ISSN 0140 3818
SSSU 26

DETERMINATION OF QUIESCENT PERIODS IN SHORT CRESTED SEAS

by P.A. Wilson

Ship Science Report No. 26

July 1986

DETERMINATION OF QUIESCENT PERIODS IN SHORT CRESTED SEAS

The normally accepted method of analysis of the motion of a ship in a seaway is via a frequency domain solution. This allows the responses of the ship to unidirectional single frequency waves to be calculated. Using the standard method of linear superposition, the statistical response of the ship to a particular spectrum of seawaves is found. This method of analysis provides a feel for the problem of some naval architecture problems, for example, deck wetness and threshold accelerations. What it does not provide is a method of solution of the ship motion in the time domain where limited periods of operation are the required asset.

With this thought in mind a time series of solution utilizing the frequency responses has been added to the suite of seakeeping programs.

One requirement in the design of a ship is that a helicopter should be able to land and take off from a platform of a moving ship. To achieve this when the ship is progressing at high speed in a moderate sea state is usually only possible during lulls in the localized motions of the heli-deck. These lulls, or as they are also known as quiescent periods, may be only determined in the time domain. The quantification of a quiescent period is usually dependent upon a number of motions being below some limit value. in this study these motions have been taken as the absolute vertical displacement, absolute vertical velocity and absolute vertical acceleration, all being simultaneously below separate limits.

The purpose of this study is to investigate the occurrences of these quiescent periods in terms of frequency and duration in short crested seas. To give another understanding of this problem the case of long crested seas has been investigated first.

Long crested seas are an over simplification of the seascape, whereby the direction of all waves is the same and the wave crests are infinitely long in a direction at right angles to propogation. Short crested seas are a notion that has been developed to allow a more realistic picture of the behaviour of the sea surface. The most popular methods of representing this short crestedness is by some directional spreading function, which has the effect of 'smearing' the energy within the sea away from the so called predominant direction. This smearing is usually limited to be up to 90^o either side of the predominant direction.

This short crested presentation of the sea surface precludes any knowledge of the group behaviour of waves whereby a 'group' of waves of neighbouring frequencies and direction propogate together. To get some idea of the groupiness of a seaway would require covariance functions and correlation functions between wave direction and wave frequency. To my knowledge these are not available. So the way ahead was only along an analagous direction to long crestedness. If a wave directional spectrum $S(\omega, \mu)$ is defined then a frequently used definition is

$$S(\omega,\mu) = G(s)\, \phi(\omega) \cos^{2s} (\theta-\mu)$$

$\phi(\omega)$ is the sea wave spectrum. A value for s is chosen often to be 1.

The value of G(s) is calculated such that

$$\int_{-\pi/2}^{\pi/2} G(s) \cos^{2s} \theta d\theta = 1.0$$

With long crested seas the amplitude can be determined from the spectrum by

$$\tfrac{1}{2}\, a^2(t) = \int_0^\infty S(\omega)\, e^{i(\omega t-\alpha)}\, d\omega$$

The only problem in this determination is that the phase α is unknown, so this value α is taken to be a random variable in the range $\varepsilon[0,\pi]$.

This is physically calculated by the process of taking a finite range of wave frequencies and summing over that range,

$$a(t) = \sum_{n=0}^{m} \sqrt{S(\omega_n)\ 2\Delta\omega_n}\ e^{i(\omega_n t - \alpha_n)}$$

where either the real or imaginary value of this function is taken to be the time series. Thus many realisations of this seaway can be produced by the different values of α_n.

If this process is continued to include the response of the ship to the waves, say $H(\omega)$, then a time series of the motion can be taken in a similar way but defining the spectrum $S(\omega)$ to be that due to the seaway and the ship.

So $S(\omega) = \phi(\omega)\ H^2(\omega)$, where $\phi(\omega)$ is the sea spectrum

To allow for wave spreading then the effective spectral response is now

$$H_1^2(\omega,\mu) = \int_{-\pi/2}^{\pi/2} \cos^{2s}(\theta-\mu)\ H^2(\omega,\theta)\ G(s)\ d\theta$$

Thus the spreading function that purports to show short crestedness is used to give a weight averaged spectral response operator.

This last equation is the one that has been used to give a first approximation of the motion of a ship in short crested seaways. It is no means absolutely correct but it is the only one that is available given directional spectra.

The process by which this time series is evaluated is best performed via a Fast Fourier Transform (FFT). This has the ability of performing the sum of a(t) in a fraction of the time compared to a direct summation evaluation of all the series terms.

There are many methods available to performing FFT, and since the overall requirement of the work is that it should be able to produce time series at many points in many spectra, means that a very efficient FFT is required. To this end a subroutine has been devised that calculates the Fourier Transorm in a mixed radix mode, depending upon the number of samples available. Typically in the programme 4096 samples are used. The programme then uses a radix 4 algorithm, rather than a radix two as most FFT programmes.

The determination of the quiescent periods has been based upon three separate motions, the vertical absolute motion, velocity, and acceleration. The latter two are developed from the first by differentiating the spectrum of the displacement, in-time, to give a frequency of encounter multiplication to the response function. The method used is to check that the time value is less than the limit, if it is satisfied a flag is given the value true. When all three flags are concurrently true the period of queiscence has started. The period of quiescent start is extrapolated backwards to give a start period. When one of the motions exceed the limit the quiescent period has been deemed to have ended. The actual end of the period is extrapolated backwards to find the point where the motion was at the limit. The actual frequency of occurrence and duration of these lulls can then be analysed.

Input Requirements

Record 1)	NWA	Number of wave angles (max 25)
	NFR	Number of wave frequencies (max 51)
	NSP	Number of ship speeds (max 10)
	NPL	Number of positions along ship length (max 10)
	NS	Number of spectra (max 10)
	DWANG	Increment in wave angle in degrees
	DFR	Increment in wave frequency in rads/sec

Record 2) (SPECM(I,J), J = 1, NS), I = 1, NFR)

Spectral ordinates in metric units for N spectra, and NFR wave frequencies

Record 3) (WAVE(I), I = 1, NFR)

Wave frequency array in radians per second

Record 4) (Speed(I), I = 1, NSP)

Ship speed array in knots

Record 5) (WAD(I), I = 1, NWA)

Wave angles array over which the calculations are performed. Values are in degrees, 0^o following seas, 180^o head seas

Record 6) (XPOS(I), YPOS(I), ZPOS(I), I = 1, NPL)

These are the arrays of the coordinates of points where the motions are calculated.

XPOS X station number 1 FP, 21 AP

YPOS distance from longitudinal centreline in metres positive to starboard

ZPOS distance from keel line measured vertically upwards in metres

Record 7) (((RESP(J,K,L) RESP(J,K,L+10) L = 1, NPL), K = 1, NFR), J = 1, NWA)

These values are the absolute heave responses at the Lth point of calculation of the total motion, for the Kth frequency at the Jth wave angle, at the point and its image point, even when the wave angle is 0^o and 180^o. These results are calculated from the total motions programme.

Record 7 has to be input for each ship speed.

Record 6a |ISN| number of conditions for limits

if ISN>0 use calculated values

if ISN<0 input limits

6b if ISN 0 input conditions for |ISN| values for the three motions.

((COND(I,J), I = 1, |ISN|), J = 1,3)

Program Organisation

The program has been organised on a subroutine basis.

The main program controls the flow of data into and out of the program.

MAIN

1) Inputs all data from channel 38.
2) Calculates the spreading function.
3) Loops over the following calculations:

   a) Ship speed
   b) Predominant wave direction
   c) Position where total motions are calculated
   d) Wave frequencies
   e) Spreading the response operators of
   f) Absolute vertical displacement
   g) Absolute vertical velocity
   h) Absolute vertical acceleration
   i) Loop over spectra

      i) interpolating NFR responses to NFFT responses
      ii) perform FFT on the three spectral responses
      iii) assess the time series for quiescent periods.

INTERP

This program linearly interpolates the NFR spectra generated in (f,h) above. The interpolated values are placed into a complex array A with a multiplication by $e^{i\phi}$ where ϕ is the random phase angle, generated from within the programme.

This phase angle is also used for velocity and acceleration calculations. To make sure that the time series is purely real the coefficients of the Fourier series are defined

$$\alpha_k = \bar{\alpha}_{n-k} \qquad \text{— is the complex conjugate.}$$

Since the mean value of this motion is zero then $\alpha_1 = 0.0$

ASSESS

This takes input from the subroutine FFT and analyses the real part Fourier time series. The assessment procedure can follow one of two methods. The first method is to specify the limits of the motions from some set proportions of the maximum encountered values of the motions. The second method is to specify the limits of the motions on input. A maximum of up to five conditions have been specified.

The method of determination of the quiescent periods is when to assign a flag a value of true when the motion is below the threshold. When all three flags are set to be true then the quiescent period is deemed to have already started. The start of the period is estimated by extrapolating backwards to the time when the motions were at the threshold values. A similar process is used to find the end of the interval.

SINT

This function is used to give a trapezoidal integration of some array Q.

Subroutine FFT

This subroutine performs multivariate complex Fourier transforms in place using mixed radix FFT algorithms.

Arrays A and B originally hold the real and imaginary components of the resulting Fourier coefficients. Multivariate data is indexed according to the FORTRAN array element successor function, without limit on the number of implied multiple subscripts. The calls for a multivariate transform may be in any order.

The sign of ISN determines the sign of the complex exponential. Thus for positive values, a Fourier transform of a time series is performed, to produce the coefficients.

The type of radix algorithm used is determined by factorising the number of samples. A maximum value of the prime factors is 23. The order of the transformation is to place the factors of n in pairs of those that are squares and those that are not.

PAW004/SBG
31.7.86