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Effective Provision of Critical Care Services

Thesis for the degree of Doctor of Philosophy

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ABSTRACT

FACULTY OF SOCIAL, HUMAN AND MATHEMATICAL SCIENCES

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Doctor of Philosophy

EFFECTIVE PROVISION OF CRITICAL CARE SERVICES

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This research aims to improve the efficiency of intensive care units (ICU) by improving patient flow. A UK ICU provides a case study for this research. Of particular interest in this work is the impact of 'late admissions', which account for 13.8% of all first-time admissions to this ICU. Patients admitted to the ICU more than a day after entering the hospital are shown to have higher mortality rates and to stay longer in the ICU.

Mortality and length of stay (LoS) are predicted to assist ICU modelling. After comparing different binary prediction models, three sets of logistic regression models have been built to predict patients' mortality from different admission groups (such as planned, unplanned, late or re-admission). The overall performance of the prediction models developed in this project is better than using ICNARC (Intensive Care National Audit and Research Centre) probability directly. LoS of individuals is found to be hard to predict. A new method for modelling LoS is tested and applied. LoS is split into three sub-parts, admission hour, nights spent in the ICU and discharge hour, for which empirical distribution functions are used.

We describe a Discrete Event Simulation (DES) model to investigate the impact of the late admission group and strategies for improving efficiency by bringing patients into the ICU earlier. Mortality prediction models and the new method of LoS modelling are incorporated into the DES input distributions. Several scenarios are investigated including varying resource number and earlier admission of patients. A key finding is that the ICU can accommodate 20% more unplanned patients based on the current situation if the late admission group can be reduced to 5% of all first-time admissions. We also consider an epidemic scenario: it is demonstrated that the ICU would only be able to cope with a mild epidemic.

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Author's declaration

I, Dandan Shi, declare that the thesis entitled "Effective Provision of Critical Care Services" and the work presented in it are my own. I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;

2. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;

3. Where I have consulted the published work of others, this is always clearly attributed;

4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;

5. I have acknowledged all main sources of help;

6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;

7. None of this work has been published before submission;

8. Parts of this work have been presented

    - Shi D, Smith, H.K., and Currie, C.S.M. "Effective Provision of Critical Care Services - Data Analysis" Poster presentation in 8th IMA Conference on Quantitative

Modelling in the Management of Health and Social Care

- Shi D, Smith, H.K., and Currie, C.S.M. "Effective Provision of Critical Care Services - Problem Identification" Oral presentation in 5th Student Conference on Operational Research

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Date:

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Definitions and abbreviations

AdH: Admission Hour

APACHE: the Acute Physiology and Chronic Health Evaluation

ANOVA: Analysis of Variance

AUROC: Area Under the curve of the Receiver Operating Characteristic

A&E: Accident and Emergency

BRI : Bristol Royal Infirmary

CART: Classification and Regression Trees

CCG: Clinical Commissioning Group

CI: Confidence Interval

DES: Discrete Event Simulation

DisH: discharge hour

ED: Emergency Department

EDF: Empirical Cumulative Distribution Function

GCS:Glasgow Coma Scale

ICNARC : Intensive Care National Audit and Research Centre

ICM: ICNARC Coding Method

ICU : Intensive Care Unit

ITU : Intensive Therapy Unit

IU: Input Uncertainty

HDU : High Dependency Unit

LoS : Length of Stay

MPM: Mortality Probability Model

NHPP: non-homogeneous Poisson process

NHS: National Health Service

PA: patients acuity

SAPS: Simplified Acute Physiology Score

SOFA: Sequential Organ Failure Assessment

Chapter 1

Introduction

Critical care is a very importance service in hospitals as it provides observation and care for patients with potentially recoverable conditions which cannot be provided safely in a general ward. Due to expensive equipment and demanding staffing resources, critical care is a high-cost healthcare provision. An intensive care unit (ICU) is a ward that provides critical care services.

The cost of an ICU consists of six components, which are capital equipment, estates, clinical support services, non-clinical support services, consumables and staff cost. In a study of eleven ICUs in the UK, staff costs were found to be the single biggest ICU cost, accounting for over 50% of the total (Seidel et al., 2006). According to the Department of Health (NHS, 2013b), an ICU bed costs around £1,100 to £1,900 per day. In contrast, the cost of a general ward bed is around £300 depending on treatments received. ICU service accounts for 15-40% of hospital budgets among developed countries (Tan et al., 2012; Barrett et al., 2011).

The utilisation of ICUs in England was constantly over 80% and sometimes close to 100% over the period 2010 to 2016, resulting in cancellation of urgent operations (NHS, 2016a).

This research investigates effective provision of critical care services. It is in collaboration with Bristol Royal Infirmary (BRI), a tertiary level hospital serving the West of England.

The BRI ICU has an overall "good" performance recognised by Care Quality Commission (CQC) in England. However, bed availability is an major issue for the ICU (CQC, 2014).

1.1 Research motivation

Two major operational difficulties in an ICU in a hospital are shortage of clinical staff and shortage of intensive care beds. The National Confidential Enquiry into Patient Outcome and Death (NCEPOD) reports that these two major difficulties have occurred in ICUs in England and Wales almost every year since the 1990s (NCEPOD, 2014).

A study based on 65 ICUs in the UK shows that higher numbers of clinical staff per bed are associated with higher survival rates in hospitals and especially in ICUs (West et al., 2014). Staffing levels and workload were also proved to be critical in ICUs in France (Neuraz et al., 2015). Multi-national research undertaken in North America and Western Europe, including Canada, the US, Belgium, France, Germany, the Netherlands, Spain and the UK, showed that the hospital mortality rate for ICU patients around 2005 was highly inversely correlated with the number of intensive care beds per 10,000 people ($\rho = -0.82$) (Wunsch et al., 2008). Moreover, the UK had the lowest level of adult intensive care beds amongst all the investigated regions with 1/5 and 1/6 of the intensive care beds per 10,000 population in the US and Germany respectively. Wallace et al. (2015) analysed the trends of ICU beds and staff in the US. Although the number of ICUs in the US is much greater than that of the UK and is still growing, practitioners and researchers in the US still believe there is a shortage of beds and staff there.

Delays in discharging patients from ICUs to general wards are not uncommon. Although beds in ICUs are precious and limited resources, there are, nevertheless, a considerable number of patients who have delayed discharge from ICUs. Lack of general ward beds accounts for a large proportion of delayed discharges. Colvin and Peden (2012) state in their audit report that congestion in general wards causes gridlock of the whole system and suggest that bed management needs to be set at least at a whole hospital level and preferably at a local area

level. To improve discharge from general wards to home, patients are recommended to recover at their home or local care home by using personalised care packages. Care packages include essential equipment and allocation of professional personnel to deliver care. However, many patients are unable to leave hospital because of the delayed preparation of care packages. Hospitals try to solve the problem by cooperating closely with local Clinical Commissioning Groups (CCGs) (BBC, 2014; NHS, 2016b). The service needs to be set within an effective whole hospital bed management system which ensures that every patient is in an appropriate location to meet their needs for staffing and equipment to support their care.

Premature discharge is also a problem for ICUs. ICUs are recommended to reserve around 30% of beds to deal with emergency admissions (CQC, 2014). However, most of the time, ICUs are not able to meet this recommendation because of lack of beds or nurses. Therefore, it may occur that patients in ICUs are discharged early if more severely ill patients need to be admitted. Patients could, therefore, stay in ICUs for a shorter time than they actually need. This group of patients is more vulnerable to readmission, which is proved to cause an increased risk of mortality and longer ICU length of stay (LoS) (Town et al., 2014). Therefore, premature discharge can cause a waste of ICU resources rather than saving resources.

According to Chalfin et al. (2007), immediately admitting patients in need to ICUs helps to lower the mortality rate and average LoS of ICU patients overall. However, some patients may not be admitted in a timely manner due to numbers of reasons, e.g. prolonged waiting in the emergency department (ED) or lack of ICU resources causing delayed admission. Goldhill et al. (2004) showed that hospital mortality is higher for patients admitted from wards to ICU rather than directly. Harris et al. (2015) analysed 12,495 ICU patients from 48 hospitals in the UK showing that delayed admissions lead to a significantly increased mortality. The concept of late admission is similar to delayed admission in that it concerns the delay in admitting patients in need. However, late admission is defined as late identification of patients' needs rather than the delay caused purely by operational effects (Restrepo et al., 2010; Renaud et al., 2012). We include both delayed admission and late admission in this research. Late admission in our context refers to two groups of admissions: first, surgical critical care admission following postoperative care on a standard ward; second, medical critical care

admission with an admission time difference between hospital and ICU admission of longer than one day.

These problems, delayed discharge, premature discharge and delayed or late admission, lead to a waste of expensive ICU resources and increase congestion in ICUs. The difficulties can potentially be lessened by better planning of resources, admissions and discharge.

This research will concentrate on providing possible solutions to these problems by better planning of ICU patients' admissions and discharges and improving use of ICU resources (i.e. beds and staff) on the basis of optimised admission and discharge.

1.2 Research objectives and questions

Modelling and analysis in this research are based on data collected in a case study of the Adult ICU in BRI. We had various discussions with the BRI ICU staff and formed some research objectives based on these discussions.

The ultimate objective of the research is to improve the efficiency of overall ICU management. This can be decomposed into following objectives

- To find out the characteristics of "late" admission and investigate the impact of late admissions and readmissions on the ICU.
- To investigate mortality and ICU length of stay prediction, incorporating the influence of late arrivals in the ICU and busyness in the general wards.
- To investigate the potential benefits and challenges of combining data mining and simulation modelling to tackle healthcare management problems – using ICU management as a test case.

Research objectives will be achieved by finding answers for the following research questions. These questions arise from Chapter 3.

- How do late-admitted patients affect the efficiency and effectiveness of the ICU?
- What factors may impact patients' outcomes and LoS?
- By applying different admission and discharge policies for patients in the ICU, what improvements in efficiency can be achieved?
- How does resource level affect ICU effectiveness and efficiency?
- How will extreme conditions (i.e. pandemic) influence the ICU?

1.3 Thesis structure

The thesis continues as follows.

Chapter 2 describes the current situation of ICUs in the UK and BRI in particular. This chapter also describes frequently-used ICU scoring models.

Chapter 3, the literature review, provides related literature in accordance with research objectives including late admission, prediction and ICU modelling. Research questions to achieve the objectives are raised from the review.

Chapter 4 provides preliminary analysis of the data with a focus on late admission. Late admission is shown to have an adverse effect on both patients' mortality and ICU LoS. An analysis of timing effects is also provided in this chapter. We find that peak time admission has a positive effect on ICU outcome, with patients admitted at peak times more likely to be discharged alive. Patients acuity (PA), a measurement to obtain the busyness of mixed ICU, is created to indicate the congestion of general wards.

Chapter 5 describes data mining for improving mortality and LoS prediction. Mortality is split into three categories for investigation: within ICU, hospital and post-ICU. The benchmark models are found to work well in general but predictability varies with different patient categories such as those admitted late. Some improvements in predictions are achieved using

regression models. Both single stage and multiple stage regression models are trialled for predicting ICU LoS but are found to have low predictability.

Chapter 6 describes a discrete event simulation (DES) model of the ICU to investigate effects of earlier admission and other scenarios. A novel approach to ICU LoS modelling is introduced. Benchmark mortality prediction models are incorporated into the DES model.

Chapter 7 reports the results of testing six sets of scenarios, including earlier admission and epidemic scenarios. Earlier admission is shown to help to improve ICU throughput.

Chapter 8 summarises conclusions drawn from former sections, discusses research limitations and points to possible future extensions to this work.

Chapter 2

Background

This chapter aims at giving essential background information of intensive care services and the hospital we collaborate with. With a knowledge of UK intensive care and established methods in ICU operation, we can then step into better achieving our objectives by reviewing more specialised research papers.

This chapter proceeds with introducing intensive care services in the UK and in the BRI. Then, widely-used ICU scoring systems are introduced.

2.1 Intensive care in the UK

ICUs, also known as critical care units (CCUs) or intensive therapy units (ITUs), are specialist hospital wards. They provide intensive care (treatment and monitoring) for people in a critically ill or unstable condition. Patients are admitted to ICUs for various reasons. Some common ones are serious accidents (usually involving severe head injuries or burns), a serious short-term condition (such as a stroke or a heart attack), a serious infection (typically sepsis or pneumonia) and major surgery. Support equipment is commonly used in ICUs to support patients' body functions and monitor their states. Typical equipment includes ventilators, feeding tubes, intravenous lines and pumps, drains and catheters and monitoring equipment

(NHS, 2014). More seriously ill patients may need several organs supported.

The National Health Service (NHS) in England divides critical care into four different levels according to patients' needs, from Level 0 to Level 3, as shown in Table 2.1.

Table 2.1: Classification of critical care levels [source: (Department of Health, 2000)]

Level 0	Patients whose needs can be met through normal ward care in an acute hospital
Level 1	Patients at risk of their condition deteriorating, or those recently relocated from higher levels of care, whose needs can be met on an acute ward with additional advice and support from the Critical Care team
Level 2	Patients requiring more detailed observation or intervention including support for a single failing organ system or post-operative care and those 'stepping down' from higher levels of care
Level 3	Patients requiring advanced respiratory support alone, or basic respiratory support together with support of at least two organ systems. This level includes all complex patients requiring support for multi-organ failure.

The guidelines for ICU staffing levels suggest that level 3 beds need a 1:1 nurse-to-patient ratio while level 2 beds need 1:2 (Bray et al., 2010; Ball and Barker, 2010; Royal College of Nursing (RCN), 2012).

Pearse et al. (2006) analysed inpatient general surgical procedures and ICU admissions in 94 NHS hospitals between January 1999 and October 2004. Valuable information was discovered from the medical outcomes of the ICUs. First of all, around 20% of high risk surgical procedures accounted for approximately 80% of death of surgical admission patients in the UK. Focusing on ICU admissions, the groups of readmitted and delayed admission patients generally had higher mortality rates compared to the group admitted to ICU directly. However, LoS in the ICU for those different groups of patients was not included in the research.

According to Jhanji (2008), mortality rates in a large NHS trust in England were distinctively high amongst patients discharged and readmitted to critical care and amongst those admitted to critical care following initial postoperative care on a standard ward. It is shown that if late admission and readmission is avoided, the mortality rate of elective patients and emergency patients can be largely improved by 80.42% and 6.44%, respectively. An analysis of UK-wide data found similar results (Pearse et al., 2006).

2.1.1 Intensive Care National Audit and Research Centre (ICNARC) Coding Method

All admissions to adult ICUs in England are reported to ICNARC. ICNARC established a coding method to simplify the reporting of ICU patients' admission reasons. There are 709 diagnostic categories according to the ICNARC coding method (ICM) (ICNARC, 2016). The ICM has been updated twice, in the years 2006 and 2015 (ICNARC, 2015b). ICU admission reasons are coded in five sections. The first section represents surgical/medical; the second denotes body system; the third gives body site; the fourth the process involved and the fifth gives the detailed condition. To illustrate this, the code 1.3.7.15.4 can be interpreted as 1: a condition where surgery has been performed (surgical code), 3: Gastrointestinal (system), 7: Liver or biliary tree (site), 15: Haemorrhage (process), 4: Acute fatty liver of pregnancy (condition) (ICNARC, 2015a).

2.2 Intensive care in BRI

Bristol Royal Infirmary (BRI) is one of eight hospitals in Bristol University NHS Foundation Trust. This trust is a centre for medical training in the South West of England. BRI is a teaching hospital affiliated to Bristol University. There are an accident and emergency (A&E) department, acute medical and surgical wards and ICUs in the hospital. Services are provided for both the city of Bristol and the whole South West region. Having foundation trust status means that profits can be re-invested to improve services (BRI, 2016). Critical care in BRI received 'good' in the rating in the CQC report of 2014 while the Bristol University NHS Foundation Trust as a whole received a 'requires improvement' rating. However, problems relating to access to critical care beds, resulting in cancelled operations and delays in transfer to critical care, were thought likely to continue due to the lack of available suitable beds (CQC, 2014).

There are three different critical care / high dependency units in BRI: the adult intensive therapy unit (ITU), the cardiac intensive care / high dependency unit (CICU) and the

coronary care unit (CCU). Both the ITU and the CICU are mixed ICUs with admission of both level 2 and level 3 patients. The CCU is a high dependency unit, treating only level 2 patients. In this research we are collaborating with the adult ITU, which has 21 beds and 16 ICU nurses in every shift. The staffing level is equivalent to 16 level 3 beds or 12 level 3 beds and 8 level 2 beds as designed. Patients are admitted to the ITU from a range of sources, from all kinds of department in the same hospital including other ICUs. The ITU can also admit patients from sources not in this specific hospital, for example from home or other hospitals. The four main sources of ITU patient admissions are the A&E department, theatre/recovery area, general wards and the imaging department. The ITU in BRI may also divert emergency patients to other hospitals if all the beds are filled. Around 1100 admissions are taken annually.

2.3 Introduction to ICU scoring systems and models

There are a number of useful and robust models for assessing the severity of ICU patients and their probability of death. The models used in BRI are Intensive Care National Audit and Research Centre (ICNARC) models and the Acute Physiology and Chronic Health Evaluation (APACHE) classification systems. The Glasgow Coma Scale (GCS), an important contributor to all the models, will be described first.

2.3.1 Glasgow Coma Scale (GCS)

The GCS is a neurological scale that records the state of consciousness of a person. It is a description of patients rather than a prediction. The assessment includes three tests: eye, verbal and motor responses. The scale reports both total score and individual elements. The total score is calculated by adding scores of the three tests. The total ranges from 3 to 15, with the lowest being 1 in each assessment and highest being 4, 5 and 6 in eye, verbal and motor responses respectively. A lower score represents a lower level of consciousness. Although GCS has been criticised for its inter-rater reliability (the inconsistency of scores

assessed by different raters for the same patient) and lack of prognostic utility (Green, 2011; Gill et al., 2004), it is still widely applied to all acute medical and trauma patients. The GCS is also used as a part of many ICU scoring systems, including the APACHE classification system, SAPS, the ICNARC score and SOFA score.

2.3.2 Sequential Organ Failure Assessment (SOFA) Score

The Sepsis-related Organ Failure Assessment score, also known as Sequential Organ Failure Assessment score (SOFA score), is an ICU score to track patients' multiple organ failure during their ICU stay. The SOFA score assesses organ dysfunction or failure in critically ill patients. The score is designed to describe patients' status rather than predict patients' outcomes, though assessment of morbidity does help to predict mortality (Vincent et al., 1996).

The score is limited to six systems to keep it simple: respiratory, coagulation, liver, cardiovascular, central nervous system (using GCS directly) and renal. The scores are assessed in each individual system and then added together. For each system, the score ranges from one to four, with a higher score related to worse organ function.

The SOFA score can also predict nurses' workload in ICUs, according to research in the BRI adult ITU (Thomas et al., 2013).

2.3.3 APACHE classification system

Acute Physiology and Chronic Health Evaluation (APACHE) is a disease severity measurement. It utilises conditions of patients within 24 hours of their admission. The model was first developed by Knaus et al. in 1981 and it then underwent major revisions in 1985 and 1991 (Knaus et al., 1981, 1985, 1991). The APACHE systems were developed and calibrated based on adult medical/surgical ICU admissions at US hospitals. ICNARC includes definitions and classifications from APACHE II and III.

The first APACHE classification system includes 34 variables and is composed of two parts: first, the APACHE II Acute Physiology Score (APS) measuring the severity of illness, ranging from 0 to 60, collected within the first 32 hours of admission; second, past medical history (Knaus et al., 1981).

APACHE II

The APACHE II score is composed of APS, past medical history including recent surgery, history of severe organ insufficiency and immunocompromised state; age, patient type (medical and surgical); and 34 disease categories. All the data are collected in the first 24 hours after ICU admission. No recalculation is allowed during the stay. The score is an integer ranging from 0 to 71, with a higher score corresponding to a higher risk of death and more severe diseases (Knaus et al., 1985).

The APS component considers 12 physiological measurements, including AaDO2 or PaO2 (depending on FiO2), temperature (rectal), mean arterial pressure, pH arterial, heart rate, respiratory rate, Sodium (serum), Potassium (serum), Creatinine, haematocrit, white blood cell count and GCS. The score ranges from 0 to 60.

A major disadvantage of the APACHE II score is that it is not comparable between different disease groups. For example, an APACHE II score of 25 points is associated with a predicted mortality rate of 73.6% if a patient is admitted for neoplasm, but only 38.9% if admitted for a seizure disorder (ClinCalc, 2016).

The APACHE II risk prediction model combines the APACHE II score with other factors, admission from theatre following emergency surgery, reason for admission and CPR within 24 hours prior to admission, to calculate the risk of hospital mortality for critical care unit admissions.

APACHE III

The APACHE III score provides a risk classification of severely ill hospitalised patients with three components, physiology, age and chronic disease. Unlike the APACHE II score, it can be recalculated every day. Daily APACHE III scores are used in the APACHE III equation to update mortality prediction of patients. First-day APACHE III score shows the highest discrimination power compared to the past APACHE models (Knaus et al., 1991).

The APACHE III score ranges from 0 to 299 with a positive correlation with patients' mortality risk (Knaus et al., 1991). A physiology component of the APACHE III includes the APS components and four more elements, urine output, Albumin, Bilirubin and Glucose. The APACHE equation is developed by combining disease type and patient's previous location, and assigning different weights to the three APACHE III score components, physiology, age and medical history.

2.3.4 Simplified Acute Physiology Score (SAPS)

SAPS was designed for use as a simplification of APACHE II. The score ranges from 0 to 163 with a predicted mortality between 0% to 100%. The score is calculated for every individual patient during his/her first 24h after admission to the ICU.

SAPS II was developed from a multicentre study involving European and North America hospitals (Le Gall et al., 1993). It includes only 17 variables: 14 physiology variables (Heart Rate, Systolic Blood Pressure, Temperature, Glasgow Coma Scale, Mechanical Ventilation or CPAP, PaO2, FiO2, Urine Output, Blood Urea Nitrogen, Sodium, Potassium, Bicarbonate, Bilirubin, White Blood Cell), age, type of admission (scheduled surgical, unscheduled surgical, or medical), and three chronic disease variables (acquired immunodeficiency syndrome (AIDS), metastatic cancer, and haematological malignancy). The area under receiver operating characteristic curve (AUROC) was 0.88 in the training sample and 0.86 in the validation sample in mortality prediction (Le Gall et al., 1993).

In contrast to the APACHE II score, the resulting value of SAPS is better at comparing patients with different diseases. There is no discrepancy of mortality prediction in a patient population with the same score and with a variety of diseases (Le Gall et al., 1993).

2.3.5 Mortality Probability Model (MPM)

The first MPM was developed in 1985, then revised in 1993 (Lemeshow et al., 1993). The most recent version was released in 2007 (Higgins et al., 2007). The model has four predictive models, at admittance, 24h, 48h and 72h.

MPM estimates mortality probability at hospital discharge using variables obtained at the time of ICU admission or during ICU stay. The admission model, MPM_0, contains 15 readily obtainable variables. In developmental and validation samples AUROC=0.837 and 0.824, respectively. The 24-hour model, MPM_{24} (developed for patients still in the ICU at 24 hours), contains five of the admission variables and eight additional variables at 24h. It also discriminated well (AUROC=0.844 and 0.836 in the developmental and validation samples, respectively). Among severity systems for intensive care patients, MPM_0 is the only model available for use at ICU admission. Both MPM_0 and MPM_{24} are useful research tools and provide important clinical information when used alone or together. MPM_{48} and MPM_{72} contain the same 13 variables and coefficients as MPM_{24}. The models differ only in the constant terms, which reflect the increasing probability of mortality with increasing LoS in the ICU (Lemeshow et al., 1994).

2.3.6 ICNARC models

The ICNARC model was first implemented in 2007 based on UK critical care statistics. The model was proved to be the best mortality risk prediction model for patients admitted to UK critical care units. The model is calibrated periodically. The most recent ICNARC model was released in January 2016 (Ferrando-Vivas et al., 2016).

ICNARC physiology score

The ICNARC physiological score takes other models into account. The score uses physiological indicator categories from other models, APACHE II, APACHE III, SAPS II, and MPM II. Also included are selected categories of extreme systolic blood pressure, categories of respiratory rate from APACHE III (removing the condition that ventilated respiratory rates between 6 and 13 are not weighted), categories of PaO2/ FIO2 from SAPS II and interaction with ventilation status, categories of arterial pH and associated PaCO2 and categories of creatinine from APACHE II (without doubling the weighting for acute renal failure). Neurological status is modelled with 13 categories for individual GCS values from 3 to 15 (assessed during the first 24h following admission to ICU) and two additional categories for patients who were either sedated or paralyzed and sedated for the entire first 24h (Harrison et al., 2007).

The ICNARC score theoretically ranges from 0 to 100 with the observed highest score being 82. Although the score is no longer considered in the calculation of mortality risk since the full ICNARC model is in use, it is still used to report severity of illness (Ferrando-Vivas et al., 2016).

Full ICNARC model

The ICNARC mortality risk prediction model is developed from age, past medical history, source of admission, cardiopulmonary resuscitation (CPR) before admission, diagnosis category and ICNARC physiology score. Past medical history is considered in five categories. Sources of admission are combined into six broad categories. 67 non surgical diagnoses and 19 interactions with the physiology score are considered directly in the model. 34 surgical diagnoses and four interactions are also modelled directly. Other diagnostic categories are allocated with coefficients according to body systems. The ICNARC model was simplified by combining adjacent categories and backward elimination. The "best" model contains 12 physiological variables and 63 other variables were included in the final model (Harrison

et al., 2007). The BRI ICNARC data is in accordance with this model.

The current collection of data in UK ICUs is widely standardised according to the requirements of ICNARC. The ICNARC risk prediction model is regularly calibrated. A new ICNARC model was calibrated with data collected between 1 April 2013 and 31 March 2014. Instead of calculating the ICNARC physiology score and then inputting the score to the risk prediction model, the latest model incorporates measured physiological data directly. However, the physiological variables included are the same as that of the previous model (Ferrando-Vivas et al., 2016).

2.4 Summary

The background chapter gives an overview of the current situation of intensive care in the UK. Definitions have been provided for levels of care in intensive care wards, informing the simulation modelling in Chapter 6. A wide range of ICU risk prediction models are considered. An understanding of the various ICU scoring systems leads on to the use made of such variables and scores in data analysis and modelling in Chapters 4, 5 and 6. Literature reviewed in Chapter 3 refers to these ICU risk prediction models. The BRI ICU data include ICNARC scores and probabilities, and APACHE scores.

Chapter 3

Literature Review

This chapter aims at discovering feasible methods by reviewing related literature to achieve our research objectives. Research questions shall also be drawn from what is learnt from the review.

A review of related literature covering all related topics is provided in this chapter. The review is ordered in accordance with our research objectives. First, late admission related problems and models are reviewed. We next review mortality and LoS prediction models used in ICUs to understand what might applied to our case study. Finally, ICU modelling as a key part is reviewed and discussed.

Late admission is the key element of our research. Apart from the general investigation of late admissions in Chapter 1, a better understanding of late admission is necessary. The focus will be on consequences of late admissions shown in medical papers and the impact of delayed admissions modelled by operational research papers. These articles also demonstrate approaches to analysing the late admission problem.

As a preliminary to an understanding of prediction models, a review of timing effect is carried out; this is widely recognised as a potential impact on patient outcomes.

An introduction of ICU scoring systems, which largely focus on predicting patients' outcome,

has been presented in Chapter 2. A further review of other methods used in ICU mortality prediction enables us to investigate the prediction in a more detailed way. These models also gives a hint of influential variables in predictions.

LoS modelling is another key part of our modelling, and so methods for ICU LoS prediction are reviewed to initiate the construction of prediction models. We also pay attention to covariates contributing to ICU LoS prediction to see how non-medical variables are involved, for example, congestion and timing effects on ICUs.

In order to construct an appropriate ICU simulation model, we would like to review frequently used ICU modelling methods, including modelling of both arrival process and ICU services. Regarding ICU services, we consider ICU admission and discharge, ICU resources management and ICU services under an extreme condition (i.e. a pandemic). Then, simulation modelling of ICUs as a whole is reviewed to give a holistic view of the application.

We would like to investigate potential benefits and challenges of combining data mining and simulation modelling to tackle healthcare management problems. We review literature in order to know current applications of combining data mining and simulation modelling in healthcare management and whether the combination can add value to a DES model.

3.1 Late admission in ICUs

We discuss firstly, a select of medical literature on the topic of late admission. Then, models of late admission in ICUs are discussed. This is an area in which there is a scarcity of literature.

The definition of late admission follows the one in Jhanji et al. (2008); Restrepo et al. (2010); Renaud et al. (2012). It considers the delays originated from both lack of ICU resources and late identification of sick patients. Compared to readmission and premature discharge problems, late admission is a problem that has been given considerably less attention than other risks face by patients Jhanji et al. (2008).

3.1.1 Medical research into late admissions

Discussion of late admission problems can be traced back to the 1990s with evaluation of triage decisions for ICU admissions. Metcalfe et al. (1997) find that patients who are refused by the first-choice ICU have high mortality, in a UK study. In the research, they recorded first time admission and admission after refusal but did not report detailed results. A similar study carried out by Sprung et al. (1999) in Israel shows that a higher mortality rate was found in patients admitted later compared to immediate admitted patients.

In clinical studies in developed countries, poor hospital discharge outcomes, higher mortality and longer hospital LoS are proved to be associated with delayed transfer of critically ill patients from the ED to the ICU (Chalfin et al., 2007; Rincon et al., 2010). Cohort studies in developing countries also shows that delay in ICU admission from ED and post anaesthesia care unit associates with higher mortality (Cardoso et al., 2011; Bing-Hua, 2014).

Goldhill et al. (2004) explored the relationship of hospital outcome and hospitalisation before ICU admission using 50,837 records from 24 ICUs located in London and Essex, during the period of Apr 1992 to Dec 2000. They categorised hospital LoS before ICU admissions into six groups: 0-3, 4-6, 7-9, 10-12, 13-15 and $\geq$16 days. They found that the longer they stayed in the hospital before their ICU admission, the higher mortality and longer LoS, after case-mix adjustment.

Studies regarding specific groups of patients, patients with community-acquired pneumonia (Renaud et al., 2012), cancer with acute respiratory failure (Mokart et al., 2013) and acute myeloid leukaemia (Lengliné et al., 2012) also show that immediate admission of patients are related with lower mortality and shorter LoS.

However, a different view was found by O'Callaghan et al. (2012). They analysed five-year (2003 to 2007) ICU admissions from A&E of a 12-bed UK-based ICU. They conclude that delayed admission (ICU admission taken after three hours of decision made) correlated with both requirement and time of advanced respiratory support but no significant differences in either mortality rates or ICU LoS could be found between the two groups (delayed and

non-delayed). It should be noticed that longer delays than three hours are more generally researched in the literature.

3.1.2 Modelling late admissions

Bountourelis et al. (2011) present a large scale DES model aiming at solving patients' delays in both transfer into and out from the ICU. The model considers all ICUs and SDUs and four other departments in a US hospital. The authors show that the model is capable enough to capture situation of the ICU including bed-blocking. They plan to develop optimisation method based on the DES model to make full use of current beds as well as carry out further analysis of patient transfer scenarios.

Chan et al. (2016) use a queueing model to study delays of providing ICU service to patients coming from EDs, and find that long delays may have adverse effects on patient outcomes and can potentially lead to longer lengths of stay. The delay here refers to the boarding delay of ED patients to the ICU because of resource shortage or treatment needed. This paper focuses on understanding the influences of delayed boarding and assumes there is no early discharging in the ICU. The result shows that increased boarding times contribute to longer ICU LoS, with a one-hour delay leading to a 5.69% increase in the stay, higher overall congestion level of ICU and lower access to care for other critical patients. An M/M(f)/s queueing model is used, where service times are exponentially distributed with mean increasing with congestion following a growth function f. It is shown that the system load with expected work grows much faster than the normal $1/(1-\rho)$ relationship in most queueing systems.

A topic related to that of late admission is that of early warning systems for ward patients. Smith et al. (2014) review early warning system scores for clinical deterioration in hospitalized patients that have been implemented for earlier admission of ward patients to ICUs. Implementation of early warning systems in wards can help health professionals to identify sicker patients. It is related to a 33% decrease in hospital mortality of ICU patients, according to research in Houston, US, involving 3090 baseline patients and 8926 intervention

patients (Berger et al., 2018).

Hu et al. (2018) propose an Early Detection of Impending Physiologic Deterioration (EDIP2) score to indicate the appropriateness of proactive admission of ward patients to ICUs. It shows that a proactive admission policy could improve patient outcomes. However, it could also lead to ICU congestion which may finally lower the effectiveness of critical care. The authors suggest use of stochastic modelling and a dynamic optimisation framework to compare alternative policies, but do not undertake such modelling.

The previous research related to the impact of delayed admission concentrates on admissions to ICU from single sources, either delayed ICU boarding of patients from EDs or on delays of ICU admissions for clinically deteriorated patients from general wards. No work has been found in modelling the delays from all departments of hospitals.

Both delayed boarding of ICU patients from ED and delayed admission from general wards affect mortality and LoS of ICU patients. Various questions are raised from the literature. As most of the reviews are based in the US, is late admission a widely-existing problem? Should all the admission sources be considered when investigating late admission? How do late-admitted patients affect the efficiency and effectiveness of the ICU?

3.2 Timing effect of healthcare and intensive care

This review of timing effects in ICUs is carried out to provide a preliminary to mortality and LoS prediction of ICU patients. There is a large body of literature regarding timing effects; a flavour of different views are given here.

There widely exists a "weekend effect" of hospital admission of patients. The weekend effect shows that patients admitted to hospital for treatment during weekends have significantly higher mortality rates compared to those admitted during weekdays (Cram et al., 2004). Several medical studies have been carried out on weekend effects all over the world. Some of the studies show that the weekend effect appears in adult ICUs (Barnett et al., 2002;

Uusaro et al., 2003) especially in the elective surgical patient group (Bhonagiri et al., 2011). Ensminger et al. (2004) claim that the weekend effect exists only in surgical ICUs, not in medical or multispecialty ICUs, according to their study in the US. However, several researchers claim that there is no significant difference of patients' mortality between weekend and weekdays for either admission or discharge; Laupland et al. (2008) showed this in a Canadian study and Ju et al. (2013) in a Chinese study. The weekend effect is of particular interest in the UK healthcare system.

Moreover, "after hours" ("night time") admission and discharge are also tested in many studies. Uusaro et al. (2003), in a study in Finland of ICUs, find that discharge in the night time has a significant negative impact on patients' outcomes. Night time admission rather than discharge was found to be detrimental to ICU patients by Ju et al. (2013). Both Laupland et al. (2008) and Bhonagiri et al. (2011) find a strong after-hour effect of both ICU admission and discharge in their studies. Neuraz et al. (2015) explained the weekend effect and the after-hour effect by the patient-to-caregiver ratios, as the largest nurse staffing shortages and highest patient-to-physician ratios occur at weekends and during the night shift.

Timing effects in healthcare systems have received wide attention over a number of years. However, different research shows different results to the question of whether timing effects exist. Moreover, reasons for any effect have not yet been clearly found. An investigation of timing effects and their contributions to prediction is thus worthwhile.

3.3 Mortality prediction

Mortality is the key and the most straight forward ICU patient outcome. Current performance measurement of ICU is mortality-based (Barbash et al., 2016). The mortality considered in this study is both mortality in the ICU and hospital mortality of patients who have had an ICU stay i.e. mortality before the ultimate hospital discharge.

As introduced in Section 2.3, there are many ICU mortality risk prediction models using

scoring systems to predict the risk of mortality of patients admitted to ICUs. These were tested and compared in a wide range of datasets. The ICNARC risk prediction model is currently the best mortality prediction model for ICU patients in the UK (Harrison et al., 2007). These risk prediction models consider both physiological factors of patients and ICU operational factors, e.g. admission sources.

Apart from ICU scoring systems, there are other widely used models in ICU mortality prediction including artificial neural networks (ANNs), regression models and tree-based models. Bayesian methods are also found incorporated in tree-based and regression models. Frequently-used ICU scoring systems are introduced in Chapter 2. This section gives models commonly used in mortality prediction, with the aim of covering different approaches. A selection of examples of each method are given here to illustrate the applications and modelling potential.

Artificial Neural Networks (ANNs) try to mimic the working of human brain. However, the transparency of ANN models is low. Therefore, results gained from an ANN model may not be incorporated well with medical experts' knowledge. However, there are several applications of ANNs to prediction in ICUs. Gholipour et al. (2015) adopted ANNs for predicting survival and LoS of patients in an ICU and ward for trauma patients. The ANN model was fed by Trauma and Injury Severity Score (TRISS) components, biochemical findings and risk factors. TRISS is a score for specifically trauma and injury patients rather than ICU patients. Three different types of data were collected to predict ICU outcome and LoS in trauma patients: i) mechanism of trauma and site involved; ii) vital signs and physical examination of patients in the ED; iii) laboratory findings. GCS and peripheral capillary oxygen saturation (SpO2) in the second type of data together with base excess and blood glucose in the third type of data showed significant differences between patient groups that survived or died. The model achieved a 0.75 sensitivity and 0.96 specificity in predicting the outcomes, and predicted 93.33% of outcomes in the test group correctly. Also, the mean LoS predicted was not significantly different from the actual mean LoS.

A logistic regression model aimed at predicting death or readmission within seven days of

discharge after clinically finishing their critical care was built by Ouanes et al. (2012) based on empirical data from four ICUs in France and Tunisia. Six variables with good predictability are age, SAPS II at ICU admission, use of a central venous catheter, Maximum Systemic Inflammatory Response Syndrome scores during ICU stay, SOFA score at discharge, and discharge at night. The model achieves an AUROC of 0.74, which is marginally better than predicting using SAPS II only. Moran and Solomon (2012) predict mortality of ventilate patients by considering intensive care occupancy together with APACHE scores.

Chang et al. (2013) observed a surgical ICU (SICU) in Taiwan to investigate hospital mortality of prolonged stay patients who were aged more than 16 and stayed more than 14 days. Those patients consumed 9.7% of total SICU admissions but 51.7% of total SICU days. Four patient factors (gender, longer pre-ICU days, larger Charlson comorbidity index and not admitted from ED at admission) and seven factors at the 14th day of ICU stay (lower GCS, lower mean arterial pressure, higher dosage of inotropes required, higher serum lactate level, higher serum bilirubin level, lower platelet count and the use of renal replacement therapy) increased hospital mortality of prolonged-stay patients. Least absolute shrinkage and selection operator (LASSO) regression and logistic regression were adopted as variable selection and prediction methods.

A different view of prediction is provided by Ramon et al. (2007), who summarise the application of data mining methods for predicting state changes of patients in an ICU. Predictions include patient mortality, LoS longer than three days, and how long a patient will be in one of the endangering states (i.e. kidney dysfunction, inflammation, severe inflammation, inflammation shock and low blood pressure needing vasopressor support). One of the challenges in this area is that the absolute values of patient attributes such as heart rate are hard to compare. To solve this problem, the authors suggest applying a two-level Bayesian approach using the deviation of patients' heart rate instead of the absolute value. The researchers also compared four different data mining methods, decision trees (DT), First Order Random Forests (FORF), Naive Bayes (NB) and Tree Augmented Naive Bayes (TAN) in carrying out different tasks. Each one has their own strengths and weaknesses.

Wong and Ismail (2016) build two sets of four multivariate logistic regression models for a Malaysian ICU. One set uses maximum likelihood estimation (MLE); the other set utilises a Bayesian Markov Chain Monte Carlo simulation approach in the development of the models. APACHE scores and GCS are included in all eight models. The AUROCs for the Bayesian models and MLE models are very similar.

Johnson et al. (2012) develop a 'forest' based on 500 two-layer trees using UK ICU data. A Bayesian Markov Chain Monte Carlo approach is used to optimise the tree-based 'forest'. They used scalar features, i.e. minimum, maximum, median, first, last, and number of values, instead of temporal features in their model for each time-based variable. The AUROC of the model is 0.86 which is significant higher than the SAPS II estimation of 0.667. However, the authors have not stated all the variables they considered neither have they indicated if they put an established ICU scoring system into their model.

Apart from the scoring systems, different prediction models have been developed according to specific needs. These have also been compared with scoring systems in discrimination ability. Sinuff et al. (2006) compare physicians' subjective discrimination between survivors and non-survivors with the performance of SAPS II, MPM, APACHE II and other computer models. Results show that ICU physicians predict mortality more accurately than scoring systems or models, in the first 24 hours of ICU admission (AUROC: 0.85 ± 0.03 vs 0.63 ± 0.06).

To summarise, to achieve a result considerably better than existing ICU scoring systems has not been demonstrated in these studies. However, it is still possible to calibrate an ICU-specific model by incorporating ICU scores and other factors. Therefore, we are interested in whether there are any other factors that could contribute significantly to the mortality prediction, for example, late admission of patients. Moreover, mortality prediction models in literature focus either on all patients in an ICU or a certain group of patients. No investigation of predictability of models for diverse groups of patients is found.

3.4 LoS prediction

Since ICU resources are extremely expensive, hospitals and patients can both benefit from optimising LoS, that is, providing patients with long enough but not excessive stays in an ICU. This section discusses research into LoS prediction in hospital wards, whether general wards or ICUs. Patient hospital length of stay refers to the number of days that an inpatient stays in a health care facility during a single admission. LoS is a strong indicator of resource consumption level in a hospital. It also helps to understand flow of patients and evaluate the operational efficiency of hospital departments. This research is seeking ways to predict patients' LoS more precisely both in the ICU and in hospital after leaving the ICU.

LoS of a patient in general reflects the severity of patients' illness. However, for ICU patients in particular, the most severe group of patients may stay a very short time as they may die very soon after their admission. For all the remaining patients, the longer they stay, the higher mortality probability they have (Martin et al., 2005). Moreover, LoS also reflects the operational efficiency of a hospital; the availability of general ward beds can impact the discharge of ICU patients. Similarly, social care and community nursing support can influence discharge of hospital patients. Hospital management style can also influence LoS of patients (Awad et al., 2016).

In the following sub-sections reviews are given of different type of LoS prediction models and their applications in the ICU. The aim is to introduce how methods are utilised in prediction and give examples of them. Various methods and their advantages and disadvantages are covered and some comparison of different methods is provided.

3.4.1 Statistical models for LoS prediction

Descriptive statistics

Average LoS is a frequently-used statistic of in-patient hospital LoS. It is easy to quantify and often used in hospital resource and capacity modelling and management. However, it is

not representative as LoS is often largely skewed. Median LoS could be more representative in some cases as LoS is recognised as right skewed (Awad et al., 2016). However, both of the two statistics give only a deterministic estimation of LoS. Outliers can be problematic in LoS prediction. Predefined lower and upper boundaries could be used to replace the outliers, which could reduce the skewness of data. The boundaries can be a certain percentile or predefined numbers calculated from robust statistical methods, for example mean $\pm$ 2*(standard deviation) (Senthilkumar and Ramakrishnan, 2012). Truncation is another way to deal with outliers. Instead of replacing outliers with other statistics, truncation excludes outliers directly. Various truncation methods have been applied to LoS modelling (Guzman Castillo, 2012). Data transformation can be employed before setting truncation boundaries.

Regression analysis

Linear regression

Linear regression models are used in modelling LoS of both hospital and ICU patients. LoS as a dependent variable is predicted by several independent variables. Moran et al. (2008) predicted log transformed ICU LoS in Australia and New Zealand using linear regression and incorporating patient demographics, severity score, surgical and ventilation status, ICU categories, and geographical locality as independent variables.

The usual approach for estimating unknown parameters in linear regression models is ordinary least squares (OLS). However, OLS is limited by several assumptions. These assumptions can easily be violated by the nature of LoS data (e.g. right-skewness). Transformations are considered to normalise the data. However, transformed LoS is not practically meaningful and requires back-transforming. There is a heteroscedasticity problem that causes a large bias after back-transforming the logarithmic LoS (Mihaylova et al., 2011).

Generalised Linear Models (GLMs)

In GLMs, dependent variables are connected to the linear equation by a link function and also the variance of data are captured in the models. The assumption of normality of

data in a linear regression can be avoided. GLMs avoid problems of data transformation (Guzman Castillo, 2012). GLMs perform better in interpretation of coefficients than using transformed data in OLS models to deal with heteroscedasticity.

Moran and Solomon (2012) compare several LoS estimation models based on data from ICU patients in Australia and New Zealand. The models include OLS models of original LoS and log–transformed LoS, and GLMs with log link function and four different distributions (Poisson, gamma, negative binomial and inverse-Gaussian) using untransformed LoS. They also considered extended GLM with flexible link and variance functions as well as multi-level or hierarchical linear mixed models (LMMs) incorporating random effects. The results show that LMMs are the best amongst all methods regarding consistency and predictability. However, the R-squareds of the LMMs range from 0.17 to 0.22, not high predictability.

Survival analysis

Survival analysis is a method for exploring time-to-event data. It is generally defined as a set of methods for analyzing data where the outcome variable is the time until the occurrence of an event of interest. Referring to LoS modelling, the event is discharge from an in-patient department e.g. an ICU (Collett, 2015).

Two main concepts in survival model are survival functions and hazard functions. While survival functions give the probability that the event of interest has not occurred by a certain duration, hazard functions are regarded as an instantaneous rate of occurrence of the event. (Rodríguez, 2007)

There are two categories of hazard functions, parametric and non-parametric estimations. Homogeneity of the study population is the basis for estimating hazard functions. A range of distributions for continuous non-negative variables are used as parametric hazard functions such as Weibull. Mason et al. (2015) selected the Weibull distribution as the baseline hazard function to analyse the LoS of patients in an ICU after cardiac surgery. Cosgrove et al. (2005) also used the Weibull distribution as a baseline hazard function to study LoS of

patients after staphylococcus aureus bacteraemia. Ravangard et al. (2011) found that the gamma distribution fitted the best to the LoS of patients in a tertiary teaching hospital in Iran. Mason et al. (2015) select the Weibull distribution as the baseline hazard function to analyse the LoS of patients in an ICU after cardiac surgery.

Non-parametric survival models are often estimated by Kaplan-Meier curves or Nelson-Aalen estimates, but these method limit the adjustments for covariates. Barton et al. (2009) applied Kaplan-Meier survival curves to determine LoS of stroke patients. Forster et al. (2012) used Kaplan-Meier estimation to study patient LoS with hospital-acquired infection.

In a proportional hazards model, increase in a covariate does not have a simply additive or linear relation with the hazard function; instead there is multiplication of the hazard rate. The Cox proportional hazards model is an approach to estimate effect parameters without any knowledge of the hazard function, based on the proportional hazards assumption. Beyersmann et al. (2006) adopt a Cox proportional hazards model in modelling prolongation of ICU LoS due to nosocomial infection. Nosocomial infection was modelled as a time-dependent covariate in a proportional hazards model.

Sá et al. (2007) apply different parametric and semi-parametric survival models for analysis of the influence of observed and unobserved covariates on patients' hospital LoS. They demonstrate that parameter estimates in LoS models are very sensitive to the assumptions regarding the hazard function.

Death is treated as a censoring point in survival analysis by Prinja et al. (2010). However, Lin et al. (2017) find that "censoring due to death" can lose implied causal inference in LoS modelling. The key advantage of a survival model is that it considers censored data. Although censoring is not a problem in most LoS analysis, survival analysis is still widely applied in LoS analysis as it is less parametric compared to OLS and GLMs (Basu et al., 2004).

Comparison of different statistical models

Verburg et al. (2014) compare different regression methods for modelling patient ICU LoS in the Netherlands. Eight different models are considered, linear regression using LoS, truncated LoS and log(LoS), GLM using Gaussian distribution and a logarithmic link function, Poisson regression, negative binomial regression, gamma regression with a logarithmic link function and Cox proportional hazard regression. In general, the Poisson model and Gaussian GLM performed the best in terms of R^2. Cox regression and linear regression of log(LoS) did better for patient ICU LoS shorter than four days. However, overall R^2s for models are not high falling in the range 0.09 to 0.20. Cyclical terms for discharge times were added to the models by the authors but with little improvement. Moreover, linear regression for log(LoS) may not be meaningful for decision-making processes (Verburg et al., 2014; Guzman Castillo, 2012).

3.4.2 Data mining methods in LoS prediction

Lim et al. (2000) compare a wide range of data mining methods for LoS and mortality prediction in both ICU and hospital considering both their accuracy and training cost. Differences in the results are generally ignorable. Interpretability of models can be a more important factor when choosing models.

Guiza Grandas et al. (2006) use four different data mining algorithms, DT, FORF, NB and TAN, on four different tasks: prediction of patient survival, prediction of patient LoS (short, medium and long), prediction of development of endangering states and prediction of recovery from endangering states. The results show that there is no one best model for all tasks. Different models suit different tasks. For LoS, the best method is TAN with an accuracy of 83%. Accuracy for other models ranges from 75% to 83%.

Milić et al. (2009) take use of a single score, either APACHE II or SOFA, on admission and on the third day of stay to predict LoS in general and cardiac ICUs in Croatia. Either scores on admission shows significant correlation with LoS. However, LoS and APACHE II/ SOFA

on the third day of stay strongly correlate, with $\rho = 0.728$ and $\rho = 0.725$ respectively.

Hachesu et al. (2013) use three classification algorithms, namely, decision tree (DT) using C5.0 algorithm, support vector machines (SVM), and artificial neural network (ANN), in predicting LoS of cardiac patients. They categorise LoS into LoS1, LoS2 and LoS3 where LoS1 $\leq$ 5 days, 6 days $\leq$ LoS2 $\leq$ 9 days, and LoS3 $\geq$ 10 days. 36 input variables are used for prediction. All three algorithms are able to predict LoS with limited different between them, but SVM has the best fit (accuracy=96.4%). There was a significant tendency for LoS to be longer in patients with lung or respiratory disorders and high blood pressure.

Lella et al. (2015) describe an unsupervised LoS prediction model, ANN combined with clustering. The developed model detects autonomously the subset of non-class attributes to be considered in these classification tasks, and the structure of the trained self-organizing network can be analysed in order to extract the main factors leading to the overcoming of a local LoS threshold. It is claimed that the model performs better than other ones commonly used in this kind of problem.

LoS appears to be highly correlated with clinically scores (Milić et al., 2009). Also, data mining methods can classify LoS into short, median and long stays with an acceptable accuracy (Lim et al., 2000; Guiza Grandas et al., 2006; Hachesu et al., 2013). However, such an inexact classification would not help much with management of ICU resources. The difference in predictability of different methods appears to be limited. Moreover, interpretability could be a more important factor when choosing between models.

3.4.3 Markov models

If probabilities of future states of a stochastic process depend only on the current state and the states happening directly before the current state, then this process is said to have the Markov property (Ross, 1996). Markov models are used to model systems with the Markov property.

A compartmental model describes how patients are transferred between different sections of

a healthcare system (Awad et al., 2016). Taylor et al. (2000) investigate the time geriatric patients spend in hospital and in the community by applying a continuous Markov model to a six-compartment model. The six stages include acute, rehabilitative, long-stay, two stages in the community and death. The study is based on an extensive 16-year dataset of UK geriatric patients. The model provides adequate fit for patients with short stay ($\leq$ 30days) but deviates much for those with long-stay.

Pérez et al. (2006) build models to predict average LoS of patients in an ICU and at each destination after discharge from the ICU. Patients are divided into six different diagnosis groups, cardiovascular, neurological, respiratory, gastrointestinal, trauma and other diagnostic groups. Different discrete time Markov models were estimated for each group. After the first day of ICU admission, patients move to eight different destinations including two absorbing states, death and discharge. The model shows good predictability (Chi-squared test) in cardiovascular, neurological and gastrointestinal patients but not in other groups. However, the Markov property is hard to verify.

Generally speaking, the accurate prediction of LoS for individual patient could be hard to achieve. However, LoS prediction may also be enhanced by determining appropriate covariates in regression models, splitting patients into homogeneous groups or incorporating cyclical terms in prediction. It worth finding out what factors may influence LoS and how we can better model LoS.

3.5 ICU modelling

Literature reviewed in this section focuses on simulation and queueing models depicting the whole ICU. We consider first these preliminaries to ICU modelling: arrival processes and service modelling, including ICU admission and discharge, ICU resource management and services under extreme conditions. Then, a review of modelling of whole ICUs is provided.

3.5.1 ICU arrival process

The modelling of arrival processes and service rates using exponential distributions is being challenged. Back to 1970, Swartzman had already claimed that non-homogenous Poisson process (NHPP) could only depict unscheduled patients (Swartzman, 1970). Kim et al. (1999) recommend that patients from different sources, including scheduled patients, are modelled separately to get a more accurate result when building a model to support capacity management. It is found that modelling patient arrivals by week is not sufficiently accurate. Kim and Whitt (2014) investigated data from call centres and hospitals and demonstrated that data rounding, mis-choosing of sub-intervals and ignoring the day-of-week effect may cause the rejection of the NHPP null hypothesis. This paper investigates the use of NHPP for modelling hospital arrivals but such a comprehensive investigation into the use of Poisson processes in illustrating ICU arrivals is not evident in the literature.

Poisson processes are popular in modelling the arrival process of ICUs. However, the justification for their use is usually not provided (Bai et al., 2016). Shmueli et al. (2003) use a homogeneous Poisson process (HPP) to model the arrival of all the ICU patients in a hospital in Israel with a single arrival rate parameter. They then compare different admission policies: first come first served (FCFS), first come first served for all referrals who can benefit more than some hurdle (FCFS-H), and first come first served for all potential admissions whose benefits exceed a bed-specific hurdle depending on the idle beds number (FCFS-BSH). They find out that the FCFS-BSH policy is the best for overall utility. HPP is adopted to modelling ICU admissions in other research. Kim et al. (1999) mix three HPPs for ward, A&E and outpatient emergency patients together via convolution. The convolution of two functions (f and g) is defined as $(f * g)(t) \equiv \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$ (Bracewell and Bracewell, 1986). An NHPP is assumed as the arrival process in modelling an ICU at a US military establishment (Masterson et al., 2004). McManus et al. (2004) fit an HPP for all patients including emergency and elective. Litvak et al. (2008) use different HPPs to model the arrival process of patients from different sources.

3.5.2 ICU services

ICU admission and discharge

McManus et al. (2004) use two years' admission, discharge, and turn-away data in a busy, 18-bed ICU in the US. They claim that a simple queueing model (M/M/c/c) is powerful enough to model ICU operation and flow. The model provides accurate results including turn-away rate and monthly responsiveness to changing demand which indicates the need for resources. The prediction works well at a high level (monthly changing demand and average turn-away rate). However, none of the elements in the simple queueing model can exactly depict what happens in an ICU.

Litvak et al. (2008) research management of the overflow of ICU patients. They proposed that several hospitals in a region jointly reserve a small number of beds for regional emergency patients, to tackle the ICU capacity problem in the Netherlands. According to the authors, the cooperation can achieve a higher acceptance of regional patients and a lower probability of cancelling operations, together with a smaller total number of ICU beds.

Dobson et al. (2010) built a model of ICU bumping, early discharging of patients who should not be discharged that early. The model helps to predict performance when bumping occurs with various arrival patterns and capacity. An algorithm is also developed to track the time in the system for each patient. Use of the algorithm avoids use of the assumption of an exponential distribution for LoS. The model also suggests the influence of surgery schedules on bumping rates.

Chan et al. (2012) develop a decision support tool to aid clinicians in discharging patients when an ICU is highly occupied ($\geq$ 75% occupancy) and new patients are waiting. The optimization is based on reducing readmissions as well as not sacrificing mortality rate given that all new patients must be given a bed immediately unless they are diverted. The discharge policy can be described as simply choosing a patient from a group to incur the least cost. The policy requires data about particular patient groups but does not require estimation of arrival rates of different classes. Ignoring future arrivals can lead to sub-optimality of the

solution. Patients were classified into five groups by prediction of death probability using medical scores (APACHE & SAPS). Each of the groups had approximately the same number of patients. In the model, both patient state and patient diversion were considered.

Armony et al. (2013) consider the role of Step Down Units (SDUs) in the US which are similar to HDUs in the UK. A queueing model of patient flow through an ICU and SDU is built to determine the necessity for and the size of an SDU. Staff resource is a key part of the model. The results show a zero-capacity SDU (nurses are released to ICU) or a sizable SDU could both be good options under different circumstances.

Kim et al. (2014) suggest an econometric model of ICU admission. The gain and cost of every ICU admission is quantified. They built three simulation models of admissions, an optimal full model considering both experts' opinions and recorded data; an optimal observable model regarding only recorded data; and a model adding an extra ICU bed without changing the admission policy. Compared to the current admission policy, the ICU can save \$8.1m, \$1.9m and \$0.4m respectively by adopting these three models. The ICU admission decision variables were decomposed into patient characteristics and instrumental variables to avoid endogeneity of the variables. The optimal full model referred to the combination of observed and unobserved patient factors. These unobserved factors are the conditions doctors may consider in making decisions. The model can be improved significantly when including these unobserved factors. However, the optimal full model is ambiguous without stating the actual variables included.

Shi et al. (2015) use simulation modelling to propose a discharge policy for an ED. They suggest that the first discharging peak of ED patients should be brought forward from between 11am and noon to 7am to 8am. An appropriate discharge timing can help to eliminate excessive waiting of patients needing resources. This idea may also be helpful in ICU discharge arrangements.

ICU resources management modelling

The two main resources in ICUs are beds and nurses, both of which are expensive and limited.

ICU nurses are required to provide careful assessment and monitoring of patient progress in order to watch for sudden or subtle changes in a patient's medical condition that might require emergency intervention. Besides, nurses take multiple roles in the ICU and patient care, for example, ICU administrative works and patient families support (Bisk, 2017).

As stated in Section 2.1, the recommended nurse-to-patient ratio is 1:1 and 1:2 for level 3 and level 2 patients respectively. Having fewer nurses at night is associated with increased risk for specific postoperative pulmonary complications and with increased resource use in patients undergoing a hepatectomy (Dimick et al., 2001). A nurse caring for more than two ICU patients at night increases the risk of several postoperative pulmonary and infectious complications and was associated with increased resource use in patients undergoing esophageal resection (Amaravadi et al., 2000). Increased nursing staffing helps to decrease the number of central line bloodstream infections in the ICU (Cimiotti et al., 2006). According to Halwani et al. (2006), an understaffed ICU can raise the chance for cross-transmission of a pathogen.

A simple queueing model proposed by McManus et al. (2004) shows that the performance is very sensitive to bed availability. The model is useful in determining the appropriate supply of beds.

The nursing resource consists of both number of the nursing staff available and their workload. Higher workload is proved to negatively impact the quality of service and eventually increase the cost of the system (Hoonakker et al., 2011). ICU nurse workload can be measured by two approaches: first, a patient-based approach considering patient characteristics and nurse-patient ratio, which is widely used in budgeting and measuring process improvements; second, an operator-based approach taking the experience of the nurses into consideration. The first approach is usually checklist-based (Miranda et al., 1996).

Baker et al. (2009) demonstrated that high patient inflow volumes to an ICU were associated

significantly with subsequent unplanned readmissions to the unit. Kolker (2009) built a simulation model to find out the recommended maximum number of elective surgeries in order to reduce the diversion of admissions from an ICU with fixed number of beds. The model suggested the number of scheduled elective surgeries should differ by days of a week and weeks of a year.

Véricourt and Jennings (2011) model the workload experienced by nurses of a single medical unit with n homogeneous patients as a closed M/M/s//n queue. The model was extended as one with general service time and non-homogeneous patients. The many-server asymptotic results showed that effective staffing policies should deviate from threshold-specific nurse-to-patient ratios. The model comprehensively considers variability and congestion in healthcare units and patients' needs in terms of their nursing time. The model is useful in not only determining nurse staffing rules but also indicating patients' outcomes.

ICU admission and discharge decisions influence the efficiency of ICUs in the reviewed research. Multiple research also confirms that the resource level of an ICU is critical to the effectiveness of the service. It will, therefore, be interesting to know if changes in admission and discharge policies and variations resource levels are key factors in the performance of the ICU we investigate. Moreover, whether the results have a potential to be generalised?

Pandemic preparedness of ICU

ICUs are often involved in the treatment of influenza pandemics (Challen et al., 2007; Nap et al., 2008; ANZIC Influenza Investigators, 2009; Kumar et al., 2009; Carr et al., 2010). The US Centers for Disease Control and Prevention (CDC) define an epidemic as "an increase, often sudden, in the number of cases of a disease above what is normally expected in that population in that area" and pandemic as "an epidemic that has spread over several countries or continents, usually affecting a large number of people" (CDC, 2012).

Two key concepts need to be specified. First, "symptomatic patients" means patients showing symptoms of disease (DH Pandemic Influenza Preparedness Team, 2011). Second, "at-

tack rate", also known as clinical attack rate, is the cumulative proportion of symptomatic patients over a specified period of time where

$$\text{attack rate} = \frac{\text{number of new cases of disease during specified time interval}}{\text{population at start of time interval}}.$$

CDC designed a spreadsheet-based simulation tool for ICUs and hospitals to prepare for influenza pandemic (CDC, 2016). The tool estimates the possible requirement of hospital beds and ICU beds. Menon et al. (2005) utilised the tool to model the impact of an influenza pandemic on critical care services in England. They concluded that current critical care facilities were far from adequate to cope with surges in demand. Careful planning in advance is needed for such situations.

To the best of our knowledge, influenza pandemic simulation models reported are all Monte-Carlo simulation models. No detailed information on performance during a pandemic period could be obtained from such models. It is of interests to BRI ICU management and to our research to know how extreme conditions (i.e. pandemics) would affect the ICU.

3.5.3 Simulation models of ICUs

Simulation methods have gained increasing popularity in past decades (Brailsford et al., 2009). In the healthcare sector, simulation modelling is commonly used in healthcare operations, healthcare system design, medical decision-making applications, infectious disease modelling and extreme events planning (Mielczarek and Uziałko-Mydlikowska, 2012). Discrete Event Simulation (DES) is one of the most widely used simulation tool in the past years (Salleh et al., 2017). DES models built for healthcare services are ususally unit and/or facility specific (Günal and Pidd, 2010).

Simulation is useful in critical care modelling as the method captures interactions in a relatively complex system where statistical modelling has difficulties (Kreke et al., 2004). In particular, DES models have great flexibility for testing different scenarios (Costa et al., 2003). ICU DES models have usually been built to aid resource planning, performance eval-

uation and decision making (Dong et al., 2012). We did a search in Web of Science Core Collection using the search, (("critical care" OR "intensive care" OR "ICU" OR "ITU") AND "simulation"), and filtered it to the area of "operations research management science". Only journals and conference proceedings are considered. Then, papers from irrelevant research areas are excluded, e.g. papers where ITU stands for International Telecommunication Union and using simulation to research ICU reimbursement policies. There were 32 papers resulting in total, including 24 journal articles and 8 conference proceedings. We note, however, this search strategy may have missed papers in critical care medicine and surgery.

The literature can be categorised into four groups:

1. papers focusing mainly on ICUs (Romanin-Jacur and Facchin, 1987; Ridge et al., 1998; Kim et al., 1999; Seung-Chul et al., 2000; Harper and Shahani, 2002; Griffiths et al., 2005; Litvak et al., 2008; Griffiths et al., 2010; Marmor et al., 2011; Barnes et al., 2011; Bountourelis et al., 2011; Gupta et al., 2013; Fournier and Zaric, 2013; Mallor and Azcárate, 2014; Kim et al., 2014; Mallor et al., 2015; Azadeh et al., 2016; Mallor et al., 2016; Hu et al., 2018)

2. papers aiming at modelling the whole hospital (Cochran and Roche, 2008; Williams et al., 2010; Helbig et al., 2015; Mancheva and Dugdale, 2015)

3. papers investigating other department of hospitals but heavily interacting with ICUs (Kim and Horowitz, 2002; Ng and Chick, 2004; Price et al., 2011; Adan et al., 2011; Neyshabouri and Berg, 2017)

4. papers introducing methodologies and using the ICU as an example (Kaplan et al., 2007; Sachdeva et al., 2007).

To better understand current research, papers are categorised according to their planning levels. High level planning, i.e. at a strategic level, includes regional planning, hospital planning and planning of a new ICU. Tactical level planning of an ICU includes capacity and resource planning. Research at an operational level includes details of bed or patient, including admission, discharge and scheduling

Strategic planning papers, which consider regional cooperation (Litvak et al., 2008; Fournier and Zaric, 2013) and planning a new ICU planning (Romanin-Jacur and Facchin, 1987), will not be discussed in detail here since our project is not concerned with such strategic plans. We will focus on tactical and operational modelling of ICUs in this section.

At a tactical level, Agent Based Simulation (ABS) is used to test communication protocols between different hospital departments (Mancheva and Dugdale, 2015) and to compare infection control methods in an ICU (Barnes et al., 2011). These two papers focus on interaction between agents, i.e. people, in both cases. This does not particular apply to our situation.

DES is widely adopted to investigate problems related to ICUs in all three planning levels. This method is used to test different policies and optimise ICU planning and operations. As this is relevant to our case study, we will continue to discuss such research.

Several pieces of research investigate how resource level affects ICU effectiveness and efficiency.

The relationship between bed occupancy and refusal or transfer of patients is complex. A simple rule of refusal with high bed occupancy is used when planning beds in the research of Ridge et al. (1998) while Harper and Shahani (2002) conclude that both bed occupancy rate and refused admission rate need taking into consideration when managers are allocating beds since the relationship between them is often overlooked. Griffiths et al. (2005) used a DES model to estimate the need for supplementary nurses during the busy period of a UK ICU. Creating a shared pool of ICU and HDU nurses could be the most effective solution. The hospital they investigated combined the two units before the publishing of their work. However, the DES model is not applicable to the new mixed-ICU. Cochran and Roche (2008) designed a capacity planning tool for four inpatient departments including an ICU in a US hospital using financial data and billing data. They claim that these two types of data are more reliable than census data in modelling bed demand. Troy and Rosenberg (2009) built a DES model to determine the need for ICU beds for surgical patients for a hospital in Canada, by controlling wait time and cancellation of operative procedures. The result required an increase of 2-4 staffed ICU beds to meet current demands. The authors also reinforced the

view that simulation is valuable in addressing patient flow. Marmor et al. (2011) develop a simulation model to support bed planning of cardiovascular surgical patients, in which ICU and SDU beds requirements are two of the three main parts of the model. The tool is used to test different surgery schedules. They summarised that "ensuring the correct number of beds are available and staffed is an important decision". They proposed a smoothed schedule (i.e. perform Saturday morning surgeries) to reduce variability in the daily schedule and therefore to smooth the demand for the ICU and SDU beds. Barado et al. (2012) built a DES model to simulate daily bed occupancy in a Spanish hospital. They claimed that the model can be used to predict bed requirements when case-mix is changing, by considering demographic variables for input distributions. The authors also believe that the model has the potential to be promoted to other units and hospitals. Zhu et al. (2012) estimated the required ICU beds for an hospital in Singapore, considering both service quality and cost-effectiveness. They have shown that a DES model captures the variations and interactions in this case. The model also made scenario tests straightforward.

Several papers investigate what improvements in efficiency can be achieved by applying different management policies, e.g. admission and discharge policies, for patients transferred into and out from an ICU.

Griffiths et al. (2010) built an Excel-based simulation model to minimise elective surgery cancellation by considering current bed occupancy level. Several 'what-if' scenarios with different policies were considered. Adan et al. (2011) take a two-stage planning procedure to schedule elective surgical patients: first, utilise goal programming to achieve a target level of resource utilisation in the ICU; second, adjust the daily schedule of elective patients for the operating room and also incorporate rules to decide admission of emergency patients and cancellation of scheduled patients. A DES model is used to investigate how hospital efficiency and patient service influence each other. Mallor and Azcárate (2014) consider management decisions, i.e. premature discharge or extended stay based on ICU occupancy, taken by clinical staff to better describe operations in a Spanish ICU. They claim that a simulation model without management decisions cannot be regarded as valid. They utilise simulation-based optimisation to estimate model parameters. Mallor et al. (2016) uses simulation-

based optimisation to determine optimal admission and discharge decision for a Spanish ICU by minimising both new patient rejection and LoS shortening of current patients if early discharge is required. Mahmoudian-Dehkordi and Sadat (2017) utilised a simulation model, specifically a system dynamics model, to evaluate ICU management policies in an Iranian hospital. They found that general ward admission was not a key reason for ICU congestion; other ICU management policies were possibly the reason for congestion. Neyshabouri and Berg (2017) use DES to test solutions of an robust optimisation approach to elective surgery scheduling based on capacity of downstream units, i.e. post-anaesthesia care unit (PACU) and ICU.

It is evident that DES models are useful in modelling various ICU operations and the method is also applicable in our case of a mixed ICU. Although simulation is widely used, none of the above literature investigated an existing mixed-ICU or late admission problems. However, the features of diversity and flexibility of DES modelling makes it suitable for modelling a mixed-ICU. Scenario tests based on a DES model for admission, discharge and resource levels can reveal the influences of different ICU operation policies. Testing influenza pandemic scenarios based on a DES model can provide a more detailed picture of the effect of a pandemic. The understanding of performance variations may contribute to ICU management in planning for and managing changing situations and resources.

DES has also been widely applied to tactical and operational problems in ICUs. Resource utilisation and ICU management policies are discussed in multiple papers. Questions relevant to our case study will be investigated in this research:

- How does resource level affect ICU effectiveness and efficiency?
- What improvements in efficiency can be achieved by applying different admission and discharge policies for patients in the ICU?

3.6 Combining data mining and simulation

Data mining tools are widely applied in the healthcare industry as the industry itself is typically 'information rich' yet 'knowledge poor' (Kaur and Wasan, 2006). Data availability in the healthcare industry is high, while the quality of data may not be high and the inter-relationships of data are largely unknown. Data mining techniques haves been implemented in disease prediction, treatment effectiveness, healthcare management, customer relationship management and fraud and abuse (Koh and Tan, 2011).

Data mining methods also have great potential to add value to a simulation model, as suggested by Ceglowski et al. (2007), who combined data mining methods with DES. They used a clustering method to categorise patients into treatment groups and then built a treatment-based DES model to simulate a hospital ED in Australia. Average LoS of patients from different categories were found to differ, which also impacted on the queueing time.

Glowacka et al. (2009) utilised a hybrid data mining / simulation model technique to model outpatient no-show problems in the US. Rule mining is used to predict patient no-show. Simulation is used to find the optimal number of scheduled patients.

Elbattah (2018) discussed how machine learning methods can be integrated in modelling practice in particular with applications in healthcare. He discussed the use of unsupervised data mining methods in pattern identification with a simulation model to represent flows. Supervised data mining methods are adopted to predict outcomes for individuals. He also discussed the possible applications based on an Irish hospital but did not carried out detailed modelling.

To the best of our knowledge, hybrid models combining data mining and simulation have not been well studied compared to pure simulation methods, while ICU models combining simulation and optimisation are quite common in reviewed literature (Adan et al., 2011; Price et al., 2011; Mallor and Azcárate, 2014; Mallor et al., 2015, 2016; Neyshabouri and Berg, 2017). We did not find specific literature discussing ICU simulation models integrated with data mining methods.

Data mining models incorporated in the simulation models are generally not very sophisticated but informative. Referring to Section 3.3, models such as ANNs and random forests could be useful but these models require huge amounts of input data and calibration. The final model could also be composed of many variables, which require input data when running simulation model. The constraints in data may cause difficulties in running DES models. Therefore, it is essential to keep the data mining model effective but concise. There could be multiple ways of connecting data mining models and ICU simulation models. It will be useful for modellers if general principles for hybrid models are investigated.

3.7 Conclusion

From our literature survey, we find that ICU-related problems have gained much attention from both practitioners and academia. However, gaps in the literature still exist. The available literature concentrates on single level ICUs rather than mixed ICUs. Patient stays are divided into either ICU stays or HDU stays and there is no need to consider changes in level of care. Therefore, no well-established method exists for the study of mixed ICUs. The BRI Adult ITU, a representative of many other ICUs in the UK, is a mixed ICU. According to data from Health and Social Care Information Centre (2013; 2014; 2015; 2016), more than 67% of ICU beds are used as flexible beds (both level 2 and level 3) in England. In our case, we do not distinguish between the different lengths of stay in ICU and HDU. The LoS in the unit is treated as a whole.

Delayed admission problems have been considered in some medical papers (Goldhill et al., 2004; Chalfin et al., 2007; Rincon et al., 2010). The medical effects of late admission have also been recognised by Restrepo et al. (2010); Renaud et al. (2012); Jhanji et al. (2008). Chan et al. (2016) examine problems caused by a single hospital department, i.e. ED. However, there is still a lack of comprehensive research into lateness in ICUs. An examination of admissions to the ICU from all wards of a hospital is missing. Moreover, none of the existing ICU simulation models consider the effect of earlier admissions of patients to the ICU.

We will analyse the consequences of late admissions with descriptive statistics and multivariate analysis in Chapter 4. Then, in Chapter 5 we customise the prediction of mortality and ICU LoS to our case study. We can conclude from the papers that a number of variables may contribute to both. Therefore, including them into prediction investigation would be interesting. Investigating the influence of late admissions and other questions using a comprehensively built DES model is what we are particularly interested in. In Chapter 6, the process of developing a DES model is charted in detail and all aspects of the model are tested. Then, we employ the DES model to test other scenarios in Chapter 7, including late admissions, changes of admission and discharge policies, variations of resource levels and the detailed influence of pandemics.

Chapter 4

Preliminary Analysis

To understand the consequences of late admissions and the current situation of the ICU, a descriptive analysis of current data will help. The literature suggests that late admissions may lead to worse outcomes in the ICU. Descriptive statistics and multivariate analysis help to understand what the structure of the data is, what population is included, what potential inferences we may get. Analysing data of interest is the first key step of our modelling.

In this chapter, details are given of analysis of data from the BRI adult ICU according to aspects highlighted in Chapters 2 and 3: readmitted patients, admission sources, mortality rates, LoS, timing effects of admission and discharge and late admissions. The literature has differing conclusions on timing effects and, therefore, we would like to investigate this for our case study.

Details are given firstly of data processing with grouping according to readmission, in Section 4.1. An overview is provided on the processed data including mortality rates and LoS for first admissions and readmissions. Admission sources are also described and mortality rates and LoS by admission sources are also analysed. A detailed analysis of late admission in the ICU is then provided with analyses by sources of patients in Section 4.5. Next, readmission is considered in Section 4.3. Moreover, timing effects are investigated in Section 4.4. Finally, busyness measurements are introduced in Section 4.6.

4.1 Data processing

This analysis is based on the currently available dataset, BRI 2008-2013 ICNARC data, which contains 6284 admission records in total. The data involves three key aspects of information: first, operational information of patients, ICU admission time and discharge time; second, medical information of patients, medical scores and patients' states; third, basic biological information, age and gender.

4.1.1 Data cleaning

In this section, a process of data cleaning is described. An indicator of ICU patients acuity (PA) will be introduced in Section 4.6.1. As PA is based on bed occupancy and patients' levels, the values of the first ten days are significantly influenced by arrivals in preceding days. To exclude the unstable period of PA, only admissions taken after 11-Jan-2008 and discharged before 10-Nov-2013 are considered. Admissions out of this period were removed (n=106). Also, records of admissions need to be complete. Patients with either no ICU outcome or hospital outcome were excluded from the analysis (n=39). According to ICU consultants in BRI, patients discharged alive from the ICU can only be discharged after eight hours of admission. Records of patients with less than eight hours stay and discharged alive were therefore excluded from the analysis (n=108). Then, we double checked readmission data and removed unmatched data i.e. where related first / subsequent admissions had been removed in previous steps (n=5). We also found a patient with two admissions, with the same hospital admission time but different hospital discharge times and different hospital outcomes. These two admissions were then removed (n=2). Also, unusual outliers were omitted in order to have a robust result (n=2). For example, when we were considering the time lags between hospital admission and ICU admission, there was one admission with an exactly nine-year stay which was thought to be a typing error, typing 2000 instead of 2009.

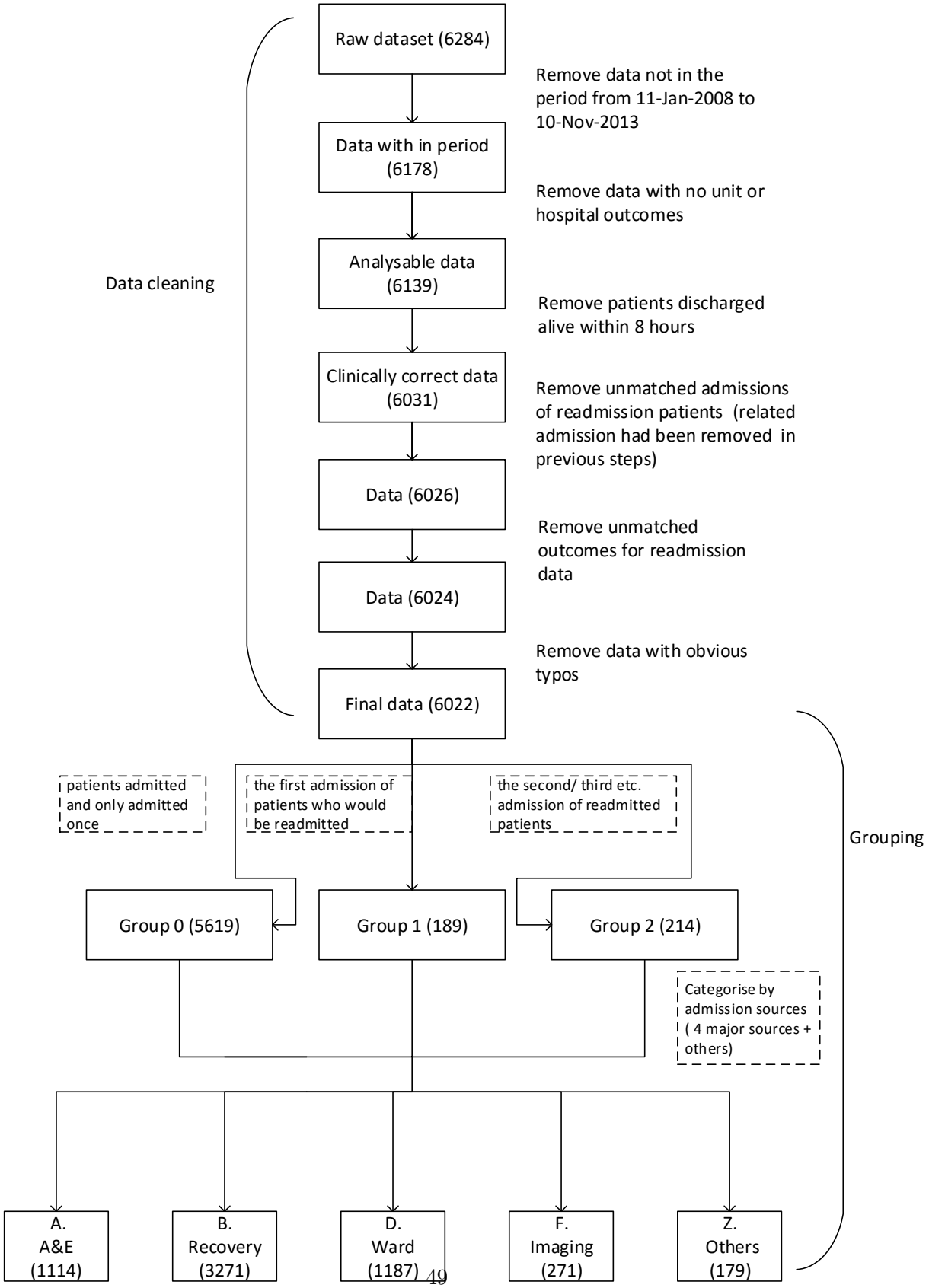


Figure 4.1: Data cleaning and grouping

Data cleaning and grouping is summarised in Figure 4.1. There are 6284 pieces of data in the dataset. 6178 pieces of admissions lay within the desired period, 6139 with both hospital and unit outcomes, 6031 with normal discharge. Unmatched readmissions were excluded from the dataset, after which 6024 pieces of data were remaining. We then removed data with obvious typing errors. 6022 out of the original 6284 pieces of data were kept for further analysis, in which there are 5808 distinct patients.

4.1.2 Grouping by admission and readmission

Before analysing the data, we categorise the data by three admission groups: "group 0" is made up of admission records of patients who were admitted only once to the ICU; "group 1" contains first admissions of patients who were admitted more than once; "group 2" contains second/third etc. admissions of all readmitted patients.

As shown in Figure 4.1, 5619 patient admission records are in group 0; 189 patient admissions are in group 1 and 214 admissions are in group 2. All the patients appearing in group 1 must appear one or more times in group 2 also since multiple readmissions are possible.

4.2 Data overview

4.2.1 Demographic and clinical characteristics of patients

Demographic and clinical descriptions of ICU admissions are summarised in Tables 4.1 and 4.2. Table 4.1 describes all the admissions. Table 4.2 describes the first admission of all the patients. "Hospital survivor" and "Hospital non-survivor" in this table mean patients discharged alive and dead from the final hospital attended, respectively.

Table 4.1: Demographic and clinical characteristics of ICU admissions

	Total	ICU survivor	ICU non-survivor
Admission, count	6022	5164	858
Age, mean (sd)	60.24 (16.95)	59.46 (17.05)	64.94 (15.55)
Actual ICU LoS in minutes mean (sd)	6898.98 (9227.36)	6952.12 (9210.26)	6579.12 (9328.69)
Actual ICU LoS in minutes median (25%-75%)	3960.00 (2050.75-7410.00)	4020.00 (2250.00-7415.25)	3569.5 (1290.00-7400.00)
ICNARC score, mean (sd)	17.15 (9.39)	14.93 (7.25)	30.51 (9.68)
ICNARC probability mean (sd)	20.85 (26.72)	14.05 (19.48)	61.78 (27.84)
APACHE II score mean (sd)	9.87 (8.76)	9.33 (8.02)	13.16 (11.78)
APACHE II mortality mean (sd)	11.89 (18.60)	9.24 (14.82)	27.79 (28.51)
ICNARC diagnosis system (Surgical)	Count	Count	Count
Total for surgical	3549	3311	238
Respiratory	689	647	42
Cardiovascular	366	322	44
Gastrointestinal	2007	1880	127
Neurological	101	100	1
Trauma	80	71	9
Genito-urinary	180	176	4
Endocrine, Metabolic, Thermoregulation	55	55	0

Continued on next page

Table 4.1 – continued from previous page

	Total	ICU survivor	ICU non-survivor
Haematological/ Immunological	6	3	3
Musculoskeletal	49	44	5
Dermatological	16	13	3
ICNARC diagnosis system (Medical)	Count	Count	Count
Total for medical	2473	1853	620
Respiratory	644	509	135
Cardiovascular	9	7	2
Gastrointestinal	6	6	0
Neurological	1	1	0
Trauma	422	292	130
Poisoning	175	127	48
Genito-urinary	612	377	235
Endocrine, Metabolic, Thermoregulation	38	35	3
Haematological/ Immunological	191	184	7
Musculoskeletal	170	142	28
Dermatological	120	112	8
Psychiatric	85	61	24

Table 4.2: Demographic and clinical characteristics of ICU patients: first admissions

	Total	Hospital survivor	Hospital non-survivor
Patients, count	5808	4685	1123
Age, mean (sd)	60.18 (17.00)	58.97 (17.17)	65.24 (15.23)
Actual ICU LoS in minutes mean (sd)	6721.31 (9004.70)	6624.77 (8799.14)	7124.03 (9809.90)
Actual ICU LoS in minutes median (25%-75%)	3895 (1979.75-7227.00)	3894.00 (2148.00-7125.00)	3937.00 (2148.00-7125.00)
ICNARC score, mean (sd)	17.12 (9.44)	14.50 (7.06)	28.03 (10.26)
ICNARC probability mean (sd)	20.68 (26.84)	12.53 (18.12)	54.67 (30.40)
APACHE II score mean (sd)	10.23 (8.71)	9.53 (7.79)	13.14 (11.34)
APACHE II mortality mean (sd)	12.32 (18.80)	9.01 (14.26)	26.12 (27.28)
ICNARC diagnosis system (Surgical)	Count	Count	Count
Total for surgical	3432	3076	356
Respiratory	660	597	63
Cardiovascular	359	303	56
Gastrointestinal	1934	1742	192
Neurological	99	93	6
Trauma	76	65	11
Genito-urinary	179	169	10
Endocrine, Metabolic, Thermoregulation	54	53	1

Continued on next page

Table 4.2 – continued from previous page

	Total	Hospital survivor	Hospital non-survivor
Haematological/ Immunological	6	3	3
Musculoskeletal	49	41	8
Dermatological	16	10	6
ICNARC diagnosis system (Medical)	Count	Count	Count
Total for medical	2376	1609	767
Respiratory	586	411	175
Cardiovascular	404	246	158
Gastrointestinal	166	101	65
Neurological	606	335	271
Trauma	38	35	3
Poisoning	191	183	8
Genito-urinary	166	123	43
Endocrine, Metabolic, Thermoregulation	119	110	9
Haematological/ Immunological	84	52	32
Musculoskeletal	9	7	2
Dermatological	6	6	0
Psychiatric	1	0	1

4.2.2 Tests of significance

We would like to briefly introduced the statistical tests used test significance of differences.

Test of equal means (two groups)

It is assumed that $(X_1, \cdots, X_n)$ and $(Y_1, \cdots, Y_m)$ are two independent identically distributed samples with means $\bar{X}$ and $\bar{Y}$ and variances S_X^2 and S_Y^2. The samples are extracted from two populations following $\mathcal{N}(\mu_X, \sigma_X^2)$ and $\mathcal{N}(\mu_Y, \sigma_Y^2)$; a test of equal means ($\mu_X = \mu_Y$) is achieved by a Welch's t-test if $n \neq m$ or $\sigma_X^2 \neq \sigma_Y^2$. If $n = m$, a test of equal variances ($\sigma_X^2 = \sigma_Y^2$) is recommended before a t-test (Zimmerman, 2004).

$\bar{X}$, $\bar{Y}$, S_X^2 and S_Y^2 are calculated as following:

$$\bar{X} = \frac{\sum_{i=1}^{n}(X_i)}{n}, \qquad \bar{Y} = \frac{\sum_{i=1}^{n}(Y_i)}{m};$$
$$S_X^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})}{n-1}, \qquad S_Y^2 = \frac{\sum_{i=1}^{m}(Y_i - \bar{Y})}{m-1}.$$

A Welch's t-test is as described below.

1. Set null hypothesis $H_0^{(t)} : \mu_X = \mu_Y$;
2. Calculate t-statistic $t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_X^2}{n} + \frac{S_Y^2}{m}}}$;
3. Compute degrees of freedom $df = \left\lfloor \frac{(\frac{S_X^2}{n} + \frac{S_Y^2}{m})^2}{\frac{S_X^2/n}{n-1} + \frac{S_Y^2/m}{m-1}} \right\rfloor$;
4. Compare t with critical value; reject $H_0^{(t)}$ if $t >$ critical value.

Test of equal means (multiple groups)

One-way analysis of variance (ANOVA) is used to test the equality of means when there are more than two groups in a sample. Here are some notations used in ANOVA.

n_i : sample size of group i

n : sample size; $n = \sum_{i=1}^{k} n_i$

k : number of groups

Y_{ij}: j^{th} response variable from group i

$\bar{Y}_{i\bullet}$: the sample mean of response variable of group i; $\bar{Y}_{i\bullet} = \frac{1}{n_i}\sum_{j=1}^{n_i} Y_{ij}$

$\bar{Y}$: sample mean of all individuals; $\bar{Y} = \frac{1}{n}\sum_{ij} Y_{ij}$

Table 4.3: ANOVA table

	SS	df	MS
between groups	$\mathrm{SS}_b = \sum_{i=1}^{k} n_i(\bar{Y}_{i\bullet} - \bar{Y})^2$	$df_b = k-1$	$\mathrm{MS}_b = \mathrm{SS}_b/df_b$
within group	$\mathrm{SS}_w = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y}_{i\bullet})^2$	$df_w = n-k$	$\mathrm{MS}_w = \mathrm{SS}_w/df_w$
total	$\mathrm{SST} = \mathrm{SS}_b + \mathrm{SS}_w$ $= \sum_{i=1}^{k}\sum_{j=1}^{n_i}(Y_{ij} - \bar{Y})^2$	$n-1$	

The F-statistic is calculated based on the table, $F^{(A)} = MS_b/MS_w$. The null hypothesis of the test can be written as $H_0^{(A)} : \mu_1 = \mu_2 = \cdots = \mu_k$. If $F^{(A)} >$ critical value of F_{df_b,df_w}, the null hypothesis is rejected.

Multiple comparison between groups

The Tukey-Kramer method can be used to carry out multiple comparisons between groups with different sample sizes.

First, the overall standard deviation of the sample (including all groups) is calculated as

$$\hat{\sigma}_\epsilon = \sqrt{\frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{N_i}(Y_{ij} - \bar{Y})^2}$$

The confidence interval (CI) of the differences between group p and group q ($p, q = 1, \cdots, k$; $p \neq q$) depending on the significance level (α) is written as

$$CI_{pq} = \left[\bar{Y}_{p\bullet} - \bar{Y}_{q\bullet} - \frac{q_{\alpha;k;n-k}}{\sqrt{2}}\hat{\sigma}_\epsilon\sqrt{\frac{1}{N_i} + \frac{1}{N_j}}, \bar{Y}_{p\bullet} - \bar{Y}_{q\bullet} + \frac{q_{\alpha;k;n-k}}{\sqrt{2}}\hat{\sigma}_\epsilon\sqrt{\frac{1}{N_i} + \frac{1}{N_j}}\right]$$

If the CI does not include zero, means of group p and group q are said to have a significant

difference at α.

Test of differences in crude ratios

A contingency table with r rows and c columns may presents frequencies of events (columns) in each category (row). A Chi-squared test is used to test differences between crude ratios in the contingency table.

The test statistic (χ^2) is calculated as:

$$\chi^2 = \sum_{j=1}^{r} \sum_{i=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

where

O_{ij}: observed numbers of event i for sample j

E_{ij}: expected numbers of event i for sample j

Yate's correction is applied when $df = 1$. Then,

$$\chi^2 = \sum_{j=1}^{r} \sum_{i=1}^{c} \frac{(|O_{ij} - E_{ij}| - 0.5)^2}{E_{ij}}$$

Compare the χ^2 statistics with critical value; if χ^2 >critical value, reject the null hypothesis of no difference.

4.2.3 Mortality rate

Table 4.4 shows a comparison of mortality rates of patients admitted once only (group 0) and those admitted more than once (group 1). The mortality rates were calculated based on the hospital outcomes rather than unit outcomes.

Table 4.4: Mortality Rate Comparison

Admission Group	0	1
Mortality Rate	18.36%	21.58%
Lower 95% CI	17.85%	18.58%
Higher 95% CI	18.87%	24.58%

Group 2 was excluded from the mortality rate calculation as they were included in group 1 for their first admissions. The mortality rate of readmitted patients is higher than that of patients admitted only once. However, there is overlap between the 95% CI of the two mortality rates. The difference between them is not significant according to a Chi-squared test. The UK nation-wide statistics have shown a larger discrepancy between these two mortality rates (Jhanji et al., 2008).

4.2.4 Length of stay

Statistics are presented in Figure 4.2 regarding LoS in the ICU, where LoS is measured in minutes. For patients discharged alive, the ICU LoS was calculated as "time when clinically ready to be discharged −ICU admission time". For patients who died in the ICU, the ICU LoS was calculated as "time of death −ICU admission time". The analysis of LoS considers all admissions including readmissions.

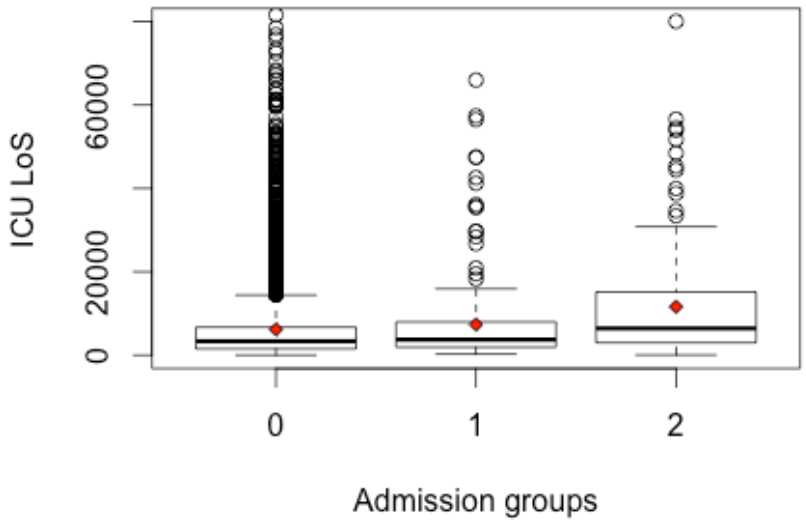


Figure 4.2: ICU LoS (minutes) based on different admission groups

The red diamonds in Figure 4.2 show the average ICU LoS of different groups of patients as defined in Section 4.1.2. Compared to group 0 and 1 admissions, group 2 admissions have a significantly longer LoS according to t-test. That is to say, readmitted patients have longer stays in the ICU than first admission patients. Meanwhile, no significant difference between LoS of group 0 and group 1 can be detected from our data.

4.2.5 Admission sources

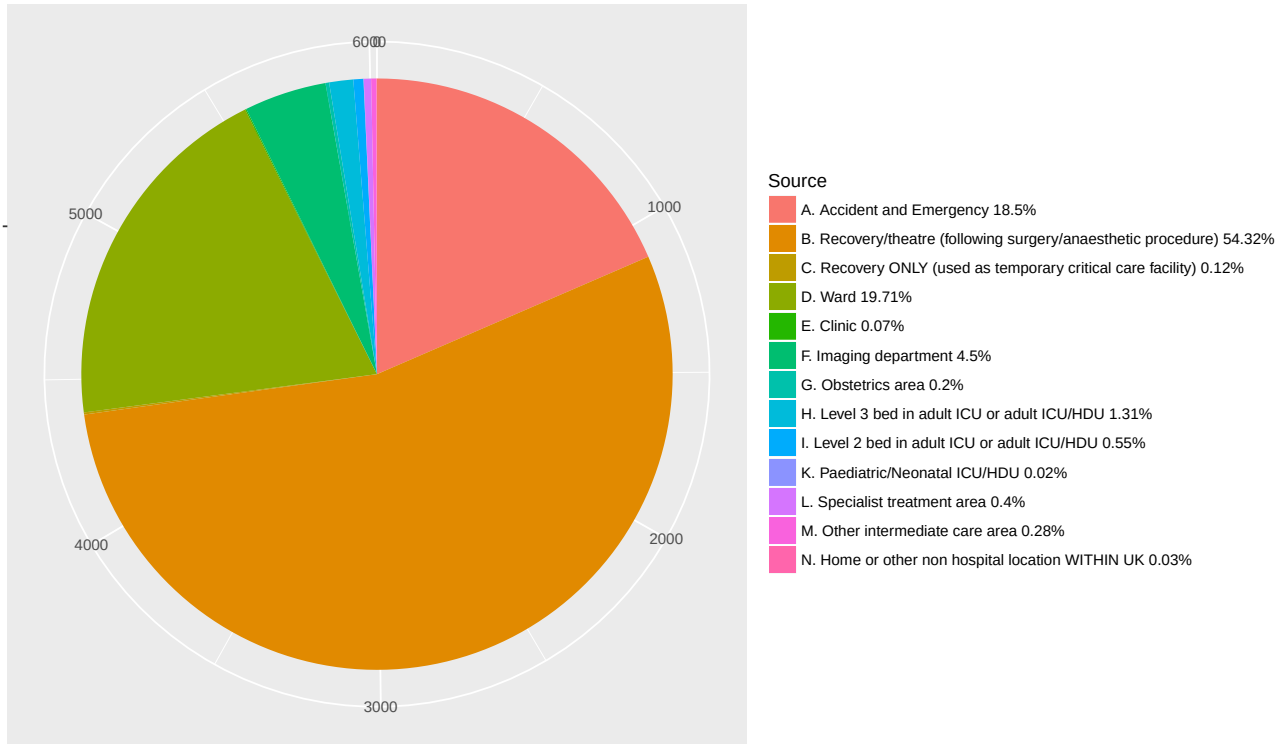
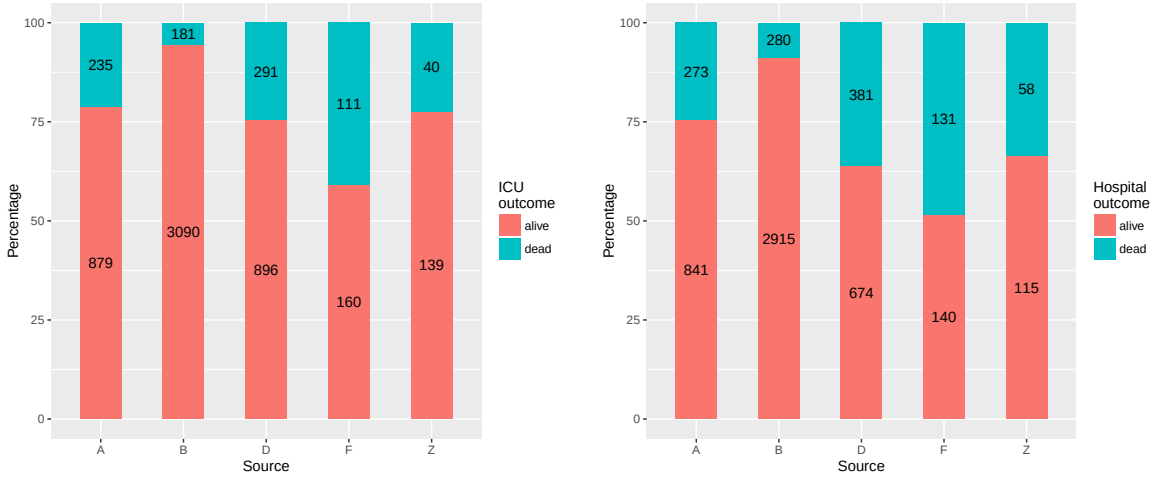


Figure 4.3: Analysis of admission sources

The pie chart in Figure 4.3 depicts the admission sources of patients. A. A&E, B. Recovery/theatre, D. Ward and F. Imaging department are the four main sources of ICU admissions, accounting for approximately 97% of ICU admissions.

In the following analysis referring to admission source, we categorise the data into five groups, the four main sources and the 'others' group.

Figure 4.4 shows mortality rates by admission source for the five main groups. Within these five groups, patients from the Imaging Department and Wards were threatened by higher probability of death while the mortality rate of patients from recovery/theatre is significantly lower than that of the other groups, according to t-tests.



(a) ICU mortality

(b) Hospital mortality

Figure 4.4: Mortality rate of different admission sources

We cannot distinguish in the data between medical and surgical ward patients. However, we know that patients coming from a surgical ward are usually admitted to the ICU via theatre/recovery. Patients coming directly from a surgical ward would be readmitted or late admitted patients. Subsequent admissions of readmitted patients are excluded from the mortality rate calculation to avoid double counting of the same patients. Patients from medical wards could be late admitted patients. The high mortality rate of patients from wards is possibly caused by late admission patients.

The mortality rate of the recovery/theatre group could be underestimated. Patients who were not admitted to the ICU directly after operations are not included in the calculation of this group. However, these patients were more likely to die according to the late admission analysis carried out later in Section 4.5.

We were also interested in LoS of patients from different sources. An one way analysis of variance (ANOVA) was conducted to observe the significance of the different mean LoS from the five re-categorised sources. The result rejects the hypothesis of equal means (p-value=2.2e-16), which means LoS of patients from different sources are significantly different.

Amongst the four main sources, A&E (A), recovery/theatre (B), ward (D) and imaging de-

partment (F), shown in Figure 4.5, D and F have a remarkably high ICU LoS. The relatively high average LoS of ICU patients admitted from the ward (D) and low in recovery/theatre (B) could be caused by the proportion of patients who were not admitted to the ICU directly after operations. These patients could not be identified as having come from recovery/theatre patients and were not included in the calculation of source B.

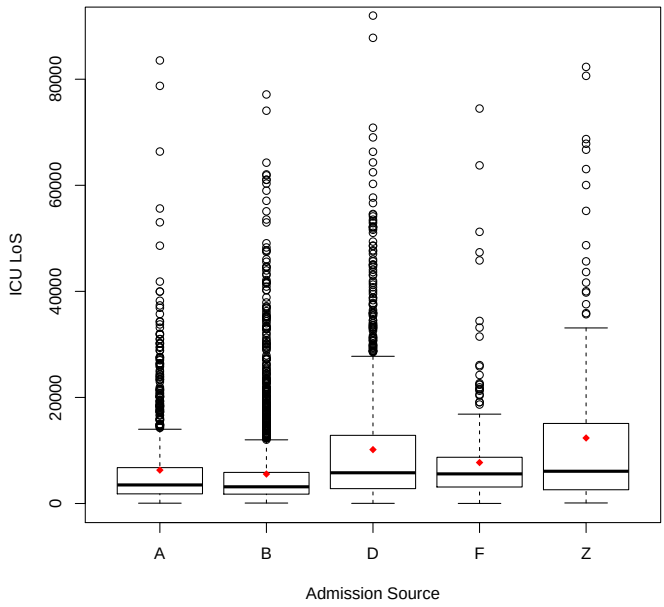


Figure 4.5: LoS of admissions from different sources

4.3 Readmission

In our research, readmission specifically means ICU readmission. It is defined as "the same person being admitted to your unit on two or more separate occasions, regardless of whether these admissions occurred during the same hospital stay" according to ICNARC Case Mix Programme team (ICNARC, 2013). Within the 214 readmissions of our data, 191 are admitted during the same hospitalisation. Readmission thus generally means return to the ICU after discharge to a hospital ward, rather than returning from home.

Compared to the national average of 4.7% readmission rate during same hospital stay, the overall readmission rate is 3.25% for the whole period in the BRI ICU. The key performance indicator of readmission, unplanned readmission within 48 hours of ICU discharge, is 0.6% in the ICU, which is much lower than the national average 1.4% (ICNARC, 2015c).

Table 4.5 shows where the readmissions come from. Patients with their first ICU admissions from source B (recovery/theatre) and D (ward) make a larger contribution to readmission than other sources. Also they are more likely to be readmitted to the ICU than patients admitted from all other sources. The subsequent admissions are mainly from source D and source B as shown in Table 4.6, which occupy 97.20% of all the readmissions.

Table 4.5: First admission sources of readmitted patients

Source	A	B	D	F	Z
Readmitted patients	13	131	38	0	7
Total patients	1114	3195	1055	271	173
Probability of readmission	1.17%	4.10%	3.60%	0.00%	4.05%

Table 4.6: Readmission sources

Readmission Sources	A	B	D	F	Z
Number of readmissions	0	76	132	0	6
Percentage of readmission from the source	0.00%	35.51%	61.68%	0.00%	2.80%

4.4 Timing effects analysis

4.4.1 Timing effects of admission day and time

We investigate the effects of admission day and time. As mentioned in several pieces of research, admission during weekends and night time can have a negative impact on patients' outcomes (mortality) (Neuraz et al., 2015; Bhonagiri et al., 2011).

To avoid double counting the result of a same patient, only the first visit was considered for patients with subsequent visits in analysing effects of admission day and time. There are

5808 patients in all, summing group 0 and group 1, see Figure 4.1.

Percentages of patients discharged from hospital alive or dead were calculated by times of admission to the ICU in hours and plotted in Figure 4.6. Using Pearson's Chi-squared test to examine the difference of mortality rates in different admission hours, we obtained $p-$value $< 2.2e-16$. The result indicates the possible timing effect of admission time.

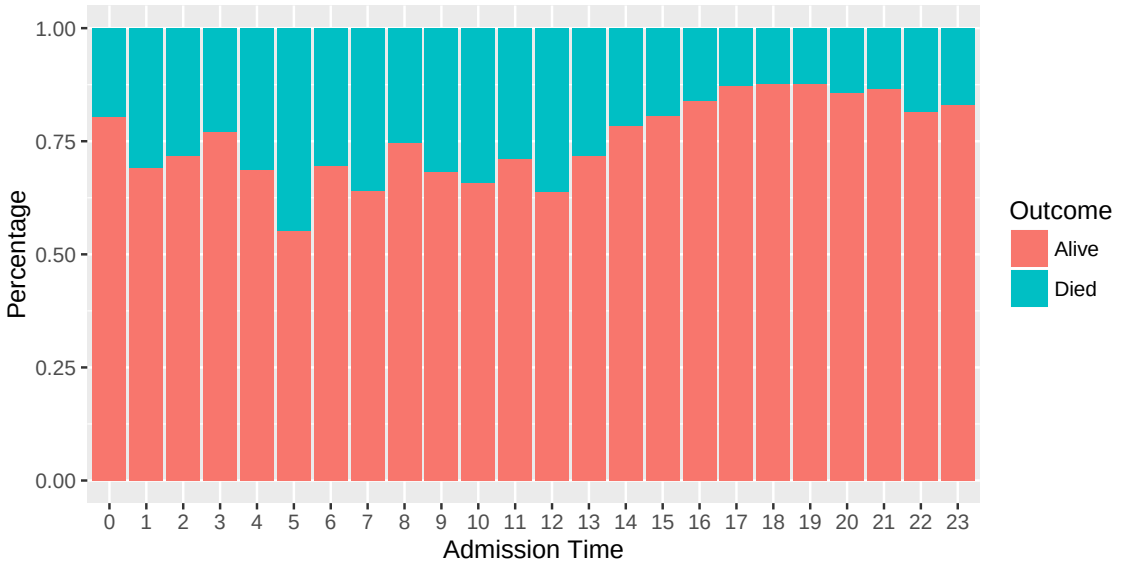


Figure 4.6: Timing effect of admission time on outcomes for all patients

Figure 4.6 shows a spike of mortality in patients admitted from 5am to 6am and a relatively low mortality rate in the afternoon and evening. The pattern could come from other confounding variables rather than the admission time. We may expect that surgeries are usually started from morning so afternoon and evening admissions could largely come from planned surgical admission and are less likely to be late admissions. We plot the figure to show the different combinations of surgery type, admission source and admission type of each hour's admissions. Figure 4.7 shows percentages of different surgery types for all patients by hour of ICU admission. "Not relevant" indicates medical patients, i.e. those not requiring surgery. "Emergency" and "Urgent" represent different degrees of urgency for performing surgery; these are "unplanned surgical patients". For "scheduled" and "elective" patients, surgery is planned in advance: relatively long for elective, short for scheduled; these are "planned

surgical patients". Figure 4.8 shows percentages of admissions by admission source and Figure 4.9 by admission type, by hour of the day. All the admissions falling in the categories 'unplanned local admission' and 'unplanned transfer in' are called 'unplanned admissions'; the other admissions are termed 'planned'.

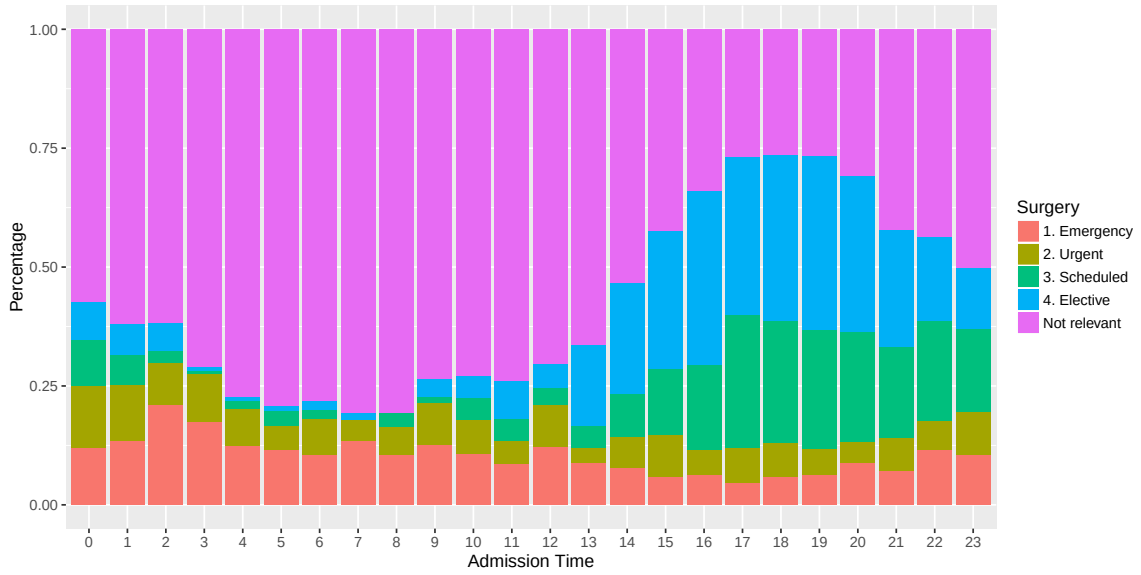


Figure 4.7: Surgery type of patients by different ICU admission time

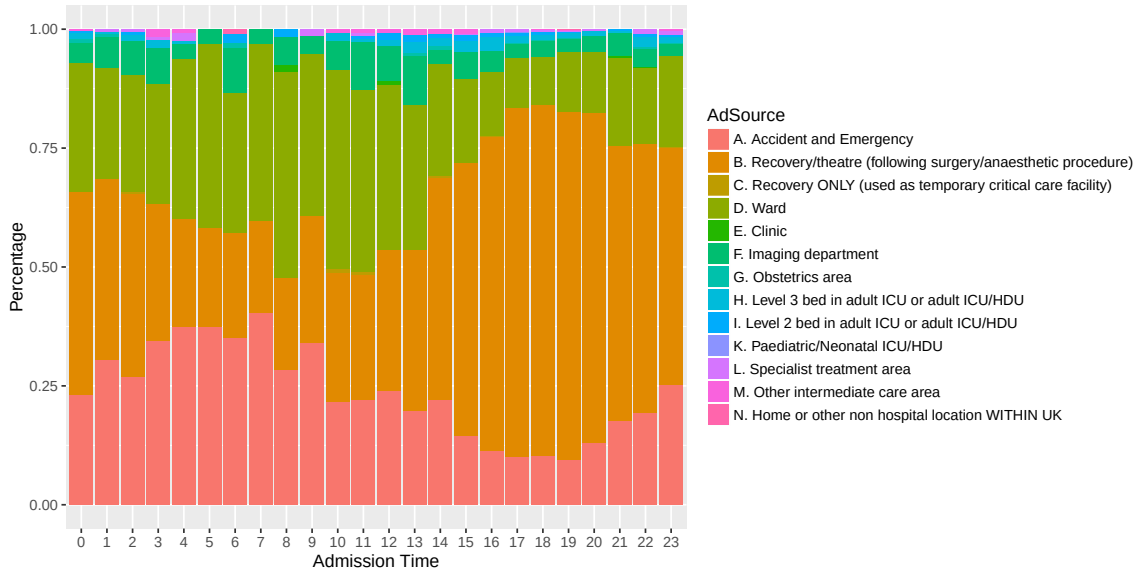


Figure 4.8: Admission source of patients by different admission time

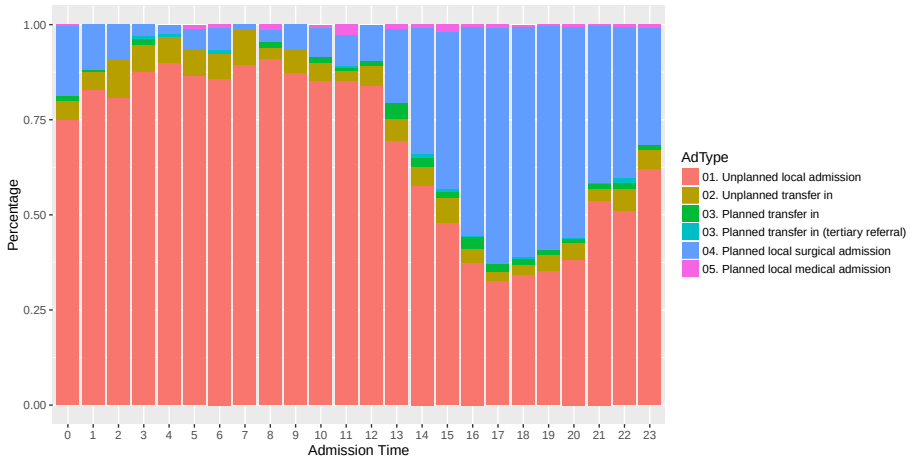


Figure 4.9: Admission type of patients by different admission time

As exhibited in Figures 4.7 and 4.9, admissions occurring at different hours are very different in composition, which may cause the different mortality rates for planned and unplanned patients.

ICNARC probability of death considers both patients' severity of illness and a large number of operational factors. In order to take the influence of these factors into consideration when analysing timing effect, ICNARC probability is included to adjust the odds ratio of mortality rate for different admission times.

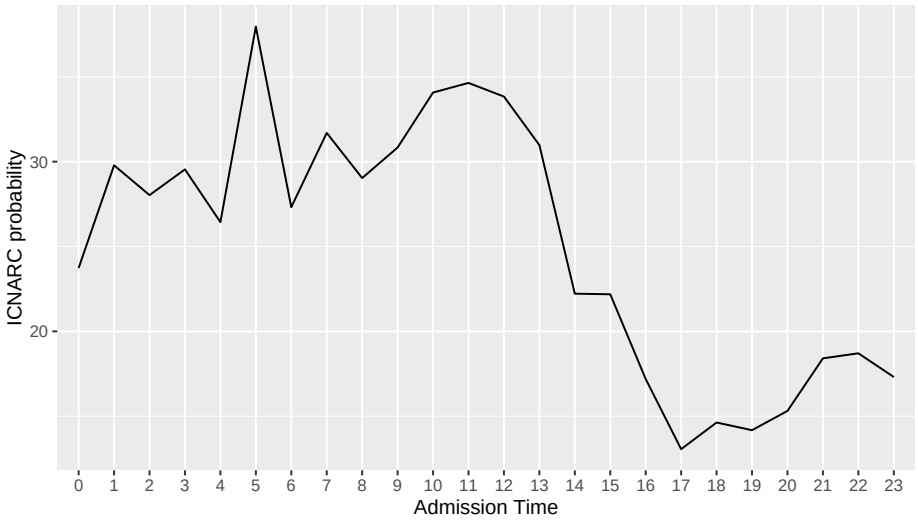


Figure 4.10: Average ICNARC probability of death of admissions in each hour

Figure 4.10 shows average ICNARC probability by hour of admission, which confirms that average ICNARC probabilities of patients vary by hour of the day. These differences suggest that ICNARC probability could be a strong predictor of mortality by admission hour. Hence, timing effect can only be confirmed after ICNARC probability adjustment.

A timing effect analysis was carried out for mortality in the ICU. To simplify the analysis in a practical manner, we categorise admissions firstly by ICU nurse shifts, and secondly by peak/non-peak times of day.

Table 4.7: Night effect examination

Admission time	Mortality rate	Odds ratio	Adjusted odds ratio
Day	0.1997	1	1
Night	0.1864	0.9331	0.9838

We categorised admission time in two groups in relation to the shift of ICU nurses, day admission (admissions between 7am and 8pm) and night admission. The adjusted odds ratio was calculated using ICNARC probability as a confounding variable. We used ICNARC probability and night effect as the two independent variables in a logistic regression. The log odds was calculated for night and day admission from the regression and then converted back to an odds ratio. We failed to prove the timing effect of admission hours, as shown in Table 4.7, with a Chi-squared test p-value of 0.85.

We also examine the difference in mortality rates of patients admitted during peak and non-peak admission times. First of all, Figure 4.11 was plotted to determine the peak admission period. All the admission data, including subsequent admissions of readmitted patients, were included in this analysis.

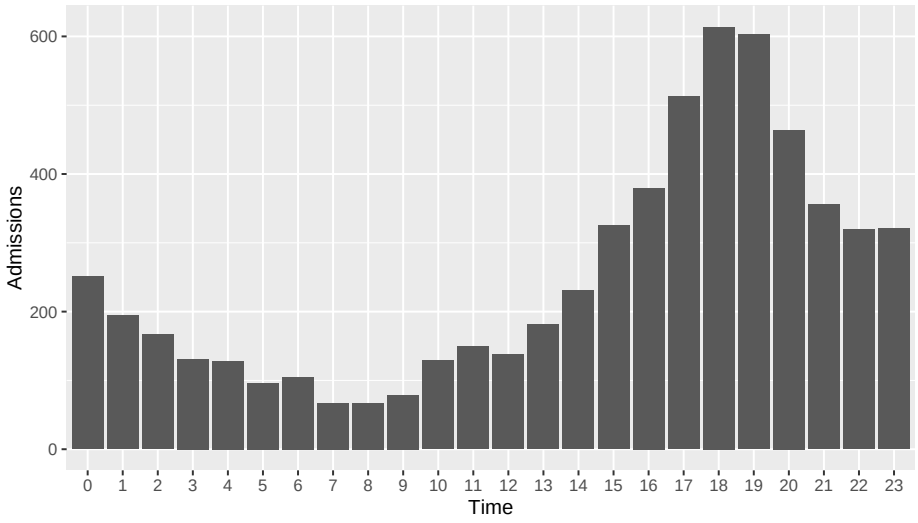


Figure 4.11: Number of admissions by hours

The time period, 2pm to 00:59am, can be observed to be the peak admission time according to Figure 4.11.

Table 4.8: Admission peak effect examination

Admission time	Mortality rate	Odds ratio	Adjusted odds ratio
non-peak	0.3110	1	1
peak	0.1498	0.4817	0.6636

As shown in Table 4.8, the adjusted odds ratio of mortality rate in peak time is 0.6636. The 95% CI of the odds ratio of adjusted peak time mortality rate is 0.5571-0.7916, with p-value$=4.7e-06$. Both of the results show that peak time admission has a significant influence on patients' medical outcome. Looking back to Figures 4.11 and 4.9, we find that the planned admissions account for most of the peak time admissions while the unplanned admissions account for most of other slots. Mortality rate is higher in the unplanned admissions. Moreover, after case mix adjustment by excluding the influence of ICNARC probability, the positive impact of admission peak on patients' mortality still exists. This specific timing effect has not been previously researched. We further research the effect in Section 5.1.2.

As well as time-of-day effect, day-of-week effect has also been discovered in worldwide research as discussed in Section 3.2. Figure 4.12 shows directly the day-of-week effect. We

first tested timing effects of admission days. The p-value of Pearson's Chi-square test equals 0.1872, meaning that we cannot reject the null hypothesis of the equality of mortality rates on different weekdays. Equivalently, no effect of admission day was discovered in this specific dataset. To be more conservative, we also calibrated the results by incorporating ICNARC probability; p-value still failed to be less than 0.1, which further confirms our results that no outstanding day of week effects could be found. Moreover, we also classed days into weekdays and weekends. The p-value obtained for weekend effect admission is 0.21. To sum up, no weekend or day-of-week effect could be discovered in our dataset.

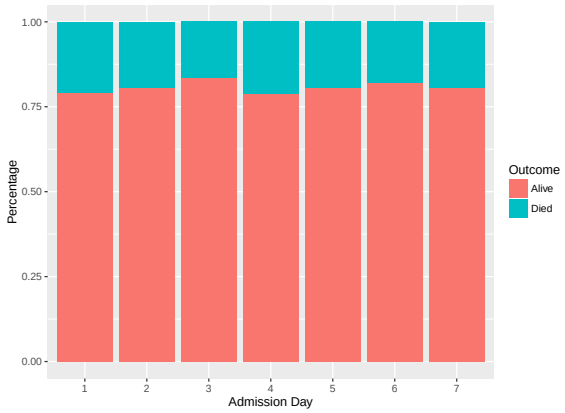


Figure 4.12: Day of week effect of admission day

4.4.2 Timing effects of discharge day and time

Only patients discharged alive from the ICU were considered in examining the timing effect of discharge.

In the dataset, the two different discharge times are stated. One is "clinically ready to discharge" time. The other one is actual discharge time.

In this section, the effect of these two different discharge times will be examined individually. The analysis procedures are exactly the same as those of timing effects analysis of admissions.

First of all, we plot the frequency chart of ICU discharge by hours, see Figure 4.13.

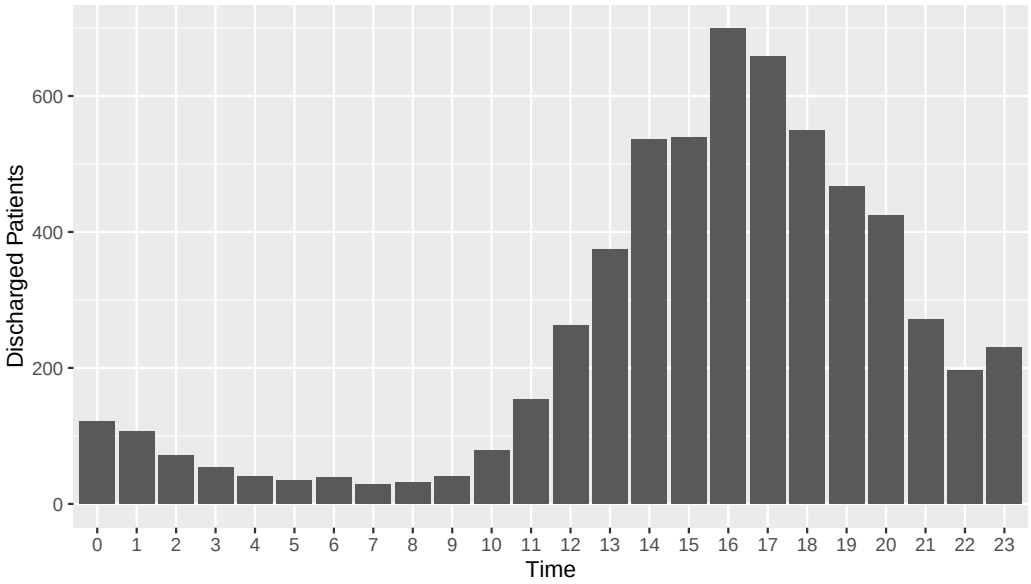


Figure 4.13: Number of discharges from the ICU by hour of the day

A day was divided into two parts in two different ways. First, a natural day was divided into day and night and then the night effect of discharge time was examined. Second, a natural day was divided into peak discharge time and non-peak discharge time. There is a clear peak for discharge, from 12:00 to 21:59. ICNARC probability was also introduced as a confounding variable. Night discharge has a negative impact on patients' mortality. However, the significance of this negative impact is not very strong (p-value=0.0520). There is no obvious peak time discharge timing effect on mortality based on an initial Chi-squared test (p-value=0.2164). No discharge timing effect has been found in any other situations we tested.

We are also interested in the effect of discharge decision timing. A frequency plot is provided in Figure 4.14.

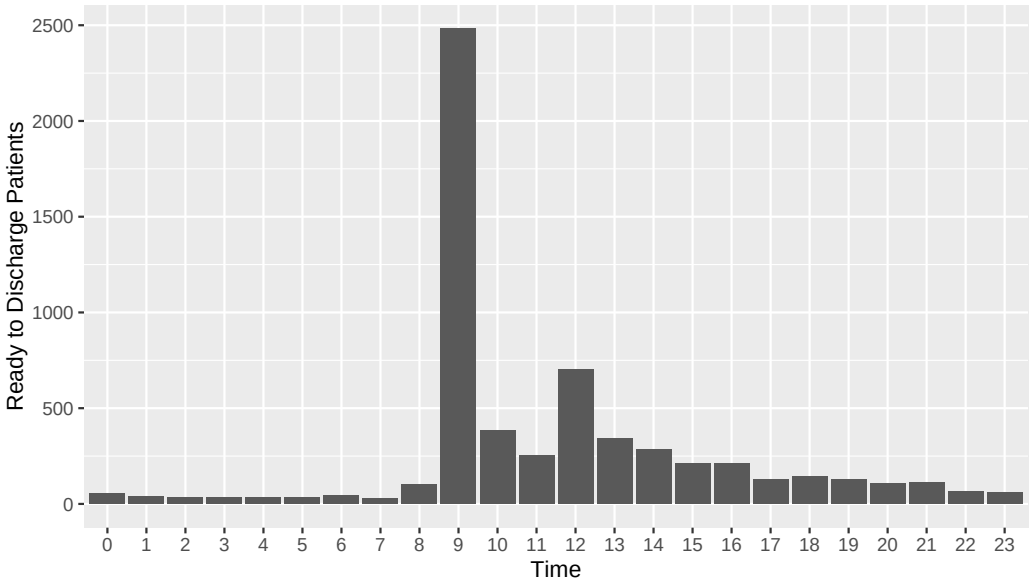


Figure 4.14: Number of discharge decisions by hours

It is clear from Figure 4.14 that the peak of making discharge decisions is: 9:00-14:59. Discharge decision timing was not found to affect patients' outcomes from all three perspectives, which are hour-based difference, day and night difference and peak and non-peak difference.

To sum up, no effect of either day-of-week, clinically ready to discharge time or actual discharge time can be observed in our data.

4.5 Late admission

An analysis follows to determine a suitable lag between hospital admission and ICU admission to define "late" admission. All the patients are included in the analysis but only their first time admissions were counted. Equivalently, only group 0 and group 1 are considered in this phase. Group 2 patients are excluded because readmitted patients naturally have relatively long lags between hospital admission and ICU admission.

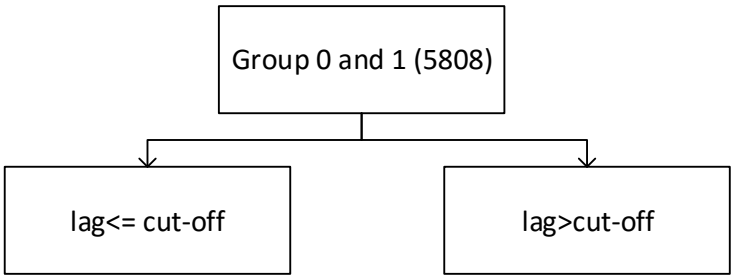


Figure 4.15: Data grouping: late admission

Lag trials were carried out to determine the lag to define late admission, see Figure 4.15. Because of the limitations of the data, lag was measured in days rather than hours or minutes. For each trial, the cut-off was set from 0 to 20 days. The cut-off divides the patients into two parts: first, patients with lags smaller than or equal to the cut-off; second, patients with lags greater than the cut-off. For example, cut-off=1 (lag=1) divides patients into two groups. One group is patients with admission lags equal to 0 or 1 day, which means this group of patients were admitted to the ICU on the same day or the day after the day that they were admitted to the hospital. We label the group as '$\leq$' group. The other group is patients with admission lags larger than 1, which means this group of patients were admitted to the ICU more than one day after the day they were admitted to the hospital. This group is labelled as '$>$' group. The influence of late admission is assessed by using a statistically significant lag cut-off, showing as 95% CIs that are not overlapped for '$\leq$' and '$>$' groups.

Late admission will be examined for different groups of patients: first, for all patients; second, for patients from operating theatres; third, for ward patients.

4.5.1 All patients

We put all the patients (all group 0 and 1 patients) together to carry out the first lag analysis. Firstly, Figure 4.16 shows the frequency distribution of lag days in the data.

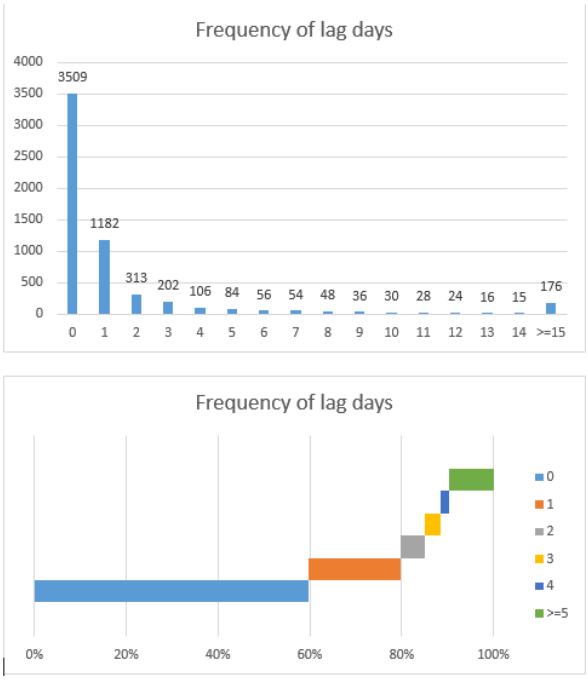


Figure 4.16: Number and percentage of patients by lag days

Figure 4.16 also demonstrates that around 40% of the ICU patients were not admitted to the ICU directly. Approximately 12% of all the admitted patients had been delayed for more than five days.

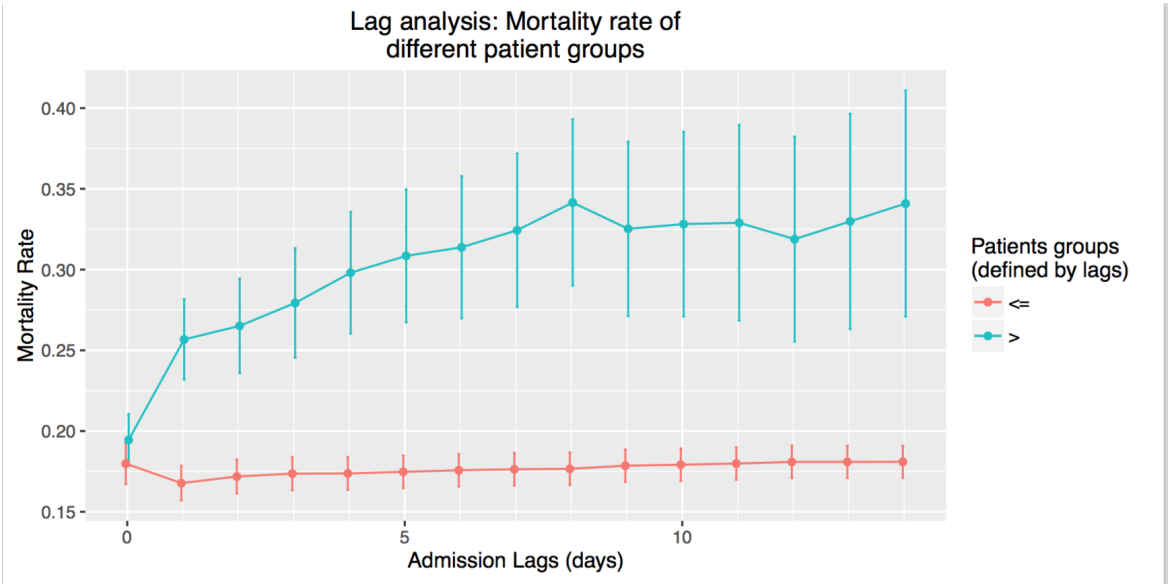


Figure 4.17: Mortality rates of patient groups of different lag trials

The study of all patients continues with analysis of mortality rates for the different patient groups defined by lags, as shown in Figure 4.17. Bars show the 95% CI of the mortality rate calculated from following approximations.

For each day of delayed admission, say n days, the "$\leq$" group contains patients admitted to the ICU within n days of hospital admission. The "$>$" group contains those admitted to the ICU more than n days of hospital admission.

It can be observed from Figure 4.17 that mortality rate undergoes a sharp increase for the patients having admission lags larger than one day. It is maintained at a more than 30% level for patients with admission lag days being more than four days. Then, it keeps going up with the increase of lags.

The actual distribution of mortality rate is assumed to be a Binomial distribution with p denoting mortality rate.

According to probability mass function (pmf) of binomial distribution,

$$P(Y = k) = \binom{N}{k} \cdot p^k (1-p)^{N-k}$$

p: the proportion of sample in a particular class

The random variable Y can be denoted as $Y \sim B(n, p)$

$$E[Y] = np$$

$$Var[Y] = np(1-p).$$

We can estimate an unbiased $\hat{p}$

$$\mu = E[\ \hat{p}\ ] = E[\frac{Y}{n}] = E[\frac{np}{n}] = p$$

and obtain the variance σ^2

$$\sigma^2 = Var[\frac{Y}{n}] = (\frac{1}{n})^2 Var[Y] = \frac{p(1-p)}{n}.$$

A common way to deal with the CI of p is to employ the Normal approximation to the Binomial distribution. If n is large enough, we can approximate $B(n, p)$ to the Normal distribution $\mathcal{N}(np,\ np(1-p))$.

The random variable $\hat{p}$ follows a distribution with expected value (μ) and variance(σ^2)

$$\hat{p} \sim [p, \frac{p(1-p)}{n}],$$

giving the CI of $\hat{p}$:

$$\left(p - z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}},\ p + z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}\right)$$

p: proportion of interest i.e. mortality rate

n: population size i.e. number of admissions

$z_{\frac{\alpha}{2}}$: critical value of Normal distribution for the given error level α ($z_{\frac{\alpha}{2}} \approx 1.96$ for $\alpha = 5\%$)

It can be observed from Figure 4.17 there is a drop in mortality for the "$\leq$" group between cut-off=0 and cut-off=1 which may result from patients who died shortly after their ICU admission. A relatively large number of patients (n=103) died shortly after their ICU admissions (within eight hours of admission). We recalculated the mortality rate of different patient groups after excluding patients dying within eight hours after admission. Table 4.9

demonstrates the full results of mortality rates of different patient groups. Row titles in the table show patient groups. Column titles in the table show the lags.

Table 4.9: Drop in mortality rate from Lag=0 to Lag=1

Group\Lag	0	0 (excl. death in 8hrs)	1	1 (excl. death in 8hrs)
$\leq$	18.87%	17.56%	17.59%	16.40%
$>$	19.97%	18.96%	26.14%	24.99%

Although patients dying within eight hours after admission increase the mortality rate of the patients whose admission lag equals to zero, Table 4.9 shows that the effect of the influence is not as great as expected since this group would appear in all "$\leq$" groups.

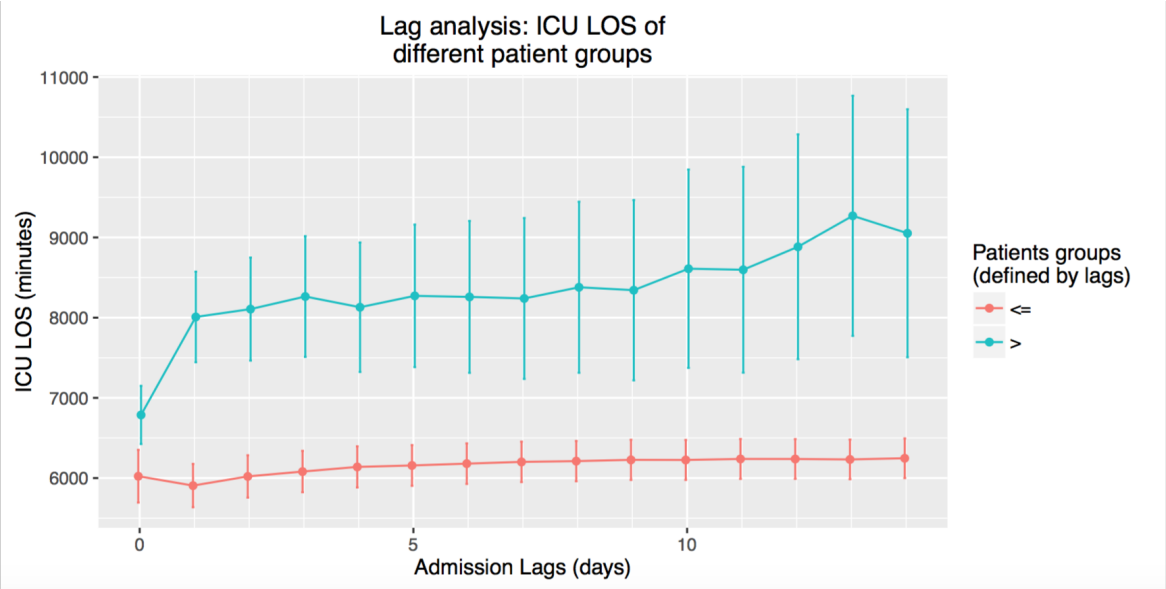


Figure 4.18: Average ICU LoS (minutes) of patient groups divided by different lag trials

Similarly, lag analysis was carried out for LoS, see Figure 4.18. Patients who were admitted immediately to the ICU are the most severely ill patients of all. It is natural that a higher mortality rate and also a longer average length of stay occurred in this group. This explains the average ICU LoS drop, from cut-off=0 to cut-off=1 shown in Figure 4.18. The bars in the Figure show the 95% CI of average LoS, which are calculated from following steps.

According to the Central Limit Theorem, we can approximate the distribution of average

LoS to a Normal distribution and then calculate the CI using the appropriate error level α.

The CI could be described as

$$\left(\mu - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}},\ \mu + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

n: population size i.e. number of admissions

μ: population mean i.e. average LoS

σ: population standard deviation i.e. standard deviation of LoS

$z_{\frac{\alpha}{2}}$: critical value of Normal distribution for the given error level α ($z_{\frac{\alpha}{2}} \approx 1.96$ for $\alpha = 5\%$)

As shown in Figure 4.17 and Figure 4.18, patients admitted later (from the second day onwards) have increases both in mortality rate and average ICU LoS. This group of patients may be delayed by some complicated symptoms which are not easy to be discovered. As discussed in Section 3.1, some early warning system scores for clinical deterioration of patients could be implemented to better discover their needs.

Moreover, we tested the differences of mortality rate and average ICU LoS between two groups of patients based on the lags using t-tests. Both of them become significant, with significance level ≤ 0.01, when cut-off equals to one. Therefore, cut-off equal to one was chosen to define late admission. A plot of ICU LoS frequencies is shown in Figure 4.19 to demonstrate differences in LoS between "non-late" and "late" admissions.

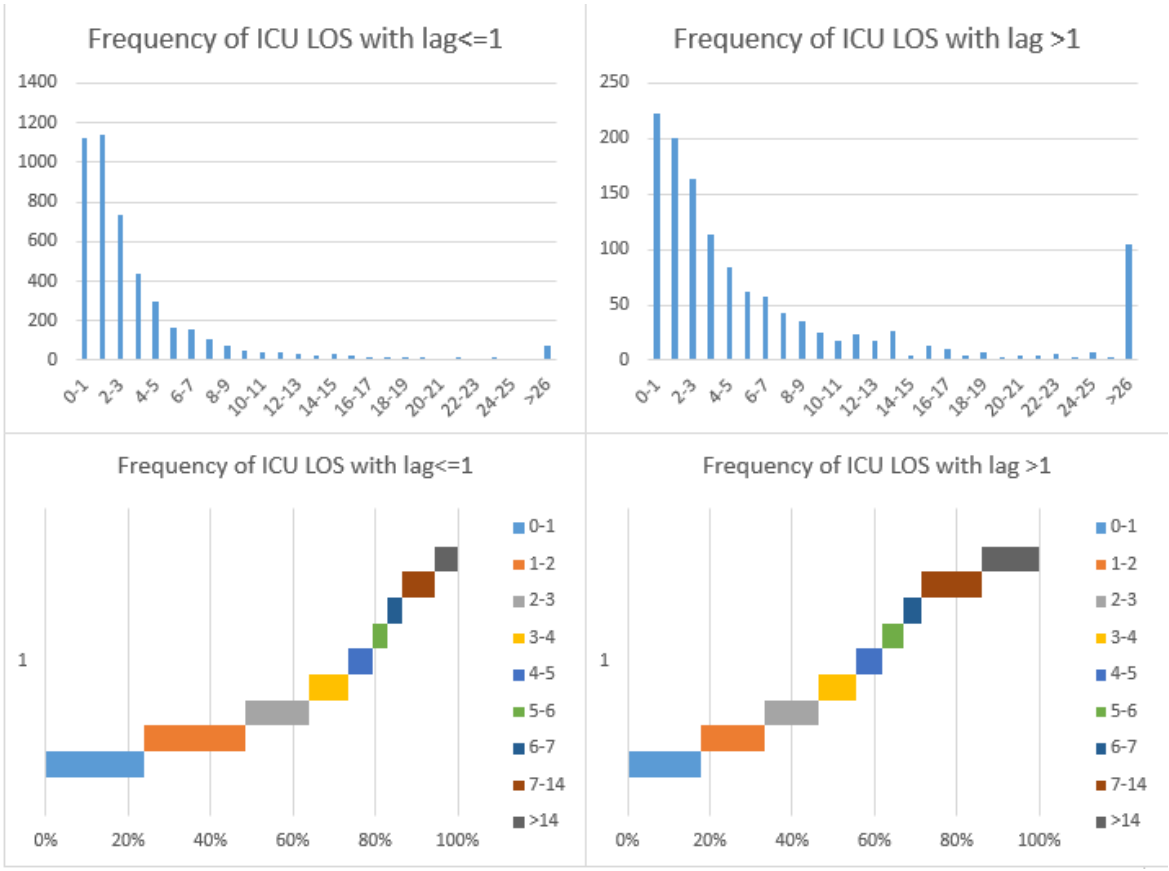


Figure 4.19: Number of patients with different lag (days) in different groups

The frequency plot in Figure 4.19 shows that ICU LoS in different groups has significant differences in not only their average value but also their distribution. The late admission group had around 30% of patients staying more than seven days in the ICU, which was twice as much as the percentage of the immediate admission group.

To better identify patients with delayed admissions, we individually plot the frequency of lag days in patients from different sources, see Figures 4.20, 4.21 and 4.22.

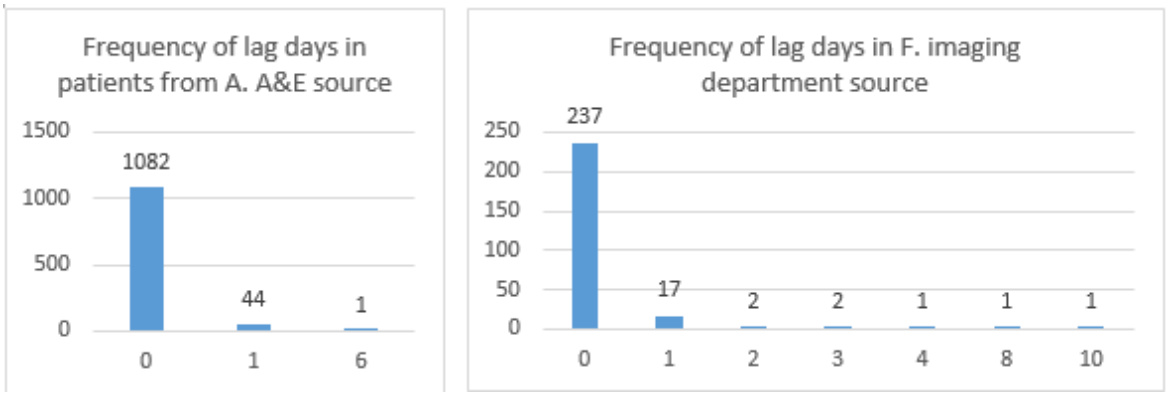


Figure 4.20: Number of patients with different lag days (A&E and imaging department)

Figure 4.20 shows that the lags of patients from A&E and the imaging department are very limited and so these are not the major sources of late admission patients in our data. Patients with very long lags of admission from the imaging department are likely to have spent those days in a general ward.

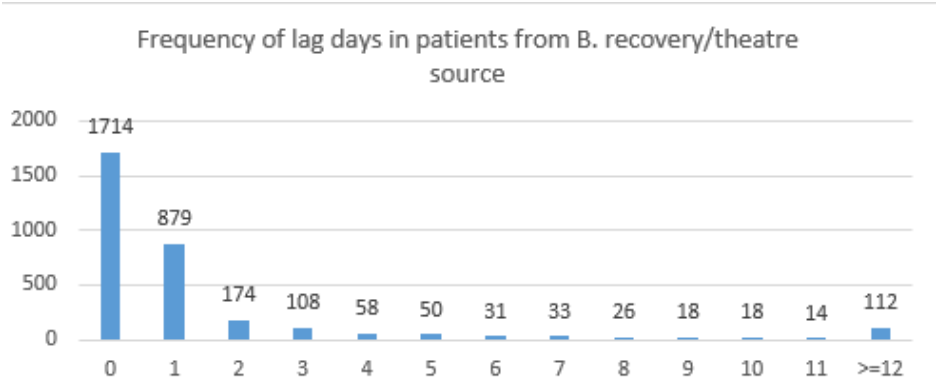


Figure 4.21: Number of patients with different lag days (recovery/theatre)

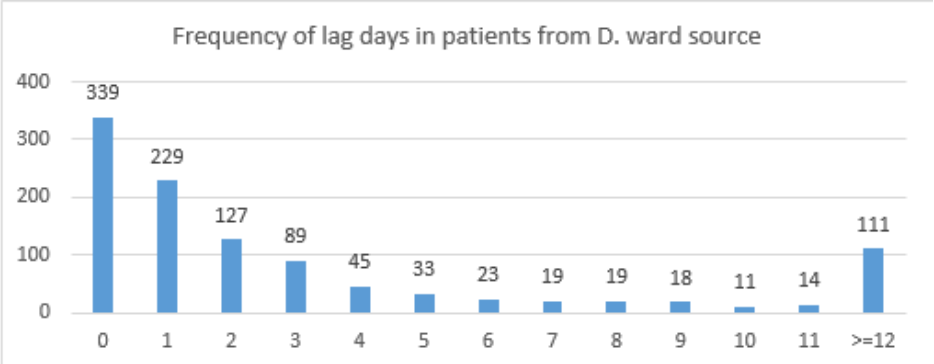


Figure 4.22: Number of patients with different lag days (ward)

Figure 4.21 and 4.22 above show that lags between hospital admissions and unit admissions mainly come from B. Recovery/ theatre and D. Ward sources. We inspect the two sources individually in the following sections.

4.5.2 Patients from recovery / operating theatre

One might expect that theatre patients do not have admission lags as they are planned to be admitted to the ICU before their surgery and they were supposed to be admitted to the hospital close to the date of their surgery. However, Figure 4.21 show that a large number of patients were delayed perhaps because of delayed surgery.

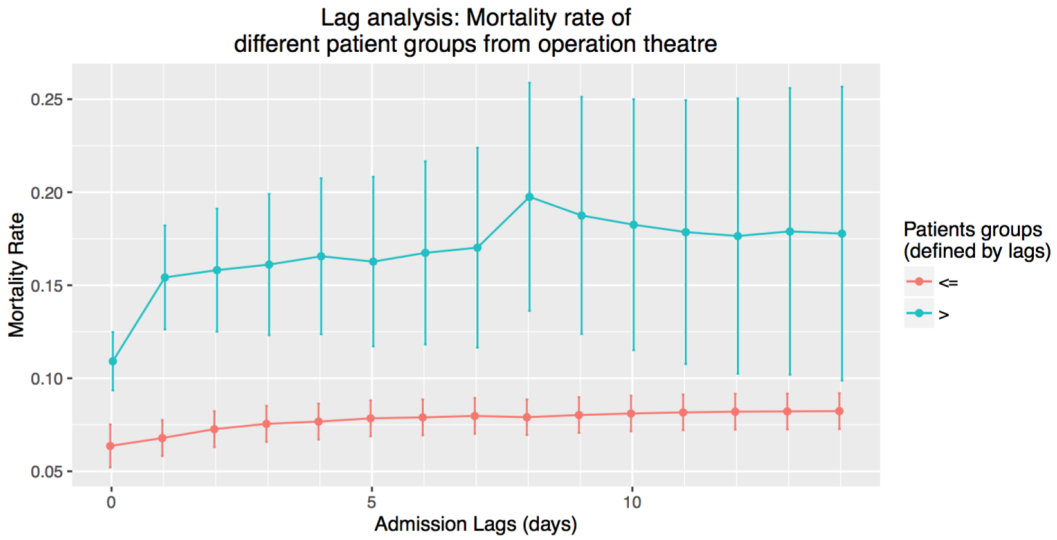


Figure 4.23: Mortality rate of different patient groups from the Recovery/ Operating theatres

The two lines in Figure 4.23 show the mortality rate of different patients groups from operating theatre. Bars show the 95% CI of the mortality rate. With the increase of lag days, the mortality rate for the '>' group increases. There is a sharp increase in mortality rate for the '>' group from lag=0 to lag=1, from 11% to 15%. The variation of the '>' group increases as lag increases, which may result from the reducing sample size.

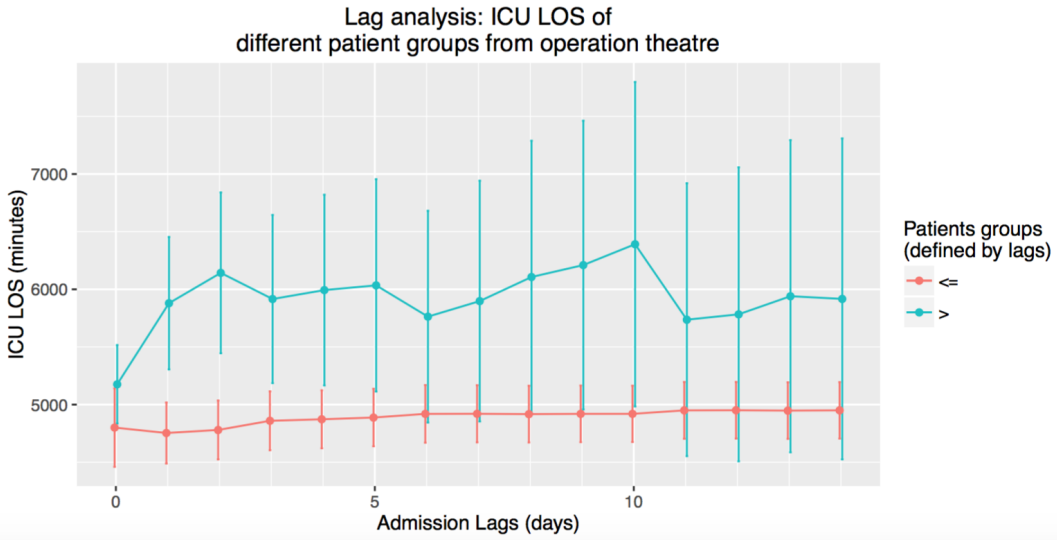


Figure 4.24: ICU LoS (minutes) of different patient groups from the Recovery / Operating theatre

Figure 4.24 show the mean ICU LoS of different groups of patients. As Figure 4.24 shows the mean length of stay has a significant difference between two groups when lag=1. However, with the increase of lag and smaller numbers of patients in the ">" group, the differences become less significant and overlaps of 95% CIs of the mean LoS start to be displayed in the large lag cutoffs. One may expect from the plot that lag=1 may divide two distinct groups of patients.

4.5.3 Patients from general wards

It could happen that surgical patients were transferred to general wards directly after their operations and then admitted later to the ICU. This transferring can cause lags between hospital admissions and ICU admissions. Besides, medical patients may be admitted from general wards to the ICU, and lags may also occur.

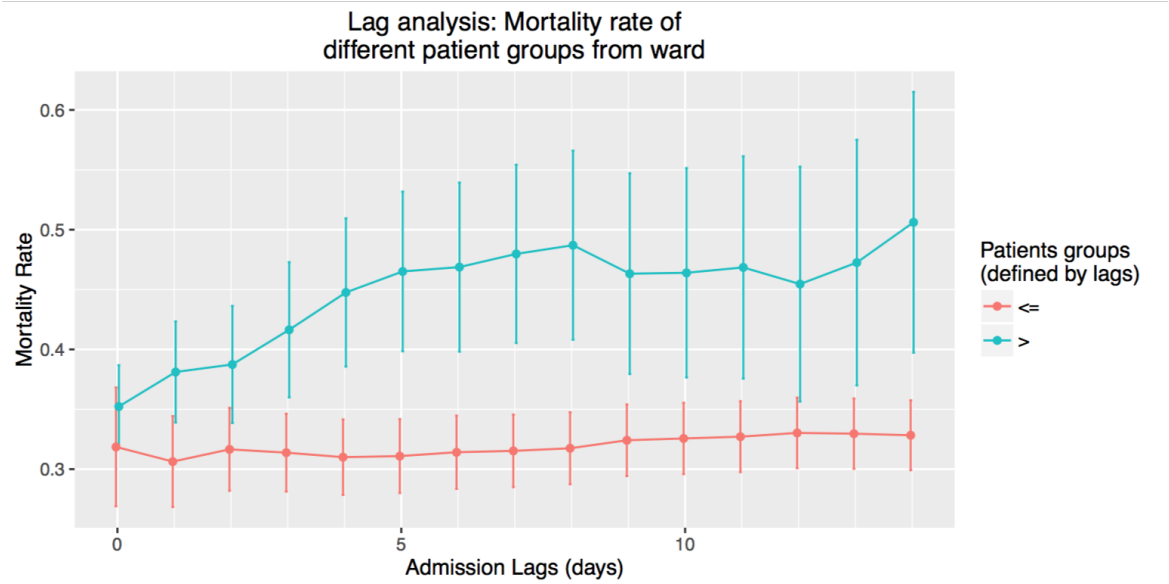


Figure 4.25: Mortality rate of different patient groups from the ward

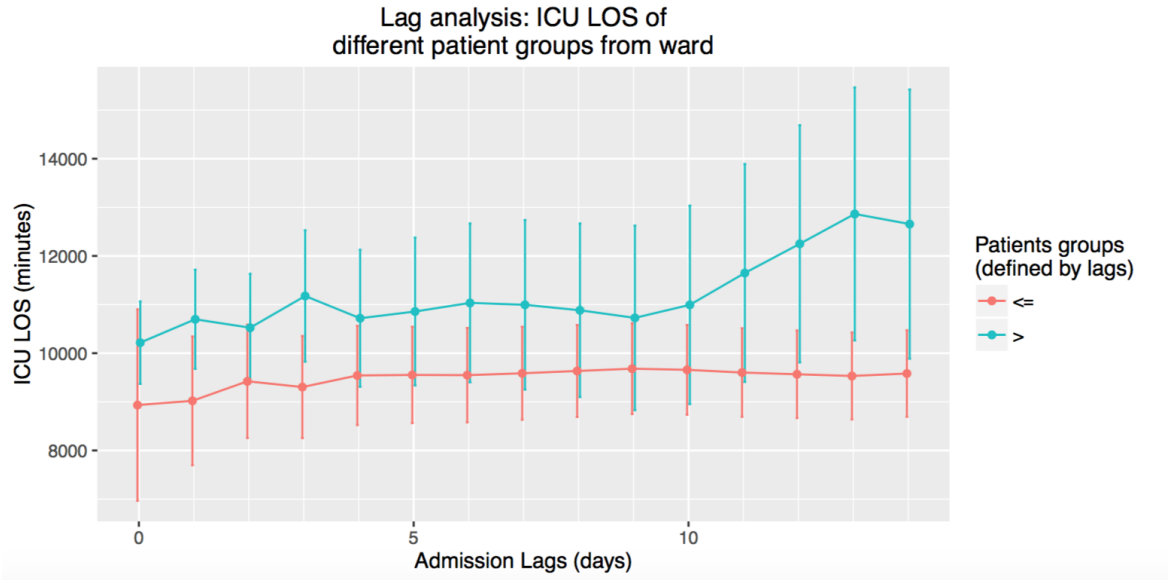


Figure 4.26: ICU LoS (minutes) of different patient groups from the ward

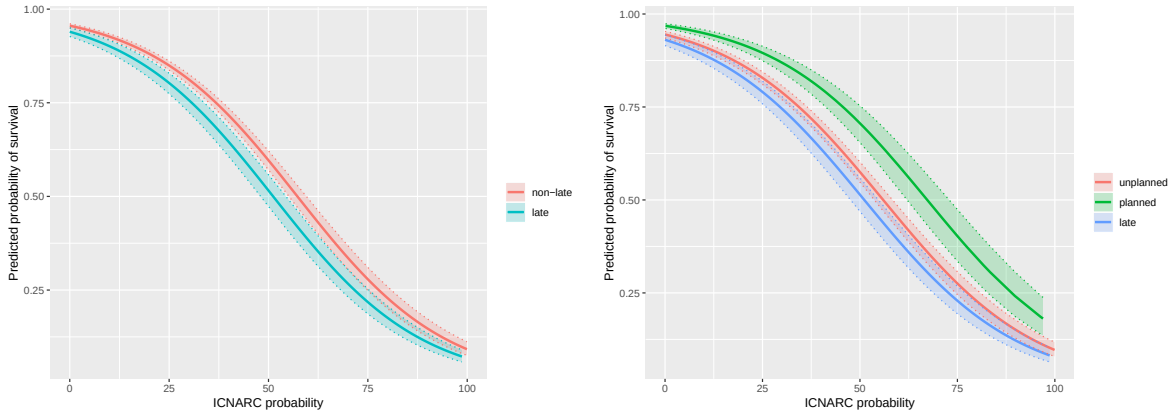
A very high mortality rate was detected in the patient group from the ward. The mortality rate is over 30% for the whole group, in which the late admission group could be threatened by a 40% probability of death. Of the patients coming from wards with an ICU admission lag of more than 14 days, 50% were dead before hospital discharge. Figure 4.26 shows that there is a significant difference in LoS between two groups when lag=1 ($p\text{-}value = 2.75 \times 19^{-4}$ by a t-test).

4.5.4 Test of confounding of late admission

To further investigate the cause of significant difference between two admission groups ,we would like to analyse possible confounding variables. As explained in Section 4.4, ICNARC probability is the main likely confounding variable.

First of all, we calculate the log odds of different groups of patients (non-late vs late and planned non-late, unplanned non-late and late) for their mortality upon hospital discharge. Log odds of hospital outcome versus ICNARC probability are calculated. These are plotted with the y-coordinate transformed back to predicted probability by the inverse logistics

function (i.e. $y = \frac{1}{1+e^{-logodds}}$) showing the probability of survival.



(a) Hospital outcomes versus ICNARC probability (non-late vs late)

(b) Hospital outcomes versus ICNARC probability (unplanned, planned and late)

Figure 4.27: Transformed log odds plot: ICNARC probability versus hospital outcomes for different patient categories

Figures 4.27a and 4.27b show that late admissions of patients lead to worse outcomes even after the adjustment of case-mix effect. ICNARC probability should be able to capture most confounding effects. We should be confident enough to conclude that late admission results in higher mortality. However, there may exist some other effects such as differences of decision makers (i.e. ICU consultants) that also contribute to the differences but are hard to verify.

4.5.5 Conclusion on late admission

We clarify the definition of late admission based on definitions from Restrepo et al. (2010); Renaud et al. (2012); Jhanji et al. (2008) and the analysis above. Late admission is defined separately for medical admission and surgical admission. Late medical admission is defined as ICU admission with the delay between hospital admission and unit admission longer than one day. Late surgical admission is defined by two rules. First, all the surgical patients admitted from wards are counted as late admissions. Second, ICU admission with the delay between hospital admission and unit admission longer than 1 day, from sources other than ward, is counted as late admission.

Our analysis shows that late admission appears to be a more severe problem in the hospital than readmission since there is a higher percentage of late admitted patients and higher mortality rate compared to the readmitted patients, while these two groups have similar average LoS. We confirmed the effects of lateness by using ICNARC probability as a confounding variable. However, It should be acknowledged that there may be other influences but in our context we assume the existence of difference between non-late and late patients' outcomes.

4.6 Measures of busyness in the ICU

In this section, to inform the prediction modelling that follows in Chapter 5, we describe measures of patient levels, and a new indicator of the busyness of both the ICU and general wards, the ICU Patients Acuity (PA). Month and day-of-week effects on these indicators are also investigated.

4.6.1 Measurements of patient levels in the ICU

As the hospital has no independent High Dependency Unit (HDUs), all the severely-ill patients stay in the ICU. The ICU has both level 3 and level 2 beds as described in Section 2.1. We analyse ICU patients' days by their levels. Typically, either patients die quickly after admission if they are critically ill, or they recover after some treatment. Thus, they often step down from level 3 to level 2 in a short period after admission. However, patients' states could also worsen from level 2 to level 3. Normally, all the patients in the ICU are level 3 and level 2 patients as they need special treatment and extra attendance, but there were also found some level 1 patients in the data. It is possible that these patients could have been discharged earlier to a general ward, but for some reason, which could be the congestion in general wards, they were kept in the ICU for a longer time.

To simplify expression, we use L1, L2 and L3 to denote level 1, level 2 and level 3 respectively in the following text.

Five different measurements are considered to measure the busyness of ICU. Beds in use (#Beds), bed occupancy rate (%Beds), number of non-L3 patients (#NonL3), percentage of non-L3 patients (%NonL3), and PA.

Beds in use and bed occupancy rate

The first two measurements take all patients as identical individuals when measuring how busy the ICU is. These measurements could be simple and effective in measuring busyness of an "all level 3 ICU" (Kim et al., 2016) but are of limited ability for a mixed ICU with both level 3 and level 2 beds..

Beds in use

$$\#Beds = absolute\ number\ of\ beds\ occupied$$

Bed occupancy rate

$$\%Beds = \frac{\#Beds}{total\ beds}$$

Non L3 patients number and percentage

The ICU currently can accommodate twelve L3 patients and eight L2 patients in total at most. If the ICU admits 13 L3 patients, they could only admit six level 2 patients due to nursing resource constraints even though there are vacant beds. In other words, 60% L3 patients is the designed level of the ICU.

Therefore, we can measure the busyness of the ICU and general wards via the occupancy rate of L3 patients and the occupancy rate of non L3 patients or L1 patients.

As mentioned before, the ICU went through two major changes in the past eight years. One of the reasons for change was to expand the ICU to meet the demands of patients, which means the ICU is really busy in admitting critically ill patients of which most are L3 patients. Although L2 patients are supposed to be in the mixed ICU, admission priority still need to

be given to L3 patients. A higher portion of non-L3 occupants in the ICU suggests that ICU patients are overall under a less risky situation. Unless there are no severe patients in waiting or the cost of an idle bed is greater than that of accommodating a less 'qualified' patient, it is inconsistent with the operation target if the ICU is majorly occupied by non-L3 patients. It could be expected that non-L3 patients may congest the ICU by unnecessary prolonged stays in the ICU and preventing more severe patients from admission because of lack of resources. If it was possible, these patients would have been discharged earlier. Once the medical states of patients meet the discharge criteria, the only reason that keep patients in the ICU is the lack of beds in general wards. Therefore, we could make a reasonable guess that non L3 patients kept in the ICU suggests the congestion level of general wards in the hospital.

Counting the number of non L3 patients, considering that the total number of patients is given, does exactly the same thing as counting the number of L3 patients. Since non-L3 patient numbers may suggest the congestion level of general ward, which is desirable in our case, we utilise non-L3 rather than L3 patient numbers and percentage as two new measurements.

Number of non-L3 patients

$$\#NonL3 = absolute\ number\ of\ non-L3\ patients\ in\ ICU$$

Percentage of non-L3 patients

$$\%NonL3 = \frac{\#NonL3}{total\ patients}$$

According to our interviews with intensivists, critical care nurses and administration personnel, lack of beds in general wards is the key reason for delayed discharge of patients. As the high proportion of non-L3 occupants in the ICU could suggest a high congestion level of general wards, it could be expected that a higher percentage of non-L3 patients in the ICU suggests a probable longer LoS of ICU patients, which will be examined in Section 5.4, LoS

prediction.

Patients acuity

In order to better define the busyness ratio of an ICU, PA is introduced. The indicator originated from bed occupancy rate and then was improved by taking human resource consumption into consideration. We assign different multipliers in accordance with mandatory nursing staff to different level of patients (Ball and Barker, 2010; Royal College of Nursing (RCN), 2012). Using a ratio rather than exact numbers allows for changes in resource levels, i.e. beds and nurses, during the data collection period. PA describes the overall acuity of patients in ICU. The value could vary from 0 to 2 in theory since a bed can be occupied by more than one patient in a day.

Patients Acuity (PA) is defined as

$$PA = \frac{1 \times (L3\ patients) + 0.5 \times (L2\ patients) + 0.25 \times (L1\ patients)}{1 \times (designed\ L3\ beds) + 0.5 \times (designed\ L2\ beds)}$$

The average ICU PA during patients' ICU stay may suggest care that patients received. Nurse-to-patients ratio can influence the outcomes of ICU patients. Busy nurses lead to unsatisfactory ICU outcomes (Numata et al., 2006; Penoyer, 2010; Kelly et al., 2014). Research shows that as well as the nurse-to-patient ratio, the intensivist-to-patient ratio also associates with the outcomes of ICU patients (Dara and Afessa, 2005; Ward et al., 2013). Although the ICU we investigate implements the mandatory nurse-to-patient ratio, PA still suggests the busyness of consultants which may also influence the care level that patients could receive.

Relationships between Measurements

Several different measurements are suggested in measuring the busyness of ICU and hospital. We use our data to examine the relationship between each of them.

Although both of %NonL3 and %Beds could suggest how busy the ICU is, no significant correlation could be observed between bed occupancy rate and percentage of non L3 patients ($\rho = 0.0668$). The relationships between PA and %Beds, PA and %NonL3 were examined independently (see Figures 4.28 and 4.29).

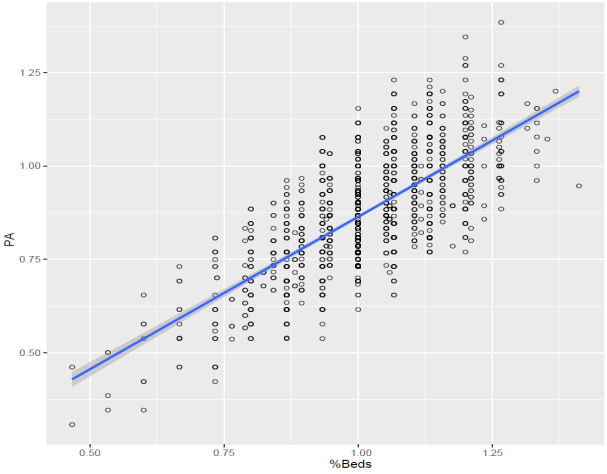


Figure 4.28: Correlation between %Beds and PA

As expected, PA exhibits significant correlation with %Beds ($\rho = 0.7011$). PA is developed based on bed occupancy rate and resource consumption level. There is a natural positive correlation between bed occupancy rate and PA.

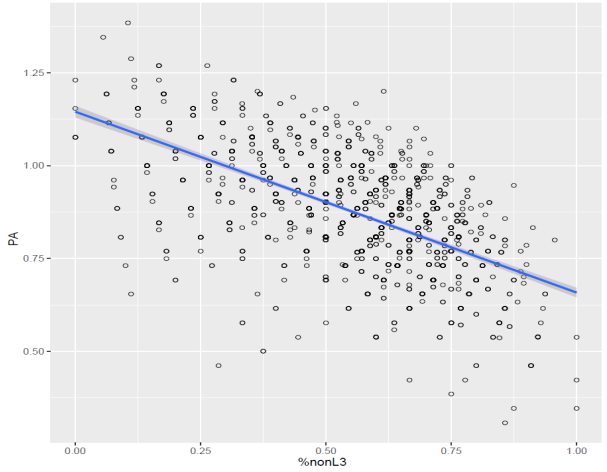
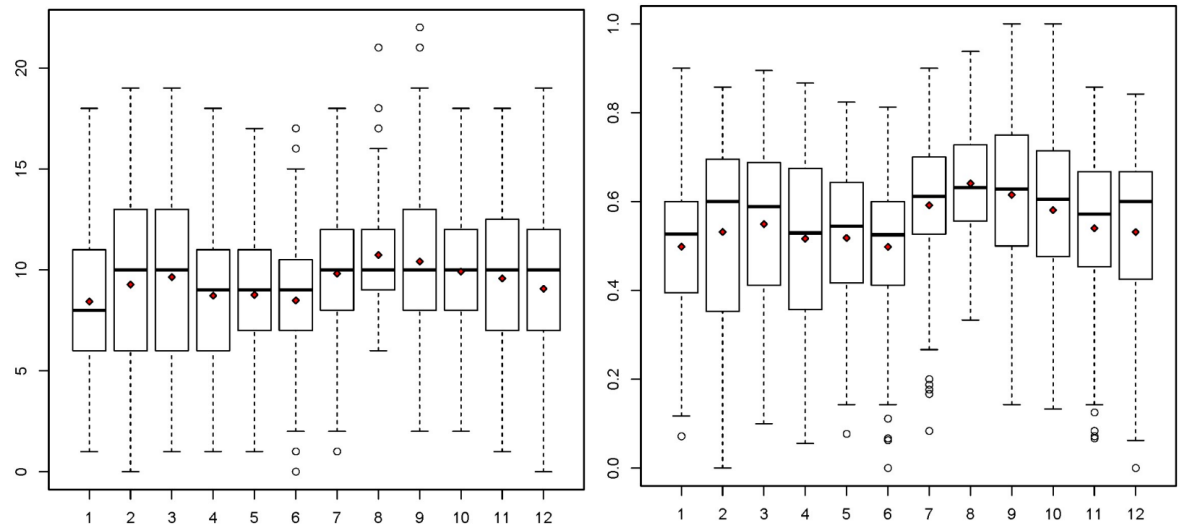


Figure 4.29: Correlation between %nonL3 and PA



(a) Boxplot of monthly #nonL3 (b) Boxplot of monthly %nonL3

Figure 4.30: Boxplot of monthly #nonL3 and %nonL3

Figure 4.29 shows that PA strongly negatively correlated with %nonL3 with $\rho = -0.6124$. The result is quite intuitive as a higher percentage of non L3 patients means a less severely ill patient group in the ICU.

From Figures 4.28 and 4.29, we demonstrate that PA is an integrated indicator as it combines both bed occupancy and patients' severity of illness.

4.6.2 Time series plots of indicators

Pattern check - monthly

Seasonality is of great interest to researchers in time series data. In our case, we would like to see if there is any seasonality in the data. In this section, we will focus on the monthly seasonality of the data. To specify the plots below, we denote January to December as 1 to 12. Box plots are recruited to check the monthly pattern of three indicators, #nonL3, %nonL3, and PA. Three box plots of monthly seasonality are provided below.

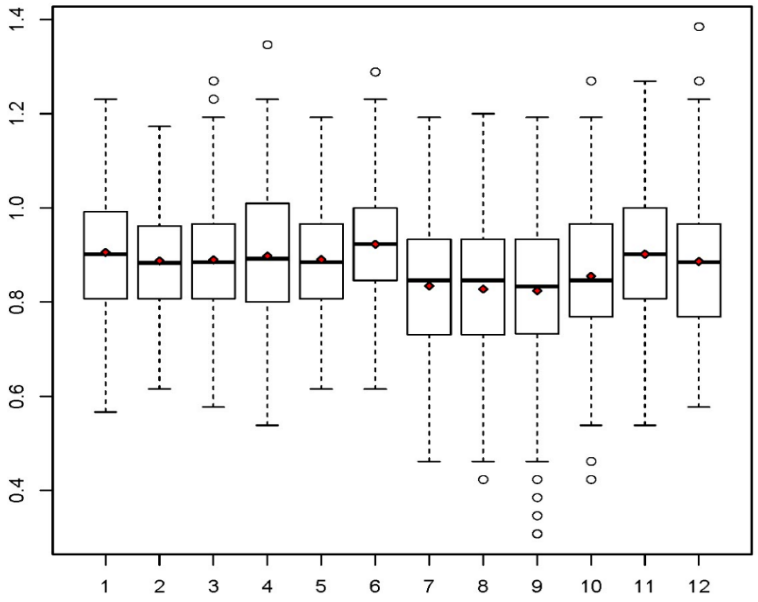


Figure 4.31: Boxplot of monthly PA

Figures 4.30a and 4.30b, both show a relatively high values of #nonL3 and %nonL3 from July to September which is consistent with the low PA shown in Figure 4.31.

Pattern check - day of week

After checking monthly patterns of the data, we are also interested in weekly seasonality of the data. Similar to the monthly seasonality plots, we denote Monday to Sunday as 1 to 7. Box plots are recruited to check the day-of-week pattern of three indicators, #nonL3, %nonL3, and PA. Three box plots of weekly seasonality are provided below.

As Figures 4.32a, 4.32b and 4.33 show, besides the slightly higher average of Thursday, we cannot observe a significant difference of #nonL3 and %nonL3 by different days of week but there is a relatively lower PA at weekends compared to weekdays.

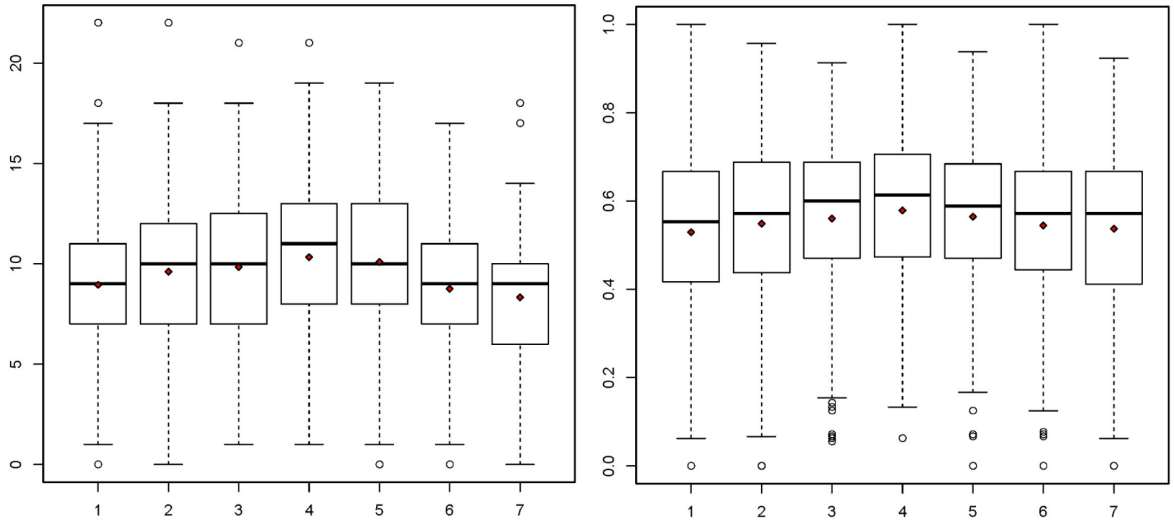
A one-way ANOVA test was used to examine the difference of PA in each day of the week and the difference of PA in each month. Tukey-Kramer method was adopted to carry out multiple comparisons between pairs of months and pairs of days.

Test results show that PA in July, August and September is significantly lower than PA in other months ($p-$value< 0.05). We thus group all July to September data as a category named 'summer' for prediction purposes. A distinctly lower level of PA in summer could be explained by less A&E admissions during summer time as A&E is a large source of ICU admissions (Baker, 2016).

PA on Saturdays and Sundays is significantly different from PA on other weekdays. Therefore, we divided the days of the week into two categories, weekdays and weekends. Low PA means busy general wards, which could result from the less nurse involvement at weekends than on weekdays in general wards.

4.7 Conclusion

We have clearly defined late admitted patients, both surgical and medical, and identified the risks of mortality and long LoS for these patients. We established relationships between late admission and mortality or LoS by analysing data and considering a confounding variable, ICNARC probability. Late admission of patients to ICU have gained little attention, while



(a) Boxplot of weekly #nonL3

(b) Boxplot of weekly %nonL3

Figure 4.32: Boxplot of weekly #nonL3 and %nonL3

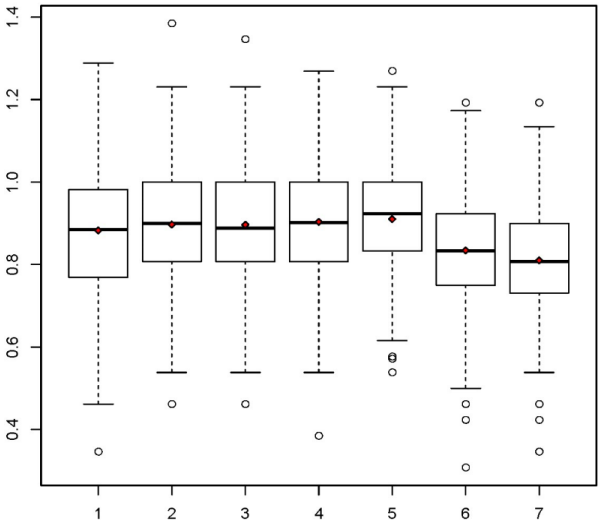


Figure 4.33: Boxplot of weekly PA

problems caused by re-admission of patients to ICU have been thoroughly studied during recent years. According to the data collected in BRI, late admission problems are at least as critical as readmission problems. The group of late admission is larger than that of readmission with the late admission rate equalling 13.84% and the readmission rate equalling 3.25%. The mortality rate of the late admission group is much higher than that of the readmission group (29.07% for late admitted surgical patients, 42.40% for late admitted medical patients, 18.08% for readmitted surgical patients and 18.00% for readmitted medical patients). LoS of late admission patients are of comparable length to readmitted patients (6.91 days for late admitted surgical patients, 7.36 days for late admitted medical patients, 7.04 days for readmitted surgical patients and 8.79 days for readmitted medical patients). The difference between the immediate admission group and the late admission group is even higher. Therefore, identifying and accommodating late admission better can significantly contribute to the performance of the ICU. We will continue to identify readmission records in our prediction models and simulation, but will not propose new policies for readmission as the problem is relatively well contained at BRI.

The profile of the late admission group is summarised. Of this group, 43.28% and 56.72% are surgical and medical admissions, respectively; 12.19% and 87.81% are planned and unplanned admissions, respectively; 60.57% and 39.4% are admitted during peak time and

off-peak time, respectively; 9.33% are admitted from source A (A&E), 3.68% from source B (Recovery / Operating Theatre), 75.25% from source D (Ward), 1.24% from source F (Imaging Department) and 10.32% from Z (other sources).

We examine the timing effects of ICU admission and discharge using ICNARC probability as a confounding variable. After excluding the influence of ICNARC probability, discharge timing does not show any effect on mortality in either day of week or time of day aspects. Admission during peak time (2pm - 00:59am) shows a positive impact on patients' outcomes. No other timing effect is detected. We further examine time effect in analysis of admission categories in Section 5.1.2.

The ICU we investigated is a mixed ICU with both L3 and L2 beds. Bed occupancy rate alone does not give a measure of ICU busyness. We have developed the measure of PA to indicate the busyness of both the ICU and general ward. As we explained in Section 4.6.1, a lower PA may indicate a busier general ward and cause longer ICU stays. Average PA at weekends is considerably lower than average PA on weekdays; this could be explained by less discharges from general wards during weekends. The average PA in July, August and September being lower than that of other months may not be linked to busy general wards but could be explained by less A&E patients during summer time as A&E is a large source of ICU admission.

The group of late admitted patients will be labelled and the timing effects will be considered in the data mining models described in Chapter 5, to see differences caused by timing effect in predicting hospital mortality and ICU LoS of ICU patients. The predictions can be finally used in building the simulation model of the ICU as described in Chapter 6.

Chapter 5

Predictions of Mortality and Length of Stay

In this chapter, in order to investigate the mortality and LoS prediction, we develop a number of predictive models. We first describe how we categorise patients and deal with variables. Then, we incorporate the findings on Patients Acuity from Section 4.6.1 and timing effects in the ICU from Section 4.4 in the improved mortality and LoS prediction models. By predicting mortality and LoS with consideration of the specific situation of the BRI ICU, we will be able to construct a more precise simulation model of the ICU in Chapter 6.

5.1 Preparation for prediction modelling

In this section, we carry out correlation analysis and determine suitable admission categories, in preparation for the prediction models to be built of mortality and LoS.

We will first calculate the correlations for continuous variables in Section 5.1.1; then use log odds plots to visualise patients' outcomes and categorise patients for mortality predictions in Section 5.1.2; finally we use the Kolmogorov-Smirnov (KS) test to detect the similarities

of ICU LoS distributions for different categories of patients in Section 5.1.3.

5.1.1 Correlations of continuous variables

Correlations for all the interval variables are calculated to help us understand the variables. The full correlation matrix is shown in Appendix B.

Strong correlations are shown in different ICU scores and mortality predictions. High correlations between ICNARC score and ICNARC probability, $\rho = 0.8741$, suggests that using both them in prediction models may cause multicollinearity. However, neither APACHE II score nor mortality prediction has a high correlation with ICNARC probability, $\rho = 0.2709$ and $\rho = 0.5901$ respectively. These relationships are shown in plots in Figure B.1 in Appendix B.

There are a number of "zeros" in the APACHE II system which may result from lack of specific indicators. Thus, "zeros" are excluded in this analysis (i.e. not for subsequent predictions) and correlations for APACHE II system and ICNARC systems are recalculated and plotted, see Figure 5.1. Both APACHE II mortality prediction and APACHE shows a strong positive correlation with ICNARC probability. Figure 5.1 exhibits the strong correlations between predictions and scores from different ICU scoring systems. Moreover, the APACHE II scoring system does not suit our data as score and predictions are incalculable for more than 2000 admissions. Thus, neither APACHE II score nor prediction will be incorporated in our prediction model.

There are strong correlations for other ICU variables (e.g. %nonL3 and PA) as shown in Appendix B. However, these high correlations are expected as explained in Sections 4.6.1. Instead of keeping every busyness related measurement, only the most significant one will be kept when modelling mortality and LoS.

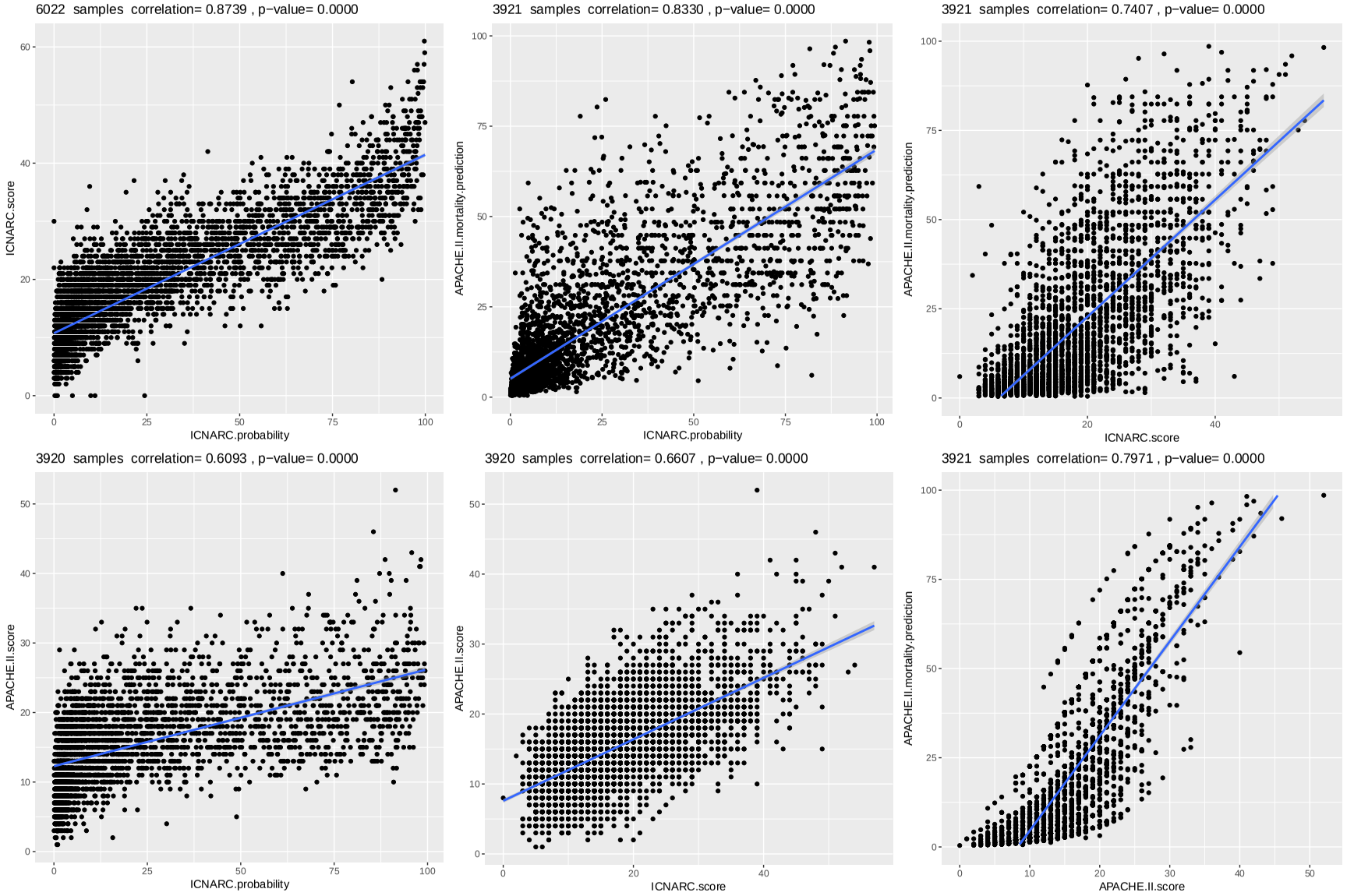


Figure 5.1: Correlations between ICU scores and mortality predictions (excluded “zero”s)

5.1.2 Log odds and admission categories

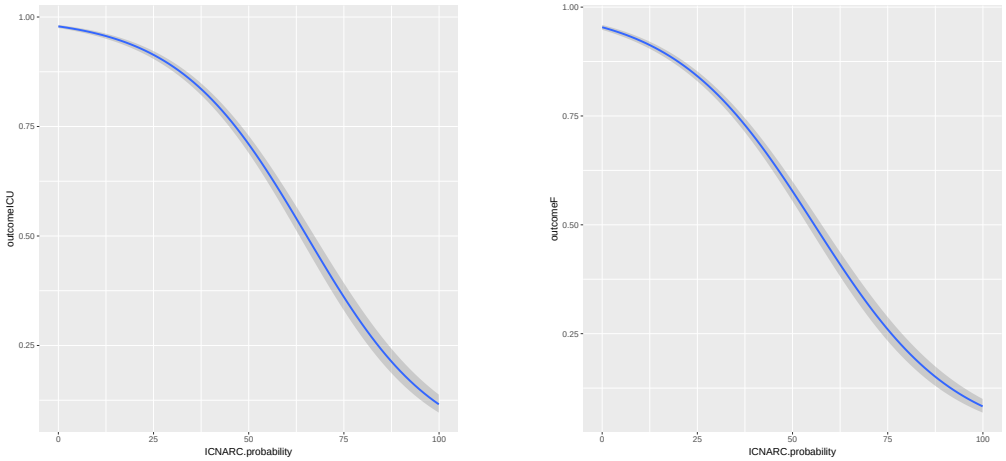
Log odds plots : ICU outcomes versus ICNARC probabilities

It would be expected that the mortality prediction using the ICNARC system is consistent for patients with different operational factors as the ICNARC scoring system has taken operational factors into considerations. However, after checking log odds for different categorical variables including surgery type, admission sources, admission type, admission timing and combinations of them, we found that operational factors led to prediction bias for admission with same ICNARC probabilities. Therefore, categorising admissions according to their operational factors is necessary.

First of all, we calculate the log odds of all the admissions (n=6022) for their mortality upon ICU discharge time. Log odds of ICU outcomes versus ICNARC probability are calculated where the outcome 1 is patients discharged alive, 0 dead. These are plotted with the y-coordinate transformed back to predicted probability by the inverse logistics function (i.e. $y = \frac{1}{1+e^{-logodds}}$). Figure 5.2a reveals that the predicted probability of survival is negatively correlated with ICNARC probability but it is not a linear correlation.

We also calculated the log odds of hospital outcomes versus ICNARC probability using all the group 0 and the last admissions in group 2 (n=5808) in Figure 5.2b. As expected there are similarities between the log odds for ICNARC probability versus hospital outcomes and ICU outcomes. The curve shown in 5.2b appears as a left-shifted version of Figure 5.2a. Patients' deaths during hospital stays after their ICU treatments will increase the mortality. That is to say, patients with the same ICNARC probability on arrival will have a higher probability of being alive at ICU discharge than at hospital discharge. Hence, the curve shifts to the left.

Furthermore, we calculate the log odds of ICU outcomes versus ICNARC probability (by groups) and plot to visualise categorical variables including surgery type, admission sources, admission type, admission timing and combinations of them. We discuss these plots in the following sections. A similar series of plots for ICNARC probability versus hospital



(a) ICU outcomes versus ICNARC probability

(b) Hospital outcomes versus ICNARC probability

Figure 5.2: Transformed log odds plot: ICNARC probability versus ICU and hospital outcomes

Note: Y coordinate is the predicted probability of being discharged alive

outcomes show similarities with plots based on ICU mortality. These hospital plots are placed in Appendix C for reference.

Log odds for surgery types

After consulting intensivists from the BRI, we decided to categorise surgery types of patients into three: one is 'emergency admission' which includes both emergency and urgent admission; one is 'elective admission' which includes both scheduled and elective patients; the other is 'not relevant' which means the admission does not have surgery (i.e. medical admission). Figure 5.3 shows the bias of using ICNARC probability only in predicting mortality. For admissions with the same ICNARC probability, the predicted mortality varied between surgery types. It could be seen from Figure 5.3, for a given ICNARC probability scheduled and elective patients tend to have better ICU outcomes than emergency and urgent and medical patients.

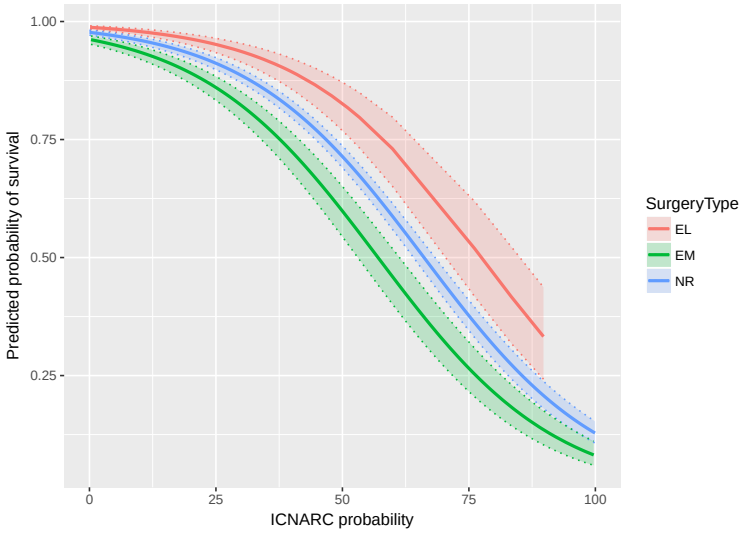


Figure 5.3: Transformed log odds plot for different surgery types
"EL" scheduled and elective, "EM" emergency and urgent, "NR" not relevant

Log odds for admission sources

Five admission source categories, A for A&E, B for operating theatre, D for general wards, F for imaging department and Z for others, are considered in in Figure 5.4. The lines are close to each other for Source A, B and Z, showing that, for the same ICNARC probability, these three admission sources will not result in much different outcomes. The curve of Source D slightly shifts towards the right and the curve of Source F significantly shifts to the right; this implies biases in predicting using ICNARC probability. ICNARC probability for patients from Source F are worse than those for patients from other sources. However, it could be seen from Figure 5.4, for a given ICNARC probability patients admitted from imaging department tend to have better ICU outcomes. We will use the five admission sources as a categorisation method to group patients and then predict mortality for each group of patients.

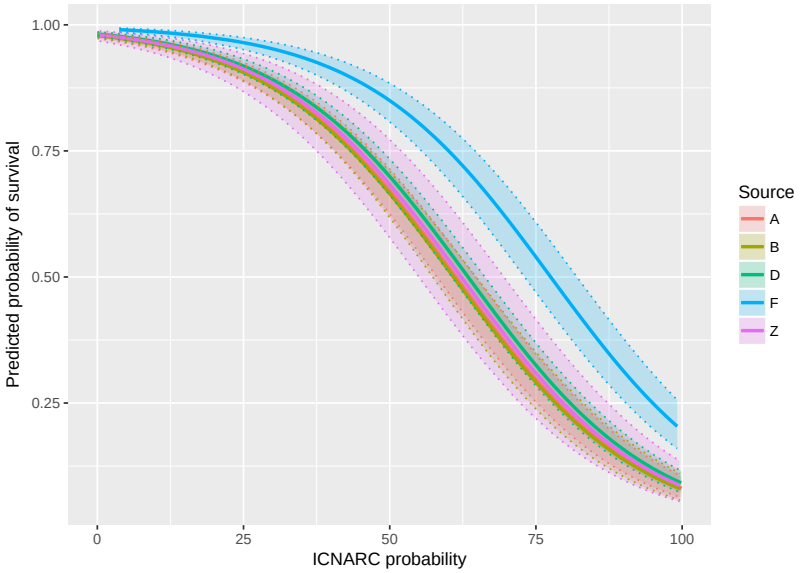
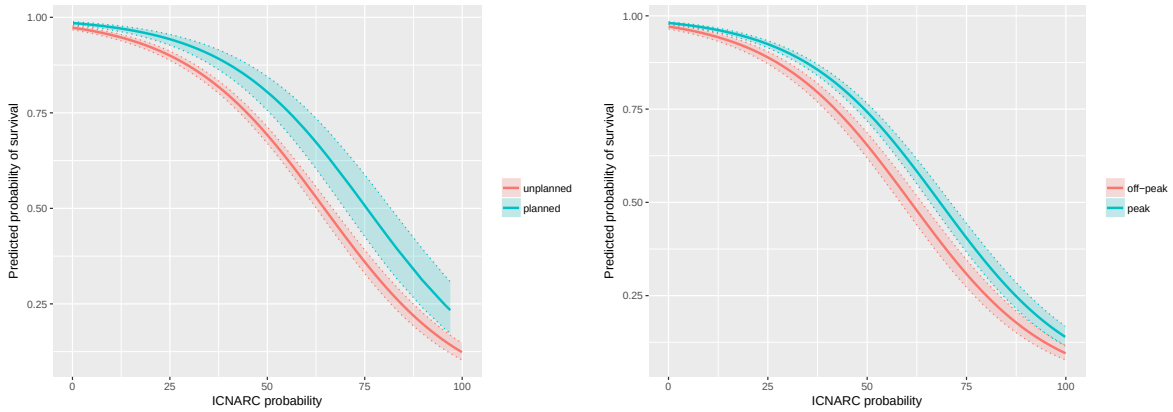


Figure 5.4: Transformed log odds plot for different admission sources

Log odds for admission type and admission timing

Planned and unplanned admissions are modelled separately in some literature (Ridge et al., 1998; Kim et al., 1999; Costa et al., 2003; Griffiths et al., 2005; Hagen et al., 2013). Planned and unplanned admissions are found to usually occur at different time (see Section 4.4.1 and Figure 4.9). Curves of planned and unplanned admissions in Figure 5.5a (1 and 0 representing planned and unplanned respectively) display a significant discrepancy. The shadowed areas in Figure 5.5a are the 95% CIs of predicted probabilities.

A log odds plot of peak admissions and non-peak admissions is displayed in Figure 5.5b. The effect of admission timing has been analysed in Section 4.4. The lines deviate from each other. Peak time admissions have positive influences on patients' outcomes. However, off-peak admissions may represent unplanned admissions, and it is not clear whether the deviation of the curves results from admission type or admission timing.



(a) Admission types

(b) Admission timing

Figure 5.5: Transformed log odds plot for admission types or admission timing

Log odds for combined factors

As mentioned in Section 5.1.2, both unplanned admissions and off peak time admissions have negative effects on ICU outcomes, so we would like to consider the planned/unplanned and peak/off-peak together. Figure 5.6 shows the results for combined factors. The green (planned peak), purple (unplanned peak) and turquoise blue (unplanned off-peak) curves show clear distances between each other. Moreover, a minimal overlapping of 95% CI has been detected. However, a large overlap of the red line (planned off-peak) and its shadow area with all the other lines has been captured in the figure. This was expected as the group of planned off-peak is relatively small (n=167).

Figures 5.4 and 5.5a show that both admission sources and planned admissions lead to biased ICNARC prediction of ICU mortality. Further grouping of admissions according to both admission type and admission sources is of interest. Figure 5.7 gives ICNARC probability versus predicted survival probability for ten groups of patients.

The predicted probability is "1" for all the A1 admissions (planned admissions from emergency department). There are 6 admissions in total for A1 group, all of them discharged alive. The lack of data reduces the credibility of the prediction, and furthermore the presence of planned admissions from emergency department is questionable. In later modelling, we

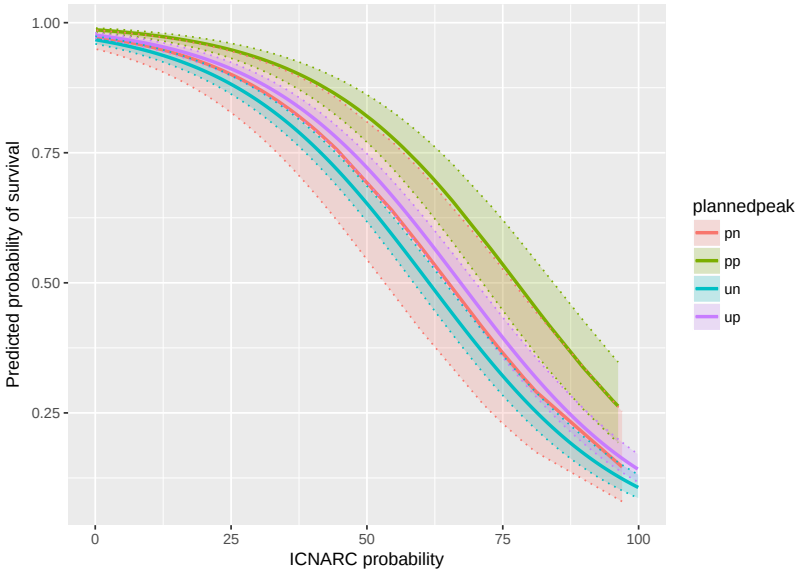


Figure 5.6: Transformed log odds plot for different admission types and timing
"pn" planned off-peak, "pp" planned peak
"un" unplanned off-peak, "up" unplanned off-peak

combine the A1 group with A0 (unplanned admission from emergency department). The line of Z0 (unplanned admissions from other sources) almost perfectly overlays the curve of D1 (planned admissions from general wards). Groups D1 and Z0 are also combined in later analysis.

5.1.3 Tests for goodness of fit

We first plot the empirical distributions (EDFs) of different groups of patients in Figure 5.8. We plotted the EDFs of non-late and late admissions, also unplanned and planned admissions. The EDFs diverge from their counterparts.

The KS test was adopted to test the empirical distribution for LoS of different groups of patients. The KS distance (D) between two empirical distributions was calculated using

$$D_{n,m} = \sup_x |F_{1,n}(x) - F_{2,m}(x)|,$$

where $F_{1,n}$ and $F_{2,m}$ are empirical cumulative distribution functions (ecdf) of two samples.

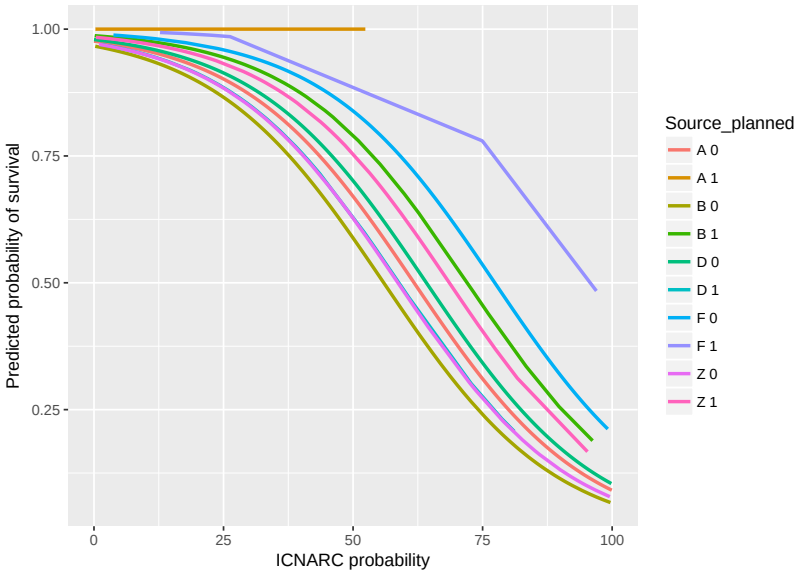
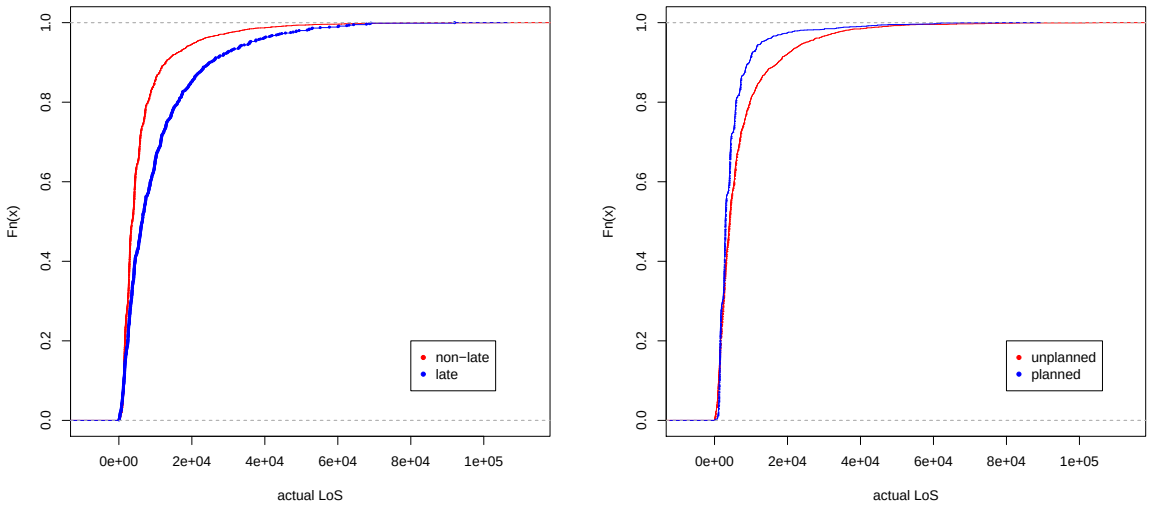


Figure 5.7: Log odds plot for different admission sources and admission types



(a) Late versus non late

(b) Planned versus unplanned

Figure 5.8: EDFs of actual LoS for different admission categories

The null hypothesis is rejected at level α if

$$D_{n,m} > c(\alpha)\sqrt{\frac{n+m}{nm}},$$

where n and m are sizes of the two empirical distributions; $c(\alpha)$ is the critical value for significance level α. $c(\alpha)$ is obtained from

$$c\left(\alpha\right) = \sqrt{-\frac{1}{2}\ln\left(\frac{\alpha}{2}\right)}.$$

If samples of two admission categories are shown to be from the same distribution, they will be put together in the modelling of LoS and they will not be combined if distributions are shown to be different.

Table 5.1 shows the results of the KS test for these admission categories. Based on the low p-values, we will reject the null hypothesis that the two tested groups are the same. That is to say, when we go on to model LoS, it is better to consider them as different groups instead of putting them all together.

Table 5.1: Results of KS tests

KS test	unplanned vs planned	non-late vs late
D	0.1600	0.2521
p-value	0.0000	0.0000

In the following sections, we will give a description of predictions in three main parts. A general picture will be given in the first place. It will list all the models carried out in the predictions and describe how we handle categorical variables with several categories in Section 5.2. Then, mortality prediction will be further divided into ICU mortality prediction, hospital mortality prediction and after-ICU mortality prediction and discussed in Section 5.3. The last part is LoS prediction. We illustrate results in detail for the 'all admissions' models. For a comparison of all models please refer to Appendix D.

5.2 Prediction models

Log odds plots of ICNARC probability versus ICU/hospital outcomes indicate that admissions from different categories have different characteristics (see Section 5.1.2). Using the same prediction model could bring bias to the result. Thus, the prediction will not merely consider all the admissions as a homogeneous group. Mortality prediction models will also be built individually for different categories of admissions; admission type, admission timing and admission sources will be adopted to categorise admissions. Every ICU admission contributes to one ICU outcome but probably more than one hospital outcome. Therefore, we put all the ICU admission data together for ICU mortality prediction but further divided hospital mortality prediction data into 'FirstAD' and 'LastAD' (see Section 5.3.2 for details). The categories for predicting ICU LoS will be based on the results of the KS tests and admission sources.

Table 5.2 shows all the models we will build in this chapter by admission categories. Different models will be built by incorporating suitable variables, where "AD" represents admission variables and "DIS" denotes discharge variables. In the table, symbol '✕' denotes 'not applicable' and symbol '/' denotes 'combined in other groups'. We have excluded the groups with insufficient numbers to build prediction models. All the variables included in the prediction models are listed in Appendix A.

Table 5.2: Prediction models (variables used)

Admission categories	**Mortality in ICU**	**Mortality in ICU and hospital**		**after-ICU Mortality**	**ICU LoS**
		FirstAD	*LastAD*	*LastAD*	
All	AD	AD	AD	AD+DIS	AD
Plannned	AD	AD	AD	AD+DIS	AD
Unplanned	AD	AD	AD	AD+DIS	AD
Unplanned peak time	AD	AD	AD	AD+DIS	AD

Continued on next page

Table 5.2 – continued from previous page

Admission categories	**Mortality in ICU**	**Mortality in ICU and hospital**		**after-ICU Mortality**	**ICU LoS**
		FirstAD	*LastAD*	*LastAD*	
Unplanned off-peak	AD	AD	AD	AD+DIS	AD
Source A (A&E)	AD	AD	AD	AD+DIS	/
Source B (Operating theatre)	AD	AD	AD	AD+DIS	/
Source D (General Ward)	/	/	AD	/	/
Source Z (Others)	/	/	AD	/	/
Source D+Z	AD	AD	AD	AD+DIS	×
Unplanned Source B	AD	AD	AD	AD+DIS	×
Unplanned Source D	AD	AD	AD	AD+DIS	×
Planned Source B	AD	AD	AD	AD+DIS	×
Non-late	AD	AD	AD	AD+DIS	AD
Non-late planned	/	AD	×	×	/
Non-late unplanned	/	AD	×	×	/
Late	AD	AD	AD	AD+DIS	AD
Non-late planned (no readmission)	AD	×	AD	AD+DIS	AD
Non-late unplanned (no readmission)	AD	×	AD	AD+DIS	AD
Readmission	×	×	AD	AD+DIS	AD

AD, admission variables; DIS, discharge variables;

×, not applicable; /, combined in other groups.

As discussed in Section 2.1 ICNARC has developed a comprehensive way to predict the

probability of mortality of patients in hospitals. Every patient admitted to an ICU has an ICNARC probability which indicates his/her probability of death. ICNARC probability is composed of the ICNARC score and GCS. Results predicted by ICNARC model are proved to be more accurate than the prediction using APACHE II score (Harrison et al., 2007).

Three different models, two logistic regression models and a decision tree, will be built for mortality prediction for each admission category for: 1) mortality in ICU (ICU mortality), 2) I. mortality in hospital of all first admissions (hospital mortality FirstAD) and II. mortality in hospital of all last admissions (hospital mortality LastAD) and 3) mortality in hospital after ICU-stay (after-ICU mortality). First, a benchmark model, a logistic regression model utilising ICNARC probability only will be built as a benchmark model for ICU and hospital mortality prediction. Second, an improved logistic regression model, using all the admission variables including ICNARC probability, will be built. The final logistic regression model will be chosen based on the variables' contributions to the reduction of residual deviation and the lowest model AIC. Third, a decision tree based on the Classification and Regression Tree (CART) algorithm will be constructed.

In a logistic regression model, Y_i represents the state of admission i. The occurrence of event 'discharged dead' and 'discharged alive' in the admission is denotes as $Y_i = 0$ and $Y_i = 1$. The probability of $Y_i = 1$ (p_i) is denoted by $\mathbf{E}[Y_i|\mathbf{X_i}]$ for given admission attributes $\mathbf{X_i}$.

A logistic regression model could be generalised to

$$\mathbf{E}[Y_i|\mathbf{X_i}] = p_i = \frac{1}{1 + e^{-\beta^{\mathbf{T}}\mathbf{X}}},$$

where

$$\beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_j \\ \vdots \\ \beta_n \end{bmatrix}, \mathbf{X} = \begin{bmatrix} 1 \\ x_{i,1} \\ \vdots \\ x_{i,j} \\ \vdots \\ x_{i,n} \end{bmatrix};$$

$x_{i,j}$ are patients' attributes; β_j are parameters estimated by maximum likelihood for independent variable $x_{i,j}$.

Classification and regression methods are adopted in the prediction of LoS in ICU. A regression model for the hospital LoS will be built with all the admission categories combined. Data cleaning follows the procedure described in Section 4.1.1. The predictions and analysis are conducted using the R 3.3.3 and rpart package.

Handling variables

Before including all the variables into the models, we need to deal with categorical variables that have numerous categories and some of the categories appear infrequently.

There are 466 distinctive admission reasons in the ICNARC methods. We counted the frequency of every admission reason and divided them into four groups, Frequent, High risk, Low risk and Others. Admission reasons under "Frequent" appear at least 200 times as the primary admission reason. "High risk" contains admission reasons appear between 10 and 199 times and average ICNARC probability over 50. "Low risk" contains admission reasons appear between 10 and average ICNARC probability less than 50. "Others" include are the other admission reasons appearing 0 to 9 times in our dataset. There are six, eight, 114 and 338 admission reasons under each group respectively. A new nominal variable, "CateReason", is created to replace admission reasons. "CateReason" contains nine categories, which are all the admission reasons in the "Frequent" group, i.e. Anoxic or ischaemic

coma or encephalopathy, Primary lung tumour, Malignant neoplasm of oesophagus, Pancreatic or pancreato-duodenal tumour, Secondary hepatic tumour and Pneumonia, no organism isolated, High risk, Low risk and Others.

5.3 Mortality prediction

5.3.1 ICU mortality prediction

There are 6022 admissions in total. Random sampling was used to split the dataset into training and testing parts, with 2/3 as a training dataset and 1/3 testing.

For the different admission categories, appropriate data points were taken from the training and testing datasets. Table 5.3 lists numbers of data points in the training and testing datasets for all the ICU mortality prediction models. Because of low numbers, we will not predict group Z1 and Z0+D1 separately.

Table 5.3: Numbers of data points for ICU mortality prediction models

Prediction Model	**Total**	**Training**	**Testing**
all	6022	4016	2006
planned	2430	1626	804
unplanned	3592	2389	1203
unplanned peak	2122	1416	706
unplanned off-peak	1470	973	497
A	1114	744	370
B	3271	2190	1081
D+Z	1366	900	466
F	271	181	90
B0	990	660	330
D0	1133	748	385
Z0+D1	151	103	48
B1	2281	1530	751
Z1	82	49	33
non late	5004	3329	1675
Non-late planned (no readmission)	2299	1538	761
Non-late unplanned (no readmission)	2705	1790	915
late	804	544	260

ICU mortality predictions: all admissions

Three different models were built for the dataset containing all admissions. A logistic regression model of predicting mortality based on ICNARC probability only was built as a benchmark. A comprehensive logistic regression model was constructed to see if mortality prediction could be improved by adding other elements. A decision tree based on the CART algorithm was also assessed.

Using maximum likelihood estimation, the benchmark model for this dataset could be written as

$$E(Y_i|X) = \frac{1}{1 + e^(0.05934x - 3.77302)}.$$

The maximum split point of the benchmark model is ICNARC.probability=63.5831.

We then built the decision tree. Figure 5.9 shows the tree where the left and right leaves represent the predicted deaths and survivals. The variables involved are also shown in Figure 5.9.

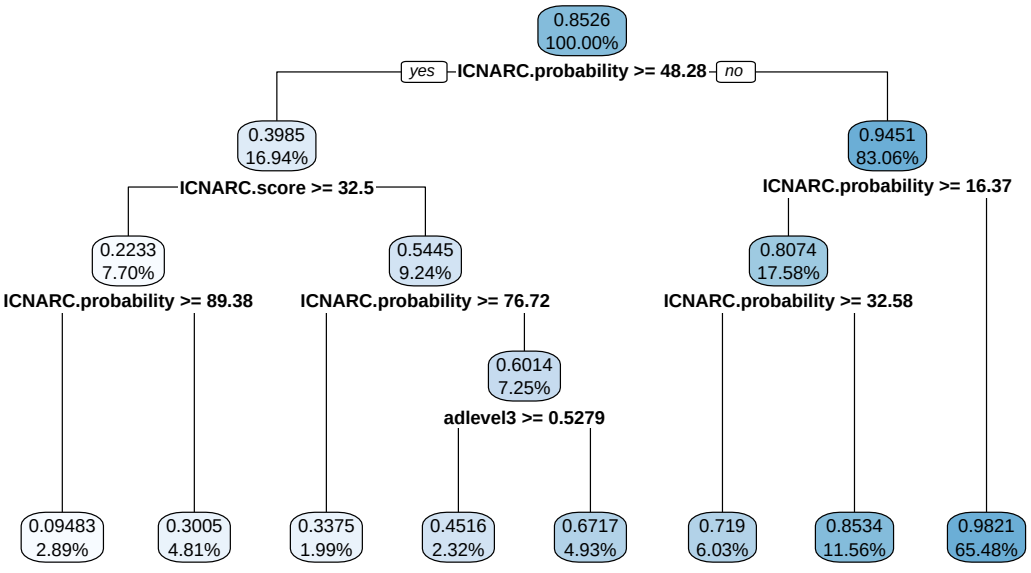


Figure 5.9: Classification tree for ICU mortality prediction (all admissions)

A comprehensive logistic regression model was built for all the admissions. The final model is selected by the minimum AIC values with avoidance of multicollinearity. As we discussed in Section 5.1.1, only one variable within a group of highly correlated variables will be kept in the final model. Table 5.4 gives the coefficients of variables and their p-values.

Table 5.4: Logit coefficients for ICU mortality prediction (all admissions)

	Estimate	P-value
(Intercept)	5.0557	0.0000
Age	-0.0246	0.0000
SourceB	0.0073	0.9702
SourceD	-0.3702	0.0254
SourceF	0.9851	0.0000
SourceZ	-0.2664	0.4170
ICNARC.score	-0.1695	0.0000
adPA	0.3228	0.0094
planned	0.5550	0.0104
CateReasonhighrisk	0.0899	0.7241
CateReasonlowrisk	1.2638	0.0000
CateReasonMalignant neoplasm of oesophagus	1.7393	0.0026
CateReasonothers	1.3779	0.0000
CateReasonPancreatic or pancreato-duodenal tumour	1.7382	0.0103
CateReasonPneumonia, no organism isolated	1.4788	0.0000
CateReasonPrimary lung tumour	1.1587	0.0108
CateReasonSecondary hepatic tumour	3.0282	0.0095

We use both AUROC and KS distance to assess the models. Table 5.5 gives AUROC and KS distances for different models. Figure 5.10 displays the ROC curves for the different models. The logistic regression model performs similarly to the benchmark model as performance measured by AUROC and KS suggests conflicting result. The decision tree from CART performed worse than the other two models.

Table 5.5: AUROC and KS distance for ICU mortality prediction models (all admissions)

	Training AUROC	Testing AUROC	Training KS	Testing KS
Benchmark	0.9164	0.9073	0.6872	0.6813
Logistic regression	0.9198	0.9119	0.6911	0.6741
Decision tree	0.9023	0.8920	0.6757	0.6561

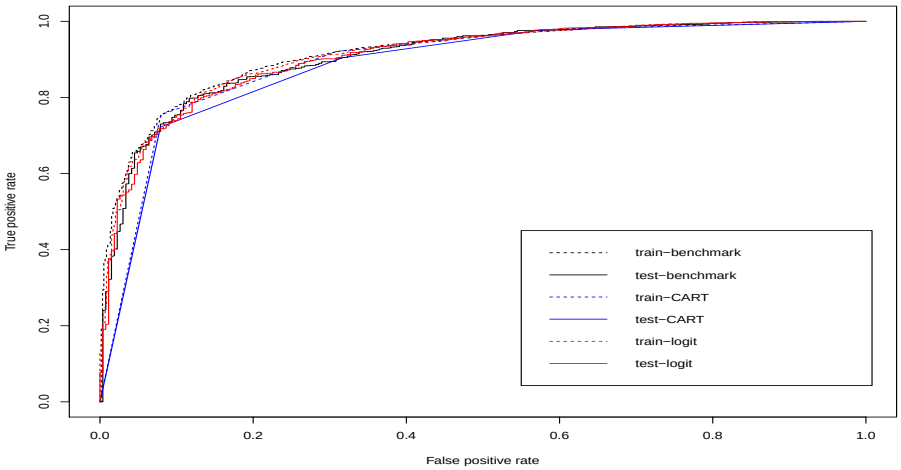


Figure 5.10: ROC curves for ICU mortality prediction models (all admissions)

5.3.2 Hospital mortality prediction

The ICNARC mortality prediction model is designed to predict hospital mortality (death both in ICU and elsewhere in hospital) of ICU patients. It has proved to be the most effective mortality probability prediction model in the UK nationwide. A study of 343,860 admissions to adult ICUs in England, Wales and Northern Ireland gave AUROC=0.870. An external validation in 23,269 admissions to Scotland ICUs shows a similar discrimination power (AUROC=0.848) (Harrison et al., 2007, 2014). Our models look in detail at a number of admission categories and investigate possible improvements in mortality prediction compared to the baseline ICNARC models.

Hospital mortality prediction requires dealing with readmissions. We cannot consider every admission in the prediction. Instead, we will carry out predictions twice for two different datasets considering either first or last admissions. First, we use only first time admissions, all patients' admissions from admission group 0 and 1. All the patients are distinct within this group. We use "FirstAD" to denote these patients. Second, we will use all the last admissions. All admissions from admission group 0 and part of admissions from group 2 will be involved. For group 0 admissions, as they are admissions for one time only patients, they are included in both of the two predictions. We denote it as "LastAD".

The same prediction models as ICU mortality prediction will be built in this section; we will create three models, benchmark, logistic regression model and decision tree using the CART algorithm, for two datasets considering each admission category. Results of these three models will be assessed and compared using ROC curves, AUROC and KS distances for discrimination power.

For the different admission categories, appropriate data points are taken from the training and testing datasets created in Section 5.3.1. Tables 5.6 and 5.7 list numbers of data points in the training and testing datasets for all the hospital prediction models.

Table 5.6: Numbers of data points of hospital mortality prediction models (FirstAD)

	Total	**Training**	**Testing**
all	5808	3872	1936
planned	2397	1602	795
unplanned	3411	2270	1141
unplanned peak	2016	1347	669
unplanned off peak	1395	923	472
A	1114	744	370
B	3195	2134	1061
D+Z	1228	813	415
F	271	181	90
B0	940	624	316
D0	1004	665	339
Z0+D1	146	101	45
B1	2255	1510	745
Z1	78	47	31
non late	5004	3329	1675
Non-late planned	2299	1538	761
Non-late unplanned	2705	1790	915
late	804	544	260

Table 5.7: Numbers of data points of hospital mortality prediction models (LastAD)

	Total	**Training**	**Testing**
all	5808	3874	1934
planned	2320	1554	766
unplanned	3488	2320	1168
unplanned peak	2059	1370	689
unplanned off peak	1429	950	479
A	1101	733	368
B	3128	2092	1036
D	1136	759	377
F	271	181	90
Z	172	109	63
D+Z	1308	868	440
B0	948	630	318
D0	1086	722	364
B1	2180	1462	718
D1	50	37	13
non late (no readmission)	4847	3220	1627
Non-late planned (no readmission)	2202	1474	728
Non-late unplanned (no readmission)	2645	1746	899
late	772	526	246
readmission	189	128	61

All Patients (FirstAD)

A logistic regression model of predicting mortality based on ICNARC probability only was built as a benchmark to assess if mortality prediction could be improved by adding other elements. The baseline model can be written as $E(Y_i|X) = \frac{1}{1+e^{(0.0576x-3.0769)}}$. The maximum split point of the benchmark model is ICNARC.probability=53.4088.

After an initial test of three decision tree algorithms, classification tree and logistic regression were recruited to predict patients' hospital mortality. For logistic regression, we used the stepwise method to select variables.

Table 5.8: Logit coefficients for hospital mortality prediction (FirstAD)

	Estimate	P-value
(Intercept)	3.7525	0.0000
Age	-0.0195	0.0000
Days.between.hospital.and.unit.admit	-0.0115	0.0407
ICNARC.score	-0.0900	0.0000
ICNARC.probability	-0.0246	0.0000
P2_system	0.0689	0.0136
adPA_1	0.5723	0.0766
CateReasonhighrisk	-0.2718	0.3074
CateReasonlowrisk	0.6668	0.0127
CateReasonMalignant neoplasm of oesophagus	1.3954	0.0113
CateReasonothers	0.5822	0.0412
CateReasonPancreatic or pancreato-duodenal tumour	0.4775	0.3204
CateReasonPneumonia, no organism isolated	1.0658	0.0021
CateReasonPrimary lung tumour	0.7247	0.0894
CateReasonSecondary hepatic tumour	2.0756	0.0153
SourceB	0.4130	0.0727
SourceD	-0.5413	0.0011
SourceF	0.8404	0.0004
SourceZ	-0.3889	0.1805
EMELEM	-0.7488	0.0003
Adpeak	0.2519	0.0292

Figure 5.11 gives the pruned CART tree. Only ICNARC probability and ICNARC score are recruited by the final pruned tree.

Logistic regression did better than the classification tree as shown in Figure 5.12 and Table

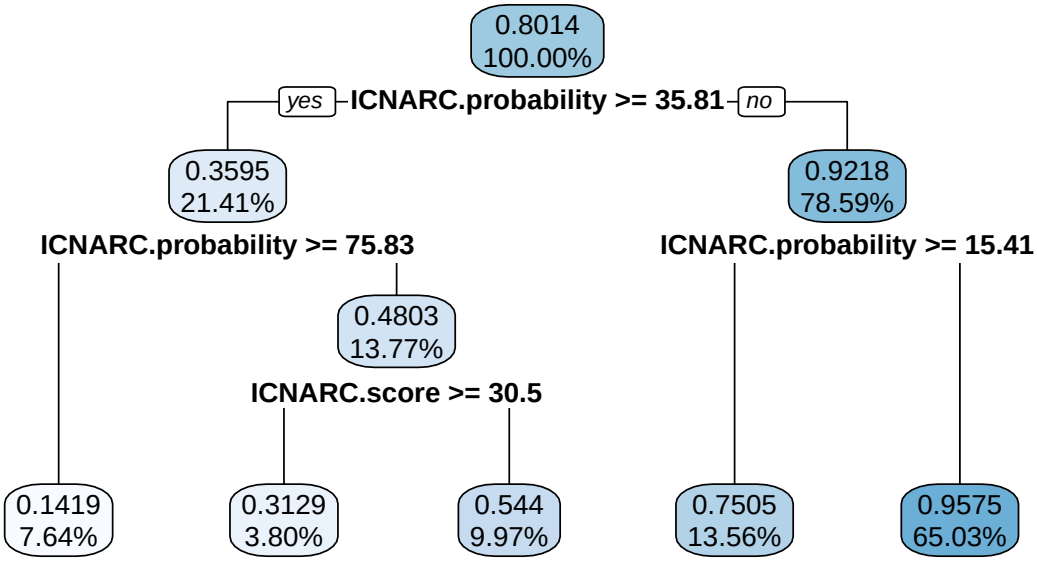


Figure 5.11: Classification tree for hospital mortality prediction (FirstAD)

5.9 regarding both AUROC and KS distance. However, the improvement is not huge.

Table 5.9: AUROC and KS distance for hospital mortality prediction models (FirstAD)

	Training AUROC	Testing AUROC	Training KS	Testing KS
Benchmark	0.8958	0.8743	0.6400	0.5923
Logistic regression	0.9016	0.8783	0.6352	0.6127
Decision tree	0.8685	0.8371	0.6378	0.5789

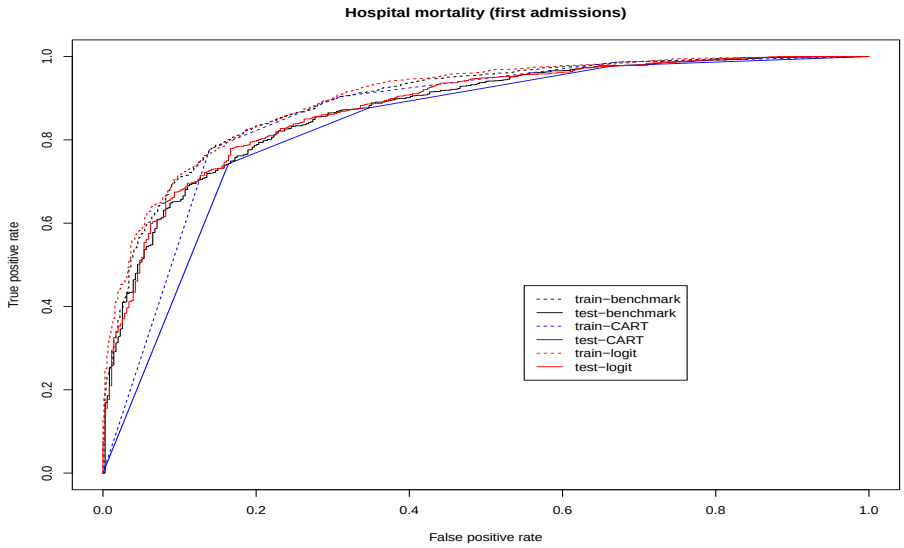


Figure 5.12: ROC curves for hospital mortality prediction models (FirstAD)

All patients (LastAD)

$E(Y_i|X) = \frac{1}{1+e^{(0.0585x-3.1565)}}$.

By using last admission only, we found a slightly different maximum split point of the benchmark model, ICNARC.probability=53.9752.

Table 5.10: Logit coefficients for hospital mortality prediction (LastAD)

	Estimate	P-value
(Intercept)	4.0607	0.0000
Age	-0.0181	0.0001
Days.between.hospital.and.unit.admit	-0.0156	0.0008
SourceB	0.0575	0.7716
SourceD	-0.3659	0.0358
SourceF	0.9077	0.0002
SourceZ	-0.2663	0.3775
ICNARC.score	-0.0925	0.0000

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Table 5.10 – continued from previous page

	Estimate	P-value
ICNARC.probability	-0.0269	0.0000
P2_system2	0.2585	0.2084
P2_system3	0.2491	0.1927
P2_system4	-0.0632	0.8144
P2_system5	-0.0641	0.8755
P2_system6	1.5753	0.0226
P2_system7	0.3944	0.1373
P2_system8	1.7889	0.0026
P2_system9	0.3410	0.3926
P2_system10	-0.1421	0.7926
P2_system11	-0.5171	0.4627
P2_system12	-13.6822	0.9664
adlevel1	6.3232	0.0094
adPA_1	0.5512	0.0933
Adpeak	0.3440	0.0030
CateReasonhighrisk	-0.7253	0.0381
CateReasonlowrisk	0.2972	0.3739
CateReasonMalignant neoplasm of oesophagus	1.3957	0.0278
CateReasonothers	0.2114	0.5482
CateReasonPancreatic or pancreato-duodenal tumour	0.5027	0.3634
CateReasonPneumonia, no organism isolated	0.7584	0.0653
CateReasonPrimary lung tumour	0.7393	0.1332
CateReasonSecondary hepatic tumour	1.9153	0.0295

Figure 5.13 shows the CART decision tree. It is much larger than the tree for first admissions. Also, it involves more variables. Admission PA and admission %L3 are included in the final tree, which indicates the influence of busyness of the unit on patients.

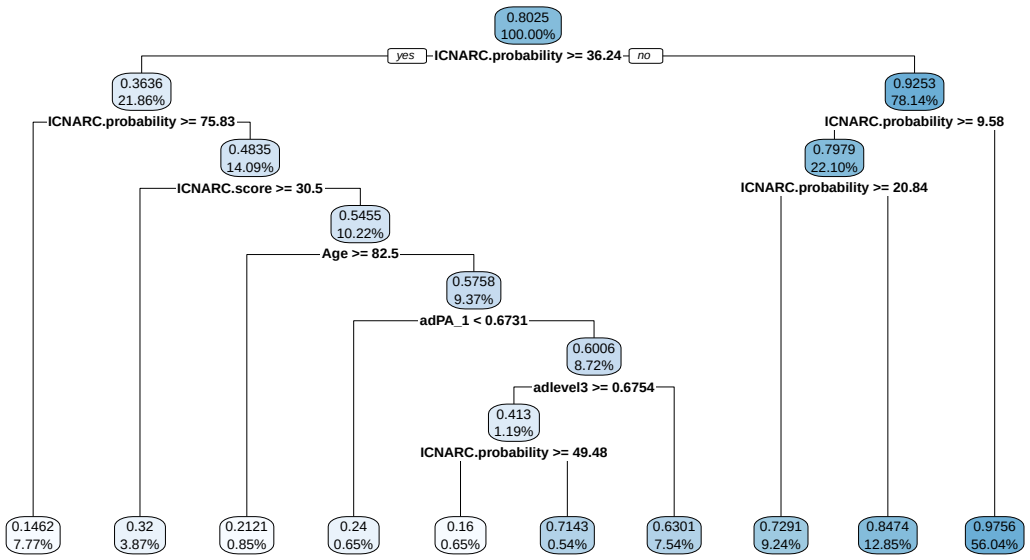

Figure 5.13: Classification tree for hospital mortality prediction (LastAD)

Table 5.11 and Figure 5.14 show the comparison of different models.

Table 5.11: AUROC and KS distance for hospital mortality prediction models (LastAD)

	Training AUROC	Testing AUROC	Training KS	Testing KS
Benchmark	0.9015	0.8849	0.6487	0.6176
Logistic regression	0.9070	0.8869	0.6521	0.6281
Decision tree	0.8942	0.8700	0.6484	0.6099

We find that the logistic regression model performs better in the training dataset but not as well in the testing dataset. This results from overfitting of the training dataset although variables in the model have been selected using AIC.

The discrimination power of the benchmark model is excellent as measured by AUROC in both two datasets. It outperforms the ICNARC development model (AUROC=0.870) and an external validation in Scotland data (AUROC=0.848) (Harrison et al., 2007, 2014).

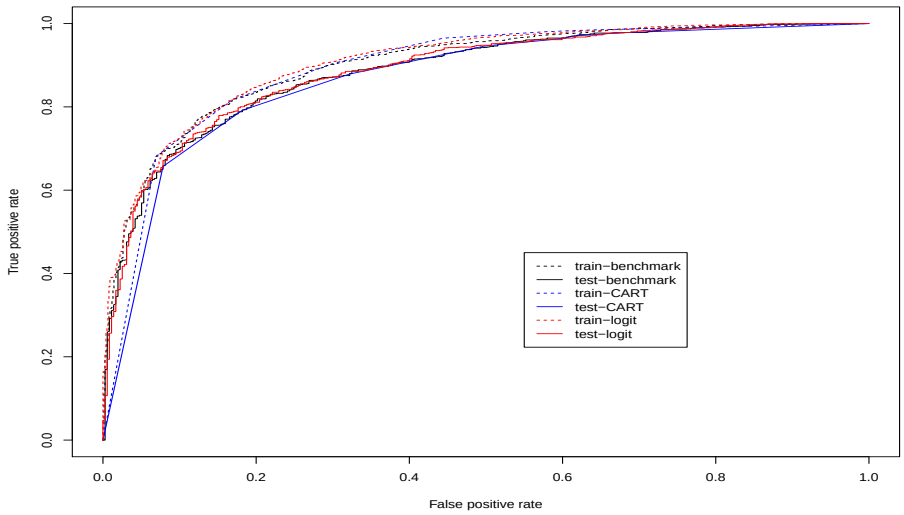

Figure 5.14: ROC curves for hospital mortality prediction models (LastAD)

5.3.3 After-ICU mortality

After-ICU / Post-ICU mortality means death after ICU discharge so deaths in ICU are not included in the prediction. We do not use first admissions of readmissions in this prediction since death after first ICU stay can occur during either their hospital stays or their subsequent stays in the ICU. Only the last admissions for readmitted patients will be modelled.

Last admissions

Patient's mortality after ICU is critical for modelling readmission. We found a split point at ICNARC.probability=96.0113 from $E(Y_i|X) = \frac{1}{1+e^{(0.0393x-3.7746)}}$. The number is very close to the largest ICNARC probability value (97.97) amongst all patients discharged alive from the ICU. This proves the effectiveness of ICU treatments but not very helpful in predicting after-ICU mortality.

Figure 5.15 shows the tree constructed for after-ICU mortality prediction. The final tree contains only one split based on ICNARC probability.

Table 5.12: Numbers of data points of after-ICU mortality prediction models

	Total	**Training**	**Testing**
all	4950	3282	1668
planned	2253	1510	743
unplanned	2697	1772	925
unplanned peak	1645	1083	562
unplanned off peak	1052	689	363
A	866	566	300
B	2947	1974	973
D+Z	977	637	440
F	160	105	55
B0	812	543	269
D0	804	522	282
B1	2135	1431	704
D1	41	31	10
non-late (no readmissions)	4202	2783	1421
non-late planned (no readmissions)	2148	1437	711
non-late unplanned (no readmissions)	2056	1346	710
late (no readmissions)	587	390	197
readmission	159	109	50

Table 5.13: Logit coefficients for after-ICU mortality prediction (LastAD)

	Estimate	P-value
(Intercept)	4.2547	0.0000
ReadmissionYes	0.7315	0.1246
SexM	-0.3928	0.0489
Age	-0.0294	0.0000
Days.between.hospital.and.unit.admit	-0.0292	0.0000
planned	0.6866	0.0798
ICNARC.probability	-0.0237	0.0000
Reason.dischargedB. Comparable critical care continuing	-1.4750	0.0203
Reason.dischargedD. More specialist critical care	-0.1531	0.8405
Reason.dischargedE. Repatriation	-2.1882	0.0000
Reason.dischargedF. Palliative care	-3.2478	0.0000
Reason.dischargedG. Self-discharge	12.2386	0.9819
EMELEM	-0.7247	0.1044
EMELNR	-0.7012	0.1216
adlevel1	12.6141	0.0197
adPA_1	1.6624	0.0017
DisReadynighteffect	0.8897	0.0713
CDispeak	0.5011	0.0364

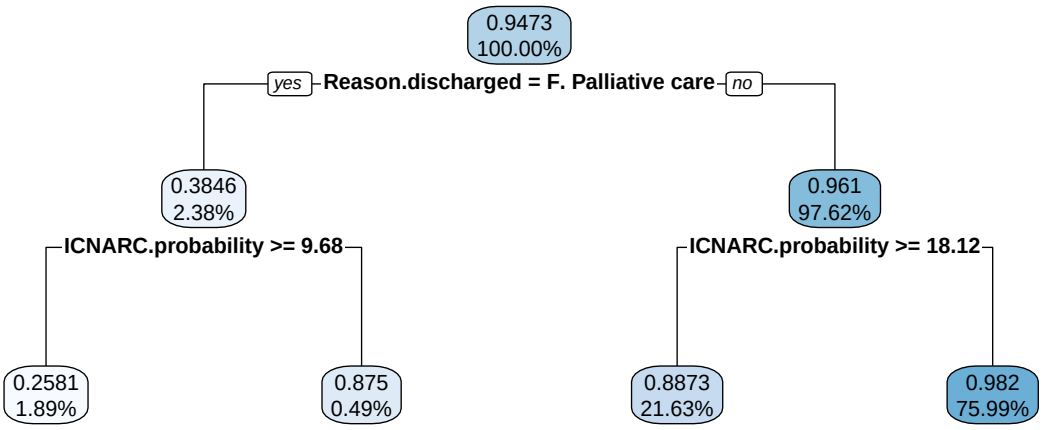

Figure 5.15: Classification tree for after-ICU mortality prediction (LastAD)

Table 5.14 and Figure 5.16 show the comparison of after-ICU mortality prediction models. The logistic regression model performed better than the other two models in the training dataset. However, the benchmark model tended to predict all the patients as alive.

Table 5.14: AUROC and KS distance for after-ICU mortality prediction models (LastAD)

	Training AUROC	Testing AUROC	Training KS	Testing KS
Benchmark	0.8164	0.7974	0.5255	0.4913
Logistic regression	0.8755	0.8235	0.5857	0.5198
Decision tree	0.7902	0.7772	0.4276	0.4771

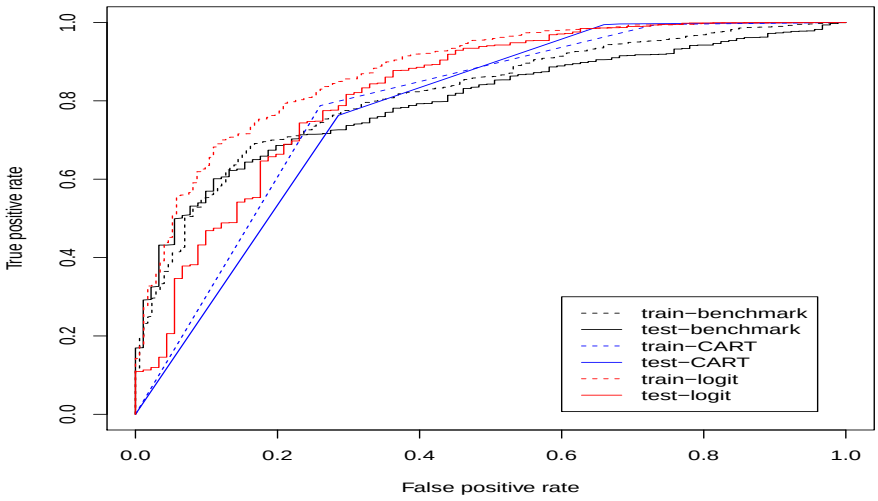

Figure 5.16: ROC curves for after-ICU mortality prediction models (LastAD)

5.3.4 Comparing the performance of mortality prediction models

We have included all the results of the predictability of the prediction models in Appendix D. We would like to highlight results which will be helpful in our future simulation model building.

For the ICU mortality prediction model, we find that the logistic regression model performs the best in the training dataset for almost all the groups of patients, although the ICNARC predictions are still excellent. However, the performance in the testing dataset is not guaranteed to be the best, although most of the time it is, as sometimes over-fitting may occur. The logistic regression models perform especially better than the benchmark models in admissions categorised by admission peaks for unplanned peak and unplanned off-peak patients. The performance of late admission prediction is also improved with both AUROC and KS in testing.

For hospital mortality predictions, the performance of the logistic regression models is similar to the benchmark models. The logistic regression models still do better in mortality prediction for late admission patients.

For after-ICU mortality prediction, the benchmark models are not as good as the benchmark ICU or hospital mortality prediction models since they based on admission variables only. The results from benchmark after-ICU mortality prediction show that the benchmark model tends to predict all the patients as 'alive', which is not true. Improvements in the after-ICU mortality prediction were shown in both CART and logistic regression models for some patient categories. These models include discharge variables so they would be expected to perform better than ICNARC models. This group of predictions is designed specifically for readmission modelling. Only patients who survived after their discharge from the ICU could be readmitted. Also, the prediction for the unplanned admissions group shows that the prediction can be based only on the operational factors at their discharge. That is to say, we do not need to consider the severity of patients' illness. This type of prediction could be valuable in simulation modelling.

Taking both simplicity and accuracy into consideration, three benchmark logistic regression models re-calibrating from current ICNARC models (i.e. using ICNARC probability as the only covariate) for planned, unplanned and late admissions respectively will be brought forward to the next Chapter.

5.4 LoS prediction

5.4.1 LoS prediction: single stage models

Two types of LoS are recorded in the dataset, clinical LoS and actual LoS. For patients discharged alive, clinical LoS was calculated as "clinically ready to discharge time - ICU admission time"; actual LoS was calculated as "actual discharge time - ICU admission time". For patients who died, clinical LoS and actual LoS are the same: "time of death - ICU admission time". Actual LoS reflects the time that a patient spends in the ICU. It is also the key factor of ICU occupancy and busyness. Thus actual LoS will be the target variable in LoS prediction. Clinical LoS will only be predicted in limited groups to observe differences between two types of LoS in terms of which variables are used in predictions. Chan et al.

(2016) include only patients who were discharged alive. To predict LoS more accurately, we use all patients and use patients' status to re-classify patients. As showed in Table 5.15, three different types of prediction which involve three different response variables are carried out in our research.

Table 5.15: Different response variables in actual LoS prediction

Response variable	Patient status groups
LoS1	All patients
LoS2	Patients discharged alive + Patients discharged died (LoS>8 hrs)
LoS3	Patients discharged alive only

The R package 'rpart' for CART was employed for variable selection. After that, linear regression was adopted for predicting LoS. Two things need to be pointed out. First, in the variable selection part, all available variables are included apart from those are highly correlated. Second, the admission reasons were categorised in the same way as in mortality prediction (see Section 5.3.1). Moreover, in LoS prediction, we also set aside one third of the dataset as a test dataset. Models were built on the training data. Out-of-sample tests were carried out using the test dataset.

The discrepancy between actual LoS and clinical LoS, namely operational delay, comes from mainly lack of beds in the general ward and also the preparation of paperwork. From the variable selection process, we find that PA is not included in the model construction for any of the clinical LoS predictions. However, the indicator contributed to building the models for predicting actual LoS of patients. From this we suppose PA reflects more on congestion of general wards rather than actually measuring overall severity of all ICU patients in a day.

The homoscedasticity assumption of OLS cannot be guaranteed by using original LoS directly, so we applied log-transformation to the LoS. We then use all the variables of importance in CART to construct the linear regression models of log-transformed LoS data and check R-squared values for all models.

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We compared our results with those from the research of Verburg et al. (2014). The R-

Table 5.16: R-squared and adjusted R-squared of log-transformed LoS prediction in both training and testing data

LoS	R-squared (training)	Adjusted R-squared (training)	R-squared (testing)	Adjusted R-squared (testing)
LoS1	0.1578	0.1334	0.0670	0.0598
LoS2	0.1835	0.1616	0.0556	0.0507
LoS3	0.3286	0.3042	0.0412	0.0392

squared for all patients and survivors are 0.149 and 0.196 using log-transformed LoS in their research, which are very similar to our results in Table 5.16. Meanwhile, we have the same problem as theirs, the heteroscedasticity in the linear regression cannot be solved even after the log-transformation of LoS data. This causes robustness problems when using the model. Therefore, to be prudent, we perform linear regression on the original LoS data and report the results of heteroscedasticity consistent (HC) coefficients and p-values instead of the result from the original linear regression using OLS estimations in Tables 5.18 and 5.20. The R-squared values for models fitted with original data are also reported in Table 5.17.

As we can see from model coefficients in predicting LoS3 in Table 5.20, discharge PA has a negative effect on ICU LoS, which is consistent with the supposition we made before. It suggests that a lower PA means a busier general ward which will eventually lead to a longer ICU LoS. However, if patients who were discharged dead are included in the prediction, PA is not a significant variable in the model shown in Tables 5.18 and 5.19.

Table 5.17: R-squared and adjusted R-squared of LoS prediction in both training and testing data (original data)

LoS	R-squared (training)	Adjusted R-squared (training)	R-squared (testing)	Adjusted R-squared (testing)
LoS1	0.1262	0.1205	0.1138	0.1253
LoS2	0.1439	0.1380	0.1398	0.1291
LoS3	0.2589	0.2524	0.2338	0.2219

Table 5.17 suggests that if only patients discharged alive are included in LoS prediction, the predictive power of models will be significantly increased. However, in modelling operation of the ICU, LoS of every patient is critical

Table 5.18: HC estimations of LoS prediction model (all patients)

	Estimate	P-value
(Intercept)	-645.4569	0.5363
ICNARC.score	226.0699	0.0000
EMELEM	1742.0405	0.0000
EMELNR	1839.5308	0.0007
SourceD	2362.6347	0.0001
SourceF	1673.5512	0.0483
SourceZ	5815.9121	0.0002
CateReasonhighrisk	-1802.4851	0.0814
CateReasonlowrisk	2631.2908	0.0014
CateReasonMalignant neoplasm of oesophagus	4303.2957	0.0001
CateReasonothers	1726.2010	0.0655
CateReasonPancreatic or pancreato-duodenal tumour	2337.5917	0.0249
CateReasonPneumonia, no organism isolated	3730.6379	0.0055
CateReasonPrimary lung tumour	2316.0894	0.0356
CateReasonSecondary hepatic tumour	703.0581	0.4393
P2_system2	-1122.4750	0.0685
P2_system3	954.0693	0.0741
P2_system4	-2001.2075	0.0026
P2_system5	-408.4263	0.6179
P2_system6	-3569.7125	0.0000
P2_system7	-1686.5200	0.0049
P2_system8	-2016.3440	0.0035
P2_system9	-4873.4496	0.0000
P2_system10	-1351.2166	0.2614
P2_system11	-4641.0006	0.0002
P2_system12	6364.8968	0.0000

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Table 5.18 – continued from previous page

	Estimate	P-value
Days.between.hospital.and.unit.admit	20.2229	0.1699

Table 5.19: HC estimations of LoS prediction model (alive or dead after 8hr of admission)

	Estimate	P-value
(Intercept)	-2516.9192	0.0616
ICNARC.score	271.2897	0.0000
EMELEM	1614.3776	0.0000
EMELNR	660.3809	0.3916
SourceD	1587.1246	0.0161
SourceF	1659.2124	0.0584
SourceZ	5713.9237	0.0005
P1	987.8538	0.1750
P2_system2	-959.1475	0.1214
P2_system3	1046.3395	0.0563
P2_system4	-1947.4843	0.0040
P2_system5	-576.0929	0.4882
P2_system6	-3405.6068	0.0000
P2_system7	-1703.2828	0.0047
P2_system8	-1799.7974	0.0081
P2_system9	-4226.8293	0.0000
P2_system10	-1453.9837	0.2327
P2_system11	-4192.0016	0.0009
P2_system12	5068.6933	0.0000
CateReasonhighrisk	-1485.4667	0.1807
CateReasonlowrisk	2941.3307	0.0005
CateReasonMalignant neoplasm of oesophagus	4507.8267	0.0001
CateReasonothers	1954.3721	0.0409

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Table 5.19 – continued from previous page

	Estimate	P-value
CateReasonPancreatic or pancreato-duodenal tumour	2713.9508	0.0104
CateReasonPneumonia, no organism isolated	3907.3260	0.0040
CateReasonPrimary lung tumour	2652.6675	0.0183
CateReasonSecondary hepatic tumour	1013.3069	0.2749
AdLate	1828.3979	0.0100

Table 5.20: HC estimations of LoS prediction model (alive patients)

	Estimate	P-value
(Intercept)	-1778.1761	0.2132
ICNARC.score	538.3599	0.0000
P2_system2	-739.9817	0.2290
P2_system3	1360.3694	0.0102
P2_system4	-1339.7250	0.0612
P2_system5	43.2905	0.9536
P2_system6	-2794.0598	0.0000
P2_system7	-1799.9773	0.0016
P2_system8	-2344.4433	0.0003
P2_system9	-4510.9813	0.0000
P2_system10	-1109.3204	0.2954
P2_system11	-3328.6514	0.0032
P2_system12	2766.5200	0.0002
adPA_1	1371.8698	0.0824
disPA	-6881.0864	0.0007
SourceB	-993.7363	0.0974
SourceD	1623.7478	0.0255
SourceF	262.2693	0.8313
SourceZ	6592.1270	0.0004

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Table 5.20 – continued from previous page

	Estimate	P-value
AdLate	1465.9221	0.0497
CateReasonhighrisk	-1320.0970	0.4278
CateReasonlowrisk	1898.8090	0.0706
CateReasonMalignant neoplasm of oesophagus	2313.1100	0.0602
CateReasonothers	1174.8592	0.3142
CateReasonPancreatic or pancreato-duodenal tumour	2088.2720	0.0903
CateReasonPneumonia, no organism isolated	3065.6165	0.0540
CateReasonPrimary lung tumour	1117.5492	0.3645
CateReasonSecondary hepatic tumour	384.5207	0.7317
EMELEM	951.3637	0.0081

The R-squared of the training dataset is not achieved in the prediction of testing dataset. It performs only slightly better than the a random guess. To better predicting LoS, we will proceed to a two-stage model.

5.4.2 LoS prediction: two-stage models

As detailed in Section 5.4, the prediction of LoS of patients discharged alive was better than the prediction of LoS of all patients. Moreover, ICU mortality can be accurately predicted by logistic regression models (see Section 5.3.1). A "two-stage" process is therefore devised for ICU LoS prediction of all patients with the first stage being prediction of ICU mortality and the second stage prediction of LoS split by whether the patient is predicted to be discharged alive or not. Such a model has a potential to predict LoS more accurately then the previous single-stage methods.

The "two-stage" approach consist of three models in total. The first stage is building a model to predict mortality of all patients. The mortality prediction model using logistic regression described in Section 5.3.1 will be adopted here. Then, two linear regression models are built

to predict LoS for patients likely to be discharged alive or discharged dead are constructed separately. Finally, R-squared for the model as a whole is calculated using predicted and original LoS values to assess the performance of the two-stage model.

The linear regression model for patients who died in the ICU achieved 0.2481 and 0.2259 of R-squared and adjusted R-squared respectively. We finally achieve 0.1909 of R-squared and 0.1852 of adjusted R-squared for the two-stage model. The model shows improvement compared to the one-step linear regression models. However, the predictability is not high enough to give accurate prediction.

5.5 Conclusion

This chapter investigated mortality prediction and LoS prediction models. In thestudy of mortality prediction, the benchmark ICNARC models are shown to be excellent in predicting patients' death in ICU and in hospital in most of the groups but not so strong in prediction after an ICU stay in any groups. However, benchmark mortality prediction of late admissions, in ICU, in hospital and after-ICU, is at a relatively lower power. This reassured the possible negative impact of late admissions on ICU mortality and LoS and worthiness of incorporation late admissions in prediction models. Therefore, we have built separate enhanced models to predict mortality of late admissions and other patient categories. Benchmark ICU mortality prediction models (unplanned, planned and late admissions) will be used in simulation modelling in Chapter 6 because of good predictability and simplified data requirement.

We found an interesting point in after-ICU mortality prediction for non-late unplanned patients that only discharge reason and operational factors (discharge time and ICU busyness) are included in the logistic regression model. However, discharge reasons are not included in our simulation in Chapter 6 so we are not able to carry out post-ICU predictions in the simulation. The focus of our simulation is on late admission rather than discharge reasons.

When modelling LoS, we use three different combinations of patients with LoS as a response variable and then assess the results of both log-transformed and original data. Models con-

cerning only patients discharged alive from ICU perform better than other models. However, modelling the real world ICU cannot ignore dead patients. The R-squared values for all prediction models are not very high. The coefficients of LoS prediction models suggest that PA has a negative effect on ICU LoS of patients. That is to say, low PA at discharge can reflect general ward busyness and result in longer LoS.

To better understand the pattern of LoS, we will proceed in Chapter 6 with distribution fitting to describe the data for the purpose of simulation modelling.

Chapter 6

Simulation Models

Chapters 4 and 5 inform the simulation model in this chapter, concerning, for example, the split of patient groups and ICU mortality prediction. To begin our investigation of the potential benefits and challenges of combining data mining and simulation modelling, in this chapter, a discrete event simulation (DES) model is described that depicts the ICU in our case study.

A description of the conceptual ICU model is provided first in Section 6.1. We then go on to describe and justify the choice of input distributions for the simulation model in Sections 6.2 to 6.4. The development of these distributions draws on the results of Chapters 4 and 5 but we also introduce a novel way of modelling LoS is in Section 6.3.2. The logic and further details of the DES model are provided in Section 6.5 and an analysis of input uncertainty of the DES model is provided in Section 6.6. A standard verification and validation process is followed for the model and described in Section 6.7. Conclusions from this chapter are provided in Section 6.8.

6.1 Conceptual model

Our research focuses on late admission and its influences. The conceptual model is illustrated in Figure 6.1. The model aims at emphasising the late admission group. There is a differentiation between planned and unplanned admissions. As discussed in Section 5.1.2, planned admissions are modelled according to an empirical distribution function (EDF), mostly arriving in peak times. Unplanned arrivals are generated by an NHPP and may occur at any time of the day. Admission sources or disease categories are not modelled but readmission to the ICU is included. Busyness in the general ward (leading to low PA in the ICU) is not accounted for. The conceptual model was verified by an ICU specialist.

As discussed in Sections 4.2.5, 4.4 and 5.1.2, most of the performance divergences for patients admitted from different sources can be captured by ICNARC probability. However, the performances of unplanned and planned admissions are very different after case-mix adjustment. Moreover, unplanned and planned arrivals to ICUs are usually modelled separately in research (see Section 5.1.2). Therefore, the unplanned arrival process and the planned arrival process are modelled separately in this research.

As described in Section 2.2, all the beds are equipped with L3 facilities; the only difference in service is the nurse-to-patient ratio. Therefore we do not distinguish L2 and L3 beds in the model.

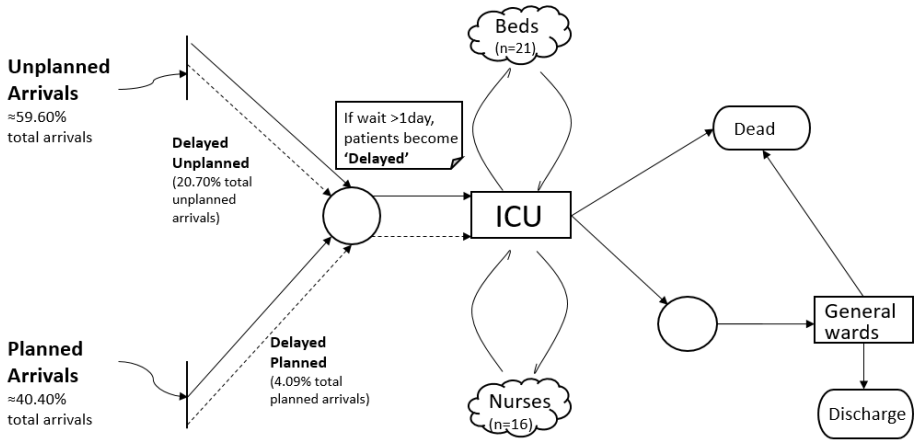


Figure 6.1: Conceptual Model

Data needed for a simulation model of the ICU can be identified from Figure 6.1. The planned and unplanned arrival processes need to be modelled separately; these are discussed in Section 6.2. As a key part of the model ICU, a new LoS modelling method is used as introduced in Section 6.3.2. A simplified version of mortality prediction is used as described in Section 6.4. LoS in the general wards for post-ICU patients is obtained from the data. An exponential distribution is used to model length of general wards' stay. Our data is, thus, sufficient to build a DES model that can model the aspects of the ICU which we are interested.

6.2 Input distributions for arrival processes

6.2.1 Unplanned arrival process

Poisson Processes (PPs) are used in modelling arrival processes in ICUs in several applications (Bai et al., 2016). In our research, a PP with a constant arrival rate cannot fit the data well because of the weekly pattern. An NHPP with piece-wise rates is therefore adopted here. The method of variance-to-mean ratio (VMR= $\frac{\sigma^2}{\mu}$, where σ is standard deviation and μ is mean) is adopted here to ensure that our data can be adequately described by the selected NHPP intervals. The VMR of the arrival number for each time interval should be around 1, for a Poisson distribution to apply (Cox and Lewis, 1966, p. 72). Taking interpretablity into consideration, three different time intervals are tried and the best one is picked here: 1) seven intervals using day of a week, 2) 14 intervals standing for 14 half days (i.e. 0:00am to 11:59am and 12:00pm to 23:59pm) in a week and 3) 14 intervals representing 14 shifts in a week.

Table 6.1: The VMRs of shift time intervals for unplanned arrivals

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Day Shift	1.2820	1.0077	1.0877	1.2161	1.0621	1.2329	1.1037
Night Shift	1.0659	0.9981	1.1659	1.1943	1.0679	1.1065	0.9730

After comparing VMRs, the division of 14 intervals representing 14 shifts is selected. Table

6.1 shows that the VMR for each interval is approximately 1 with this interval division, which means an NHPP using shifts as time intervals is suitable to model unplanned arrivals in our data. The VMRs for other two interval divisions are listed in Table F.1 and Table F.2 in Appendix F.

An arrival rate is estimated for each shift, where day shifts start from 08.00 and end at 19.59 and night shifts account for the period from 20.00 to 07.59 on the following day. Parameters of exponential distributions for the DES model are listed in Table 6.2; the inter-arrival times (hours) equal to 1/ mean hourly number of unplanned arrivals (i.e. $\frac{1}{\lambda/12}$, where λs are average arrival numbers in the intervals).

Table 6.2: Average inter-arrival time (hours) for exponential distribution for unplanned admissions

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Day Shift	13.9770	15.0744	15.5234	15.3924	14.1313	17.5962	15.6567
Night Shift	14.9508	15.3277	15.7922	13.8707	14.8780	15.1240	13.4118

6.2.2 Planned arrival process

For planned admissions, seven empirical distribution functions (EDFs) for number of patients arriving on different days of the week are used for modelling arrivals. Next, the arrivals are assigned to different admission times according to an EDF of admission time for planned arrivals, which is the same for each day of the week.

Assuming $(X_1, \cdots, X_n)$ are independent identically distributed random variables (the numbers of daily planned arrivals), EDFs are calculated using

$$\hat{F}_n(t) = P(X \leq t) = \frac{\text{number of elements in the sample} \leq t}{n},$$

with values listed in Table 6.3. For a more straightforward illustration, frequency plots are shown in Figures F.1 and F.2

Table 6.3: EDFs of numbers of planned arrivals

	Cumulative probability of numbers of arrivals						
# of arrivals (t)	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
$t < 0$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$0 \leq t < 1$	0.2303	0.1842	0.1908	0.1711	0.2007	0.7566	0.8651
$1 \leq t < 2$	0.5888	0.4737	0.5592	0.4375	0.5263	0.9737	0.9836
$2 \leq t < 3$	0.8487	0.8224	0.8289	0.7763	0.8092	1.0000	0.9967
$3 \leq t < 4$	0.9770	0.9539	0.9539	0.9474	0.9770	1.0000	1.0000
$4 \leq t < 5$	0.9934	0.9967	0.9967	0.9901	0.9967	1.0000	1.0000
$t \geq 5$	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Similarly, assuming $(X_1, \cdots, X_n)$ are independent identically distributed random variables, admission hour of planned patients, in this case, the EDF is described in Table F.3 and relative frequencies are plotted in Figure 6.2.

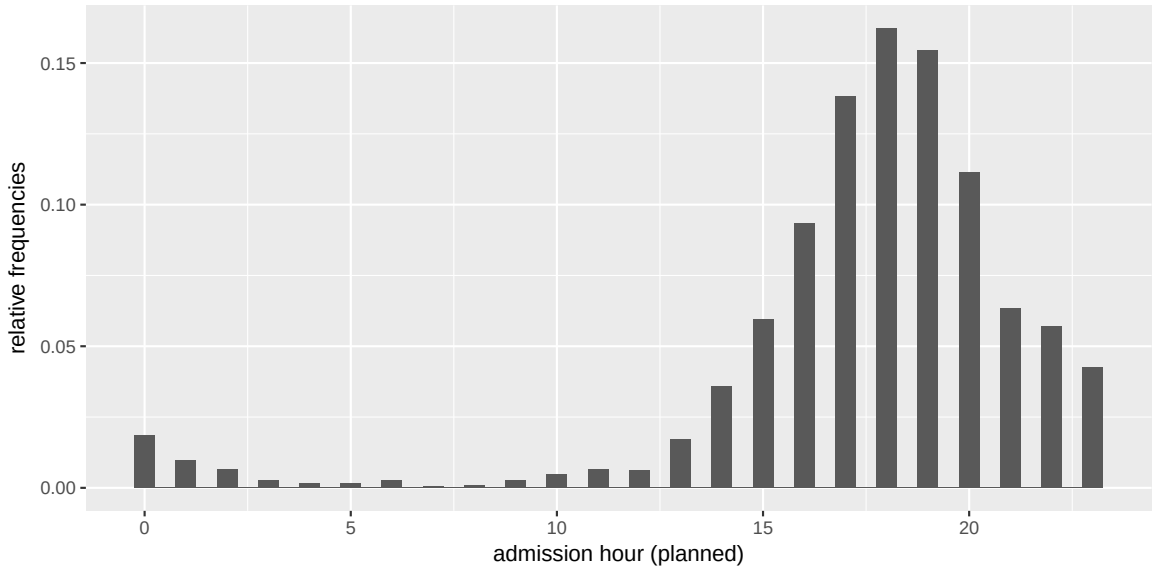


Figure 6.2: Admission hours of planned arrivals

6.3 LoS modelling

As shown in Section 5.4, predicting LoS on an individual patient basis does not work well in our data. A collective way is therefore tried in this chapter. The method of fitting

distributions to the data is tried first. Then, we propose a new model to sample LoS.

6.3.1 LoS distributions

LoS in ICUs is captured by exponential distributions as discussed in Chapter 3. In this research, several distributions were fitted to LoS and tested using goodness of fit. The distribution fitting was for six categories of patients: planned, unplanned, late, readmissions, late and readmission combined and all patients combined. Figure 6.3 shows the frequency plot, PP plot, QQ plot and cdf plot for all patients.

Figure 6.3 illustrates that none of the tested distributions fit the data well. A goodness of fit test was also carried out to statistically prove the results. A Chi-square test, KS test and CVM test rejected the null hypothesis of no significant difference at 0.01 level for all the tested distributions. Therefore, the tested distributions cannot capture the LoS pattern in our data well.

However, the PP plot in Figure 6.3 shows an interesting feature. The multiple spikes of LoS indicate a cyclical characteristic of LoS, which result from peaks in admission numbers and discharge numbers during the day, as noted in Chapter 4 (see Figures 4.11 and 4.13).

6.3.2 LoS modelling using sub parts

As a result of its cyclic behaviour, we propose modelling LoS in three parts as shown in Figure 6.5: admission time, nights spent in ICU and discharge time. We use "*subLoS*" to denote the separate components of the LoS. The LoS can be expressed as

$$LoS = subLoS1 + subLoS2 + subLoS3\,,$$

where

$$subLoS1 = 24 - AdH,\ subLoS2 = (nights - 1) * 24,\ subLoS3 = DisH\,.$$

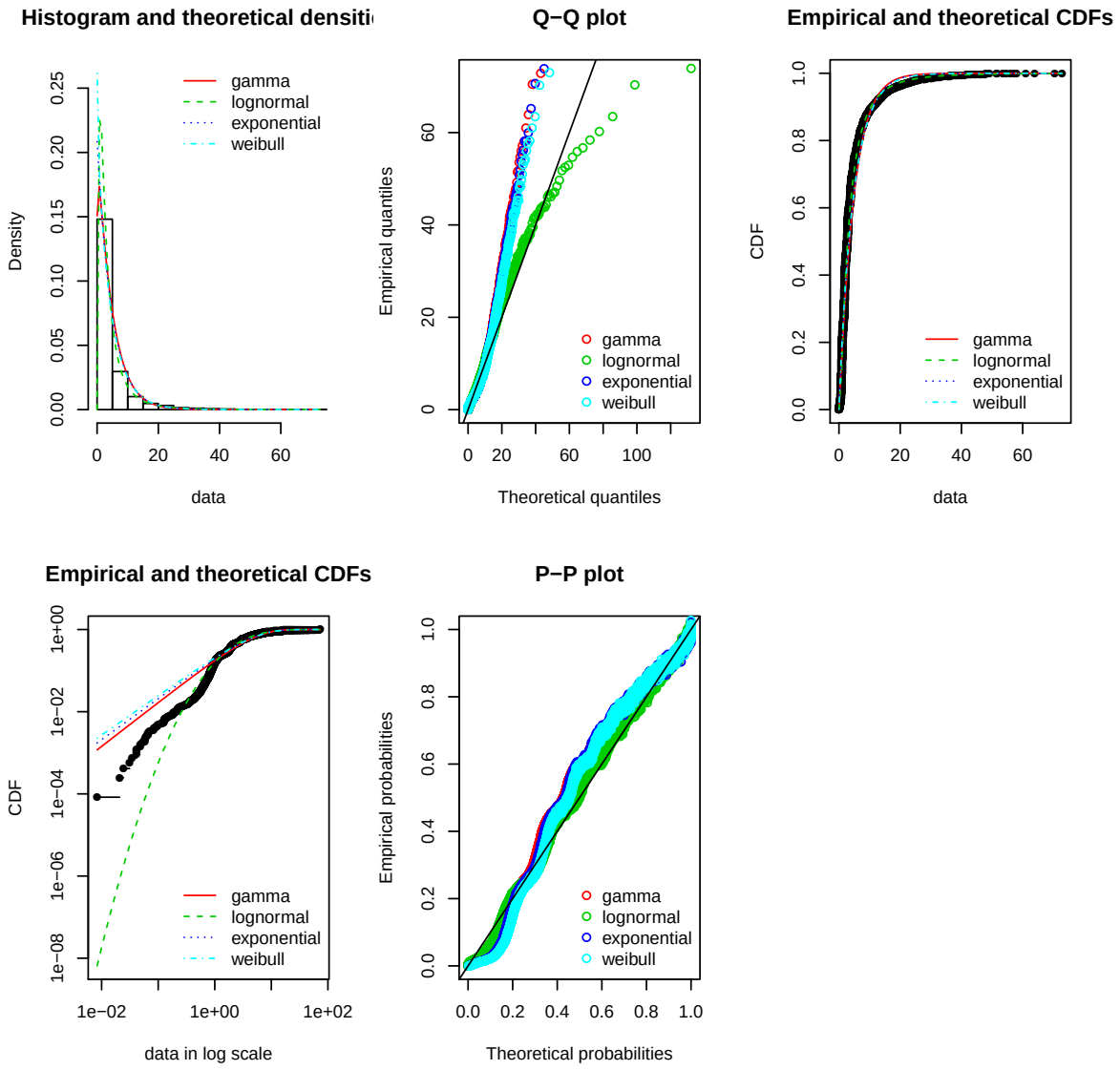


Figure 6.3: Distribution fitting

"AdH" means admission hour in a day. "Nights" denotes nights spent in the ICU. "DisH" means discharge hour in a day.

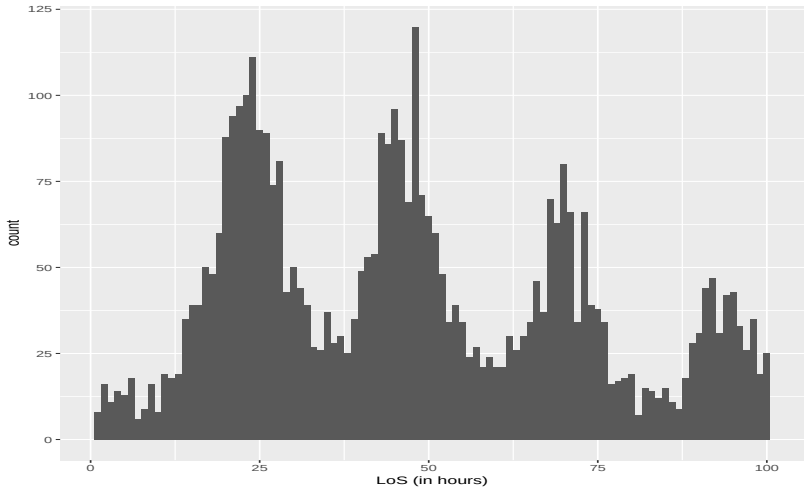


Figure 6.4: Frequency plot of LoS in hours

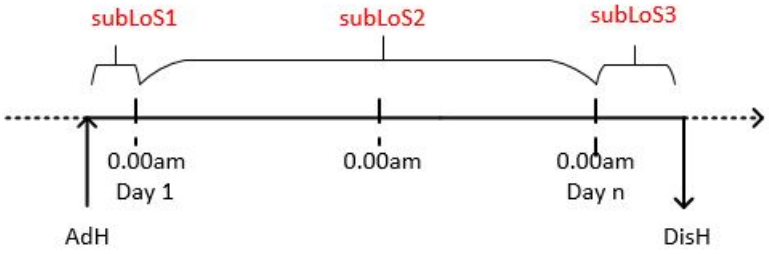
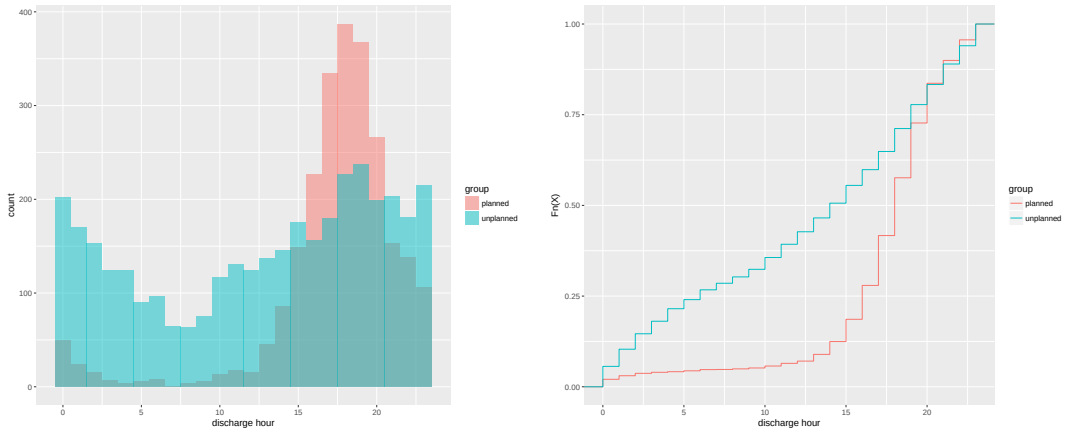


Figure 6.5: LoS modelling

The original LoS modelling problem is consequently divided into three sub-problems: modelling the arrival time, nights in ICU and the discharge time. No suitable distribution has been found to fit any of the three *subLoS*. Consequently, we sample from the EDFs for *subLoS*. Total LoS is obtained by summing up the three *subLoS*. When setting up the EDFs for the three subLoS calculations, we group patients according to different characteristics (e.g. arrival process or discharge states).

Modelling AdH is the first problem to solve. Figure 6.6a is a frequency plot of total admissions at different hours in a day. Occurrence of planned admissions is mainly in the afternoon, while unplanned admissions are spread over the day. AdH in data is captured using two

different EDFs (Figure 6.6b) for planned and unplanned arrivals. A KS test of the EDFs gives a p-value=0.0000, which means these two groups are from two different distributions. When building the DES model, the EDF in Table F.3 is used to obtain AdH for planned patients. AdH for unplanned arrivals is obtained from arrivals when they occur following the NHPP with parameters stated in Table 6.2.



(a) Frequency of ICU Admission Time (b) EDFs of ICU Admission Time

Figure 6.6: ICU Admission Time Modelling

Next, nights in the ICU are modelled. Figure 6.7 illustrates the four groups used in investigating LoS distributions, planned non-late, unplanned non-late, late and re-admissions. According to a KS test, the difference between readmissions and late admissions is not statistically significant (p-value > 0.1). Therefore, nights in the ICU are modelled by three EDFs for planned non-late admissions, unplanned non-late admissions and late/re-admissions.

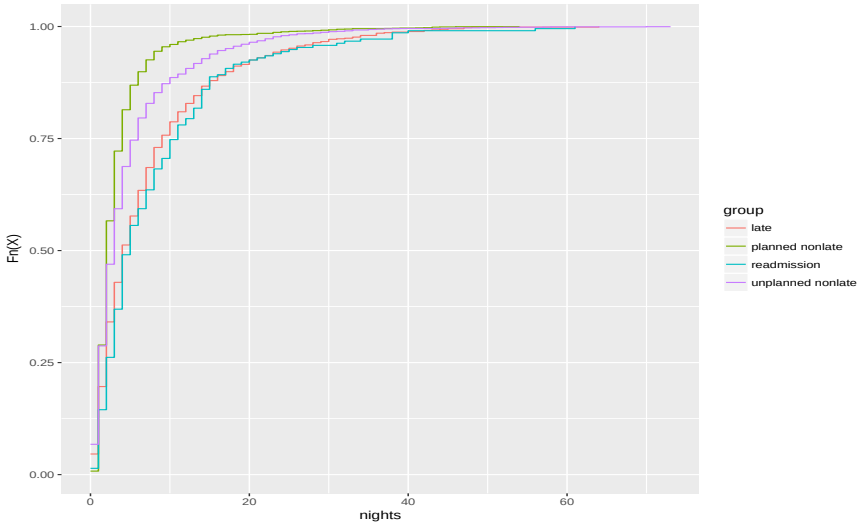
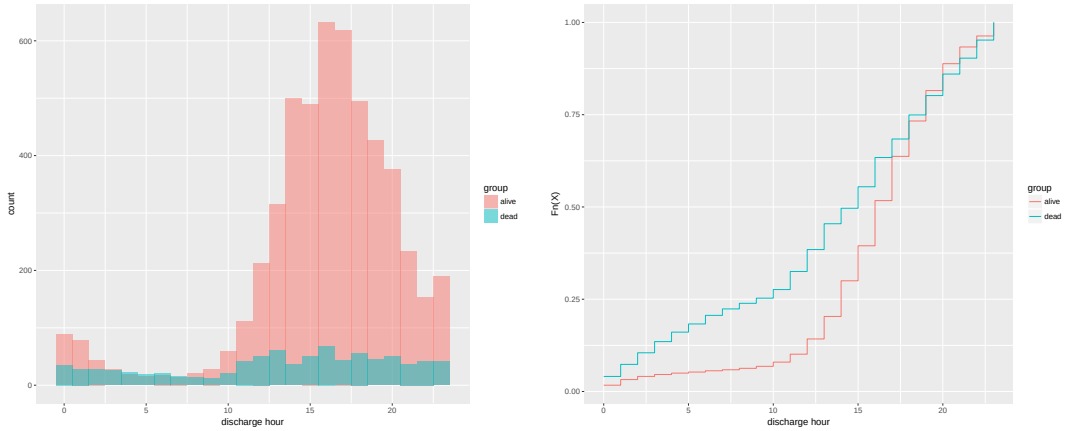


Figure 6.7: EDFs for Nights in ICU

Finally, discharge times are investigated. Grouping patients into planned/unplanned admission does not provide as much information as ICU outcomes when modelling discharge time. As shown in Figure 6.8a, patients who died in the ICU can be discharged at any time of the day, while patients who survived after ICU treatments tend to be discharged at a fixed period of the day. In addition, mortality prediction in ICU has a good performance as demonstrated in Section 5.3.1. Therefore, ICU outcomes (i.e. alive/dead) are selected to group patients. Two EDFs of ICU discharge time are illustrated in Figure 6.8b.



(a) Frequency of ICU Discharge Time

(b) EDFs of ICU Discharge Time

Figure 6.8: ICU Discharge Time Modelling

Consequently, we model the three *subLoS* using different grouping methods:

- *subLOS*1 (arrival time) split into two groups: planned and unplanned.
- *subLOS*2 (number of nights) split into three groups: planned, unplanned and late/re-admissions.
- *subLOS*3 (discharge time) split into two groups: discharged alive and discharged dead.

The proposed method was tested and compared to the original data. Cumulative distributions of original LoS and simulated LoS are shown in Figure 6.9. A considerable overlap is observed, showing that the method samples LoS correctly.

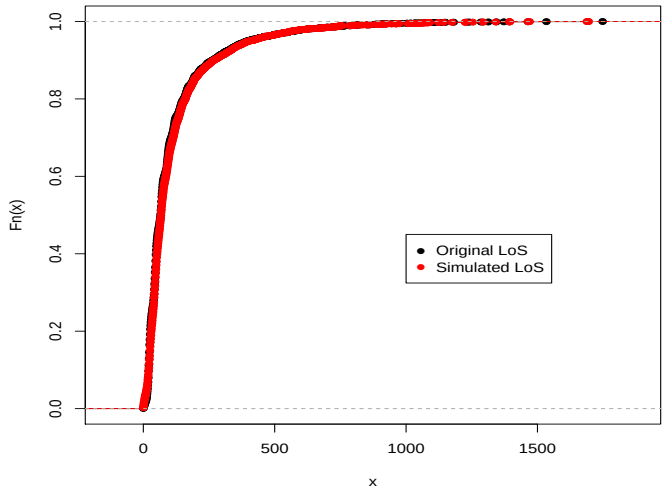


Figure 6.9: EDFs for original and simulated LoS

Furthermore, compared to directly sampling from the LoS EDF of ICU patients, this three-part method keeps flexibility for scheduling planned patients arrivals and testing the effect of discharge timing for patients.

6.4 Prediction models in the DES model

This section describes prediction models incorporated in the DES model. First, ICU mortality prediction and hospital mortality prediction are summarised. Then, a patients' starting critical care level (patients' severity) prediction model is built.

6.4.1 Mortality prediction

Models for mortality prediction have been discussed thoroughly in Section 5.3. For the sake of simplicity of variables, we use the benchmark models here; admissions are first split into planned, unplanned and late/re-admitted as suggested in Section 5.2. To summarise, The probability that a patient is discharged alive is given by

$$\mathbf{E}[Y_i = 1|\mathbf{X_i}] = p_i = \frac{1}{1 + e^{\beta_1 x_i - \beta_0}},$$

where $\mathbf{x}_i$ is the ICNARC probability of patient i and β_0, β_1 are parameters to be estimated from the data. Table 6.4 lists the values of β_0, β_1 for the three regression models for reference purposes.

Table 6.4: Parameters for the ICU Mortality Prediction Logistic Regression Models

Model	β_0	β_1
Planned	4.5425	0.0785
Unplanned	3.4210	0.0539
Late/re-admitted*	3.2723	0.0495

* using the model built for late admission

The predictability of the models were assessed by AUROC, which are 0.9073, 0.9073 and 0.8692 respectively. These three models are incorporated into the simulation model to generate the ICU mortality of patients.

Hospital mortality of all patients is also modelled using a new simplified logistic regression model (training AUROC=0.8990, testing AUROC=0.8792), which includes both the

ICNARC probability and the effect of readmission as factors:

$$\mathbf{E}[Y_i = 1|\mathbf{X_i}] = \frac{1}{1 + e^(0.0394 * ICNARC - 0.1540 * Readmission - 3.7699)}.$$

6.4.2 Setting the initial critical care level

Each ICU nurse can take care of at most two patients depending on the patients' levels. As discussed in Section 2.1, a patient's level indicates the severity of their condition and the Nurse-to-patient ratios are 1:1 and 1:2 for level 3 and level 2 patients respectively. Nurse requirements therefore vary for each patient according to their level. A patient's initial critical care level refers to the level of a patient at admission. Hence, we build a logistic regression model to predict patients' levels on admission to the ICU. The best fitted model selected using AUROC is described as:

$$\mathbf{E}[Y_i = L3|X] = \frac{1}{1 + exp^(0.0443 * ICNARC - 0.3654 * LateorRe - 0.8970 * planned - 0.2886)}$$

with ICNARC probability, admission type (planned/unplanned) and effect of late or readmission used as independent variables of the model. The training and test AUROC for the model are 0.8321 and 0.7957 respectively.

6.5 DES model description

6.5.1 DES model building

Sargent (2013) provides a comprehensive process of building a valid simulation model. We summarise the process as problem formulation, conceptual model building and validation, data validation, computer model building and verification and operational validation. Figure 6.10 shows how each stage contributes to the model building process.

The STRESS guidelines proposed by Monks et al. (2017) are used to support the model

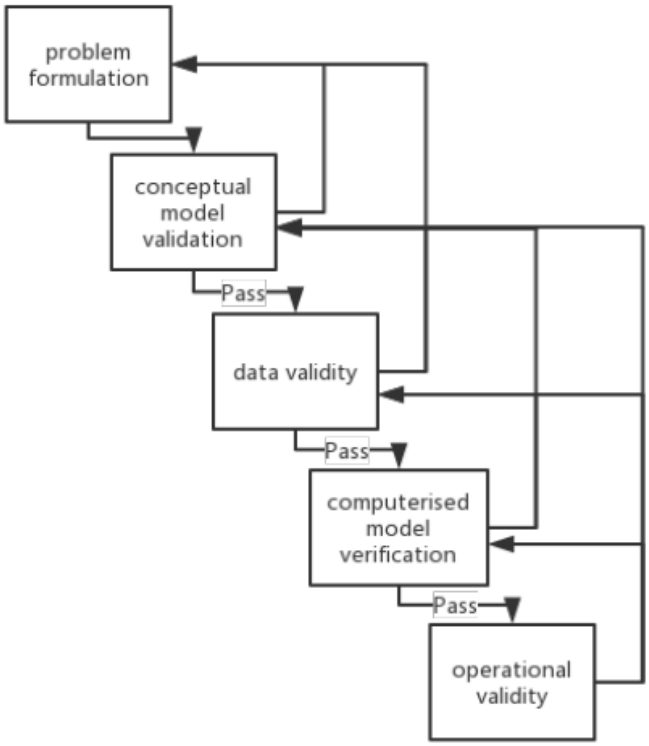


Figure 6.10: DES model building, verification and validation

description and ensure that we include sufficient details to enable reproducibility. These guidelines require authors to specify the objectives of a DES model first and then give detailed logic, data sources, data pre-processing methods and input parameters for the model. Software used, run length and sampling method for the simulation also needs to be stated.

A computer model was built and implemented in Simul8 2017 professional under Windows 10. The logic for mortality predictions and occupancy checking was coded and realised by the Visual Logic module included in Simul8. The data analysis part was conducted using R 4.1.1.

Problems identification has been achieved in Chapter 4, where we found that late admission is a severe problem and has a potential to be solved by better planning.

6.5.2 Model details

A DES model is built to study the behaviour of the ICU. The model focuses on the effect of late admissions and the potential impacts of bringing patients into the ICU more promptly. The model is described by the flow chart in Figure 6.11.

The validated model is described below. Model validation will be further discussed later in Section 6.7. The model can be divided into 4 major parts: ICU arrival, pre-ICU sampling, ICU service and post-ICU service.

The system runs for twenty four hours per day, seven days per week, with the same quantities of resources. There are no staff or bed reductions at any time. The simulation time unit in the model is "hour". ICNARC data from BRI between January 2008 and November 2013 are used to estimate distributions and parameters in the ICU model.

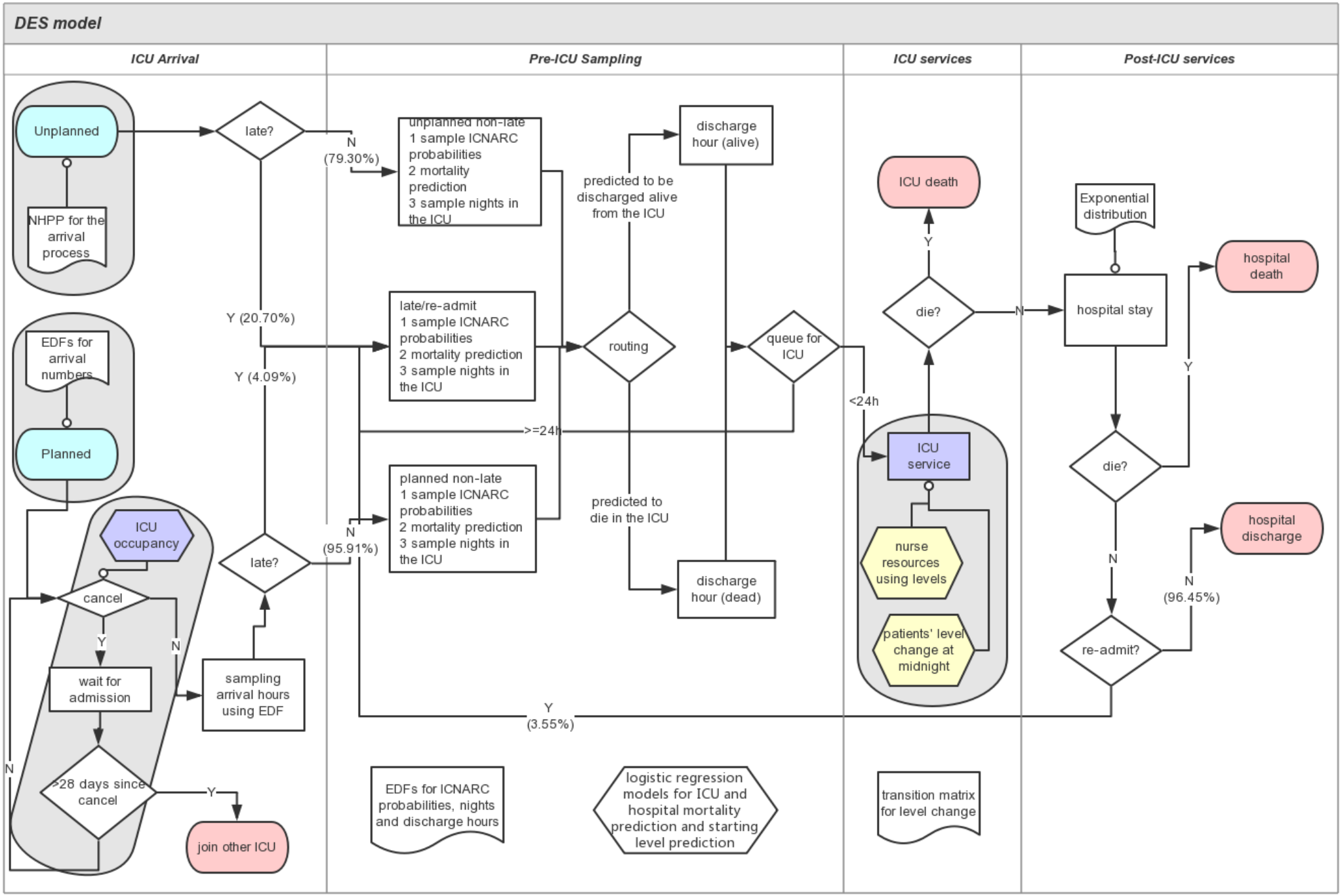


Figure 6.11: Detailed ICU model DES model flowchart

ICU arrivals

Two independent entry points are used to give different arrival rates to planned and unplanned patients as discussed in Section 6.2. The NHPP with piecewise rates shown in Table 6.2 is used to model the weekly cycle of unplanned admissions. For planned admissions, we use the empirical distribution of daily admission numbers on a weekday basis to estimate the number of admissions per day, as shown in Table 6.3. Arrivals are then assigned to different arrival hours according to the EDF of arrival timing for planned arrivals (see Table F.3).

We introduce a cancellation policy for planned arrivals: if the ICU has less than two idle beds or equivalent nurses on a particular day, all the planned patients for that day will be cancelled. The model follows NHS policy that cancelled planned patients will be readmitted within 28 days after cancelling (NHS, 2015). The length of time to readmission is estimated using a discrete uniform distribution with values being integers. The lower bound and upper bound of the distribution equal 1 and 28 (days) respectively. For any $k \in [1, 28]$, the cumulative distribution function (CDF) of the distribution can be written as, $F(k) = \frac{\lfloor k \rfloor}{28}$.

If a patient has to wait for more than 28 days from their first cancellation, the patient will be diverted to another hospital. Otherwise, the maximum number of planned admissions that can be admitted on any day is the total idle beds minus two reserved for unplanned arrivals.

The anticipated number of empty beds, e, is given by $e = max\{r - (n - 18), 0\}$ where

n is the number of occupied beds;
r is the number of beds expected to be released in the next 24h.

The total cancellations in a day, m, are given by $m = max\{(p + c - e), 0\}$, where

c is the number of patients who have been cancelled once and are waiting for readmission;
p is the number of planned arrivals for that day.

The cancellation priority is: first, admit all patients; second, admit all patients who have been cancelled once and cancel some planned patients; third, cancel all planned patients and

some patients who have been cancelled once already; fourth, cancel all planned patients and all patients who have been cancelled before.

Algorithm 1 is implemented at every midnight to decide on the cancellation of patients (see the first block from the left in Figure 6.11). "X_i" is used to denote the i^{th} patient in the ICU.

Algorithm 1 Planned patient cancellation

```
r ← 0
for i = 1 to n do
  if remaining nights of X_i ≤ 1 then
    r ← r + 1
  end if
end for
e ← max(r − (n − 18), 0)
Sample for p
if p + c ≤ e then
  admit all patients
else if c ≤ e < p + c then
  admit all cancelled patients and (e − c) number of planned arrivals
else
  admit e (≥ 0) number of cancelled patients and cancel all the other patients
end if
```

Pre-ICU sampling

Patients are routed to queues on a percentage basis: 79.30% of unplanned arrivals are routed to 'queue for unplanned'. The remaining unplanned arrivals travel to 'queue for late/readmission'. Similarly, 95.91% of planned arrivals go to 'queue for planned'. The others are sent to 'queue for late'. The severity and impact of late admission has been discussed in Section 4.5. Therefore, a separate queue is created for these patients.

ICNARC probabilities are sampled for different arrival categories (unplanned, planned and late) as soon as patients arrive. The predictions of the initial critical care level of a patient and of ICU mortality are explained in Sections 6.4.2 and 6.4.1. The levels are reassessed at midnight every day. Patients can move between L2 and L3.

ICU service times are sampled from EDFs using the modelling method described in Section 6.3.2. Sampling of 'nights spent in the ICU' is achieved by creating three dummy work-centres: 'unplanned nights', 'planned nights' and 'late nights', and attaching the sampled values from EDFs to every patient. Every individual patient is routed to a dummy work-centre either 'discharge hour (alive)' or 'discharge hour (dead)' to get a discharge time from EDFs.

All the patients join the queue for ICU services after getting the sampled ICU LoS. Patients who are late will be given the first priority. All the other patients will be routed into the ICU on a first come first served (FCFS) basis when spaces become available. Moreover, patients waiting longer than one day (24 hours) will renege from the current queue and join the 'queue for late/readmission' and get re-sampled data.

ICU service

As soon as the ICU has adequate beds and nurses to serve more patients, patients in the queue will join the ICU. At every midnight, in accordance with practice in the ICU, the severity of each patient in the ICU model will be reassessed. According to the data, patients' level change at midnight is calculated as the state chart shown in Figure 6.12.

Figure 6.12: State chart of patients' level change

The transition matrix can be expressed as:

$$\mathbf{P} = \begin{array}{c} \\ L2 \\ L3 \end{array} \begin{array}{c} \begin{array}{cc} L2 & L3 \end{array} \\ \begin{pmatrix} 0.9584 & 0.0416 \\ 0.1668 & 0.8332 \end{pmatrix} \end{array}$$

The transition matrix is used to assign new levels to patients in the ICU at midnight. Nurses required for patients will also be altered according to their new levels.

Post-ICU service

All patients who are discharged alive from the ICU will spend a certain amount of time in general wards: the LoS there follows an exponential distribution with average service time equal to 348h. The post-ICU mortality prediction model as detailed in Section 5.3.3 and summarised in Section 6.4 is modelled in this stage to indicate each patient's ultimate outcome. Among patients who are predicted to be discharged alive from the hospital, 3.55% of them will be readmitted to the ICU. That is to say, they will join the "queue for late/readmission" and get re-sampled statistics including ICNARC probability, nights in ICU and discharge hour.

6.6 Input uncertainty

Input uncertainty (IU) arises from fitting a limited amount of real-world data to obtain input distributions for simulations models. Variation of simulation outputs comes from two origins, IU and stochastic variation of activities within simulation models (Barton, 2012). Stochastic variation is the part that is the major interest: IU can be regarded as inaccuracy in results. Therefore, IU needs to be considered for reliable simulation results and also to indicate whether the amount of data available is sufficient.

For convenience purposes, all input distributions are summarised in Table 6.5. As Table 6.5 suggests, most of the distributions come from real-world data. IU are likely to incur

in distribution fitting and sampling. Amongst all the listed distributions, waiting time of cancelled patients is modelled using the NHS rule. Thus, it is treated as a true distribution rather than an estimation. The exponential distribution used to describe hospital LoS will not be tested for ICU either, as this input will not affect the ICU output. IU of all the other input distributions will be examined.

Table 6.5: Summary of the DES model input distributions

	Input	Model	Source
Arrival	unplanned arrivals	NHPP (14 rates)	real-world
	arrival numbers (planned)	7 EDFs	real-world
	arrival hour (planned)	EDF	real-world
	waiting time of cancelled patients	discrete uniform	NHS rule
Pre-ICU	ICNARC probability (unplanned)	EDF	real-world
	ICNARC probability (planned)	EDF	real-world
	ICNARC probability (re-admitted)	EDF	real-world
	ICNARC probability (late)	EDF	real-world
	nights in ICU (unplanned)	EDF	real-world
	nights in ICU (planned)	EDF	real-world
	nights in ICU (late/re-admitted)	EDF	real-world
	discharge hour (alive)	EDF	real-world
	discharge hour (dead)	EDF	real-world
Post-ICU	hospital LoS	Exponential	real-world

The objective of a simulation model is to give $\mathbb{E}[Y(\mathbf{F})]$, where $\mathbf{F}$ is the collection of L true input distributions $\{F_1, F_2, \cdots, F_L\}$ of the DES model. However, $Y(\mathbf{F})$ is usually approximated using $Y(\hat{\mathbf{F}})$ where $\hat{\mathbf{F}} = \{\hat{F_1}, \hat{F_2}, \cdots, \hat{F_L}\}$ fitted from real-world data; the l^{th} marginal distribution of $\hat{\mathbf{F}}$ is denoted by $\hat{F_l}$. Real world observations follow $\{X_{l1}, X_{l2}, \cdots, X_{lm}\} \overset{i.i.d}{\sim} \hat{F_l}$. Running n replications of the simulation model using input model $\hat{\mathbf{F}}$, the output of the j^{th} replication is denoted by $Y_j(\hat{\mathbf{F}}) = \eta(\hat{\mathbf{F}}) + \varepsilon_j$, where $\eta(\hat{\mathbf{F}}) = \mathbb{E}[Y_j(\hat{\mathbf{F}})]$ is the expected value of the simulation output, given input models $\hat{\mathbf{F}}$ and ε_j. Error terms $\{\varepsilon_1, \cdots, \varepsilon_j, \cdots, \varepsilon_n\}$ are i.i.d. with $mean = 0$ and $variance = \sigma^2$.

IU is formally defined by Song et al. (2014) as $\sigma_I^2 = Var[\eta(\hat{\mathbf{F}})]$, "the variance in the system mean due to having estimated $\mathbf{F}$". σ_I^2 is approximated using $\hat{\sigma}_I^{2\star}$ via direct bootstrapping.

6.6.1 IU from the whole model

Ankenman and Nelson (2012) introduce Algorithm Quick (see Algorithm 2) to quantify the overall IU of a simulation model. This method can be applied to both parametric and non-parametric models. Therefore, it is adopted to quantify the overall IU of the DES model. The influence of IU can be quantified using a ratio $\gamma = \frac{\sigma_I}{\sigma/\sqrt{n}}$. The ratio is interpreted as standard deviation due to IU in the units of standard error of simulation estimated $\eta(\hat{\mathbf{F}})$. The smaller the ratio is, the less significant the IU is relative to stochastic error. If the IU is taken into consideration, the length of CI will be magnified by $\sqrt{1+\gamma^2}$ (Song and Nelson, 2015).

Algorithm 2 Algorithm Quick

for $l = 1$ to **n do**
  Given real-world data:$\{X_{l1}, X_{l2}, \cdots, X_{lm}\}$
  for $b = 1$ to B **do**
    (a) Generate bootstrap samples: $X_{l1}^{\star(b)}, X_{l2}^{\star(b)}, \cdots, X_{lm}^{\star(b)} \overset{i.i.d}{\sim} \hat{F}$ for $b = 1, 2, \cdots, B$ using real-world data
    (b) Fit $X_{l1}^{\star(b)}, X_{l2}^{\star(b)}, \cdots, X_{lm}^{\star(b)} \overset{i.i.d}{\sim} \hat{F_l^{\star b}}$
    (c) Simulate R replications of $Y_j(\hat{F_l}^{\star(b)})$ using $\hat{F_l}^{\star(b)}$
  end for
end for
Calculate $\gamma = \frac{\sigma_I}{\sigma/\sqrt{n}}$

σ_I^2 is estimated using $\hat{\sigma}_I^{2\star} = \frac{1}{B-1}\sum_{b=1}^{B}(\eta(\hat{\mathbf{F}}^{\star(b)}) - \bar{\eta}^{\star})^2$, where $\bar{\eta}^{\star} = \sum_{b=1}^{B}\frac{\eta(\hat{\mathbf{F}}^{\star(b)})}{B}$;

σ^2 is estimated using input models $\hat{\mathbf{F}}$;

n is the number of replications required by the DES model.

Given capacity of N runs in total for testing IU, B and R should be such that $BR = N$. The required number of B is recommended to be $B = 2L + 2$ (Song and Nelson, 2015). After B and N are decided, R can be obtained using $R = \lfloor N/B \rfloor$.

ICU annual throughput, late admission rate and ICU and hospital mortality rates are key outputs of the DES model. Thus, four γ ratios were calculated to measure the influence of IU in these aspects. In terms of the estimation of variance from random sampling (σ^2), the minimum requirement of runs per trial is 39 to achieve 2.5% precision (equivalent to

95% CI for two tailed distributions) with input distributions $\hat{\mathbf{F}}$. To save runs, 5% precision is required for experiments with bootstrapped values. Thus, 15 runs per trial was used to estimate σ_I^2.

As illustrated in Table 6.5, there are 14 input distributions including an NHPP with 14 different rates estimated from real-world observations. The NHPP with 14 piece-wise constant arrival rates can be decomposed into 14 distinct stationary PPs (Morgan et al., 2016). Therefore, the total number of input distributions is 31. The minimum requirement of $B = 31 \times 2 + 2 = 64$ was used to quantify the overall IU in the DES model. Consequently, $n = 39$ was used to get baseline statistics. $B = 64$, $R = 15$ and $N = 960$ were used for bootstrapping and estimating IU. Direct bootstrapping sampling as described in Barton (2012) is used to obtain $\hat{\mathbf{F}}^{\star(b)}$.

Figure 6.13 illustrates how input distributions (blue nodes) influence simulation outputs (red nodes). An arrow from a source node to a target node denotes an direct effect of the source on the target. The effect of the input can be passed by intermediate nodes to a node not directly linked.

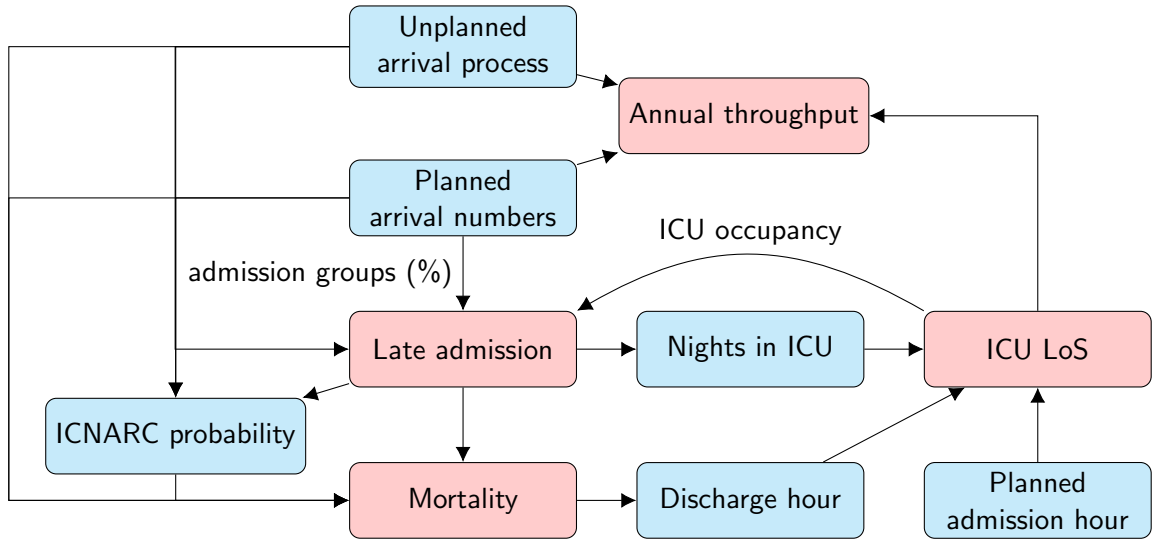


Figure 6.13: Influences of input distributions

Annual throughput is influenced by all the input distributions while other outputs are affected by some of input distributions. Both hospital and ICU mortality are influenced by arrival

processes and ICNARC probability. Percentage of late admissions is influence by arrival processes and LoS. LoS is sampled for different groups of patients without involving ICNARC probability. Therefore, the γ ratio for annual throughput is likely to be larger than the γ ratios for other outputs.

Table 6.6 shows the ratios of all five simulation outputs. For annual throughput, a γ ratio of 2.8 shows that the IU error is around three times the standard error of the point estimator. For hospital and ICU mortality rates the input error is around twice the stochastic error. The γ ratio of approximately 1 for late admission rate is the smallest amongst all five ratios. When estimating ICU LoS the error of IU is almost three times the stochastic error.

Table 6.6: IU from all input distributions

	σ_I	σ	γ
Annual throughput	12.1802	26.7072	2.8481
Hospital mortality	4.5567×10^{-3}	1.2275×10^{-2}	2.3183
ICU mortality	3.8993×10^{-3}	1.1090×10^{-2}	2.1958
Late admission	1.7807×10^{-3}	1.0156×10^{-2}	1.0950
ICU LoS	2.4454	5.4895	2.7820

Because of the γ ratios observed, an analysis follows of IU of individual inputs.

6.6.2 IU from unplanned arrival process

The quantification of IU originating from the NHPP was also tested using Algorithm 2. In order to test the IU from NHPP only, other input distributions of the model remained the same in every replication. $B = 14 \times 2 + 2 = 30$, $R = 15$ and $N = 450$ were used to estimate this particular ICU model.

All the generated work items (patients) are admitted to the ICU. The number of unplanned arrivals is influenced by the NHPP only. IU originating from the NHPP also has an impact on other outputs via indirect links. Therefore, the total number of unplanned arrivals is added to the result as it is a better indicator of the IU from the NHPP compared to other indicators.

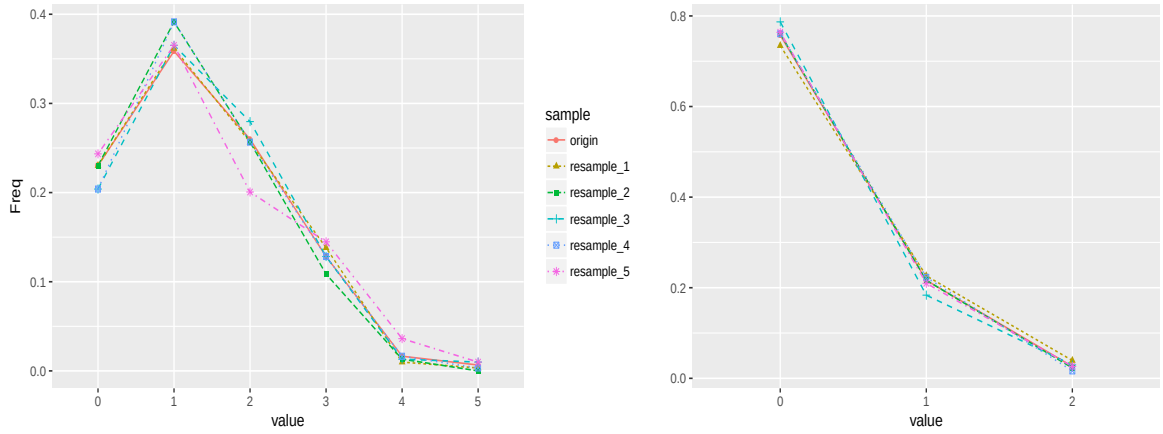
Table 6.7 lists all the standard deviations of bootstrapping and baseline results. The standard deviation of total unplanned arrivals obtained from 14 bootstrap-generated NHPPs is about twice as large as the standard error of the baseline model. As expected, γ ratios of outputs other than unplanned arrivals are smaller than the γ ratios caused by all input distributions.

Table 6.7: IU from NHPP

	σ_I	σ	γ
Annual throughput	12.1802	26.7072	1.6774
Hospital mortality	2.0960×10^{-3}	1.2275×10^{-2}	1.0664
ICU mortality	1.9480×10^{-3}	1.1090×10^{-2}	1.0970
Late admission	1.5556×10^{-3}	1.0156×10^{-2}	0.7013
ICU LoS	0.2360	5.4895	0.2685
Unplanned arrivals	7.9616	24.7022	2.0128

6.6.3 IU from planned arrival process

First, IU of the planned arrival process is visualised by plotting sampling with replacement for number of arrivals on Mondays (6.14a) and Saturdays (6.14b) in Figure 6.14 and admission hours in Figure (6.15). Plots of the other weekdays are in Appendix G.



(a) Monday (b) Saturday

Figure 6.14: Planned arrivals re-sampling

As Figure 6.14a shows, the pink line (resample_5) is diverted from the red line (original value) despite the relatively large sample size (n=305). The differences between bootstrapping

samples for arrivals on Saturdays is not notable.

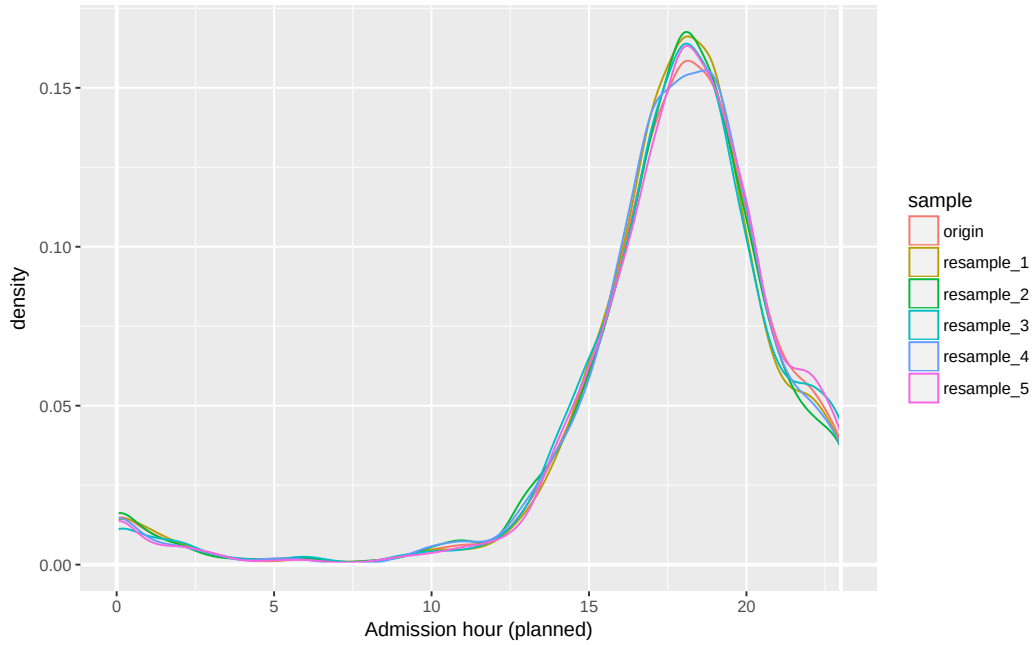


Figure 6.15: Planned admission hour re-sampling

Figure 6.15 demonstrates some shift in the bootstrapping samples of admission hours. However, no distinct shift is observed directly from the figure due to the large sample size (n=2299) and concentration of the original data.

IU generated from the planned arrival process is quantified using Algorithm Quick. $B = 8 \times 2 + 2 = 18$, $R = 15$ and $N = 270$ were used to estimate this part of IU. Results including the number of planned arrivals are reported in Table 6.8.

Table 6.8: IU from planned arrival process

	σ_I	σ	γ
Annual throughput	6.9339	26.7072	1.6214
Hospital mortality	1.8372×10^{-3}	1.2275×10^{-2}	0.9347
ICU mortality	1.9259×10^{-3}	1.1090×10^{-2}	1.0845
Late admission	1.5877×10^{-3}	1.0156×10^{-2}	0.9763
ICU LoS	0.3519	5.4895	0.4003
Planned arrivals	6.7774	15.9121	2.6599

It is shown in Table 6.6 that the impact of planned arrival process IU on the number of

planned arrivals is twice as large as the stochastic error resulting from simulation. Similar to the IU of NHPP, the effect on other outputs are smaller than the effect of IU caused by all input distributions.

6.6.4 IU of nights in the ICU

A similar approach to the analysis of IU from planned admission process (Section 6.6.3) is adopted to investigate IU from sampling nights in the ICU. In addition, bootstrap CIs of four statistics, mean, variance, maximum, and percentage of longer than 20 nights, are calculated. Figure 6.16 illustrates five bootstrap sample sets of unplanned nights in the ICU. Figures for planned and late/re-admission are in Appendix G.

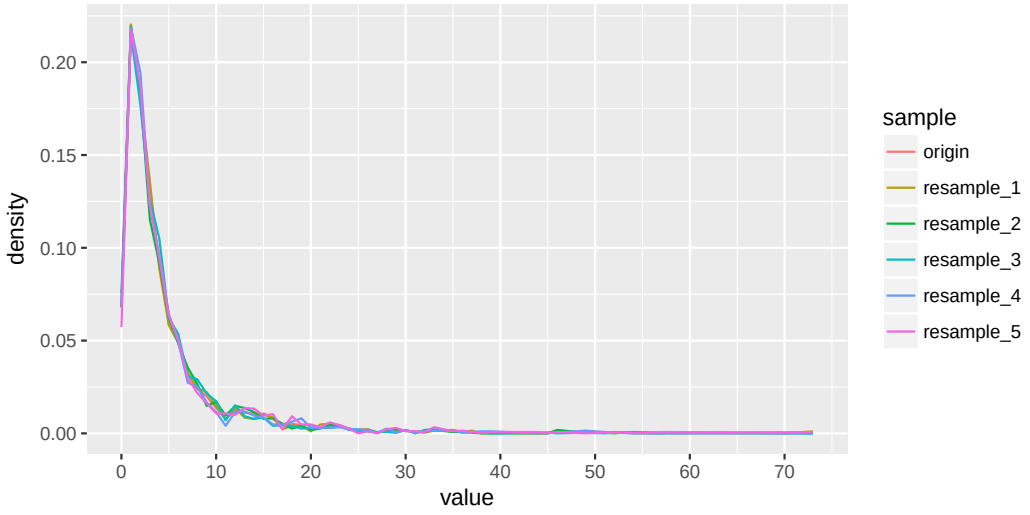


Figure 6.16: Nights (unplanned) in the ICU re-sampling

No significant difference between bootstrapped samples and original samples can be observed directly from Figure 6.16. The calculation of statistics was fulfilled by the "boot" package in R. 1000 replications were assigned to obtain statistics. The results are reported in Table 6.9. Results for planned and late/re-admission nights are in Appendix G.

As shown in Tables 6.9, G.1 and G.2, statistics obtained from non-parametric bootstrap samples are very close to the original statistics. IU is not expected to be a critical problem in this particular part of the simulation, number of nights in the ICU.

Table 6.9: Bootstrap statistics of nights (unplanned) in the ICU

	original	bias	standard error	95% CI
Mean	4.8651	5.4166×10^{-3}	0.1246	(4.6155, 5.1038)
Variance	42.2403	0.0126	3.7122	(34.9518, 49.5035)
Max	73	-2.741	5.2095	(65.5305, 85.9515)
Nights>20 (%)	0.0351	1.6377×10^{-4}	3.3385×10^{-3}	(0.0284, 0.0415)

A further quantification using Algorithm Quick was conducted; $B = max(3 \times 2 + 2, 10) = 10$, $R = 15$ and $N = 150$ were used to estimate the influence of IU. Results are listed in Table 6.10.

Table 6.10: IU from nights

	σ_I	σ	γ
Annual throughput	3.8214	26.7072	0.8936
Hospital mortality	1.6281×10^{-3}	1.2275×10^{-2}	0.8283
ICU mortality	1.8627×10^{-3}	1.1090×10^{-2}	1.0489
Late admission	1.1270×10^{-3}	1.0156×10^{-2}	0.6930
ICU LoS	2.0159	5.4895	2.2934

Although IU from sampling nights in the ICU is not expected to be a serious problem, the results show that the IU contributes error more than twice the stochastic error to the total error of ICU LoS. The number of bootstrap replications (B) just meets the minimum requirement, which may lead to an imprecise estimation.

6.6.5 IU of discharge hours

The quantification of IU of discharge hour EDFs is based on 15 replications for each bootstrap sample with total $B = max(2 \times 2 + 2, 10) = 10$. Results are presented in Table 6.11. As expected, discharge hour inputs do not show as large an impact on the results as other inputs (i.e. nights and arrival processes). Nevertheless, the IU cannot be eliminated.

Table 6.11: IU from discharge hour

	σ_I	σ	γ
Annual throughput	3.1757	26.7072	0.7426
Hospital mortality	1.5486×10^{-3}	1.2275×10^{-2}	0.7879
ICU mortality	1.3826×10^{-3}	1.1090×10^{-2}	0.7786
Late admission	1.2406×10^{-3}	1.0156×10^{-2}	0.7629
ICU LoS	0.2297	5.4895	0.2613

6.6.6 IU from ICNARC probability

ICNARC probability is a continuous variable with theoretical value ranging from 0 to 100. It is a key influencing factor of mortality in the DES model. The IU is evaluated using visualisation, statistical calculation and quantification. Figure 6.17 shows density curves of original ICNARC probability of unplanned non-late admissions and its bootstrap samples. Plots for ICNARC probability of other admission categories are in Appendix G.

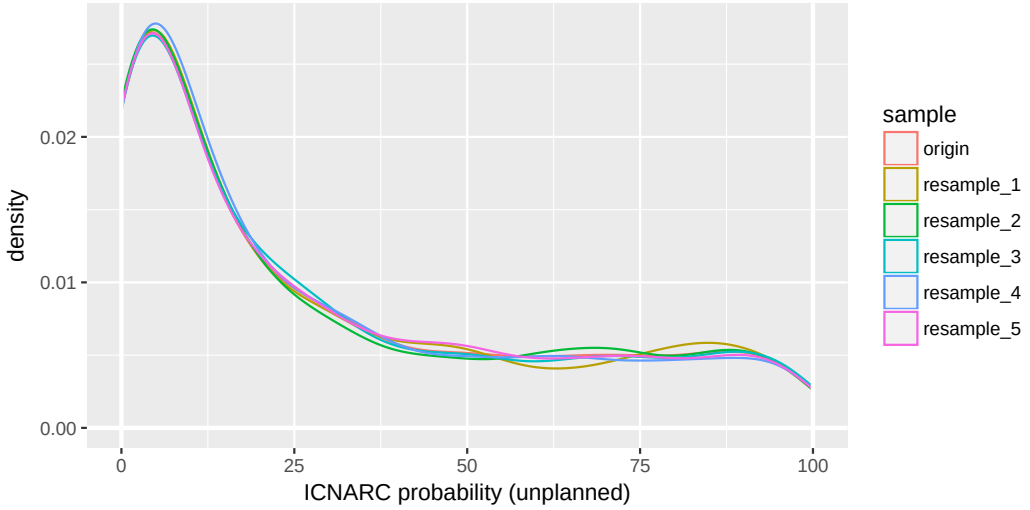


Figure 6.17: ICNARC probability (unplanned non-late) re-sampling

As shown in Figure 6.17, the dispersion of densities is observable in the tail range of bootstrap samples. This dispersion may distort the generation of ICU and hospital mortality rates.

Table 6.12 lists statistics obtained from running 1000 replications of bootstrapping.

As shown in Tables 6.12, G.3, G.4 and G.5, given large enough bootstrapping iterations,

Table 6.12: Bootstrap statistics of ICNARC probability (unplanned)

	original	bias	standard error	95% CI
Mean	29.6643	-0.0114	0.5720	(28.5546, 30.7968)
Variance	905.4737	-0.3986	21.3406	(864.0454, 947.6991)

statistics calculated using non-parametric bootstrap samples are very close to the original statistics regarding ICNARC probability. Furthermore, the impact of these particular inputs on simulation outputs other than mortality is passed via discharge hours (see Figure 6.13), which do not have a large contribution to IU (see Section 6.6.5). IU is not likely to be a key issue for these particular inputs.

For the purpose of quantification, $B = max(4 \times 2 + 2, 10) = 10$, $R = 15$ and $N = 150$ were used to estimate impact of IU from all ICNARC probabilities. Table 6.13 presents σ_I, σ and γ ratios of different model outputs.

Table 6.13: IU from ICNARC

	σ_I	σ	γ
Annual throughput	2.8419	26.7072	0.6645
Hospital mortality	4.6880×10^{-3}	1.1090×10^{-2}	2.3851
ICU mortality	4.3657×10^{-3}	1.2275×10^{-2}	2.4585
Late admission	9.5346×10^{-4}	1.0156×10^{-2}	0.5863
ICU LoS	0.1022	5.4895	0.1163

IU's impact on error when estimating mortality is more than twice the simulation stochastic error. Meanwhile, the impact on other output is relatively limited as expected. Error of mortality caused by the IU would be likely to decrease as suggested by Table 6.12 if a more generous bootstrapping budget (1000 runs) was given.

6.6.7 Summary of the IU Results

It has been demonstrated that IU due to insufficient data causes problems in the simulation outputs. In particular, IU has a relatively high impact on annual throughput and ICU LoS. As more input data would required to solve the problem, we are unable to tackle it currently.

However, the general conclusions gained from the model should still be valid as IU will not affect the mean value and tendency.

6.7 Verification and validation

The whole process of DES model building was explained in Section 6.5.1. As well as the conceptual model validation, we also walked through Figure 6.11 with an ICU consultant to make sure the details of the model is in line with the real system. In this section, a comprehensive verification and validation process will be described, covering the last two stages, verification and operational validity of computer models, see Figure 6.10.

6.7.1 Verification and validation of computer models

Regarding verification and validation of the computer model, two type of tests are recommended: static testing and dynamic testing (Sargent, 2013). Static testing is used to verify a computerised model while dynamic testing is designed to validate its functionality.

Static testing techniques, code review and walkthrough, were adopted to verify the DES model. The code review was conducted by the author to ensure the correctness of code logic by giving explanations to current code. The syntax was checked by Visual Logic in Simul8 automatically. Code documentations were also created for record purposes.

As commercial software, Simul8 is reliable for standard procedures and usage. However, some procedures in our DES model were realised by using some unusual routines since no established procedure could be found. Dynamic testing focused on these non-standard procedures and procedures involving coding.

Dynamic testing for the DES model was performed in a series of unit tests for individual parts of the DES model. Before initiating systematic dynamic testing, a single run was observed to check the general usability and flow, taking advantage of Simul8 visualisation,

where the behaviour of every work item (patient) in the model can be observed directly. Some abnormal results were observed in an early stage of dynamic testing. A new routine (planned patient cancellation process) was added to solve the problem.

Regarding unit testing, the model is split into five parts to validate the functionality and correctness: first, the implementation of NHPP; second, sampling of planned arrivals; third, cancellation of planned patients; fourth, the change of requirement of resources in line with patients' levels; fifth, mortality prediction models embedded in the DES model. These five units (shaded in Figure 6.11) were tested independently by creating simplified models for each part.

The implementation of the piece-wise constant NHPP was achieved by using a time dependent distribution with three daily time slots which re-occur every day; distinct parameters are obtained for different weekdays. This was tested by comparing simulated arrival numbers with theoretical expected arrival numbers based on the data.

In the DES model, sampling planned arrivals is achieved by generating one batch at every midnight and then setting the batch number, which can be any non-negative integer, following EDFs shown in Table 6.3. To test this process, it was simplified by setting the batch size for each day to one, increasing by one to seven in subsequent days. The simplified model was run for one week and monitored for arrival times of each item. The total number of arrivals equalled 28 and the arrival pattern followed the settings.

The algorithm for cancellation of planned patients (Algorithm 1) was tested separately by giving fixed values to related parts and checking the cancellation process visually.

In the DES model, change of requirement of resources in line with patients' levels is composed of patients' level change and nurse requirement change. The former is achieved using software features. The latter is implemented using routing out and in and choosing resource according to a level label. This was tested by isolating the ICU service part and simplifying it to a single-server system, one ICU bed only, with two resources, $2 \times (0.5$ nurses). Two work items with fixed operation time of 48h each were generated from the start point with predetermined

changes, the first work item from L2 to L3 and the second work item from L3 to L2, happening at midnight. A slow motion simulation was run for 96h to observe the behaviour. The utilisation of the resource was 75%; the change in numbers of nurses could be visually observed while running.

Mortality prediction models were tested by setting labels used in prediction (ICNARC probability and admission categories) to a predefined value and tracing route-out behaviour of work items.

6.7.2 Operational validity

Each run of the model lasts 8760 hours (365 days). We use the trial calculator in Simul8 to find the appropriate number of iterations (39) to run to ensure that the variations of completed jobs are within 5%. Three trials with three different random number sets were run for comparison purposes. The validation process and scenarios were tested using each random number set. General validity of the model, annual throughput and arrivals, ICU and hospital mortality rates, ICU LoS and resource utilisations are checked to investigate the operational validity of the model. Plots and statistical tests are adopted to compare the model outputs with the real world system.

General validity check

Since the data populating distributions to the model only include admissions to the ICU, it is, therefore, anticipated that the ICU in the model should be able to serve all the generated arrivals promptly. The number of patients being late due to queueing for ICU for more than 24 hours is very small (around 2 per run).

Annual throughput

Annual throughput in the model equals 1029, which is similar to the throughput of the actual ICU (1031 per year). The average total arrivals in the model is 993 per run (year). The average total arrivals to the real ICU is 994 excluding readmissions. Table 6.14 gives the statistics calculated from the simulation model output and the original data.

Table 6.14: Operational validity - throughput and admission

	Model	95%CI	Real-world system	t-statistic	p-value
Annual throughput	1029	(1024, 1034)	1031	0.0394	0.9712
Annual arrival	993	(989, 997)	996	0.0719	0.9701

Results indicate that differences of annual admissions and the number of first-time patients in the simulation model and real data are very small. Differences could not be detected by a Welch's t-test.

Mortality

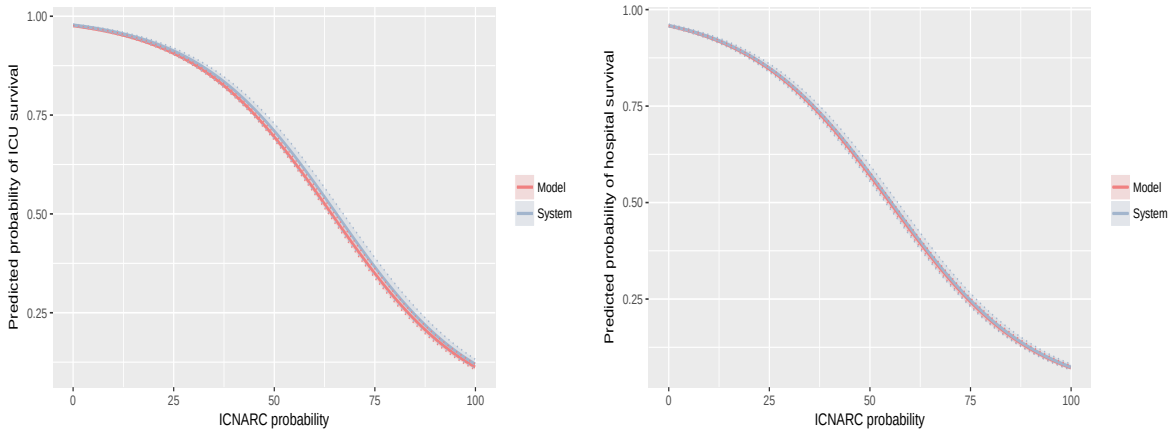
We use a Chi-squared test with Yates' correction to compare the model and actual ICU and hospital mortality rates. Relevant statistics are reported in Table 6.15. Model mortality rates are 14.67% and 19.53% while the actual ICU and hospital mortality rates are 14.25% and 19.39%. Neither difference is statistically significant, with p-value > 0.1.

Table 6.15: Operational validity - mortality

	Model	95%CI	System	χ^2	p-value
ICU mortality rate	14.67%	(14.15%, 15.19%)	14.25%	0.7165	0.3973
Hospital mortality rate	19.53%	(19.13%, 19.93%)	19.39%	0.0528	0.8183

We validate the predictions of the mortality rates using log-odds plots. Transformed log-odds plots in Figure 6.18 show a considerable overlap of prediction in the model and the real world in both ICU (6.18a) and hospital (6.18b) mortality predictions.

Regarding mortality in the ICU and hospital, the model provides a good simulation for both the overall mortality rates and mortality prediction for individuals.



(a) ICU (b) Hospital

Figure 6.18: Operational validity - mortality prediction

ICU LoS

The model predicts a mean LoS of 114.80 hours with 95% CI= (113.29, 116.31), compared with the system's mean LoS of 114.98 hours. A Welch's t-test suggests that the difference is not significant, with t-statistic = 0.0871 and p-value = 0.9306.

A boxplot is employed to explore the different between ICU LoS of the DES model and the real world, as shown in Figure 6.19. These two boxes look very similar to each other.

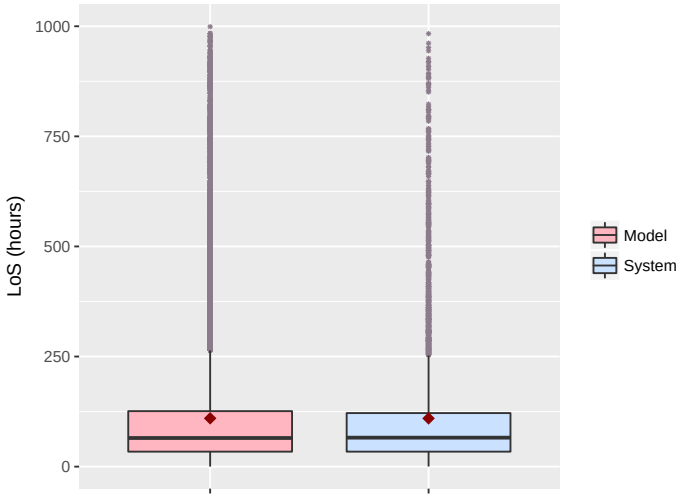


Figure 6.19: Boxplot for comparison of model and system LoS

Overnight bed occupancy

Overall bed utilisation will not be checked as it is equivalent to a combination of annual throughput and ICU LoS, i.e.

$$utilisation_{bed} = \frac{(average\ LoS) \times (total\ number\ of\ admissions)}{24h \times (number\ of\ ICU\ beds)}.$$

Our final check is, therefore, to compare the overnight bed occupancy of the model with that observed in the ICU data. The number of occupied beds, instead of an occupancy rate, is used to examine the occupancy.

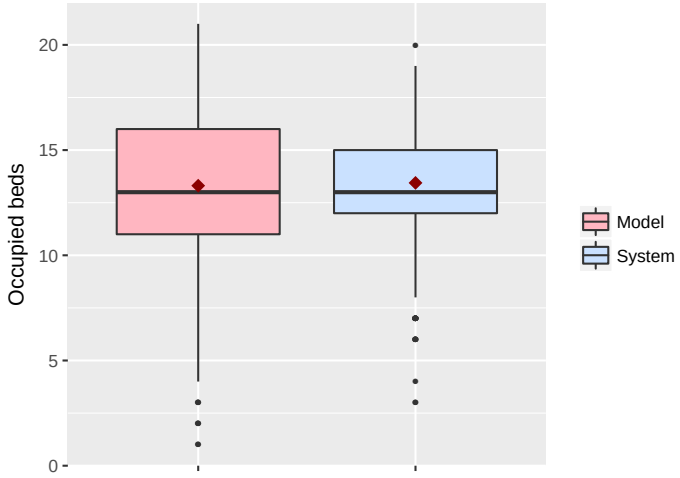
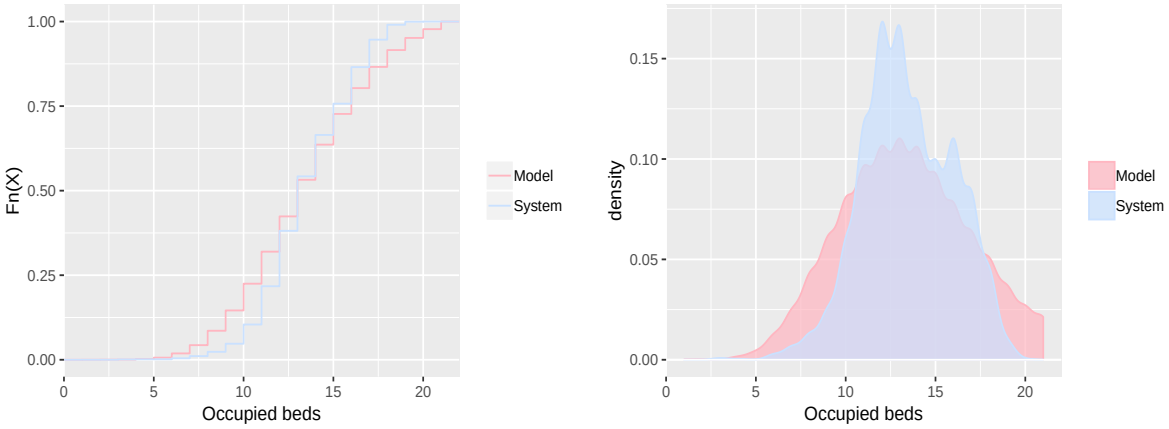


Figure 6.20: Boxplot for comparison of model and system overnight bed occupancy

The average number of beds occupied overnight is 13.41 (13.30,13.52) in the model, very similar to the observed average occupancy of 13.44 ICU beds. Differences in the average overnight occupied beds are not statistically significant, based on a t-test with t-statistic=0.5266 and p-value=0.595.

However, one may identify from the boxplot (Figure 6.20) that the model shows a larger dispersion than the real system in terms of bed occupancy. For a more thorough analysis, we compare the distributions of occupied beds simulated from the model with the real situation. Both EDFs and smoothed distributions are plotted in Figure 6.21. The distribution of num-

bers of overnight occupied beds simulated from the model looks more flat compared to the original distribution (need to compare skewness & kurtosis). A KS test and a Mann–Whitney U test are used to test the difference between these two distributions; p-values for both tests are smaller than 0.05, which indicate that these two samples are not drawn from the same population. We have to clearly point out that this difference may cause inaccuracy of simulation results. The ICU is less likely to run at a low occupancy in reality compared to the model (i.e. the cumulative distribution with number of occupied beds less than 12 is more likely to occur in the simulation model than in reality).



(a) EDFs

(b) Smoothed distributions

Figure 6.21: Comparison of model and system overnight bed occupancy

6.8 Conclusion

This chapter described the DES model building, IU analysis and model verification and validation of the ICU. In designing of the embedding of data mining models within the simulation, we found that it was important to consider the degree of complexity to which we should extend the models. If the data mining were to rely on large number of variables, both the model building and running cost will be high. We, therefore, chosen simplified versions, i.e. the benchmark models of the data mining models built in Chapter 5.

In the preparation of DES model inputs, we split the problem into arrival process fitting,

LoS modelling and modelling of other input. Modelling of arrival process was further divided into unplanned arrivals modelling and planned arrivals modelling. We fitted an NHPP with 14 intervals representing 14 shifts of staff for the unplanned arrivals of the ICU and tested the NHPP using VMRs to ensure suitability. The planned arrival process is modelled by eight EDFs where seven of them depict the arrival numbers of day of a week and the other one describes arrival time.

Neither distribution fitting or prediction (Chapter 5) worked well for LoS modelling in our case. A novel method to modelling LoS, a three-part model, was described in Section 6.3.2. LoS was modelled using three elements, arrival time, nights in ICU and discharge time. Each element is modelled individually using suitable splits. In our case, arrival time is split into unplanned and planned. Nights are sampled separately for unplanned, planned and late (and re-) admissions. Discharge time modelling is split into ICU survivors and non-survivors. This method models LoS accurately and keeps variability of admission and discharge time.

In addition to the mortality prediction model described in Section 5.2, a patients' initial level prediction model was also introduced.

The DES model was described in detail in Section 6.5.2. A flowchart of the DES model was shown in Figure 6.11. An algorithm of planned patient cancellation (Algorithm 1) was introduced to control the late admissions.

A thorough analysis of IU was provided in Section 6.6. IU has a relatively high impact on annual throughput and ICU LoS. It will not affect the mean values nor the general conclusions but may increase the width of the confidence intervals. More input data, especially for nights spent in the ICU, are needed to get more precise estimations.

A comprehensive verification and validation process was recorded in Section 6.7. Both static and dynamic testing have been used to ensure that the computer model serves our aims. For the purpose of operational validation, several statistics from the DES model have been checked and compared with the original data. Model validation results show that the key statistics gained from the current model are close to those calculated from the original data.

Chapter 7

Simulation Results

In order to investigate the effect of various admission, discharge and staffing policies, we have designed a series of scenarios. The validated DES model depicting the current situation of the ICU is used as a baseline model for the scenario tests. In this chapter, we first describe the design of scenarios. Then, the results of the simulation experiments are reported in Sections 7.1 to 7.6. A conclusion to the scenario analysis is provided in Section 7.7.

Six sets of scenarios were designed;

1. ICU performance under increased unplanned arrival rates are described in Section 7.1;
2. Varying available resources including removing and adding ICU beds and nurses are discussed in Section 7.2;
3. Section 7.3 gives results of reducing the proportion of the late admission group;
4. The scenarios of combinations of increased arrival rates and earlier admission are described in Section 7.4;
5. We also examine a set of scenarios of changing discharge times in Section 7.5;
6. An epidemic scenario designed to simulate possible critical situations, is detailed in

Section 7.6.

7.1 Scenario set 1: arrival number increasing

A scenario test of increasing ICU arrivals was carried out in the first place to find out the potential of the ICU. To increase the total arrivals, the baseline rates of the NHPP (Table 6.2) were multiplied by a factor between 1.05 and 2.00, which means the total unplanned arrival numbers is factor times that of the baseline scenario. For example, 1.20X means multiply current unplanned arrival rates by 1.20 which results in a 20% increase in unplanned arrival numbers. In the simulation model, the variation is achieved by $\frac{\text{inter-arrival time}}{factor}$.

The means and 95% CIs of key outputs, late admission percentages, annual throughput, ICU LoS, mortality rates, and resource utilisations are calculated based on 39 runs and plotted in Figures 7.1 - 7.5. The baseline scenario is marked using blue colours in all these plots. Error bars represent 95% CIs of means.

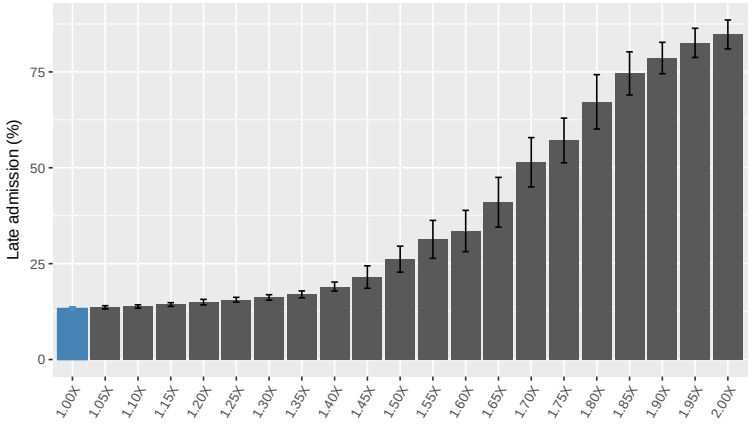


Figure 7.1: Variation of late admissions while increasing unplanned arrivals

Increasing admissions means that queues increase for ICU beds (see Figure 6.11) and therefore numbers of late admissions increase. As shown in Figure 7.1, the late admission group as a percentage of all admissions grows rapidly when the arrival rates are more than 30% of the current level. The group will grow to the largest group of admissions, which accounts for approximately 20% of total arrivals. Late admission may further lead to a prolonged ICU

stay (Figure 7.3) and higher probability of mortality (Figure 7.4).

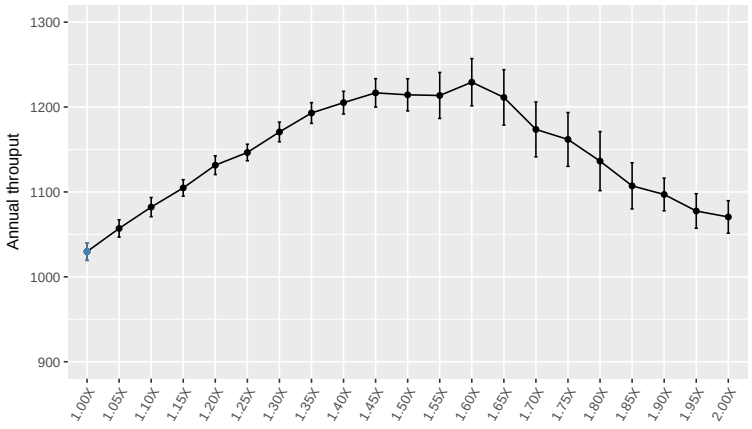


Figure 7.2: Variation of annual throughput while increasing unplanned arrivals

Figure 7.2 exhibits annual throughput under different scenarios (different arrival rates). The highest point of the figure shows that the ICU can accommodate approximately 1215 admissions per year if all the increases in admissions are unplanned admissions. An increase of throughput is observed with increased arrival rates when the factor is smaller than 1.45. When the factor is larger than 1.60, annual throughput declines when arrivals increase. With the growth of arrivals, the queue of patients waiting for admission to the ICU expands in both size and waiting times. More patients become 'late' while they are waiting for an ICU admission. These patients will have a prolonged stay in the ICU (see Figure 7.3). Therefore, the annual throughput is worsened.

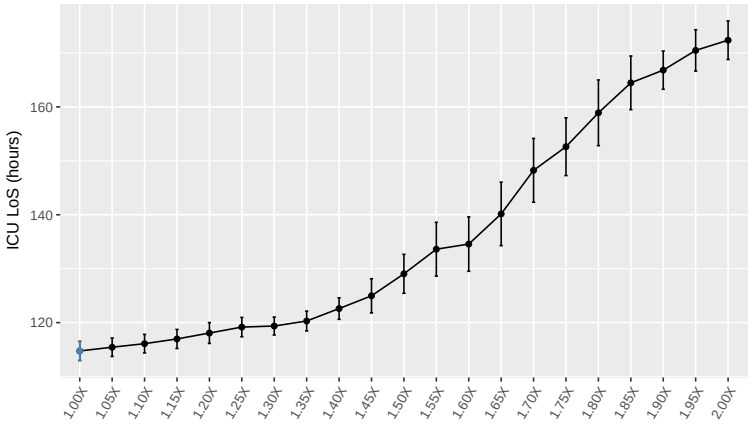


Figure 7.3: Variation of ICU LoS while increasing unplanned arrivals

An increasing trend is observed in the ICU LoS when the arrival rates are increased (see Figure 7.3). As explained, the increasing number of late admitted patients induces prolonged stays in the ICU. A relatively steady trend is observed initially. No statistically significant difference can be observed between groups of 1.00X to 1.15X arrivals according to an one-way ANOVA test. This indicates that the ICU has the potential for treating around 1100 patients while maintaining the current service standard.

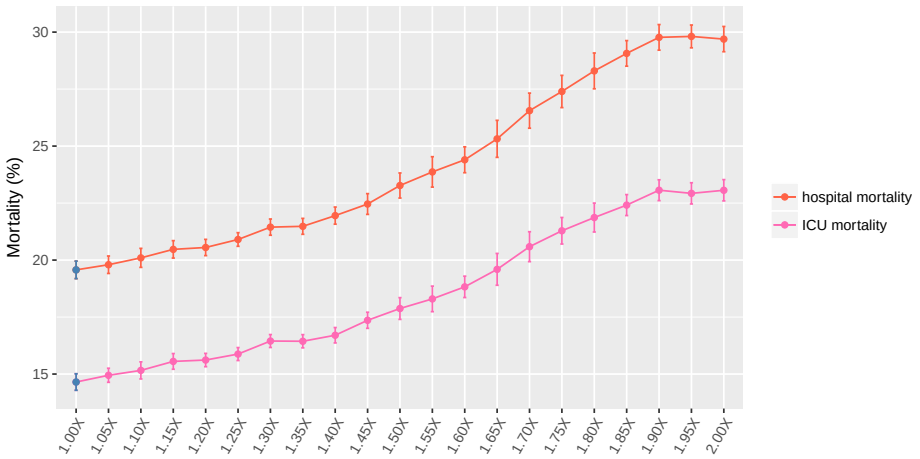
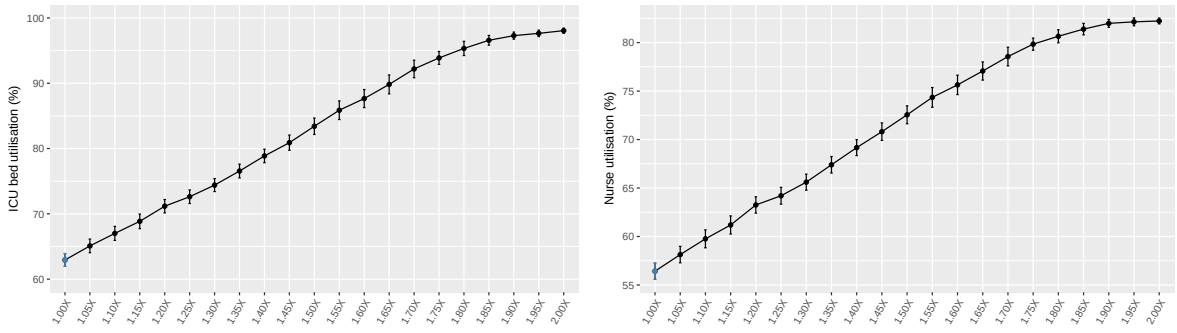


Figure 7.4: Variation of mortality rates while increasing unplanned arrivals

Figure 7.4 presents ICU and hospital mortality rates under different arrival rates. As expected, an increasing trend in mortality is demonstrated because of the increase in late admissions. Differences of mortality rates between the baseline scenario and 1.20X scenario are significant. This difference originates from the growing proportion of unplanned patients who are more prone to death compared to planned patients. If mortality rates are compared when excluding planned patients, the differences between 1.00X and 1.20X are not significant according to a Chi-square test.

We also analysed resource utilisations of the ICU. Figure 7.5a demonstrates that with the increase of arrival rates, the utilisation of ICU beds will grow to almost 100%. The utilisation of nurse resources reaches around 83% due to the nurse-to-patient ratio of different levels of patients from 1.75X arrival rates and higher (see Figure 7.5b).



(a) ICU bed utilisation (b) Nurse utilisation

Figure 7.5: Variation of resource utilisations while increasing unplanned arrivals

7.2 Scenario set 2: resource change

This section analyses the influence of resource changes on the operational efficiency of the ICU. Both ICU beds and ICU nurses are considered in this set of scenarios.

First, we examine the situation of removing or adding ICU beds but maintaining nurse numbers unchanged (n=16). Scenarios of setting the number of beds from 14 to 23 were tested. Simulation results of percentage of late admissions, annual throughput, ICU LoS and mortality rates from different bed numbers are plotted in Figures 7.6 - 7.9. The error bars in these figures show the 95% CIs of the values. The calculation of means (points) and CIs (error bars) are based on 39 runs of every trial.

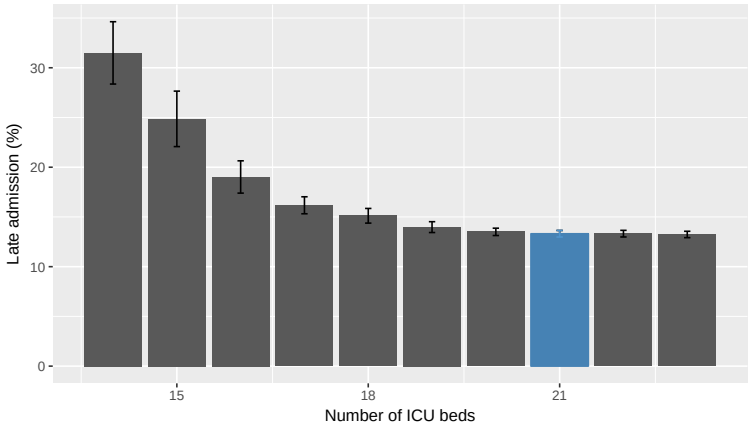


Figure 7.6: Variation of late admissions while changing number of beds

The bar chart (Figure 7.6) presents the variation of percentage of late admissions while changing bed numbers. The baseline 21-bed scenario is highlighted in blue. It is clear that inadequate numbers of ICU beds lead to a severe delay of patients. The delay will eventually result in a prolonged ICU stay (Figure 7.8) and higher ICU mortality (Figure 7.9).

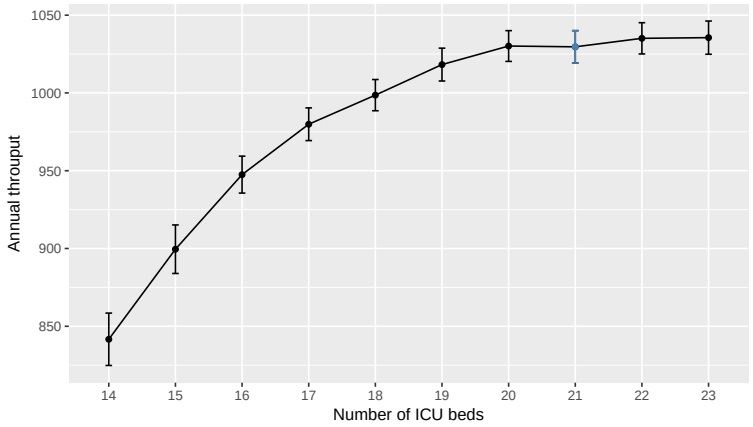


Figure 7.7: Variation of annual throughput while changing number of beds

Figure 7.7 shows the influence of ICU beds on annual throughput. As expected, the decrease in the number of beds causes decrease in the throughput. However, throughput of the 20-bed and 21-bed scenarios are very close as the nurse resource is designed to serve a 20-bed ICU (see Section 2.2). Variation of the throughput is not significant if an extra bed is added ($p\text{-}value > 0.90$). This results from the number of arrivals staying the same as currently in our model. The data include only patients who were admitted to the ICU but not patients who had been turned away.

Figure 7.8 shows the variation of ICU LoS with changing bed numbers. It appears to be a trend of increasing LoS as numbers decrease from the baseline of 21. In simulation, this is caused by the prolonged waiting time due to the lack to beds, which can be inferred from Figure 7.6. Adding extra beds above the baseline number makes only slight decrease of LoS due to the fixed proportion of late admissions in the simulation (see Section 6.5.2).

ICU mortality rates, hospital mortality rates and their 95% CIs are demonstrated in Figure 7.9. The trends of the lines are similar to the trends shown in Figure 7.8. The decrease in both mortality rates with the increase of ICU beds is mainly as a consequence of less late

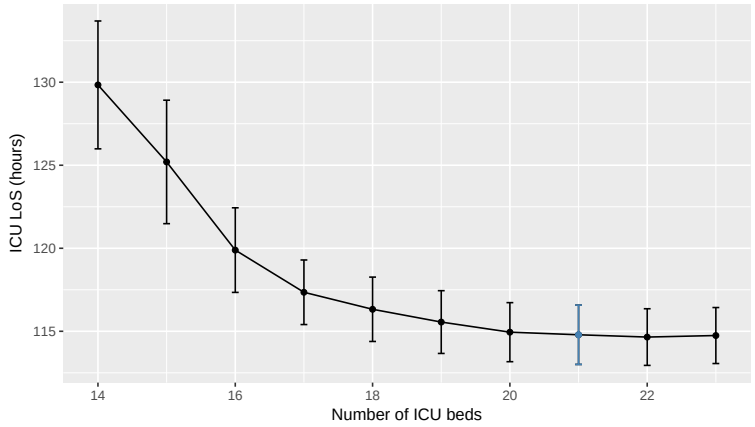


Figure 7.8: Variation of ICU LoS while changing number of beds

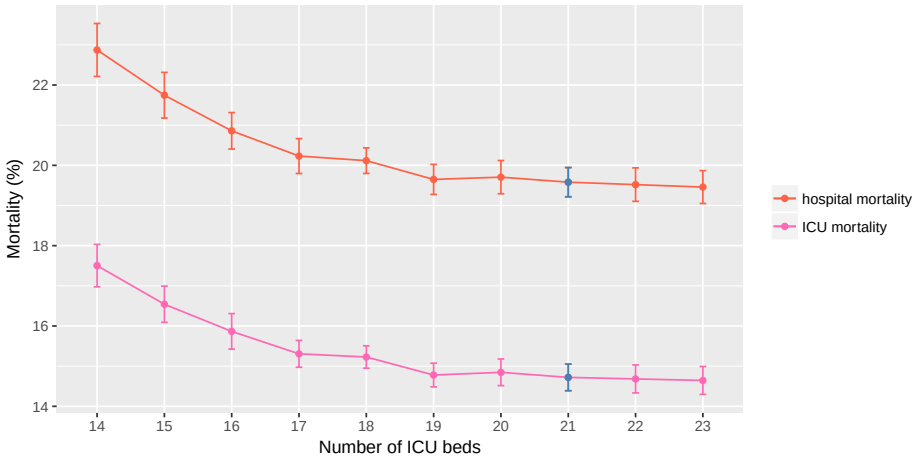


Figure 7.9: Variation of mortality rates while changing number of beds

admissions.

After examining the influence of ICU beds, we then check the influence of the nurse resource. Initially, there are 16 nurses in total. Each can serve either one L3 patient or two L2 patients. The same set of results as for the bed number analysis are shown in Figures 7.10 to 7.13.

Percentages of late admission under different numbers of nurses are shown in Figure 7.10. With dropping nurse numbers by one or two, no significant increase in percentage of late admission could be observed. If nurse numbers keep reducing, the group of late admitted patients will grow dramatically.

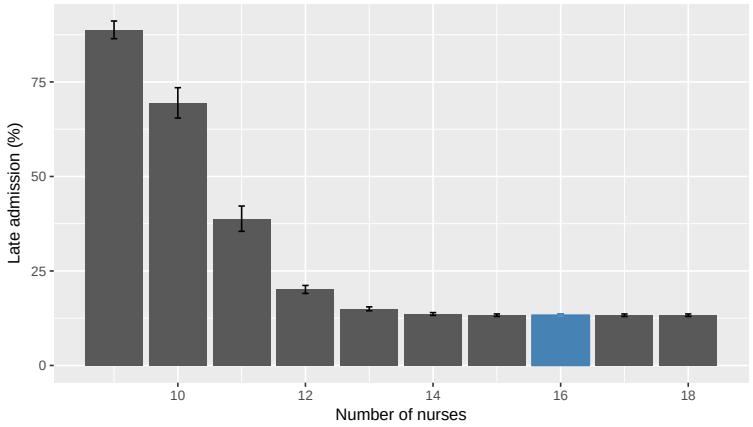


Figure 7.10: Variation of late admissions while changing number of nurses

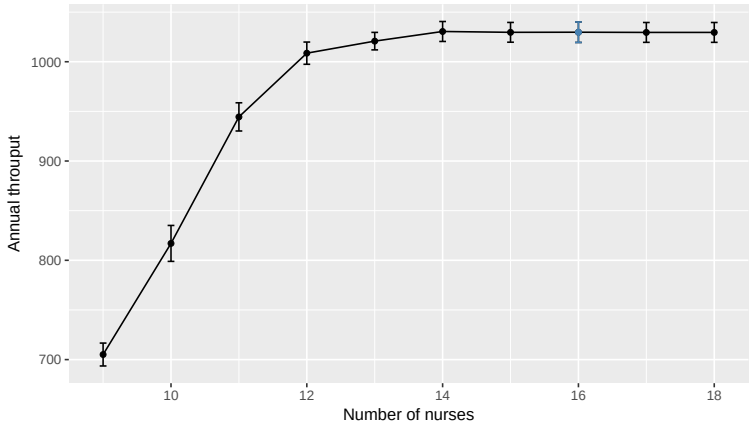


Figure 7.11: Variation of annual throughput while changing number of nurses

Figure 7.11 demonstrates that annual throughput under scenarios between 12 and 18 nurses does not differ significantly. A further drop of nurse numbers from 12 and downwards will rapidly reduce the annual throughput, which shows the criticality of the nurse resource in that range.

ICU LoS shows a stable pattern while the ICU is staffed with more than 13 nurses, as in Figure 7.12, which can be inferred earlier from Figure 7.10. A high proportion of the late admission group lengthens the ICU LoS.

Mortality rates in Figure 7.13 shows similar trends to ICU LoS (Figure 7.12) and late admission (Figure 7.10), where the nurse number equalling 13 is a borderline scenario. Further

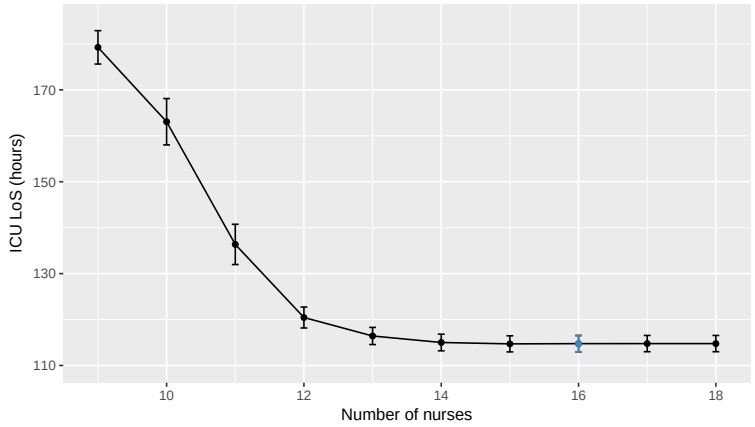


Figure 7.12: Variation of ICU LoS while changing number of nurses

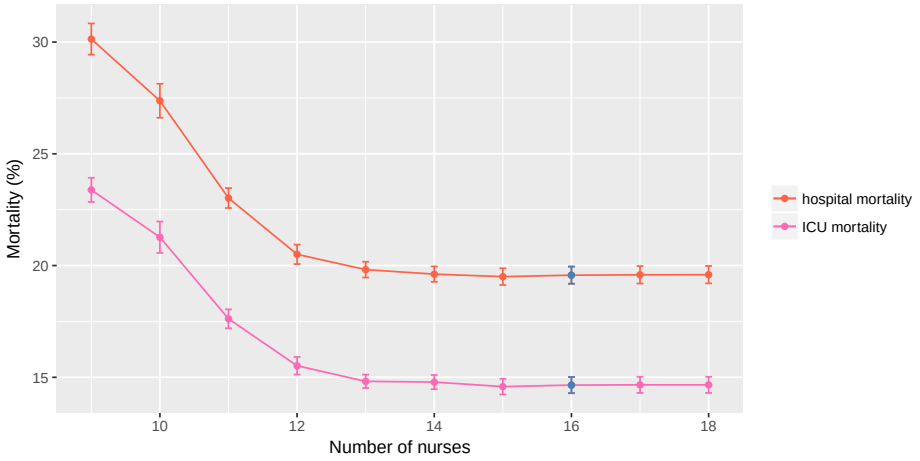


Figure 7.13: Variation of mortality rates while changing number of nurses

decreasing from the borderline scenario leads to a fast worsening of performance.

Comparing the influence of nurse numbers with that of bed numbers, it can be discovered that the decrease of bed numbers will be reflected in the worsened performance immediately while the decrease of nurse numbers will not impact negatively on the ICU performance until the number is reduced by more than three. Therefore, in the current situation, beds are more critical to the ICU as it is just enough for current arrivals. However, if both resources are reduced in the ICU, nurses will demonstrate a larger influence on ICU operation.

7.3 Scenario set 3: earlier admission

We assume that a change in admission timing could influence the mortality and ICU LoS accordingly although there may exist some not yet investigated confounding variables. This set of seven scenarios consist of base case, five varied unplanned late percentage and an optimal scenario of prompt admission of all patients. The base case is 20.70% late admission for unplanned patients and 4.09% for planned. The details of the other scenarios are listed in Table 7.1. For example, the "85%-15% (unplanned)" scenario has 15% late admission of unplanned patients and 4.09% of planned patients. Only percentage of late admissions varied; all the other inputs maintain the same value as the validated baseline model.

Table 7.1: Description of scenarios of earlier admission

	Scenarios	80%-20% (unplanned)	85%-15% (unplanned)	90%-10% (unplanned)
Late (%)	unplanned	20%	15%	10%
	planned	4.09%	4.09%	4.09%
	Scenarios	95%-5% (unplanned)	100%-0% (unplanned)	100%-0% (all-optimal)
Late (%)	unplanned	5%	0%	0%
	planned	4.09%	4.09%	0%

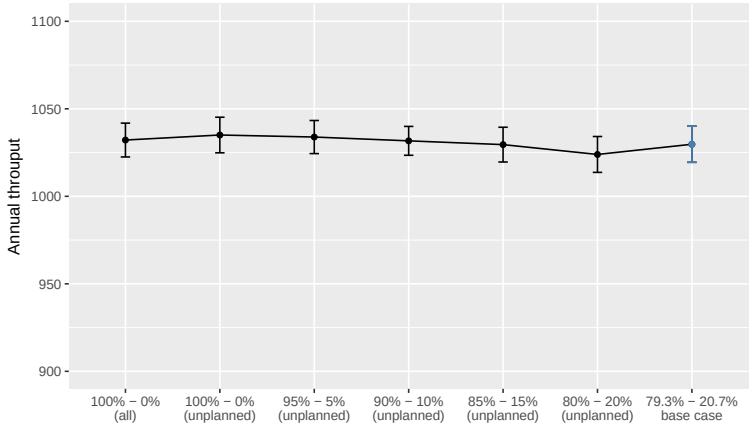


Figure 7.14: Variation of annual throughput under earlier admission scenarios

Annual throughput in the different scenarios does not exhibit much difference as shown in Figure 7.14. An one-way ANOVA test indicates that the differences between each groups are not significant. As explained earlier, due to the limitation of the data, no more arrivals

can be generated under base conditions.

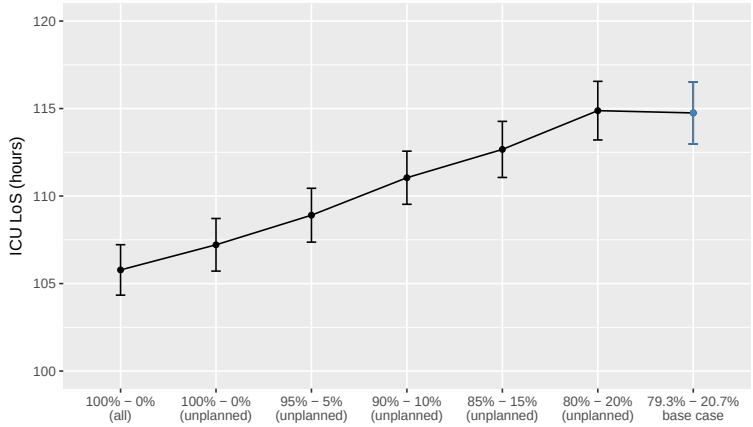


Figure 7.15: Variation of ICU LoS under earlier admission scenarios

From Figure 7.15, it can be observed that ICU LoS would be shortened if more patients could be admitted on time. There is a great potential in saving resources and increasing throughput as indicated by the shorter LoS.

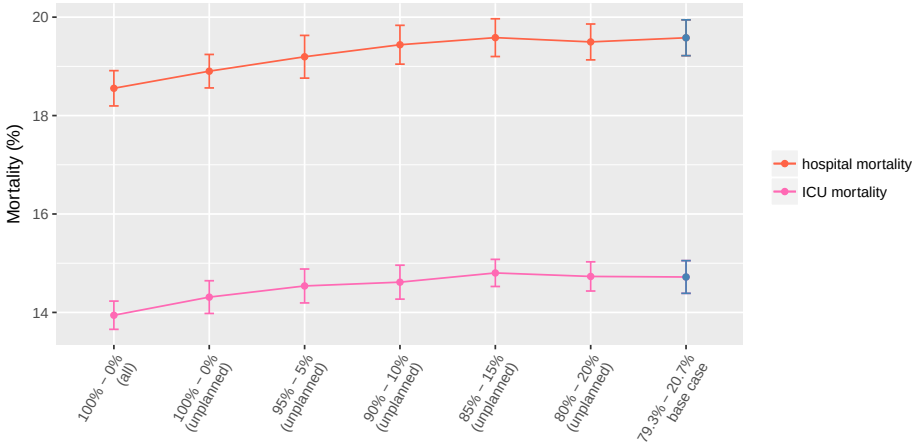
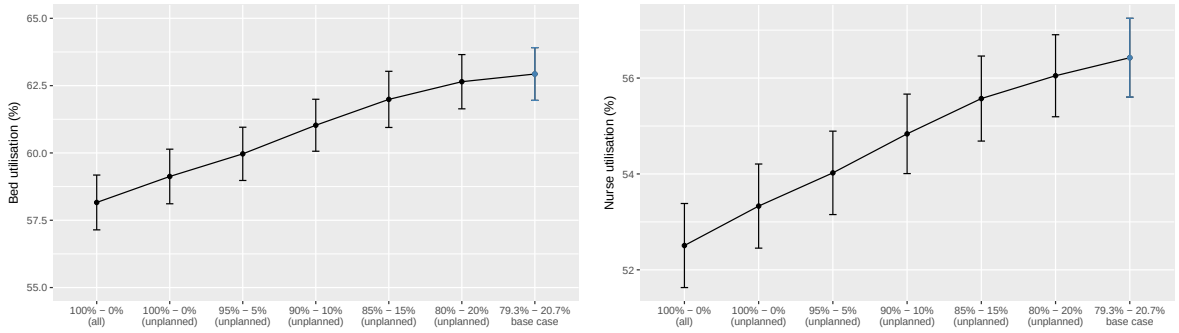


Figure 7.16: Variation of mortality rates under earlier admission scenarios

Mortality rates do not show a dramatic decrease in earlier admission scenarios as shown by Figure 7.16. However, the difference between the optimal case and the base case is significant in both ICU and hospital mortality rates. It is worth pointing out that since patients admitted by the ICU are critically ill, the mortality rate is not expected to be zero. Therefore, the marginal improvement in chance of survival is still important.



(a) ICU bed utilisation

(b) Nurse utilisation

Figure 7.17: Variation of resource utilisations under earlier admission scenarios

The trends of the utilisation plots for beds and nurse in Figures 7.17a and 7.17b are in line with the line chart for LoS (Figure 7.15). In the ideal scenario, a larger number of admissions can be treated without increasing current workload.

7.4 Scenario set 4: earlier admission under increased unplanned arrival rates

The previous section examined the earlier admission scenarios. Higher annual throughput and better performance are expected if the late admission problem may be alleviated. We further extend the scenarios by combining increased unplanned ICU arrivals with earlier admission. In this section only 90%-10% (unplanned) and 95%-5% (unplanned) scenarios will be considered as we expected an improvement in late admission but it is very hard to eliminate the problem entirely. When plotting Figures 7.18 to 7.21, each line represents a different set of scenarios with arrival rate increment factors shown as 'X' values. In consistency with other scenarios, the blue line here shows the baseline scenarios.

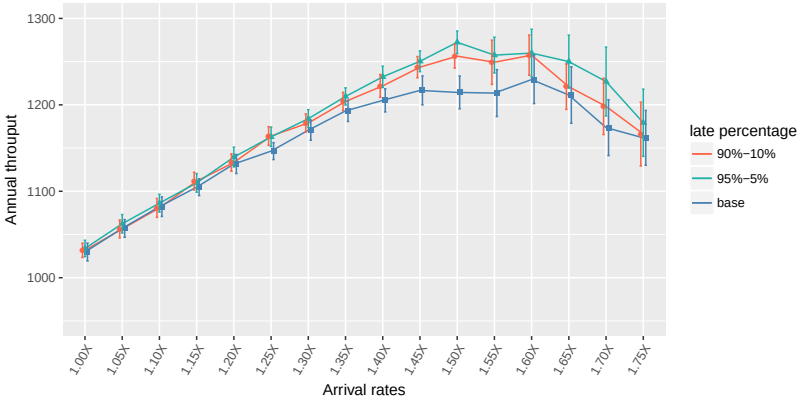


Figure 7.18: Variation of annual throughput under increased arrival rates and earlier admission

It is shown in Figure 7.18 that annual throughput has a potential to be increased from 1210 to 1270 under the 1.50X factor of the NHPP rates. Similar tendencies of growth and decline are observed in all three groups with the growth rate higher in earlier admission groups. The 95% CIs of annual throughput with arrival rates from 1.45X to 1.55X are not overlapped, which indicates a significant difference between them.

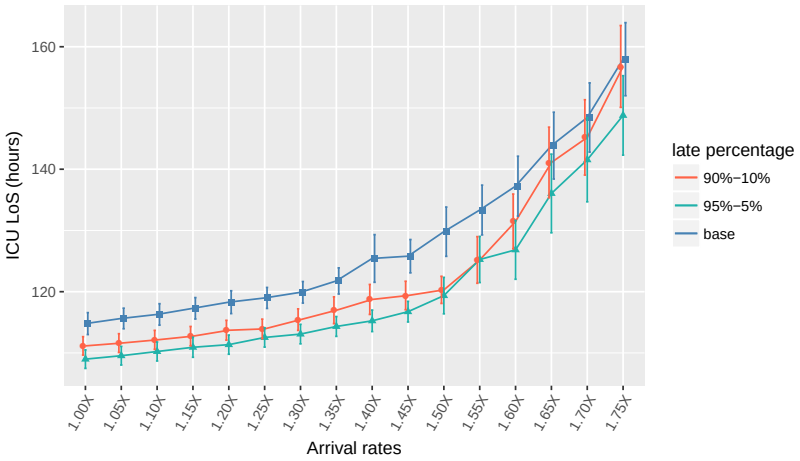
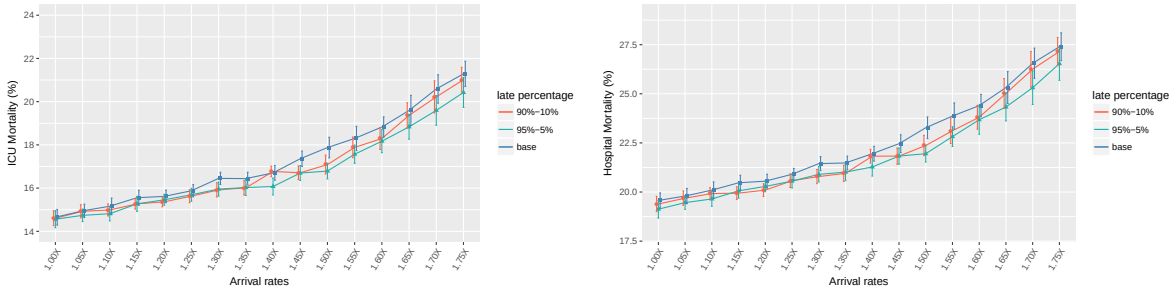


Figure 7.19: Variation of ICU LoS under increased arrival rates and earlier admission

The increasing trends of ICU LoS in all three groups observed from Figure 7.19 are the same while the differences in LoS is significant between earlier admissions and base case when the factor multiplying baseline NHPP rates is not larger than 1.55. The ICU LoS of the 90%-10% and 95%-5% groups under factors 1.30X and 1.45X respectively is very close to the value of

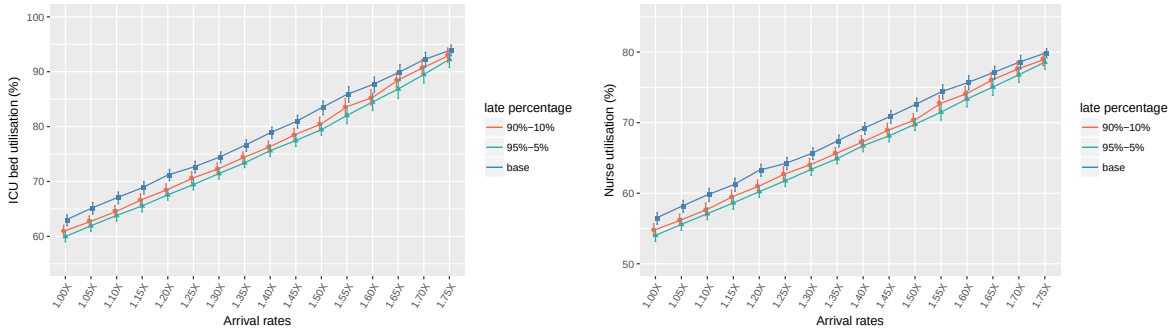
ICU LoS in the baseline case, which implies the potential of ICU performance under earlier admissions.



(a) ICU mortality (b) Hospital mortality

Figure 7.20: Variation of mortality rates under increased arrival rates and earlier admission

Figure 7.20 demonstrates ICU and hospital mortality rates under different circumstances. Both mortality rates improve marginally under earlier admission conditions. The improvements under the maximum throughput arrival rates (1.30X to 1.55X) are significant for both ICU and hospital mortality rates.



(a) ICU bed utilisation (b) Nurse utilisation

Figure 7.21: Variation of resource utilisations under increased arrival rates and earlier admission

The line and the error bars of the base case (the blue line) in Figure 7.21a) are not overlapped with most of the parts of the lines depicting earlier admissions, meaning that the bed utilisation is significantly reduced if patients are admitted earlier. Figure 7.21b presents a similar trend for nurse utilisation as Figure 7.21a.

7.5 Scenario set 5: varying discharge time

Previous scenarios are focused on admissions rather than discharge. In this section, we would like to find out how discharge time influences ICU operations. Patients should be discharged within four hours of their clinically ready-to-be discharged time. Delayed discharge is observed in about half of all discharges. We set three different scenarios for three different optimal discharge times as shown in Table 7.2. The EDFs of nights spent in the ICU are changed to reflect the scenario under consideration, making use of the data available on the time that a patient is clinically ready for discharge.

Table 7.2: Scenarios of optimal discharge hours

	ICU non-survivors	ICU-surviors	
		discharged within 4hr limit	discharged out of 4hr limit
base	actual time	actual time	actual time
sce 1	actual time	actual time	clinically ready time
sce 2	actual time	actual time	clinically ready time + $2hrs$
sce 3	actual time	actual time	clinically ready time + $4hrs$

In order to analyse the effect of varying discharge time thoroughly, increases of the NHPP rates are taken into consideration. Optimal discharge time are demonstrated in different scenarios using different colours of lines in Figures 7.22 to 7.26. The multiplication factors of the rates are presented in the horizontal axis.

Figure 7.22 gives the mean values and 95% CIs of annual throughput under various conditions. No significant differences of annual throughput can be observed in all the tested scenarios while the number of annual arrivals is close to current level. Therefore, prompt discharge will not substantially affect annual throughput in this particular case. However, while the ICU is under pressure (e.g. continues high numbers of arrivals lasting for a period), prompt discharge makes a significant difference to the potential annual throughput.

Four nearly parallel lines for LoS are shown in Figure 7.23. The blue line is not overlaid with any of the other lines, indicating that ICU LoS can be reduced significantly if patients are discharged on time. The lines are parallel mainly because of the different settings of the scenarios (i.e. 2hrs difference between each optimal discharge time group).

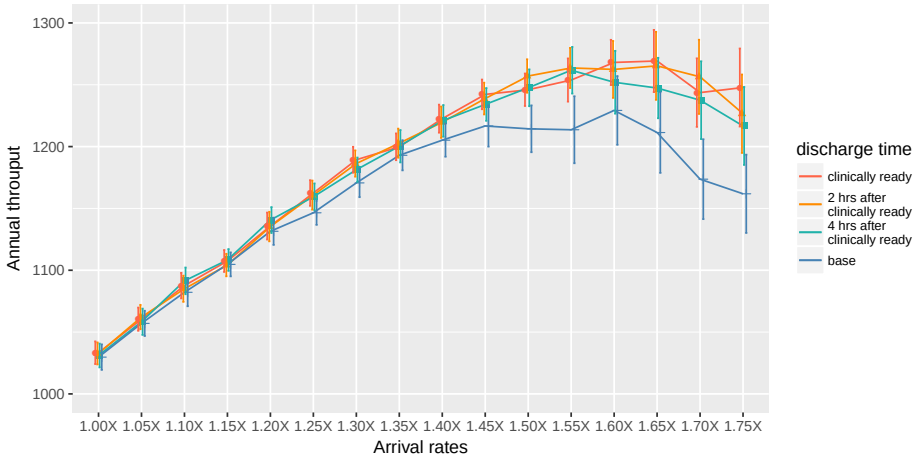


Figure 7.22: Variation of annual throughput under optimal discharge times

As shown in Figure 7.24, the percentages of late admissions are close between groups when the increases of unplanned arrival rates are smaller than 1.15. With the increase of arrival rates, variations of late admissions also increase (i.e. larger 95% CI), possibly resulting from the increasing dependency on the stochastic process of arrivals. The current discharge time (base case) generates substantially more late patients while the ICU is under pressure. Discharge on time is especially important while sharp increase of arrivals happens.

ICU and hospital mortality rates in this set of scenarios are demonstrated in Figures 7.25a and 7.25b. No statistically significant between-group differences can be detected from either of the mortality rates. Similar mortality rates arise from similar late admission percentages.

Both Figures 7.26a and 7.26b show parallel and non-overlapping lines and 95% CIs, suggesting that if patients could be discharged within the required time limit, the workload of the ICU can be brought down significantly. This reduction can also be implied from the ICU LoS in Figure 7.23.

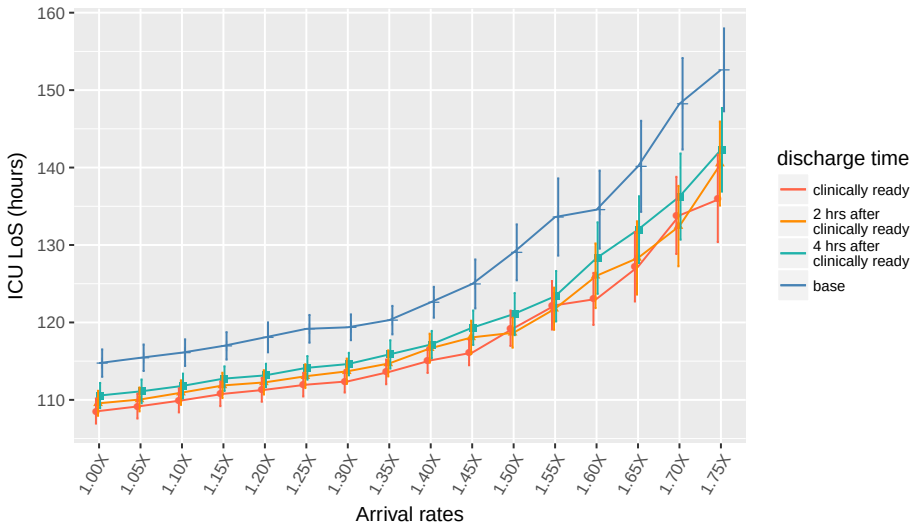


Figure 7.23: Variation of ICU LoS under optimal discharge times

7.6 Scenario set 6: epidemic

As reviewed in Section 3.5.2, ICUs are involved in pandemics extensively. In this section, a number of scenarios of epidemics are designed to mimic potential impacts of an influenza pandemic situations. According to the UK Department of Health contingency and response plans for an influenza pandemic (Department of Health, 2005, 2012), the worst case could be an attack rate as high as 50% with a fatality rate of 2.4%, lasting for 15 weeks. Between 1% and 4% of symptomatic patients would be admitted to hospitals and up to 25% of them would require L3 critical care. The pandemic preparedness strategy shows that demand for critical care services will not be met even at maximum expansion in a relatively mild case (DH Pandemic Influenza Preparedness Team, 2011). A likely scenario is a 25% attack rate in an 8-week period with 0.37% fatality. A combination of high attack rates (circa 50%) and a long pandemic duration (i.e. $>$ 8 weeks) or severe cases (inferred by a high fatality rate) is unlikely but hard to predict.

Workforce during an influenza pandemic is a major issue. In the absence of vaccination, 40-70% of staff may not be able to work during an influenza pandemic according to Anderson et al. (2003). In the case in Liverpool in 1957, 12% to 19% of nurses were absent in most hospitals during a four-week period. The highest absence rate was 1/3 (Department of

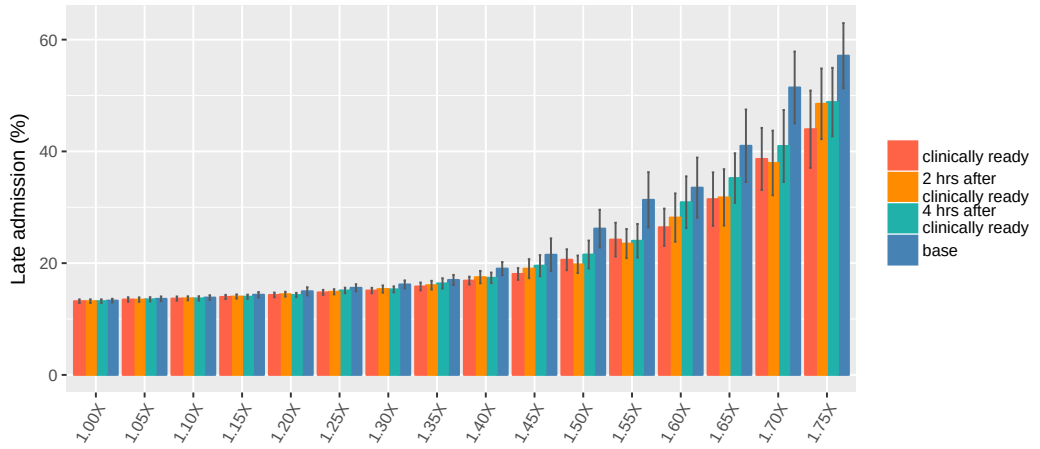
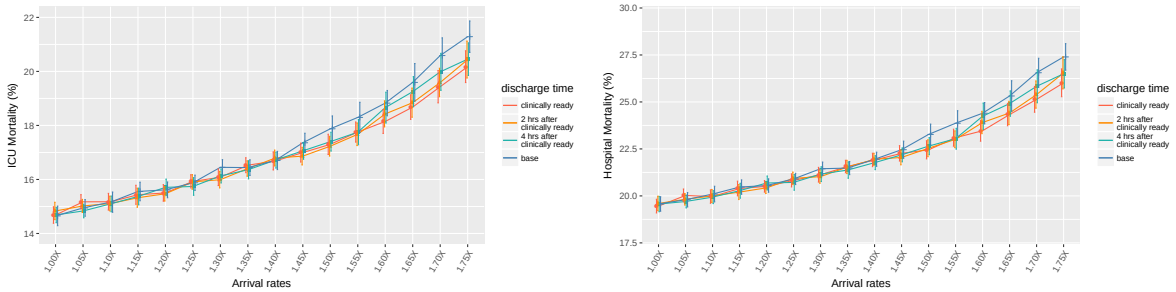


Figure 7.24: Variation of late admissions under optimal discharge times



(a) ICU mortality

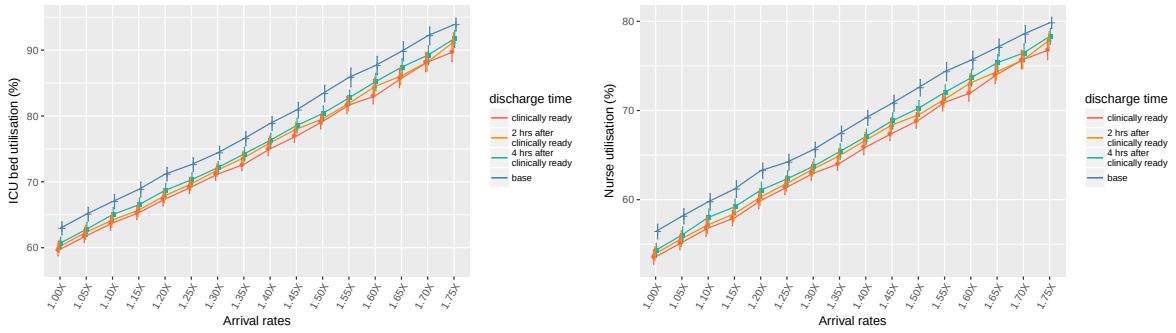
(b) Hospital mortality

Figure 7.25: Variation of mortality rates under optimal discharge times

Health, 2005). During the Severe Acute Respiratory Syndrome (SARS) period in Toronto, 73 ICU beds, approximately 1/3 of the ICU beds in the city, were closed due to secondary transmission (Booth and Stewart, 2003).

Several versions of three epidemic scenarios are described in Table 7.3 according to historical records of influenza pandemics, assuming that influenza pandemic-related statistics (attack rate, admission rate, etc.) are the same all over the UK (Department of Health, 2005, 2012; CDC, 2016). The scenarios are indexed in the first column.

Population data is also needed in the scenario design. We use a regional population projection for 2026 (Table 7.4) to estimate admissions and arrivals. The latest population census was in 2011. Population projections by decade for 2016 to 2046 by area are based on the latest



(a) ICU bed utilisations

(b) Nurse utilisations

Figure 7.26: Variation of resources' utilisations under optimal discharge times

Table 7.3: Epidemic scenarios

	scenario	attack rate (%)	hospital admission (%)	ICU admission* (%)	period (weeks)	staffing (%)
1a	mild	15	1	15	15	100
1b	mild	15	1	15	15	90
2a	likely	25	2.5	15	8	100
2b	likely	25	2.5	15	8	80
2c	likely	15	4	25	8	100
2d	likely	50	1	15	8	100
3a	worst	50	4	25	6	100
3b	worst	50	4	25	6	70

*ICU admission rate $= \frac{\text{total ICU admissions}}{\text{total hospital admissions}}$

census (Office for National Statistics, 2017).

Table 7.4: Population projection for 2026 of areas served

	total	≤16 years old	>16 years old
Bristol, City of	496,807	103,105	393,702
North Somerset	231,585	44,852	186,733
South Gloucestershire	302,489	60,697	241,792
all areas	**1,030,881**	**208,654**	**822,227**

ICU LoS of influenza patients is modelled by an exponential distribution with $mean = 173.1234\,hours$. The value was calculated from median ICU LoS of adult influenza patients in a US hospital using $mean = \frac{median}{log2}$ (Bramley et al., 2012).

The arrivals are assumed to be spread out unevenly during the whole period. The listed

scenarios are tested under two different patterns of arrivals. First, influenza patient arrivals follow a PP with a constant daily arrival rate equalling total ICU admissions divided by pandemic duration (dashed lines in Figure 7.27). Second, arrivals obey an NHPP with a 3% daily increase and a 3% daily decrease in the arrival rate before and after the peak time (solid step lines in Figure 7.27). The peak time is assumed to be the mid-point of a pandemic duration. All these settings follow suggestions from FluSurge2.0 (CDC, 2016). The tool was not used directly as it does not support attack rates, periods or staffing level variations or investigate behaviour of an ICU directly.

BRI serves the southwest of England with a focus on Bristol, North Somerset and South Gloucestershire, which have around one million residents (NHS, 2017a). Three NHS trusts are equipped with ICU beds in these three areas. There are 214 ICU beds in total, within which 108 and 106 are paediatric/neonatal (PICU) and adult critical care beds respectively (NHS, 2017b). PICUs normally treat patients up to the age of 16 (NHS, 2013a).

Five additional assumptions for the scenarios are made:

- PICUs will not treat patients over 16 years old (even when adult ICUs are not able to cope with all admissions)
- Resources in PICUs are enough to treat all influenza pandemic patients (tested using FluSurge), which means adult ICUs will only treat patients over 16 years old.
- Patients in all age groups have similar admission rates and ICU LoS (no admission data for different age groups could be found).
- Pandemic cases treated by the BRI Adult ICU are in proportion to critical care bed numbers (i.e. 21/106 of total influenza pandemic ICU admissions in served areas)
- Staffing levels are assumed to steadily decrease from full attendance to the minimum staffing level specified in Table 7.3 before the beginning of a peak week, maintaining the minimum level during three days before and after the peak time and then gradually growing back to full attendance during the week after a pandemic. Detailed staffing

levels are plotted in Figure 7.28.

All the input parameters for the epidemic scenarios are summarised in Table 7.5 and Figures 7.27 and 7.28 based on these assumptions.

Table 7.5: Total arrivals and staffing numbers (epidemic scenarios)

scenario	symptomatic patients	hospital admissions	ICU admissions	BRI ICU admissions	minimum nurses
1a	123,334	1,233	185	37	16
1b	123,334	1,233	185	37	14
2a	205,557	5,139	771	153	16
2b	205,557	5,139	771	153	13
2c	123,334	4,933	1,233	244	16
2d	411,114	4,111	617	122	16
3a	411,114	16,445	4,111	814	16
3b	411,114	16,445	4,111	814	11

Assuming either a PP or an NHPP, the daily arrival rate of ICU admitted pandemic patients can be calculated. Arrival patterns of different scenarios are plotted in Figure 7.27.

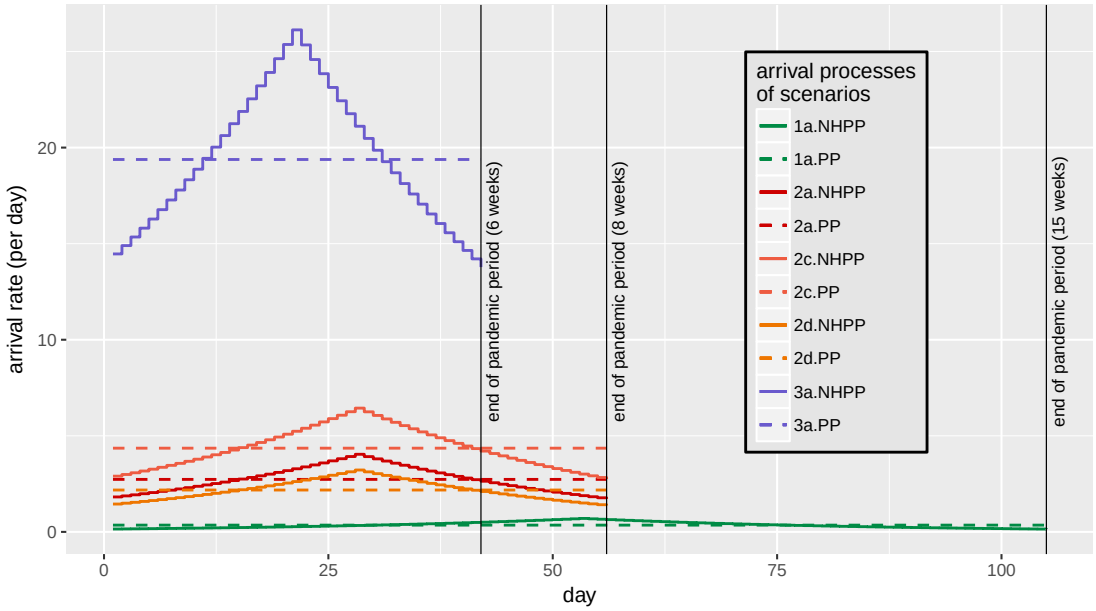


Figure 7.27: Arrival rates of the ICU (epidemic scenarios)

In an M/M/c queue representing a PP, assuming the arrival rate is λ, the service rate is μ, server number is c; the utilisation ratio (ρ), also known as traffic intensity, is obtained

using $\rho = \frac{\lambda}{c\mu}$. $\rho \geq 1$ suggests an infinite queue. Table 7.6 illustrates different ρ of different scenarios and server numbers.

Table 7.6: Traffic intensities of different scenarios

	λ	μ	$\rho_{c=6}$	$\rho_{c=10}$	$\rho_{c=16}$	$\rho_{c=21}$
1a,b	0.3524	0.1386	**0.4237**	**0.2542**	**0.1589**	**0.1210**
2a,b	2.7321	0.1386	3.2847	1.9708	1.2317	**0.9385**
2c	4.3571	0.1386	5.2383	3.1430	1.9644	1.4967
2d	2.1785	0.1386	2.6191	1.5715	**0.9822**	**0.7483**
3a,b	19.3810	0.1386	23.3007	13.9804	8.7378	6.6574

The results in the last four columns in Table 7.6 show traffic intensities of different scenarios while different numbers of beds are available. Scenario with number of servers $c = 6$ refers to available resources based on current utilisation of beds and nurses. $c = 10$ considers reducing current arrivals to a 60% level, to reserve resources to serve pandemic arrivals. $c = 16$ is the full capacity of the ICU while considering both nurses and beds. $c = 21$ requires an involvement of more nurses to make all 21 beds into ICU beds.

Table 7.6 shows that, under the most optimistic condition (21-bed), the ICU cannot cope with scenarios '3a' and '2c'. The '2a' scenario may also overload the ICU sometimes as the traffic intensity is close to 1. Under all the other conditions ($c = 6, 10\, or\, 16$), the ICU is not likely to be able to deal with the pandemic arrivals perfectly for any the pandemic scenarios except the mild one.

To generate NHPP and PP pandemic arrivals, a new start point is added to the DES model. The thinning method is used to generate NHPP arrivals. Consider a piecewise-constant NHPP with rate function $\lambda(t)$, $t \geq 0$ and $\lambda_M = max(\lambda(t))$ in an interval $(0, T]$. The thinning method for generating arrivals is shown in Algorithm 3. The new start point will only generate arrivals from the 31^{st} day till the end of pandemic period.

It should be noted that staffing level change is tested only under the NHPP arrivals as it is a more practical arrival pattern compared to a PP. We index scenarios using PP and NHPP, for example "1a.PP" means a mild scenario with a constant staffing level and PP arrivals.

We made four more assumptions when programming epidemic scenarios:

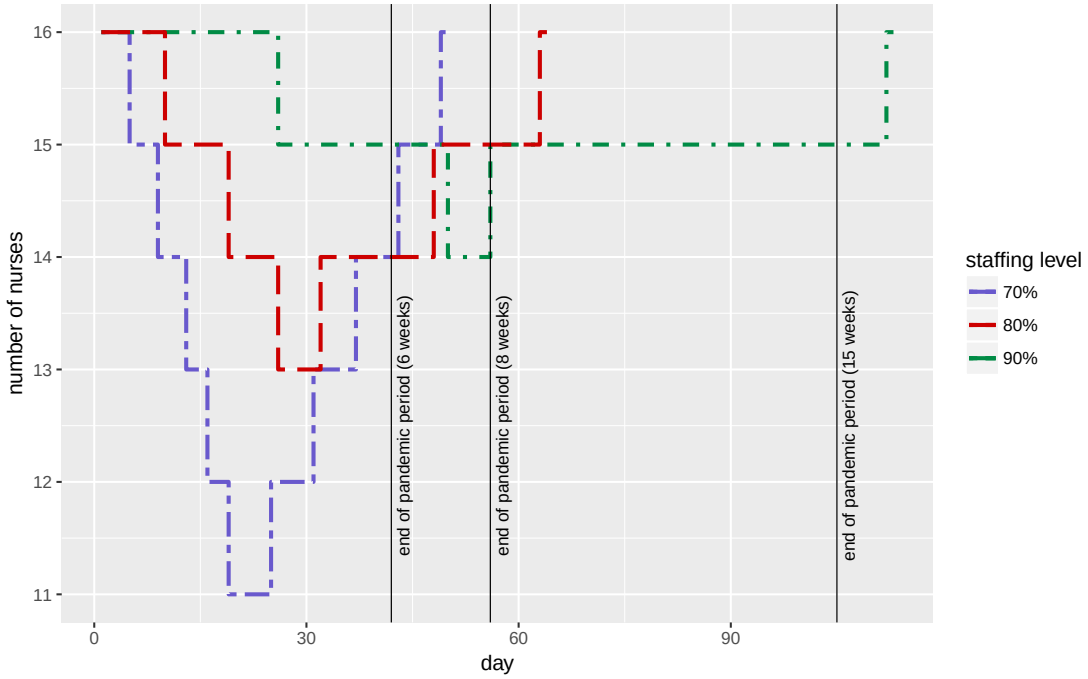


Figure 7.28: Staffing levels at the ICU during pandemics

- Pandemic patients have the highest priority amongst all arrivals
- Cancellation of planned patients using Algorithm 1 will also consider pandemic arrivals.
- Pandemic patients waiting for more than 3 days (72hrs) will renege the queue.
- The pandemic period starts from the second month of a simulation (i.e. 720hrs after warm-up period). Putting the pandemic period in an early stage of the simulation allows us to investigate the influence of an epidemic on the ICU.

We focus on the impact of pandemic on the operation of the ICU. For the pandemic part, the reneging rate and service size will be checked. We ran all the scenarios to confirm:

- How many pandemic patients can the ICU treat during pandemic period as well as how many pandemic patients will leave without treatment?
- How long do pandemic patients wait before being admitted to an ICU?

Algorithm 3 Thinning

(1) Initialise $t = 0$
(2) Generate an exponential random variable e with rate λ_M
(3) Set $t = t + e$
(4) Generate a uniform random variable $u \sim U(0, 1)$
if $t \leq T$ **then**
  if $u \leq \lambda(t)/\lambda_M$ **then**
    Accept the arrival at time t
    GoTo (2)
  else
    Reject the arrival
    GoTo (2)
  end if
end if

- How will the pandemic period affect ICU performance on treating regular patients (i.e. the differences in ICU mortality, ICU LoS and late admissions of pandemic-included and pandemic-excluded performance of regular patients)?

- How many patients will be cancelled?

- How long does the ICU takes to recover from a pandemic?

In the first place, the queuing behaviour of pandemic arrivals are examined. Figure 7.29 shows arrivals and services, and Figure 7.30 summarises queue behaviour of pandemic patients. Detailed results of each scenario are attached in Appendix H.

Each stacked column in Figure 7.29 represents total pandemic arrivals of a scenario, where the plum coloured part denotes admitted patients and the grey colour denotes patients waiting for more than 72hrs and leaving the queue without treatment. Values in the figure illustrate the percentage of treated pandemic arrivals (i.e. admitted pandemic patients/all pandemic arrivals). It can be observed that

1. if pandemic patients are prioritised, the ICU can admit all the pandemic arrivals in the mild scenario;

2. in the other scenarios, when arrival rates vary (i.e. NHPP arrivals), not as many patients

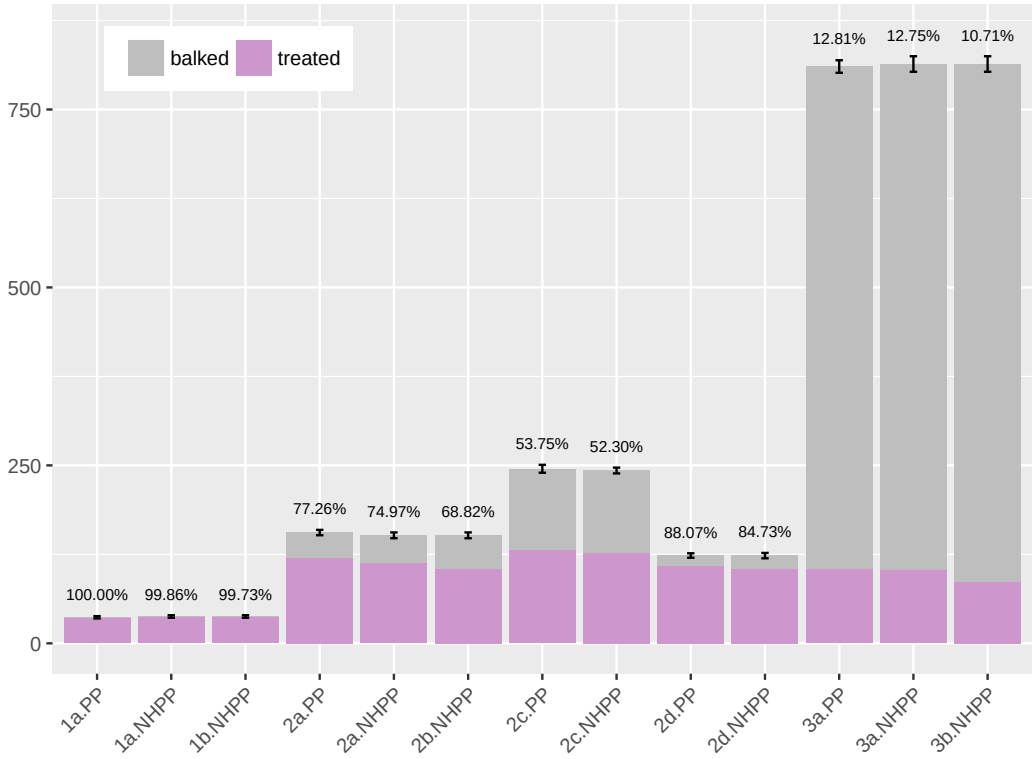


Figure 7.29: ICU admission of pandemic patients

can be treated as under a constant arrival rate;

3. decreasing in the number of nurses has a significant negative effect on the pandemic throughput in the 1a,b and 3a,b scenarios, shown by t-tests between NHPP arrivals with and without changing nurse levels (p-values$= 3.23 \times 10^{-4}$ and 9.96×10^{-7} respectively).

Error bars in Figure 7.30 show 95% CIs of average waiting time in hours of pandemic patients admitted to the ICU. In the worst scenarios (3a,b), patients need to wait three days before admission. In the likely scenarios (2a,b,c,d) the average waiting time is still over 24hrs.

Table 7.7: Percentage of pandemic patients waiting for less than 24hrs (%)

	1a	1b	2a	2b	2c	2d	3a	3b
PP	99.13	×	19.55	×	6.40	35.9	1.34	×
NHPP	98.31	94.59	22.01	18.94	7.90	33.65	1.34	1.34

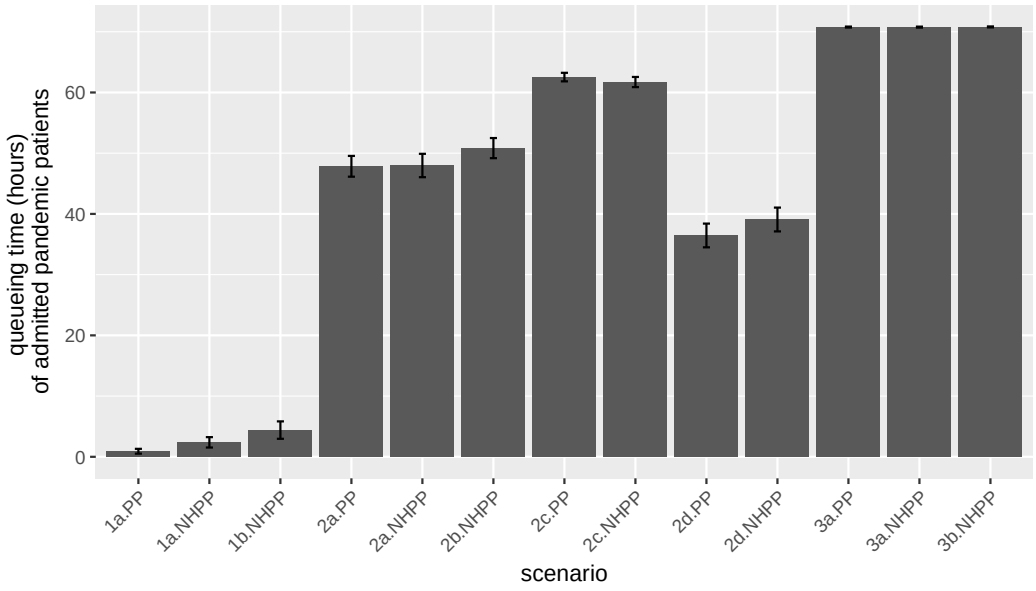


Figure 7.30: Queueing time of admitted pandemic patients

Table 7.7 exhibits the percentage of ICU-admitted pandemic patients who wait less than 24hrs to be admitted to the ICU. Except in the mild scenarios, most patients wait for more than 24hrs before admissions if no premature discharge is considered.

In the case of the BRI ICU, it is clear that the ICU would be under great pressure during a pandemic period except under the mild scenarios. These findings can be compared with previous estimations which find in the UK that current ICU capacities are not able to accommodate all possible arrivals (pandemic and non-pandemic), not even all pandemic patients only, according to DH Pandemic Influenza Preparedness Team (2011) and estimations based on FluSurge (Menon et al., 2005). In the US, Rubinson and O'Toole (2005) also claim that without careful pre-event planning, ICU resources will quickly be overwhelmed and further prevent people benefiting from them in.

Besides short-term influence on the ICU, the impact of an epidemic on the ICU's relatively long-term operation is also important.

Figure 7.31 shows total annual throughput of the ICU in different epidemic scenarios. The annual throughput increases under mild scenarios since the ICU can admit and treat more

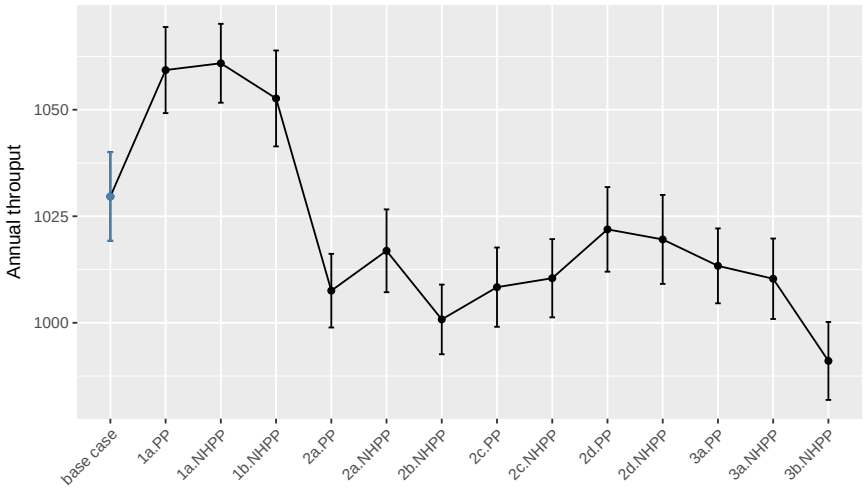


Figure 7.31: Annual throughput in different epidemic scenarios

patients. This could be also confirmed by Section 7.1 which shows that the ICU has the potential to admit more patients. However, the annual throughput decreases significantly in all the other scenarios since the explosive arrivals of pandemic patients may lead to late admission of regular patients.

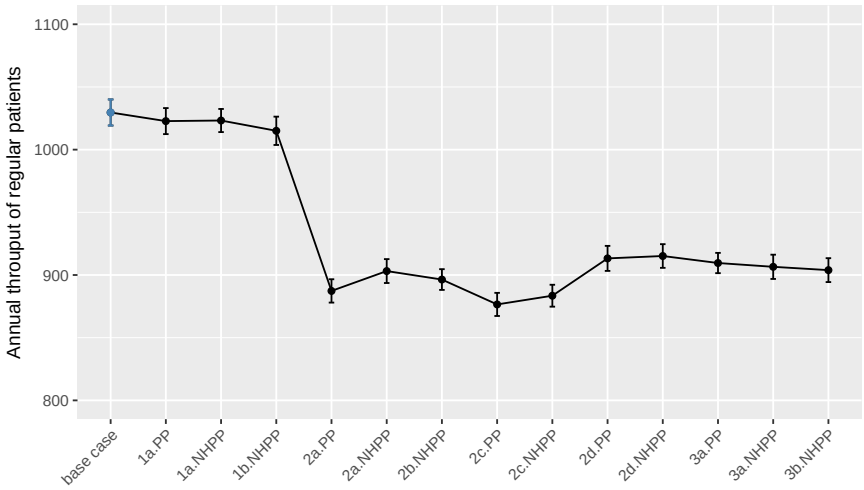


Figure 7.32: Annual throughput of regular patients in different epidemic scenarios

Figure 7.32 shows the annual throughput of regular patients. No statistically significant differences can be observed between the mild scenarios and the base case. This indicates that the ICU is likely to cope with a mild epidemic in its area and still serve regular patients well. However, significant drops of throughput of regular patients could be observed in all

the other scenarios, which implies negative influences on regular ICU services due to the epidemic.

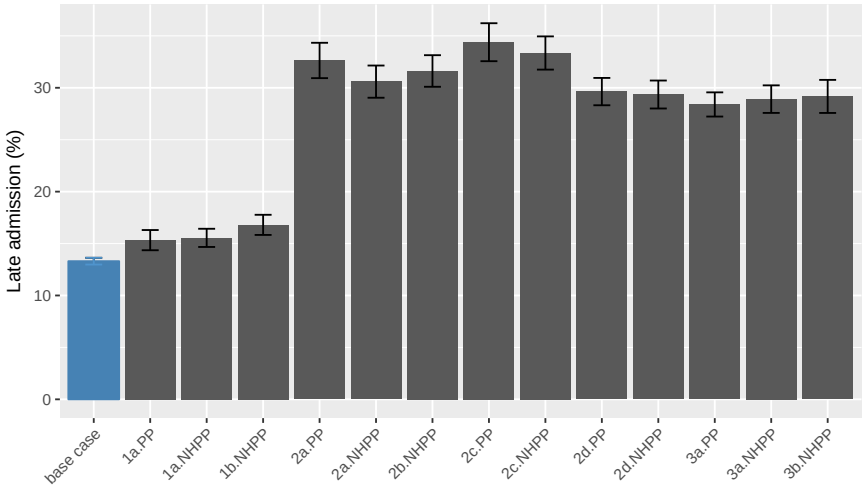


Figure 7.33: Percentage of late admissions of regular patients

Percentages of late admitted regular patients are shown in Figure 7.33. The late admission group accounts for a larger percentage compared to the base case. This group is doubled in all the epidemic scenarios except the mild case.

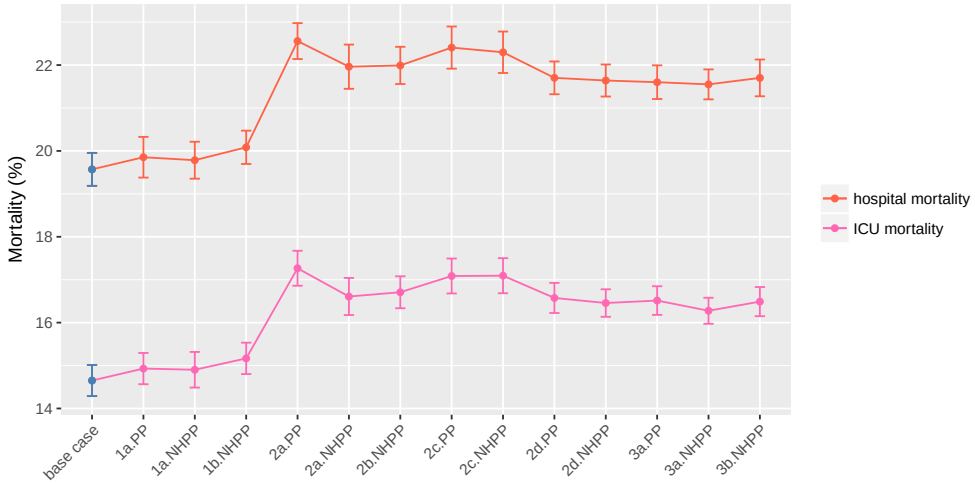


Figure 7.34: Mortality rates of regular patients in different epidemic scenarios

ICU and hospital mortality rates of regular ICU patients are plotted in Figure 7.34. Significantly higher mortality rates can be observed in all the likely (2a,b,c,d) and worst (3a,b)

scenarios. This could also be inferred from an increase of the late admission group (Figure 7.33).

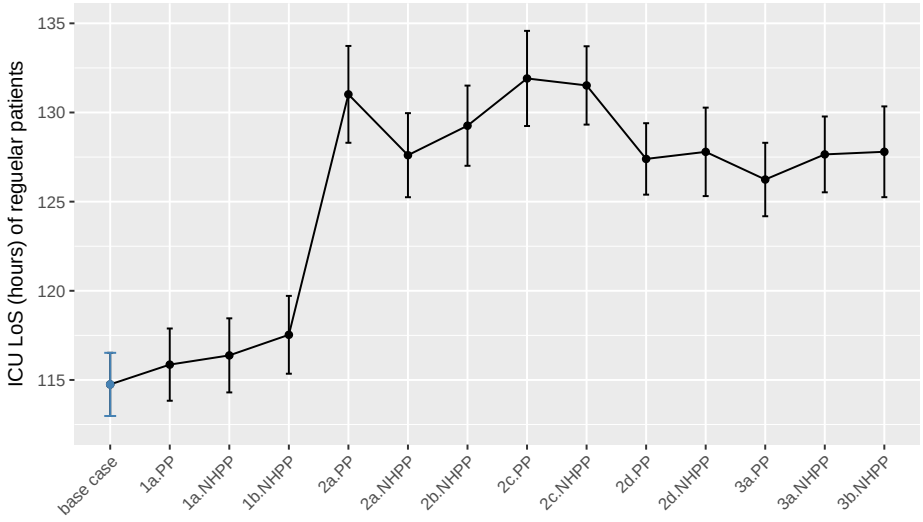
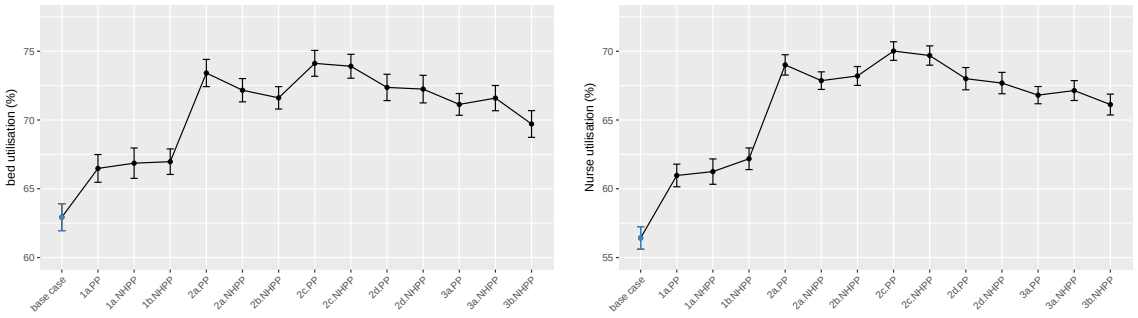


Figure 7.35: LoS of regular patients in different epidemic scenarios

We expect an increase in LoS as a result of the increase in late admission. It is confirmed by Figure 7.35, which illustrates LoS of regular patients in different scenarios. The occurrence of an epidemic is likely to bring continuous pressure to the ICU. The admission of regular patients needs to be well scheduled in the early phase of an epidemic.



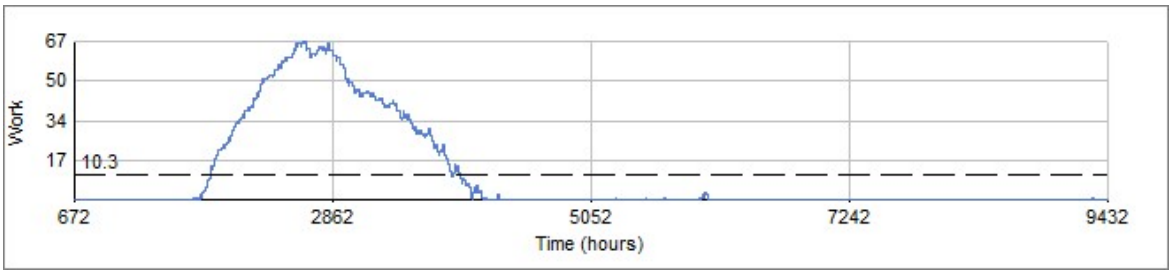
(a) ICU bed utilisation (b) Nurse utilisation

Figure 7.36: Resource utilisation in different epidemic scenarios

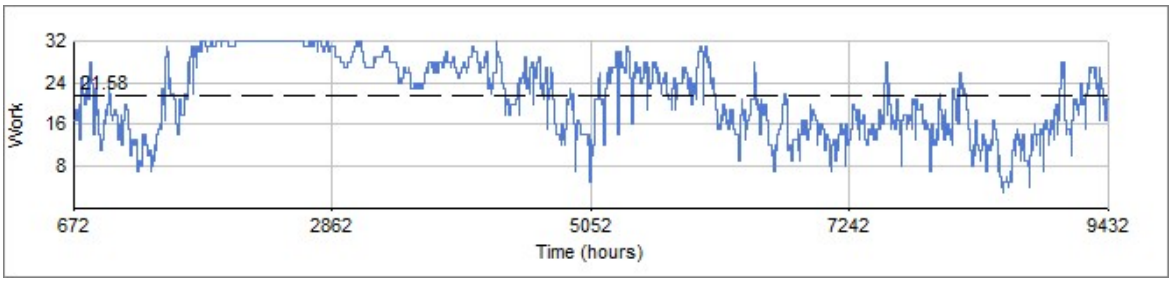
The differences in resource utilisation of different scenarios as exhibited in Figure 7.36 show similar patterns to both LoS (Figure 7.35) and mortality rates (Figure 7.34), which are highly correlated with late admission (Figure 7.33).

In addition to general analysis, the impact of an epidemic on the ICU is investigated in detail by analysing the queue of regular patients waiting for ICU admission and resource utilisation hour by hour of scenario 2a with NHPP arrivals.

(a) Regular patients waiting for ICU admission



(b) Nurse in use



(c) ICU bed in use

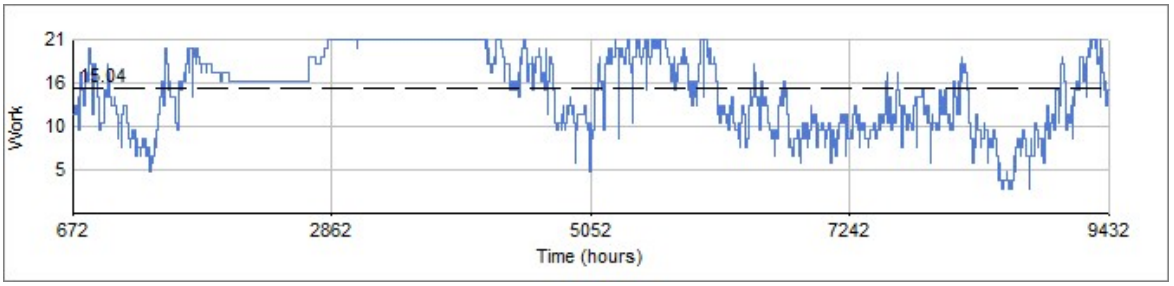


Figure 7.37: Scenario 2a with NHPP arrivals

Figures 7.37a to 7.37c show the details of the queue for the ICU and resource utilisation over a year. The pandemic period is 1392hrs to 2734hrs. With the increasing demand of pandemic patients, the queue of regular patients expands during the pandemic period. The queue takes a similar period to recover. The utilisation of ICU bed is close to 100% after the pandemic period (8 weeks) for about two and a half months. The nurse utilisation is also

high. These figures infer that the ICU takes at least as long a time as the pandemic period to recover to normal operations.

7.7 Conclusions

Six sets of scenarios were designed and tested in this Chapter. Tests of increased ICU arrivals in Section 7.1 show that the ICU can accommodate up to approximately 1220 admissions every year but both mortality rates and LoS will increase.

As discussed in Section 7.2, the number of ICU beds rather than nurses is the more critical resource to the ICU in the current situation since the number of bed is just enough to meet current demands. However, the marginal negative effect of reducing nurse resource is larger than that of reducing bed numbers.

Regarding admission policies, with everything else kept unchanged, bringing in patients earlier can save lives and resources as shown in Section 7.3. Moreover, Section 7.4 demonstrates that the ICU can accommodate 20% more unplanned patients based on the current situation without sacrificing service standard if the late admission group can be reduced to 10% of all first-time admissions.

In terms of discharge, controlling the discharge time of patients to be within four hours of discharge decisions does not have a significant impact on ICU throughput, as shown in Section 7.5.

Results from epidemic scenarios (Section 7.6) show that the ICU can only treat pandemic arrivals in the mild scenario. If the scenario worsens, a large proportion of pandemic arrivals will leave without receiving proper treatment. Moreover, the ICU could take a long time to recover from an epidemic, which may require careful planning.

Chapter 8

Conclusion

Five research questions as follows are proposed in Chapter 1.

- How do late-admitted patients affect the efficiency and effectiveness of the ICU?
- What factors may impact patients' outcomes and LoS?
- By applying different admission and discharge policies for patients in the ICU, what improvements in efficiency can be achieved?
- How does resource level affect ICU effectiveness and efficiency?
- How will extreme conditions (i.e. pandemic) influence the ICU?

Full answers to all these questions are provided by research in previous chapters. The conclusion chapter summarises those findings. Contributions of this work in achievement of questions one and two and ICU modelling are listed and explained in Section 8.1. We summarise the features of the analysis that are of particular interest to ICU managers, which mainly come from questions three to five, in Section 8.2. Limitations of the research are stated in Section 8.3. Possible future work is discussed in Section 8.4, given that more and wider data could be accessed.

8.1 Contributions to ICU modelling methodology

The main contributions to ICU modelling are listed here and we describe each of these in more detail throughout the rest of the section.

1. Identification and investigation of late admissions to the ICU

2. The analysis and modelling of a mixed ICU: this includes introduction of a new measure for a mixed ICU, PA in the ICU, which indicates how busy the general wards are where data on these wards are limited.

3. Mortality prediction: the differing predictability of benchmark models for different patient categories. Improving the prediction of mortality for late-admitted patients over the benchmark models.

4. Improvements to the modelling of LoS in the ICU by splitting the LoS into three components as described in Section 6.3.

5. Combining data mining and simulation models to provide an enhanced description of the system.

8.1.1 Identification and investigation of late admissions to the ICU

To the best of our knowledge, ours is the most comprehensive definition of lateness of admissions, for use in both prediction and simulation modelling.

As a result of a unique analysis of mortality and LoS data, we defined late admissions in a novel manner according to admission types and lag days. Late medical admission is defined as ICU admission with delay between unit admission and hospital admission longer than one day. Late surgical admission is defined by two rules. First, all the surgical patients admitted from general wards are counted as late admissions. Second, ICU admission with the delay between unit admission and hospital admission longer than one day, from sources other than

ward and recovery, is counted as late admission. Therefore, each patient can be categorised in either the immediate or late admission groups. LoS for the immediate and late admission groups has been calculated. The late admission group of patients occupies a significantly large portion of resources although the size of the group is considerably smaller than the immediate admission group.

This novel analysis have shown that late admission impacts on patients' outcomes and consequently that this is a potential direction for improving ICU performance. Moreover, lateness in admissions has a broader application in healthcare. This issue could be relevant in hospital admission, operation scheduling, patients' rehabilitation and so on, as we discuss in Section 8.4.

8.1.2 Analysis and modelling of a mixed ICU

Our modelling is unlike most other research into intensive care units, in that it concerns a mixed ICU, rather than separate intensive care and high dependency units. Such a mixed ICU is often present in UK hospitals. Moreover, unlike other research our modelling combines prediction with simulation to study the effects of aspects such as lateness on outcomes.

An ICU, as a part of a hospital, cannot be well observed or studied by isolated approaches. Researchers need to take a holistic view of the ICU and its surrounding wards. As analysis of the BRI discharge data suggests, a large majority of delayed discharges result from lack of general ward beds. We introduce a new measure suitable for a mixed ICU known as patients' acuity (PA) to estimate the busyness of the ICU and also indicate the busyness of the general wards. Busyness measurements are then included in both ICU mortality and LoS prediction.

A lower PA in the ICU may indicate congested general wards which can cause longer ICU stays. The PA in the ICU gives a new and simple way to assess busyness of general wards in a hospital when data from general wards is limited as in this study. However, we have no hospital data to validate PA and so it would require further research to be used as a measurement.

We have built a discrete event simulation model for the ICU (see Figure 6.11). The model shows a good validity as assessed in Section 6.7. The late admission group has been incorporated into the model using the results of the data mining to inform parameterisation. Two independent entry points are created to capture different patterns of unplanned and planned patients. Dummy work centres are created for setting ICNARC probabilities for different groups of patients and fulfilling pre-ICU sampling as mentioned in Section 6.5.2. An NHPP with periodic intensity was used to model the weekly cycle of unplanned admissions. We divided a week into 14 intervals that represent 14 shifts in a week. An arrival rate was estimated for each shift as shown in Table 6.2. For planned admissions, we used the empirical distribution of daily admission numbers on a weekday basis. The number of arrivals in the day were simulated and assigned to different arrival hours according to EDFs of arrival timing for planned arrivals on different weekdays. These distributions have been described in Section 6.2.2. The model details were described in Section 6.5.2

Six sets of scenarios have been tested using the validated model. The impact of ICU resources, beds and nurses, are investigated first in Section 7.2. The influence of the late admission group on operations of the ICU was examined in Sections 7.1, 7.3 and 7.4. A wider admission of patients could change the arrival process of patients and also impact on the throughput and survival rates of the ICU. Discharge timing is investigated in Section 7.5. Epidemic scenarios are designed and examined in Section 7.6.

8.1.3 Mortality prediction of ICU admissions

Our research gives a more comprehensive view of ICU mortality prediction than previous research. To the best of our knowledge, previous research has either kept all patients together or has focused on a particular patient group. Our approach of splitting ICU patients into different categories has demonstrated that the performance of the benchmark ICNARC mortality prediction models varies greatly for different patient groups. Logistic regression models have been built which show improvements on benchmark models for late admission groups and most of ICU mortality prediction.

8.1.4 Novel approach to modelling of LoS

When modelling LoS, we use three different combinations of patients with LoS and log-transformed LoS as response variables and then assess the results of six models. Models concerning all the ICU population or on the basis of a two-stage method (i.e. predict mortality first and then LoS) have not achieved a high predictability. The R-squared results gained from the testing dataset are poor. We also tried fitting distributions for LoS, but no parametric distribution appears to fit LoS well. A cyclical pattern was discovered while fitting distribution; therefore, an alternative method of modelling LoS is proposed.

LoS is modelled in a novel manner in our research by splitting it into three parts, admission time, nights spent in the ICU and discharge time, modelled separately. Different grouping methods are used in the three components. The method has been described in detail in Section 6.3. We tested the proposed LoS modelling method, using EDFs of arrival timing, nights and discharge timing for all patients. The EDFs of original LoS and simulated LoS are plotted in Figure 6.9, which shows the simulated LoS matched the original data well.

8.1.5 Hybrid data mining and simulation models

Our study contributes to the literature on the value of combining data mining with simulation modelling. In the models we build here, predictive models are used to categorise patients and route them through the system. Outcome prediction for individuals was also considered in Elbattah (2018) in a simulation model, but was not modelled in detail. In our research, four outcome prediction benchmark models are encountered in the simulation model to route entities later. A model for initial critical care level prediction was also embedded.

A Data mining methods embedded simulation model describe a system more precisely comparing to a pure stochastic simulation model. It is not only informative in the system level, but also gives some insights for individual entities in the system, which is important for decision making. In real life systems with heterogeneous individuals, in particular, will be benefited from data mining embedded simulation models. Many areas in healthcare are laid

into this categories.

From our view, When incorporating data mining methods into a simulation model, one may take extra caution in two aspects, the amount of details one would like to include and the values one could gain from the model. With the growing of accessible details, modellers are usually tempted to include all the information. However, data mining models embedded in the simulation model are preferred to be simple but informative. Over complicated data mining model requires extensive data and running cost, which may not worth. Moreover, system with homogeneous individuals, for example a manufacturing system, may only gain limited values from these kind of models. Modellers need to think of the balance between the value gained and cost of model building.

8.2 Contributions to ICU Management

This research makes a number of contributions that are of interest to ICU managers. The main contributions to management are listed here and detailed in following subsections.

1. An analysis of the timing effect of admissions and discharges on ICU and hospital outcomes.

2. Determining the influence of late admission and different admission policies on LoS, ICU bed turnover and ICU throughput.

3. Showing the effect of varying ICU resources.

4. Demonstrating impact of different discharge policies.

5. Designing and testing epidemic scenarios and investigating the influence on normal ICU operations.

6. Insights into ICNARC probability.

8.2.1 Timing effect of ICU admission and discharge

Timing effects have been examined after excluding the impact of case-mix effects by using ICNARC probablity as a confounding variable. A positive impact of timing on ICU mortality are found in peak time admissions (2:00pm - 00:59am). Commonly found day-of-week effects or discharge timing effects were not found to be significant in our data. This analysis is likely to be specific to the BRI ICU because of different operation policies applied in different ICUs

8.2.2 Influence of late admission and different admission policies

The effect of late admission has been shown to be as severe as that of readmission on outcomes. Also, as admission lag increases, the trend in mortality and LoS likewise increases. Late admission is likely to be a widely exist problem in all the ICUs, although the exact relationship between admission timing and patients outcome may slight vary.

We highlight some key findings from the scenario tests. A higher admission rate in the ICU has been shown increase the pressure on ICU resources in the short term but tests show that it may have an overall positive impact in the mid/long term. This is because, for instance, a wider direct admission of post-operative patients to the ICU may reduce late admissions that are often associated with long lengths of stay and intensive use of resources.

The impact of wider admissions is considered in Section 7.1. Increasing current arrival rates of unplanned patients by 10% will not cause significant differences in service time (LoS) or service quality (mortality).

Earlier admission, reducing the percentage of late group, has also been evaluated in Section 7.3. It would not improve annual throughput given current arrivals due to the limitation of the data. However, LoS would be shortened significantly and it shows a great potential in resource saving by assuming change in admission timing influences the mortality and ICU LoS.

A combination of wider and earlier admissions is discussed in Section 7.4. It would increase

the annual throughput more than solely wider admission. Both hospital and ICU mortality rates will also be improved significantly. ICU LoS will not be lengthened for a wider admission if patients could be brought in earlier.

8.2.3 Effects of changes of ICU resources

Two critical ICU resources, beds and nurses, have been analysed in Section 7.2. Resource allocation pattern are different for different ICUs, insights drawn from this analysis is ICU specific. In general, the number of ICU beds is on the borderline to meet current service demand. However, if both resources are further reduced in the ICU, it is demonstrated that nurse numbers will have a larger influence on ICU operation.

A decrease in the number of beds would cause decrease in throughput. Results show that inadequate numbers of ICU beds could lead to a severe delay to admission of patients, which would eventually result in a prolonged ICU stay and higher ICU mortality.

No significant increase in the percentage of late admissions could be observed while dropping nurse numbers by just one or two from the current baseline of 16. For scenarios between 12 and 18 nurses, annual throughput does not differ significantly. Further decreasing from the borderline scenario of twelve nurses leads to a fast worsening of performance. Variations of ICU LoS and mortality are caused by late admissions resulting from reduced resources.

8.2.4 Impact of prompt ICU discharge

The ICU experiences delays in discharges currently, mainly because of lack of general ward beds. We analyse three possible discharge time scenarios: immediately when clinically ready, and 2 and 4hours after clinically ready. Performance of the ICU has been analysed in these scenarios. Since delayed discharge is a widely existed problem, the results should be applicable to other ICUs although we use our ICU as a case study.

It is shown that all three scenarios for prompt discharge would not affect annual through-

put substantially with current arrival rates. However, if the ICU is under pressure (e.g. a continuous high level of arrivals lasting for a period), prompt discharge is shown to make a significant difference to the potential annual throughput.

The current discharge scenario (base case) generates more late patients than prompt discharge while the ICU is under pressure. Discharge on time is especially important if a sharp increase of arrivals happens; ICU LoS and work load can then be reduced significantly.

8.2.5 Impact of epidemic scenarios

Our research provides comprehensive testing of epidemic scenarios through DES modelling. Previous research has only considered overall resource needs using Monte-Carlo Simulations. The insights gained from the epidemic scenarios should be very useful to all ICU practitioners and policy makers.

Instead of using tools and scenarios established for situations elsewhere (in the US), three categories of epidemic scenarios, mild, likely and worst (shown in Table 7.3) are specifically designed according to UK data in our research.

In the case of the BRI ICU, it is clear that the ICU would be under great pressure during a pandemic period. Except in the mild scenarios (1a, b), most patients wait for more than 24hrs before admissions if no premature discharge is considered (see Section 7.6).

An epidemic will also affect the normal operation of the ICU. Generally, the ICU is likely to cope with the mild epidemic scenarios and still serve regular patients well. For more serious epidemics, the annual throughput decreases significantly in all scenarios since the explosive arrivals of pandemic patients causes late admission of regular patients. In terms of throughput, significant drops of throughput of regular patients could be observed in all likely and worst scenarios, which implies negative influences on regular ICU services due to an epidemic. The late admission group accounts for a larger percent in epidemic scenarios compared to the base case. This group is doubled in all the epidemic scenarios except the mild case. Significantly higher mortality rates and longer ICU LoS of regular patients can

be observed in all the likely (2a,b,c,d) and worst (3a,b) scenarios.

One of the most likely scenarios (scenario 2a) is investigated in detail both during and after the epidemic. The utilisation of ICU beds is close to 100% after the pandemic period (8 weeks) for about two and a half months. The ICU takes at least as long a time as the pandemic period to recover to normal operations.

The epidemic scenarios demonstrated that the current resource is not enough to deal with any epidemic more than a mild one. Moreover, it will take a considerable period of time for the ICU to recover to its normal operation level after a pandemic period.

The occurrence of an epidemic is likely to bring continuous pressure to the ICU. The admission of regular patients needs to be well re-scheduled in the early phase of an epidemic.

8.2.6 Insights into ICNARC probability

We have shown the effects of admission source on predicting outcome: it is of interest that certain groups have better predicted probability of survival for the same ICNARC probability. In particular, the effects of timing and patient categories on outcome have been demonstrated. Peak/non-peak, planned/unplanned groups have differing probability of survival for the same ICNARC probability. However, there is no evidence for a weekend effect.

8.3 Limitations

Limitations of this research are detailed in this section to give insights into the possible improvements and extensions of this research.

In the preliminary analysis (Chapter 4), we found effects of late admission on mortality and ICU LoS. Although we have analysed possible confounding variables, it is hard to exhaust the complete list; for example, the effect due to decision makers is hard to measure. Moreover,

late admission is defined by the lag between hospital admission and ICU admission, which is counted by days. This may not be precise enough; a more accurate measurement (i.e. by hours or minutes) has the potential to achieve a better result. Another limitation in the chapter is that we proposed PA to indicate the busyness of the whole hospital. However, validation of PA was not possible in this research. To confirm the usability of the measurement, we need extra data to validate PA.

In terms of prediction models in Chapter 5, two limitations need to be pointed out, ICNARC-based mortality prediction and the limited predictability of individual ICU LoS. Mortality prediction is ICNARC based, which will be a limitation when implementing the model for non-UK ICUs. However, the re-calibration of a well-established scoring system is usually regarded as a good way to build a risk prediction model (Ankerst, 2016); researchers may substitute a re-calibrated country-specific model for ICNARC as appropriate. Prediction of ICU LoS for individual patients did not achieve a good performance. As shown in Section 5.4, the R-squared of the prediction methods tested are generally not good. Several prediction methods have been tried in this section. The next step for LoS prediction could be use supplementary data with more effort in variable selection.

The performance of the DES model described in Chapter 6, is generally good. However, we would like to flag up two limitations which could benefit from more attention: the dispersion of overnight bed occupancy and the unavailability of ICU LoS for individual patients. Overnight bed occupancy generated by the model is more dispersed than that calculated from system all ward data. The prediction of LoS for individual patient was found to be hard to achieve as described in 8.1.4. This meant LoS has to be sampled from EDFs, which makes the model less informative. Both of these limitations can cause inaccuracy of the model.

In the results chapter, Chapter 7 all the results are based on the model and the statistical analysis. Whether a result is significant or not generally means the significance shown in the DES model and measured by statistical methods. However, significance in the real world may the otherwise so these results should be filled with caution. These effects need to be

doubly confirmed by observing ICU operation.

8.4 Future possible extensions to this work

The findings of this research on late ICU admission have the potential to be incorporated into a whole hospital bed management simulation to demonstrate benefits of earlier admission to the ICU, and prompt discharge from the ICU, for the whole hospital. As earlier admission to ICUs are enabled, shortened post-ICU stays and reduced mortality are likely to result.

Delayed admission for many healthcare procedures has been shown in recent research to reduce the probability of a successful outcome. The approach taken by this research, of using analyses of LoS and mortality to define "lateness" and analyse its effects, could be applicable in other areas. For example, in A&E, prolonged waiting times for patients admitted to hospital leads to a higher mortality (Higginson, 2012). For surgical patients, a delayed operation is associated with an increase in morbidity and mortality (Nyholm et al., 2015; Haltmeier et al., 2015). Vidal et al. (2012) found that a longer fracture to hospital admission time rather than admission to surgery time decreases survival of hip fracture patients. Delayed rehabilitation transfer impacts negatively on both finance and performance (The King's Fund, 2018). Research focusing on solving such problems of delayed transfers of care as a part of integrated systems has great potential for improving both patient and hospital outcomes.

With the accumulation of post-ICU hospital stay data, the prediction of post-ICU mortality could be improved. We found an interesting point that in some post-ICU mortality predictions (e.g. non-late planned admission group), only discharge reason and operational factors are included in the logistic regression model. This gives a direction for how post-ICU mortality prediction modelling could be simplified and the results incorporated into a simulation model. It is not necessary to consider patients' medical states if they are normally discharged. Every admission can be labelled with its operational characteristics and the operational factors used to predict post-ICU mortality. Such prediction shows a great potential

to be used in the ICU discharge control and hospital bed management.

The methods demonstrated in this research have made a number of important contributions to research in both the modelling and management of ICUs. Moreover, this work points the way to benefits of studying and managing integrated health systems, as a means of sustaining services that are under pressure from increasing demand.

Appendices

Appendix A

Full Variable List for All models

Variable Name	**Variable Description**
Abnormal.delay.caused.by	reason for abnormal discharge
adbusyness	PA of admission day
AdDateHosp	hospital admission day
AdDateICU	ICU admission day
AdLate	late admission (late/immediate)
admission.group	admission group by readmission; see Section 4.1.2
adnonlevel3	%nonL3 of admission day
Adpeak	peak time admission; see Figure 4.11
Adshift	nurse staffing shift when admission
AdTimeICU	ICU admission time
AdWeffect	weekend effect of admissions, 1 for weekends, 0 for weekdays
Age	age of patient when admitted
APACHE.II.mortality.prediction	probability of death when using APACHE II mortality prediction
APACHE.II.score	APACHE II score
Body.removed.at..time.	body removed time (For patients who died only)

Body.removed.on..date.	body removed date (For patients who died only)
CateReason	admission reasons re-categorised by their frequency and severity
CCMDS.Level.3.day	show which day of the stay is level 3 day
ClinicalLoS	clinical ICU LoS
Date.of.ultimate.hospital.discharge	hospital discharge date (for the last hospital that patient stays)
Days.between.hospital.and.unit.admit	lag days between hospital admision and ICU admission
Destination..name.	discharge destination
disPA_1	PA of the day before discharge
disPA	PA of discharge day
Discharge.delay.abnormal	if the delay of discharge is abnormal
disnonlevel3	%nonL3 of discharge day
Dispeak	peak time discharge; see Figure 4.13
DispeakReady	peak time discharge decision making; see Figure 4.14
Disshift	nurse staffing shift when discharge
EMEL	Emergency (EM), Elective (EL), NR (medical)
HousedWithin	hospital type that patient comes from
ICNARC	ID of each ICU admission
ICNARC.probability	probability of death when using ICNARC mortality prediction
ICNARC.score	ICNARC score
ICUadH	hour of ICU admission
ICUdisH	hour of ICU discharge
ICUdisHReady	hour of the discharge decision making
OpDelay	operational delay (RealLoS-ClinicalLoS)
outcomeHosp	patient's outcome of ultimate hospital discharge
outcomeICU	patient's outcome when discharged from the ICU
patient.type	patient type

ReadyDisDate	clinical ready to discharge date
ReadyDisTime	clinical ready to discharge time
RealDisDate	real discharge date
RealDisTime	real discharge time
RealLoS	actual ICU LoS (we also call it actual LoS in text)
Reason.Primary	primary admission reason
Reason.Secondary	secondary admission reason
Source	admission sources; see Figure 4.2.5
Sex	gender of patients
WdayAd	day of week (admission)
WdayDis	discharge (day of week)

Appendix B

Correlation Matrix

	Age	**LagDays**	**ICNARC score**	**ICNARC probability**
Age	1	0.024	0.1065	0.2301
LagDays	0.024	1	0.0277	0.0391
ICNARC score	0.1065	0.0277	1	0.8741
ICNARC probability	0.2301	0.0391	0.8741	1
APACHE.II score	0.1631	-0.0821	0.3063	0.2709
APACHE.II mortality prediction	0.1079	-0.0399	0.5243	0.591
adnonlevel3	0.0313	-0.0015	-0.0712	-0.0614
adlevel1	0.0116	-0.0068	-0.0096	-0.0069
adlevel3	-0.0313	0.0015	0.0712	0.0614
adPA	0.0032	0.0077	0.0541	0.041
disnonlevel3	0.0115	-0.006	-0.0422	-0.0524

dislevel1	0.0177	0.0012	-0.0215	-0.0222
dislevel3	-0.0115	0.006	0.0422	0.0524
disPA	0.0105	0.0045	0.0423	0.0319
disnonl3_1	0.0219	-0.0042	-0.0385	-0.0472
disPA_1	-0.001	0.0029	0.0371	0.0318
disreadynonl3_1	0.0216	-0.0028	-0.0411	-0.0496
disreadyPA_1	0.0052	0.0032	0.0389	0.0332
disreadynonl3	0.01	-0.0062	-0.0415	-0.0519
disreadyPA	0.0036	0.0057	0.0494	0.0366
RealLoS	0.0076	0.0651	0.2447	0.1777
ClinicalLoS	0.0065	0.0647	0.2566	0.1897
OpDelay	0.0135	0.0121	-0.1194	-0.1275
	APACHE.II score	**APACHE.II mortality prediction**	**adnonlevel3**	**adlevel1**
Age	0.1631	0.1079	0.0313	0.0116
LagDays	-0.0821	-0.0399	-0.0015	-0.0068
ICNARC score	0.3063	0.5243	-0.0712	-0.0096
ICNARC probability	0.2709	0.591	-0.0614	-0.0069
APACHE.II score	1.0000	0.7832	0.3761	-0.0122
APACHE.II mortality prediction	0.7832	1	0.1701	-0.019
adnonlevel3	0.3761	0.1701	1	-0.0118
adlevel1	-0.0122	-0.019	-0.0118	1
adlevel3	-0.3761	-0.1701	-1	0.0118

adPA	-0.0855	-0.0239	-0.6265	0.0365
disnonlevel3	0.3715	0.165	0.6988	-0.0503
dislevel1	-0.0191	-0.0282	-0.0213	0.2565
dislevel3	-0.3715	-0.165	-0.6988	0.0503
disPA	-0.0775	-0.0156	-0.4161	0.0712
disnonl3_1	0.3688	0.1635	0.7749	-0.0455
disPA_1	-0.0742	-0.0127	-0.4612	0.0799
disreadynonl3_1	0.3691	0.1625	0.782	-0.0432
disreadyPA_1	-0.0779	-0.015	-0.4692	0.0827
disreadynonl3	0.3711	0.1661	0.7029	-0.0504
disreadyPA	-0.0847	-0.0182	-0.4192	0.0696
RealLoS	0.0776	0.118	-0.0275	0.0146
ClinicalLoS	0.0732	0.1214	-0.0336	0.0108
OpDelay	0.0627	-0.0288	0.0719	0.0487
	adlevel3	**adPA**	**disnonlevel3**	**dislevel1**
Age	-0.0313	0.0032	0.0115	0.0177
LagDays	0.0015	0.0077	-0.006	0.0012
ICNARC score	0.0712	0.0541	-0.0422	-0.0215
ICNARC probability	0.0614	0.0410	-0.0524	-0.0222
APACHE.II score	-0.3761	-0.0855	0.3715	-0.0191
APACHE.II mortality prediction	-0.1701	-0.0239	0.165	-0.0282
adnonlevel3	-1	-0.6265	0.6988	-0.0213
adlevel1	0.0118	0.0365	-0.0503	0.2565
adlevel3	1	0.6265	-0.6988	0.0213

adPA	0.6265	1.0000	-0.4387	0.052
disnonlevel3	-0.6988	-0.4387	1	-0.0055
dislevel1	0.0213	0.0520	-0.0055	1
dislevel3	0.6988	0.4387	-1	0.0055
disPA	0.4161	0.4660	-0.6221	0.0189
disnonl3_1	-0.7749	-0.4893	0.8505	-0.0097
disPA_1	0.4612	0.5691	-0.5354	0.0308
disreadynonl3_1	-0.782	-0.4943	0.8401	-0.0044
disreadyPA_1	0.4692	0.5797	-0.5338	0.0246
disreadynonl3	-0.7029	-0.4422	0.9886	-0.0082
disreadyPA	0.4192	0.4708	-0.6193	0.0125
RealLoS	0.0275	0.0039	0.0066	0.0381
ClinicalLoS	0.0336	0.0048	0.0004	0.0291
OpDelay	-0.0719	-0.0104	0.0755	0.1128
	dislevel3	**disPA**	**disnonl3_1**	**disPA_1**
Age	-0.0115	0.0105	0.0219	-0.001
LagDays	0.006	0.0045	-0.0042	0.0029
ICNARC score	0.0422	0.0423	-0.0385	0.0371
ICNARC probability	0.0524	0.0319	-0.0472	0.0318
APACHE.II score	-0.3715	-0.0775	0.3688	-0.0742
APACHE.II mortality prediction	-0.165	-0.0156	0.1635	-0.0127
adnonlevel3	-0.6988	-0.4161	0.7749	-0.4612
adlevel1	0.0503	0.0712	-0.0455	0.0799
adlevel3	0.6988	0.4161	-0.7749	0.4612

adPA	0.4387	0.466	-0.4893	0.5691
disnonlevel3	-1	-0.6221	0.8505	-0.5354
dislevel1	0.0055	0.0189	-0.0097	0.0308
dislevel3	1	0.6221	-0.8505	0.5354
disPA	0.6221	1	-0.5037	0.6319
disnonl3_1	-0.8505	-0.5037	1.0000	-0.6075
disPA_1	0.5354	0.6319	-0.6075	1
disreadynonl3_1	-0.8401	-0.4996	0.9854	-0.5968
disreadyPA_1	0.5338	0.6208	-0.6065	0.9696
disreadynonl3	-0.9886	-0.6135	0.8583	-0.5381
disreadyPA	0.6193	0.9689	-0.5109	0.6537
RealLoS	-0.0066	0.0075	0.0036	-0.0289
ClinicalLoS	-0.0004	0.0094	-0.0026	-0.0267
OpDelay	-0.0755	-0.0215	0.0760	-0.0299
	disreadynonl3_1	**disreadyPA_1**	**disreadynonl3**	**disreadyPA**
Age	0.0216	0.0052	0.01	0.0036
LagDays	-0.0028	0.0032	-0.0062	0.0057
ICNARC score	-0.0411	0.0389	-0.0415	0.0494
ICNARC probability	-0.0496	0.0332	-0.0519	0.0366
APACHE.II score	0.3691	-0.0779	0.3711	-0.0847
APACHE.II mortality prediction	0.1625	-0.015	0.1661	-0.0182
adnonlevel3	0.782	-0.4692	0.7029	-0.4192
adlevel1	-0.0432	0.0827	-0.0504	0.0696
adlevel3	-0.782	0.4692	-0.7029	0.4192

adPA	-0.4943	0.5797	-0.4422	0.4708
disnonlevel3	0.8401	-0.5338	0.9886	-0.6193
dislevel1	-0.0044	0.0246	-0.0082	0.0125
dislevel3	-0.8401	0.5338	-0.9886	0.6193
disPA	-0.4996	0.6208	-0.6135	0.9689
disnonl3_1	0.9854	-0.6065	0.8583	-0.5109
disPA_1	-0.5968	0.9696	-0.5381	0.6537
disreadynonl3_1	1	-0.6131	0.8481	-0.5029
disreadyPA_1	-0.6131	1	-0.5393	0.6319
disreadynonl3	0.8481	-0.5393	1	-0.6218
disreadyPA	-0.5029	0.6319	-0.6218	1.0000
RealLoS	0.0021	-0.0373	0.0067	-0.0009
ClinicalLoS	-0.0041	-0.0341	0.0005	0.0045
OpDelay	0.0758	-0.0429	0.0761	-0.0659
	RealLoS	**ClinicalLoS**	**OpDelay**	
Age	0.0076	0.0065	0.0135	
LagDays	0.0651	0.0647	0.0121	
ICNARC score	0.2447	0.2566	-0.1194	
ICNARC probability	0.1777	0.1897	-0.1275	
APACHE.II score	0.0776	0.0732	0.0627	
APACHE.II mortality prediction	0.118	0.1214	-0.0288	
adnonlevel3	-0.0275	-0.0336	0.0719	
adlevel1	0.0146	0.0108	0.0487	
adlevel3	0.0275	0.0336	-0.0719	

adPA	0.0039	0.0048	-0.0104	
disnonlevel3	0.0066	0.0004	0.0755	
dislevel1	0.0381	0.0291	0.1128	
dislevel3	-0.0066	-0.0004	-0.0755	
disPA	0.0075	0.0094	-0.0215	
disnonl3_1	0.0036	-0.0026	0.076	
disPA_1	-0.0289	-0.0267	-0.0299	
disreadynonl3_1	0.0021	-0.0041	0.0758	
disreadyPA_1	-0.0373	-0.0341	-0.0429	
disreadynonl3	0.0067	0.0005	0.0761	
disreadyPA	-0.0009	0.0045	-0.0659	
RealLoS	1	0.9967	0.1436	
ClinicalLoS	0.9967	1	0.0627	
OpDelay	0.1436	0.0627	1	

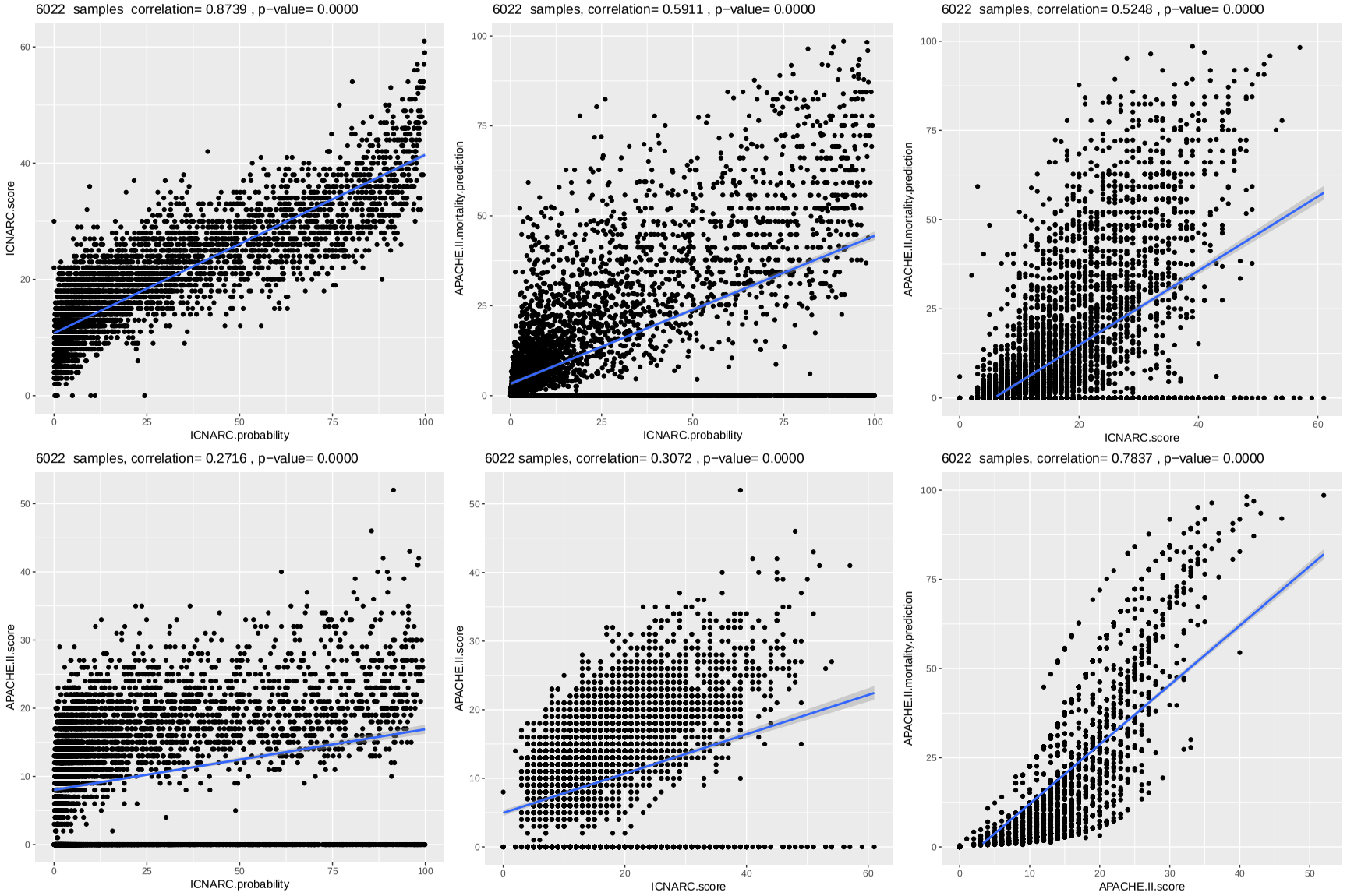


Figure B.1: Correlations between ICU scores and mortality predictions

Appendix C

Log Odds Plots for Hospital Outcomes

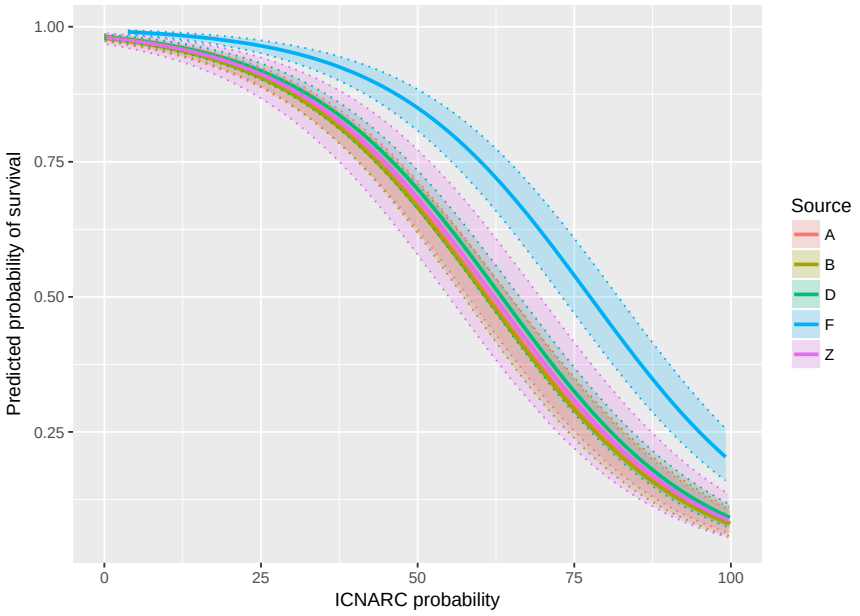


Figure C.1: Transformed log odds plot for different admission sources

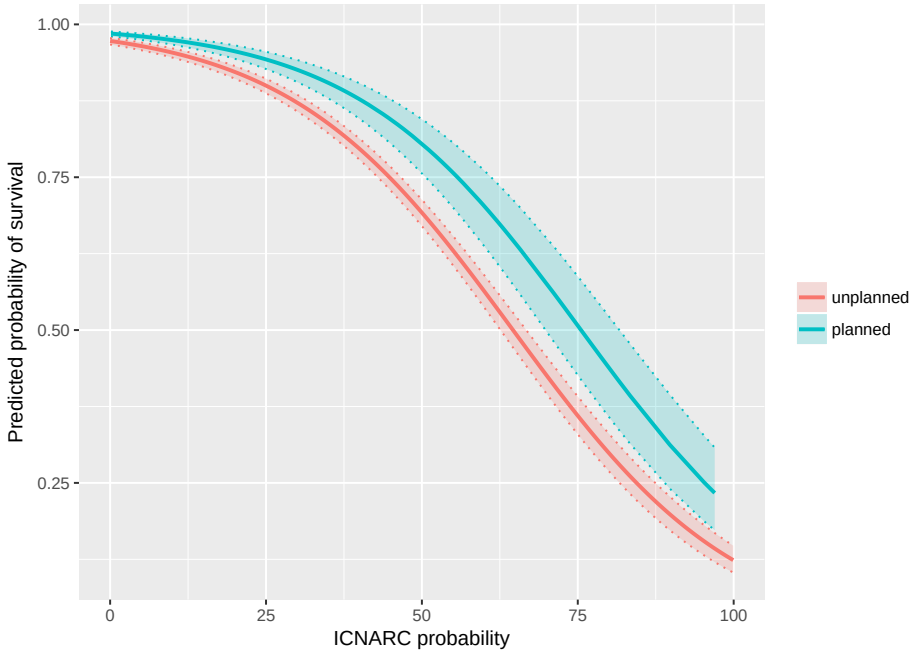


Figure C.2: Transformed log odds plot for different admission types

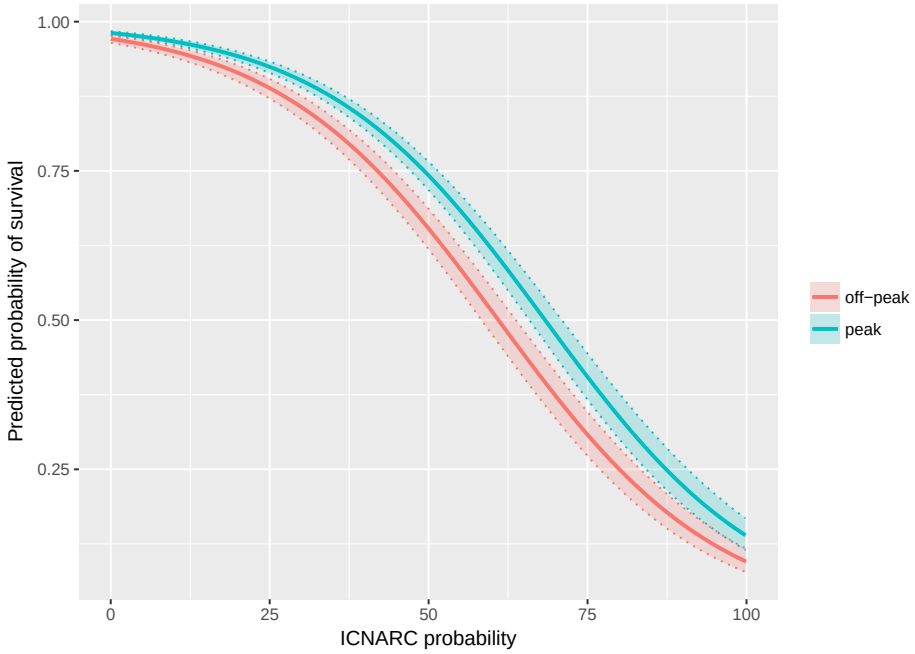


Figure C.3: Transformed log odds plot for different admission timing

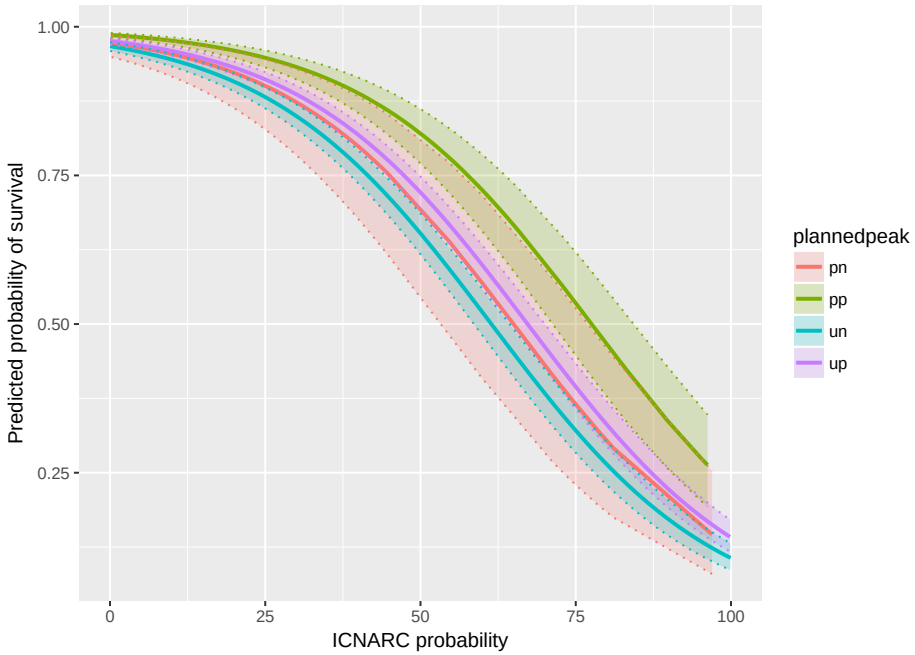


Figure C.4: Transformed log odds plot for different admission types and admission timing

Appendix D

Results for Mortality Prediction Models

This appendix includes results ortality prediction models. Each dataset is predicted by benchmark, logit and CART. AUROC and KS distance of all models are listed in Tables. Three benchmark models adopted in the simulation model are also given here.

D.1 ICU mortality

Table D.1: AUROC of all ICU mortality prediction models

Models	training benchmark	logit	CART	testing benchmark	logit	CART
planned	0.9170	0.9444	0.9023	0.8760	0.8853	0.8920
unplanned	0.8811	0.8878	0.8734	0.8776	0.8828	0.8701
unplanned peak	0.8832	0.9215	0.9023	0.8656	0.9083	0.8920
unplanned off-peak	0.8771	0.8825	0.8571	0.8899	0.9005	0.8706
source A	0.9223	0.9374	0.9111	0.895	0.8864	0.8788
source B	0.9251	0.9273	0.8606	0.9168	0.9090	0.8663
source D+Z	0.8403	0.8577	0.8360	0.8356	0.8271	0.8135
B0	0.8919	0.8926	0.8846	0.9049	0.8915	0.8728
D0	0.8359	0.8415	0.8364	0.8544	0.8537	0.8166
B1	0.9084	0.9302	0.7936	0.8336	0.8442	0.7660
non-late	0.9328	0.9332	0.7137	0.9161	0.9219	0.8944
non-late planned (no readmission)	0.9164	0.9250	0.8250	0.9073	0.8674	0.8037
non-late unplanned (no readmission)	0.9164	0.9068	0.8944	0.9073	0.8834	0.8713
late admissions	0.8356	0.8425	0.8265	0.8692	0.8758	0.7748

Table D.2: KS distance of all ICU mortality prediction models

Models	training benchmark	logit	CART	testing benchmark	logit	CART
planned	0.6777	0.7282	0.6757	0.6585	0.6721	0.6561
unplanned	0.6139	0.6314	0.6200	0.5979	0.6125	0.5907
unplanned peak	0.6414	0.6910	0.6757	0.5744	0.6689	0.6561
unplanned off-peak	0.5944	0.6076	0.5559	0.6195	0.6446	0.5703
source A	0.6997	0.7433	0.6804	0.6675	0.6862	0.6347
source B	0.6974	0.7149	0.6911	0.7247	0.6810	0.6984
source D+Z	0.5456	0.5882	0.5244	0.5085	0.5150	0.4742
B0	0.6495	0.6762	0.6495	0.7174	0.6908	0.6311
D0	0.5378	0.5757	0.5212	0.5489	0.5865	0.5225
B1	0.6797	0.7387	0.5709	0.5267	0.6432	0.5198
non-late	0.7322	0.7284	0.7306	0.7188	0.7175	0.6812
non-late planned	0.6872	0.7039	0.6270	0.6813	0.6426	0.5893
non-late unplanned	0.6872	0.6641	0.6453	0.6813	0.6119	0.5908
late	0.5243	0.5662	0.5809	0.5888	0.6239	0.5309

D.1.1 Planned Admissions (non-late)

$$E(Y_i|X) = \frac{1}{1 + e^(0.0785x - 4.5425)}.$$

D.1.2 Unplanned admissions (non-late)

$$E(Y_i|X) = \frac{1}{1 + e^(0.0539x - 3.4210)}.$$

D.1.3 Unplanned admissions (late)

$$E(Y_i|X) = \frac{1}{1 + e^(0.0495x - 3.2732)}.$$

D.2 Hospital mortality (FirstAD)

Table D.3: AUROC of all hospital mortality prediction models (FirstAD)

	training			testing		
Models	benchmark	logit	CART	benchmark	logit	CART
planned	0.8379	0.861	0.8685	0.8255	0.7979	0.8371
unplanned	0.864	0.8702	0.8455	0.8497	0.8563	0.8166
unplanned peak	0.8621	0.8811	0.8706	0.8391	0.8325	0.7974
unplanned off-peak	0.8647	0.8718	0.848	0.8609	0.862	0.8232
source A	0.9133	0.9251	0.9064	0.8995	0.8941	0.8785
source B	0.8579	0.8712	0.7771	0.8569	0.8486	0.7591
source D+Z	0.8331	0.8368	0.8243	0.7671	0.7707	0.7348
B0	0.8236	0.8314	0.8076	0.8518	0.8362	0.7975
D0	0.8267	0.8307	0.822	0.7925	0.7914	0.7688
B1	0.8047	0.8121	0.7789	0.7626	0.7558	0.6551
non-late	0.903	0.9111	0.8986	0.882	0.8808	0.8675
non-late planned	0.8222	0.854	0.8001	0.7893	0.7952	0.6882
non-late unplanned	0.8761	0.8873	0.8644	0.8557	0.8586	0.8349
late	0.8143	0.8227	0.8082	0.797	0.7941	0.7614

Table D.4: KS distance of all hospital mortality prediction models (FirstAD)

Models	training benchmark	logit	CART	testing benchmark	logit	CART
planned	0.5725	0.5613	0.6378	0.4855	0.4564	0.5789
unplanned	0.5656	0.5743	0.5259	0.5478	0.5595	0.4738
unplanned peak	0.5727	0.6025	0.6107	0.5296	0.5286	0.4953
unplanned off-peak	0.5655	0.5847	0.5278	0.5701	0.5832	0.4839
source A	0.6689	0.7204	0.6658	0.7044	0.6852	0.6189
source B	0.5532	0.5856	0.5321	0.5522	0.5647	0.4901
source D+Z	0.507	0.5123	0.4988	0.3781	0.3931	0.3464
B0	0.5299	0.5441	0.5262	0.568	0.5404	0.4834
D0	0.4987	0.5157	0.4986	0.4343	0.4238	0.3933
B1	0.5243	0.4702	0.5243	0.4166	0.4148	0.2833
non-late	0.6652	0.6703	0.6652	0.6137	0.6284	0.6029
non-late planned	0.5507	0.53	0.5507	0.4423	0.4809	0.341
non-late unplanned	0.595	0.6163	0.5946	0.5622	0.5582	0.5334
late	0.4781	0.4958	0.4742	0.4377	0.4762	0.3791

D.3 Hospital mortality (Last AD)

Table D.5: AUROC of all hospital mortality prediction models (LastAD)

Models	training benchmark	logit	CART	testing benchmark	logit	CART
planned	0.8610	0.8835	0.7031	0.8653	0.8560	0.6793
unplanned	0.8610	0.8835	0.7031	0.8531	0.8562	0.8320
unplanned peak	0.8632	0.8792	0.8679	0.8404	0.8438	0.8040
unplanned off-peak	0.8651	0.8727	0.8645	0.8674	0.8736	0.8504
source A	0.9170	0.9282	0.9098	0.8991	0.8951	0.8780
source B	0.8816	0.8916	0.8050	0.8812	0.8795	0.7862
source D+Z	0.8259	0.8326	0.8192	0.7905	0.7923	0.7530
source D	0.8177	0.8298	0.8144	0.8035	0.8028	0.7646
source Z	0.8795	0.9305	0.8023	0.7185	0.6870	0.6109
B0	0.8362	0.8466	0.8155	0.8558	0.8364	0.8026
D0	0.8195	0.8261	0.8165	0.8087	0.8040	0.7675
B1	0.8289	0.8414	0.7153	0.8047	0.8008	0.6598
non-late	0.9157	0.9227	0.9019	0.8939	0.8936	0.8723
non-late planned (no readmission)	0.8461	0.8563	0.6762	0.8276	0.8204	0.6254
non-late unplanned (no readmission)	0.8842	0.8950	0.8729	0.8600	0.8622	0.8414
late	0.8154	0.8228	0.8109	0.8071	0.8074	0.7749
readmission	0.7108	0.8145	0.8043	0.8570	0.742	0.7582

Table D.6: KS distance of all hospital mortality prediction models (LastAD)

Models	training benchmark	logit	CART	testing benchmark	logit	CART
planned	0.6048	0.6031	0.4063	0.5765	0.5692	0.3586
unplanned	0.5667	0.5778	0.5667	0.5554	0.5638	0.5222
unplanned peak	0.574	0.5922	0.597	0.5295	0.5437	0.4645
unplanned off-peak	0.5691	0.5756	0.5716	0.5924	0.6072	0.5787
source A	0.6788	0.7285	0.6758	0.7032	0.6856	0.6171
source B	0.6129	0.6358	0.5837	0.609	0.6072	0.5415
source D+Z	0.4999	0.5149	0.4888	0.4221	0.4407	0.3866
source D	0.4883	0.506	0.4804	0.4443	0.4773	0.4062
source Z	0.6394	0.7371	0.6046	0.3707	0.3217	0.2219
B0	0.5359	0.5402	0.5313	0.5883	0.5184	0.4877
D0	0.7905	0.4972	0.4862	0.4615	0.4601	0.4226
B1	0.5587	0.5439	0.4201	0.4878	0.4762	0.6109
non-late	0.6928	0.6952	0.6641	0.6418	0.6543	0.6082
non-late planned (no readmission)	0.5383	0.5842	0.3524	0.5037	0.5139	0.2509
non-late unplanned (no readmission)	0.6057	0.5278	0.6053	0.5748	0.5831	0.5411
late	0.4815	0.4818	0.4752	0.4601	0.5036	0.3946
readmission	0.4201	0.5136	0.5445	0.5789	0.5147	0.505

D.4 After-ICU mortality

Table D.7: AUROC of all after-ICU mortality prediction models (LastAD)

Models	training benchmark	logit	CART	testing benchmark	logit	CART
planned	0.7492	0.7772	0.7255	0.8441	0.7454	0.6513
unplanned	0.7454	0.8650	0.7951	0.7375	0.8377	0.7876
unplanned peak	0.7531	0.8721	0.7398	0.7412	0.8491	0.7154
unplanned off-peak	0.7566	0.8893	0.7584	0.7294	0.7809	0.7179
source A	0.8142	0.9388	0.7617	0.8782	0.8499	0.6735
source B	0.7727	0.8022	0.6960	0.7927	0.7625	0.6455
source D+Z	0.7283	0.8757	0.7715	0.6611	0.7767	0.7098
B0	0.6870	0.8132	0.5961	0.6969	0.6371	0.6191
D0	0.7193	0.9053	0.7914	0.6660	0.7687	0.7491
B1	0.6768	0.7490	0.6768	0.7635	0.5928	0.5835
non-late	0.8170	0.8605	0.8018	0.8004	0.8327	0.7942
non-late planned (no readmission)	0.6909	0.7415	0.6949	0.7855	0.4771	0.6431
non-late unplanned (no readmission)	0.7685	0.8464	0.808	0.7416	0.8614	0.8208
late	0.7190	0.8823	0.8053	0.6713	0.7362	0.6261
readmission	0.8225	0.8670	0.8111	0.7920	0.8174	0.7983

Table D.8: KS distance of all after-ICU mortality prediction models (LastAD)

Models	training benchmark	logit	CART	testing benchmark	logit	CART
planned	0.448	0.4613	0.4216	0.5394	0.4083	0.3755
unplanned	0.3879	0.5446	0.4424	0.3761	0.5514	0.4517
unplanned peak	0.4293	0.5911	0.4718	0.4124	0.6247	0.4193
unplanned off-peak	0.3863	0.6483	0.4878	0.4039	0.5918	0.4077
source A	0.5336	0.7776	0.5160	0.7537	0.6114	0.3536
source B	0.4497	0.5248	0.3762	0.5079	0.4172	0.2757
source D+Z	0.3326	0.5794	0.5280	0.2933	0.4378	0.4300
B0	0.3285	0.4566	0.1921	0.4316	0.3308	0.2381
D0	0.3449	0.6160	0.5709	0.3049	0.4783	0.4932
B1	0.3370	0.4851	0.3375	0.4793	0.2866	0.1676
non-late	0.5291	0.6012	0.5429	0.5002	0.5513	0.5036
non-late planned (no readmission)	0.3575	0.4084	0.3747	0.4942	0.1425	0.2797
non-late unplanned (no readmission)	0.4133	0.5166	0.4685	0.4003	0.5602	0.5094
late	0.3456	0.6264	0.5758	0.2955	0.4750	0.2551
readmission	0.5357	0.5849	0.5471	0.4923	0.5220	0.4974

Appendix E

Results for LoS Prediction Models

E.1 LoS1

LoS1: all admissions

Table E.1: HC estimations of LoS1 prediction (planned admissions)

	Estimate	P-Value
(Intercept)	-439.0046	0.7002
ReadmissionYes	5216.8399	0.0866
ICNARC.score	448.9349	0.0000
adPA_1	-2585.1745	0.1211
adlevel1	19281.8693	0.0307
disPA	2012.5825	0.2303
AdLate	5730.7949	0.0015
Adnighteffect	899.7385	0.0231

Table E.2: HC estimations of LoS1 prediction (unplanned admissions)

	Estimate	P-Value
(Intercept)	-2193.1400	0.3042
ReadmissionYes	1468.6409	0.1401
ICNARC.score	181.7433	0.0000
P1	1855.0978	0.0238

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Table E.2 – continued from previous page

	Estimate	P-Value
P2_system2	-1788.4168	0.0285
P2_system3	2190.0812	0.0106
P2_system4	-2584.9272	0.0015
P2_system5	-504.0537	0.5453
P2_system6	-3862.0837	0.0000
P2_system7	-1950.4142	0.0084
P2_system8	-2931.0640	0.0014
P2_system9	-4122.2051	0.0000
P2_system10	-1260.9492	0.3897
P2_system11	-3793.2969	0.0036
P2_system12	4714.4886	0.0000
adnonlevel3	-2661.6695	0.1024
disnonlevel3	4587.8506	0.0040
dislevel1	25797.8627	0.0012
SourceB	-1275.9844	0.1525
SourceD	1200.6238	0.0700
SourceF	1645.9345	0.0548
SourceZ	4079.9872	0.0249
AdLate	1645.4522	0.0206
Adnighteffect	-898.0642	0.0215
Adpeak	1020.6546	0.0090
CateReasonhighrisk	-1799.7172	0.0968
CateReasonlowrisk	2038.6945	0.0166
CateReasonMalignant neoplasm of oesophagus	4443.7181	0.0638
CateReasonothers	883.8303	0.3771
CateReasonPancreatic or pancreato-duodenal tumour	1753.7319	0.7082
CateReasonPneumonia, no organism isolated	3275.0813	0.0206

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Table E.2 – continued from previous page

	Estimate	P-Value
CateReasonPrimary lung tumour	2523.5668	0.1711
CateReasonSecondary hepatic tumour	-3852.8535	0.0444
EMELEM	1884.0035	0.0065

Table E.6: HC estimations of LoS1 prediction (non-late unplanned admissions)

	Estimate	P-Value
(Intercept)	1090.7654	0.4532
ICNARC.score	179.0387	0.0000
P2_system2	-1444.0783	0.1272
P2_system3	1935.9305	0.0548
P2_system4	-1866.8751	0.0521
P2_system5	-587.1093	0.5284
P2_system6	-3854.6837	0.0000
P2_system7	-1933.7139	0.0238
P2_system8	-3575.4481	0.0000
P2_system9	-4234.4027	0.0000
P2_system10	-1306.3303	0.3084
P2_system11	-2837.4182	0.0437
disnonlevel3	2514.9650	0.0307
dislevel1	25006.2389	0.0015
SourceB	-2556.7553	0.0017
SourceD	1429.5945	0.0714
SourceF	1784.6302	0.0466
SourceZ	3483.1890	0.0910
Adnighteffect	-930.6813	0.0293
CateReasonhighrisk	-1253.3722	0.2887
CateReasonlowrisk	2750.9457	0.0037

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Table E.6 – continued from previous page

	Estimate	P-Value
CateReasonMalignant neoplasm of oesophagus	4693.5802	0.1815
CateReasonothers	1762.9331	0.1258
CateReasonPancreatic or pancreato-duodenal tumour	21407.0812	0.1618
CateReasonPneumonia, no organism isolated	3090.6286	0.0644
CateReasonPrimary lung tumour	3756.5433	0.0484
CateReasonSecondary hepatic tumour	-2773.4819	0.1912
EMELEM	1508.9002	0.0294

Table E.3: HC estimations of LoS1 prediction (unplanned peak admissions)

	Estimate	P-Value
(Intercept)	-1261.6386	0.6059
ICNARC.score	177.3176	0.0000
P1	1795.0106	0.0887
P2_system2	-2601.0603	0.0044
P2_system3	3008.6898	0.0045
P2_system4	-2948.5709	0.0010
P2_system5	431.5099	0.7125
P2_system6	-3976.4983	0.0000
P2_system7	-1843.9867	0.0219
P2_system8	-2981.6722	0.0092
P2_system9	-4888.4383	0.0000
P2_system10	-1172.9922	0.5343
P2_system11	-4394.7610	0.0097
P2_system12	4010.6691	0.0000
disnonlevel3	4067.4502	0.0071
dislevel1	25416.9137	0.0156
SourceB	189.8091	0.8626
SourceD	1137.3009	0.1908
SourceF	1245.0272	0.2894
SourceZ	6080.7106	0.0249
AdLate	2380.1204	0.0145

Table E.4: HC estimations of LoS1 prediction (unplanned off-peak admissions)

	Estimate	P-Value
(Intercept)	3245.8672	0.2138
ReadmissionYes	2534.4617	0.1043
ICNARC.score	399.0663	0.0000
ICNARC.probability	-104.5019	0.0001
P1	1718.8234	0.0655
P2_system2	-2096.5840	0.1436
P2_system3	1156.1528	0.3666
P2_system4	-4224.3067	0.0002
P2_system5	-2566.0761	0.0214
P2_system6	-4895.0182	0.0000
P2_system7	-2372.8027	0.0714
P2_system8	-3805.0947	0.0099
P2_system9	-4046.1930	0.0019
P2_system10	-2919.1225	0.0555
P2_system11	-2868.4239	0.0421
adnonlevel3	-1203.4091	0.3866
dislevel1	26972.8675	0.0219
AdLate	1810.5530	0.0058
Adnighteffect	-1383.3667	0.0251
CateReasonhighrisk	-2359.2592	0.0537
CateReasonlowrisk	-1619.8509	0.1478
CateReasonMalignant neoplasm of oesophagus	1081.0154	0.7477
CateReasonothers	-3260.8797	0.0108
CateReasonPancreatic or pancreato-duodenal tumour	-7066.6024	0.0001
CateReasonPneumonia, no organism isolated	1563.4565	0.4189
CateReasonPrimary lung tumour	750.2530	0.8315
CateReasonSecondary hepatic tumour	-7817.5009	0.0000

Table E.5: HC estimations of LoS1 prediction (non-late planned admissions)

	Estimate	P-Value
(Intercept)	1348.8895	0.2062
ICNARC.score	389.1853	0.0000
adlevel1	17974.7529	0.0578
adPA_1	-1729.8700	0.1142
dislevel1	10744.8414	0.0900
Adnighteffect	799.7125	0.0283

Table E.7: HC estimations of LoS1 prediction (late admissions & readmissions)

	Estimate	P-Value
(Intercept)	2474.6929	0.0726
ReadmissionYes	2894.3000	0.0295
ICNARC.score	248.9715	0.0000
dislevel1	32867.8597	0.0889
SourceB	-1510.2313	0.3952
SourceD	1644.5010	0.2119
SourceF	-1785.4097	0.4104
SourceZ	7689.5402	0.0056
Adpeak	1911.4268	0.0303
AdWeffect	-1698.9116	0.0600

Table E.8: R-squared of LoS1 Prediction Models

	R-squared (training)	Adjusted R-squared (training)	R-squared (testing)	Adjusted R-squared (testing)
planned	0.1597	0.1561	0.1378	0.1303
unplanned	0.1209	0.1086	0.0884	0.0627
unplanned peak	0.1158	0.1031	0.0887	0.0621
unplanned off-peak	0.1452	0.1217	0.0540	0.0017
non-late planned	0.1019	0.0990	0.0899	0.0838
non-late unplanned	0.1077	0.0940	0.0706	0.0423
late & readmission	0.0734	0.0611	0.0412	0.0344

E.2 LoS2

LoS2: all admissions discharged alive and admissions discharged died after 8hr of admissions

Table E.9: HC estimations of LoS2 prediction (planned admissions)

	Estimate	P-Value
(Intercept)	-5069.9925	0.3728
ICNARC.score	464.9260	0.0000
adlevel1	20510.1318	0.0236
SourceB	-1360.6109	0.6894
SourceD	2676.2084	0.5111
SourceF	-1979.8834	0.6752
SourceZ	7197.8671	0.0803
Adnighteffect	1011.4391	0.0090
CateReasonhighrisk	1962.8286	0.7697
CateReasonlowrisk	5150.7691	0.4486
CateReasonMalignant neoplasm of oesophagus	7423.0020	0.2767
CateReasonothers	4835.6766	0.4754
CateReasonPancreatic or pancreato-duodenal tumour	5926.3393	0.3849
CateReasonPneumonia, no organism isolated	1370.7508	0.8472
CateReasonPrimary lung tumour	4777.7570	0.4839
CateReasonSecondary hepatic tumour	4604.6274	0.4988

Table E.10: HC estimations of LoS2 prediction (unplanned admissions)

	Estimate	P-Value
(Intercept)	-2226.6918	0.2862
ICNARC.score	226.3710	0.0000
P1	1929.4104	0.0206
P2_system2	-1560.9896	0.0612
P2_system3	2059.2333	0.0170
P2_system4	-2641.6329	0.0014
P2_system5	-562.4987	0.5000
P2_system6	-3791.6358	0.0000
P2_system7	-2193.4743	0.0030
P2_system8	-2894.5055	0.0014
P2_system9	-4096.5438	0.0000

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Table E.10 – continued from previous page

	Estimate	P-Value
P2_system10	-1388.8386	0.3484
P2_system11	-3737.8326	0.0037
P2_system12	4831.3053	0.0000
dislevel1	24865.6025	0.0020
SourceB	-1069.2258	0.2295
SourceD	1263.1642	0.0574
SourceF	1641.5857	0.0643
SourceZ	4018.8742	0.0329
AdLate	1771.3225	0.0140
Adnighteffect	-789.0015	0.0464
Adpeak	1033.9584	0.0086
CateReasonhighrisk	-1577.7042	0.1784
CateReasonlowrisk	2279.6928	0.0095
CateReasonMalignant neoplasm of oesophagus	4956.1280	0.0371
CateReasonothers	1148.8451	0.2676
CateReasonPancreatic or pancreato-duodenal tumour	3136.4658	0.5332
CateReasonPneumonia, no organism isolated	3449.6735	0.0159
CateReasonPrimary lung tumour	2535.7563	0.1746
CateReasonSecondary hepatic tumour	-3688.6147	0.0743
EMELEM	1961.6960	0.0039

Table E.14: HC estimations of LoS2 prediction (non-late unplanned admissions)

	Estimate	P-Value
(Intercept)	2825.1148	0.2506
ICNARC.probability	55.0022	0.0000
P2_system2	-1138.7478	0.2549
P2_system3	1837.0462	0.0737

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Table E.14 – continued from previous page

	Estimate	P-Value
P2_system4	-2034.1077	0.0405
P2_system5	-770.5406	0.4292
P2_system6	-4128.4268	0.0000
P2_system7	-1897.0862	0.0290
P2_system8	-3477.4225	0.0001
P2_system9	-4139.8923	0.0000
P2_system10	-1484.9921	0.2642
P2_system11	-2409.5210	0.0978
adnonlevel3	-4226.8478	0.0246
adPA_1	-3348.5814	0.0287
disnonlevel3	5924.8535	0.0014
dislevel1	24610.3466	0.0023
disPA	3490.8101	0.0262
SourceB	-2889.4687	0.0009
SourceD	1191.1994	0.1384
SourceF	2147.9370	0.0202
SourceZ	3266.5389	0.1273
Adnighteffect	-989.2815	0.0225
CateReasonhighrisk	-524.2728	0.6926
CateReasonlowrisk	3722.1052	0.0006
CateReasonMalignant neoplasm of oesophagus	5483.0504	0.1333
CateReasonothers	2579.0674	0.0410
CateReasonPancreatic or pancreato-duodenal tumour	22665.3513	0.1224
CateReasonPneumonia, no organism isolated	4132.9678	0.0164
CateReasonPrimary lung tumour	4259.6939	0.0353
CateReasonSecondary hepatic tumour	-1687.9946	0.4804
EMELEM	1642.3606	0.0237

Table E.11: HC estimations of LoS2 prediction (unplanned peak admissions)

	Estimate	P-Value
(Intercept)	-2028.9007	0.4115
ICNARC.score	218.2505	0.0000
P1	1944.5203	0.0678
P2_system2	-2147.2728	0.0212
P2_system3	2967.2722	0.0054
P2_system4	-2977.8541	0.0010
P2_system5	393.1968	0.7360
P2_system6	-3931.4739	0.0000
P2_system7	-1943.0799	0.0153
P2_system8	-3063.0243	0.0068
P2_system9	-4319.7784	0.0002
P2_system10	-1201.3848	0.5275
P2_system11	-4184.7424	0.0228
P2_system12	3706.3141	0.0002
disnonlevel3	3736.5807	0.0140
dislevel1	22909.7420	0.0280
SourceB	425.0103	0.7022
SourceD	1314.6399	0.1324
SourceF	1178.0966	0.3344
SourceZ	6409.0780	0.0230
AdLate	2200.2073	0.0251

Table E.12: HC estimations of LoS2 prediction (unplanned off-peak admissions)

	Estimate	P-Value
(Intercept)	5771.4034	0.0000
ICNARC.score	214.9902	0.0000
Days.between.hospital.and.unit.admit	48.1673	0.1318
P2_system2	-3251.1759	0.0100
P2_system3	-1306.2308	0.2203
P2_system4	-5470.7438	0.0000
P2_system5	-3478.1613	0.0005
P2_system6	-4363.6039	0.0000
P2_system7	-3692.8790	0.0017
P2_system8	-3989.4280	0.0019
P2_system9	-6933.6650	0.0000
P2_system10	-3842.4240	0.0045
P2_system11	-5275.4391	0.0010
dislevel1	30113.9367	0.0123
SourceB	-184.6192	0.8560
SourceD	1531.9059	0.0643
SourceF	-133.7342	0.9063
SourceZ	-242.7018	0.8739
Adnighteffect	-1561.8638	0.0102

Table E.13: HC estimations of LoS2 prediction (non-late planned admissions)

	Estimate	P-Value
(Intercept)	-5315.6386	0.2722
ICNARC.probability	212.1659	0.0000
adlevel1	17974.8402	0.0544
SourceB	4538.8507	0.2710
SourceD	9029.1670	0.1037
SourceF	-1506.5462	0.7606
SourceZ	6341.0117	0.1909
Adnighteffect	957.1692	0.0088
CateReasonhighrisk	4576.5898	0.5429
CateReasonlowrisk	3740.2379	0.6294
CateReasonMalignant neoplasm of oesophagus	7106.8574	0.3613
CateReasonothers	3827.3671	0.6213
CateReasonPancreatic or pancreato-duodenal tumour	4329.4761	0.5780
CateReasonPneumonia, no organism isolated	62.3912	0.9936
CateReasonPrimary lung tumour	3786.2141	0.6271
CateReasonSecondary hepatic tumour	3124.8120	0.6879

Table E.15: HC estimations of LoS2 prediction (late admissions & readmissions)

	Estimate	P-Value
(Intercept)	1172.4021	0.4793
ICNARC.score	276.7212	0.0000
disnonlevel3	3719.8814	0.0914
dislevel1	32584.9076	0.1006
SourceB	1090.0885	0.4553
SourceD	2211.7599	0.0868
SourceF	-1913.3834	0.3672
SourceZ	8344.0028	0.0036
AdWeffect	-1719.7953	0.0602

Table E.16: R-squared of LoS2 Prediction Models

	R-squared (training)	Adjusted R-squared (training)	R-squared (testing)	Adjusted R-squared (testing)
planned	0.1854	0.1778	0.1690	0.1532
unplanned	0.1281	0.1168	0.1034	0.0799
unplanned peak	0.1257	0.1129	0.1103	0.0838
unplanned off-peak	0.1285	0.1116	0.0723	0.0365
non-late planned	0.0967	0.0878	0.0763	0.0576
non-late unplanned	0.0988	0.0831	0.0592	0.0265
late & readmission	0.1303	0.1203	0.0613	0.0377

E.3 LoS3

LoS3: all admissions discharged alive

Table E.17: HC estimations of LoS3 prediction (planned admissions)

	Estimate	P-Value
(Intercept)	-2596.7652	0.6933
ICNARC.score	460.5571	0.0000
adlevel1	12315.7402	0.1171
SourceB	-710.5379	0.8486
SourceD	4873.2081	0.2743
SourceF	92.6470	0.9870
SourceZ	9278.6475	0.0442
Adnighteffect	1033.5056	0.0034
CateReasonhighrisk	-817.8441	0.9221
CateReasonlowrisk	1954.3247	0.8074
CateReasonMalignant neoplasm of oesophagus	3743.3899	0.6411
CateReasonothers	1613.9216	0.8399
CateReasonPancreatic or pancreato-duodenal tumour	3005.5783	0.7081
CateReasonPneumonia, no organism isolated	-472.5579	0.9535
CateReasonPrimary lung tumour	1063.3437	0.8946
CateReasonSecondary hepatic tumour	1606.7119	0.8412

Table E.18: HC estimations of LoS3 prediction (unplanned admissions)

	Estimate	P-Value
(Intercept)	-3236.2235	0.1276
ICNARC.score	551.8265	0.0000
P1	1793.1423	0.0357
P2_system2	-2039.4448	0.0111
P2_system3	2415.9790	0.0048
P2_system4	-2871.1843	0.0001
P2_system5	-110.7909	0.8811
P2_system6	-3345.5565	0.0000
P2_system7	-3087.7421	0.0000
P2_system8	-4371.6745	0.0000
P2_system9	-5628.5518	0.0000
P2_system10	-147.5933	0.9120
P2_system11	-3452.1493	0.0051
P2_system12	1096.9791	0.1646
adPA	-1836.7662	0.1557
dislevel1	25177.7595	0.0016
SourceB	344.5349	0.7020
SourceD	1313.4691	0.0629
SourceF	-938.2964	0.4505
SourceZ	5132.4576	0.0235
AdLate	1546.5178	0.0432
Adpeak	1003.4322	0.0200

Table E.19: HC estimations of LoS3 prediction (unplanned peak admissions)

	Estimate	P-Value
(Intercept)	-1675.7920	0.5195
ICNARC.score	535.3744	0.0000
P1	1932.3523	0.0755
P2_system2	-1576.9885	0.0881
P2_system3	3284.4466	0.0029
P2_system4	-2249.3671	0.0285
P2_system5	427.5900	0.6685
P2_system6	-3517.2465	0.0002
P2_system7	-2451.9245	0.0018
P2_system8	-4714.5394	0.0000
P2_system9	-4586.1451	0.0002
P2_system10	1253.9494	0.4602
P2_system11	-3305.2025	0.0319
P2_system12	1852.6582	0.0623
adlevel1	-18243.5778	0.0530
dislevel1	25964.3307	0.0115
disPA	-2906.0300	0.0652
SourceB	295.4345	0.7931
SourceD	1146.7816	0.2172
SourceF	435.4293	0.7931
SourceZ	8618.7113	0.0053
AdLate	2120.4003	0.0377

Table E.20: HC estimations of LoS3 prediction (unplanned off-peak admissions)

	Estimate	P-Value
(Intercept)	-291.7821	0.8208
ICNARC.score	585.2170	0.0000
P2_system2	-2657.6543	0.0748
P2_system3	390.8547	0.7410
P2_system4	-4138.7584	0.0000
P2_system5	-851.8743	0.4066
P2_system6	-3507.1583	0.0009
P2_system7	-4285.5190	0.0001
P2_system8	-3931.7359	0.0030
P2_system9	-7562.8804	0.0000
P2_system10	-4021.9252	0.0216
P2_system11	-2210.0150	0.1971
dislevel1	28510.7589	0.0413
SourceB	-554.0524	0.6443
SourceD	1476.8889	0.1295
SourceF	-3446.4756	0.0484
SourceZ	-2439.9990	0.1762
Adnighteffect	-1173.9985	0.0935

Table E.21: HC estimations of LoS3 prediction (non-late planned admissions)

	Estimate	P-Value
(Intercept)	-5697.2946	0.3826
ICNARC.probability	267.6023	0.0000
adPA_1	-1714.6804	0.0669
dislevel1	9839.1209	0.0878
disPA	1402.4113	0.1924
SourceB	6841.0427	0.2514
SourceD	10664.7436	0.1186
SourceF	-692.5031	0.9240
SourceZ	9227.3691	0.1885
Adnighteffect	998.6089	0.0018
CateReasonhighrisk	5150.8331	0.6245
CateReasonlowrisk	1673.0144	0.8661
CateReasonMalignant neoplasm of oesophagus	4436.5750	0.6551
CateReasonothers	1670.1713	0.8661
CateReasonPancreatic or pancreato-duodenal tumour	2592.7678	0.7942
CateReasonPneumonia, no organism isolated	-1948.6858	0.8444
CateReasonPrimary lung tumour	1146.1702	0.9082
CateReasonSecondary hepatic tumour	1436.8145	0.8851

Table E.22: HC estimations of LoS3 prediction (non-late unplanned admissions)

	Estimate	P-Value
(Intercept)	-2065.3335	0.3383
ICNARC.probability	191.7318	0.0000
P2_system2	-926.1391	0.4049
P2_system3	2276.9627	0.0388
P2_system4	-1289.0519	0.2449
P2_system5	-390.8986	0.6773
P2_system6	-2989.6805	0.0003
P2_system7	-1922.2688	0.0122
P2_system8	-3116.4204	0.0004
P2_system9	-5033.0823	0.0000
P2_system10	-512.4822	0.6781
P2_system11	-1584.7300	0.3076
adPA_1	-2138.1138	0.1240
dislevel1	29119.4910	0.0012
Adnighteffect	-902.2622	0.0711
CateReasonhighrisk	1138.0407	0.5959
CateReasonlowrisk	7180.6632	0.0000
CateReasonMalignant neoplasm of oesophagus	10441.7518	0.0077
CateReasonothers	6491.4538	0.0001
CateReasonPancreatic or pancreato-duodenal tumour	25960.6091	0.0805
CateReasonPneumonia, no organism isolated	6830.8335	0.0007
CateReasonPrimary lung tumour	8697.8933	0.0001
CateReasonSecondary hepatic tumour	3707.6124	0.0933
EMELEM	1362.5733	0.0447
EMELNR	3159.1158	0.0002

Table E.23: HC estimations of LoS3 prediction (late admissions & readmissions)

	Estimate	P-Value
(Intercept)	-3688.2875	0.0507
ICNARC.score	665.7807	0.0000
SourceB	1032.4004	0.4820
SourceD	2389.1071	0.0795
SourceF	-5444.5773	0.0667
SourceZ	9846.7441	0.0008
AdWeekday2	788.5010	0.6357
AdWeekday3	3998.9219	0.0512
AdWeekday4	3927.6491	0.0625
AdWeekday5	154.2060	0.9311
AdWeekday6	-888.2272	0.5398
AdWeekday7	-1223.7807	0.4223

Table E.24: R-squared of LoS3 Prediction Models

	R-squared (training)	Adjusted R-squared (training)	R-squared (testing)	Adjusted R-squared (testing)
planned	0.2246	0.2172	0.1829	0.1669
unplanned	0.2430	0.2343	0.2122	0.1946
unplanned peak	0.2550	0.2409	0.2261	0.1970
unplanned off-peak	0.2566	0.2384	0.1639	0.1247
non-late planned	0.1878	0.1735	0.1403	0.1108
non-late unplanned	0.1303	0.1203	0.1136	0.0928
late & readmission	0.1303	0.1203	0.1136	0.0928

Appendix F

Arrivals at the ICU

Table F.1: The VMRs of day time intervals for unplanned arrivals

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1.656	1.0963	1.1513	1.1804	1.1819	1.1804	1.1243

Table F.2: The VMRs of half day time intervals for unplanned arrivals

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Day Shift	0.9813	1.0838	0.9511	1.2278	1.0789	1.0376	1.1048
Night Shift	1.2575	1.0214	1.1661	1.1293	1.1382	1.3000	1.0199

Table F.3: EDF of admission hour of planned arrivals

Arrival Hour (t)	0	1	2	3	4	5
$P(X \leq t)$	0.0206	0.0305	0.0370	0.0399	0.0416	0.0440
Arrival Hour (t)	6	7	8	9	10	11
$P(X \leq t)$	0.0473	0.0477	0.0494	0.0519	0.0572	0.0646
Arrival Hour (t)	12	13	14	15	16	17
$P(X \leq t)$	0.0708	0.0893	0.1247	0.1860	0.2794	0.4169
Arrival Hour (t)	18	19	20	21	22	23
$P(X \leq t)$	0.5761	0.7272	0.8366	0.8996	0.9564	1.0000

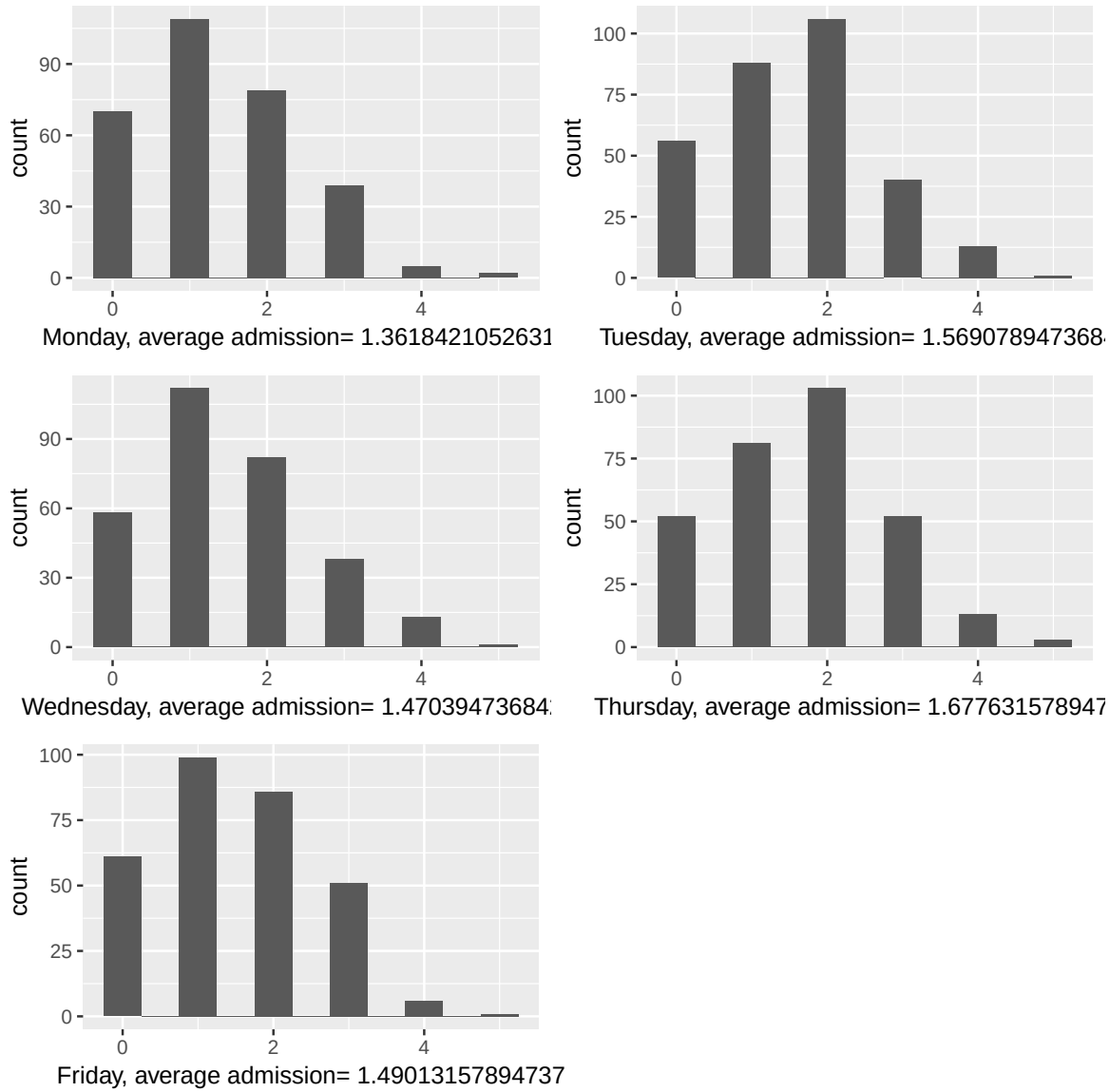


Figure F.1: Numbers of planned arrivals (weekdays)

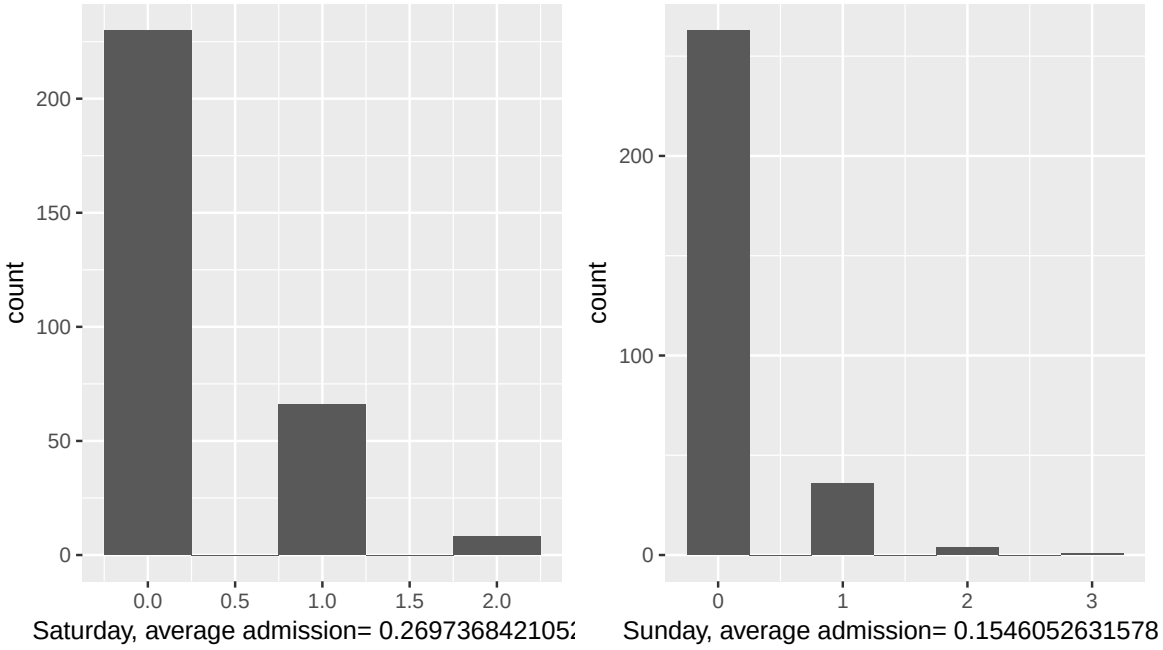


Figure F.2: Numbers of planned arrivals (weekends)

Appendix G

Input Uncertainties

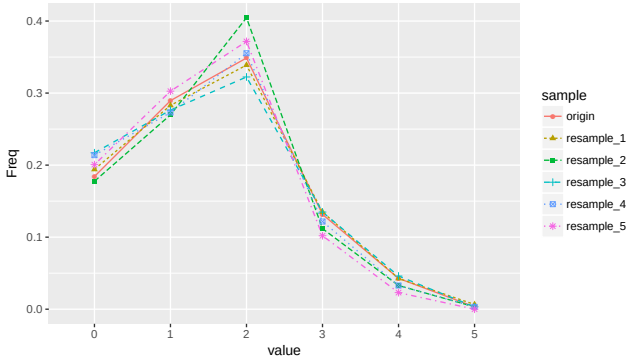


Figure G.1: Planned arrival sampling (Tuesday)

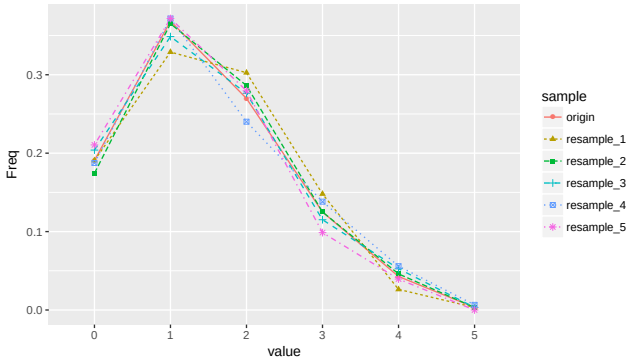


Figure G.2: Planned arrival sampling (Wednesday)

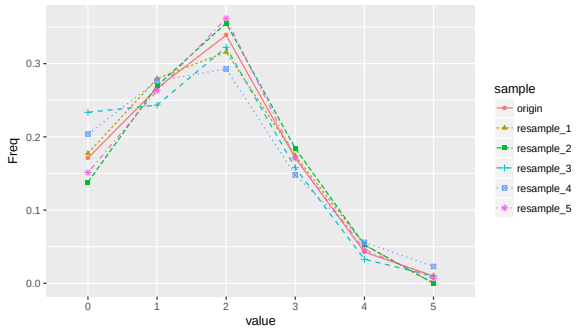


Figure G.3: Planned arrival sampling (Thursday)

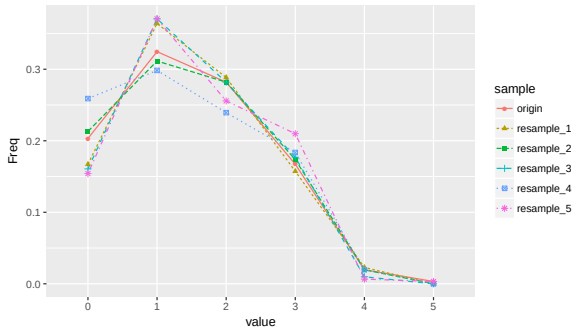


Figure G.4: Planned arrival sampling (Friday)

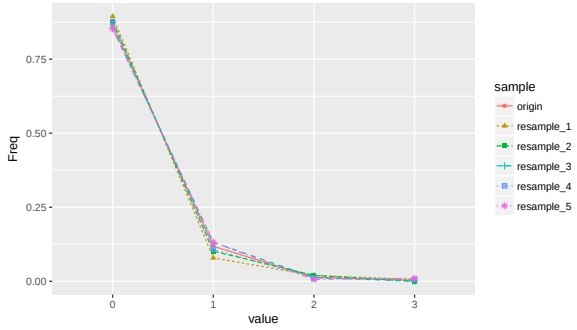


Figure G.5: Planned arrival sampling (Sunday)

Table G.1: Bootstrap statistics of nights (planned) in the ICU

	original	bias	standard error	95% CI
Mean	3.4441	2.7207×10^{-3}	0.0953	(3.2601, 3.6335)
Variance	20.5577	-0.0809	2.6876	(15.3710, 25.9062)
Max	54	-3.2250	4.3247	(48.7487, 65.7013)
Nights>20 (%)	0.0174	6.0461×10^{-5}	2.6613×10^{-3}	(0.0122, 0.0227)

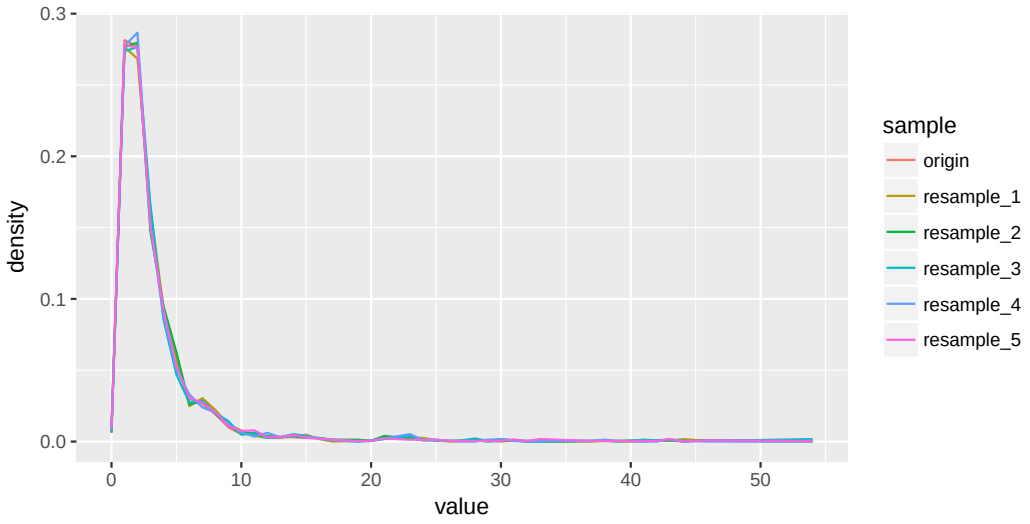


Figure G.6: Nights (planned) in the ICU re-sampling

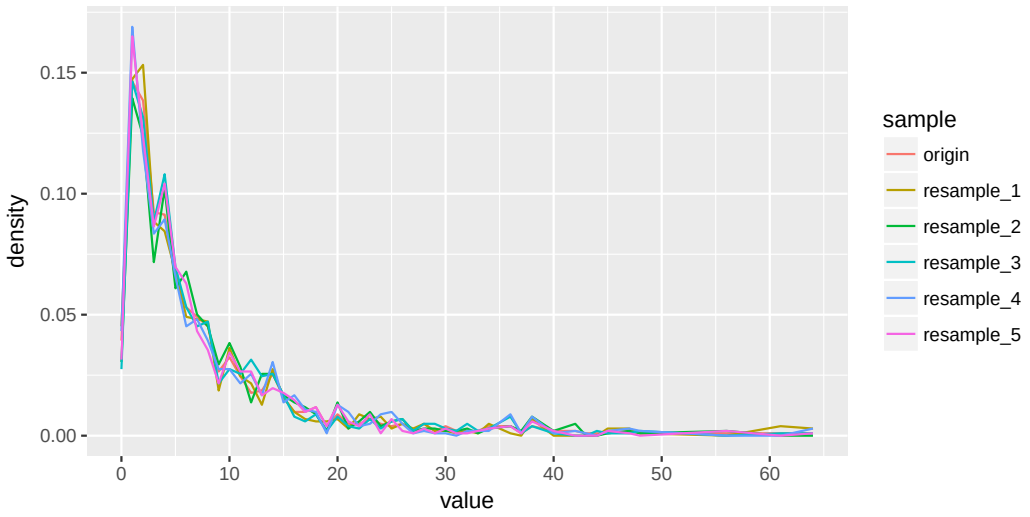


Figure G.7: Nights (late/re-admission) in the ICU re-sampling

Table G.2: Bootstrap statistics of nights (late/re-admission) in the ICU

	original	bias	standard error	95% CI
Mean	7.4185	2.7809×10^{-3}	0.2725	(6.8816, 7.9498)
Variance	73.2210	-0.1693	7.2913	(59.0996, 87.6810)
Max	64	-2.3930	4.2674	(58.0290, 74.7570)
Nights>20 (%)	0.0747	2.4754×10^{-4}	8.2999×10^{-3}	(0.0581, 0.0907)

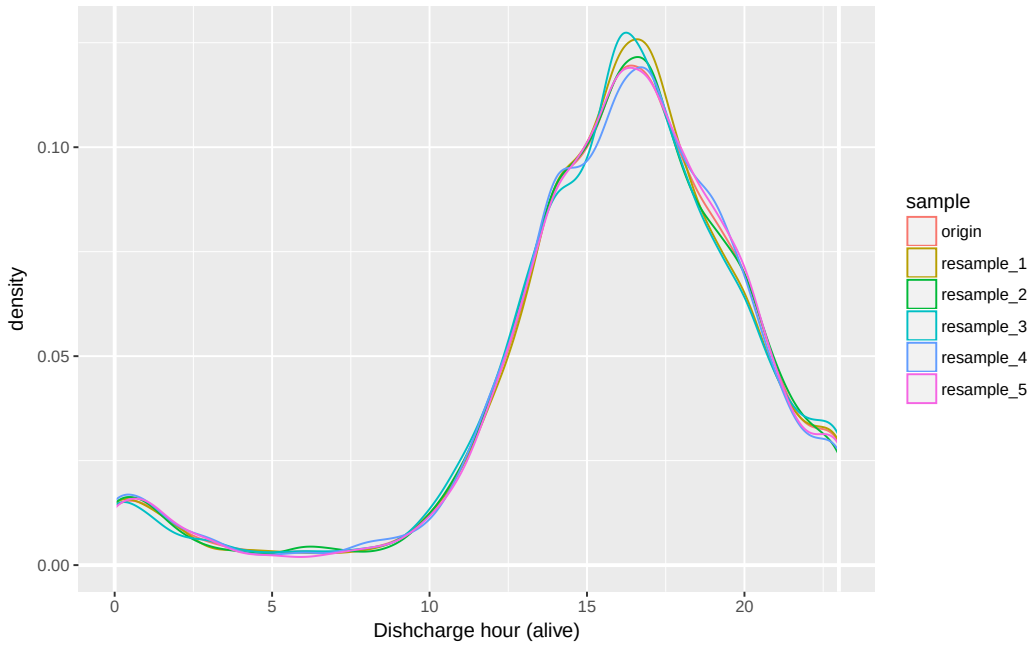


Figure G.8: Discharge hour (survivors) in the ICU re-sampling

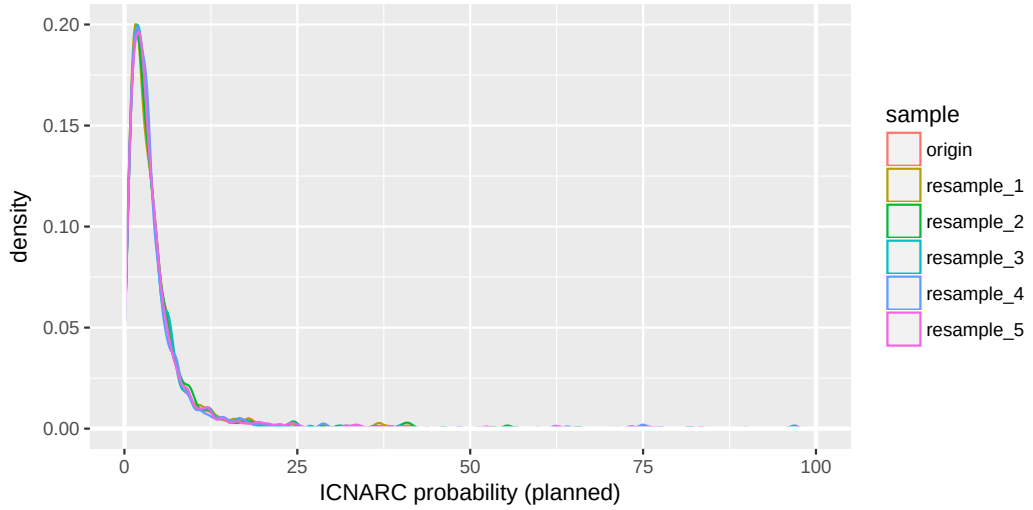


Figure G.9: ICNARC probability (planned) re-sampling

Table G.3: Bootstrap statistics of ICNARC probability (planned)

	original	bias	standard error	95% CI
Mean	5.1179	-4.1697×10^{-3}	0.1786	(4.7721, 5.4721)
Variance	72.5388	-0.2034	10.2213	(52.7087, 92.7756)

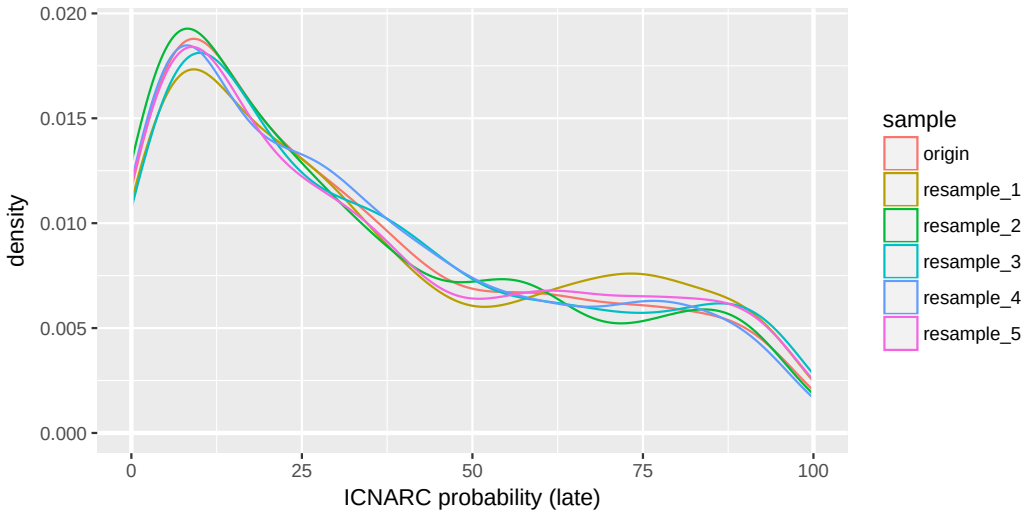


Figure G.10: ICNARC probability (late) re-sampling

Table G.4: Bootstrap statistics of ICNARC probability (late)

	original	bias	standard error	95% CI
Mean	34.9370	-0.0308	1.0011	(28.5546, 30.7968)
Variance	785.3171	-1.2199	31.0932	(725.5955, 847.4785)

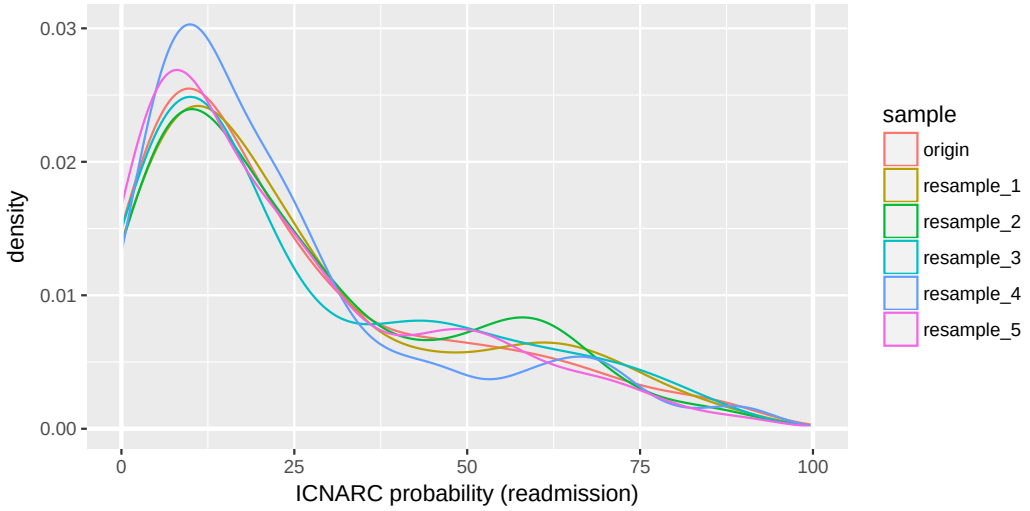


Figure G.11: ICNARC probability (readmission) re-sampling

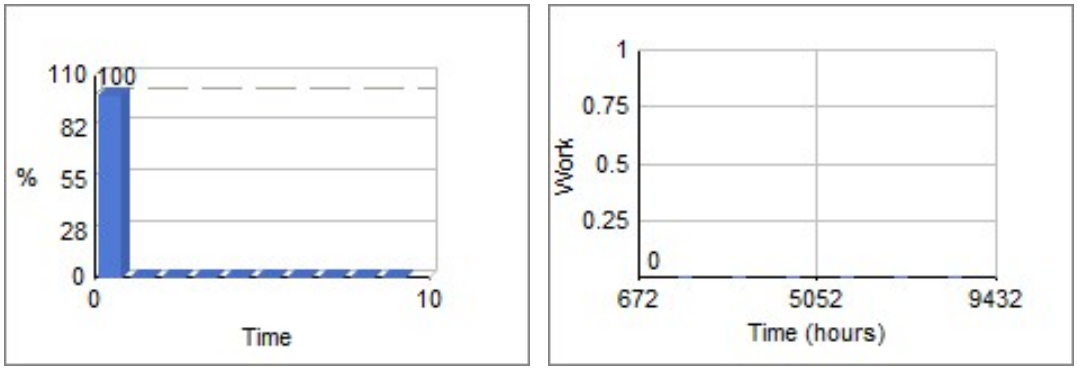
Table G.5: Bootstrap statistics of ICNARC probability (readmission)

	original	bias	standard error	95% CI
Mean	25.5304	-5.1682×10^{-5}	1.5787	(22.4364, 28.6246)
Variance	513.4311	-3.0625	50.5814	(417.3559, 615.6312)

Appendix H

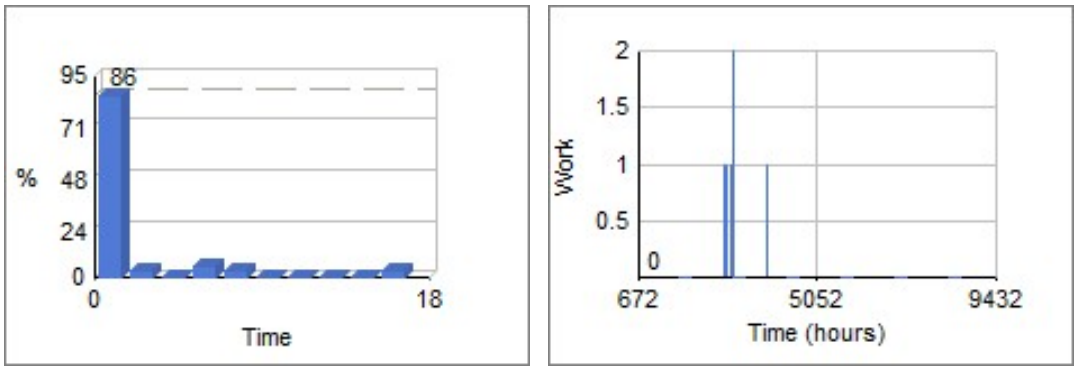
Results for Scenario Tests

H.1 Scenarios 6: serving pandemic arrivals



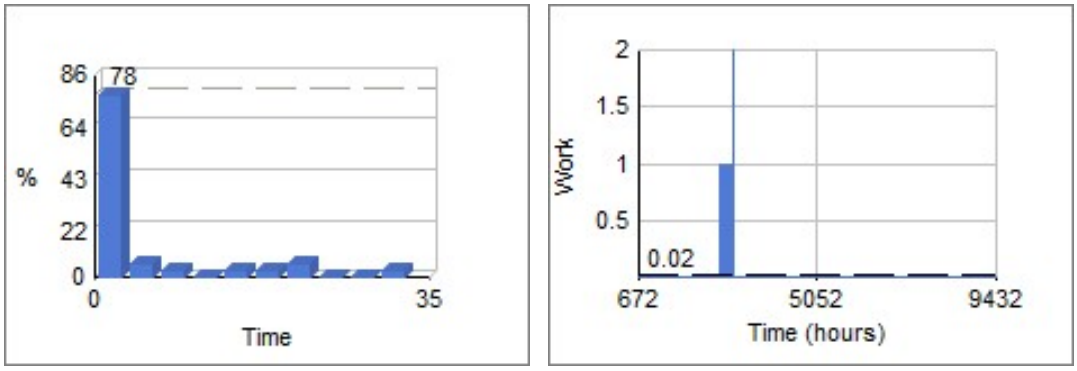
(a) Queue time (b) Queue size

Figure H.1: Queue of pandemic arrivals (scenario[mild] with PP arrival)



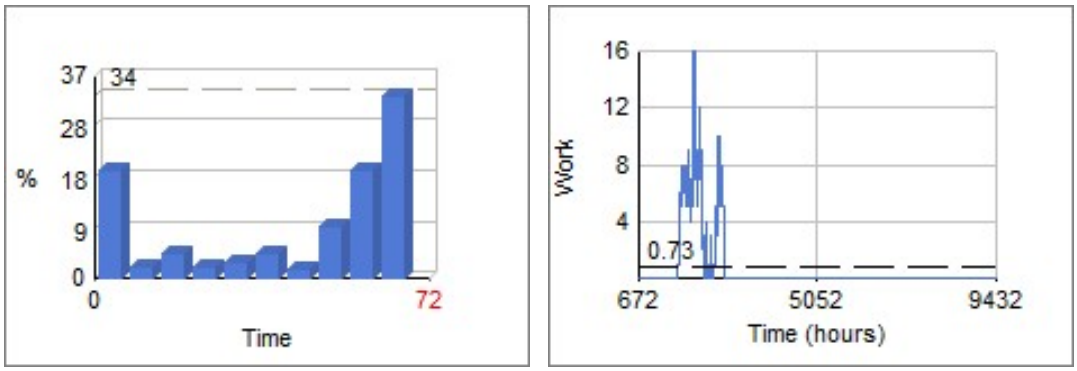
(a) Queue time (b) Queue size

Figure H.2: Queue of pandemic arrivals (scenario[mild] with NHPP arrival)



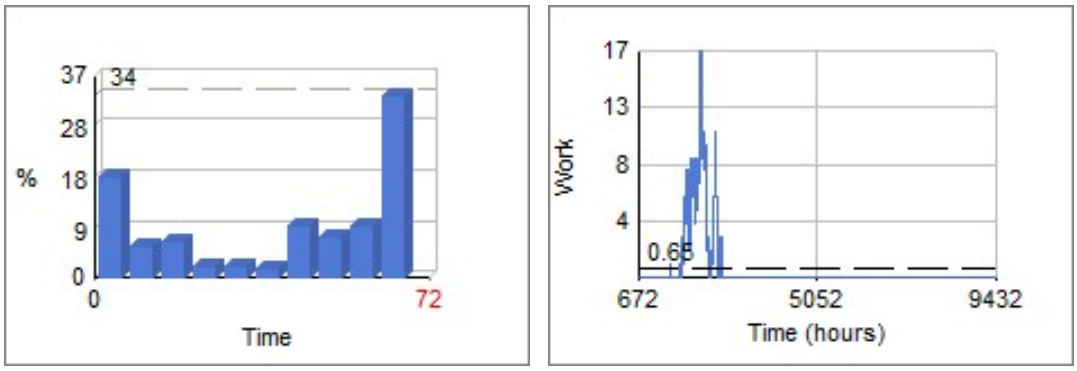
(a) Queue time (b) Queue size

Figure H.3: Queue of pandemic arrivals while changing nurse number (scenario[mild] with NHPP arrival)



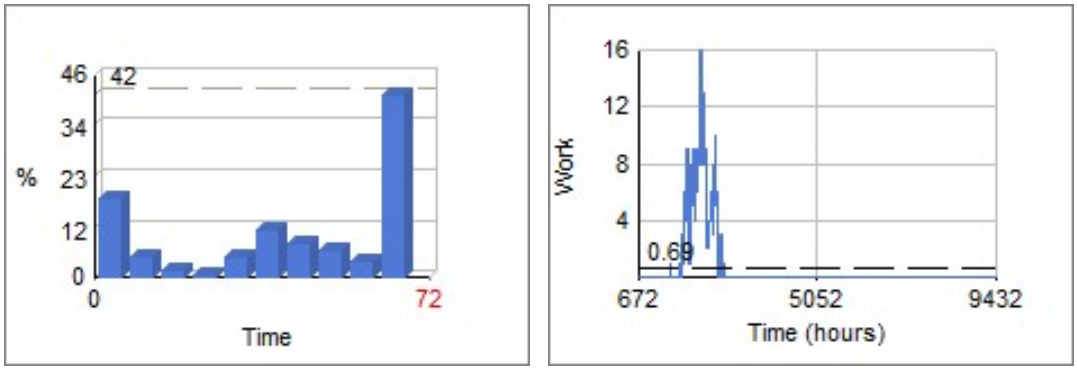
(a) Queue time (b) Queue size

Figure H.4: Queue of pandemic arrivals (scenario[likely1] with PP arrival)



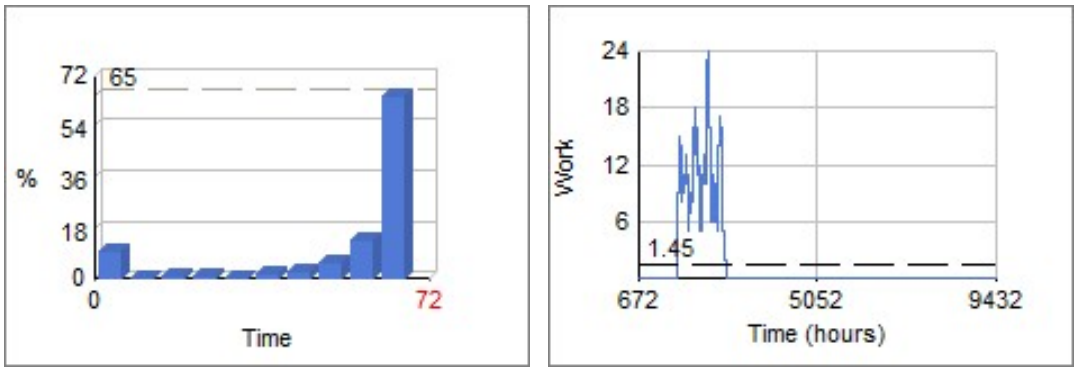
(a) Queue time (b) Queue size

Figure H.5: Queue of pandemic arrivals (scenario[likely1] with NHPP arrival)



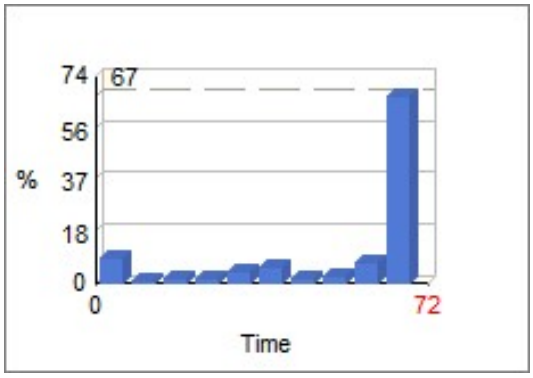
(a) Queue time (b) Queue size

Figure H.6: Queue of pandemic arrivals while changing nurse number (scenario[likely1] with NHPP arrival)

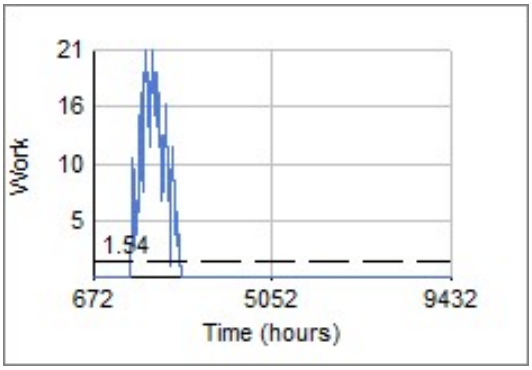


(a) Queue time (b) Queue size

Figure H.7: Queue of pandemic arrivals (scenario[likely2] with PP arrival)

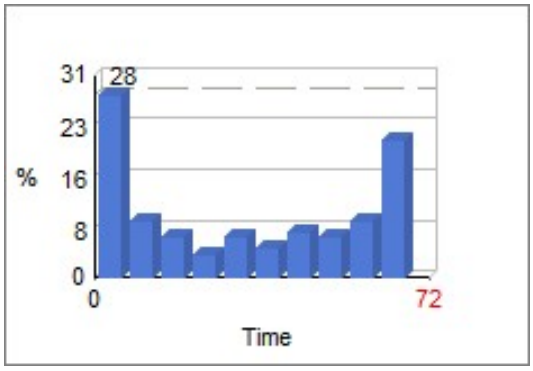


(a) Queue time

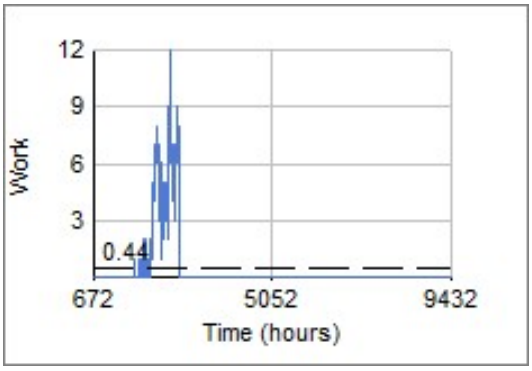


(b) Queue size

Figure H.8: Queue of pandemic arrivals (scenario[likely2] with NHPP arrival)

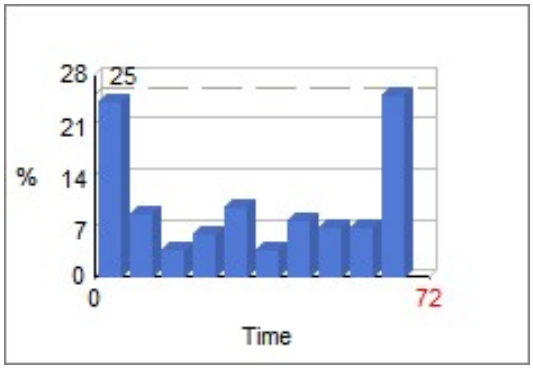


(a) Queue time

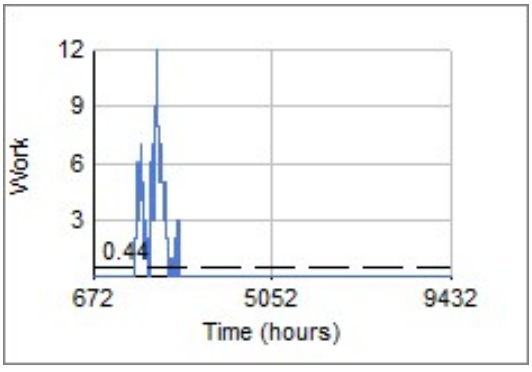


(b) Queue size

Figure H.9: Queue of pandemic arrivals (scenario[likely3] with PP arrival)

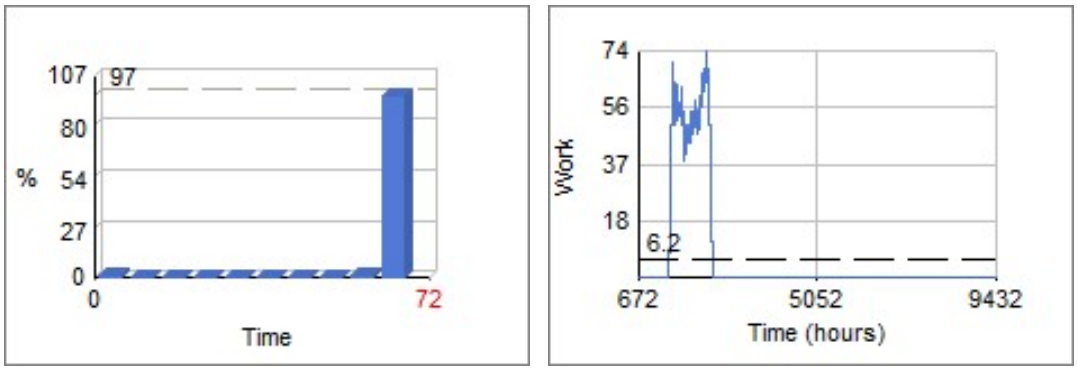


(a) Queue time



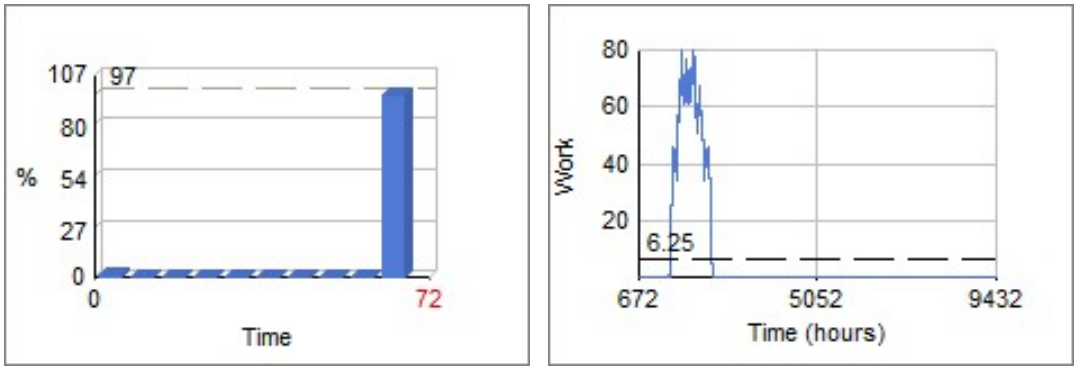
(b) Queue size

Figure H.10: Queue of pandemic arrivals (scenario[likely3] with NHPP arrival)



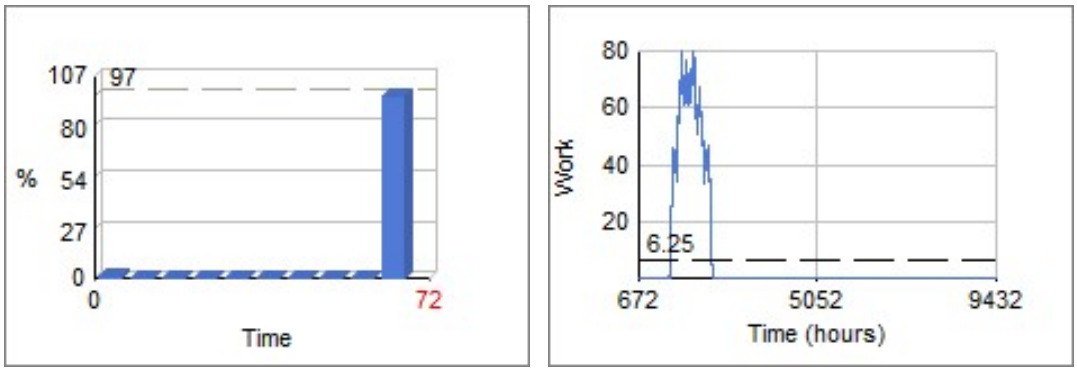
(a) Queue time (b) Queue size

Figure H.11: Queue of pandemic arrivals (scenario[worst] with PP arrival)



(a) Queue time (b) Queue size

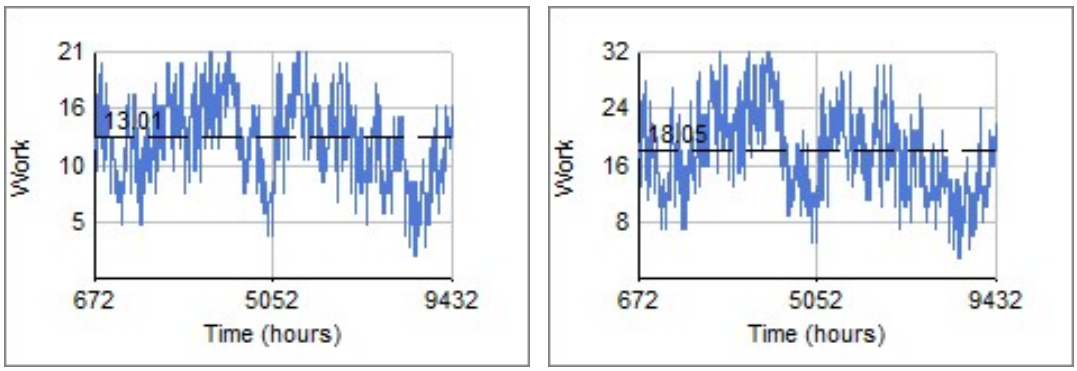
Figure H.12: Queue of pandemic arrivals (scenario[worst] with NHPP arrival)



(a) Queue time (b) Queue size

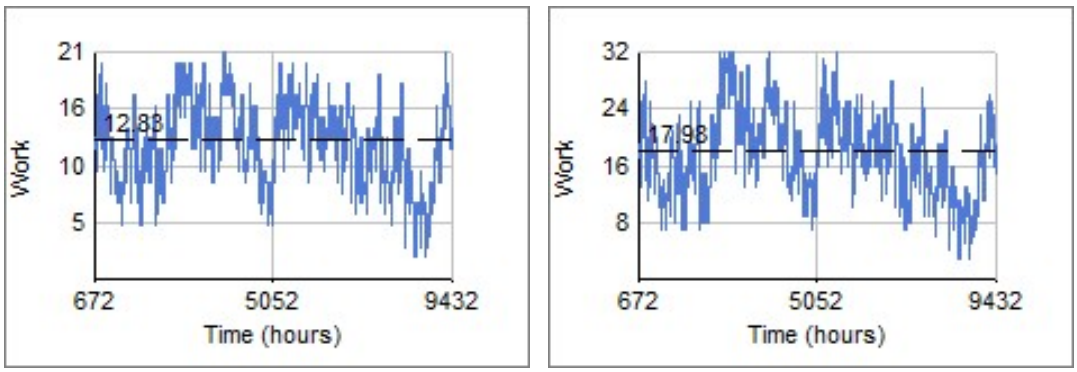
Figure H.13: Queue of pandemic arrivals while changing nurse number (scenario[worst] with NHPP arrival)

H.2 Scenarios 6: influence on the ICU (epidemic-mild)



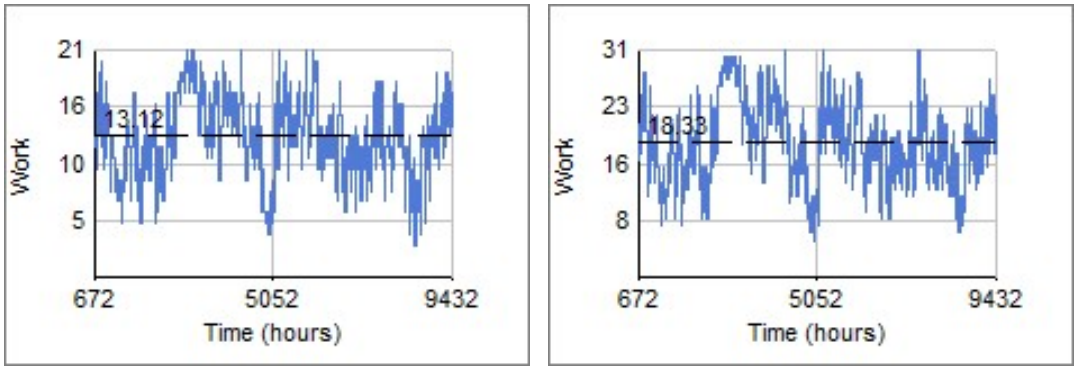
(a) Bed (b) Nurse

Figure H.14: Use of resources (scenario[mild] with PP arrival)



(a) Bed (b) Nurse

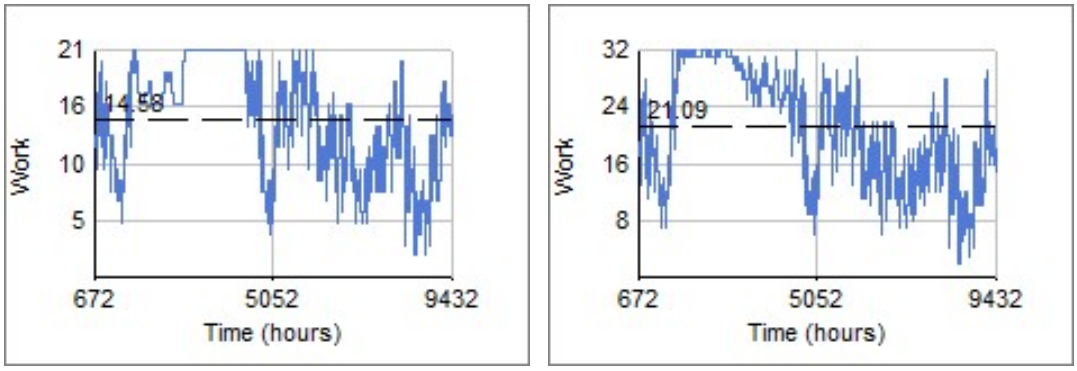
Figure H.15: Use of resources (scenario[mild] with NHPP arrival)



(a) Bed (b) Nurse

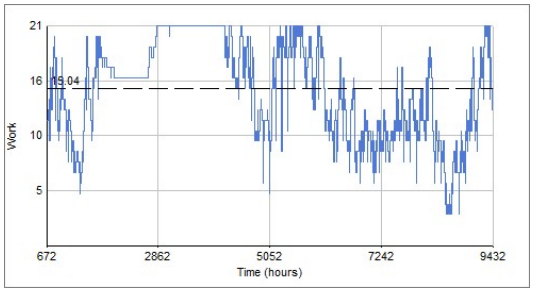
Figure H.16: Use of resources while changing nurse number (scenario[mild] with NHPP arrival)

H.3 Scenarios 6: influence on the ICU (epidemic-likely1)

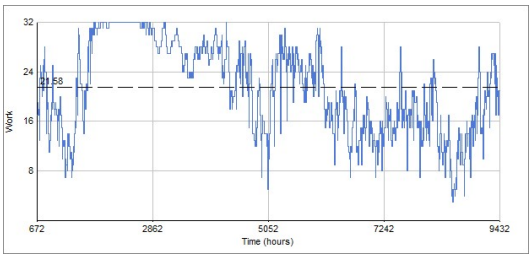


(a) Bed (b) Nurse

Figure H.17: Use of resources (scenario[likely1] with PP arrival)

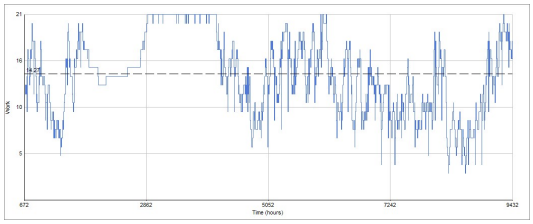


(a) Bed

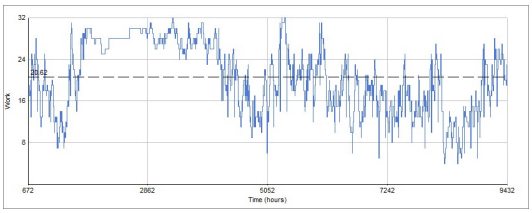


(b) Nurse

Figure H.18: Use of resources (scenario[likely1] with NHPP arrival)



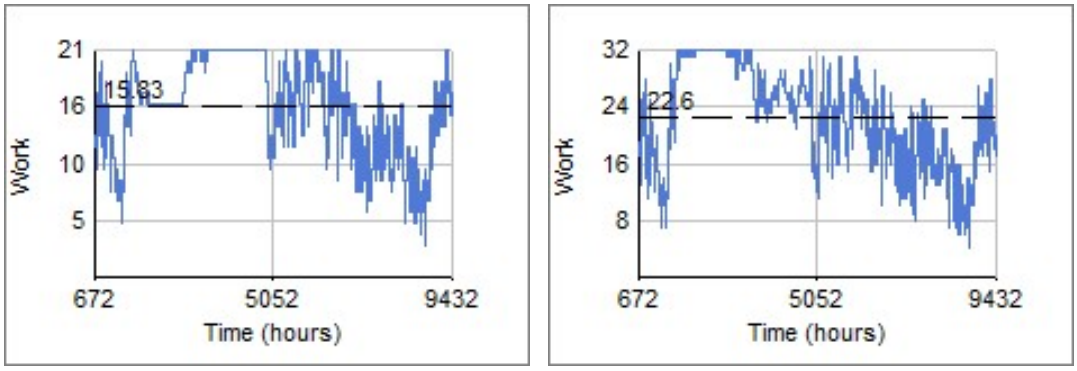
(a) Bed



(b) Nurse

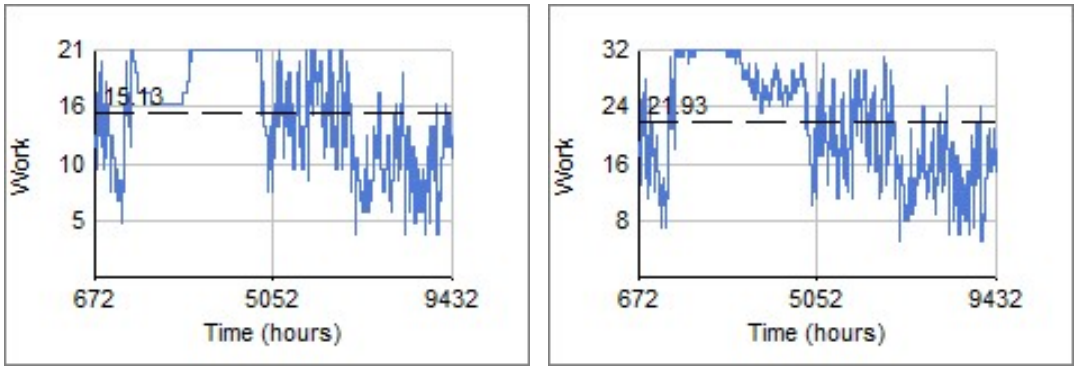
Figure H.19: Use of resources while changing nurse number (scenario[likely1] with NHPP arrival)

H.4 Scenarios 6: influence on the ICU (epidemic-likely2)



(a) Bed (b) Nurse

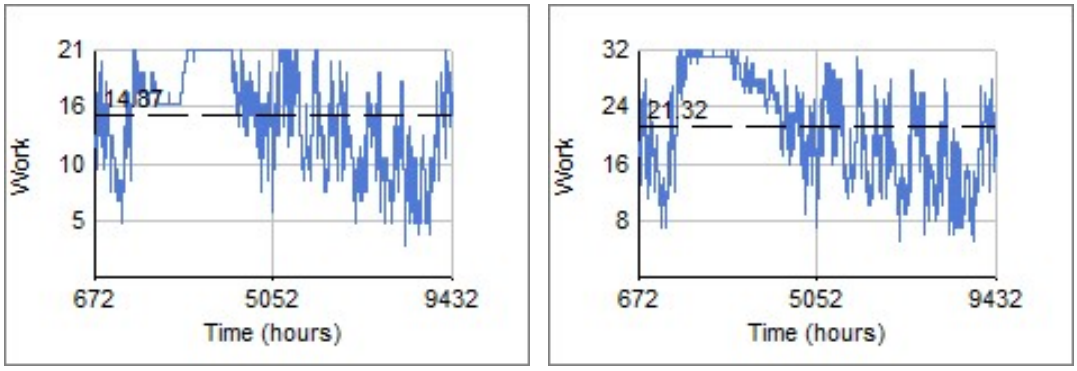
Figure H.20: Use of resources (scenario[likely2] with PP arrival)



(a) Bed (b) Nurse

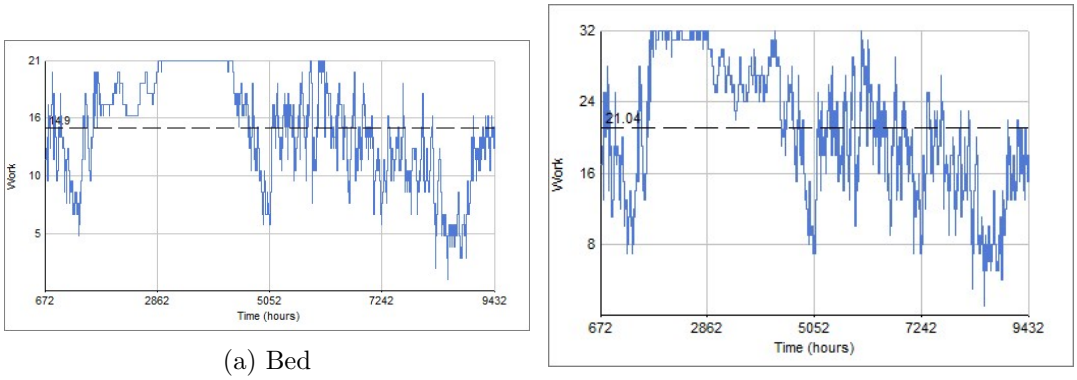
Figure H.21: Use of resources (scenario[likely2] with NHPP arrival)

H.5 Scenarios 6: influence on the ICU (epidemic-likely3)



(a) Bed (b) Nurse

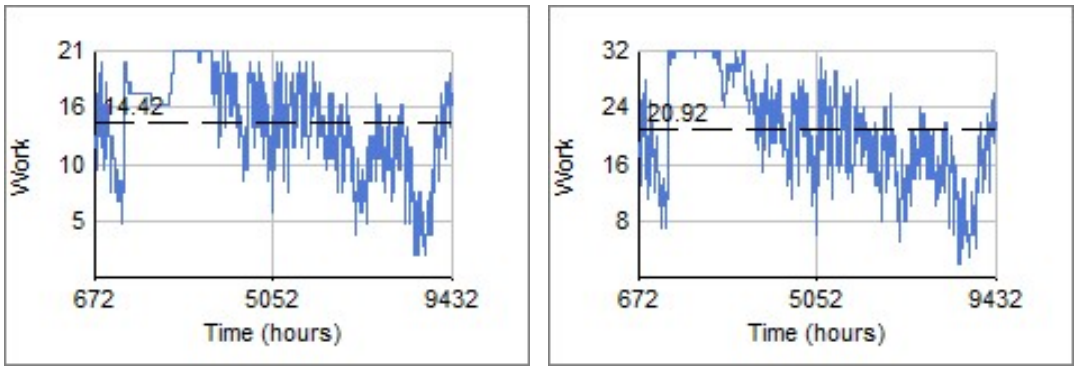
Figure H.22: Use of resources (scenario[likely3] with PP arrival)



(a) Bed (b) Nurse

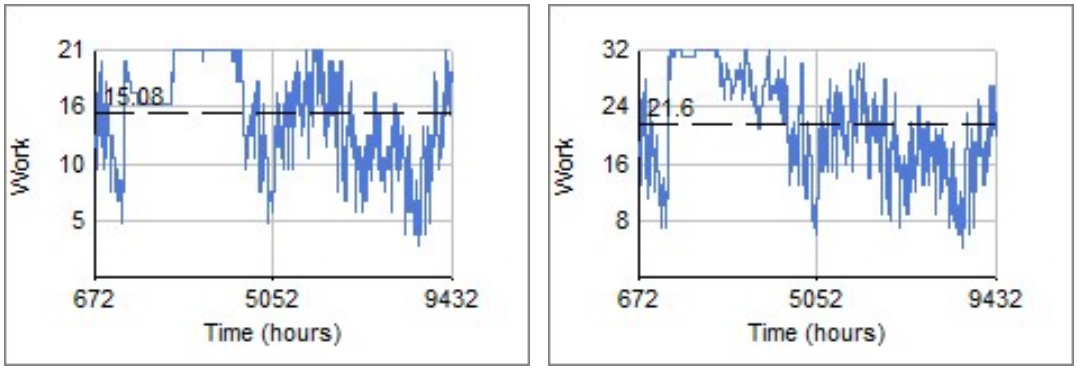
Figure H.23: Use of resources (scenario[likely3] with NHPP arrival)

H.6 Scenarios 6: influence on the ICU (epidemic-worst)



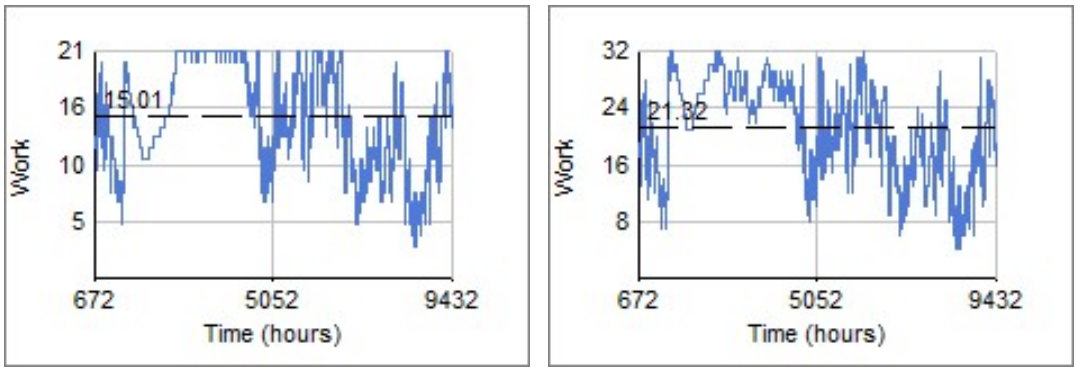
(a) Bed (b) Nurse

Figure H.24: Use of resources (scenario[worst] with PP arrival)



(a) Bed (b) Nurse

Figure H.25: Use of resources (scenario[worst] with NHPP arrival)



(a) Bed (b) Nurse

Figure H.26: Use of resources while changing nurse number (scenario[worst] with NHPP arrival)

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