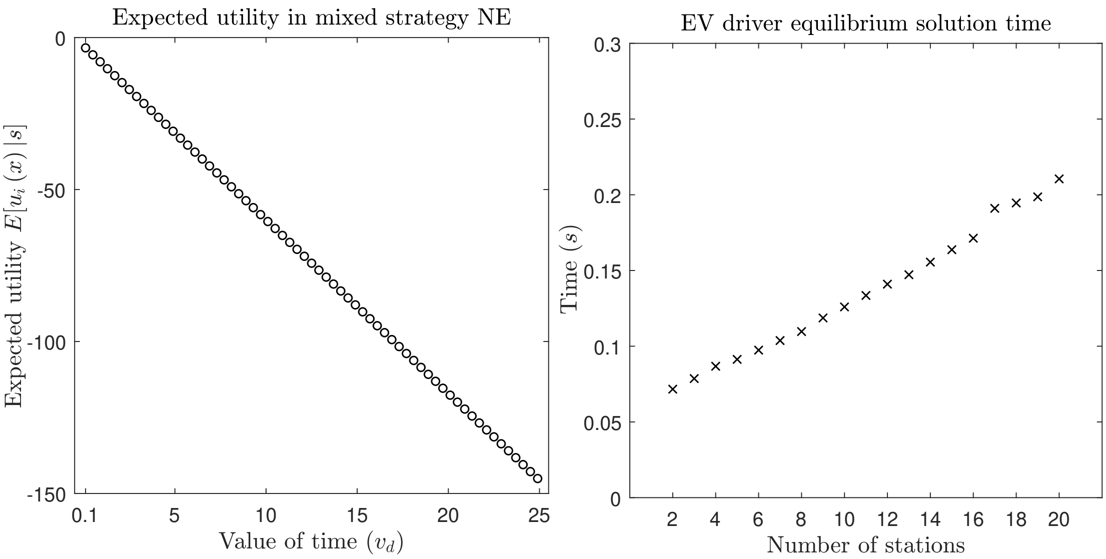


(b)

Figure 4.9: Expected queuing time in mixed strategy NE (left) and expected utility of playing in mixed strategy NE (right) for a varying number of drivers.

Utilising the same setting and additionally considering $n = 30$ drivers, the value of time v_d is explored. At this stage, the value of time does not really affect driver behaviour and its purpose is for station investors to later get a more realistic perspective on prices. It is expected that an increasing value of time will scale time costs upward in a linear fashion, and indeed it is so as seen in Figure 4.10, since utility decreases in a linear fashion for an increasing value of time.

Last, it is time to discuss the computational complexity of the mixed strategy NE in station choices. The symmetric equilibrium is linear with respect to most parameters, except for station capacities which are to the power -1. It is thus fairly easy to solve the system of Equation (3.8) using a symbolic solver, that is without replacing any parameter. Even so, some parameters for which the numbers are known already can be replaced which makes solving even quicker. For the purpose of this thesis, Matlab's symbolic toolkit was used and the process was run on a single core of a quad-core Intel i7-4790k at 4.8GHz. The run times for an increasing number of stations can be seen in Figure 4.10b. For this experiment, the solution for each number of stations was iterated 500 times and the times were averaged. As expected the increase in computational time is linear. Small problems of 2-7 stations can be solved under $100ms$ and even 20 stations take about $210ms$ to solve. Consequently, the symmetric mixed NE in drivers' station choices is not



(a) (b)

Figure 4.10: Expected utility for a varying value of time (left) and mixed strategy NE complexity (right).

expected to pose any problems computationally throughout the rest of this work. Now that the parameters of the EV drivers' station choice have been explored, the station investors' capacity choice will be evaluated in the following section.

4.3 Evaluation of the Stations' Equilibrium in Capacities

This section will discuss findings that concern the station investors' pure strategy Nash equilibrium in capacities as part of a SPE solution. The evaluation follows the general idea that two station investors consider building one station each, either at the same or at different locations. Scenarios in which respective parameters are the same for both investors will be called symmetric scenarios, and scenarios in which some parameters differ for each investor will be called asymmetric scenarios. Parameter settings will follow the reference settings in Section 4.1, and deviations from these will be clarified.

The initial hypothesis, given the game defined in Section 3.3, is that investors should generally play symmetric pure strategy NE in symmetric scenarios and asymmetric pure strategy NE in asymmetric scenarios. That is, in symmetric scenarios investors should make the same capacity choice while in asymmetric scenarios stations should choose different capacities. Second, it is likely that pure strategy Nash equilibria may not exist in some cases and this has to be investigated in more detail. Toward this Section 3.5.2 already presented a thorough discussion on some qualitative aspects of the station investors' capacity game. Next, Section 4.3.1 evaluates SPEs with regard to building costs for stations, while in Section 4.3.2 SPEs are evaluated for variation in charging fees. Last, Section 4.3.3 discusses other parameters that affect station choices less directly.

So far, capacity competition was analysed in Section 3.5.2 using arbitrary costs and profit to show some theoretical results on equilibrium existence that are of note, and to avoid looking at confusing utility matrices. With the reference settings of Section 4.1, and prices set to $f_1 = f_2 = 10$, there is one SPE in which investors play capacities $(c_1 = 3, c_1 = 3)$ (see Figure 4.13a), and drivers select either station with 0.5 probability.

4.3.1 Building cost

First, equilibria will be analysed with regard to building cost. For the experiment that follows, the charging units' building cost for station 2 is kept at $b_2 = £36000$ while the cost for station 1, b_1, is varied. To show more detailed plots in how the equilibria are affected, it was chosen to present this experiment for $n = 100$ drivers.

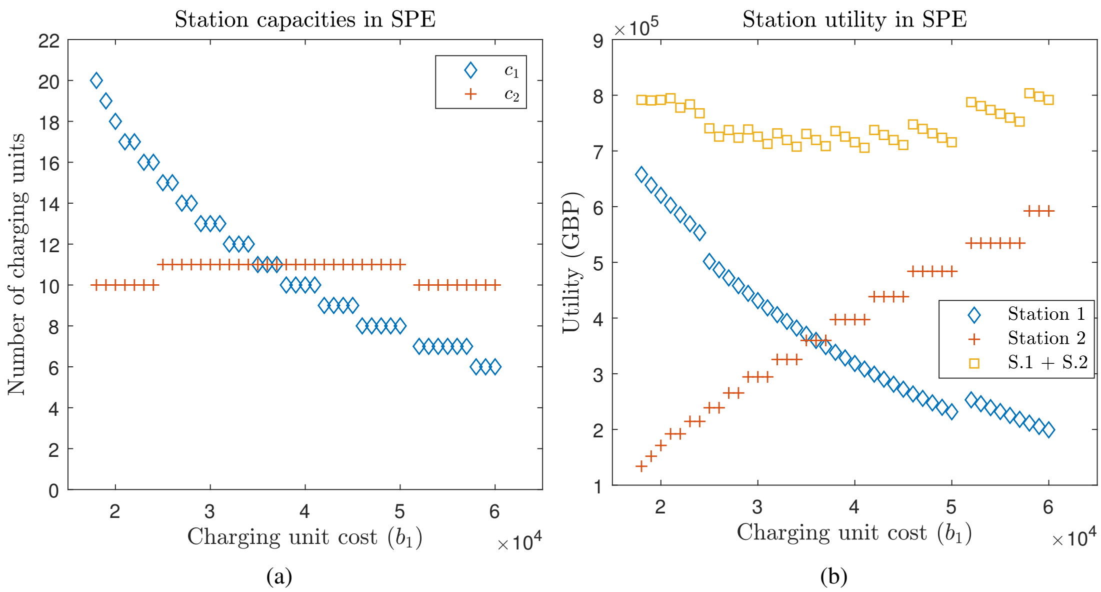


Figure 4.11: Pure strategy equilibria in capacities (left) and station utility (right) for a varying building cost in station 1.

Equilibria in capacities in general were found to exist, except in very few cases such as for $b_1 = £51000$. Looking at the equilibrium capacities in Figure 4.11a, the capacity of station 1, c_1, decreases as the building cost b_1 increases and shows a light exponential decay. This type of decay in this case means that an investor is more inclined toward reducing capacity, because of an increase in building cost, when building cost is small. Station 2, on the other hand who buys charging units at a constant price will generally keep the same capacity. There is an increase as the capacity of station 1 gets closer, and this is reasonable because when the capacities are closer an increase by station 2 will yield a good increase in the probability of choosing station 2.

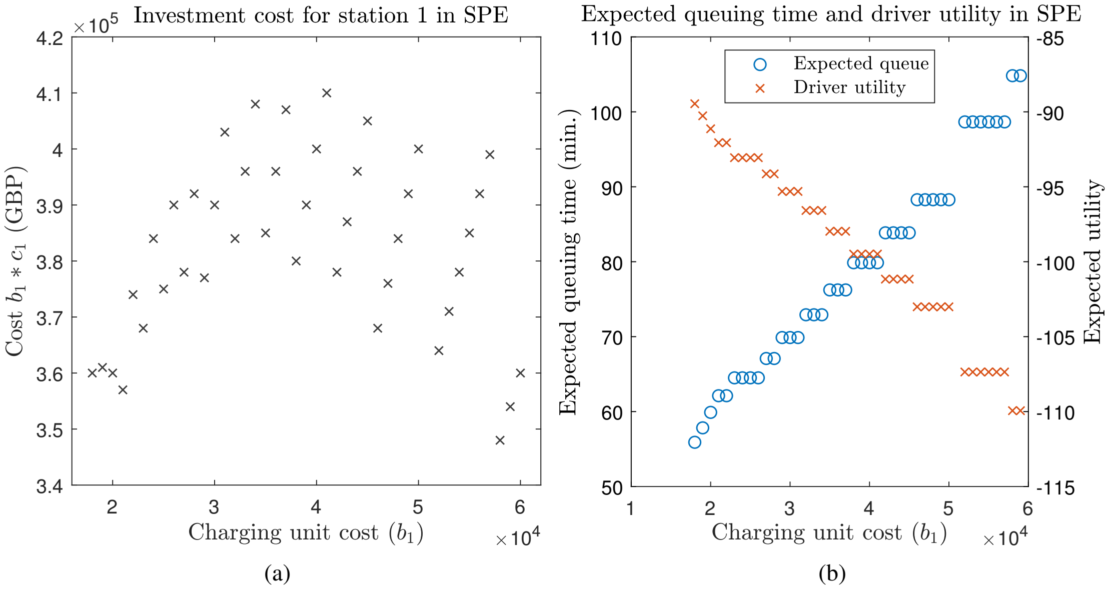
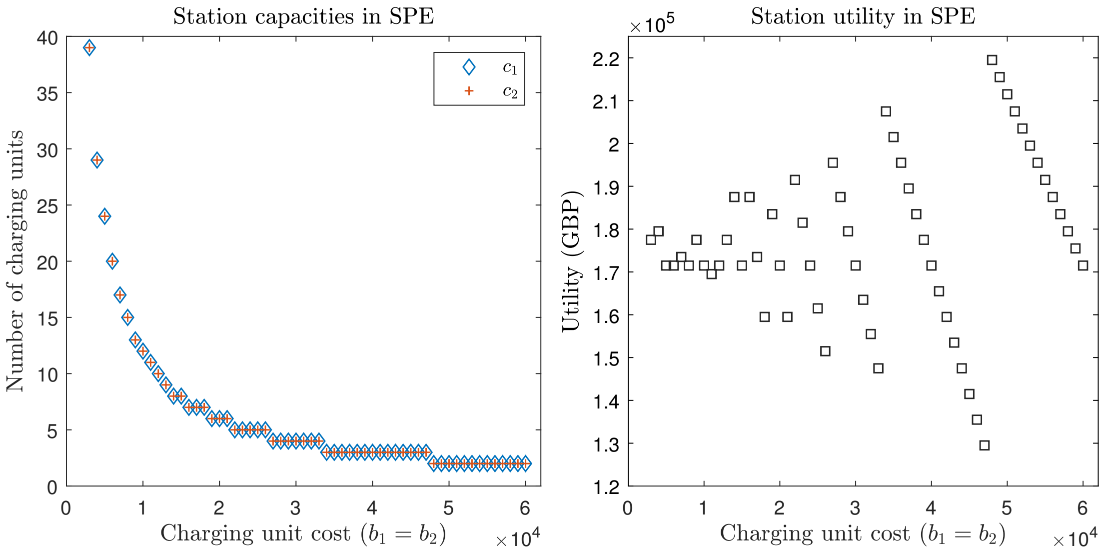


Figure 4.12: Investment cost for station 1 (left) and Expected queuing time and utility for drivers (right) for a varying building cost in station 1.

As c_1 decreases further below c_2, station 2 also reduces capacity to $c_2 = 10$ as station 1 is much less competitive at a high b_1. Station 1's utility (Figure 4.11b) shows a similar exponential decay as c_1, while the utility of station 2 increases due to an increase in EV traffic toward station 2. There is a jump in utility loss for station 1 at $b_1 = £26000$, which is the same point at which station 2 increases capacity to $c_2 = 11$. Aggregate station utility reduces slightly after $b_1 = £26000$ and increases again after $b_1 = £56000$. This is expected as now station 2 invests more in that interval, and station 1 also invests more to remain competitive as seen in Figure 4.12a. Finally the situation for the drivers is shown in Figure 4.12b, and expected queuing time for the drivers generally increases linearly for an increasing cost b_1, while expected utility decreases at the same rate, as queuing time is the only factor for driver utility that changes in this experiment. Utility and queuing time are, of course unaffected when an increase in building cost does not motivate a change in capacities.

The building cost is now explored by varying the cost for both investors ($b_1 = b_2$). The number of drivers is set to $n = 30$ for this experiment. Station capacities in Figure 4.13a show a steep exponential decay with an increasing building cost, and both stations play the same strategy in NE as expected since parameters are the same for both stations. Note that the strategy for $b_1 = b_2 = £1000$ has been omitted to increase the detail shown in the figure, but it is
$(c_1 = 60, c_2 = 60)$.

It is noted that because of the approximation in queuing time stations here can play capacities that exceed the number of drivers, as expected queuing time never becomes exactly zero. It is reasonable to assume that capacity competition would end when both stations' capacities would equal to n. This is not so much of an issue in this case, because it happens when building costs



(a) (b)

Figure 4.13: Pure strategy equilibria in capacities (left) and station utility (right) for a varying building cost in both stations.

are extremely low (i.e. £1000 − £2500) which exaggerates the problem as an investor can invest only a very small fraction of income to shift the probability of drivers coming to their station.

Station utility in Figure 4.13b is shown only for one station as they are identical, and shows that investors generally try to maintain net profit by adjusting capacities. Toward higher building cost, for example after 25000, utility decreases linearly with an increasing building cost which is reasonable as investors maintain capacities for some interval of increase, until they adjust capacity down and utility increases, following the same trend again. With regard to driver utility and expected queuing time shown in Figure 4.14, these are quasi-linear with respect to building cost. This is not unexpected, given that it was shown earlier in Section 4.2 that queuing time increases exponentially with increasing capacities, but in this case capacities decay exponentially so the result is expected to be much closer to linear. It is also noted that building cost intervals for which utility and queuing time remain unaffected, coincide with those in which capacities remain unaffected. Last, in both the experiments with building cost,

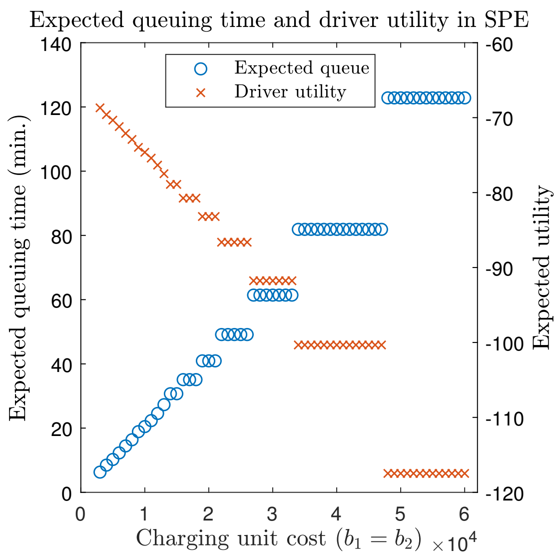
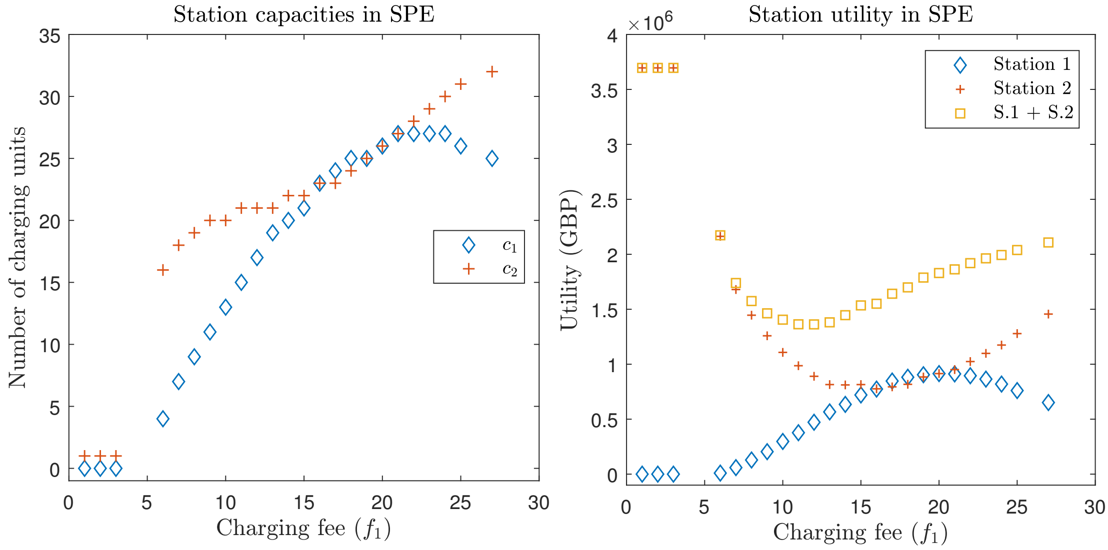


Figure 4.14: Expected queuing time and driver utility for a varying building cost in both stations.

evidence was not found to suggest that building cost might affect the complexity of calculating the Nash equilibrium in capacities.

4.3.2 Charging fees

Similar experiments were carried out with varying fees, where other parameters are identical for both investors. In the experiment that follows, the fee of station 1 is varied, while the fee of station 2 is set to $f_2 = £20$, and the number of drivers to $n = 30$. Varying one fee shows again that equilibria may not exist for some particular fee combinations. As a result, data for some points are missing from the plots and also it is not possible to examine a very wide difference in fees.



(a) (b)

Figure 4.15: Pure strategy equilibria in capacities (left) and station utility (right) for a varying charging fee at station 1.

With regard to station capacities, Figure 4.15a shows that station 1, which offers a lower fee initially, starts with a lower capacity in NE. This is expected as f_1 is very low, so increasing capacity to attract more drivers cannot outweigh the cost of investment. As f_1 increases up to $f_1 = £19$ capacity increases almost linearly, which is reasonable as the profit margin for station 1 becomes higher and this motivates an increase in investment to attract more customers. At the same time, station 2 also increases capacity to remain competitive. When the fees are about equal around $£20$, stations play similar or the same capacity, but from $f_1 = £22$ forward station 1 reduces capacity again.

The competition leads station 2 to increase capacity even further, and when f_1 is higher than f_2, station 2 offers both a lower fee and a higher capacity which station 1 cannot keep up with. This is also reflected in station utilities in Figure 4.15b, where station 1 has an advantage in utility for a slightly lower fee than $f_2 = £20$ but starts losing utility from $f_1 = £21$ onward. At that point, the cost saved from keeping capacity at $c_1 = 27$ outweighs the profit gained if station 1 increased capacity. This leads to reduced profit which reduces even further and causes station 1 to start decreasing capacity. In Figure 4.15a it is noted that for $f_1 = 1, 2, 3$ the equilibrium is for station 1 to be closed, as these prices are below or at the charging cost h and result in negative utility for station 1. In that case, station 2 has no incentive to increase capacity and plays $c_2 = 1$.

Expected queuing time and the utility for drivers are shown in Figure 4.16. Expected queuing time improves as f_1 increases, and this is in line with the earlier finding that both investors increase capacity. However, there is a turning point for queuing time at $f_1 = £23$ and from $f_1 = 24$ queuing time increases again. At that point, drivers prefer going to station 2 and the tradeoff for getting a lower price is higher queuing time due to more drivers going to station 2, despite the fact that $c_2 > c_1$. Utility for drivers shows similar behaviour, increasing initially, but has a maximum value at $f_1 = 16$ despite an increase in capacity by station 1 at $f_1 = £17$. From there it starts decreasing again due to increasing prices that outweigh any gains in queuing time.

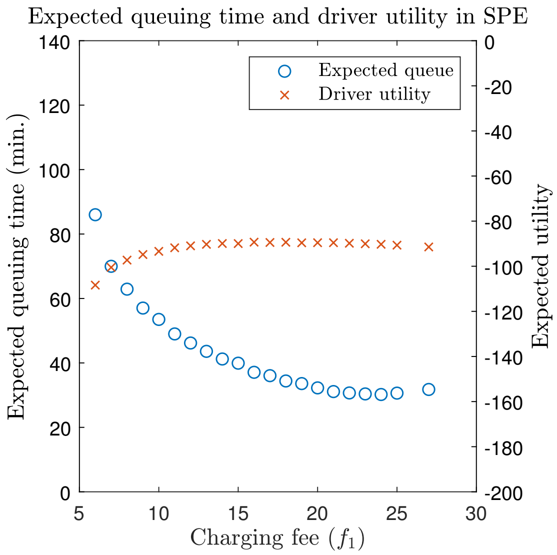
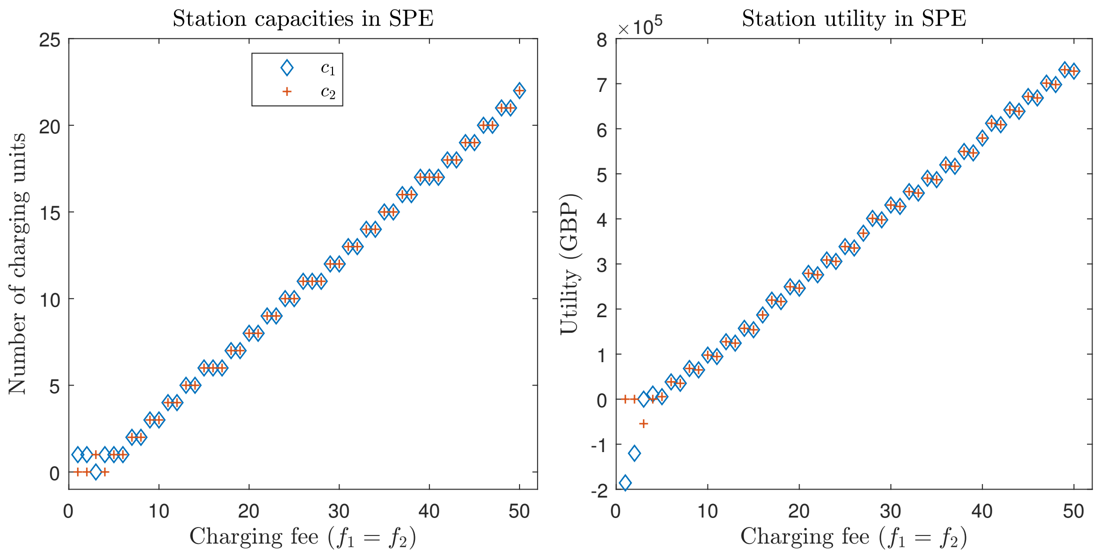


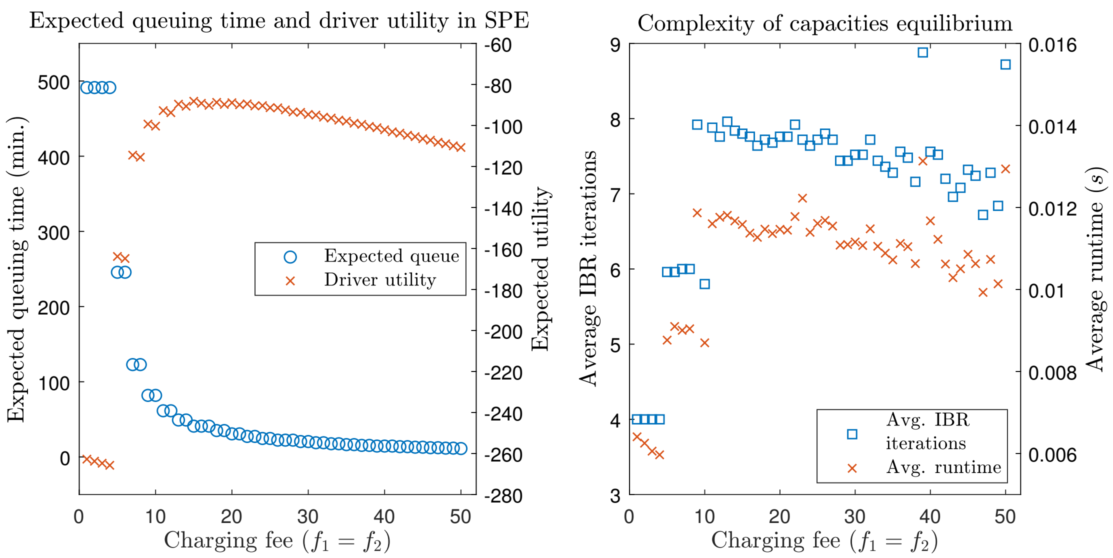
Figure 4.16: Expected queuing time and driver utility for a varying fee at station 1.



(a) (b)

Figure 4.17: Pure strategy equilibria in capacities (left) and station utility (right) for a varying charging fee at both stations.

Now charging fees are varied at the same time ($f_1 = f_2$), while the number of drivers is set to $n = 30$. Equilibrium capacities in Figure 4.17a show a linear increase, with a linear increase in station utility shown in Figure 4.17b. This is sensible, as increasing prices produces increasing profit margins for stations, which means investors are now able to increase capacity in order to attract more customers for a wider range of strategies of the other player. We note that utility initially is negative and this is the result of the example behaviour in Table 3.2. For those prices $(1, 2, 3, 4)$, utility is negative which results in two asymmetric equilibria in which one station is closed. Expected queuing time (Figure 4.18a) decreases exponentially for increasing fees, as is expected from a linear increase in capacities. While queuing time moves asymptotically toward 0, driver utility (Figure 4.18a) increases exponentially initially, but has a maximum value for $f_1 = f_2 = 15$. After that point it decreases quasi-linearly with increasing fees which is expected given that improvements in queuing time are minimal as fees increase.



(a) (b)

Figure 4.18: Expected queuing time and driver utility (left), and complexity of equilibrium in capacities (right) for a varying charging fee at both stations.

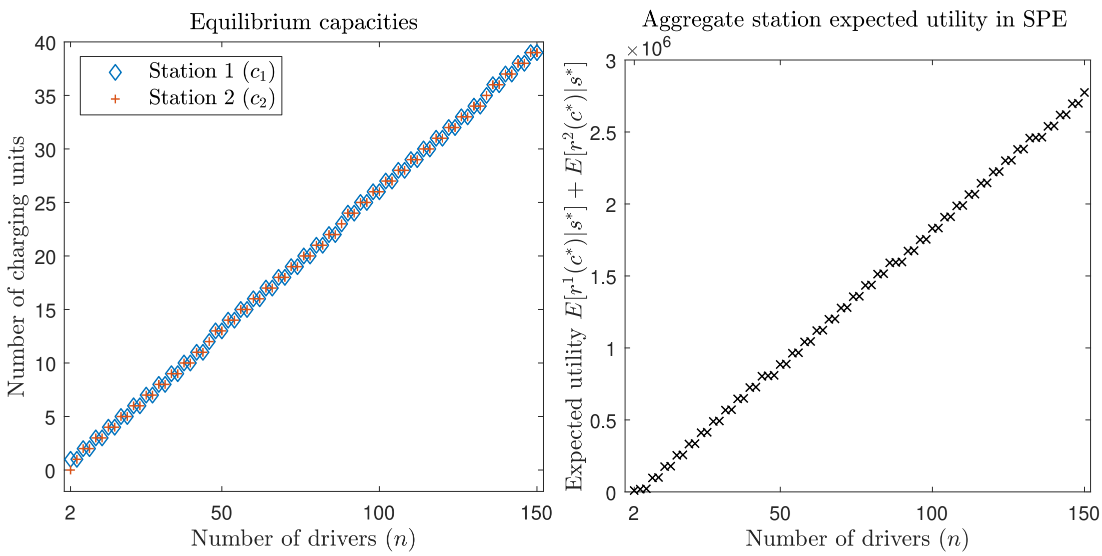
With regard to complexity, there is not enough evidence in the case of only varying f_1 to show there is a correlation between the capacities' equilibrium complexity and increasing fees. However, in the case where both fees are varied there is a much clearer picture which is shown in Figure 4.18b. For very low fees, the equilibrium is easier to find while there is a jump in computational time around $f_1 = f_2 = 10$. A reason for this is that when fees are very low, utilities can be negative for a wider range of capacities. If we visualise a utility matrix for very low fees, a very large portion of the matrix will feature only negative utilities for a station given the other station's strategy. This happens because for a high capacity by one investor the other investor cannot divert enough drivers to pay even for a capacity of 1. Therefore, positive utilities are focused toward the top-left part of the matrix. At the same time, a random initialisation of the IBR algorithm is very likely to initialise stations in such a part where utilities will only be negative no matter the strategy. Thus the first investor will maximise by playing a capacity of 0, and given that the next investor will play a capacity of 1, which sets investors close to where the equilibrium will be in only two iterations. On the other hand, investors will need more iterations to get close to the equilibrium when prices are high.

Other than the increase in computational time initially, it becomes slightly easier afterward to find the equilibrium for increasing prices beyond $f_1 = f_2 = £10$, but not significantly easier. As prices are higher, profit margins for the investors also increase which means that it is less likely for the maximum for an investor to move for a small change in capacity by the other investor. This reduces the number of iterations needed to find the equilibrium slightly, but not significantly so. Having examined building costs and charging fees, in the next section other parameters which do not influence stations' decisions as directly are examined.

4.3.3 Other parameters

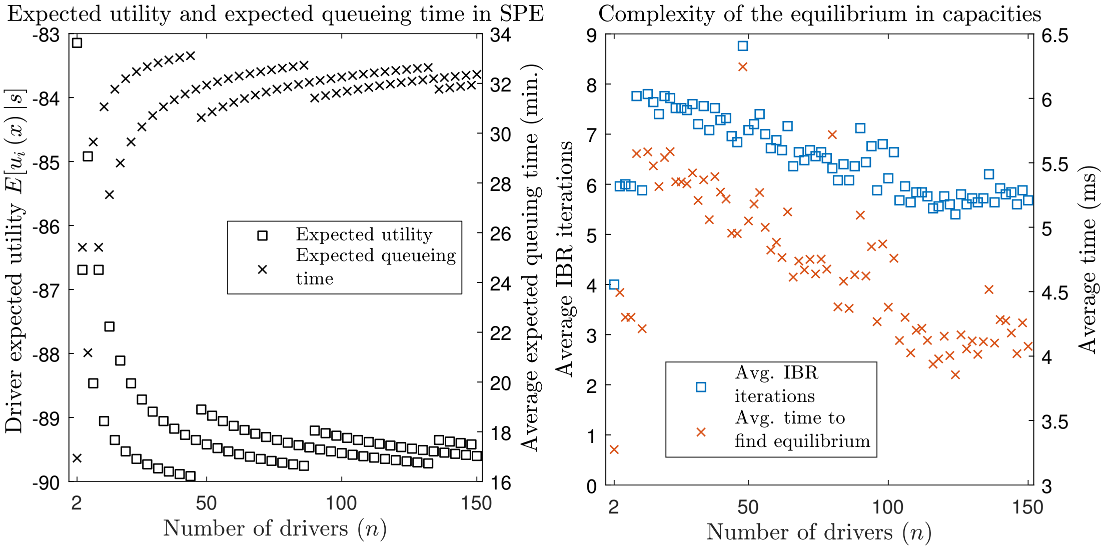
This section will explain what happens to the stations' equilibrium in capacities with variations in the number of drivers, the number of stations, travel times, charging unit power output and the value of time.

Starting with varying the number of drivers, fees for this experiment were set to $f_1 = f_2 = £20$. Capacities in Figure 4.19a show a linear increase, while stations for $n = 2$ drivers play an asymmetric equilibrium $(c_1 = 1, c_2 = 0)$. The reverse is also an equilibrium, but is not shown here for clarity. The stations' utilities (Figure 4.19b) also show a linear increase with an increasing number of drivers, which is reasonable since fees are constant.



(a) (b)

Figure 4.19: Capacities (left) and station utility (right) in subgame-perfect equilibrium for a varying number of drivers.



(a) (b)

Figure 4.20: Expected queuing time and driver utility (left), and computational complexity (right) in subgame-perfect equilibrium for a varying number of drivers.

Expected queuing time (Figure 4.20a) increases exponentially with the number of drivers, which is expected for an almost linear increase in capacities by stations. Driver utility in the same figure decreases exponentially at a similar rate with the increase in queuing time. It is noted that utilities and queuing time seem to be varying ‘discontinuously’ for slightly less or slightly more drivers. This behaviour is expected and the reason for that is that investors do not change capacities for every increase in the number of drivers. This can be seen in Figure 4.19a where stations keep the same capacities usually for 2 points on the plot. Each point represents an increase of 2 drivers from the previous[1], which means that stations increase capacity approximately every 4 additional drivers. This causes utility and queuing time for neighbouring points in the plot to vary more significantly, as when capacity increases utility improves, and when capacity is the same utility decreases.

The complexity of computing the equilibrium in Figure 4.20b with the IBR algorithm shows similar characteristics to that for varying station fees that was shown in Figure 4.18b for similar reasons, as the number of drivers also scales profits upward and makes investors’ maxima more concrete, which helps reduce the iterations needed.

With regard to travel times, a symmetric increase in travel times (i.e. $t_1 = t_2 = 2$) will not affect the equilibrium for the stations at all because, as it was shown in Figure 4.8a in Section 4.2.3, this situation does not affect the drivers’ probabilities of station choice and drivers select stations with the same probability. An increase in travel times does scale driver utility down, but there is nothing to add in this scenario to what was explained in Section 4.2.3. An asymmetric increase in travel time for one station does, however, affect the probabilities of station choice. Unfortunately there exist many cases when travel times are different where equilibria do not

[1]It was not possible to show plots for an increase by 1 driver each time and keep plots coherent.

exist, and so coherent plots are not possible. This issue is resolved with the introduction of prices in Chapter 5, but for now different travel times enable asymmetric equilibria. More specifically, the investor whose station is in an advantageous route (e.g. $t_1 < t_2$) will play a higher capacity ($c_1 > c_2$) than the investor in the longer route, and this is expected as the station in the long route needs a much higher capacity to attract the same number of drivers, something that is very costly.

On the same note, a symmetric increase in charger output will only scale driver utilities but will not affect station choice for the drivers and shows identical characteristics to the ones discussed in Section 4.2.1. An asymmetric increase, though, yields some interesting results that will now be discussed. In this experiment, the number of drivers is set to $n = 30$ and prices are at $f_1 = f_2 = £20$. The charging units' output in station 1 (O_1) is varied while in station 2 output is kept at a constant $O_2 = 50kW$ for each charging unit.

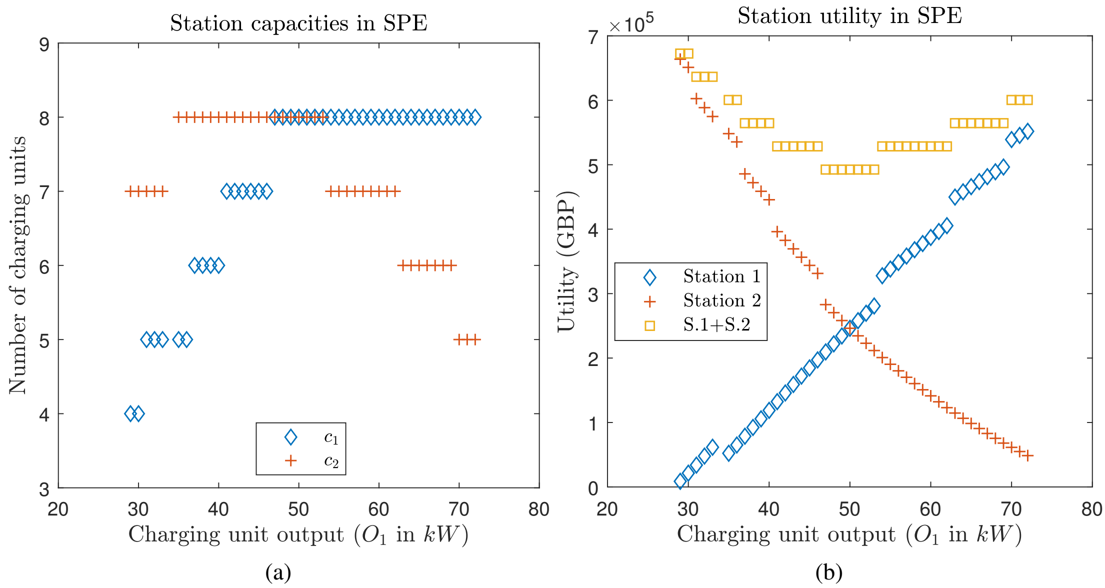


Figure 4.21: Equilibrium capacities (left), and station utility (right) in subgame-perfect equilibrium for a charging unit output at station 1.

As seen in Figure 4.21a, the investor of station 1 is at a disadvantage when O_1 is lower than $O_2 = 50kW$, and plays lower capacities than station 2. It might seem logical at this point that the reasonable thing to do would be for station 1 to play a higher capacity to try to counter the disadvantage. Then again, if we reason about station 2, station 2 can also increase capacity and it is certain that an increase by station 2 will attract more drivers than an increase from station 1 will, given station 2 offers better charging time. Therefore, station 1 is at quite a disadvantage and ends up playing a low capacity.

This situation at first leads to station 2 absorbing most of the drivers, having more capacity and better charging units; in Figure 4.21b we can see that aggregate station profit for $O_1 = 29, 30kW$ mostly consists of station 2's profit, while station 1's profit is marginal. As O_1 increases, the situation improves for station 1 who increases capacity, and at $36kW$ station 2 also increases capacity to remain competitive. The utility of station 1 increases and the utility of station 2 decreases at approximately the same rate, while aggregate station utility decreases up to $50kW$. This is logical given that investors invest more and more in capacity for the same number of drivers. Stations play a symmetric equilibrium when O_1 is near $50kW$, and at exactly $50kW$ stations have the same utility which is expected. However, when O_1 is higher than $O_2 = 50kW$, station 2 gradually reduces capacity as an increasing portion of drivers is lost to station 1.

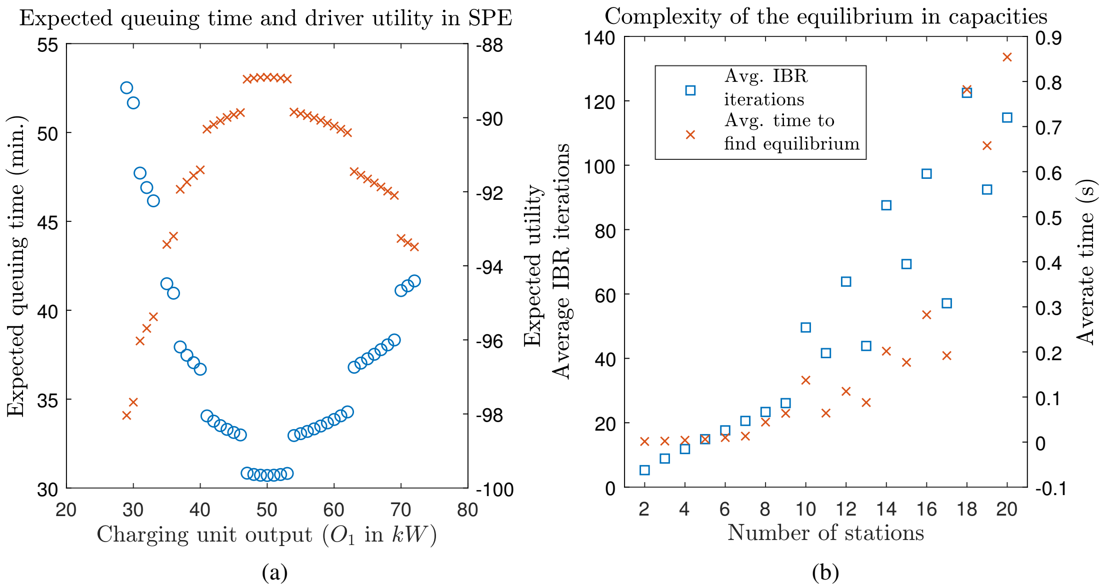


Figure 4.22: Expected queuing time and driver utility for a varying charger output at station 1 (left), and computational complexity of the capacities equilibrium for an increasing number of stations (right).

Expected queuing time for the drivers shown in Figure 4.22a decreases rapidly with an increase in O_1. This is reasonable given that station 1 charges EVs increasingly faster, and average capacity improves until the two stations have similar outputs. Then, after $O_1 = 53kW$ station 2 decreases capacity significantly and queuing time increases again despite a decrease in charging time at station 1. This is an expected outcome as the capacity of station 1 remains constant, while increasingly more drivers go to station 1, and the improvement in output is not enough yet to alleviate queues. Driver utility consequently improves and then decreases at the same rates.

Moving on to the value of time parameter v_d, this does not affect station choices at this stage other than scaling driver utility up or down. This parameter is expected to matter later on in improvements introduced in Chapter 5 and will be analysed there.

Last, to investigate the complexity of locating an equilibrium in capacities, a symmetric scenario is run for an increasing number of stations. The experiment is run 1000 times for each number of stations and the result is averaged. Figure 4.22b shows these results, where it is observed that the complexity of finding the equilibrium with the IBR algorithm increases exponentially with the number of stations, something that is expected given that the strategy space also increases exponentially with the number of stations. That being said, it is much better than the complexity of finding it traditionally. The game's complexity is affected only by station capacities and the size of the strategy set of the players. In the two-station examples here, the capacity limit was set to $\Theta = 100$ thus players consider 101 capacities. This means that if we use a traditional matrix method for finding the equilibrium, it would be necessary to maximise utility for each investor 101 times. In each maximisation step the investor considers 101 capacities, which means we would need to calculate 101×101 utilities times 2 investors. On the other hand, the IBR finds most equlibria doing only $7 - 8$ maximisations for two stations, that is 8×101 utilities which is significantly better. Even with 20 stations, computational time is under $1s$ which indicates that the model up to now is not significantly complex computationally.

Chapter 5

Competitive Pricing and Extraneous Competition

This chapter will expand on EV charging station competition, by examining the choice of prices station investors make, in addition to the capacity choice discussed in Chapter 4. Furthermore, extraneous competition will be considered in the form of an alternative option for drivers, outside the station investor system. The empirical evaluation with regard to competitive pricing is presented in Section 5.1, and the outside option is discussed in Section 5.2.

5.1 Evaluation of SPE Including Prices

In this section, the competitive pricing model for investors, which was presented in Section 3.2, will be evaluated. It will be done so in the context of the SLCOP problem, where charging station investors select capacities and then prices for their stations, and then EV drivers choose stations based on these. The theoretical analysis performed in Section 3.5.1 has shown that when the stations' capacities are taken for granted and when stations are on the same route, equilibrium prices will converge asymptotically to the cost for stations to recharge each EV, h, with an increasing charging unit output. However, it is not yet as clear what will happen when stations are in addition allowed to decide capacities. In addition, when stations are in different routes Equation (3.16) indicates that a station in a longer route will have to offer a lower price than a station in a shorter route for the same capacities, and it is interesting to study the behaviour of pricing in that case.

The empirical evaluation that will follow will enrich these theoretical findings by considering the station investors' price game as part of a SPE solution for station capacities and prices, and driver station choices. As in Chapter 4, symmetric and asymmetric two-station scenarios will be utilised. Determining the parameter values for the model follows the same logic explained in 4.1 which, to reiterate, results in the following reference settings. The value of time is set

Table 5.1: Capacities for an increasing number of stations in SPE

m	c_1	c_2	c_3	c_4	c_5	c_6	c_7
2	1	1	–	–	–	–	–
3	4	4	4	–	–	–	–
4	4	4	4	4	–	–	–
5	3	3	3	4	3	–	–
6	3	3	3	3	3	3	–
7	3	3	3	3	3	3	0

to $v_d = £15.6/$*half-hour*, the number of drivers to $n = 30$, charging unit building costs will be $b_1 = b_2 = 36000$, and one-time building costs to $o_1 = o_2 = £30000$. Travel times will be $t_1 = t_2 = 3 + 1/3$ *half-hours*, charging unit power output is $50kW$ with 85% efficiency, which sets charging time at $R_1 = R_2 = 1.1294$ *half-hours* assuming linear power output over time. The price stations buy electricity at is $£0.1/kWh$ which means that the cost for stations to recharge each EV is $h = £2.8235$. Last, profit normalisation is $w = 2190$. Any deviation these values will be clearly stated.

5.1.1 General observations

Under the settings outlined earlier in a two-station scenario, the SPE is for the two investors to play capacities $(c_1 = 1, c_2 = 1)$ and fees $(f_1 = £208.512, f_2 = £208.512)$. This seems odd at first, but is not particularly so. Now stations have the option to set prices and capacities. If we take into account that all drivers have to recharge somewhere, it is reasonable that investors minimise the cost of investment and set prices so as to maximise profit. It just goes to show that with the reference settings there is not enough competition in a two-station scenario to force investors to increase capacity. Profit margins in this case are very high and stations can compete by undercutting prices alone. Increasing the number of stations to 3 introduces more competition which shrinks profit margins and forces investors to consider capacities as well.

Equilibrium capacities for a symmetric scenario with the reference settings are shown in Table 5.1 for an increasing number of stations, and equilibrium prices are shown in Table 5.2. It is evident then by looking at those tables, that as more competition is introduced stations compete more on the choice of capacity as well, and that more competition also leads to better prices and service for drivers. That is up to a market of 6 independent stations, as from 7 stations and on it is an equilibrium for only 6 of the stations to be open. However, in that case, all combinations of 6 stations being open (or $z - 6$ stations being closed) are equilibria, as the reference settings are symmetric for investors (e.g. for 7 stations there are 7 equilibria, for 8 stations 28 and so on).

A noteworthy finding at this point, if we look at the row for 5 stations in Tables 5.1 and 5.2, is that the fourth station has different capacity and price than the other stations. With the inclusion of

Table 5.2: Prices for an increasing number of stations in SPE

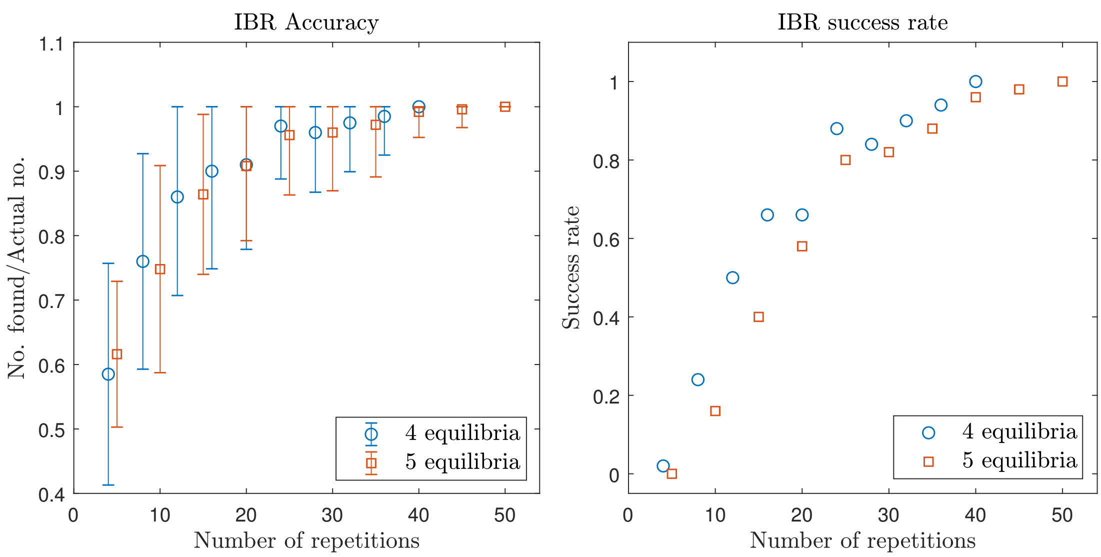
m	f_1	f_2	f_3	f_4	f_5	f_6	f_7
2	£208.51	£208.51	–	–	–	–	–
3	£28.53	£28.53	£28.53	–	–	–	–
4	£19.96	£19.96	£19.96	£19.96	–	–	–
5	£18.82	£18.82	£18.82	£19.39	£18.82	–	–
6	£16.53	£16.53	£16.53	£16.53	£16.53	£16.53	–
7	£16.53	£16.53	£16.53	£16.53	£16.53	£16.53	–

a choice in charging price, and the discontinuity in the capacities domain, it is now possible for investors to reach asymmetric equilibria even in problems where the parameters are the same for all stations. These asymmetric equilibria, again because the scenario is symmetric for investors, will be many. In this particular case for 5 stations, it is also an asymmetric equilibrium for any other investor to choose a capacity of 4 and a price of £19.39, that is there exist 5 asymmetric equilibria, while a symmetric equilibrium for investors does not exist. In the case where there are many equilibria, graphs from now on will show the asymmetric equilibrium which yields the lowest utility for firms, as this is considered the measure for efficiency (see Section 3.6). Furthermore, in the case where the equilibrium is asymmetric, and the scenario is symmetric for investors, only the equilibrium where firm 1 has the highest capacity will be shown.

With regard to the complexity of computing the equilibrium in prices symbolically, this increases slowly with the number of stations but may show an upper limit of 7-8 stations depending on the methodology followed. The derivatives of station utilities include m capacity terms, each of which include a station's capacity c_j to the power $m-1$, O(mc_j^{m-1}). In the case of an analytical solution, this is solved immediately after the equilibrium for drivers (after line 8 in Algorithm 1 in Section 3.4.2). However, for larger settings it is also possible to solve the prices equilibrium entirely numerically, by replacing all variables into station utilities each time an investor calculates the utility for a given strategy and solving the prices equilibrium there. This calculation takes about $0.05s$ for 12 stations and needs to be performed Θ times[1] each time an investor maximises utility given the strategy of the other investors. If we look back at Figure 4.22b in Section 4.3.3, the IBR needs about 65 iterations to find the capacities equilibrium for 12 stations. This means that to find a SPE by calculating the prices equilibrium numerically for 12 stations with a capacity limit of $\Theta = 10$, we need on average $0.05 \times 10 \times 65 \approx 33s$. Assuming a further 120 repetitions of the IBR to find all equilibria, we need $33s \times 120 = 66\textit{min.}$, without any parallelisation. While this is not very quick, it is reasonable time for a problem which is not time critical. The equilibrium in prices, which could be said to be time critical (i.e. investors adjust prices each day given capacities and expected traffic) is very quick to compute numerically for specific settings.

[1]For a capacity of 0 the analytical solution is not defined and utility is hard-coded to 0

If we want to ensure that we find all or most equilibria through the experiments, the following methodology is to be followed. First, the threshold K in Algorithm 1, which sets the number of repetitions for the IBR algorithm (each repetition locates one SPE), is to be set at a high value, for example $K = 1000$, to find all equilibria. Alternatively, many short runs can help determine the actual number of equilibria. From there, it has been determined empirically that if ψ is the actual number of equilibria in the game, a value $K = 10\psi$ is a value that guarantees finding all equilibria. For example a two-station scenario that is presented later in Section 5.2.3 has been used. When one-time building costs are the same and equal to $\pounds 30000$, the game has four equilibria. When the building cost for the first firm is then increased to $o_1 = \pounds 350000$, the game has five equilibria. This game is run in each of the two settings for an increasing number of repetitions K, 100 times for each K. Two metrics are recorded in this experiment. The first is the accuracy of the algorithm, that is the proportion of equilibria it finds compared to the true number of equilibria. The second is the success rate of the algorithm, that is for each K (across its 100 instances) the number of times all equilibria were found is recorded, and the success rate tells us how likely it is for the algorithm to find all equilibria at that K. The accuracy of the IBR is seen in Figure 5.1a, and the success rate in Figure 5.1b. We note that both show logarithmic increase with an increasing number of repetitions, therefore for very large generic problems that can have many equilibria, it may be good enough to set a K which guarantees a success rate smaller than 1.



(a) (b)

Figure 5.1: Accuracy (left), and success rate (right) of the IBR Algorithm in finding equilibria for an increasing number of repetitions.

One issue that exists so far is that it is not possible to measure the efficiency of the investors' equlibrium. That is so, because the optimum centralised strategy for investors would always be for one station to be open, setting the maximum allowed price or an infinite price. This renders measuring efficiency in terms of the utility obtained in SPE versus the optimum utility that can be obtained pointless. With these in mind, the next section will present an evaluation of SPEs with regard to building costs.

5.1.2 Building cost

When it comes to the cost of building capacity, a symmetric increase in building costs will not cause any change in the equilibrium for 2 stations, as the capacities equilibrium is already $(c_1 = 1, c_2 = 1)$. The only effect in that case is that when building cost is very high stations will go to an equilibrium in capacities of $(c_1 = 1, c_2 = 0)$ and the reverse, as was explained in Section 4.3.1. This raises an interesting question of what the price is in that case. Because the model does not utilise a limit in prices, the price in that case for the open station should be infinite. If a prices limit is utilised, then the utility for that investor will have a maximum value at the highest allowed price, since drivers do not have any other option. This behaviour is relatively problematic, but is resolved with the inclusion of an outside option for customers.

Prices in SPE are not affected in a two-station symmetric scenario where building cost is varied, because as we saw in Equation (3.16) the equilibrium in prices is indifferent to station building costs. Thus prices are not affected directly by the building cost of charging units. However, they are affected indirectly through the capacity choice the stations will make under different building costs.

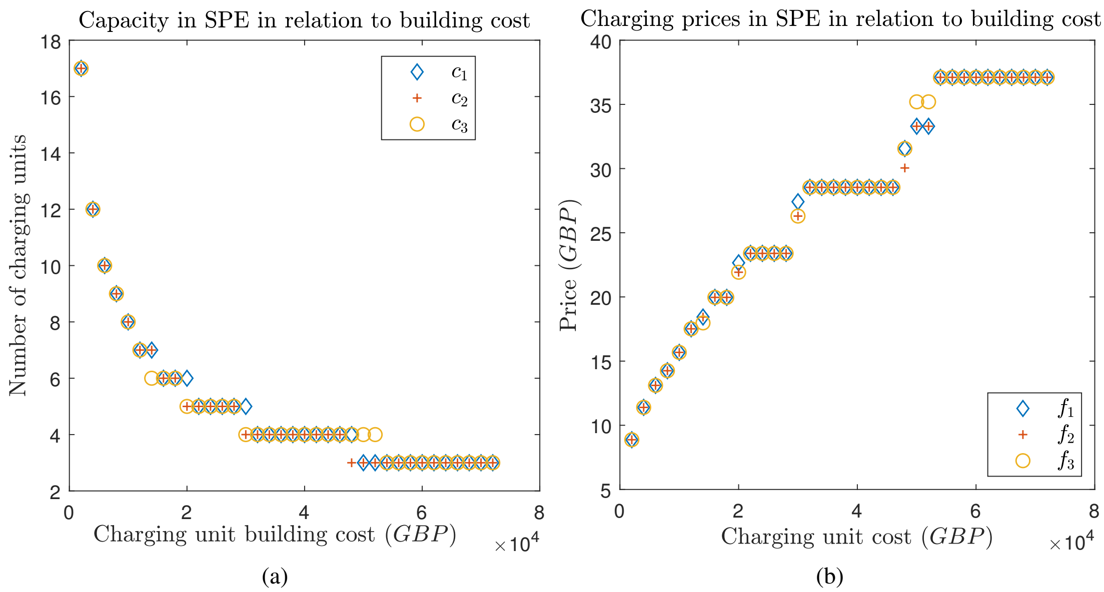
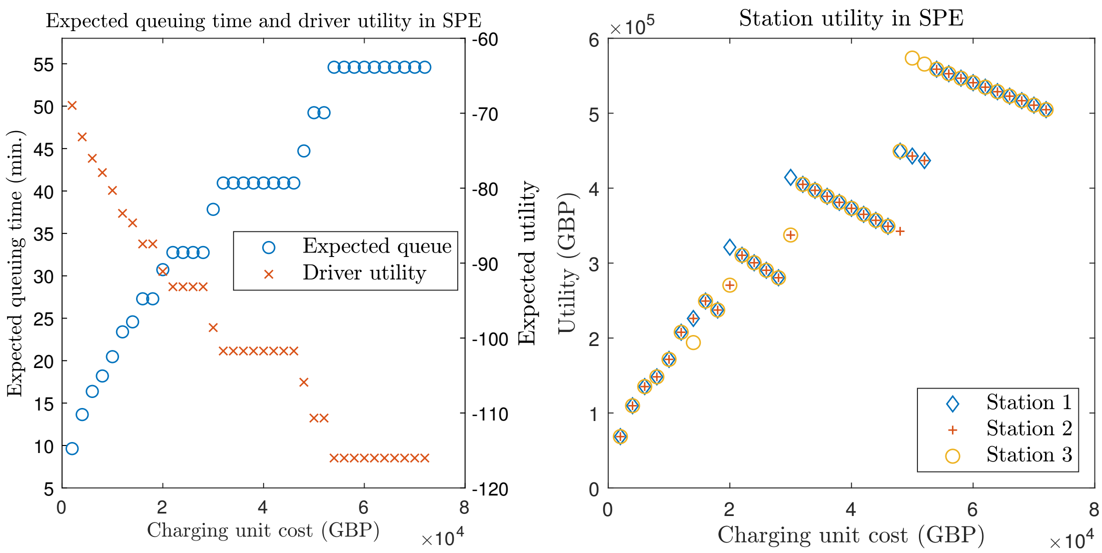


Figure 5.2: Capacities (left) and prices (right) at stations for a varying building cost in a 3-station symmetric scenario.

More interesting is to examine a 3-station symmetric scenario, where competition on investment levels is more intense. The building cost for all investors is varied by the same amount ($b_1 = b_2 = b_3$). This scenario shows that capacities (Figure 5.2a) decrease exponentially with an increasing building cost and this behaviour is in line with the finding in Figure 4.11a in Section 4.3.1 where increasing building cost with set prices showed similar behaviour.

However, now that investors can decide prices, we notice that investors are more inclined to maintain capacity and adjust it less often, since they now also increase prices quasi-linearly (shown in Figure 5.2b) when they adjust capacity. It is noted that investors now may reach asymmetric equilibria despite all parameters being equal for all investors. These asymmetric equilibria in the symmetric scenario come in triads for 3 stations, that is each asymmetric combination of prices and capacities that is a SPE, is also a SPE if different investors play these strategies. These are omitted here for clarity. With increasing building cost, investors will either reduce capacity and increase price, or maintain both capacity and price. This behaviour is expected, as it was discussed in the previous section that for a given capacity combination building cost does not affect the equilibrium price. Finally it is observed that when investors have different capacities, investors with the lower capacity ask for a lower charging fee, while investors with higher capacity ask for a higher fee. This results from the fact that investors respond to the behaviour of EV drivers by lowering price for a lower capacity to attract more drivers, in an attempt to minimise losses in utility.



(a) (b)

Figure 5.3: Expected queuing time and driver utility (left), and station utilities (right) for a varying building cost in a 3-station symmetric scenario.

With regard to drivers, expected queuing time shows a quasi-linear growth in Figure 5.3a while driver expected utility decreases also quasi-linearly and this reasonable as prices and expected queuing time show similar growth rates. Last, in Figure 5.3b, it is worth noting that despite the fact that investors generally invest more with an increasing building cost[2], station utility improves as prices become quite high and EV drivers will definitely recharge. It is now time

[2]e.g. for a cost of £4000 per unit each investor invests $12 \times$ £4000 $=$ £48000 while for a cost of £40000 each investor invests $4 \times$ £40000 $=$ £160000

to examine the charging units' power output in the next section which yields some interesting results.

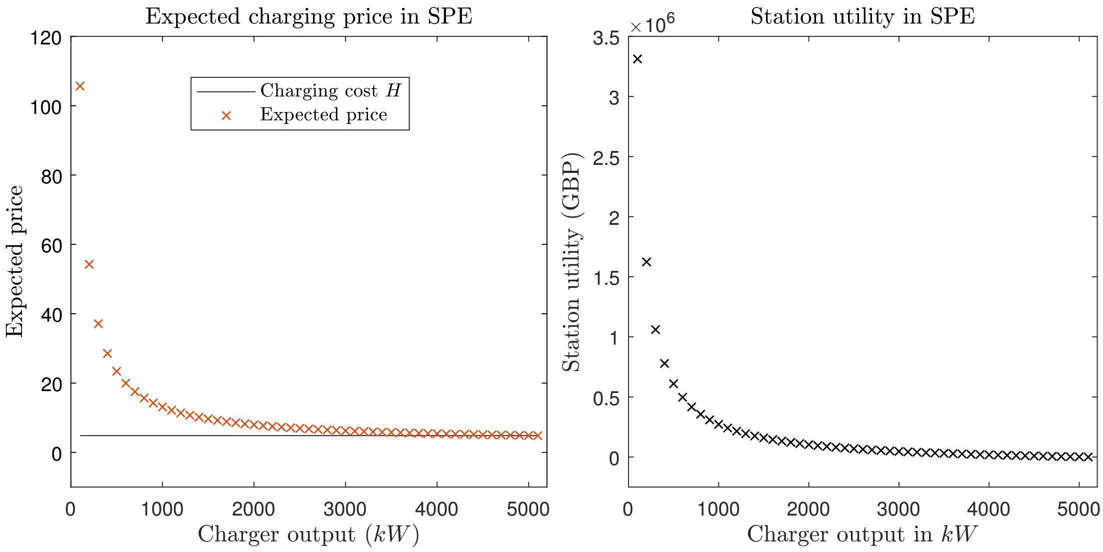
5.1.3 Charging unit power output

It was shown in Theorem 3.9 in Section 3.5.1 that if we take stations' capacities for granted, an increasing power output (lower service time) makes the equilibrium in prices converge to marginal charging cost h. This is set to $h = \pounds 2.8235$ in these experiments and is the cost of electricity for stations to recharge each EV. Intuitively, this means that drivers will have to pay more because it is not possible to satisfy charging demand immediately (i.e. to charge an EV instantly). However, the prices equilibrium is indifferent to building costs thereby making the matter of what happens now that stations can decide prices *and* capacities interesting.

The experiment that will follow utilises a two-station symmetric scenario where the output of charging units in both stations is increased identically. We expect from the findings so far that in this case stations will play capacities at 1. With regard to charging fees, when these were explored in Section 4.3.2, it was found that when stations were forced to have a very low fee that is not enough to cover up for expenses, they may actually end up having negative utility in equlibrium and this was imposed by the requirement for at least one station to be open. Now that stations decide prices, this behaviour is not expected to show here. Playing a price lower than the costs for a capacity of 1 will always be dominated by playing a capacity of 0 and ending up with 0 utility, in which case the other player will play 1 and 'set' an infinite price.

Regarding marginal cost, now that investors choose both capacities and prices, this is not only the cost of electricity but also the cost of infrastructure. If the cost of infrastructure is shared among all drivers who will recharge at stations and is added to h, this should give us the marginal cost that equilibrium prices should converge to with an increasing output. Taking into account the normalised number of drivers for profits, in this symmetric scenario it would be $H = h + \frac{c_1 b_1 + c_2 b_2 + o_1 + o_2}{nw}$.

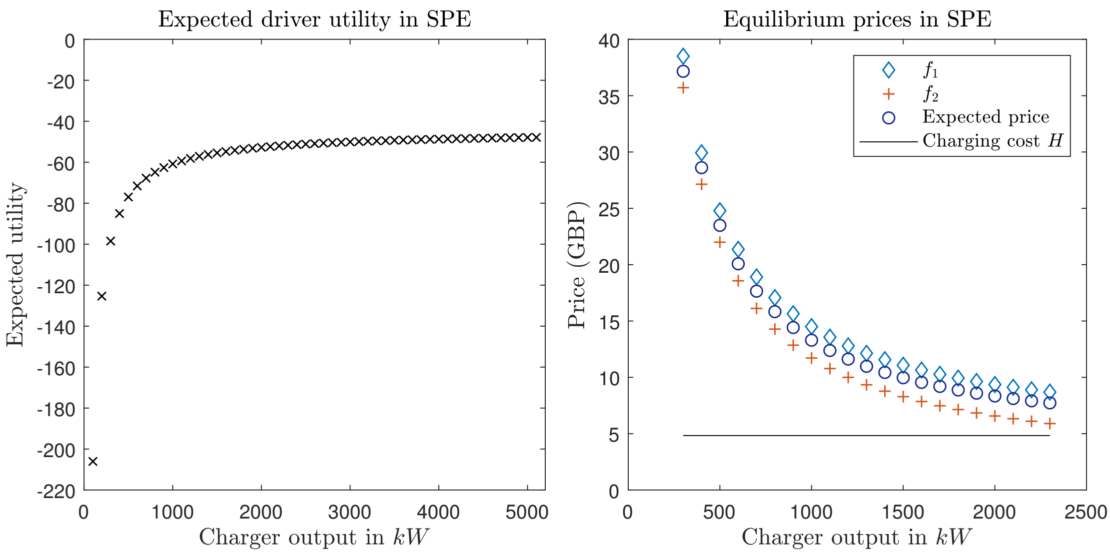
With the reference settings and given we expect stations to play capacities of 1 this is $H = 2.8235 + \frac{2 \cdot 36000 + 60000}{30 \cdot 2190} = 2.8235 + 2.0091 = 4.8326$. Results show that stations will indeed play capacities of 1 as expected. Equilibrium prices in Figure 5.4a converge to the charging cost H of charging each EV, and decay exponentially with an increasing charger output. Alternatively, reading the plot from right to left, it means that prices are expected to divert upward of the marginal charging cost H with exponential growth as the output of charging units decreases. Note that the plot shows the expected charging price, but that is equal to each of the stations' prices, since stations play the same price and thus drivers select stations with 0.5 probability each. The minimum price shown in the plot is $\pounds 4.84$ for an output of $5100kW/hour$. More detailed results show that stations will play a minimum price of $\pounds 4.8327$ for an output of $5118.838kW$, after which an increase by $0.0001kW$ will result in one or the other station closing.



(a) (b)

Figure 5.4: Equilibrium prices (left), and station utility (right) for a symmetric increase in the output of charging units in a 2-station symmetric scenario

Station utility (Figure 5.4b shows for one station, but they are the same) also decreases exponentially and approaches 0. Driver expected utility in Figure 5.5a shows steep logarithmic increase, which is expected given that queues and prices decrease exponentially. In Figure 5.5b we can additionally observe the equilibrium prices for a symmetric increase in output, when station 1 is placed on a more favourable route ($t_1 = 3, t_2 = 3 + 1/3$). Station 1 who has an advantage



(a) (b)

Figure 5.5: Driver expected utility (left) for a symmetric increase in the output of charging units in a 2-station symmetric scenario. Also equilibrium prices for an increasing output when stations are placed on different routes.

offers a higher charging fee than station 2, and this is expected from the analysis in Section 3.5.1. We notice that prices show similar exponential decay, but never quite reach H. This is because for station 2 the strategy of undercutting price is dominated earlier by a capacity choice of 0, because a large portion of drivers chooses to go to station 1. After that there is only one asymmetric equilibrium of station 1 being open.

5.1.4 Travel times

This section will analyse the SPE in capacities, prices and driver station choice with regard to travel times of the routes stations are placed at. A symmetric increase in travel times will not affect station choices when all other parameters are the same for both stations. This is expected as in that case drivers will select stations with 0.5 probability anyway, regardless of the magnitude of travel time. More interesting is the case where stations are placed at different routes. To experiment on this, a two-station scenario is considered where all parameters except for travel times are the same for both stations. Travel time for station 2 is kept at a constant $t_2 = 3 + 1/3$, while t_1 is varied, and results are examined in relation to the ratio of travel times t_1/t_2.

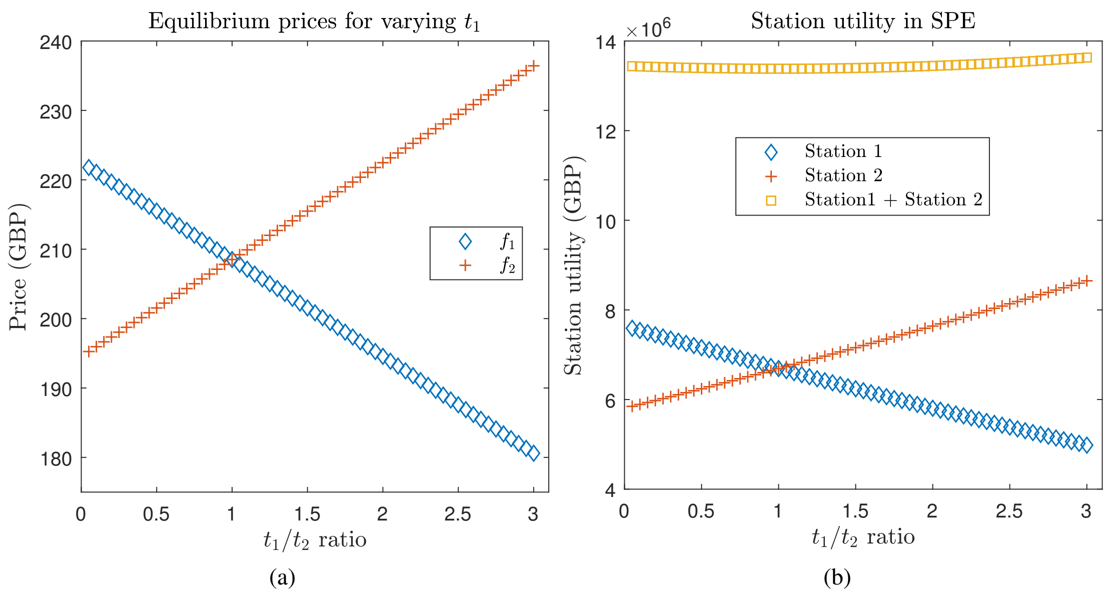


Figure 5.6: Equilibrium prices (left) and station utility (right) for a varying t_1/t_2 ratio.

The first conclusion that should be noted here is that the inclusion of the equilibrium in prices eliminates the issue of the existence of the equilibrium in capacities that was identified in the previous chapter. However, for two stations investors still both play a capacity of 1. Equilibrium prices in Figure 5.6a show that station 1 starts from a higher price than station 2, which is in line with the findings in Section 3.5.1 that a firm with an advantageous access cost can set a higher price in equilibrium than other investors. The price of station 1 decreases linearly as t_1 increases, while station 2's price increases at the same rate. At $t_1 = t_2$ ($t_1/t_2 = 1$), both stations play

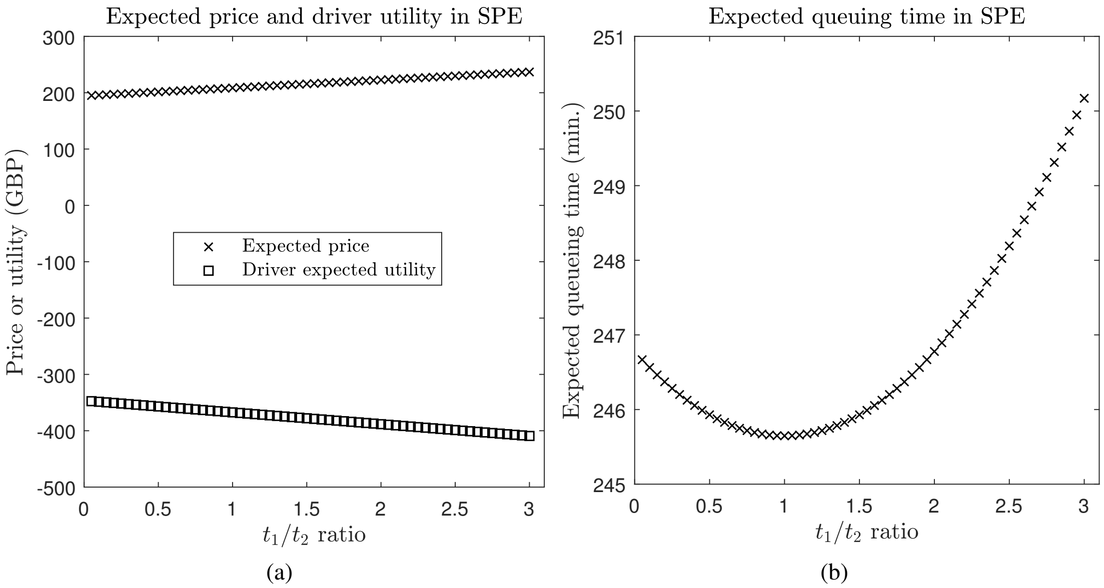


Figure 5.7: Equilibrium price and driver expected utility (left) and expected queuing time (right) for a varying t_1/t_2 ratio.

a symmetric SPE, that is the same capacities and prices. Station utility (Figure 5.6b) increases linearly for station 1, and decreases linearly for station 2, while aggregate station utility improves slowly with an increasing t_1.

Expected queuing time in Figure 5.7b shows a second order polynomial behaviour, where it decreases steeply with an increasing t_1 when $t_1/t_2 < 1$, and then increases steeply when $t_1/t_2 > 1$. This is in line with customer behaviour as it was found in Section 4.2.3 Figure 4.8a, and is reasonable given that stations will not alter capacities in this experiment. Consequently, when t_1 is very small drivers select station 1 with a high probability which increases queues dramatically, and the same happens with station 2 when t_1 is very high. The rate of decrease and increase, however, is far less steep than the one shown in Figure 4.8a which is a positive sign that the stations' adjustments in prices result in a lower rate of change in probabilities as t_1 increases. This is confirmed also by the expected utility for drivers in Figure 5.7a, which now shows a slow linear decrease at approximately the same rate expected price (also in Figure 5.7a) increases, rather than the steeper linear descent that was shown in Figure 4.8b for a varying travel time at station 1.

5.1.5 Value of time and number of drivers

So far, building costs, charging unit output and travel times have been examined. In the previous chapter, the value of time was treated as a minor parameter given that stations could not alter prices, so the value of time had no effect. However, now that investors can also choose prices the value of time needs to be examined more closely. The fact that with two stations under the reference settings the SPE results in capacities of 1, so far has not been an issue when examining

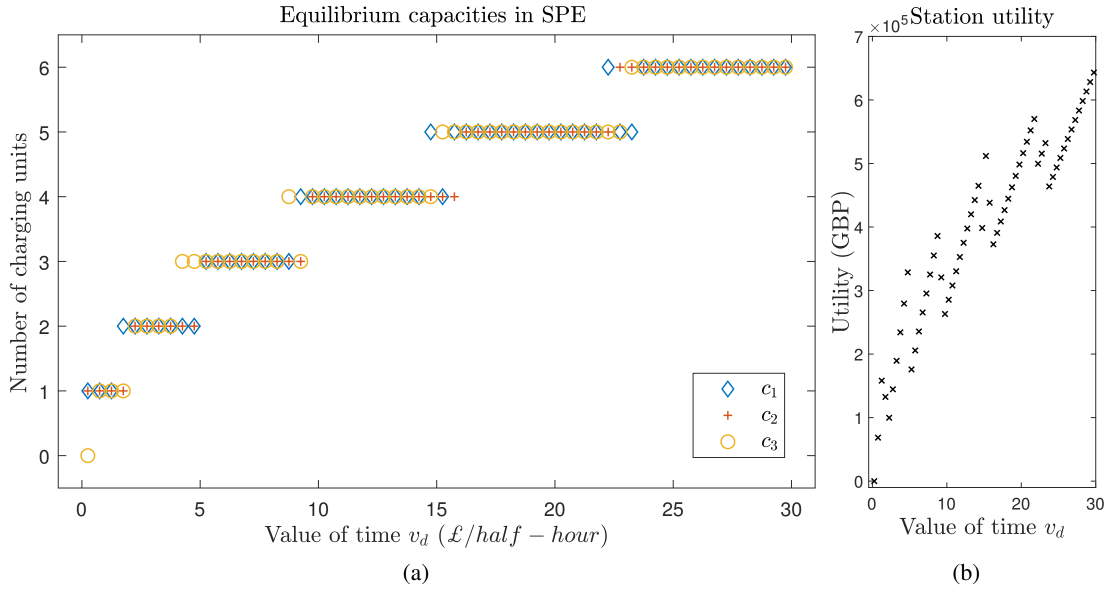


Figure 5.8: Equilibrium capacities (left) and station utility (right) for a varying value of time, in a three-station symmetric scenario.

some parameters; on the contrary it shows more clearly what is happening with equilibrium prices. However, for examining the value of time a three-station example will be utilised. That is so because the magnitude of equilibrium prices is expected to depend on the value of time as is evident from Equation 3.16, which in turn is expected to affect equilibrium capacities significantly. Consequently, it would be better to utilise a scenario which promotes competition in capacities.

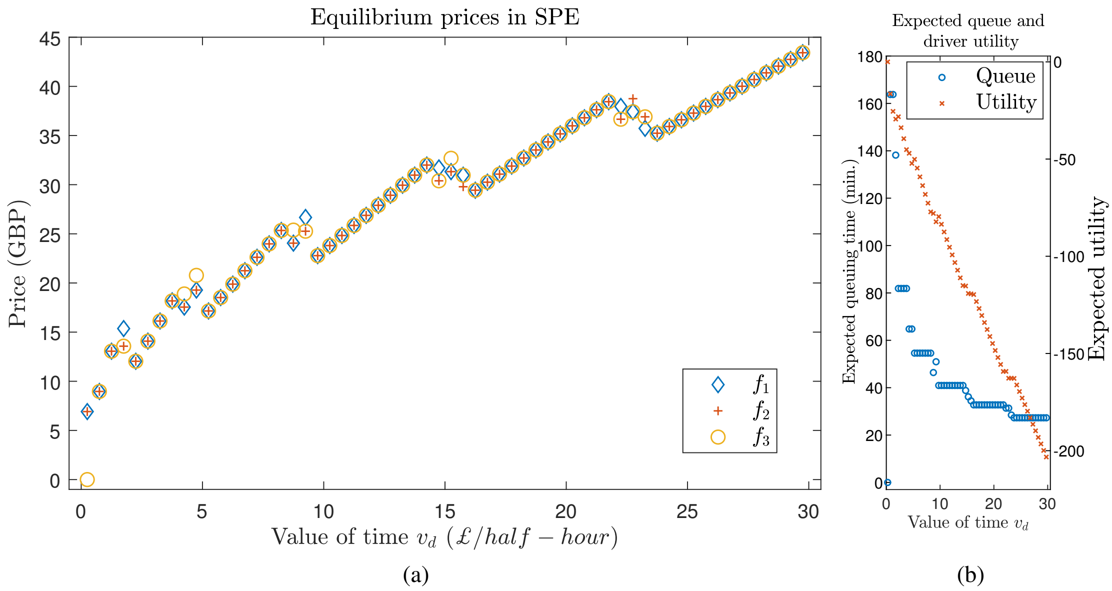


Figure 5.9: Equilibrium prices (left) and expected queuing time and driver utility (right) for a varying value of time, in a three-station symmetric scenario.

Utilising the reference settings, a symmetric scenario with 3 stations where the value of time v_d is varied is now presented. Capacities in Figure 5.8a increase almost linearly with an increasing value of time, but investors are less inclined to increase capacity as the value rises. Prices in Figure 5.9a show an almost linear increase in general and there are asymmetric equilibria in many cases. An interesting point to make is that stations that increase capacity sometimes also increase price at the same time (e.g. station 1 increases both at $v_d = 9.75$). The value of time is, in essence, a value that helps drivers weigh time costs with monetary costs. The result is that the more drivers are willing to spend for travelling quicker, they will indeed travel quicker and pay more in doing so.

The station investors, however, take advantage of this behaviour. This can be seen by the fact that whereas expected queuing time (Figure 5.9b) decreases at a mild exponential rate, albeit with large plateaus where there is no impovement, driver utility (in Figure 5.9b again) also decreases at steep linear rate. At the same time, station aggregate[3] utility in Figure 5.8b also generally increases linearly. Of course, when stations increase capacity there is a drop in utility but this is slowly recompensed by increasing prices as v_d increases, when stations do not change capacity. Therefore, it is safe to conclude at this point that generally the more drivers are willing to pay to save time, they will do so at an increasingly high cost.

To investigate this more thoroughly, the slope of the drivers' utility loss is calculated. Given that loss in utility is fairly linear, this should provide an adequate approximation. Two points in drivers' expected utility are $(1.25, -25.46)$ and $(29.75, -203.3)$. This results in a slope of -6.24 which means drivers lose 6.24 utility for each increase in value of time by 1. Figures here are shown in increments of 0.5, so to put them on the same scale, 3.12 utility is lost every 0.5 increment. This, however includes losses due to changes in fees, changes in queuing time, and changes due to scaling constant time costs with an increasing value of time. The expected queuing time is shown here in minutes, but is actually calculated in half-hours and is represented in half-hours in driver utility. Expected queuing cost every 0.5 increment is then subtracted from the next increment. These are multiplied by the corresponding value of time. Then, if we subtract these from the rate of loss we get the rate of utility loss taking into account improvements due to less queuing, scaled with the correct value of time for each increment. However, this still includes loss in utility due to scaling

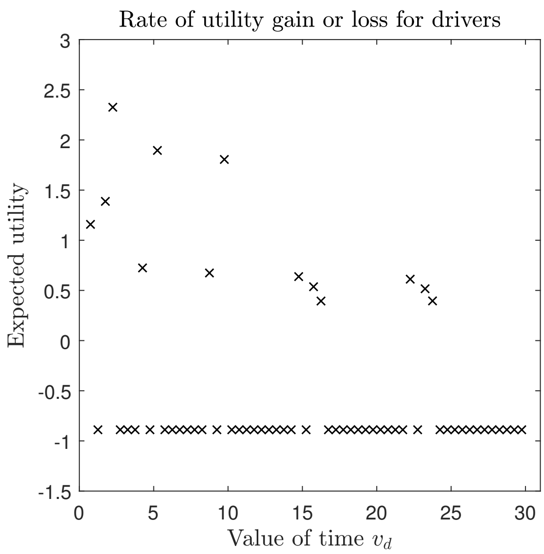


Figure 5.10: Rate of loss in drivers' expected utility due to increasing charging fees.

[3]It was chosen to show only the aggregate utility because stations' utilities are very similar.

the constant costs in travel time and charging time due to an increasing value of time. Travel time and charging time are $3+1/3$ and 1.1294 in half-hours, this means that drivers lose 4.4627 every increment of 1 for the value of time, or 2.2313 every 0.5 increment of v_d, which is also subtracted from the rate of loss. This results in the rate of loss due to changes in charging fees, which can be seen in Figure 5.10 and shows how much utility is gained or lost for every 0.5 increment of v_d due to charging fees, accounting for improvements in queuing time. While there are some significant gains in utility when queues improve, in general there is a significant loss in utility due to consistently increasing fees. Thus the original assessment was correct in that drivers keep losing utility from increasing fees despite improvements in queuing time, which explains the fact that the station investors generally increase profit with an increasing value of time.

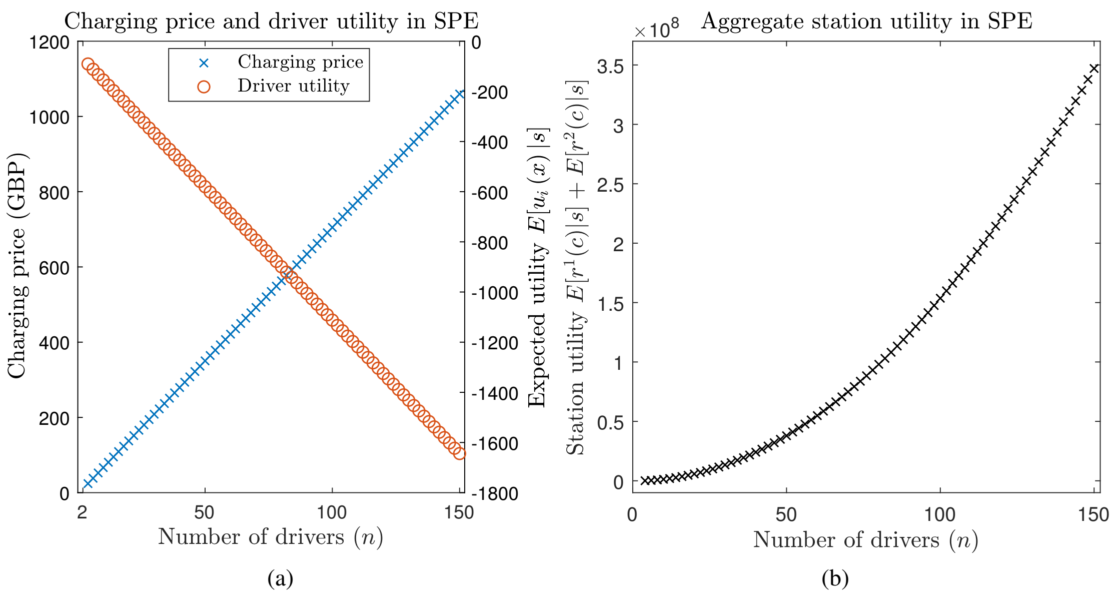


Figure 5.11: Equilibrium prices (left) and station utility (right) for a varying number of drivers.

The last parameter that will be examined is the number of drivers. This is done in a two-station setting, with reference settings and varying the number of drivers. SPE capacities are still at 1 for both stations regardless of the number of drivers, which will provide a more clear picture of equilibrium prices. From Figure 5.11a we note that charging prices[4] increase linearly with the number of drivers and this is expected from Equation (3.16) for a constant capacity and constant charging time. Station utility (Figure 5.11b) shows an exponential increase, which is logical given that the increasing number of drivers is multiplied with an increasing price. Expected queuing time (Figure 5.12) shows a linear increase, and expected utility for driver i shows a linear decrease. Expected queuing time has been defined as linear with respect to the number of drivers, and driver utility is the added cost of prices and queuing which are both linear, so by all means the model behaves as expected in this experiment.

[4]Only shown for one station, the other plays the same price.

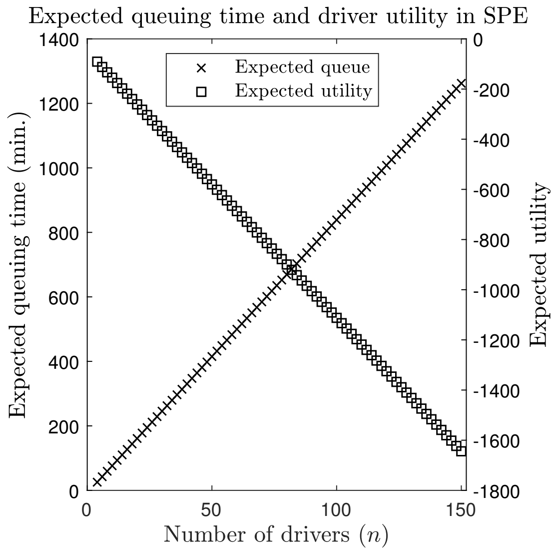


Figure 5.12: Expected queuing time and driver utility for a varying number of drivers.

5.2 Evaluation of SPE With Extraneous Competition

Up to now, evaluation has been carried out under the assumption that demand for the product (in this case charging demand) must be satisfied in its entirety. The addition of an outside option for customers in Section 3.1.4 relaxes this assumption. This is expected to change the station investors' competition, who now have to compete with a third party to maximise profit in addition to competing with other investors. That being said, investors do not really compete on the same level with the outside option as with other investors, as the provider of the outside option does not participate in investment level or pricing competition. However, it is interesting to examine competition in the new setting, where now an uncertain portion of demand will be satisfied.

To reiterate, the reference settings that have been used so far are as follows. The value of time for driving is set to $v_d = \pounds 15.6/$*half-hour*, the number of drivers to $n = 30$, charging unit building costs are set to $b_1 = b_2 = \pounds 36000$, and one-time building costs to $o_1 = o_2 = \pounds 30000$. Travel times will be $t_1 = t_2 = 3 + 1/3$ *half-hours*, charger output is $50kW$ with 85% efficiency which sets charging time at $R_1 = R_2 = 1.1294$ *half-hours*. The price stations buy electricity at is $\pounds 0.1/$*kWh* and the cost for stations to recharge each EV is $h = \pounds 2.8235$.

In the SLCOP problem that is being used as a paradigm in this thesis, it was chosen that the outside option represent a train option that drivers will also consider when deciding on their journey. Keeping in mind that the general setting considered is a trip from central Southampton to central London, this would be a realistic alternative for drivers who commute between these two areas. The value of time for taking the train has been determined to be $v_T = \pounds 18.1/$*half-hour* for the chosen trip's length and mode of transport (Department for Transport, 2015). The time needed to travel with the train is set to $t_T = 4$ *half-hours*, that is 2 hours, and it includes 20 minutes to commute to and from origin and destination train stations. This is a realistic value for someone performing the chosen trip. Finally, the fee for the journey with the train has been set to $f_T = £21.9$ which was a realistic ticket price when the experiments were carried out. It has been determined that a value of $D = 0.95$ will provide a satisfactory level of service by stations given these settings, and this will be further explained in the next section.

Table 5.3: Subgame-perfect equilibria with reference settings

c_1^*	c_2^*	f_1^*	f_2^*	s_i^{1*}	s_i^{2*}
9	6	$£24.80$	$£23.10$	0.57	0.43
6	9	$£23.10$	$£24.80$	0.43	0.57
8	7	$£24.08$	$£23.52$	0.52	0.48
7	8	$£23.52$	$£24.08$	0.48	0.52

Given these settings, in a symmetric two-station scenario there are four subgame-perfect equilibria that are asymmetric for station investors. These are shown in Table 5.3. Because the scenario is symmetric for station investors, if a strategy where station 1 plays different capacity and price than station 2 is an equilibrium, then the same strategies are an equilibrium when station 2 plays the strategy station 1 played and vice versa. It is noted also that a higher capacity at a station is accompanied by a higher charging price, while a lower capacity also yields a lower charging price. This behaviour is expected from Equation (3.16) in Section 3.5.1 where we see that a station's capacity is more important in that station's equilibrium price. This results in a lower price for a station with lower capacity. These two different combinations that result in four SPEs yield different utilities for station investors and in this case the sum of utilities is smaller for the equilibria in the last two rows than for the SPEs in the top two rows. For the purpose of presenting results in symmetric scenarios, the worst equilibria for stations will be presented on plots (explained in Section 3.6). Because such asymmetric equilibria always come in pairs in two-station symmetric scenarios, only the equilibrium where station 1 plays the higher capacity will be shown. With these in mind, customer disappointment D is explained in detail in the next section.

5.2.1 Determining a value for customer disappointment *D*

The parameter D was used in Section 3.1.4 to model the customers' disappointment at not using the service firms offer, and now the logic behind this will be further explained. In the SLCOP problem, the option for drivers to not use the stations has been introduced in the form of a train option. In reality, however, firms consider a variety of factors when setting prices and deciding the magnitude of investment. Although realistic settings for the train have been set, the idea that charging demand may not be satisfied entirely necessitates that station investors have a perspective on what level of service would be satisfactory.

Intuitively, when EV drivers are more disappointed for not using the EV, this should make them more inclined toward using it and a higher value of D should result in more drivers using their

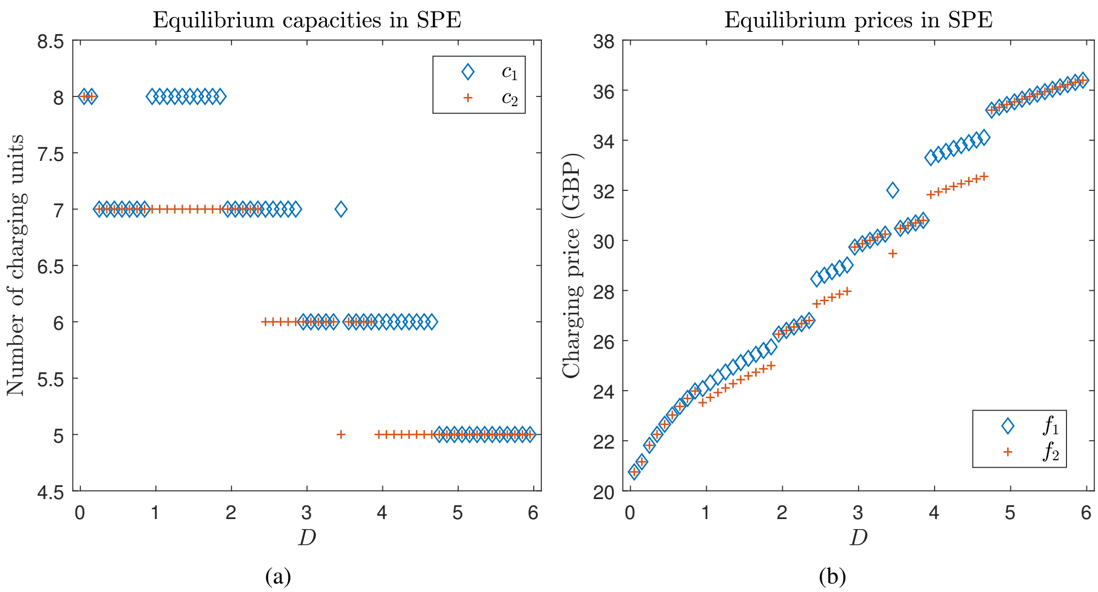
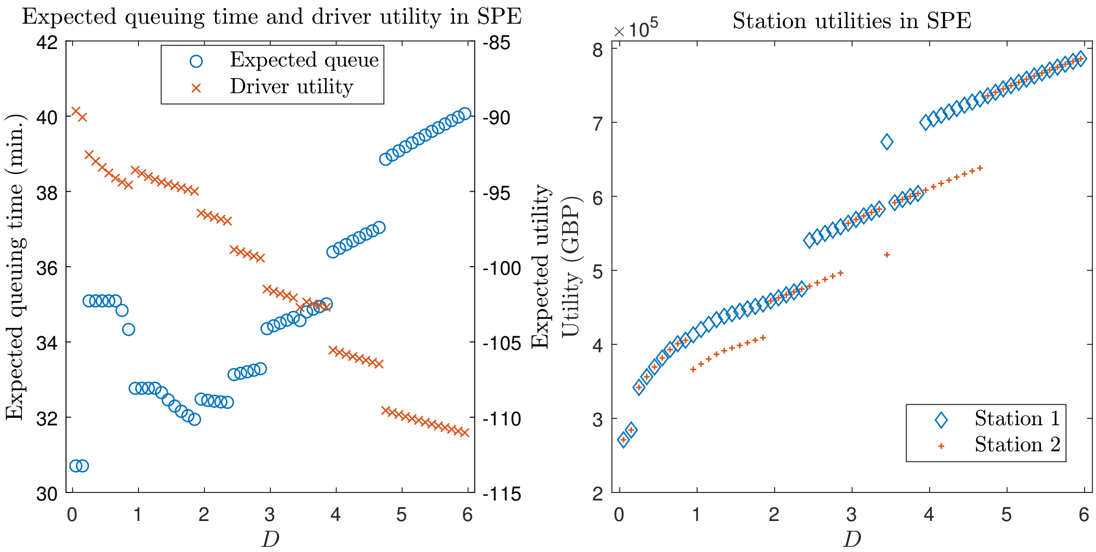


Figure 5.13: Equilibrium capacities (left) and prices (right) for a varying dissatisfaction.

EVs. Technically, by looking at Equations (3.7) and (3.8) it is straightforward to deduce that for constant capacities and prices at stations, an increasing D will turn more drivers toward using their EVs. That is so because in NE the expected utilities for each action available to driver i are equal and a lower utility for using the train will result in the equilibrium probabilities for choosing stations to increase. Thus D could be also seen as the drivers' dissatisfaction with having to use the train. However, station investors can take action whereas the train does not actively compete with stations in prices or investment levels. Hence it is expected that an increasing D will effectively set a worse benchmark for stations, and a decreasing D a better one. The analysis on the value of time in Section 5.1.5 showed that stations will increasingly take advantage of an opportunity to increase profits the greater the opportunity is, at the expense of the drivers. An increasing D that makes drivers more averse to the train provides such an opportunity, thereby station services are expected to show increasing deterioration the more drivers are inclined to use their EVs.

An exploration of D reveals that indeed investors reduce capacity (Figure 5.13a) and increase prices in Figure 5.13b with an increasing D. This increases queuing time and decreases the utility for drivers (both in Figure 5.14a), while the utility for the stations in Figure 5.14b increases as they invest less and charge more for their services. At the same time, the probability that drivers will take the train s_i^T (Figure 5.15) also rises. This initially seems to pose a causality dilemma on whether it is D that causes drivers to use the train more and this causes a worse service, or is it that drivers divert to the train because stations offer a worse service; it tends to be both, but this dilemma is distracting. In reality, an increasing D biases drivers toward using their EVs more by



(a) (b)

Figure 5.14: Expected queuing time and driver utility (left) and station utility (right) for a varying level of dissatisfaction D.

setting lower utility for the train. This in turn gives investors headroom to increase prices and decrease capacity, and investors act to take advantage of the lower benchmark in utility. In turn, this causes increasingly more drivers to take the train as services deteriorate. This can be seen in Figure 5.15 where the probability of taking the train climbs every time investors decrease capacity or increase prices significantly. Then, it slowly reduces due to that bias even though prices keep increasing, until the next change in capacity or major change in prices. This is why conceptually D is seen as the dissatisfaction for not using the EV; D will calibrate the drivers' inclination to use the EVs, and this will in turn calibrate the investors' behaviour with the outside option as a reference point to reality. This will make investors consider more realistic capacities and prices instead of competing secluded for the sake of competition. That said, an interesting aspect of the problem that emerges by looking at these results is that, just as with the value of time, the more EV drivers are inclined toward using their EVs, the more stations will take advantage of this increasing their utility at the expense of the drivers. This indicates that the EV drivers' cost minimisation and the station investors' profit maximisation are mutually exclusive concepts, meaning that constant parameter

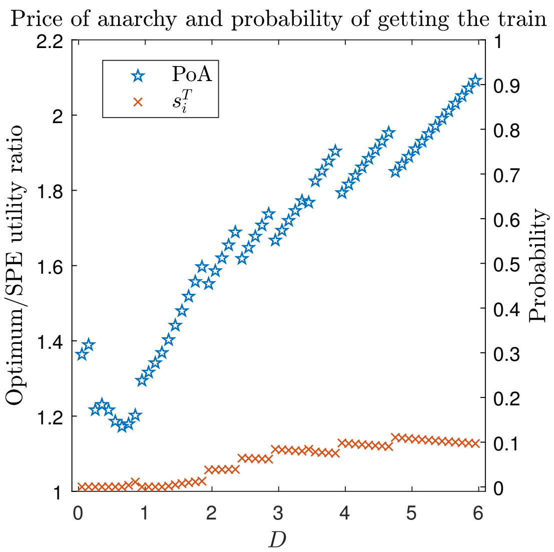


Figure 5.15: Probability of opting to use the train s_i^T for a varying level of dissatisfaction.

allocations which improve the situation for stations, will worsen the utility for drivers and vice versa. This makes the use of subgame-perfect equilibrium solutions for allocating stations all the more important compared to single-minded optimisation.

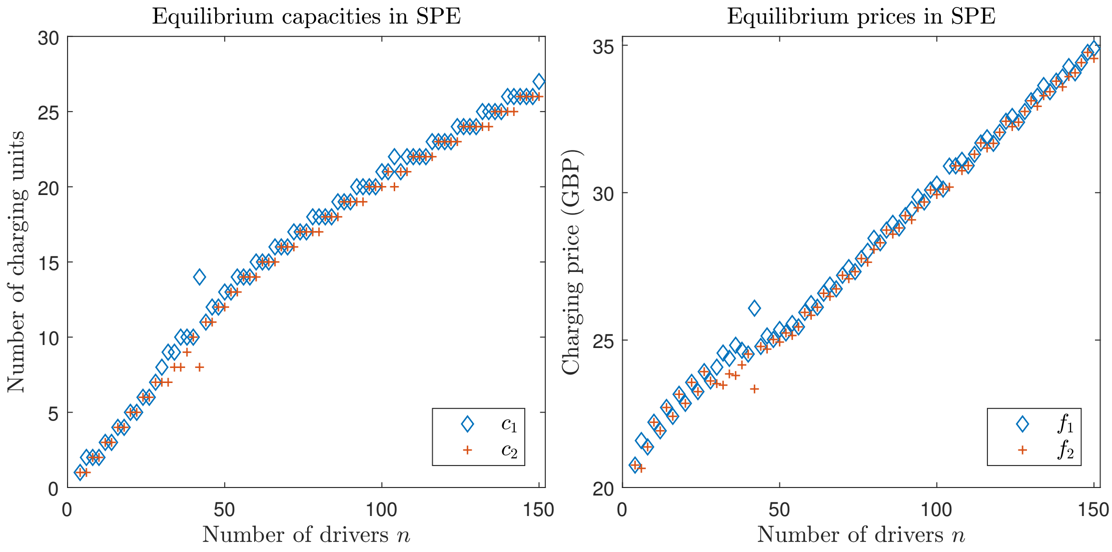
It is still a question, however, what value should be set to D. The point of this approach is for stations to offer prices and capacities that scale with a realistic economy regardless of the number of stations that compete with each other. In addition, one of the objectives of allocating station capacities and prices is for EV drivers to be using their EVs comfortably. With these in mind, it is necessary to find the point at which the probability of taking the train starts to rise. Then, a value of D slightly lower than that point is the most suitable. The logic here is that with good service by stations, no EV driver who wants to perform the trip with the EV should have to get the train instead. Therefore, setting D to account for this logic will result in capacities and prices EV drivers are satisfied with. If D is set exactly on that margin, some small increase in traffic would cause some drivers to take the train, therefore D has to be a little lower so as to allow for small fluctuations in EV traffic. With the reference settings outlined in the start of Section 5.2, the margin where the probability of taking the train becomes non-zero is $D = 1.25$, so a value of $D = 0.95$ will be used for $n = 30$ drivers and this should allow investors some headroom in competition for reasonable variations of other parameters. The next section will now examine the robustness of the model for variations in EV traffic to stations.

5.2.2 Fluctuations in traffic toward stations

In the previous section a hypothesis was posed in order to determine the value of parameter D. It was assessed that D should be set at a point which allows headroom to stations for fluctuations in EV traffic, so that the level of service does not steer excess drivers away from stations. This implicitly poses the hypothesis that an increasing number of drivers will show an increasing tendency to use the train because the model was calibrated for lower peak traffic. This section will test this hypothesis by keeping the reference settings in a symmetric two-station scenario, setting $D = 0.95$ and varying the number of drivers n.

Results show that SPE capacities in Figure 5.16a and prices (Figure 5.16b) will increase with an increasing number of drivers which is expected so far, but we notice that the slope of the quasi-linear increase in capacities decreases at $n = 60$, while the slope of prices increases at the same point. At the same time, expected queuing time starts increasing dramatically while driver utility decreases at a steep rate (both in Figure 5.17a) from $n = 60$ and on. Station utility in Figure 5.17b increases linearly, as is expected, up to $n = 60$ but then shows a slight exponential increase, as prices increase at a higher rate but the rate of investment slows down.

In Figure 5.18a it is noticed that the probability of using the train s_i^T rises from $n = 44$ onward. Actually, a more detailed look at the data reveals that the rise starts to happen at $n = 39$ drivers. Therefore, the hypothesis that an increasing number of drivers will cause drivers to use the train

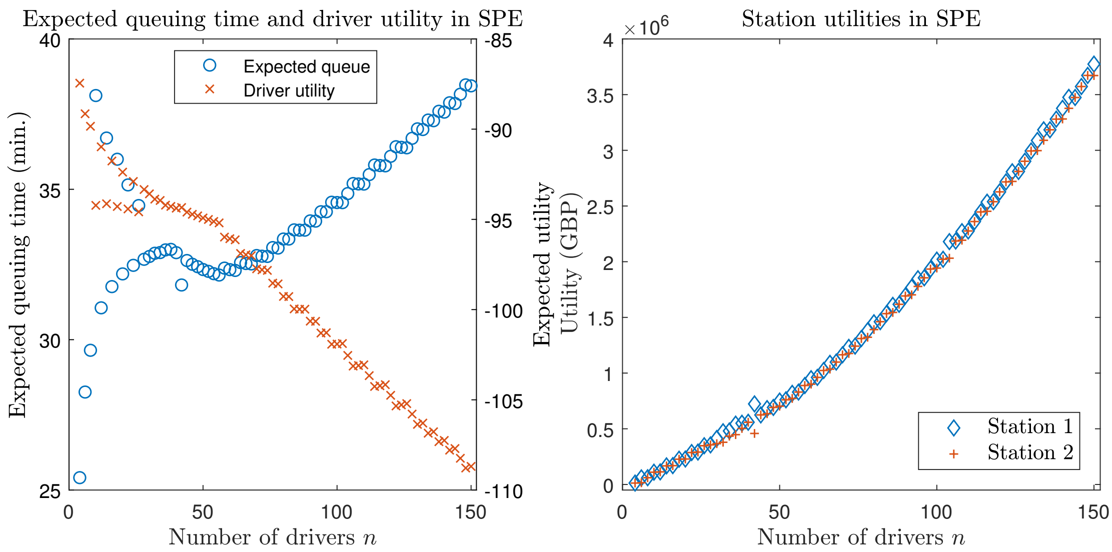


(a) (b)

Figure 5.16: Equilibrium capacities (left) and prices (right) for a varying number of drivers.

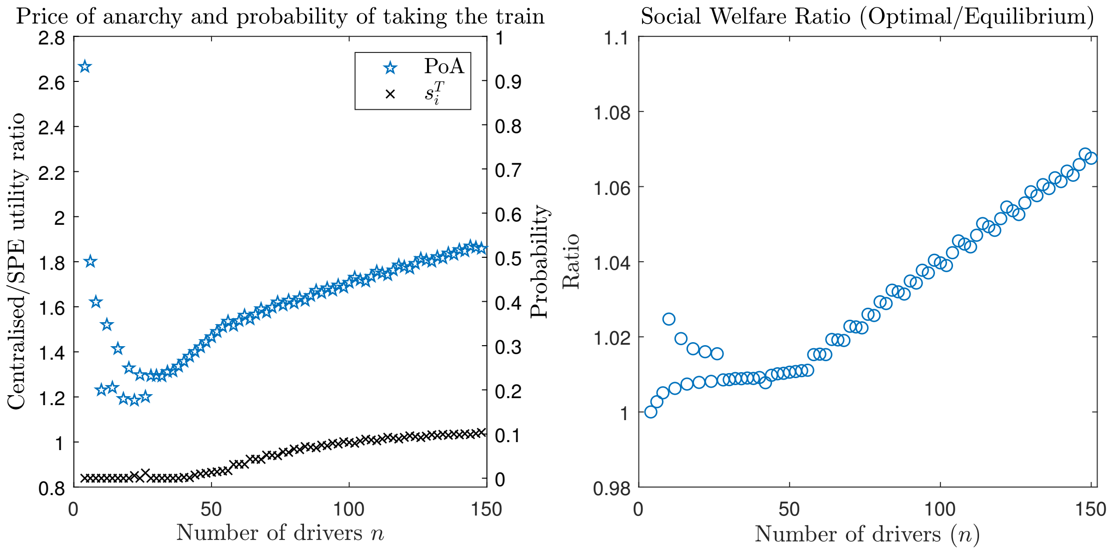
more if the model is calibrated for less drivers, is correct. In addition, the chosen value $D = 0.95$ provides headroom for a 30% increase in peak traffic before drivers start using the train.

It is also noted that the point at which the slopes of capacities, prices and utilities changes significantly is the point at which the probability of taking the train also starts climbing more rapidly. This means that D sets a limit beyond which station investors start taking advantage of



(a) (b)

Figure 5.17: Expected queuing time and driver utility (left) and station utility (right) for a varying number of drivers.

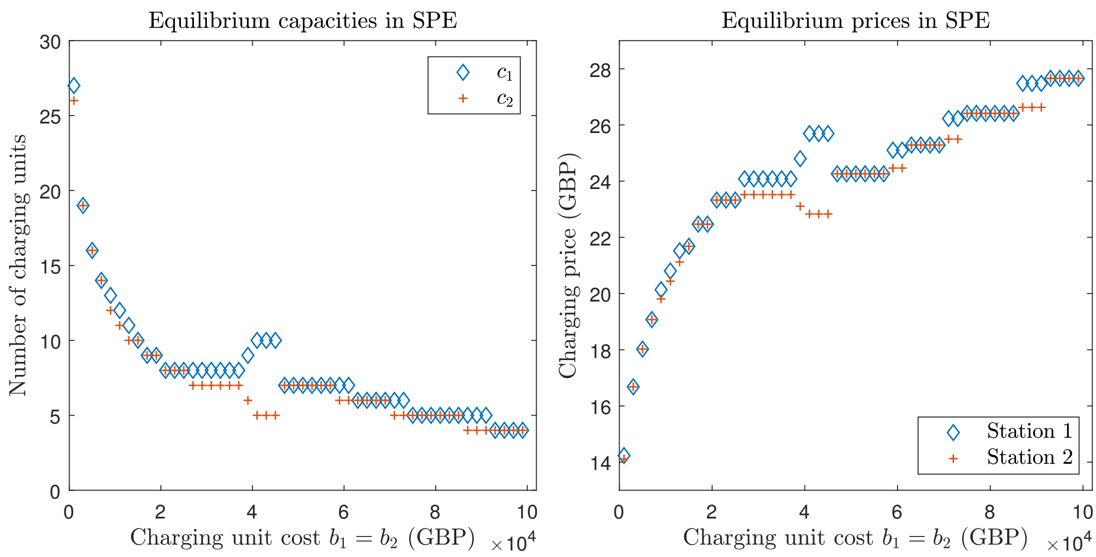


(a) (b)

Figure 5.18: Price of anarchy (PoA) and probability of opting to use the train (s_i^T) (left), and Social Welfare Ratio (right) for a varying number of drivers.

drivers increasingly and therefore it was correct to set D a little lower than the margin of taking the train. This is shown also by the fact that the price of anarchy for stations (Figure 5.18a) is relatively unaffected around 30 drivers. For $n = 30$, the price of anarchy is $PoA = 1.3$ thus the global centralised optimum allocation is 30% more efficient for stations than the SPE. A ratio of $1/PoA$ will show how efficient the SPE is in comparison to the centralised optimum. For a range of traffic from $20 - 40$ drivers, which is a reasonable fluctuation in peak traffic with $n = 30$ as a reference point, the worst-case efficiency of the SPE solution is $78.29\% \pm 3.02\%$. For a smaller fluctuation from $25 - 35$ drivers it is $79.18\% \pm 2.22\%$. Therefore, reasonable fluctuations in peak traffic do not cause severe behavioural anomalies from investors. Stations that have played in SPE for $n = 30$ drivers peak traffic can therefore micro-adjust prices daily to account for fluctuations, rather than having to build more charging units or to reduce capacity. This is also reflected in the Social Welfare Ratio (Figure 5.18b) which remains largely unaffected for small fluctuations in traffic. Moreover, the SWR shows that the SPE solution is very efficient for system-wide welfare. The optimal solution yields at most 2.4% better system-wide utility compared to the SPE, up to a peak traffic of 56 drivers, with most cases showing an increase well under 1%.

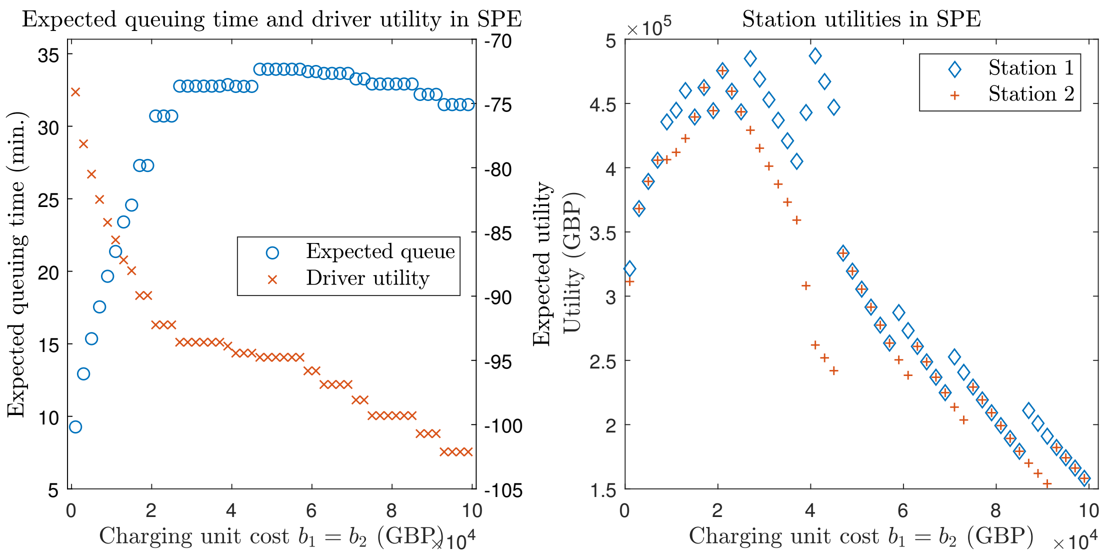
5.2.3 Fluctuations in building costs



(a) (b)

Figure 5.19: Equilibrium capacities (left) and prices (right) for a varying cost of building charging units.

On a similar note, the model's robustness toward fluctuations in building costs is examined. For this, a symmetric experiment with varying cost for building charging units will be used ($b_1 = b_2$). In Figure 5.19a it is observed that capacities decrease with an increasing building cost while prices (Figure 5.19b) increase. This is in line with the general behaviour that has been identified so far in Section 5.1.2. This results in increasing queuing time and decreasing utility



(a) (b)

Figure 5.20: Expected queuing time and driver utility (left) and station utility (right) for a varying cost of building charging units

for drivers (both in Figure 5.20a). Station utility in Figure 5.20b, however, now shows different behaviour. For increasing costs up to £41000 station utility increases, as was observed before, but now beyond that point it generally decreases rapidly. This is explained by the fact that stations now tend to maintain capacity more beyond £41000, and also do not alter equilibrium prices as often, which results in decreasing utility with an increasing building cost.

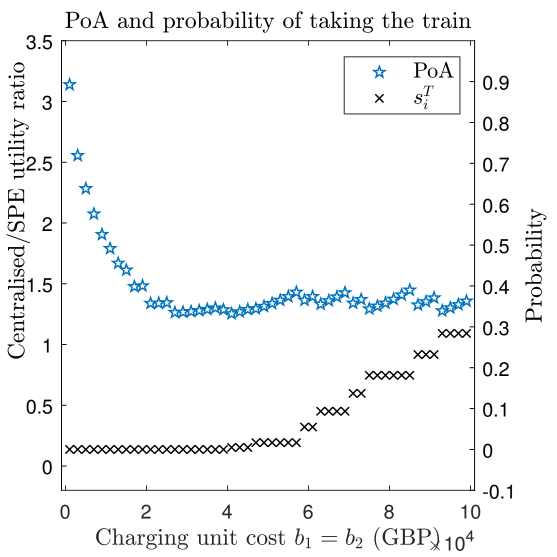


Figure 5.21: Price of anarchy (PoA) and probability of opting to use the train (s_i^T) for a varying cost of building charging units.

This behaviour is a result of the fact that beyond a building cost of £41000 per charging unit the probability of taking the train s_i^T in Figure 5.21 starts increasing, therefore stations do not reduce capacity and increase prices as rapidly as they did without the train option in Section 5.1.2. This sort of behaviour is exactly what was intended with introducing an outside option for drivers; station investors are more aware of the drivers' needs now that drivers do not have to use the stations. The price of anarchy in Figure 5.21 is the same *PoA* $= 1.3$ for $b_1 = b_2 = 36000$ and is largely unaffected by reasonable fluctuations in building cost. For a range between £31000−£41000 in building cost with £36000 as a reference point, average worst-case efficiency of a SPE solution is $77.94\% \pm 1.02\%$ therefore the calibration performed allows for reasonable fluctuations in building costs without investors making spasmodic, unpredictable decisions.

Last, it is noted that despite the fact driver utility is decreasing, queuing time is slowly improving for very high building costs, and that is because station capacity changes slowly, which in combination with the rapid increase in s_i^T means that queues are improving because drivers opt to use the train more.

As concerns one-time building costs, a symmetric increase in $o_1 = o_2$ will not have an effect up to $o_j = $ £396000 in the stations' equilibrium. That is so because it scales station utilities similarly. However, at $o_j = $ £396500 station utilities will decrease very much and this will cause one station to close, and the other to maximise against the train with a capacity of 15 and a charging price of £34.311. An asymmetric increase where only o_1 for station 1 is varied while $o_2 = $ £30000, reveals that up to $o_1 = $ £204000 nothing changes in SPEs[5], even though station 1's utility decreases.

[5]Shown in Table 5.3 at the start of Section 5.2

c_1^*	c_2^*	f_1^*	f_2^*
9	6	$£24.80$	$£23.10$
6	9	$£23.10$	$£24.80$
8	7	$£24.08$	$£23.52$
7	8	$£23.52$	$£24.08$
0	15	–	$£34.31$

Table 5.4: SPEs for $£205000 \leq o_1 \leq £374000$

c_1^*	c_2^*	f_1^*	f_2^*
9	6	$£24.80$	$£23.10$
8	7	$£24.08$	$£23.52$
7	8	$£23.52$	$£24.08$
0	15	–	$£34.31$

Table 5.5: SPEs for $£375000 \leq o_1 \leq £399000$

c_1^*	c_2^*	f_1^*	f_2^*
9	6	$£24.80$	$£23.10$
8	7	$£24.08$	$£23.52$
0	15	–	$£34.31$

Table 5.6: SPEs for $£400000 \leq o_1 \leq £442000$

$£443000 \leq o_1 \leq £499000$			
c_1^*	c_2^*	f_1^*	f_2^*
9	6	$£24.80$	$£23.10$
0	15	–	$£34.31$
$o_1 \geq £500000$			
c_1^*	c_2^*	f_1^*	f_2^*
0	15	–	$£34.31$

Table 5.7: SPEs for $o_1 \geq £443000$

However, from $£205000$ it is also an equilibrium for station 1 to be closed (Table 5.4). A further increase past $£375000$ (Table 5.5) eliminates the equilibrium where station 1's capacity is 6, and a further increase beyond $£400000$ (Table 5.6) eliminates the SPE where station 1's capacity is 7. Last, after $£443000$ (Table 5.7) the only SPE where station 1 is open is with capacities $(c_1 = 9, c_2 = 6)$ and beyond $£500000$ station 1 will not be open anymore in SPE.

This behaviour is expected, as one-time building cost does not influence equilibrium prices, therefore the maxima of stations for capacity combinations remain the same, only the utility moves up or down the y axis with a decreasing or increasing o_1. Hence, as o_1 increases SPEs with lower capacities for the disadvantaged station are eliminated as they turn to yielding negative utility.

5.2.4 Fluctuations in travel time

When it comes to route travel times, if those are increased symmetrically investors will start reducing prices. This is expected because the profit margin for investors reduces, as a larger portion of driver utility consists of travel time costs which investors can do nothing for. Investors thus have to counter this by reducing prices to compete with the train option. Capacities also decrease as a result of lower income, but a symmetric increase in travel time is not very interesting itself.

More interesting is the situation where the travel time in only one of the two routes varies. This is so because small fluctuations in the travel time ratio t_1/t_2 can conceptually represent situations in which the travel time for one route can vary, for example due to traffic congestion which the model in this thesis does not address explicitly. Therefore, it will be interesting to see whether a varying travel time ratio will provoke aggressive or more mild adjustments in capacities and prices. A more mild adjustment in capacities especially will indicate that the magnitude of investment that has been made using the calibrated model is robust against travel time fluctuations. Therefore, investors need only micro-adjust prices each day to account for traffic congestion during peak hours. For the experiment that will follow, the reference settings at the beginning of Section 5.2 will be used again in a two-station asymmetric scenario where $t_2 = 3 + 1/3$ while t_1 will be varied.

Results show that station 2 will start from a much lower capacity than station 1 (Figure 5.22a) as it is heavily disadvantaged when t_1 is miniscule. At the same time, station 1 who has much better capacity and is also on a very favourable route to start with, will ask for a very high charging price (Figure 5.22b).

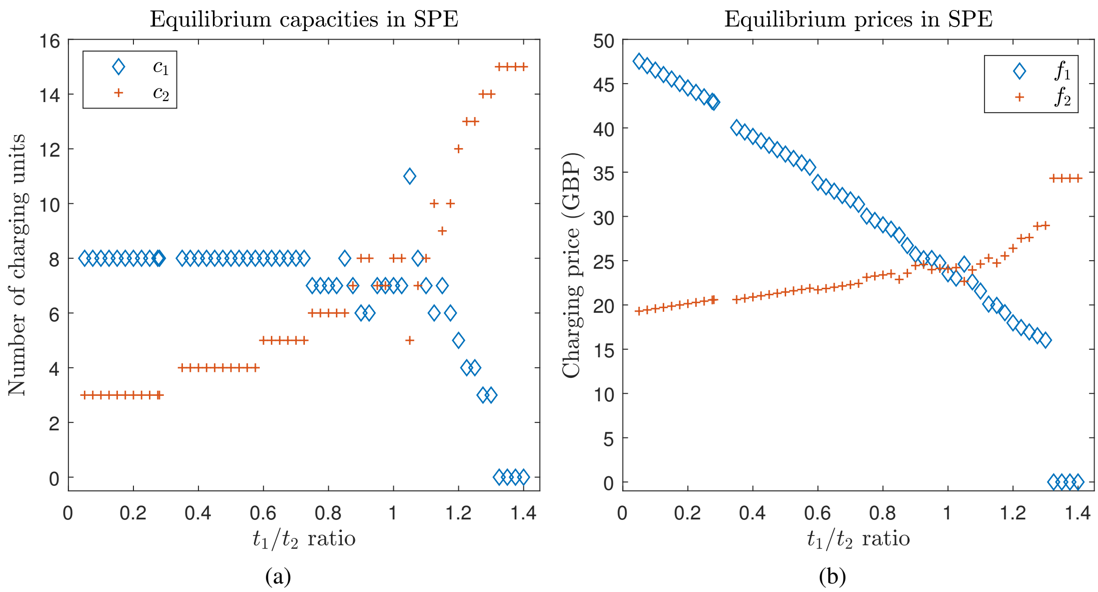
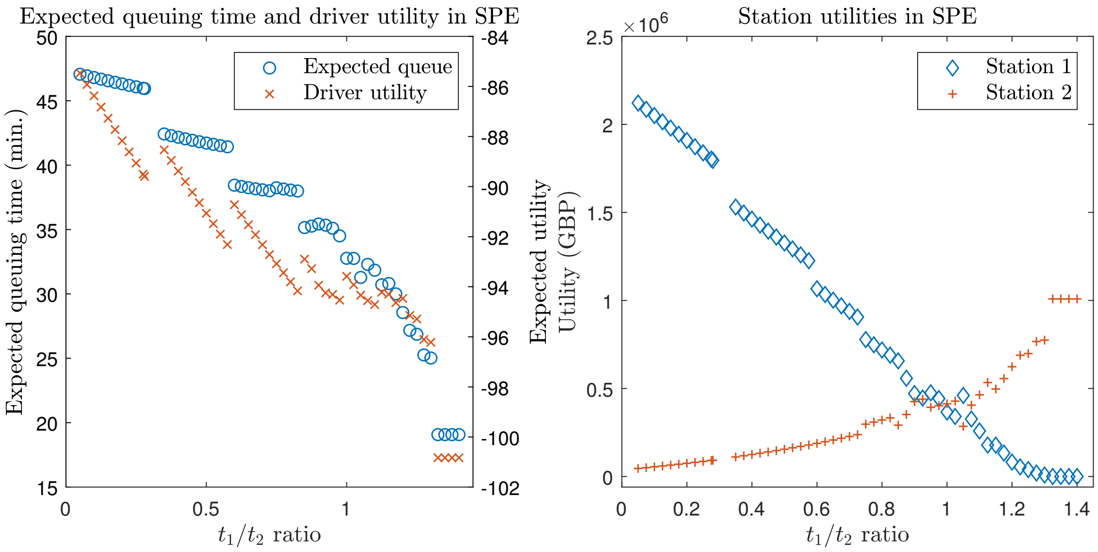


Figure 5.22: Equilibrium capacities (left) and prices (right) for a varying travel time at station 1.

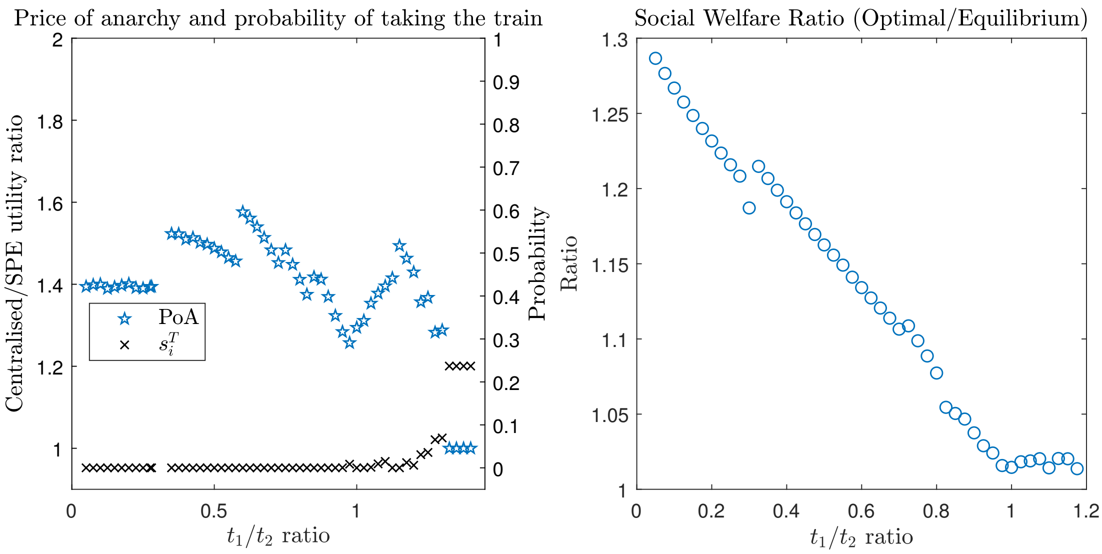
As t_1 gets closer to t_2 station 1 maintains capacity and reduces the charging price because station 2 becomes more competitive. This also gives room to station 2 to increase capacity and price. When t_1/t_2 enters more realistic levels around 1, stations engage in more close competition on investment levels. At $t_1/t_2 = 0.9$ station 1 drops capacity and station 2 increases it, resulting in station 1 having lower capacity than station 2. It is noted at this point that this is the worst SPE shown here, but it is also a SPE at $t_1/t_2 = 0.9, 0.925$ for both stations to have a capacity of 7, that is the same capacities as in $t_1/t_2 = 0.875, 0.95, 0.975$. As average capacity increases, the expected queuing time decreases but so does the utility for drivers (both in Figure 5.23a). This is reasonable, because even though queues improve expected travel time increases considerably with an increasing t_1. Moreover, this will also cause more drivers to shift toward station 2 that



(a) (b)

Figure 5.23: Expected queuing time and driver utility (left) and station utility (right) for a varying travel time at station 1.

has a longer travel time. Of course, the utility of station 1 in Figure 5.23b decreases rapidly as the advantage in travel time is lost. This is attributed to an increasing t_1 which will turn more drivers to station 2, and a decreasing price at station 1. At $t_1/t_2 = 1.2$, the travel time for station 1 is the same as the travel time for the train. As t_1 goes beyond the train's travel time, station 1 reduces capacity rapidly and station 2 increases capacity rapidly until station 1 cannot compete any more and does not open at all. Station 2 then maximises against the train. This is logical



(a) (b)

Figure 5.24: Price of anarchy (PoA) and probability of opting to use the train (s_i^T) (left), and Social Welfare Ratio for a varying travel time at station 1.

since now station 1 also loses customers to the train, for which the probability rises as seen in Figure 5.24a.

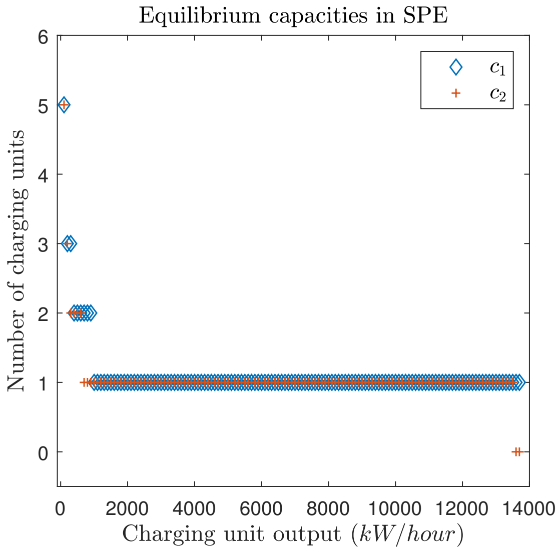
With regard to the SPEs' efficiency, the PoA for stations (Figure 5.24a) does fluctuate but not overly so. Worst-case efficiency of the SPE is $72.85\% \pm 3.47\%$ for a ratio range $0.8 - 1.2$. This represents about a +-20 minute fluctuation which is reasonable to consider for two stations on similar routes, where one route might show traffic congestion. Overall worst-case efficiency across the whole range $0.05 - 1.3$ (excluding the extrema where station 2 maximises) is $71.1\% \pm 4.7\%$. Investor behaviour is generally not spasmodic, and in the cases where capacities were swapped near $t_1/t_2 = 0.9$ there also exist SPEs where the investors maintain the capacities. Consequently, the model is reasonably robust to fluctuations in travel time. As regards system-wide utility in SPE, the Social Welfare Ratio in Figure 5.18b shows that when the two routes have comparable travel time SPEs are quite efficient compared to optimal station allocations. For example, in the travel time ratio range $0.8 - 1.2$ the SPE solution is at least 92.85% efficient. However, for larger travel time differences efficiency reduces significantly, albeit great travel time differences among routes represent less realistic station competition scenarios.

5.2.5 Charging unit power output

The last experiment that will be presented in this chapter is to explore a symmetric increase in charging unit output. It was first shown theoretically in Section 3.5.1 that equilibrium charging prices will converge asymptotically to marginal cost $h = 2.8235$ as charging demand gets closer to being satisfied immediately. That cost is the cost of electricity for stations to recharge each EV. Furthermore, in Section 5.1.3 it was shown empirically that charging prices in SPE will converge asymptotically to marginal charging cost $H = 4.8326$, which is h plus the cost of building the stations shared among the drivers that will recharge ($H = h + \frac{c_1 b_1 + c_2 b_2 + o_1 + o_2}{nw}$). However, in that experiment competition was not enough for two investors to invest in more capacity, hence SPE capacities for both stations were at 1.

It is hypothesised now that with the extension to include the outside option the convergence for very high charging output will also be H. That must be so, as a very high charging output means that investors will not have incentive to build more capacity, and will still end up playing capacities of 1. This also means that initially the marginal cost H will be different due to higher capacities and will start decreasing until the stations play capacities of 1 where it will be $H = 4.8326$ again.

Indeed, experiments show that stations start normally with higher capacities (Figure 5.25a) for low outputs and decrease capacity as the output increases. At $700kW$ station 2 reduces capacity to 1, and at $1000kW$ station 1 also does the same. From there, prices (Figure 5.25b) start an asymptotic movement toward $H = 4.8326$. At $13575kW$ charging prices are at $£4.8334$ and immediately after station 2 closes and station 1 maximises price. Note that in the end, it is also a SPE for station 1 to close and for station 2 to maximise instead.



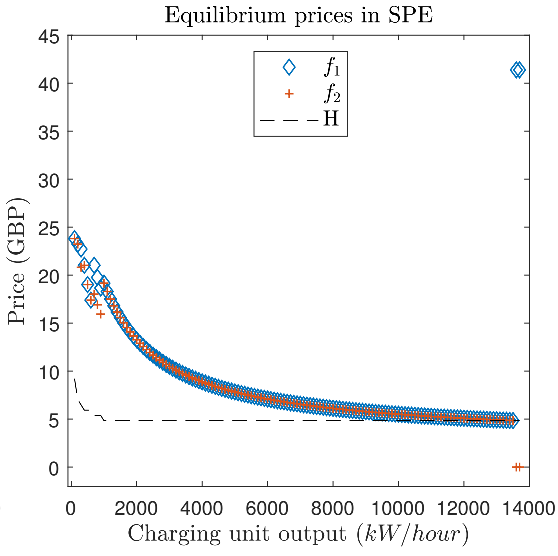


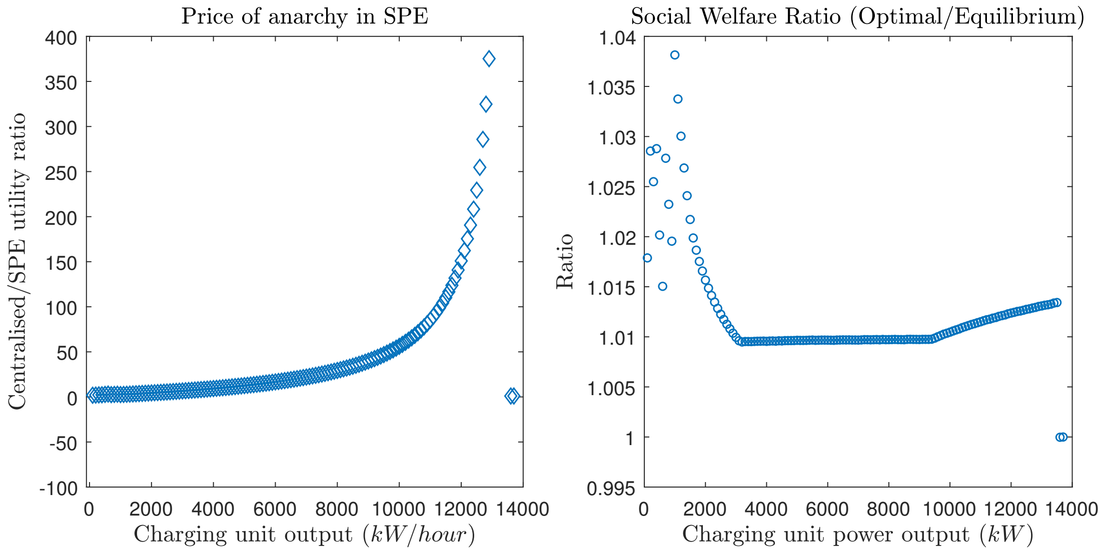
Figure 5.25: Equilibrium capacities (left) and prices (right) for an increasing charging unit power output.

It is worth to mention that the exponential decay of prices shown here is much slower than the one seen earlier in Section 5.1.3 and prices converge to H at more than double the power output. This can be attributed to two reasons. First, the stations start from much higher capacities. Until output is so high that they reach capacities of 1, convergence has not started. Second, the inclusion of the outside option forces stations to start from lower prices when capacities become 1[6]. This means that investors have a lower profit margin when convergence starts, therefore they cannot undercut prices as aggressively at first. The PoA (Figure 5.26a) increases exponentially with an increasing output which is reasonable. As output increases, SPE utility is decreasing at the exponential rate prices decrease, while stations could increase prices instead now that charging time is lower. However, because demand gets closer to being satisfied at once, charging prices become an increasingly deciding factor in the EV drivers' station choice. Consequently, investors reduce price because a strategy of price increase can always be responded to by a small undercut in price by the competitor, which will result in a large shift of drivers to the competitor. In Bertrand competition, this is exaggerated even further because in addition it is assumed that all customers will buy from the firm with the lowest price. Nevertheless, even at an output of $2000kW$, which is about 20 times better[7] than current charging technology, prices are still in the region of $\pounds 15$ which is three times the marginal charging cost. In the end, this goes to show that charging prices will be significantly higher than the marginal charging cost H because of the inability to satisfy charging demand immediately, or almost immediately.

Regarding system-wide utility in SPE, Figure 5.26b shows that SPEs are very efficient regardless of charging unit power output. Specifically, a centralised allocation for stations has been found

[6] About £20 here vs. about £208 without the outside option.

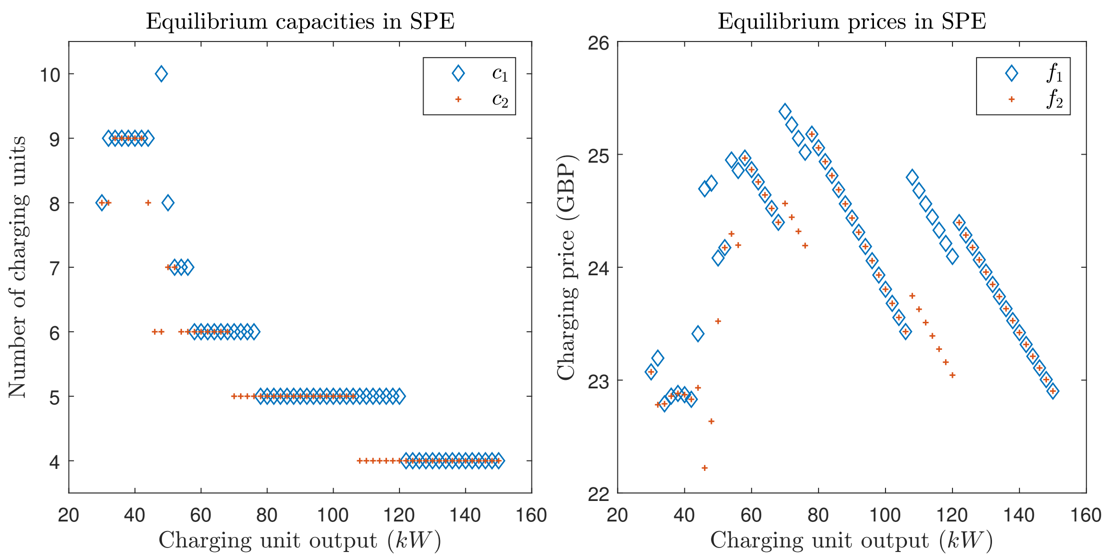
[7] e.g. Tesla's supercharger has an output of $120kW/hour$ and is already significantly better than common rapid chargers.



(a) (b)

Figure 5.26: Price of anarchy (PoA) and probability of opting to use the train (s_i^T) (left), and Social Welfare Ratio (right) for a varying charging unit power output

to be at most 3.8% more efficient than SPEs and in conjunction with the PoA for stations in Figure 5.26a we can conclude that what is lost in efficiency for stations is largely gained by the drivers. A point of criticism on the methodology followed in this experiment could be that the cost of charging units is constant. However, an increasing output is intended as a technological time-line; a reasonable assumption to make is that better technology becomes more accessible in time. In addition, inflation in cost and in drivers' buying power is going to be similar. Even then,



(a) (b)

Figure 5.27: Equilibrium capacities (left) and prices (right) for a varying charging unit output.

it is certain that prices will converge toward the marginal cost at the time, which is impossible to estimate given that such high charging unit power outputs are, perhaps, many decades away.

A more microscopic look into the same experiment reveals that we are considerably far away from a reduction in prices due to quicker satisfaction of demand, even assuming that faster technology will become more affordable. In fact, if stations are able to purchase charging units of up to $75kW$ at the price of $50kW$ units, prices (Figure 5.27b) will still rise. However, despite a reduction in capacity (Figure 5.27a) and an increase in prices driver utility (Figure 5.28a) will keep improving as better technology becomes more available due to a reduction in queuing times. After prices peak around $75kW$ they start reducing again while stations lose utility (shown in Figure 5.28b) that had peaked for $75kW$. However, even for a power output of $150kW$, prices are still not significantly lower than for $40 - 50kW$ charging units, and that is assuming all the units cost the same.

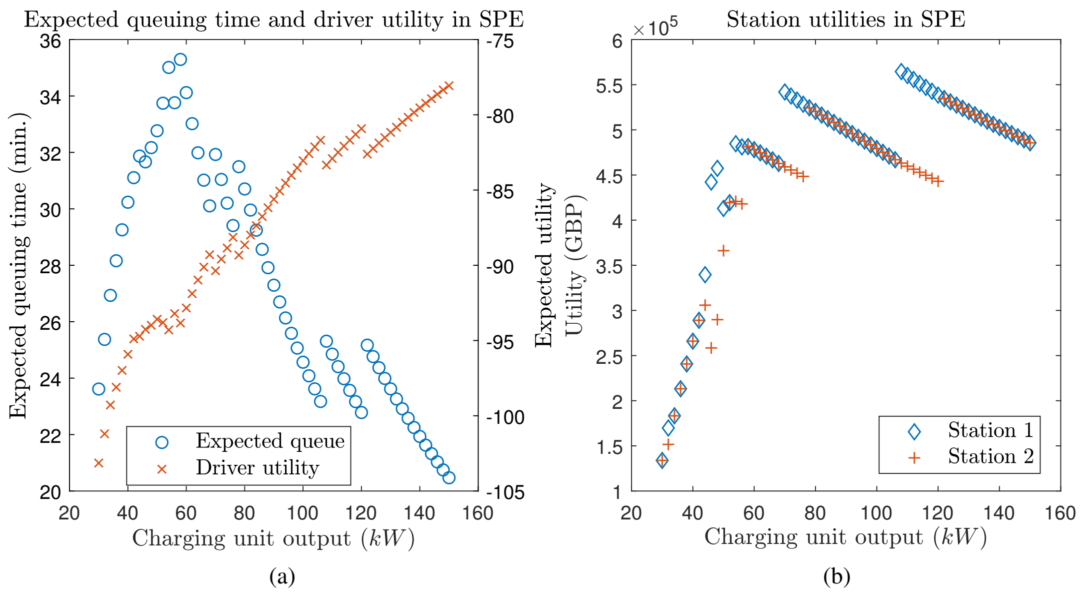


Figure 5.28: Expected queuing time and driver utility (left) and station utility (right) for a varying charging unit output.

Chapter 6

Locations, Speed of Service and Subsidies

This chapter will present further evaluation with regard to the investor location and speed of service choices, which will follow the SLCOP problem that has been analysed throughout this thesis. Furthermore, the model will be utilised to examine subsidies to charging station investors as incentives to expand rapid charging stations. First, Section 6.1 will evaluate the investors' choice of location and charging unit power output, and subsidies toward charging stations will be discussed in Section 6.2.

6.1 Evaluation of Competition in Locations and Speed of Service

The inclusion of the ability for investors to choose locations and the speed of service in the first stage of the extensive-form game which was explained in Section 3.3, first of all increases the computational complexity of the model significantly. While the customers' mixed NE in firm choice is straightforward to calculate, the equilibrium in prices is not as simple. As was explained in Section 5.1.1, the equilibrium in prices is a problem always linear with respect to the firms' fees, but it is polynomial with respect to capacities. Consequently, solving it symbolically becomes significantly more difficult with an increasing number of stations. Two things are of note in tackling the complexity of the equilibrium in prices. First, it is necessary to solve symbolically only once for a particular number of investors and locations, and then it can be used to explore various instances with the same number of locations and investors. Second, it is not necessary to solve it symbolically at all, and it can be solved numerically instead every time an investor evaluates a pure strategy c_L^k for maximising utility.

This brings us to the actual computational burden, which is the pure strategy NE in capacities, locations and speed of service. In cases where the number of available server types, and especially the number of locations is large, the strategy space C_k for the investors becomes very large. Each investor will reason selecting out of $\Theta + 1$ available capacities for each available location, and this is multiplied by the size of the server type options G. It is beneficial for the IBR algorithm to parallelise the calculation of utilities for investor k within the same iteration, using a numerical solution to the prices equilibrium. This may still, however, require many calculations of the equilibrium in prices. For example, if there are 5 locations available to each of 5 investors, the capacity limit is $\Theta = 15$ and investors have 2 server type options, an investor will have to calculate $(\Theta + 1)^5 2 = 2 \cdot 16^5 = 2097152$ utilities each iteration of the IBR, which requires about 30 hours for one iteration. One straightforward way to reduce this complexity is to consider a capacity limit that is large enough to contain the Nash equilibrium, but small enough to provide for better computational time. For example, we saw in Section 5.1.1 and Table 5.1 that 6 investors will play capacities of 3, therefore a capacity limit of 5 is enough to contain the equilibrium and at the same time to not push it artificially toward the bottom-right end of the utility matrix. This would mean the investor would now have to calculate 'only' 6250 utilities which can be done in about 6 minutes without parallelisation.

Still, computational complexity of the full model is significant even with a careful choice of Θ, but this is an issue mostly when examining generic problems. In reality, a situation where a planner would be called to determine SPEs for several investors who all consider building on all the available locations at the same time is rather unlikely for a variety of reasons. First, investors are bound to have constraints on locations, budget and server choices for reasons such as personal preference or a good deal with a supplier and so on. Furthermore, it is still unlikely that several investors, even with constraints, will have to decide at the same time. For example physical firms like charging stations can directly compete only with other firms within a certain range. Therefore, it is unlikely (a) that there will be several investors considering the same locations and (b) that investors will decide the magnitude of investment at the same time. A likely real-world use case of this model is when an investor wants to build a firm considering some locations, but some competition already exists. A more realistic use is also for providing daily counsel on prices to some investors, something that can be calculated quickly. In the end, the complexity of this model is an issue when examining large, generic theoretical problems but these have little to do with realistic use cases.

With these in mind, Section 6.1.1 will present an evaluation of location competition and Section 6.1.2 will discuss competition in the speed of service, in the context of the EV SLCOP problem.

6.1.1 Evaluating location choice

In order to evaluate the investors' location choices it is instructive to first examine the equilibrium in prices for two competing firms, that was found in Theorem 3.9 in Section 3.5.1. This, of course, assumed that investors can have only one station, and the outside option was not considered yet, but some qualitative characteristics that can be extracted more easily remain. The equilibrium in prices had been found to be:

$$f_1^* = h - \frac{1}{3}vd(t_1 - t_2) + Rv_d(n-1)\frac{2c_1 + c_2}{6c_1c_2}$$
$$f_2^* = h - \frac{1}{3}vd(t_2 - t_1) + Rv_d(n-1)\frac{c_1 + 2c_2}{6c_1c_2}$$
$$v_d, c_1, c_2 > 0 \quad n > 1$$

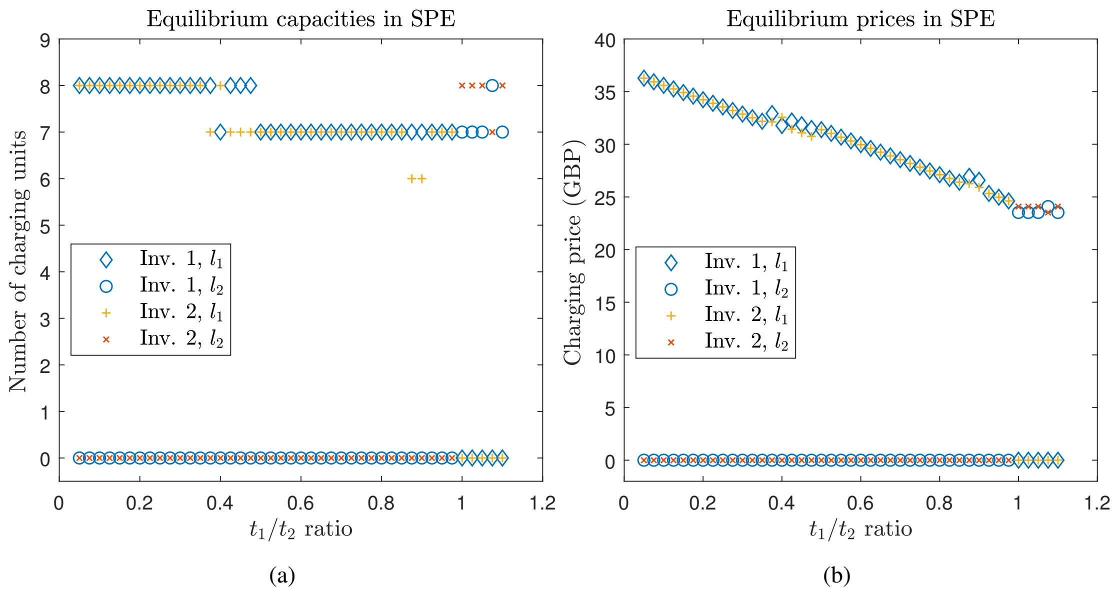


Figure 6.1: Equilibrium capacities (left) and prices (right) for a varying t_1/t_2 ratio, where investors can build up to one station in each of two routes (l_1, l_2).

Now if we consider in the context of charging station competition that the two investors have stations that are located on different routes, it is evident that if t_1 is higher, that is if station 1 is on a longer route, the price at station 1 will be lower than the price at station 2 if capacities are the same.

This implicitly indicates that investors may be biased toward competing on the same route rather than different routes when the other parameters for stations are the same. To test this hypothesis, an experiment is conducted with two investors, Investor 1 and Investor 2 and two different routes, l_1 and l_2. Each investor can choose to build up to one station at each route. The travel time for the first route t_1 will be varied, while the travel time for the second route will be constant at $t_2 = 3 + 1/3$.

Looking at the capacities investors have chosen in SPE (Figure 6.1a) shows that both investors have chosen to build at route l_1 when t_1 is smaller than t_2. While capacities and prices (Figure 6.1b) generally decrease for an increasing t_1/t_2 ratio, it is noted that there are now many situations where the investors play symmetric equilibria, while sometimes they play asymmetric equilibria. In the case where they play different strategies, the opposite strategies are also SPEs.

Now that investors choose to build on the same route the problem becomes symmetric for investors even when route travel times are different. as the other parameters are the same for both investors. The investors' utilities in Figure 6.2 generally decrease with an increasing t_1/t_2 ratio, until the point when $t_1/t_2 = 1$. At that point, it is an SPE for an investor to play the strategies shown at any one location, but past 1 both investors choose to build at route l_2 as now it is the shortest route. This confirms the hypothesis that investors will prefer to compete on the same route for the same parameters, and SPEs where investors build on different routes do not exist except for when $t_1 = t_2$.

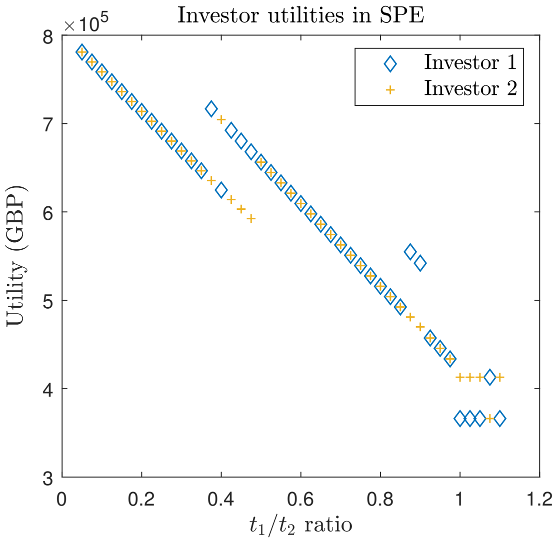


Figure 6.2: Investor utilities for a varying t_1/t_2 ratio, where investors can build up to one station in each of two routes (l_1, l_2).

Variation on the number of drivers n does not show any indication that it might affect the choice of location, and so do most other parameters. The other two parameters that can affect the choice of location are charging unit building costs and one-time building cost. The building cost will not be examined explicitly here, as it will be examined later in Section 6.2.2 with subsidies, and the results are equivalent. With regard to one-time building cost, this shows very predictable behaviour. When this is increased in one route for both investors it shows identical behaviour to the above experiment with travel time, that is as building cost for the faster route increases there is a point where investors both choose to build at the slower route. When the building cost varies asymmetrically, for only one investor in the faster route, that investor will at some point build on the slower route instead. The second investor has no reason to follow and remains in the first route. Finally, there have been no cases in which investors build in both routes, even when one-time building cost is 0. This seems odd at first, but is reasonable given that a slower route will be accompanied by reduced price and therefore the investor could just build one more unit at the faster route.

Having shown that investors prefer to compete on the same, rather than different routes, the next section will evaluate the station investors' choice in the type of charging units for their stations.

6.1.2 Evaluating the speed of service choice

When it comes to the decision on the speed of service, in the context of the SLCOP this is assumed to be represented by the choice of charging unit power output, which will influence service time. Therefore, the cost of building charging units is the obvious candidate to examine.

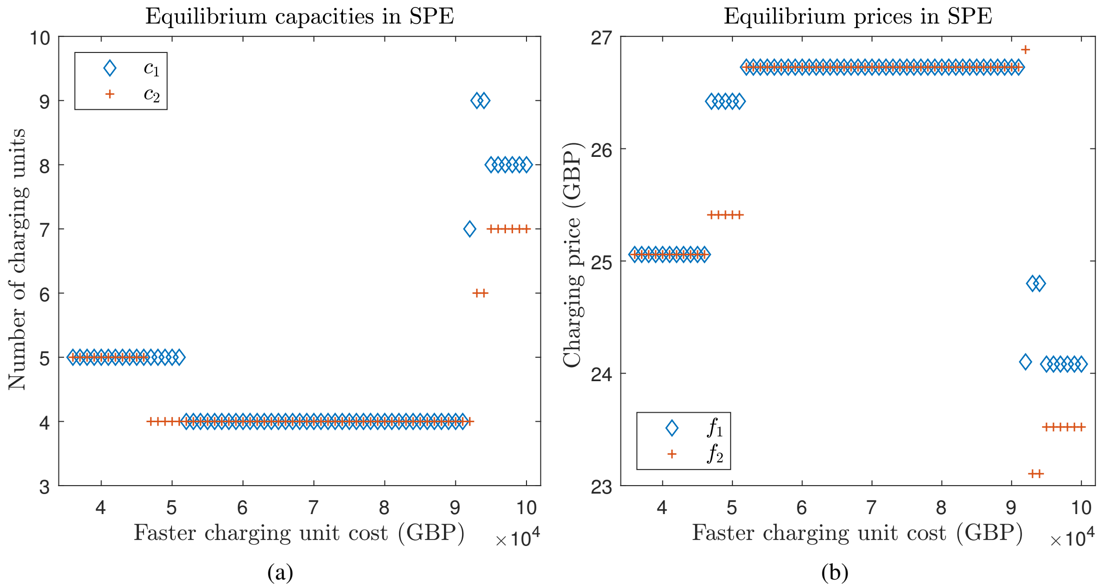
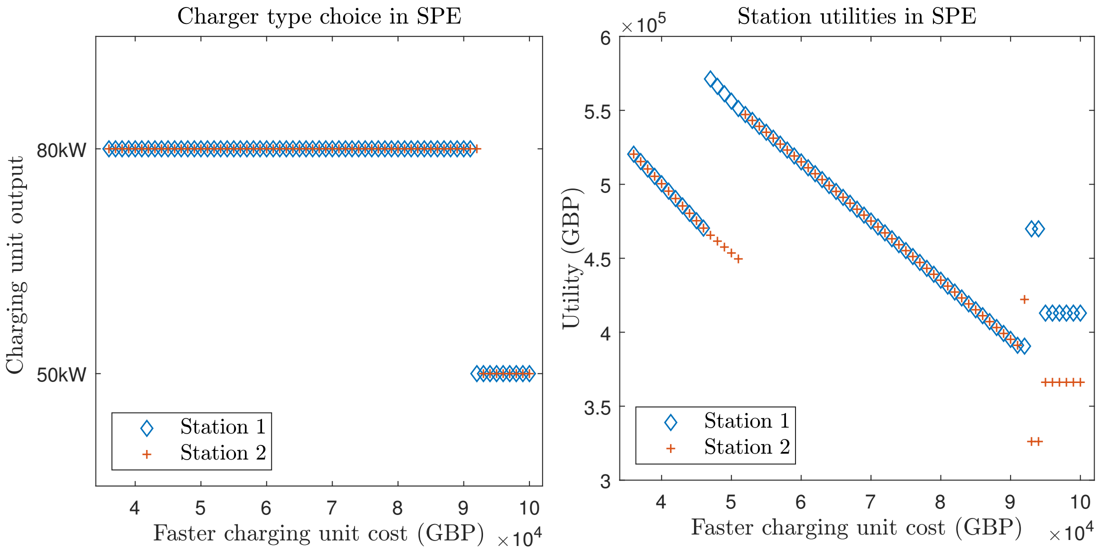


Figure 6.3: Equilibrium capacities (left) and prices (right) for a varying cost of the faster, $80kW$ charging unit.

For the purpose this experiment, a two-station symmetric scenario will be considered. This will include two types of charging units; one type are the units that have been used throughout all the experiments and are part of the reference settings, with a power output of $50kW$ and their building cost is set to $£36000$ for both investors. The other type of units that will be considered feature an $80kW$ power output and their cost will be varied symmetrically for both investors. The building cost of the faster charging units has been explored thoroughly, but for better presentation of results only costs above $£36000$ will be shown. The investors' behaviour up to that point is fairly similar to other building cost explorations that have been presented before, and more interesting is the situation where investors switch from one type to the other.

Investors both choose the $80kW$ charging units (Figure 6.4a) when their cost is comparable, and continue to do so up to a cost of $£92000$ where station 1 switches to the slower, $50kW$ units. Up to that point, as the building cost increases first station 2 reduces capacity (Figure 6.3a) and increases price (Figure 6.3b) at $£47000$, and in response station 1 sets a much higher price now that it offers better service. At a cost of $£52000$ station 1 also reduces capacity and both stations set an even higher price, playing the same strategies until $£92000$, when station 1 chooses to use the $50kW$ charging units. At that point, station 1 also reduces price and station 2 further increases price, which results in station 1 maintaining utility (Figure 6.4b) and station 2 gaining utility. It might seem odd at first that station 2 gains utility while increasing price, at the same time station 1 reduces price considerably.

A more thorough analysis shows that now 52% of drivers will go to station 2 despite station 1 setting a much lower price, because now charging time for driver i at station 2 is about 21 minutes, while at station 1 it is about 34 minutes. As charging unit cost for the fast unit increases further, station 2 also switches to the more affordable $50kW$ units. What is note-



(a) (b)

Figure 6.4: Station charging unit type choice (left) and station utility (right) for a varying cost of the faster, $80kW$ charging unit.

worthy, however, is that the expected utility for driver i (Figure 6.5) when both stations are using the $50kW$ units is highly comparable to the utility before investors switched, and for quite a wide range of cost of the fast charging unit. Also, queuing time with the faster units is higher from $\pounds 52000$ and on than with the slower units. This means that investors take advantage of the faster charging time offered to driver i in order to build less charging units and ask for a higher charging price (remember, charging time for i is not included in expected queuing time). This is also reflected in the fact that utility for the stations (Figure 6.4b) is generally much better with the fast charging units than with the slower ones even though they are more expensive, while utility for drivers is about the same.

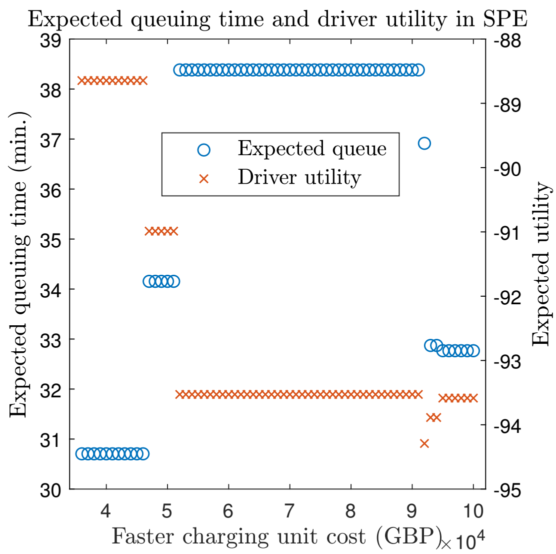


Figure 6.5: EV driver expected queuing time and expected utility for a varying cost of the faster, $80kW$ charging unit.

That said, it was impossible to determine a realistic cost for units other than $50kW$ DC chargers. Several companies in the UK were contacted, but none was willing to share information on costs and maintenance without a formal request for a study of costs for a particular location.

6.2 Subsidising Station Investors

This section will present how the model can be applied to examine subsidies to charging stations as incentives for rapid-charging station expansion. Subsidising stations can affect utility for investors and drivers, and provides for an interesting use-case of the model. Two forms of subsidies that are immediately of interest are subsidies for the cost of installing charging units and subsidies for the cost of electricity at stations. For the purpose of the experiments presented here, the reference settings used are as follows.

The value of time for driving is set to $v_d = £15.6/\textit{half-hour}$, the number of drivers to $n = 30$, charging unit building costs are set to $b_1 = b_2 = £36000$, and one-time building costs to $o_1 = o_2 = £30000$. Travel times will be $t_1 = t_2 = 3 + 1/3$ *half-hours*, charger output is $50kW$ with 85% efficiency which sets charging time at $R_1 = R_2 = 1.1294$ *half-hours*. The price stations buy electricity at is $£0.1/\textit{kWh}$ and the cost for stations to recharge each EV is $h = £2.8235$. Last, the value of time for using the train is set to $v_T = £18.1/\textit{half-hour}$, the travel time for the train is $t_T = 4$ *half-hours*, the fee for the train will be $f_T = £21.9$ and driver disappointment for not using the EV is $D = 0.95$. Any deviation from these parameters will be clarified explicitly.

Before evaluating subsidies, however, some performance metrics will need to be defined first. Hence, Section 6.2.1 provides a discussion on measuring the subsidies' performance, and defines suitable metrics. Then, Section 6.2.2 presents an evaluation of subsidies for installing charging units. Last, Section 6.2.3 discusses the performance of subsidies toward the cost of electricity at stations.

6.2.1 Metrics for the performance of subsidies

In order to determine the efficiency of a subsidy, it is necessary to consider both station investors and drivers. With regard to station investors, the difference between utility with the subsidy and without the subsidy is divided by the total cost of the subsidy. Since the utility for investors is net profit, this will essentially show how much money stations gain or lose per pound spent in subsidies. A useful property of the model is that by using the value of time parameter v_d to convert the time costs in driver i's utility, the same can be determined for drivers. Then, if the gain or loss for driver i is multiplied by nw, the same normalisation for station profits, the total gain for drivers can be examined in the same order of magnitude as the stations' gain and the total cost of the subsidy. Finally, adding the total gain for drivers and stations will indicate how much extra or less money is generated in system-wide utility due to the subsidy.

Definition 6.1 (Efficiency of charging unit subsidy for investors)**.** Let σ denote the amount that is going to be subsidised for the purchase of each charging unit. If each of z investors considers building stations at μ locations, the total cost of the subsidy is $\sigma \sum_{k=1}^{z} \sum_{j=1}^{\mu} c_j^k$. Then, if $\mathbb{E}_{su}[r^k(c)|s]$ is investor k's utility with the subsidy and $\mathbb{E}_0[r^k(c)|s]$ is investor k's utility without the subsidy, the efficiency of the subsidy for all investors is:

$$\epsilon_I = \frac{\sum_{k=1}^{z} \mathbb{E}_{su}[r^k(c)|s] - \sum_{k=1}^{z} \mathbb{E}_0[r^k(c)|s]}{\sigma \sum_{k=1}^{z} \sum_{j=1}^{\mu} c_j^k} \tag{6.1}$$

Having defined a measure of efficiency for investors, a measure for drivers is also necessary.

Definition 6.2 (Efficiency of charging unit subsidy for drivers)**.** Let σ denote the amount that is going to be subsidised for the purchase of each charging unit. If each of z investors consider building stations at μ locations, the total cost of the subsidy is $\sigma \sum_{k=1}^{z} \sum_{j=1}^{\mu} c_j^k$. Then, if $\mathbb{E}_{su}[u_i(x)|s]$ is driver i's expected utility with the subsidy and $\mathbb{E}_0[u_i(x)|s]$ is i's expected utility without the subsidy, the efficiency of the subsidy for all drivers is:

$$\epsilon_N = nw \frac{\mathbb{E}_{su}[u_i(x)|s] - \mathbb{E}_0[u_i(x)|s]}{\sigma \sum_{k=1}^{z} \sum_{j=1}^{\mu} c_j^k} \tag{6.2}$$

With regard to subsidies for electricity costs at stations, the metrics are slightly different.

Definition 6.3 (Efficiency of electricity cost subsidy for investors)**.** Let σ denote the amount that is going to be subsidised for each *kWh* a station consumes. If E is the energy requirement in kW for a charging station to recharge each EV, then the total cost of the subsidy is the added normalised cost of all the drivers who have recharged multiplied by the amount of the subsidy. If s_i^T is the probability of drivers taking the train, the total cost of the subsidy then is $w(1-s_i^T)nE\sigma$. Moreover, if $\mathbb{E}_{su}[r^k(c)|s]$ is investor k's utility with the subsidy and $\mathbb{E}_0[r^k(c)|s]$ is investor k's utility without the subsidy, the efficiency of the subsidy for all investors is:

$$\epsilon_I = \frac{\sum_{k=1}^{z} \mathbb{E}_{su}[r^k(c)|s] - \sum_{k=1}^{z} \mathbb{E}_0[r^k(c)|s]}{w(1 - s_i^T)nE\sigma} \tag{6.3}$$

And a redefinition of the metric for drivers is now necessary.

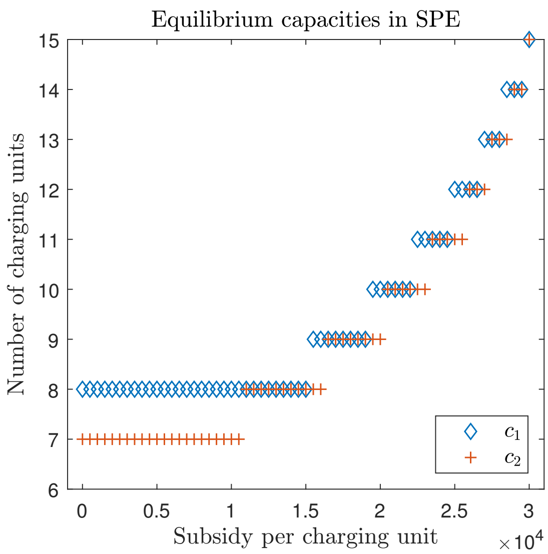
Definition 6.4 (Efficiency of electricity cost subsidy for drivers)**.** Let σ denote the amount that is going to be subsidised for each *kWh* a station consumes. If E is the energy requirement in kW for a charging station to recharge each EV, then the total cost of the subsidy is the added normalised cost of all the drivers who have recharged multiplied by the amount of the subsidy. If s_i^T is the probability of drivers taking the train, the total cost of the subsidy then is $w(1 - s_i^T)nE\sigma$. Then, if $\mathbb{E}_{su}[u_i(x)|s]$ is driver i's expected utility with the subsidy and $\mathbb{E}_0[u_i(x)|s]$ is i's expected utility without the subsidy, the efficiency of the subsidy for all drivers is:

$$\epsilon_N = nw \frac{\mathbb{E}_{su}[u_i(x)|s] - \mathbb{E}_0[u_i(x)|s]}{(1 - s_i^T)nwE\sigma} = \frac{\mathbb{E}_{su}[u_i(x)|s] - \mathbb{E}_0[u_i(x)|s]}{(1 - s_i^T)E\sigma} \tag{6.4}$$

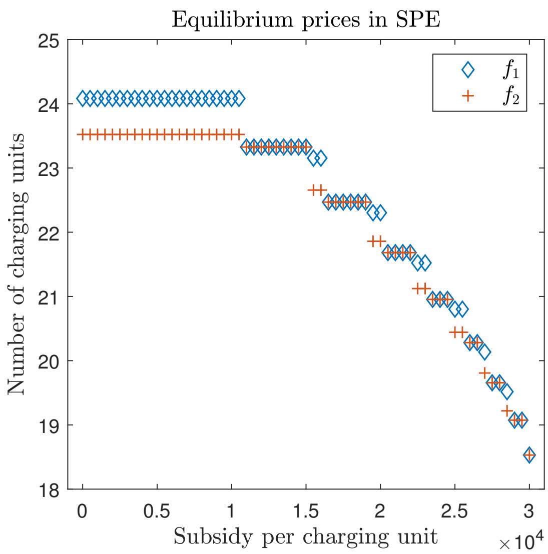
Note that while the metrics in performance given for drivers in Definitions 6.2 and 6.4 are sound, in order to be able to say that e.g. £1 is generated for drivers for every pound of the subsidy, this implicitly requires that the probability of taking the train s_i^T is zero. This is so because the expected utility for playing in mixed strategy s may include the weighed utility of using the train. While driver disappointment D could be broadly considered as a monetary quantity, the correct equivalent deduction in case some drivers take the train is that 1 utility is generated for every pound of the subsidy. This will not be a problem here, as the model has been calibrated so that no drivers want to use the train even before the subsidy.

6.2.2 Subsidising the cost of charging units

Regarding subsidising the cost of charging units, the first experiment that will be carried out is a two-station symmetric scenario using the reference settings outlined in the start of Section 6.2. Each of two investors can have at most one station on the same route as the other investor, and the purchase of charging units is subsidused by the amoun σ for both investors. The results show that SPE capacities (Figure 6.6a) and prices (Figure 6.6b) are unaffected up to a subsidy of £10500 per charging unit. Consequently, up to that amount driver utility and expected queuing time (both in Figure 6.7a) are also unaffected, but stations of course gain the subsidy as utility (Figure 6.7b). Therefore, the subsidy generates £1 for stations for each pound paid in subsidies (Figure 6.8a), which means that it is completely absorbed by stations. However, from a subsidy amount of £11000 onward stations start increasing capacity and decreasing prices which shows that the subsidy is beginning to have an effect, as expected queuing time and driver utility improve.

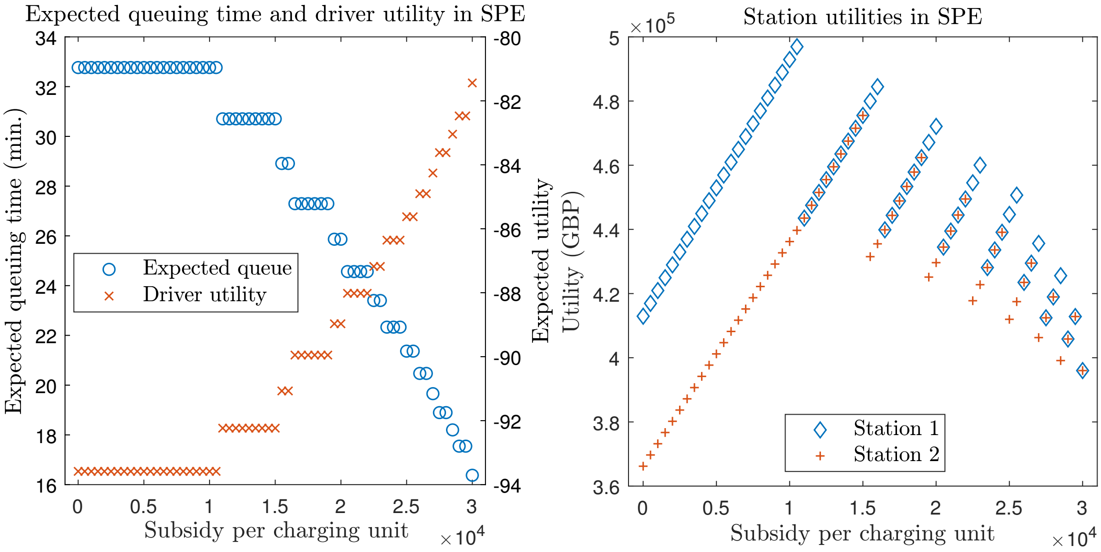


(a)



(b)

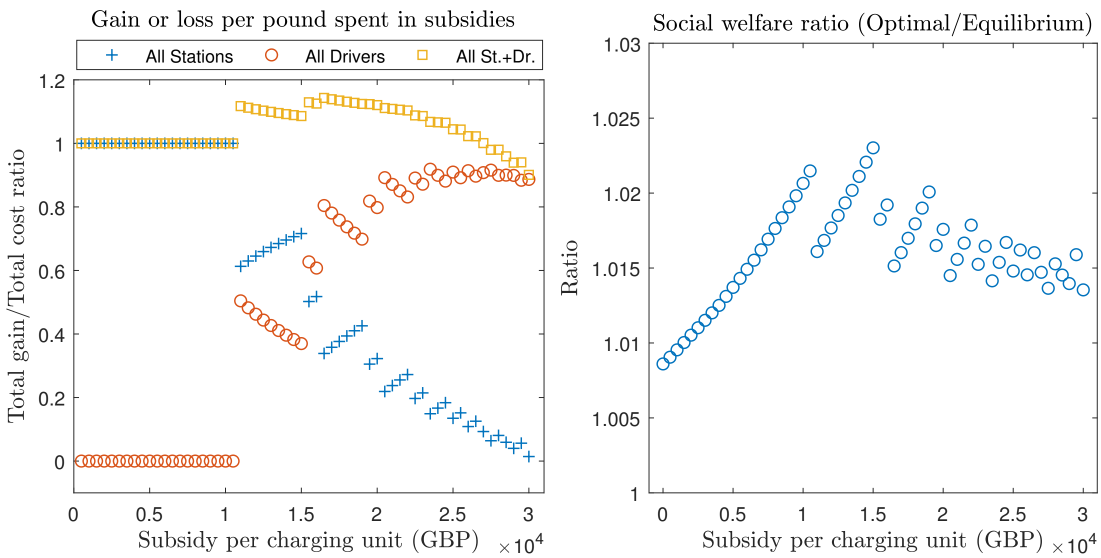
Figure 6.6: Equilibrium capacities (left) and prices (right) for an increasing subsidy per charging unit.



(a) (b)

Figure 6.7: Expected queuing time and driver utility (left), and station utility (right) for an increasing subsidy per charging unit.

Stations lose utility as the subsidy increases, compared to their peak utility at $\sigma = \pounds 10500$, but still have higher utility than without the subsidy. To produce Figure 6.8a, the methodology and metrics from Section 6.2.1 were used. Thus the gain for all stations is that of Equation (6.1) and for all drivers that of Equation (6.2). This analysis shows that subsidies for the cost of charging units can be very effective. This is so because the subsidy gives incentive to investors to increase capacity which improves queuing times, and at the same time they offer better prices.

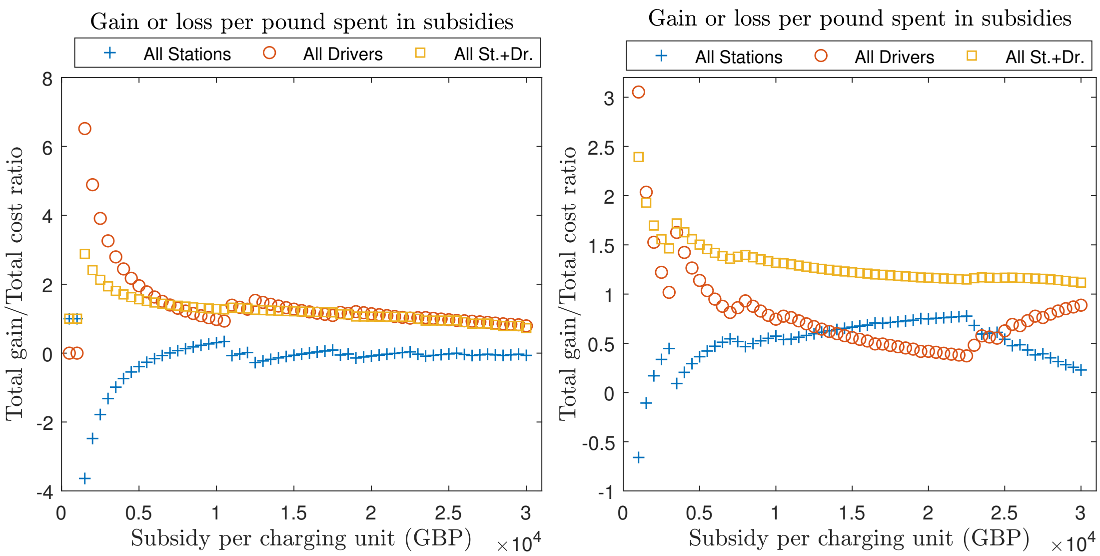


(a) (b)

Figure 6.8: Subsidy efficiency (left), and Social Welfare Ratio (right) for an increasing subsidy per charging unit.

Therefore, stations gain utility because they do not implement 100% of the subsidy and drivers gain utility from both improved queuing times and prices, and the subsidy can generate more than $£1$ in system-wide utility for each pound spent in the subsidy. At the same time, the Social Welfare Ratio (Figure 6.8b) shows that although SPE efficiency generally reduces due to the subsidy, especially when stations are subsidised and do not improve capacity, SPEs remain highly efficient for the station-driver system showing worst-case efficiency of 97.75% compared to optimal station allocation.

Additional results for fewer and more drivers in Figure 6.9, with D calibrated accordingly, show similar qualitative characteristics. In the case of few drivers (Figure 6.9a), drivers benefit much more than stations and stations can have losses in some cases, but in most cases these are few or marginally zero. For high peak traffic (Figure 6.9b), the situation is comparable to Figure 6.8a, only now system-wide utility gain is even greater.



(a) $n = 15$

(b) $n = 60$

Figure 6.9: Subsidy efficiency for an increasing subsidy per charging unit, considering peak traffic of $n = 15$ drivers (left) and $n = 60$ drivers (right).

With this methodology, it is straightforward to determine optimal subsidy levels according to various criteria. For example, for a peak traffic of $n = 30$ drivers Figure 6.8 shows that the optimal system-wide utility is generated at a subsidy level $\sigma = £16500$ per charging unit where each pound spent in subsidies generates $\epsilon_I + \epsilon_N = 1.143$ pounds in utility. Peak efficiency for the drivers is at $£23500$ and for investors at $£15000$.

A further experiment is performed with two station investors and two available routes (locations) l_1 and l_2. The travel time for route l_2 is set to the reference $t_2 = 3 + 1/3$ while for l_1 it is set to a more advantageous $t_1 = 3$. Both stations will be subsidised equally by an amount σ per charging unit they build, but only for building on the disadvantageous location l_2. As is seen in Figure 6.10a investors initially build stations at l_1 when the subsidy is smaller, and are indifferent to the subsidy up to an amount of $\sigma = £14000$ per charging unit. This means that there is no subsidy yet from which drivers can benefit. In Figure 6.11 this is represented by a *zero* ratio to put the points on the plot, but in reality the efficiency is not defined, as the total cost of the subsidy is also 0. However, from a subsidy amount $\sigma = £15000$ and on, station investors choose to build on route l_2, increasing capacity (Figure 6.10a) and decreasing prices (Figure 6.10b).

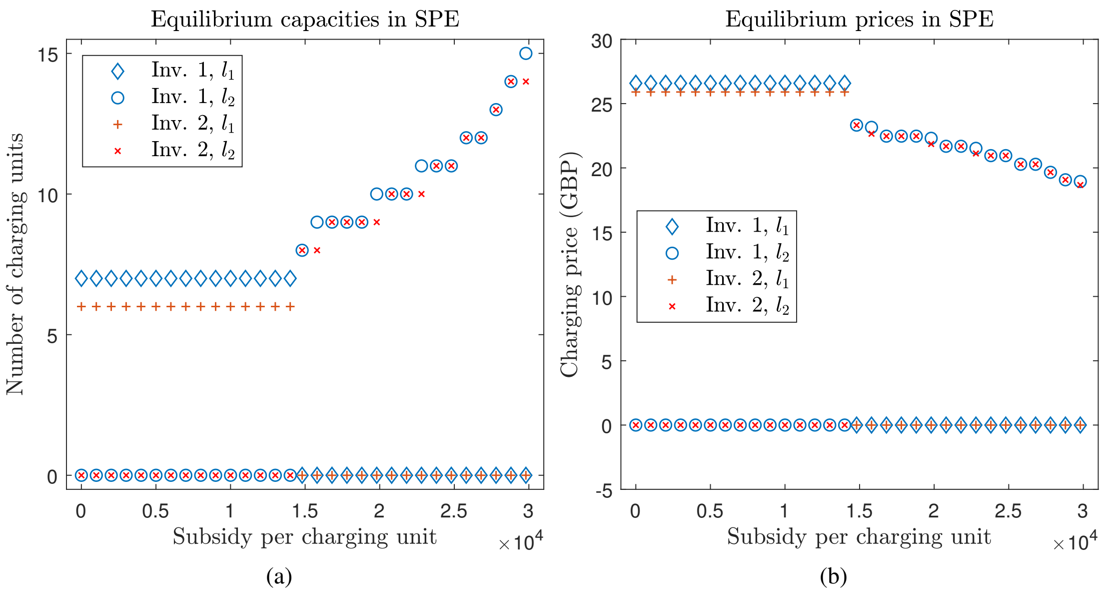


Figure 6.10: Equilibrium capacities (left), and prices (right) for an increasing subsidy per charging unit only for l_2.

While prices generally decrease at a slow rate, at the point where stations choose the slower route there investors have to reduce price considerably to compete with the train, because time costs for drivers increase. This means that whereas drivers gain from the subsidy (Figure 6.11), compared to unsubsidised stations in the faster route, stations lose utility. This indicates that subsidising disadvantageous routes may be detrimental to investors. To investigate this further, only Investor 2 is now subsidised for building at the slower route l_2. This can represent a situation where a station is located already in the faster of two routes, and the slower route is subsidised to expand service toward that route. The results show that Investor 1 will reduce capacity (Figure 6.12a) at a subsidy level of $£14500$ induced by an undercut in price (Figure 6.12b) by Investor 2. At $\sigma = £15000$ Investor 2 will choose to build at the slower, subsidised route. Investor 1 will remain in the favourable

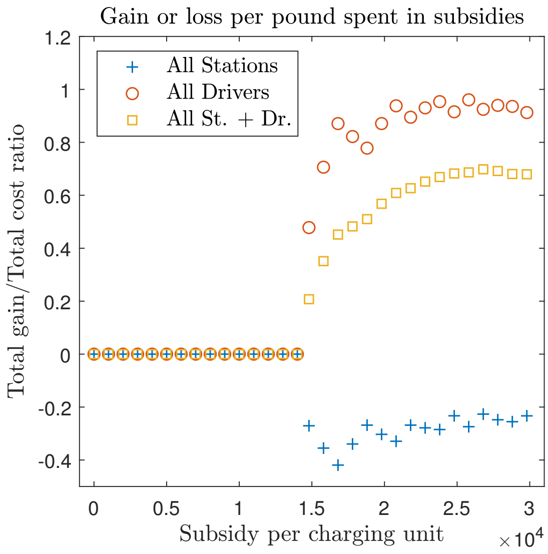
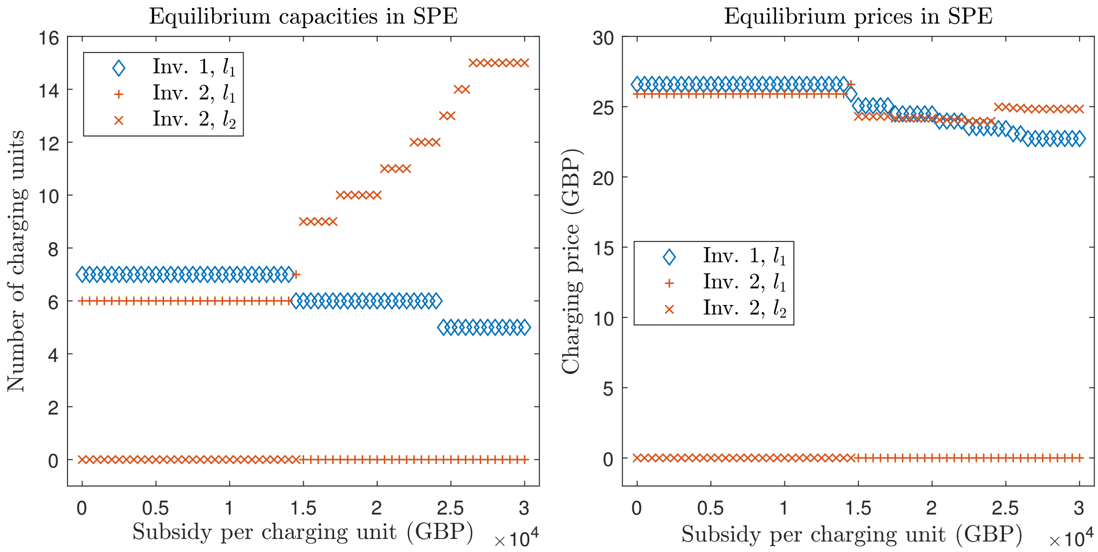


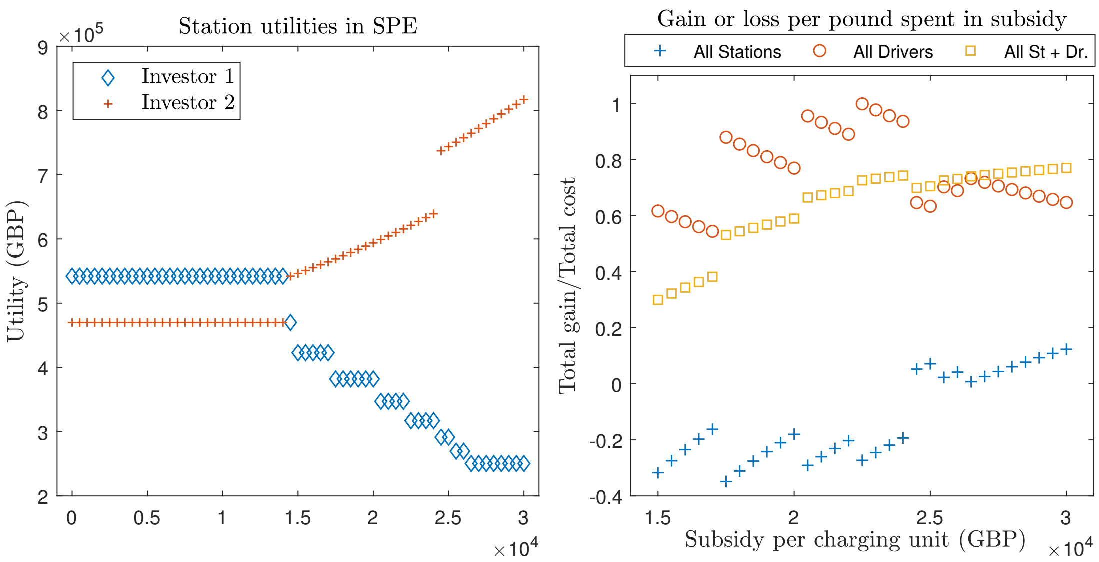
Figure 6.11: Subsidy efficiency for an increasing subsidy per charging unit only for l_2.



(a) (b)

Figure 6.12: Equilibrium capacities (left) and prices (right) for an increasing subsidy per charging unit only for Investor 2 in route l_2.

route, but as Investor 2 is now subsidised at increasing levels Investor 1 has to reduce price to remain competitive. This is followed by a further decrease in Investor 1's capacity as more drivers shift toward Investor 2 who increases capacity for an increasing subsidy level. This behaviour results in considerable gain in utility for drivers (Figure 6.13b) for each pound of the subsidy, but Investor 1 suffers severely. This can be seen by the fact that although Investor 2 gains utility (Figure 6.13a), Investor 1 loses considerable utility which results in the overall effect of the



(a) (b)

Figure 6.13: Station utility (left), and efficiency of the subsidy (right) for an increasing subsidy per charging unit only for Investor 2 in route l_2.

subsidy (Figure 6.13b) being loss for stations. Toward very high subsidy levels, the subsidy produces a slight positive effect for stations, but this is only heavily favourable for Investor 2, while Investor 1 has lost considerable profits. In the end, subsidising investors or routes asymmetrically has to be carried out with caution because as we saw it can put both investors at a disadvantage, or provide a heavy disadvantage to pre-existing stations.

6.2.3 Subsidising the cost of electricity for stations

The other type of subsidy that is now going to be examined is subsidising the cost of electricity for stations. This will be examined as a subsidy toward the price stations purchase each *kWh* of electrical power. The first experiment will be a symmetric scenario with the reference settings, where two stations on the same route are subsidised. Results show that very small subsidies can result in an improvement for drivers and stations, but the subsidy is generally absorbed by stations mostly (Figure 6.13) who also increase profits at the expense of drivers. At a subsidy level of £0.012 (12% of the reference price of £$0.1/kWh$), station 1 reduces capacity (Figure 6.14a) to 7 and both stations increase prices (Figure 6.14b). Furthermore, at 0.018 there is an increase in average capacity back to the initial levels, but this is followed by a spike in expected price (Figure 6.14b). Because station 2 now has a lower capacity but a much lower price, drivers prefer station 2 which increases expected queuing time (Figure 6.15a). Further on, stations adjust capacity to 7 something that increases queuing time even more, and although prices reduce linearly with an increasing subsidy, drivers are not able to gain the lost utility back until almost the entire price per *kWh* is subsidised. Station utilities (Figure 6.15b) show that station utility increases in general, something that is also reflected at a good gain ratio in Figure 6.16a for stations.

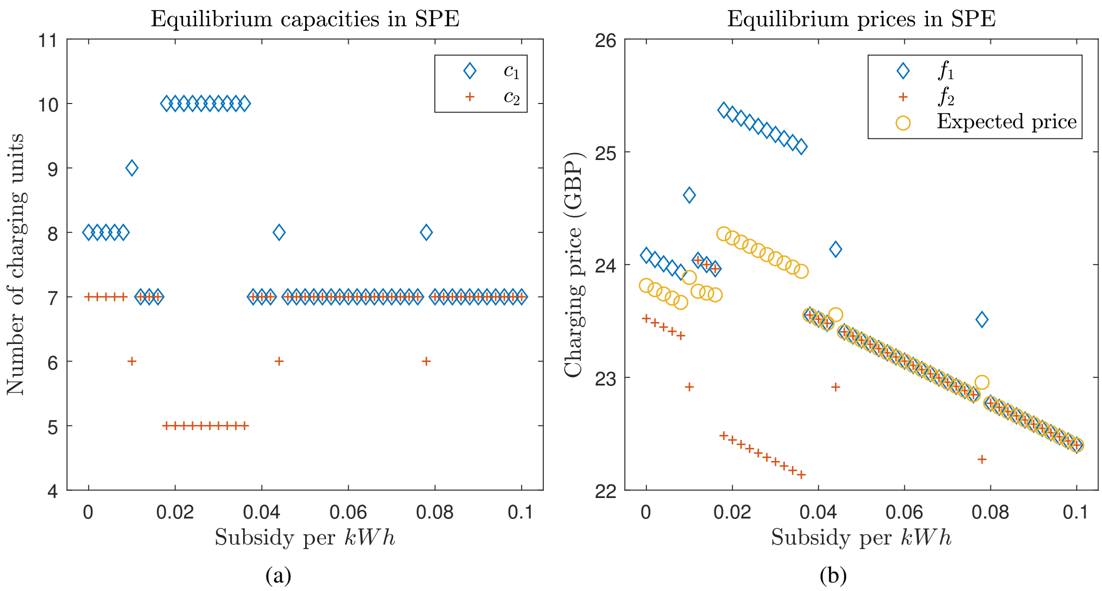
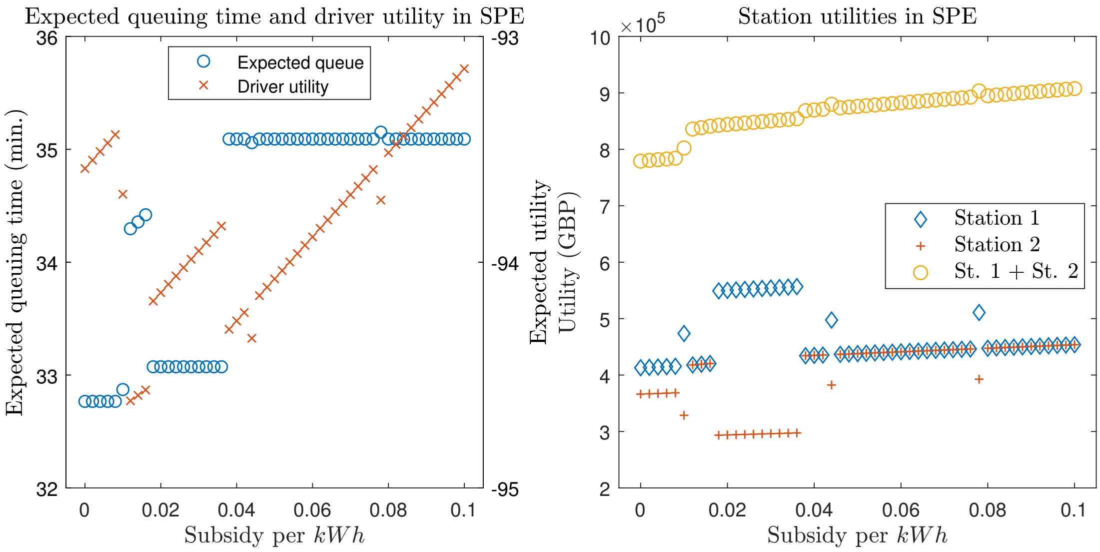


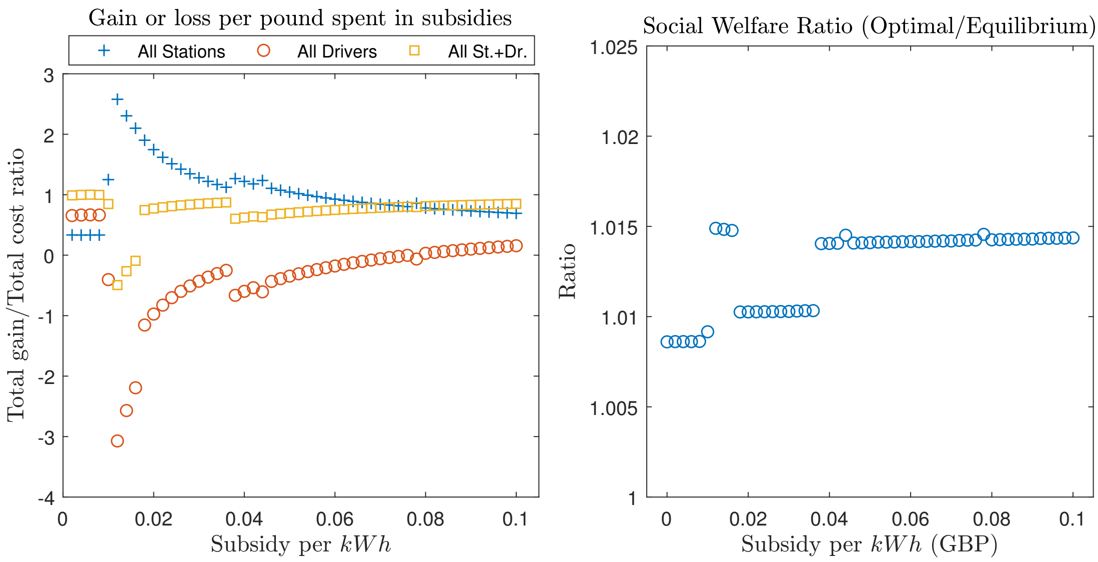
Figure 6.14: Equilibrium capacities (left) and prices (right) for an increasing subsidy per *kWh* of electrical power used.



(a) (b)

Figure 6.15: Expected queuing time and driver utility (left), and station utility (right) for an increasing subsidy per *kWh* of electrical power used.

Although drivers do gain some utility for a very low subsidy, in general they lose considerable utility compared to an unsubsidised state. Overall, electricity price subsidies are of very small magnitude compared to charging unit subsidies, therefore any gains for the drivers, when they do gain, are minimal. Subsidising electricity cost does reduce system-wide SPE efficiency slightly (Figure 6.16b), but generally it remains high with SPEs being at least 98.52% efficient compared to optimal station allocation.



(a) (b)

Figure 6.16: Subsidy efficiency (left), and Social Welfare Ratio (right) for an increasing subsidy per *kWh* of electrical power used.

It is also a question whether it is a sound policy to subsidise almost the entire cost of electricity for stations, when charging prices are much higher than marginal cost, for a minimal gain by drivers. Furthermore, the risk that stations might end up lowering overall capacity means that electricity subsidies can be counter intuitive. Experiments on lower and higher peak traffic show similar behaviour, and no evidence could be found that subsidising the price stations buy electricity at could have a consistently positive effect.

Chapter 7

Conclusions and Future Work

This chapter will bring the thesis to a close with a summary of the work presented throughout the thesis, coalescing the conclusions drawn in Section 7.1, and a brief discussion on future directions in Section 7.2.

7.1 Conclusions

This thesis has presented and evaluated a model for competing firm investors that takes into account the behaviour of customers. Customers have been modelled as self-interested agents who make stochastic choices over available firms, so that the expected cost of acquiring the desired service is minimised. These costs include the fee for obtaining the service, as well as expected congestion at firms and firm access costs, and this behaviour addresses Requirement 1 that was set in Section 1.3. In order to determine expected congestion cost at firms, the number of customers who are expected to choose each firm, as well as the firms' capacities and service times are taken into account which addresses Requirements 2 and 3.

Firm investors have also been modelled as self-interested agents who choose locations, capacities the speed of service and prices at their firms, with the goal of maximising expected profit as was required in Requirement 4. The expected profit for firms takes into account potential customer decisions, capacities and fees at firms, as well as the cost of building firms which addresses Requirement 5. As was necessitated by Requirement 6, the building cost of a firm depends on an one-time building cost the investor pays to build at the selected location, and the firm's number and type of servers.

These decisions by investors and customers are made in a series in which investors first decide locations, capacities and the speed of service. Then, they decide prices and last customers select firms. This sequence of decisions has been modelled as an extensive form game, which is solved by obtaining subgame-perfect equilibria through backward induction. That is, it is assumed that at each stage players are able to observe the initialisation and the events that transpired in previous stages. The mixed strategy Nash equilibrium in customers' choices is then solved first, then the investors' pure strategy Nash equilibrium in prices is solved second, and the pure NE in locations, capacities and speed of service is solved last. A combination of pure strategy equilibria in investor choices and mixed NE in customer choices obtained this way is a subgame-perfect equilibrium. This approach is novel in that it combines elements from different domains. The problem that is solved is, in essence, similar to network pricing games only significantly more complex (i.e. parameter-wise), and this can help encompass a more wide category of problems. To do so, elements from spatial competition and Stackelberg games have been utilised to allow for a solution that allows for concrete theoretical and empirical analysis.

The model is evaluated in the context of competing EV charging station investors. Drivers behave as expected, choosing stations with the same probability when the parameters for the stations are the same for all stations. In addition, expected queuing time increases exponentially with decreasing capacities and this is expected from (3.1). With regard to charging fees, however, expected queuing time is a second degree polynomial when the charging fee at one station is unilaterally adjusted. This means that there can be situations in which increasing the fee at one station may actually result in lower queuing times if the fee in that station was too low. The drivers' symmetric mixed strategy NE in station choice shows only linear increase in computational time with an increasing number of stations, and this is satisfactory given that the intent was to design a model for customers that will not add a computational hurdle to the complex firm competition.

With regard to the firms' pure strategy Nash equilibria in capacities, these have been found to always exist when the problem for stations is symmetric, that is when respective parameters are the same for all investors. However, cases of non-existence have also been shown and discussed, and non-existence generally occurs when some parameters differ greatly from one investor to the other, that is when one investor is heavily advantaged. Non-existence is, in principle, an issue but it is expected that the inclusion of the price choice for investors will help alleviate the problem. When firms can only choose capacities, if all capacity choices for an investor yield negative utility for a given strategy by the opponent, the disadvantaged investor can do nothing to alter this situation, and this results in the maxima for investors to never coincide. With the inclusion of a decision in prices, however, the disadvantaged investor can then undercut price to draw more drivers toward the station, thus creating maxima in the utility that did not exist before. Furthermore, it has been shown that the cost of building charging units, as well as charging fees are significant factors in the investors' capacity choice. This can prove a major disadvantage for an investor that cannot get as good a price on charging units as others or cannot offer competitive fees. Last, the Iterated Best-Response algorithm that was chosen to compute the investors' equilibria in capacities converges quickly up to this point, but complexity increases exponentially with the number of stations. This indicates that computational time may increase significantly when evaluating location and speed of service choices for investors.

A significant finding regarding equilibrium prices that was shown theoretically in Section 3.5.1 and confirmed empirically in Section 5.1.3, is that competing charging station investors will choose charging prices that are considerably higher than the marginal cost for stations to recharge an EV. This was shown to happen due to the fact that charging demand cannot be satisfied immediately in the EV charging problem, unless charging units with power output in the thousands of kW are used. The theoretical analysis of equilibrium prices also showed that the fact that stations may be placed on different routes induces product differentiation through route travel times. This means that stations in more disadvantageous routes have to ask for a lower charging fee from EV drivers, and this finding was also confirmed empirically in Section 5.1.3. Finally, in Section 5.1.5 it was shown that when EV drivers are increasingly willing to pay more in order to save time from their journey, station investors will also increasingly take advantage of that behaviour. This leads to a considerable loss in utility for drivers, even though queuing times improve, due to much higher charging fees.

However, although the inclusion of prices reinforces the existence of subgame-perfect equilibria, there are some issues that the inclusion of competition in prices introduces. First of all, because drivers have to recharge it is not possible to measure the efficiency of subgame-perfect equilibria for investors. This is because the globally optimal strategy for stations is to set capacity at 1 and set infinite prices. Second, if the number of firms is very small, profit margins may be high enough for investors to compete by undercutting prices alone. This means that very few investors can set arbitrarily high prices and may not have incentive to increase station capacities. This again stems from the fact that it is necessary for EV drivers to recharge.

The introduction of an outside option for customers in Section 3.1.4 relaxes the assumption that demand has to be satisfied and solves the aforementioned problems. It gives a perspective to investors on what would be acceptable services and forces them to consider realistic prices. This is done through a parameter that models the customers' disappointment at not being able to use the desired service. The logic behind this is that an increasing disappointment reduces the expected utility for the outside option, therefore makes customers more inclined toward choosing the firms. Just as with the situation when EV drivers are increasingly willing to pay more, an increasing inclination to use the EV resulted in stations taking advantage and deteriorating services increasingly. This in turn causes an increasing number of drivers to use the outside option (in the SLCOP this was represented by a train option). In order to give investors a more realistic perspective of satisfactory service, therefore, disappointment is set close to the margin when no driver wants to use the outside option. Empirical results showed that subgame-perfect equilibria obtained this way are robust against fluctuations in peak EV traffic, building costs and route travel times. This means that if a subgame-perfect equilibrium is implemented by investors, fluctuations in EV traffic or travel times can be accounted for by only adjusting prices, in which case the efficiency of the new SPE is very close to the efficiency of the SPE that was implemented. Furthermore, SPEs show very good efficiency compared to optimal allocations when it comes to system-wide social welfare. Specifically, SPEs showed a worst-case efficiency of 93% in reasonable competition scenarios where stations compete on the same or different routes whose travel time differs by up to 20%, and in many cases efficiency was much better.

An empirical evaluation of location and speed of service choices in the context of the SLCOP confirms that station investors prefer to compete on the same, rather than on different routes when parameters for investors such as building cost are the same or similar. With regard to the choice of charging units, it is shown empirically that a choice of more expensive units with higher power output may, depending on the cost difference from slower units, actually increase queuing times and prices. This is because investors take advantage of the fact that EVs now charge more quickly, thus they can reduce the number of charging units and increase price, since drivers make tradeoffs between time and monetary costs.

Finally, the model was applied to examine the effect of subsidies to charging station investors as incentives to expand rapid-charging station networks. Toward this, the proposed model allowed to define subsidy metrics which show how much money is gained or lost for drivers and station investors, for each pound spent in subsidies. This approach in turn allows to determine optimal subsidy levels according to criteria such as system-wide gain, gain for drivers or gain for stations. Empirical results show that subsidies toward the purchase of charging units can be very effective, with a positive impact on drivers and station investors, and can generate a gain of more than $\pounds 1$ for stations and drivers together for each pound spent in the subsidy. On the other hand, subsidising the cost of electricity for stations can be far less effective, as drivers need to pay a charging fee which is significantly higher than the actual cost of recharging an EV. Furthermore, subsidising electricity can, in some cases, provide incentive to investors to reduce capacities and increase prices, producing an adverse effect for drivers. In any case, the efficiency of SPEs in system-wide social welfare reduces slightly when subsidising stations, but still remains very high -in many cases over 98% compared to allocations optimal for welfare. Last, it was shown that subsidising the purchase of charging units for a more disadvantageous route can result in losses to investors in the case where all investors are subsidised by the same amount. In addition, when an unfavourable location is subsidised for only one investor, this can introduce unfair competition and prove catastrophic to other investors that already have stations in more favourable locations.

Overall, this work has performed extensive analysis on firm competition. The proposed model combines several aspects of spatial competition, network price games, Stackelberg games to address problems where customers may experience uncertain congestion at firms. The model compromises realism in some aspects for abstraction, but this was necessary in order to perform theoretical, qualitative and quantitative analysis using a single model. The abstraction level of the model makes it versatile and it has been used to conceptualise a variety of real-world situations. Specifically, using the theoretical model it was shown that charging prices for EVs will be significantly higher than the actual cost to recharge EVs because of limitations in charging technology. Moreover, the model was used to show that within the current technological window in EV charging, a choice of better charging units by stations can potentially result in worse queuing times and higher charging prices. Analysis on subsidies produced both qualitative and quantitative conclusions. For example, it was shown that with the proposed model it is possible to determine metrics for the efficiency of various forms of subsidies. Using these metric, subsidies toward purchasing charging units were found to be very effective. On the other hand, subsidies for the cost of electricity at stations do not have a significant effect and can even make the situation worse for drivers, leading to increased prices and queues. In addition, optimal levels for subsidies were determined using the same model. Last, the model can already be used effectively to consult investors on prices and investment levels given certain constraints and pre-existing competition. Now that the purpose of theoretical and game-theoretic analysis has been served, there is room for future extensions which will be discussed in the following section.

7.2 Future Work

An interesting direction for future work is to relax the assumptions that were made in order to promote theoretical and qualitative analysis which have served their purpose. The customer flows considered in this thesis are influenced by the presence and characteristics of firms, so they are a step forward from static traffic flows that many firm competition models consider. However, the model can be scaled to better fit more realistic, and larger markets. For example, today the EV charging market is not at a level yet where there are several independent investors with very few stations each. It is mostly comprised of oligopolies, and until charging technology becomes more accessible it is reasonable to assess it may remain so. The model already considers heterogeneous investors, but to direct this work toward large-scale use in more heterogeneous settings, it would be beneficial to also consider heterogeneous customers. This would not alter most theoretical and empirical findings which are of qualitative nature, but may shift quantitative results such as optimal subsidy magnitude and provide for more realistic prices and capacities.

Thus, future work will firstly involve relaxing the assumptions that guarantee symmetric mixed Nash equilibria. In the context of the SLCOP problem, this will enable considering real-world data on vehicle trips, road networks and existing stations. This way, asymmetric mixed strategy Nash equilibria can be examined with drivers that have different utilities, perform different trips and have different station choices available. While it will not be straightforward to use closed-form solutions for these in larger settings, it is also possible to simulate more heterogeneous settings and asymmetric stochastic behaviour, or to use techniques such as evolutionary learning. Secondly, although the model is a significant advancement in firm competition, the full model is computationally complex. The increased complexity stems from the fact that investors consider a range of capacities and server type choices for each of the potential locations, something which increases exponentially with the number of available locations. Whereas realistic use-cases are bound to be significantly less complex due to constraints investors may have[1], complexity is an issue that needs to be addressed to promote large-scale application. For example, more efficient algorithms for pure NE identification can be explored, or pure NE could be approximated.

[1]i.e. it is unlikely that multiple investors will be called to consider building stations on all the available locations at the same time, while considering many types of charging units

Appendices

Appendix A

Expanded mathematical formulations and proofs

A.1 Customers' equilibrium with two firms

According to equation (3.8), finding the equilibrium for customers choosing over two firms means solving the system:

$$\begin{aligned} &\mathbb{E}[u_i^1(x)\,|s_{-i}^*] = \mathbb{E}[u_i^2(x)\,|s_{-i}^*] \\ &s_{-i}^{*1} + s_{-i}^{*2} = 1 \end{aligned} \tag{A.1}$$

For simplicity and without loss of generality we will assume that service time is the same at both firms and equal to R. By substituting expected utility from (3.4), and by substituting $s_{-i}^{*2} = 1 - s_{-i}^{*1}$, the first equation of system (A.1) becomes:

$$\begin{aligned}
&-v_d\left(t_1 + \frac{s_{-i}^1(n-1)}{2c_1}R + R\right) - f_1 = -v_d\left(t_2 + \frac{\left(1-s_{-i}^{*1}\right)(n-1)}{2c_2}R + R\right) - f_2 \Leftrightarrow \\
&-v_d(t_1+R) - f_1 - v_d\frac{s_{-i}^1(n-1)}{2c_1}R = -v_d(t_2+R) - f_2 - v_d\frac{\left(1-s_{-i}^{*1}\right)(n-1)}{2c_2}R \Leftrightarrow \\
&-v_d(t_1+R) - f_1 - v_d\frac{s_{-i}^1(n-1)}{2c_1}R = -v_d\left(t_2 + R + \frac{(n-1)}{2c_2}R\right) - f_2 + v_d\frac{s_{-i}^{*1}(n-1)}{2c_2}R \\
&\Leftrightarrow -v_d\frac{s_{-i}^{*1}(n-1)}{2c_1}R - v_d\frac{s_{-i}^{*1}(n-1)}{2c_2}R = -v_d\left(t_2 + R + \frac{(n-1)}{2c_2}R\right) - f_2 + v_d(t_1+R) \\
&+ f_1 \Leftrightarrow -s_{-i}^{*1}v_d\frac{(c_1+c_2)(n-1)}{2c_1c_2}R = -v_d\left((t_2-t_1) + \frac{(n-1)}{2c_2}R\right) - (f_2-f_1) \\
&\Leftrightarrow s_{-i}^{*1}\frac{(c_1+c_2)(n-1)}{2c_1c_2}Rv_d = \frac{2c_2v_d(t_2-t_1) + v_d(n-1)R + 2c_2(f_2-f_1)}{2c_2} \Leftrightarrow \\
&s_{-i}^{*1} = \frac{c_1v_dR(n-1) + 2c_1c_2v_d(t_2-t_1) + 2c_1c_2(f_2-f_1)}{v_d(c_1+c_2)(n-1)R}
\end{aligned}$$

Then, for firm 2 the equilibrium probability is:

$$s_{-i}^{*2} = 1 - s_{-i}^{*1} = 1 - \frac{c_1 v_d\left(n-1\right)R + 2c_1c_2v_d\left(t_2 - t_1\right) + 2c_1c_2\left(f_2 - f_1\right)}{v_d\left(c_1 + c_2\right)\left(n-1\right)R} \Leftrightarrow$$

$$s_{-i}^{*2} = \frac{v_d\left(c_1 + c_2\right)\left(n-1\right)R}{v_d\left(c_1 + c_2\right)\left(n-1\right)R} - \frac{c_1 v_d\left(n-1\right)R + 2c_1c_2v_d\left(t_2 - t_1\right) + 2c_1c_2\left(f_2 - f_1\right)}{v_d\left(c_1 + c_2\right)\left(n-1\right)R} \Leftrightarrow$$

$$s_{-i}^{*2} = \frac{c_2 v_d R(n-1) + 2c_1c_2v_d(t_1 - t_2) + 2c_1c_2(f_1 - f_2)}{v_d(n-1)(c_1 + c_2)R}$$

A.2 Two firms' equilibrium in prices

Simple notation is going to used for this example, as investors are tied to particular locations therefore j can be used to indicate the investor, the investor's firm and that particular location. For firm 1, we use the expected utility from Equation (3.11), and after substituting the probability from (3.9) this becomes:

$$\mathbb{E}[r_j^k(f)|s] = nw\frac{c_1v_dR(n-1)+2c_1c_2v_d(t_2-t_1)+2c_1c_2(f_2-f_1)}{v_dR(n-1)(c_1+c_2)}(f_1-h)-b_1c_1-o_1$$
$$= nw\frac{c_1v_dR(n-1)+2c_1c_2v_d(t_2-t_1)+2c_1c_2(f_2-f_1)}{v_dR(n-1)(c_1+c_2)}f_1-$$
$$nw\frac{c_1v_dR(n-1)+2c_1c_2v_d(t_2-t_1)+2c_1c_2(f_2-f_1)}{v_dR(n-1)(c_1+c_2)}h-b_1c_1-o_1 \textit{ (Split } 2c_1c_2(f_2-f_1))$$
$$= nw\frac{c_1v_dR(n-1)+2c_1c_2v_d(t_2-t_1)+2c_1c_2f_2}{v_dR(n-1)(c_1+c_2)}f_1-\frac{nw2c_1c_2}{v_dR(n-1)(c_1+c_2)}f_1^2-$$
$$nw\frac{c_1v_dR(n-1)+2c_1c_2v_d(t_2-t_1)+2c_1c_2f_2}{v_dR(n-1)(c_1+c_2)}h+\frac{nw2c_1c_2h}{v_dR(n-1)(c_1+c_2)}f_1-b_1c_1-o_1$$
$$= -\frac{nw2c_1c_2}{v_dR(n-1)(c_1+c_2)}f_1^2+nw\frac{c_1v_dR(n-1)+2c_1c_2v_d(t_2-t_1)+2c_1c_2(f_2+h)}{v_dR(n-1)(c_1+c_2)}f_1-$$
$$nw\frac{c_1v_dR(n-1)+2c_1c_2v_d(t_2-t_1)+2c_1c_2f_2}{v_dR(n-1)(c_1+c_2)}h-b_1c_1-o_1$$

This utility is a second-degree polynomial in f_1 and is continuously differentiable in $(-\infty,+\infty)$. Firm 1's partial derivative with respect to f_1 then is:

$$\frac{\partial\mathbb{E}[r^1(f)|s^*]}{\partial f_1} =$$
$$-\frac{nw4c_1c_2}{v_dR(n-1)(c_1+c_2)}f_1+nw\frac{c_1v_dR(n-1)+2c_1c_2v_d(t_2-t_1)+2c_1c_2(f_2+h)}{v_dR(n-1)(c_1+c_2)}$$
$$= nw\frac{c_1v_dR(n-1)-2c_1c_2v_d(t_1-t_2)+2c_1c_2(f_2+h)-4c_1c_2f_1}{v_dR(n-1)(c_1+c_2)}$$

Following the same methodology for firm 2, results in the partial derivatives of firms with respect to their price being:

$$\begin{aligned}\frac{\partial\mathbb{E}[r^1(f)|s^*]}{\partial f_1} &= nw\frac{c_1v_dR(n-1)-2c_1c_2v_d(t_1-t_2)+2c_1c_2(f_2+h)-4c_1c_2f_1}{v_dR(n-1)(c_1+c_2)}\\ \frac{\partial\mathbb{E}[r^2(f)|s^*]}{\partial f_2} &= nw\frac{c_2v_dR(n-1)-2c_1c_2v_d(t_2-t_1)+2c_1c_2(f_1+h)-4c_1c_2f_2}{v_dR(n-1)(c_1+c_2)}\end{aligned} \tag{A.2}$$

From Equation (A.2) it is straightforward to deduce, by setting the numerator of the derivatives to 0, that each firm's utility has exactly one critical point in $\mathbb{R}$ which are:

$$\begin{aligned}f_1^0 &= \frac{2c_2h-2c_2v_d(t_1-t_2)+v_dR(n-1)}{4c_2}+\frac{1}{2}f_2\\ f_2^0 &= \frac{2c_1h-2c_1v_d(t_2-t_1)+v_dR(n-1)}{4c_1}+\frac{1}{2}f_1\end{aligned} \tag{A.3}$$

Now the second partial derivatives of firm utilities are straightforward to find and are:

$$\begin{aligned}\frac{\partial^2 \mathbb{E}[r_1(f)|s]}{\partial f_1^2} &= -\frac{4nc_1c_2w}{v_dR(n-1)(c_1+c_2)} < 0 \\ \frac{\partial^2 \mathbb{E}[r_2(f)|s}{\partial f_2^2} &= -\frac{4nc_1c_2w}{v_dR(n-1)(c_1+c_2)} < 0\end{aligned} \tag{A.4}$$

Both second partial derivatives in Equation (A.4) are negative for any price in $(-\infty, +\infty)$, therefore station utilities are concave down in $(-\infty, +\infty)$. Moreover, the gradient of stations' utilities on the 'f_j^2' term, that can be seen in Equation (A.2), is negative. Therefore $\lim_{f_j \to -\infty} \mathbb{E}[r_j(f)|s^*] = -\infty$ and $\lim_{f_j \to +\infty} \mathbb{E}[r_j(f)|s^*] = -\infty$. Hence f_1^0, f_2^0 in Equation (A.3) are global maxima of firm 1 and 2's utilities respectively, in $(-\infty, +\infty)$. Finally, by setting f_2 in Equation A.3 to f_2^0 and f_1 in to f_1^0 (maximum given that the other firm is also going to maximise), we solve the simple linear system and obtain the equilibrium prices shown in equation (3.16). Of course, the solution is governed by the same boundary conditions as the probabilities due to substitution, which were explained in Section 3.1.5.

A.3 Proof for concavity in firm's expected utility

The first derivative of firm utility from equation 3.23, with respect to c_j is:

$$
\begin{gathered}
\mathbb{E}[r^k(c)|s]' = \left(\left(\frac{p}{c_j + \sum\limits_{-j \in I} c_{-j}} - b_j \right) c_j - \cancelto{0}{o_j} \right)' = \\
\left(\frac{p}{c_j + \sum\limits_{-j \in I} c_{-j}} - b_j \right) c_j' + \left(\frac{p}{c_j + \sum\limits_{-j \in I} c_{-j}} - \cancelto{0}{b_j} \right)' c_j = \left(\frac{p}{c_j + \sum\limits_{-j \in I} c_{-j}} - b_j \right) + \\
\frac{\cancelto{0}{p'} \left(c_j + \sum\limits_{-j \in I} c_{-j} \right) - p \cancelto{1}{\left(c_j + \sum\limits_{-j \in I} c_{-j} \right)'}}{\left(c_j + \sum\limits_{-j \in I} c_{-j} \right)^2} c_j = \\
\frac{p}{c_j + \sum\limits_{-j \in I} c_{-j}} - b_j - \frac{c_j p}{\left(c_j + \sum\limits_{-j \in I} c_{-j} \right)^2} = \frac{\left(c_j + \sum\limits_{-j \in I} c_{-j} \right) p - c_j p}{\left(c_j + \sum\limits_{-j \in I} c_{-j} \right)^2} - b_j = \\
\frac{p \sum\limits_{-j \in I} c_{-j}}{\left(c_j + \sum\limits_{-j \in I} c_{-j} \right)^2} - b_j
\end{gathered}
\tag{A.5}
$$

A critical point of firm j's expected utility is thus one that satisfies:

$$\mathbb{E}[r^k(c^*)|s]' = 0 \Leftrightarrow \frac{p\sum_{-j\in I} c_{-j}}{\left(c_j + \sum_{-j\in I} c_{-j}\right)^2} - b_j = 0 \Leftrightarrow p\sum_{-j\in I} c_{-j} - b_j\left(c_j + \sum_{-j\in I} c_{-j}\right)^2 = 0$$

$$\Leftrightarrow p\sum_{-j\in I} c_{-j} - b_j\left(c_j^2 + 2c_j\sum_{-j\in I} c_{-j} + \left(\sum_{-j\in I} c_{-j}\right)^2\right) = 0 \Leftrightarrow$$

$$p\sum_{-j\in I} c_{-j} - b_j c_j^2 - 2b_j\sum_{-j\in I} c_{-j}c_j - b_j\left(\sum_{-j\in I} c_{-j}\right)^2 = 0 \Leftrightarrow$$

$$b_j c_j^2 + 2b_j\sum_{-j\in I} c_{-j}c_j + b_j\left(\sum_{-j\in I} c_{-j}\right)^2 - p\sum_{-j\in I} c_{-j} = 0$$

The discriminant of this second-degree polynomial is:

$$D = \left(2b_j\sum_{-j\in I}\right)^2 - 4(b_j)\left(b_j\left(\sum_{-j\in I} c_{-j}\right)^2 - p\sum_{-j\in I} c_{-j}\right) =$$

$$\cancel{4b_j^2\left(\sum_{-j\in I} c_{-j}\right)^2} \cancel{-4b_j^2\left(\sum_{-j\in I} c_{-j}\right)^2} + 4b_j p\sum_{-j\in I} c_{-j} = 4b_j p\sum_{-j\in I} c_{-j} > 0$$

and therefore it has two roots in $\mathbb{R}$:

$$c_j^1 = \frac{-2b_j\sum_{-j\in I} c_{-j} + \sqrt{4b_j p\sum_{-j\in I} c_{-j}}}{2b_j}$$

and

$$c_j^2 = \frac{-2b_j\sum_{-j\in I} c_{-j} - \sqrt{4b_j p\sum_{-j\in I} c_{-j}}}{2b_j} < 0$$

Of these, c_j^2 is clearly negative, and therefore outside the $(0, \Theta)$ interval we are interested in. In addition, for c_j^1 to be a valid root, assuming profit p is positive ($f_j \geq h_j$) it must be:

$$c_j^1 > 0 \Leftrightarrow \frac{-2b_j \sum\limits_{-j\in I} c_{-j} + \sqrt{4b_j p \sum\limits_{-j\in I} c_{-j}}}{2b_j} > 0 \Leftrightarrow -2b_j \sum\limits_{-j\in I} c_{-j} + \sqrt{4b_j p \sum\limits_{-j\in I} c_{-j}} > 0 \Leftrightarrow$$

$$\sqrt{4b_j p \sum\limits_{-j\in I} c_{-j}} > 2b_j \sum\limits_{-j\in I} c_{-j} \Leftrightarrow 2\sqrt{p} > 2\sqrt{b_j \sum\limits_{-j\in I} c_{-j}} \Leftrightarrow p > b_j \sum\limits_{-j\in I} c_{-j} \Leftrightarrow$$

$$\textit{from equation } (3.17)\ (f_j - h_j)nw > b_j \sum\limits_{-j\in I} c_{-j} \Leftrightarrow f_j > \frac{b_j \sum\limits_{-j\in I} c_{-j}}{nw} + h_j$$

Therefore firm j's expected utility has one critical point in $(0, \Theta)$, provided that $f_j > \frac{b_j \sum\limits_{-j\in I} c_{-j}}{nw} + h_j$.

Continuing on from equation (A.5), the second derivative of firm utility is:

$$\mathbb{E}[r^k(c)|s]'' = \left(\frac{p \sum\limits_{-j\in I} c_{-j}}{\left(c_j + \sum\limits_{-j\in I} c_{-j}\right)^2} - \cancelto{0}{b_j} \right)' =$$

$$\frac{\cancelto{0}{\left(p \sum\limits_{-j\in I} c_{-j}\right)'} \left(c_j + \sum\limits_{-j\in I} c_{-j}\right)^2 - p \sum\limits_{-j\in I} c_{-j} \left(\left(c_j + \sum\limits_{-j\in I} c_{-j}\right)^2 \right)'}{\left(c_j + \sum\limits_{-j\in I} c_{-j}\right)^4} =$$

$$\frac{-2p \sum\limits_{-j\in I} c_{-j} \left(c_j + \sum\limits_{-j\in I} c_{-j}\right)}{\left(c_j + \sum\limits_{-j\in I} c_{-j}\right)^4} < 0\ \forall c_j \in (0, \Theta)$$

Thus equation 3.23 is concave down in $(0, \Theta)$, and has a maximum value in $(0, \Theta)$ when $f_j > \frac{b_j \sum\limits_{-j\in I} c_{-j}}{nw} + h_j$.

A.4 Equilibrium condition expanded formula

$\forall j \in I: \quad \mathbb{E}[r^k((c_j^*, c_{-j}^*))|s] \geq \mathbb{E}[r^k((c_j^* + \alpha, c_{-j}^*))|s] \Leftrightarrow$ *from* (3.18) *and* (3.21)

$$\frac{c_j^*}{c_j^* + \sum\limits_{-j \in I} c_{-j}^*} p - b_j c_j^* - \cancel{o_j} \geq \frac{c_j^* + \alpha}{c_j^* + \alpha + \sum\limits_{-j \in I} c_{-j}^*} p - b_j (c_j^* + \alpha) - \cancel{o_j} \Leftrightarrow$$

$$\frac{c_j^*}{c_j^* + \sum\limits_{-j \in I} c_{-j}^*} p - \cancel{b_j c_j^*} \geq \frac{c_j^* + \alpha}{c_j^* + \alpha + \sum\limits_{-j \in I} c_{-j}^*} p - \cancel{b_j c_j^*} - b_j \alpha \Leftrightarrow$$

$$\frac{c_j^*}{c_j^* + \sum\limits_{-j \in I} c_{-j}^*} p - \frac{c_j^* + \alpha}{c_j^* + \alpha + \sum\limits_{-j \in I} c_{-j}^*} p \geq -b_j \alpha \Leftrightarrow (\times(-1))$$

$$\frac{c_j^* + \alpha}{c_j^* + \alpha + \sum\limits_{-j \in I} c_{-j}^*} p - \frac{c_j^*}{c_j^* + \sum\limits_{-j \in I} c_{-j}^*} p \leq b_j \alpha \Leftrightarrow$$

$$p \left(\frac{c_j^* + \alpha}{c_j^* + \alpha + \sum\limits_{-j \in I} c_{-j}^*} - \frac{c_j^*}{c_j^* + \sum\limits_{-j \in I} c_{-j}^*} \right) \leq b_j \alpha \Leftrightarrow$$

$$b_j \alpha \geq p \frac{(c_j^* + \alpha) \left(c_j^* + \sum\limits_{-j \in I} c_{-j}^* \right) - c_j^* \left(c_j^* + \alpha + \sum\limits_{-j \in I} c_{-j}^* \right)}{\left(c_j^* + \alpha + \sum\limits_{-j \in I} c_{-j}^* \right) \left(c_j^* + \sum\limits_{-j \in I} c_{-j}^* \right)} \Leftrightarrow$$

$$b_j \alpha \geq p \frac{\cancel{c_j^{*2}} + c_j^* \cancel{\sum\limits_{-j \in I} c_{-j}^*} + \cancel{\alpha c_j^*} + \alpha \sum\limits_{-j \in I} c_{-j}^* - \cancel{c_j^{*2}} - \cancel{\alpha c_j^*} - c_j^* \cancel{\sum\limits_{-j \in I} c_{-j}^*}}{\left(c_j^* + \sum\limits_{-j \in I} c_{-j}^* \right)^2 + \alpha \left(c_j^* + \sum\limits_{-j \in I} c_{-j}^* \right)} \Leftrightarrow$$

$$b_j \alpha \geq p \frac{\alpha \sum\limits_{-j \in I} c_{-j}^*}{\left(c_j^* + \sum\limits_{-j \in I} c_{-j}^* \right)^2 + \alpha \left(c_j^* + \sum\limits_{-j \in I} c_{-j}^* \right)}$$

A.5 EV driver's expected utility linearity with a two-station example

The goal of this section is to show that the driver i's expected utility for playing in mixed strategy Nash equilibrium is expected to be linear with respect to one station's fees given the fee at the other station. We will assume that drivers are called to choose between two stations, station 1 and station 2 and for simplicity it is assumed that charging time is the same at both stations and equal to R. The expected utility of the mixed strategy was defined in Equation (3.5), and by replacing the expected utility of station choice $\mathbb{E}[u_i^j(x)\,|s_{-i}]$ from Equation (3.7) it becomes:

$$\begin{aligned}&\mathbb{E}[u_i(x)\,|s] = \sum_{j=1}^{m} s_i^j \mathbb{E}[u_i^j(x)\,|s_{-i}] = \\ &= s_i^1\left(-v_d\left(t_1 + s_{-i}^1\frac{n-1}{2c_1}R + R\right) - f_1\right) + s_i^2\left(-v_d\left(t_2 + s_{-i}^2\frac{n-1}{2c_2}R + R\right) - f_2\right)\end{aligned} \tag{A.6}$$

Given that drivers are going to play in a symmetric mixed strategy Nash equlibrium, then the probability that driver i will choose route j is equal to the probability other drivers choose route j, that is $s_i^j = s_{-i}^j$. Also we have that $s_i^1 + s_i^2 = 1 \Leftrightarrow s_i^2 = 1 - s_i^1$. So, by replacing $s_{-i}^1 = s_i^1$ and $s_{-i}^2 = s_i^2 = 1 - s_i^1$, Equation (A.6) becomes:

$$\begin{aligned}
&s_i^1\left(-v_d\left(t_1 + s_i^1\frac{n-1}{2c_1}R + R\right) - f_1\right) + \\
&+ (1 - s_i^1)\left(-v_d\left(t_2 + (1 - s_i^1)\frac{n-1}{2c_2}R + R\right) - f_2\right) = \\
=& - v_d t_1 s_i^1 - v_d\frac{n-1}{2c_1}R(s_i^1)^2 - v_d R s_i^1 - f_1 s_i^1 - v_d t_2(1 - s_i^1) - v_d\frac{n-1}{2c_2}R(1 - s_i^1)^2 \\
&- v_d R(1 - s_i^1) - f_2(1 - s_i^1) = \\
=& - v_d t_1 s_i^1 - v_d\frac{n-1}{2c_1}R(s_i^1)^2 - v_d R s_i^1 - f_1 s_i^1 - v_d t_2 + v_d t_2 s_i^1 - v_d\frac{n-1}{2c_2}R(1 + (s_i^1)^2 - 2s_i^1) \\
&- v_d R + v_d R s_i^1 - f_2 + f_2 s_i^1 = \\
=& - v_d t_1 s_i^1 - v_d\frac{n-1}{2c_1}R(s_i^1)^2 - \cancel{v_d R s_i^1} - f_1 s_i^1 - v_d t_2 + v_d t_2 s_i^1 - v_d\frac{n-1}{2c_2}R - v_d\frac{n-1}{2c_2}R(s_i^1)^2 \\
&+ 2v_d\frac{n-1}{2c_2}R s_i^1 - v_d R + \cancel{v_d R s_i^1} - f_2 + f_2 s_i^1 = \\
=& - v_d(t_1 - t_2)s_i^1 - v_d(n-1)R\left(\frac{1}{2c_1} + \frac{1}{2c_2}\right)(s_i^1)^2 - (f_1 - f_2)s_i^1 + 2v_d\frac{n-1}{2c_2}R s_i^1 - v_d R \\
&- f_2 - v_d\frac{n-1}{2c_2}R - v_d t_1 =
\end{aligned}$$

$$
\begin{aligned}
&= - v_d(t_1 - t_2)s_i^1 - \frac{v_d(n-1)(c_1+c_2)R}{2c_1c_2}(s_i^1)^2 - (f_1 - f_2)s_i^1 + 2v_d\frac{n-1}{2c_2}Rs_i^1 - v_dR \\
&\quad - f_2 - v_d\frac{n-1}{2c_2}R - v_dt_1
\end{aligned} \tag{A.7}
$$

Given that Equation (A.7) is differentiable on the fees in $(-\infty, +\infty)$, we partially differentiate for f_1 to get the rate of change of expected utility for a varying price from station 1, given the price of the other station. Of course, the probability s_i^1 is also a function of f_1 and thus continuing from Equation (A.7) this partial derivative is:

$$
\begin{aligned}
&= \Bigg(- v_d(t_1 - t_2)s_i^1(f_1) - \frac{v_d(n-1)(c_1+c_2)R}{2c_1c_2}(s_i^1(f_1))^2 - (f_1 - f_2)s_i^1(f_1) \\
&\quad +2v_d\frac{n-1}{2c_2}Rs_i^1(f_1) - \underbrace{v_dR - f_2 - v_d\frac{n-1}{2c_2}R - v_dt_1}_{\to 0} \Bigg)' = \\
&= - v_d(t_1 - t_2)s_i^{1\prime}(f_1) - 2\frac{v_d(n-1)(c_1+c_2)R}{2c_1c_2}s_i^1(f_1)s_i^{1\prime}(f_1) - \big((f_1 - f_2)s_i^1(f_1)\big)' \\
&\quad + 2v_d\frac{n-1}{2c_2}Rs_i^{1\prime}(f_1) = \\
&= - v_d(t_1 - t_2)s_i^{1\prime}(f_1) - 2\frac{v_d(n-1)(c_1+c_2)R}{2c_1c_2}s_i^1(f_1)s_i^{1\prime}(f_1) - \Big(s_i^1(f_1) + (f_1 - f_2)s_i^{1\prime}(f_1)\Big) \\
&\quad + 2v_d\frac{n-1}{2c_2}Rs_i^{1\prime}(f_1) = \\
&= - v_d(t_1 - t_2)s_i^{1\prime}(f_1) - 2\frac{v_d(n-1)(c_1+c_2)R}{2c_1c_2}s_i^1(f_1)s_i^{1\prime}(f_1) - s_i^1(f_1) - (f_1 - f_2)s_i^{1\prime}(f_1) \\
&\quad + 2v_d\frac{n-1}{2c_2}Rs_i^{1\prime}(f_1).
\end{aligned} \tag{A.8}
$$

However, from Equation (3.9), we have for s_i^1 in equilibrium that:

$$
\begin{aligned}
s_i^1(f_1) &= \frac{c_1v_dR(n-1) + 2c_1c_2v_d(t_2 - t_1) + 2c_1c_2(f_2 - f_1)}{v_d(n-1)(c_1+c_2)R} \Leftrightarrow \\
\Leftrightarrow s_i^1(f_1) &= \frac{c_1v_dR(n-1) + 2c_1c_2v_d(t_2 - t_1) + 2c_1c_2f_2}{v_d(n-1)(c_1+c_2)R} - \frac{2c_1c_2}{v_d(n-1)(c_1+c_2)R}f_1 \\
\Rightarrow s_i^{1\prime}(f_1) &= \left(\underbrace{\frac{c_1v_dR(n-1) + 2c_1c_2v_d(t_2 - t_1) + 2c_1c_2f_2}{v_d(n-1)(c_1+c_2)R}}_{\to 0} - \frac{2c_1c_2}{v_d(n-1)(c_1+c_2)R}f_1 \right)'
\end{aligned}
$$

$$\Rightarrow s_i^{1\prime}(f_1) = -\frac{2c_1c_2}{v_d(n-1)(c_1+c_2)R} \tag{A.9}$$

Now, continuing from Equation (A.8) by replacing $s_i^{1\prime}(f_1)$ from Equation A.9 we have:

$$\begin{aligned}
&- v_d(t_1 - t_2)\left(-\frac{2c_1c_2}{v_d(n-1)(c_1+c_2)R}\right) \\
&\cancel{-}2\,\cancel{\frac{v_d(n-1)(c_1+c_2)R}{2c_1c_2}}\left(\cancel{-}\,\cancel{\frac{2c_1c_2}{v_d(n-1)(c_1+c_2)R}}\right)s_i^1(f_1) - s_i^1(f_1) \\
&- (f_1 - f_2)\left(-\frac{2c_1c_2}{v_d(n-1)(c_1+c_2)R}\right) + 2v_d\frac{n-1}{\cancel{2c_2}}R\left(-\frac{\cancel{2}c_1\cancel{c_2}}{v_d(n-1)(c_1+c_2)R}\right) \\
= &- \frac{-2c_1c_2v_d(t_1-t_2)}{v_d(n-1)(c_1+c_2)R} + 2s_i^1(f_1) - s_i^1(f_1) - \frac{-2c_1c_2(f_1-f_2)}{v_d(n-1)(c_1+c_2)R} \\
&- \frac{2c_1v_d(n-1)R}{v_d(n-1)(c_1+c_2)R} \\
= &- \frac{2c_1c_2v_d(t_2-t_1)}{v_d(n-1)(c_1+c_2)R} - \frac{2c_1c_2(f_2-f_1)}{v_d(n-1)(c_1+c_2)R} - \frac{2c_1v_d(n-1)R}{v_d(n-1)(c_1+c_2)R} + s_i^1(f_1) \\
= &- \frac{2c_1c_2v_d(t_2-t_1)}{v_d(n-1)(c_1+c_2)R} - \frac{2c_1c_2(f_2-f_1)}{v_d(n-1)(c_1+c_2)R} - \frac{c_1v_d(n-1)R}{v_d(n-1)(c_1+c_2)R} \\
&- \frac{c_1v_d(n-1)R}{v_d(n-1)(c_1+c_2)R} + s_i^1(f_1) \\
= &- \frac{c_1v_d(n-1)R + 2c_1c_2v_d(t_2-t_1) + 2c_1c_2(f_2-f_1)}{v_d(n-1)(c_1+c_2)R} + s_i^1(f_1) - \frac{c_1\cancel{v_d(n-1)R}}{\cancel{v_d(n-1)}(c_1+c_2)\cancel{R}}
\end{aligned} \tag{A.10}$$

At this point we notice from Equation (3.9) that the large, first, fraction is actually the probability $s_i^1(f_1)$. Then, Equation (A.10) becomes:

$$-s_i^1(f_1) + s_i^1(f_1) - \frac{c_1}{c_1+c_2} = -\frac{c_1}{c_1+c_2}$$

This is a constant for given capacities, therefore driver i's expected utility will decrease linearly with an increasing price at station 1, given the price at station 2, following a $-\frac{c_1}{c_1+c_2}$ gradient. Similarly, the partial derivative of expected utility with respect to f_2 is $-\frac{c_2}{c_1+c_2}$ and can be found following similar methodology.

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