Effective breadth for top-hat stiffened composite structures

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ABSTRACT: Structural design often considers single beams instead of larger structures. To allow the single stiffener to be considered instead of the plate requires an assumption that the stress in the structure is carried through the stiffeners and not the attached plate, and therefore the stresses in each member do not interact.However, this assumption is not completely realistic so effective breadth has been developed to calculate an area of plate, carrying a uniform stress ensuring that the stresses are close to those in the larger structure.It is commonly used to design uniformly loaded structures such as ships, bridges and aircraft; allowing the replacement of complex and computationally expensive structural units with smaller monodimensional elements. Despite the effective breadth having been widely investigated for metallic structures specific derivations for composites are still limited, as they are still based on these original definitions. Almost every study that has been performed leads to the creation of a new formula but these studies tend not to compare back to the original larger structural units. This paper investigates the use of effective breadth for composite top-hat stiffened structures by comparing a number of definitions of effective breadth. It is shown that there is a wide variation in the different definitions and that comparison to realistic structural units is important, to ensure that the individual beams are replicating the behaviour of the full structure. The position of the stiffener is important, with intersection stresses calculated accurately but edge stresses giving poorer results, and a new formula is proposed to account for this.

Keywords: Stiffened Structures, Grillage Analysis, Composites, Rule Development, Finite Element Analysis,

# Effective breadth definitions for composites

Classical beam theory assumes that the stress distribution across the stiffened plate is uniform. However, for a plate in bending this is not the case due to the transmission of shear from the stiffeners to the plate. In a structure the stress distributions look similar to those illustrated in Figure 1a and to replicate the same stress the effective breadth, as illustrated by Figure 1b, is defined as a width over which the stress can be considered uniform.



y

x

Figure 1: Effective breadth concept a) real structure b) effective breadth

The effective breadth should not, but often is, mistaken for effective width with each concept considering a different type of loading. The phrase effective width is used when axial loads, leading to the buckling of the plate, are applied and effective breadth is used when the bending loads are applied. Different approaches have been adopted to calculate effective breadth, which has led to a number of empirical definitions and classification design rules, most of which have been derived for steel structures and these are reviewed here.

Von Karman et al. [1] were among the first who studied the effective breadth. The definition given is closer to the effective width and definitions established for a buckling plate are used for plates under bending. Schade [2] was the first to make a clear distinction between the two concepts and to analytically define the effective breadth. He demonstrates that the effective breadth depends on the loading and the boundary conditions but not on the geometry. Through this analytical study, an approximation of the effective breadth is proposed and a general analytical definition is expressed as follows:

|  |  |
| --- | --- |
|  | (1) |

where be is the effective breadth σmax is the maximum stress and σx is the stress in the x-direction. Timoshenko and Goodier [3] confirm Schade’s work by developing an analytical model for an infinitely long continuous beam on equidistant supports. They use a series solution form of the Airy’s stress function for the plane stress distribution, chosen to satisfy the boundary conditions and the principle of minimum strain energy in the beam to get values for the effective breadth. However, this study is not completely realistic as a displacement incompatibility is added at the flange-web intersection to solve their equation system. The first investigation into structural level analysis is by Clarkson [4] who investigates a parametric analysis of the effective breadth on a steel grillage structure and validates this analytical model against experiments. Unfortunately, most of these results are useable only in some specific cases of loading with some particular boundary conditions. Mansour [5] determines the influence of the boundary conditions and of the loading on the effective breadth. The effective breadth is given as a function of the L/B ratio by fitting these functions to the stresses plotted from computational methods and depends on the boundary conditions and the number of stiffeners; this is the first time that number of stiffeners is introduced. Moffatt and Dowling [6] perform a parametric finite element analysis on steel box girders. Their results and values for the shear lag are still used as a basis for the formulation of the British shear lag rules used in civil engineering.

More recently Tenchev [7] improves the effective breadth analysis by performing a parametric study considering several beam boundary conditions and beam cross-sections. A formula is proposed for longitudinally stiffened plates with high Young’s modulus to shear modulus ratio, making his formula one of the first useable for fibre reinforced composite laminated plates. Wang and Rammerstorfer [8] use a finite strip analysis to present a numerical investigation of the effective breadth. Their approach allows the study of the webs and the flanges using a plate model. Pavazza et al. [9] applies an analytical analysis to three types of deck platings for ships: without bulkheads, with a single central bulkhead and for ships with two symmetrical bulkheads; the results are validated against a finite element analysis. Katsikadelis and Sapountzakis [10] conduct an analytical study on the effective breadth by taking into account the resulting in-plane forces and deformations of the plate, the axial forces and deformations of the beam, due to the combined response of the system. Paik and Thayamballi [11] propose another formula for the effective breadth, assuming that the plate lateral deflection is proportional to where is the deflection wave length depending on the rigidity of the stiffeners and the type of load application, for stiff frames one may take , the length of the plate. A simple approximation of the effective breadth is also presented for design purposes. Boote [12] was one of the first to investigate the effective breadth of composite materials conducting a parametric study of a single top-hat stiffened fibre reinforced panel using finite element modelling. A mathematical regression is used to develop a formula dependent for the geometric parameters of the panel. The approach is based on one originally developed by Boote [13] for steel structures. Hughes and Paik [14] consider the effective breadth based on maximum bending moment, rather than maximum stress, for design purposes. Two different cases of loading are treated, uniform load and point loads. The shape of the stiffened plate is also taken into account by the shape coefficient derived directly from Schade’s calculations. Tigkas and Theodoulides, [15] perform a finite element analysis on steel T-beam reinforced panels and propose a formula inspired from Schade’s work. Recently, Ghelardi *et al.* [16] studied the effective breadth for composite materials using the same approach as Boote [12]. The Schade formula is modified by replacing the mid-plane stress distribution by the strain distribution of the most deformed ply. This approach is proposed as a more accurate evaluation of the effective breadth for the design of composite plates. This approach is extended in Boote et al. [17] for yacht hulls made of Glass Reinforced Polymers. It develops several parameters that can be adapted to the specific stiffener configuration being analysed, based on a regression from 150 FEA simulations where the breadth, L/b and thickness are investigated. The ranges for this regression are: breadths from 400-600mm, in increments of 50mm, L/b from 1.5-4, in increments of 0.5, and thicknesses of 7.70-13.89m,m in increments of 1.54mm.

Table 1: Formulas for effective breadth evaluation

|  |  |  |
| --- | --- | --- |
| **Definition – Formula** |  | **Approach** |
| Von Karman et al. [1], 1932:  thickness of plate, = Young’s modulus, maximum stress |  | Steel T-beam reinforced panel  Effective width formula applied for the effective breadth |
| Schade [2], 1951:   |  |  |  | | --- | --- | --- | |  | Approximation : |  |   plate element length, stress in the x-direction, breadth |  | Steel T-beam  One-direction stiffened panel  Analytical study |
| Timoshenko and Goodier [3], 1951:  Infinitely long continuous beam on equidistant supports, all spans are equally loaded by loads symmetrical with respect to the middle of the spans:  Infinitely long continuous beam with equal concentrated forces at the middle of the spans:  poisson’s ratio |  | Steel T-beam reinforced panel  Infinitely long T-beam  Analytical study : Fourier series |
| Clarkson [4], 1965: |  | Experiment on steel grillage  Parametric analysis |
| Mansour [5], 1970:   |  |  | | --- | --- | | Stiffener height,  Web thickness | where | |  | Longitudinally stiffened panel.  T-beam stiffened panel |
| Tenchev [7], 1995:  Stiffener width, = Normal stress at plate-stiffener intersection |  | Analytical study |
| Paik and Thayamballi [11], 2003:  = deflection wave length, can be approximated to be : |  | Analytical approach  Shape of the deflection assumed depending on the rigidity of the stiffeners and the loadings |
| Boote [12], 2003:  Base width of the stiffener |  | FEA  Composite  Parametric study.  Infinite panel stiffened longitudinally and transversely. |
| Tigkas and Theodoulides [15], 2012: |  | FEA  T-beams  Steel |
| Ghelardi et al. [16], 2014  Strain along the beam |  | Longitudinally top-hat composite stiffened panel.  FEA parametric analysis |
| Boote et al. [17]    0.863654  0.72488  4.7102 | | FEA parametric analysis |

Classification societies have also developed definitions for the effective breadth to ensure the safety of vessels and it is assumed that the definitions provide a more conservative estimate and are simpler to apply. Many of these formulas depend on the thickness of the plate and are directly derived from the Von Karman et al. [1] formula or depend on the L/B ratio and are derived from Schade [2]. Some classifications societies have adapted their approaches to suit composite materials and, except for the DNV composite formula, they all take into account the thickness, which is an important parameter for the ‘effective width’. Table 2 shows some of the different approaches that have been adopted and consequently the definitions that have been established.

Table 2: Formulas for effective breadth evaluation by the classification societies

|  |  |  |
| --- | --- | --- |
|  | **Material** | **Definition Formula** |
| Lloyd’s Register [18] | Non Specified |  |
| Lloyd’s Register [19] | Composite materials | For single skin construction : |
| Germanischer Lloyds [20] | Non Specified | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |  | 0.36 | 0.64 | 0.82 | 0.91 | 0.96 | 0.98 | 1 | 1 | |  | 0.20 | 0.37 | 0.52 | 0.65 | 0.75 | 0.84 | 0.89 | 0.9 |   for simply supported primary supporting members  =   for primary supporting members with both ends constraint  : uniformly distributed loads or else by not less than 6 equally spaced single loads  : 3 or less single loads |
| ABS [21] | Fibre Reinforced Plastic | For single skin: |
| ABS  [21] | Steel and aluminium | Steel Members :  Aluminium Members |
| R.I.Na  [22] | Non specified |  |
| DNV  [23] | Composite materials | For a simply supported beam:  = Shear modulus of flange laminate  For a beam with fixed ends:  outside the inflexion points (i.e. at the ends) is to be taken as 0.67 time the effective breadth calculated above |
| DNV  [23] | Steel and light alloy | Where  for an uniform load  for a point load |
| ISO 12215 [24] | Composite materials |  |
| Class NK Rules [25] and Korean Rules [26] | Steel and Composite materials |  |
| China Classification Society [27] | Composite | The lesser of the following: or  net breadth of secondary members |

Of the available literature only Boote [12] and Ghelardi et al. [16] have dealt with composite stiffened panels specifically. In the first case, an infinite panel has been considered which doesn’t allow for studies on the influence of the number of stiffeners and the boundary conditions. In the second case, a panel stiffened in only one direction has been studied. However, the accuracy of these methods is difficult to determine from the literature and therefore this paper investigates the effective breadth for large, finite, composite panels by comparing a number of these definitions for structures with a relatively large span. A finite element analysis model has been verified against experiments and used to compare a small one-stiffened panel and a larger multi-stiffened panel.

# Structural model development

The accuracy of effective breadth definitions when used on composite structures is investigated by using finite element method and simple beam theory calculations. The distribution of stress and strain are obtained from finite element models alongside the maximum stress of the composite structure. The maximum stress of a given composite structure is also calculated from simple beam theory by applying effective breadths calculated from various definitions and compared to the one obtained from the finite element method. Based on this comparison, existing effective breadth definitions are evaluated.

Finite element analysis is implemented in ABAQUS with a model adapted from that used in Yetman et al.'s [28] study. This model is material and geometrically non-linear and solved using the ABAQUS/standard, implicit, solver.This is performed using an automated script to generate an equivalent single layer from 8 node shell elements (S8), minimizing the computational effort required whilst providing an accurate assessment of the stiffened plate. These elements were selected over solid elements as the through-thickness stresses are expected to be low, and as the stiffeners are in bending the shell elements will provide a more accurate stiffness, as solid elements are likely to “lock” under these conditions.The stiffeners and plate are connected by multi-point constraints which restricts the degrees of freedom of the plate nodes to zero and ties them to those of the connected flange nodes, effectively pinning them together. Figures 2 and 3 represent the topology for the single stiffened panel and the grillage respectively.

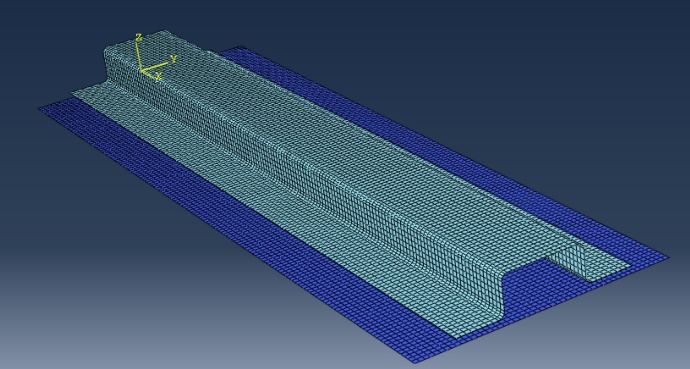


Figure 2: Topology of the single stiffened panel

Figure 3 Grillage topology

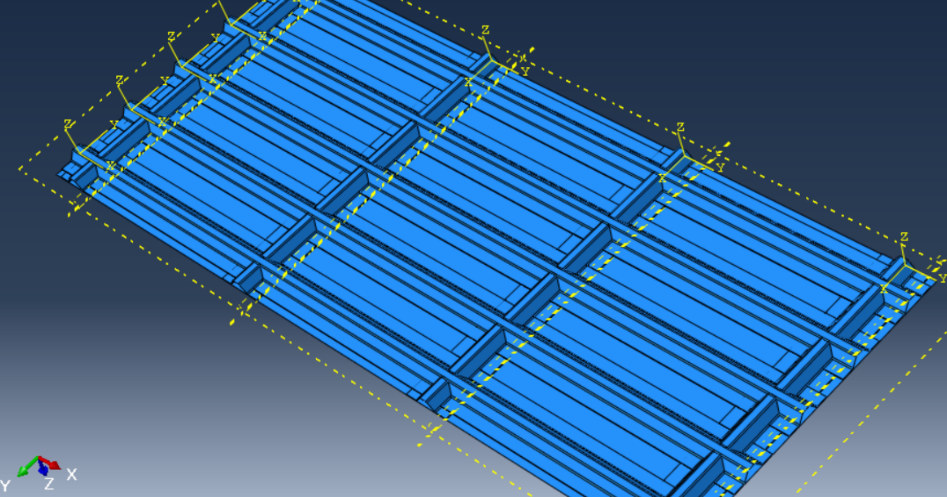


Figure 3: Grillage topology

To verify the model the results are compared to the experiments and modelling performed by Eksik *et al.* [29] on a top-hat stiffened cross panel. Table 3 shows the material used for each layer, with 10 layers for the plate and 4 extra layers for the stiffeners, the material properties of the plies are outlined in Table 4 and the locations of strain gauges are shown in Figure 4

Table 3: Composite lay-up for the Eksik *et al.* [29] top-hat stiffened cross panel

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Geometry** | **Layer** | **Zone** | **Material** | **Thickness (mm)** | **Angle (°)** |
|  | 1 | Plate | CSM 300 | 0.75 | - |
| 2 | Plate | CSM 600 | 1.50 | - |
| 3 | Plate | WR 600 | 1.00 | 0/90 |
| 4 | Plate | CSM 600 | 1.50 | - |
| 5 | Plate | CSM 600 | 1.50 | - |
| 6 | Plate | WR 600 | 1.00 | 0/90 |
| 7 | Plate | CSM 450 | 0.80 | - |
| 8 | Plate | CSM 600 | 1.50 | - |
| 9 | Plate | WR 600 | 1.00 | 0/90 |
| 10 | Plate | CSM 450 | 0.80 | - |
| 11ec | Web, Flange&  Table | CSM 600 | 1.50 | - |
| 12 | Web, Flange&  Table | CSM 600 | 1.50 | - |
| 13 | Table | UD 1600 | 2.00 | 0 |
| 14 | Web, Flange&  Table | CSM 600 | 1.50 | - |

Table 4: Materials properties for the Eksik *et al.* [29] top-hat stiffened cross panel

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Material** | **Young’s Moduli (MPa)** | | | **Shear Moduli (MPa)** | | | **Poisson’s Ratio** | | |
| **E1** | **E2** | **E3** | **G12** | **G13** | **G23** | **ν12** | **ν13** | **ν23** |
| Chopped Strand Mat  (CSM) 300g/m2 | 8000 | 8000 | 8000 | 3100 | 3100 | 3100 | 0.3 | 0.3 | 0.3 |
| Chopped Strand Mat  (CSM) 600g/m2 | 6800 | 6800 | 6800 | 2600 | 2600 | 2600 | 0.3 | 0.3 | 0.3 |
| Chopped Strand Mat  (CSM) 450g/m2 | 7300 | 7300 | 7300 | 2800 | 2800 | 2800 | 0.3 | 0.3 | 0.3 |
| Woven Rowing  (WR) 600g/m2 | 14800 | 14800 | 3000 | 3400 | 3600 | 3600 | 0.092 | 0.092 | 0.092 |
| Unidirectional (UD) 1600g/m2 | 24600 | 7300 | 7300 | 4200 | 3500 | 3500 | 0.39 | 0.39 | 0.39 |

In order to simulate the boundary conditions the four plate corners are clamped and the stiffener’s ends and plate edges are pinned. Uniform pressures, with 0.025 MPa increments, are applied until a maximum pressure of 0.3 MPa. The micro strains at each location are computed from the FEA model.

800

120

850

50

120

43.37

100

130

43.75

Y

X

SG1

SG2

SG3

SG6

SG5

SG4

SG7

SG8

SG9

SG10

SG13

SG12

SG11

SG14

Unloaded face

Loaded face

120

120

120

120

Figure 4: Strain gauges locations

Verification results are shown in Figure 5 for SG1 and SG5 in comparison to the experimental and linear FEA results from Eksik et al. [29]. Figure 6 shows the micro stains at SG12, which is located at the web of the stiffener. A good correlation is shown between the FEA models and the experiments with some discrepancies which are assumed to be due to the uncertainties in the boundary conditions, the manufacturing quality, and small differences in strain gauge location. The discrepancies between the two FEA results are based on the linear models used in Eksik et al. [29], compared to the results presented in this paper which use a non-linear solver, which gives a more accurate prediction.All of the strain locations from Figure 4 are replicated with a good level of accuracy.

Figure 5: Comparison of x directional micro strain at SG1 and SG5

A convergence analysis is also conducted by measuring the strains, stresses, and displacements at all of the locations, Figure 7 shows the stress convergence results for SG 14 which show a similar trend to the results for the stress, strain and displacement at the other locations. The results show that model converges for 150000 elements, which are selected to be used in the analysis.

Figure 6: Comparison of y directional micro strain SG12

Figure 7: Convergence of stress for SG 14

# Maximum stress comparison

The maximum stresses occur at the intersections in composite grillages when the deflection of the plate is relatively large, while the maximum stresses occur at the edges when the deflection is relatively small. Therefore, this section is divided into three parts, the first part analyses the case where the maximum stress occurs at the intersection, the second part analyses the case that maximum stress occurs at the edges and the final part analyses the maximum stresses from large grillages. The loads applied are 0.01MPa, 0.03MPa, and 0.1MPa, L/B ratios that used are 1.5, 2.0, 2.5 and 3.0, within these parameters the model behaves like a beam in bending and are standard ratios for boatbuilding.

## Maximum stress at the stiffener intersection

The effective breadth definitions are divided into two categories, definitions developed from state-of the-art literature, Figure 8, and those provided by classification societies, Figure 9. In these cases the maximum stresses from the finite element results are obtained from the intersection between the stiffener and the plate, while the stress for the effective breadth is determined using beam theory and calculated in the x-direction of the plate. The results from the different definitions show a similar trend to each other with most of the definitions underestimating or slightly overestimating the maximum stress at low L/B ratio and overestimating it at high L/B ratio, as shown in Figure 8. This occurs as when the deflection of the model is small, local bending of the plate dominates the structure and the maximum stress remains at a constant level. At higher L/B the stress increases and it is this trend that effective breadth definitions have been developed to follow. Beam theory does not take such effects into account. The results for Von Karman’s definition show a rapid increase in stress with higher L/B ratio showing a large separation giving the largest error at L/B of 3 at 13.79MPa. This profile is similar to the definitions developed by Paik and Theodoulides and Clarkson. Of the definitions the results are normally conservative but Ghelardi et al., Tenchev and Schade all underestimate the stress at low L/B ratios. The Schade approximation follows the FEA most closely, giving a consistent separation and a slightly conservative value of 35% but as the stresses are low this only corresponds to a stress of 1.3 MPa.

Figure 8: Maximum stress comparison of literature definitions at the intersection for a load of 0.01MPa

The classification societies definitions all provide a higher stress prediction than the FEA model; however there is a large range in the errors produced. The ISO 12215 provides the safest results, generally giving the highest predicted stress with an increasing separation between the rules and the FEA at high L/B resulting in an over prediction of the stress by 17.92MPa or 215%. The ABS and R.I.Na rules provide a more efficient structure, with a closer prediction of the stress where ABS provides a minimum error of 4% a L/B of 1.5. All the rules provide closer stress predictions at lower L/B ratios, as shown in Figure 9, with DNV providing the worst prediction at L/B of 1.5 at an error of 5.28 MPa or 97% despite providing the most similar trend to the FEA.

Figure 9: Maximum stress comparison for classification societies at the intersection for a load of 0.01MPa

Results computed from loads at 0.03MPa are shown in Figures 10 and 11. Maximum stresses computed from all these definitions are above the maximum computed from the finite element analysis, which indicates these definition provide conservative estimates for small panels. Once again the academic definitions provide accurate assessments at a L/B of 1.5, with a maximum error for the Boote et al . approximation of 118% or 6.3MPa and a minimum error for the Tenchev definition of -2.8% or 0.15MPa, but at higher L/B these estimates are worse, with the maximum error shown by the von Karman definition of 197.7% or 28.75 MPa and a minimum error for the Schade approximation of 4.62 MPa or 31.77%; Tenchev provides the best prediction overall. The classification society rules all provide more conservative results with ABS providing a minimum error of 1.28MPa or 23.9% at L/B 1.5 and R.i.Na providing a minimum error of 8.95MPa or 23.49%. However, the maximum errors are similar to the academic definitions with DNV giving a maximum error of 12.11MPa or 126.35% and ISO 12215 giving an error of 26.5MPa or 182.25%.

Figure 10: Maximum stress comparison of literature definitions at the intersection for a load = 0.03MPa

Figure 11: Maximum stress comparison for classification societies at the intersection for a load of 0.03MPa

At higher loads the maximum stress of the FEA analysis reduces after a L/B ratio higher than 2, shown in Figures 12 and 13. In this loading condition, the panel and stiffener are over loaded, the model is not in a pure bending condition, strains in some parts of the midsection of the plate are positive and the two ends start to pull the plate away in opposite directions. Therefore, the compression stress at the top of surface reduces. In this case, the maximum tension stress is not considered as it not included in the concept of effective breadth. Results from this loading condition show that effective breadth calculated from various definitions are still safe when this extreme load is applied.

Figure 12: Maximum stress comparison for literature definitions at the intersection for a load of 0.1MPa

Schade’s approximation is the most efficient method for calculating effective breadth of composite structures at the intersection between the stiffeners. For the other definitions and classification rules a reasonable safety factor needs to be applied when the load and L/B ratio are relatively low. When the L/B ratio is above 3 and a fixed boundary is applied, the beam might not be able to be considered in pure bending. In this case, the effective breadth approaches a value of 1.

Figure 13: Maximum stress comparison for classification societies at the intersection for a load of 0.1MPa

## Maximum stress occurring at the edges

Due to local bending, there is a situation where the maximum stress occurs at the edges. Normally, this happens when the plate is too thick or a relatively low L/B ratio is applied. In order to simulate this situation, a small panel with a 20mm thick plate is examined. In this case, the most critical ply is the bottom ply which is in tension. The maximum stresses are obtained from the finite element model at the edges of this ply and the effective breadth is calculated. Definitions are grouped in the same manner as the previous section.

The results computed from the 0.01MPa loading condition are plotted in Figures 14 and 15. The results from the other loading conditions are similar to the 0.01MPa condition and are therefore not documented. The finite element results show that the maximum stresses obtained from different L/B ratios are almost constant, and only slightly increase when L/B ratio is 3. However, the results computed from beam theory increase along with the growth of L/B ratio. In this case, by applying effective breadths to the beam theory cannot predict precise maximum stresses, the results are underestimated when the L/B ratio is less than 2, except in the case of the Schade and Boote et al. criteria, and overestimated when the L/B ratio is higher. Therefore, the effective breadths applied to beam theory needs to be modified for these conditions.

Figure 14: Maximum stress comparison of literature definitions at the edges for a load of 0.01MPa

The original Schade’s definition is modified where the predicted maximum stresses from Schade’s match the finite element results at L/B = 2. At this point the effective breadth applied to beam theory is appropriate, before this point it is too large and after this point it is too small. Thus a formula is developed and shown in equation 2;

. (2)

In this formula C is a constant determined by the L/B ratio where the beam theory estimation curve matches the finite element curve; K is a constant determined by the slope of the beam theory estimation curve.

Figure 15: Maximum stress comparison for classification societies at the edges for a load of 0.01MPa

The value of the constant, C, is determined by loads applied to the structures. However, the numerical value of L/B ratio when Schade’s curve matches the finite element curve increases slightly if higher loads are applied. In this case, C can be considered as 2.3. Constant K is used to adjust the difference of the slope between the beam theory curve and the finite element curve; these values are closer to each other when higher loads are applied. The deflection of the whole panel increases when higher loads are applied so that the plate contributes more to the resistance, at this point the maximum stresses at the edges are close to that expected from beam theory. Once the loads increase further the plate can be considered to be thin, despite its 20mm thickness. At this point the whole panel can be considered the same as at the intersection. When the maximum stress occurs at the edges, the constant K can be considered to be 0.35. By applying equation 2, the effective breadths are modified and provide more accurate results. The maximum stresses computed from beam theory using the modified effective breadths are shown in Figures 16 and 17. Figure 16 has larger errors, 8% at an effective breadth of 1.5 and up to around 40%, though the difference in stress is less than 2 MPa. Figure 17 shows that the accuracy is higher at larger loads, with an average error of 11.8% and a maximum error of 14.6% at the higher L/B ratios.

Figure 16: Modified formula stress prediction at the edges for a load of 0.01MPa

Figure 17: Modified formula stress prediction at the edges for a load of 0.03MPa

## Maximum stresses obtained from grillages

Similar to most of the existing studies, the effective breadth has been investigated using a single stiffened panel to represent a single stiffener of a grillage. Although the above comparisons show that the maximum stresses obtained from beam theory match the maximum stresses obtained from single stiffened panels well, it is not clear whether beam theory prediction can match the maximum stresses obtained from grillages. This section analyses the behaviour of grillages and compares the maximum stresses to beam theory estimations.

A grillage with 5 longitudinal stiffeners and 4 transverse stiffeners is developed, shown in Figure 18. The red dotted line shows the location of the single panel and the yellow dotted line indicates the place where the maximum stresses occur. The boundary conditions of the real grillage is neither totally fixed nor simply supported and therefore the boundary condition applied to the grillage model is shown in Figure 18. A comparison is made between this boundary condition and a clamped condition. In these conditions the stresses at the boundary are large and therefore viewed as being unrealistic; these results are therefore not included.The green lines represent the three directional displacements are zero and free to rotate. The red circles represent that the four corners of the grillage are totally fixed. In this case, the original Eksik et al.’s composite lay-up is applied, as it is taken from an actual ship structure, in order to ensure a reasonable thickness is applied to the grillage. Loads of 0.01Mpa and 0.02Mpa are applied because these two loading conditions are enough to show the differences in the analysis, but are considered to be low for most ship applications. Tests are repeated at L/B ratios of 1.5, 2, 2.5, and 3.0.

X

Y

Figure 18: Grillage configuration in comparison to the beam

Results computed from various models and loading conditions show that the grillage behaves differently from a small panel. One of the stress distributions is shown in Figure 19. The entire plate experiences tension at the midsection of grillage, this might be because boundary conditions at the edges are too rigid but these replicate the connection to the girder, which are a relatively fixed condition, and so are judged to be reasonable. Once loads are applied, the elongation of the plate is larger than the compression due to bending. Therefore, strains at midsection of grillage are positive so that the entire plate is in tension. While the plate experiences mostly compression at the midsection of a small panel. Hence, stiffeners in the grillage cannot be considered to be in pure bending. In this case, the maximum stresses occur at ply1 on the outside of the plate. However, the boundaries at the single stiffener are more realistic than those in the previous studies as they are formed by the other stiffeners in the grillage and utilising the large structural unit increases the realism of this boundary condition.

Figure 19: Strain distribution across the centre of the large grillage at L/B ratio of 2.5 and a load of 0.01 MPa

The maximum stresses obtained from a grillage with a load of 0.01MPa are plotted against different effective breadth definitions in Figures 20 and 21. The maximum stresses from the grillage are larger than the stresses obtained from the different definitions in most of the cases. Although beam theory results are more similar to the grillage results at high L/B ratios, and in some cases provide conservative estimates, the beam theory calculations cannot predict the maximum stresses of the grillage for most cases. As the load is larger the difference at lower L/B ratios is less, but never reaches the value from the grillage. At higher L/B ratios then the prediction is better, as the grillage stresses reduce and the different definitions have a steep increase in stress with increasing L/B. The difference in results is because the predicted stresses are compression stresses, while the maximum stresses in the grillage are in tension, particularly for composite materials. In this case there are further difficulties than just the prediction as the tensile strength and compression strength are also different for composite materials.

Figure 20: Maximum stress comparison to the grillage using literature definitions at a load of 0.01MPa

Figure 21: Maximum stress comparison to the grillage using classification society definitions at a load of 0.01MPa

# Recommendations

Simple beam theory and effective breadth cannot be used for estimating the maximum stress of composites for several reasons. First, simple beam theory is only suitable for predicting the maximum stress of models that behave like beams in bending, while small panels with fixed end boundary conditions only can be considered as a beam within a certain range of parameters, these ranges are determined by the stiffener sizes and loading conditions. Large grillages behave like beams under tensile loads. These phenomena restrict the usage of simple beam theory and effective breadth. Second, uncertainties in the composite lay-up can lead to difficulties in calculating the effective breadth. It is hard to determine which ply in the plate is the most critical ply. Generally, the outmost ply is selected to calculate the effective breadth. However, parametric studies show that it cannot represent the effective breadth of the whole panel. Third, local bending of composite plates have a large influence on the edges. Therefore, the maximum stress might not occur at the intersection.

For the case that that the maximum stress occurs at the intersection Schade’s [2] approximation formula is suggested. In this case, the model is considered as a beam in bending and the stiffener dominates the structure. The maximum stress comparison shows that Schade’s approximation’s results are closest to the finite element model. This indicates that the effectiveness of composite materials to resist a load are determined by the entire plate, the same as for isotropic materials. For the case where the maximum stress occurs at the edges, the maximum stress is caused by local bending of the composite plate. Based on the verification it appears that the deflection shapes accurately replicate those in a stiffened structure. However, the models do not consider the foam core inside of the stiffener, and while it is common not to model this component due to its small influence on the structural rigidity, further investigations into the interaction between the foam core and the composite stiffener are necessary to ensure the validity of the local bending.

Equation 2 modifies the effective breadth of the composite plate at low and high L/B ratios to increase the predicted maximum stress at low L/B ratio and decreases the predicted maximum stress at high L/B ratio. The modified results match finite element results well. Simple beam theory might not be suitable for estimating the maximum stress of large grillages. The most critical panel in a grillage is the central one. In the longitudinal direction, this panel experiences tension stresses rather than compressive ones and for most L/B ratios the effective breadths do not predict the stresses accurately and follow a different trend. This could be an issue with large composite structures where there are larger gaps between bulkheads and these structures need to be considered with appropriate methods.

# Conclusions

Effective breadthis commonly used to design uniformly loaded structures in the marine, civil and aerospace industries by simplifying the structure to a smaller monodimensional element. Despite its wide usage specific derivations for composites are still limited with most based on those for metallic structures. This paper investigates a number of definitions for composite top-hat stiffened structures taken from literature and classification societies. Three main recommendations are made:

1. when the maximum stress occurs at the intersection the current Schade formulation is correct;
2. if the maximum stress occurs at the edge of the plate the current formulations are inaccurate and a new formulation is determined;
3. the single beam theory calculations do not match grillage theory and comparisons to larger structures should be the focus of future research in this area.

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#### **References**

[1] T. von Kármán, E. Sechier, & H. Donnell, The strength of thin plates in compression. *Transactions of the*[*American Society of Mechanical Engineers*](http://en.wikipedia.org/wiki/American_Society_of_Mechanical_Engineers)*,* 54:53-57, 1932*.*

[2] H. Schade, “The effective breadth of stiffened plating under bending loads,” *Transactions of The Society of Naval Architects and Marine Engineers,* vol. 59, 1951.

[3] S. Timoshenko and J. N. Goodier, “Effective Width of Wide Beam Flanges,” in *Theory of Elasticity*, McGraw-Hill Book Company, 1951, pp. 171-177.

[4] J. Clarkson, The Elastic Analysis of Flat Grillages with Particular Reference to Ship Structure, Cambridge at the University Press, 1965.

[5] A. Mansour, “Effective flange breadth of stiffened plates under axial tensile load or uniform bending moment,” *Journal of Ship Research,* vol. 14, pp. 8-14, March 1970.

[6] K. Moffatt and P. Dowling, “Shear Lag in steel Box girder bridges,” *The Structural Engineer,* 1975.

[7] R. Tenchev, “Shear Lag in Orthotropic Beam Flanges and PLates with Stiffeners,” *Journal of Solids Structures,* vol. 33, pp. 1317-1334, 1995.

[8] X. Wang and F. Rammerstorfer, “Determination of Effective Breadth and Effective Width of Stiffened Plates by Finite Strip Analysis,” in *Thin Walled Structures,* Vol.26, No. 4, Elsevier Science Ltd, 1996, pp. 261-286.

[9] R. Pavazza, B. Plazibat and A. Matokovic, “On the Effective Breadth Problem of Deck Plating of Ships with Longitudinal Bulkheads,” *International Shipbuilding Progress,* vol. 48, pp. 51-85, 2001.

[10] J. T. Katsikadelis and E. J. Sapountzakis, “A realistic estimation ot the effective breadth of ribbed plates,” *International Journal of Solids and Structures,* vol. 39, 2002.

[11] J. Paik and A. Thayamballi, “Effective Breadth/Width of Attached Plating,” in *Ultimate Limit State Design of Steel-Plated Structures*, John Wiley and Sons, 2003, pp. 51-58.

[12] D. Boote, “Parametric evaluation of the effective breadth for GRP beams with FEM calculation,” P*roceedings of the 12th International Congress of the International Maritime Association of the Mediterranean, IMAM 2007,* Volume 2, 2008, Pages 979-988.

[13] D. Boote and D. Mascia, On the Effective Breadth in Stiffened Platings, Ocean Engineering, Vol.18, No.6, 1991.

[14] O. F. Hughes and J. K. Paik, “Shear Effects and Other Departures from Simple Beam Theory,” in *Ship Structural Analysis and Design*, Jersey City, New Jersey, The Society of Naval Architects and Marine Engineers, 2010.

[15] I. Tigkas and A. Theodoulides, “On the effective breadth of plating,” in *Sustainable Maritime Transportation and Exploitation of Sea Resources*, 2012, pp. 281-288.

[16] S. Ghelardi, M. Gaiotti and C. M. Rizzo, “On the shear lag effective breadth concept for composite hull structures,” *Ships and Offshore Structures,* 2014.

[17] D. Boote, G. Vergassola, L. Strano "A Numerical Determination of Effective Breadth of Grp Stiffened Plates." *The Transactions of the Royal Institution of Naval Architects Part B: International Journal of Small Craft Technology*, vol. 158, p. 49-61, doi: 10.3940/rina.ijsct.2016.b1.187",

[18] Lloyd's Register, “Rules for the Classification of Trimarans; Scantling Determination,” January 2006.

[19] Lloyd's Register, “Rules and Regulations for the Classification of Special Service Craft; Part 8 Hull construction in composite,” 2011.

[20] Germanischer Lloyd, “Rules for Classification and Construction, Ship Technology, Special Craft, Chapter1, Section 2 Hull Strcutures,” 2003.

[21] ABS, “Guide for building and classing, high speed craft, Part 3, Chapter 1, Section 2,” 2014.

[22] R.I.Na, “Rules for the Classification of Pleasure Yachts, Part B Hull and Stability,” 2011.

[23] DNV, “Rules for Classification of High Speed, Light Craft and Naval Surface Craft,” January 2013

[24] ISO 12215, “Hull Construction Scantlings – Part 5: Design Pressure for monohulls, design stresses, scantlings determination”, 2004

[25] Class NK, “Rules for High Speed Craft”, 2015

[26] Korean Register, ”Rules for the Classification of Steel Ships”, Part 10 Hull Structure and Equipment of Small Steel Ships, 2010

[27] China Classification Society, “Rules for Construction of Yachts”, 2008.

[28] J.E.Yetman A.J. Sobey J.I.R. Blake and R.A. Shenoi "Investigation into skin stiffener debonding of top-hat stiffened composite structures” Composite Structures, vol. 132, pp. 1-32, 2015

[29] O. Eksik, R.A. Shenoi, S.S.J. Moy, H.K. Jeong, “Experiments on Top-Hat-Stiffened Panels of Fiber-Reinforced-Plastic Boat Structures,” *Marine Technology,* vol. 44, pp. 1-15, January 2007.