R&D Investments under Endogenous Cluster Formation *

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Abstract

We provide an innovative theory-based explanation for the positive relationship between firms' R&D intensity and their degree of R&D cooperation. We show that, when oligopolistic firms decide on long-term R&D investment before forming research clusters among competitors, investment incentives are increased by the desire to become a member of an attractive cluster. This can result in over-investment compared to the welfare optimum and compared to a scenario where research clusters are ex-ante fixed. Thereby, as a theoretical contribution, we fully characterize the equilibria of the unanimity game on cluster formation with heterogeneous firms.

JEL Classifications: C71, C72, L13, O30

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1 Introduction

R&D Cooperations between competing firms[1] play a crucial role in many industries (see e.g. Hagedoorn, 2002; Powell et al., 2005; Roijakkers and Hagedoorn, 2006). Firms cooperate on R&D by e.g. forming research joint ventures, exchanging information, or sharing laboratories and facilities. The main motivations for firms to enter such R&D cooperations are knowledge and technology transfers from the partners and therefore

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[1] Although empirical studies show that many R&D cooperations between firms are vertical (i.e. with suppliers or customers), also a large number of horizontal cooperations between competing firms is observed, where this type of cooperation is most frequent in high-technology sectors (see Miotti and Sachwald, 2003). Examples for horizontal R&D cooperations include the Global Hybrid Cooperation between GM, Daimler, Chrysler, and BMW for the development of hybrid cars, the cooperation between Sony and Samsung for the development of TFT-LCD screens, or the cooperation between Lenovo and NEC to develop tablet computers.

the choice of the partners is of crucial importance (see Cassiman and Veugelers, 2002; Miotti and Sachwald, 2003; Krammer, 2016; Li et al., 2019).

Concerning partner choice, an important trade-off has been documented across many industries: on the one hand, firms which are far from the technological frontier, have a high incentive to enter R&D cooperations, as pointed out e.g. in Belderbos et al. (2004); on the other hand, the R&D capabilities[2] of firms have significant positive impact on their rate of participation in R&D consortia, as demonstrated in Sakakibara (2002) based on Japanese data. These R&D capabilities are directly related to investments in R&D. Relying on empirical observations in different industries Maritan (2001) describes the build-up of such capabilities through investments related to skill upgrading and improving the flexibility and quality of equipment. Also, the investments in physical equipment and training associated with the establishment of an R&D lab can be seen as capability enhancing long-term investments. Spillovers arising in an R&D cooperation strongly depend on the capabilities of the involved firms (e.g. Jo and Lee, 2014)), which means that the size and direction of such spillovers in many instances depend on investment decisions made by firms before the consortium was built.[3] This might explain why, those firms which invest a lot in R&D also engage in more cooperations, although firms with lower R&D capabilities potentially have more to gain from R&D cooperations and hence should have larger incentives for joining them.

We explore this phenomenon by developing a theory of long-term investments in R&D in the presence of the opportunity to form R&D cooperations with horizontally related firms. Our model is characterized by two crucial features: first, we explicitly focus on long term investments in R&D, or expressed differently, investments which enhance R&D capabilities, and thereby taking place before firms engage in R&D cooperations. Second, we assume that firms are in control of their cooperation structure, meaning that access to their research can be limited to the respective cooperation partners. We provide a theoretical analysis of which research clusters form and how the structure of clusters is affected by the (long-term) investments in R&D. Furthermore we show how both, cluster structure and R&D investments in equilibrium, compare to the welfare optimum and to the case of cluster structures which are exogenously given and unaffected by the investment choice.

The theoretical literature on R&D investments instead has solely focused on a different aspect of R&D: short term investments which are adapted to the environment, in particular to the size of the spillover parameter and the cooperation structure. Standard models of innovation incentives in the presence of knowledge spillover to competitors (see e.g. D'Aspremont and Jacquemin, 1989; Kamien et al., 1992) predict that an increase in the intensity of the knowledge exchange (typically captured by a spillover parameter) reduces the R&D investments of the firms.[4] The intensity of the knowledge

[2]See Mitchell and Skrzypacz (2015) for a recent treatment of the importance of firm capabilities for their success in innovation.

[3]This is confirmed, for example, by Okamuro et al. (2011) using data from Japanese start-ups. They show that the experience of the start-up founder with product or process innovation and the level of R&D expenditures at start-up have a significant positive impact on the firm's propensity to enter an R&D cooperation with another firm.

[4]Consistent with the literature on R&D networks, to be reviewed below, in this paper we interpret R&D cooperations as an agreement to share (parts of) the R&D results with the partners. The literature on R&D joint ventures initiated by D'Aspremont and Jacquemin (1989); Kamien et al. (1992) typically also considers the effect of cooperating by jointly determining the level of R&D investments of

exchange can also be explicitly modeled by the number of cooperating partners where again a negative correlation to (short term) R&D investments is predicted (see e.g. Goyal and Moraga-Gonzalez, 2001; Greenlee, 2005). The reason for this negative correlation predicted in theoretical models is that firms when deciding on R&D investments before determining competition strategies take into account the effects their R&D has on other firms' competition strategies. The more R&D partners, the more spillovers are created, hence the smaller each firm's investment.[5]

These theoretical results, seem at odds with empirical findings: Studies based on data from numerous countries and sectors have consistently found a positive relationship between the R&D intensity and the degree of R&D cooperation of firms (see e.g. Veugelers, 1997; Kaiser, 2002; Becker and Dietz, 2004; Franco and Gussoni, 2014), as well as between R&D capability and R&D cooperation (Sakakibara, 2002). To explain these empirical findings, theory often reverts to absorptive capacity of firms. For instance, by extending their previous model assuming that the absorptive capacity of firms is positively influenced by own R&D, Kamien and Zang (2000) show that an increase in the spillover parameter leads to stronger R&D incentives as long as the elasticity of the absorptive capacity with respect to own R&D is sufficiently large. In light of these results and, more generally, in the extensive literature on absorptive capacity started by Cohen and Levinthal (1989), the empirical evidence about the positive relationship between R&D investments and R&D cooperation has been mainly interpreted as evidence that firms need own R&D activities to profit from R&D cooperations.

However, the formation of an R&D cooperation typically requires the agreement of all partners[6], which means that the R&D capability of a firm, determined by previous R&D expenditures, does not only influence the incentives of the firm to enter R&D cooperations, but also determines whether potential partners are willing to enter such an agreement with the firm. This aspect of the formation of R&D cooperations has so far been neglected in the theoretical literature and this paper makes a first step to fill this gap. In particular, in line with our discussion above concerning the role of R&D capabilities for cluster formation, we consider a Cournot oligopoly where firms make an R&D investment before they form R&D clusters. Firms within the same cluster receive spillovers from all cluster members and the sum of own R&D and incoming spillovers determines the marginal production costs of a firm. Although in the main body of the paper we restrict attention to cost reducing process R&D, which is in accordance with the majority of the theoretical literature on R&D cooperation, we show in Appendix A that all our findings also apply to a model in which firms engage in quality improving product innovation such that the type of innovation (cost reduction or quality improvement) is not important for our results.

all partners with the goal of maximizing joint profits of the partners. In the empirical literature these different types of cooperations usually cannot be distinguished. Many studies are based on European Community Innovation Survey (CIS) data and in CIS questionnaires cooperations are defined in a broad sense including an informal exchange of information.

[5]When firms, instead, jointly choose R&D and competition strategies, then only the direct (cost reducing) effects of R&D on own quantities (and not the effects on other's competition strategies) are considered which in turn leads to a positive correlation between R&D investments and the degree of R&D cooperation, see e.g. Hsieh et al. (2018) and König et al. (2019).

[6]Note that knowledge spillovers between firms might also arise without the consent of the knowledge source, e.g. through labor flows; see Gersbach and Schmutzler (2003) for a treatment of this channel in a setting related to ours.

The main innovative aspect of our analysis is that we explicitly consider interplay between the firms' R&D decisions and the process by which the R&D clusters among potentially heterogeneous firms with respect to R&D investments are formed. In the main part of our analysis we consider a scenario in which only such clusters are formed where all members agree to the membership of all other firms in the cluster. Our approach captures that firms choosing a high level of R&D investment do not only thereby reduce their production costs, but become more attractive for potential partners since members of a cluster with high investing firms will receive a larger amount of spillovers. Thereby the empirically observed phenomenon that R&D capabilities of firms have significant positive impact on their rate of participation in R&D consortia while low investors have a larger incentive to enter these consortia is well captured by our model. In order to focus on this aspect of the choice of own R&D investment, we abstract from any dependence of a firm's absorptive capacity on own R&D spending.

1.1 Main Findings and Policy Implications

Formally, we consider a non-cooperative three stage game. In the first stage firms choose between two levels (high/low) of cost-reducing R&D investments. The endogenous cluster formation is modeled in the second stage. To capture a process in which agreement of all firms is needed for the establishment of a cluster, we employ a non-cooperative game which is a version of the unanimity game first introduced in Bloch (1995)[7]. Knowledge spillovers occur in all clusters[8] and firms compete with respect to quantities in the third stage.

The timing of the model is motivated by the fact that we focus on long-term (irreversible) investments in R&D which enhance R&D capabilities. Because of this nature of R&D investments, we assume that the cluster structure can adjust to these investments and therefore the cluster structure is formed after the choice of R&D investments. In Section 5.2 we allow R&D investments to be increased after the formation of clusters which does not affect the outcomes of the game. In line with most of the literature, we assume that quantities are easiest to adjust and are therefore chosen after R&D investments and cluster structure have been settled. The timing of the game therefore reflects the nature of our long term R&D investments in the presence of cluster formation and quantity competition.

Our model allows for a closed form analytical solution of subgame perfect equilibria. We find that with respect to the emerging structure of the R&D clusters and under weak conditions,[9] all firms are arranged in exactly two clusters, where one of these clusters may be heterogeneous, i.e. consisting of both low and high investors. Investing high increases the probability to participate in the more attractive cluster consisting of a larger number of high investors and thereby to profit from the corresponding spillovers.

[7]Our reason for using the unanimity game for modeling the cluster formation process is that it is one of the few non-cooperative coalition formation games in the literature, which, on the one hand, captures the need for agreement by all cluster members and, on the other hand, allows for generically (almost) unique equilibrium predictions about the shape and size of the emerging clusters.

[8]In line with the focus of our analysis on the effect of endogenous cluster formation and consistent with much of the related literature we use a simple reduced form representation of spillover generation and abstract from issues related to the governance of the interaction in R&D cooperations (see e.g. Bhaskaran and Krishnan (2009); Bhattacharya et al. (2015)).

[9]We show in Appendix B that many of our results also hold when the weak assumptions are relaxed.

For a large range of the number of high investing firms in the population, this effect is stronger the more other firms in the industry choose a high R&D level, and, based on this effect, strategic complementarities between the R&D investment decisions of the firms arise. However this holds only up to a threshold where a large fraction of the industry engages in high R&D investments. Beyond this point an increase of the number of R&D intensive competitors reduces the return on investment in R&D of a firm. In such a heated market scenario, where competitor's invest heavily in R&D, choosing low own R&D investment and relying on spillovers from the other firms in the cluster can be more profitable for a firm, although this implies that the firm will end up in the less attractive cluster. The dominant effects determining the firm's optimal R&D strategy in this range are the decrease in the firm's market share induced by increased R&D activities of the competitors, as well as the fact that due to the large number of firms with high R&D activity, several of these firms are also present in the less attractive cluster.

With respect to industry-level patterns, we show that whereas for sufficiently small and sufficiently large investment costs a unique equilibrium pattern with all respectively none of the firms investing high arises, for a large intermediate range of investment costs a no–investment equilibrium co-exists with an equilibrium where a large fraction or even all firms choose high level of R&D.

Because of the impact of investment on cluster membership, we find that firms have substantially higher investment incentives compared to scenarios where the cluster structure is exogenously given. In particular, there is a range of investment cost values such that in the unique equilibrium of the game with exogenous consortia no firm invests although the only equilibrium profile under endogenous formation of consortia implies full investment. Our baseline model assumes for reasons of simplicity that the level of R&D investment cannot be adjusted after the cluster formation stage, however we also show that our results stay intact if we add another investment stage to the game, such that firms have another opportunity to invest after the profile of R&D clusters has been determined.

Comparing equilibrium outcomes with the welfare optimum, it turns out that the emerging clusters are too small from a welfare perspective. Due to the strategic complementarity between firms' R&D decisions, distortions of investment incentives relative to the social optimum in both directions can occur. On the one hand, for a considerable range of investment costs over–investment arises in a sense that there is an equilibrium with high investment of all or at least a large fraction of the firms, whereas no investment would be optimal from a welfare perspective. On the other hand, for smaller values of investment costs, profiles without any investment can emerge in equilibrium although welfare is maximized if all firms choose a high R&D level.

In order to examine the importance of the institutional framework underlying the cluster formation, we complement the analysis of cluster formation under the unanimity game with a variant of the model that treats cluster formation as an open membership game. The important difference between these approaches is that under the open membership game, contrary to the unanimity game, a firm cannot restrict the set of firms joining its cluster. We show that in such an institutional setting all subgame perfect equilibria induce the formation of a single cluster containing all firms. Due to this, the investment incentive stemming in our baseline model from the desire of a firm to join the more attractive cluster is not present under the open membership game and we show that the incentives to invest are substantially lower than in a setting where cluster formation is determined by the unanimity game. Hence, the ability of a cluster's members to restrict entry of other firms is a crucial factor for our results.

Recognizing the potential distortions of R&D incentives in both directions together with the insight that endogenous R&D cluster formation induces strategic complementarities between firms' R&D investments, gives rise to several policy implications. First, our results suggest that in the case of under–investment, sketched above, a small change in investment costs, e.g. due to R&D subsidies, can induce an abrupt increase in the level of R&D investment and vice versa. Second, our insight that the process through which clusters are formed is crucial for determining R&D incentives, has important implications for the design of public programs aiming to foster the formation of R&D cooperations. In particular, our results suggest that public measures facilitating unrestricted highly competitive formation of R&D clusters (e.g. by providing information about potential partners or providing public support programs imposing weak eligibility criteria on the consortia), in general increase the incentives for R&D investments in that industry.

1.2 Contributions to the Literature

The present paper substantially extends the theoretical literature on R&D cooperations since it is the first contribution to provide a general analytical characterization of emerging R&D cooperation structures in an oligopoly setting where firms choose of R&D efforts before competing in the market.

There is a body of literature which studies the formation of cooperation structures between competitors. Most closely related to our model are Goyal and Moraga-Gonzalez (2001) and Greenlee (2005) who also consider settings where both the choice of R&D effort and the formation of cooperation structures are endogenous. Goyal and Moraga-Gonzalez (2001) restrict attention to binary cooperations and characterize stable R&D networks in this setting under the assumption that all firms have an identical number of cooperation partners. A general analysis, not relying on the assumption of a regular R&D network, is provided only for the special case of three firms. Greenlee (2005), instead, provides a partial analytical characterization together with a numerical analysis of the shape of R&D consortia generated through the unanimity game in a setting where firms endogenously choose their R&D effort.

Such models where R&D investments are chosen before quantities in the competition stage, therefore, have analytical tractability issues. Some papers in this context sidestep these problems, and are then able to focus on the formation of the cooperation structure. König et al. (2019) assume that after the R&D network is formed, R&D efforts and quantities are chosen simultaneously. In a similar setting, Hsieh et al. (2018) study a model where quantities are chosen before the determination of R&D efforts. In both cases R&D efforts do not strategically influence Cournot quantities which are characterized to depend only on the Bonacich centrality of the network. König et al. (2019) then show that all equilibrium networks are nested split graphs while Hsieh et al. (2018) provide equilibrium selection by analyzing stochastically stable networks. The local complementarities arising from cooperation ensure a positive correlation between R&D efforts and size of cooperation.

All these contributions differ from our setup by assuming that the firm's choice of R&D investment occurs after the cooperation structure has been settled. In this sense these papers deal with short term R&D decisions (with different adjustment assumptions), whereas we are concerned about the decision about long term capacity enhancing investments. Thereby we study the strategic effects of such R&D investments on the cooperation structure and competition.

Our contribution also extends the paper by Bloch (1995), where the outcome of the unanimity game is characterized in a Cournot oligopoly setting where marginal costs of a firm are entirely determined by the pure size of its consortium. In particular, investments in R&D are not modeled in Bloch (1995). In our setting, the analysis in Bloch (1995) corresponds to a scenario where all firms have identical levels of R&D investment. We show in the more general case of firms with potentially heterogeneous investments that different structures emerge, but reproduce the findings of Bloch (1995) as a special case of our analysis. Incorporating endogenous and potentially heterogeneous investment levels, our results can also be used to understand the robustness of the qualitative insights from Bloch (1995) with respect to heterogeneity of firms' investments. More generally, we extend the analysis of equilibria in the unanimity game to a setting with heterogeneous players.

Moreover, there are several studies on the formation of bilateral R&D collaborations between homogeneous firms which abstract from endogenous determination of R&D investments. It is shown in Goyal and Joshi (2003), König et al. (2012) and Dawid and Hellmann (2014) that group structures (where all firms within a group are connected) emerge which resembles the structure that emerges from the cluster formation cases. In an analogous framework, Westbrock (2010) studies efficient networks and concludes that the welfare maximizing structures may have similar structures where, however, the sizes of groups differ from the stable structures.

The paper is organized as follows. Our model is introduced in Section 2, in which we also characterize the equilibrium outcome of the Cournot competition stage. Section 3 provides an analysis of the equilibria in the cluster formation stage and the resulting equilibrium investment patterns are examined in Section 4. In Section 5, we compare our findings to the case of exogenously given clusters and show that our results are robust with respect to the addition of a second investment stage after cluster formation. In Section 6 we provide a welfare analysis of our findings and in Section 7 we consider the scenario in which cluster formation is done according to the open membership game. We conclude in Section 8. In Appendix A we briefly outline a variant of our model where firms invest in product rather than process innovation, to which our results also apply. We elaborate on the robustness of our results when the assumption of two investment levels (which do not differ too much) is relaxed in Appendix B. All proofs are given in Appendix C.

2 The Model

An oligopoly of a set $N = \{1, \ldots, n\}$ of ex ante identical[10] firms engage in a three stage game. Firms first choose permanent R&D efforts, then form R&D clusters and finally

[10] At the end of Section 4 we briefly discuss the effect of heterogeneous investment cost ξ.

compete in the market by choosing quantities of a homogeneous product.[11]

When investing in R&D, firms make long-term and irreversible investment decisions, like building facilities, investing in a lab, or committing a budget to a permanent R&D fund. For simplicity, we assume that the investment decision is binary, such that firms can either invest high or low.[12] We denote by $x(i) \in \{\underline{x}, \bar{x}\}$ the R&D effort of firm i. Choosing to invest high, $x(i) = \bar{x} > \underline{x} > 0$, implies costs of $\xi > 0$, whereas the costs of low effort $\underline{x}$ are normalized to zero. In what follows we denote by $\mathbf{x} = (x(1), \ldots, x(n))$ the profile of R&D effort.

Firms may cooperate with other firms to lower their production costs. To do so, firms form clusters where research is shared. Each firm can only participate in one such cluster, or can stay single. Hence, the cluster structure or profile of R&D clusters[13], denoted as $\mathbf{A} = (A_1, \ldots, A_K)$, is a partition of the set of firms, i.e. $A_k \subseteq N \; \forall k = 1, \ldots, K$, $\bigcup_{k=1}^{K} A_k = N$, $A_k \cap A_j = \emptyset \; k, j = 1, \ldots, K, j \neq k$. The cluster to which firm i belongs will be referred to as $A(i)$.

We assume that the marginal production cost is constant and that R&D has a cost reducing effect and is shared within the respective clusters. That is, incoming spillovers in their cluster contribute to the cost reduction of firms. Thus the marginal cost of firm i is given by

$$c(i, \mathbf{x}, \mathbf{A}) := \bar{c} - \gamma\Big(x(i) + \beta \sum_{\substack{j \in A(i) \\ j \neq i}} x(j)\Big), \; i = 1, .., n, \tag{1}$$

where $\bar{c}$ is the base cost (pre-innovation cost) level, the parameter $\gamma > 0$ measures the marginal effect of R&D effort on marginal costs and $0 < \beta < 1$ captures the intensity of knowledge exchange within a cluster. We assume that the difference between the reservation price on the market α and marginal costs in the absence of R&D spillovers and high investments $\bar{c} - \underline{x}$ is large enough to ensure that firms produce strictly positive quantities in equilibrium for any pattern of R&D investments and any set of clusters, i.e. we assume $\alpha - (\bar{c} - \underline{x}) > \gamma(n-1)\left(1 + \beta(n-2)\bar{x} - \underline{x}\right)$.[14] Whenever the context is clear, we will also denote $c(i) = c(i, \mathbf{x}, \mathbf{A})$ to save notation.

Producing quantities of the homogeneous product $q(i)$, $i \in N$, firms face a linear inverse demand given by

$$P(Q) = \alpha - Q, \; \alpha > 0,$$

where P denotes the price and $Q = \sum_{i=1}^{n} q(i)$ total quantity.

Since we focus on long-term or permanent R&D investments, cluster formation can adapt much faster. Hence, we model the timing by the following three stages.

Stage 1: Effort Choice

[11] When we interpret R&D as product innovation rather than process innovation, products are differentiated while marginal costs are homogeneous, see Appendix A. Both model formulations lead to the same results.

[12] In Appendix B we show that our main findings can also be derived in settings with a continuous range of investment choices.

[13] In order to avoid confusion with the variables denoting firms' marginal cost we denote the clusters by A_k rather than C_k. This notation is motivated by Bloch (1995), where what we call clusters is denoted as associations.

[14] To see that this assumption indeed guarantees positive Cournot quantities for all investments and clusters, observe that (2) becomes minimial, if $c(i, \mathbf{x}, \mathbf{A})$ is maximal and $\sum_{j \neq i} c(j, \mathbf{x}, \mathbf{A})$ is minimal which is obtained if i stays singleton and invests $\underline{x}$, while all others join one cluster and invest $\bar{x}$. The assumption ensures that even in this worst case, firm i still produces strictly postive quantities.

All firms simultaneously choose their R&D effort $x(i) \in \{\bar{x}, \underline{x}\}$. The effort profile $\mathbf{x}$ becomes public knowledge at the end of the stage.

Stage 2: Cluster Formation
Firms non-cooperatively form R&D clusters. To model the cluster formation process we employ the unanimity game introduced in Bloch (1995). The unanimity game models the cluster formation process as a sequential game where firms propose clusters according to a given rule of order. We assume that the rule of order, i.e. a permutation of firms $\rho : N \to N$, is chosen from the set $\Pi = \{\rho : N \to N | \rho(i) < \rho(j) \text{ if } x(i) > x(j)\}$ with equal probability.[15] The lowest firm in order ρ then proposes a set of firms as the first cluster. All firms included in the proposal are then asked according to the order ρ whether they agree to join the cluster. If all firms in the proposal agree to join, the cluster forms, the firms leave the game, and the lowest remaining firm in the order ρ proposes the next cluster. If one of the firms in the proposal disagrees to join, then all firms remain in the game and the next proposal is made by the firm who first disagreed to join. This procedure is repeated until all firms have joined a cluster. The resulting cluster profile $\mathbf{A}$ becomes public knowledge. Furthermore, for sake of simplicity we abstract from discounting between stages of the unanimity game.

Stage 3: Quantity Choice
Firms simultaneously choose quantities given the profile of marginal costs determined by the R&D effort choices and the formed clusters, see (1). Standard calculations yield that under the assumption of a sufficiently large α the Cournot equilibrium in the 3rd stage is given by

$$q^*(i, \mathbf{x}, \mathbf{A}) = \frac{\alpha - (n+1)c(i, \mathbf{x}, \mathbf{A}) + \sum_{j \in N} c(j, \mathbf{x}, \mathbf{A})}{n+1} \tag{2}$$

and the profits read $\pi^*(i, \mathbf{x}, \mathbf{A}) = (q^*(i, \mathbf{x}, \mathbf{A}))^2 - \xi \mathbb{1}_{x(i)=\bar{x}}$. To abbreviate notation we will also denote firm i's quantities and profits by $q^*(i)$, and $\pi^*(i)$, respectively.

In order to analyze the game described above we focus on the subgame perfect equilibria of the game and therefore apply backward induction. With respect to the unanimity game in general, Bloch (1996) shows that there exists a subgame perfect equilibrium with the property that all firms always accept a proposal as long as rejecting would not result in a strictly higher payoff.[16] In what follows we restrict attention to this type of subgame perfect equilibrium in the unanimity game.

[15]Our assumption that firms with high R&D effort propose clusters before low investors, substantially simplifies the following analysis without changing much of the results. To see this suppose a low effort firm is the first proposer and the first proposal is different from the first equilibrium cluster characterized by Proposition 1. Note that any firm included in this proposal can reject and propose instead the first cluster according to Proposition 1. Hence only if the intersection of the first proposal by the low effort firm and the equilibrium cluster according to Proposition 1 is empty, then such a proposal can be accepted (since the first proposal of Proposition 1 is optimal for both involved types of firms). Thus, the first proposal of a low investor must be a subset of the second equilibrium cluster (if it is different from the first cluster of Proposition 1). Intuition tells us that this cannot be optimal when only two clusters form. Hence the main insights will not change while relaxing this assumption would greatly complicate analysis because of the many additional subgames to be considered.

[16]This observation follows from Proposition 2.4 in Bloch (1996) where it is shown that every subgame perfect equilibrium of the unanimity game with discounting is also a subgame perfect equilibrium in the game without discounting if the discount factor is sufficiently close to 1.

3 Cluster Formation

When forming the R&D clusters according to the unanimity game, interesting effects arise. Firms face the trade-off between achieving a cost advantage through the incoming spillovers and allowing other firms a cost advantage by reducing the cost of other cluster members while sharing the research within the cluster. This tradeoff is also present in Bloch (1995). In our model, because firms are heterogeneous with respect to their R&D effort chosen in the first stage, the net effect under this tradeoff depends on the profile of the cluster and the investment level of the considered firm.

To understand above effects, let us inspect the payoff implied by the Cournot quantities in the third stage (2), resulting from a given pattern of investment $\mathbf{x}$ and given cluster structure $\mathbf{A}$. In what follows we denote by h respectively l the number of high (low) investors in the firm population. Whenever we refer to these numbers excluding firm i we indicate this as h^{-i}, respectively l^{-i}, while a subscript A restricts the respective numbers to cluster $A \in \mathbf{A}$. Plugging (1) into (2) and simplifying, we get,

$$\begin{aligned}\pi(i) = \tfrac{1}{(n+1)^2}\Big[&\alpha - \bar{c} + \gamma(nx(i) - h^{-i}\bar{x} - l^{-i}\underline{x}) \\ &+ \gamma\beta\Big((n - h^{-i}_{A(i)} - l^{-i}_{A(i)})(h^{-i}_{A(i)}\bar{x} + l^{-i}_{A(i)}\underline{x}) + h^{-i}_{A(i)}(\bar{x} - x(i)) - l^{-i}_{A(i)}(x(i) - \underline{x}) \\ &- \sum_{A_k \neq A(i)} (h_{A_k}((h_{A_k} - 1)\bar{x} + l_{A_k}\underline{x}) + l_{A_k}(h_{A_k}\bar{x} + (l_{A_k} - 1)\underline{x}))\Big)\Big]^2 - \xi \mathbb{1}_{x(i)=\bar{x}}. \qquad (3)\end{aligned}$$

Since Cournot quantities are anticipated in the third stage, firms try to optimize (3) in the cluster formation process. A closer inspection of (3) turns out to be very useful for understanding the logic of the cluster formation process. First, note that the expression on the right hand side of the first line only captures the effects of the direct cost reductions generated by the R&D investments of all firms and as such is independent from the cluster profile. The effects of spillovers on the profit of firm i is given in the second and third line. The second line corresponds to the spillovers arising in the cluster of firm i, and consists of a positive term stemming from spillovers received by firm i and two negative terms describing the spillovers obtained by the other firms in the cluster. Finally, the third line depicts the effects of the spillovers in all other clusters on firm i's profit, having a cost reducing effect for other firms and, via the price channel, a negative effect for firm i's profit. Moreover, the third line also includes the costs of investment and therefore contains only negative terms.

When a firm $i \in N$ is selected to propose a cluster and contemplates which firms to include in the proposal, the marginal effect of adding an additional firm which otherwise might end up in a different cluster plays a crucial role. Hence, consider the impact of moving one firm j from a cluster $A(j) \neq A(i)$ to cluster $A(i)$. Since such a move does not affect investment costs of firm i and profit net of investment costs is the square of firm i's quantity we can restrict attention to the induced change in equilibrium quantity $q^*(i)$. This change in quantity in response to a move of firm j from $A(j)$ to $A(i)$ can be calculated to be

$$\begin{aligned}\Delta q^*(i) \;=\; \tfrac{\gamma\beta}{n+1}\Big(&nx(j) - (h_{A(i)} + l_{A(i)} - 1)x(j) - (h_{A(i)}\bar{x} + l_{A(i)}\underline{x}) + (h^{-j}_{A(j)} + l^{-j}_{A(j)})x(j) \\ &+ (h^{-j}_{A(j)}\bar{x} + l^{-j}_{A(j)}\underline{x})\Big). \qquad (4)\end{aligned}$$

Adding firm j from a cluster $A(j)$ to $A(i)$ has an effect on both i's and j's spillovers, as well as on all firms' spillovers within the respective clusters. First, firm i experiences additional spillovers by adding j where the size depends on the R&D effort of j captured by the first term in the brackets of (4). However, all other firms within i's cluster are also enjoying these spillovers which are given by the second term and firm j receives the spillovers from the whole cluster (third term). These two terms are negative since a cost reduction of other firms lead to higher quantities of these firms, thus, lower the price and decrease the equilibrium quantities (and hence profit) of i. Note that both of these terms increase in absolute value with the size of $A(i)$ since increasing the number of firms in i's cluster means that more firms receive the additional spillovers and j receives more spillovers from those firms. The last two terms of (4) describe the effects of the reduction in spillovers for the remaining members of cluster $A(j)$ and of firm j loosing spillovers from its former cluster. These two effects are positive for the profit of firm i and their size increases with the size of cluster $A(j)$.

Three important observations can be made. First, it is easy to see that $\Delta q^*(i)$ is independent of $x(i)$, implying that whenever it is optimal for a firm to invite an additional firm to its cluster, the same also holds true for all other firms in the same cluster, regardless of their choices of R&D effort. Second, $\Delta q^*(i)$ is an increasing function of $x(j)$, which means that all firms in $A(i)$ prefer to invite a firm j with high R&D effort compared to a member of $A(j)$ with low R&D effort. Third, the incentive to invite a firm j to the own cluster decreases with the size of the own cluster but increases with the size of the current cluster of firm j.

The three observations discussed above provide a clear intuition for the potential structure of the cluster profile in equilibrium.[17] Due to the fact that firms always prefer high R&D firms to join their cluster compared to low R&D firms, it is intuitive that low R&D firms are only included in a cluster proposal if no more high R&D firms are available. Hence, there can be at most one cluster containing heterogeneous firms, i.e. containing both high and low investors. Furthermore, using (4) it can be easily derived that any homogeneous cluster not limited by the number of available firms (i.e. a cluster where the proposal would not change even if an additional firm of that type would become available) will consist of at least $\lceil \frac{n+1}{2} \rceil$ members, which immediately implies that there cannot be more than one such homogeneous cluster. Together with the observation that there can be at most one mixed cluster this implies also that there cannot be more than three clusters forming in equilibrium.

To simplify the following analysis we from now on assume that the heterogeneity between firms with respect to their R&D investment is not too large. As will be shown in Proposition 1, only two clusters emerge under this assumption.

Assumption 1. *The ratio of R&D effort between high and low investors ($\bar{x}/\underline{x}$) is bounded above by* 2.

Given that we only consider firms who are active in R&D and (apart from their R&D choice) are symmetric, restricting the analysis to scenarios where the variance in R&D levels is not too large does not seem to be overly restrictive. Furthermore, as demonstrated in Appendix B, although the technical complexity would substantially

[17] Although the intuition is very straightforward, the derivation of the subgame equilibria of the unanimity game is quite involved, see proof of Proposition 1.

increase, the qualitative mechanisms driving our results would hardly be affected if we relax Assumption 1.

Proposition 1. *For any profile of investment* $\mathbf{x}$*, there exists a stationary SPE of the cluster formation game. All SPE result in the formation of two clusters* $\mathbf{A} = (A_1, A_2)$*. The number of high and low investors in each cluster are generically unique and are a function of the total number of high investors* h *such that*

$$h_{A_1}(h) = \begin{cases} h & \text{if } h \leq \tilde{\tilde{h}} \\ \left\lceil \frac{(2n+h-1)\bar{x}+(n-h)\underline{x}}{4\bar{x}} \right\rceil & \text{else} \end{cases}$$

$$l_{A_1}(h) = \begin{cases} \left\lceil \frac{(3(n-h)-1)\underline{x}-h\bar{x}}{4\underline{x}} \right\rceil & \text{if } h \leq \tilde{h} \\ 0 & \text{else} \end{cases}$$

where $\tilde{h} = \frac{(3n-1)\underline{x}}{3\underline{x}+\bar{x}}$, $\tilde{\tilde{h}} = \frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}}$. *Furthermore,* $h_{A_2}(h) = h - h_{A_1}(h)$ *and* $l_{A_2}(h) = n - h - l_{A_1}(h)$.

Proposition 1 implies that essentially three different types of cluster constellations can emerge. If the number of high investors is small, then all these high investors together with a subset of the low investors form the first cluster and all remaining low investors join for the second cluster. If, on the contrary the number of high investors is sufficiently large, then the first cluster contains only high investors and the second cluster is mixed between high and low investors. For an intermediate range of the number of high investors the two types of investors sort into two homogeneous clusters. It is quite intuitive that the thresholds separating the first scenario from the case where all high investors join the same cluster decreases with the size of the ratio $\bar{x}/\underline{x}$ since the incentives for high investors to include a low investor in their cluster decrease. Similarly, the threshold separating the case with two homogeneous clusters from the scenario where the second cluster is mixed, also decreases with $\bar{x}/\underline{x}$. The intuition for this observation is that the incentives of the members of the first cluster to include an additional high investor, thereby preventing this high investor from receiving spillovers from the low investors in the second cluster, decreases as $\bar{x}/\underline{x}$ becomes larger.

In order to gain some additional intuition about the implications of a change in the number of high investors for the size and structure of the emerging clusters let us distinguish between the cases where the homogeneous cluster consists only of low respectively high investors. First, if the homogeneous cluster has only low investors and the other cluster is mixed, an increase of the number of high investors reduces the number of low investors in the mixed cluster, where this reduction is so strong that the overall size of that cluster is weakly[18] reduced. The fact that the inclusion of one additional high investor in the cluster might trigger a reduction of the number of low investors by more than one can be explained as follows. The outgoing spillovers of the low investors in the cluster remain the same, whereas the spillovers they receive increase due to the exchange of a low with a high investor. Hence, the incentive to have the low investors in the cluster decreases. Secondly, considering the cases where a mixed

[18] Due to the fact that all cluster sizes are integers they change in discrete steps. Throughout the paper we refer to stepwise decreasing (increasing) functions as weakly decreasing (increasing).

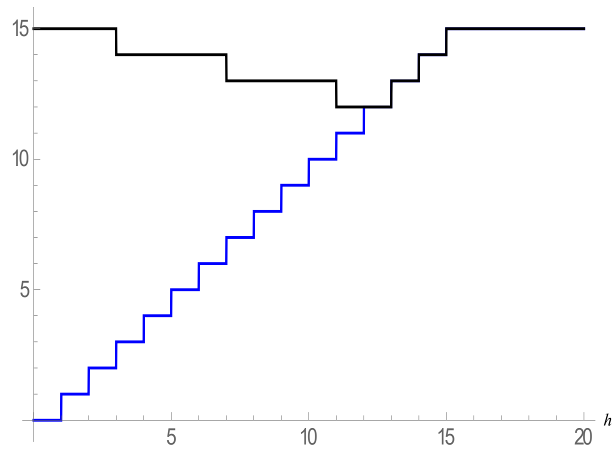


Figure 1: The size of the first cluster (black line) and the number of high investors in that cluster (blue line).

cluster coexists with a homogeneous cluster of high investors, an increase of the number of high investors induces a (weak) increase in the size of the homogeneous cluster and a (weak) decrease of the number of high investors in the mixed cluster. The underlying rationale is similar to above, namely that due to the exchange of a low investor with a high investor in the mixed cluster, the incentives for members of the homogeneous high investment cluster to transfer one additional high investor to their cluster also increase. The size and structure of the first cluster as a function of the number of high investors is illustrated in Figure 1.[19]

Finally, we note that for the case where all investments are homogeneous (i.e. either $\bar{x} = \underline{x}$ or $h = 0$ or $h = n$) the size of the first cluster is given by $\lceil \frac{3n-1}{4} \rceil$, which corresponds to the findings in Bloch (1995), where coalition formation in homogeneous populations is analyzed.

4 Effort Choice

In the investment stage, all firms simultaneously choose their R&D effort. In general, the profit of a firm induced by a certain investment profile $\mathbf{x}$ is stochastic due to our assumption that all sequences of proposal orders in the cluster formation game, which satisfy the assumption that high investors propose prior to low investors, have equal probability. Denoting by $\mathbb{E}(\pi(i, x(i), h^{-i})$ the expected profit of firm i with investment level $x(i) \in \{\underline{x}, \bar{x}\}$ if h^{-i} of its competitors choose high R&D investment, it is optimal for firm i to invest high if and only if $\Delta\pi(h^{-i}) := \mathbb{E}(\pi(i, \bar{x}, h^{-i})) - \mathbb{E}(\pi(i, \underline{x}, h^{-i})) > \xi$.

Two main effects determine the investment incentives of a firm: first, the implications of own investment for the expected attractiveness of the firm's cluster, and second, the expected profit increase for a given cluster allocation. Proposition 1 highlights that under our Assumption 1 two clusters emerge. Taking this into account, the expected

[19] In all figures in this paper we use the default parameter setting: $n = 20, \alpha = 35, \bar{c} = 4, \beta = 0.2, \gamma = 0.2, \underline{x} = 1, \bar{x} = 2$.

payoff difference between high and low investment can be written as

$$\Delta\pi(h^{-i}) = p_{A_1}(\bar{x}, h^{-i}+1)\pi_{A_1}(\bar{x}, h^{-i}+1) + (1 - p_{A_1}(\bar{x}, h^{-i}+1))\pi_{A_2}(\bar{x}, h^{-i}+1) - p_{A_1}(\underline{x}, h^{-i})\pi_{A_1}(\underline{x}, h^{-i}) - (1 - p_{A_1}(\underline{x}, h^{-i}))\pi_{A_2}(\underline{x}, h^{-i}),$$

where $p_{A_1}(x,h)$ denotes the probability of a firm with investment x to end up in the cluster A_1 and $\pi_A(x,h)$ gives the profit in cluster A of a firm with investment x, if a total number of h firms have chosen high investment. We can rearrange to get

$$\Delta\pi(h^{-i}) = \left(p_{A_1}(\bar{x}, h^{-i}+1) - p_{A_1}(\underline{x}, h^{-i})\right)\left(\pi_{A_1}(\bar{x}, h^{-i}+1) - \pi_{A_2}(\bar{x}, h^{-i}+1)\right) + \mathbb{E}_{p_{A_1}(\underline{x},h^{-i})}\left(\pi(\bar{x}, h^{-i}+1)\right) - E_{p_{A_1}(\underline{x},h^{-i})}\left(\pi(\underline{x}, h^{-i})\right) + 2\pi_{A_2}(\underline{x}, h^{-i}). \quad (5)$$

Here, the term $\mathbb{E}_{p_{A_1}}(\pi(x,h)) = p_{A_1}\pi_{A_1}(x,h) + (1-p_{A_1})\pi_{A_2}(x,h)$ denotes the expected payoff of investing x for a given (fixed) probability p_{A_1} to end up in cluster A_1.

The first of the two main effects is captured in the first line of (5). Ceteris paribus, firms prefer to become a member of the larger cluster with more high investors (i.e. $\pi_{A_1} - \pi_{A_2} > 0$), since this generates stronger incoming spillovers for a firm compared to the smaller cluster with fewer high investors. Clearly, the probability p_{A_1} for a firm to end up in this preferred cluster A_1, depends both on the level of investment of the firm, as well as, the investment pattern of all its competitors. The probability for a firm to end up in the more attractive cluster A_1 can be directly derived from Proposition 1.

$$\begin{array}{llll} p_{A_1}(\bar{x}, h^{-i}+1) = 1, & p_{A_1}(\underline{x}, h^{-i}) = \frac{l_{A_1}(h^{-i})}{n-h^{-i}} & \text{if } h^{-i} \le \tilde{h} \\ p_{A_1}(\bar{x}, h^{-i}+1) = 1, & p_{A_1}(\underline{x}, h^{-i}) = 0 & \text{if } \tilde{h} < h^{-i} < \tilde{\tilde{h}} \\ p_{A_1}(\bar{x}, h^{-i}+1) = \frac{h_{A_1}(h^{-i}+1)}{h^{-i}+1}, & p_{A_1}(\underline{x}, h^{-i}) = 0 & \text{if } h^{-i} \ge \tilde{\tilde{h}}, \end{array} \quad (6)$$

where $h_{A_1}, l_{A_1}, \tilde{h}$, and $\tilde{\tilde{h}}$ are given in Proposition 1. It is easy to see that both $l_{A_1}(h^{-i})/(n-h^{-i})$ and $h_{A_1}(h^{-i}+1)/(h^{-i}+1)$ are (weakly) decreasing functions of h^{-i}. This establishes that $p_{A_1}(\bar{x}, h^{-i}+1) - p_{A_1}(\underline{x}, h^{-i})$ is a weakly increasing function of h^{-i} for $h^{-i} \le \tilde{h}$, but (weakly) decreasing for $h^{-i} \ge \tilde{\tilde{h}}$. Hence, the increase in the probability of ending up in the more attractive cluster, which is induced by high investment, becomes larger the more competitors choose high investment as long as this number does not become so large that high investors might end up in the second cluster. For this range of competitors with high investment the consideration of the probability to become a member of the stronger cluster introduces strategic complementarities into the R&D investment choice of the firms.

However, investment incentives are not entirely driven by the effect of R&D investment on the probability to join the stronger cluster. The expected change of firms' market profit for a given probability to end up in A_1 respectively A_2 influences investment incentives as well. Formally, this is expressed by $\mathbb{E}_{p_{A_1}(\underline{x},h^{-i})}(\pi(\bar{x}, h^{-i}+1)) - \mathbb{E}(\pi(\underline{x}, h^{-i})) > 0$, see (5). The strength of this second effect essentially depends on the expected change in firms' output due to high investment and also the expected level of output, because investment reduces the firm's unit costs of production.

The following Proposition shows that the strategic complementarity sketched above is indeed the dominant force in a sense that for a large range of investment costs extreme patterns (no investment or full investment) prevail in equilibrium and that such extreme equilibria might also co-exist.

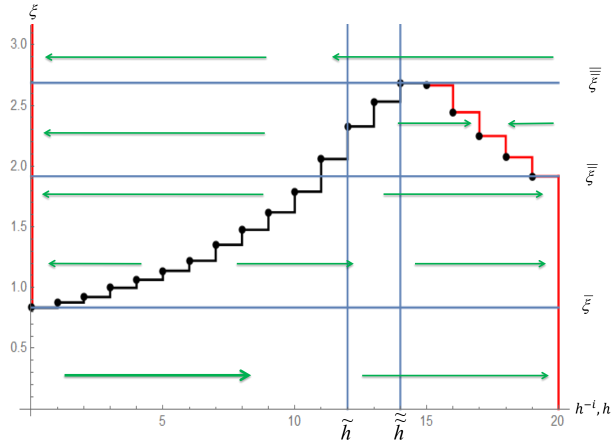


Figure 2: Best response and equilibria on the investment stage.

Proposition 2. *If $\underline{\beta} := \frac{4\underline{x}}{(n+6)\underline{x}+\bar{x}} < \beta < 1/2$, then there exist thresholds $\bar{\xi}, \bar{\bar{\xi}}, \bar{\bar{\bar{\xi}}}$ with $\max[\bar{\xi}, \bar{\bar{\xi}}] < \bar{\bar{\bar{\xi}}}$ such that*

- *For $\xi < \bar{\xi}$ there is a unique equilibrium (up to permutation of firms) where the number of firms investing $\bar{x}$ is given by $\bar{h}(\xi) > 0$. The function $\bar{h}$ is constant in ξ with $\bar{h}(\xi) = n$ for $\xi \leq \bar{\bar{\xi}}$ and weakly decreasing (step-function) in ξ for $\xi > \bar{\bar{\xi}}$.*
- *For $\bar{\xi} \leq \xi \leq \bar{\bar{\bar{\xi}}}$ an equilibrium where $\bar{h}(\xi)$ firms invest $\bar{x}$ co-exists with an equilibrium where all firms invest $\underline{x}$.*
- *For $\xi > \bar{\bar{\bar{\xi}}}$ there is a unique equilibrium where all firms invest $\underline{x}$.*

The proposition is illustrated in Figure 2, which depicts the best response for a firm on the investment stage depending on the number of high investors among the competitors for different values of investment costs ξ. In particular, the black dots indicate the values of $\Delta\pi(h^{-i})$ for all $h^{-i} = 0, .., n-1$. A green arrow to the left indicates that low investment is the best response, whereas an arrow to the right stands for a best response of high investment. The red lines correspond to equilibria in the investment stage, i.e. combinations of ξ and h values for which the investment decision of all firms is optimal.[20] The black increasing step-function indicates the minimal value of h^{-i} above which for a given value of ξ investing high becomes optimal. The figure shows that the qualitative properties of the profit difference $\Delta\pi(h^{-i})$ is indeed closely related to the difference in the probability to end up in the more attractive cluster. In particular, it can be seen that the incentive to invest increases with h^{-i} for $h \leq \tilde{h}$ and decreases for $h^{-i} \geq \tilde{\tilde{h}}$ where $\tilde{h}$ and $\tilde{\tilde{h}}$ are the boundaries from Proposition 1. Whereas $\bar{\xi} < \bar{\bar{\xi}}$ holds for the illustration in Figure 2, in general this inequality cannot be established and therefore Proposition 2 has been formulated without assuming any order between $\bar{\xi}$ and $\bar{\bar{\xi}}$.

[20]It should be noted that when interpreting the red solid lines the argument on the horizontal axis is the *total number* of high investors in the population h, rather than h^{-i}.

Taken together, Propositions 1 and 2 also allow to characterize the equilibrium cluster structure under endogenous investment. If $\xi \notin [\bar{\bar{\xi}}, \bar{\bar{\bar{\xi}}}]$ then all firms choose the same investment and the first cluster is of size $\lceil \frac{3n-1}{4} \rceil$, while the other cluster is composed of the remaining firms. The high investment equilibrium exists as long as $\xi \leq \bar{\bar{\xi}}$ while the no-investment equilibrium exists for $\xi \geq \bar{\xi}$. For $\xi \in [\bar{\bar{\xi}}, \bar{\bar{\bar{\xi}}}]$, additional to the no-investment equilibrium, there is an equilibrium consisting of one cluster with high-investing firms with size smaller than $\lceil \frac{3n-1}{4} \rceil$ and a second cluster containing either only low investors or a mix of firms with both investment levels. Hence, only in this case it is possible to observe a cluster composed of firms with heterogeneous investment levels.

Proposition 2 assumes that the spillover parameter β is in an intermediate range ($\underline{\beta} < \beta < 1/2$). To understand the implications of a very low spillover parameter $\beta << \underline{\beta}$ on the investment incentives, one can consider the extreme case of $\beta = 0$. In such a scenario, R&D investment decreases only the firm's own marginal production costs but generates no spillovers to other firms. It is well known (see e.g. Qiu, 1997) that under Cournot competition with process innovation, investments are strategic substitutes. Hence, for sufficiently small β the firms' investment incentives are decreasing in h^{-i} and, hence, generically a unique equilibrium emerges. On the other hand, if the spillovers become very large ($\beta >> 1/2$), then the incentives stemming from the spillovers in the first (larger) cluster become dominant as the difference in spillovers between the two clusters increase. In such a scenario the main effect of an increase in h^{-i} is that the number of high investors in the first cluster grows. Hence, an increase in h^{-i} increases the spillovers in the larger cluster, where the size of that effect is increasing in β. Thus, investing high becomes more profitable the larger h^{-i} since it increases the probability of being included in the large cluster. For large β this effect is so strong that strategic complements are satisfied over the whole range of h^{-i}. In this case, only equilibria with no investment and with full investment exist (and they might also co-exist). The most interesting case of the spillover parameter β, which allows also for equilibria with partial investment, is covered in Proposition 2.

In Appendix B we show numerically that the qualitative insights of Proposition 2 about the potential co-existence of low and high investment equilibria still apply also if Assumption 1 is violated, although for very large ratios between $\bar{x}$ and $\underline{x}$ three clusters might emerge in equilibrium. Furthermore, we also show analytically in Appendix B that our characterization of equilibria carries over to a variation of the game with a continuous range of R&D effort $[\underline{x}, \bar{x}]$ and an appropriate effort cost function $\chi(x)$. This shows that our assumption of a binary choice of effort level is not essential for our results. However, as becomes clear from the discussion in Appednix B, the shape of the cost function has an important effect on the equilibrium constellations arising, if continuous effort is considered.

5 Extensions

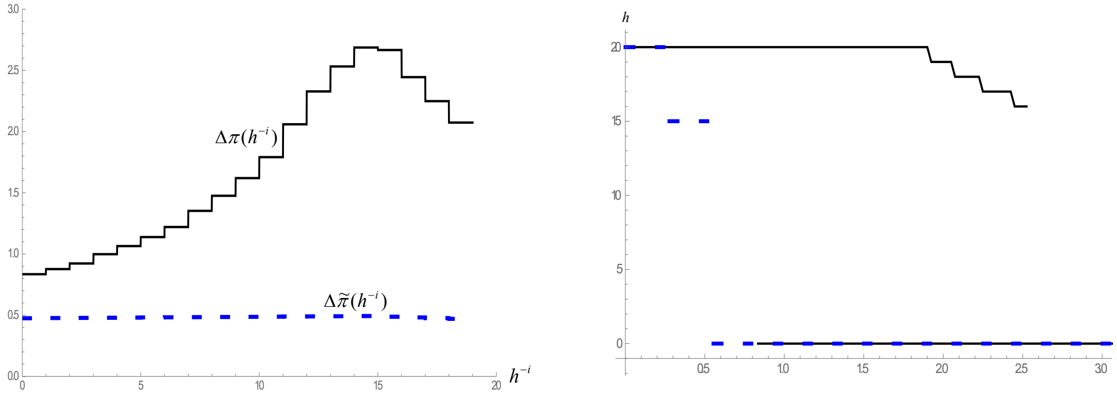
5.1 Comparison with Effort Choice under Exogenously Given Clusters

The discussion above suggests that the desire to end up in the more attractive larger cluster is the main driving force for the investment behavior of firms. To further illustrate this point we compare the investment incentives in our model in which cluster formation is endogenous with such incentives in a setting in which the allocation of firms to the two clusters is ex-ante fixed. We assume that at most two clusters form, and focus on the maximal possible investment incentives across all possible cluster structures. Formally, we define by $\tilde{\pi}_{\tilde{A}_k}(x, \tilde{\mathbf{A}}, \mathbf{x}(-i))$ the market profit of a firm with investment level x in cluster $\tilde{A}_k, k = 1, 2$ if the profile of clusters is $\tilde{\mathbf{A}} = (\tilde{A}_1, \tilde{A}_2)$ and the investment profile of firm i's competitors $\mathbf{x}(-i)$. The maximal possible investment incentives of a firm given a number h^{-i} of other high investors can be written as

$$\Delta\tilde{\pi}(h^{-i}) := \max_{\tilde{\mathbf{A}}: h^{-i}_{\tilde{A}_1} + h^{-i}_{\tilde{A}_2} = h^{-i}} \left[\tilde{\pi}_{\tilde{A}_1}(\bar{x}, \tilde{\mathbf{A}}, \mathbf{x}(-i)) - \tilde{\pi}_{\tilde{A}_1}(\underline{x}, \tilde{\mathbf{A}}, \mathbf{x}(-i)) \right].$$

Although an analytical characterization of these maximal investment incentives under exogenous cluster allocation of firms is very involved, in Figure 3(a) they are compared numerically to the incentives under endogenous cluster formation. It can be clearly seen that the incentives are substantially larger under endogenous cluster formation. The gap is so large that for a certain range of investment costs ξ the best response of the considered firm under exogenous cluster allocation is to choose $\underline{x}$ regardless of the investment pattern of the competitors, whereas under endogenous cluster formation it is $\bar{x}$ for all values of h^{-i}. Extensive numerical robustness checks have shown that the property, that the maximal investment incentives under exogenous cluster allocation are always below the minimal investment incentives under endogenous cluster formation, holds across the entire admissible parameter space, i.e. for all parameter constellations satisfying Assumption 1 and yielding non-negative marginal costs and non-negative quantities for all possible investment patterns and cluster profiles.

In order to allow for a more thorough comparison between scenarios with endogenous and exogenous cluster formation, in what follows we will sometimes refer to a scenario with ex-ante given clusters, where the cluster sizes are identical to the ones emerging as equilibrium size under endogenous cluster formation. Given the strategic complementarity between R&D investments of firms in the same cluster (for sufficiently large β) three potential equilibrium constellations might arise under such an exogenous cluster scenario. In addition to equilibria with no investment respectively full investment we can also have equilibria where all firms in the larger cluster A_1 invest, whereas all firms in the smaller cluster A_2 choose $x = \underline{x}$. The number of the high investors in the different types of equilibria under endogenous and exogenous cluster formation is illustrated in Figure 3(b). The figure shows that also under exogenous cluster allocation different equilibria might co-exist. Furthermore, the figure highlights that there is a range of investment cost values for which the unique equilibrium under endogenous cluster formation is high investment for all firms, but if clusters of identical size were fixed before the investment stage, then the unique equilibrium would be that all firms choose low investment.



(a) (b)

Figure 3: Investment incentives (a) and equilibrium number of high investors (b) under endogenous (black) and exogenous (blue-dashed) cluster formation.

5.2 Adding an Ex-Post Investment Option

So far, we have assumed that firms can decide about their investment only before the cluster formation stage. In some contexts it might be reasonable to assume that the high investment can still be implemented after the R&D cluster has been formed, but before firms compete on the market. In particular, the literature on R&D investments (see e.g. Goyal and Moraga-Gonzalez, 2001; Greenlee, 2005) consider only such short term, flexible investments after cooperations have been formed. Although we rather think of long term investments, we now want to allow for an additional investment opportunity after cooperations have formed to see whether firms actually have an incentive to commit to investments before formation of the cooperation structure. Thus, we consider an extension of our game where all firms which have chosen low investment in Stage 1 are given the opportunity to revise their investment decision in an additional stage added between the cluster formation stage and the quantity choice stage. The new stage structure reads:

- Stage 1': Initial Effort Choice, $x_i \in \{\underline{x}, \bar{x}\}$
- Stage 2': Cluster Formation
- Stage 3': Effort Adjustment Stage, if $x_i = \underline{x}$ then firm i has the option to switch to $x_i = \bar{x}$.
- Stage 4': Quantity Choice

The cost of high effort is ξ, regardless of whether the high effort is invested in Stage 1' or Stage 3'. The reason that we only consider upwards changes in the investment level at the Effort Adjustment Stage is that we interpret the firms' investment as sunk once it has been carried out, as discussed in the Introduction. For reasons of simplicity we stick to a setting with two investment levels, which means that high investors cannot increase their R&D level at the adjustment stage.

In order to understand the implication of the Effort Adjustment Stage on equilibrium behavior, it should be noted that all clusters have been formed before the second investment opportunity arises. Hence, the investment incentives of firms at this stage correspond to that with exogenous cluster allocation discussed in the previous subsection where we considered the maximal investment incentives for any exogenous coalition structure by numerical analysis. In particular, investment in Stage 3' can only arise if the costs of effort, ξ, is below the investment incentives under exogenous cluster formation. Given the insight from the previous subsection, that these incentives are always below the minimal incentives under endogenous cluster formation, the addition of Stage 3' can influence equilibrium behavior only if ξ is in a range where in the equilibrium of the original game all firms invest high, see Figure 3(a).

In such a scenario, full investment in the Initial Effort Choice Stage is also the unique equilibrium of our extended game. The main reason for this observation is the insight that even with a second investment opportunity at the cluster formation stage, competitors always prefer a high investor from Stage 1' to a low investor, which might invest in Stage 3', to be included in their cluster. This is due to the fact that the high investor already committed to investment and hence would strengthen the competing cluster while a low investor might not invest if ending up in the smaller cluster. Thus even if the number of firms in the own cluster, which end up with high investment after Stage 3' remains unchanged due to investment in Stage 3', replacing a high investor with a low investor is not desirable for a firm. Hence, no firm has an incentive to deviate from the high investment in Stage 1', since any such deviation would imply that the firm would end up in the small cluster for sure, which reduces the firm's expected payoff. Therefore, the additional effort adjustment stage keeps the equilibria identified in our original game intact regardless of the considered parameter setting.

5.3 Heterogenous Firms

Finally, let us briefly consider a scenario where, contrary to our baseline setting, firms are heterogeneous with respect to the R&D investment cost level ξ. Such heterogeneity might, for example, be based on differences with respect to the level of past R&D activities. For simplicity, let us consider the case where $\bar{n} < n$ firms have investment costs ξ_1 whereas the investment costs of the remaining $n - \bar{n}$ firms is given by $\xi_2 > \xi_1$. In what follows we argue that such heterogeneity may lead to an additional type of equilibrium compared to those described in Proposition 2. Such an equilibrium occurs when all firms with $\xi = \xi_1$ have incentives to invest high if they assume that $\bar{n} - 1$ competitors choose $\bar{x}$ whereas all firms with $\xi = \xi_2$ have incentives to invest low if they assume that $\bar{n}$ competitors choose high R&D. In this equilibrium $\bar{n}$ firms with low investment costs choose $\bar{x}$ and no other firm invests high. If $\bar{n}$ is not too large this implies that in equilibrium the large cluster A_1 consists of high and low investors, whereas the small cluster A_2 contains only firms with low R&D level. Such a scenario cannot occur as equilibrium outcome for homogeneous investment costs. Considering Figure 2 the scenario sketched here corresponds to a value of ξ_1 below the inverse U-shaped step-function for $h^{-i} = \bar{n} - 1$ and ξ_2 above the value of that step-function for $h^{-i} = \bar{n}$.

6 Welfare Analysis

In light of the different investment patterns and cluster profiles emerging under endogenous and exogenous cluster formation the question arises how welfare, consumer surplus and firm profits are affected and how these patterns compare to the social optimum. Given our linear demand function consumer surplus is given by

$$CS = \left(\sum_{i=1}^{n} q(i)\right) - \tfrac{1}{2}\left(\sum_{i=1}^{n} q(i)\right)^2 - P\left(\sum_{i=1}^{n} q(i)\right)$$

and we obtain for the social welfare function

$$W = \sum_{i=1}^{n} \pi(i) + CS = \sum_{i=1}^{n} (q(i))^2 - h\xi + \left(\sum_{i=1}^{n} q(i)\right)^2 /2. \tag{7}$$

Maximizing this function with respect to the investment pattern and the profile of clusters yields the following Proposition.

Proposition 3. *The following characterizes consumer surplus and welfare maximizing outcomes:*

(i) Consumer surplus is maximal if and only if all firms invest $\bar{x}$ and all join the same cluster.

(ii) If $\alpha - \bar{c}$ sufficiently large, then for all ξ the unique welfare maximizing cluster contains all firms.

(iii) If ξ is sufficiently low, then social welfare is maximized if and only if all firms invest $\bar{x}$ and all join the same cluster.

(iv) If ξ is sufficiently large, then social welfare is maximized if and only if all firms invest $\underline{x}$ and all join the same cluster.

Consumer surplus is maximized if the market price is minimized, which under Cournot competition corresponds to the minimization of average marginal costs. Hence, for consumer surplus to be maximal, R&D effort and spillovers must be maximized. Therefore, a single cluster in which all firms invest high is optimal from a consumer surplus perspective (point (i) of Proposition 3).

Considering welfare, the trade-off between the costs of R&D investments and their return in terms of cost reduction has to be considered. If all firms have identical R&D efforts, then from a social perspective the total cost reduction is clearly maximal if all firms join the same cluster, which maximizes spillovers. This explains parts (iii) and (iv) of Proposition 3. If firms are heterogeneous with respect to their R&D effort, including low investors in a cluster of high investors has not only the spillover induced positive effect discussed above, but also induces a larger output for the low investor compared to a scenario where it would stay in isolation.[21] Hence, it is no longer obvious that a single cluster is welfare maximizing. However, part (ii) of Proposition 3 shows that

[21] This effect is closely related to the well-known fact that reduction of marginal costs of firms with low market shares in Cournot competition can be welfare reducing, see Lahiri and Ono (1988).

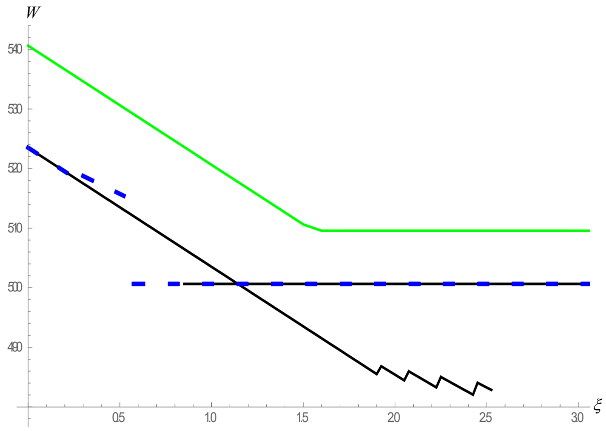


Figure 4: Maximal welfare (green) and welfare under equilibria with endogenous (black) and exogenous (blue-dashed) cluster formation.

the direct spillover effect always dominates if the market size is sufficiently large and therefore under such a condition the generation of a single cluster always maximizes welfare.

Combining Proposition 3 with Proposition 1 shows that the profile of clusters emerging in equilibrium is generically inefficient. This insight is also illustrated in Figure 4, in which the welfare maximum is compared to social welfare of the different types of equilibria under exogenous and endogenous cluster formation. Equilibrium welfare is always strictly below the maximum and it is obvious that this inefficiency stems from the profile of clusters since at least for very low and very high investment costs the welfare maximizing investment pattern coincides with that arising in equilibrium.

Comparing the welfare generated in equilibria with endogenous and exogenous cluster formation, Figure 4 shows that the effect of endogenous cluster formation on welfare is ambiguous. On the one hand, as discussed above, there is a range of investment cost values where under endogenous cluster formation there exists a unique equilibrium with high investment whereas under exogenous cluster formation only low investment is done. In such a scenario welfare is substantially larger under endogenous cluster formation. On the other hand, there is also a range of investment cost levels where under exogenous cluster allocation of firms only the firms in the large cluster invest high whereas all other invest low. Such an investment profile generates higher welfare compared to the full investment profile emerging under endogenous cluster formation because a large share of output is produced by the low cost firms in the larger cluster and for the relatively low output produced in the small cluster the saved investment costs outweigh the aggregate reduction in production costs that would result from full investment of the small cluster firms.

Furthermore, Figure 4 shows that in the upper range of investment cost levels, for which an equilibrium with high investment exists under endogenous cluster formation, such an equilibrium generates welfare which is not only substantially below the welfare maximum but also below that of the unique equilibrium under exogenous cluster formation, which corresponds to the zero investment equilibrium. Welfare maximization requires zero investment in this parameter range, which means that endogenous cluster

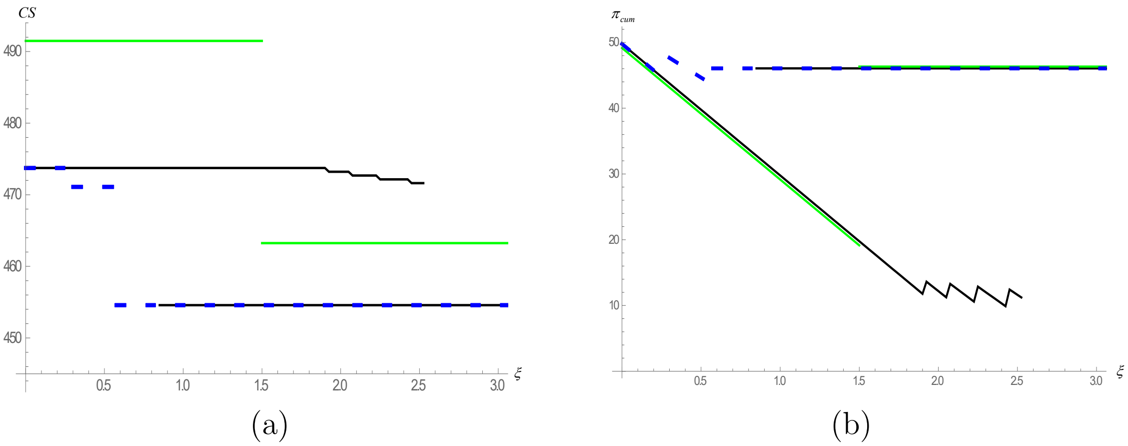


Figure 5: Consumer surplus (a) and total firm profits (b) under welfare maximizing choice of investments and profile of clusters (green) as well as under equilibria with endogenous (black) and exogenous (blue-dashed) cluster formation.

formation can yield massive over-investment in equilibrium. Intuitively this inefficiency is triggered by the tournament like structure. All firms have strong incentives to end up in the larger cluster due to the endogeneity of the difference in payoffs between the clusters driven by the strategic complementarity.[22]

Figure 5(b) shows that total industry profit, and therefore also average firm profit, is for a certain range of ξ-values strictly larger if firms are ex-ante allocated to clusters than if clusters are formed endogenously. This is quite intuitive, because under exogenous cluster allocation firms avoid the strategic over-investment, which arises under endogenous cluster formation. Hence, in principle, by cooperatively determining the cluster and investment profile and committing to transfers between firms allocated to the more and to the less attractive cluster, firm could increase their ex-ante expected profit compared to a scenario with endogenous cluster formation[23]. However, such a scenario, in which all firms in the industry cooperatively determine the amount each of them invests in long term R&D capabilities and firms in the large cluster commit to long term transfers to firms in the small cluster, which keep their capabilities low, would not only be highly problematic from an anti-trust perspective, but also seems to require too much commitment from the involved parties to be feasible and strategically stable.[24]

[22]Lazear and Rosen (1981) show in the framework of labor contracts that tournament schemes, in which the firm chooses the price structure and prices are independent from workers' investment, can induce efficient investment. In a related setting with endogenous determination of the price structure and asymmetric information about investment Zabojnik and Bernhard (2001) show that underinvestment in equilibrium results. The main difference between our setting and these contributions is that the payoffs obtained in the two clusters are positively affected by own investment and marginal returns from investment are larger in the cluster generating higher payoffs.

[23]It should be noted that, as in the previous figures, the dashed blue line in Figure 5(b) refers to the exogenous cluster profile with the largest investment incentive for firms, which in general differs from the cluster and investment profile maximizing total industry profit. Numerical evidence suggests that the total industry profit under the profit maximizing profile is strictly larger than that under equilibrium cluster and investment profiles for all considered values of the investment cost ξ.

[24]Without a full commitment of the firms in the large cluster not to accept an additional firm, even if it has high capability, firms which according to the chosen cluster allocation should invest low and

From a consumer perspective, as should be expected, the high investment incentives under endogenous cluster formation are desirable and consumer surplus is for all values of investment costs (weakly) larger in the case of endogenous cluster formation (see Figure 5(a)) than with exogenous clusters.

7 Open Membership

So far we have assumed that firms are able to exclude competitors from their cluster. To examine how important this feature is, we now consider an institutional setting in which firms cannot limit access to their cluster. In such an environment, the open membership game introduced in Yi (1997, 1998) may be the accurate way to model the cluster formation process of the second stage. In the open membership game, the firms simultaneously pick one of n possible addresses. All firms announcing the same address form one cluster.

Formallly, the set of pure strategies of each firm (in the second stage) is given by $S = \{a_1, ..., a_n\}$ and payoff is obtained from (3). The cluster profile resulting from a profile $\mathbf{s}^{-i} \in S^{n-1}$ of the competitors of i is denoted by $\mathbf{A}^{OM}(s^{-i})$.

To distinguish equilibrium quantities and efforts of the open membership game from the unanymity game, we denote these vectors by $\mathbf{x}^{OM}$, and $\mathbf{q}^{OM}$, respectively. Solving for the third stage, we obviously get that equilibrium quantities $q_i^{OM}(i, \mathbf{x}^{OM}, \mathbf{A}^{OM})$ can be calculated according to (2).

To analyze cluster formation according to the open membership game, we use (3) to calculate the best response of firm $i \in N$ with R&D investments $x_i \in \{\underline{x}, \bar{x}\}$ given the choices of other firms $\mathbf{s}^{-i}$ resulting in a cluster structure $\mathbf{A}^{-i} = (A_1^{-i}, \ldots, A_n^{-i})$ by,

$$BR_i^{OM}(\mathbf{s}^{-i}) = \{a_j \mid j \in \underset{j \in \{1,..,n\}}{\arg\max}(n(h_{A_j^{-i}}\bar{x} + l_{A_j^{-i}}\underline{x}) - (h_{A_j^{-i}} + l_{A_j^{-i}})x_i)\}. \tag{8}$$

Taking into account that the payoff of joining a cluster is an increasing function of the number of high respectively low investors in that cluster (under our condition that $\bar{x} \leq 2\underline{x}$), it is therefore straightforward to see that in any equilibrium, all high investors choose the same cluster and all low investors choose the same cluster. The following proposition establishes that it is indeed the unique equilibrium of the second stage (up to permutation of addresses) that all firms, independently of their investment in the first stage, choose the same cluster.

Proposition 4. *For any profile of investment* $\mathbf{x}$*, all firms choose the same cluster in the open membership game.*

In the proof of Proposition 4, we show that it is not an equilibrium that all high investors choose one cluster and all low investors choose a different cluster if $(2n-1)\underline{x} > \bar{x}$ which is obviously implied by our Assumption 1 that $\bar{x} \leq 2\underline{x}$. From the structure of the best replies (8) the result then immediately follows.

Comparing this proposition with part (ii) of Proposition 3 shows that the cluster profile emerging under the open membership game is always socially optimal. In particular, this implies that under very small and very large investment costs, when

stay in the small cluster, would have incentives to deviate and to invest high in order to join the large cluster (and firms in that cluster would have incentives to let the high capability firm join the cluster).

investment profiles under the open membership game coincide with the welfare maximizing profile (i.e. no investment respectively full investment) the equilibrium outcome under the open membership game is efficient.

Thus, independent of the investment in the first stage, one cluster comprising all firms always forms. For the investment incentives in the first stage of the open membership game, this means that there is no uncertainty concerning cluster membership. We can then directly calculate the investment incentives, by $\Delta\pi^{OM}(h^{-i}) := \pi^{OM}(i, \bar{x}, h^{-i})) - \pi^{OM}(i, \underline{x}, h^{-i})$, where $\pi^{OM}(i, x(i), h^{-i}) := \pi(i, \mathbf{x}, \mathbf{A})$ with $\mathbf{A} = \{N\}$ and $\mathbf{x}$ is such that h^{-i} other firms invest high, $n - 1 - h^{-i}$ other firms invest low, and firm i invest $x(i)$.

Compared to the unanimity game, see (5), this implies that one of the two effects determining the investment incentives in the first stage vanishes: increasing the investment, when the open membership game follows in the second stage, does not affect cluster membership and hence does not increase the probability to become a member of a more attractive cluster. Since the observed complementarity of investments in the first stage was due to the probability effect, we may expect that investments are strategic substitutes when open membership game follows the investment stage. This is confirmed in the following corollary.

Corollary 1. *Let $\beta < 1/2$. Then $\Delta\pi^{OM}(h^{-i})$ is decreasing in h^{-i}.*

We can also compare the magnitude of the investment incentives when the open membership game is used for cluster formation to those when the unanimity game is played in the second stage. Because of the missing positive incentives due to investment dependent cluster membership, we may expect lower investments in the first stage when cluster formation is according to the open membership game. This also becomes immediate from Figure 3(a) where the investments incentives in the unanimity game are compared to maximal investments under exogenous clusters (where the maximum is taken over all cluster structures yielding one or two clusters). Since the cluster structure of the open membership game is invariant to investments in the first stage (and yields one cluster), the investment incentives must be below the maximal investments under exogenous clusters in Figure 3(a). This immediately implies that incentives under the open membership are always lower than under the unanimity game.

To summarize, endogenous cluster formation per se does not induce higher investment compared to the ones under exogenously given cluster membership. It is crucial whether firms are able to exclude others from their clusters. Hence, there is a trade-off in terms of innovation when going from an exclusive to open membership institutional setting: on the one hand, spillovers are maximized in the open membership game since the complete cluster of all firms always forms. On the other hand, the incentives to invest in R&D are considerably reduced. This reasoning implies that for values of the investment costs, for which the induced investment patterns under the unanimity game and the open membership game coincide, welfare is larger under the open membership game. For intermediate values of ξ the trade-off between higher investment incentives and smaller cluster size under the unanimity game does not allow for general statements about the relative size of welfare under the two cluster formation regimes.

8 Conclusions

The main contribution of this paper is to improve our understanding of the strategic relationship between firms' (long term) R&D investment decisions and their participation in R&D clusters. From a theoretical perspective, we go beyond the current state of the literature by developing and analyzing a framework which allows to characterize the equilibrium profiles of both R&D investment and R&D cluster formation in a setting where R&D efforts have a strategic effect on thne competition stage. Our analysis shows that in equilibrium generically unique cluster profiles emerge which are characterized by a strong heterogeneity between clusters with respect to size and R&D investment while within clusters, the heterogeneity of R&D levels is small. In particular, it is shown that in case of heterogeneous firm investments the majority of high investors is always included in the largest cluster. Overall, our model predicts a positive relationship between the level of firms' (long term) R&D activity and the number of cooperation partners, and therefore is able to provide a theory-based explanation for a large set of empirical findings pointing towards such a positive relationship (e.g Veugelers, 1997; Becker and Dietz, 2004).[25] Aditionally, our model makes the empirically testable prediction that R&D cooperations are stratified in a sense that the variance of R&D levels within clusters is lower than that in the entire population.

Furthermore, we show in this paper that the endogenous cluster formation process implies stronger investment incentives, compared to a scenario where allocation of firms to clusters is ex-ante fixed, and generates strong strategic complementarities with respect to the firms' investment decisions. These long term investements (before the cooperation structure is formed) have a strategic effect which is not modelled in the literature so far. By choosing high R&D investments firms increase the probability to participate in the more attractive cluster resulting in some cases in overinvestment as a strategic device. A similar effect occurs in Petrakis and Tsakas (2018) where R&D cooperation formation can be used as a strategic device to prevent market entry of another firm even though R&D cooperation itself is not beneficial. In our model, firms choose high R&D investments because of the strategic implications for coalition formation.

The strategic complementarities in our model imply that for a large range of investment cost values a no-investment equilibrium co-exists with an equilibrium in which (almost) all firms choose a high R&D level. Welfare maximization would require a full investment profile for a substantial part of the investment cost range where the no-investment equilibrium exists. These insights have clear managerial and policy implications.

From a managerial perspective our analysis highlights that endogenous cluster formation is an important driver of strategic complementarity between competitors' R&D activities. Hence, for a firm to choose the level of its R&D investments and to decide on its strategic reaction to the (long-term) R&D activities of its competitors, it is crucial to have a clear understanding of how flexible the R&D cooperation structures in the industry are, i.e. whether the cooperation structures are essentially fixed or there is still room for endogenous formation of R&D clusters.

From an innovation policy perspective the observation that firms which anticipate

[25]For short term R&D efforts, where these investments do not affect quantities a similar finding has been established already in e.g. Hsieh et al. (2018) and König et al. (2019).

that their R&D level influences their cluster membership invest more, provides a potential justification for policy measures, like technology and cooperation platforms, which foster the exchange of information between firms and the continuous adjustment of cooperation structures. Furthermore, this observations suggests that public support schemes for R&D cooperations that restrict the freedom of choice of cooperation partners through restrictive eligibility conditions in general reduce incentives for (long-term) R&D investments. Furthermore, our analysis suggests that in scenarios where no-investment and full investment equilibria coexist, the introduction of a (potentially small) public R&D subsidy, which moves the level of R&D investments required from the firms below the threshold $\bar{\xi}$ can have a strong positive effect by inducing a transition to the equilibrium where all firms invest high.

Our analysis is based on a number of simplifying assumptions whose implications should be critically examined. If we would allow firms to enter individual cooperation agreements with selected competitors rather than joining a cluster, the resulting analysis would require the characterization of equilibrium network structures among general profiles of heterogeneous firms. This technically and conceptually demanding task is left for future research. Also, in this paper we have abstracted from the effects of R&D investment on a firm's absorptive capacity. Considering such effects might substantially affect the qualitative findings obtained here. Again, future work should be able to address this issue.

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Appendix

A An Oligopoly Model with Product Innovation

Here, we briefly outline an oligopoly model where products are vertically differentiated and R&D activities of firms lead to changes in product quality due to product innovation. We show that this simple model formulation yields equilibrium profit functions of firms which have a completely analogous functional form as the ones resulting from the process innovation model used in the main body of the paper. Hence, all results concerning firm investment and formation of clusters derived in the paper are also valid in this product innovation setting.

Like in the main body of the paper, we consider an oligopoly of a set $N = \{1, ..., n\}$ of ex ante identical firms which engage in a three stage game. Firms first choose permanent R&D efforts, then form R&D clusters and finally compete in the market by choosing quantities of their product. The R&D effort $x(i) \in \{\underline{x}, \bar{x}\}$ of firm i is invested in product innovation and influences the quality of the product. Choosing to invest high, $x(i) = \bar{x} > \underline{x} \geq 0$, implies costs of $\xi > 0$, whereas the costs of low effort $\underline{x}$ are normalized to zero. Firms form clusters in the same way as described in Section 2 and the quality of the product of firm i is then given by

$$u(i) = \bar{u} + \gamma\Big(x(i) + \beta \sum_{\substack{j \in A(i) \\ j \neq i}} x(j)\Big).\ i = 1, .., n, \tag{9}$$

To simplify notation we normalize $\bar{u}$ to zero. Marginal production costs of the firms, which are assumed to be constant and identical across firms, are denoted by $\bar{c} > 0$.

Demand on the market is generated by a representative consumer with the utility function (expressed in monetary units)

$$U(q(1), \ldots, q(n)) = \sum_{j \in N} (\alpha + u(j))q(j) - \frac{1}{2}\left(\sum_{j \in N} q(j)\right)^2 - \sum_{j \in N} p(j)q(j),$$

where quality and prices are given parameters from the consumer's perspective.

In the third stage of the game all product qualities are common knowledge and firms simultaneously choose their quantities. Prices are then adjusted such that the market clears, which means that the vector of chosen quantities $(q(1), \ldots, q(n))$ maximizes the consumer's utility function. The corresponding first order conditions yield

$$p(i) = \alpha + u(i) - \sum_{j \in N} q(j).$$

Taking this into account, the market profit of firm i can be written as

$$\tilde{\pi}(i) = \left(\alpha + u(i) - \sum_{j \in N} q(j) - \bar{c}\right) q(i)$$

and standard calculations yield the equilibrium quantities

$$q^*(i) = \frac{\alpha - \bar{c} + (n+1)u(i) - \sum_{j \in N} u(j)}{n+1}$$

and market profits

$$\tilde{\pi}^*(i) = \frac{\left(\alpha - \bar{c} + (n+1)u(i) - \sum_{j\in N} u(j)\right)^2}{(n+1)^2}.$$

Inserting (9) into this expression yields that the overall profit of the firm is given by (3) and hence coincides with the one derived in the process innovation model considered in the main body of the paper. Therefore, all derived results also hold for the product innovation model sketched here.

B Robustness

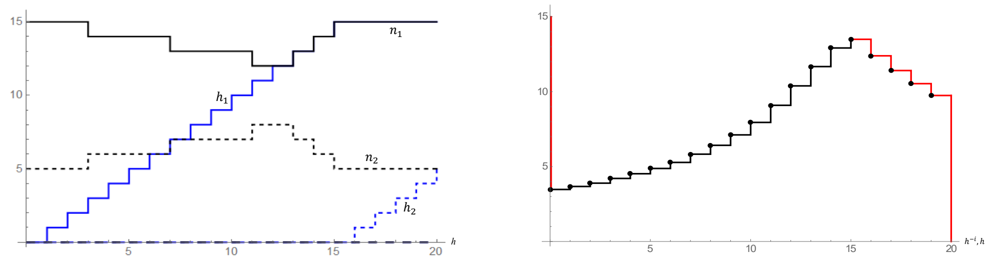
B.1 Large ratio of $\bar{x}/\underline{x}$

In order to derive our analytical results, we have restricted our analysis to scenarios where in equilibrium only two clusters emerge. This is ensured by the condition $\bar{x} \leq 2\underline{x}$, stated in Assumption 1. In this part of the appendix, we check numerically in how far our main results (qualitatively) stay intact if we allow for higher values of the ratio $\bar{x}/\underline{x}$. Our numerical procedure first determines in a given parameter setting for all combinations of the number of high investors, h, and sizes of the first cluster, $n_1 = |A_1|$ the equilibrium number of high and low investors in the second cluster (h_2, l_2) and in a potential third cluster (h_3, l_3).[26] These values are then used to determine the equilibrium size of the first cluster for any value of h. The resulting equilibrium cluster constellations are used to determine the expected payoffs of agent i under $x_i = \bar{x}$ and $x_i = \underline{x}$ for any given number h_{-i} of high investors among its competitors, which then yields the investment incentives of agent i as a function of h_{-i}. In our robustness check we keep all parameter values at their default values apart from $\bar{x}$ and α. The value of α has to be increased in order to guarantee that also for values of $\bar{x} > 2\underline{x}$ the Cournot equilibrium quantities of all firms stay positive regardless of the investment pattern and cluster constellation (see Footnote 14). In particular, we increase the market size from the default value $\alpha = 35$, used in the body of the paper, to $\alpha = 140$.

In Figure 6 we show the sizes and numbers of high investors in all equilibrium clusters as well as the investment incentives for this enlarged market size and the default value $\bar{x} = 2$. As can be seen in panel (a) changing α has no impact on the constellation of equilibrium clusters, such that for $\bar{x} = 2$ we obtain exactly the same pattern as depicted in Figure 1. Comparing panel (b) to Figure 2 shows that the investment incentives are scaled up compared to the default case, but the structure of the best response function of agent i and the resulting equilibria are qualitatively the same. This is hardly surprising since under this parameter constellation Proposition 2 still applies.

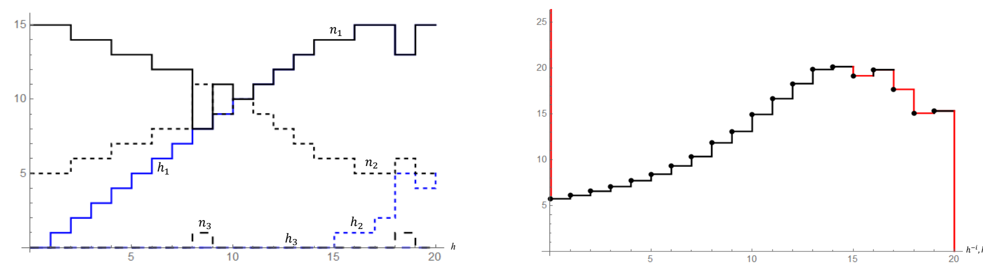
In Figures 7 and 8 we depict the equilibrium cluster constellations and best response functions for $\bar{x} = 3$ and $\bar{x} = 6$. In both cases three cluster emerge for certain numbers of high investors. As expected, the third cluster always consists only of agents with low investments. Considering the best response functions, it becomes clear that in both considered parameter settings for certain ranges of the investment costs there are equilibria of the game with three clusters. In particular, for $\bar{x} = 3$ there is an equilibrium

[26] Note that maximally three clusters can form in equilibrium.



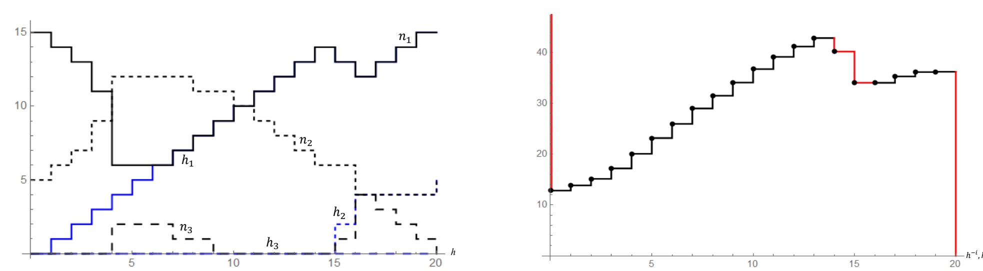
(a) (b)

Figure 6: (a) Sizes (n_i, black lines) and number of high investors (h_i, blue lines) of all clusters, and (b) the best response function for $\alpha = 140$ and $\bar{x} = 2$.



(a) (b)

Figure 7: (a) Sizes (n_i, black lines) and number of high investors (h_i, blue lines) of all clusters, and (b) the best response function for $\alpha = 140$ and $\bar{x} = 3$.



(a) (b)

Figure 8: (a) Sizes (n_i, black lines) and number of high investors (h_i, blue lines) of all clusters, and (b) the best response function for $\alpha = 140$ and $\bar{x} = 6$.

with $h = 18$ and a cluster constellation $n_1 = h_1 = 13, n_2 = 6, h_2 = 5, n_3 = 1, h_3 = 0$. Similarly, for $\bar{x} = 6$ an equilibrium with $h = 15$ and three cluster exists, where $n_1 = h_1 = 13, n_2 = 6, h_2 = 2, n_3 = 1, h_3 = 0$. Intuitively, for such large values of $\bar{x}$ the high investor proposing the second cluster has lower incentives to include low investors into the own cluster, which implies that a third small cluster of low investors might emerge. In spite of the fact that for $\bar{x} > 2\underline{x}$ there can be equilibria with three clusters, the main qualitative properties of the equilibrium investment patters, as described in Proposition 2, still apply. In particular, also for $\bar{x} = 3$ and $\bar{x} = 6$ for low values of investment costs ξ there is a unique equilibrium with full investment, whereas for large values of ξ the unique equilibrium induces no investment. Furthermore, there is an intermediate range of ξ such that the no-investment equilibrium co-exists with an equilibrium where either all or at least a large fraction of agents invest high.[27] The number of high investors in the high investment equilibrium weakly decreases with ξ.

Another assumption underlying our analysis is that firms choose between two possible investment levels. Relaxing this assumption implies that the potential number of equilibrium clusters increases, in particular for k investment levels up to $2k-1$ clusters might emerge.[28] A full analysis of the equilibrium cluster constellations in such settings with large k, even by numerical means, seems hardly feasible.

B.2 Continuous range of investment choices

In this section we show that the equilibrium constellations characterized under the assumption of binary investment choices caries over to settings with a continuous investment range under appropriate investment cost functions. In particular, we assume that $x_i \in [\underline{x}, \bar{x}]$ for all firms $i = 1, ..., n$ and the investment cost function is characterized by fixed costs F of investing above $\underline{x}$. In particular, we assume a cost function of the form $\chi(x) = sign(x - \underline{x})F + \tilde{\chi}(x - \underline{x})$ with $F < \xi$ and $\tilde{\chi}\frac{\xi - F}{\bar{x} - \underline{x}} > 0$. Hence, $\chi(\underline{x}) = 0$ and $\chi(\bar{x}) = \xi$. If we interpret $\underline{x}$ as the standard level of R&D that can carried out without a dedicated R&D lab, F can be interpreted as the fixed costs of establishing dedicated R&D facilities.

First, we consider a scenario where ξ is such that under our default setting with binary choice there is a no-investment equilibrium. This means that $\Delta\pi(0) < 0$, or, put differently

$$\pi_{\tilde{A}_1}(\bar{x}, 0) - \xi < p_{A_1}\pi_{A_1}(\underline{x}, 0) + (1 - p_{A_1})\pi_{A_2}(\underline{x}, 0), \tag{10}$$

where $\tilde{A}_1$ is the resulting first cluster in a scenario with $h = 1$ and (A_1, A_2) is the equilibrium cluster constellation for $h = 0$. Analogous to the notation in the proof of Proposition 2, we denote by $\pi_A(x, h^{-i})$ the Cournot profit of a firm with investment x in cluster A if h^{-i} other firms choose $x_j = \bar{x}$ and $n - h^{-i} - 1$ choose $x_j = \underline{x}$. Furthermore, we assume that

$$F > \pi_{A_1}(\underline{x}, 0) - \pi_{A_2}(\underline{x}, 0). \tag{11}$$

[27] As can be seen in Figure 8 for $\bar{x} = 6$ there is a range of ξ-values where the low investment equilibrium co-exists with *two different* high investment equilibria.

[28] This upper bound for the number of potential clusters follows from our observation that any homogeneous cluster whose size is not restricted by the number of firms with equal investment has a size of at least $\lceil \frac{n+1}{2} \rceil$ and clusters never include firms with a certain investment level as long as firms with higher investment would still be available to be included in that cluster.

Under this assumption we have

$$p_{A_1}\pi_{A_1}(\underline{x},0)+(1-p_{A_1})\pi_{A_2}(\underline{x},0)>\pi_{A_1}(\underline{x},0)-F=lim_{\epsilon\to 0}(\pi_{A_1}(\underline{x}+\epsilon,0)-\chi(\underline{x}+\epsilon)).$$

Furthermore, since A_1 is the optimal cluster for a low investor in a population with $h=0$, it follows by continuity that this is also the optimal cluster for a firm i with $x_i=\underline{x}+\epsilon$ under $h_i=0$ if ϵ is sufficiently small. Therefore

$$p_{A_1}\pi_{A_1}(\underline{x},0)+(1-p_{A_1})\pi_{A_2}(\underline{x},0)>lim_{\epsilon\to 0}(\pi_{\hat{A}_1}(\underline{x}+\epsilon,0)-\chi(\underline{x}+\epsilon)) \quad (12)$$

for any cluster $\hat{A}_1$. Furthermore, for any fixed cluster $\hat{A}_1$ the profit $\pi_{\hat{A}_1}(x,0)$ is convex in x and, due to the linearity of $\chi(x)$ and $\chi(\bar{x})=\xi$, it follows that

$$\pi_{\hat{A}_1}(x,0)-\chi(x)\leq\max[\pi_{\hat{A}_1}(\bar{x},0)-\xi,lim_{\epsilon\to 0}(\pi_{\hat{A}_1}(\underline{x}+\epsilon,0)-\chi(\underline{x}+\epsilon)]$$

for all $x\in(\underline{x},\bar{x}]$. Since $\tilde{A}_1$ is the optimal cluster under $x=\bar{x}$ we have $\pi_{\tilde{A}_1}(\bar{x},0)-\xi\geq\pi_{\hat{A}_1}(\bar{x},0)-\xi$ for any cluster $\hat{A}_1$. Together with (10) and (12) this implies

$$p_{A_1}\pi_{A_1}(\underline{x},0)+(1-p_{A_1})\pi_{A_2}(\underline{x},0)>\max_{x\in(\underline{x},\bar{x}]}\left[(\pi_{\hat{A}_1}(x,0)-\chi(x)\right]$$

for any cluster $\hat{A}_1$. Hence, $x_i=\underline{x}$ is the optimal investment of firm i if $h_i=0$, and therefore the no-investment constellation with $h=0$ is an equilibrium.

Second, we consider a scenario where x_i is such that there exists an equilibrium with $h=n$ under the setting with binary investment choice. Any deviation of a single firm to an investment level $x_i<\bar{x}$ implies that this firm will end up in the second smaller cluster and the convexity of $\pi_{A_2}(x,n-1)$ with respect to x implies that, if such a deviation is profitable, the optimal deviation is to choose $x_i=\underline{x}$. The fact that $h=n$ is equilibrium in the binary choice scenario however implies that such deviation is not profitable. Hence $h=n$ is also an equilibrium under continuous effort choice.

Third, we consider a scenario where ξ is such that there exists an equilibrium with $|A_1|<h<n$ under the setting with binary investment choice. For the h firms choosing $x_i=\bar{x}$ the same arguments just used for the case of $h=n$ show that there is no profitable deviation also in the case with continuous effort. For a firm i choosing $x_i=\underline{x}$ we know from the analysis with binary choice that a deviation to $x_i=\bar{x}$ is not profitable. Any deviation to $x<\bar{x}$ would still imply that the firm with probability 1 will end up in the second cluster A_2. Convexity of $\pi_{A_2}(x,h)-\chi(x)$ with respect to x again implies that no effort choice in $(\underline{x},\bar{x})$ can be optimal and therefore $x_i=\underline{x}$ is the optimal choice for firm i.

Finally, using again the convexity of $\pi_A(x,h)-\chi(x)$, it is easy to see that no equilibria exist where some firms invest $x_i\in(\underline{x},\bar{x})$. So, overall we have shown that under the considered cost structure and the assumption in (11), the equilibria under continuous effort choice coincide with those we have characterized under binary choice. Clearly the shape of the cost function χ is crucial for obtaining this result. In particular, it is easy to see that in the absence of fixed costs of investing above $\underline{x}$, i.e. for $F=0$, there is no equilibrium with $h=0$ since in such a setting it is always a profitable deviation for a firm to invest marginally above $\underline{x}$, which guarantees that this firm in the cluster formation stage obtains a spot in the larger first cluster. The equilibria with $h>0$ however also exist in the absence of fixed costs.

C Proofs

Proof of Proposition 1. We show the result in three steps. First, in Lemma 1 we use backward induction to calculate the number $\tilde{m}$ such that all remaining firms join one cluster and deduce from that the maximial number m such that a proposal is made which (upon acceptance) results in a number of remaining firms smaller or equal $\tilde{m}$ implying that two coalitions form. That proposal is made under the assumption that all other firms will join one coalition. Second, we show in Lemma 2 that any rational proposal is accepted by all firms included in the proposal. Finally, we show in Lemmas 3 and 4 that it is indeed optimal to propose a cluster such that all remaining firms join one coalition if the difference between high and low effort firms is bounded, i.e. $\bar{x} \leq 2\underline{x}$. In particular it is shown, that it is not profitable for the proposer to suggest a smaller cluster in order to induce the remaining firms to split up into more than one cluster after formation of the proposed cluster. This implies that under Assumption 1 always two clusters form. The size of these clusters and the number of high and low investors in each of them then follow directly from setting $m = n$ and $h_m = h$ in Lemma 1.

Lemma 1. *Assume that all cluster proposals are accepted. Then, for m remaining firms such that among these h_m invest high and l_m invest low and numbers $l^*(h_m, l_m) := \left\lceil \frac{(n-1-h_m+2l_m)\underline{x}-h_m\bar{x}}{4\underline{x}} \right\rceil$ and $h^*(h_m, l_m) := \left\lceil \frac{(n-1+2h_m+l_m)\bar{x}+l_m\underline{x}}{4\bar{x}} \right\rceil$, the following cluster is proposed under the assumption that all players outside the proposal join one cluster.*

- *If $0 < h_m \leq \frac{(n-1-2l_m)\underline{x}}{\bar{x}+\underline{x}}$, then a coalition of all remaining players is proposed.*
- *If $\frac{(n-1-2l_m)\underline{x}}{\bar{x}+\underline{x}} \leq h_m \leq \frac{(n-1+2l_m)\underline{x}}{\bar{x}+\underline{x}}$, then a coalition of h_m high investors and l^* low investors is proposed.*
- *If $\frac{(n-1+2l_m)\underline{x}}{\bar{x}+\underline{x}} \leq h_m \leq \frac{(n-1+l_m)\bar{x}+l_m\underline{x}}{2\bar{x}}$, then a coalition of h_m high investors and no low investors is proposed.*
- *If $\frac{(n-1+l_m)\bar{x}+l_m\underline{x}}{2\bar{x}} \leq h_m$, then a coalition of h^* high investors and no low investors is proposed.*
- *If $h_m = 0$ and $l_m \leq \left\lceil \frac{n+1}{2} \right\rceil$ a coalition of all remaining players is proposed.*
- *If $h_m = 0$ and $l_m > \left\lceil \frac{n+1}{2} \right\rceil$ a coalition l^* of low investors is proposed.*

Proof of Lemma 1. Suppose $n-m$ firms have already formed clusters and let it be firm i's turn to propose the next cluster. Denote the cluster structure that has been formed before i proposes by $\mathbf{A}(-i)$. Since by assumption all proposals have been accepted and because it is assumed that the rule of proposal order ρ is such that high effort firms have a lower rank than low effort firms, i is either a high investor or there are only low investors left in the game.

First, suppose that $h_m > 0$, i.e. i is a high investor. Since by assumption the firms outside the proposal form one cluster, i faces the optimization problem to propose an optimal cluster $A(i)$ consisting of $\tilde{h}$ high investors and $\tilde{l}$ low investors such that the other $m - \tilde{h} - \tilde{l} \geq 0$ firms form one cluster, denoted by $\bar{A}$ such that the cluster structure is given by $\mathbf{A} = (\mathbf{A}(-i), A(i), \bar{A})$.

Since maximizing profit is equivalent to maximizing quantities and, in the maximization problem, quantities of i are only influenced by the spillovers from the last two clusters $A(i)$ and $\bar{A}$, we get

$$\begin{aligned}
\underset{A(i)\subset N\setminus A(-i)}{\arg\max}\ \pi(i,\mathbf{x},\mathbf{A}) &= \underset{A(i)\subset N\setminus A(-i)}{\arg\max}\ q(i,\mathbf{x},\mathbf{A}) \\
= \underset{\tilde{h}\le h_m,\tilde{l}\le l_m}{\arg\max}\ &\Big[n((\tilde{h}-1)\bar{x}+\tilde{l}\underline{x})-(\tilde{h}-1)((\tilde{h}-1)\bar{x}+\tilde{l}\underline{x}))-\tilde{l}(\tilde{h}\bar{x}+(\tilde{l}-1)\underline{x}) \\
&-(h_m-\tilde{h})\big((h_m-\tilde{h}-1)\bar{x}+(l_m-\tilde{l})\underline{x}\big)-(l_m-\tilde{l})\big((h_m-\tilde{h})\bar{x}+(l_m-\tilde{l}-1)\underline{x}\big)\Big].
\end{aligned}$$

The proposer chooses first from the high effort firms and then from the low effort firms since a high effort firm is always preferred to a low effort firm and it is assumed that the remaining firms form one cluster.

Suppose first that the proposal includes only high effort firms, i.e. $A(i)=(h_{A(i)},0)$ which implies $\bar{A}=(h_m-h_{A(i)},l_m)$. Since marginal profit of adding other firms to the own cluster $A(i)$ is decreasing in the size of the cluster $|A(i)|$, we get the optimal number of high effort firms in $A(i)$ is the largest integer h^* such that marginal profit of adding the last firm is positive. Thus, we get h^* as the largest integer satisfying,

$$\begin{aligned}
& \pi(i,\mathbf{x},((h^*,0),(h_m-h^*,l_m)))-\pi(i,\mathbf{x},((h^*-1,0),(h_m-h^*+1,l_m)))>0 \\
\Leftrightarrow\quad & n\bar{x}-(h^*-2)\bar{x}-(h^*-1)\bar{x}+(2(h_m-h^*)+l_m)\bar{x}+l_m\underline{x}>0 \\
\Leftrightarrow\quad & \frac{1}{4\bar{x}}\bar{x}(n+3+2h_m+l_m)+l_m\underline{x}>h^*.
\end{aligned}$$

Hence, the optimal coalition size is given by

$$h^* := \left\lceil \frac{(n-1+2h_m+l_m)\bar{x}+l_m\underline{x}}{4\bar{x}} \right\rceil$$

To be consistent with the assumption that no low investors are selected, we need $h^*\le h_m$ which is equivalent to $\frac{(n-1+2h_m+l_m)\bar{x}+l_m\underline{x}}{4\bar{x}}\le h_m$ since h_m is an integer. Hence,

$$h^*\le h_m \ \Leftrightarrow\ h_m\ge \frac{(n-1+l_m)\bar{x}+l_m\underline{x}}{2\bar{x}} =: h_m^3$$

Thus, for $h_m\ge h_m^3$, i proposes the cluster $A(i)=(h^*,0)$.

Now consider the case that $h^*<h_m$. Therefore, choosing h^* high investors for the cluster $A(i)$ is no longer feasible. Since high effort firms are more attractive as partners, i will, hence, select all high effort firms. Additionally low investors may also be included. Again, since marginal profit of adding other firms to the own cluster $A(i)$ is decreasing in the size of the cluster $|A(i)|$, we get the optimal number of low effort firms l^* by solving,

$$\begin{aligned}
& \pi(i,\mathbf{x},((h_m,l^*+1),(0,l_m-l^*-1)))-\pi(i,\mathbf{x},((h_m,l^*),(0,l_m-l^*)))=0 \\
\Leftrightarrow\quad & n\underline{x}-(h_m+l^*-1)\underline{x}-h_m\bar{x}-l^*\underline{x}+2(l_m-l^*-1)\underline{x}=0 \\
\Leftrightarrow\quad & \tfrac{1}{4\underline{x}}(n-1-h_m+2l_m)\underline{x}-h_m\bar{x}=l^*
\end{aligned}$$

Thus, the optimal coalition size is given by

$$l^* := \left\lceil \frac{(n-1-h_m+2l_m)\underline{x}-h_m\bar{x}}{4\underline{x}} \right\rceil$$

To ensure that the number selected is feasible, we need $0 \leq l^* \leq l_m$ which is equivalent to $0 \leq \frac{(n-1-h_m+2l_m)\underline{x}-h_m\bar{x}}{4\underline{x}} \leq l_m$ since 0 and l_m are integers. Hence,

$$
\begin{aligned}
0 \leq l^* &\Leftrightarrow h_m \leq \frac{(n-1+2l_m)\underline{x}}{\bar{x}+\underline{x}} &&=: h_m^2 \\
l_m \geq l^* &\Leftrightarrow l_m \geq \frac{(n-1)\underline{x}-h_m(\bar{x}+\underline{x})}{2\underline{x}} \\
&\Leftrightarrow \frac{h_m(\bar{x}+\underline{x})}{2\underline{x}} \geq \frac{(n-1-2l_m)\underline{x}}{2\underline{x}} \\
&\Leftrightarrow h_m \geq \frac{(n-1-2l_m)\underline{x}}{\bar{x}+\underline{x}} &&=: h_m^1
\end{aligned}
$$

Thus, for $h_m^1 \leq h_m \leq h_m^2$, i proposes the cluster $A(i) = (h_m, l^*)$. It follows that for $h_m \geq h_m^2$ no low effort firms are included in $A(i)$, which implies that for $h_m^2 \leq h_m \leq h_m^3$ i proposes the cluster $A(i) = (h_m, 0)$. On the other hand, for $h < h_m^1$, i's proposal includes all remaining firms, i.e. $A(i) = N \setminus \mathbf{A}(-i)$. To see this we determine under which conditions a singleton low effort firm j with $|A(j)| = 1$, is accepted in a cluster of size $|A(i)| = m-1$. Note that this corresponds to the case where the incentive to add j is minimal since by (4) the incentive is decreasing in the size of $A(i)$ and decreasing in the investment $x(j)$ while increasing in the size of $A(j)$. Hence, we get from (4) that $\Delta q^*(i) > 0$ if and only if

$$
m < \tfrac{n+3}{2} - \tfrac{1}{2}h_m\left(\tfrac{\bar{x}}{\underline{x}} - 1\right). \tag{13}
$$

Direct calculations show that this inequality is satisfied for $h < h_m^1$.

Finally, the case where there are no high effort firms, $h_m = 0$ corresponds to the case when there are no low effort firms since in both cases all firms are symmetric, completing the proof of Lemma 1.

□

Lemma 2. *Suppose that $\bar{N} \subset N$ have already formed clusters and it is i's turn to propose the next cluster. If firm i proposes a payoff maximizing cluster $A(i) \subset N$, in the sense that i's payoff is maximized among all continuation payoffs following any accepted proposal $\tilde{A}(i) \subset N$, then proposal $A(i)$ is accepted by all firms $j \in A(i)$.*

Proof. Suppose that at some point of the game a firm i is to propose a cluster. In other words, either i is the first to propose or it proposes after the clusters $A_1, \ldots, A_l$ have already been formed. Now, firm i, which is the first of the remaining firms according to the rule of order, proposes a cluster $A(i)$, which upon acceptance results in a continuation subgame with a set $\tilde{N} \subseteq N$ of firms. Furthermore, we assume that $A(i)$ is chosen in a way that it maximizes the profit for firm i induced under the assumption that its current proposal is accepted and the subgame perfect equilibrium is followed in the continuation subgame. For further reference we observe that this optimality property implies that $q(i; \mathbf{x}, \mathbf{A}) \geq q(i; \mathbf{x}, \tilde{\mathbf{A}})$, where $\mathbf{A}$ denotes the cluster profile induced by $A(i)$ and $\tilde{\mathbf{A}}$ a cluster profile induced by some alternative proposal $\tilde{A}(i)$ at the current stage.

Consider now a firm $j \in A(i)$, $j \neq i$ with $x(i) = x(j)$. Clearly, the payoff of j under this proposal is identical to that of i. Assume that j rejects the proposal. This would only be rational if j could obtain a strictly higher payoff by offering an alternative proposal $\tilde{A}(j)$.[29] If $i \in \tilde{A}(j)$ then the payoff of i in this alternative proposal would be identical to that of j, which implies that the original proposal $A(i)$ would not be optimal for i, which is a contradiction to our assumption. If $i \notin \tilde{A}(j)$ then consider instead the proposal $\tilde{A}(i) = \tilde{A}(j) \setminus \{j\} \cup \{i\}$ by firm i. Comparing the subgames after $\tilde{A}(j)$ and $\tilde{A}(i)$ have been accepted, it turns out that both are identical up to permutation of players, since the number of high and low effort firms remaining are identical, and the rule of order is preserved since we assumed that high effort firms have a lower rank than low effort firms. Hence, we can conclude that the payoff of i in $\tilde{A}(i)$ is identical to the payoff of j in $\tilde{A}(j)$. This again yields a contradiction to the assumption that $A(i)$ is the optimal proposal for firm i.

Consider now a firm $j \in A(i),\ j \neq i$ with $x(i) \neq x(j)$. Given our assumption that high investors propose before low investors in the rule of order we must have $x(i) = \bar{x}$ and $x(j) = \underline{x}$. Assume that j rejects the proposal. This would only be rational if j could obtain a higher payoff by offering an alternative proposal $\tilde{A}(j)$. Similar to above we distinguish between the cases where $i \in \tilde{A}(j)$ and $i \notin \tilde{A}(j)$.

In case $i \in \tilde{A}(j)$ let us denote by $\mathbf{A}$ and $\tilde{\mathbf{A}}$ the unique[30] cluster profiles induced by the acceptance of proposals $A(i)$ and $\tilde{A}(j)$. Further, denote by $\Delta q(i) := q(i, \mathbf{x}, \tilde{\mathbf{A}}) - q(i, \mathbf{x}, \mathbf{A})$, respectively $\Delta q(j)$ the differences in quantities for the two firms between the cases where $\tilde{A}(j)$ is accepted and $A(i)$ is accepted. Since j rejects $A(i)$, it must strictly prefer the outcome induced by $\tilde{A}(j)$ and since profits (net of investment costs) are given by the square of the quantities, we must have $\Delta q(j) > 0$.

Thus,

$$
\begin{aligned}
&(n+1)\Delta q(i) \\
&= -n\Big(c(i;\mathbf{x},\tilde{\mathbf{A}}) - c(i;\mathbf{x},\mathbf{A})\Big) + c(j;\mathbf{x},\tilde{\mathbf{A}}) - c(j;\mathbf{x},\mathbf{A}) + \sum_{m\neq i,j} c(m;\mathbf{x},\tilde{\mathbf{A}}) - c(m;\mathbf{x},\mathbf{A}) \\
&= \gamma\beta\Big[-n\Big((h_{\tilde{A}(j)}-1)\bar{x} + l_{\tilde{A}(j)}\underline{x} - (h_{A(i)}-1)\bar{x} - l_{A(i)}\underline{x}\Big) + \Big(h_{\tilde{A}(j)}\bar{x} + (l_{\tilde{A}(j)}-1)\underline{x} \\
&\quad - h_{A(i)}\bar{x} - (l_{A(i)}-1)\underline{x}\Big)\Big] + \sum_{m\neq i,j} c(m;\mathbf{x},\tilde{\mathbf{A}}) - c(m;\mathbf{x},\mathbf{A}) \\
&= \gamma\beta\Big[-(n-1)\Big((h_{\tilde{A}(j)} - h_{A(i)})\bar{x} + (l_{\tilde{A}(j)} - l_{A(i)})\underline{x}\Big)\Big] + \sum_{m\neq i,j} c(m;\mathbf{x},\tilde{\mathbf{A}}) - c(m;\mathbf{x},\mathbf{A}) \\
&= (n+1)\Delta q(j) > 0.
\end{aligned}
$$

Therefore, we obtain a contradiction to the assumption that proposing $A(i)$ is optimal for firm i.

As a next step we consider the case where $i \notin \tilde{A}(j)$, but there exists a firm $k \in \tilde{A}(j)$ with $x(k) = \bar{x}$. In case $k \in A(i)$ we immediately obtain $\Delta q(k) < 0$ since k is of the same type as i and therefore $A(i)$ must have been optimal for k. This implies that also $\Delta q(j) < 0$, which contradicts the optimality of $\tilde{A}(j)$ for j. Consider now the case where $k \notin A(i)$. For proposal $\tilde{A}(j)$ to be strictly preferred by firm j to

[29] Note that is shown in Bloch (1996) that there exists a subgame perfect equilibrium with the property that all firms always accept a proposal as long as rejecting would not result in a strictly higher payoff (see Proposition 2.4 in Bloch (1996)).

[30] By unique, we mean up to a permutation of firms which invest identically. Thus quantities of i and j are uniquely determined. We get the uniqueness by backward induction and acceptance of proposals in case of indifference.

$A(i)$ we must have $\Delta q(j) > 0$. Analogous to above, this implies $\tilde{\Delta} q(k) > 0$, where $\tilde{\Delta} q(k)$ denotes the difference in quantity for firm k between proposal $\tilde{A}(j)$ and proposal $A(k) = (A(i) \setminus \{i\}) \cup \{k\}$. Hence $A(k)$ would not be optimal for firm k, but since firm k is of the same type as firm i this would contradict that $A(i)$ is optimal for firm i.

Finally, consider the case where $i \notin \tilde{A}(j)$ and there does not exist $k \in \tilde{A}(j)$ with $x(k) = \bar{x}$. In other words, the counter proposal consists of only low effort firms. As above denote by $\tilde{N}$ the remaining firms (before i's proposal) and $\tilde{h}$ and $\tilde{l}$ the number of high respectively low remaining investors. Assume that $\tilde{h}$ is low enough such that i's optimal proposal $A(i)$ (conditional on acceptance) also contains low investors (otherwise a homogeneous coalition is proposed which is always accepted, see above).

We show that no counterproposal $\tilde{A}(j)$ with $x(k) = \underline{x}$ for all $k \in \tilde{A}(j)$ increases j's payoff by induction over the remaining low investors $\tilde{l}$ for given $\tilde{h}$. Clearly for $\tilde{l} = 1$ such an $\tilde{A}(j)$ yields $|\tilde{A}(j)| = 1$ and thus lower profits. In what follows, we show that under the assumption that for l low investors with $l < \tilde{l}$ no such profitable counterproposal $\tilde{A}(j)$ exists, no profitable counterproposal $\tilde{A}(j)$ also exists for $\tilde{l}$ low investors. To the contrary, suppose that for $l = \tilde{l}$ there is a profit increasing counterproposal $\tilde{A}(j)$ which is hence accepted. After formation of $\tilde{A}(j)$ all proposals are accepted by assumption above and, hence, cluster sizes are given by Lemma 1. It is easy to see that with $|\tilde{N} \setminus \tilde{A}(j)| \geq n/2$ the profit of the members of $\tilde{A}(j)$ would increase if they add a high investor to their cluster. Following identical arguments to above this would imply that the profit of j in $A(i)$ would be higher than in $\tilde{A}(j)$.

Based on this we restrict attention to the case where $|\tilde{N} \setminus \tilde{A}(j)| < n/2$. Again, by induction hypotheses, after formation of $\tilde{A}(j)$, all proposals are accepted and we are in the case of Lemma 1. If all firms in $\tilde{N} \setminus \tilde{A}(j)$ join one cluster $\tilde{A}(i) = \tilde{N} \setminus \tilde{A}(j)$, then $\tilde{A}(j)$ clearly cannot be optimal since $(\tilde{A}(j) \setminus \{k\} \cup \{i\})$ yields higher profits for all firms in $\tilde{A}(j) \setminus \{k\}$. This follows from the fact that $x(k) = \underline{x}$ for all $k \in A(j)$ and $x(i) = \bar{x}$ and the observation that for any firm exchanging a low investor in the own cluster with a high investor from another cluster increases the firm's quantity. Since we know that $A(i)$ generates the highest profits for i (and, hence, for j) among all mixed clusters, this implies that $A(i)$ yields higher profits compared to $\tilde{A}(j)$ for firm j. Hence, consider the formation of two clusters among the firms in $\tilde{N} \setminus \tilde{A}(j)$. Lemma 1 then implies that these are $\tilde{A}_2 := (\tilde{h}, l^*(\hat{l}))$ where $l^*(\hat{l}) := \frac{(n-1-\tilde{h}+2(\tilde{l}-\hat{l}))\underline{x}-\tilde{h}\bar{x}}{4\underline{x}}$ and $\tilde{A}_3 := (0, \tilde{l} - \hat{l} - l^*(\hat{l}))$. Let $\tilde{\mathbf{A}} := (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3)$ denote the resulting cluster structure in the game of remaining firms $\tilde{N}$ induced by the proposal $\tilde{A}_1 := \tilde{A}(j) = (0, \hat{l})$. Consider the alternative counterproposal $A'(j) = (1, \hat{l} - 1)$ which results in the cluster structure $\mathbf{A}' := (A'_1, A'_2, A'_3)$ with $A'_1 := A'(j)$. Hence by Lemma 1 , $A'_2 := (\tilde{h} - 1, l^*(\hat{l} - 1)$ and $\tilde{A}_3 := (0, \tilde{l} - \hat{l} - l^*(\hat{l} - 1))$. Note that from Lemma 1 we get that $l^*(\hat{l} - 1) = l^*(\hat{l}) + \frac{3}{4} + \frac{1}{4}\frac{\bar{x}}{\underline{x}}$. Calculating $\Delta q(j) := q(j, \mathbf{x}, \tilde{\mathbf{A}}) - q(j, \mathbf{x}, \mathbf{A}')$, we get:

$$\begin{aligned}\Delta q(j) =& \tfrac{\gamma}{(n+1)} \Big(ns(j, \mathbf{x}, \tilde{\mathbf{A}}) - \sum_{k \in \tilde{A}_1, k \neq j} s(k, \mathbf{x}, \tilde{\mathbf{A}}) - \sum_{k \in \tilde{A}_2} s(k, \mathbf{x}, \tilde{\mathbf{A}}) - \sum_{k \in \tilde{A}_3} s(k, \mathbf{x}, \tilde{\mathbf{A}}) \\ & - ns(j, \mathbf{x}, \mathbf{A}') + \sum_{k \in A'_1, k \neq j} s(k, \mathbf{x}, \mathbf{A}') + \sum_{k \in A'_2} s(k, \mathbf{x}, \mathbf{A}') + \sum_{k \in A'_3} s(k, \mathbf{x}, \mathbf{A}') \Big)\end{aligned}$$

where $s(i, \mathbf{x}, A) = \beta \sum_{k \in A(i)\setminus\{i\}} x(k)$ denotes the spillovers experienced by $i \in N$ in an cluster structure $\mathbf{A}$. We get for the difference in total spillovers in each cluster:

$$
\begin{aligned}
\Delta S(A_1) :=& ns(j, \mathbf{x}, \tilde{\mathbf{A}}) - \sum_{k \in \tilde{A}_1, k \neq j} s(k, \mathbf{x}, \tilde{\mathbf{A}}) - ns(j, \mathbf{x}, \mathbf{A}') + \sum_{k \in A_1', k \neq j} s(k, \mathbf{x}, \mathbf{A}') \\
=& \beta\Big(n(\hat{l}-1)\underline{x} - (\hat{l}-1)^2\underline{x} - n\big((\hat{l}-2)\underline{x} + \bar{x}\big) \\
& + (\hat{l}-2)\big((\hat{l}-2)\underline{x} + \bar{x}\big) + (\hat{l}-1)\underline{x}\Big) \\
=& -\beta(n - \hat{l} + 2)(\bar{x} - \underline{x}) \\
\Delta S(A_2) :=& -\sum_{k \in \tilde{A}_2} s(k, \mathbf{x}, \tilde{\mathbf{A}}) + \sum_{k \in A_2'} s(k, \mathbf{x}, \mathbf{A}') \\
=& -\beta\Big(\tilde{h}\big((\tilde{h}-1)\bar{x} + l^*(\hat{l})\underline{x}\big) + (l^*(\hat{l}) - 1)\big(\tilde{h}\bar{x} + (l^*(\hat{l}) - 1)\underline{x}\big) \\
& + (\tilde{h}-1)\big((\tilde{h}-2)\bar{x} + l^*(\hat{l}-1)\underline{x}\big) + (l^*(\hat{l}-1) - 1)\big((\tilde{h}-1)\bar{x} \\
& + (l^*(\hat{l}-1) - 1)\underline{x}\big)\Big) \\
=& \beta\tfrac{1}{4}(\bar{x} - \underline{x})\Big(-\tfrac{n}{2} + \tilde{h}(\tfrac{3}{2}\tfrac{\bar{x}}{\underline{x}} - \tfrac{5}{2}) - \tilde{l} + \hat{l} + \tfrac{5}{2} - \tfrac{3}{4}\tfrac{\bar{x}-\underline{x}}{\underline{x}}\Big) \\
\Delta S(A_3) :=& -\sum_{k \in \tilde{A}_3} s(k, \mathbf{x}, \tilde{\mathbf{A}}) + \sum_{k \in A_3'} s(k, \mathbf{x}, \mathbf{A}') \\
=& \beta\Big(\big(l - \hat{l} - l^*(\hat{l})\big)\big(l - \hat{l} - l^*(\hat{l}) - 1\big)\underline{x} \\
& - \big(l - \hat{l} - l^*(\hat{l}-1) + 1\big)\big(l - \hat{l} - l^*(\hat{l}-1)\big)\Big)\underline{x} \\
=& \beta\Big(\big(l^*(\hat{l}-1) - l^*(\hat{l}) - 1\big)\big(l^*(\hat{l}-1) + l(\hat{l}) - 2(\tilde{l} - \hat{l})\big)\Big) \\
=& -\tfrac{\beta}{4}(\bar{x} - \underline{x})\Big(-\tfrac{n}{2} + \tilde{h}(\tfrac{\bar{x}}{2\underline{x}} + \tfrac{1}{2}) + \tilde{l} - \hat{l} - \tfrac{1}{4} - \tfrac{\bar{x}}{4\underline{x}}\Big)
\end{aligned}
$$

Thus we get:

$$
\begin{aligned}
\Delta q(j) =& \tfrac{\gamma}{(n+1)}(\Delta S(A_1) + \Delta S(A_2) + \Delta S(A_3)) \\
=& \tfrac{\gamma\beta}{(n+1)}\tfrac{\bar{x}-\underline{x}}{4}\Big(-4n + \tilde{h}(\tfrac{\bar{x}}{\underline{x}} - 3) + 6\hat{l} - 2\tilde{l} - \tfrac{7}{2} - \tfrac{1}{2}\tfrac{\bar{x}}{\underline{x}}\Big)
\end{aligned}
$$

Thus, for $\frac{\bar{x}}{\underline{x}} - 3 < 4$ the above bracket is clearly negative since $n > \tilde{h} + \hat{l}$ and $\tilde{l} > \hat{l}$. Hence, if $\frac{\bar{x}}{\underline{x}}$ is not too large, in particular $\bar{x} < 7\underline{x}$, then $\Delta q(i) < 0$ and thus the counterproposal cannot have been optimal, implying that for small enough $\bar{x}$ every proposal will be accepted.

□

Lemma 3. *If $h < \lceil \frac{n-1}{2} \rceil$, then every equilibrium of the unanimity game results in the formation of two clusters.*

Proof. First note that by assumption on the rule of order, the first proposer is a high investor. Further by assumption of the Lemma, $h < \lceil \frac{n-1}{2} \rceil$, implies that all high investors will be included in the first proposal, since otherwise marginal utility of adding a high effort firm is always positive which cannot be optimal since all proposals are accepted, see derivation in (13). This implies, that at most three coalitions form, since after all high effort firms joined the first cluster A_1, there are only low effort firms left which form at most two coalitions, see also Bloch (1995). Since there may be also low effort firms included in the first proposal, we get the following cluster structure: $\mathbf{A} = (A_1, A_2, A_3)$ with $A_1 = (h, l_1)$, $A_2 = (0, l_2)$ and $A_3 = (0, l_3)$. We show here that the last cluster is empty, i.e. $l_3 = 0$ for $\bar{x} \leq 2\underline{x}$.

Given $l - l_1$ remaining low effort firms after the first coalition forms, we can calculate the size of A_2 to be the largest integer such that

$$\pi_{A_2}\Big(i, \mathbf{x}, \mathbf{A}\Big) - \pi_{A_2}\Big(i, \mathbf{x}, \big(A_1, A_2 - (0,1), A_3 + (0,1)\big)\Big) \quad > 0$$

and therefore the optimal value of l_2 is given by $l_2^*(l_1) := \left\lceil \frac{n+2(l-l_1)-1}{4} \right\rceil$, see Lemma 1. Note that $l_3^* = l - l_1 - l_2^*(l_1)$. It is easy to see that, given $l - l_1$ remaining firms after the first proposal, if $l_i^*(l_1+1) < l_i^*(l_1)$, $i \in \{2,3\}$, then $l_j^*(l_1+1) = l_j^*(l_1)$ and $l_j^*(l_1+2) < l_j^*(l_1+1)$ as well as $l_i^*(l_1+2) = l_i^*(l_1+1)$ $j \in \{2,3\}$, $j \neq i$. Thus, when firms are added to the first cluster and A_2 and A_3 are non-empty, then these firms are added in alternating order from A_2 and A_3.

Hence the first firm i in order ρ (implying $x(i) = \bar{x}$) chooses l_1 as the largest integer such that

$$\begin{aligned} \pi_{A_1}\Big(i, \mathbf{x}, \big((h, l_1), A_2, A_3\big)\Big) - \pi_{A_1}\big(i, \mathbf{x}, \big((h, l_1 - 2), A_2 + (0,1), A_3 + (0,1)\big)\big) > 0 \\ \Leftrightarrow \quad 2\left(n\underline{x} - h(\bar{x} + \underline{x}) - 2l_1\underline{x} + l_2\underline{x} + l_3\underline{x} + 3\underline{x}\right) > 0 \end{aligned}$$

This implies

$$l_1^*(h) := \max\left\{0, \left\lceil \frac{n\underline{x} - h(\bar{x}+\underline{x}) + l\underline{x}}{3\underline{x}} \right\rceil\right\}, \tag{14}$$

which is solved by substituting $l_3 = l - l_1 - l_2$. Note that it is necessary for three coalitions to form that $l_3^*(h) = n - h - l_1^*(h) - l_2^*(l_1^*(h)) > 0$ which implies that $n - h - l_1^*(h) = n - |A_1| > \left\lceil \frac{n+1}{2} \right\rceil$ (compare also to Lemma 1). Note that (14) implies that the size of A_1, given by $h + l_1^*(h) = \left\lceil \frac{2n\underline{x} - h(\bar{x} - \underline{x})}{3\underline{x}} \right\rceil$, is decreasing in h (for an illustration, see also Figure 1). Hence choosing h maximal under the assumption $h < \left\lceil \frac{n-1}{2} \right\rceil$ yields the minimal size of A_1 which implies $h = \frac{n-2}{2}$ and n is even for $|A_1|$ to be minimal under our assumption. We then get this size of A_1 by calculating $l_1^*(h)$:

$$\begin{aligned} l_1^*(\tfrac{n-2}{2}) &= \max\left\{0, \left\lceil \tfrac{1}{3}\Big(n - \tfrac{n-2}{2}\Big(\tfrac{\bar{x}}{\underline{x}} + 1\Big) + n - \tfrac{n-2}{2}\Big)\right\rceil\right\} \\ &= \max\left\{0 \left\lceil \tfrac{1}{6}\Big(2(n+2) - \Big(\tfrac{\bar{x}}{\underline{x}}\Big)(n-2)\Big)\right\rceil\right\} \end{aligned}$$

which ist obviously positive due to $\bar{x} \leq 2\underline{x}$, see Assumption 1. Hence $|A_1| = h + l^*(h) > \left\lceil \frac{n-1}{2} \right\rceil$ and hence no three cluster outcome can be supported as an equilibrium for $h < \left\lceil \frac{n-1}{2} \right\rceil$.

□

Lemma 4. *If $h \geq \left\lceil \frac{n-1}{2} \right\rceil$, then every equilibrium of the unanimity game results in the formation of two clusters.*

Proof. First, note that for $\lceil\frac{n-1}{2}\rceil \leq h \leq \lceil\frac{n+1}{2}\rceil$ the first cluster A_1 will include all high effort firms (see Lemma 1 by setting $l_m = n-h$ and using $\bar{x} \geq \underline{x}$), implying that no high effort and only $l_m < n - \lceil\frac{n-1}{2}\rceil = \lfloor\frac{n+1}{2}\rfloor$ low effort firms remain after the first proposal. These form one coalition by Lemma 1, see also derivation in (13). Hence, three coalitions are only possible if $h > \lceil\frac{n+1}{2}\rceil$ such that the first proposal does not include all high effort firms.

Thus, consider the formation of three clusters A_1, A_2, and A_3 such that A_1 consists of only high effort firms $A_1 = (h_1, 0)$ with $h_1 \geq \lceil\frac{n+1}{2}\rceil$ and, hence, $h - h_1 \leq \frac{n-1}{2} \leq \frac{(n-1+l)\bar{x}+l\underline{x}}{2\bar{x}}$. Therefore, A_3 cannot include any high effort firms by Lemma 1.

To summarize, the only way that three coalitions can be supported in equilibrium is to have $A_1 = (h_1, 0)$, $A_2 = (h-h_1, l_2)$ and $A_3 = (0, l_3)$ if $h \geq \lceil\frac{n-1}{2}\rceil$. To the contrary, suppose that these three coalitions indeed form. Denoting by $h_2(h_1)$, $l_2(h_1)$, and $l_3(h_1)$ the number of high respectively low effort firms in coalition 2 and 3 for a given h_1, we get (trivially) $h_2(h_1) = h - h_1$ and, in equilibrium, by Lemma 1, $l_2(h_1) = \left\lceil\frac{(n-1-(h-h_1)+2l)\underline{x}-(h-h_1)\bar{x}}{4\underline{x}}\right\rceil$ and, trivially, $l_3(h_1) = n - h - l_2(h_1)$. Again for $\mathbf{A} = (A_1, A_2, A_3)$ to be an equilibrium outcome, the first proposal must be such that it maximizes payoff under the expectation that these three coalitions form. Note that the quantity of the proposing firm i (lowest ranked firm in order ρ) choosing h_1 is given by

$$q_{A_1}\big(i,\mathbf{x},\mathbf{A}\big) = \frac{\gamma\beta}{n+1}\Big[n(h_1-1)\bar{x} - (h_1-1)^2\bar{x} - h_2(h_1)\big[(h_2(h_1)-1)\bar{x} + l_2(h_1)\underline{x}\big] - l_2(h_1)[h_2(h_1)\bar{x} + (l_2(h_1)-1)\underline{x}] - l_3(h_1)(l_3(h_1)-1)\underline{x}\Big] + C,$$

where C is a constant which is independent from the cluster profile $\mathbf{A}$. Since profit is strictly increasing in the quantity, the optimal choice of h_1 is determined by the first order condition

$$\begin{aligned}
&\frac{\partial q_{A_1}(i,\mathbf{x},\mathbf{A})}{\partial h_1} = 0\\
\Leftrightarrow\quad & 0 = n\bar{x} - 2(h_1-1)\bar{x} + (h-h_1)\big[\bar{x} - \tfrac{\bar{x}+\underline{x}}{4}\big] + \big[(h-h_1-1)\bar{x} + l_2(h_1)\underline{x}\big]\\
&\quad - \tfrac{\bar{x}+\underline{x}}{4\underline{x}}\big[(h-h_1)\bar{x} + l_2(h_1)\underline{x} - \underline{x}\big] + l_2(h_1)\big[\bar{x} - \tfrac{\bar{x}+\underline{x}}{4}\big] + 2(l - l_2(h_1))\tfrac{\bar{x}+\underline{x}}{4} - \tfrac{\bar{x}+\underline{x}}{4}\\
\Leftrightarrow\quad & 0 = (n+1-2h)\bar{x} + (h-h_1)\big[4\bar{x} - \tfrac{\bar{x}+\underline{x}}{4}\tfrac{\bar{x}+\underline{x}}{\underline{x}}\big] + l_2(h_1)\big[\underline{x} - 4\tfrac{\bar{x}+\underline{x}}{4} + \bar{x}\big] + 2l\tfrac{\bar{x}+\underline{x}}{4}\\
\Leftrightarrow\quad & 0 = (n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x} + (h-h_1)[4\bar{x} - \tfrac{(\bar{x}+\underline{x})^2}{4\underline{x}}]\\
\Leftrightarrow\quad & h_1^*(h) = \tfrac{4\underline{x}}{16\bar{x}\underline{x}+(\bar{x}+\underline{x})^2}\big[(n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x}\big] + h
\end{aligned}$$

And hence,

$$\Leftrightarrow\quad h_2^*(h) = -\tfrac{4\underline{x}}{16\bar{x}\underline{x}+(\bar{x}+\underline{x})^2}\big[(n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x}\big]$$

As pointed out above, we need $h_2^*(h) > 0$ if $h \geq \frac{n-1}{2}$, in order to have $\mathbf{A}$ as an equilibrium outcome. Hence,

$$\begin{aligned}
& 0 > (n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x}\\
\Leftrightarrow\quad & h > \tfrac{(3\bar{x}+\underline{x})n+2\bar{x}}{5\bar{x}+\underline{x}} =: \bar{h}
\end{aligned} \tag{15}$$

Moreover, we must have $h_2^* > h_m^1 = \frac{(n-1-2l)\underline{x}}{\bar{x}+\underline{x}}$ in order to have A_3 non-empty, see Lemma 1. Using $l = n-h$, we get the condition

$$h_2^*(h) - h_m^1 = -\tfrac{4\underline{x}}{16\bar{x}\underline{x}+(\bar{x}+\underline{x})^2}\big[(n-h)(\bar{x} + \tfrac{\bar{x}+\underline{x}}{2}) - (h-1)\bar{x}\big] - \tfrac{((n-1)-2(n-h))\underline{x}}{\bar{x}+\underline{x}} > 0 \tag{16}$$

The left-hand side of (16) is non-increasing in h if

$$0 \geq \tfrac{5\bar{x}+\underline{x}}{2} - 2\tfrac{16\bar{x}\underline{x}+(\bar{x}+\underline{x})^2}{4\underline{x}}\tfrac{\underline{x}}{\bar{x}+\underline{x}} \qquad \Leftrightarrow \qquad 3\underline{x} \geq \bar{x}$$

Hence, for $3\underline{x} \geq \bar{x}$, (15) and (16) cannot be simultaneously satisfied. To see this, note we must have $h \geq \bar{h}$ by (15) and we get $h_2^*(\bar{h}) = 0$ implying that the left-hand side of (16) is negative for $h = \bar{h}$. Since it is, moreover, decreasing in h for $3\underline{x} \geq \bar{x}$, (16) can then not be satisfied. Thus, for $3\underline{x} \geq \bar{x}$ and $h \geq \frac{n-3}{2}$, there cannot exist three coalitions which are supported by a subgame perfect equilibrium. □

To wrap up, we have first characterized optimal cluster profiles for m remaining firms if firms expect all other firms to join one cluster. That proposals are indeed accepted by all players is shown by backward induction in Lemma 2 under the condition that $7\underline{x} > \bar{x}$. Finally, in Lemmas 3 and 4, it is shown that for $2\underline{x} > \bar{x}$, three (and trivially also more) clusters cannot be supported by a subgame perfect equilibrium. Hence, any subgame perfect equilibrium consist of two clusters, and, hence, the sizes and composition of these two clusters are given in Lemma 1. Setting $h_m = h, l_m = n-h$ in Lemma 1 yields the expressions for the cluster sizes as well as $\tilde{h}, \tilde{\tilde{h}}$ in the Proposition. This completes the proof.

□

Proof of Proposition 2. We show the proposition in three steps by considering the investment incentives of firms, i.e. the marginal return on investment. First, if the number of other high effort firms h^{-i} is low then incentives are increasing in h^{-i} for large enough values of β which is shown in Lemma 5. For large values of h^{-i}, the investment incentives are decreasing (Lemma 6) if β is not too large, while for intermediate values, there is a unique maximum (Lemma 7). Together these Lemmas imply the Proposition.

Lemma 5. *If $\beta > \underline{\beta} := \frac{4\underline{x}}{(n+6)\underline{x}+\bar{x}}$, then for $h^{-i} \leq \frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}} - 1$ expected return on investment is increasing in h^{-i}.*

Proof. It follows from (6) that for $h^{-i} \leq \frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}} - 1$ all high investors participate in the first cluster, i.e. $p_{A_1}(\bar{x}, h^{-i}+1) = 1$. Since $l_1 = l_{A_1}(h^{-i}+1)$, the profit of a high investor is given by:

$$\begin{aligned}\pi(\bar{x}, h^{-i}) =& \tfrac{1}{(n+1)^2}\Big[\alpha - \bar{c} + \gamma(n-h^{-i})(\bar{x}-\underline{x}) + \gamma\underline{x} + \gamma\beta\Big((n-h^{-i})(h^{-i}\bar{x} + l_1\underline{x}) \\ & - l_1((h^{-i}+1)\bar{x} + (l_1-1)\underline{x}) - (n-h^{-i}-l_1-1)(n-h^{-i}-l_1-2)\underline{x}\Big)\Big]^2 - \xi\end{aligned}$$

The derivative with respect to h^{-i} yields:

$$\tfrac{\partial\pi(\bar{x},h^{-i})}{\partial h^{-i}} = \tfrac{2\gamma q(\bar{x})}{(n+1)^2}(\bar{x}-\underline{x})\Big[-1 + \tfrac{\beta}{4}\Big(n + h^{-i}\big(\tfrac{\bar{x}}{\underline{x}} - 1\big) + 6 + \tfrac{\bar{x}}{\underline{x}}\Big)\Big]$$

where $q(\bar{x})$ denotes the optimal quantity of a high investment firm. If instead firm i chooses low investment, she will join A_1 with probability $p_{A_1}(\underline{x}, h^{-i}) = \frac{l_1}{n-h^{-i}}$ and A_2 with probability $1 - p_{A_1}(\underline{x}, h^{-i}) = \frac{n-h^{-i}-l_1}{n-h^{-i}}$. The resulting payoff from low investment in A_1 weighted with the probability of being in A_1 is hence

$$\hat{\pi}(\underline{x}, h^{-i}, A_1) = \tfrac{1}{(n+1)^2}p_{A_1}(\underline{x}, h^{-i})\Big[\alpha - \bar{c} - \gamma h^{-i}(\bar{x}-\underline{x}) + \gamma\underline{x} + \gamma\beta\Big((n-(l_1-1))(h^{-i}\bar{x} + (l_1-1)\underline{x})$$

$$- h^{-i}((h^{-i}-1)\bar{x} + l_1\underline{x}) - (n-h^{-i}-l_1-1)(n-h^{-i}-l_1-2)\underline{x}\Big)\Big]^2,$$

where $l_1 = l_{A_1}(h^{-i})$. The derivative of $\hat{\pi}(\underline{x}, h^{-i}, A_1)$ with respect to h^{-i} yields:

$$\begin{aligned}\frac{\partial\hat{\pi}(\underline{x},h^{-i},A_1)}{\partial h^{-i}} =& 2\frac{1}{(n+1)^2}\gamma(\bar{x}-\underline{x})p_{A_1}(\underline{x},h^{-i})q(\underline{x},A_1)\Big[-1+\frac{\beta}{4}\Big(n+h^{-i}(\frac{\bar{x}}{\underline{x}}-1)+7\Big)\Big] \\ &+ \frac{\partial p_{A_1}(\underline{x},h^{-i})}{\partial h^{-i}}q(\underline{x},A_1)^2\end{aligned}$$

where $q(\underline{x}, A_1)$ denotes the quantity produced by a low effort firm in A_1. Considering now the payoff from low investment in A_2, weighted with the probability of being in A_2, we obtain

$$\begin{aligned}\hat{\pi}(\underline{x},h^{-i},A_2) =& \frac{1}{(n+1)^2}\big(1-p_{A_1}(\underline{x},h^{-i})\big)\Big[\alpha-\bar{c}-\gamma h^{-i}(\bar{x}-\underline{x})+\gamma\underline{x} \\ &+\gamma\beta\Big(n-(n-h^{-i}-l_1-1))((n-h^{-i}-l_1-1)\underline{x}) \\ &-h^{-i}((h^{-i}-1)\bar{x}+l_1\underline{x})-l_1(h^{-i}x+(l_1-1)\underline{x})\Big)\Big]^2,\end{aligned}$$

where, again, $l_1 = l_{A_1}(h^{-i})$. The derivative of $\hat{\pi}(\underline{x}, h^{-i}, A_2)$ with respect to h^{-i} yields,

$$\begin{aligned}\frac{\partial\hat{\pi}(\underline{x},h^{-i},A_2)}{\partial h^{-i}} =& 2\frac{1}{(n+1)^2}\gamma(\bar{x}-\underline{x})(1-p_{A_1}(\underline{x},h^{-i}))q(\underline{x},A_2)\Big[-1+\frac{\beta}{4}\Big(-n+h^{-i}(\frac{\bar{x}}{\underline{x}}-1)+5\Big)\Big] \\ &- \frac{\partial p_{A_1}(\underline{x},h^{-i})}{\partial h^{-i}}q(\underline{x},A_2)^2\end{aligned}$$

Note that expected payoff from choosing low investment is given by $\mathbb{E}\big(\pi(\underline{x},h^{-i})\big) = \hat{\pi}(\underline{x},h^{-i},A_1)+\hat{\pi}(\underline{x},h^{-i},A_2)$. Thus the expected return on investment $\Delta\pi := \pi(\bar{x},h^{-i}) - \mathbb{E}\big(\pi(\underline{x},h^{-i})\big)$ changes with h^{-i} according to

$$\begin{aligned}\frac{\partial\Delta\pi}{\partial h^{-i}} =& \frac{\partial\pi(\bar{x},h^{-i})}{\partial h^{-i}} - \frac{\partial\mathbb{E}\big(\pi(\underline{x},h^{-i})\big)}{\partial h^{-i}} = \frac{\partial\pi(\bar{x},h^{-i})}{\partial h^{-i}} - \Big(\frac{\partial\hat{\pi}(\underline{x},h^{-i},A_1)}{\partial h^{-i}} + \frac{\partial\hat{\pi}(\underline{x},h^{-i},A_2)}{\partial h^{-i}}\Big) \\ =& \frac{2}{(n+1)^2}\gamma(\bar{x}-\underline{x})\Big[\big(q(\bar{x})-\mathbb{E}(q(\underline{x}))\big)\Big(-1+\frac{\beta}{4}\Big(n+h^{-i}\big(\frac{\bar{x}}{\underline{x}}-1\big)+6+\frac{\bar{x}}{\underline{x}}\Big) \\ &+p_{A_1}(\underline{x},h^{-i})q(\underline{x},A_1)\frac{\beta}{4}(\frac{\bar{x}}{\underline{x}}-1)+\big(1-p_{A_1}(\underline{x},h^{-i})\big)q(\underline{x},A_2)\frac{\beta}{4}(2n+1)\Big] \\ &- \frac{\partial p_{A_1}(\underline{x},h^{-i})}{\partial h^{-i}}\big(q(\underline{x},A_1)^2-q(\underline{x},A_2)^2\big)\end{aligned}$$

where $\mathbb{E}(q(\underline{x})) = p_{A_1}(\underline{x},h^{-i})q(\underline{x},A_1)+\big(1-p_{A_1}(\underline{x},h^{-i})\big)q(\underline{x},A_2)$ is the expected quantity of a low effort firm. We clearly have that the quantity produced by a high effort firm in A_1 exceeds the expected quantity of a low effort firm, i.e. $q(\bar{x}) > \mathbb{E}(q(\underline{x}))$. Hence, $\frac{\partial\Delta\pi}{\partial h^{-i}}$ is positive if $\beta > \frac{4\underline{x}}{(n+7)\underline{x}+(h^{-i}+1)(\bar{x}-\underline{x})}$ since $\frac{\partial p_{A_1}(\underline{x},h^{-i})}{\partial h^{-i}} = -\frac{\big(3+\frac{\bar{x}}{\underline{x}}\big)(n-h^{-i})-4l_1}{4(n-h^{-i})^2} < 0$. Expected return on investment is hence increasing in h^{-i} for $h^{-i} \leq \frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}} - 1$ and $2\underline{x} \geq \bar{x}$ under the condition of $\beta > \frac{4\underline{x}}{(n+7)\underline{x}+(h^{-i}+1)(\bar{x}-\underline{x})}$. This expression is maximized for $h^{-i} = 0$ yielding $\underline{\beta} := \frac{4\underline{x}}{(n+6)\underline{x}+\bar{x}}$. Note that the latter is only a sufficient condition. □

Lemma 6. *If $\beta < 1/2$, then for $h^{-i} \geq \frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}}$ expected return on investment is decreasing in h^{-i}.*

Proof. If $h^{-i} \geq \frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}}$ then by Proposition 1, we have that two clusters $A_1 = (h_1, 0)$ and $A_2 = (h - h_1, l)$ form with $h_1 := h_{A_1}(h) = \left\lceil \frac{(2n+h-1)\bar{x}+(n-h)\underline{x}}{4\bar{x}} \right\rceil$. When i chooses $x(i) = \bar{x}$, she will be included in A_1 with probability $p_{A_1}(\bar{x}, h^{-i}+1) = \frac{h_{A_1}(h^{-i}+1)}{h^{-i}+1}$ (see (6)). In this case the payoff of a high investor in A_1 weighted with the probability of being in A_1 will be

$$\begin{aligned}\hat{\pi}(\bar{x}, h^{-i}, A_1) =& p_{A_1}(\bar{x}, h^{-i}+1)\tfrac{1}{(n+1)^2}\Big[\alpha - \bar{c} + \gamma(n-h^{-i})(\bar{x}-\underline{x}) + \gamma\underline{x} \\ &+ \gamma\beta\Big((n-(h_1-1))(h_1-1)\bar{x} - ((h^{-i}+1)-h_1)\big((h^{-i}-h_1)\bar{x} + (n-h^{-i}-1)\underline{x}\big) \\ &- (n-h^{-i}-1)\big((h^{-i}+1-h_1)\bar{x} + (n-h^{-i}-2)\underline{x}\big)\Big)\Big]^2 - p_{A_1}(\bar{x}, h^{-i}+1)\xi\end{aligned}$$

Taking the derivative with respect to h^{-i} yields,

$$\begin{aligned}\tfrac{\partial\hat{\pi}(\bar{x},h^{-i},A_1)}{\partial h^{-i}} =& p_{A_1}(\bar{x}, h^{-i}+1)\tfrac{2q(\bar{x},A_1)}{n+1}\gamma(\bar{x}-\underline{x})\Big[-1 + \tfrac{\beta}{4}\Big(n(\tfrac{\underline{x}}{\bar{x}}-2) + h^{-i}(1-\tfrac{\underline{x}}{\bar{x}}) + 6 - \tfrac{\underline{x}}{\bar{x}}\Big)\Big] \\ &+ \tfrac{\partial p_{A_1}(\bar{x},h^{-i}+1)}{\partial h^{-i}}\big(q(\bar{x},A_1)^2 - \xi\big)\end{aligned}$$

When i chooses $x(i) = \bar{x}$, she could also end up in A_2 which happens with probability $1 - p_{A_1}(\bar{x}, h^{-i}+1)$. In this case the expected payoff will be

$$\begin{aligned}\hat{\pi}(\bar{x}, h^{-i}, A_2) =& (1-p_{A_1}(\bar{x}, h^{-i}+1))\tfrac{1}{(n+1)^2}\Big[\alpha - \bar{c} + \gamma(n-h^{-i})(\bar{x}-\underline{x}) + \gamma\underline{x} \\ &+ \gamma\beta\Big((n-h^{-i}+h_1)\big((h^{-i}-h_1)\bar{x} + (n-h^{-i}-1)\underline{x}\big) \\ &- (n-h^{-i}-1)\big((h^{-i}-h_1+1)\bar{x} + (n-h^{-i}-2)\underline{x}\big) - h_1(h_1-1)\bar{x}\Big)\Big]^2 \\ &- (1-p_{A_1}(\bar{x}, h^{-i}+1))\xi\end{aligned}$$

Taking the derivative with respect to h^{-i} yields,

$$\begin{aligned}\tfrac{\partial\hat{\pi}(\bar{x},h^{-i},A_2)}{\partial h^{-i}} =& (1-p_{A_1}(\bar{x}, h^{-i}+1))\tfrac{2q(\bar{x},A_2)}{n+1}\gamma(\bar{x}-\underline{x})\Big[-1 + \tfrac{\beta}{4}\Big(n\tfrac{\underline{x}}{\bar{x}} + h^{-i}(1-\tfrac{\underline{x}}{\bar{x}}) + 8 - \tfrac{\underline{x}}{\bar{x}}\Big)\Big] \\ &- \tfrac{\partial p_{A_1}(\bar{x},h^{-i}+1)}{\partial h^{-i}}\Big(q(\bar{x},A_2)^2 - \xi\Big)\end{aligned}$$

Finally, if i invests low, she will be in A_2 for sure, i.e. $p_{A_1}(\underline{x}, h^{-i}) = 0$. Payoff is then given by

$$\begin{aligned}\pi(\underline{x}, h^{-i}) =& \tfrac{1}{(n+1)^2}\Big[\alpha - \bar{c} - \gamma h^{-i}(\bar{x}-\underline{x}) + \gamma\underline{x} + \gamma\beta\Big((h^{-i}+1)\big((h^{-i}-h_1)\bar{x} \\ &+ (n-h^{-i}-1)\underline{x}\big) - (h^{-i}-h_1)\big((h^{-i}-h_1-1)\bar{x} + (n-h^{-i})\underline{x}\big) - h_1(h_1-1)\bar{x}\Big)\Big]^2\end{aligned}$$

Taking the derivative with respect to h^{-i} yields,

$$\tfrac{\partial\pi(\underline{x},h^{-i})}{\partial h^{-i}} = \tfrac{2q(\underline{x},A_2)}{n+1}\gamma(\bar{x}-\underline{x})\Big[-1 + \tfrac{\beta}{4}\Big(n(\tfrac{\underline{x}}{\bar{x}}) + h^{-i}(1-\tfrac{\underline{x}}{\bar{x}}) + 7\Big)\Big].$$

Note that expected payoff from choosing high investment is given by $\mathbb{E}(\pi(\bar{x}, h^{-i})) = \hat{\pi}(\bar{x}, h^{-i}, A_1) + \hat{\pi}(\bar{x}, h^{-i}, A_2)$. Thus the expected return on investment $\Delta\pi := \mathbb{E}(\pi(\bar{x}, h^{-i})) - \pi(\underline{x}, h^{-i})$ changes with h^{-i} according to

$$\tfrac{\partial\Delta\pi}{\partial h^{-i}} = \tfrac{\partial\mathbb{E}(\pi(\bar{x},h^{-i}))}{\partial h^{-i}} - \tfrac{\partial\pi(\underline{x},h^{-i})}{\partial h^{-i}} = \tfrac{\partial\hat{\pi}(\bar{x},h^{-i},A_1)}{\partial h^{-i}} + \tfrac{\partial\hat{\pi}(\bar{x},h^{-i},A_2)}{\partial h^{-i}} - \tfrac{\partial\pi(\underline{x},h^{-i})}{\partial h^{-i}}$$

$$
\begin{aligned}
=&\big(\mathbb{E}(q(\bar{x}))-q(\underline{x})\big)\tfrac{2}{(n+1)^2}\gamma(\bar{x}-\underline{x})\Big[-1+\tfrac{\beta}{4}\Big(-(n-h^{-i})(1-\tfrac{\underline{x}}{\bar{x}})+8-\tfrac{\underline{x}}{\bar{x}}\Big)\Big]\\
&-\tfrac{2}{(n+1)^2}\gamma(\bar{x}-\underline{x})\tfrac{\beta}{4}\Big[p_{A_1}(\bar{x},h^{-i}+1)q(\bar{x},A_1)(n+2)-(1-p_{A_1}(\bar{x},h^{-i}+1))q(\bar{x},A_2)n\\
&+q(\underline{x},A_2)(n+1-\tfrac{\underline{x}}{\bar{x}})\Big]+\tfrac{\partial p_{A_1}(\bar{x},h^{-i}+1)}{\partial h^{-i}}\big(q(\bar{x},A_1)^2-q(\bar{x},A_2)^2\big)\\
\overset{(*)}{>}&\big(\mathbb{E}(q(\bar{x}))-q(\underline{x})\big)\tfrac{2}{(n+1)^2}\gamma(\bar{x}-\underline{x})\Big[-1+\tfrac{\beta}{4}\Big(-(n-h^{-i})(1-\tfrac{\underline{x}}{\bar{x}})+8-\tfrac{\underline{x}}{\bar{x}}\Big)\Big]\\
&+\tfrac{\partial p_{A_1}(\bar{x},h^{-i}+1)}{\partial h^{-i}}\big(q(\bar{x},A_1)^2-q(\bar{x},A_2)^2\big)
\end{aligned}
$$

where, again $\mathbb{E}(q(\bar{x}))$ is the expected quantity produced by a high effort firm. The last inequality $(*)$ holds since $q(\bar{x},A_1)>q(\bar{x},A_2)$ and, furthermore, the fact that $p_{A_1}(\bar{x},h^{-i}+1)$ is decreasing in h^{-i} and, hence, for all $h^{-i}<n-1$ it holds that $p_{A_1}(\bar{x},h^{-i}+1)\geq p_{A_1}(\bar{x},n)=\lceil\frac{3n-1}{4n}\rceil>1-\lceil\frac{3n-1}{4n}\rceil\geq 1-p_{A_1}(\bar{x},h^{-i}+1)$ (see Proposition 1). Thus, if $\beta<1/2$ then all terms above are non-positive which implies the statement of Lemma 6. □

Lemma 7. *For $h^{-i}\in\Big[\frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}},\frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}}-1\Big]$ the expected return on investment is increasing in h^{-i} on the entire interval, decreasing in h^{-i} on the entire interval or has a unique local maximum in the interior of the interval.*

Proof. In order to show the claim of the proposition we prove that the change of the return on investment is concave in h^{-i} on the considered interval. It follows from Lemma 1 that for $h^{-i}\in\Big[\frac{3(n-1)\underline{x}}{3\bar{x}+\underline{x}},\frac{(2n-1)\bar{x}+n\underline{x}}{3\bar{x}+\underline{x}}-1\Big]$ there are two clusters where all high investors are in the first and all low investors are in the second cluster. Taking this into account the return on investment is given by

$$
\begin{aligned}
\Delta\pi=&\pi(\bar{x},h^{-i}+1,A_1)-\pi(\underline{x},h^{-i},A_2)\\
=&\tfrac{1}{(n+1)^2}\Big[\alpha-\bar{c}+\gamma(n-h^{-i})(\bar{x}-\underline{x})+\gamma\underline{x}+\gamma\beta((h^{-i}(n-h^{-i})\bar{x}-(n-h^{-i}-1)(n-h^{-i}-2)\underline{x})\Big]^2\\
&-\tfrac{1}{(n+1)^2}\Big[\alpha-\bar{c}-\gamma h^{-i}(\bar{x}-\underline{x})+\gamma\underline{x}+\gamma\beta(n(n-h^{-i})\bar{x}-h^{-i}(h^{-i}-1)\bar{x}-(n-h^{-i}-1)^2\underline{x}\Big]^2.
\end{aligned}
$$

Considering the derivative with respect to h^{-i} and collecting terms yields

$$
\begin{aligned}
\tfrac{\partial\Delta\pi}{\partial h^{-i}}=\tfrac{2\gamma}{(n+1)}\Big[&-(\bar{x}-\underline{x})(q(\bar{x},A_1)-q(\underline{x},A_2))+\beta\Big(n\bar{x}(q(\bar{x},A_1)+q(\underline{x},A2))\\
&+((2n-3)\underline{x}-2h^{-i}(\bar{x}+\underline{x}))(q(\bar{x},A_1)-q(\underline{x},A2))+(\bar{x}-\underline{x})q(\underline{x},A_2)\Big)\Big]
\end{aligned}
$$

Furthermore, we have

$$\tfrac{\partial q(\underline{x},A_2)}{\partial h^{-i}}=-\gamma(\bar{x}-\underline{x})-\gamma\beta(n\underline{x}+(2h^{-i}-1)\bar{x}-2(n-h^{-i}-1)\underline{x})<0,$$

because $h^{-i}>\frac{3(n-1)}{3\bar{x}+\underline{x}}$ implies $(n\underline{x}+(2h^{-i}-1)\bar{x}-2(n-h^{-i}-1)\underline{x})>0$. Moreover,

$$\tfrac{\partial(q(\bar{x},A_1)+q(\underline{x},A_2))}{\partial h^{-i}}=-2\gamma(\bar{x}-\underline{x})-\gamma\beta((n-4h^{-i}+1)\bar{x}+(3n-4h^{-i}-5)\underline{x})<0,$$

where $((n-4h^{-i}+1)\bar{x}+(3n-4h^{-i}-5)\underline{x})<0$ again follows from $h^{-i}>\frac{3(n-1)}{3\bar{x}+\underline{x}}$ in combination with $\bar{x}\leq 2\underline{x}$. Finally,

$$\tfrac{\partial(q(\bar{x},A_1)-q(\underline{x},A_2))}{\partial h^{-i}}=\gamma\beta((n-1)(\bar{x}+\underline{x})>0.$$

Taking these observations into account we obtain

$$\begin{aligned}\frac{\partial^2 \Delta\pi}{\partial (h^{-i})^2} =& \frac{2\gamma}{(n+1)}\Bigg[-\underbrace{(\bar{x}-\underline{x})}_{>0}\underbrace{\frac{\partial(q(\bar{x},A_1)-q(\underline{x},A_2))}{\partial h^{-i}}}_{>0} + \beta\Bigg(n\bar{x}\underbrace{\frac{\partial(q(\bar{x},A_1)+q(\underline{x},A2))}{\partial h^{-i}}}_{<0} \\ &+ ((2n-3)\underline{x}-2h^{-i}(\bar{x}+\underline{x}))\underbrace{\frac{\partial(q(\bar{x},A_1)-q(\underline{x},A_2))}{\partial h^{-i}}}_{>0} + \underbrace{(\bar{x}-\underline{x})}_{>0}\underbrace{\frac{\partial q(\underline{x},A_2)}{\partial h^{-i}}}_{<0}\Bigg)\Bigg] \\ <& 0,\end{aligned}$$

where we have used that $h^{-i} > \frac{3(n-1)}{3\bar{x}+\underline{x}}$ induces $((2n-3)\underline{x} - 2h^{-i}(\bar{x}+\underline{x})) < 0$. □

Lemmas 5–7 then imply that the investment incentives have the shape that is depicted in Figure 2 such that the investment incentives have a unique local maximum. For the sake of the argument, we denote the number of firms which invest high as h^* where this local maximum of $\pi(\bar{x}, h^{-i}+1) - \pi(\underline{x}, h^{-i})$ is attained. Hence, if costs ξ are low, i.e. $\xi < \bar{\xi} := \pi(\bar{x}, 1) - \pi(\underline{x}, 0)$ then because return of investment dominates its cost, even if no other firm invests, there is a unique equilibrium. The equilibrium is such that $h(\xi)$ firms invest high and $n - h(\xi)$ invest low, where $h(\xi) = n$ if $\xi < \bar{\bar{\xi}} := \pi(\bar{x}, n) - \pi(\underline{x}, n-1)$ or $h(\xi)$ solves $\min\{h \in \{h^*, \ldots, n\} : \pi(\bar{x}, h+1) - \pi(\underline{x}, h) < \xi\}$ else. For $\xi > \bar{\xi}$ there is also an equilibrium where no firm invests, since $\pi(\bar{x}, 1) - \pi(\underline{x}, 0) < \xi$. Finally, if $\xi > \bar{\bar{\bar{\xi}}} := \pi(\bar{x}, h^*+1) - \pi(\underline{x}, h^*)$, then there is no equilibrium where $h(\xi)$ firms invest high, since investment cost exceed the maximal gains of investment for all values of h^{-i}, which concludes the proof.

□

Proof of Proposition 3. (i) Suppose that there are K clusters A_k, $k \in \{1, ..., K\}$ and denote $X := \sum_{i\in N} x(i)$, $X_k = \sum_{i \in A_k} x(i)$, and $a_k := |A_k|$.Thus, $\sum_{k=1}^K a_k = n$. Note that maximizing consumer surplus $CS = Q^2/2$ is equivalent to minimizing the sum of all marginal costs $C(\mathbf{x}, \mathbf{A}) = \sum_{i=1}^n c(i, \mathbf{x}, \mathbf{A})$, since $(n+1)Q(\mathbf{x}, \mathbf{A}) = n\alpha - C(\mathbf{x}, \mathbf{A})$. Then we get for total cost:

$$C(\mathbf{x}, \mathbf{A}) = \sum_{i=1}^n \Big(\bar{c} - \gamma x(i) - \gamma\beta \sum_{j \in A(i), j\neq i} x(j)\Big) = n\bar{c} - \gamma X - \gamma\beta \sum_{k=1}^K \Big((a_k)^2 - 1\Big) X_k.$$

Clearly C is minimized if $x(i) = \bar{x}$ for all $i \in N$ and further $a_1 = n$. Thus, a single cluster where all firms choose high investments is maximizing consumer surplus.

(ii) Fix some profile of investment $x = (x(1), ..., x(n))$ and denote by $\tilde{s}(i, \mathbf{x}, \mathbf{A}) = \gamma(x(i) + \beta \sum_{j \in A(i)\setminus\{i\}} x(j))$ the cost reduction of firm i due to own R&D investment and incoming spillovers for a profile of clusters $\mathbf{A} \in \mathcal{A}$. Thus, $c(i, \mathbf{x}, \mathbf{A}) = \bar{c} - \tilde{s}(i, \mathbf{x}, \mathbf{A}))$. Denote by $\tilde{S}(\mathbf{x}, \mathbf{A}) := \sum_{j\in N} \tilde{s}(j, \mathbf{x}, \mathbf{A})$ and, as above, $C(\mathbf{x}, \mathbf{A}) := \sum_{j \in N} c(j, \mathbf{x}, \mathbf{A})$. This implies that given an investment profile $\mathbf{x}$, we can write welfare, consisting of the sum of firm profits and consumer surplus, for a cluster structure $\mathbf{A} \in \mathcal{A}$ as

$$W = \sum_{i=1}^n (q(i, \mathbf{x}, \mathbf{A}))^2 - h\xi + Q^2(\mathbf{x}, \mathbf{A})/2$$

$$
\begin{aligned}
&= \tfrac{1}{(n+1)^2}\left[\sum_{i=1}^{n}(\alpha-(n+1)c(i,\mathbf{x},\mathbf{A})+C(\mathbf{x},\mathbf{A}))^2+(n\alpha-C(\mathbf{x},\mathbf{A}))^2/2\right]-h\xi\\
&= \tfrac{1}{(n+1)^2}\left[\sum_{i=1}^{n}\left(\alpha-\bar{c}+\gamma\left((n+1)(x(i)+\tilde{s}(i,\mathbf{x},\mathbf{A}))-\tilde{S}(\mathbf{x},\mathbf{A})\right)\right)^2+(n(\alpha-\bar{c})+\tilde{S}(\mathbf{x},\mathbf{A}))^2/2\right]\\
&\quad -h\xi\\
&= \tfrac{1}{(n+1)^2}\Bigg[(n+2)(\alpha-\bar{c})\tilde{S}(\mathbf{x},\mathbf{A})+n(\alpha-\bar{c})^2(1+n/2)-\tilde{S}(\mathbf{x},\mathbf{A})^2(n+3/2)\\
&\qquad +(n+1)^2\sum_{i=1}^{n}\tilde{s}(i,\mathbf{x},\mathbf{A})^2\Bigg]-h\xi
\end{aligned}
$$

Considering the last expression and taking into account that $\tilde{s}(i,x,A)\leq\gamma\bar{x}(1+\beta(n-1))$ it is obvious that for sufficiently large $(\alpha-\bar{c})$ maximizing W is equivalent to maximizing $\tilde{S}(\mathbf{x},\mathbf{A})$. Since every member of a cluster generates spillovers to all cluster members, we have

$$
\tilde{S}(\mathbf{x},\mathbf{A})=\gamma\left(\sum_{j=1}^{n}x(j)+\beta\sum_{k=1}^{K}\left((a_k-1)\sum_{i\in A_k}x(i)\right)\right)=\gamma\left(\sum_{j=1}^{n}x(j)+\beta\sum_{i=1}^{n}(a(i)-1)x(i)\right).
$$

and therefore maximizing $\tilde{S}(\mathbf{x},\mathbf{A})$ is equivalent to maximizing $\sum_{i=1}^{n}(a(i)-1)x(i)$ over all profiles of clusters $\mathbf{A}\in\mathcal{A}$. It is straightforward to see that

$$
\sum_{i=1}^{n}a(i)x(i)\leq n\sum_{i=1}^{n}x(i)
$$

and, since the right hand side corresponds to the case of a single cluster containing all firms, we have shown that welfare is maximized for such a cluster.

(iii) We show that a single cluster with full investment strictly maximizes welfare for $\xi=0$, which by continuity implies the claim of this part of the Proposition. For $\xi=0$ welfare can be written as

$$
W(\mathbf{x},\mathbf{A})=U(Q(\mathbf{x},\mathbf{A}))-\sum_{i=1}^{n}c(i,\mathbf{x},\mathbf{A})q(i,\mathbf{x},\mathbf{A}),
$$

where $U(.)$ is the utility function of the representative consumer. Denote by $\bar{\mathbf{x}}=(\bar{x},\dots,\bar{x})$ the investment profile where all firms invest high and by $\bar{\mathbf{A}}$ the cluster structure in which all firms are in the same cluster. Since under $(\bar{\mathbf{x}},\bar{\mathbf{A}})$ each firm has maximal own R&D investment as well as maximal incoming spillovers, it is easy to see that

$$
\sum_{j\in N}c(j,\bar{\mathbf{x}},\bar{\mathbf{A}})=nc_{\min}<\sum_{j\in N}c(j,\mathbf{x},\mathbf{A})
$$

for all $(\mathbf{x},\mathbf{A})\neq(\bar{\mathbf{x}},\bar{\mathbf{A}})$, where $c_{min}=c(j,\bar{\mathbf{x}},\bar{\mathbf{A}})$ ist the minimal marginal cost value that can be reached by any firm. Note that $c(j,\bar{\mathbf{x}},\bar{\mathbf{A}})$ is identical across all firms j. Due to $Q(\mathbf{x},\mathbf{A})=\frac{n\alpha}{n+1}-\frac{1}{n+1}\sum_{j\in N}c(j,\mathbf{x},\mathbf{A})$ we conclude that

$$
Q(\bar{\mathbf{x}},\bar{\mathbf{A}})>Q(\mathbf{x},\mathbf{A}).
$$

Furthermore, $U'(\tilde{Q}) = p(\tilde{Q}) > c_{min}$, for all $\tilde{Q} \in [Q(\mathbf{x}, \mathbf{A}), Q(\bar{\mathbf{x}}, \bar{\mathbf{A}})]$ and therefore

$$\begin{aligned} W(\bar{\mathbf{x}}, \bar{\mathbf{A}}) &= U(Q(\bar{\mathbf{x}}, \bar{\mathbf{A}})) - c_{min} Q(\bar{\mathbf{x}}, \bar{\mathbf{A}}) \\ &> U(Q(\mathbf{x}, \mathbf{A})) - c_{min} Q(\mathbf{x}, \mathbf{A}) \\ &\geq U(Q(\mathbf{x}, \mathbf{A})) - \sum_{j \in N} c(j, \mathbf{x}, \mathbf{A}) q(j, \mathbf{x}, \mathbf{A}) \end{aligned}$$

for all $(\mathbf{x}, \mathbf{A}) \neq (\bar{\mathbf{x}}, \bar{\mathbf{A}})$, where the last inequality follows from $c(j, \mathbf{x}, \mathbf{A}) \geq c_{min}$ for all $j \in N$.

(iv) For ξ large enough, any benefit of investment is dominated by the costs, hence all firms must invest low in the welfare maximum. Given this investment pattern, it is easy to see that no profile of clusters can generate a lower value of marginal production costs than what is obtained by all firms if a single cluster is formed. Taking this into account, an analogous argument to that used in the proof of part (iii) establishes that the generation of a single cluster containing all firms maximizes welfare.

□

Proof of Proposition 4. Since the expression on the right hand side of (8), determining the best response, is increasing both with respect to $h_{A_j^{-i}}$ and $l_{A_j^{-i}}$, the open membership game is a coordination game such that the pure strategy Nash equilibria are such that all investors of the same type choose the same address. Wlog let high investors choose A_1 and note that by (8) it is always an equilibrium if all low investors also choose A_1. It remains to show, that it is not an equilibrium that all low investors choose a different address, say wlog A_2. Suppose to the contrary that all low investors choose A_2. For the strategy profile supporting this cluster structure to be an equilibrium, no high investor $i \in A_1$ shall have profitable deviation by switching to A_2, which, by (8), is equivalent to,

$$\begin{aligned} & \pi(i, \mathbf{x}, (A_1, A_2)) \geq \pi(i, \mathbf{x}, (A_1 \setminus \{i\}, A_2 \cup \{i\})) \\ \Leftrightarrow & (n-1)(h-1)\bar{x} \geq n(n-h)\underline{x} - (n-h)\bar{x}) \\ \Leftrightarrow & \qquad\qquad h \geq \tfrac{n^2 \underline{x} - \bar{x}}{(n-2)\bar{x} + n\underline{x}}. \end{aligned}$$

Further, no low investor $j \in A_2$ shall have profitable deviation by switching to A_1 which, by (8), is equivalent to,

$$\begin{aligned} & \pi(j, \mathbf{x}, (A_1, A_2)) \geq \pi(j, \mathbf{x}, (A_1 \cup \{j\}, A_2 \setminus \{j\})) \\ \Leftrightarrow & (n-1)(n-h-1)\underline{x} \geq nh\bar{x} - h\underline{x} \\ \Leftrightarrow & \qquad\qquad h \leq \tfrac{(n-1)^2 \underline{x}}{n\bar{x} + (n-2)\underline{x}} \end{aligned}$$

It is easy to see that both conditions cannot hold simultaneously if $\frac{(n-2)\bar{x} + n\underline{x}}{n\bar{x} + (n-2)\underline{x}}(n-1)^2 \underline{x} < n^2 \underline{x} - \bar{x}$ which is implied by $(2n-1)\underline{x} > \bar{x}$ which holds by assumption for $n \geq 2$. Hence all firms choose the same address in equilibrium of the second stage. □

Proof of Corollary 1. Since by Proposition 4 the complete cluster always forms we can calculate the investment incentives by:

$$\Delta \pi^{OM}(h^{-i}) = q^{OM}(\bar{x}, h^{-i})^2 - q^{OM}(\underline{x}, h^{-i})^2 - \xi, \tag{17}$$

where $q^{OM}(x(i), h^{-i})$ is the equilibrium quantity of a firm investing $x(i)$ while all firms belong to the same cluster, i.e. $\mathbf{A} = \{N\}$ and h^{-i} other firms invest high. From Equation 2, we then get that $q^{OM}(\bar{x}, h^{-i}) = q^{OM}(\underline{x}, h^{-i}) + \frac{\gamma}{n+1}(n - \beta(n-1))(\bar{x} - \underline{x})$. From (17) ít then follows that,

$$\Delta\pi^{OM}(h^{-i}) = \tfrac{\gamma(n-\beta(n-1))(\bar{x}-\underline{x})}{n+1}\Big(2q^{OM}(\underline{x}, h^{-i}) + \tfrac{1}{n+1}\gamma(n - \beta(n-1))(\bar{x} - \underline{x})\Big) - \xi. \quad (18)$$

Quantities of a low investor in the grand cluster simplify to,

$$q^{OM}(\underline{x}, h^{-i}) = \tfrac{1}{n+1}\big(\alpha - \bar{c} - \gamma h^{-i}(\bar{x} - \underline{x})(1 - 2\beta) + \gamma\underline{x}(1 + (n-1)\beta)\big).$$

Hence for $\Delta\pi^{OM}(h^{-i})$, we get from (**??**),

$$\begin{aligned}\Delta\pi^{OM}(h^{-i}) = \tfrac{\gamma(n-\beta(n-1))(\bar{x}-\underline{x})}{(n+1)^2}\big(&2(\alpha - \bar{c}) - \gamma(\bar{x} - \underline{x})(2h^{-i}(1 - 2\beta) - n + \beta(n-1)) \\ &+ 2\gamma\underline{x}(1 + (n-1)\beta)\big) - \xi.\end{aligned}$$

Taking the derivative with respect to h^{-i} then gives

$$\frac{\partial\Delta\pi^{OM}(h^{-i})}{\partial h^{-i}} = -2\tfrac{\gamma^2(\bar{x}-\underline{x})^2}{(n+1)^2}\big(1 - 2\beta\big)\big(n - \beta(n-1)\big)$$

which is obviously negative for $\beta < 1/2$. □