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Flow resistance network analysis of the back-pressure of automotive mufflers

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Complexity of mufflers generally introduces considerable pressure drop, which affects the engine performance adversely. Not much literature is available for pressure drop across perforates. In this paper, the stagnation pressure drop across perforated muffler elements has been measured experimentally and generalized expressions have been developed for the pressure loss across cross-flow expansion and cross-flow contraction elements. A flow resistance model available in the literature has been made use of to analytically determine the flow distribution and thereby the pressure drop of mufflers. A generalized expression has been derived here for evaluation of the equivalent flow resistance for parallel flow paths. Expressions for flow resistance across perforated elements, derived by means of flow experiments, have been implemented in the flow resistance network. The results have been validated with experimental data. Thus, the newly developed integrated flow resistance networks would enable us to determine the normalized stagnation pressure drop of commercial automotive mufflers, thus enabling an efficient flow-acoustic design of silencing systems.

Keywords: Muffler, Flow resistance network, Back pressure

To improve the performance metrics of mufflers, one makes use of various acoustic elements, namely, sudden expansion/contraction, perforated pipes and baffles, flow reversing end chambers, etc. Though introduction of such elements helps in designing better mufflers in terms of noise attenuation, yet these elements exert substantial back-pressure on the pistons of the reciprocating engines. Back-pressure is defined as the difference between the mean exhaust pressure and the ambient pressure and is due to a drop in stagnation pressure across various perforated elements and the sudden area discontinuities. Increase in back-pressure leads to decrease in thermodynamic efficiency as well as the net power available. This induced back-pressure does not pose a serious problem for luxury vehicles. However, for the economy vehicles, where the specific fuel consumption is a major criterion, the minimization of back-pressure is important. Hence, proper care should be taken while designing the mufflers to obtain adequate transmission loss or insertion loss with minimal back-pressure. Therefore, prediction of back-pressure analytically is as important as acoustical performance.

Though substantial literature is available on acoustic characterization of complex mufflers, sufficient information on the pressure drop characterization of complex mufflers is not available. Pure analytical investigations of such complex configurations are not reported in the literature. Munjal *et al.*[1] did some experimental investigation for particular types of mufflers like plug muffler, end chambers with three interacting perforated ducts, and evolved some empirical expressions for the stagnation pressure drop. Later, Panigrahi and Munjal[2] developed expressions for the cross-flow expansion and contraction using CFD analysis. Elnady *et al.*[3] have developed a network for analyzing the flow distribution across mufflers. The same network has been applied in calculating the pressure drop across mufflers with elements connected in series[4].

The flow resistance network makes use of the flow networks modeling (FNM) methodology presented by Ellison[5,6] for a first-order thermal analysis and expedient thermal design of the cooling system of complex electronic systems[7,8]. A commercial software is now available for analysis and design of liquid cooling systems using FNM methodology[8,9]. However, Elnady *et al.*[3,4] are probably the first researchers to adapt it for prediction of flow distribution and pressure drop across complex automotive mufflers.

Elnady *et al.*[3,4] have defined flow resistance R_f as $\Delta p/Q^2$ in Ref. [3], but $\Delta p/Q$ in Ref. [4]. Thus, R_f is

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constant in Ref. [3], but proportional to the flow discharge rate Q in Ref. [4]. Therefore, the notation of Ref. [3] has been adopted in the present paper. However, R_f being defined as $\Delta p/Q^2$ has interesting implication for the case of parallel flow path (R_1 being parallel to R_2 in the electrical analogous circuit). An expression has been derived in the present paper for evaluation of the equivalent flow resistance for this nonlinear case. Using the resultant expressions for combining flow resistances in parallel as well as in series, the total flow resistance and thence back-pressure have been calculated without having to solve a set of nonlinear equations simultaneously. Thus, the work presented in this paper seeks to extend Elnady *et al.*'s flow network approach[3] to analytically determine the pressure drop across any given arbitrary complex muffler configuration.

A systematic experimental study has been taken up first to determine the back-pressure characteristics of the cross flow perforated muffler elements, and empirical relations have been developed to calculate the stagnation pressure drop across the cross-flow expansion and contraction elements. The expressions so developed have been implemented in the flow resistance network for complex muffler configuration including the one investigated by Elnady *et al*[3]. Using these expressions, pressure drop calculations have been done for plug mufflers and resultant values have been compared with the measured values.

Experimental Setup

In any commercial muffler, one can perform a notional dissection to arrive at a number of basic constituent elements, namely, sudden expansion, sudden contraction, cross-flow perforated tubes, flow reversal elements, etc[10]. Acoustically, these elements behave differently with respect to their pressure drop characteristics. Some of these elements have very serious detrimental effects on the specific fuel consumption of the engine though acoustically they are very effective for use in a muffler. Three such elements, namely, cross-flow expansion, cross-flow contraction and plug mufflers, which are generally used in commercial mufflers, have been investigated here. These are shown in Fig. 1.

The objective of the experiment is to evolve empirical expressions relating the physical parameters, open area ratio (OAR) and porosity (σ) of the muffler, to the total pressure drop across the cross-flow perforated elements of the muffler. OAR is defined as the ratio of the total area of the pores or holes of the perforate to the cross-sectional area of the perforated elements[1]. Porosity is related to the centre-to-centre distance between the holes. OAR and porosity are in turn related to each other as:

$$OAR = \frac{4L_p\sigma}{d} = n_h\left(\frac{d_h}{d}\right)^2 \qquad \ldots (1)$$

where L_p is length of the perforate, σ is porosity of the perforate, d is diameter of the inner pipe (d = 50 mm), d_h is diameter of each hole (d_h = 3 mm in all configurations), and n_h is the total number of the holes in the perforate.

Knowing the OAR, porosity and diameter of the pipe and holes, the other parameters like length of the perforate and number of holes can be calculated from Eq. (1). Incidentally, the empirical expressions developed in Ref. [1] were deficient in as much as the effect of porosity was not considered independently of the open area ratio.

The experimental setup to measure the total or stagnation pressure drop across various configurations

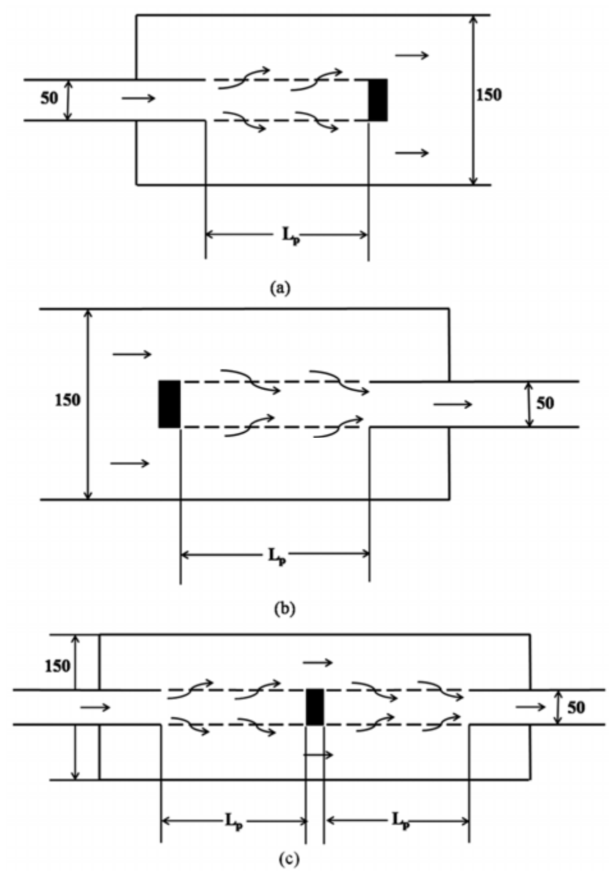

Fig. 1— Dimensional details of perforated elements (all dimensions are in mm) (a) cross-flow expansion, (b) cross-flow contraction and (c) plug muffler

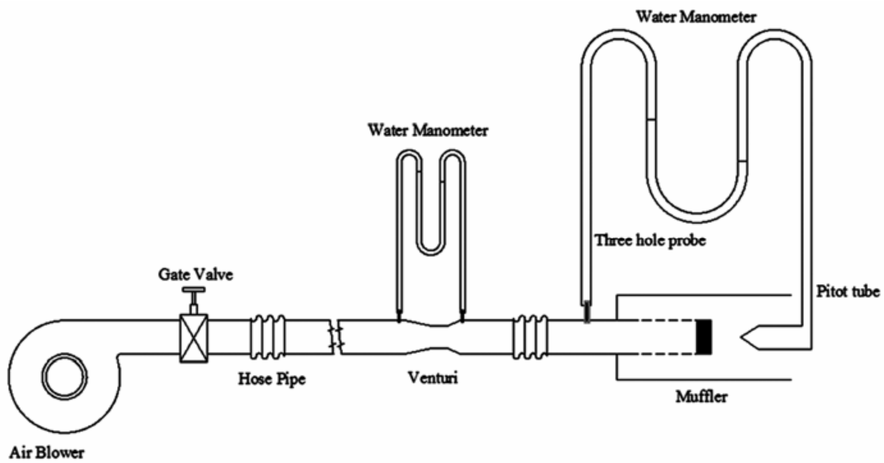

Fig. 2—Experimental setup for measurement of pressure drop across cross-flow elements

Table 1—Normalized total pressure drop across cross-flow expansion with OAR=0.306 and porosity = 7%.

	Flow calculation			Pressure drop calculation				Normalized
Serial No.	Height of water column (cm)	Mach number	Velocity (m/s)	h_1 (cm)	h_2 (cm)	Difference h (cm)	ΔP=ρgh (Pa)	stagnation pressure drop $\Delta P/H$
1	8	0.044	15.26	75.5	99.5	24	2354.4	16.5
2	16	0.063	21.58	65.9	113.6	47.7	4679.37	16.4
3	24	0.077	26.42	50.4	121	70.6	6926.15	16.2
4	32	0.09	30.5	42.2	136.2	94	9240	16.5
								Average $\Delta P/H$ = 16.4

of the perforated mufflers is shown in Fig. 2. Air is supplied by a centrifugal type blower driven by an induction motor. Rate of flow of air supply is controlled by a gate valve. Air flow measurement is done with the help of a venturi meter. Making use of the theory of venture tubes, air flux is computed from the static and total pressure measurements, and is given by[11]:

$$Q = 0.995\left(\frac{\pi d_t^2}{4}\right)\sqrt{\frac{2(\Delta P)}{\rho_0(1-\beta^4)}} \qquad \dots (2)$$

where the ratio of the throat diameter to the pipe diameter,

$$\beta \equiv d_t / d = 0.4$$

and pressure drop,

$$\Delta P = \rho_w g h$$

Here, ρ_0 is density of the air medium, ρ_w is density of water, h is difference in the water columns, d_t is diameter of throat of the venturi, and d is diameter of the inlet venturi pipe. All variables are in SI units.

Difference in water columns, h, was calibrated against the mean flow Mach number, $M = u/c_0$, for ready use during experiments. For a desired Mach number, the control valve was adjusted so as to obtain the corresponding value of h. Total pressure drop across the muffler elements is measured by means of a U-tube water manometer. A three-hole probe is connected in the upstream of the muffler element and a pitot tube is connected to the downstream. These two are connected to the U-tube manometer, the differential head of which is used to evaluate the pressure drop across the muffler element. All experimental readings are tabulated and the measured stagnation pressure drop ΔP is then normalized as:

Normalized stagnation pressure drop

$$\equiv \frac{\Delta P}{H}, \quad H = \frac{1}{2}\rho u^2 \qquad \dots (3)$$

where H is the dynamic head in the inner pipe, with u representing the incoming space-averaged flow velocity.

Cross-Flow Expansion

The measured data for the normalized pressure drop across a cross-flow expansion element have been listed for different values of the open area ratio and porosity in Table 1. The details of other configurations along with the measured normalized pressure drop have been listed in Table 2.

It can be observed that the pressure drop remains constant with the mean flow Mach number. The results of the experimental values have been plotted with respect to the open area ratio. By using the curve fitting techniques[12], the following empirical expression has been derived for the normalized pressure drop as a function of the open area ratio (OAR) and porosity (σ):

$$\Delta P/H = 3.252\ (OAR)^{-1.391}\sigma^{0.018},$$
$$0.3 < OAR < 2.2,\ 0.055 < \sigma < 0.21 \qquad \ldots (4a)$$

Normalized total pressure drop as a function of the open area ratio for cross-flow expansion is shown in Fig. 3. It may be noted from Eq. (4a) that the normalized stagnation pressure drop has a relatively weak dependence on porosity outside OAR which is proportional to porosity. Thus, for practical purposes, Eq. (4a) may be simplified to

$$\Delta P/H = 3.136\ (OAR)^{-1.391},\ 0.3 < OAR < 2.2 \qquad \ldots (4b)$$

Cross-Flow Contraction

Results of measurement of the normalized stagnation pressure drop across a cross-flow contraction element for different open-area ratios and porosities have been given in Table 3. Details of other configurations along with the measured normalized pressure drop are given in Table 4. It can be observed that cross-flow contraction elements offer relatively less pressure drop as compared to the cross-flow expansion element.

Making use of the experimental data and the standard curve fitting techniques[12], the following empirical expression has been derived for the normalized stagnation pressure drop as a function of the open area ratio and porosity:

$$\Delta P/H = 2.31\ (OAR)^{-1.5818}\sigma^{0.019},$$
$$0.31 < OAR < 1.66,\ 0.055 < \sigma < 0.13 \qquad \ldots (5a)$$

The normalized total pressure drop as a function of open area ratio for cross-flow contraction is shown in Fig. 4. It may be observed from Eq. (5a) that the normalized pressure drop across a cross-flow contraction element has a weak dependence on porosity outside OAR which is proportional to

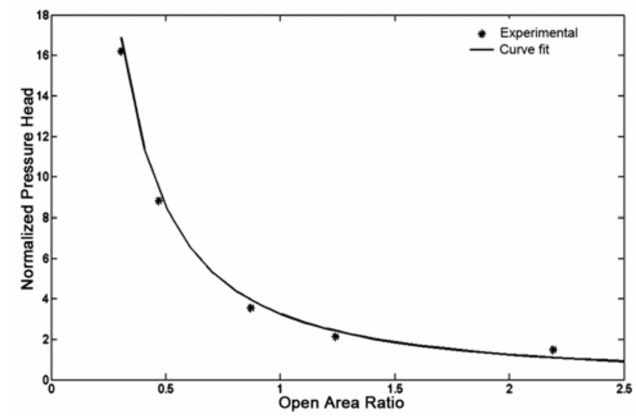

Fig. 3—Normalized total pressure drop versus open area ratio for cross-flow expansion

Table 2—Normalized stagnation pressure drop across cross-flow expansion for different values of OAR and porosity.

Serial No.	Open area ratio (OAR)	Porosity (σ)	Average normalized stagnation pressure drop $\Delta P/H$
1	0.306	7 %	16.4
2	0.470	12 %	8.83
3	0.871	12 %	3.55
4	1.242	5 %	2.19
5	2.192	20.6%	1.2

Table 4—Normalized total pressure drop across cross-flow contraction for different values of OAR and porosity

Serial No.	Open Area Ratio (OAR)	Porosity (σ)	Average normalized stagnation pressure drop $\Delta P/H$
1	0.306	7 %	14.43
2	0.396	12 %	9.7
3	0.871	12 %	2.7
4	1.2	5 %	1.77
5	1.656	13.6 %	1.364

Table 3—Normalized total pressure drop across cross-flow contraction with OAR = 0.306 and porosity = 7%.

Serial No.	Flow calculation			Pressure drop calculation				Normalized stagnation pressure drop
	Height of water column (cm)	Mach number	Velocity (m/s)	h_1 (cm)	h_2 (cm)	Difference (cm)	$\Delta P = \rho g h$ (Pa)	$\Delta P/H$
1	8	0.044	15.26	79.4	100.2	20.8	2039.18	14.3
2	16	0.063	21.58	69.8	111.7	41.9	4107.46	14.4
3	24	0.077	26.42	59.4	122.6	63.2	6199.33	14.5
4	32	0.09	30.5	46.1	130.3	84.2	8265	14.5

Average $\Delta P/H$ = 14.425

velocity. Thus, for practical purposes, Eq. (5a) may be simplified to

$$\Delta P/H = 2.208\ (OAR)^{-1.5818},\ 0.31 < OAR < 1.66 \qquad \dots (5b)$$

Plug Muffler

The pressure drop across a plug muffler has been found out through experiments. Details of the configurations along with the measured normalized pressure drop are given in Table 5. The observed values have been compared with the values computed by implementing the expressions (4) and (5) in the flow resistance network for the constituent cross-flow elements.

Flow Resistance Networks

In a flow resistance network, the muffler is divided into a number of elements, each of which is represented as a lumped resistance in the network. Analogous to voltage (or electromotive force) and current, stagnation pressure and volume velocity have been used here as the state variables[3-9]. Flow is assumed to be incompressible. The frictional losses have been neglected and the volume flux continuity holds good in the network. The resistance block chosen may be a straight pipe, area discontinuity or a perforate, as applicable. Assuming incompressible flow, Kirchhoff's laws can be applied. As per Kirchhoff's first law, the volume flow into a junction must equal the volume flow out of the junction, and following the second law, for any closed loop path around a circuit, the sum of the voltage (or stagnation pressure) gains and voltage (or stagnation pressure) drops equals zero.

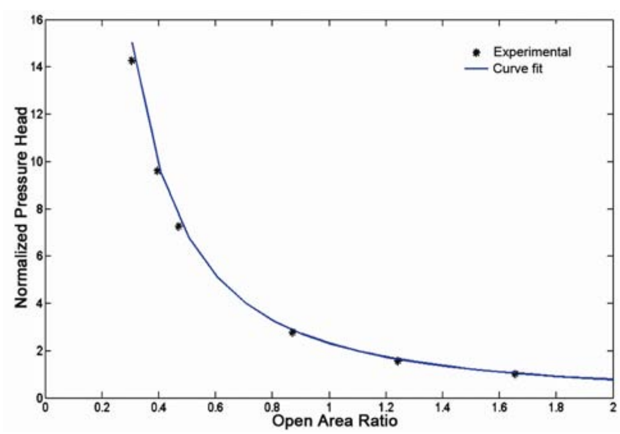

Fig. 4—Normalized stagnation pressure drop versus open area ratio for cross-flow contraction

To implement this network, Bernoulli's equation and energy equation are used. The energy equation is very similar to Bernoulli's equation. The difference is the presence of the head loss term. The energy equation can be expressed as[4]:

$$P_{s1} + \frac{1}{2}\rho u_1^2 + \rho g z_1 = P_{s2} + \frac{1}{2}\rho v_2^2 + \rho g z_2 \qquad \dots (6)$$

where P_s is the fluid static pressure, ρ is the fluid density, z is the height above datum, g is the gravitational acceleration, and h_e is the head loss. Subscript (1) denotes the inlet side and (2) denotes the outlet side. For gaseous flows, $\rho g(z_1 - z_2)$ is negligible. The stagnation pressure is the sum of the static pressure head and velocity head,

$$P = P_s + \frac{1}{2}\rho u^2 \qquad \dots (7)$$

Eq. (6) then reduces to

$$P_1 = P_2 + \rho g h_e \qquad \dots (8)$$

Eq. (6) can also be written as a function of flow resistance term (R) and volume velocity (Q) of the flow[3,5-7]:

$$P_1 = P_2 + RQ|Q|,\quad Q = Au \qquad \dots (9a)$$

Thus,

$$\Delta P \equiv P_1 - P_2 = RQ|Q| = \rho g h_e \qquad \dots (9b)$$

Table 5—Measured and predicted values of the plug muffler configurations.

Serial No.	Cross-flow expansion			Cross-flow contraction			$(\Delta P/H)_{plug}$ (calculated)	$(\Delta P/H)_{plug}$ (experiment)
	OAR	Porosity	$(\Delta P/H)_{CFE}$	OAR	Porosity	$(\Delta P/H)_{CFC}$		
1	0.308	7%	15.95	0.308	7%	14.15	30.1	27.9
2	0.396	12%	11.6	0.396	12%	10.5	22.6	20.6
3	0.871	12%	3.8	0.871	12%	2.7	6.5	4.1
4	0.308	7%	15.95	0.871	12%	2.7	18.65	18.28
5	0.871	12%	3.8	0.308	7%	14.15	17.95	14.6
6	0.871	12%	3.8	1.242	5.5%	1.55	5.35	4.6
7	1.242	5.5%	2.28	0.871	12%	2.7	4.96	4.43

Eqs (9a) and (9b) imply the following definition of flow resistance, ***R***:

$$R = \frac{\Delta P}{Q|Q|} \qquad \ldots (10)$$

Use of $Q|\ Q|$ instead of Q^2 in the denominator of Eq. (10) is to keep track of the direction of flow in the flow networks.

The flow resistance expressions for area expansion, contraction and hard pipe are evaluated from the corresponding expressions for the loss coefficients[11]. The expressions for perforated elements have been evaluated from the empirical pressure loss expressions (4) and (5). The various resistance blocks (elements) used in the network and their resistance values are given in Table 6.

Calculation of Pressure Drop Using Flow Resistance Networks

To develop a methodology for constructing the network for a given muffler configuration and analyzing the network to determine the overall pressure drop across a complex muffler, let us first consider a simple expansion chamber muffler as shown in Fig. 5.

The equivalent flow resistance circuit for the muffler of Fig. 5 is shown in Fig. 6. Here, R_{exp} and R_{con} represent the lumped resistances due to sudden expansion and sudden contraction, respectively. Conservation of mass is valid in the approach and frictional pressure drop across the chamber wall is neglected. For incompressible flow, the outgoing flow rate would be equal to the incoming flow rate.

Table 6—Flow resistances for a number of elements.

Element	Resistance
Open pipe[8]	$R_{open} = \begin{cases} \dfrac{\rho}{2A^2} & \text{for outflow} \\ \dfrac{\rho}{4A^2} & \text{for inflow} \end{cases}$
Area expansion[3]	$R_{\exp} = \frac{1}{2}[1-n]^2 \frac{\rho}{A^2}$
Area contraction[3]	$R_{con} = \frac{1}{2}\left[1-\frac{1}{n}\right]\frac{\rho}{A^2}$
Cross-flow expansion (Eq. 4b)	$R_{c\_e} = \frac{1}{2}\left(3.136(OAR)^{-1.391}\right)\frac{\rho}{A^2}$
Cross-flow contraction (Eq. 5b)	$R_{c\_c} = \frac{1}{2}\left(2.208(OAR)^{-1.5818}\right)\frac{\rho}{A^2}$
Hard pipe[3, 8]	$R_{pipe} = \frac{1}{2}\frac{\rho}{A^2}\frac{4fL}{D}$
Baffles[8]	$R_{baf} = \frac{1}{2}\frac{1}{(OAR)^2 C_D^2}\frac{\rho}{A^2}$

The resistances due to sudden expansion and contraction are in series, as seen from the network in Fig. 6. The total resistance or the equivalent resistance of the circuit is the sum of the two resistances and hence the total pressure drop can be calculated from the equivalent resistance thus obtained as shown below:

$$R_{eq} = R_{\exp} + R_{con} \qquad \ldots (11)$$

and the stagnation pressure drop across the chamber of Fig. 5 is given by

$$\Delta P = R_{eq} Q_0^2 = (R_{\exp} + R_{con}) Q_0^2 \qquad \ldots (12)$$

Thus, the resistances in series can simply be added as in an electrically equivalent circuit. When it comes to resistances in parallel, however, there is a complication due to the nonlinearity in Eq. (10). Let us consider a general circuit with resistances in parallel as shown in Fig. 7, and derive the relation for the equivalent resistance.

By Kirchhoff's first law, the volume velocity entering a node should be equal to that leaving a node. Hence,

$$Q_0 = Q_1 + Q_2 \qquad \ldots (13)$$

From Kirchhoff's second law, for the closed loop,

$$Q_1|Q_1|R_1 - Q_2|Q_2|R_2 = 0 \qquad \ldots (14)$$

Assuming the flow to be positive, we get

$$Q_1^2 R_1 = Q_2^2 R_2 \qquad \ldots (15)$$

The total pressure drop across the muffler can be defined in terms of the equivalent resistance

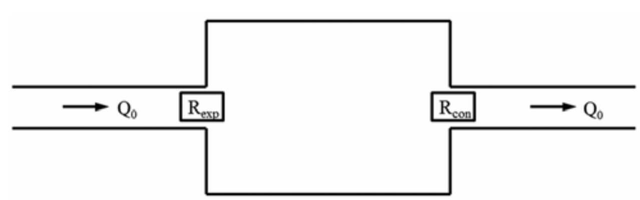

Fig. 5—A simple expansion chamber muffler

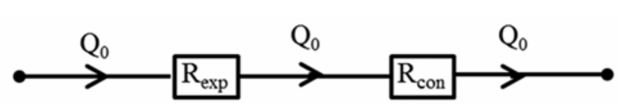

Fig. 6—Equivalent flow resistance network for simple expansion chamber

(see Fig. 7). But this should be equal to the pressure drop across the inner loop and also the outer loop. Hence, we get

$$\Delta P = \Delta P_{inner\ loop} = \Delta P_{outer\ loop} \qquad \dots (16)$$

or

$$Q_0^2 R_{eq} = Q_1^2 R_1 = Q_2^2 R_2 \qquad \dots (17)$$

From Eq. (17), we get

$$Q_1 = Q_0 \sqrt{\frac{R_{eq}}{R_1}} \qquad \dots (18)$$

and

$$Q_2 = Q_0 \sqrt{\frac{R_{eq}}{R_2}} \qquad \dots (19)$$

Substituting Eqs. (18) and (19) in Eq. (13) we get

$$Q_0 = Q_0 \sqrt{\frac{R_{eq}}{R_1}} + Q_0 \sqrt{\frac{R_{eq}}{R_2}} \qquad \dots (20)$$

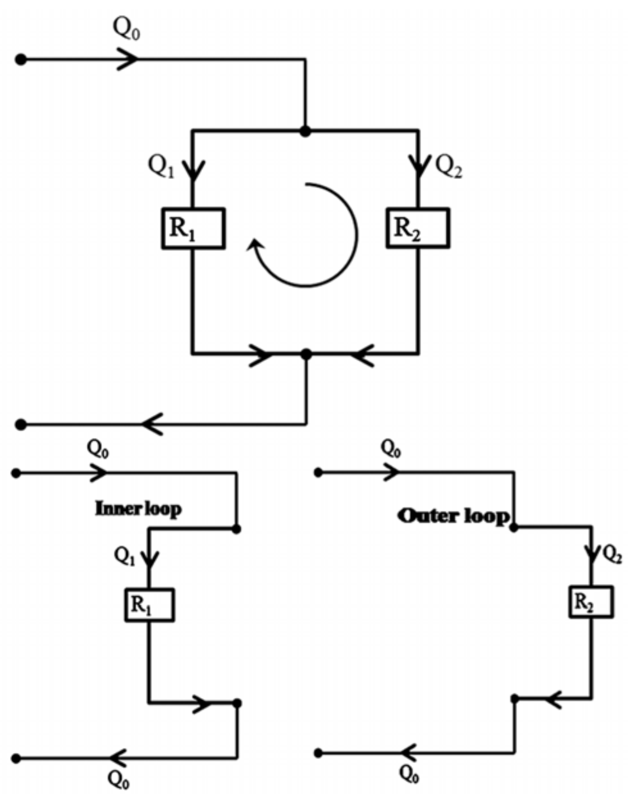

Fig. 7—A network with resistances in parallel

Therefore,

$$\sqrt{\frac{1}{R_{eq}}} = \sqrt{\frac{1}{R_1}} + \sqrt{\frac{1}{R_2}} \qquad \dots (21a)$$

or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2 + 2\sqrt{R_1 R_2}} \qquad \dots (21b)$$

Incidentally, Eq. (21b) indicates that if $R_1 = R_2 = R_0$ (say), then $R_{eq} = R_0/4$, not $R_0/2$, which would be the value for linear element circuits. However, as $Q_1 = Q_2 = Q_0/2$, $R_1 = R_2$ = four times that with respect to Q_0. Consequently, back-pressure would remain unchanged. In particular, for ***n*** parallel paths of total area of cross-section equal to that of the incoming or outgoing pipe, the stagnation pressure drop due to wall friction will be equal to that of the single pipe of the same length and equivalent area of cross-section.

Now, the resistances of any complex muffler can be converted into one single equivalent resistance using formulae (11) and (21) for resistances in series and parallel, respectively. Once the equivalent resistance is obtained, the total pressure drop can be easily calculated.

Application of Flow Resistance Network to Mufflers

A chamber with three interacting ducts

An expansion chamber with three interacting ducts, shown in Fig. 8, can be represented in an equivalent resistance network as shown in Fig. 9. The network consists of resistances due to sudden area changes (expansion and contraction) and resistances due to the flow through perforates.

Q_0 is the input volume flow which is usually measured upstream of the muffler. It is related to the flow velocity ***u*** and Mach number ***M*** by

$$Q_0 = uA_0 = McA_0 \qquad \dots (22)$$

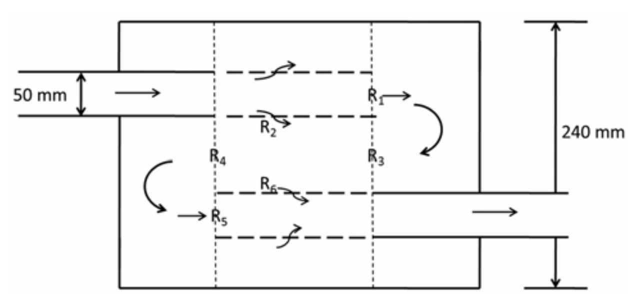

Fig. 8—Schematic of a three duct interacting element

where c is the speed of sound and A_0 is the inlet duct cross-sectional area. Assuming incompressible flow, Kirchhoff's first law can be applied. For the analogous circuit shown in Fig. 9, this gives

$$Q_0 = Q_1 + Q_2 \quad \ldots (23)$$

$$Q_0 = Q_3 + Q_4 \quad \ldots (24)$$

Application of the Kirchhoff's second law to the two closed loops in Fig. 9 yields

$$R_1 Q_1 |Q_1| + R_3 Q_1 |Q_1| - R_2 Q_2 |Q_2| = 0 \quad \ldots (25)$$

$$R_4 Q_4 |Q_4| + R_5 Q_4 |Q_4| - R_6 Q_3 |Q_3| = 0 \quad \ldots (26)$$

Equations (23)-(26) represent a set of four non-linear equations. This system has four equations and four unknowns (Q_1, Q_2, Q_3 and Q_4). Following Elnady *et al*.[3] the equations would have to be solved simultaneously to get the flow distribution.

The network has six resistances. The resistances R_1 and R_4 are due to sudden area expansion, R_3 and R_5 due to sudden contractions, and R_2 and R_6 due to resistance across perforates. The resistance due to wall frictional losses has been neglected here. The resistances can be calculated using the formulae given in Table 6.

Once the resistances are known, the total pressure drop can be calculated by determining the equivalent resistance of the circuit. The given circuit can be brought down to a circuit with equivalent resistance (Fig. 10).

Now, use of Fig. 9 and Eqs (11) and (21b) yields

$$R_{eq} = R_{eq1} + R_{eq2} \quad \ldots (27)$$

where

$$R_{eq1} = \frac{(R_1 + R_3) R_2}{(R_1 + R_3) + R_2 + 2\sqrt{(R_1 + R_3) R_2}} \quad \ldots (28)$$

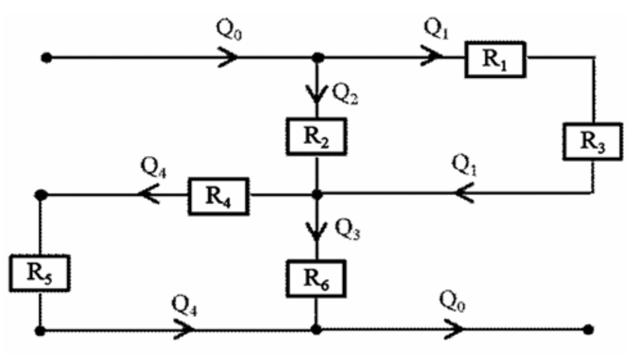

Fig. 9—Flow resistance network for three duct interacting element of Fig. 8

and

$$R_{eq2} = \frac{(R_4 + R_5) R_6}{(R_4 + R_5) + R_6 + 2\sqrt{(R_4 + R_5) R_6}} \quad \ldots (29)$$

The total pressure drop can now be calculated as

$$\Delta P = R_{eq} Q_0^2 \quad \ldots (30)$$

The pressure drop thus calculated has been compared with the measured values. The given configuration has been validated for three open area ratios: 0.306, 0.397, 0.871 in Figs 11, 12 and 13,

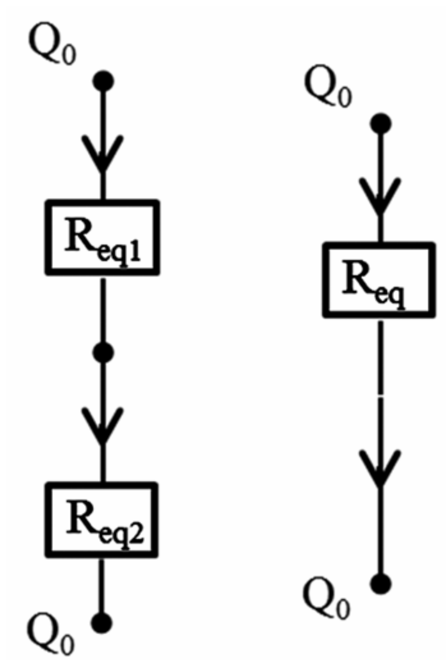

Fig. 10—Reduction of the electrical analogous circuit of Fig. 9 into an equivalent flow resistance network

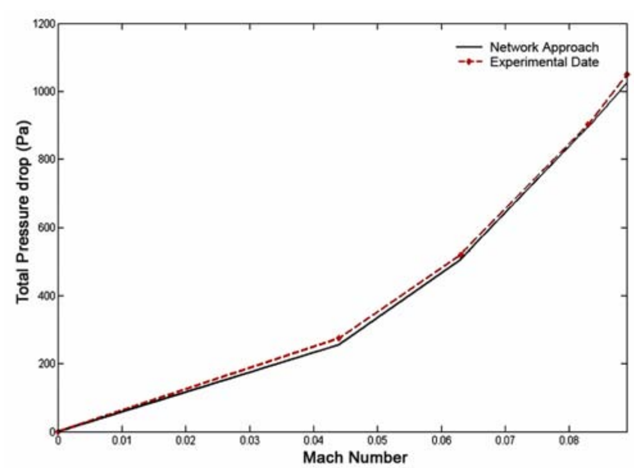

Fig 11—Pressure flow curve of a three duct muffler with OAR = 0.306

respectively. It is clear that predictions of the network approach presented here compare well with those observed experimentally.

Application to a complex muffler configuration

The flow resistance network is now applied to a complex muffler configuration shown in Fig. 14, adopted from Elnady *el al.*[3]. It can be represented in an equivalent flow resistance network[3] as shown in Fig. 15. The network consists of lumped flow resistances due to sudden area changes (expansion R_1 and contraction R_4) and resistance due to flow through perforates (R_2, R_8 and R_9) and baffles (R_3, R_5 and R_6). The resistance due to wall frictional losses has been neglected here.

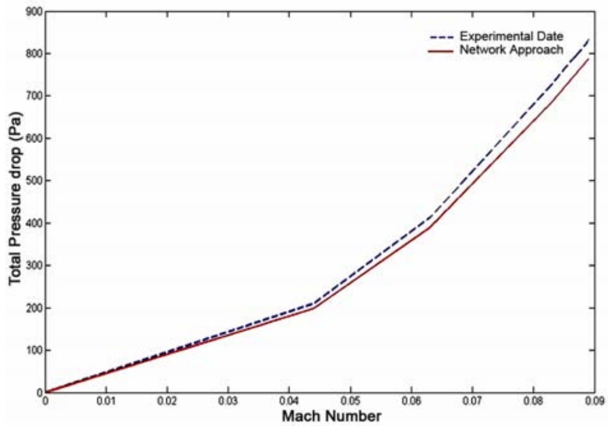

Fig. 12—Pressure flow curve of a three duct muffler with OAR = 0.397

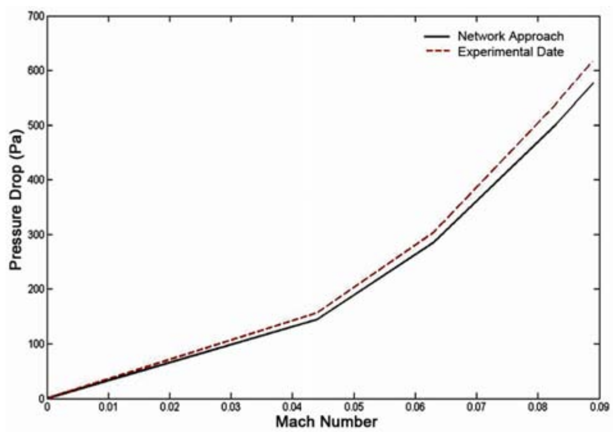

Fig. 13—Pressure flow curve of a three duct muffler with OAR = 0.871

For the circuit given above, applying Kirchhoff's first and second law, equations at the junctions can be written as[3],

$$Q_0 = Q_1 + Q_2 \qquad \dots (31)$$

$$Q_1 = Q_3 + Q_4 \qquad \dots (32)$$

$$Q_3 = Q_5 + Q_6 \qquad \dots (33)$$

$$Q_7 = Q_2 + Q_5 \qquad \dots (34)$$

$$R_1Q_1|Q_1| + R_3Q_1|Q_1| + R_4Q_3|Q_3| + R_5Q_5|Q_5| - R_2Q_2|Q_2| = 0 \qquad \dots (35)$$

$$R_8Q_6|Q_6| - R_5Q_5|Q_5| - R_6Q_7|Q_7| - R_9Q_7|Q_7| = 0 \qquad \dots (36)$$

$$R_7Q_4|Q_4| - R_8Q_6|Q_6| - R_4Q_3|Q_3| = 0 \qquad \dots (37)$$

The '*fsolve*' built-in function in Matlab can be used to solve this system in the form of $f(x) = 0$, where

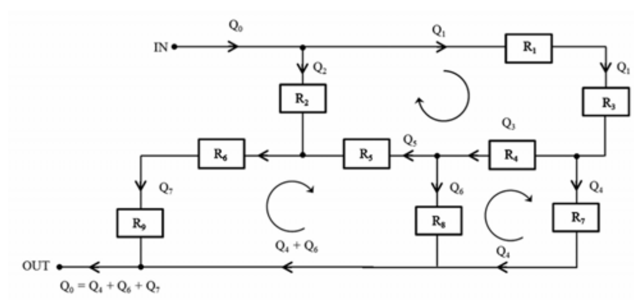

Fig.15—Flow resistance network for the muffler of Fig. 14

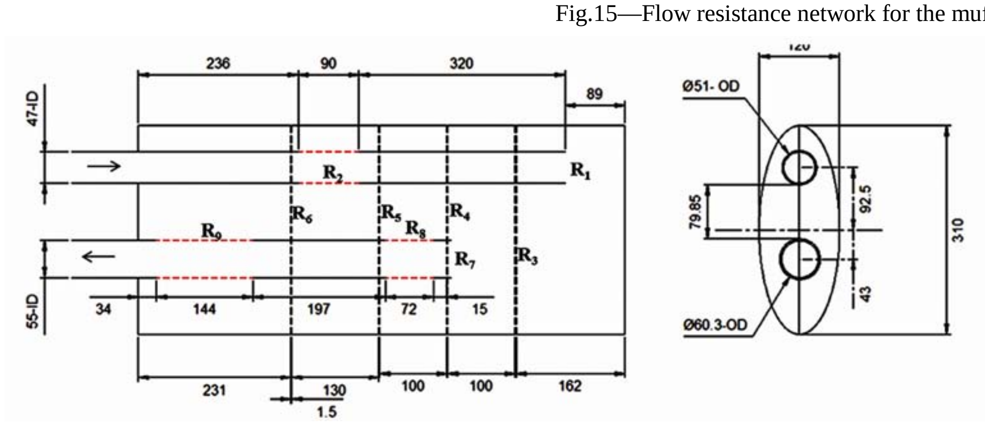

Fig. 14—A complex muffler configuration

$$f\left(\bar{Q}\right)=\begin{bmatrix} Q_0-Q_1-Q_2 \\ Q_1-Q_3-Q_4 \\ Q_3-Q_5-Q_6 \\ Q_7-Q_2-Q_5 \\ R_1Q_1\left|Q_1\right|+R_3Q_1\left|Q_1\right|+R_4Q_3\left|Q_3\right|+R_5Q_5\left|Q_5\right|-R_2Q_2\left|Q_2\right| \\ R_8Q_6\left|Q_6\right|-R_5Q_5\left|Q_5\right|-R_6Q_7\left|Q_7\right|-R_9Q_7\left|Q_7\right| \\ R_7Q_4\left|Q_4\right|-R_8Q_6\left|Q_6\right|-R_4Q_3\left|Q_3\right| \end{bmatrix} \quad \ldots (38)$$

and

$$\bar{Q}=\left[Q_1,Q_2,Q_3,Q_4,Q_5,Q_6,Q_7\right]^T \quad \ldots (39)$$

Equations (31) to (37) represent a set of seven non-linear equations, which can be solved in Matlab for volume velocities (Q_1 to Q_7) in terms of Q_0. This gives the volume velocities (flow distribution) at each element. Once the volume velocities are known, the back-pressure can be calculated readily by considering any loop of the circuit and calculating the pressure drop across that inner loop. Considering the loop with resistances R_2, R_6 and R_9 of Fig. 15 yields the following expression for back-pressure Δp:

$$\Delta P=R_2Q_2\left|Q_2\right|+R_6Q_7\left|Q_7\right|+R_9Q_7\left|Q_7\right| \quad \ldots (40)$$

The muffler configuration of Fig. 14 (adopted from Ref. 3) was fabricated and tested for stagnation pressure drop in the laboratory. The calculated pressure drop has been compared with the measurements. There is a good agreement between the two, as can be observed from Fig. 16. The baffle's resistance depends on the coefficient of discharge (C_D), which in turn depends on Mach number. The C_D value decreases with the increasing velocity. But this has been taken as constant in the analysis, possibly leading to a discrepancy at higher Mach flows.

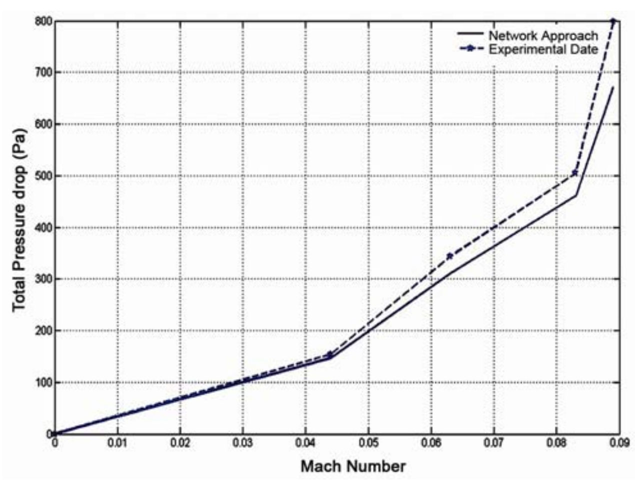

Fig. 16—Pressure flow curve for the muffler of Fig. 14

Conclusions

The stagnation pressure drop has been measured across various configurations of mufflers, namely, cross-flow expansion element, cross-flow contraction element and plug muffler. It is observed that the normalized stagnation pressure drop is almost constant for different Mach numbers. Hence, we can take mean of the normalized stagnation pressure drop for the range of Mach numbers for better results.

The normalized total pressure drop versus open area ratio (OAR) is plotted for cross-flow expansion and cross-flow contraction, respectively. To derive an empirical relation between non-dimensional pressure head, open area ratio and porosity, a least squares fit has been employed, and appropriate empirical relations have been derived for both the cross-flow elements.

It has been observed that for typical mean flow Mach number $M \lesssim 0.2$ the stagnation pressure drop normalized with respect to dynamic head remains constant with Mach number, but varies with open area ratio and porosity. Also, cross-flow expansion elements offer relatively more pressure drop as compared to the cross-flow contraction element. This phenomenon can also be observed in the plug muffler configurations with different open area ratios. Interestingly, the plug muffler offer higher pressure drop when the lower open area ratio elements are placed at the expansion end.

The electrical network technique based on flow resistances[3] for calculation of flow distribution has been generalized to calculate the total stagnation pressure drop in muffler systems. The network has been validated with experimental values of two complex muffler configurations. The results show reasonably good agreement with experiments, thereby validating the flow network approach.

The present work, thus, enables us to predict the total back-pressure of the complex multiply-connected muffler configurations, without having to solve a set of nonlinear equations simultaneously for most automotive muffler configurations.

Acknowledgements

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