**Authors:** Dongchen Zhao and Christian Bokhove

**Title:** Book review: What is the most important source of teachers’ knowledge？Lianghuo Fan (2014) *Investigating the Pedagogy of Mathematics: How do Teachers Develop Their Knowledge?*

**Affiliations and addresses of the authors:**

Dr Dongchen Zhao

Department of Education, Harbin Normal University

1 Shida Road, Linmin Developing Zone, Hulan District, Harbin City, Heilongjiang Province, China

Post code: 150025

Dr Christian Bokhove

Southampton Education School, University of Southampton

Building 32, University of Southampton, University Road, Southampton, United Kingdom

Post code: SO17 1BJ

**Corresponding author:** Dr Dongchen Zhao

Email: [jykyff2016@sina.com](http://mailto:jykyff2016@sina.com/" \t "_blank)

[dongchenzhao@163.com](mailto:dongchenzhao@163.com)

Mobile phone: +86-13009800431

ORCID: 0000-0002-1629-1004

**Book Review: What is the most important source of teachers’ knowledge?**

**Lianghuo Fan (2014) *Investigating the Pedagogy of Mathematics: How do Teachers Develop Their Knowledge?***

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Since the 1980s efforts to improve the quality of education have increasingly led to more attention from policymakers, school administrators, teacher educators, and researchers for teachers’ professional development. One of the key topics concerning this issue, involves teachers’ knowledge, partly popularised by Shulman’s (1986) seminal work on Pedagogical Content Knowledge. Related research has grown rapidly since the mid-1980s, mainly around three aspects: what knowledge should teachers have, what knowledge do teachers *actually* have, and how do teachers develop their knowledge? In contrast with the first two aspects, the third aspect has been studied less systematically, especially in the last two decades. As a result, there still much to explore. Professor Lianghuo Fan’s book, *Investigating the Pedagogy of Mathematics: How do Teachers Develop Their Knowledge?*, contributes to the literature.

**1 Overview of this book**

Fan’s book, focusing on the growth of teachers’ knowledge, presents two studies on how mathematics teachers develop their pedagogical knowledge. The first study was conducted in the city of Chicago of the United States (hereafter called “the Chicago study”) and the second one was conducted in Singapore (hereafter called “the Singapore study”). Both studies address two research questions: (1) Are there different sources for teachers to gain their pedagogical knowledge? (2) If the answer to the first question is “yes,” how do these different sources contribute to the development of teachers’ pedagogical knowledge? The book is organized into two parts. In Part I, which consists of 9 chapters, the author gives a detailed account of the Chicago study. In Part II, two chapters are devoted to the Singapore study and a comparison between the Chicago and Singapore studies. The structure of this book is as below.

***Part I The Chicago Study:***

Chapter 1. Introduction   
Chapter 2. Review of the Literature  
Chapter 3. A Conceptual Framework of the Study  
Chapter 4. Research Design and Procedures   
Chapter 5. Findings of the Chicago Study (I): Pedagogical Curricular Knowledge  
Chapter 6. Findings of the Chicago Study (II): Pedagogical Content Knowledge   
Chapter 7. Findings of the Chicago Study (III): Pedagogical Instructional Knowledge   
Chapter 8. Findings of the Chicago Study (IV): Some Other Issues  
Chapter 9. Conclusions, Implications, and Recommendations

***Part II*** ***The Singapore Study:***

Chapter 10. The Singapore Study  
Chapter 11. Comparison and Conclusion

The first chapter is an introduction to the Chicago study. The author firstly outlines the background of the study by highlighting the educational reforms in the United States since the 1980s which seek knowledge of teacher improvement, and then states the significance and the research questions of the study.

Chapter 2 systematically reviews the related studies on teacher knowledge. Although the focus of the Chicago study is on how teachers develop their pedagogical knowledge, this chapter covers three major issues in the field of teacher knowledge study which correspond to the three aspects introduced in the first paragraph of this review. Therefore, readers can gain a broad background on teacher knowledge study from the literature review, and also can understand the position of Fan’s research in this context as compared with existing research work.

In Chapter 3, the author clarifies some key constructs and establishes a conceptual framework to guide the Chicago study. In this study, *knowledge* is understood from the dynamic relation of the knower (the subject of knowledge), the known (the object of knowledge), and the knowing (the interaction of the subject and the object): “A subject’s knowledge of an object is defined as a mental result of certain interaction of the subject and the object” (p. 38). From the perspective of this study, *teachers’ knowledge* refers to the “knowledge possessed by teachers,” which means that the knowers are teachers and that the known is what teachers know. Accordingly, *teachers’ pedagogical knowledge* is then generally defined as what teachers know about *how* to teach, which includes teachers’ knowledge of both curriculum and method of teaching. More specifically, in mathematics, teachers’ pedagogical knowledge is further classified into three components based on the NCTM’s *Professional Standards for Teaching Mathematics* (NCTM, 1991, pp. 151-153): (1) Pedagogical Curricular Knowledge (PCrK): knowledge of teaching materials and resources, including technology; (2) Pedagogical Content Knowledge (PCnK): knowledge of ways to represent mathematics concepts and procedures; and (3) Pedagogical Instructional Knowledge (PIK): knowledge of general teaching strategies and classroom organizational models.

In the study, sources of teachers’ knowledge are understood to be the means by which the teachers developed their knowledge. Taking all stages of a teacher’s career into account, the Chicago study establishes a framework with three sources of teachers’ knowledge, namely teachers’ experience as learners before being accepted into formal preservice training, teachers’ preservice training experience, and teachers’ inservice experience. These are then further subdivided into teachers’ experience in primary and secondary schools, preservice training experience, in-service training experience, organized professional activities, daily exchanges with colleagues, reading professional journals and books, teachers’ own teaching experience and reflection, and so on (for more details see Fan, 2014, p. 51).

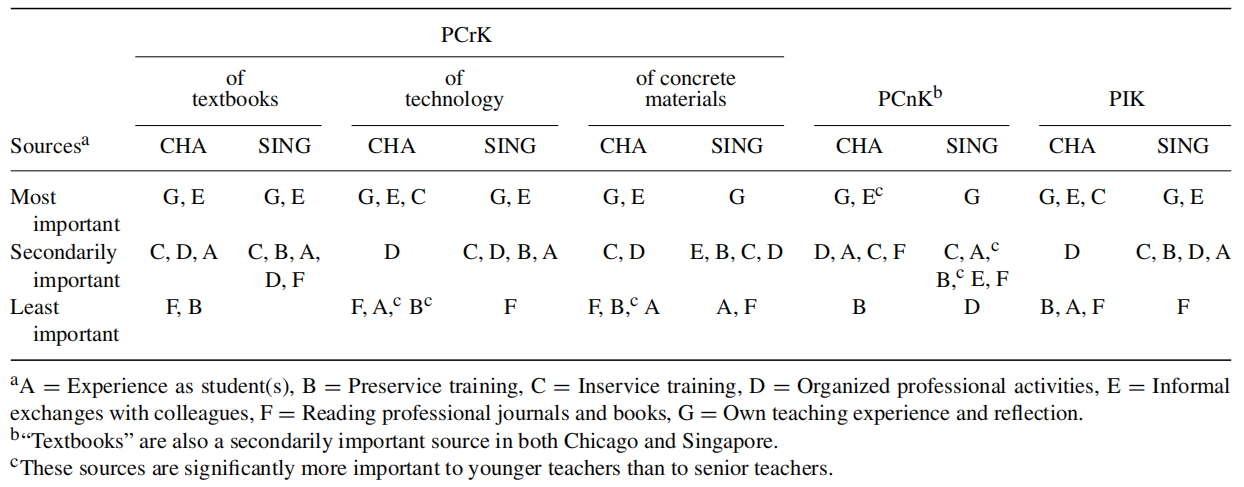
Chapter 4 describes in detail the methodological features of the Chicago study, including the targeted population and sample, research instruments, data collection procedure, methods employed to process and analyze the data, and the strengths and limitations of the methodology. Based on the review in Chapter 2 and the conceptual framework established in Chapter 3, a multiple-stage methodology was adopted. Elements of the methodology consist of the questionnaire survey in the first stage, classroom observation in the second stage, and interviews in the last stage. 77 mathematics teachers from three sample schools were selected as the subjects of this study, with all participating in a questionnaire survey. The questionnaire of 22 questions was designed based on the framework introduced in Chapter 3 and included questions on teachers’ background information, questions on how teachers’ specific experiences contributed to the development of their pedagogical knowledge, and finally how teachers’ specific pedagogical knowledge was developed from their different experiences. A subsample consisting of nine teachers was observed twice for their mathematics teaching, with the aim of identifying how teachers teach in their classrooms, and what kind of pedagogical knowledge they utilize in their teaching, so that questions could be raised during the interviews about how these teachers developed their pedagogical knowledge. Aligned to the framework of pedagogical knowledge in this study, the classroom observations focus on how teachers use the teaching resources and which ones they use (including textbooks, concrete materials, and technologies), how they represent mathematical concepts or procedures to students, employ teaching strategies, and organize the classroom. Following these classroom observations, structured interviews were conducted with the 9 teachers. The interviews focus on why the teachers taught the lessons the way they did, and how they acquired the knowledge they employed to plan the lessons. In addition to the 9 teachers, 3 chairs of mathematics departments (one chair from each sampled school) also were interviewed for the study. For the mathematics chairs, the interviews focus on inservice training and professional activities in their schools, providing further information about the school environments in which teachers develop their pedagogical knowledge, and aiding the interpretation of teachers’ responses from the questionnaires and interviews. Both quantitative and qualitative methods were employed to analyze the data. Quantitative methods were mainly used on the data collected from the questionnaire to gain insights into the general patterns about how teachers developed their pedagogical knowledge. Qualitative methods were mainly used on the data collected from classroom observations and interviews to provide an in-depth account on how individual teachers developed their pedagogical knowledge.

Chapters 5 to 8 provide the main findings of the Chicago study. The results identifying how teachers develop their PCrK, PCnK, and PIK are reported respectively in Chapters 5, 6, and 7. Chapter 8 addresses other questions which are not discussed in previous three chapters but related to the development of teachers’ pedagogical knowledge, such as how did teachers use or act on specific sources, and what were their main difficulties in improving pedagogical knowledge. Chapter 9 provides a summary of the Chicago study and its main conclusions. The study reveals that teachers have many different sources for developing their pedagogical knowledge (including PCrK, PCnK, and PIK). Teachers derive their knowledge of how to teach mathematics from their experiences as students, their preservice training, their in-service teaching experience, and even from their social and parenting experiences. Taken together, all sources contribute to the development of teachers’ pedagogical knowledge, but to varying extents. Based on log-linear regression models, the order of importance of different sources was divided into three levels: most important, secondarily important, and least important. The Chicago results (p. 212) show that, taking into account all the components of teachers’ pedagogical knowledge together, “teachers’ own teaching experience and reflection” and their “daily exchanges with colleagues” are the two most important sources of pedagogical knowledge; inservice training and the organized professional activities are secondarily important. However, teachers’ “experience as learners in primary and secondary schools,” “preservice training,” and “reading professional journals and books” are the least important sources. Based on these findings, Fan discusses the implications of the studies for teacher educators, school administrators, and teachers for promoting the development of teachers’ knowledge. One of the implications for teachers is that in order to develop their pedagogical knowledge they need to reflect on their own teaching practices intentionally and carefully, to gradually establish a store of pedagogical knowledge, to exchange ideas with their colleagues regularly, and to have open minds and be active learners (p. 215).

In Part II of the book, Chapter 10 presents an overall report of the Singapore study. The Singapore study is largely a replication of the Chicago study. The research questions, conceptual frameworks, research instruments, and methods for data collection and analysis between the two studies are intended to be similar. 73 mathematics teachers from six sample schools participated in the questionnaire survey. 18 of these teachers, including 2 heads of mathematics departments, were observed for classroom teaching, and were then interviewed. The results of the Singapore study are again reported, based on the three components of teachers’ pedagogical knowledge, i.e., PCrK, PCnK, and PIK. In Chapter 11, the author juxtaposes the Singapore study with the Chicago one. The results of each study are compared to reveal the similarities and differences of teachers’ pedagogical knowledge development between Chicago and Singapore. As a concluding chapter, Chapter 11 refines the conclusions in Chapters 9 and 10. On page 270 of the book, the author uses a table (see Table 1) to summarize the findings of both studies.

Table 1

*An* *Overview of the Results of the Chicago (CHA) and Singapore (SING) Studies*



*Note:* Reprinted from *Investigating the pedagogy of mathematics: How do teachers develop their knowledge?* by Lianghuo Fan. Copyright 2014 Imperial College Press.

The table shows the results of both studies based on statistical analysis. From these results, we can see that despite the differences in social, cultural, and educational backgrounds between Singapore and Chicago, the conclusions of the Singapore study are mostly consistent with those of the Chicago study. Overall, teachers’ “own teaching experience and reflection” is the most important source for them to develop all the three components of pedagogical knowledge. The second most important source is teachers’ informal exchanges with colleagues. For the other sources, there exist a difference in importance in teachers’ developing one or more components of their pedagogical knowledge. In most cases, inservice training and organized professional activities are sources of secondary importance; and “reading professional journals and books” is the least important source. Table 1 also shows some minor differences between the Chicago study and the Singapore study. For example, Chicagoan teachers consider preservice training experience as the least most important source for them to gain their pedagogical knowledge, while Singaporean teachers view it as more important, yet still secondarily important source. By contrast, Singaporean teachers’ evaluation of the importance of daily exchanges with colleagues about their knowledge of concrete materials and PCnK is not as high as Chicagoan teachers. The last section of Chapter 11 contains the concluding remarks of the whole book. The author rethinks the importance of each source, discusses the implications of the two studies for mathematics teacher education, and formulates opportunities for future research on teachers’ knowledge development.

**2 What is special about this book?**

Fan’s book provides a wealth of information on the Chicago and Singapore studies. As readers, what impresses us most are the research design and conclusions in this book.

The research design of this study is very elaborate and rigorous, especially in sampling strategies and research methods. In terms of sampling, the sample sizes, 77 in Chicago and 73 in Singapore, at first glance seem quite small, but via well-designed sampling, the samples could represent the target population. In the Chicago study, the targeted school population consists of the 25 best public high schools, making up 12.9% of all the public high schools in the metropolitan area of Chicago, Illinois. The criterion for determining “the best schools” was the students’ average scores of each school in the Illinois Goal Assessment Program (IGAP) mathematics test. Three schools were selected from the population by stratified random sampling; among which one was selected randomly from the best eight schools in terms of the IGAP average score, another from the second best eight schools, and the other from the remainder of the school population. The same sampling strategy was utilized in the Singapore study; and six schools were selected from all 152 secondary schools in Singapore. In the two studies, all the mathematics teachers who were at that time teaching regular mathematics classes in the sample schools were taken as the subjects of the studies. The distribution of samples in gender, age, and teaching experience appears roughly balanced. For example, in the Chicago study, 77 teachers participated in a questionnaire survey, and 69 of them returned the questionnaire. The respondent teachers include 31 males and 38 females, among them, 17 teachers with 0~5 years’ mathematics teaching experience, another 17 teachers with 6~15 years’ mathematics teaching experience, and the final 35 teachers with 16 or more years’ mathematics teaching experience (for more details about participants’ background in both studies see pp. 295-296, 305-306). As “stratification may produce a smaller bound on the error of estimation than would be produced by a simple random sample of the same size” (Scheaffer, Mendenhall, Ott, & Gerow, 2011, p. 117), coupled with the sample teachers’ multiple backgrounds in major demographic variables, the samples in the two studies could well represent the target population.

In terms of research methods, both the Chicago study and the Singapore study used questionnaire surveys, classroom observations, and interviews. The purpose of the questionnaire was to understand what kind of pedagogical knowledge teachers have from their own perspective; the classroom observations were aimed at understanding what kind of pedagogical knowledge teachers *actually* use during their teaching; and the interviews aimed at exploring how teachers develop their pedagogical knowledge in a specific context. Together, these three methods collect information from different perspectives and cover both the tacit and explicit aspects of teacher knowledge, as well as the context, process, and outcome of knowledge growth, aiming to present a complete picture of how teachers develop their pedagogical knowledge. Furthermore, the inclusion of two diverse locations, Chicago and Singapore, offers an effective comparison, giving credence to some generalizability. Studies using such mixed methods have been conducted before, but seem to be particularly effective in this study, with the methods and instruments complementing each other. The research design therefore can provide a model for those intending to conduct similar research on teachers’ knowledge.

The conclusions of Fan’s investigation are also noteworthy. On the one hand, some of the findings identified above corrected our own previous understandings. For example, one of the conclusions is that the contribution of “preservice training” to the development of teachers’ pedagogical knowledge is not the “most important”. This conclusion may be unexpected by teacher educators who work in preservice teacher training, and it might indicate the necessity for improving the quality of preservice teacher training programs. On the other hand, other findings in this book provide an evidence base for existing views in teacher development. For instance, Fan articulates that “teachers’ own teaching experience and reflection” is the most important source of teacher’s pedagogical knowledge. This could serve as support for Schön’s (1983) proposal of the teacher as a reflective practitioner, and for Posner’s (1989) formula of “Experience + Reflection = Growth” (p. 21). Fan’s finding regarding “exchanges with colleagues” also supports Hargreaves’s (1994, p. 238) advocacy of teachers’ “collaborative culture,” as well as the concept of “professional learning community” proposed by Hord (2004) from the perspective of teachers’ knowledge development. There also are similarities with Wenger’s “communities of practice” (Wenger, 1998) and the role of peer support networks in preservice teachers (e.g., Bokhove & Downey, 2018). Given the fact that the development of teachers’ professional knowledge continues to be an important challenge in teacher development, these themes are pertinent. Fan’s findings have both academic and practical significance for promoting teachers’ knowledge development.

**3 What further thinking could this book lead to?**

This book also pushes our thinking forward. As suggested at the end of Chapter 9, “it is natural as well as interesting to ask similar questions in other domains of teachers’ knowledge” (p. 216). For example, how do teachers in general develop their subject matter knowledge? Subject knowledge is undoubtedly necessary knowledge for teachers, but its development might use different sources from pedagogical knowledge development. It seems logical that mathematics courses during and prior to preservice training should greatly impact on the development of teachers’ mathematics knowledge. However, the degree to which this assumption holds true needs to be verified by empirical studies. Besides subject knowledge, another critical issue we would identify is knowledge of or about technology. Nowadays, technology is becoming an increasingly important part of education. As Artigue (2015, p. 361) stated, we have “entered the digital era, and that integrating this reality into mathematics education is no longer an option.” More specifically, in a recent book focusing mathematics and technology, Aldon, Hitt, Bazzini, and Gellert (2017) outlined four aspects of the growing importance of technology in mathematics education: (a) as a pedagogical tool, (b) as a means of communication, (c) as a means of information sharing about teaching and learning, and (d) professional development. Fan’s Chicago study and Singapore study also examined mathematics teachers’ knowledge of technology, but this was only as part of PCrK (Pedagogical Curricular Knowledge, i.e., knowledge of instructional materials and resources).

Not giving much attention to the role of technology may be an underdeveloped point in Fan’s book. However, Fan’s aspects of teachers’ pedagogical knowledge could usefully be augmented or even synthesized with existing frameworks for technology use in mathematics education, such as those Ruthven (2014, pp. 390-391) highlights, like TPACK (Technological Pedagogical Content Knowledge), instrumentation, and the “structuring features of classroom practice framework.” In addition to the reconceptualization of teachers’ knowledge, future studies could also extend its target population to other types of teachers. The participants in Fan’s study were mathematics teachers from high schools. What might be the findings if the teachers were from elementary or middle-school levels, or they taught other subjects, such as Languages, other STEM (Science, Technology, Engineering and Mathematics) subjects or the Arts? These issues are worth further investigation.

A more fundamental issue for future research and practice is how we are meant to evaluate and maximize the role of each source of teachers’ knowledge. The actual role of each source revealed by Fan’s studies does not necessarily represent the best role it could play. For example, in this book, “preservice training” was found to make a more limited contribution to the development of teachers’ pedagogical knowledge than expected. Should this, however, mean that “preservice training” can be neglected? Does it mean that the focus of teacher education should be shifted from “preservice training” to “in-service practice”? It is likely that the answer to this question is not a simple “yes” or “no.”. Given the fact that Fan’s book highlights differences in the contribution of “preservice training” between Chicago and Singapore, it seems plausible that it also differs between countries. Many cross-cultural comparative studies on mathematics classroom have pointed out that teaching is a cultural activity (e.g., Clarke, Emanuelsson, Jablonka, & Mok, 2006; Clarke, Keitel, & Shimizu, 2006; Hiebert et al., 2003; Stigler & Hiebert, 1999). Therefore, teachers’ pedagogy should also have relevant cultural attributes, and the construction of their pedagogical knowledge would inevitably be influenced by the social and cultural context in which the teachers study and work. Preservice training is not an abstract concept, but rather a concrete practice, which is carried out in specific ways and in specific contexts. Its contribution to teachers’ knowledge development varies, depending on the ways it is implemented, its context, as well as the competencies of practitioners—both the trainer and the trainee. Given the above, what Fan’s conclusions offer, what may not be a final or universal judgement, but a “mirror” for researchers and teacher educators to reflect on the practice in their own context, to make their own judgements, and then to make full use of the most effective sources and improve the less effective ones.

This point of view is also true for other sources of teachers’ knowledge development. Take, for example, the “teachers’ own teaching experience and reflection.” Although this source was looked upon as the most important by the sampled teachers, not all teacher made favorable comments. In other words, the role each source plays in developing teachers’ knowledge differs between individual teachers. According to the definition of knowledge in Fan’s book, teachers’ knowledge is understood as a mental result of certain interactions between the subject (teacher) and the object (including the sources of knowledge). Another question that then arises concerns how the teacher interacts with each source and which eventually becomes a mental result, and how knowledge becomes integrated into teachers’ repertoires. This issue essentially belongs to the mechanism of teachers’ knowledge development and involves cognitive, institutional, as well as cultural aspects. This seems to be a more complicated issue. Just as Boyd et al*.*’s(2013) argument, cited by Fan at the end of his book, “what is less clear is how teachers develop such [mathematical] knowledge for teaching” (p. 1012). From this perspective, Fan’s book has brought to us not only his research itself but a prompt for us to further unpick the development of teachers’ knowledge. It should be noted, though, that—given this contextualized and individualized nature of teachers’ professional development—Fan’s suggestion for teachers developing their pedagogical knowledge is not a silver bullet for every situation. Ultimately, it will be up to the individual teacher in every classroom in every school in every country, to develop their own knowledge for teaching. For future research this could mean that, in addition to more large-scale empirical investigations, contextually-situated and interpretation-oriented case studies of individual teachers, or specific groups of teachers, are also needed. For teachers who hope to improve their knowledge, this means that they need to know themselves as well as their situations, and to be willing to examine both the results and processes of their knowledge development.

Finally, we would like to mention the impact of the Chicago study in this book on education research in China. As Fan points out in the introductory part of his book, the Chicago study stems from his doctoral dissertation at the University of Chicago, USA. The dissertation was translated into Chinese and published in 2003 by East China Normal University Press, titled *A Study on the Development of Teachers’ Pedagogical Knowledge*. This Chinese book has received much attention from researchers in China during the past 15 years. According to the China National Knowledge Infrastructure (CNKI), the most well-known Chinese database in the Chinese mainland, from January 1, 2003, to December 31, 2018, a total of 1302 Chinese articles (not including monographs, but including 135 doctoral dissertations) cited Fan’s (2003) book. A survey of the above Chinese articles by the first author of this review found that some Chinese researchers had investigated similar questions by referring to or adapting Fan’s (2003) conceptual framework or research methods; among which were 15 studies published in Chinese Social Science Citation Index (CSSCI) journals. For example, Zhang, Dong, and Jing (2011) investigated the development of knowledge of 92 Information and Technology teachers in high schools; Li, Wan, and Yang (2012) investigated the development of mathematics pedagogical content knowledge (MPCK) of 41 mathematics teachers in rural middle schools; and Liu, Zhang, and Sun (2018) investigated 131 English teachers’ knowledge development in primary schools. Most of the above 15 studies found that teachers’ own teaching experience and reflection and their exchanges with colleagues were the most important sources of teachers’ knowledge development, which is consistent with Fan’s conclusions. However, the importance of other sources varied according to the types of knowledge, disciplines, and the levels of school. We can see that, over the past decade or more, many studies in the Chinese mainland have been inspired by Fan’s (2003) book and have expanded Fan’s studies. With the 2014 publication of the English version of *Investigating the Pedagogy of Mathematics: How Do Teachers Develop Their Knowledge?*, more readers or researchers have been able to become acquainted with Fan’s work and to draw inspiration from it. One of the recent cases may be Wei, Chen, and Chen’s (2019) article which referred to Fan’s (2014) theoretical framework of knowledge sources as well as the research instruments. Given that studying the sources of teachers’ knowledge and the importance of different sources both have academic and practical significance for promoting teachers’ knowledge development, and that Fan’s studies (Chicago Study & Singapore Study) are systematic in both findings and methods, Fan’s book, although not particularly new, is worthy of attention of future study in this field.

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