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OPTIMAL PRELIMINARY PROPELLER DESIGN USING NONLINEAR CONSTRAINED MATHEMATICAL PROGRAMMING TECHNIQUE

By D. Radojčić

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ABSTRACT

Presented is a nonlinear constrained optimization technique applied to optimal propeller design at the preliminary design stage. The optimization method used is Sequential Unconstrained Minimization Technique – SUMT, which can treat equality and inequality, or only inequality constraints. Both approaches are shown. Application is given for Wageningen B-series and Gawn series propellers. The problem is solved on an Apple II microcomputer. One of the advantages of treating the constrained problem is that the user's knowledge about propellers is not essential, the process is automatic. More realistic propellers are found when design constraints such as D_{max}, N_{min}, and/or Ae/Ao_{min} are applied. Treating blade area ratio as an independent variable shows that, for some cases, higher BAR may be a better choice than lower BAR value.

CONTENTS

1. INTRODUCTION

The objective is to present a possible technique for choosing the optimal propeller characteristics with the help of a microcomputer. The paper does not give full theoretical explanations, but shows the results obtained when a constrained nonlinear programming method is applied to the propeller problems.

Since the presentation of B-series propeller results in the polynomial form (initially in [1] and in the final form in [2]), almost all open-water propeller design computer programs have used them. It is not only that the Wageningen B-series polynomials are attractive for computer applications, but the results obtained are reasonably correct. Therefore, these polynomials form the basis for the optimization process.

The method presented has been developed from the following previous attempts at propeller performance optimization:

Triantafyollou in [3], and similarly Bernitsas and Ray in [4] and [5], treat the choice of optimal propeller characteristics as an unconstrained mathematical programming problem. With the help of classical optimization technique of Lagrange multipliers, and after the elimination of the single multiplier as it was not of any practical interest, two equations with two unknowns (J and P/D) were derived. This yielded a single stationary point which gives the maximum propeller efficiency η_o. Cavitation was not considered. Therefore, iterations were necessary before a final result was obtained (the same as would be done if the calculations were carried out by hand).

Markussen [6] incorporated a cavitation equation prior to treating the optimal propeller design problem with the help of Lagrangian multipliers. After the elimination of multipliers, three simultaneous equations in three unknown variables, namely, blade area ratio - BAR or Ae/Ao, pitch ratio - P/D, and advance coefficient - J, followed. This was solved with the iterative process of Newton-Raphson. For constant BAR this technique reduces to the same as given in [3] and [4 and 5].

It should be noted that in both cases the unconstrained optimization technique was used, i.e. there were no boundaries on the main variables J, P/D, and Ae/Ao. Therefore they could have any values between $-\infty$ and $+\infty$, although feasible boundaries or constraints, naturally, do exist. Practically, it follows that the above approaches are useful only if the optimal solution is somewhere inside the boundaries, which is, unfortunately, not known in advance. This further means that the process of choosing an optimal propeller must be controlled throughout.

Incorporating the following constraints:

$$0<J<J_{K_T=0}$$

$$0.5<P/D<1.4$$

$$0.3<Ae/Ao<1.05$$

which are valid for the Wageningen B-series, and if the same method of Lagrange multipliers was used, it would be necessary to introduce slack variables and solve 16 equations with 16 unknown values, although only three are of practical interest. The other possibility is the elimination of the constraints by the transformation of the variables. This would simplify the problem (reduce the number of constraints to be considered), but usually complicates it by introducing extra local minima [7].

These were the reasons for the application of the most complicated mathematical programming technique - nonlinear constrained optimization, which in this particular case gave very satisfactory results.

Since the published propeller polynomials are used extensively, they will be briefly explained. The nonlinear optimization technique used, and the application of the optimization on the propeller design problems are given. The results included show that it is not always necessarily true that the propellers with the lowest BAR are most efficient.

2. POLYNOMIAL EXPRESSIONS OF PROPELLER SERIES

Wageningen B-series propeller polynomial expressions for thrust and torque coefficients, for two to seven blades and Reynolds number 2×10^6, were published by Osterveld and Oossanen in [2]. The polynomials were given in the following form:

$$K_T=\sum_{1}^{39} C_T(J)^s(P/D)^t(Ae/Ao)^u(Z)^v$$

$$K_Q=\sum_{1}^{47} C_Q(J)^s(P/D)^t(Ae/Ao)^u(Z)^v$$

The coefficients are given in tabular form and hold good for blade thickness ratio $(t/c)_{0.75R}=f(Z,Ae/Ao)$. Correction for Reynolds number effect is given as $K_T, K_Q=f(J,P/D,Ae/Ao,Z,\log(Rn))$. The effect of change in blade thickness can be taken into account with the correction of Rn. However, from the diagrams in [2] it can be seen that the effect of Reynolds number is not significant. That is, the correction of K_T and K_Q may be omitted in the preliminary design stage. Similarly, Figure 1, taken from [8], shows the effect of 100% thickness change on the propeller efficiency η_o and pitch.

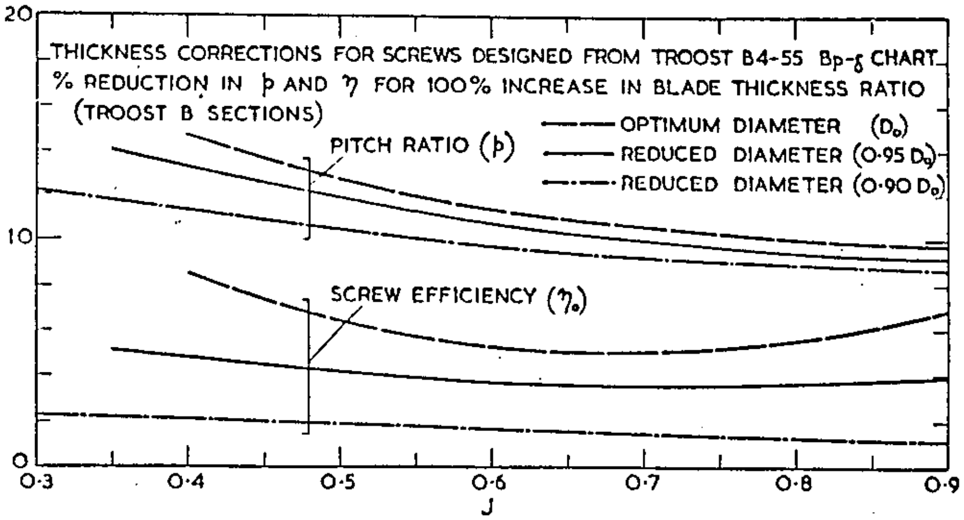

Figure 1

It should be noted that with the help of regression analysis the fairing of the B-screw series test results has been completed and that the area of applicability extended.

The Gawn three bladed series was presented in paper [9] in the polynomial form in an identical way to the B-series. Regression was used only to calculate the difference in K_T and K_Q between B-series and Gawn series. That is the reason the terms of the polynomials are the same, but of different magnitude. It is stated in [9] that the applicability of Gawn polynomials is in the range

$$3<Z<4$$

$$0.6<P/D<1.6$$

$$0.5<Ae/Ao<1.1$$

Correction for change of Reynolds number or thickness ratio is not mentioned in the above paper.

The same confidence should not be given to the Gawn polynomials as to the B-series polynomials. The extreme values should be avoided, particularly for low J values. Taking into account that this is the only analytical representation of the segmental section Gawn series, the polynomials are very helpful, particularly since large scale Gawn diagrams are not widely available.

3. OPTIMIZATION

Every design problem may be considered as optimization. However, optimization as a mathematical programming technique is relatively new, and is still developing. Optimization routines are essential parts of many computer programs, specifically CAD. They are used in almost every branch of science, from engineering to chemistry and economics.

Many books have dealt with this subject, some of which are given in references [10] to [14].

There are several classifications of optimization, for example:
- linear vs nonlinear problem
- constrained vs unconstrained problem
- one variable vs n-variable problem
- continuous vs discrete function
- search vs gradient methods.

The problem in hand is nonlinear, with equality and inequality constraints, multivariable, and continuous. A gradient method was chosen since textbooks dealing with optimization recommend them as more efficient. However, search methods, particularly Hook and Jeeves, are widely used by engineers even when the function is continuous. Perhaps this is because they are easier to understand and therefore to program.

Search methods do not need partial derivatives, but since less information is given, more function evaluations are necessary. It is likely that the effort involved in evaluating the expressions for first partial derivatives will pay off in the efficiency and reliability. In the propeller problem case, although only three variables are present, the polynomials are relatively long, so the number of function evaluations should be minimized. Since it was thought that more complicated problems could be solved with the same optimization routine, a more powerful method was the obvious choice.

One of the methods which can cope with the above problems, and which can be utilized on the microcomputer, is Sequential Unconstrained Minimization Technique - SUMT. Developed in the sixties by Fiacco and McCormick, it is referenced in most books dealing with optimization methods. SUMT is today considered as an "old fashioned" method, but, as is cited in [13] "recently developed methods tend to be so complex that it is unlikely that the typical user will have the time or inclination to write his own computer program".

Taking all this into consideration, plus additional reasons outside the scope of this paper (for example the various published test cases solved with various optimization methods) the SUMT was chosen and will be briefly explained in the next section.

3.1. Sequential Unconstrained Minimization Technique - SUMT

SUMT is a penalty function method, i.e. it is necessary to transform a constrained problem into an unconstrained one which gives the same solution. When the constraint is violated a high value is given to the original objective function by a penalty term. If the constrained problem is to minimise $f(X)$, then a transformed unconstrained function to be minimized would be $f^*(X)=f(X)+P(X)$, where $P(X)$ is the penalty term. The penalty term has a special form which enables it to have a high value when one of the constraints is violated.

If only inequality constraints are present $c_i(X)\geqslant 0$, $i=1,2,\ldots m$, the transformed unconstrained function may be like this:

$$f^*(R,X)=f(X)+R[1/(c_i(X))].$$

If $c_i(X)>0$ and R are sufficiently small, then $f^*(R,X)\approx f(X)$. However, if $c_i(X)\leqslant 0$ than a penalty term will be relatively high, and the unconstrained optimization routine will move the solution in a feasible direction. SUMT actually solves the sequences of the unconstrained problem $f^*(R,X)$, for a sequence of R values which tend to zero. More about SUMT can be found in [15].

For practical solution of the propeller optimization problem a routine given in Reference [12] was adapted to handle the equality as well as inequality constraints. The flow chart for SUMT is given in Figure 2.

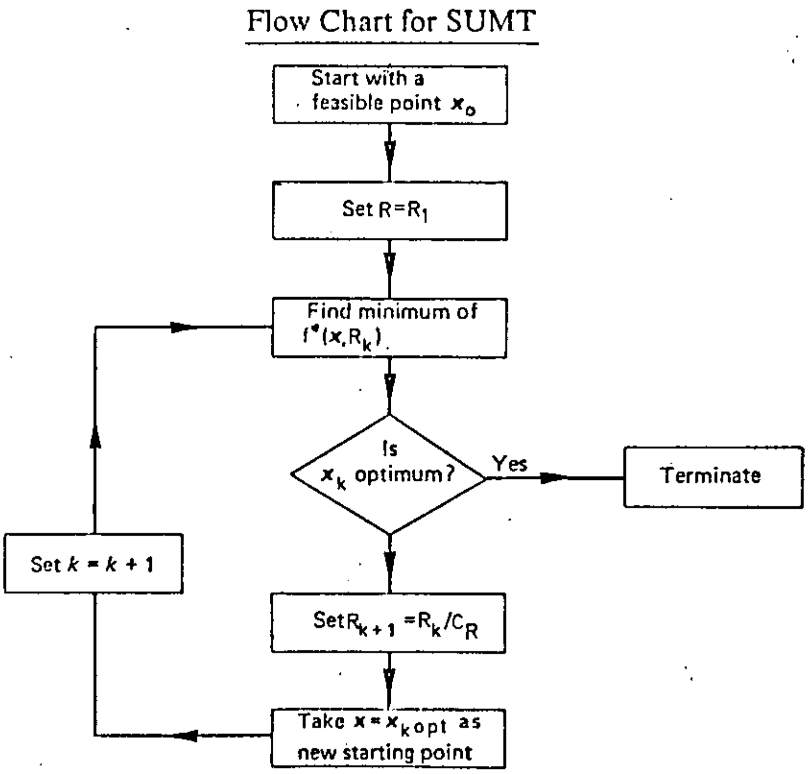

Figure 2

In the SUMT routine the unconstrained minimization is handled by the Davidon-Fletcher-Powell technique which is a gradient based method. To ensure that the search is always in the feasible region a step length L is divided by a constant C_L until the new point is inside the boundaries.

$$P_{new} = P_{old} + (L/C_L)\cdot(\text{search direction}).$$

The minimizations in each sequence (for R=const.) are carried out until $(f^*_{old} - f^*_{new})/f^*_{old} \leqslant E_1$. Complete procedure terminates when $R_k \leqslant E_2$. Other termination criteria may be chosen if desired.

Obviously, the whole procedure is to some extent dependent on R, C_R, L, C_L, E_1, and E_2.

After carrying out a certain number of unconstrained minimizations, since sequences of X and f*converge to X_{opt} and f_{opt} respectively, the result obtained so far can be used to estimate the minimum of the constrained problem by applying extrapolation. The extrapolation routine enables the iterations to be stopped (for example if calculating time is critical) and to improve the result so far obtained.

SUMT's main disadvantage is the fact that as $R_k \longrightarrow 0$ it becomes increasingly difficult to solve the unconstrained minimization problem. Therefore, the SUMT should be considered as an approximate method. But, as cited in [11] "the simplicity of the penalty methods will continue to attract the unsophisticated user".

4. PRELIMINARY PROPELLER DESIGN

4.1 Solution by Hand

In order to demonstrate the formulation of the preliminary propeller design problem, a solution by hand will be shown first. There are several approaches to the problem and the one chosen is the same as in References [4 and 5].

The basis for this approach are the K_T-K_Q-J charts for all four main options given in Table 1.

Option	Given values (input)	Calculated values (output)
1	R_t, D, V	N_{opt}, $(J, P/D, A_e/A_o, \eta_o)$
2	P_d, D, V	N_{opt}, $(J, P/D, A_e/A_o, \eta_o)$
3	R_t, N, V	D_{opt}, $(J, P/D, A_e/A_o, \eta_o)$
4	P_d, N, V	D_{opt}, $(J, P/D, A_e/A_o, \eta_o)$

Table 1

Later it will be shown that other possible options may be derived from these four. Since fundamentally, solution of the four problems is the same, although the reasons for using each of them are obviously different, they will be treated together.

For the sake of demonstration the example given in [4 and 5] will be presented here. The same example will be used throughout this paper.

Elimination of the unknown variables from

$$K_T = R_t/\varrho(1-t)N^2D^4, \qquad K_Q = (P_d \cdot \eta_r)/\varrho 2\pi N^3 D^5, \qquad J = V(1-w)/N \cdot D$$

is shown in Table 2. Graphical representation is given in the Figure 3.

Option 1	Option 2	Option 3	Option 4
$K_T/J^2 = \underbrace{R_t/[\varrho(1-t)(1-w)^2V^2D^2]}_{\text{const.}}$	$K_Q/J^3 = \underbrace{P_d\eta_r/[\varrho 2\pi(1-w)^3V^3D^2]}_{\text{const.}}$	$K_T/J^4 = \underbrace{R_t n^2/[\varrho(1-t)(1-w)^4V^4]}_{\text{const.}}$	$K_Q/J^5 = \underbrace{P_d\eta_r n^2/[\varrho 2\pi(1-w)^5V^5]}_{\text{const.}}$
Example from [4 and 5]	$C_b=0.65$ $V/\sqrt{L}=0.8$ $L/B=7.25$ $B/T=2.5$ $L=400'=122m$ $R_t=61900$ lb = 275345 N DHP=3782 ⇒ $P_d=2820kw$ N=77RPM D=18'=5.4864m $w=0.252$ $t=0.155$ $\eta_r=1.018$ $A_e/A_o=0.65$ Z=5 V=16kn		
$K_T = 0.278 \cdot J^2$	$K_Q = 0.0641 \cdot J^3$	$K_T = 0.3611 \cdot J^4$	$K_Q = 0.0822 \cdot J^5$

Table 2

The intersection of the K_T or K_Q curve with the parabola at each P/D value gives a corresponding value of η_o. The value of P/D that gives the maximum η_o defines the optimal propeller.

4.2. Mathematical Programming Formulation

Mathematical programming formulation for the preliminary propeller design problem would be:

maximize $\eta_o = (J \cdot K_T)/(2\pi \cdot K_Q)$ where $K_T, K_Q = f(J, P/D, Ae/Ao, Z)$,

$Rn = 2 \times 10^6$, $t/c = f(Ae/Ao, Z)$

subject to: Equality constraint $K_c = C \cdot J^p$ where

$$K_c = \begin{cases} K_T & \text{if } R_t \text{ is given} \\ K_Q & \text{if } P_d \text{ is given} \end{cases} \quad \text{and} \quad p = \begin{cases} 2 & \text{for option 1} \\ 3 & - \quad 2 \\ 4 & - \quad 3 \\ 5 & - \quad 4 \end{cases}$$

Inequality constraints (range constraints)

$$0.5 < P/D < 1.4$$
$$0.3 < Ae/Ao < 1.05$$

WAGENINGEN B-SERIES PROPELLERS
FOR 5 BLADES AE/AO= 0.650
P/D=0.50 TO 1.40

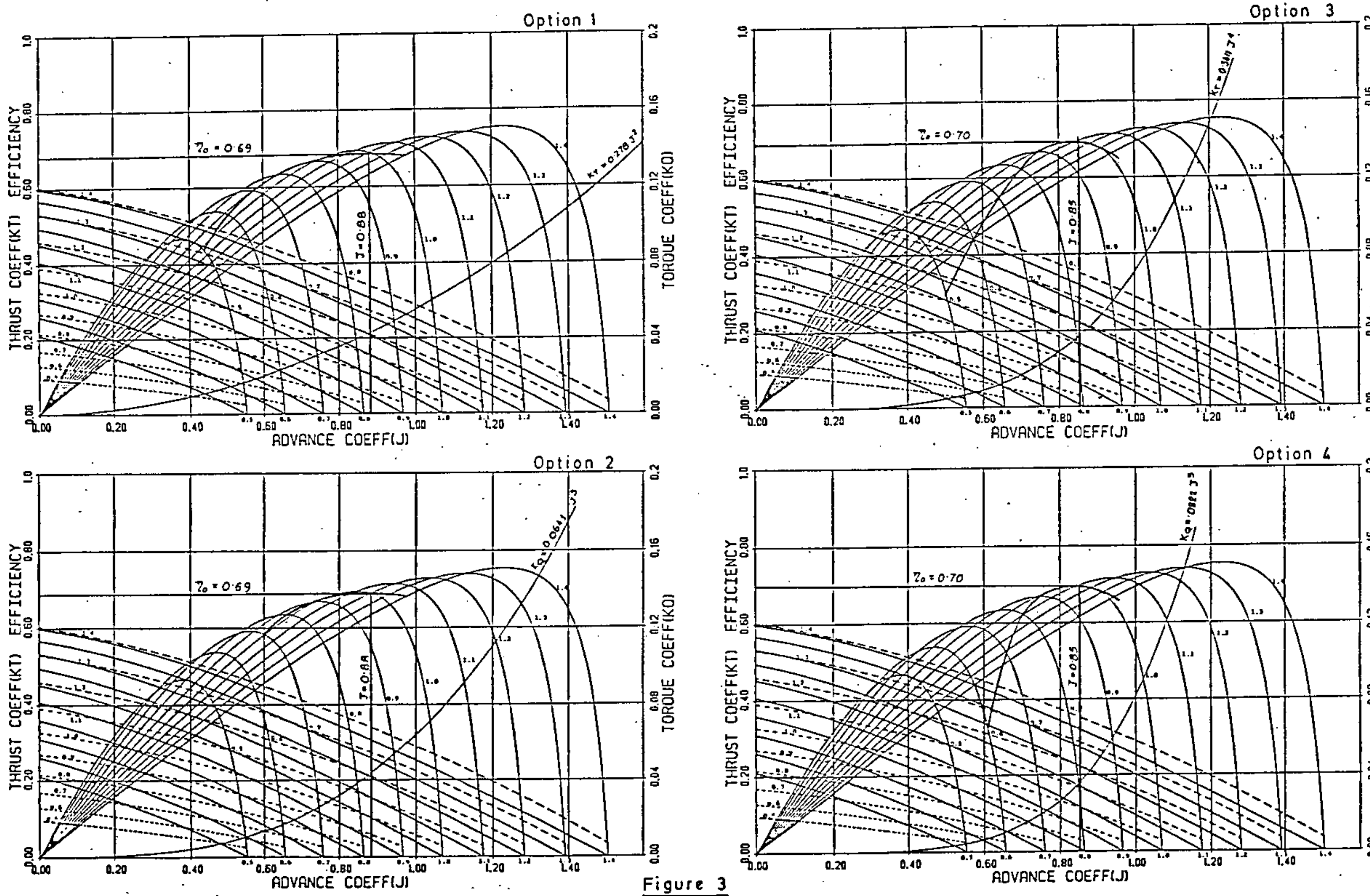

Figure 3

It should be noted that in the "solution by hand" Ae/Ao=const. which is a special case of the above problem.

The mathematical formulation which is more convenient for SUMT application should be:

FORMULATION A

minimize $f(X_1, X_2, X_3) = -\eta_o = -(J \cdot K_T)/(2\pi \cdot K_Q)$

(where $X_1 = J$, $X_2 = P/D$, $X_3 = Ae/Ao$)

subject to: Inequality constraints

$J \geqslant 0$	1
$K_T \geqslant 0$	2
$P/D - 0.5 \geqslant 0$	3
$1.4 - P/D \geqslant 0$	4
$2 - J \geqslant 0$	5
$Ae/Ao - 0.3 \geqslant 0$	6
$1.05 - Ae/Ao \geqslant 0$	7

Equality constraint

$K_c = C \cdot J^p$ 8

Inequality constraints 1 and 2 are necessary for feasible solutions; 3 and 4 are range constraints. If Ae/Ao=const. (special case) these four statements would be sufficient. However certain starting point values in combination with relatively high R could produce J=9, although inequality constraints 1 and 2 were not violated. That is the reason for constraint 5 which assumes that for B-series J<2. Constraints 6 and 7 are range constraints for the general propeller optimization problem.

Different formulation would follow if the equality constraint is treated as an inequality constraint. Mathematically this would be:

$$K_c - C \cdot J^p \geqslant 0$$
$$-K_c + C \cdot J^p \geqslant 0.$$

This approach is only of a theoretical value and cannot be applied to SUMT. However, from the naval architect's point of view, the optimal propeller cannot produce greater thrust than is needed, otherwise it would not be optimal.

Therefore the relation $K_c - CJ^p \geqslant 0$ will practically always be satisfied as $K_c - CJ^p = 0$. Figure 4 includes the curve K_{Topt} (K_T when $\eta_o = \eta_{max}$) and the parabola $K_c = CJ^p$. To the left side of the intersection between these, the propeller efficiency η_o is larger than it would be if it was necessary to satisfy the relation $K_c = CJ^p$. On the right side of the intersection, which is of practical interest, the largest value of η_o will be obtained only when $K_c = CJ^p$. This is similar for K_{Qopt}.

The left side is not of practical interest, since if $K_c = CJ^p$ were satisfied, η_o would be on the right side of η_{max} for each P/D value. Therefore, although the problem is stated as $K_c - CJ^p \geq 0$, practically it would always be satisfied as $K_c - CJ^p = 0$. To ensure solution is valied for all cases (not only for the sake of optimization but from the naval architect's point of view) the gradient $\partial\eta/\partial J$ should be positive.

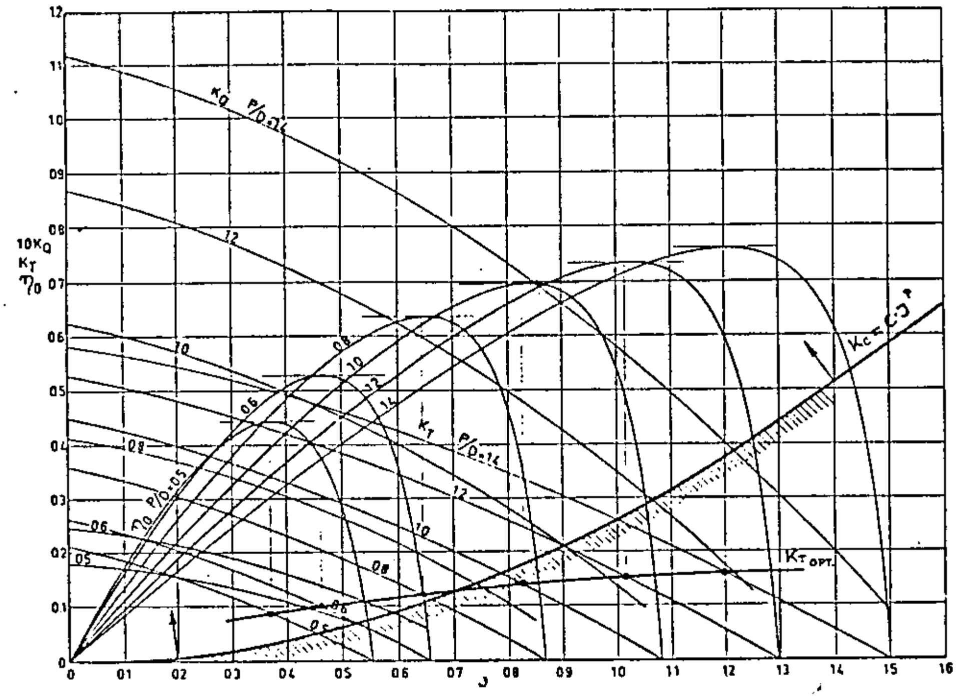

Figure 4

Mathematical formulation, with the constraints of the inequality type, follows:

FORMULATION B

$$\text{minimize } f(X_1, X_2, X_3) = -\eta_o = -(J \cdot K_T)/(2\pi \cdot K_Q)$$

subject to inequality constraints only

$J \geq 0$	1
$P/D - 0.5 \geq 0$	2
$1.4 - P/D \geq 0$	3
$Ae/Ao - 0.3 \geq 0$	4
$1.05 - Ae/Ao \geq 0$	5
$K_c - CJ^p \geq 0$	6
$2 - J \geq 0$	7

Formulation B is generally sufficient with only the first 6 constraints. The 7th constraint, which is usually nonactive, is added since it may be of practical interest to restrict the maximum value of J.

This approach is completely opposite to that in references [3], [4 and 5], and [6]. There the constraints were only of the equality type, whilst in Formulation B, they are only of the inequality type. In Formulation A the constraints are of inequality and equality type.

4.3. SUMT Transformation

Transformation of the constrained problem given in Formulation A, to the unconstrained form, applicable to SUMT is:

$$f^*(R,X) = \underbrace{-\frac{J}{2\pi}\cdot\frac{K_T}{K_Q}}_{f(X)} + R\underbrace{\left(\frac{1}{J} + \frac{1}{K_T} + \frac{1}{P/D - 0.5} + \frac{1}{1.4 - P/D} + \frac{1}{2 - J} + \frac{1}{Ae/Ao - 0.3} + \frac{1}{1.05 - Ae/Ao}\right)}_{P_1(X)}$$

$$+ \frac{1}{\sqrt{R}}\underbrace{(K_c - C\cdot J^p)^2}_{P_2(X)}$$

Partial derivatives necessary for the D-F-P gradient method follow:

$$\frac{\partial f(X)}{\partial J} = -\left(\frac{1}{2\pi}\cdot\frac{K_T}{K_Q} + \frac{J}{2\pi}\cdot\frac{\frac{\partial K_T}{\partial J}K_Q - K_T\frac{\partial K_Q}{\partial J}}{K_Q^2}\right) \qquad 1$$

$$\frac{\partial f(X)}{\partial P/D} = -\left(\frac{J}{2\pi}\cdot\frac{\frac{\partial K_T}{\partial P/D}K_Q - K_T\frac{\partial K_Q}{\partial P/D}}{K_Q^2}\right) \qquad 2$$

$$\frac{\partial f(X)}{\partial Ae/Ao} = -\left(\frac{J}{2\pi}\cdot\frac{\frac{\partial K_T}{\partial Ae/Ao}K_Q - K_T\frac{\partial K_Q}{\partial Ae/Ao}}{K_Q^2}\right) \qquad 3$$

$$\frac{\partial P_1(X)}{\partial J} = -\left(\frac{1}{J^2} + \frac{\frac{\partial K_T}{\partial J}}{K_T^2} - \frac{1}{(2-J)^2}\right) \qquad 4$$

$$\frac{\partial P_1(X)}{\partial P/D} = -\left(\frac{\frac{\partial K_T}{\partial P/D}}{K_T^2} + \frac{1}{(P/D - 0.5)^2} - \frac{1}{(1.4 - P/D)^2}\right) \qquad 5$$

$$\frac{\partial P_1(X)}{\partial Ae/Ao} = -\left(\frac{\frac{\partial K_T}{\partial Ae/Ao}}{K_T^2} + \frac{1}{(Ae/Ao - 0.3)^2} - \frac{1}{(1.05 - Ae/Ao)^2}\right) \qquad 6$$

$$\frac{\partial P_2(X)}{\partial J} = 2\cdot(K_c - C\cdot J^p)\left(\frac{\partial K_c}{\partial J} - p\cdot C\cdot J^{p-1}\right) \qquad 7$$

$$\frac{\partial P_2(X)}{\partial P/D} = 2\cdot(K_c - C\cdot J^p)\frac{\partial K_c}{\partial P/D} \qquad 8$$

$$\frac{\partial P_2(X)}{\partial Ae/Ao} = 2\cdot(K_c - C\cdot J^p)\frac{\partial K_c}{\partial Ae/Ao} \qquad 9$$

The above transformation which is used throughout this paper, is most widely used and is sometimes called SUMT-1967 version. Other transformations may be used, one of them SUMT-1970, which generally did not seem to be any better than the original transformation. For SUMT-1970 the unconstrained function is of this form:

$$f^*(R,X) = -\frac{J}{2\pi}\cdot\frac{K_T}{K_Q} - R\left(\ln(J) + \ln(K_T) + \ln(P/D-0.5) + \ln(1.4-P/D) + \ln(2-J) + \ln(Ae/Ao-0.3) + \ln(1.05-Ae/Ao)\right)$$

$$+ 1/\sqrt{R}\,(K_c - C\cdot J^p)^2$$

Obviously, the first partial derivatives are somewhat simpler in this case.

Transformation of the constrained problem given in the Formulation B would be:

$$f^*(R,X) = \underbrace{-\frac{J}{2\pi}\cdot\frac{K_T}{K_Q}}_{f(X)} + \underbrace{R\left(\frac{1}{J} + \frac{1}{P/D-0.5} + \frac{1}{1.4-P/D} + \frac{1}{Ae/Ao-0.3} + \frac{1}{1.05-Ae/Ao} + \frac{1}{K_c - C\cdot J^p} + \frac{1}{2-J}\right)}_{P(X)}$$

Here the first three derivatives would be the same as in the previous case (Formulation A) but instead of derivatives 4 to 9 only three new partial derivatives follow:

$$\frac{\partial P(X)}{\partial J} = -\left(\frac{1}{J^2} + \frac{\frac{\partial K_c}{\partial J} - p\cdot C\cdot J^{p-1}}{(K_c - C\cdot J^p)^2} - \frac{1}{(2-J)^2}\right) \qquad 4'$$

$$\frac{\partial P(X)}{\partial P/D} = -\left(\frac{1}{(P/D-0.5)^2} - \frac{1}{(1.4-P/D)^2} + \frac{\frac{\partial K_c}{\partial P/D}}{(K_c - C\cdot J^p)^2}\right) \qquad 5'$$

$$\frac{\partial P(X)}{\partial Ae/Ao} = -\left(\frac{1}{(Ae/Ao-0.3)^2} - \frac{1}{(1.05-Ae/Ao)^2} + \frac{\frac{\partial K_c}{\partial Ae/Ao}}{(K_c - C\cdot J^p)^2}\right) \qquad 6'$$

This is much simpler and less liable to ill-conditioning when $R_k \to 0$. However, the choice of the starting point, which must be inside the boundaries, is more difficult to determine.

4.4. Practical Application

The above approach enables practical bounds or constraints to be applied to propeller design problems. From the naval architect's point of view this can be very useful and simple:

- Maximum allowable propeller diameter, when choosing optimal propeller diameter (options 3 and 4), may be incorporated to give a new constraint (instead of constraint 1) as $J_{min} \geqslant V(1-w)/N \cdot D_{max}$.

- Minimum allowable RPM (options 1 and 2) may be incorporated to give a new constraint (instead of constraint 5 in Formulation A and 7 in Formulation B) as $J_{max} \leqslant V(1-w)/N_{min} \cdot D$.

- Minimum allowable blade area may be taken into account with the help of Keller's cavitation criteria (transformation as given in [6]) with the change of the constraints 6 and 4 in formulations A and B respectively, as $Ae/Ao_{min} \geqslant B(K_T/J^2)+k$, where

$$B = [(1.3+0.3Z)\rho V_a^2]/(p_o-p_v) = \text{const.}$$

$$k = \begin{cases} 0 & \text{for fast twin-screw ship} \\ 0.1 & \text{for other twin-screw ship} \\ 0.2 & \text{for single-screw ship} \end{cases}$$

(in [6] the above relation was treated as an equality constraint).

Other constraints may easily be changed as well. For example, the ratio of P/D_{max} or Ae/Ao_{max} may be restrained for technologicall – manufacturing reasons, or if the propellers from stock are considered, etc.

The general flow chart for preliminary propeller design is given in Figure 5.

5. RESULTS

The same input data as for the "hand solution method" is used throughout.

In Table 3 the results obtained for all 4 options are given. Defaulted constraints were used.

Table 4 shows the iterations for option 1.

Table 5 shows the result obtained for option 3, whilst Table 6 gives the results obtained with the Wolfson Unit Program [16] as a comparison.

Table 7 is similar to Table 5, but for option 2.

Table 8 shows the results when D=const. and N=const. for option 3. This is obtained with the constraint $D_{max} \leqslant 5.16$ m, i.e. $J \geqslant 0.93$. It can be seen that for Z=2 to 4, $R_{K_T} < R_t$. R_{K_T} is the thrust obtainable with a particular propeller. Therefore, the results are correct only for Z=5, 6, and 7. In other words, propellers with 2, 3, and 4 blades cannot provide the necessary thrust for the 16kn vessel with $D_{max} \leqslant 5.16$m and N=77 RPM.

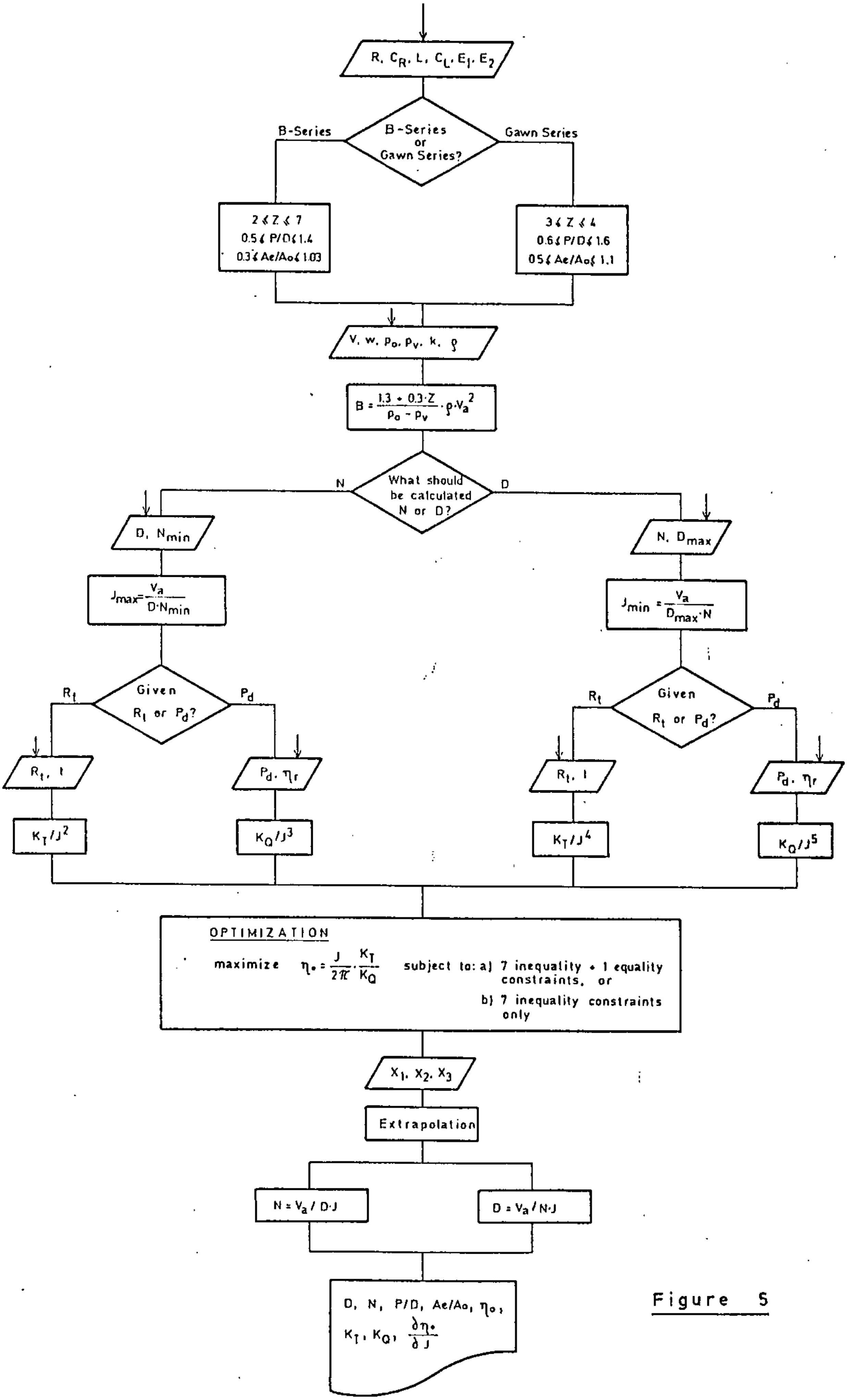
R, C_R, L, C_L, E_1, E_2
B-Series
B-Series or Gawn Series?
Gawn Series
2 ≤ Z ≤ 7
0.5 ≤ P/D ≤ 1.4
0.3 ≤ Ae/Ao ≤ 1.03
3 ≤ Z ≤ 4
0.6 ≤ P/D ≤ 1.6
0.5 ≤ Ae/Ao ≤ 1.1
V, w, p_o, p_v, k, ρ
B = (1.3 + 0.3·Z)/(p_o − p_v) · ρ·V_a^2
N
What should be calculated N or D?
D
D, N_min
N, D_max
J_max = V_a / D·N_min
J_min = V_a / D_max·N
R_t
Given R_t or P_d?
P_d
R_t, t
P_d, η_r
K_T/J^2
K_Q/J^3
K_T/J^4
K_Q/J^5
OPTIMIZATION
maximize η_o = J/2π · K_T/K_Q
subject to: a) 7 inequality + 1 equality constraints, or
b) 7 inequality constraints only
X_1, X_2, X_3
Extrapolation
N = V_a / D·J
D = V_a / N·J
D, N, P/D, Ae/Ao, η_o, K_T, K_Q, ∂η_o/∂J

Figure 5

	Option 1	Option 2	Option 3	Option 4		
J	0.8617	0.8682	0.8302	0.8315	Last iteration	$R_1=0.1$
P/D	1.1805	1.1850	1.0882	1.0865		$C_R=10$
A_e/A_o	0.6398	0.6432	0.6792	0.6589		L=2
$-\eta_o=f(X)$	-0.6849	-0.6868	-0.6909	-0.6916		$C_L=2$
$f^*(R,X)$	-0.6849	-0.6867	-0.6909	-0.6915		$E_1=1E-05$
J	0.8617	0.8679	0.8302	0.8316	Extrapolation	$E_2=1E-08$
P/D	1.1805	1.1852	1.0882	1.0872		$(J)_o=0.3$
A_e/A_o	0.6398	0.6432	0.6792	0.6574		$(P/D)_o=0.8$
η_o	0.6849	0.6866	0.6909	0.6915		$(A_e/A_o)_o=0.4$
D m	5.4864	5.4864	5.7789	5.7691		Z = 5
N RPM	78.141	77.580	77.000	77.000		
No. of iterat.	22	22	27	33		

Table 3

Option 1

R	Iterat.	J	P/D	A_e/A_o	$\eta_o=-f(X)$	$f^*(R,X)$
0.1	0 3	0.3 0.7801	0.8 1.1462	0.4 0.7638	0.3812 0.6513	2.2486 1.1791
0.01	4 7	0.7182 0.8428	1.0657 1.1206	0.6725 0.6652	0.6406 0.6901	-0.4131 -0.5016
1E-03	8 11	0.8417 0.8565	1.1193 1.1578	0.6638 0.6437	0.6899 0.6880	-0.6658 -0.6677
1E-04	12 14	0.8575 0.8613	1.1598 1.1756	0.6486 0.6409	0.6880 0.6859	-0.6814 -0.6836
1E-05	15 16	0.8634 0.8622	1.1794 1.1801	0.6401 0.6401	0.6860 0.6853	-0.6842 -0.6849
1E-06	17 18	0.8622 0.8618	1.1802 1.1804	0.6399 0.6399	0.6853 0.6850	-0.6847 -0.6850
1E-07	19 20	0.8619 0.8617	1.1804 1.1805	0.6399 0.6399	0.6850 0.6850	-0.6849 -0.6849
1E-08	21 22	0.8617 0.8617	1.1805 1.1805	0.6398 0.6398	0.6850 0.6849	-0.6849 -0.6849
Extrap.	-	0.8617	1.1805	0.6398	0.6849	-

Table 4

Option 3

	Z=2	Z=3	Z=4	Z=5	Z=6	Z=7
J	0.7092	0.7763	0.7999	0.8302	0.8578	0.9304
P/D	0.8616	0.9816	1.0267	1.0882	1.1425	1.3429
A_e/A_o	0.3001	0.3003	0.5308	0.6792	0.7223	1.0499
η_o	0.8150	0.7269	0.6937	0.6909	0.6859	0.6817
D m	6.7648	6.1804	5.9976	5.7789	5.5927	5.1565
Iterat.	29	35	29	27	25	34

Table 5

Option 3

$\frac{A_e}{A_o}$	D [m]/P/D/η_o					
	Z=2	Z=3	Z=4	Z=5	Z=6	Z=7
0.30	6.775/0.877/0.771					
0.35		6.139/0.998/0.728				
0.40			5.877/1.062/0.691			
0.45				5.709/1.102/0.689		
0.50		6.214/0.985/0.713			5.390/1.227/0.676	
0.55			6.139/0.983/0.692			5.465/1.183/0.670
0.60				5.690/1.118/0.694		
0.65		5.893/1.072/0.678			5.671/1.112/0.685	
0.70			6.027/1.029/0.690			5.390/1.215/0.674
0.75				5.840/1.073/0.689		
0.80					5.578/1.153/0.685	
0.85						5.241/1.288/0.677
0.90				5.802/1.096/0.675		
0.95					5.465/1.206/0.677	
1.00						
1.05						

Table 6

Option 2

	Z=2	Z=3	Z=4	Z=5	Z=6	Z=7
J	0.6250	0.7402	0.9521	0.8679	0.9112	0.9878
P/D	0.8513	1.0053	1.4003	1.1852	1.2474	1.4000
A_e/A_o	0.3000	0.3000	0.2997	0.6432	0.7398	1.0501
η_o	0.7588	0.7072	0.6931	0.6866	0.6871	0.6994
N RPM	107.725	90.959	70.723	77.580	73.891	68.166
Iterat.	37	29	33	22	22	34

Table 7

	Z=2	Z=3	Z=4	Z=5	Z=6	Z=7	
J	0.9300	0.9300	0.9300	0.9308	0.9300	0.9361	E_1=1E-05
P/D	1.4000	1.4009	1.4000	1.3946	1.3478	1.3609	E_2=1E-07
A_e/A_o	1.0501	1.0453	1.0181	0.2935	0.7170	1.0487	J ⩾ 0.93
η_o	0.6992	0.6607	0.6432	0.6689	0.6695	0.6818	N=77 RPM
D m	5.1587	5.1588	5.1586	5.1541	5.1587	5.1251	V_a=6.16 m/s
R_{K_T}	262752	268144	275125	275902	275833	275296	R_t=275345 N

Table 8

Table 9 shows the result for identical conditions as in Table 8, but with SUMT-1970 transformation. The difference in Ae/Ao for Z=5 was checked for various L values. From this it was concluded that SUMT-1970 was more sensitive to L values than SUMT-1967. Further work showed that in this particular case η_o is not sensitive to Ae/Ao variation.

	Z=5	Z=6	Z=7	
J	0.9300	0.9300	0.9343	E_1=1E-05
P/D	1.3840	1.3478	1.3551	E_2=1E-08
A_e/A_o	0.3899	0.7114	1.0497	J ⩾ 0.93
η_o	0.6688	0.6694	0.6817	N=77 RPM
D m	5.1586	5.1585	5.1350	V_a=6.16 m/s
R_{K_T}	275345	275345	275345	R_t=275345 N

Table 9

For Gawn propeller series the results are given in the Table 10.

	Option 1		Option 3		
	Z=3	Z=4	Z=3	Z=4	(Formulation B)
J	0.8198	0.8701	0.8240	0.8526	$(J)_o$=0.4
P/D	1.1304	1.1889	1.0918	1.1332	$(P/D)_o$=1.2
A_e/A_o	0.5000	0.5000	0.5000	0.5000	$(A_e/A_o)_o$=0.6
η_o	0.6924	0.6971	0.7004	0.6992	R_1=0.1
D m	5.4864	5.4864	5.8220	5.6271	C_R=10
N RPM	82.132	77.385	77.000	77.000	L=2
Iterat.	32	35	33	32	C_L=2

Table 10

Table 11 shows the result for Gawn propeller series when the diameter is restrained as in the Table 8.

	Z=3	Z=4	
			(Formulation B)
J	0.9300	0.9300	$(X)_o$=0.95; 1.55; 1.00
P/D	1.3491	1.3935	E_1=1E-05; E_2=1E-08
A_e/A_o	1.0999	0.5000	J ⩾ 0.93
η_o	0.6266	0.6714	N=77 RPM
D m	5.1587	5.1586	V_a=6.16 m/s
R_{K_T}	275755	275603	R_t=275345 N

Table 11

6. CONCLUSIONS AND RECOMMENDATIONS

Four points should be made before any conclusions are drawn:

- The answers are, naturally, dependent on the mathematical models used which in these cases are based on B-series and Gawn series polynomials.
- In all examples an impractical starting point was chosen to show the ability of SUMT. Usually it is not difficult to choose a starting point which is closer to the optimum.
- The calculating time in some cases was relatively long, but a microcomputer was used, not a mainframe as in [3], [4 and 5], and [6].
- Since the constraints were obeyed, and only a feasible solution could be obtained, the process of choosing the optimal propeller was automatic. Therefore, the user does not require knowledge of propeller design.

The conclusions relating to the optimization technique applied are given below:

- Generally, all optimization methods find only the local minima. However, this disadvantage was not observed here.
- SUMT's parameters C, C_L, E_1, and E_2 may influence the answer if they are not chosen with care.
- It would be good practice to follow the iterations; specifically the difference between f(X) and f*(R,X).
- If the iteration process stops prematurely, a change of some of the SUMT's parameters may help.
- It appears that for the propeller design problem SUMT-1967 transformation is more convenient than SUMT-1970, although it may be useful to check the answer with the other method.
- If Formulation B is used (only inequalities) it may be difficult to choose the starting point. At this time a separate routine for choosing the starting point has not been developed. However, mathematically it is more convinient to have a problem with inequalities only, rather than inequality and equality constraints.
- Of use to the naval architect, various constraints may be easily applied.

It should be noted that instead of checking whether $\partial\eta/\partial J$ is positive and R_{K_T} is greater than R_t, two new inequality constraints could be created and added to the inequalities already mentioned:

$$\partial\eta/\partial J \geqslant 0$$
$$R_{K_T} - R_t \geqslant 0.$$

However, this was not done at this stage since it was felt that they would complicate the transformed objective function, and in any case would usually be non active.

With reference to the propellers, the conclusions are:

- It is not necessarily true that the propeller with the lower BAR is better than the one with higher BAR. This is particularly the case for propellers with a higher number of blades.
- The widely accepted recommendation to choose the lowest BAR possible is only true if BAR obtained from the cavitation criteria is higher than BAR obtained using the above method.
- Optimum BAR (for η_o optimal) is higher for a greater number of blades.
- In some cases the change of BAR does not significantly influence the open water propeller efficiency η_o.

The above recommendations probably have not been highlighted enough in books on naval architecture.

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