THE LANGUAGE OF FRIENDSHIP: DEVELOPING SOCIOMATHEMATICAL NORMS IN THE SECONDARY SCHOOL CLASSROOM

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This paper reports on a study of friendship groups as they learned mathematics in small groups in a secondary school classroom. It examines the role that discussions between friends have on their ability to negotiate taken-as-shared meanings (or sociomathematical norms). Transcripts of peer talk in a low attaining group of 14-15 year olds are analysed for evidence of the sociomathematical norms evident in a study by Cobb et al *(1995) with 6-8 year olds. Findings suggest that similar negotiations are evident, despite the differences in age, but that an additional sociomathematical norm related to mathematical efficiency in written communication is identified.*

The Nature of Friendship

The study of friendship is undertaken in three fields of study – anthropology, psychology and sociology. Each offers its own perspective on the nature and function of friendships. Despite the multitude of studies, Allan (1996) notes that there is a lack of firmly agreed and socially acknowledged criteria for what makes a person a friend. From an anthropological perspective, Pahl (2000) offers a definition of friendship which fits the research setting described here:

> Friendship is a relationship built upon the whole person and aims at a psychological intimacy, which in this limited form makes it, in practice, a rare phenomenon, even though it may be more widely desired. It is a relationship based on freedom and is, at the same time, a guarantor of freedom. A society in which this kind of relationship is growing and flourishing is qualitatively different from a society based on the culturally reinforced norms of kinship and institutional roles and behaviour (pp163-4).

Bell and Coleman (1999) similarly argue an anthropological stance that a Western view of friendship is a matter of choice and that “friendship becomes a special relationship between two equal individuals involved in a uniquely constituted dyad” (p8). However, the research undertaken here is with friendship groups of between three and six individuals. Allan (1989) suggests that even in the dyadic context, friendships are a matter of opportunity, dependent on class, gender, age, ethnicity and geography. This is reflected in the discussions amongst friends in the research study.

In psychological studies, there is a linking of developmental stages in friendship with Piagetian stages of development. For example, in developing notions of empathy and the ability to see the point of view of another, Erwin (1993) outlines Selman’s (1980) model of the stages of development in ‘role-taking’. Note that this ‘role-taking’ is different from that related to work in groups. I outline the final two of five stages, as these pertain best to the age of the students in the study in this paper. Selman’s fourth stage is called ‘Mutual role-taking’ and occurs at approximately 10-12 years of age. This involves the child in being able to recognise the relationship of their own perspective to that of another and in appreciating that others are also aware of their perspective. The fifth stage, ‘Social and conventional system role-taking’, begins between 12 to 15 years and continues into adulthood. This is when general social considerations, rules and norms are taken into account and reflected upon. The complexity and subjectivity of other people are recognised, as are their consistent patterns of personality and behaviour. Given the age of the students in this wider study (11-15 years), it is expected that these two stages will be evident in the discussions.

Sociological studies examine the impact of friendships on individuals and in social contexts. Adams and Allan (1998) state that friendships cannot happen in a social or economic vacuum:

> Relationships have a broader basis than the dyad alone; they develop anad endure within a wider complex of interacting influences which help to give each relationship its shape and structure. If we are to understand fully the nature of friendships, these relationships need to be interpreted from a perspective which recognises the impact of this wider complex (pp2-3).

Gottman and Parker (1986) describe the particular social skills which are developed within friendships. The final six of these are:

> conform, cooperate and compete
>
> take risks
>
> develop communication skills
>
> develop negotiation skills and tact
>
> resolve conflicts
>
> develop shared meanings for group interaction (p282)

These six skills are particularly relevant to the study of friendships in mathematics classrooms. In other studies of young children working in friendship groups, these skills are similarly identified. Schneider (2000) reports Nelson and Aboud’s (1985) study which found that friends explained their opinions and criticised their partners more often than non-friends. They argued that “higher levels of disagreement led to more cognitive change than did compliance” and concluded that “friends who experience conflict undergo more social development than non-friends do in conflict” (p76). The reasons given for this were that friends were likely to change in favour of the more mature solution, whereas in non-friend pairs, either in the pair was likely to change. This has implications for friends working in groups in mathematics classrooms, as there may be a parallel in friends opting for the more mathematically different, mathematically sophisticated, mathematically efficient or mathematically elegant solution.

Research on Sociomathematical Norms

In order to explain the nature and development of sociomathematical norms in classrooms, I intend to focus on the work of six researchers (three American and three German), undertaking research from psychological and sociological perspectives on the same data collected over a period of 10 weeks in second and third-grade (6-8 year olds) US classrooms during a year-long classroom experiment in inquiry-based classrooms. Bauersfeld, Cobb, Krummheuer, Voigt, Wood and Yackel, define the study as a ‘teaching experiment classroom’. Lessons typically consisted of a teacher-led introduction to a problem as a whole class activity, cooperative small-group work in pairs, and follow-up whole class discussion where children explain and justify solutions to each other. Recordings were taken of all small-group sessions and whole-class discussions on an arithmetic topic and these tapes were analysed. Small-group interactions were analysed on the basis of their “taken-as-shared” mathematical meanings that were established within the group (Cobb 1995). The teacher actively guided this establishing process. Cobb describes small-group norms as including:

> explaining one’s mathematical thinking to the partner, listening to and attempting to make sense of the partner’s explanations, challenging explanations that do not seem reasonable, justifying interpretations and solutions in response to challenges, and agreeing on an answer and, ideally, a solution method (p 104)

Interactions between children were identified as univocal explanation (in which one child assumed the authoritative position) or multivocal explanation (in which explanations and solutions were joint). A definition of authority was only accepted if the non-authoritative child accepted the authority of the other. Some children found multivocal explanations difficult because they had not established a ‘taken-as-shared’ basis for their discussion. However, only multivocal explanation was considered productive in its outcome. Direct collaboration in which roles were assigned to meet the end need was deemed non-productive. Indirect collaboration in which children appeared to be working independently whilst talking aloud, was considered productive because children found what each other were saying significant for them at the time.

These six authors assert that in a mathematical environment, the social norms that are interactively established in groups in any setting take on particular features specific to mathematics. These were recognised from tape recordings of lessons by identifying regularities in the patterns of social interactions. The authors argue that, whilst children should be challenging each other’s thinking and justifying their own thinking in any area of the curriculum, in mathematics there are particular norms set up within groups as to what is taken-as-shared meaning about acceptable mathematical explanation and justification.

The premise upon which sociomathematical norms are established is that children understand that the basis for explanation is mathematical rather than status-based (e.g. explaining for authority). Yackel and Cobb (1996) argue that these norms are established in stages of development. The first is explaining as a description of procedure, i.e. instructing how to do an act; the second is explaining as describing actions on a real (mathematical) object; the third is accepting this second stage as an object of reflection and deciding if it is valid for others. These can be interpreted as stages of computation, conceptual explanation and reflective action.

This exploration of a sociomathematical norm as determining an acceptable mathematical explanation serves to illustrate other sociomathematical norms identified. These include what counts as mathematically different, mathematically sophisticated, mathematically efficient and mathematically elegant. In negotiating sociomathematical norms, children become increasingly autonomous, the authors argue. They provide evidence of increased learning opportunities through listening and challenging the explanations of others.

I argue that friendship groups in mathematics classrooms of 11-15 year olds, in particular, offer the opportunities for these sociomathematical norms to be negotiated effectively. The following, from a study of friendship groups, offers evidence for the stages of developing sociomathematical norms and suggests differences because of the relative ages of student participants in the study.

The Study of Friendship Groups

Students in this study (Edwards, 2003) attended an inner-city comprehensive secondary girls' school of 1087 pupils in the south of England. This population represented a full social and ethnic mix, with the majority of girls of white background, though there is a significant minority of 22% Asian girls and a total ethnic minority of 28%. The department operated a problem-solving curriculum based on the activities of the Graded Assessment in Mathematics (GAIM) project. These activities were introduced as a whole-class discussion, with students and teacher making possible suggestions for routes for exploration. Most of the subsequent work was in small groups of two to six students, though the class was sometimes drawn together at various points to enable a student to explain a discovery or the teacher to make a teaching point from something that has arisen. The teacher circulated amongst the small groups, supporting thinking, and assisting the direction of the activity. Small-group organisation was on a self-selected friendship basis but some groups were reorganised or split if they become mathematically unproductive.

Recordings of whole-class and small-group interactions were taken over a period of eight weeks for a high attaining Year 9 group (13-14 year olds) for all lessons covering two GAIM activities. A low attaining Year 10 class (14-15 year olds) was recorded for some of its lessons over a period of two weeks using the same GAIM activity undertaken by a middle attaining Year 7 class (11-12 year olds) and this Year 7 class was recorded over the same period of time. The recordings were taken in the third term of schooling when these groups had been working together for approximately 24 school weeks. Evidence from a Year 10 group is presented here.

Evidence from work in friendship groups

The full transcript of the lesson for F, R and Z (Year 10) from which this example is taken gives strong evidence for the three levels of establishing sociomathematical norms for mathematical explanation. Their levels of questioning and understanding develop from procedural through conceptual to bordering on reflective. F, R and Z are completing an activity in which they are agreeing a solution for finding the number of possible half time scores for a Hockey match, given any final score. Initially, they focus on procedure:

Z Now two times three ..

R two times one is two

Z Yeah

F Add two......

R add four

Z Yeah, both those ... equals six …two add two .. two times two is four, is it ..? Yeah Add that, add that is nine … two times three is six .. Oh, maybe not

R Yeah but that’s not ... that’s the unacceptable one, innit.

Z I’ll just see this one

R I’ll ask her. Miss? (T arrives)

T two times one is two

Z Is it that, Miss, look … two times one is two, add two, add two, equals six … two times two is four add that add that is nine …What am I doing about ..? two times three is six Oh that doesn’t work But it does work over here …three times one is three add that add that equals eight

T Does it work for this one?

Later, after an intervention from the teacher, they then focus on the reasons why they need to have the solution they have derived. This demonstrates the conceptual level described by Yackel and Cobb (*ibid*).

T Think about *why* you need to add one each time ... What have you got there?

Z Four sets of group, um, four sets of goals, ohh

R I know Miss

Z What is it?

R We can add one to 0 to get our next .. things and then one, to .. you add another one to one to get two

F Yeah but *why*? The reason *why*, not what you do

R Yeah, *why*, right What she's asking us this bit, yeah, *why* do we need to add one to that. The reason is that we need to add one to that first to get that

F Which means, in many more words, is you need to add one to get your answer.

R No

Z But she said *why* didn't she

R That ain't the answer. That ain't the answer. That's not answer.

F No, but the answer to why .. is why you have to add one to get the answer

R Is because you add one

The final stage of this development of sociomathematical norms is demonstrated clearly when these girls are considering the impact of the written communication of their solution. They are writing the reason why they needed to add one to each number in their solution: Half time scores = (n+1) x (y+1). They are attempting to write their verbal description of needing to add one each time because they are including zero in the total. Although they are not at the stage of fully reflecting on this communication, they are at the stage of recognising its impact and importance. They are using their explanation as an object for a focus for activity.

F The .. reason .. why .. you .. add .. one ... [as she writes]

R To what .. what do we add one to?

F Add ... one ...

Z To ..

F Add one, right, to each goal

R To each set

F Yeah, to each goal number

R To .. each .. goal .. number ... [writing]

Z Is .. because ..

F Because ..

Z If it was ..

F Because .. hang on .. because we started off with zero

R We always included zero

F Because .. we .. start .. off .. with .. zero .. and we have to add one all the time. That's it

Z Because we start off with zero and what?

F We have to .. add

R Zero. Have you got ...

Z What?

F Add ... include zero

R And we need to add one

F And we have to add one

R We have to move .. to goals

Z To make it up to another number, add one

R No, later on .. Cos it’s the next ..

F We have to add on zero, start with the zero because

At this point they are confused about the difference between adding on zero and starting with zero. However, the continued extract shows that this confusion is only a function of the writing as, between the three students, they sort out an acceptable written explanation.

Z Start off with zero

F Because ..

Z Zero, and to add on another number

R One, add one because ..

Z Read that

F Add one to the set of goals, hang on, we .. add .. one .. to .. the .. set .. of .. goals .. because ..

R We need to move onto the next one ...

F Because we started ..

R We need to go onto the next number

F With ..

Z Yeah

F Zero Zero [reading] We add one to the set of goals because we started with zero and ..

Z We need to go onto the next number

F We .. need .. to .. move .. on .. to ..the .. next .. number .. which .. is ..what .. we .. started .. with. What do you think of that?

These low attaining Year 10 girls who are working towards their algebraic solution: Half time scores = (n+1) x (y+1) know that they are refining their mathematical efficiency through symbolism. This provides an example of a different sociomathematical norm being established than those identified by Yackel and Cobb and is similarly identified in the Year 7 and Year 9 groups. Although this is not an example suggested by Yackel and Cobb, I believe that it is, equally, an example of a sociomathematical norm at this age level because the students establish a taken-as-shared meaning for this important element of communicating mathematics. The reason it may not be identified in Yackel and Cobb’s work is because their research was done with elementary school children where written recording of work may not be a focus of activity.

Talking aloud is a significant and prevalent feature of all the groups studied. Noddings (1990) suggests that the level of elaboration required by talking aloud forced the student to concentrate on the problem. In the extract above, the teacher was not present during most of the time these pupils talked aloud as they wrote their solutions. However, the extent to which the purpose of talking aloud, in this case, is to keep them focused on the problem is debatable. I believe the purpose is related more to refining their own constructions of the solution.

The development of explanation and justification is an essential component of group work if students are to benefit from the trust established in friendships. In all the recordings there is a drive by group participants to generate a solution that they knew would work. Much of this knowing comes from questioning each other, arguing and justifying decisions to each other. Throughout the recordings there is also evidence of enjoyment in the form of laughter about mathematical situations that arise and a gentle banter about own performance or ability or that of another's. Rodgers (1995) argues in support of this enjoyment when she says "All the evidence points to the fact that the use of humour and laughter are very useful in dissipating the tensions created by learning difficulties" (p 36). The familiarity of friends in the context of mathematics groupings is a mechanism by which tensions relating to mathematics are more easily addressed (Edwards, 2004).

Discussion

The sociomathematical norms identified in this study are almost all based on mathematical explanation, as are those of Cobb *et al* in their study. Norms of mathematical difference, mathematical sophistication, and mathematical elegance are not identified, though examples of mathematical efficiency in communicating are identified in the older age groups.

The difference in age groups in this study and that of Yackel and Cobb raise issues of comparability. The level of mathematical language used in secondary classrooms is already more sophisticated than that in elementary classrooms. This makes analysis of small group talk to determine whether the group is establishing taken-as-shared meaning about mathematical sophistication more complex. Similarly, the complexity of the problems posed in each of the studies is very different, and this has repercussions for the level of language used and thence the type of sociomathematical norms which will be established. It also makes the norms more difficult to identify. However, the norms in this study were consistently identifiable over three age groups at the secondary level.

It is interesting that, in the most established friendship groups (Year 10), negotiations of sociomathematical norms were found to be as equally identifiable as in the less established working groups (Year 7 and Year 9). Whilst Cobb *et al* assert that there is mathematical specificity to any sociomathematical norms that are interactively established in groups in any setting, it may well be the case that these norms may also be context specific and therefore generate a need for groups to establish new taken-as-shared meanings in each of these contexts. Thus, established friendship groups are combining a mutually shared understanding of some established sociomathematical norms but, in a new mathematical context, are needing to generate and negotiate new norms. Since the study undertaken by Cobb *et al* was in a classroom where the teacher and class were undergoing a change in pedagogy and methodology towards social constructivism, it would be possible that the sociomathematical norms established in these conditions may not apply to a classroom where this mode of working is already an established norm. However, there is sufficient evidence in this small study to contradict this assumption. Indeed, a further sociomathematical norm was identified which I shall term *mathematical evidence*. This is demonstrated by the taken-as-shared meanings for the effective written communication of mathematical understanding.

Friendship groups appear to provide the necessary conditions for students to successfully challenge and justify ideas. The evidence to confirm Nelson and Aboud's (*ibid*) findings that friendships offer an environment in which learning leads to greater cognitive change for social situations may be transferable to mathematical learning. This is confirmed by Zarjac and Hartup (1997) who found that friends were better co-learners than non-friends. Whilst there is evidence in the Year 10 example in the study described here, the wider evidence from all three age groups confirms that friends are deferring to the more acceptable and efficient mathematical explanations.

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