

# The seismology of Love: An effective model for the neutron star tidal deformability

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## Abstract

We develop a phenomenological, physically motivated, model for the effective tidal deformability of a neutron star binary, adding the frequency dependence (associated with the star's fundamental mode of oscillation) that comes into play during the late stages of an inspiral. Testing the model against alternative descriptions, we find that it provides an accurate representation of the tidal deformability. The simplicity of the model makes it an attractive alternative for a gravitational-wave data analysis implementation as it facilitates an inexpensive construction of a large number of templates covering the relevant parameter space.

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## I. INTRODUCTION

The inspiral and merger of binary neutron stars has long been considered a bread-and-butter source for advanced gravitational-wave detectors. Hence, the release of pent-up excitement following the spectacular GW170817 event [1, 2] came as no surprise. After all, these events have the potential to unlock scientific mysteries from astrophysics (confirming binary mergers as the origin of short gamma-ray bursts and explaining the cosmic generation of heavy materials) and cosmology (through inferred values of the Hubble constant) through to nuclear physics (as the imprint of matter on the gravitational-wave signal helps constrain the equation of state relevant to the extreme conditions represented by neutron stars). Remarkably, the GW170817 event led to progress in all these directions [3–7] and the number of relevant analyses and discussions is already overwhelming (and hence impossible to do justice in a limited space).

The analysis of the GW170817 data has led to a surprisingly tight constraint on the neutron star tidal deformability, commonly expressed in terms of the dimensionless parameter [8, 9]

$$\Lambda_l = \frac{2}{(2l-1)!!} \frac{k_l}{\mathcal{C}^{2l+1}} \quad (1)$$

where  $l$  is the relevant multipole and  $\mathcal{C} = GM_\star/Rc^2$  is the (dimensionless) compactness of the star ( $M_\star$  is the mass and  $R$  the radius and in the following we use geometric units where  $c = G = 1$ ). Given that the tidal imprint enters, formally, at the 5th post-Newtonian order [8], the mass of the two binary partners can be inferred from lower order post-Newtonian terms in the signal. A constraint on  $\Lambda_l$  can then be turned into a constraint on the neutron star radius [7]. The recent analysis of [10] suggests  $400 \lesssim \tilde{\Lambda}_2 \lesssim 800$  (for a suitably averaged quadrupole tidal deformability  $\tilde{\Lambda}_2$ , depending on the mass ratio, and with the lower limit somewhat model dependent). These results provide the context for this brief paper. We outline a phenomenological (yet, physically motivated) model for the effective tidal deformability, adding frequency dependence that comes into play during the late stages of an inspiral. The simplicity of the model makes it an attractive alternative for a data analysis implementation, which requires inexpensive construction of a large number of templates covering the relevant parameter space.

## II. A SIMPLE PHENOMENOLOGICAL MODEL

We take as our starting point the discussion in [11], where the tidal response of a star is expressed in terms of the star’s normal modes of oscillation. The original analysis aimed to provide an idea of the systematic “error” associated with the assumption that a deformed neutron star is described by a barotropic (essentially chemical equilibrium) matter model rather than a model in which the matter composition is frozen as the system spirals through the sensitivity band of a gravitational-wave detector (as the timescale associated with nuclear reactions is much longer than that of the inspiral). The results demonstrate that, for a simple polytropic model in Newtonian gravity, the dynamical contribution to the tide is dominated by the excitation of the fundamental mode (the f-mode) of the star. This result was established a long time ago [12, 13] in work aimed at quantifying the role of mode resonances on the gravitational-wave signal, but the discussion in [11] adds a twist to the story. The results demonstrate that the sum over modes converges to the usual Love number in the static limit. Again, this result could have been anticipated. As long as the modes form a complete set, they can be used as a basis to describe any dynamical response of the star.

Let us now make pragmatic use of the results from [11] and build a simple model for the effective tidal deformability. The basic idea is to include only the f-mode contribution to the mode sum and accept the contribution from other modes as a systematic error. Based on the stratified Newtonian models considered in [11] we expect this systematic error to be below the 5% level. This level of uncertainty is much smaller than our ignorance of (say) the neutron star equation of state, so the relation we write down should be precise enough for a “practical” construction of gravitational-wave templates.

In essence, we start from a parameterised version of the Newtonian result [11]

$$k_l^{\text{eff}} = -\frac{1}{2} + \frac{A_f}{\tilde{\omega}_f^2 - \tilde{\omega}^2} [1 - \tilde{\omega}^2 B_f] [1 - \tilde{\omega}_f^2 B_f]^{-1} \quad (2)$$

where  $A_f$  depends on the overlap integral between the f-mode and the tidal driving, while  $B_f$  involves the ratio of the horizontal and radial mode eigenfunctions at the star’s surface. The scaled (see below) f-mode frequency is  $\tilde{\omega}_f$  while  $\tilde{\omega}$  is the similarly scaled frequency associated with the Fourier transform (see [11] for discussion) . In order for the relation (2)

to return the usual Love number,  $k_l$ , in the static,  $\tilde{\omega} \rightarrow 0$ , limit we must have

$$A_f [1 - \tilde{\omega}_f^2 B_f]^{-1} = \tilde{\omega}_f^2 \left( k_l + \frac{1}{2} \right) \quad (3)$$

This may not seem very helpful, but, in fact, it is. We know from the work in [14] (see also [15, 17]) that there exists a robust universal relation between the f-mode frequency and the tidal deformability<sup>1</sup>

$$\bar{\omega}_f = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 \quad (4)$$

where  $\bar{\omega}_f \equiv M_\star \omega_f$  and  $y = \ln \Lambda$  (and the  $a_i$  parameters are listed in table I). The scaling with the mass is different from that in [11] but this is easily addressed, since

$$\tilde{\omega}_f^2 = \frac{R^3}{GM_\star} \omega_f^2 = \frac{R^3}{GM_\star} \left( \frac{c^3}{GM_\star} \right)^2 \bar{\omega}_f^2 = \left( \frac{Rc^2}{GM_\star} \right)^3 \bar{\omega}_f^2 = \frac{\bar{\omega}_f^2}{\mathcal{C}^3} \quad (5)$$

(and similar for  $\tilde{\omega}$ ). Given (1) and (4) we can express the right-hand side of (3) in terms of  $k_l$  (or, equivalently,  $\Lambda$ ) and  $\mathcal{C}$ . However, another robust relation (equation (4) in [19]) connects the compactness  $\mathcal{C}$  to the tidal deformability;

$$\mathcal{C} \approx 3.71 \times 10^{-1} - 3.91 \times 10^{-2} y + 1.056 \times 10^{-3} y^2 \quad (6)$$

so we actually have a one-parameter expression for the combination of the coefficients on the left-hand side of (3). This is the key step, leading to

$$k_l^{\text{eff}} \approx -\frac{1}{2} + \frac{\bar{\omega}_f^2}{\bar{\omega}_f^2 - \bar{\omega}^2} \left( k_l + \frac{1}{2} \right) \left[ 1 - \frac{\bar{\omega}^2 \epsilon}{\mathcal{C}^3 l} \right] \quad (7)$$

where we have used  $B_f = \epsilon/l$  (motivated by the fact that  $B_f = 1/l$  for a homogeneous stellar model, and it is worth noting that  $\epsilon \approx 0.9$  for the polytropic models considered in [11]).

$l$	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$
2	$1.820 \times 10^{-1}$	$-6.836 \times 10^{-3}$	$-4.196 \times 10^{-3}$	$5.215 \times 10^{-4}$	$-1.857 \times 10^{-5}$
3	$2.245 \times 10^{-1}$	$-1.500 \times 10^{-2}$	$-1.412 \times 10^{-3}$	$1.832 \times 10^{-4}$	$-5.561 \times 10^{-6}$

TABLE I: The coefficients required for the empirical formula (4) for  $l = 2$  and  $l = 3$  (reproduced from [14]).

<sup>1</sup> The use of this relation should be “safe”, but one must be aware that it involves a conservative view of the equation of state, e.g. the absence of sharp phase transitions [18]. However, if this relation breaks then so do related assumptions, like the I-Love-Q relations [15] that are used to break degeneracies in gravitational inspiral waveforms. In this case, one might be able to make progress by separately constraining the tidal deformability and the f-mode frequency [16].

We now have an explicit analytic formula for the effective Love number in terms of the result in the static limit,  $k_l$ , the frequency  $\omega$  and the (to some extent) free parameter  $\epsilon$ . Moreover, even though the result was based on a Newtonian analysis, it makes use of fully relativistic relations for the static Love number and the mode frequency.

In order to make a connection with the gravitational-wave signal, we need to relate  $\omega$  to the gravitational-wave frequency. This step is, inevitably, phenomenological (at this point). Intuitively, it would make sense (based on the usual logic that the gravitational waves are emitted at twice the orbital frequency) to try the replacement  $\omega \rightarrow 2\Omega$ . We can test this idea against the results for the dynamical tide from [20, 21], which are similar in spirit as they introduced the notion of an effective tidal deformability. However, the main focus of [20, 21] was to extend the effective-one-body framework to account for the dynamical tide. In addition to this, [21] provides an approximate analytical formula based on a two-timescale analysis. This result has been tested against numerical relativity simulations (most recently in [22]) and it appears to perform well in these comparisons [23]. Hence, it provides a natural benchmark against which to test our closed-form expression.

In order to carry out this comparison, we focus on the example used in [21]; an equal mass neutron star binary with  $M_\star = 1.350M_\odot$  and  $R = 13.5$  km, leading to  $\mathcal{C} = 0.148$  and  $\Lambda_2 = 1111$ . This comparison, illustrated in figure 1, shows that (7) predicts a faster than expected rise towards the mode resonance.

It turns out that the replacement  $\omega \rightarrow \Omega$  fares slightly better, suggesting that we may be able to make progress by introducing a second free parameter,  $\delta$ , such that we have

$$k_l^{\text{eff}} \approx -\frac{1}{2} + \frac{\bar{\omega}_f^2}{\bar{\omega}_f^2 - \delta(2\bar{\Omega})^2} \left(k_l + \frac{1}{2}\right) \left[1 - \frac{(2\bar{\Omega})^2 \epsilon}{\mathcal{C}^3 l}\right] \quad (8)$$

Tuning the two parameters,  $\delta$  and  $\epsilon$ , we can obtain an accurate representation of the results from [21] throughout the relevant frequency range (up to close to merger, see below). This is also illustrated in figure 1. Given the simplicity of (8), this is promising and it would be interesting to explore to what extent the parameters of the model depend on the stellar parameters (and the matter equation of state). Progress in this direction could be made by testing (8) against numerical merger simulations (along the lines discussed below).

However, it turns out that we can reduce the freedom of the model by removing one of the two parameters. We do this by accounting for the redshift of the mode frequency. In order to do this, we note that, the results used in (4) incorporate the gravitational redshift (for a

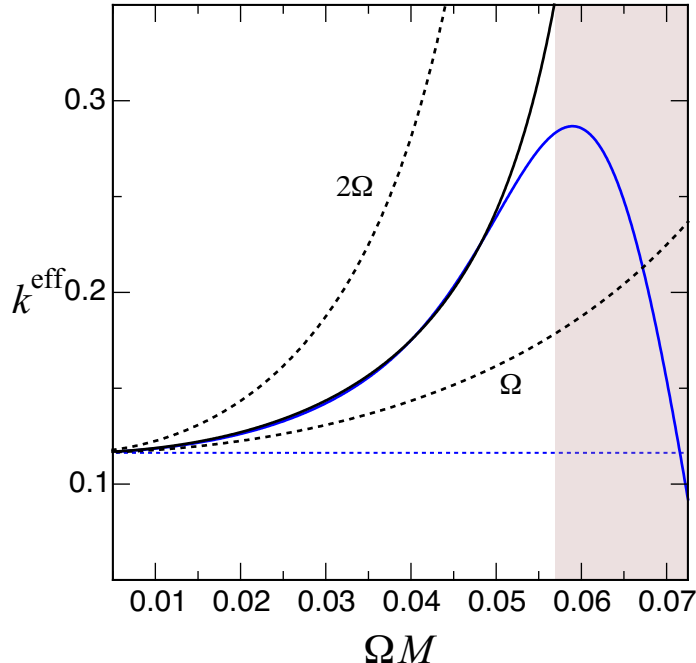


FIG. 1: Comparing the effective Love number  $k_l^{\text{eff}}$  from (8) to the results of [21] for  $l = 2$ . The dashed horizontal line represents the static Love number ( $k_l^{\text{eff}}$  in the  $\Omega \rightarrow 0$  limit). The results of [21] are shown as a (blue) solid curve. Estimates from (8) corresponding to the replacement  $\bar{\omega} \rightarrow 2\bar{\Omega}$  and  $\bar{\omega} \rightarrow \bar{\Omega}$  (as discussed in the main text) are shown as dashed curves, while a (rough) fit to the data from [21] is shown as a solid (black) curve (corresponding to taking  $\delta = 0.8$  and  $\epsilon = 1.05$  in (8)). Finally, we indicate the region beyond the (approximate) merger frequency,  $\Omega M \gtrsim 0.057$  in this case, by the coloured region in the figure.

non-rotating star) at infinity, but in the tidal problem we need the frequencies to be related at a finite orbital separation (and this should impact on the value of, in particular, the  $\delta$  parameter). Somewhat simplistically (we are, after all, only outlining the idea here), the correction we are interested in should have two terms. First of all, we need the gravitational redshift (for a non-rotating star) at a given distance. Secondly, we need to account for the fact that the star is orbiting its companion. We estimate the first term by recalling the text-book result for a signal emitted at  $r_0$  and observed at  $r_1$

$$\frac{\omega_1}{\omega_0} = \left( \frac{1 - 2M_*/r_0}{1 - 2M_*/r_1} \right)^{1/2} \quad (9)$$

leading to the usual redshift for a mode “emitted” at  $R$  and observed at infinity

$$\omega_f^2 = (1 - 2\mathcal{C})\omega_0^2 \quad (10)$$

where  $\omega_0$  is the mode frequency as measured close to the star. This factor is (implicitly) incorporated in (4).

Connecting to the tidal problem, we need the mode oscillation frequency as viewed by an observer at a distance corresponding to the orbital separation  $a$ , such that

$$\Omega^2 = \frac{GM}{a^3} \longrightarrow a = \left(\frac{GM}{\Omega^2}\right)^{1/3} \quad (11)$$

where  $M$  is the total mass of the system. Taking  $r_1 = a$  we get

$$\begin{aligned} \frac{\omega_1}{\omega_0} &= \left(1 - \frac{2GM_\star}{Rc^2}\right)^{1/2} \left[1 - \frac{2GM_\star}{c^2} \left(\frac{\Omega^2}{GM}\right)^{1/3}\right]^{-1/2} \\ &= (1 - 2\mathcal{C})^{1/2} \left[1 - \left(\frac{2M_\star}{M}\right)^{1/3} (2\bar{\Omega})^{2/3}\right]^{-1/2} \end{aligned} \quad (12)$$

and it follows that

$$\omega_1 = \left[1 - \left(\frac{2M_\star}{M}\right)^{1/3} (2\bar{\Omega})^{2/3}\right]^{-1/2} \omega_f \quad (13)$$

The second factor follows from the usual Lorentz factor. We need

$$\left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \left[1 - \left(\frac{\Omega a}{c}\right)^2\right]^{-1/2} = \left[1 - \left(\frac{M}{2M_\star}\right)^{2/3} (2\bar{\Omega})^{2/3}\right]^{-1/2} \quad (14)$$

Notably, the two factors are the same in the case of equal masses (when  $M = 2M_\star$ ).

Assuming for simplicity (although the results are easy to extend to the general case) that the masses are equal, we have the overall factor

$$\gamma = \left[1 - (2\bar{\Omega})^{2/3}\right]^{-2} \quad (15)$$

which, when combined with (7), leads to the final result (assuming  $\omega \rightarrow \Omega$ , as in the previous argument and leaving out the unknown parameter  $\delta$  in favour of the “known” redshift factor  $\gamma$ )

$$k_l^{\text{eff}} \approx -\frac{1}{2} + \frac{\bar{\omega}_f^2}{\bar{\omega}_f^2 - \gamma\bar{\Omega}^2} \left(k_l + \frac{1}{2}\right) \left[1 - \frac{\bar{\Omega}^2 \epsilon}{\mathcal{C}^3 l}\right] \quad (16)$$

This is the final result.

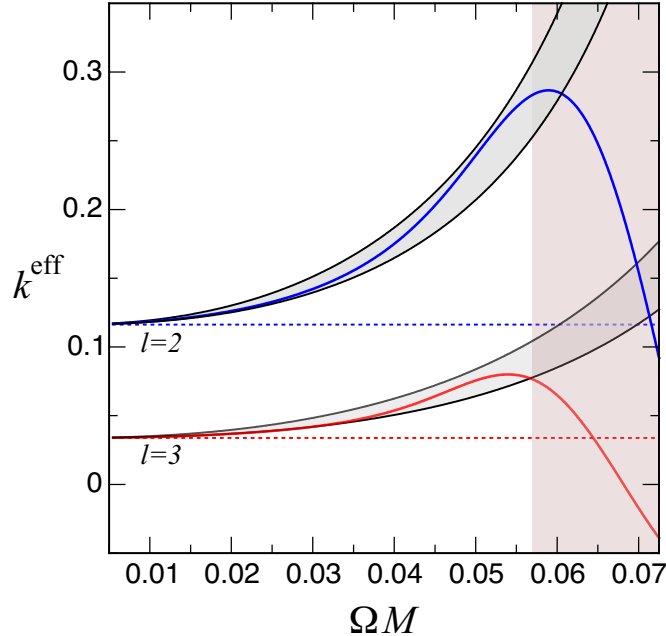


FIG. 2: Comparing the effective Love number  $k_i^{\text{eff}}$  from (16) to the results of [21], for both  $l = 2$  and  $l = 3$  (with the redshift factor included). The dashed horizontal lines represent the static Love number ( $k_i^{\text{eff}}$  in the  $\Omega \rightarrow 0$  limit). The results of [21] are shown as solid curves (blue for  $l = 2$  and red for  $l = 3$ ). Estimates from (16) are shown for the range  $\epsilon = 0.5 - 1$  with the latter representing the lower edge of the filled band in each case. Finally, we indicate the region beyond the (approximate) merger frequency,  $\Omega M \gtrsim 0.057$  in this case, by the shaded region in the figure.

The effective Love number obtained from (16) is compared to the results from [21] in figure 2, for both  $l = 2$  and  $l = 3$ . As our formula (16) leaves  $\epsilon$  as a free parameter, we show results for the range  $\epsilon = 0.5 - 1$ . From the Newtonian calculations one would expect  $\epsilon$  to lie closer to the upper end of this range and, as is clear from the result in the figure, the corresponding curve provides an excellent match to the result from [21] below  $\Omega M = 2\bar{\Omega} \approx 0.03$ . Above this orbital frequency, the effective Love number from [21] rises faster than ours, almost reaching the  $\epsilon = 0.5$  curve as the system is near merger. The essence of the comparison is that (16) performs remarkably well. It may be phenomenological in origin, but there can be little doubt that (16) (and, indeed, the less constrained model (8)) provides an effective (and very simple) representation of the required behaviour.

It is worth pointing out that, while the two sets of results diverge for larger values of  $\Omega M$



in figures 1 and 2, the corresponding frequencies are close to (or indeed above) the merger frequency. As the picture of two separate, tidally deformed, bodies breaks down there is no reason to expect the model to make sense beyond this point. The merger region is indicated by the shaded region in figure 2. In order to estimate the merger frequency, we have simply take the corresponding orbital separation to be the sum of the neutron star radii. That is, we need  $a = R_1 + R_2 = 2R$  for equal mass systems. Then we have

$$\Omega M \approx \left(\frac{M}{a}\right)^{3/2} \approx \left(\frac{M_\star}{R}\right)^{3/2} = \mathcal{C}^{3/2} \quad (17)$$

For the model used in figure 2 the estimated merger frequency would then be  $\Omega M \approx 0.057$ . A more precise estimate could be obtained from the results of [24] but this would not change our conclusions.

### III. THE STATE OF PLAY

The favourable comparison to the results from [21] suggest that our simple relations for the effective tidal deformability provide useful alternatives. In fact, the evaluation of the large set of results required to span the parameter space relevant for gravitational-wave searches should not be computationally costly. In particular, it ought to be straightforward to combine (16) with any current waveform model that implements the static tide. As we reflect on the options, it is relevant to consider the status of other efforts in this direction. In particular, the all-important benchmarking of phenomenological (computationally efficient) models against (computationally costly) nonlinear numerical simulations. This is relevant for many reasons. Perhaps most importantly, while an absolute requirement for a description of the complex merger dynamics, numerical simulations are unlikely to be able to track the many thousand binary orbits required to model a system that evolves through the detector sensitivity band [25]. This will always require an approximate description. For well-separated binaries, this need is satisfied by post-Newtonian (essentially point particle) results but this description becomes less reliable during the late stages of inspiral. This is largely due to the emergence of finite size effects, like the tidal deformability. The importance of the problem, both for signal detection and the extraction of parameters from an observation, has driven the development of reliable alternatives, like the effective-one-body framework that forms the basis for the work in [20, 21].

An important point, which appears to be commonly overlooked, involves the natural parameters to use in a phenomenological model. The typical approach connects with post-Newtonian logic by expressing the results in terms of the dimensionless parameter

$$x = (\Omega M)^{2/3} \quad (18)$$

This is an obvious choice for the main contribution to the gravitational-wave signal, which depends on the orbital motion (and involves higher order “corrections” to the quadrupole formula). It is not so obvious that this parameter “makes sense” during the late stages of inspiral, for which numerical simulations are viable (recall that it is rare that binary neutron star simulations are carried out for more than the last 10-20 orbits and it is only very recently that such simulations have been carried out with sub-radian precision in the accumulated gravitational-wave phase [22, 26–28]). There is no reason whatsoever that the matter effects would be naturally expressed in terms of a parameter based on the orbital dynamics. This point is illustrated by (16) which encodes the matter dynamics (for each binary companion) in terms of the f-mode oscillation frequency. The dimensionless variable  $\bar{\omega}_f = \omega_f M_\star$  appears naturally in the model. It is not at all clear that it “makes sense” to expand the result in powers of  $x$ . It is worth keeping this point in mind.

As a measure of the current level of uncertainty, it is useful to compare different suggested models for the tidal deformability. This kind of comparison is straightforward, as several proposed models are given in closed form. However, one has to be careful because the associated assumptions may impact on the result. Basically, we need to compare apples with apples. As will soon become clear, this turns out to be less straightforward.

A relevant comparison, with immediate implication for gravitational-wave astronomy, involves the accumulated phase associated with the tide. Roughly speaking, one would expect to be able to distinguish between models that differ by at least half a cycle (one radian) in the inspiral signal. Effectively, for the quadrupole contribution to an equal mass binary signal we need to integrate an expression of form [9, 25]

$$\frac{d\Phi_T}{dx} = -\frac{65}{2^5} \frac{k_2}{C^5} x^{3/2} f(x) \quad (19)$$

for the tidal contribution to the phase,  $\Phi_T$ . The Newtonian prefactor is the same for all models, but the factor  $f(x)$  differs. Since the models we are considering can be described analytically, it is easy to work out the required integrals. This yields the phase  $\Phi_T$  as a

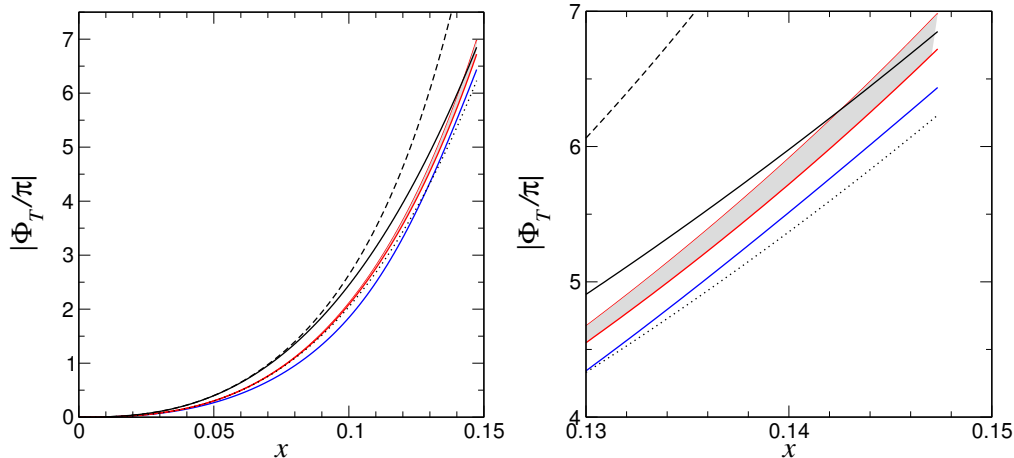


FIG. 3: Comparing the impact of the tidal deformability in different approximate models (all for  $l = 2$ ), essentially summarising the current state of the art. From top to bottom the curves represent: The post-Newtonian model from [29] (dashed black curve), the fit to the numerical data from [27] (solid black curve), the range obtained using  $\epsilon = 0.5 - 1$  in (16) (red solid lines, grey filled region), the fit to numerical data from [28] (solid blue curve) and the result obtained ignoring the dynamical contribution to the tide (black dotted line). The right panel provides a zoomed in version for the final stages of inspiral.

function of  $x$  (say) which provides some idea of the difference between tidal prescriptions. Figure 3 provides such a comparison, for the same stellar model as in figure 2.

The results in figure 3 provide a useful illustration of several key issues. First of all, we can compare the impact of the effective tidal deformability from (16) to the static tide. This is fairly straightforward, and figure 3 provides indicative results. The effective tide leads to a slight (sub-radian) change in the gravitational-wave phasing. This would be a small effect, possibly distinguishable by high signal-to-noise detections. The comparison with other models in the literature is slightly more involved as we have to make choices. In principle, it is useful to implement as much “known” post-Newtonian information as possible. In practice, this involves deciding which post-Newtonian model we should take as our baseline. The problem is that one would expect post-Newtonian estimates to lead to larger neutron stars and hence an enhanced tidal effect [30]. This expectation is illustrated by the results from the post-Newtonian model from [29] (see their equation (31)), which differ dramatically from fits based on numerical simulations for  $x \gtrsim 0.1$ . Still, the numerical

fits from [27] and [28] (see also [30, 31]) are based on different choices at low frequencies. The former limit to the results from [29] while the latter limit to the integrated version of (19) with a fixed  $k_2$  and an  $f(x)$  such that the relevant factor in the phase is  $(1 + c_1 x)$  with  $c_1 = 1817/364$  (in agreement with the post-Newtonian correction from [32]). In our comparison, we have taken this as our baseline and simply replaced the fixed  $k_2$  with  $k_2^{\text{eff}}$  from (16). As is clear from the figure, this leads to a predicted gravitational-wave phase which compares well with the fits to simulations during the late stages of inspiral. Moreover, we know that the recent numerical simulations of [22] suggest a good agreement with the effective tidal formulation from [21] (and by implication from figure 2, our expression). Of course, the issue of the “correct” form for  $f(x)$  requires further thinking. It may need some care, as one would anticipate numerical dissipation to enhance the energy loss in the system, leading to a faster inspiral in simulations and this may (to some extent) mimic the tide. It could be that the simulations do not yet have the level of precision we need for a true comparison. The results from [22] would seem to support this. However, one has to be careful. It is notable that, even though the two numerical-relativity inspired descriptions in figure 3 agree to sub-radian precision, the formulas from [27] and [28] are made to match different models in the low-frequency limit. Keeping in mind that the simulations involve only 10-20 orbits, one should really focus on the region above  $x \approx 0.1$  in the figure. This brings us to a key point, where further thinking may also be needed. A given numerical simulation does not automatically represent the past history of a binary inspiral. The initial data does not have the required “memory” (and may, for example, involve some level of unwanted eccentricity [23, 33]). As simulations become more precise (with differences at the sub-radian level required for the results to be reliable enough for gravitational-wave data analysis) the matching to the low-frequency part of the inspiral signal inevitably comes to the fore. A better understanding of the physics associated with tidal response should be valuable for this effort.

#### IV. FINAL REMARKS

We have introduced a phenomenological, physically motivated, model for the effective tidal deformability of a neutron star binary, adding frequency dependence that comes into play during the late stages of inspiral. A comparison against alternative descriptions (like

the results from [21]), suggests that we have at hand a simple, yet accurate, description of the tidal imprint (similar in spirit, but distinct from the recent effort in [34]). This should make the model an attractive alternative for an implementation of the matter effects in gravitational-wave data analysis algorithms. At the same time, the (somewhat) unexpected robustness of the model (recall that the free parameter  $\epsilon$  has hardly any influence on the gravitational-wave phasing) suggests interesting avenues for further exploration. For example, it ought to be straightforward to extend the logic to rotating systems by making use of the phenomenological relations from [35] which encode the effect that spin has on the fundamental mode. Of course, in doing this it is worth keeping in mind that merging neutron star binaries are likely to be old enough that the stars will have had plenty of time to spin down. The merger of neutron stars with significant spin would likely have to result from some alternative formation scenario. Regardless, this extension of the model should be straightforward.

At a more formal, and much more challenging level, it would be relevant to extend the mode-sum approach from [11] to general relativity. This would be important, as it should allow an actual derivation of a result along the lines of (16) (rather than the present argument, which was based on analogy). However, the problem is technically difficult because the relativistic modes are now quasi-normal (with inevitable damping due to gravitational-wave emission) and they are known not to be complete (as the scattering of waves by the spacetime curvature will lead to a late-time power-law tail [36]). This should still be a priority effort as one would also arrive at a description for the detailed role of mode resonances, for which a relativistic description is required in order for the use of a realistic matter description to make sense. The importance of this kind of development is clear, but the technical challenge should not be underestimated.

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