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**The influence of vehicle-track dynamic coupling on the fatigue failure  
of coil springs within the primary suspension of metro vehicles**

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# **The influence of vehicle-track dynamic coupling on the fatigue failure of coil springs within the primary suspension of metro vehicles**

Steel coil springs are commonly used in the primary suspension of rail vehicles, usually in the form of two concentric springs. They exhibit strong internal resonances, which can lead to high vibration amplitudes within the spring itself. In some metro vehicles, large numbers of spring failures have occurred due to fatigue fracture in working conditions. The cause of these failures is investigated by studying the vehicle/track interaction, the modal response of the coil springs and the stresses occurring within them in working conditions. A finite element model is used to determine the modal parameters of the primary suspension. The resulting dynamic stiffness matrix is then included in a multi-body vehicle model and coupled to a model of the track. This coupled model is used to investigate the effect of the dynamic properties of both the springs and the track on the stresses in the springs. The springs exhibit strong internal resonances at around 50-60 Hz, at which very large stresses occur in both springs. This frequency range coincides with the P2 resonance frequency (wheelset mass bouncing on the track stiffness) for the standard slab track system used on this metro system. For other track systems, the P2 resonance occurs at a different frequency and the stresses are lower. These results are confirmed with field test data. From the stresses the weakest position in the inner spring is identified, which is found to correspond to the position of common breakages found in field observations. Some guidelines are proposed for reducing the vibration and stress, so that the fatigue fracture incidents can be reduced.

Keywords: metro vehicle; coil springs; vehicle-track dynamics; resonance frequency; dynamic stiffness; fatigue analysis

## **1 Introduction**

Steel coil springs are widely used as vibration isolation elements in railway vehicle suspensions, particularly within the primary suspension. They are key components for maintaining vehicle safety and passenger comfort. Recently, however, large numbers of breakages of coil springs within the primary suspension of metro vehicles have been observed during operation. Currently, springs are designed according to the EN13906

standard [1]. Their static strength is calculated with a quasi-static model. When fatigue calculations are carried out, the maximum stress is calculated according to the static displacements or loads in the axial and transverse directions. However, the internal resonances within coil springs that can be excited at higher frequencies are not taken into account. Neglecting this high frequency excitation occurring in working conditions can lead to improper design. The dynamic stiffness of resilient elements can be measured using direct or indirect measurement methods [2-3], but these have not been applied to coil springs. To determine the dynamic response at higher frequencies, either analytical formulae or the finite element method can be used to model the springs [4].

Nowadays, most studies of fatigue breakage of coil springs focus on material and processing procedures [5-6]. While it is important to improve these qualities to increase the service life, in addition the reasons for the occurrence of the high stresses should be investigated, especially the effects of resonances within the springs themselves [7-9]. For automotive vehicle dynamics, some research has been performed into the spring dynamic response and wave motion [10]. Lee and Thompson [11] used the dynamic stiffness matrix method to calculate the axial and transverse dynamic stiffnesses of an automotive suspension spring, which showed a significant dynamic stiffening. Sun et al. [12] applied this approach to study the effect of the dynamic stiffening of a spring in a high-speed rail vehicle on the vibration transmission; they also performed measurements of the dynamic stiffness, which showed a good agreement with the model. Ling et al. [13] carried out an experimental and numerical study to investigate the vibration induced by rail corrugation. The result indicated that the presence of short pitch rail corrugation may reduce the fatigue life of the coil spring.

For the fatigue analysis of suspension components in the railway industry, most research has concentrated on rubber springs [14]. Wang [15] calculated the stress in a

primary suspension spring of a high-speed train with the measured load as input; this showed that the first active circle had the largest contact stresses. However, there are no previous studies including both the vehicle-track coupled dynamic system and the models of the coil spring and other suspension components in detail to discuss the main reasons for spring fatigue failure. The primary suspension is located between the wheelset and the bogie frame and is thus close to wheel/rail contact. It is important to understand the dependence of spring fatigue failure on the vehicle/track coupled dynamics as well as the influence of the dynamic properties of the spring itself.

This study focuses on one particular metro system for which a large number of coil spring fatigue failures has occurred. Analysis of the spring breakages showed that many of the breaks are typical fatigue fractures. The springs were examined by a test institute, which found that they met the requirements for both material and mechanical properties. The aim of this paper is to investigate the cause of these failures by studying the vehicle/track interaction, the modal response of the coil springs and the stresses occurring within the springs. For this purpose, a finite element model is established in Section 2 to determine the dynamic stiffness of the primary suspension coil springs. The vibration spectrum of the springs when located in the bogie is studied in Section 3 by incorporating these dynamic stiffnesses within a vehicle/track interaction model, including the suspension, bogie, car body and track structure. A parametric analysis of both vehicle and track systems is then carried out in Section 4 to investigate the main factors influencing the coil spring fatigue failures. The stress distribution in the inner spring is obtained to identify the positions within the spring that are most susceptible to failure, and the results are compared with field test results. Finally, some guidelines are proposed for reducing the vibration of coil springs within the primary suspension, so that the fatigue fracture incidents could be reduced. A novel feature of this work is the

inclusion of the point and transfer dynamic stiffness of the spring set with rubber pad, derived from an FE model, into an otherwise relatively simple coupled vehicle-track dynamic model. This allows the displacements and dynamic stresses in the spring to be determined while keeping the rest of the model sufficiently simple to allow clear interpretation of the results. The reasons for spring fatigue can be readily identified with this model.

## **2 Dynamic model of coil spring set with rubber pad**

### ***2.1 Statistical analysis of fatigue fractures***

The primary suspension of the vehicles studied here consists of two concentric steel coil springs, wound in opposite directions, and mounted on a rubber pad. A statistical analysis has been carried out of spring breakages occurring in the field. It was found that there is no obvious difference between the frequency of fracture incidents for motor or trailer bogies, or between those occurring at the top or bottom end of the spring. The results showed that more than 90% of fractures are located at positions between 1.0 and 1.6 circles from one end. No fractures were found at positions more than two circles from the end. Moreover, 95% of fractures occurred on the inner spring of the spring sets and only 5% on the outer spring. Analysis of the fractures of inner springs showed that they mostly occur either due to fretting contact between coils (around 55%), or due to fracture on the inner side (around 40%). These account for more than 95% in total, whereas only a few occur due to multi-source fractures. It can be seen from this analysis that cracks occur due to contact between the first and the second circle, but there are also many fatigue cracks initiated on the inner side of the coil.

## ***2.2 Model of spring set in series with rubber pad***

The helical coil springs from the metro vehicles studied here are shown schematically in Figure 1(a). These are the outer and inner springs of the coil spring set from the primary suspension. The coil spring set is mounted on top of an annular rubber pad, the cross-section of which is shown in Figure 1(b). The upper end of the spring set is bolted to the bogie frame; the bottom of the rubber pad is located on the upper surface of a radial arm. The parameters describing the geometric structure and the mechanical properties of both springs are listed in Table 1. The active coils are those that are not in contact with the bearing surfaces at the two ends. The static height is the height under the combined nominal preload of 22 kN. Compared with the outer spring, the inner spring has a smaller diameter, thinner wire and more coils.

Figure 1. Geometric parameters; (a) helical springs; (b) cross-section of rubber pad.

Normally, it is assumed in suspension design that a helical spring only takes axial loads and the influence of its helical angle can be ignored. This allows a simplified estimate of the static spring stiffness [16] to be obtained as follows:

$$k_s = \frac{Gd^4}{8nD^3} \quad (1)$$

where  $G$  is the shear modulus,  $d$  is the diameter of the spring wire,  $D$  is the diameter of the spring to the wire centre and  $n$  is the number of active coils.

Due to their inertia, coil springs have a series of internal resonances. The first vertical natural frequency of a coil spring, when both ends are fixed, can be estimated from the following simplified equation [17]:

$$f_{s1} = \frac{d}{2\pi n D^2} \sqrt{\frac{G}{2\rho}} \quad (2)$$

where  $\rho$  is the density. The first natural frequency when one end is fixed and the other end is free is approximately half that for the fixed-fixed conditions.

Table 1. Parameters of coil spring set

The vertical stiffness of the rubber pad  $k_r$  can be obtained as follows [18]:

$$k_r = \frac{A_r \mu_r E_r}{h_r} \quad (3)$$

where

$$A_r = \pi(D_1^2 - D_2^2) / 4 \quad (4)$$

$$\mu_r = 1.2 \times (1 + 1.65 S_r^2) \quad (5)$$

and

$$S_r = \frac{D_1 - D_2}{4h_r} \quad (6)$$

$A_r$  is the area of the upper surface,  $\mu_r$  is the vertical shape coefficient,  $D_1$  and  $D_2$  are the outer and inner diameters of the hollow rubber pad, which are 225 mm and 90 mm respectively,  $h_r$  is the height of the rubber pad under load, which is 50 mm, and the Young's modulus  $E_r$  is 5 MPa. The vertical stiffness  $k_r$  according to Eqs (3)-(6) is 7.0 MN/m, which is more than 20 times greater than the combined static stiffness of the coil springs. Damping can be included by using a complex stiffness form, i.e. multiplying the stiffness by the factor  $(1+i\eta_r)$  where  $i$  is the imaginary unit. According to the

material properties and hardness of the rubber, the damping loss factor  $\eta_r$  is set to be 0.25.

### ***2.3 Modal analysis***

To allow for their detailed geometry, as shown in Figure 1, finite element models of both the outer and inner springs and the rubber pad are established using 8-noded solid elements (type C3D8R in Abaqus). There are 37506 elements in the outer spring model, 35569 elements in the inner spring model and 21566 elements in rubber pad model.

Modal analysis of the individual springs has been carried out based on the Lanczos method with a maximum analysis frequency of 200 Hz for the geometry corresponding to the preloaded height for two different sets of boundary conditions. In one, all six degrees of freedom of all the nodes on the contact surfaces at the two ends are constrained; in the other, one end is constrained while the other end is free. It can be expected that the modal frequencies from the fixed-fixed condition will correspond to maxima in the dynamic stiffness, whereas those from the fixed-free condition will correspond to minima.

Table 2. Modal frequencies of the coil springs for fixed-fixed conditions (units: Hz)

The modal results for the fixed-fixed boundary conditions are listed in Table 2. In the frequency range below 200 Hz, there are three vertical modes of the inner spring, with modal frequencies 58.2 Hz, 114.8 Hz and 166.4 Hz. For the outer spring the corresponding frequencies are slightly lower, with four modal frequencies occurring below 200 Hz: 51.9 Hz, 99.3 Hz, 135.2 Hz and 186.1Hz. The corresponding mode shapes are shown in Figure 2 and are coloured according to the maximum principal stress. As well as these vertical modes, lateral and torsional modes occur, with pairs of closely spaced natural frequencies.



For the fixed-free boundary condition, in the frequency range below 200 Hz there are four vertical modes for the inner spring, with modal frequencies 27.3 Hz, 81.6 Hz, 135.3 Hz and 178.1 Hz and five vertical modes of the outer spring, with modal frequencies 23.8 Hz, 73.6 Hz, 112.1 Hz, 139.2 Hz and 192.2 Hz.

Figure 2. First three vertical mode shapes of inner spring and first four vertical mode shapes of outer spring with fixed-fixed boundary conditions; these are coloured with the maximum principal stress. Left: inner spring; right: outer spring

## ***2.4 Dynamic stiffness***

The dynamic stiffnesses of each of the springs have next been calculated using the above FE models. To calculate the dynamic stiffness, all six degrees of freedom of the nodes at one end are constrained; at the other end the vertical degree of freedom is left free and the other degrees of freedom are constrained. At each frequency a harmonic vertical force of unit amplitude is applied at the free end, distributed over all nodes of the support surface. The frequency range of excitation is from 0 to 200 Hz with an interval of 0.2 Hz. This procedure is then repeated with the excitation at the other end. The axial displacement of the excited end and the total reaction force at the fixed end are obtained from which the dynamic point stiffnesses  $k_{11}$ ,  $k_{22}$  and the dynamic transfer stiffness  $k_{12} = k_{21}$  can be found.

Figure 3(a,b) shows the magnitude of the point and transfer dynamic stiffnesses for the two individual springs. A material damping loss factor of 0.001 has been assumed. The dynamic stiffness tends to a constant quasi-static value at low frequencies, 120.2 kN/m for the inner spring and 213.1 kN/m for the outer spring, which are similar to the values obtained according to Equation (1) as listed in Table 1, with differences of less than 5%. Above about 10 Hz, the point stiffness decreases and the transfer stiffness increases.

The first dips in the point stiffness of the inner and outer springs are at 27.3 Hz and 23.8 Hz, which correspond to the first natural frequencies of the spring with fixed-free boundary conditions. Similarly, the first peaks in the point stiffness occur at 58.2 Hz and 51.9 Hz, which correspond to the first natural frequencies for fixed-fixed boundary conditions (Table 2). The same peaks occur in the transfer dynamic stiffness; however, as shown in the figure, the transfer dynamic stiffness differs from the point stiffness apart from close to the peaks. For the inner spring there are three main peaks in this frequency range, and for the outer spring four, corresponding to the modes in Figure 2. However, there are also other modes that are excited at higher frequencies. The stiffness values vary over a very large range, between  $10^3$  N/m and more than  $10^8$  N/m.

Figure 3. Dynamic stiffness of individual springs and spring set: (a) inner spring; (b) outer spring; (c) spring set; (d) spring set with rubber pad.

As the two springs are installed within the primary suspension in the vehicle in parallel with each other, their dynamic stiffness values are summed to obtain the total dynamic stiffness of the spring set, shown in Figure 3(c). In the same way as for the individual springs, the point stiffness of the spring set remains constant below 10 Hz. It then changes dramatically at high frequencies, with values varying by several orders of magnitude. The peaks found for the individual springs are also found in the combined stiffness, whereas the dips are modified. The first dip in the point stiffness occurs at around 23 Hz.

The spring set is then combined in series with a finite element model of the rubber pad. The point and transfer dynamic stiffnesses of the combined system are shown in Figure 3(d); these will be used in a vehicle-track coupled dynamic model in the next section.

The two point stiffnesses,  $\bar{k}_{11}$  at the top and  $\bar{k}_{22}$  at the bottom, differ from one another,

whereas by reciprocity the transfer stiffness  $\bar{k}_{21} = \bar{k}_{12}$ , is the same for both directions.

Compared with the results for the spring set alone, the stiffness limit at low frequencies is reduced only slightly, from 320 kN/m to 310 kN/m. However, the highest values of dynamic stiffness are eliminated due to the resilience of the rubber pad. The dynamic amplification at high frequencies is at most only around a factor of 100, with maximum values of about 30 MN/m. This shows how the inclusion of the rubber pad can reduce the stiffening effect of the coil springs in the suspension at high frequencies.

### **3 Coupled dynamic model of metro vehicle and track**

#### ***3.1 Vehicle dynamic model***

For many railway vehicles, the coil spring set is placed above centre of the axle box. However, the vehicle considered in this study has an offset spring mounted off a radial arm, as shown in Figure 4. Such designs are used to achieve a more compact design in the vertical direction. The radial arm can rotate as the spring is compressed. Moreover, at the pivot of the radial arm there is a rubber bushing which introduces some flexibility. It can be seen that, as well as the coil spring set with its rubber pad, this rubber bushing, and the vertical hydraulic damper all provide stiffness and/or damping in the vertical direction. The mechanical parameters of these components must be chosen to obtain a proper combined dynamic stiffness for this primary suspension design. For the offset spring arrangement in Figure 4, account should also be taken of the rotation of the radial arm.

Figure 4. Primary suspension with offset spring set

A 2D multi-body dynamic model of the vehicle has been created with 14 degrees of freedom. These include bounce and pitch of the car body and the two bogie frames, the

vertical displacement of the four wheelsets and the rotation of the corresponding radial arms about the axlebox. The parameters used in the simulations are listed in Table 3.

The dynamic stiffness matrix of the spring set is introduced into the vehicle model. The model is assumed to be symmetric about the vehicle centreline and only the response of one side is considered. Using this model, forces are applied at each wheel in turn and, from the responses, a 4×4 matrix of receptances (displacement per unit force frequency response function) at the wheels is obtained at each frequency.

Table 3. Vehicle parameters (one vehicle)

### ***3.2 Track dynamic model***

Two different track forms are considered, which are modelled by a continuously supported rail. The foundation beneath the rail consists of:

- (1) a single-layer fastener system on sleepers embedded in the slab;
- (2) a two-layer foundation with sleepers supported by a rubber ‘boot’ pad.

The rail is represented by an infinite Timoshenko beam; its damping is considered by introducing a loss factor  $\eta_r=0.01$  into the Young’s modulus and shear modulus. The mass per length of the rail used here is 60 kg/m and the vertical bending stiffness is 6.45 MN.m<sup>2</sup>. For the first track form, the sleepers and slab are assumed to be rigid and the stiffness of the support per unit length is given by:

$$s = \frac{k_p}{d} \quad (7)$$

where  $k_p$  is the fastener stiffness, and  $d$  is the sleeper spacing which is 0.6 m. For this single layer track two different rail fastener stiffnesses are used,  $k_p = 60$  and 20 MN/m.

In the second model, the sleepers are considered as rigid masses with a second layer of springs beneath them. The frequency-dependent stiffness of the support per unit length of rail is given by [19]:

$$s = \frac{(-\omega^2 M_s + k_b)k_p}{(-\omega^2 M_s + k_p + k_b)d} \quad (8)$$

where  $M_s = 35$  kg is the sleeper mass, and  $k_b$  is the resilient boot pad stiffness. For the booted sleeper slab track, only a single value of rail fastener stiffness is used,  $k_p = 60$  MN/m. The stiffness of the resilient boot pad is  $k_b = 50$  MN/m; the sleeper spacing is again 0.6 m. The damping of the fastener system and boot pad are expressed in the form of loss factors  $\eta_p$  and  $\eta_b$ , which are introduced into the stiffnesses,  $k_p$  and  $k_b$ , by using the complex form with damping loss factors of 0.25 in each case.

To couple the wheels to the track, Green's functions [20] are required for the response of the supported Timoshenko beam at a position  $x$  for a unit force acting at  $\xi$ . When  $x = \xi$ , the Green's function gives the point receptance; otherwise it is the transfer receptance according to the distance between  $x$  and  $\xi$ . The point receptances of the three track types are shown in Figure 5 together with the point receptance of the wheel. From this figure it can be seen that the receptance for each track is approximately independent of frequency (i.e. stiffness-controlled) at low frequencies. The point receptance for the two normal slab systems has a single peak, the frequency of which depends on the fastener stiffness. In contrast, the resilient booted sleeper track has two peaks, the first of which is close to the peak for the normal slab track with stiffness 20 MN/m. The wheel receptance, in contrast, is mass-controlled over this frequency region apart from resonance peaks around 50-60 Hz, which can be attributed to the coil springs.

Figure 5. Track and wheel point receptance magnitudes.

## 4 Dynamic stress analysis of coil springs

### 4.1 Dynamic stress calculation and comparison

The dynamic stiffness of the spring set including the rubber pad, described in Section 2.2 (in preloaded condition, with parameters given in Table 1) is included in the vehicle model described in Section 3.1. It is not the intention in this study to carry out a full fatigue analysis but to determine the dynamic stress response, which gives an indication of the most likely locations of fatigue failure. The model is therefore used here to calculate the dynamic stress in the springs. Figure 2 already gave the stress distribution in some low frequency modes. For simplicity the track irregularity spectrum of the American AAR standard class 5 [21] is considered as the input here. The train speed is assumed to be 90 km/h. The vibration responses of both the bogie frame above the spring set and the radial arm below it are calculated with the vehicle-track coupled dynamic model, and used to determine the forces acting on spring set. The track model corresponds initially to the normal slab track with fastener stiffness 60 MN/m.

Using the dynamic stiffnesses of the combined spring set (Figure 3(d)), the forces acting on the top of the spring set  $F_{bs}$  and on the bottom of the rubber pad  $F_{rs}$  can be derived as:

$$\begin{cases} F_{bs} = \bar{k}_{11}z_{bs} - \bar{k}_{12}z_{rs} \\ F_{rs} = \bar{k}_{21}z_{bs} - \bar{k}_{22}z_{rs} \end{cases} \quad (9)$$

where  $z_{bs}$  is the vertical displacement of the bogie frame directly above the springs and  $z_{rs}$  is the vertical displacement at the bottom of the rubber pad. As seen in Figure 3(d) the transfer dynamic stiffness is equal for the two directions,  $\bar{k}_{21} = \bar{k}_{12}$ , whereas the point stiffnesses,  $\bar{k}_{11}$  at the top and  $\bar{k}_{22}$  at the bottom, are different from each other.

To determine the dynamic stress during operation, two FE models of the spring set are used, as indicated in Figure 6. First, the left-hand model is built in which the upper surface of the spring set is fixed and the bottom surface of the rubber pad is free in the vertical direction but constrained in other directions. The dynamic stress response is calculated at each frequency with the FE model for a unit force applied at the bottom of the rubber pad. Afterwards the results are scaled to correspond to a force  $F_{rs1} = -\bar{k}_{22}z_{rs}$ . A reaction force  $F_{bs1} = -\bar{k}_{12}z_{rs}$  is generated at top of the spring set. Then the dynamic stress of both inner and outer springs is determined at locations 1.2 circles from either end, indicated with red dots in Figure 6. Second, the right-hand model is built in which the upper surface of the spring set is free in the vertical direction and the bottom of the rubber pad is constrained in all directions. Here, after scaling, the force  $F_{bs2} = \bar{k}_{11}z_{bs}$  is used as input on the top of spring set and a reaction force  $F_{rs2} = \bar{k}_{21}z_{bs}$  is generated at bottom of the pad. The dynamic stress at the same positions as before is obtained from this model. Then using superposition, the stress results from the two models are combined to give the total dynamic stress at these positions.

Figure 6. Stress calculation with forces input in turn at the bottom and top of the spring set with rubber pad and combined model based on superposition.

Figure 7 shows the predicted stress at the positions 1.2 circles from either end of the two springs during running. For both positions, the maximum dynamic stress of the inner spring is much larger than that of the outer spring, which agrees with the statistical analysis of Section 2.1, where it was found that more than 95% of fractures occur on the inner springs. For the position close to the bottom, the largest stress peaks of the inner and outer springs occur at around 58 Hz and 51 Hz, corresponding to the first vertical resonance frequencies with fixed-fixed boundary conditions. The same peaks, with

similar amplitudes, occur for the position close to the top; there is also another peak at around 63 Hz, which is a bending resonance of the combined system of spring set and rubber pad.

Field measurements have also been made [22] in which strain gauges were attached to the inner spring at a position about 1.2 circles from the lower end. The measured dynamic stress of the inner spring at this position from [22] is also shown in Figure 7(a); the calculated and measured stress spectra show good overall agreement. Between 40 and 80 Hz, the dynamic stress is quite large, especially around the peak at 58 Hz. This is around the first fixed-fixed resonance frequency of the inner coil spring. At higher frequencies, the dynamic stress is lower because the excitation reduces. There are some small peaks at around 42 Hz, 69 Hz and 82 Hz in the measurement results corresponding to modes in other directions that are not observed in the predictions which only include vertical degrees of freedom.

Figure 7. Dynamic stress at 1.2 circle positions of inner and outer spring. (a) Lower positions; (b) upper positions. Measured data from [22], used with permission.

Repeating this analysis for other positions on the spring, Figure 8 shows the stress distribution of the inner spring at positions on the inside of the coil. The largest stress occurs at the two ends in the first active circle, which gives a good agreement with the field results in section 2.1 in terms of the spring position which suffers the most damage.

Figure 8. Stress distribution along length of the spring

#### ***4.2 Influence of track dynamics***

In this section, the influence of the track dynamics is investigated by comparing the results obtained for the three track structures with different sleepers and fasteners, i.e.



the single-layer slab track with two values of fastener stiffness, and the booted sleeper track with fastener stiffness 60 MN/m. The receptance of these track structures was shown in Figure 5 along with the wheel receptance.

Figure 9 shows the wheel/rail contact force for the different track systems. For the normal slab track with fastener stiffness 60 MN/m, the contact force is larger than the others between 50 Hz and 150 Hz. For each track, the contact force has a peak between 30 and 60 Hz. This is the P2 frequency, which can be identified from Figure 5 as the frequency at which the magnitudes of the wheel and rail receptance are equal. This occurs at around 61 Hz for the normal slab track with fastener stiffness 60 MN/m, 38 Hz for the slab track with fastener stiffness 20 MN/m, and 42 Hz for the resilient booted sleeper slab track. The peaks in Figure 9 are close to these frequencies.

Figure 9. Wheel rail contact force.

The calculated dynamic stresses for the three track types are shown in Figure 10(a) for the inner spring. The peak stress occurs at around 60 Hz for all three track types but the stress is largest for the normal slab track with fastener stiffness 60 MN/m; at the peak frequency around 60 Hz this is 1.5 times greater than for the ‘booted sleeper’ and twice as large as for the stiffness 20 MN/m.

Figure 10. Dynamic stress results for different track systems; (a) calculation; (b) measurement (from [22], used with permission).

Figure 10(b) shows dynamic stresses that were measured [22] as the train ran over track containing both fastener stiffnesses, 60 MN/m and 20 MN/m. It can be seen clearly that the stress is larger and the peak frequency range becomes wider around 60 Hz for the fastener system with 60 MN/m, as also found in the predictions in Figure 10(a). This can be attributed to the fact that the P2 resonance for this fastener stiffness is close to 60

Hz, leading to a peak in the wheel/rail contact force (Figure 9).

#### ***4.3 Influence of rail corrugation***

The metro line on which the stress measurements were taken [22] also experiences long wavelength rail corrugation. Figure 11 shows one-third octave band roughness spectra obtained on track sections corresponding to different train speeds; in each case they have been plotted against frequency for the corresponding train speed. All these locations use the normal slab track system with fastener stiffness of 60 MN/m. There are peaks in each spectrum in the frequency range around 60 Hz, which are close to the P2 resonance frequency; long wavelength corrugation is often associated with the P2 resonance [23] and that seems to be the case here. The roughness spectrum from the AAR type 5 track corresponding to a running speed of 90 km/h, as used in the earlier calculations, is also shown in the figure converted to one-third octave form. The roughness of the corrugated rail is considerably larger, especially at frequencies above 20 Hz. As shown in Figure 7, the dynamic stress peaks occur in the frequency range between 40 and 80 Hz, which corresponds to the corrugation frequency range, as shown in Figure 11. As a result, the corrugation leads to much larger stresses at these frequencies, approximately 10 times (i.e. 20 dB) greater than the amplitude for the AAR type 5 roughness. It can therefore be expected that this also contributes to the fatigue failure of the inner spring. These calculation results show a good agreement with measured results in reference [22], which identified that abnormal vibration of the springs was mainly caused by the resonance of the coil spring excited by both the P2 force and rail corrugations.

Figure 11. Rail roughness comparison between AAR 5 and corrugation roughness.

## 5 Conclusions

To study fracture failures of the primary suspension springs of a metro vehicle, the inner and outer coil spring are modelled with the finite element method to obtain their dynamic stiffness and stresses due to a unit force. The vehicle and track responses are calculated with a vehicle-track coupled dynamic model including detailed representations of the primary suspension components: radial arm, pivot bush, coil spring set in series with rubber layer, and vertical damper. With this model, the vehicle and track dynamics are analysed to determine their influence on the stresses in the springs.

It is shown that the coil springs exhibit strong internal resonances, the first of which occurs at 58 Hz for the inner spring and 52 Hz for the outer spring, which lead to high vibration amplitudes within the springs themselves. Large dynamic stresses occur at these frequencies. The maximum stresses occur at around 1.2 circles from both ends of each spring. For the normal slab track with a relatively stiff fastener system, the P2 resonance frequency also occurs at about 60 Hz. This is therefore close to the first internal resonance frequency of the inner coil spring, leading to high amplitude stresses and a higher likelihood of fatigue failure. The rubber pad which is used under the spring set can reduce the peaks in the dynamic stiffness but cannot add damping to the internal vibration of the coil spring. Track structures with a lower fastener stiffness or with resilient booted sleepers have a lower P2 resonance frequency which can reduce the forces on the coil spring in the sensitive frequency range around 60 Hz. This can help to alleviate the stresses and thereby the fatigue failures of the spring. The P2 resonance frequency is also associated with rail corrugation in some areas. The wavelength changes with the train speed, giving a constant frequency range around 60 Hz, which again coincides with the spring resonance frequency. The rail corrugation thus also

contributes to the high stresses in the spring.

In conclusion, it is important that the first internal resonance frequency of the coil springs does not coincide with the dominant excitation frequencies of wheel/rail vibration, especially the P2 resonance. This coincidence is the main reason for fatigue failures of the inner coil spring for the metro vehicle studied here, contributing most to the breakages occurring at the critical position of inner spring between 1.0 and 1.6 circles. The situation is exacerbated by the rail corrugation in this frequency range. It is therefore important to take account of the track design in designing the vehicle suspension to avoid this kind of coincidence. It is suggested for example to increase the first modal frequency of the spring or to add additional damping to the springs.

All data published in this paper are openly available from the University of Southampton repository at <http://dx.doi.org/10.5258/SOTON/D1007>.

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