Conflicted Voters: A Spatial Voting Model with Multiple Party Identifications

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Abstract In this paper, we develop a unified spatial model of turnout and voting behaviors in which citizens can identify with one or two parties. We show the existence of a conflicted voter’s curse: If there is no position that reconciles the ideological views of both parties, it is always rational for citizens that identify with two parties to abstain even if they are a majority. In a two-candidate electoral competition, the conflicted voter’s curse implies that candidates converge to the center of the political domain if and only if conflicted voters are pivotal and the parties have shared ideological views. Otherwise, we show that candidates may converge or diverge depending upon the degree of party polarization and whether candidates care about ideology or not. Our analysis suggests that the behavior of conflicted voters may be relevant for electoral outcomes and public choice.

Keywords: Spatial Voting; Party identification; Conflicted voters; Electoral competition; Party polarization.

JEL classification: D03; D72; P16.

“Any party which is both responsible and reliable will probably have an ideology which is relatively coherent and immobile. In other words, its ideology will not be internally contradictory but will be at least loosely integrated around some social Weltanschauung. And the party will not radically shift its policies and doctrines overnight, but will only slowly change their nature.”

(Downs 1957, 109)

1 Introduction

After the seminal contributions of Hotelling (1929), Downs (1957), and Black (1958), the spatial theory of voting became a cornerstone to the study of elections. The aim of this literature is to study the electoral outcomes that emerge from the interaction between two economic agents: candidates (key actors) and citizens (fixed role). Citizens are assumed to have exogenous preferences over a uni or multidimensional ideological space, and candidates compete for election by adopting positions in this space. Amid the numerous contributions to this literature the term “candidate” is often interchanged with “party”, and most papers abstract from any distinction between their behaviors and objectives. In particular, most spatial voting models rule out the influence of party ideologies upon citizens’ preferences. Nevertheless, in countries where two parties dominate the political sphere, empirical evidence suggests that citizens’ ideological preferences are often driven by their party identifications1.

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1See e.g. Bartels (2002), Evans and Andersen (2004), Goren (2005), Carsey and Laymain (2006), and Dancey and Goren (2010). This is particularly salient in the U.S. See Milazzo et al. (2012) for an account of its lower salience in British politics.
When party identification is strong it often implies voting for “one’s” party’s nominee (straight ticket voting). However, recent empirical evidence suggests that an interpretation of party identification that allows for different strengths and directions may be more appropriate to explain citizens’ behavior\(^2\). Notably, citizens with moderate or mixed views seem to be of particular relevance. For instance, in the United States, one of the most prominent cases of a two-party system in which party identification is an important predictor of voting behavior, about half of the citizens declare themselves to be moderates or are unable to place themselves on an ideological scale (see e.g. Fiorina et al. 2006; Fiorina and Abrams 2008; Fiorina et al. 2008). According to the Pew Research Center (2014) data on the responses to 10 ideological values questions, 39% of Americans take a roughly equal number of liberal and conservative positions\(^3\). Evidence from this survey and other sources (e.g. Hetherington 2008) also suggests that these citizens are less likely to turnout than strong partisans and that their voting behavior is less consistent than the one of citizens that have a strong identification to a party\(^4\).

In this paper, we build a simple spatial voting model consistent with these observations. We consider two cases in terms of the relationship between candidates and party ideologies. In the first, non-ideological candidates maximize plurality independent of party ideologies. Candidates are only interested in the electoral outcome and can adopt any available strategy in the political domain. In the second case, each candidate cares about the ideology of “his” party\(^5\). In this case, ideological candidates maximize plurality bounded to the strategies close to the ideology of their respective party. The aim of the paper is then twofold: (i) to put forward an extended spatial voting model, based on party ideologies and citizens’ identifications, to study turnout and voting behaviors; and (ii) to characterize the electoral equilibrium of a two-candidate electoral competition in a political domain with two parties. By doing so, we provide a unified theory of turnout and voting behaviors with meaningful predictions about citizens’ and candidates’ behaviors.

We consider a multidimensional ideological space and represent party ideology as a (preferred) ideological point and a (surrounding) acceptance region. Any position within this region is perceived to be accepted by the party and any position outside (of it) is perceived to be rejected. The fixed and exogenous nature of party ideology intends to capture the parties’ ideological “coherence and immobility” as defended by Downs (1957). Similarly, these ideologies can be thought to represent the “silent ideology” of commonly held interests shared by the party’s elite (see Flinn and Wirt 1965). Finally, this representation is also consistent with the ideological views of other social groups that influence turnout and voting behaviors such as religion and ethnicity. The model is then suitable to study the effects of the ideologies and the identifications on diverse social identity groups on political behavior.

We follow Hershey (2015) and see party identification as a social identity motive. Taking parties or groups’ ideologies as reference points, citizens form their spatial preferences based on the ideological cues (the ideological point and acceptance region) provided by their single or multiple identifications. That is, citizens can identify with, and focus upon the ideological cues of one or two parties simultaneously. On the one hand, we represent (unconflicted) partisans as citizens that identify with a single party. On the other hand, we represent conflicted partisans as citizens that identify with two parties. A citizen anticipates an identity gain in voting for a candidate that is accepted by a party she identifies with. Voting is anticipated to be costly if it implies the betrayal of any of the citizen’s party identifications: if by the act of turnout and voting a citizen would vote for a candidate that adopts a position that is not accepted by all the parties she identifies with. In this sense, citizens are betrayal averse. They wish to be able to support a position that respects the single ideology or reconciles the multiple ones they believe in. These are interpreted as intrinsic gains and losses that can mobilize a citizen to turn out or abstain, independent of her probability to influence the outcome of the election.

To illustrate, take the example of the Democratic and Republican parties in the United States. These parties provide two reference points in terms of values and worldviews that influence citizens’ stands on different issues. Citizens use their identifications to focus on either the liberal (Democratic) worldview, the conservative (Republican) worldview, or both. In this sense, Democrats focus on and adopt a liberal

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\(^2\) See Katz (1979) and Weisberg (1980) for early accounts of the “multidimensional” nature of party identification.

\(^3\) This is down from about half of the American public in surveys conducted in 1994 and 2004. Meanwhile, the overall share of American citizens who express consistently liberal or consistently conservative opinions has doubled over the past two decades from 10% to 21%.

\(^4\) In general, partisans are more likely to vote than “mixed voters”, follow more regularly the news, are more interested in politics, and participate more than mixed voters in political activities (see Pew Research Center 2014). For instance, while 78% and 58% of consistent conservatives and consistent liberals say they always vote respectively, only 39% of those who hold a mix of conservative and liberal views describe themselves as regular voters.

\(^5\) To avoid awkward wording, we refer to candidates in the masculine and citizens in the feminine.
ideology while Republicans adopt a conservative one (what has been referred to by Levendusky 2009 and others as the partisan sort). Citizens with mixed or moderate views, on the other hand, hold seemingly conflicting views that are congruent with both ideologies. We suggest that this conflict is resolved in favor of a weak identification with the two ideologies: Conflicted partisans form the ideological preferences that give rise to their voting behavior based on the ideological cues given by the two parties. These citizens wish, if voting, to be able to reconcile both views and endorse a compromise between the two ideologies. If this is not possible, then voting is associated with a sense of betrayal and a psychological cost.

With two parties, this implies that an individual citizen turns out and votes if and only if there is a candidate position that is accepted by all the parties she identifies with. Then, if this is the case, a partisan votes for the candidate that is closest to the ideological point of the single party she identifies with. This is the available choice that procures her the highest identity gain. For conflicted partisans, their ideological preferences depend upon the ideologies of the two parties. We show that a conflicted voter’s curse emerges: If there is no position that reconciles the ideological views of both parties, it is always rational for conflicted voters to abstain even if they are, as a group, a majority. This curse is due to the high degree of perceived polarization in party ideologies, and holds no matter what position either of the candidates adopts. It provides theoretical support for the view that party/elite polarization and ideological conflict in a democracy tend to discourage moderates’ participation and political engagement (see e.g. Mutz and Reeves 2005; Fiorina et al. 2006; Rogowski 2014).

Using this approach, we address the questions of whether an electoral equilibrium exists, where such an equilibrium is located, and how the location is related to the ideological positions and acceptance regions of the two parties. We show that with non-ideological candidates, a small number of (pure strategy Nash) equilibrium pairs exist for most preference distributions. In particular, candidates converge to the position of the “median voter” if an overlap region between the parties’ acceptance regions exists. If an overlap region does not exist, candidates converge as long as the partisans of one party are more numerous than the partisans of the other. This suggests that candidates behave independently of conflicted partisans in the absence of shared ideological views, even if they would otherwise be pivotal for the electoral outcome.

Ideological candidates converge if and only if an overlap region exists and conflicted partisans are pivotal. If conflicted partisans are not pivotal but an overlap region exists, the majoritarian candidate - the candidate affiliated to the party with which a majority of partisans identifies with - can adopt any position around the ideology of his party. The minoritarian candidate moves towards the center of the political domain to gain the support of conflicted partisans even though he is certain to lose the election. This suggests that majoritarian candidates enjoy more strategic flexibility than minoritarian candidates. Finally, if an overlap region does not exist, ideological candidates can adopt any position within the acceptance region of their respective party. This indicates that the absence of shared ideological views may lead ideological candidates to adopt quite unpredictable behaviors.

Taken together, these results suggest that a high degree of perceived party/elite polarization may deter turnout as well as the convergence of candidates to the center of the political domain. In particular, turnout increases and candidates converge to a moderate position if and only if the parties share ideological views and conflicted partisans are pivotal. In this sense, the centripetal forces most prevalent in the traditional spatial voting models seem to be stronger in less polarized contexts whereas centrifugal forces are expected to dominate in more polarized political systems (see also Pardos-Prado and Dinas 2010). Our results suggest that this may be due in part to the behavior of conflicted voters.

Our model provides, in the Downsian tradition, a rational intrinsic motivation that reconciles the spatial theory of voting with positive turnout rates. In this sense, it is most related to theories that have used non-consequentialist motivations to rationalize voting and turnout behaviors in a single framework. For instance, Brennan and Hamlin (1998) analyze a spatial voting model in which turnout and voting behaviors are rationalized by the will to express support for one or the other candidate. Other authors have considered “relational goods” and the strategic calculus of groups as the major drivers of both turnout and

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6This means that under some circumstances, a partisan of one party may vote for the candidate affiliated to a different party. To substantiate this possibility, one may recall again the example of the United States, in which most, if not all elections, have featured large numbers of partisans voting against their party’s nominee (Green et al. 2002). The so-called “Reagan Democrats” were an expression of that. In the U.K., “Essex Men” was the connotation given to the citizens that voted across their partisanship in the 1979, 1983, and 1987 general elections.

7These models, like ours, are in part motivated by the fact that the rational voting model is not consistent with positive turnout rates and that exogenous explanations of turnout (e.g. an individual sense of civic duty in Riker and Ordeshook 1968) deprive the rational model of a coherent and predictive rationality (Aldrich 1993; Green and Shapiro 1994). See Dhillon and Peralta (2002), Mueller (2003), Feddersen (2004), and Geys (2006) for general reviews. See Shayo and Harel (2012) for a recent discussion and experimental evidence on non-consequentialist voting.
voting behaviors (e.g. Morton 1987, 1991; Ublauer 1989). Still others have proposed rationalizations based on party activism (e.g. Aldrich 1983, 1989). To the best of our knowledge, there is no previous spatial voting framework that uses (multiple) social identifications to explain voting and turnout behaviors\(^8\). Our theoretical framework also differs from previous contributions in the way that it models party ideologies and the focus on conflicted voters’ behavior. Consequently, both the exogenous parameters and empirical predictions of our model remain significantly different from previous studies. The remainder of the paper is organized as follows. The next section is devoted to the formal model. In Section 3 we present the equilibrium results for the two-candidate electoral competition with two parties. We divide Section 3 into two subsections: In the first we characterize the electoral equilibrium with non-ideological candidates, and in the second we characterize it with candidates that care about ideology. We discuss the underlying behavioral theory as well as limitations and extensions to the model in Section 4. We summarize the implications of our analysis in Section 5.

2 Model

In this section we introduce our model of turnout and voting behaviors. In subsection 2.1 we introduce the general setting. Subsection 2.2 is devoted to party ideologies. In subsection 2.3 we present the spatial domain. As in the traditional spatial models of electoral competition, we assume that (1) all citizens have identical perceptions of candidates’ strategies and that (2) candidates know citizens’ preferences. We divide Section 3 into two subsections: In the first we characterize the electoral equilibrium with non-ideological candidates, and in the second we characterize it with candidates that care about ideology. The candidates’ objectives are presented together with the analysis of the electoral equilibrium in Section 3.

2.1 Setting

Let \( X \subseteq \mathbb{R}^m \) denote a set of alternatives such that each \( x \in X \) is a column vector \( x = (x_1, \ldots, x_m) \). We interpret these alternatives as vectors of positions on policy issues, such as the level of income tax, as well as on non-policy issues (under the control of candidates), such as perceived image or personality\(^9\). This excludes non-policy issues that are not under the control of candidates such as age or ethnicity. We call \( X \) the political domain.

We consider a finite set of \( N \) citizens and an electoral competition between two candidates 1 and 2. Let \( \theta_1 = (\theta_{11}, \ldots, \theta_{1m}) \) and \( \theta_2 = (\theta_{21}, \ldots, \theta_{2m}) \) denote the strategies that candidates choose in the political domain. As in the traditional spatial models of electoral competition, we assume that (1) all citizens have identical perceptions of candidates’ strategies and that (2) candidates know citizens’ preferences.

2.2 Party Ideologies

There are two parties \( b \) and \( r \), the blue party and the red party respectively. A party \( p \in \{b, r\} \) is characterized by two parameters. The first of these is an ideological point \( \delta_p \) in the political domain:

\[
\text{P 1} \text{ For all } p \in \{b, r\}, \quad \delta_p = (\delta_{p1}, \delta_{p2}, \ldots, \delta_{pm}) \in X.
\]

The ideological point \( \delta_p \) represents the citizens’ (and candidates’) estimation of the party \( p \)'s ideological preferred position on each of the \( m \) dimensions. This shared perception can be seen as the reference ideological point socially attached to the party. In this sense, the ideological point can be thought of as the public view of the party’s coherent set of stands on different issues. Alternatively, it can be interpreted as the “silent ideology” of commonly held interests shared by the “like-minded men and women” that run the party (see Flinn and Wirt 1965). The second parameter is an (political) acceptance region \( A_p \) delimited by a distance \( d_p \) in \( \mathbb{R}^{++} \) to \( \delta_p \):

\[
\text{P 2} \text{ For all } p \in \{b, r\}, \quad A_p(\delta_p, d_p) = \{ x \in X : ||x - \delta_p|| \leq d_p \}.
\]

where \( ||\cdot|| \) denotes the Euclidean distance. For each \( p \in \{b, r\} \), the acceptance region \( A_p \) represents the positions within a “threshold” distance \( d_p \) that are perceived to be accepted by the party or acceptable with respect to the party’s ideology. The different threshold distances can be interpreted, for instance,

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\(^8\)See Shayo (2009) for a model that uses social identity to rationalize the political economy of income redistribution.

\(^9\)See Hinich and Ordeshook (1969) and Brennan and Hamlin (1998) for discussions about the interpretation of the various dimensions on models of spatial voting.
as the perceived “range of opinion” allowed by the two parties. In this sense, a smaller acceptance region corresponds to a party being perceived as more demanding, i.e., a party that is perceived to accept less discrepancy with respect to its preferred ideological position.

This means that whenever a candidate adopts a position within this distance it is perceived as acceptable with respect to the party’s ideology, while if a candidate adopts a position outside this distance the party is perceived to reject the candidate’s ideological stand. Then, it may be the case that a red ideological candidate adopts an ideological position that is in the acceptance region of the blue party. This is not to be interpreted as if the blue party supports the red candidate. Rather, it is interpreted as if the position adopted by the red candidate is perceived as an acceptable position according to the standards of the ideology of the blue party.

Finally, if there is an intersection between $A_b$ and $A_r$ we call it an overlap region and denote it by $O_{br}$. This region, if it exists, corresponds to the ideological positions in the political domain that are accepted by both parties. It can be interpreted as the ideological common ground or shared views of both parties or elites: the positions which reconcile both ideologies. Figure 1 represents these concepts in a two-dimensional domain.

2.3 Citizens’ Preferences

Each citizen $i \in N$ has complete and transitive preferences over $X$, that are represented by a utility function $u_i : X \to \mathbb{R}$. Let $P_i \subseteq \{b, r\}$ denote the subset of parties that citizen $i$ identifies with. Then, for all $i \in N$, either $P_i = \{b\}$ (citizen $i$ identifies with the blue party), $P_i = \{r\}$ (citizen $i$ identifies with the red party), or $P_i = \{b, r\}$ (citizen $i$ identifies with the two parties). We call partisan a citizen that identifies with a single party: from these, we distinguish blue partisans and red partisans. A citizen that identifies with the two parties is referred to as conflicted partisan (or bi-partisan).

The strength of these identifications is individual specific. It is numerically measured by the weight(s) $I_{ip} \in [0, 1]$ that citizen $i$ attaches to each party $p$ she identifies with. The higher the $I_{ip}$ the higher the identification is interpreted to be.

A citizen evaluates alternatives according to her weighted party identifications. In order to specify these preferences, we proceed as follows. First, we distinguish the three intrinsic sources of utility benefits and losses with which a citizen evaluates the alternatives in the political domain. Second, we combine these three sources to specify the utility of voting for a given position in $X$. The first source, $u_{ip}^A(x)$, is citizen $i$’s (simple) Euclidean preferences with respect to the ideological points $\delta_p$ of all the parties she identifies with:

U 1 For all $i \in N$, all $p \in P_i$, and all $x \in X$,

$$u_{ip}^A(x) = -I_{ip} \cdot ||\delta_p - x||,$$

10They are intrinsic since citizens have an effective choice on if and how they vote, but they do not have an effective choice between alternative policy outcomes since their single vote has a negligible probability of being pivotal (see Brennan and Hamlin 1998, 155-6).
with anticipated regret of an action the parties a citizen identifies with. One way of making sense of this is that while betrayal is associated there may be a foregone identity gain for not having voted for a candidate that is accepted by one of one cannot vote without betraying a candidate that adopts a position that is accepted by a given party (reflected in inaction with anticipated regret of this assumption captures the intuition that these citizens want to support an outcome that can be from inaction or omission (e.g. Kahneman and Tversky 1982; Crockett et al. 2014). Additionally, to non-monetary effects (e.g. Galanter and Pliner 1974; Crockett et al. 2014).

Tversky and Kahneman 1991; Bowman et al. 1999) and with the mild evidence in support of its extension gains. This is in accordance with the strong evidence on loss aversion on pecuniary/monetary items (e.g. Kahneman and Tversky 1982; Crockett et al. 2014). For instance, Crockett et al. (2014) show that people require more compensation to increase pain (in the form of small shocks) than they are willing to pay to decrease it by the same amount. See Dhar and Wertenbroch (2000) and Vendrik and Woltjer (2007) for loss aversion for hedonic and utilitarian attributes and on life satisfaction with respect to relative income respectively.

U1 entails that positions closer to the preferred ideological point of “one’s” party are preferred to positions which are farther from this point in what respects the identification with this party. This can be interpreted as the (psychological) loss associated with positions that do not fully represent one’s identifications. On the one hand, for a partisan this is a close formulation to the one used in traditional analysis of spatial voting. The main difference is that here the Euclidean preferences are not with respect to an “ideal point”. Instead, they are with respect to her party’s preferred ideological point. In this sense, the ideology of her party is used as a cue on the formation of her preferences. On the other hand, a conflicted partisan exhibits Euclidean preferences with respect to the ideological points of the two parties. The two parties provide ideological cues that a conflicted voter i weights according to (i) the individual weight I_i^p of each party and (ii) the size of the parties’ acceptance regions given by d_p. First, all else being equal, any “move” towards the ideological point of a party of higher weight (i.e., of stronger identification) than another entails a higher utility gain than a similar move towards the ideological point of the party with lower weight. Second, all else being equal, any move towards the ideological point of a party with a smaller acceptance region (i.e., a more demanding party) than another entails a higher utility gain than a similar move towards the ideological point of the party with a larger acceptance region.

The remaining two sources correspond to the psychological gains and losses associated with following or betraying one’s identifications. Voting is anticipated to generate identity gains or losses depending on whether or not the act of voting can be done in accordance with the perceived behavioral prescriptions of the parties/groups one identifies with. These prescriptions are given by the parties’ cues (the ideological points and acceptance regions) and generate utility benefits and losses, u_{ip}^B(x) and u_{ip}^C(x), as follows:

**U 2** For all i ∈ N, all p ∈ P_i, and all x ∈ X,

\[ u_{ip}^B(x) = \begin{cases} I_i^p & \text{if } x \in A_p \\ 0 & \text{otherwise.} \end{cases} \]

**U 3** For all i ∈ N, all p ∈ P_i, and all x ∈ X,

\[ u_{ip}^C(x) = \begin{cases} -c_i & \text{if } x \notin \bigcap_{p \in P_i} A_p \\ 0 & \text{otherwise.} \end{cases} \]

with \( c_i > I_i^p \) for all \( p \in P_i \). U2 can be interpreted as the identity gain that a citizen receives if she is to vote for a candidate that adopts a position that is accepted by a party she identifies with. The magnitude of this benefit depends upon the weight the party has for her. Note that this gain can be associated with following the prescription of a single party in the case of partisans, or of one or two parties in the case of bi-partisans. U3 represents the cost of betrayal that a citizen may incur with the act of voting. In particular, voting is anticipated to be costly if it implies the betrayal of any of the citizen’s party identifications: if by the act of turnout and voting a citizen would vote for a candidate that adopts a position that is not accepted by all the parties she identifies with.

We assume the cost of betrayal to be greater than the identity gain associated with voting for a candidate that adopts a position that is accepted by a given party (reflected in \( c_i > I_i^p \) for all \( p \in P_i \)). This means that citizens are betrayal averse: they give higher weight to identity losses than to identity gains. This is in accordance with the strong evidence on loss aversion on pecuniary/monetary items (e.g. Tversky and Kahneman 1991; Bowman et al. 1999) and with the mild evidence in support of its extension to non-monetary effects (e.g. Galanter and Pliner 1974; Crockett et al. 2014)\(^\dagger\).

For conflicted partisans, this entails that they anticipate to bear a psychological cost whenever they cannot vote without betraying one party but not necessarily the other. Then, in the case of abstention there may be a foregone identity gain for not having voted for a candidate that is accepted by one of the parties a citizen identifies with. One way of making sense of this is that while betrayal is associated with anticipated regret of an action against one’s identification, the foregone identity gain is associated with anticipated regret of inaction (not having voted for the given candidate). For that reason, betrayal aversion is in accordance with the evidence that the regret experienced from action is more intense than that from inaction or omission (e.g. Kahneman and Tversky 1982; Crockett et al. 2014). Additionally, this assumption captures the intuition that these citizens want to support an outcome that can be

\(^\dagger\)For instance, Crockett et al. (2014) show that people require more compensation to increase pain (in the form of small shocks) than they are willing to pay to decrease it by the same amount. See Dhar and Wertenbroch (2000) and Vendrik and Woltjer (2007) for loss aversion for hedonic and utilitarian attributes and on life satisfaction with respect to relative income respectively.
reconciled with both ideologies they have a personal attachment to. In other words, conflicted partisans find ideological acquaintance in the worldview of the two parties and wish, if voting, to endorse a compromise between the two. This seems especially sensible given the evidence on their mixed and moderate views that support a weak and not very disparate identification with respect to both parties\(^{12}\).

For all \( p \in P_i \), citizen \( i \) evaluates all alternatives \( x \in X \) according to the combination of these three intrinsic utility sources, such that \( u_i(x) = \sum_{p \in P_i} [u^A_{ip}(x) + u^B_{ip}(x) + u^C_{ip}(x)] \). Then, citizen \( i \)'s utility of voting for (a candidate at) position \( x \) can be written as follows:

\[
\begin{align*}
  u_i(x) &= \sum_{p \in P_i} (I^p_i - \frac{I^p_i}{d_p} \cdot ||\delta_p - x||) \text{ if } ||\delta_p - x|| \leq d_p \text{ for all } p \in P_i \\
  &= \sum_{p \in P_i} (\frac{-I^p_i}{d_p} \cdot ||\delta_p - x||) + I^q_i - c_i \text{ if } ||\delta_q - x|| \leq d_q \text{ for only one } q \in P_i \text{ and } \#P_i = 2 \\
  &= \sum_{p \in P_i} (\frac{-I^p_i}{d_p} \cdot ||\delta_p - x||) - c_i \text{ otherwise.}
\end{align*}
\]

(1)

### 2.4 Turnout and Voting Decisions

For any combination of strategies \( (\theta_1, \theta_2) \) chosen by the two candidates, citizen \( i \) either votes for one of these two candidates or abstains. Let \( t_i \in \{0, 1\} \) and \( v_i \in \{1, 2\} \) represent citizen \( i \)'s turnout and voting decisions respectively. The **turnout decision** \( t_i \) takes the value \( t_i = 1 \) if citizen \( i \) participates in the election and \( t_i = 0 \) otherwise. The **voting decision** \( v_i \) available to citizen \( i \) in case of turnout is between candidate \( 1 \) (\( v_i = 1 \)) and candidate \( 2 \) (\( v_i = 2 \)). Then, citizen \( i \) solves the following maximization problem:

\[
\max_{t_i \in \{0, 1\}, v_i \in \{1, 2\}} t_i \cdot \max_i [u_i(\theta_{v_i})] \tag{2}
\]

This decision problem can be seen as a two-stage optimization problem, where in the first stage the citizen decides whether or not to participate in the election and in the second stage she decides whom to vote for. Solving it backwards, it is straightforward to see that citizen \( i \)'s optimal voting decision is to vote for the candidate \( k \in \{1, 2\} \) that proposes the alternative associated with higher utility:

\[
v_i(u_i, \theta_k) = \begin{cases} 
1 & \text{if } u_i(\theta_1) > u_i(\theta_2) \\
2 & \text{if } u_i(\theta_1) < u_i(\theta_2). 
\end{cases} \tag{3}
\]

and in case of indifference to vote with equal probability for either candidate. Indeed, if a citizen is indifferent between two candidates that are accepted by the party/parties she identifies with, there is no reason for this citizen to abstain. Instead, she can maximize her utility by randomly selecting one of the two candidates: much as the hot and hungry sun-bather who is close and equidistant from two ice-cream sellers chooses randomly from whom to buy an ice-cream (see Brennan and Hamlin 1998, 157). Then, it follows from (1), (2), and (3) that citizen \( i \)'s optimal turnout decision is to decide whether or not to participate in the election according to the following rule:

**Proposition 1** For all \( i \in N \) and \( k \in \{1, 2\} \):

\[
t_i(u_i, \theta_k) = \begin{cases} 
1 & \text{if } \exists k \text{ such that } ||\delta_p - \theta_k|| \leq d_p \text{ for all } p \in P_i \\
0 & \text{otherwise.}
\end{cases}
\]

*Proof* See appendix (for all proofs).

\(^{12}\)Betrayal aversion can also be thought of as an ordinal assumption. Even if there is a significant cardinal difference in the identifications, this difference may not be strong enough to outweigh the aversion to the negative emotion of active betrayal of one’s own identification. Still, this assumption may be less adequate for contexts in which conflicted partisans are expected to attach very different weights to the two parties. A weaker version of betrayal aversion would be one that would hold for each party separately (i.e., \( c^i_p > I^p_i \) for all \( p \in P_i \)). This is equivalent with respect to partisans. However, it would change the behavior of bi-partisans. Below we comment on the differences that would follow from this weaker aversion.
In words, a citizen turns out if and only if there exists a candidate that is accepted by all the parties she identifies with. Otherwise, the citizen abounds. This implies that the possible candidates’ positions for which a partisan may cast a vote for are within the acceptance region of her party. In the case of a bi-partisan, the possible candidates’ positions for which she may cast a vote for are within the intersection of the acceptance regions of the two parties, i.e., within the overlap region $O_{br}$. This underlies the conflicted voter’s curse: If there is no position that reconciles the ideological views of both parties, it is always rational for conflicted partisans to abstain. This behavior is rational irrespective of candidates’ positions, and even if conflicted partisans are, as a group, a majority. This means that conflicted partisans may abstain even though they could, together, change the outcome of the election for their preferred outcome.

Conflicted partisans are ideologically mixed. The internal conflict that these mixed views create increases with the level of polarization: the more the parties are distant, the more difficult it is for these citizens to justify their beliefs. The conflicted voter’s curse suggests that a crucial element is the overlap between both ideologies. If the overlap is nonexistent/empty, citizens anticipate that they are not able to reconcile both ideologies through the act of voting and opt for inaction, that is, to abstain. The betrayal aversion that underlies this behavior is a different reason for abstention than the most common indifference and alienation hypotheses. In particular, a citizen abstains if all the candidates are sufficiently far from the ideological points of all the parties she identifies with. Then, partisans abstain much as citizens abstain in accordance to alienation while conflicted partisans feel alienated as soon as they cannot, through voting, resolve their conflict due to multiple identifications. If there is no position available that represents a compromise and reconciles the ideologies of the two parties, then all positions seem unsatisfactory and conflicted partisans abstain.

3 Electoral Equilibrium

In this section we look at the consequences of party ideologies and citizens’ preferences on the choices that candidates offer to the citizens. In subsection 3.1 we characterize the candidates’ equilibrium strategies when candidates are non-ideological, i.e., when they are unrestricted in their strategy choices. In subsection 3.2 we characterize the candidates’ equilibrium strategies when candidates are ideological, which is reflected in restricted choices on the basis of their party affiliations.

Let $N_b = \#\{i : P_i = \{b\}\}$, $N_r = \#\{i : P_i = \{r\}\}$, and $N_{br} = \#\{i : P_i = \{b, r\}\}$. Note that $N_b + N_r + N_{br} = N$. We impose the two following conditions in our analysis:

A 1 $N_b > 0$, $N_r > 0$ and $N_{br} > 0$.

A 2 For any $i \in N_{br}$, $\frac{I_{br}}{I_b} \neq \frac{I_{br}}{I_r}$.

The first condition requires that there exists at least one blue partisan, one red partisan, and one bi-partisan. The second condition requires that the weight of the two parties, when adjusted by the size of the acceptance region, be different for all conflicted partisans. It captures the intuitive idea that it is possible to distinguish bi-partisans according to the party they lean towards (e.g. citizens who lean Democratic and citizens who lean Republican). None of these conditions change our results substantially, but they entail considerable gains in parsimony.

We start our analysis by establishing the circumstances under which a citizen $i$ has a “satisfactory” ideological preferred point $x_{i}^*$ (bliss point), and, if it exists, where that bliss point (the point that gives the highest utility) is located. Such a point is satisfactory in the sense that voting for a candidate in that position entails a greater utility than abstaining. Unconflicted and conflicted partisans differ in this respect. Let $x_{i}^* \in O_{br}$ denote the points within any non-empty overlap region that minimize the distance to $d_r$ and $d_b$ respectively.

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13We have implicitly assumed that in the case that a citizen is indifferent between turning out and abstaining the tie is broken in favor of participation; this entails no loss of generality.

14If one assumed instead $c_i > I_i$ for all $i \in P_i$ (see footnote 12), then the conflicted voter’s curse would hold if $c_i > I_i$ and $c_i > I_r$ for all conflicted partisans. Otherwise, it would hold or not depending upon the adjusted weights that each conflicted partisan attached to each party (as would be the case if one assumed $c_i > I_i$ for all $i \in P_i$ or $c_i < I_i$ for all $i \in P_i$). Accordingly, these alternative specifications would entail less general analytic results concerning both the citizens’ and candidates’ behaviors. Nonetheless, to ascertain the contextual determinants responsible for the existence and form(s) of betrayal aversion is a relevant and open empirical question.

15Under indifference a citizen abstains if all the candidates assume a sufficiently similar position and there is not sufficient difference in the payoffs of voting for each candidate (see e.g. Hinich et al. 1972). Under alienation a citizen abstains if all the candidates are sufficiently far from her ideal point (see e.g. Hinich and Ordeshook 1969).
Lemma 1  (a.1) For all $i \in N_b \cup N_r$ (partisans) there always exists $x^*_i \in X$ s.t. $x^*_i = \arg\max_{t_i \in [0,1], x \in X} t_i[u_i(x)]$;

(a.2) For all $i \in N_b$ (blue partisans) $x^*_i = \delta_b$ and for all $i \in N_r$ (red partisans) $x^*_i = \delta_r$;

(b.1) For all $i \in N_{br}$ (bi-partisans) there exists $x^*_i \in X$ s.t. $x^*_i = \arg\max_{t_i \in [0,1], x \in X} t_i[u_i(x)]$ if and only if $O_{br} \neq \emptyset$;

(b.2) If $O_{br} \neq \emptyset$, $x^*_i = \alpha_b$ for all $i \in N_{br}$ s.t. $\frac{\frac{\delta_b}{\delta_r}}{\frac{\delta_r}{\delta_b}} > \frac{\frac{\delta_b}{\delta_r}}{\frac{\delta_r}{\delta_b}}$ and $x^*_i = \alpha_r$ for all $i \in N_{br}$ s.t. $\frac{\frac{\delta_b}{\delta_r}}{\frac{\delta_r}{\delta_b}} < \frac{\frac{\delta_b}{\delta_r}}{\frac{\delta_r}{\delta_b}}$.

Lemma 1 shows several interesting features of citizens’ behavior implied by our assumptions. First, partisans always have the same bliss point irrespective of the parties’ polarization. This point is at the preferred ideological position of their party. Second, conflicted partisans only have a satisfactory ideological preferred position if the overlap region is not empty. When this is the case, Lemma 1 shows that the bliss points of conflicted partisans are in between the preferred ideological positions of the two parties. This means that whenever party polarization is weak, all citizens have a satisfactory ideological preferred position that takes one of four points in the political domain. Figure 2 illustrates the positions of citizens’ bliss points in a two-dimensional political domain.

3.1 Non-ideological Candidates

With unrestricted strategies, candidates can adopt any position in the political domain. In our framework, this means that candidates have “no color”. This is the standard assumption in spatial models of electoral competition. Let $V_k(\theta_1, \theta_2)$ denote the number of citizens that vote for candidate $k$ and $Pl_k(\theta_1, \theta_2)$ denote candidate $k$’s (expected) plurality given candidates’ strategies $\theta_1$ and $\theta_2$. Then, $Pl_1(\theta_1, \theta_2) = V_1(\theta_1, \theta_2) - V_2(\theta_1, \theta_2)$ and $Pl_2(\theta_1, \theta_2) = V_2(\theta_1, \theta_2) - V_1(\theta_1, \theta_2)$. Candidates choose a strategy from $X$ that maximizes plurality\(^{16}\). We consider that the candidates’ payoffs are their (expected) pluralities, which yields a zero-sum game since $Pl_1(\theta_1, \theta_2) = -Pl_2(\theta_1, \theta_2)$. Then, a strategy combination $(\theta^*_1, \theta^*_2)$ is a (pure strategy Nash) equilibrium pair if and only if:

\[
Pl_1(\theta^*_1, \theta^*_2) \geq Pl_1(\theta^*_1, \theta_2) \geq Pl_1(\theta_1, \theta^*_2) \text{ for all } \theta_1, \theta_2 \in X
\]

and if $(\theta^*_1, \theta^*_2) = (\theta^*, \theta^*)$ then $(\theta^*, \theta^*)$ is said to be a convergent equilibrium. When the number of citizens is odd, the electoral competition game admits a convergent equilibrium as follows:

Theorem 1  If $N$ is odd and $O_{br} \neq \emptyset$, then there exists a convergent equilibrium $(\theta^*, \theta^*)$ in one of the strategies $\theta^* \in \{\delta_b, \delta_r, \alpha_b, \alpha_r\}$.

\(^{16}\)This is consistent with a plurality rule election in which each citizen is allowed to vote for one and only one candidate and the candidate with most votes wins the election. See Aranson et al. (1974) for a discussion of different candidates’ objective functions and support for the rationality of maximizing expected plurality in a two-candidate plurality rule election.
This means that when an overlap region exists and \( N \) is odd then candidates converge to one of the four potential positions where the citizens’ bliss points may be located. Which one depends upon the position of the “median voter”: That point is a Condorcet winner from the candidates’ point of view, i.e., a point that wins or ties against any other alternative with majority rule. As it can be seen from Figure 2.a., this result hinges on the fact that all \( x^* = \alpha_b, \alpha_r, \delta_b, \delta_r \) are on a line segment. The location of the convergent equilibrium is restricted to one of the four positions depicted in the figure.

This is the only case in which non-ideological candidates necessarily converge to the position of the traditional median voter. Accordingly, this result can be seen as a qualification of the “median voter theorem”. In particular, it provides a sufficient (and necessary) condition for it to hold: that of weak polarization among the perceived ideological positions of the two parties. Further, this result differs from the traditional median voter theorem in that it points out four sensible positions where this convergence might occur, i.e., Theorem 1 pinpoints four reference positions in the political domain in which the median voter may be positioned \emph{ex-post}. Finally, this result indicates different levels of turnout depending upon whether candidates converge to the center of the political domain (\( \alpha_b \) or \( \alpha_r \)) or to one of the ideological poles (\( \delta_b \) or \( \delta_r \)). In the former case every citizen turns out while in the latter only the partisans that are a majority do so.

There are cases for which an overlap region exists and a convergent equilibrium is not the solution of the electoral competition game. Still, it is possible to identify circumstances under which a limited number of equilibrium pairs exist. The essential condition is that there is a majority made of partisans of one party and bi-partisans that lean towards this party (e.g. strong and lean Democrats are a majority):

\[
B_1 \quad N_b + \# \{ i \in N_{br} : \frac{\mu_i^b}{\mu_i^r} > \frac{\mu_i^r}{\mu_i^b} \} \neq N_r + \# \{ i \in N_{br} : \frac{\mu_i^b}{\mu_i^r} < \frac{\mu_i^r}{\mu_i^b} \}.
\]

This condition allows us to show that whenever an overlap region exists and candidates are unrestricted in their strategy choices then the existence of a small number of equilibrium pairs is quite general:

**Theorem 2** Suppose \( O_{br} \neq \emptyset \). Then, there exists at least one and no more than four equilibrium pairs \((\theta^*_1, \theta^*_2)\) such that \( \theta^*_1, \theta^*_2 \in \{\delta_b, \delta_r, \alpha_b, \alpha_r\} \) if and only if \( B_1 \) holds.

Condition \( B_1 \) is necessary and sufficient for Theorem 2. Nevertheless, if \( B_1 \) does not hold, the set of dominant positions is still considerably restricted. Although it is not possible to determine a precise number of equilibrium pairs, the location of the equilibrium pairs is restricted to the points within the overlap region that are on the line segment that connects the ideological points of both parties \( \delta_b \) and \( \delta_r \). All these points can guarantee a tie against each other (for some cases one of the ideological points as well), and candidates can adopt any of these positions at equilibrium.

If instead there is no overlap region, candidates almost always converge even though it is not to the position of the traditional median voter. In this case, the electoral competition game admits a convergent equilibrium as follows:

**Theorem 3** Suppose \( O_{br} = \emptyset \). Then, there exists a convergent equilibrium \((\theta^*, \theta^*)\) in one of the strategies \( \theta^* \in \{\delta_b, \delta_r\} \) if and only if \( N_b \neq N_r \).

In words, if there is no overlap between the two ideologies then candidates converge as long as the number of blue partisans is \emph{not equal} to the number of red partisans. Their point of convergence is the preferred ideological point of the party that a greater number of citizens identify with (see Figure 2.b). Otherwise (in the unlikely case that \( N_b = N_r \)), candidates may converge to the preferred ideological point of one of the parties or diverge by each adopting a distinct position at one of the two parties’ ideological points \( \delta_b \) and \( \delta_r \).

An empty overlap region can be interpreted as a case of high degree of polarization among the perceived ideologies of the two major parties or among the elites that compose them. In particular, it corresponds to a case in which no position reconciles the ideological views of both parties or elites. This result suggests that if candidates are not restricted on their strategy choices, then high degrees of polarization may not stop them from converging to the preferred ideological point of one of the parties in order to maximize plurality. Turnout decreases not only due to the conflicted voters’ abstention, but also due to the abstention of the partisans that are in minority.

When candidates do not care about ideology, and contrary to the traditional Downsian models, our framework does not predict the existence of a unique equilibrium. In this sense, our model rationalizes the empirical observation that candidates may diverge even if they are only interested in maximizing plurality. At the same time, our model captures the lower participation of citizens with moderate preferences that
in the traditional spatial voting models with abstention would be the most likely to vote. And contrary to exogenous explanations such as a sense of civic duty, the social identity motive in our model rationalizes different levels of turnout depending upon the candidates’ positions. Finally, it is worth noting that these results are not dependent upon the existence of an *ex-ante* distribution of ideal points and that nonetheless there exists a small number of equilibrium pairs for most preference distributions. This differs from many of the results of other multidimensional spatial voting models with abstention that are for uni-modal or bi-modal distributions of preferences (e.g. Davis et al. 1970; Hinich et al. 1973).

### 3.2 Ideological Candidates

In the previous subsection, we have kept the assumption of most spatial voting models in terms of the absence of restrictions upon candidates’ strategy choices. However, the assumption that candidates can adopt any position in the political domain can be unsatisfactory in many circumstances (see e.g. Downs 1957; Kramer and Klevorick 1974; Matthews 1979; Samuelson 1984)\(^{17}\). Among other examples, candidates’ party affiliations/identifications is one that seems rather salient. In many countries candidates are formally linked to parties and depend upon their support. Candidates may also want to be, and gain from being, ideologically coherent, credible, or loyal to their party. This is the case in the United States; it is also in many European countries, as the evidence from the Manifesto Research Group suggests: according to their comparative coding of party platforms in 19 European countries, candidates often vary their policies within ideologically delimited areas\(^{18}\).

Our framework conveys a natural way to restrict the set of possible strategies available for candidates that is consistent with these observations: each candidate is affiliated with one party and his strategies are restricted to the acceptance region of this party. Formally, we let candidates 1 and 2’s *strategy spaces* \(X_1, X_2 \subseteq X\) be \(X_1 = A_b\) and \(X_2 = A_r\). This can be interpreted as if candidate 1 is affiliated with party \(b\) and candidate 2 is affiliated with party \(r\). We refer to them as the blue and red candidate respectively.

Like non-ideological candidates, ideological candidates are interested in maximizing plurality. But now they choose a strategy to maximize this objective from \(X_k\) that is different from \(X\). That is, they maximize plurality bounded by “their color”. This means that a candidate can present himself to the election to maximize his plurality, even though, *a priori*, he has no possibility of winning the election. This seems a rational behavior in line with the conflict between maximizing plurality and ideological coherence. We consider that the (expected) pluralities are the candidates’ payoffs, which yields again a zero-sum game. Now, a strategy combination \((\theta_1^*, \theta_2^*)\) is a (pure strategy) equilibrium pair if and only if:

\[
Pl_1(\theta_1^*, \theta_2) \geq Pl_1(\theta_1^*, \theta_2^*) \geq Pl_1(\theta_1, \theta_2^*) \quad \text{for all } \theta_1 \in X_1 \text{ and all } \theta_2 \in X_2
\]

and if \((\theta_1^*, \theta_2^*) = (\theta^*, \theta^*)\) then \((\theta^*, \theta^*)\) is also said to be a convergent equilibrium. The next result requires that we specify two additional conditions. First, conflicted partisans need to be *pivotal*: this means that they are numerous enough, as a group, to modify the result of the election:

**C 1** \(N_b \geq \left|N_b - N_r\right|\).

Second, it is not the case that either of the parties’ ideological points is within the overlap region. This excludes cases of very strong overlap between the two ideologies (as in Figure 3.a):

**C 2** \(\delta_b, \delta_r \notin O_{br}\).

When the number of citizens is odd, the electoral competition game between ideological candidates admits a convergent equilibrium under the following conditions:

**Theorem 4** Suppose \(N\) is odd and C2 holds. Then, there exists a convergent equilibrium \((\theta^*, \theta^*)\) in one of the strategies \(\theta^* \in \{\alpha_b, \alpha_r\}\) if and only if \(O_{br} \neq \emptyset\) and C1 holds.

In words, if the overlap is not very strong, candidates converge if and only if the parties share ideological views and conflicted partisans are pivotal. In this case, candidates adopt a moderate position at the center of the political domain that leads every citizen, including conflicted partisans, to turn out.

\(^{17}\)For instance, in order to maintain credibility candidates may be constrained to strategies near a previously adopted position (Samuelson 1984), or restricted to strategies near a *status quo* such as a convergent equilibrium (Kramer and Klevorick 1974 and Coughlin and Nitzan 1981).

\(^{18}\)See e.g. Budge et al. (1987) for one of the original studies and Adams (2001) for a review.
Contrary to non-ideological candidates, the condition that conflicted partisans are pivotal is now essential for convergence. Indeed, Theorem 4’s conditions are sufficient, and with an exception, necessary for convergence. The exception is a political domain characterized by a strong overlap between the two ideologies as in Figure 3a. In this case, a convergent equilibrium may still exist even if conflicted partisans are not pivotal. In the figure’s example, this would be the case if the blue partisans were a majority. In that case, both candidates would con- verge to the blue party’s ideological point \( \delta_b \). For any other case a convergent equilibrium does not exist. This means that, as expected, convergence is harder with ideological candidates than with non-ideological candidates.

Nonetheless, even if conflicted partisans are not pivotal it is still possible to determine a restricted set of equilibrium pairs. The next result requires that there exists at least one bi-partisan leaning towards one party and one leaning towards the other party:

**Theorem 5** Suppose \( O_{br} \neq \emptyset \), \( C1 \) does not hold, \( C2 \) and \( C3 \) hold. Then, if \( N_b > N_r + N_{br} \) the equilibrium pairs \( (\theta^*_1, \theta^*_2) \) are such that \( \theta^*_1 \in ||\delta_b - \theta^*_1|| < ||\delta_b - \alpha_b|| \) and \( \theta^*_2 = \alpha_b \). If \( N_r > N_b + N_{br} \) the equilibrium pairs \( (\theta^*_1, \theta^*_2) \) are such that \( \theta^*_1 = \alpha_r \) and \( \theta^*_2 \in ||\delta_r - \theta^*_2|| < ||\delta_r - \alpha_r|| \).

Since bi-partisans are not pivotal, then partisans of one party are the majority. Suppose that these are the red partisans. In this case, the blue candidate adopts strategy \( \alpha_r \) in order to maximize his plurality, even though it will be negative. It is rational for this candidate to do so since he cannot move beyond the range of opinion perceived to be allowed by his party, and such a position at the center of the political domain guarantees the support of blue partisans and conflicted partisans against the dominant positions of the red candidate. Indeed, the red candidate can adopt any position within \( A_r \) as long as this position is not farther from \( \delta_r \) than \( \alpha_r \). Any of these strategies guarantees the support of the red partisans that is sufficient to win the election. These are the positions within the grey region in Figure 3b.

This illustrates an interesting feature of the electoral competition between two ideological candidates when conflicted voters are not pivotal. While it is in the best interest of a minoritarian candidate to adopt a position at the center to gain the support of conflicted partisans, a majoritarian candidate has the flexibility to adopt several positions around the ideology of his party. On the one hand, the minoritarian candidate attracts all bi-partisans including the ones leaning towards the other party (e.g. voters who lean Republican vote for the Democratic candidate). On the other hand, the majoritarian candidate enjoys a majoritarian partisan core of support. This allows him to adopt, inclusively, what in our framework can be seen as more extreme positions (the points on the grey region North-West of \( \delta_p \) in the example of Figure 3b). This suggests the potential for progressive behavior of majoritarian candidates and the more constrained and moderate behavior that minoritarian candidates are induced to adopt.

If instead the parties are perceived to share no ideological views, then ideological candidates can adopt any position within the acceptance regions of their respective parties, i.e.:
Theorem 6 Suppose $O_{br} = \emptyset$. Then, the equilibrium pairs $(\theta_1^*, \theta_2^*)$ are such that $\theta_1^* \in A_b$ and $\theta_2^* \in A_r$.

This suggests that candidates' behavior may be quite unpredictable in cases in which no position is perceived to reconcile the views of both parties. It implies a causal arrow from strong party or elite polarization to higher political/strategic flexibility of candidates. In particular, candidates may be able to adopt any position, even more extreme ones, as long as it is not perceived as firmly rejected by their party. This illustrates the relationship between polarization, ideological coherence, and the possibility for candidates to take progressive or more extreme stands.

With ideological candidates, all citizens turn out when polarization is perceived to be weak (Theorems 4 and 5). Otherwise, turnout is composed of the partisans of each party (Theorem 6). Conflicted voters, due to the high degree of party polarization, abstain irrespective of the candidates' positions. It thus follows that conflicted partisans may even be a majority and candidates (ideological and non-ideological alike) may still adopt positions far away from the center of the political domain. Indeed, our results suggest that if the perceived party polarization is strong then candidates favor positions closer to the preferred position of “their” partisans. This is consistent with the candidates’ typical focus on “base voters”. That is, candidates ignore conflicted partisans in favor of their electoral core of support. These results also suggest that the median, mean, and mode of the electorate may have no bearing on the outcome of the election.

Our model rationalizes both centripetal and centrifugal forces within a limited set of possible positions. We modify several of the assumptions found in the traditional Downsian model. In this respect, we follow the several attempts that have been made to modify this model to make it compatible, among other things, with the evidence that candidates in most democratic countries generally adopt different political stands (e.g. Adams 2001). According to Grofman (2004, 40-1), who reviews and defends this so-called neo-Downsian agenda, it allows us to build toward an “institution-specific and voter preference-distribution-specific theory of party competition” that has testable implications in terms of comparative statics.

Since turnout and candidates’ ideological positions vary from one election to the other, it is important to identify variables that differentiate elections. Our exercise is useful in this sense since it identifies the perception of the parties’ ideological positions and their polarization, the distribution of citizens’ identifications, and candidates’ party affiliations as such variables. Different values of these parameters imply different testable implications concerning voting and turnout behaviors and candidates’ centripetal or centrifugal movements.

4 Discussion

We have advanced a self-regarding motivation to political behavior - a social-identity motive with individual identity gains and losses - that is consistent with other-regarding behavior such as acting on behalf of the interests of the members of one’s social networks (see Gintis 2016). By doing so, we intended to provide an individually rational non-altruistic explanation of both turnout and voting behaviors. The aim of this section is to substantiate the assumptions and identify the limitations of this explanation.

4.1 A Behavioral Voting Theory

We start by pointing out additional theoretical and empirical support for some of the social cognitive processes that underlay our model. In particular, we focus on the following assumptions: (i) party identification and social identity conflicts are relevant phenomenons, (ii) some citizens behave as if they have multiple party identifications, (iii) citizens with two identifications will tend to adopt a compromise behavior, and (iv) the absence of shared ideological views by the two parties induces conflicted partisans to abstain. While we indicate additional empirical evidence in support of these hypotheses, our model aims to be a behavioral voting theory that points towards new and open avenues for empirical research.

\footnote{See Enelow and Hinich (1984) for an introduction and a defense of the spatial theory of voting as a “complete theory of voting”. See e.g. Roemer (2001) and Degan and Merlo (2011) for relevant extensions and empirical applications of this framework. See Green and Shapiro (1994) for a criticism of the value of the research on party competition models in the Downsian tradition.}
4.1.1 Party Identification and Social Identity Conflicts

The primary, and to some extent uncontroversial hypothesis in our model, is the long held idea that party identification is a strong influence in citizens’ preferences and ultimate choices (see Campbell et al. 1954 and 1960 for seminal contributions). If questions concerning the stability and primacy of this effect continue to be a focus of attention and controversy, there is by now a considerable amount of evidence on its pervasive influence (e.g. Green et al. 2002; Hershey 2015). Furthermore, the influence of a sense of attachment to different groups is backed by several experimental results that suggest that even induced social identities may influence behavior (e.g. Chen and Chen 2011). In the electoral context, experiments have shown that even minimal group identification both increases voter turnout (Schram and Sonnemans 1996; Robalo et al. 2013) and is a significant influence on how citizens make their political choices (Feddersen et al. 2009; Bassi et al. 2011).

In our model, citizens can identify not only with one but with several groups. If, to the best of our knowledge, there is no direct experimental evidence either supporting or discouraging this hypothesis in the political domain, recent experiments suggest that conflicting identities are relevant for other economically meaningful preferences (e.g. Benjamin et al. 2010; LeBoeuf et al. 2010). These studies lend support to this hypothesis by showing that it is sufficient to make one identity more salient than another (the Asian or the American identity of Asian-American subjects) to trigger different behavioral responses in terms of patience (Benjamin et al. 2010) and cooperation (LeBoeuf et al. 2010). This goes in line with earlier research suggesting that the self-concept is not monolithic but multifaceted, including as many social identities as social categories one might consider relevant to oneself (e.g. Turner 1985).

4.1.2 Multiple Party Identifications

Political behavior seems a natural instance for identity conflicts to emerge. This is indirectly supported by the evidence on the conflict among electoral values and attitudes as well as on the conflict among identifications on successive elections. For instance, Bassili (1995) finds that Canadian “conflicted partisans” take more time than unconflicted partisans, measured as response latencies to two questions, to express both their voting intentions and their party identifications. This suggests a real cognitive conflict between different identifications that can influence both turnout and voting behaviors. Hawkins and Nosek (2012) show that implicit party identification predicts political judgments among self-proclaimed independents. Regardless of the policy details, those who were self-proclaimed independents and implicitly Democratic preferred policies labeled as if coming from the Democratic party, and those who were self-proclaimed independents and implicitly Republican preferred policies labeled as if coming from the Republican party. This backs the co-existence of affects to different parties, since they can be, at least in principle, not only explicit (conscious) but also implicit (unconscious).

Niemi et al. (1987) estimate that during the 1970s and 1980s, 20% or more of Americans were split-level identifiers, i.e., citizens that identify with a different party at different levels of government (e.g. identify with a different party at the national and state levels). According to a national sample of campaign contributors and two national cross-section samples, these citizens were less likely to participate in politics than consistent identifiers (Niemi et al. 1987). Split-level identifiers were on average over 20% of citizens during the successive Canadian federal and provincial elections of 1974, 1979, and 1980 (Clarke and Stewart 1987), and up to 20% of the partisans of the main left party in the French presidential elections of 2007 (Perrineau 2007).

Taken as a whole, this evidence suggests that multiple identifications might be common and behaviorally relevant in different electoral contexts. Although most of the existing evidence is indirect and/or

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20On the topic of stability, see e.g. Campbell et al. (1960), Fiorina (1981), Green and Palmquist (1990), and Bartels (2002) for competing views.

21Social identity has been applied in economics to explain various types of economic behavior such as gender discrimination (Akerlof and Kranton 2000), contract theory (Akerlof and Kranton 2005), and equilibrium selection in cooperative games (Chen and Chen 2011). However, most, if not all theoretical contributions have focused on identity groups that are defined with respect to a single identification. See Kirman and Teschl (2006) and Davis (2007) for discussions of Akerlof and Kranton’s (2000) framework and the inclusion of multiple identities.

22In Bassili (1995) “conflicted partisans” are citizens that identify with one party but intend to vote for the candidate of a different party.

23See Green et al. (2002) for some evidence that multiple identifications are not common in the Italian electoral context. Uslaner (1989) reports a similar share of split-level identifiers in the 1979 Canadian elections as Clarke and Stewart (1987), but finds evidence suggesting that these citizens participated as much as consistent partisans in that elections.
for multiple party identifications in two different electoral instances, one can certainly envision that conflicting identifications are relevant for voting and turnout behaviors in one election. In addition, multiple identifications may or may not be the result of shared ideological views between the two parties. Very often, social identifications are ingrown human responses based on a sense of belonging to several social categories (see e.g., Turner 1982). Then, we expect multiple identifications to hold even in the absence of shared ideological views. In this light, conflicted voters are bi-conceptual that relate to two political worldviews, leaning either towards one view or towards the other irrespective of party polarization. Future empirical research may shed some light on how citizens that behave as if they have multiple (weak) identifications relate to different party ideologies and different levels of party polarization and change.

4.1.3 Compromise Heuristic

We believe that this view has particular bite in large elections, where voters seem to rely on cues and heuristics to overcome their information shortfalls (Downs 1957; Sniderman et al. 1991; Lupia and McCubbins 1998; Lau and Redlawsk 2001). The political choices that citizens are asked to make are in general complex and the information they have to reach to a decision thin (Sniderman 2000). Heuristics are “methods for arriving at satisfactory solutions with modest amounts of computation” (Simon 1990, 11), that in the electoral context allow citizens to simplify the choice itself and arrive to a rational/reasoned choice in a cost effective way (Sniderman et al. 1991; Lupia and McCubbins 1998, 2000). Party ideologies are visible and efficient cues - easy conveyable ideological stands - that provide guidance in the political landscape. The two parties provide narratives or worldviews through which citizens can understand the world of politics, and by doing so they constrain the political choices - the “choice set” - available to citizens (Sniderman 2000).

Party identification can then be seen, in part, as a judgmental heuristic (Sniderman 2000). It influences how citizens consider and weight different cues. In our model, this can be divided into two effects. First, party identification serves as an attention filter in the sense that citizens, when forming their preferences, focus on the cues given by their parties (e.g., read the news/opinion articles written by the elite of their party). These cues are seen by the voters as the behavioral prescriptions of the parties for their voting behavior. Second, one’s self-image (linked to one’s social identity) is a personal reference point that turns the act of voting as a possibility to incur in identity gains or losses, that is, to enhance or hurt one’s identity or self-concept. The more (less) compatible is a citizen’s potential vote with the ideologies of the groups she identifies with, the higher the identity gains (losses) that turn out and voting may entail. Voting, in this light, is an act that can be connected with intrinsic utility gains and losses that depend on the candidates’ positions and the behavioral prescriptions of the parties.

It follows that conflicted partisans (i) focus on the cues of the two parties and (ii) derive identity gains or losses with respect to the prescriptions of the two parties. Taken together, these hypotheses imply that the bi-partisans’ identity gains and losses depend upon whether the candidates’ positions are or not in line with the behavioral prescriptions (ideological cues) given by the two parties. Our model suggests that this results in what can be seen as a compromise heuristic: A simple process of compromise based upon the cues of the two parties that replaces complex and mixed views. According to this view, conflicted voters use a cognitive shortcut to aggregate their mixed stated preferences on different issues into a moderate revealed preference that reconciles their two identifications.

Further experimental investigation should clarify whether voters could actually resort to such a heuristic. Several reasons can in principle underlie such behavior. For instance, a compromise heuristic could be used to simplify a difficult choice (see Lau and Redlawsk 2001), to justify a decision to oneself and to others (see Simonson 1989), or reduce the cognitive dissonance created by two conflicting affects (see Festinger 1957). All these processes seem more likely to occur whenever conflicted partisans are able to endorse a position that is a compromise and reconciles the multiple ideologies they identify with.

24 But not necessarily to be resilient to strong changes in party polarization. Still, this is not implied by the static absence of shared ideological views. See the brief discussion on the “dynamics of party identifications” in subsection 4.2.

25 This cognitive shortcut is different, but in many ways related, to the compromise heuristic in Simonson (1989). The author finds that an alternative tends to gain market share when it becomes a compromise or middle option in a set. Here, conflicted partisans exhibit a preference to reconcile two ideologies and endorse, whenever they can, a compromise alternative that is in between the two more salient alternatives of the political choice set.
4.1.4 Conflicted Voter’s Curse

This compromise is, however, not always possible. This is the case whenever the two parties do not provide options which reconcile their ideological views (empty overlap region). In this situation, conflicted partisans cannot reconcile both worldviews through the act of voting. Then, it seems more difficult to justify to oneself the act of voting. It appears, as well, that a reconciliation of the potential dissonant cognitions is harder to accomplish. Finally, the anticipation of regret becomes salient when it is inevitable to betray one’s identifications through the act of voting (see subsection 2.3). If one interprets, in the sense of Schelling (1985), such identity gains and losses as “mental consumptions”, then social identity losses loom larger than gains whenever voting is not an opportunity to affirm one’s identity or to reconcile one’s multiple identifications. If parties do not provide a compromise option, conflicted voters resent both parties and are more likely to abstain.

4.2 Limitations and Extensions

Our model intends to be a simple theoretical framework with precise implications about citizens’ and candidates’ behaviors in large elections. It is thus a theory limited in several respects. At the same time, it is a theory prone to several extensions. We focus on (i) the treatment of the multiple issues/dimensions of the political domain, (ii) non-partisans and (iii) party loyalty, (iv) negative identifications, (v) probabilistic voting, and (vi) the dynamics of party identifications and (vii) of party ideologies.

4.2.1 Multiple Issues/Dimensions

One limitation of our model is the restrictive nature of preferences and perceptions, in particular when considering multiple and varied issues/dimensions. For instance, the loss function in U1 implies linear costs and that all citizens share the same level of concern over all issues. Additionally, the perceptions of the parties and candidates’ positions are also shared by all citizens. However, most citizens or groups attach different levels of concern to distinct issues, and citizens from different groups have different perceptions on parties and candidates’ positions (see e.g. Hetherington 2008). A more general formulation could attach different preferences and perceptions to different groups of citizens according to their identifications, and use weighted Euclidean preferences instead of simple Euclidean preferences. We refrained from doing so in order to focus on essentials and have general analytic results for m dimensions.

Still, we believe the model to be best suitable for political contests that concentrate over few dimensions. Our results hinge on the reduction of the dimensionality of the political domain. Citizens form their preferences based on the ideological cues given by the two parties, which shrinks the multidimensional domain into a uni-dimensional bi-polar axis. This seems to be adapted to the political debate in many countries in which the increase in party polarization has led to the reduction of the dimensionality of the political debate to a single party conflict dimension. Many issues once distinct are absorbed by a “left-right” conflict, and both citizens and politicians position themselves with relation to the ideologies of the two major parties. Such a reduction would be more difficult to defend in contexts where the debate is highly multi-dimensional and varied.

In such contexts, an alternative representation would be for mixed and moderate voters to adopt the view of one party on some issues and the view of the other party on other issues. This would picture these citizens as “eclectic voters”, rationally endorsing a mix of policies from two different ideologies. Then, it would be possible to escape the one-dimensional feature of many political taxonomies and represent the different groups that seem to compose the political center. If having more detailed models of the political center is an exciting line of future research, the non-direct link between stated and revealed preferences should not be overlooked. For instance, heuristics and cues are commonly used by voters to arrive at their revealed preferences (Lau and Redlawsk 2001). The compromise heuristic that we have put forward in subsection 4.1.3 represents one way in which some groups of mixed and moderate voters may aggregate their stated preferences into revealed preferences; and other heuristics such as partisan stereotypes (Lodge and Hamill 1986; Rahn 1993) may also play an important role in explaining the turnout and voting behaviors of the different groups that compose the political center.
4.2.2 Non-partisans

When it comes to the political sphere, we concur with the authors that hold that the rational economic maximizer is not a common actor (e.g. Green and Shapiro 1994; Gintis 2016). In our framework, we have excluded this actor by imposing that each citizen identifies at least with one group. In terms of partisan affiliation, this means that we let nonpartisans/independents outside of the model. This exclusion is backed by the evidence, at least for countries such as the U.S., that nonpartisans are a small share of the total population and an even smaller share of the voting population. In 2014, only around 13% of American mixed voters stated to have no leaning, which amounts to few more than the 5% of solid liberals and the 9% of solid conservatives that stated the same (Pew Research Center 2014). In addition, even citizens that describe themselves as political independents tend to lean towards a given party (Hawkins and Nosek 2012). This evidence suggests that most individuals have a partisan imprint in countries where two groups dominate the political sphere.

4.2.3 Party Loyalty

Still, an alternative specification of the model could include the co-existence of party/ideological cues and citizens’ ideal points. For partisans, a way of doing this would be to have a distribution of ideal points within the acceptance region of their party. For conflicted partisans, the ideal points could be distributed in the center of the political domain (between the ideological points of the two parties) and party identifications would only matter when the overlap region would be non-empty. Otherwise, they would vote solely based on their exogenous ideal point. This would make our model closer to the neo-Downsian models of party loyalty (e.g. Adams 2001; Merrill and Adams 2001). In these models, party identification produces an *ex-post* bias on the citizens’ preferences in favor of their party’s nominee, in a way that leads them to vote for the candidate of their party as long as the candidate of the opposite party is not too much closer to their ideal point. The reasons underlying this partisan bias are in general similar to the ones we have put forward here for party or group identification. Still, in our model citizens have what can be seen as an ideological bias. Citizens’ ideological preferences are influenced by the ideological cues of the parties they identify with irrespective of the party’s nominee’s identity. This means that the citizens’ ideological preferences are biased *ex-ante* by their party identifications. Candidates, either from one party or the other, need to be closer than the other candidate to the citizens’ bliss points if they wish to gain their support. In accordance, our view of party identification does not imply either straight ticket voting or partisan stereotypes; instead, it suggests an ideological bias on citizens’ preferences that do not allow them to vote for candidates, irrespective of their affiliation, that do not respect the prescriptions of the party/parties they identify with.

4.2.4 Negative Identifications

Another dimension that is absent in our model is the potential influence of negative identifications, partisan antipathies, and out-group cues to citizens’ preferences. Hostility to the opposing party is likely to be a motivator for voter turnout, and can, at least in principle, affect citizens’ ideological preferences. For instance, experimental evidence from surveys in the American context suggests that out-party cues, i.e., the endorsement of a position by the elite of the party one does not identify with, may be more powerful than in-party cues in motivating value expression (Goren et al. 2009) and polarization in public opinion (Nicholson 2012). This is in line with the view that party identification represents a meaningful group affiliation as long as it implies not only “positive sentiment for one’s own group, but also negative sentiment toward those identifying with opposing groups” (Iyengar et al., 2012, 2). If more evidence is needed to understand the relative importance of out-party and in-party cues and their relationship, adding this feature to the model is an interesting extension.

4.2.5 Probabilistic Voting

A relevant extension is to treat the vote as a non-degenerate random variable (see e.g. Coughlin 1984). For illustration, say that we assume that the probability of a citizen to vote for a given candidate increases with the (expected) utility of voting for this candidate as long as this utility is superior to that of voting for the other candidate, otherwise the probability of voting for the former candidate is zero (see e.g. Hinich et al. 1973). In our framework, partisans would then vote with the highest probability for a candidate that
adopts a position at the preferred ideological point of their party. At the same time, it would be reasonable to assume that they would have a negligible probability of turnout for any candidate at the frontier of the acceptance region of their party. As for bi-partisans, their turnout probability would increase with lower ideological polarization, but they would continue to have a negligible probability of turnout with high degrees of polarization. Instead, an alternative specification could use probabilistic voting to address the discontinuity implicit in the citizens’ turnout behavior.

Without being exhaustive, we call attention to some of the differences in electoral outcomes that would emerge with the adoption of probabilistic voting. First, and for the two types of candidates (ideological and non-ideological), candidates would be less likely to converge to the center of the political domain. This would hold since the conflicted partisans’ turnout probability would be lower than that of the partisans in the case of a weak overlap. Then, candidates would be more likely to stick closer to the ideological point of one of the parties to secure the partisans’ support. Second, turnout would be higher the lower the party polarization would be (given that an overlap region exists). This would be mostly driven by bi-partisans’ higher levels of participation. Third, turnout and electoral equilibrium locations would depend upon the weight that citizens attach to each group. This would imply, among other things, that in the absence of an overlap region each candidate would stick to the ideological point of one party in order to secure the highest turnout probability of partisans. This contrasts with our result for ideological candidates that suggests an unpredictable behavior in the case that parties share no ideological views.

4.2.6 Dynamics of Party Identifications

Another extension concerns the dynamics and evolution of citizens’ party identifications. This has deserved a lot of attention in the literature, and future research could be carried out to study it within our framework. Even if slowly, party ideologies evolve over time, and partisan realignment is an important phenomenon. For instance, Milazzo et al. (2012) find evidence that citizens’ decision rules are an endogenous function of parties’ policy positions. In our model, the weight associated to each identification could be endogenous to the parties’ respective ideologies (positions and size of the acceptance regions). Then, conflicted partisans could lean towards one or the other party from one election to the next in response to the behavior of the parties. In principle, it would be also possible to incorporate changes in identifications (partisan realignment) due to radical shifts of party ideologies. In this perspective, partisan preferences would be seen as a learning process (Key and Cummings 1966), that change according to the citizens’ perceptions of the social groups and whether they include themselves among these groups or not (see Green et al. 2002).

4.2.7 Dynamics of Party ideologies

Finally, our model can be used to analyze parties’ best ideological strategies from one election to the other. Parties compete to win public power, and a central aspect of this competition is their effort to define the terms of political choice (Sniderman 2000). As an illustration, say that citizens and candidates’ preferences and behavior are given (e.g. from the previous election); parties could then compete on the definition of their ideologies (ideological point and acceptance region) in order for their candidate to win the election. We conjecture that such an analysis would predict, among other things, that the party with a greater partisan core of support (call it the majoritarian party) would adopt a different strategy than the minoritarian party. On the one hand, the best strategy of the majoritarian party would be to try to secure a win by avoiding the perception that the two parties share ideological views (avoid an overlap region). On the other hand, and in particular if conflicted partisans were pivotal, it would be in the best interest of the minoritarian party to enlarge the political choices and find shared ideological views with the majoritarian party. If successful, conflicted partisans could lean towards the candidate of the minoritarian party and he could either tie or win against the candidate of the majoritarian party. These strategies, and their success, would depend on parameters such as the stability with respect to previously adopted positions and costs of changing ideological views. A detailed analysis of this problem is left for future research; but it can, at least in principle, connect our framework with Roemer’s (2001) model of political competition in which the dynamics within the parties are focal.
5 Conclusion

In this paper, we propose a unified rational theory of turnout and voting behaviors. We combine spatial voting with a social-psychology theory of group identification, and ask how the interaction between parties, candidates, and citizens affects the choices that ideological and non-ideological candidates offer to the citizens. The contribution of this paper is two-fold: (i) to propose a new formal framework, based on party ideologies and citizens’ group identifications, to study citizens’ turnout and voting behaviors; and (ii) to characterize ideological and non-ideological candidates’ behavior in an election with two candidates and two parties. Our model illustrates behavioral regularities of the partisans’, conflicted partisans’, and candidates’ behavior vis-à-vis the ideologies of parties or other groups. One of the main strengths of our results is that they report parsimonious propositions about citizens’ and candidates’ observable behaviors. This endows the model with sensible testable implications with respect to an original set of exogenous parameters.

Our results suggest that the particular behavior of conflicted voters may be relevant for electoral outcomes and public choice. The theoretical treatment of the behavior of these voters is one of the main novelties of the paper, which opens several empirical demanding questions. Given the difficulty of disentangling different underlying reasons behind citizens’ and candidates’ behaviors, one potential fruitful way to test these implications is through laboratory or field experiments. One could then test, among other things, different strengths of betrayal aversion for subjects with multiple identifications.

In any case, our results remain suggestive, and by no means do we wish to claim that all the predictions (neither all the assumptions) will be empirically verified. Instead, future research can take them as a benchmark and find specific forms and regularities depending on contextual factors. These predictions serve as intuitions of what could emerge, on average or as trends, in elections where two groups play a fundamental role in determining the behavior of citizens (and candidates). Seen in this light, it is possible to summarize our main implications in a few claims.

Claim 1 If there exist no shared ideological views then conflicted voters are more likely to abstain.

This captures the rational abstention of citizens that feel dissatisfied with all the possible positions available in the political domain due to excessive party polarization. While we do not expect, as our model would predict, that all bi-partisans would abstain in such a case, we find it useful to have such a yardstick for future research on the contextual determinants that may increase or lower such abstention. Furthermore, the conflicted voter’s curse rationalizes how a group of citizens may abstain even if they could, as a group, change the outcome of the election in favor of their preferred outcome.

Claim 2 If there exist shared ideological views and conflicted voters are pivotal then candidates tend to converge to the center of the political domain.

This highlights sufficient conditions for candidates to converge to a position that is in between the two parties’ ideologies. If these conditions hold, our model predicts that both non-ideological and ideological candidates behave similarly to the traditional spatial voting model and converge to the position of the “median voter”. Furthermore, our results pinpoint two reference positions in the center of the political domain in which this median voter may be positioned ex-post.

Claims 1 and 2 also expose why politics attracting the median voter might work better in times of lower political conflict. In these times, mixed and moderate voters are more likely to support a candidate that tries to conciliate the narratives of both parties. In times of greater perceived polarization, and contrary to the traditional spatial voting theory, our model suggests that it is rational for candidates to focus on their partisan core of support.

Claim 3 If there exist shared ideological views and conflicted voters are not pivotal then majoritarian ideological candidates can adopt a progressive behavior while minoritarian ideological candidates tend to adopt a position at the center of the political domain.

This means that if party polarization is weak and there are few conflicted partisans then majoritarian candidates tend to have strategic flexibility to adopt any position around the preferred ideology of their party. At the same time, it is rational for minoritarian candidates to adopt a moderate position at the center of the political domain in order to attract conflicted partisans to turn out and vote for them.

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26See Palfrey (2009) and Morton and Williams (2008) for defenses and reviews of this line of research in political economy.
Claim 4 If there exist no shared ideological views then ideological candidates can adopt any position around their party’s ideology.

This suggests that candidates’ behavior may be quite unpredictable in cases of strong perceived polarization in party ideologies. Claims 3 and 4 illustrate the relationship between party polarization, ideological coherence, and the possibility of candidates to take progressive or more extreme positions. These claims also highlight the tension between the loyalty to a party (or ideological credibility) and the will to maximize plurality.

The identification to parties or other identity groups, as a source of an ideological bias, is one possibility to reconcile turnout and voting behaviors with other types of social-cooperative behaviors. Only by resorting to behavioral theories that explain the origin, strength, and extent of the underlying reasons of these behaviors can we hope to have ex-ante rationalizations of the act of voting (Mueller 2003). Our model is a modest attempt at this, which we hope can encourage further research on multiple party identifications, ideological bias, and the behavior of conflicted voters.

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Appendix

PROOF OF PROPOSITION 1:

Proof For any \( i \in N \), suppose that it does not exist \( k \in \{1,2\} \) such that \(|\delta_p - \theta_k| \leq d_p \) for all \( p \in P \).

Proof (a.1 and a.2) Suppose \( i \in N \). From (1) we have that \( u_i(\alpha_k) > u_i(x) \) for all \( x \in X \) such that \( \delta_k \neq x \), since any deviation from \( \delta_k \) to \( x \) entails \( u_i(x) - u_i(\delta_k) = \frac{T^b}{x^b}||\delta_k - x|| < 0 \) if \( x \in A_b \) and \( u_i(x) - u_i(\delta_k) = -\frac{T^b}{x^b}||\delta_k - x|| + T^b + c^b < 0 \) otherwise.

Given Proposition 1, it follows that for all \( i \in N \) there always exists \( x^*_i \in X \) such that \( x^*_i = \arg\max_{t_i \in (0,1), x \in X} t_i[u_i(x)] = \delta_k \).

An analogous reasoning proves that for all \( i \in N \) there always exists \( x^*_i \in X \) such that \( x^*_i = \arg\max_{t_i \in (0,1), x \in X} t_i[u_i(x)] = \delta_k \).

(b.1 and b.2) Suppose \( i \in N_{br} \). Since \( \frac{T^b}{x^b} \neq \frac{T^b}{x^b} \) for any \( i \in N_{br} \), then either \( \frac{T^b}{x^b} > \frac{T^b}{x^b} \) or \( \frac{T^b}{x^b} < \frac{T^b}{x^b} \) for all \( i \in N_{br} \).

Assume \( \frac{T^b}{x^b} > \frac{T^b}{x^b} \), and that \( O_{br} \neq \emptyset \). From (1) we have that \( u_i(\alpha_k) > u_i(x) \) for all \( x \in X \) such that \( \alpha_k \notin x \), since any deviation from \( \alpha_k \) to \( x \) entails \( u_i(x) - u_i(\alpha_k) = \frac{T^b}{x^b}||\delta_b - x|| + \frac{T^b}{x^b}||\delta_b - x|| - \frac{T^b}{x^b}||\delta_b - \alpha_b|| + \frac{T^b}{x^b}||\delta_b - \alpha_b|| < 0 \) if \( x \in O_{br} \), \( u_i(x) - u_i(\alpha_k) = \frac{T^b}{x^b}||\delta_b - x|| + \frac{T^b}{x^b}||\delta_b - x|| - \frac{T^b}{x^b}||\delta_b - \alpha_b|| + \frac{T^b}{x^b}||\delta_b - \alpha_b|| < 0 \) if \( x \notin A_b \cup A_r \). It follows that for all \( i \in N \), such that \( \frac{T^b}{x^b} > \frac{T^b}{x^b} \) if \( O_{br} \neq \emptyset \) then there exists a \( x^*_i \in X \) such that \( x^*_i = \arg\max_{t_i \in (0,1), x \in X} t_i[u_i(x)] = \alpha_k \).

PROOF OF THEOREM 1:

Proof Since \( O_{br} \neq \emptyset \), it follows from Lemma 1 that for all \( i \in N \) there exists \( x^*_i \in X \) such that \( x^*_i = \arg\max_{t_i \in (0,1), x \in X} t_i[u_i(x)] \) and \( x^*_i \in \{\delta_b, \delta_r, \alpha_b, \alpha_r\} \). From P1-P2 and (1) we have that \( \delta_k, \delta_r, \alpha_b, \) and \( \alpha_r \) can all be connected with a line segment...
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For any $x \in \mathbb{R}$, let $N^*_x$ denote the number of $x^*_i$ that are on one side of the line with respect to $x$ including the ones that lie on $x$ and $N^*_x$ denote the number of $x^*_i$ that are on the other side of the line with respect to $x$ including the ones that lie on $x$. Then, since $N$ is odd, there exists $x \in \left\{ \delta_0, \delta_1, \alpha_x, \alpha_x \right\}$ such that $N^*_x \geq \frac{N}{2}$ and $N^*_x \leq \frac{N}{2}$. This point guarantees a tie while any unilateral deviation from this point either entails a loss or no strict advantage. It follows that there exists a convergent equilibrium $(\theta^*_1, \theta^*_2) = (\theta^*, \theta^*)$ at one of the strategies $\theta^* = (\delta_0, \delta_1, \alpha_x, \alpha_x)$.

**Proof of Theorem 2:**

Given Theorem 1, there are two remaining cases in which $O_{br} \neq \emptyset$ and $B_1$ holds. The first is when either $N_b = N_b + b$ or $N_b = N_b + b$. Suppose that $N_b = N_b + b$. It follows from $N_b = N_b + b$ and $N_b + N_b + b = N$ that $N_b = \frac{N}{2}$ and $N_b + b = \frac{N}{2}$. Lemma 1, A1, and B1 imply that there is at least one $i \in N_{br}$ such that $x^*_i = \alpha_x$. Then $\delta_0$ and $\delta_1$ guarantee a tie against each other while any unilateral deviation from one of these points either entails a loss or no strict advantage. This is the case since any $x \in A_1 \setminus \{O_{br}\}$ loses against $\delta_0$ and $\delta_1$ while any $x \in A_2$ such that $x \neq \alpha_x$, $\alpha_x$ guarantees at least $N_b + 1 = \frac{N}{2} + 1$ votes against any distinct $x \in A$. Thus, while $\delta_0$ and $\delta_1$ guarantee a tie against each other any deviation from one of these points does not guarantee a tie. An analogous reasoning proves that if $N_r = N_r + r$, then $\delta_0$ and $\delta_1$ guarantee a tie against each other while any deviation from one of these points does not. Therefore there exist four equilibrium pairs $(\theta^*_1, \theta^*_2)$ such that $\theta^*_1 \neq \theta^*_2 \in \{\delta_0, \alpha_x\}$ if $N_b = N_b + b$ and $\theta^*_1 \neq \theta^*_2 \in \{\delta_1, \alpha_x\}$ if $N_r = N_r + r$.

The second case is when $N$ is even and neither $N_b = N_b + b$ nor $N_r = N_r + r$ holds. Given B1, either $N_b + \#(i \in N_{br}) : \frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$ or $N_r + \#(i \in N_{br}) : \frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$ if $\theta^* = (\delta^*, \delta^*)$ at one of the strategies $\theta^* = (\delta_0, \delta_1, \alpha_x, \alpha_x)$. Assume there are $i, j \in N_{br}$ such that $\frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$ and $\frac{d^*_j}{\theta_j} < \frac{\#_j}{\theta^*}$. It follows that any point on $O_{br}$ is on the line segment that connects $\delta_0$ to $\delta_1$ can guarantee a tie against any other of these points. This implies that all the possible combinations of these points are equilibria. Note now that there are no $i, j \in N_{br}$ such that $\frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$ and $\frac{d^*_j}{\theta_j} < \frac{\#_j}{\theta^*}$. Then, A1 and $N_b + \#(i \in N_{br}) : \frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$ imply that either $N_b = N_b + b$ or $N_r = N_r + r$ holds. If $N_b = N_b + b$ then $\delta_0$ and $\delta_1$ guarantee a tie against any other of these points. The analogous holds if $N_r = N_r + r$. It follows now that there exists a convergent equilibrium $(\theta^*_1, \theta^*_2) = (\theta^*, \theta^*)$ at $\theta^* = (\delta^*, \delta^*)$ if $\theta^* = (\delta^*, \delta^*)$ at $\theta^* = (\delta^*, \delta^*)$ if $\theta^* = (\delta^*, \delta^*)$ if $\theta^* = (\delta^*, \delta^*)$ holds. Then, both $\delta_0$ and $\delta_1$ guarantee a tie against each other while any unilateral deviation from one of these points entails a loss or no strict advantage. Thus, there exist four equilibrium pairs $(\theta^*_1, \theta^*_2)$ such that $\theta^*_1 \neq \theta^*_2 \in \{\delta_0, \alpha_x\}$.

**Proof of Theorem 3:**

Since $O_{br} = \emptyset$, it follows from Lemma 1 that for all $i \in N_{br}$ it does not exist $x \in X$ such that $x = \arg\max_{t \in \{0, 1\}, i \in X} t_i[u_i(x)]$ (i.e., $t_i = 0$ maximizes utility; see also Proposition 1). Lemma 1 also implies that $x^*_i = \delta_0$ for all $i \in N_b$ and $x^*_i = \delta_1$ for all $i \in N_r$. Suppose that $N_b \neq N_r$. Then $\delta_0$ guarantees a tie while any unilateral deviation from this point either entails a loss or no strict advantage. The analogous holds if $N_r \neq N_b$. It follows now that there exists a convergent equilibrium $(\theta^*_1, \theta^*_2) = (\theta^*, \theta^*)$ at $\theta^* = (\delta^*, \delta^*)$ if $\theta^* = (\delta^*, \delta^*)$ at $\theta^* = (\delta^*, \delta^*)$ holds. Then, both $\delta_0$ and $\delta_1$ guarantee a tie against each other while any unilateral deviation from one of these points entails a loss or no strict advantage. Thus, there exist four equilibrium pairs $(\theta^*_1, \theta^*_2)$ such that $\theta^*_1 \neq \theta^*_2 \in \{\delta_0, \alpha_x\}$.

**Proof of Theorem 4:**

From C1 and N odd, we have $N_{br} > |N_b - N_r|$. Lemma 1 and $O_{br} = \emptyset$ imply that for all $i \in N$ there exists $x^*_i \in X$ such that $x^*_i = \arg\max_{t \in \{0, 1\}, i \in X} t_i[u_i(x)]$ and $x^*_i \in \{\delta_0, \delta_1, \alpha_x, \alpha_x\}$, and from P1-P2 and (1) we have that $\delta_0$, $\delta_1$, $\alpha_x$, and $\alpha_x$ can be connected with a line segment $L = \{(1 - l)\delta_0 + l \delta_1 : l \in \mathbb{R}^+\}$. Since $N$ is odd there exists $x \in X$ such that $N^*_x \geq \frac{N}{2}$ and $N^*_x \leq \frac{N}{2}$ (see proof of Theorem 1). Then, since $N_{br} > |N_b - N_r|$, this point is a convergent equilibrium $(\theta^*_1, \theta^*_2) = (\theta^*, \theta^*)$ at one of the strategies $\theta^* = (\alpha_x, \alpha_x)$.

Suppose now that $N$ is even. Then, it is possible that $N_b + \#(i \in N_{br}) : \frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$ and $N_r + \#(i \in N_{br}) : \frac{d^*_i}{\theta_i} < \frac{\#_i}{\theta^*}$ and $|N_b - N_r| = 0$ (and $O_{br} = \emptyset$). In that case, a convergent equilibrium is not the solution of the electoral competition game (see proof of Theorem 2). Now suppose that C1 does not hold, i.e., $N_b < |N_b - N_r|$. Then either $N_b > N_r + N_{br}$ or $N_r > N_b + N_{br}$. Assume $N_b > N_r + N_{br}$ and that there are $i, j \in N_{br}$ such that $\frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$ and $\frac{d^*_j}{\theta_j} < \frac{\#_j}{\theta^*}$ given that $O_{br} = \emptyset$, C2, and $X = A_2$, the closest position to $\delta_0$ that candidate 2 can adopt is $\alpha_x$. Then, since $N_b > N_r + N_{br}$, candidate 1 guarantees a win and the same maximal plurality at any $x \in A_0$ that is at a lower distance from $\delta_0$ than $\alpha_x$. Candidate 2 maximizes plurality at $\alpha_x$, since $\alpha_x$ guarantees at least $N_b + 1 = \frac{N}{2} + 1$ votes against any $x \in A_2$ and $N_r + N_{br}$ against the dominant positions of candidate 1. Any unilateral deviation from these points entails no strict advantage or a decrease in plurality. All possible combinations of these points are equilibrium pairs: There exist a continuum of equilibrium pairs $(\theta^*_1, \theta^*_2)$ such that $\theta^*_1 \in \{\delta_0 - \theta^*_0 \leq \| \delta_0 - \alpha_x \|$ and $\theta^*_2 \in \{\alpha_x \}$. An analogous result holds if there is no $i \in N_{br}$ such that $\frac{d^*_i}{\theta_i} < \frac{\#_i}{\theta^*}$ (but there exists $i \in N_{br}$ such that $\frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$). Now suppose there is no $i \in N_{br}$ such that $\frac{d^*_i}{\theta_i} > \frac{\#_i}{\theta^*}$. Then, candidate 2 guarantees the same maximal plurality as $O_{br}$, which is on the line segment that connects $\delta_0$ to $\delta_1$, while candidate 1 guarantees the same maximal plurality at the same set of points as before. All possible combinations of these points are equilibrium pairs. If we assume instead that $N_r > N_b + N_{br}$, then analogous results to these hold.
Suppose now that $O_\emptyset = \emptyset$. Given that $X_1 = A_2$ and $X_2 = A_1$, candidate 1 cannot adopt any position $x \in A_2$ and candidate 2 cannot adopt any position $x \in A_1$. It follows that candidate 1 guarantees the same maximal plurality at any $x \in A_2$ and candidate 2 guarantees the same maximal plurality at any $x \in A_1$. All possible combinations of these points are equilibrium pairs: There exist a continuum of equilibrium pairs $(\theta_1^*, \theta_2^*)$ such that $\theta_1^* \in A_2$ and $\theta_2^* \in A_1$.

**PROOF OF THEOREM 5:**

*Proof* See proof of Theorem 4.

**PROOF OF THEOREM 6:**

*Proof* See proof of Theorem 4.

References


