Electrodynamics beyond common multipoles

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Multipole expansion is one of the main analytical tools of the modern theoretical physics. In electrodynamics it allows one to describe the radiation properties of a charge-current excitation of any spatial complexity, and is routinely used in nanophotonics for simplifying the analysis of light scattering by nanostructures and metamaterials. The dynamic multipole expansion is commonly presented as a series of terms of two distinct types – the so-called electric and magnetic multipoles, which correspond to elementary radiation sources formed by oscillating charges and circulating currents, respectively. However, as it was indicated by several theoretical groups [1-3] and confirmed experimentally [4], the complete multipole expansion of electromagnetic sources must contain more types of multipole terms than it is generally acknowledged. The most celebrated example of those additional and, so far, exotic multpoles can be visualised as current loops oscillating on a surface of a torus along its meridians, and is known as the dynamic toroidal dipole.

In this talk I am going to revisit the toroidal dipole, its role in the metamaterial response and use for sensing applications. I will also discuss the problem of discriminating between the toroidal and electric dipoles in an optical experiment, as well as detecting their non-radiating combination known as the dynamic anapole [5]. Lastly, I will describe some other, even less familiar multipoles termed as the mean-square radii [1, 2], which not only contribute to electromagnetic scattering on a par with common multipoles, but can also constitute non-trivial non-radiating excitations akin to the dynamic anapole.

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