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TEACHERS OF MATHEMATICS WORKING AND LEARNING IN COLLABORATIVE GROUPS
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A COMPARATIVE ANALYSIS OF DIFFERENT MODELS OF MATHEMATICS TEACHER COLLABORATION AND LEARNING

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In this paper, we employ an analytical framework to examine the design features, teachers' learning processes, and outcomes expected in three selected models of teachers collaboration and learning, namely the Action-Education model, the Learning Study model, and a Community-Centered model for teacher learning. Based on our analysis, we outline the affordances and limitations of the selected models from the three perspectives: analytical vs. holistic ways of thinking of the relationship between research and practice in teacher's collaboration and learning; cooperation vs. collaboration in interactions between researchers, knowledgeable others and teachers; and the tacit nature of knowledge for teaching mathematics. Through our analysis we suggest a flexible approach to the use of such models, reconsidering them as tools for which it is essential to propose sets of practical principles that can inform teachers of the choice of models as tools for particular purposes, as well as more generally informing the design, evaluation and research of professional learning.

ICMI Study 25 seeks to better understand and address the challenges in the relationship between mathematics teachers' collaboration and learning (MTC&L). Across the world, various forms of teacher collaboration have been developed to support and study teachers learning, including Lesson Study, Action Research, and Design Research (Chen & Zhang, 2019). To date, however, issues that have remained unclear include the boundaries of these for supporting MTC&L at a scientific level (e.g., Wood, 2017; Ding et al., 2019) and the relationship between theory and practice in these various models (e.g., Huang & Shimizu, 2016; Kempe, 2019; Morris & Hiebert, 2011). Collaborative work across professional communities is highly valued to develop a deeper understanding of the interface of theoretical and practical principles of research and practice that is the key to generating knowledge to improve teaching. In this paper, we aim to contribute to theme A (Theoretical perspectives on studying mathematics teacher collaboration) by focusing on the following two questions:

- What is illuminated by the different perspectives and methodologies and what needs further investigation?
- What are promising research designs and data collection and analysis methods to study teacher collaboration?

From a literature review of studies of MTC&L across professional communities, we selected three models that have been conducted in different cultural contexts for a more in-depth analysis. The three models, each designed for supporting (and studying) inservice teacher collaboration and learning, are: (1) The Action-Education (AE) Model (Gu & Gu, 2016), a combination of *Keli study* (exemplary lesson development) practiced by researchers and teachers in schools in China and action research; (2) Learning Study (LS) (Lo & Marton, 2012), a combination of Lesson Study and design study originally conducted in Hong Kong; (3) The Community-Centered (CC) model for teacher learning (Borko et al., 2005), a university-based summer institute program for supporting MTC&L in the U.S..

We recognize the different notions of the framework/model/form in the literature. In this paper we adopt the terminology used by the authors of the models according to the analytical framework by Boylan *et al*. (2018). In what follows, we summarise the selected studies and use the analytical framework to analyse each one. Finally, we discuss the findings of our analysis of the selected studies and propose further issues to be tackled in our future work.

Research design of the different models

Gu and Gu (2016) examine how the AE model works to improve teaching through, in particular, the nature of the work of the *knowledgeable other* (in their study, mathematics teaching research specialists, TRS) in mentoring teachers' practice during post-lesson debriefs in *Keli study* in China. Influential in China as a school-based form of Teacher Professional Development (TPD) that is widely applied (Ding et al., 2019), it aims to update teachers' theoretical ideas of teaching and learning, to support them to design new learning activities, and to improve classroom practice through *Keli study* in the context of the ongoing national curriculum and pedagogy reforms in the country.

Lo and Marton's (2012) LS model focuses on using variation theory as a source of pedagogical principles within teachers' practice. First carried out in Hong Kong in 1999, and subsequently applied in other parts of the world, including Sweden, Brunei, and the UK, the original conception of LS, as explained by Lo and Marton (2012), was to allow the research team to learn the potential value of variation theory. They consider the LS an appropriate model as it allows researchers and teachers to see how the objects of learning are dealt with in the classroom.

Borko et al. (2005) develop the CC model of guiding the design of a university-based TPD programme focusing on cultivating in-service middle-grade teachers' understanding of mathematics teaching, and learning. This model explicitly brought together constructivist and situated perspectives on teachers' learning that Cobb et al. (2017) see as an example of university-based TPD design studies in the U.S. that situate teachers' activity with respect to the TPD learning environment.

Table 1: An analytical framework for analysing models of professional learning

Categories	Questions to be focused in the analysis
Components & relationships	To what extent do the components of the model map onto the components of the focus PD programme or activity? Are there important aspects of the PD programme or activity that are not easily accounted for by the model? What are the change processes that underlie the PD programme or activity? Do these accord with the model?
Scope	Is the programme focused on the micro, meso or macro scale? What outcomes are the foci of the development programme or activity? Is the focus on discrete PD episodes or broader than that? What is the context of the PD? Does it require a systemic perspective?
Theory of learning	What theory of learning is espoused by the programme or activity, or is expected to be relevant? How far is the model congruent with this?
Location of agency	How is agency conceived within the programme - is it focussed on individual teacher agency, or does it include broader conceptions?

Analytical framework for analyzing the models of professional learning

Given the questions that we address in this paper, we employ the analytical framework by Boylan et al. (2018), which is based on a critical analysis of a number of models of theorising the nature and process of teachers' professional learning. Their framework focuses on categories of model components, purposes, scope, and explicit and implicit theories of learning and change processes, etc. (see Table 1). We first analyse the three selected models according to the categories and questions in

Table 1, and then we accordingly reflect on, and discuss, the different perspectives and methodologies of the models. Note that the category of *philosophical paradigms* included in Boylan et al. (2018) is not used in our analysis because the selected models do not address this aspect in detail.

Analyzing the three models of teachers' collaboration and learning

Given our focus, in this section we report our analysis of the features of the selected models, the teachers' learning that occurred in the processes of the collaboration, and their learning outcome.

Model 1. The Action-Education (AE) model

Components and relationships. The AE model includes three phases of teaching action and two reflections between the teaching actions on developing a *Keli study*. The three teaching actions are: (1) Existing action, focusing on a teacher's previous personal teaching experience; (2) New design, focusing on the new design of the *Keli*; (3) New action, focusing on the new classroom practice. A fundamental feature of the AE model is *Keli* - one or more cycles of planning, delivering, debriefing, revising and re-teaching of the exemplary lesson in a school setting. The *Keli* group is a professional learning community consisting of teachers and teaching research specialists (TRS) who usually work in various layers of teaching research system in China and have considerable experience and expertise in teaching mathematics and working with mathematics teachers (Gu & Gu, 2016).

In Gu and Gu (2016), two dimensions of TRS' mentoring are closely studied: one is of *mentoring content* (namely the types of teachers' knowledge such as mathematical knowledge, pedagogical knowledge, and practical knowledge) and the other is the *mentoring interaction* between the TRS and practicing teachers. The research team identified a four-core component model for conceptualizing practical knowledge (comprising goal analysis, task design, formative assessment, and behavior improvement) and provide an explanation of the relationships among these four components.

Four types of mentoring strategies were identified to account for the nature of conversations between TRS and teachers: general comments, comments on anticipated problems, responses to teachers' questions, and dialogues with teachers. Gu and Gu (2016) found that in their study, Chinese TRS mainly focused on discussing *practical knowledge* when mentoring practicing teachers during post-lesson debriefing. Interestingly, their study indicates a close yet complicated relationship of the three types of professional knowledge for teaching mathematics; that is, Chinese TRS did not tend to discuss theories and knowledge at a general and abstract level. They tried to help teachers to understand mathematical knowledge and pedagogical knowledge through analyzing concrete instructional cases that embrace mathematical and pedagogical ideas (which are regarded as parts of practical knowledge). In terms of the four-core component model, their study shows that Chinese TRS pay greater attention to task design and implementation by focusing on teaching behavior improvement and less attention to goal analysis and formative assessment.

Scope. The study is, to an extent, a mixture of micro (teachers' moment-to-moment learning experience), meso (teachers in the context of their school-based professional development activities), and macro (researchers with motivation for a national TPD programme in the wider context of curriculum and pedagogy reforms). The initial study on mentoring activities within the AE model took place in an elementary school in Zhejiang province. Two teachers were selected to develop the lesson subtraction with two-digit numbers through a typical three cycles of a Keli study. Four TRS mentored the entire cycle of the Keli study. Pre- and post-tests were given to students immediately

before and after each lesson, and all the lessons and debriefs were videotaped. After each lesson, interviews with the teachers and selected students were audio recorded. In the later stages of the study, the research team organized over 20 TRS (including those who mentored the practicing teachers) to watch the videotaped mentoring meetings and to try to explain the mentoring activities in terms of their purposes, actions, intentions, and effects. The discussions about the nature of the videotaped debriefing meetings were also videotaped. The nature and the model of practical knowledge characterized in Gu and Gu (2016) is being continuously refined as the AE model has become a form for the school-based teaching research activities and TRS usually works in different layers of teaching research system in the country (Chen & Zhang, 2019; Ding et al., 2019; Huang & Shimizu, 2016).

Theory of learning. The AE model refers to two Chinese classic theoretical ideas of human learning: one is of the *wisdom of action*, which refers to the practical knowledge that integrates subject knowledge with pedagogical knowledge in the context of purposefully improving action; and the other the *unity of knowing and acting*, which is rooted in the Ancient Chinese philosopher's (Wang Shouren, 1472-1529) epistemological theory of learning. Their study largely examined the features of practical knowledge, which is considered by the researchers as a combination of *knowledge-in-practice and knowledge-of- practice*) (for details of the literature references see Gu & Gu 2016). Their study suggests that TRS' practical knowledge is closely related to PCK, but it is built on content and pedagogical knowledge beyond a combination of them.

Location of agency. The study examines the features of practical knowledge and its relationship with PCK and the expertise of mathematics teacher educators (in the Chinese context, TRS) that are considered as a key to support individual teachers' effective learning and improving their teaching mathematics in their classrooms. Their study illustrated that the TRS tended to comment on lessons in general and address anticipated problems based on their previous experience, and pay less attention to address issues raised by the teachers or to engage in dynamic dialogue with them. This finding is different from our early studies to which the AE model was largely referred (e.g., Ding et al., 2014, 2015). This shows a complicated feature of the *Keli* group collaboration in which knowledgeable others collaborate or cooperate with teachers. We return to this issue in the discussion section.

Model 2. The Learning Study (LS) model

Components and relationships. Broadly speaking, the LS model (Lo & Marton, 2012) adopts the Japanese *Lesson Study* model (Stigler and Hiebert, 1999) that involves teachers (with or without researchers) working together through one or more cycles of planning a lesson, and then teaching, observing, evaluating, and modifying the lesson by the team. Noticeably, however, the researchers also tried to reformulate the Japanese lesson study model as a form of 'design experiment'. That is, the research lessons in LS are based on a specific theoretical framework of learning, that of Variation Theory, and the research team wish to learn how well the theory can work.

Lo and Marton (2012) used two lesson episodes extracted from their LS to show how variation theory serves as a guiding principle of pedagogical design. The first case examines the relationship of the following components of the LS model: the last episode of the research lesson (topic is Cantonese Opera) of both cycle 1 and cycle 2, and pre- and post-tests of students' learning outcomes between the two cycles. One of the key components of teachers' collaboration and learning in the team is that in the course of a LS, teachers practiced the pedagogical principle implied by variation theory in their

classrooms, and were supported to develop a deeper understanding of the pedagogical principle and its application. In this case, teachers learned that developing a lesson plan with a variation pattern design was not sufficient. Enactment of the lesson must allow the variation pattern to be experienced by the students. The teaching strategy was thus needed to enable the intended pattern of variation to be experienced by the students. The second case shows the relationship of a research lesson plan and its implementation (topic is of the electrochemical series at secondary school), and students' interview of learning outcomes. In this case, the researchers identified an important aspect of teachers' teaching activity that was not easily accounted by the theory of variation. That is, when the object of learning is complex and more than one critical feature must be discerned simultaneously, it is not always clear how to act out the patterns of variation in the lesson to bring about the desired effect. The 'ingenuity' of people in teaching practice leads the researchers to think alternatively about "The science of the art of teaching?" in the debates about teaching as a science or as an art.

Scope. The LS is also a mixture of micro, meso, and macro (regional scope). According to Lo and Marton (2012), over 300 learning studies have been developed through various projects of the Hong Kong Institute of Education, and many schools have developed learning studies on their own. Learning study has been found to improve student learning, reduce the gap between the high and low achievers, and contribute to teachers' professional development and the learning of researchers.

Theory of learning. Lo and Marton (2012) suggest that variation theory serves as a guiding principle of pedagogical design, and could be applied as an importance research approach of developing a strong theoretical mode of professional interactions that build teachers' learning and commitment to future inquiry and maintain their focus on student learning. Variation theory brings the focus of the LS sharply on the object of learning and provides a theoretical grounding to understand some of the necessary conditions of learning.

Location of agency. Lo and Marton (2012) shows researchers and teachers' efforts to address the links between practical knowledge of teaching strategies shared in teachers collaboration and learning in Japanese lesson study and the implicit and unclear nature of the pedagogical theories that might underpin such knowledge building in LS in a different context.

Model 3. The Community-Centered (CC) model for teacher learning

Components and relationships. Borko et al. (2005) designed a two-week long university-based summer institute for MTC&L that comprised 60 contact hours of meeting time structured around four major types of activities: solving mathematical problems; examining children's thinking; reading and discussing current literature; and reflecting on one's own learning. Borko et al. (2005) shared their efforts to develop (and research) the community-centered (CC) model for enhancing teachers' knowledge of algebra. The CC model connects two constructs that are central components of their TPD program. One construct—teacher learning communities—recognizes the impact of sociocultural factors upon teacher learning. The other construct—knowledge for teaching (teachers' mathematical and pedagogical knowledge)—focuses on teacher change. These researchers consider that a unique strength of their model is the emphasis on the symbiotic relationship between the two primary goals—community and mathematics understanding.

Borko et al. (2005) considered four features of classroom life that are fundamental to establishing and maintaining a successful learning community: safe environments, rich tasks, students' explanations

and justifications, and shared processing of ideas. The first vignette given in their article, derived from an activity that occurred on the first day of the institute, illustrated the goals of the instructor of establishing a safe environment and creating a culture to support the sharing of ideas. The second vignette, occurred on the fourth day of the summer institute, depicted an activity in which the instructors deliberately guided teachers to develop algebraic knowledge and reasoning. Nevertheless, these researchers raised up several key questions about the challenges of the improvement and expansion of the CC model. For instance, how might the model be brought to some larger scale? How dependent is this model on the skills and temperaments of the instructors? What kinds of supports must be in place for teachers over time, enabling them to build upon the growth they experienced in this professional development program?

Scope. This study is meso in scope. Sixteen teachers from three different school districts participated in the summer institute. Thirteen of them were teaching at the middle school level, and three were elementary school teachers. The institute was taught collaboratively by two mathematics educators, who were university lecturers and members of the research project.

Theory of learning. The TPD programme and its research are firmly rooted in a situative perspective on teacher learning (i.e., it recognizes the contextual influences on knowledge construction for TPD).

Location of agency. The teachers worked collaboratively with their colleagues throughout the institute. While they addressed a wide range of algebra problems (often from contemporary curricular programs), the professional developers selected these problems with the focus on the goals of the program. Borko et al. (2005) developed the vignettes and the analysis that focused primarily on the ways in which the instructors created a professional learning community with the teachers, and how this community contributed to the development of teachers' knowledge of algebra. Borko et al. (2005) further showed that analyses of pre- and post-institute algebra content tests and interviews, teachers' daily reflections, and their final papers provide initial evidence that the summer institute had an impact on participating teachers.

Discussion and Conclusion

To discuss the different perspectives illuminated by the comparison of the three selected models, we summarize, in Table 2, our analysis of the features of the selected models.

We discuss three perspectives of the selected models and the value of the comparative perspectives:

- Analytical vs. holistic ways of thinking of the relationship between research and practice in MTC&L;
- Cooperation vs. collaboration in interactions between researchers, knowledgeable others and teachers;
- The tacit nature of knowledge for teaching mathematics.

First, both AE and LS models emphasize the cycles of the Lesson Study approach that aims to generate knowledge to improve teaching. As recognized by Morris and Hiebert (2011, p. 8), the outcome of Lesson Study, "an instructional product", has the potential to "guide actions towards helping students to achieve the learning goals". Moreover, as shown in Table 2, both AE and LS models address simultaneously lesson plan design and implementation as a whole professional learning and knowledge-generating process. Huang and Shimizu (2016) further argue that both *Lesson Study* and *Learning Study* provide evidence about how theory can be used to guide teaching and how teaching experiments can further refine theory. Note that the western TPD design studies share this analytical way of thinking of the relationship between theory and practice. As explained in

Cobb et al. (2017), pragmatically, TPD design studies involve supporting teachers in improving specific aspects of their instructional practice. Theoretically, TPD design studies involve developing, testing, and revising conjectures about both the process by which teachers develop increasingly sophisticated instructional practices and the means of supporting that development.

Table 2: Goals, learning processes and outcomes of the models

	AE	LS	CC
Goals	Update teachers' theoretical ideas and action on curriculum reform-based mathematics teaching and learning.	How well the VT theory has worked in teachers' practice of learning objects.	Cultivate teachers' understanding of algebraic thinking, teaching, and learning.
Learning processes	Three phases of teaching actions and two reflections on one or more cycles of *Keli* study.	Cycles of lesson plan, teaching and observation, evaluation, and modification guided by VT.	Four types of teaching activities: problem solving; understanding children's thinking; reading literature; and reflection.
Learning outcomes	Core elements of practical knowledge & its relationship such as task design and lesson implementation.	Lesson design and implementation of necessary conditions for learning according to VT.	Content knowledge, mathematics-specific pedagogical knowledge, and recognition of the importance of learning community.
Contexts	School-based teacher research activities system in China.	Projects of the HK Institute of Education in collaboration with schools in Hong Kong.	Pull-out sessions in a university in the U.S.

Here, we should draw researchers and practitioners' attention to the holistic way of thinking of the relationship between theory and practice that has yet remained to refine and understand in the AE model. That is, as noted by Gu and Gu (2016), Chinese TRS tend to help teachers to understand mathematical knowledge and pedagogical knowledge through analyzing concrete instructional cases that would otherwise be difficult for teachers to understand if given the theories and knowledge at a general and an abstract level. We believe that this professional interactions in which theory is interpreted and understood in teachers' teaching practice and practice is deliberately guided and controlled by theory should be a main focus in our future study of MTC&L. In so doing, it is likely to help teachers to overcome the gap between theory and practice and develop teachers' knowledge and reflection skills for making their own teaching theories (Ding et al., 2019; Kempe, 2019).

Secondly, findings from the analysis of the three models lead to further questions that need to be investigated in future work: Whose knowledge is the focus in the TPD models? What is expected to be learnt or improved through the models, and how does this occur? Kempe (2019) highlights the issue of teacher-researcher collaboration through different forms of practice and research. For instance, design research is mainly university-driven (e.g., the CC model analysed above), while lesson study can be teacher-driven. Kempe (2019) explains that the LS model emphasizes a teacher-researcher collaboration because both have a common object of research. That is, it is research *with* teachers, rather than *on* teachers and focuses on constructing knowledge concerning objects of learning as well as teaching-learning relationships. The AE model shares the same strengths of LS model. It is recognised that closing the research-practice 'gap' can actively involve teachers in a genuine process of collaboration where there are shared and common object of research (e.g., Ding et al., 2019; Kempe, 2019). Nevertheless, in line with Borko et al. (2005) and Huang and Shimizu

(2016), strong leadership by *knowledgeable others* is evident in all the three models, and the obstacle that needs to be overcome is to do with the question of what variation may be necessary to enable the further development of these models so as to be accessed by less experienced, or new, teachers (Ding et al., 2019). This leads to the final point we wish to make.

It is important to develop new research designs to collect data to enable researchers to overcome the challenges to understand and characterize the tacit nature of professional practical knowledge for teaching mathematics and the difficulty to disseminate it by the existing models (Kempe, 2019; Morris & Hiebert, 2011; Stigler & Hiebert, 2016). For example, Marton's variation theory is a general theory for learning, and in the work of Gu and Gu (2016) there remain a number of questions about the relationship between mentoring content and mentoring model. Such things mean that there is a problem to understand the types of knowledge, mathematical, pedagogical or practical knowledge. In our future work, we consider that it is necessary to understand not only the tacit nature of practical knowledge for teaching, but also people's attitudes and thinking of such knowledge for teaching mathematics rooted in their philosophical underpinnings (Boylan et al., 2018). We support Boylan et al.'s (2018) suggestion for a flexible approach to the use of models, reconsidering them as tools, and it is essential to propose a set of practical principles that can inform teachers of the choice of models as tools for particular purposes, as well as more generally to inform the design, evaluation and research of professional learning.

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