Efficiency and Fairness of Resource Utilisation under Uncertainty

Doctoral Consortium

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ABSTRACT

The problem of multi-agent resource allocation is important and well-studied within AI and economics. The general assumption is that the amount of each resource is known beforehand. However, many real-world problems, the exact amount of each resource may not be known at the time of decision making, e,g. in the case of weather dependent renewable energy production. This work considers a homogeneous divisible resource where the available amount is given by a probability distribution. In general, a model for efficient usage under fairness and the possibilities of manipulation is studied. Firstly, the notion of ex-ante envy-freeness, where, in expectation, agents weakly prefer their allocation over every other agent's allocation is introduced. For this case the tension between fairness and social welfare is considered. The price of envy-freeness is at least $\Omega(n)$, where n is the number of agents and the problem of optimising ex-ante social welfare subject to ex-ante envy-freeness is strongly NP-hard. Additionally, the possibility for an integer program to calculate the optimal ex-ante envy-free allocation for linear satiable valuation functions is presented.

KEYWORDS

Fair allocation, Social choice theory, Auctions and mechanism design

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1 INTRODUCTION

Multi-agent resource allocation has been studied under a wide variety of characteristics and aims [2]. In these allocation problems there is a resource which has to be divided among a number of interested parties or agents. Among the principle aims are efficiency, fairness and preventing manipulation [1, 4]. The main objective from a societal perspective is efficiency, often represented by social welfare. The principal idea of social welfare is that the resource is distributed to give it to the agents who can get the most value out of them. Conversely, most agents are self interested and might try to manipulate the allocation process to benefit or expect the process to be fair. Manipulation is possible since in many problems only the agents know how valuable the resource is to them and how good they can use it. For fairness a wide number of measures has been proposed; one prevailing notion is envy-freeness [4]. This natural measure requires that any agents is at least as content with the own allocation as with any other agent's allocation.

Considerable attention has been given to heterogeneous divisible resources and indivisible items [4]. Conversely, despite its wide applicability, including electricity, estate, storage space, bandwidth or time [2, 5], less attention has been given to homogeneous divisible resources [5]. One reason is that for a fixed amount of resource efficient solutions can be found easily and envy-freeness admits only the solution that gives every agent the same amount (equal share). However, the amount of resource might not be known since many real-world problems exhibited uncertainty in some form [11]. For example, a group of households sharing a photovoltaic installation on their roofs have to decide who will get the energy at different times of the day. The available electricity from the photovoltaic installation is weather dependent which means the amount that will be available is not known. Simultaneously, the agents may need to plan how much electricity they have available and if they require to acquire electricity from a different source. Nevertheless, even in an uncertain setting considering fairness ex-post does not change the possibilities of finding envy-free and efficient allocations.

This work studies the possibility of efficient algorithms and the trade-offs between efficiency and fairness in the presence of uncertainty. Specifically, a measure of fairness is introduced to consider fairness ex-ante. In the context of fairness and no manipulation, the possibilities of efficiency in settings of an uncertain amount of resource is considered. This is extended by the intractability of finding optimal efficient and fair allocations[3].

2 RESOURCE ALLOCATION PROBLEM

$n \in \mathbb{N}$ agents are interested in a resource. The amount of the resource is uncertain which is represented by a random variable $X \in [0,1]$ ($X : \Omega \to [0,1]$) with a finite number of events $m := |\Omega|$ with $\Omega \subsetneq [0,1]$ and probability mass function f. For simplicity the notation for events is overloaded by letting $\omega := X(\omega)$ for $\omega \in \Omega$. The resource allocation to the agents is indicated by the allocation functions $a_i : \Omega \to [0,1]$ for every agent $i \in [n]$. All allocation functions together form an allocation $A = (a_1, a_2, \dots, a_n)$. The validity of an allocation A is indicated by positivity ($a_i(\omega) \geq 0$ $\forall i \in [n], \forall \omega \in \Omega$), and that the allocation functions respect the maximal amount ($\sum_{i \in [n]} a_i(\omega) \leq \omega \ \forall \omega \in \Omega$).

An agent $i \in [n]$ values the amount of received resource according to a privately known valuation function $v_i : [0,1] \to \mathbb{R}$.

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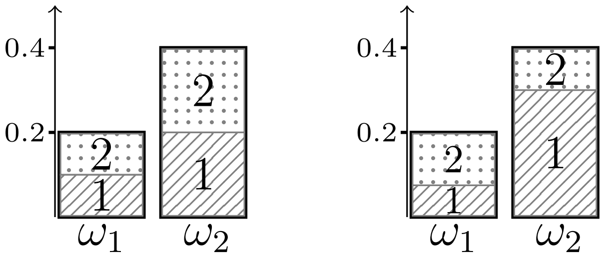

(a) The equal share allocation.

(b) An ex-ante envy-free allocation.

Figure 1: The example's allocations. The bars are the events ω_1 and ω_2 with their respective available amount of 0.2 and 0.4. The allocations to the agents are patterned as diagonal stripes for the first agents and dots for the second agent.

All valuation functions are monotonically increasing from zero ($v(0) = 0$) and satisfy non-negativity ($v(x) \geq 0\ \forall x \in [0, 1]$). Moreover, the expected valuation, denoted by $V_i : [0, 1]^{\Omega} \rightarrow \mathbb{R}$ for $i \in [n]$, is the agent's utility for a given allocation function, i.e. $V_i(a_j) := \sum_{\omega \in \Omega} v_i(a_j(\omega))f(\omega)$ for any $j \in [n]$. Finally, the social welfare of an allocation A is the sum of utilities, i.e. $W(A) := \sum_{i \in [n]} V_i(a_i)$.

3 FAIR ALLOCATION OF RESOURCES

Firstly, ex-ante envy-freeness is considered since ex-post envy-freeness does not allow allocations different from equal share. An allocation is ex-ante envy-free if an agent's utility from the own allocation is weakly greater than the utility from any other agent's allocation, $V_i(a_i) \geq V_i(a_j)$ for all $i, j \in [n]$. Moreover, the overall aim is to find an ex-ante envy-free efficient allocation.

3.1 Example Allocations

This setting allows allocations other than equal share as the following example illustrates. Assume there are two events ω_1 and ω_2 with probability $^2/_3$ and $^1/_3$, and production amount of 0.2 and 0.4, respectively. The two agents' valuation functions are $v_1(x) = \frac{5}{0.3} \cdot x$ for $0 \leq x < 0.3$, $v_1(x) = 5$ for $x \geq 0.3$, and $v_2(x) = \frac{1}{0.2} \cdot x$ for $0 \leq x < 0.2$ and $v_2(x) = 1$ for $x \geq 0.2$. The equal share allocation achieves a social welfare of $2\frac{8}{9}$ in this case (see Figure 1a). However, there is an allocation that is envy-free and allows a social welfare of $3\frac{1}{12}$. In this allocation the first agent gets $0.075kWh$ and $0.3kWh$ in the first and second event, respectively, and the second agent's allocation is the remaining amount (see Figure 1b). This shows that ex-ante fairness allows different solutions.

3.2 Efficiency, Effects and Complexity of Envy-Freeness

While the example shows that other solutions than equal share are possible, generally, the worst case of equal share remains. Hence, the social welfare can depend mostly on one agent. In other words, the price of envy-freeness [2] which is the ratio of the best efficient allocation over the ex-ante envy-free allocation can have a lower bound of $\Omega(n)$.

Moreover, while the difficulty of calculating the overall efficient allocation is not more difficult than without uncertainty, calculating the ex-ante envy-free efficient allocation is strongly NP-hard by reduction from 3-partition. This holds already for continuous and concave valuation functions and a uniform probability distribution.

3.3 Calculating an Optimal Solution

The intractability hinders the calculation of optimal allocations. However, in the specific case of linear but satiable functions, finding ex-ante envy-free efficient allocations can be formulated as an integer program allowing calculation for reasonably sized instances.

More explicitly, for valuation functions $v_i(x) = {}^{u_i}/_{q_i} \cdot x$ for $x < q_i$ and $v_i(x) = u_i$ otherwise, with $i \in [n]$, *saturation amount* $q_i \in [0, 1]$ and *maximal value* $u_i \in \mathbb{R}^+$ the utility can be expressed as $V_i(a) = \frac{u_i}{q_i} \cdot \sum_{\omega \in \Omega} \min\{a(\omega_j), q_i\} f(\omega)$, and the envy-freeness constraint can be expressed as $EF(i, j) := \sum_{\omega \in \Omega} \big(\min\{a_i(\omega), q_i\} - \min\{a_j(\omega), q_i\}\big) f(\omega) \geq 0$ for agents $i, j \in [n]$.

With these valuation functions is it straightforward to represent the problem as the following optimisation program with decision variables x_{ij} for $i \in [n]$ and $j \in [m]$.

$$\max \quad \sum_{i \in [n]} V_i\left((x_{ij})_{j \in [m]}\right) \tag{1}$$

$$\text{s.t.} \quad \sum_{i \in [n]} x_{ij} \leq \omega_j \qquad \forall j \in [m] \tag{2}$$

$$EF(i, k) \geq 0 \qquad \forall i, k \in [n] \tag{3}$$

$$x_{ij} \geq 0 \qquad \forall i \in [n], j \in [m] \tag{4}$$

The optimisation function and the envy-freeness constraint are not linear in this formulation. Nevertheless, it is possible through a number of standard transformations to convert the program into an integer program.

4 CONCLUSION

In general, the results of this work affect the wide array of related problems and areas, including cake cutting [8, 9], estate/land division [7], divisible auctions, divisible task scheduling [6] and packing problems [10]. For example, similar to this work, inherent uncertainty might enrich the solution space in fair division of indivisible items.

In particular, the complexity and the non-polynomial mathematical program for the optimal allocation for a very specific case form a foundation that invites further research. A logical next step is finding a polynomial time approximation algorithm for the general case or in a first instance for the specific case of valuations as in Section 3.3. Moreover, from the broader scope of the presented problem the manipulation has not been addressed yet. Hence, the following step is to consider the effects and possibilities for the case of strategic agents.

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