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# THE USE OF SYMMETRY IN SEAKEEPING CALCULATIONS

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## THE USE OF SYMMETRY IN SEAKEEPING CALCULATIONS

For some ten years seakeeping programs have existed that are used fairly routinely to calculate the principal rigid body motions of a ship. During this timescale the main interest of naval architects has centred, in these studies, around statistical responses of the motion of the centre of gravity of the ship. The normal method of calculation of these statistical responses has been via the usual linear superposition technique. In this instance the response of the ships motions is calculated for unidirectional seas for the ship moving at constant speed, and constant angle to the sea. To generate this statistical response the usual method has been via a power spectral density function that describes a given sea state using typically two bulk parameters. The normal parameters are modal period and significant wave height.

Obviously the need of the naval architect has progressed from this rather simplistic standpoint to much wider horizons of calculation. One area that is of interest is to calculate the motion of an arbitrary point on the ship relative to the sea surface. This may be required, for instance, to assess the probable deck wetness of the bow area of a new ship design. This calculation may be required for all ship headings from following seas to head seas and then back to following seas on the opposite ship side. Another reason for the need to calculate the motion of a point on the ship at `all angles` is to allow for what is termed short crested seas.

In the first paragraph the ordinary sea responses are calculated assuming long crestedness. This means the seas are unidirectional. Thus equation 1 gives the method of calculation

$$H^S(\mu) = \int_0^\infty H^2(\omega,\mu)\, S(\omega)\, d\omega \tag{1}$$

$S(\omega)$ is the power spectral density function, which could be typically the Bretschneider spectrum

$$S(\omega) = \frac{AB}{\omega^5} e^{-B/\omega^4} \tag{2}$$

$$A = 0.25\, H_{1/3}^2 \tag{3}$$

$$B = \frac{691.16}{T^4} \tag{4}$$

$H(\omega,\mu)$ is the response in heave, say, for all wave frequencies $\omega$, and wave angles $\mu$. T is the mean period.

However for short crested seas the following calculation is performed:

$$H^{s}(\mu) = \int_{-\pi/2}^{\pi/2} \int_{0}^{\infty} H^{2}(\omega,\ \mu-\alpha)\ S(\omega)\ Sp(\alpha)\ d\alpha d\omega \qquad (5)$$

$\alpha \qquad \omega$

$Sp(\alpha)$ is the spreading function.

This allows for the responses of seas, that are not in the direction of motion of the ships having some effect on the rigid body motion. In particular, the unidirectional response of a ship in roll in head seas is zero. In short crested seas the ship roll response in the predominant direction of head seas is then non-zero. This is because the unidirectional roll responses in directions other than head and following seas is non-zero in general. These will contribute to the head sea case in the short crested case, via the formulation in equation 5.

To calculate the motion at a point in spread seas requires the calculation of the motion at all wave headings. This can be a costly time consuming exercise. The use of symmetry usually allows naval architects to calculate the motions from following seas to head seas and them use symmetry to produce the same response for the equivalent wave angle on the opposite side of the ship. Thus for example, the responses of the centre of gravity at an angle $\mu$ to the wave system are numerically the same magnitude as for a wave angle $2\pi-\mu$. Figure 1 shows this effect, which is purely because of the longitudinal symmetry of the ship.

At a general point P with coordinate (x, y, z) relative to the centre of gravity of the ship, the total motion is found via the vector sum:

$$\underset{\sim}{T} = \underset{\sim}{b} + \underset{\sim}{\Omega} \times \underset{\sim}{r} \qquad (6)$$

$\underset{\sim}{b}$ is the body displacement vector along the three axes.

$\underset{\sim}{\Omega}$ is the body rotational vector along the three axes.

Figure 2 gives the conventions and names for the six components.

The vector $$\underset{\sim}{b} = (Su, Sw, He) \qquad (7)$$

and $$\underset{\sim}{\Omega} = (Ro, Pi, Ya) \qquad (8)$$

$$\underset{\sim}{r} = (x, y, z) \qquad (9)$$

The components in both equations 7 and 8 have an assumed dependence of $e^{i(\omega_e t+\varepsilon)}$, where $\varepsilon$ is the phase angle of the motion relative to the forcing wave. $\omega_e$ is the encounter frequency of the waves to the ship.

$$\omega_e = \omega - \frac{\omega^2 V}{g} \cos\mu$$

$\omega$ is the wave frequency

$V$ is the ship speed

$\mu$ is the heading angle.

Thus the components of motion from equation 6 are

$$(Su + z\ Pi - y\ Ya,\ Sw + x\ Ya - z\ Ro,\ He + y\ Ro - x\ Pi)$$

Thus at the point P (x, y, z) the absolute heave motion is

$$He + y\ Ro - x\ Pi \qquad (10)$$

If the wave that is forcing the ship to move is given by

$$A\ e^{-ik(x\cos\mu + y\sin\mu)}\ e^{i\omega_e t} \qquad (11)$$

where k is the wave number and equal to $\omega^2/g$ for deep water. Then surpressing the time variation, the absolute heave motion relative to the sea surface is

$$RV = He + y\,Ro - x\,Pi - A\,e^{-ik(x\cos\mu + y\sin\mu)} \qquad (12)$$

define a function $F(x, y, \mu)$ as

$$F(x, y, \mu) = RV - He + x\,Pi \qquad (13)$$

$$\therefore \quad F(x, y, \mu) = y\,Ro - A\,e^{-ik(x\cos\mu + y\sin\mu)} \qquad (14)$$

Since, as previously stated, the ship is symmetrical then this allows the motion of the ship to be classed into two groups, symmetric and antisymmetric. The symmetric group are Heave, Pitch and Surge, thus for heading angles between $(0,\pi)$ the ship responds exactly the same as for angles $(\pi, 2\pi)$. The anti-symmetric group, (Roll, Sway and Yaw) are such that the phase of the motion is opposite for an equivalent angle in the ranges $(0,\pi)$ and $(\pi,2\pi)$. To summarise for two equivalent wave angles $\mu$, and $2\pi-\mu$.

$$He(\mu) = + He\,(2\pi-\mu)$$

$$Ro(\mu) = - Ro\,(2\pi-\mu)$$

$$Pi(\mu) = + Pi\,(2\pi-\mu)$$

$$Sw(\mu) = - Sw\ (2\pi-\mu)$$

$$Ya(\mu) = - Ya\ (2\pi-\mu)$$

$$Su(\mu) = + Su\ (2\pi-\mu)$$

Thus returning to equation 14, for these two wave angles:

$$F(x,\ y,\ \mu) = y\ Ro(\mu) - A\ e^{-ik(x\cos\mu\ +\ y\sin\mu)}$$

$$F(x,\ y,\ 2\pi-\mu) = y\ Ro(2\pi-\mu) - A\ e^{-ik(x\cos\mu\ -\ y\sin\mu)}$$

Thus substituting for $R_o(2\pi-\mu)$ gives

$$F(y,\ 2\pi-\mu) = -y\ Ro(\mu) - A\ e^{-ik(x\cos\mu\ -\ y\sin\mu)}$$

$$\therefore \quad F(y,\ 2\ -\mu) = F(-y,\mu) \qquad (15)$$

The physical reality of equation 15 is obvious if Figure 3 is studied. If the points P and S are equi-distant either side of the ship centreline then the relative vertical motion at P in waves of direction $\mu$, is exactly the same as for S in waves of heading angle $2\pi-\mu$.

Thus the calculation for spread responses for short crested seas in predominant directions of angles greater than 90 degrees can make use of the relationship given in equation 15. This obviously reduces the computation time by about a factor of 2.

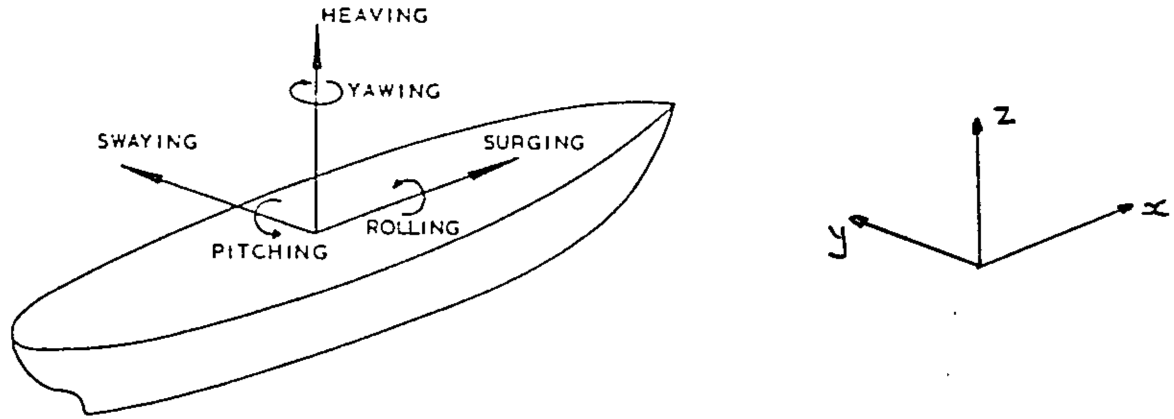


FIGURE 1

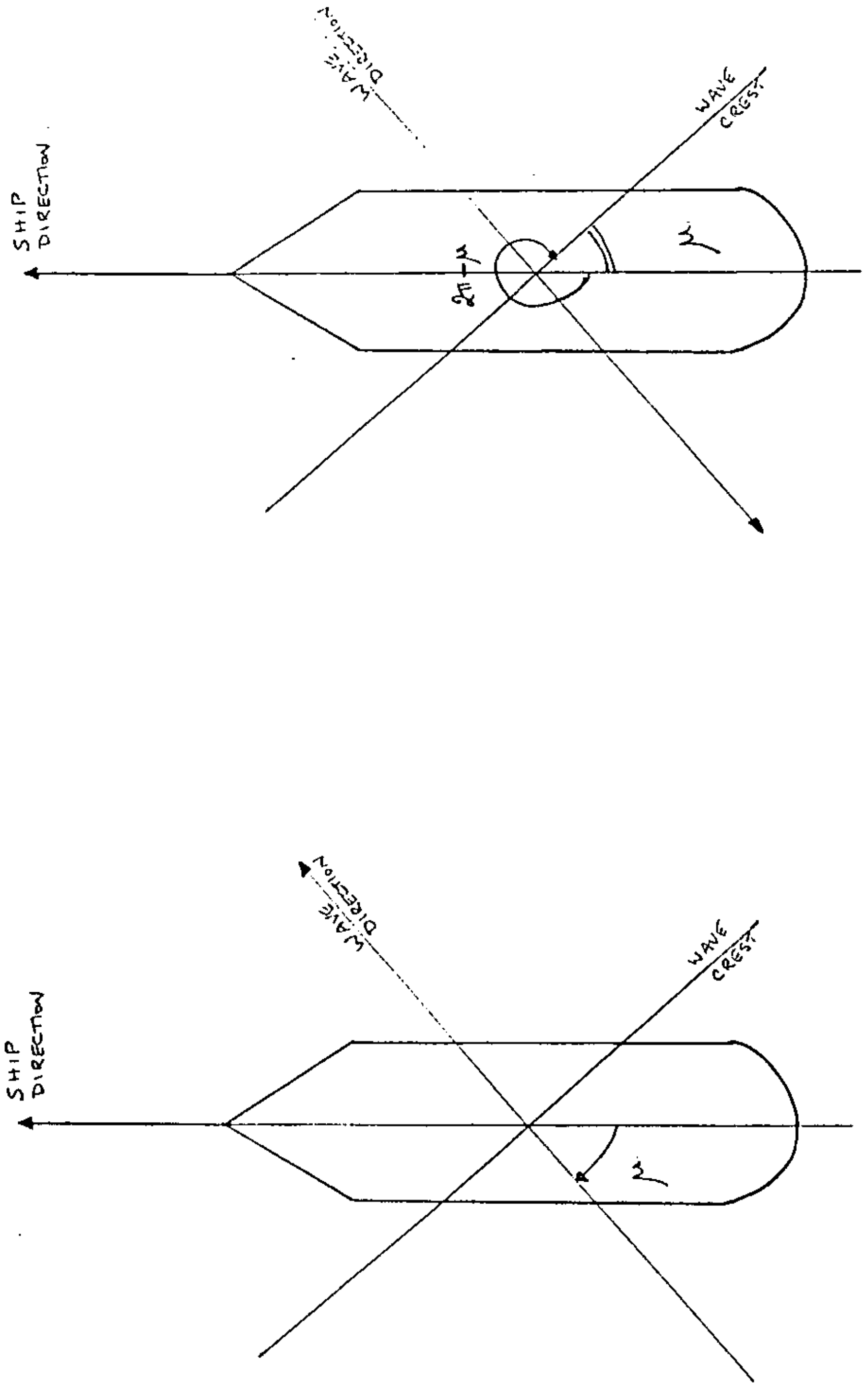
SHIP DIRECTION
WAVE DIRECTION
WAVE CREST
2π − μ
μ
SHIP DIRECTION
WAVE DIRECTION
WAVE CREST
μ

FIGURE 2

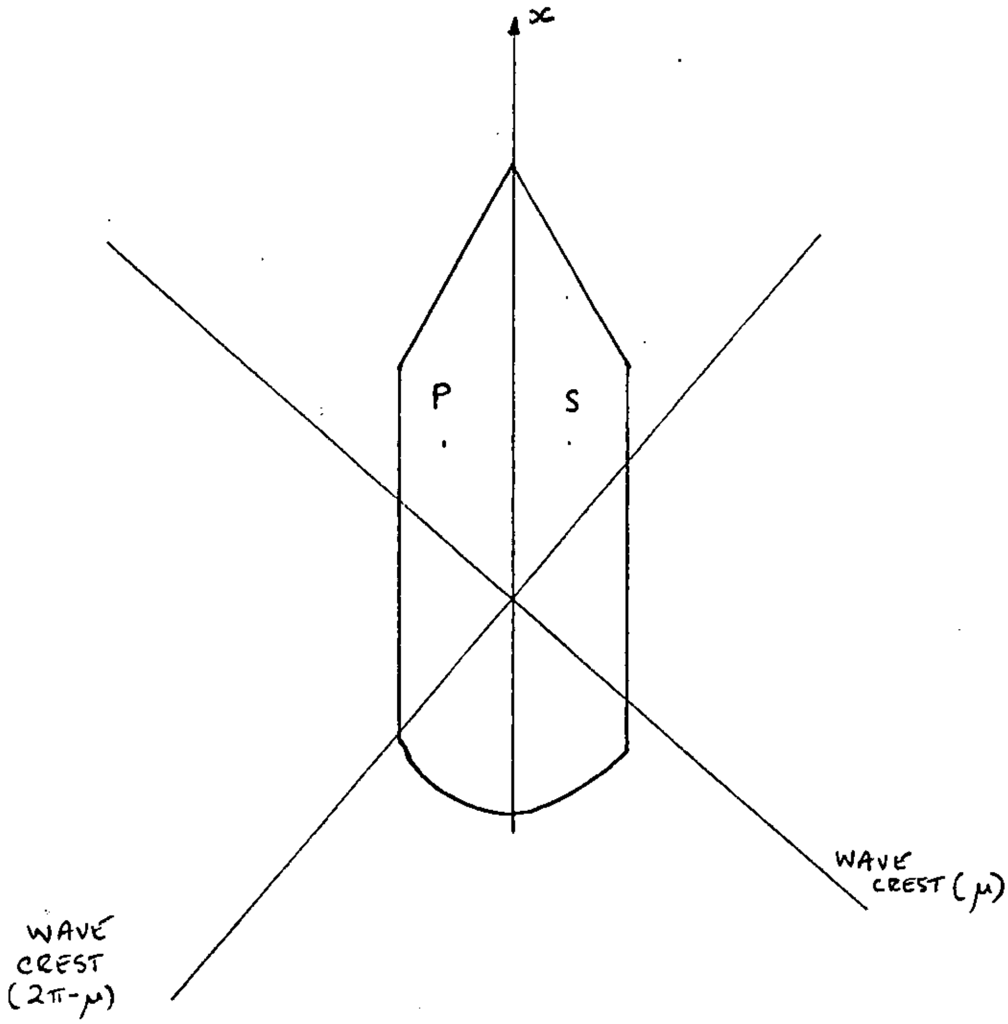


FIGURE 3