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Littlest inverse seesaw model

A.E. Cárcamo Hernández [a,b,*], S.F. King [a,b]

[a] Universidad Técnica Federico Santa María and Centro Científico-Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile

[b] School of Physics and Astronomy, University of Southampton, SO17 1BJ Southampton, United Kingdom

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Abstract

We propose a minimal predictive inverse seesaw model based on two right-handed neutrinos and two additional singlets, leading to the same low energy neutrino mass matrix as in the Littlest Seesaw (LS) (type I) model. In order to implement such a Littlest Inverse Seesaw (LIS) model, we have used an S_4 family symmetry, together with other various symmetries, flavons and driving fields. The resulting LIS model leads to an excellent fit to the low energy neutrino parameters, including the prediction of a normal neutrino mass ordering, exactly as in the usual LS model. However, unlike the LS model, the LIS model allows charged lepton flavor violating (CLFV) processes and lepton conversion in nuclei within reach of the forthcoming experiments.

1. Introduction

The existence of three fermion families, as well as their particular pattern of masses and mixing angles is not explained in the Standard Model (SM), and makes it appealing to consider a more fundamental theory addressing these issues. This problem is especially challenging in the neutrino sector, where the tiny values of the neutrino masses and large mixing angles between generations suggest a different kind of underlying physics than what should be responsible for the quark mass and mixing pattern. Whereas the small quark mixing angles decrease from one

* Corresponding author.
E-mail addresses: antonio.carcamo@usm.cl (A.E. Cárcamo Hernández), king@soton.ac.uk (S.F. King).

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generation to the next, in the lepton sector two of the mixing angles are large, and one mixing angle is small.

The tiny neutrino masses might well originate from a type I seesaw mechanism [1–5], but in general this is hard to test experimentally. A minimal version of the type I seesaw mechanism, involving just two right-handed neutrinos (2RHN), was first proposed by one of us [6,7], where we noted that the lightest neutrino is massless. Such a model with two texture zeros in the Dirac neutrino mass matrix, proposed somewhat later [8], is consistent with cosmological leptogenesis [9–16], but not compatible with the normal hierarchy (NH) of neutrino masses, favored by current data [15,16]. On the other hand the originally proposed 2RHN model with one texture zero [6,7], actually predicts a NH.

The Littlest Seesaw (LS) model is a special case of 2RHN models with one texture zero, which involves just two independent Yukawa couplings [17–24], leading to a highly predictive scheme characterized by near maximal atmospheric mixing and CP violation, with an approximate $\mu-\tau$ reflection symmetry [25,26] but with additional predictions arising from tri-maximal nature of the first column of the PMNS matrix as well as a predicted reactor angle.

All type I seesaw models, including the LS model above, predict very tiny branching ratios for the charged lepton flavor violating (LFV) decays, such as $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$, several orders of magnitude lower than their corresponding projective experimental sensitivity. These very tiny branching ratios for the charged lepton flavor violating (LFV) decays can be significantly enhanced by several orders of magnitude if one considers low scale seesaw models [27–32]. Thus if charged lepton flavor violating decays are observed in the future, it will provide indubitable evidence of Physics Beyond the Standard Model and their observation will shed light in the dynamics responsible for the smallness of neutrino masses and the nature of lepton mixing.

In this paper, motivated by such considerations, we propose a fusion of the LS model and the inverse seesaw model [33], which we refer to as the Littlest Inverse Seesaw (LIS) model. The neutrino mass matrix of the LIS model, which involves two right-handed neutrinos plus two additional singlets, is given by:

$$M_\nu = \begin{pmatrix} 0_{3\times 3} & m_D & 0_{3\times 2} \\ m_D^T & 0_{2\times 2} & M \\ 0_{2\times 3} & M^T & \mu \end{pmatrix}, \tag{1.1}$$

where $0_{n\times m}$ are $n \times m$ dimensional submatrices consisting of all zeroes and the other submatrices in the flavor basis have the structure:

$$m_D \sim \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}, \qquad M \sim \begin{pmatrix} 1 & 0 \\ 0 & z \end{pmatrix}, \qquad \mu \sim \begin{pmatrix} 1 & 0 \\ 0 & \omega \end{pmatrix}, \qquad \omega = e^{\frac{2\pi i}{3}}. \tag{1.2}$$

The light active neutrino mass matrix arising from the inverse seesaw formula $m_\nu = -m_D (M^T)^{-1} \mu M^{-1} m_D^T$ takes the same form as the usual LS model [17–24]:

$$m_\nu = m_{\nu a} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_{\nu b}\omega \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix} \tag{1.3}$$

The above mass matrix structures are motivated by the phenomenological success of the low energy mass matrix in Eq. (1.3) which is identical to that of the usual LS model, involving two right-handed neutrinos, but in this case arising from the inverse seesaw model, including the two additional singlets. Such an extension allows CLFV decays, such as $\mu \to e\gamma$, at observable rates, since in the inverse seesaw model small neutrino masses are explained by the smallness of the μ matrix,[1] which allows Dirac masses to be large even for TeV scale values of M. This is the first low scale seesaw model leading to a successful fit of the 6 physical observables of the neutrino sector with only 2 effective free parameters. In our model the small masses for the light active neutrinos are generated from an inverse seesaw mechanism. In order to achieve the above mass matrices, we appeal to standard approaches to the flavor puzzle based on symmetries, as follows.

The flavor puzzle of the SM indicates that New Physics has to be advocated to explain the observed SM fermion mass and mixing pattern. This is the so called flavor puzzle, which is not explained by the SM and provides motivation for building models with additional scalars and fermions in their particle spectrum and with extended symmetries which can be continuous or discrete and their breaking produces the observed pattern of SM fermion mass and mixing pattern. Several discrete groups have been employed in extensions of the SM to tackle SM fermion flavor puzzle. In particular the discrete group S_4 [35–58], together with the groups A_4 [41,57,59–98], T_7 [99–108], $\Delta(27)$ [109–133] and T' [134–155], is the smallest group containing an irreducible triplet representation that can accommodate the three fermion families of the Standard model (SM). These groups have been widely used in several extensions of the SM since they are particular promising in providing a viable and predictive description of the observed SM fermion mass spectrum and mixing parameters. In the present article, we shall employ S_4, together with other auxiliary symmetries, in order to achieve the above mass matrices of the LIS model, together with a diagonal charged lepton mass matrix.

The current article is organized as follows. In section 2 we explain our model. In section 3 we present our results in terms of neutrino masses and mixing. The implications of our model in the lepton flavor violating decays $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ and lepton conversion in nuclei are studied in section 3. We conclude in section 5. A description of the S_4 discrete group is presented in Appendix A. The superpotential that determines the vacuum configuration for the S_4 doublet and triplet scalars of our model is presented in Appendix B.

2. The model

We consider an S_4 flavor model for leptons where the masses for the light active neutrinos are generated from an inverse seesaw mechanism [30,33,156–160]. The implementation of the inverse seesaw mechanism in our model relies in the inclusion of four gauge singlets right handed Majorana neutrinos, which is the minimal amount of gauge singlet right handed Majorana neutrinos needed to implement a realistic inverse seesaw mechanism as pointed out for the first time in Ref. [160]. The leptonic and scalar spectrum of our model with their assignments under the $S_4 \times U(1) \times Z_3 \times Z_6 \times Z_9 \times U(1)_R$ symmetry are shown in Table 1.

The scalar spectrum of our model is composed of the $SU(2)_L$ Higgs doublets H_u, H_d and several gauge singlet scalar fields, which are grouped into one S_4 singlet, i.e., ρ, three S_4 doublets, i.e., φ, ϕ, η and five S_4 triplets, i.e., χ, ξ, σ_μ, σ_τ, σ_e. The gauge singlet scalar fields σ_μ, σ_τ, σ_e only participate in the charged lepton Yukawa interactions and whose inclusion is crucial to get a diagonal charged lepton mass matrix. On the other hand, the remaining gauge singlet scalars, i.e.,

[1] An example of a dynamical explanation for the smallness of the μ parameter of the inverse seesaw and its connection with Dark matter is provided in Ref. [34].

Table 1
Leptonic and scalar field assignments under the $S_4 \times U(1) \times Z_3 \times Z_6 \times Z_9 \times U(1)_R$ symmetry.

	l_L	l_{1R}	l_{2R}	l_{3R}	ν_{1R}	ν_{2R}	N_R	H_u	H_d	ρ	φ	ϕ	η	σ_μ	σ_τ	σ_e	χ	ξ	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	Φ	Δ	Θ	Ξ
S_4	$\mathbf{3}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}'$
$U(1)$	1	2	2	2	1	1	−2	0	−1	0	−3	−1	−1	0	0	0	0	0	2	2	0	0	1	3	0	0	9	3	3	0	0	0	0
Z_3	0	−1	−1	0	0	−1	0	0	0	0	0	0	−1	1	0	1	0	1	2	0	−1	−1	−2	2	−1	0	0	−2	−2	−2	0	−2	−1
Z_6	0	3	1	1	0	0	0	0	1	0	−1	0	0	−2	−2	−4	0	0	0	0	4	0	0	0	2	2	3	−1	1	4	4	2	4
Z_9	−4	4	0	−2	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$U(1)_R$	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

φ, ϕ, η, χ and ξ only appear in the neutrino Yukawa terms, which yield a viable and very predictive mass matrix for light active neutrinos. Thus, the inclusion of these scalar fields is necessary to have a highly predictive model for the lepton sector with only two effective parameters in the light active neutrino sector that allows to successfully reproduce the six experimental values of the physical observables of the neutrino sector, i.e., the two neutrino mass squared splittings, the three leptonic mixing angles and the leptonic Dirac CP violating phase.

We additionally introduce several driving fields, which are grouped into eleven S_4 singlets, i.e., X_k ($k = 1, 2, \cdots, 11$) and four S_4 triplets, i.e., Φ, Δ, Θ and Ξ. These driving fields are crucial for determining the vacuum alignments of the S_4 doublets and triplets in our model (to be specified below) that give rise to a diagonal SM charged lepton mass matrix and to a highly predictive and viable light active neutrino mass matrix, having only two free effective parameters.

In our model, the SM gauge symmetry is supplemented by the inclusion of the $S_4 \times U(1) \times Z_3 \times Z_6 \times Z_9 \times U(1)_R$ symmetry. We choose S_4 since it is the smallest non abelian group having doublet, triplet and singlet irreducible representations, thus allowing us to naturally accommodate the three families of the SM left handed leptonic fields into a S_4 triplet and the four gauge singlet right handed Majorana neutrinos into two S_4 singlets and one S_4 doublet, which is crucial to have highly predictive model that successfully describes lepton masses and mixings. The $Z_3 \times Z_6$ discrete symmetry allows to get a diagonal SM charged lepton mass matrix and Dirac neutrino mass matrix that yields a predictive and viable light active neutrino mass matrix. Thus, the leptonic mixing in our model arises from the neutrino sector. The Z_9 discrete symmetry sets the SM charged lepton mass hierarchy. It is worth mentioning that despite its extended particle spectrum and symmetries, each introduced field and symmetry plays its own role (described above) in predicting viable textures for the lepton sector that allows to successfully reproduce the experimental values of the six physical neutrino sector observables with only two effective parameters in the light active neutrino sector. This is achieved without the need to introduce hierarchy between the Yukawa couplings. Furthermore, the spontaneous breaking of the $S_4 \times Z_3 \times Z_6 \times Z_9$ at very high energy, gives rise to the SM charged lepton mass hierarchy. Besides that, the spontaneous breaking of the $U(1)$ global symmetry, which is assumed to take place at the TeV scale is crucial to generate a renormalizable and a non renormalizable mass terms involving gauge singlet right handed Majorana neutrinos, required for the implementation of the inverse seesaw mechanism that produce small masses for light active neutrinos. Note we have introduced a $U(1)_R$ symmetry under which the chiral supermultiplets containing the SM fermions have charge equal $+1$, whereas the driving fields X_k ($k = 1, 2, \cdots, 11$), Φ, Δ, Θ and Ξ have $U(1)_R$ charge equal to $+2$ and the remaining scalar fields are neutral under this symmetry. As a consequence of that $U(1)_R$ charge assignment, the aforementioned driving fields can only appear linearly in the superpotential and do not feature Yukawa interactions with SM fermions. The inclusion of the aforementioned driving files, whose corresponding superpotential is given in Appendix B, is necessary for achieving the following VEV configuration of the S_4 doublets and triplet scalars in our model:

$$\begin{aligned}
&\langle\varphi\rangle = v_{\varphi}(1,\omega), \qquad \langle\phi\rangle = v_{\phi}(0,1), \qquad \langle\eta\rangle = v_{\eta}(1,0), \qquad \langle\chi\rangle = v_{\chi}(0,1,1), \\
&\langle\xi\rangle = v_{\xi}(1,3,1), \qquad \left\langle\sigma_{\mu}\right\rangle = v_{\sigma_{\mu}}(0,1,0), \qquad \langle\sigma_{\tau}\rangle = v_{\sigma_{\tau}}(0,0,1), \\
&\langle\sigma_{e}\rangle = v_{\sigma_{e}}(1,0,0),
\end{aligned} \tag{2.1}$$

where $\omega = e^{\frac{2\pi i}{3}}$.

Since the spontaneous breaking of the $S_4 \times Z_3 \times Z_6 \times Z_9$ discrete group gives rise to the hierarchy of charged lepton masses, we set the vacuum expectation values (VEVs) of the different gauge singlet scalars with respect to the Wolfenstein parameter $\lambda = 0.225$ and the model cutoff Λ, as follows:

$$v_\phi \sim v_\eta \sim v_\varphi \sim \mathcal{O}(1)\ \text{TeV} << v_\chi \sim v_\xi \sim v_{\sigma_e} \sim v_{\sigma_\mu} \sim v_{\sigma_\tau} \sim v_\rho \sim \lambda\Lambda. \tag{2.2}$$

Here, for the sake of simplicity, the VEVs v_φ, v_ϕ, v_η, v_ρ, v_χ, v_ξ, v_{σ_μ} and v_{σ_τ} are assumed to be real. As it will be shown in section 3, the assumption of Eq. (2.2) will allow to explain the SM charged lepton mass hierarchy since it will relate the SM charged lepton masses with different powers of the Wolfenstein parameter times $\mathcal{O}(1)$ coefficients. It is worth mentioning that the model cutoff scale can be interpreted as the scale of the UV completion of the model, e.g. the masses Froggatt-Nielsen messenger fields. Furthermore, notice that the gauge singlet scalar fields ϕ, η and φ are assumed to get VEVs at the TeV scale in order to have TeV scale sterile neutrinos, thus allowing to have a model testable at colliders. Thus, the hierarchy in the VEVs of the gauge singlet scalar fields shown in Eq. (2.2) is motivated in order to have TeV scale sterile neutrinos and to explain the SM charged lepton mass hierarchy. Such two scale VEV hierarchy can be explained by having appropriate relations between the different mass coefficients of the bilinear terms of the scalar potential and the VEVs of such scalar fields. To show this explicitly, we consider the simplified case of two singlet scalar fields S_1 and S_2, whose VEVs satisfy the hierarchy $v_{S_2} >> v_{S_1}$. The corresponding scalar potential involving such fields takes the form:

$$V = -\mu_{S_1}^2 |S_1|^2 - \mu_{S_2}^2 |S_2|^2 + \lambda_1 |S_1|^4 + \lambda_2 |S_2|^4 + \lambda_3 |S_1|^2 |S_2|^2 . \tag{2.3}$$

Its minimization yields the following relations:

$$\mu_{S_1}^2 = 2\lambda_1 v_{S_1}^2 + \lambda_3 v_{S_2}^2, \qquad \mu_{S_2}^2 = 2\lambda_2 v_{S_2}^2 + \lambda_3 v_{S_1}^2. \tag{2.4}$$

Consequently, the VEV hierarchy $v_{S_2} >> v_{S_1}$, can be justified by requiring $\mu_{S_2}^2 \simeq 2\mu_{S_1}^2$ and considering the case where the quartic scalar couplings satisfy $\lambda_i \simeq \lambda$ $(i = 1, 2, 3)$. A straightforward but tedious extension of the aforementioned argument will give rise to large a set of relations between the different mass coefficients of the bilinear terms of the scalar potential and the VEVs of the large number of gauge singlet scalar fields of our model that will yield the VEV hierarchy shown in Eq. (2.2).

With the above particle content, we have the following relevant charged lepton and neutrino Yukawa terms:

$$-\mathcal{L}_Y^{(l)} = y_1^{(l)} \left(\bar{l}_L H_d \sigma_e\right)_{\mathbf{1}'} l_{1R} \frac{\rho^8}{\Lambda^9} + y_2^{(l)} \left(\bar{l}_L H_d \sigma_\mu\right)_{\mathbf{1}} l_{2R} \frac{\rho^4}{\Lambda^5} + y_3^{(l)} \left(\bar{l}_L H_d \sigma_\tau\right)_{\mathbf{1}} l_{3R} \frac{\rho^2}{\Lambda^3} + H.c \tag{2.5}$$

$$\begin{aligned}-\mathcal{L}_Y^{(\nu)} &= y_1^{(\nu)} \left(\bar{l}_L H_u \chi\right)_{\mathbf{1}} \nu_{1R} \frac{\rho^4}{\Lambda^5} + y_2^{(\nu)} \left(\bar{l}_L H_u \xi\right)_{\mathbf{1}} \nu_{2R} \frac{\rho^4}{\Lambda^5} \\ &+ y_{1\nu N} \bar{\nu}_{1R} \left(\phi N_R^C\right)_{\mathbf{1}} \frac{1}{\Lambda} + y_{2\nu N} \bar{\nu}_{2R} \left(\eta N_R^C\right)_{\mathbf{1}} + y_N \left(\overline{N}_R N_R^C\right)_{\mathbf{2}} \varphi \frac{H_u H_d}{\Lambda^2} + H.c\end{aligned} \tag{2.6}$$

3. Lepton masses and mixings

From the charged lepton Yukawa terms, we find that the charged lepton mass matrix is diagonal and the SM charged lepton masses are given by:

$$
\begin{aligned}
m_e &= y_1^{(l)} \frac{v_{\sigma_e} v_\rho^8}{\sqrt{2}\Lambda^9} v_{H_d} = a_1^{(l)} \lambda^9 \frac{v}{\sqrt{2}}, \qquad m_\mu = y_2^{(l)} \frac{v_{\sigma_\mu} v_\rho^4}{\sqrt{2}\Lambda^5} v_{H_d} = a_2^{(l)} \lambda^5 \frac{v}{\sqrt{2}}, \\
m_\tau &= y_3^{(l)} \frac{v_{\sigma_\tau} v_\rho^2}{\sqrt{2}\Lambda^3} v_{H_d} = a_3^{(l)} \lambda^3 \frac{v}{\sqrt{2}},
\end{aligned} \tag{3.1}
$$

where $a_1^{(l)}$, $a_2^{(l)}$ and $a_3^{(l)}$ are real $\mathcal{O}(1)$ dimensionless parameters and we have assumed that $v_{H_d} \sim v/\sqrt{2}$, being $v = 246$ GeV the electroweak symmetry breaking scale.

Regarding the neutrino sector, from the Eq. (2.6), we find the following neutrino mass terms:

$$
-\mathcal{L}_{mass}^{(\nu)} = \frac{1}{2} \begin{pmatrix} \overline{\nu_L^C} & \overline{\nu_R} & \overline{N_R} \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C \end{pmatrix} + H.c, \tag{3.2}
$$

where the neutrino mass matrix is given by:

$$
M_\nu = \begin{pmatrix} 0_{3\times 3} & m_D & 0_{3\times 2} \\ m_D^T & 0_{2\times 2} & M \\ 0_{2\times 3} & M^T & \mu \end{pmatrix}, \tag{3.3}
$$

where $0_{n\times m}$ are $n \times m$ dimensional submatrices consisting of all zeroes and the other submatrices in the flavor basis have the structure:

$$
\begin{aligned}
m_D &= v_{H_u} \begin{pmatrix} 0 & b \\ a & 3b \\ a & b \end{pmatrix}, \qquad M = m_N \begin{pmatrix} 1 & 0 \\ 0 & z \end{pmatrix}, \\
\mu &= \frac{y_N v_{H_u} v_{H_d} v_\varphi}{\Lambda^2} \begin{pmatrix} 1 & 0 \\ 0 & \omega \end{pmatrix}, \qquad \omega = e^{\frac{2\pi i}{3}}, \\
a &= y_1^{(\nu)} \frac{v_\chi v_\rho^4}{\Lambda^5} = x_1^{(\nu)} \lambda^5, \qquad b = y_2^{(\nu)} \frac{v_\xi v_\rho^4}{\Lambda^5} = x_2^{(\nu)} \lambda^5, \\
m_N &= y_{1\nu N} v_\phi, \qquad z = y_{2\nu N} \frac{v_\eta}{v_\phi}.
\end{aligned} \tag{3.4}
$$

The above mass matrices in Eqs. (3.3), (3.4) have precisely the desired LIS structure given in Eqs. (1.1), (1.2) in Section 1.

As shown in detail in Ref. [161], the full rotation matrix that diagonalizes a neutrino mass matrix of the form of Eq. (3.3) is given by:

$$
\mathbb{R} = \begin{pmatrix} R_\nu & R_1 R_M^{(1)} & R_2 R_M^{(2)} \\ -\frac{(R_1^\dagger + R_2^\dagger)}{\sqrt{2}} R_\nu & \frac{(1-S)}{\sqrt{2}} R_M^{(1)} & \frac{(1+S)}{\sqrt{2}} R_M^{(2)} \\ -\frac{(R_1^\dagger - R_2^\dagger)}{\sqrt{2}} R_\nu & \frac{(-1-S)}{\sqrt{2}} R_M^{(1)} & \frac{(1-S)}{\sqrt{2}} R_M^{(2)} \end{pmatrix}, \tag{3.5}
$$

where

$$
S = -\frac{1}{4} M^{-1} \mu, \qquad R_1 \simeq R_2 \simeq \frac{1}{\sqrt{2}} m_D^* M^{-1} = \frac{1}{\sqrt{2} m_N} m_D^*. \tag{3.6}
$$

The light active masses arise from an inverse seesaw mechanism and the physical neutrino mass matrices are:

$$m_\nu = m_D\left(M^T\right)^{-1}\mu M^{-1} m_D^T, \qquad M_\nu^{(1)} = -\frac{1}{2}\left(M + M^T\right) + \frac{1}{2}\mu,$$
$$M_\nu^{(2)} = \frac{1}{2}\left(M + M^T\right) + \frac{1}{2}\mu, \tag{3.7}$$

where m_ν corresponds to the active neutrino mass matrix whereas $M_\nu^{(1)}$ and $M_\nu^{(2)}$ are the exotic neutrino mass matrices.

Note that the physical neutrino spectrum is composed of three light active neutrinos and four exotic neutrinos. The exotic neutrinos are pseudo-Dirac, with masses $\sim \pm\frac{1}{2}\left(M + M^T\right)$ and a small splitting μ. Furthermore, R_ν, $R_M^{(1)}$ and $R_M^{(2)}$ are the rotation matrices which diagonalize m_ν, $M_\nu^{(1)}$ and $M_\nu^{(2)}$, respectively. Since in our model $M_\nu^{(1)}$ and $M_\nu^{(2)}$ are diagonal, $R_M^{(1)}$ and $R_M^{(2)}$ are equal to the 2×2 identity matrix, the rotation matrix $\mathbb{R}$ of Eq. (3.5) can be rewritten as follows:

$$\mathbb{R} = \begin{pmatrix} R_\nu & R_1 & R_2 \\ -\frac{(B_2^\dagger + B_3^\dagger)}{\sqrt{2}} R_\nu & \frac{(1-S)}{\sqrt{2}} & \frac{(1+S)}{\sqrt{2}} \\ -\frac{(B_2^\dagger - B_3^\dagger)}{\sqrt{2}} R_\nu & \frac{(-1-S)}{\sqrt{2}} & \frac{(1-S)}{\sqrt{2}} \end{pmatrix}. \tag{3.8}$$

Furthermore, using Eq. (3.5) we find that the neutrino fields $\nu_L = (\nu_{1L}, \nu_{2L}, \nu_{3L})^T$, $\nu_R^C = \left(\nu_{1R}^C, \nu_{2R}^C\right)$ and $N_R^C = \left(N_{1R}^C, N_{2R}^C\right)$ are related with the neutrino mass eigenstates by the following relations:

$$\begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C \end{pmatrix} = \mathbb{R}\Omega_L \simeq \begin{pmatrix} R_\nu & R_1 & R_2 \\ -\frac{(B_2^\dagger + B_3^\dagger)}{\sqrt{2}} R_\nu & \frac{(1-S)}{\sqrt{2}} & \frac{(1+S)}{\sqrt{2}} \\ -\frac{(B_2^\dagger - B_3^\dagger)}{\sqrt{2}} R_\nu & \frac{(-1-S)}{\sqrt{2}} & \frac{(1-S)}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \Omega_L^{(1)} \\ \Omega_L^{(2)} \\ \Omega_L^{(3)} \end{pmatrix},$$
$$\Omega_L = \begin{pmatrix} \Omega_L^{(1)} \\ \Omega_L^{(2)} \\ \Omega_L^{(3)} \end{pmatrix}, \tag{3.9}$$

where $\Omega_{jL}^{(1)}$ $(j = 1, 2, 3)$, $\Omega_{kL}^{(2)}$ and $\Omega_{kL}^{(3)}$ $(k = 1, 2)$ are the three active neutrinos and four exotic neutrinos, respectively.

Using Eq. (3.7), the light active neutrino mass matrix arising from the inverse seesaw mechanism takes the form:

$$m_\nu = m_{\nu a}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_{\nu b}\omega \begin{pmatrix} 1 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{pmatrix}, \qquad m_{\nu a} = \frac{a^2 y_N v_{H_u}^3 v_{H_d} v_\varphi}{y_{1\nu N}^2 v_\phi^2 \Lambda^2},$$
$$m_{\nu b} = \frac{b^2 z^2 y_N v_{H_u}^3 v_{H_d} v_\varphi}{y_{1\nu N}^2 v_\phi^2 \Lambda^2}. \tag{3.10}$$

The low energy neutrino mass matrix in Eq. (3.10) is of the highly predictive LS form given in Eq. (1.3) which gives a good fit to low energy neutrino data using the parameter values discussed for example in [23]. The neutrino mass squared splittings, light active neutrino masses, leptonic mixing angles and CP violating phase for the scenario of normal neutrino mass hierarchy can be very well reproduced with only two effective free parameters, whose values are given by [23]:

Table 2
Model and experimental values of the light active neutrino masses, leptonic mixing angles and CP violating phase for the scenario of normal (NH) neutrino mass hierarchy. The experimental values are taken from Refs. [162,163].

Observable	Model	bpf $\pm 1\sigma$ [162]	bpf $\pm 1\sigma$ [163]	3σ range [162]	3σ range [163]
Δm_{21}^2 [10^{-5} eV^2]	7.38	$7.55_{-0.16}^{+0.20}$	$7.39_{-0.20}^{+0.21}$	$7.05-8.14$	$6.79-8.01$
Δm_{31}^2 [10^{-3} eV^2]	2.48	2.50 ± 0.03	$2.525_{-0.031}^{+0.033}$	$2.41-2.60$	$2.431-2.622$
$\theta_{12}^{(l)}(^\circ)$	34.32	$34.5_{-1.0}^{+1.2}$	$33.82_{-0.76}^{+0.78}$	$31.5-38.0$	$31.61-36.27$
$\theta_{13}^{(l)}(^\circ)$	8.67	$8.45_{-0.14}^{+0.16}$	$8.61_{-0.13}^{+0.12}$	$8.0-8.9$	$8.22-8.98$
$\theta_{23}^{(l)}(^\circ)$	45.77	$47.9_{-1.7}^{+1.0}$	$49.7_{-1.1}^{+0.9}$	$41.8-50.7$	$40.9-52.2$
$\delta_{CP}^{(l)}(^\circ)$	-86.67	-142_{-27}^{+38}	217_{-28}^{+40}	$157-349$	$135-366$

$$m_{\nu a}\simeq 26.57\text{ meV},\qquad m_{\nu b}\simeq 2.684\text{ meV}. \tag{3.11}$$

Thus, using the numerical value for $m_{\nu b}$ given by Eq. (3.11) and considering $v_{H_u}\sim v_{H_d}\sim\frac{v}{\sqrt{2}}\sim$ 174 GeV, $v_\phi\sim v_\varphi\sim 1$ TeV, $y_{\nu N}\sim y_N\sim 1$, $b\sim\lambda^5$, with $\lambda=0.225$ and $v=246$ GeV, we estimate our model cutoff as $\Lambda\sim 3\times 10^5$ GeV, in order to naturally reproduce the smallness of the light active neutrino masses.

In addition, we find that the light active neutrino masses are:

$$m_1=0,\qquad m_2=8.59\text{ meV}\qquad m_3=49.81\text{ meV}. \tag{3.12}$$

From Table 2, it follows that the neutrino mass squared splittings, i.e., Δm_{21}^2 and Δm_{31}^2, the leptonic mixing angles $\theta_{12}^{(l)}$, $\theta_{23}^{(l)}$, $\theta_{13}^{(l)}$ and the Dirac leptonic CP violating phase are consistent with neutrino oscillation experimental data for the scenario of normal neutrino mass hierarchy. It is remarkable that our model relies on only two effective parameters in the light active neutrino sector that allows to successfully reproduce six neutrino physical observables: neutrino mass squared splittings, leptonic mixing angles and Dirac leptonic CP violating phase. Let us note that, for the inverted neutrino mass hierarchy, the obtained leptonic mixing parameters are very much outside the 3σ experimentally allowed range. Consequently, our model is only viable for the scenario of normal neutrino mass hierarchy.

4. Charged lepton flavor violating decays

In this section we will discuss the implications of our model in the lepton flavor violating decays $\mu\to e\gamma$, $\tau\to\mu\gamma$ and $\tau\to e\gamma$. As mentioned in the previous section, the physical sterile neutrino spectrum of our model is composed of four TeV scale neutrinos, which are practically degenerate. These heavy sterile neutrinos mix the active ones, with mixing angles of the order of $\frac{1}{\sqrt{2}m_N}(m_D)_{in}$ ($i=1,2,3$ and $n=1,2$). The admixture of the heavy sterile neutrinos in the left-handed charged current $SU_{2L}\times U_{1Y}$ weak interaction, gives rise to the $l_i\to l_j\gamma$ decay at one loop level, whose Branching ratio takes the form [28,164,165]:

$$Br\left(l_i\to l_j\gamma\right)=\frac{\alpha_W^3 s_W^2 m_{l_i}^5}{256\pi^2 m_W^4\Gamma_i}\left|G_{ij}\right|^2,$$

$$G_{ij}=\sum_k\left(\mathbb{R}^*\right)_{ik}\left(\mathbb{R}\right)_{jk}G_\gamma\left(\frac{m_{N_k}^2}{m_W^2}\right)\simeq 2\left(R_1R_1^T\right)_{ij}G_\gamma\left(\frac{m_N^2}{m_W^2}\right)$$

$$= \frac{\left(m_D^* m_D^\dagger\right)_{ij}}{m_N^2} G_\gamma \left(\frac{m_N^2}{m_W^2}\right),$$

$$G_\gamma = -\frac{2x^3 + 5x^2 - x}{4(1-x)^2} - \frac{3x^3}{2(1-x)^4} \ln x. \tag{4.1}$$

Thus, the Branching ratios for the $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ decays in our model are respectively given by:

$$Br(\mu \to e\gamma) = \frac{9\alpha_W^3 s_W^2 b^4 v_{H_u}^4 m_\mu^5}{256\pi^2 m_W^4 \Gamma_\mu m_N^4} \left| G_\gamma \left(\frac{m_N^2}{m_W^2}\right)\right|^2,$$

$$Br(\tau \to \mu\gamma) = \frac{9\alpha_W^3 s_W^2 b^4 v_{H_u}^4 m_\tau^5}{256\pi^2 m_W^4 \Gamma_\tau m_N^4} \left| G_\gamma \left(\frac{m_N^2}{m_W^2}\right)\right|^2,$$

$$Br(\tau \to e\gamma) = \frac{\alpha_W^3 s_W^2 b^4 v_{H_u}^4 m_\tau^5}{256\pi^2 m_W^4 \Gamma_\tau m_N^4} \left| G_\gamma \left(\frac{m_N^2}{m_W^2}\right)\right|^2, \tag{4.2}$$

being $\Gamma_\mu = 3 \times 10^{-19}$ GeV and $\Gamma_\tau = 2.27 \times 10^{-12}$ GeV the total muon and tau decay widths, respectively. Fig. 1 shows the allowed parameter space in the $m_N - bv_{H_u}$ and $m_N - x_2^{(\nu)}$ and $m_N - \tan\beta$ planes consistent with the LFV constraints. The plots of Fig. 1 were obtained by randomly generating the parameters m_N, bv_{H_u} and $x_2^{(\nu)}$ (keeping in mind that $b = x_2^{(\nu)}\lambda^5$ (see Eq. (3.4))) in a range of values where the Branching ratio for the $\mu \to e\gamma$ decay is below its upper experimental limit of 4.2×10^{-13}. To choose the region where bv_{H_u} was varied, we chose a scenario where $v_{H_u} = 200$ GeV with the dimensionless coupling $x_2^{(\nu)}$ in the range $1 \lesssim x_2^{(\nu)} \lesssim \sqrt{4\pi}$, where the upper bound of $\sqrt{4\pi}$ for $x_2^{(\nu)}$ corresponds to the maximum value allowed by perturbativity. In what regards the third plot of Fig. 1, we have set $x_2^{(\nu)}$ equal to unity and we have varied $\tan\beta = \frac{v_{H_u}}{v_{H_d}}$ in the range $5 \lesssim \tan\beta \lesssim 50$ as done in Ref. [166].

As seen from Fig. 1, the obtained values for the branching ratio of the $\mu \to e\gamma$ decay are below its experimental upper limit of 4.2×10^{-13} since these values are located in the range $8 \times 10^{-14} \lesssim Br(\mu \to e\gamma) \lesssim 1.8 \times 10^{-13}$, for a large region of parameter space of our model. Furthermore, let us note that the branching ratio for the $\mu \to e\gamma$ decay has a low sensitivity with $\tan\beta$ when it is varied in the range $5 \lesssim \tan\beta \lesssim 10$.

In the same region of parameter space, we found that the branching ratios for the $\tau \to \mu\gamma$ and $\tau \to e\gamma$ decays are in the ranges $2 \times 10^{-13} \lesssim Br(\tau \to \mu\gamma) \lesssim 1.6 \times 10^{-12}$ and $2 \times 10^{-14} \lesssim Br(\tau \to e\gamma) \lesssim 1.8 \times 10^{-13}$, respectively, which is well below their upper experimental limits of 4.4×10^{-9} and 3.3×10^{-9}, respectively. Consequently, our model is highly consistent with the constraints arising from lepton flavor violating decays for a large region of parameter space. Given that future experiments such as Mu2e and COMET are expected to measure or bound lepton conversion in nuclei with much better precision than the radiative rare lepton decays, we proceed to determine the constraints imposed by lepton conversion in nuclei on the model parameter space. It is worth mentioning that the branching ratio for the $\mu^- - e^-$ conversion takes the form [165]:

$$CR(\mu - e) = \frac{\Gamma\left(\mu^- + Nucleus(A,Z) \to e^- + Nucleus(A,Z)\right)}{\Gamma\left(\mu^- + Nucleus(A,Z) \to \nu_\mu + Nucleus(A,Z-1)\right)} \tag{4.3}$$

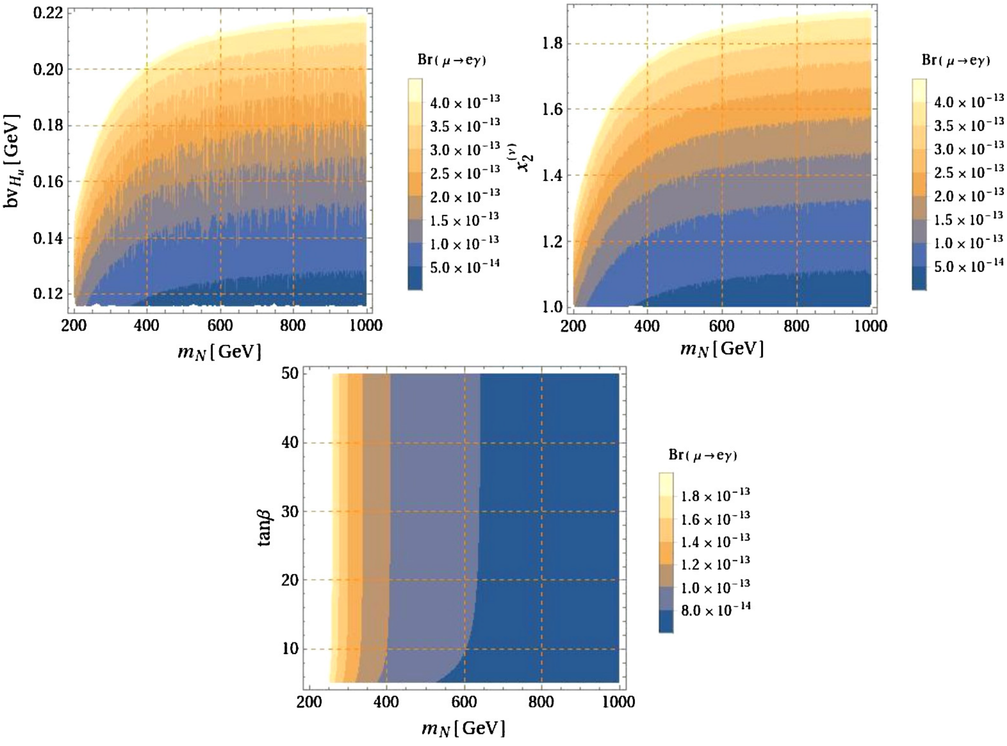


Fig. 1. Allowed parameter space in the $m_N - bv_{H_u}$, $m_N - x_2^{(\nu)}$ and $m_N - \tan\beta$ planes consistent with the LFV constraints. In the third plot $x_2^{(\nu)}$ has been set equal to unity. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Using an Effective Lagrangian approach for describing lepton flavor violating processes as done in [167] and considering the low momentum limit where the off-shell contributions from photon exchange are negligible with respect to the contributions arising from real photon emission, the dipole operators dominate the conversion rate thus yielding the following relations [165,167]:

$$CR\,(\mu Ti \to eTi) \simeq \frac{1}{200} Br\,(\mu \to e\gamma) \qquad CR\,(\mu Al \to eAl) \simeq \frac{1}{350} Br\,(\mu \to e\gamma) \tag{4.4}$$

It is worth mentioning that the Effective field theory treatment used in [167], is valid for supersymmetric models like the one discussed in this paper.

Fig. 2 shows the $CR\,(\mu Ti \to eTi)$ (left plot) and $CR\,(\mu Al \to eAl)$ (right plot) parameters as function of the sterile neutrino mass m_N for different values of the dimensionless coupling $x_2^{(\nu)}$. The black horizontal line in the left plot corresponds to the expected sensitivity of $\sim 10^{-18}$ of the CERN Neutrino Factory that will use Titanium as target [168]. On the other hand, the black horizontal line in the right plot corresponds to the expected sensitivities of $\sim 10^{-17}$ of the next generation of experiments such as Mu2e and COMET [169], where the Aluminum will be used as a target instead. In these plots we have set $\tan\beta = 5$. These plots show that the next generation experiments where the Titanium and Aluminum will be used as targets, will rule out the part of the model parameter space where $x_2^{(\nu)} \gtrsim 0.2$ and $x_2^{(\nu)} \gtrsim 0.4$, respectively,

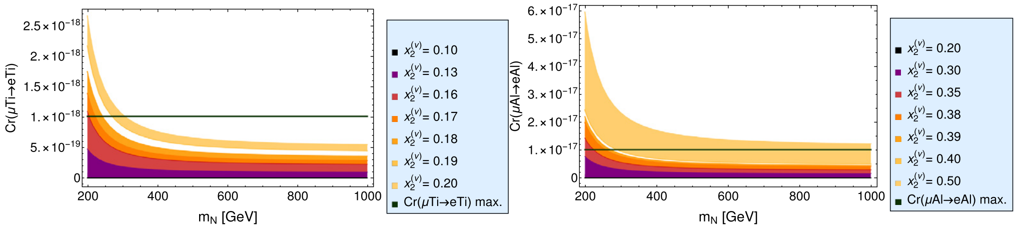


Fig. 2. $CR(\mu Ti \to eTi)$ (left plot) and $CR(\mu Al \to eAl)$ (right plot) as function of the sterile neutrino mass m_N for different values of the dimensionless coupling $x_2^{(\nu)}$. The black horizontal line in each plot corresponds to the expected sensitivities of the next generation of experiments that will use Titanium [168] and Aluminum [169] as targets, respectively. Here we have set $\tan\beta = 5$.

for sterile neutrino masses larger than about 300 GeV. Consequently, a precise measurement of lepton conversion in nuclei by future experiments will be crucial to set constraints on the active-sterile neutrino mixing angles, which will be crucial to determine the allowed region of parameter space of inverse seesaw models. Finally to close this section, it is worth mentioning that our results regarding the charged lepton flavor violating processes are not generic features of low scale seesaw models. The S_4 flavor symmetry and the different auxiliary cyclic symmetries introduced in our model, allows to get defined predictions for the branching ratios for the lepton flavor violating decays $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$. Depending on the discrete symmetries assignments one can have, for instance sizeable $\tau \to e\gamma$ decay, but strongly suppressed $\mu \to e\gamma$ and $\tau \to \mu\gamma$ processes as shown in the A_4 flavor model of Ref. [98].

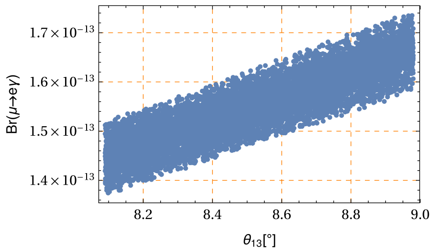
Fig. 3 shows the correlations of the Branching ratio for the $\mu \to e\gamma$ decay with the leptonic mixing angles as well as with the leptonic Dirac CP violating phase. To obtain these Figures, the lepton sector parameters were randomly generated in a range of values where the neutrino mass squared splittings, leptonic mixing angles and leptonic Dirac CP violating phase are inside the 3σ experimentally allowed range. The plots in Fig. 3 show that the Branching ratio for the $\mu \to e\gamma$ decay increases when the reactor θ_{13} and atmospheric θ_{23} mixing angles as well as the leptonic Dirac CP violating phase δ_{CP} take larger values. On the other hand, the Branching ratio for the $\mu \to e\gamma$ decay decreases as the solar mixing angle θ_{12} is increased.

5. Conclusions

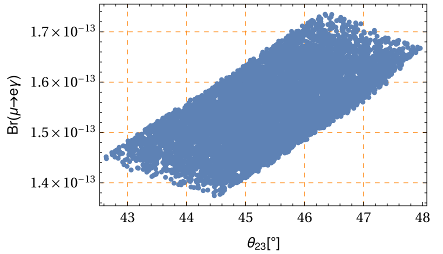
We have proposed a minimal predictive inverse seesaw model based on two right-handed neutrinos and two additional singlets, which yields the same low energy neutrino mass matrix as in the Littlest Seesaw (LS) (type I) model. The model is called the Littlest Inverse Seesaw (LIS) model and yields the mass matrix structures as shown in the Introduction.

In order to implement the LIS model, we have used an S_4 family symmetry, supplemented by the $U(1) \times Z_3 \times Z_6 \times U(1)_R$ group. The charged lepton mass hierarchy is produced by the spontaneous breaking of the $S_4 \times Z_3 \times Z_6$ discrete group at very high energies. The nature of the inverse seesaw mechanism is guaranteed by renormalizable and non-renormalizable mass terms involving gauge singlet right handed Majorana neutrinos. These terms are generated after the spontaneous breaking of the $U(1)$ global symmetry at the TeV scale.

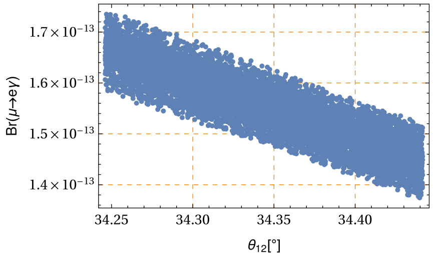
The resulting LIS model proposed here is the first low scale seesaw model which incorporates the successful predictions of the LS model, including the prediction of a normal neutrino mass ordering, all arising from only two effective free parameters. However there is one crucial phenomenological difference between the LS and the LIS models: the LIS model allows



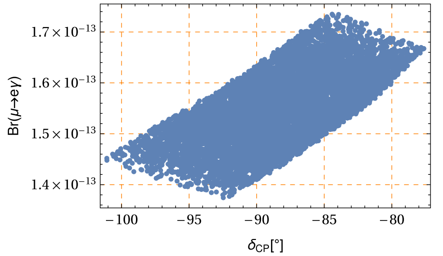
(a) Correlation between $Br(\mu \to e\gamma)$ and the reactor mixing angle θ_{13}.



(b) Correlation between $Br(\mu \to e\gamma)$ and the atmospheric mixing angle θ_{23}.



(c) Correlation between $Br(\mu \to e\gamma)$ and solar mixing angle θ_{12}.



(d) Correlation between $Br(\mu \to e\gamma)$ and the leptonic Dirac CP violating phase δ_{CP}.

Fig. 3. Correlations between $Br(\mu \to e\gamma)$ and the different lepton sector observables.

charged lepton flavor violating (CLFV) processes within the reach of future experimental sensitivity. In addition, we have studied the implications of our model in the lepton conversion in nuclei. We have found that in order that our model's predictions for the $CR(\mu Ti \to eTi)$ and $CR(\mu Al \to eAl)$ effective parameters be lower than the expected sensitivities of the next generation of experiments that will use Titanium and Aluminum as targets, the effective neutrino Yukawa coupling $x_2^{(\nu)}$ has to be lower than about 0.2 and 0.4, respectively, for sterile neutrino masses larger than around 300 GeV.

In summary, the LIS model predicts branching ratios for the charged lepton flavor violating processes: $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ in the ranges $8 \times 10^{-14} \lesssim Br(\mu \to e\gamma) \lesssim 1.8 \times 10^{-13}$, $2 \times 10^{-13} \lesssim Br(\tau \to \mu\gamma) \lesssim 1.6 \times 10^{-12}$ and $2 \times 10^{-14} \lesssim Br(\tau \to e\gamma) \lesssim 1.8 \times 10^{-13}$, which will all present a target for the forthcoming CLFV experiments.

Declaration of competing interest

We certify that we do not have affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript.

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Appendix A. S_4 symmetry

The S_4 is the smallest non abelian group having doublet and singlet irreducible representations. S_4 is the group of permutations of four objects, which includes five irreducible representations, i.e., $\mathbf{1}, \mathbf{1}', \mathbf{2}, \mathbf{3}, \mathbf{3}'$ fulfilling the following tensor product rules [170]

$$\mathbf{3}\otimes\mathbf{3}=\mathbf{1}\oplus\mathbf{2}\oplus\mathbf{3}\oplus\mathbf{3}', \qquad \mathbf{3}'\otimes\mathbf{3}'=\mathbf{1}\oplus\mathbf{2}\oplus\mathbf{3}\oplus\mathbf{3}', \qquad \mathbf{3}\otimes\mathbf{3}'=\mathbf{1}'\oplus\mathbf{2}\oplus\mathbf{3}\oplus\mathbf{3}', \tag{A.1}$$

$$\mathbf{2}\otimes\mathbf{2}=\mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{2}, \qquad \mathbf{2}\otimes\mathbf{3}=\mathbf{3}\oplus\mathbf{3}', \qquad \mathbf{2}\otimes\mathbf{3}'=\mathbf{3}'\oplus\mathbf{3}, \tag{A.2}$$

$$\mathbf{3}\otimes\mathbf{1}'=\mathbf{3}', \qquad \mathbf{3}'\otimes\mathbf{1}'=\mathbf{3}, \qquad \mathbf{2}\otimes\mathbf{1}'=\mathbf{2}. \tag{A.3}$$

Explicitly, the basis used in this paper corresponds to Ref. [170] and results in

$$(\mathbf{A})_{\mathbf{3}}\times(\mathbf{B})_{\mathbf{3}}=(\mathbf{A}\cdot\mathbf{B})_{\mathbf{1}}+\begin{pmatrix}\mathbf{A}\cdot\Sigma\cdot\mathbf{B}\\ \mathbf{A}\cdot\Sigma^*\cdot\mathbf{B}\end{pmatrix}_{\mathbf{2}}+\begin{pmatrix}\{A_yB_z\}\\ \{A_zB_x\}\\ \{A_xB_y\}\end{pmatrix}_{\mathbf{3}}+\begin{pmatrix}[A_yB_z]\\ [A_zB_x]\\ [A_xB_y]\end{pmatrix}_{\mathbf{3}'}, \tag{A.4}$$

$$(\mathbf{A})_{\mathbf{3}'}\times(\mathbf{B})_{\mathbf{3}'}=(\mathbf{A}\cdot\mathbf{B})_{\mathbf{1}}+\begin{pmatrix}\mathbf{A}\cdot\Sigma\cdot\mathbf{B}\\ \mathbf{A}\cdot\Sigma^*\cdot\mathbf{B}\end{pmatrix}_{\mathbf{2}}+\begin{pmatrix}\{A_yB_z\}\\ \{A_zB_x\}\\ \{A_xB_y\}\end{pmatrix}_{\mathbf{3}}+\begin{pmatrix}[A_yB_z]\\ [A_zB_x]\\ [A_xB_y]\end{pmatrix}_{\mathbf{3}'}, \tag{A.5}$$

$$(\mathbf{A})_{\mathbf{3}}\times(\mathbf{B})_{\mathbf{3}'}=(\mathbf{A}\cdot\mathbf{B})_{\mathbf{1}'}+\begin{pmatrix}\mathbf{A}\cdot\Sigma\cdot\mathbf{B}\\ -\mathbf{A}\cdot\Sigma^*\cdot\mathbf{B}\end{pmatrix}_{\mathbf{2}}+\begin{pmatrix}\{A_yB_z\}\\ \{A_zB_x\}\\ \{A_xB_y\}\end{pmatrix}_{\mathbf{3}'}+\begin{pmatrix}[A_yB_z]\\ [A_zB_x]\\ [A_xB_y]\end{pmatrix}_{\mathbf{3}}, \tag{A.6}$$

$$(\mathbf{A})_{\mathbf{2}}\times(\mathbf{B})_{\mathbf{2}}=\{A_xB_y\}_{\mathbf{1}}+[A_xB_y]_{\mathbf{1}'}+\begin{pmatrix}A_yB_y\\ A_xB_x\end{pmatrix}_{\mathbf{2}}, \tag{A.7}$$

$$\begin{pmatrix}A_x\\ A_y\end{pmatrix}_{\mathbf{2}}\times\begin{pmatrix}B_x\\ B_y\\ B_z\end{pmatrix}_{\mathbf{3}}=\begin{pmatrix}(A_x+A_y)B_x\\ (\omega^2A_x+\omega A_y)B_y\\ (\omega A_x+\omega^2A_y)B_z\end{pmatrix}_{\mathbf{3}}+\begin{pmatrix}(A_x-A_y)B_x\\ (\omega^2A_x-\omega A_y)B_y\\ (\omega A_x-\omega^2A_y)B_z\end{pmatrix}_{\mathbf{3}'}, \tag{A.8}$$

$$\begin{pmatrix}A_x\\ A_y\end{pmatrix}_{\mathbf{2}}\times\begin{pmatrix}B_x\\ B_y\\ B_z\end{pmatrix}_{\mathbf{3}'}=\begin{pmatrix}(A_x+A_y)B_x\\ (\omega^2A_x+\omega A_y)B_y\\ (\omega A_x+\omega^2A_y)B_z\end{pmatrix}_{\mathbf{3}'}+\begin{pmatrix}(A_x-A_y)B_x\\ (\omega^2A_x-\omega A_y)B_y\\ (\omega A_x-\omega^2A_y)B_z\end{pmatrix}_{\mathbf{3}}, \tag{A.9}$$

with

$$\begin{aligned}\mathbf{A}\cdot\mathbf{B}&=A_xB_x+A_yB_y+A_zB_z,\\ \{A_xB_y\}&=A_xB_y+A_yB_x,\\ [A_xB_y]&=A_xB_y-A_yB_x,\\ \mathbf{A}\cdot\Sigma\cdot\mathbf{B}&=A_xB_x+\omega A_yB_y+\omega^2A_zB_z,\\ \mathbf{A}\cdot\Sigma^*\cdot\mathbf{B}&=A_xB_x+\omega^2A_yB_y+\omega A_zB_z,\end{aligned} \tag{A.10}$$

where $\omega=e^{2\pi i/3}$ is a complex square root of unity.

Appendix B. The S_4 flavored superpotential

In order to obtain the VEV configuration for the S_4 doublet and triplet scalars shown in Eq. (2.1), we consider the following $S_4 \times U(1) \times Z_3 \times Z_6$ invariant superpotential:

$$
\begin{aligned}
W = & \kappa_1 (\eta\eta)_{\mathbf{1}} X_1 + \kappa_2 (\phi\phi)_{\mathbf{1}} X_2 + \kappa_3 \left(\sigma_\mu \sigma_\tau\right)_{\mathbf{1}} X_3 + \kappa_4 (\sigma_e \sigma_\tau)_{\mathbf{1}} X_4 \\
& + \kappa_5 \left(\sigma_e \sigma_\mu\right)_{\mathbf{1}} X_5 + \frac{\kappa_6}{\Lambda}\left[(\eta\eta)_{\mathbf{2}} \phi\right]_{\mathbf{1}'} X_6 \\
& + \frac{\kappa_7}{\Lambda}\left[(\chi\chi)_{\mathbf{3}} \sigma_\mu\right]_{\mathbf{1}} X_7 + \frac{\kappa_8}{\Lambda}\left[(\chi\chi)_{\mathbf{3}} \sigma_\tau\right]_{\mathbf{1}} X_8 + \frac{\kappa_9}{\Lambda}\left[(\varphi\varphi)_{\mathbf{2}} \varphi\right]_{\mathbf{1}'} X_9 \\
& + \frac{\kappa_{10}}{\Lambda}\left[\left(\sigma_\mu \sigma_\mu\right)_{\mathbf{2}} \varphi\right]_{\mathbf{1}} X_{10} + \frac{\kappa_{11}}{\Lambda}\left[(\xi\xi)_{\mathbf{2}} \varphi\right]_{\mathbf{1}'} X_{11} + \kappa_{12} \left(\sigma_\mu \sigma_\mu\right)_{\mathbf{3}} \Phi \\
& + \kappa_{13} (\sigma_\tau \sigma_\tau)_{\mathbf{3}} \Delta + \kappa_{14} (\sigma_e \sigma_e)_{\mathbf{3}} \Theta + \frac{\kappa_{15}}{\Lambda}\left[(\chi\chi)_{\mathbf{2}} \sigma_e\right]_{\mathbf{3}'} \Xi \\
= & 2\kappa_1 \eta_1 \eta_2 X_1 + 2\kappa_2 \phi_1 \phi_2 X_2 + \kappa_3 \left(\sigma_{1\mu}\sigma_{1\tau} + \sigma_{2\mu}\sigma_{2\tau} + \sigma_{3\mu}\sigma_{3\tau}\right) X_3 \\
& + \kappa_4 (\sigma_{1e}\sigma_{1\tau} + \sigma_{2e}\sigma_{2\tau} + \sigma_{3e}\sigma_{3\tau}) X_4 \\
& + \kappa_5 \left(\sigma_{1e}\sigma_{1\mu} + \sigma_{2e}\sigma_{2\mu} + \sigma_{3e}\sigma_{3\mu}\right) X_5 + \frac{\kappa_6}{\Lambda}\left(\eta_2^2 \phi_2 - \eta_1^2 \phi_1\right) X_6 \\
& + \frac{\kappa_7}{\Lambda}\left(\sigma_{1\mu}\chi_2\chi_3 + \sigma_{2\mu}\chi_1\chi_3 + \sigma_{3\mu}\chi_1\chi_2\right) X_7 \\
& + \frac{\kappa_8}{\Lambda}(\sigma_{1\tau}\chi_2\chi_3 + \sigma_{2\tau}\chi_1\chi_3 + \sigma_{3\tau}\chi_1\chi_2) X_8 + \frac{\kappa_9}{\Lambda}\left(\varphi_2^3 - \varphi_1^3\right) X_9 \\
& + \frac{\kappa_{10}}{\Lambda}\left[\varphi_2\left(\sigma_{1\mu}^2 + \omega\sigma_{2\mu}^2 + \omega^2\sigma_{3\mu}^2\right) + \varphi_1\left(\sigma_{1\mu}^2 + \omega^2\sigma_{2\mu}^2 + \omega\sigma_{3\mu}^2\right)\right] X_{10} \\
& + \frac{\kappa_{11}}{\Lambda}\left[\varphi_2\left(\xi_1^2 + \omega\xi_2^2 + \omega^2\xi_3^2\right) - \varphi_1\left(\xi_1^2 + \omega^2\xi_2^2 + \omega\xi_3^2\right)\right] X_{11} \\
& + 2\kappa_{12}\left(\sigma_{2\mu}\sigma_{3\mu}\Phi_1 + \sigma_{1\mu}\sigma_{3\mu}\Phi_2 + \sigma_{1\mu}\sigma_{2\mu}\Phi_3\right) \\
& + 2\kappa_{13}(\sigma_{2\tau}\sigma_{3\tau}\Delta_1 + \sigma_{1\tau}\sigma_{3\tau}\Delta_2 + \sigma_{1\tau}\sigma_{2\tau}\Delta_3) \\
& + 2\kappa_{14}(\sigma_{2e}\sigma_{3e}\Theta_1 + \sigma_{1e}\sigma_{3e}\Theta_2 + \sigma_{1e}\sigma_{2e}\Theta_3) \\
& + \frac{\kappa_{15}}{\Lambda}\left[\left(\chi_1^2 + \omega\chi_2^2 + \omega^2\chi_3^2\right) - \left(\chi_1^2 + \omega^2\chi_2^2 + \omega\chi_3^2\right)\right]\sigma_{1e}\Xi_1 \\
& + \frac{\kappa_{15}}{\Lambda}\left[\omega^2\left(\chi_1^2 + \omega\chi_2^2 + \omega^2\chi_3^2\right) - \omega\left(\chi_1^2 + \omega^2\chi_2^2 + \omega\chi_3^2\right)\right]\sigma_{2e}\Xi_2 \\
& + \frac{\kappa_{15}}{\Lambda}\left[\omega\left(\chi_1^2 + \omega\chi_2^2 + \omega^2\chi_3^2\right) - \omega^2\left(\chi_1^2 + \omega^2\chi_2^2 + \omega\chi_3^2\right)\right]\sigma_{3e}\Xi_3
\end{aligned} \tag{B.1}
$$

Notice that there are two scales for the VEVs of the gauge singlet scalar fields of our model, i.e., the TeV scale and the large scale $\approx \lambda\Lambda$, with $\lambda = 0.225$. The singlet scalar fields having TeV scale VEVs are charged under the global $U(1)$ symmetry, whereas the remaining scalar singlets are neutral under this symmetry and do acquire VEVs at the large scale. Because of this reason higher order terms in the superpotential will not affect the stability of the VEVs.

From the superpotential given above, we find the following potential minimum conditions:

$$
\begin{aligned}
v_{\eta_1} v_{\eta_2} &= 0, \\
v_{\phi_1} v_{\phi_2} &= 0,
\end{aligned} \tag{B.2}
$$

$$
\begin{aligned}
v_{\sigma_{1\mu}} v_{\chi_2} v_{\chi_3} + v_{\sigma_{2\mu}} v_{\chi_1} v_{\chi_3} + v_{\sigma_{3\mu}} v_{\chi_1} v_{\chi_2} &= 0, \\
v_{\eta_2}^2 v_{\phi_2} - v_{\eta_1}^2 v_{\phi_1} &= 0,
\end{aligned} \tag{B.3}
$$

$$v_{\sigma_{1\tau}} v_{\chi_2} v_{\chi_3} + v_{\sigma_{2\tau}} v_{\chi_1} v_{\chi_3} + v_{\sigma_{3\tau}} v_{\chi_1} v_{\chi_2} = 0, \tag{B.4}$$
$$v_{\sigma_{1\mu}} v_{\sigma_{1\tau}} + v_{\sigma_{2\mu}} v_{\sigma_{2\tau}} + v_{\sigma_{3\mu}} v_{\sigma_{3\tau}} = 0,$$
$$v_{\varphi_2}^3 - v_{\varphi_1}^3 = 0, \tag{B.5}$$
$$v_{\sigma_{1e}} v_{\sigma_{1\tau}} + v_{\sigma_{2e}} v_{\sigma_{2\tau}} + v_{\sigma_{3e}} v_{\sigma_{3\tau}} = 0, \tag{B.6}$$
$$v_{\sigma_{1\mu}} v_{\sigma_{1e}} + v_{\sigma_{2\mu}} v_{\sigma_{2e}} + v_{\sigma_{3\mu}} v_{\sigma_{3e}} = 0, \tag{B.7}$$
$$v_{\varphi_2} \left(v_{\sigma_{1\mu}}^2 + \omega v_{\sigma_{2\mu}}^2 + \omega^2 v_{\sigma_{3\mu}}^2 \right) + v_{\varphi_1} \left(v_{\sigma_{1\mu}}^2 + \omega^2 v_{\sigma_{2\mu}}^2 + \omega v_{\sigma_{3\mu}}^2 \right) = 0, \tag{B.8}$$
$$v_{\varphi_2} \left(v_{\xi_1}^2 + \omega v_{\xi_2}^2 + \omega^2 v_{\xi_3}^2 \right) - v_{\varphi_1} \left(v_{\xi_1}^2 + \omega^2 v_{\xi_2}^2 + \omega v_{\xi_3}^2 \right) = 0, \tag{B.9}$$
$$v_{\chi_1} v_{\zeta_1} + v_{\chi_2} v_{\zeta_2} + v_{\chi_3} v_{\zeta_3} = 0,$$
$$v_{\xi_1} v_{\zeta_1} + v_{\xi_2} v_{\zeta_2} + v_{\xi_3} v_{\zeta_3} = 0, \tag{B.10}$$
$$v_{\sigma_{2\mu}} v_{\sigma_{1\mu}} = 0,$$
$$v_{\sigma_{3\mu}} v_{\sigma_{2\mu}} = 0, \tag{B.11}$$
$$v_{\sigma_{1\mu}} v_{\sigma_{3\mu}} = 0,$$
$$v_{\sigma_{2\tau}} v_{\sigma_{1\tau}} = 0, \tag{B.12}$$
$$v_{\sigma_{3\tau}} v_{\sigma_{2\tau}} = 0,$$
$$v_{\sigma_{1\tau}} v_{\sigma_{3\tau}} = 0, \tag{B.13}$$
$$\left[\left(v_{\chi_1}^2 + \omega v_{\chi_2}^2 + \omega^2 v_{\chi_3}^2 \right) - \left(v_{\chi_1}^2 + \omega^2 v_{\chi_2}^2 + \omega v_{\chi_3}^2 \right) \right] v_{\sigma_{1e}} = 0, \tag{B.14}$$
$$\left[\omega^2 \left(v_{\chi_1}^2 + \omega v_{\chi_2}^2 + \omega^2 v_{\chi_3}^2 \right) - \omega \left(v_{\chi_1}^2 + \omega^2 v_{\chi_2}^2 + \omega v_{\chi_3}^2 \right) \right] v_{\sigma_{2e}} = 0, \tag{B.15}$$
$$\left[\omega \left(v_{\chi_1}^2 + \omega v_{\chi_2}^2 + \omega^2 v_{\chi_3}^2 \right) - \omega^2 \left(v_{\chi_1}^2 + \omega^2 v_{\chi_2}^2 + \omega v_{\chi_3}^2 \right) \right] v_{\sigma_{3e}} = 0. \tag{B.16}$$

Combining Eqs (B.2) and (B.3) we find:

$$v_{\eta_2} = v_{\phi_1} = 0, \qquad v_{\eta_1} \neq 0, \qquad v_{\phi_2} \neq 0, \qquad \text{or} \qquad v_{\eta_1} = v_{\phi_2} = 0,$$
$$v_{\eta_2} \neq 0, \qquad v_{\phi_1} \neq 0 \tag{B.17}$$

Furthermore, from Eq. (B.5), we get:

$$v_{\varphi_2} = v_{\varphi_1}, \qquad v_{\varphi_2} = \omega^{\pm 1} v_{\varphi_1}, \qquad \text{or} \qquad v_{\varphi_1} = \omega^{\pm 1} v_{\varphi_2} \tag{B.18}$$

We proceed to choose the solution:

$$\langle \varphi \rangle = v_{\varphi} (1, \omega), \qquad \langle \phi \rangle = v_{\phi} (0, 1), \qquad \langle \eta \rangle = v_{\eta} (1, 0). \tag{B.19}$$

Then, using the above given VEV configuration for φ, Eqs. (B.8) and (B.9) take the form:

$$v_{\sigma_{1\mu}}^2 + v_{\sigma_{3\mu}}^2 = 0, \qquad v_{\xi_1}^2 - v_{\xi_3}^2 = 0. \tag{B.20}$$

Restricting to real solutions for the components of the VEV patterns for the S_4 scalar triplets, from Eqs. (B.20), (B.11) and (B.12), we find:

$$\langle \sigma_{\mu} \rangle = v_{\sigma_{\mu}} (0, 1, 0). \tag{B.21}$$

Thus, replacing Eq. (B.21) in Eqs. (B.6) and (B.7) yield the following VEV pattern for the S_4 scalar triplet σ_e:

$$\langle \sigma_e \rangle = v_{\sigma_e} (1, 0, 0) \,. \tag{B.22}$$

Furthermore, from Eq. (B.20), we find:

$$\langle \xi \rangle = v_\xi (\pm 1, r, \pm 1) \,, \qquad \text{or} \qquad \langle \xi \rangle = v_\xi (\pm 1, r, \mp 1) \,. \tag{B.23}$$

We choose the following solution:

$$\langle \xi \rangle = v_\xi (1, 3, 1) \,. \tag{B.24}$$

Replacing the VEV pattern of the S_4 triplet σ_μ given by Eq. (B.21) in Eqs. (B.5), (B.12) and (B.13), we get:

$$\langle \sigma_\tau \rangle = v_{\sigma_\tau} (0, 0, 1) \,. \tag{B.25}$$

Furthermore, combining Eqs. (B.3), (B.4), (B.21) and (B.25) yield the following relations:

$$v_{\chi 1} v_{\chi 3} = 0, \qquad v_{\chi 1} v_{\chi 2} = 0, \tag{B.26}$$

which implies one of the following solutions:

$$v_{\chi 1} = 0, \qquad v_{\chi 2} \neq 0, \qquad v_{\chi 3} \neq 0, \qquad \text{or} \qquad v_{\chi 1} = v_{\chi 2} = v_{\chi 3} = 0. \tag{B.27}$$

We choose the nontrivial solution:

$$v_{\chi 1} = 0, \qquad v_{\chi 2} \neq 0, \qquad v_{\chi 3} \neq 0. \tag{B.28}$$

Besides that, from Eqs. (B.22) and (B.14), we find:

$$v_{\chi 2}^2 - v_{\chi 3}^2 = 0. \tag{B.29}$$

Consequently, the S_4 triplet χ has the following VEV configuration:

$$\langle \chi \rangle = v_\chi (0, 1, 1) \,. \tag{B.30}$$

Thus, the potential minimum conditions yield the following VEV patterns for the S_4 doublets and triplet scalars of our model:

$$\begin{gathered} \langle \varphi \rangle = v_\varphi (1, \omega) \,, \qquad \langle \phi \rangle = v_\phi (0, 1) \,, \qquad \langle \eta \rangle = v_\eta (1, 0) \,, \qquad \langle \chi \rangle = v_\chi (0, 1, 1) \,, \\ \langle \xi \rangle = v_\xi (1, 3, 1) \,, \qquad \left\langle \sigma_\mu \right\rangle = v_{\sigma_\mu} (0, 1, 0) \,, \qquad \langle \sigma_\tau \rangle = v_{\sigma_\tau} (0, 0, 1) \,, \\ \langle \sigma_e \rangle = v_{\sigma_e} (1, 0, 0) \,. \end{gathered} \tag{B.31}$$

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