

Gravitational waves from transient neutron star f-mode oscillations

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During their most recent observing run, LIGO/Virgo reported the gravitational wave (GW) transient S191110af, a burst signal at a frequency of 1.78 kHz that lasted for 0.104 s. While this signal was later deemed non-astrophysical, genuine detections of uncertain origin will occur in the future. Here we study the potential for detecting GWs from neutron star fluid oscillations, which have mode frequency and duration matching those of S191110af and which can be used to constrain the equation of state of nuclear matter. Assuming that such transient oscillations can be excited to energies typical of a pulsar glitch, we use measured properties of known glitching pulsars to estimate the amplitude of GWs produced by such events. We find that current GW detectors may observe nearby pulsars undergoing large events with energy similar to Vela pulsar glitch energies, while next generation detectors could observe a significant number of events. Finally, we show that it is possible to distinguish between GWs produced by rapidly rotating and slowly rotating pulsars from the imprint of rotation on the f-mode frequency.

Introduction The new era of gravitational wave (GW) astronomy began with the detection of binary black hole and binary neutron star (NS) mergers in the last several years by the advanced GW detectors of Laser Interferometer Gravitational-wave Observatory (LIGO) and Virgo [1, 2]. Searches are ongoing for more black hole and NS mergers, as well as for NS-black hole mergers and other GW sources such as transient signals associated with supernovae and fast radio bursts (FRBs). Like in traditional electromagnetic astronomy, there may be occasions when “rare” GW signals are detected whose properties are not well-understood or modeled at the time of discovery. For example, the phenomenon of FRBs was not unequivocally known to have an astrophysical origin when the first one was found in 2007 [3], and the astrophysical nature of FRBs is still not known even after more than one hundred events have been detected [4–7].

In the GW regime, a somewhat analogous signal to FRBs was reported recently. The GW transient candidate S191110af was detected on 2019 November 10 by LIGO/Virgo and consists of a signal at 1.78 kHz that lasted for 0.104 s [8]. Follow-up analysis over the next few days identified instrumental artifacts in the data, which led to retraction of S191110af as a genuine astrophysical signal [9]. In the intervening time, it was pointed out that the frequency and burst duration of S191110af are consistent with the fundamental stellar oscillation mode (f-mode) of a NS of mass $M = 1.25 M_{\text{Sun}}$ and radius $R = 13.3$ km [10, 11]. In addition, the results of [12, 13] were used to estimate that a f-mode could produce a GW signal-to-noise ratio (SNR) ~ 10 [11]. GW-producing f-mode oscillations can be triggered by transient events internal to the NS [13–16], such as a sudden phase transition, magnetic field reorganization, or pulsar glitch (a

sudden change $\Delta\nu_s$ of NS spin rate ν_s due to starquakes or more likely angular momentum exchange between normal and superfluid components within the star; [17, 18]). However, evidence of glitching pulsars that could be responsible for S191110af was not found [11]. The effectiveness of different proposed mechanisms is also unclear since their energy may not be released as GWs.

GWs associated with the f-mode have been of great interest because the mode frequency depends on the dynamical timescale of NSs and hence is a probe of NS density, mass, and radius [10, 12, 19]. A f-mode GW signal can appear in newborn NSs [20], magnetars [21], and NS mergers, during both pre-merger [22–26] and post-merger [27–30] phases. Here we expand on the brief analysis of [11] and study detectability of GWs produced by a f-mode in rotating NSs, assuming that the mode is excited to a level corresponding to the energy associated with typical pulsar glitches [14]. This is sensible because we know that pulsars exhibit transients at this level, even though there is no established connection between mode excitation and observed glitches. The advantage is that, by considering normal isolated NSs that are well-studied, we have a well-defined source population with known properties, and we avoid the uncertainties of speculating on and modeling unknown sources. Moreover, this allows us to consider a question that may become relevant in the future: How do we distinguish astrophysical transients from detector noise if both are associated with exponentially damped sinusoidal signals?

Model for GW source Consider a stellar oscillation with frequency ν_{gw} that is induced at time $t = 0$ and damps on a timescale τ_{gw} . Following [31, 32], the GW amplitude from such an oscillation is then zero for $t < 0$

and

$$h(t) = h_0 e^{-t/\tau_{\text{gw}}} \sin(2\pi\nu_{\text{gw}}t) \quad \text{for } t > 0. \quad (1)$$

The peak amplitude h_0 can be determined by first noting that the GW luminosity of a source at distance d is [33]

$$\frac{dE_{\text{gw}}}{dt} = \frac{c^3 d^2}{10G} \left(2\pi\nu_{\text{gw}} h_0 e^{-t/\tau_{\text{gw}}} \right)^2. \quad (2)$$

We then integrate equation (2) over $0 < t < \infty$ to obtain the total GW energy emitted E_{gw} and solve for h_0 to find

$$h_0 = \frac{1}{\pi d \nu_{\text{gw}}} \left(\frac{5G E_{\text{gw}}}{c^3 \tau_{\text{gw}}} \right)^{1/2} = 4.85 \times 10^{-17} \left(\frac{1 \text{ kpc}}{d} \right) \times \left(\frac{E_{\text{gw}}}{M_{\text{Sun}} c^2} \right)^{1/2} \left(\frac{1 \text{ kHz}}{\nu_{\text{gw}}} \right) \left(\frac{0.1 \text{ s}}{\tau_{\text{gw}}} \right)^{1/2}. \quad (3)$$

Now consider the oscillation mode is excited to a level corresponding to a pulsar glitch, such that the GW energy E_{gw} is supplied by the energy of the glitch

$$E_{\text{glitch}} = 4\pi^2 I \nu_s \Delta\nu_s = 3.95 \times 10^{40} \text{ erg} \left(\frac{\nu_s}{10 \text{ Hz}} \right) \left(\frac{\Delta\nu_s}{10^{-7} \text{ Hz}} \right), \quad (4)$$

where NS moment of inertia $I \sim 10^{45} \text{ g cm}^2$. Substituting equation (4) into equation (3), the peak GW amplitude is

$$h_0 = 7.21 \times 10^{-24} \left(\frac{1 \text{ kpc}}{d} \right) \left(\frac{\nu_s}{10 \text{ Hz}} \right)^{1/2} \left(\frac{\Delta\nu_s}{10^{-7} \text{ Hz}} \right)^{1/2} \times \left(\frac{1 \text{ kHz}}{\nu_{\text{gw}}} \right) \left(\frac{0.1 \text{ s}}{\tau_{\text{gw}}} \right)^{1/2}. \quad (5)$$

Thus for a given oscillation mode frequency ν_{gw} and damping time τ_{gw} , the peak GW amplitude h_0 depends on distance d to the pulsar, the pulsar spin frequency ν_s , and glitch size $\Delta\nu_s$. In this calculation, we consider the f-mode oscillation to be efficient at extracting energy at the level of glitches and driving the emission of GWs. In reality, such a process is likely to be at least somewhat inefficient. However, a factor of, e.g., ten lower energy that is converted to GWs (from a glitch or other process) only reduces the GW amplitude by a factor of three since $h_0 \propto \sqrt{E_{\text{gw}}}$. On the other hand, g-modes are known to be significantly less efficient than f-modes at producing GWs [20, 34, 35].

Pulsar and glitch distributions For our nominal GW sources, we use 552 glitches from 188 pulsars in the Jodrell Bank Glitch Catalogue [36][37]. The Jodrell Bank Glitch Catalogue lists the relative spin frequency change $\Delta\nu_s/\nu_s$ for each detected glitch. We use the ATNF Pulsar Catalogue [38] to supplement the glitch data with each pulsar's spin frequency ν_s , distance d , and sky position. Note that the default distance in the ATNF Pulsar Catalogue is derived from each pulsar's dispersion measure

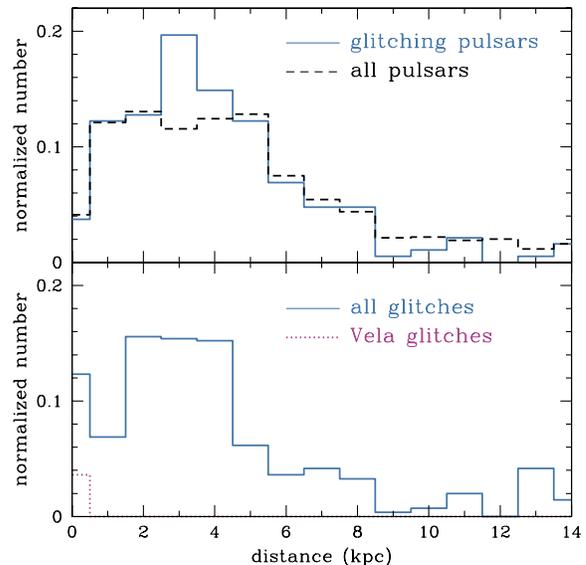


FIG. 1. Top: Normalized distributions of distance for all pulsars (dashed line) and for all glitching pulsars (solid line). Bottom: Normalized distribution of distance for all glitches (solid line). Dotted line is the distribution for Vela glitches.

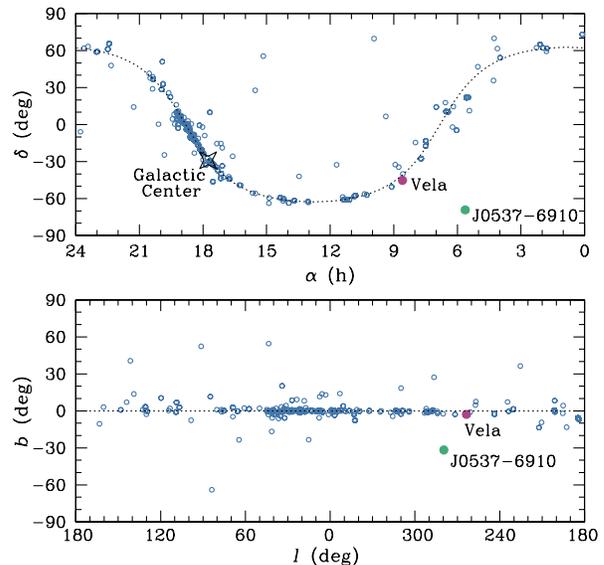


FIG. 2. Position of glitching pulsars in right ascension α and declination δ (top) and Galactic longitude l and latitude b (bottom). The Galactic plane is denoted by dotted lines, and positions of the Galactic Center and pulsars Vela and PSR J0537-6910 are labeled.

[39], although in some cases an independent distance is known. Since we are not focused on most individual pulsars but on the overall population, distance errors are not important.

The top panel of Figure 1 shows the normalized distance distributions of all ~ 2700 known pulsars (with a distance) in the ATNF Pulsar Catalogue and 188 glitch-

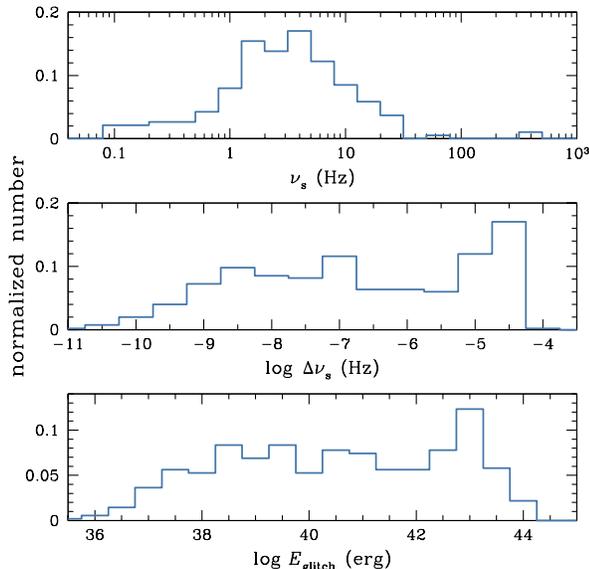


FIG. 3. Normalized distributions of spin frequency ν_s (top) for all 188 glitching pulsars and glitch size $\Delta\nu_s$ (middle) and glitch energy E_{glitch} (bottom) for all 552 glitches.

ing pulsars in the Jodrell Bank Glitch Catalogue. The bottom panel shows the distance distribution of the 552 glitches. We see that a majority of known pulsars and glitching pulsars are at distances $d < 6$ kpc. This is due in large part to observational selection effects. The Vela pulsar, at a distance of 287 pc [40], contributes significantly to the very nearby glitch population. Meanwhile, PSR J0537–6910, also known as the Big Glitcher [41], is in the Large Magellanic Cloud at a distance of 50 kpc and thus is not shown in Figure 1. We also need to keep in mind that the population of “seismically active” NSs which emit GWs could be dominated by objects that have not yet been detected.

Figure 2 shows the position of each glitching pulsar. Glitching pulsars are clearly clustered in the Galactic plane, as expected for relatively young pulsars. Because source localization by only GW detectors is generally poor, we may not be able to determine definitively whether an individual GW burst originates from a source in the Galactic plane. However, such a determination may be possible for a population of burst sources (such as the glitching pulsars) if they all contain localization regions that overlap with the Galactic plane or even cluster near the Galactic Center [42].

The top panel of Figure 3 shows the distribution of spin frequency ν_s for the 188 glitching pulsars, and the middle and bottom panels show the distributions of glitch size $\Delta\nu_s$ and glitch energy E_{glitch} , respectively, for the 552 glitches. Most glitching pulsars have relatively low spin frequencies, i.e., $\nu_s \approx 1 - 30$ Hz. Glitch size has a broad range $\Delta\nu_s \sim 10^{-9} - 10^{-5}$ Hz [36, 43, 44], which leads to a broad range of glitch energies $E_{\text{glitch}} \sim 10^{37} - 10^{44}$ erg \sim

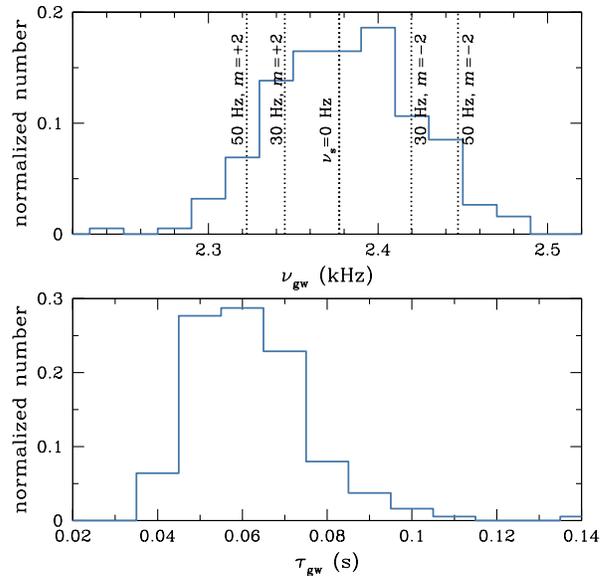


FIG. 4. Normalized distributions of f-mode frequency ν_{gw} (top) and damping time τ_{gw} (bottom), derived using a Gaussian mass distribution peaked at $M = 1.4M_{\text{Sun}}$ with $0.15M_{\text{Sun}}$ width, the BSk24 EOS for the radius, and non-rotating $\nu_{\text{gw}}(M, R)$ and $\tau_{\text{gw}}(M, R)$ relations of [45]. Vertical dotted lines indicate (inertial) frame ν_{gw} for labeled spin frequency and oscillation mode order m and using spin corrections of [45] with $\nu_{\text{K}} = 1$ kHz.

$10^{-17} - 10^{-10} M_{\text{Sun}} c^2$.

Results With the known properties of our model source population described above, we compute the amplitude of GWs emitted from a damped f-mode triggered by the energy equivalent to a pulsar glitch [see equation (5)]. First, we must determine the f-mode frequency and damping time. Early works [10, 12, 19] show that ν_{gw} and τ_{gw} are related to NS mass M and radius R in a way that is approximately independent of nuclear equation-of-state (EOS). Subsequent work verified these relations [45] and find alternative relations that depend on M and moment of inertia I [46, 47].

For simplicity, we use the f-mode frequency and damping time relations of [45] to M and R in the non-rotating limit (see below). We randomly assign a mass to each of the 188 glitching pulsars, where M is drawn from a Gaussian distribution centered at $M = 1.4M_{\text{Sun}}$ with a width of $0.15M_{\text{Sun}}$. The radius is then determined from the mass using the BSk24 EOS [48], which is a modern nuclear EOS that we choose simply as an example. The BSk24 EOS generates NSs whose mass and radius satisfy the $M-R$ constraints from *NICER* [49, 50] and produces a maximum NS mass that exceeds the highest observed NS mass [51]. Figure 4 shows the resulting distributions of f-mode frequency and damping time. We note that the peak f-mode frequency and damping time for the somewhat softer APR EOS [52] are at 2.5 kHz and 0.05 s, respectively. Since $\nu_{\text{gw}} \propto \sqrt{M/R^3}$ and $\tau_{\text{gw}} \propto R^4/M^3$

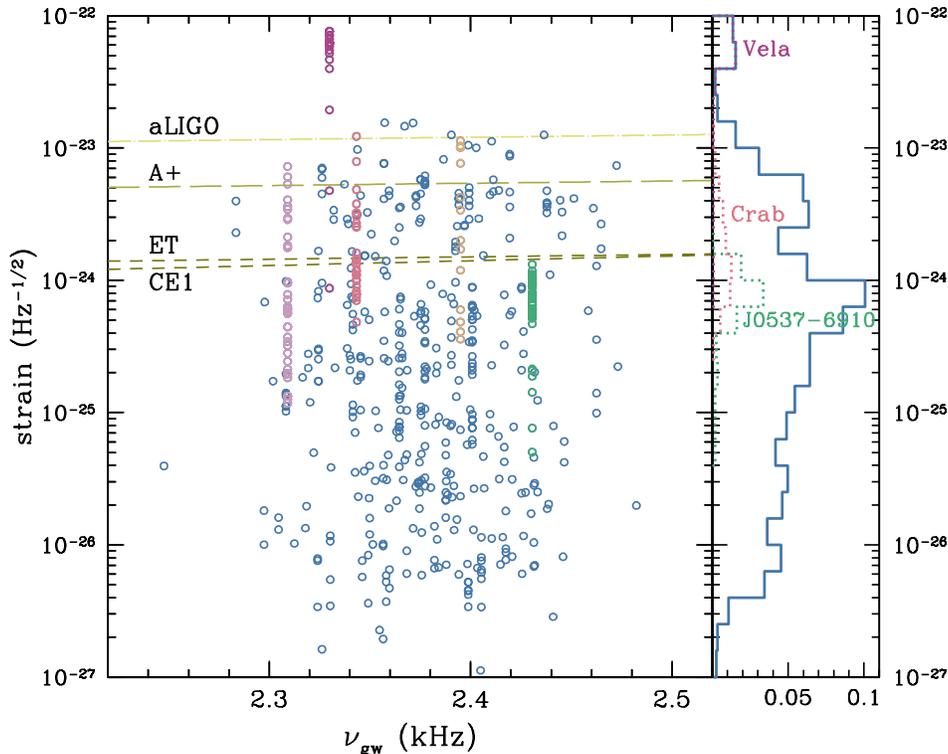


FIG. 5. GW spectrum. Each circle denotes the peak strain ($=h_0\sqrt{\tau_{\text{gw}}}$) of a burst of GWs emitted at ν_{gw} from a damped f-mode oscillation excited to a level equivalent to the energy of one of the 552 known glitches in the Jodrell Bank Glitch Catalogue. Nearly horizontal dashed lines are sensitivities of Advanced LIGO (aLIGO), A+, Einstein Telescope (ET), and Cosmic Explorer (CE1). Right panel: Normalized strain distribution. Bursts attributed to glitches of PSR B1737–30, Vela and Crab pulsars, PSR J0205+6449, and PSR J0537–6910 are highlighted (from left to right).

[10, 19, 45], our assumed mass distribution with width $\sim 10\%$ produces f-mode and damping time distributions with width $\sim 5\%$ and $\sim 30\%$, respectively. On the other hand, there is generally not much difference between radii of different masses around $1.4M_{\text{Sun}}$ for a given EOS, e.g., the radius differs by $< 1\%$ in the mass range $M = 1.1\text{--}1.7M_{\text{Sun}}$ for BSk24 ($< 3\%$ for APR). Therefore radius variations do not contribute significantly to variations of ν_{gw} and τ_{gw} .

The leading order spin corrections to ν_{gw} are $\approx (0.2 - 0.4)(\nu_s/\nu_K)$ [45], where Kepler frequency $\nu_K \approx \sqrt{GM/R^3}/3\pi \sim 1$ kHz [53]. Almost all glitching pulsars have a relatively low spin frequency ($\nu_s \lesssim 30$ Hz; see top panel of Figure 3), such that they would have f-mode frequency corrections of $< 1\%$. There are two glitching pulsars with $\nu_s \approx 327$ Hz, but these have only been observed to glitch once and are not expected to glitch again for a long time (> 100 yr) given their low spin-down rate, and each glitch was also very small in size, i.e., $\Delta\nu_s \sim 10^{-9}$ Hz. The only other fast-spinning glitching pulsar is PSR J0537–6910 with $\nu_s = 62$ Hz but is at a distant 50 kpc. Thus based on current observational evidence, it appears safe to ignore rotational effects. On the other hand, we can see from the top panel of Figure 4 that

a burst whose frequency is markedly distinct from the distribution average could originate from a pulsar with $\nu_s > 50$ Hz. Therefore detection of such a GW burst could indicate a fast-spinning pulsar, especially since a (currently unknown) population of active GW-emitting NSs may not share all the same properties as glitching pulsars. It is also possible for f-modes of different spherical harmonic to be excited. The frequency difference between rotation-induced $m=0, \pm 1, \pm 2$ for $l=2$ modes is approximately the spin frequency and would likely be resolvable for NSs with $\nu_s > 1/T_{\text{obs}}$, where T_{obs} is time over which a GW search is performed. The sub-second duration f-modes considered here imply frequency splitting could be seen in bursts from pulsars with $\nu_s > 1$ Hz.

With a characteristic f-mode frequency ν_{gw} and damping time τ_{gw} assigned to each of the 188 known glitching pulsars, as well as their measured spin frequency ν_s and distance d , and the glitch size $\Delta\nu_s$ of each of the 552 measured glitches, we calculate peak GW amplitude h_0 using equation (5). Figure 5 shows the resulting peak GW strain ($= h_0\sqrt{\tau_{\text{gw}}}$), as well as the spectral noise density $\sqrt{S_h}$ of LIGO and next generation GW detectors [54–57]. While the glitch size of most measured glitches and distance to each corresponding pulsar pro-

duce $h_0\sqrt{\tau_{\text{gw}}} < 10^{-24} \text{ Hz}^{-1/2}$, about 20% of glitches would be strong enough to produce a GW signal that is observable by current and next generation detectors. For example, bursts with the energy expected from a Vela glitch can reach $\text{SNR} = h_0\sqrt{\tau_{\text{gw}}/2S_h} \sim 5$ in advanced LIGO data and ~ 40 using third generation detectors. Bursts from a Crab-level glitch could have $\text{SNR} \sim 2$ using A+ and ~ 6 using third generation detectors.

Discussion It is important to note that, since the true nuclear EOS is unknown at this time, other model EOSs can yield average ν_{gw} and τ_{gw} much lower or higher than the 2.4 kHz and 0.06 s obtained for the BSk24 EOS, although their dispersions for a given EOS would be similar to those shown in Figure 4. Thus detection of bursts with average ν_{gw} significantly different from 2.4 kHz does not invalidate our results but may indicate a different EOS than the one considered here is preferred. One can envision measuring bursts clustered around a particular frequency due to f-mode oscillations (glitch-excited or by other means), as well as burst signals at other frequencies due to entirely different types of GW sources. We should expect GW bursts from f-mode oscillations to obey the $\nu_{\text{gw}}(M, R)$ and $\tau_{\text{gw}}(M, R)$ relations of [10, 19, 45] and $\nu_{\text{gw}}(M, I)$ and $\tau_{\text{gw}}(M, I)$ relations of [46, 47]. For example, bursts with higher ν_{gw} should have shorter τ_{gw} . Most should also have localization regions that overlap with the Galactic plane. An interesting avenue for future research is investigating data analysis strategies based on an expected excess of transient events in the relevant frequency range.

The GW strains shown in Figure 5 would seem to suggest that GW bursts from systems like the Vela pulsar are essentially the only ones that could be measured by current detectors, due to the pulsar's proximity (287 pc) and large glitches ($\Delta\nu_s \gtrsim 10^{-5} \text{ Hz}$). However, Vela glitches are relatively infrequent for GW searches, occurring every 3–4 yr. Thus one might expect the contribution of this type of burst source to the total number of unmodeled transients detected by LIGO/Virgo/KAGRA to be low. However, our knowledge of the number of (nearby) glitching pulsars and the number of glitches each pulsar undergoes is limited because monitoring and timing each pulsar are crucial to being able to measure glitches. While there are only 15 known glitching pulsars at $< 1 \text{ kpc}$, there are actually more than 250 known pulsars at these distances (see Figure 1). Some of these latter pulsars could have undergone (electromagnetically unobserved) glitches and thus could contribute to the number of GW bursts. An advantage of GW observations is that they are not limited to observing pulsars whose electromagnetic emission is beamed towards us or that are electromagnetically-bright. Thus there is potential for the type of GW source described here to form a sizable fraction of transient signals detected by current and future GW detectors. It may even be possible to constrain the number of glitching pulsars with GW data. Finally it is important to reit-

erate that there is currently no clear evidence for glitch-induced f-mode oscillations. Nevertheless, these events provide a convenient known source population with measured parameters and an illustration of the energies required to produce detectable GW signals.

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