UNIVERSITY OF SOUTHAMPTON

SOUTHAMPTON BUSINESS SCHOOL

Centre for Decision, Analytics and Risk Research

Testing Price Efficiency of Fundamental Information in Financial Markets using a Multi-Stage Modelling Methodology Employed in Racetrack Betting Market Studies

by

Gyanendra Vinesh Rao

Primary Supervisor: Professor Johnnie Johnson

Supervisor: Professor Ming C Sung

Thesis Submitted for the degree of

Doctor of Philosophy

2017

TESTING PRICE EFFICIENCY OF FUNDAMENTAL INFORMATION IN FINANCIAL MARKETS USING A MULTI-STAGE MODELLING METHODOLOGY EMPLOYED IN RACETRACK BETTING MARKET STUDIES

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

SOUTHAMPTON BUSINESS SCHOOL

Centre for Decision, Analytics and Risk Research

Doctor of Philosophy

TESTING PRICE EFFICIENCY OF FUNDAMENTAL INFORMATION IN FINANCIAL MARKETS USING A MULTI-STAGE MODELLING METHODOLOGY EMPLOYED IN RACETRACK BETTING MARKET STUDIES

by

Gyanendra Vinesh Rao

This paper applies multi-stage modelling methods from sports betting market research to test for semi-strong form efficiency in financial markets. Specifically, modelling methods from racetrack betting market studies are applied to the UK equities market to determine the extent to which publicly available fundamental information is priced. Fundamental variables are modelled using the logit regression technique to study monthly returns. The out-of-sample results reject the null hypothesis and confirm that financial markets are not semi-strong form efficient for all the securities when multi-stage modelling techniques are utilised. A Kelly betting strategy yields positive returns in the out-of-sample period, outperforming benchmarks, including FTSE-100, suggesting that the multi-stage racetrack betting modelling methodology extracts information that has not been priced by the financial markets.

List of Contents

List of Tables

List of Figures

Declaration of Authorship

I, Gyanendra Vinesh Rao, declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research.

TESTING PRICE EFFICIENCY OF FUNDAMENTAL INFORMATION IN FINANCIAL MARKETS USING A MULTI-STAGE MODELLING METHODOLOGY EMPLOYED IN RACETRACK BETTING MARKET STUDIES

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
2. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
3. Where I have consulted the published work of others, this is always clearly attributed;
4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
5. I have acknowledged all main sources of help;
6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
7. None of this work has been published before submission.

Signed: ……………………………………………………………………………

Date: ……………………………………………………………………………

Acknowledgement

My sincerest gratitude to my supervisory team, for their continuous support.

Special acknowledgement to Professor Johnnie Johnson whose support throughout with his patience, guidance and helpful comments were invaluable in writing this thesis.

Chapter 1: Introduction

This study extends the principles and techniques developed in sports betting market research to test for semi-strong form efficiency of fundamental information in the wider financial markets. Specifically, multi-stage modelling methods from racetrack betting market studies are applied to the UK equities market to determine the extent to which publicly available fundamental information is priced. The rationale for applying multi-stage racetrack betting methodology to financial markets is the existence of equivalents in the two markets; the overarching framework of information efficiency, market structures with a presence of exchange-based trading and the comparative behaviour of market participants. In addition, there are similarities in fundamental information which could be applied in the modelling process.

Racetrack betting markets offer ideal opportunities for economic analysis and "*provide a clear view of pricing issues that are more complex elsewhere"* (Sauer, 1998, p. 2021). These markets have long been used as a proxy to understand pricing and speculative behaviour, for example, Figlewski (1979) on information aggregation and bettors' ability take into account factual and subjective information; Johnson, Jones, and Tang (2006) on the ability of bettors to incorporate all published information, obscure and transparent, simultaneously; Golec and Tamarkin (1998) on bettor preferences for skewness and risk aversion; Hausch, Ziemba and Rubinstein (1981) on price inefficiency and availability of profitable betting opportunities. Markets in racetrack betting benefit from having a well-defined termination point where all information is revealed and outcome known.

In comparison, price discovery in financial markets is an ongoing process where information is revealed over extended periods and compounded in prices on a continuous basis. Untangling the various market signals and discerning price behaviour from the information revealed is a more complex process, compared to racetrack betting. A collection of pricing models have emerged over the past five decades to test for information efficiency and return predictability, for example Capital Asset Pricing Model (CAPM), (Sharpe W. F., 1964), (Lintner, 1965), Dividend Model (Gordon, 1962), Arbitrage Pricing Model, (Roll, 1977), Three-Factor Model (Fama & French, 1993)) to the Five-Factor Model (Fama & French, 2015).

There is a general acknowledgment in the finance literature that prices are not fully explained by existing models. Two schools of thought are predominant in this regard; those that argue markets are efficient (for example, (Fama E. F., 1991), (Malkiel B. G., 2003)) and an investor is simply compensated for holding a risky asset, equal to a risk premium plus a return on a risk-free asset. The other school of thought demonstrates, with empirical evidence, that markets do not follow the risk-return principles of market efficiency (for example (De Bondt & Thaler, 1985), (Cochrane J. , 2007)), and that returns are predictable. There is also a third emerging view on market efficiency which states that markets adapt and evolve (Lo A. , 2004) thereby allowing for the two beliefs of market efficiency to be reconciled and coexist.

Predictability of returns suggests that markets are inefficient and challenges contemporary thinking of the past few decades. However, there is no real evidence in the finance literature that statistical or technical models could be developed to predict equity returns. Cochrane (2005, p. 390) suggests that any such technical systems are *"close to useless".*

This paper applies multi-stage modelling techniques developed in racetrack betting markets to test for semi-strong form efficiency of fundamental information and whether a technical system is feasible in financial markets. There are no known studies, to my knowledge, that apply techniques developed in racetrack betting market research to UK equities. The next chapter details the thematic organisation of the thesis and how literature review in the two disciplines of study has been organised.

Chapter 2: Organisation of Thesis and Literature Review

The first question that arises is how should the thesis be organised thematically. It could be agreed at the outset that the study of sports betting markets and financial markets are independent disciplines where the pursuit of either field does not necessarily overlap or merge. There is evidence from literature as well which is devoid of papers presenting an integrated or interdependent view of the two markets, although there are suggestions in racetrack betting studies (for example, Hausch, Ziemba, & Rubinstein, 1981) of similarities between the two markets.

Financial markets studies are undertaken within a framework of finance and economics where the overriding principle could be said to facilitate efficient allocation of resources and capital. The depth, diversity and complexity of literature within this discipline are enormous. Several areas within this field are subject to specialised treatment in literature. For example, asset pricing is one theme that is explored as this supports the resource allocation process. Similarly, and in comparison, racetrack betting studies have been undertaken within a framework of decision sciences and gambling studies where one focus is to understand behaviour and decision-making processes of individuals. Racetrack betting markets studies also have a deep vein of literature. For example, the favourite long-shot bias (FLB) is one subject that has been explored to explain individual decision-making behaviour.

A clearly defined framework is therefore required to demonstrate the basis for applying multi-stage modelling methods in racetrack betting to financial markets so that the aim of this thesis is achieved. A multi-stage modelling approach in financial markets would also help validate the true extent of efficiency in a market where fundamental information sources are diverse and released at differing frequencies. Sung & Johnson (2007) in their study of racetrack betting markets noted that a one-step model overestimated degree of market efficiency, as the two-step model yields substantially higher profits. In financial markets, companies announce financial results annually and provide quarterly or half-yearly updates. Similarly, macroeconomic data is released by the Official of National Statistics on a monthly, quarterly or annual basis, depending on the data type. For example, retail data is released monthly and Gross Domestic Product (GDP) on a quarterly basis. Prices, on the other hand, are available on a continuous basis when the market is open for trading. Given the varying frequencies of fundamental data, there is every likelihood that prices would reveal more information on a security than fundamental variables available at a lesser frequency, were this information combined in a single model. In addition, the efficient markets model suggests (for example, (Malkiel B. G., 2003)) that financial and macroeconomic data should already be incorporated in prices.

By "harnessing" the principles from racetrack betting and employing these in financial markets, one also indirectly questions the paradigm of independent study of the two financial markets. A related question therefore would be whether a shift in paradigm is needed to one of an interdependent or a more integrated perspective where the two markets are viewed from the same prism, and would this alternative paradigm better describe how assets are priced and markets behave?

The question of how the review of literature in the two disciplines and the rest of the paper should be approached then must be answered so that the pros and cons become evident. A comparative and integrated view of literature and modelling methodologies is one option for presenting the analysis on racetrack betting and financial markets. However, the lack of previous papers and precedence on where these two fields of study have been combined suggests that an integrated framework for presenting the analysis of the two markets poses more of a danger of misinterpretation than providing clarity. The enormity of financial markets literature also presents the risk of racetrack betting literature being overwhelmed given the relatively higher volume of economic and finance papers that have been published. In addition, there is the potential problem of reaching predisposed conclusions in an integrated analysis when approached from the perspective of one field of study.

The alternative approach is to present the literature on racetrack betting and that of financial markets as two independent strands. This structure has the benefit of not requiring a predisposition towards either racetrack betting or financial markets. A rationale, however, is first needed to identify the fundamentals that must exist to enable the application of racetrack betting methodologies that are feasible to financial markets. The organisation principle of this paper is based on this approach and on the thesis that the paradigms of racetrack betting and financial markets could be viewed from the same prism.

In this paper, I propose the null hypothesis that securities markets are semi-strong form efficient for publicly available fundamental information and that a multi-stage modelling methodology will identify the true extent of market efficiency compared to a single-stage models as noted in in literature (for example single-stage CAPM models). The aim of the thesis is not to simply use the multi-stage approach to see if the market is efficient. Rather use the multi-stage modelling approach to see if the market is efficient, and if not then the extent of market inefficiency. It is anticipated that a multi-stage approach will extract maximum information of value from fundamental variables. New information release in financial markets vary in frequency and depends on variable type; financial statements are released annually, macroeconomic data monthly and prices are updated continuously. Combining this information of varying frequencies into a single model would most likely obscure the importance of variables of a lesser frequency and nature of interrelationships, where these variables have a circular economic relationship. A multi-stage modelling methodology is therefore considered a more appropriate compared to a single-stage model. Studies in racetrack betting suggest that markets show a higher level of market inefficiency compare to a single-stage model. For example, Sung & Johnson, (2007) compared a single stage and two-stage logit model and found that the latter exhibited a higher level of profitability compared to the single stage model. Similar results were noted in subsequent racetrack betting studies (for example, (Lessmann S. , Sung, Johnson, & Ma, 2012), (Sung & Lessman, 2012) that utilised multi-stage modelling methodology and confirmed the existence of market inefficiency.

I utilise a multi-stage racetrack betting methodology and logistic regression technique to develop a price prediction model, and then forecast one-month direction of future prices for a sample of FTSE-100 securities. I first develop three base fundamental models for each security; a financial statement, macroeconomic and price model. A limited set of fundamental variables that had been previously identified in literature are used as inputs to develop the base models. Data for the period 2005 to 2011 (7 years) and 2,850 trades per security, on average, are utilised to determine model parameters.

The fundamental variables, as a first step, are transformed and pre-processed using linear regression. The resulting output variables are then included as inputs to determine the base logit models and then combined into a final stage prediction model for each security. An independent data sample, from 2012 to 2013 and on average 522 trades per security, is used to determine the final prediction logit models.

A daily trading strategy is implemented based on the final model probabilities and returns analysed for the out-of-sample period, for the years 2014-2015 (522 trades per security on average). Securities are bought and sold after a monthly holding period where final model probabilities exceed 0.5. Similarly, a short strategy is executed where final model probabilities are less than 0.5. The out-of-sample portfolio returns after transaction costs are significant in comparison to a benchmark model suggesting that not all securities in the UK equity markets are semi-strong form efficient.

To achieve the aims of this thesis this paper has been organised as follows: Chapter 3 provides a rationale for applying racetrack betting methodologies to financial markets. The chapter also discusses the notion of information efficiency. Market efficiency[1] is central to this study and the definitions of efficiency are fundamental to both, financial markets and racetrack betting markets. The predominant view in finance literature for many decades has been; if markets are efficient then prices will not be predictable, as prices reflect all available information to value a security and follow a random walk (Fama , 1965). Section 3.1 discusses this view. Section 3.2 the discusses prices in discrete and continuous time and Section 3.3 then compares the pricing and payoff structure in the two markets. Section 3.4 discusses the structural and organisational features of betting and wider financial markets. The main contribution of this chapter is to demonstrate that racetrack betting and financial markets could be viewed from the same prism; which reflects the overall rationale for applying the multi-stage racetrack betting methodology to financial markets.

Chapters 4 and 5 provide a literature review. Chapter 6 details the research methodology and Chapter 7 presents the empirical results. Chapter 4 discusses literature on modelling methods in finance. This chapter has been grouped into three broad categories; Consumption-based models; Dividend-based models and Linear/Non-linear models. Section 4.1 provides an overview of fundamental information and equity prices in financial markets. Section 4.2 discusses rational economic decision-making behaviour of individuals and their risk preferences, given that economic theory on how assets are priced is based on individuals' collective assessments of risks, consumption preferences, and their decision-making process.

The CAPM-based models are then discussed in sections 4.3 to 4.5, including an analysis of the underlying assumptions of these models. Finance literature also provides evidence that

[1] The terms, information efficiency and market efficiency are read as having the same meaning in this paper.

there are naturally occurring cognitive biases in individuals when processing information and these influence their rational economic decision-making behaviour (for example, Kahneman & Tversky, 1979), suggesting that models and modelling methods must consider these cognitive biases. These behavioural biases underpin studies in behavioural finance and provide evidence which contradicts CAPM. Behavioural finance is now a significant field with widely published literature in this discipline. The literature review for behavioural finance therefore has been limited to the contradictory evidence it provides to asset-pricing models, rather than offering a detailed behavioural finance critique.

Section 4.6 discusses stochastic and volatility models. Section 4.7 then analyses other linear and non-linear modelling techniques and models developed in financial markets. Empirical evidence on effectiveness of these models in calibrating fundamental information is presented in the context of price efficiency and security return predictability. Finally, section 4.8 summarises the modelling methods in financial markets and concludes this chapter.

The main contribution of chapter 4 is to present a collection of models and modelling methods that have been developed to test efficiency of fundamental information and prices in financial markets. The aim of this chapter, however, is not to offer an exhaustive list of models (and variations) that have been developed in financial markets as the literature is enormous, but to provide the key strands of modelling methodologies that have been developed to test for market efficiency for fundamental information. The linkages of these models to economic theory and individual decision-making behaviour are also explored. Chapter 4 highlights the transformation of the early predominant view that markets followed a random walk to one where returns are predictable. Cochrane (2011, p. 1047) suggests that *"Now we have a zoo of new factors"* that explain return predictability.

Chapter 5 is a literature review of modelling techniques employed in racetrack betting market studies. This section is organised as follows: Section 5.1 discusses the FLB, a well-documented deviation in market efficiency. Sections 5.2 and 5.3 describe the modelling and wagering techniques that are prevalent in racetrack betting market studies, and empirical evidence is presented for these models. Section 5.4 summarises the key methodological principles of racetrack betting market studies that could be applied to model prices in financial markets. The key contribution of chapter 5 is to demonstrate the evolution of the model development process from betting strategies to complex multi-stage modelling methodology for predicting winners in racetrack betting markets.

The overall contribution of Chapters 4 and 5 is also to facilitate a comparative analysis of financial and racetrack betting market literature and the framework in which the modelling methodologies have evolved in the two markets. For example, in finance literature, models have been developed with the underlying assumption that markets and individuals behave rationally. In comparison, racetrack betting markets ascribe to the view that there are biases in the behaviour of market participants.

Chapter 6 outlines the research methodology adopted to apply the multi-stage racetrack betting principles to financial markets. Section 6.1 discusses the appropriate research design paradigm and framework applicable for this field of study. Section 6.2 develops the hypothesis to be tested. The data-gathering process, including sources from which fundamental information was obtained and the criteria for security sample selection for empirical analysis, is then discussed next in Section 6.3. Sections 6.4 to 6.14 discuss in detail the core principles adopted, with worked examples, to apply modelling methodologies from racetrack betting markets to financial markets. Section 6.15 details the calculation methodology for the dependent variables. Section 6.16 discusses the Kelly strategy deployed for trading. Sections 6.17 and 6.18 detail the performance measures and returns statistics used for analysis of the empirical results. Finally, section 6.19 provides a summary of this section. The major contribution of this chapter is to develop and propose a methodology that could be followed to apply the multi-stage racetrack betting principles to financial markets.

Chapter 7 presents the empirical findings and is divided into three parts. Part I details the returns of the benchmark models and the portfolio strategies employed to measure performance of the multi-stage modelling methodology. Part II, for an example security, details the calculations and final results. Finally, Part III presents a detailed analysis of portfolio returns and model-fit statistics. Additional statistical results to supplement the analysis in Part III are included in the Appendix.

Chapter 8 begins with a discussion and analyses of results with respect to the thesis hypothesis and consistency of findings with previous literature. Possible areas for further research are identified. The contributions of chapter 7 & 8 are therefore empirical. Finally, Chapter 8 concludes this thesis and comments on the merits of applying a multi-stage racetrack betting methodology to the wider financial markets.

Chapter 3: Rationale for Applying Racetrack Betting Methodology to Financial Markets

The main aim of this chapter is to provide a comparative analysis of the two markets from an organisational perspective and demonstrate the feasibility of applying the principles from racetrack betting to financial markets.

The chapter is organised as follows: Section 3.1 defines market efficiency and is applicable to both financial and racetrack betting markets. Section 3.2 discusses prices in discrete and continuous time. Section 3.3 then compares the prices and payoffs structures in the two markets. The key structural and organisational features in the two markets and the behaviour of market participants in these two markets are then reviewed in section 3.4. Section 3.5 discusses the modelling processes in the two markets. Section 3.6 is a summary of this chapter.

The next section discusses market efficiency.

3.1 Market Efficiency

Markets are considered information efficient if prices fully incorporate an information set. This definition is universal to both, racetrack betting and financial markets.

Market efficiency has been defined in a number of studies; Malkiel (1992):

> *"A capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Formally the market is said to be efficient with respect to some information set……if security prices would be unaffected by revealing that information to all participants. Moreover, efficiency with respect to an information set …implies that it is impossible to make economic profits by trading on the basis of that information set."* (Campbell, Lo, & Mackinlay, 1997, p. 20)

This definition is based on the original market efficiency notion that asset prices fully reflect all available information. Fama (1970) classified market efficiency into three forms; weak-form, where historical information is reflected in current prices; semi-strong form, where prices reflect all publicly available information, and strong-form in which prices reflect all available information including privately held information. Jensen (1978, p. 97) summarised market efficiency as follows:

"A market is efficient with respect to information set θ_t if it is impossible to make economic profits by trading on the basis of information set $\theta_{t.}$"

For weak-form efficiency, information set, $\theta_{t,}$ is taken to be the information contained in the past price history of the asset as of time t and a weak-form efficient market is one where it is not possible to predict returns based on historical information. In racetrack betting, this implies that analysis of historical price information will not yield profitable results since odds reflect all historical information.

In the semi-strong form, θ_t represents all publicly available information at time t and in the strong-form, θ_t, represents all publicly and privately held information at time t. Semi-strong form efficiency implies that profits cannot be made using any publicly available information. In racetrack betting, this implies that analysis fundamental information relating to horse and jockey performance would not result in profitable trading information.

Finally, the strong-form of market efficiency implies that prices reflect all information relevant to the firm, including privately held information by insiders such as company executives. In racetrack betting markets, this implies that prices reflect information held by jockeys, trainers or owners who may possess inside knowledge on the true winning probability of the horse, as well as publicly available information.

Prices are therefore unbiased estimates of the true value of an asset or the winning probability of a horse in an efficient market. In other words, an efficient market means that prices cannot deviate from their true value, and any errors are random and unbiased.

The definitions of information efficiency provide a common framework for the analysis of prices, market structure and participants' behaviours. This has been acknowledged in racetrack betting literature in a number of studies (for example, (Hausch, Ziemba, & Rubinstein, 1981), (Sung & Johnson, 2005), (Edelmen, 2007)). In general, financial markets have been accepted as being efficient in processing price information ((Fama E. F., 1991) and (Sauer, 1998)). If financial markets were efficient then asset prices would follow a random walk model or a martingale. The random walk model, first proposed by Bachelier (1900) in his thesis '*Theory de La Speculation',* was analysed by Fama (1965) in his seminal paper, *The Behaviour of Stock-Market Prices*.

The price of a security in a random walk model could be described by a martingale (Campbell, Lo, & Mackinlay, 1997, p. 31), as follows:

$$P_t = \mu + P_{t-1} + \epsilon_t \qquad \epsilon_t \sim IID\ (0, \sigma^2) \qquad \text{EQ. 3-1}$$

Where

P_t Represents the price observed at the beginning of time, t.

μ Represents the expected change in price or drift

ϵ Represents the error term that is independently and identically distributed (IID) with a mean of zero and variance of σ^2

Prices are therefore considered as having no memory to derive any patterns or serial correlations and would not yield any useful information. Arbitrage opportunities therefore do not exist to allow investors to achieve above-average returns without accepting above-average risk, as prices adjust to new information without a delay. Malkiel (2003, p. 59) suggests an investor would therefore be better off holding a broad-based market index fund than an active portfolio. Investment managers will not outperform the market in the long run as information is immediately reflected in prices. Since prices follow a random walk, future prices cannot be predicted using historical information. Predictive models would therefore not be successful.

The overall empirical evidence on information efficiency, however, when taken in its entirety over the past few decades presents mixed results, and at best, is inconclusive. There is a large body of evidence which shows that stock returns are predictable and that fundamental variables, such as dividend/price ratio and term premium, predict a significant amount of variation in stock returns (Cochrane J. H., 2005). Stock returns have predictable time variation and cross-sectional components, supporting the view that security prices do not follow a random walk. The notion that stock returns are predictable over the long term has been referred to as a "*New Fact in Finance*" (Cochrane J. H., 1999), (Cochrane J. , 2007). The most prominent fundamental variables studied in the literature include: dividend-price ratio and dividend yields (for example (Ball R. , Anomalies in Relationships Between Securities' Yields and Yield-Surrogates, 1978), (Campbell & Shiller, 1998), (Campbell & Yogo, 2006), earnings to price ratio (Lamont, 1998), book-to-market ratio (for example (Kothari, Shanken, & Sloan, 1995), (Pontiff & Schall, 1998)), macroeconomic variables such as interest ((Campbell J. Y., 1987) and inflation rates ((Campbell & Vuolteenaho, 2004), or a combination of macroeconomic and firm variables (Abarbanell & Bushee, 1998) . Asset managers have also been shown to earn more than market returns, after controlling for market risks.

In comparison, prices in racetrack betting markets are discrete events and literature also presents similar evidence. There is also a distinction with respect to payoffs and return expectation in the two markets. In financial markets $E(R_{it)}\ > 0$ in financial markets whereas in racetrack betting markets $E(R_{it)} < 0$.

Where

$E(R_{it)}$ Represents return/payoff on asset / horse i in time t / race t

With the upper bound of zero for expected returns, profitable opportunities on average in racetrack betting do not exist (Sauer, 1998, p. 2021). *"Why do people trade in these markets?"* (Sauer, 1998, p. 2021) given that expected returns are negative, compared to financial markets where equity securities have positive expected returns. A number of explanations have been put forward for this behaviour; risk-seeking (Weitzman, 1965), differing utility functions (Asch, Malkiel, & Quandt, 1984), market inefficiency (Edelman, 2003).

In summary, in an efficient financial market, prices follow a random walk and do not exhibit a behaviour of return predictability. These prices are in continuous time. Investors' returns expectations are, however, positive on all securities. These returns are compensations for underlying security risks and any excess amounts would suggest the existence of market inefficiency. In comparison, in an efficient racetrack betting market, prices are characterised by FLB but also do not exhibit a behaviour of return predictability. These prices are discrete. Bettors' returns expectations are, however, negative where only the winner of the race and placegetters (second and third) are in the money. These differences in efficiency characteristics in the two markets have resulted in different sets of criteria determining efficiency in the two markets. Modelling methodologies have therefore evolved differently in the two markets to examine market efficiency. In racetrack betting markets, models demonstrate the feasibility of earning positive returns to confirm the existence of inefficiency, given these markets have negative return expectations. In contrast, returns greater than an acceptable benchmark is required to be demonstrated to confirm the existence of financial market inefficiency, given the positive return expectations. Racetrack betting have long been used as a proxy to understand pricing and speculative behaviour and how individuals aggregate publicly available information.

There is evidence in racetrack betting literature that multi-stage modelling methodologies present opportunities for profitable betting (for example, (Benter, 1994), (Sung & Johnson, 2007), (Edelmen, 2007) by extracting information that has not been priced by the market. Market efficiency studies in racetrack betting suggest the existence of the well-known anomaly in the betting literature, FLB, where returns to bets on short-odds exceed returns on long-odds. Thaler & Ziemba (1988, p. 163) viewed the FLB as an "*empirical regularity*".

Although prices in financial markets are continuous, prices have been modelled in financial literature in both discrete (for example (Cox, Ross, & Rubinstein, 1979) and continuous time (for example (Merton R. , 1992). The next section discusses the nature of prices in discrete and continuous time.

3.2 Price in Discrete and Continuous Time

Given a sample space, S, and an event, E which is a subset of the sample space the function for a random variable, X, could then be denoted by:[2]

$$E[X] = \int_{-\infty}^{\infty} x dF(x) \qquad \text{EQ. 3-2}$$

Where

F Represents the distribution function of the random variable, X

x Represents a real number between the specified interval range, $-\infty$ and ∞.

$\underline{X} = X(t)$ is then a stochastic process and a collection of random variables and either discrete or continuous.

Where

t Represents time.

The random variable, X, is said to be discrete if the possible values assigned to each outcome are countable. The mean of a discrete variable is then defined by:

$$E[X] = \sum_{X} xP\{X = x\} \qquad \text{EQ. 3-3}$$

Where

P Represents the probability distribution function of the event, E

The random variable, X, is said to be continuous if the possible values to assigned to each outcome do not have any 'jumps' in the process, compared to a discrete variable. In other words, x, could take any value in the specified range. The mean of a continuous variable is then defined by:

[2] Standard probability treatment in advanced texts. Adapted from Ross S. M (Stochastic Processes, 1996) – Chapter 1.

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx \qquad \text{EQ. 3-4}$$

A discrete function is not differentiable as it is not continuous and has different probability distributions that describe these compared to continuous functions. In general, a Binomial or Poisson distribution are used to describe discrete probability events, whereas continuous functions are described by normal distributions.

Asset pricing models in financial markets have predominantly been modelled in continuous time since Merton, (1971) (1992), as it is considered to provide a better approximation of security prices and describing economic behaviour. Finance literature, as a result, has shown a rapid development of mathematical models in continuous time to price securities and derivatives (for example, (Black & Scholes, 1973), (Duffie, 1996), (Duffie, Pan, & Singleton, 2000), (Singleton K. J., 2006). Although prices have been described in continuous time, in practice, however, security prices become discrete when trades are executed by market participants. Security prices have also been modelled in discrete time (Cox, Ross, & Rubinstein, 1979) where price follow a 'jump' process, going up or down in a binary model.

$$S_{i,t+1} = f(\mathrm{Y_i}) \qquad \text{EQ. 3-5}$$

Where

S_{it} Represents security i in time t

$$\mathrm{Y_i} = \begin{cases} 1 - \text{upward movement in price} \\ 0 - \text{downward movement in price} \end{cases}$$

In comparison, prices in racetrack betting markets are discrete events as there are well-defined termination points when outcome becomes known. The hypothesis to be tested requires a modelling and estimation techniques that will correctly determine the direction of price movements rather than prediction of specific values on a continuum function of prices. Modelling methodologies in racetrack betting markets therefore could be considered appropriate for financial market applications when prices in both markets are measured as discrete events. Although different distribution functions may describe prices in racetrack betting and financial markets, the point at which execution of trades occur are discrete events suggesting that modelling methodologies in betting markets could be applied to financial markets.

Singleton (2006, p. 2) notes that discrete and continuous time modelling of prices are simply choices of estimation strategy and influenced by the extent to which *"a complete economic environment is* *specified, data limitations as well as the computational complexity of solving and estimating a model".* In other words, a model estimation process is influence by the required level of accuracy as well as the type of variable predicted. Where securities trading occurs continuously assumptions of underlying data generation process that is described in continuous time, and therefore a stochastic process, such as random walk would most likely be considered appropriate. In this study however, the prediction is the directional movement of prices in discrete time rather than a detailed specification of prices in continuous time. It is therefore considered that racetrack betting and financial markets have the necessary mutual attributes to enable application of racetrack betting methodologies to financial markets. Prices and payoffs in both markets are also outcomes of probabilistic events in a market for contingent claims, dependent on events occurring and states realised. The next section compares the prices and payoff structure in the two markets.

3.3 Prices and Payoffs – in a Market of Contingent Claims

The underlying proposition of a contingent claims model in financial markets is that for each state of the world a state price exists where prices are the "*weighted sum or expectation of the security's state-contingent dividend"* (Duffie, 1996, p. 2), or an Arrow-Debreu security, which pays out a dollar if a particular state is obtained, otherwise nothing. Racetrack betting markets have also been described as markets for contingent claims and bets described as financial assets (Hausch, Ziemba, & Rubinstein, 1981), (Shin H. S., 1992) (Bruce & Johnson, 2001; Plott, Wit, & Yang, 2003). Shin (1992) describes racetrack betting market as a market for bets in n-horse race which corresponds to a "*market for contingent claims with n states of the world, in which the i^{th} state corresponds to the outcome in which the i^{th} horse wins the race"*. Prices in the betting markets are therefore representative of an Arrow-Debreu security where there is a payoff when the state is realised. The primary difference between the two markets is that the payoff in racetrack betting is a natural outcome of a race, a one-period event in complete markets where all information is fully revealed. By contrast securities in financial markets are continuously priced meaning there is no defined termination point (except stocks have a lower bound of zero and the security becomes worthless at zero). However, from a trader perspective, an arbitrary termination point exists when a decision to sell the security (at which point the outcome (gain or loss) becomes known) is made. The loss in racetrack betting is the initial outlay on a losing horse whereas in financial markets a loss is realised when a security is sold at a lower value than the purchase prices. The assumption here is that in financial markets securities are bought and sold for the purposes of trading and not held for dividends or indefinitely. Similarly, a gain is a winning ticket in betting markets and a gain in financial markets is where a security is sold at a price higher than the purchase price.

Contingent claims and state prices are well established in in finance literature. State probabilities are determined for each state, which collectively determine the final price of the security; as a function of payoff for each state multiplied by probability for each state (Harrison & Kreps, 1978), (Cochrane J. H., 2005), (Duffie, 1996). A security, S, therefore is a set of payoffs where one payoff is associated with each possible state in a market for contingent claims and is represented by a state price vector of the form:

$$S = \langle P^1, P^2, P^3, P^n \rangle \qquad \text{EQ. 3-6}$$

Where

P^1, P^2, P^3, P^n Represents states of uncertainty, one of which will be revealed as true

Where

P Represents the probability of the consumption states occurring and the security price is a sum of the weighted consumption probabilities.

n Represents the various consumption states occurring; 1, 2, 3……. n

The asset price then is a state-price-weighted average of the payoffs for each state of nature and is shown as a probability weighted average of the payoffs.

Given that a security is a claim on future dividends streams the price of a security is then given by sum of states of the future dividend payoffs (Duffie, 1996, p. 5):

$$\frac{q_i}{\gamma_0} = E\ (D_{i)} = \sum_{j=1}^{s} \gamma_j D_{ij} \qquad \text{EQ. 3-7}$$

Where

q_i Represents the price of security i

D Represents dividends paid by security I when the j[th] state is revealed

$j = \{1 \ldots \ldots s\}$ Represents the finite states one of which will be revealed true

$\gamma = \{1 \ldots \ldots s\}$ Represents the vector of probabilities for security i

γ_0 Represents the sum of the vector of probabilities

The price of a security is then postulated to be a function of future payoffs of the security, as follows (Cochrane J. H., 2005, p. 6):

$$P_{it} = E_t(M_{t+1}, X_{i,t+1}) \quad \text{EQ. 3-8}$$

Where

i Represents asset i

t Represents the time, t

P_{it} Represents price of asset i in time t

$X_{i,t+1}$ Represents random asset payoff of asset i in time t+1

M_{t+1} Represents stochastic discount factor or SDF or risk premium

E_t Represents expectations operator

In other words, the discount factor and the asset payoffs determine the price of a security, and the current price is a function of future payoffs $(t + 1)$ of the security.

For a dividend paying security, the payoff in t+1 would be the equal to the price received in t+1 when selling the security plus any dividends distributions received.

$$X_{i,t+1} = P_{i,t+1} + Dividends \quad \text{EQ. 3-9}$$

Prices in racetrack betting could also be shown as a representation of the asset pricing model:

$$P_{it} = \in_t (W_{t+1}, X_{i,t+1}) \quad \text{EQ. 3-10}$$

Where

P_{it} Represents price of asset i in time t or the final public odds/price of a horse before the start of a race

W_{t+1} Represents the stochastic discount factor (SDF), except that for horse races are single period events. The discount factor will be equal to 1 for horse race events, compared to M_{t+1} in financial markets where the stochastic discount factor values are >0 and ≤1, depending on the riskiness of the asset.

X_{t+1} Represents actual win probability or the actual payoff

$\in_t$ Represents expectations operator

The positive expected return on assets in financial markets is discounted by M_{t+1} to take into account the risk premium for holding a risky asset whereas in racetrack betting the return expectation is simply negative. This distinction in payoff expectations has resulted in pricing in the two markets evolving differently. In financial markets literature (for example, (Cochrane J. H., 2011), (Duffie, 1996) has focussed on determining the appropriate stochastic discount factor or risk premium for future cash flows of the underlying asset to determine future asset prices. By contrast, in racetrack betting markets the objective is to simply determine the win probability of a horse, given that expected return is less than zero (for example, (Bolton & Chapman, 1986), (Benter, 1994), (Lessmann S. , Sung, Johnson, & Ma, 2012).

Although pricing in securities markets are multi-period events in incomplete markets, security prices have also been viewed as a single period event with expected probabilities of prices going up or down. By comparison the outcomes of horserace events are binary, win or loss. Therefore, security prices when viewed as discrete single period events, could be viewed as equivalent probabilistic events to racetrack betting markets. Cox, Ross, & Rubinstein (1979) demonstrated that securities and options could be priced by modelling as upward and downward movement of stock prices of over multi-period horizon in discrete time, as an alternative to the Black & Scholes (1973) option pricing formula.

$$S_{n+1} = \begin{cases} uS_n & p \\ dS_n & 1-p \end{cases} \qquad \text{EQ. 3-11}$$

Where

S_{n+1} Represents the price of stock, S at n

u Represents the upward movement in price

d Represents the downward movement in price

$p \; and \; 1-p$ Represent the respective probabilities of the upward and downward movements in stock prices.

Since these early models a large body of mathematical and finance literature, as well as books (for example, (Singleton K. J., 2006), (Duffie, 1996)), have been written on jumps and diffusion models to identify the possible paths and distributions followed by security prices. In summary, both markets exhibit uncertainty in the value of the final payoff, hence the probabilistic nature of outcomes and a contingent claims market. In addition, models in both markets can be expressed as a common structure where prices are a function of expected payoffs. The returns on financial assets in both markets also exhibit behaviour correlation to other assets. The next section discusses the correlation of asset returns.

3.3.1 Correlation of Returns and Holding a Portfolio of Risky Assets

One of the cornerstones of CAPM and portfolio theory is that security returns are correlated which is determined by the covariance of the securities in the portfolio and given by:

$$COV_{ij......n} = E[(r_i - E(r_i))\ x\ \left(r_{j.....n} - E(r_{j....n})\right)] \qquad \text{EQ. 3-12}$$

Where

$COV_{i,j,....n}$	Represents the measure to which the securities move together relative to the individual mean values.
$i, j \ldots..n$	Represents securities $i, j \ldots..n$
r_i	Represents rate of return on security i
E	Represents the expectations operator

Racetrack betting markets also exhibit return correlation in the tote market where payoffs are relative to prices of other horses in the field. The payoffs are a function of the amount bet on the winning horse where proceeds are redistributed from the losing horses to the winning horse. The gross return to a winning $1 bet on horse i in the tote market is given by (Sauer, 1998, p. 2023):

$$R_i = QW/W_i \qquad \text{EQ. 3-13}$$

Where

W_i	Represents total dollars bet on horse i to win
$W = \sum i = 1, n$	Represents total wagered on all n horses
Q	Represents fraction of the wagering pool returned to winning bettors and $Q = 1 - \tau$
τ	Represents track take

This feature, however, is restricted to the pari-mutuel market as in the parallel bookmaker market bettors receive fixed odds and returns are not correlated to the total wagers placed. Arguably, the existence of FLB and bookmaker requirements to make a profit would suggest that prices offered on each horse in this market are also to some extent correlated. The difference being that in the tote market return correlation is explicit. The prices offered in betting markets and the existence of FLB, where horses with a low probability of being placed but are offered long odds, suggest that these payoffs are expected returns on a riskier asset.

Betting exchanges is yet another significant venue where bets are placed by persons against each other on a future outcome at a given price with others who are willing to offer that price, thereby allowing individuals to act as traditional bookmakers as well as bettors. Individuals in this market are able to bet on an event occurring (back) or not occurring (lay). Bets therefore can be placed on a winning or losing horse and could be compared to a long/short trading strategy in the financial markets. The individual's payoff therefore is a function of the bettor's information model on the event occurring; horse winning or losing. Studies show (Smith, Paton, & Williams, 2006) that person-to-person exchanges have a lower level FLB existence compared to the tote and book maker markets.

Busche & Hall (1988, p. 338) place racetrack betting in the context of *risky portfolio decisions*, where individuals consider their respective utility functions when placing bets. Ali (1979)) considers valuation of a bet as being similar to valuation of a security as in both cases future earnings are unknown and investors (bettors) bid against each other to determine prices or returns on security. It could therefore be argued that wagers on horse are simply a portfolio of risky assets. Hausch, Ziemba, & Rubinstein (1981, p. 435) note "*that the track is similar in many ways to the stock market markets as basic strategies are either fundamental or technical in nature"*.

These similarities in payoff structure in the two markets, where the outcome of events is probabilistic and decision-making within a context of risks and uncertain environment, suggest a level of standardisation and homogeneity to warrant the application of racetrack betting methodologies to financial markets. There similarities in prices and payoff structure in the two markets, however, may not be ample reason on its own. A comparative market structure and how participants behave in these markets would considerably strengthen the argument and validate the premise that racetrack betting methodology could be applied to financial markets. The next section discusses the structural and organisation features of the two markets.

3.4 Structural and Organisational Features of Betting and Wider Financial Markets

The global market size for trading in securities is in excess of US$212 trillion with the market for equity more than US$54 trillion (Source: QVM LLC, 2013). This turnover significantly eclipses the sports betting market. UK horserace betting in 2015-2016 was in excess of £5bn (Source: UK Gambling Commission). Regulatory jurisdictions of the two markets are different. The Gambling Commission regulates all betting activities in the UK. On the other hand, the Financial Conduct Authority (FCA), in conjunction with other organisations such as Companies House, London Stock Exchange regulate the activities of UK financial markets.

A common feature in both markets is the existence of a standardised form of security. Duffie (1996, p. 22) defines an equity security as a *"claim to an adapted dividend process where dividend is paid by the security at time t"*. Therefore, equity holders are in essence the owners of the company and receive dividends and any capital distributions from the company. Equities are securities issued by companies and provide the holder *"a residual interest in the assets of an entity after deducting all of its liabilities"*[3].

A fundamental difference, however, is that in racetrack betting market a security only comes into existence when a bettor agrees to place a bet at the chosen trading venue. By contrast, equity markets are secondary exchanges in which securities traded are pre-issued by the company raising equity capital. Financial market securities are therefore tradable securities whereas a bet placed at the tote is not tradable but can only be redeemed. The existence of securities in financial markets and non-existence of securities in racetrack betting has resulted in the profit maximisation functions to evolve differently. In equity markets, traders have additional costs of managing inventory of security holdings and of managing exposures on those securities. By comparison, bookmakers have to manage exposures only when bets are taken at the different odds offered. The advent of person-to-person betting exchanges, however, has blurred this distinction as traders in betting exchanges may back or lay a horse and are therefore faced with a similar exposure management exercise as traders in financial markets.

Literature (for example (Snyder, 1978), (Asch, Malkiel, & Quandt, 1984), (Hausch, Ziemba, & Rubinstein, 1981), (Losey & Talbot, 1980) describes racetrack betting and financial markets as being perfectly competitive, having a large number of participants, availability of extensive market

[3] IAS32 - Financial Instruments: Presentation - Definitions

information and the existence of professional investors/bettors/analysts who rely on publicly available information when making decisions to place a bet/ buy or sell securities or make recommendations. In financial markets, brokerage firms and security analysts utilise publicly available information to forecast future asset prices and make investment recommendations. Similarly, in racetrack betting markets, handicappers use public information and publish their expert opinions and odds in the weekly/daily racing magazines, newspapers and the track programs. These participants are both informed (professional bettors and in the case of financial markets, referred to as strategic traders) and uninformed (leisure punters in racetrack betting or liquidity traders in case of financial markets, who trade according to liquidity needs, and noise traders). There is no exposure management required in tote market as payoffs are a function of monies received. The next section gives an overview of equity financial market operations.

3.4.1 Overview of Equity Financial Markets and Operations

Market operations for exchange traded equities are conducted via intermediaries. There are two distinct but blurred intermediaries within the equities market; a market maker who posts continuous bid and offer prices at which the market maker is willing to buy or sell a security, or, at which the investing public can trade. Market makers perform a similar function to bookmakers in the racetrack betting market by providing a process at which trades can occur or bets can be placed. The other intermediaries are brokers who act as agents between the investors/traders and market makers. Investors/traders place orders with their respective brokers/market makers and these orders are executed continuously in real-time. Transaction costs are incurred on each trade. Price paid includes price of security and a transaction cost. An equity trader therefore incurs two sets of transaction costs; costs for buying a security and costs for selling a security. A minimum quantity is required to be bought or sold, usually in lots of 10's, 100's or 1000's depending on the price of security.

Two distinct markets operate concurrently for trading equities; order-driven market where all orders of both buyers and sellers are displayed, detailing prices at which traders are willing to buy or sell a security, and the amount of security that they are willing to buy or sell at that price. The advantage of the order-driven market is transparency and the disadvantage is that there is no guarantee the order will be executed. The second type of market is a quote-driven market which only displays bid (price at which a dealer will buy security) and ask/offer (prices at which securities are sold by the dealer) prices are placed by designated market makers, who are willing to accept the security at that time for the displayed prices. The advantage of quote driven market is the liquidity it presents to traders as market makers are required to fulfil the quoted prices. A trader therefore is able to execute trades immediately. The major drawback of the quote driven market is that there is no transparency of prices in the market. Therefore, more than a single price exists for a given security, given the two markets.

A spread exists between the bid and ask prices which is an indicator of liquidity, depth and efficiency in the price formation process of that security. A narrow spread indicates a more efficient security price microstructure with a high level of liquidity compared to a wide bid-offer spread. Amihud & Mendelson (1980) showed that expected returns are a decreasing function of liquidity because market makers must be compensated for the higher transaction costs that market makers bear in less liquid markets. Stoll (1978) and Ho & Stoll, (1981) reported similar findings and showed that, for market makers to be willing to make a market and hold an inefficient portfolio of securities, compensation is required. The market maker's profit is a function of demand and supply of the particular security and own inventory holdings in the security. During a trading period, market makers face a series of auctions and run the risk of either holding cash or being out of inventory. Market makers therefore face liquidity risks and inventory holding costs. Price are actively adjusted in relation to inventory and at the end of the trading day, excess inventory of securities must be liquidated or stored overnight at a cost. Amihud and Mendelson (1980) described the market maker's problem as being able to set the bid and ask prices and fulfilling a succession of buy and sell orders, while maximising profit and probability of survival.

In addition to the demand and supply functions for a security, a market maker is confronted with both informed and uninformed traders. The market maker minimizes losses against the informed traders, given the superior information advantage, and offsets the loss by setting bid and asks prices so that there is a gain against the uninformed traders. Hence, a positive bid-ask spread emerges in favour of the market maker (Glosten & Milgrom, 1985) (O'Hara, 1995). These prices are updated as the market maker learns from the trading process. Easley & O'Hara (1992) noted that the time-horizon of bid and ask prices arriving in the market and the sequence of prices conveys information. In addition, volumes traded also convey information as large trading volumes imply informed trading. French & Roll (1986) investigated the variability of stock returns over trading and non-trading days and found that the variance of stock returns from open to close of trading were often five times larger than the variance of close-to-open returns. In addition, on an hourly basis, the variance during trading periods was at least twenty times larger than the variance during non-trading periods. French and Roll (1986) concluded that at most 12 percent of the daily return variance was caused by the trading process itself (mispricing) and the remaining attributable to information factors. French and Roll (1986) also found evidence that most of the volatility of stock returns was caused by informed traders whose private information was impounded in prices when exchanges were open.

The microstructure literature (for example (Stoll, 1978), (Ho & Stoll, 1981), argues that information risk, due to asymmetric information, impact on prices. Investors may possess a better quality of information on the outlook of the security than market markers. These market makers therefore face potential ruin if the resulting information and consequential results are in favour of the trader. The setting of the spreads between the bid and offer prices by the market-makers prices is such that there is a gain against the uninformed traders and an overall profit emerges (Easley & O'Hara, 1992). These prices are updated as the market maker learns from the trading process.

There have been a number of studies on market microstructure of securities, efficiency of price formation process and transaction costs; for example Glosten & Milgrom (1985) on the formation of bid-ask prices, Amihud and Mendelson (1980) on market maker trades, divergence in actual and desired inventory levels resulting in adjustment of stock prices; Madhavan & Smidt (1993) on changes in inventory; Harsbrouck (2007) on market microstructure; Coval and Shumway (2005) on the relationship between each trader's profit in the morning and risk taken in the afternoon, and whether traders exhibited loss aversion behaviour. A detailed review of market microstructure and how prices are formed within this microcosm is beyond the scope of this paper. In summary, the market microstructure of securities influences the choice of market, final prices paid/received by a trader and transaction costs incurred. This influences the final returns and is an additional factor to the fundamental elements that determine equity prices.

The availability of continuous prices in financial markets, however, has led to the growth of independent derivative markets, such as the options, futures, spread-trading and over-the-counter (OTC) markets, where market participants are able to obtain exposure to movements in prices of the physical security without the need to hold the physical security. This feature is unique to the financial markets where securities only have an arbitrary expiry date when the decision to buy or sell is made. By comparison, tote tickets are expunged at the end of a race when values are realised.

Bookmakers also exhibit a similar behaviour in racetrack betting (for example (Shin H. S., 1991), (Shin H. S., 1992), (Schnytzer, Lamers, & Makropoulou, 2010) to protect against informed traders and manage profit. A FLB in prices emerges as a result. The influence of market microstructure on securities is similar to the one faced by racegoers who may be faced with different odds provided by bookmakers and the tote on the same horse. Therefore, the bettors' final returns are a function of the market in which the bets are placed. The next section gives an overview of racetrack betting market operations.

3.4.2 Overview of Racetrack Betting Market Operations

The racetrack betting market is a market for placing bets on horses in a race, where winnings are distributed to the backers of the winner (and in some markets to those who selected the second and third horse to finish). The remaining placeholders lose the amount bet.   The winners and losers are determined when the horse race finishes and final positions determined.

In the UK, three parallel racetrack betting markets coexist: the bookmarker market, the pari-mutuel or tote market and online betting exchanges. The bookmaker and the tote market have a long history and have been around for at least a century in the UK, as well as in most countries where betting is allowed. Online betting exchanges are recent entrants to the betting market and in the UK have been in operation since 2000. A punter is therefore able to place a bet at any of the three venues. For popular races, for example, the Melbourne Cup, in Australia, Aintree Grand National in the UK, Kentucky Derby in the US, bettors are able to place bets well in advance of the race and up to at least a day.

The betting market in general opens for at least 30 minutes before the start of a race, when the odds become available on each horse in the race and closes when the race starts. The odds represent the winning probability that a horse will win a particular race and is the betting price of that horse. For example, in the bookmaker market, a 9-to-1-odds represents a 10% winning probability (1 in 10 chance the horse will win) and £1 bet will yield a profit of £9 if the horse wins and a loss of a £1 if the horse loses. The pay-out however is different in the pari-mutuel market where a 9-to-1 odds will return £9 and yield a net profit of £8. A number of betting options are available to bettors in the pari-mutuel market; a win bet where the prediction is that a horse will win the race outright; a place bet where a horse will finish 2nd or 3rd; as well as exotic wagers such as quinella, where the bet is placed on the 1st and 2nd horses but not necessarily in the same finishing order, or a trifecta where a bet is placed on 1st, 2nd and 3rd in an any order. In person-to-person betting exchanges, on the other hand, a trader could back or lay a horse. This feature allows for betting possibilities on winning as well as losing horses.

The bookmaker market consists of market makers where a punter places bets at fixed odds and winning proceeds are pre-determined at the time when bets are placed. The bookmakers determine the odds to be offered to the bettors. The bookmaker market is considered a supply-driven market

(Bruce & Johnson, 2000), given that there is no direct interaction with the punters in the bookmakers' odds formation process. Large bets on, for example a previously long-odds horse, or increase in demand for bets on a specific horse would most likely result in market makers revise expectations and odds offered on these horses. The bookmaker market also exists in a number of other countries, for example in Australia and New Zealand. Legal bookmakers are however non-existent in countries where the activity is considered illegal, for example, US and Canada. In these markets, the tote is the dominant form for racetrack betting. Tote markets are common in most countries; US, UK, Canada, Hong Kong, Japan, Australia and New Zealand.

The tote market is considered a demand driven market (Bruce & Johnson, 2000) as the odds offered vary and change according to betting patterns on horses. These odds are inversely related to amount of wagers placed on the horses; the higher the amounts bet on a horse the lower the odds on that particular horse. Race favourites therefore have more money placed in wagers and have lower odds offered. These, therefore, represent the horses which the market considers to have the highest winning probability. By comparison the proceeds from a winning bet are known in the bookmaker market at the time the bet is struck but these are not known in the tote market until the close of the market. In the tote market, deductions are made from gross betting proceeds (the track-take: usually between 15% and 20% depending on the jurisdiction) to cover the overheads of the tote operator prior to distribution on winning bets. The tote market is a demand driven market as the proceeds are simply a redistribution of amount bets on the horses according to a pre-set formula, and there is a complete absence of a supply side.

In the tote market, a dollar bet on horse h, if horse h wins, will return (Bolton & Chapman, 1986, p. 1041) r_h:

$$r_h = \frac{(1-\delta)W}{w_h} - 1 \qquad \text{EQ. 3-14}$$

Where

δ Represents the track-take

W Represents total size of the betting pool

w_h Represents total amount bet on horse h

The expected payoff on horse h is then given by $p_h(r_h + 1)$

Where

p_h Represents the true winning probability associated with horse h and

π_h Represents the public's estimate of a horse's winning probability and is based on the total amount wagered by the public, and is given by $\pi_h = w_h / W$

Payoffs are therefore negative when $\pi_h = p_h$ as the tote takes a percentage as track take and positive when $\pi_h \neq p_h$, or in other words the public estimate of winning probability is incorrect.

The third market type, the online person-to-person betting exchanges, is where individuals are able to place bets against each other. Individuals in these markets are therefore allowed to act as bookmakers and bettors. These bets are at fixed odds. The online exchanges provide opportunities to place bets on winning and as well as losing horses for bettors. In addition, these markets are used by bookmakers to hedge any losing positions. Betting exchanges are, however, illegal in certain jurisdictions such as the US.

A question arises whether these markets which offer odds on the same horses in a race are efficient in pricing or if there are arbitrage opportunities available in these markets. There have been a number of studies examining whether arbitrage opportunities exist for the different prices offered in these parallel markets. Gabriel and Marsden (1990), (1991) investigated the comparative efficiency of Tote and bookmaker market in the UK. They compared returns to winning bets in the tote market with those offered by bookmakers at starting prices for the 1978 British horse racing season. Gabriel and Marsden (1990), (1991) found that returns to winning bets in the Tote market were higher on average than those offered by bookmakers, suggesting that these markets were not efficient. The difference in returns for winners at bookmaker odds of 10-1 or less was less than 8.9 percent and this difference increased to 29 percent when long shot winners were included. Sauer's (1998, p. 2041) suggested that these *"result calls for explanation*".

In a subsequent study Bruce and Johnson (2000) confirmed these findings. However, they noted that the odds' inefficiencies were limited to a specific range. The Starting Prices in the tote and bookmaker prices for 2109 races in the UK from June to August 1996 were compared. Bruce and Johnson (2000) observed that bookmaker odds were significantly higher than Tote prices, and that this difference increased as odds lengthened. The study found that the tote market provided better odds in comparison to bookmaker markets for odds beyond 10/1 and that bettors' returns were superior in the bookmaker market for the lower odds range. Their explanation was that there was an imbalance between supplier and buyer power in the bookmaker market. Given that the bookmaker odds are a supply driven market, Bruce and Johnson (2000, p. 427) concluded that *"results were strongly supportive of a significant role for the influence of bookmakers' decisions in* *explaining the existence of the favourite longshot bias in UK horserace betting markets".* Cain, Peel, & Law (2003) later confirmed Bruce and Johnson (2000) findings that bookmakers paid more generously than Tote on low-odds winners, and less generously on high-odd winners.

Peirson & Blackburn (2003) provide market structural and customer product preference explanations for the price differentials in the two markets. In particular, they point out that in the bookmaker market bets are made at known odds and therefore expected returns are known, compared to the tote market where final prices are not known until close of betting. Returns at tote also depend on final amounts bet on a winning horse and all horses.

In a later study, Smith & Williams (2010), analysed bookmaker odds over a 10-year period from 1996 to 2005. Their results suggest that the degree of FLB has declined since the entry of betting exchanges into racetrack betting market. The study split the data into two sample periods, 1996-2000 and 2001-2005. The sample period 2001-2005 included data for the first five years of betting exchange operations, Betfair. Smith & Williams (2010) found that the second sample period, 2001-2005, had a lower degree of FLB compared to the earlier sample period. An earlier study (Smith, Paton, & Williams, 2006) also noted that person-to-person betting exchanges had a lesser degree of FLB compared to bookmaker and tote markets.

The interaction of supply and demand for securities impact profitability in the financial and betting markets differently. In racetrack betting markets once all bets are placed, before start of a race (i.e. given initial security price), the result and corresponding payoffs depend only on the event (i.e. race being run). This differs to financial markets where investors' profits depend not only on the initial price paid for a security (supply uncertainty), but also what some other investor is willing to pay for the security when the investor decides to sell. The degree of uncertainty for both demand and supply of security, which is a function of how investors value the outlook of a security held when stocks are bought and sold, could significantly influence profits realized. By contrast, there is no demand uncertainty at the race track as final pay-outs are adjusted at the tote and the bookmaker market is a supply driven market, where the bookmaker pays out at pre-set odds (Bruce & Johnson, 2001). Person-to-person betting exchanges however have a similar supply and demand uncertainty of available odds as securities in financial markets which would impact profitability. The next section compares bookmaker and bettor behaviour in financial and racetrack betting markets.

3.4.3 Investor and Bettor Behaviour

A fundamental feature of both, racetrack betting and financial markets is the existence of heterogeneous population of market participants with differing utility profiles and propensity to risks; exhibiting risk-taking, biases and rational decision-making behaviour. There exist individuals who are willing to engage in speculative activity and bear risks. In racetrack betting a bettor by placing a bet either with the tote, bookmaker or betting exchange creates (and bears) risks which previously did not exist. The FLB is an established characteristic of racetrack betting markets and has been discussed in literature where bettor behaviour is described in terms of utility and risk preferences (for example, (Weitzman, 1965), (Rosett, 1965), (Asch, Malkiel, & Quandt, 1984)). Consequently, there are bettors who are willing to hold bets on long-odds horses in anticipation of a large payoff, if the horse wins.

A comparative long-shot view in financial markets is where individuals' trade in risky securities, for example, buying stocks in a mineral exploration company with probability that "oil" would be discovered. Harrison & Kreps (1978, p. 324) refer to this as investors exhibiting speculative behaviour. Specifically, where investors are" *willing to pay more for a security given the stock could be resold in the market, than they would pay if obliged to hold it forever",* compared to a dividend-paying stock. These speculators place a higher value on ownership of the security than to ownership of future dividend streams.

"*Investors would then bid up the price of the stock in anticipation of future opportunities of selling it at higher prices than they themselves would be willing to pay. Investors therefore are willing to pay a "speculative" premium because of anticipated capital gains* (Harrison & Kreps, 1978, p. 324)*".*

In addition to the existence of risky decision-making behaviour, both racetrack betting and financial markets show the presence of biases in decision making behaviour (for example (Golec & Tamarkin, 1998) in racetrack betting and (Barberis, Huang, & Santos, 2001) in financial markets). Lastly, both markets include participants who exhibit rational behaviour. In financial markets, the existence of economically rational individuals has been the cornerstone of pricing and rational decision-making behaviour and has been well discussed in literature (for example (Markowitz, 1952)) and in the development of CAPM. Similarly, literature in racetrack betting also shows that individuals make rational decisions based on their differing utility functions (for example (Asch, Malkiel, & Quandt, 1984). The next section discusses modelling methodologies in the two markets.

3.5 Modelling Methodology in Financial and Racetrack Betting Markets

There are three common factors that influence modelling process in the two markets; the underlying process by which data is generated; modelling techniques deployed and the nature of variables examined – fundamental variables and prices.

Campbell (2000, p. 1515) noted that in financial markets "*data are generated naturally rather than experimentally, and so researchers cannot control the quantity of data or the random shocks that affect the data"*. Prices in financial markets are generated real time as results of interactions between buyers (traders) and sellers (market markers), each with their own levels of information symmetry.

Similarly, in racetrack betting markets odds are determined dynamically, in real-time, in the bookmaker, tote market and betting exchanges. Racetrack betting studies are conducted in a *"naturalistic environment",* in field settings (Bruce & Johnson, 2003), and therefore provide a strong foundation for application of research to financial markets. Bruce & Johnson (2000) noted that there are limitations to calibrations in a laboratory environment as *"the dynamics of the market structure cannot be created in a laboratory environment"* compared to a field setting. The availability of wide and diverse source of data on actual betting decision making behaviour provide an overwhelming advantage in understanding human decision-making behaviour under uncertainty and price discovery process, and are comparable to the financial market environment.

Although data is generated in naturally in both markets, the length of data available in the two markets varies. The limited racing-life spans of horses means that race track data covers short time periods and generally a greater number of races over which a model is estimated. For example, Bolton and Chapman (1986) estimated the logit model utilising data from 200 races, Gu, Huang, & Benter's (2003) probit model was based on a study of 6,726 Hong Kong races and Sung and Johnson (2007) utilised data from 1675 flat races during the period January 1995 to August 2000 to determine the effectiveness of one-step and two-step logit models. However recent studies (for example (Lessmann, Sung, & Johnson, 2010)) have utilised data for longer periods, of up to 10 years and 4,276 races.

By contrast, financial market data for securities cover longer time periods than racetrack data, given that companies are around for longer periods compared to horses. Financial market studies have also employed varying lengths of data and periods of study have ranged from 10 to over 100 years. De Bondt and Thaler's study (1985) of winners and losers covered a five-year rolling period from 1926 to 1982 (57 years) and Jegadeesh and Titman's study (1993) of winners and losers covered monthly rolling periods from 1965 to 1989 (25 years). Ou & Penman (1989) logit model study utilised data from 1965 to 1984 using 8 years (1965 – 1972) and 5 years (1973-1977) respectively for model parameter specification and 1973-1983 for model testing. Foerster and Sapp (2005) study included data over a 130-year period from 1871 to 2003.

Economies and companies, however, undergo structural transformations, cycles and move across industries. Variables that may have significant explanatory information content for the in-sample period may become redundant in the holdout period. Prices, for example, are expected to experience a significantly higher level of volatility in the current era of multiple trading platforms and global market participants, compared to a few decades ago when they were only traded was limited to exchanges and with far less international interest. Yet another example is British Telecom, a fixed lines communications provider but is now also competing as a media giant. Variables which previously would have been significant may now be no longer relevant. Schwert (2002) showed that size, value, weekend, and dividend yield effect had weakened or disappeared after papers which highlighted them were published. Consequently, (Schwert, 2002, p. 47) found that *"many of the well-known anomalies in the finance literature do not hold up in different sample periods"* as practitioners learn quickly about any true predictable pattern and exploit to the extent that strategies are no longer profitable. A similar observation has also been noted in early racetrack betting studies. Benter (1994) noted that profits for fundamental variable based models disappear over time. This not only suggests that sample periods must have a level of uniformness to be able to draw any conclusions but also market evolves as it becomes efficient with respect to previously identified anomalies.

Linear, non-linear, classification and regression techniques (CART) techniques have been utilised to understand data in both markets, although the underlying methodology and application processes have been somewhat different.

Finally, modelling methods in both markets demonstrate the significance of fundamental variables and prices in determining predictability of prices. Although both are financial markets requiring dynamic decision making under uncertainty, the racetrack betting markets have the advantage of being complete financial markets where all information is revealed just before a race starts and the outcome when the race finishes reveals the 'correct price'. The next section provides a summary of the rationale to apply racetrack betting methodology to financial markets.

3.6 Summary

This chapter has presented the rationale for applying racetrack betting methodology to financial markets. The interpretation and validity of results on information efficiency, to a considerable extent, depends on whether the two financial markets are comparable such that replication of studies can be effectively implemented. There is a range of similarities between the two types of market that provide a compelling case for application of empirical analysis and findings in racetrack betting markets to the wider financial markets. In particular; (a) the behavioural characteristics of participants in both markets, where the individuals are willing to undertake risks and engage in speculative behaviour, (b) the asset pricing models, where payoffs and payoff structures are a function of some underlying variable and there is correlation of asset returns to other assets in the portfolio, and (c) market microstructure similarities; the existence of standardised tradable securities, market makers (although the two markets have evolved differently) and exchange-based trading.

The price-setting behaviour of bookmakers in racetrack betting and market-makers in financial markets exhibits profit management and protection against traders with insider information. Market microstructure models recognize that information about companies' fundamentals may be unequally distributed between market participants.

A unified framework of information efficiency describes both markets. These factors therefore would suggest that racetrack betting modelling methodologies are transferable to financial markets. A logical extension therefore should be the application of modelling methodologies in one market to the other market to determine price efficiency. In this case, whether multi-stage modelling methodologies in racetrack betting markets are applicable to the equities markets.

However, any adaptation of racetrack betting methodology to financial markets (or vice versa) could not be a direct replication, as there are unique features which distinguish the two markets and have to be taken into account. Therefore, any crossover of methodology will require a level of modelling variation and modification to ensure an effective transformation and translation of methods to the other market. Logic would suggest that given the equivalences in the two markets, models developed in racetrack betting markets where return expectations are less than zero most likely calibrate variables differently compared to models developed in financial markets where expected returns are positive. The next two chapters present a literature review of modelling methods in the two markets. Chapter 4 discusses modelling methods in financial markets and chapter 5 reviews modelling techniques employed in racetrack betting markets.

Chapter 4: Modelling Methods Employed in Financial Market Studies

The aim of this chapter is to discuss the key approaches to modelling fundamental information and prices in finance literature to test for market efficiency, and the underlying framework for these models. There is a wide and diverse body of empirical literature in finance that tests for market efficiency and it is not the objective to discuss of all these papers, rather to detail the key strands in model development. This chapter demonstrates that there is a gap in finance literature in developing multi-stage modelling methodologies for analysis of fundamental information in financial markets.

Section 4.1, as an introduction, provides an overview of the nature of fundamental information and its economic linkages to prices. This section also provides the rationale for organising modelling methodologies in financial markets into the four broad categories; consumption-based models, dividend models, stochastic and volatility models, linear and non-linear models.

Section 4.2 discusses the underlying economic theory and the existences of rational individual decision-making behaviour who take into account risk preferences. Section 4.3 discusses CAPM, empirical evidence and the behavioural view of prices. Section 4.4 discusses factor based models. Models based on dividends and earning are then discussed next in Section 4.5. Section 4.6 discusses asset pricing models where pricing does not consider individual risk preferences and volatility is a key pricing variable. Linear and non-linear models are discussed in Section 4.7. Finally, section 4.8 concludes on the findings of modelling methods in finance.

4.1 Introduction – Fundamental Information and Models

The perennial question in finance is “what determines equity security prices?”

It is well established that investors rely on a firms’ financial statements as a primary source of fundamental information to determine profitability. The firm’s earnings in turn reveal the extent to which cash flows are available for reinvestment to drive future growth and the surplus available for distribution as dividends to equity holders. A firm’s earnings and future earnings’ capability are therefore key fundamental variables that investors analyse to determine security prices. Dividend policy becomes relevant to determine distributions available in the future to shareholders. The firms' earnings are in turn dependent on macroeconomic factors such as aggregate demand, consumption and spending by households, and overall investment activity. This economic linkage is a circular relationship, where microeconomic decision-making behaviour of individuals and consumption influence aggregate demand and production decisions of firms, which in turn influence profitability and security prices.

The relationship between stock prices and firm's earnings has been well documented since the seminal paper, *"An Empirical Evaluation of Accounting Income Numbers"* by Ball and Brown (1968) which noted that accounting earnings are valued positively by investors. Beaver, Lambert and Morse (1980) noted that earnings reflect information in prices with a lag. Beaver, Lambert and Ryan (1987) found that leading price changes predict earnings better than accounting earnings. Ou & Penman (1989) found that publicly available financial statement information on future earnings is incorporated in prices with a lag. Lamont (1998) considered the role of earnings and dividends as predictive variables for stock price and found that both contain information on future returns above and beyond information contained in prices. However, information value was limited to short variation in stock expected returns and price was the only relevant variable in forecasting long-horizon returns. Chan, Chan, Jegadeesh, & Lakonishok (2006) studied the quality of earnings, accrual accounting and impact on stock returns and found that accruals were reliably, negatively related to future stock returns.

Dividends have also been subject to a number of studies (for example (Gordon, 1962), (Gwilym, Seaton, Suddason, & Thomas, 2006), (Rangvid, Schmeling, & Schrimpf, 2014) on whether dividend announcements and growth rates impact prices). Similarly, links to macroeconomic factors have been noted (for example Lettau & Ludvigson (2001) for consumption, Campbell & Yogo (2006) on interest rates}.

On the other hand, market efficiency and the random walk model imply that information is reflected in security prices and that fundamental analysis will not reveal information that is not priced. This school of thought has focussed on prices and risk-related variables to understand price efficiency (for example (Sharpe W. F., 1964), (Fama E. F., 1991)) and determine what is a fair price of a security.

Modelling methodologies have also developed along similar lines depending on whether markets are perceived to be efficient or inefficient and can be grouped into three overlapping but distinct categories; consumption-based asset pricing models where the primary focus is on security prices and risk measures, dividend models that focus on dividend policies and dividend growth rates to determine prices; and fundamental models which utilise financial statement and macroeconomic data to predict firm earnings or expected returns.

The organising principle I have employed for literature review has been to classify the modelling methodologies into the three broad categories:

(i) **Consumption-based Asset Pricing Models** – The consumption-based models are sub-grouped into three broad categories.

   a. **The CAPM model**, which is probably the most 'celebrated' and widely published (and empirically tested) asset pricing model. Its theoretical foundations are embedded in individual rational economic decision making for consumption decisions, hence consumption models. The CAPM subscribes to the view that securities earn market returns plus a risk premium for holding a risky security.
   b. **Factor-based models** where factors are identified that determine security prices. Market-related and fundamental variables such as size (Banz, 1981), debt to equity ratio (Bhandari, 1988) are used to determine market efficiency or simply factors are identified that determine prices (Ross S. , 1976).
   c. **Stochastic and Volatility Models** The stochastic and volatility models are the development of asset pricing in continuous time and has resulted in the development of derivative pricing models (for example Vasicek model and bond pricing (Vasicek, 1977), pricing options (Black & Scholes, 1973) and derivatives-pricing.

(ii) **Dividend-based Models** – the dividend models are based on the premise that investors receive a stream of dividends on a security, which can be priced. The price of a security therefore could be determined by valuing this stream of dividends and predicting future dividend growth rates ((Gordon, 1962)), (Rangvid, Schmeling, & Schrimpf, 2014)).

(iii) **Fundamental linear and non-linear models** – The fundamental linear and non-linear models are a collection of security valuation models that utilise financial statement, macroeconomic, industry and related data as sources of fundamental information to predict asset prices. The fundamental models subscribe to the view that analysis of publicly available information will yield price-sensitive information thereby revealing mispriced assets. Fundamental models include residual income models (Ohlson, 1995), and earnings models (Ou & Penman, 1989), which attempt to predict returns or utilise earnings as a proxy to forecast prices.

Although these models have been grouped into the three categories for analysis, the overarching feature is that these models could all be considered consumption-based factor models and have a different emphasis on the underlying variable. For example, the factor priced in CAPM is risk and volatility models also price risk. The economic foundations of risks are in individual utility behaviour. Similarly identifying dividend-relevant information is integral to dividend-based models where dividends (as a factor) are future consumption streams of a dividend-paying security. Likewise, a multitude of macroeconomic and microeconomic factors are relevant in linear and non-linear models where the macroeconomic information is related to aggregate consumption factors in the economy. A consumption-based decision-making framework therefore relates the three groups of models for pricing a security. Where these models differ with respect to pricing an asset, are the relative importance of the variable components considered relevant in determining price efficiency.

An array of advanced econometric techniques has been deployed in the empirical analysis of these models and are widely discussed in post-graduate textbooks (for example Campbell, Lo, & Mackinlay (1997), Cochrane J. H (2005), Singleton (2006) detail these methodologies}. These techniques are generally regression-based and include linear regression, time series regression, cross-sectional regression, ARCH and GARCH Models with embedded maximum likelihood functions and General Methods of Moments (GMM) to determine variable coefficients. In addition, non-linear techniques such as Markov chains, neural networks have also been applied (for example (Mills & Markellos, 2011)). The next sections discuss CAPM. First, individual consumption decision making behaviour is analysed as it is the underlying framework for CAPM models.

4.2 Risk Preferences and Individual Decision-Making Behaviour – The Rational Individual

A cornerstone of models in financial markets is the existence of a rational individual who when confronted with risky alternative choices X and Y and a budget constraint will prefer a consumption basket that will maximise utility and minimise risk[4] according to his/her risk preference where

$$E[\mu\ (x)] \geq\ E[\mu\ (y)] \qquad \text{EQ. 4-1}$$

Where

$E[.]$	Represents expectation under individual's probability beliefs, which are assumed to be objective
μ	Represents the utility function of the individual
x and y	Represent consumption vectors for various states of x and y with a probability P that gives the relative likelihood of the various states.

If $x\ \geq y$ and $y\ \geq z$ then $x\ \geq z$ then these preferences of x over y and z are continuous over the y and z functions. Additionally, any revisions in expectations from new information are processed according to Bayes' rules where probability is given by

$$P\ (X/Y) = \frac{P(Y/X)P(X)}{P(Y)} \qquad \text{EQ. 4-2}$$

Where

$P(X)$	Represents initial or prior probability of X
$P(Y)$	Represents initial or prior probability of Y
$P\ (X/Y)$	Represents conditional probability of X given Y
$P\ (Y/X)$	Represents conditional probability of X given Y

[4] Standard microeconomic textbook treatment – von Neumann-Morgenstern Utility function

The risk-averse individual, who prefers a certain outcome to a gamble, will hold a risk-free asset instead of a risky asset, and would be unwilling to take part in a fair gamble. This individual will only invest in a risky asset if return is greater than the return on a risk-free asset, for which the individual will require a risk premium to be paid. Therefore, given a choice of two assets with the same level of expected returns the investment with less risk will be selected. A risk-neutral individual will be indifferent to levels of risk in an asset, as risk does not impact the decision-making process. The expected return on the asset is the only consideration. Similarly, a risk-seeking individual prefers more risk to less and may accept negative expected rates of return. A risk-loving trader therefore would accept a lower expected return with higher variance and the possibility (with a low probability) to earn a significant payoff.

The rational individual, given the risk profile, is assumed to have the ability to incorporate all available information, perform a cost-benefit analysis, and revise expectations in a cohesive and consistent manner (independent of any framing) to achieve his/her consumption goals. The rational individual will therefore hold only those combinations of assets for future consumption that are mean-variance efficient or assets which will maximise the mean return of assets and minimise risk (variance or standard deviation) of those asset returns (Markowitz, 1952). Given the individuals' consumption, risk preferences and decision-making behaviour, a logical extension has been that prices of securities are functions of these attributes. Price of a security therefore could be described as a function of risk where the investor demands a premium or offer a discount to hold the security.

Campbell (2000) noted that the stochastic discount factor was linearly related to a set of common shocks and that asset returns could be described by a linear factor model. Specifically, if the economy has a representative agent with a well-defined utility function, then the stochastic discount factor is related to the marginal utility of aggregate consumption.

Literature documents the development of two different asset pricing methodologies based on these attributes: (i) the Capital Asset Pricing Model (CAPM) (Sharpe W. F., 1964) and (Lintner, 1965), where the assumptions of rational behaviour of expected utility (i.e. updating of expectations for new information and risk preferences) have been the building blocks of investment decision making, and (ii) where assets are priced in a contingent claims market as a function of risks, payoffs and payoff-states being realised (Harrison & Kreps, 1978). The next section discusses CAPM.

4.3 Capital Asset Pricing Models (CAPM)

Mean-variance analysis (Markowitz, 1952) , where investors would optimally hold a mean variance efficient portfolio, laid the framework for the CAPM ((Sharpe W. F., 1964) and (Lintner, 1965)). Markowitz's contribution to capital market theory has been described in terms of an investors' efficient frontier. Markowitz (1952) made the following assumptions about the individual investors' decision-making process:

i. Investors are rational and risk averse with homogenous expectations.
ii. Investors prefer more consumption to less and face a concave utility function
iii. Investors will maximise returns for a given level of risk and minimise risk for a given level of return. In other words, given two securities with exactly same returns, the investor will prefer the security with a lower risk.
iv. Investors have all available information at their disposal without any costs to help in the decision-making process and have an investment horizon of one period only.

Markowitz's (1952, p. 79) efficient frontier portfolio states that

"there is a portfolio which gives both maximum expected return and minimum variance, and it commends this portfolio to the investor".

The asset returns are assumed to be normally distributed. However not all the risks for an asset can be diversified and eliminated and the variance in asset return, as defined by the standard deviation of returns, will remain. Markowitz (1952) refers to this as systematic (non-diversifiable) and unsystematic risk (diversifiable). Similarly, the portfolio risks remain which is a function of variances and co-variances of the individual securities included in the portfolio.

Sharpe (1964) and Lintner (1965) extended mean-variance efficient frontier to derive the CAPM. Two additional assumptions were made to derive CAPM:

i. Markets were frictionless. In other words, there were no transaction costs for buying and selling a security. No taxes were assumed.

ii. The existence of risk free assets where individuals could borrow and lend at the risk-free rate.

The final CAPM model and total return on a security is then given by:

$$r_i = r_f + \beta_i \left(r_M - r_f\right) + \varepsilon_i \quad \text{EQ. 4-3}$$

Where

r_i	Represents rate of return on security i
r_f	Represents risk free rate
β_i	Represents systematic risk or beta for security i
r_M	Represents return on market portfolio where $i \epsilon M$, hence return on security i is related to the market portfolio
ε_i	Represents random error term on asset i

The returns of security r_i are correlated to the market portfolio, r_M where r_i is a function of r_M and $r_i \in r_M$.

β_i is defined as

$$\beta_i = \frac{COV_{i,j,\dots.n}}{\sigma_{i,j,\dots.n}} \quad \text{EQ. 4-4}$$

Where

$i, j \dots..n$	Represents securities $i, j \dots..n$
$COV_{i,j,\dots.n}$	Represents the measure to which the securities move together relative to the individual mean values and is given by:

$$COV_{ij\dots\dots n} = E[\left(r_i - E(r_i)\right) x \left(r_{j\dots..n} - E\left(r_{j\dots.n}\right)\right)] \quad \text{EQ. 4-5}$$

$\sigma_{i,j\dots\dots n}$ Represents the standard deviation and is given by $\sqrt{variance\ or\ \sigma^2}$

A security or a portfolio of securities considered efficient if no other security or portfolio of securities offers higher expected returns with the same level (or lower) of risk. Investors therefore consider the probability distribution of expected returns over some holding period and the volatility in expected returns (when selecting between investment alternatives) as determined by its variance as follows (Reilly & Brown, 1997, p. 253):

$$\sigma^2 = \sum_{i=1}^{n} [Ri - E(Ri)]^2 \, Pi \qquad \text{EQ. 4-6}$$

Where

σ^2 Represents Variance

Ri Represents return on asset i

$E(Ri)$ Represents expected return on asset i

Pi Represents probability of return on asset i

E Represents the expectations operator

In other words, CAPM states that asset returns are correlated to market returns (r_M) and the investor is simply compensated with a risk premium ($\beta_i \left(r_M - r_f\right)$) for holding a risky security compared to risk-free asset (r_f). A return on a security therefore is a function of the riskiness of the security for which the investor is paid a risk premium. Investors are assumed to base their decisions solely on expected returns and risk, which solely explains return on assets and therefore the price of an asset.

CAPM suggests that returns are not predictable and that risk measures provide a good explanation of why some securities have higher returns than others. Professional managers therefore will not be able to consistently outperform the market and can earn only risk adjusted returns. The CAPM is a single period model and does not consider investor horizons for multi-period investments. Subsequent studies to CAPM have extended CAPM to include multi-period investing (Kazemi, 1991); aggregate (macroeconomic) consumption (Breedan, 1979), (Duffie & Zame, 1989); CAPM with different borrowing and lending rates; CAPM with transaction costs (Magill & Constantinedes, 1976). The next section discusses empirical evidence related to CAPM.

4.3.1 CAPM Empirical Evidence

Early empirical studies have supported the CAPM and explain results within the CAPM framework. Black, Jensen, & Scholes (1972) in a study of monthly returns on securities listed on NYSE found that there was positive relationship between beta and monthly returns for the period 1931 to 1965. In a similar study of monthly security returns for the period 1926 - 1968 of stocks listed on NYSE, Fama & MacBeth (1973, p. 633) concluded that on *"average there seems to be a positive trade-off between return and risk".* Blume & Friend (1973) also reported a linear relationship between risk and return in a study of US securities for the 1955 to 1968. However later studies suggest a lack of evidence to support for CAPM and its theoretical extensions.

Ball ((1978, p. 103) noted stock price anomalies after earnings announcements by companies where excess security returns were "*correlated with the sign of the deviation of earnings from its expectation"*. Ball (1978) concluded that the securities markets were inefficient. In addition, Ball (1978) suggested that CAPM was not able to explain prices as it did not include the fundamental variable; earnings announcements. The CAPM model, when applied to a portfolio of equities did not explain the *"process generating securities' yields in equilibrium" and that "earnings and dividend variables proxy for the underlying determinants of equilibrium yields"* (Ball R. , Anomalies in Relationships Between Securities' Yields and Yield-Surrogates, 1978, p. 111). The earnings announcement anomaly persisted well beyond the announcement period of post one-month.

Basu (1983, p. 129) also questioned the *"descriptive validity"* of CAPM after his findings that earnings to price ratio was a significant variable in explaining security returns. In a study of US equity returns from 1963 to 1980 Basu (1983) found that firms with higher price to earnings ratio earned a higher risk-adjusted returns, even after controlling for the size effect. Fama & French (1992) noted that beta did not have the explanatory power for stock returns. In addition, they found that size, earnings to price ratio, leverage, and book-to-market equity explains returns better. Mehra & Prescott (1985, p. 146) found that returns on equities were far too high and returns on risk free rate too low in relation to the CAPM framework. The *"average real return and high average return on equity cannot simultaneously be rationalized"* suggesting that the equity risk premium puzzle was not supported by consumption data.

Roll (1977) questioned whether CAPM was empirically testable as the composition of the market portfolio is not known and is difficult to identify. *"The theory is not testable unless the exact composition of the true market portfolio is known and used in the tests."* (Roll, 1977, p. 130). Roll suggested that the market portfolio would include other assets such as fixed income, foreign exchange, liquid or illiquid assets.

Ross (1976) proposed a factor based model, the Arbitrage Pricing model as an alternative to CAPM. The behavioural finance literature has consistently criticised CAPM, questioning the theoretical foundations of individual decision-making behaviour and whether the individual is truly rational in obeying Bayes rule to optimise choice. The next section discusses behavioural view of decision making behaviour.

4.3.2 Risk Preferences and Individual Decision-Making Behaviour – A Behavioural Perspective

Behavioural finance is now a significant discipline with a widely published body of literature. It is not the intention to provide a detailed critique of the behavioural finance literature. Rather I highlight the empirical findings and behavioural studies that contradict the CAPM framework.

It is now well-established that individuals have heuristic biases. Kahneman & Tversky (1979) show that processing multiple layers of information is impacted by limitations in cognitive processing capabilities. Malhotra (1982) noted that consumers have finite limits to processing and absorbing information and, when information overload occurs it results in poor decision making. Decision-making is therefore subject to errors resulting from mismatches between cognitive processing abilities and levels of complexity. Individuals as a result execute simpler strategies relying on "rules of thumb" in the decision-making processes. Anchoring behaviour, mental accounting and cultural influences help define these rules of thumb. Ariely (2006) noted anchoring and arbitrary coherence behaviour where individuals have a monetary and non-monetary anchor values established in their mind. This behaviour shaped not only current prices of similar goods and services but also future prices coherent to the initial anchoring. The individuals appeared to be influenced by the first prices to which they were exposed, for a product and this represented as an anchor for subsequent prices. Ariely (2006) noted that anchoring behaviour did not decline and individuals responded more to anchor values than subsequent information signals.

Thaler (1999) noted that individuals kept a mental account of all financial activities to organise, evaluate, keep track and report results of transactions. This process consists of three components; how outcomes are perceived and experienced, how decisions are subsequently made and how decisions are evaluated. These factors have received more consideration in the decision-making process in comparison to other economic factors. Thaler's key proposition is that mental accounting is not neutral and there are influences which impact choice and decision making because mental accounting *"violates the economic notion of fungibility"* (1999, p. 185).

Kahneman and Tversky (1973) showed that decision-making behaviour was governed by the use of heuristics and individuals ignore base rate (objective) information. In a subsequent series of studies Kahneman & Tversky (1979), (1984), (1992) showed that individuals were less willing to gamble with profits than with losses, as individuals were more averse to loss when faced with outcomes of equal probability. Kahneman & Tversky concluded that individuals were less than rational and not uniformly risk averse, and in certain instances, were risk seeking. In a related study, Thaler & Johnson (1990) showed that when faced with sequential gambles, people were more risk-taking if they earned money on prior gambles than if they lost. Similarly, Grether (1980) also noted a reliance on representativeness to determine probability estimates resulting in incorrect probability judgements.

These behavioural studies therefore suggest that representative errors in probability judgements are likely to transmit to prices in financial markets. There is the likelihood that individuals' computational abilities will be distorted because they do not behave according to an economic cost-benefit model to maximise utility and minimise risk to meet consumption objectives. These biases will cause individuals to systematically make incorrect decisions when analysing and incorporating information. The next section reviews empirical evidence which suggest support for these biases.

4.3.3 Empirical Evidence from the Behavioural Finance Literature

The studies examined here consist of a collection of investigations that suggest markets are not rational. These studies have found evidence of anomalies in market behaviour that are contrary to CAPM. For example, daily, weekly and monthly returns have been analysed to this effect.

4.3.3.1 Day-of-the-Week, Month Effect, Winners and Losers

Keim (1983) studied daily returns for January for seasonal biases and found that daily abnormal return distributions in January had large means relative to the other 11 months. The study found that the relation between abnormal returns and size was always negative and more prominent in January than any other month, even in years when, on average, large firms earn larger risk-adjusted returns than small firms.

Lakonishok & Lev (1982), Keim & Stambaugh (1984), found evidence that US stock returns were negative over the weekend, where large stock decreases tended to occur between close of Friday's trading and opening of the market on Monday. Jaffe & Westerfield (1985) found evidence of weekend effect in the US as well as in Canada, Japan, Australia and UK.

A momentum strategy of picking winners and losers has also been subject to a number of studies. De Bondt & Thaler (1985), (1987) constructed a 'winners' and 'losers' portfolio by focussing on US securities which had either made extreme gains or losses over periods of up to five years, from 1926 to 1982. De Bondt & Thaler (1985) found that losers outperformed the market by 19.6% in the subsequent 3 years, whereas winners under-performed the market by 5.0%, a difference of 24.6% over the fifty years. The study concluded that investors overreacted to earnings, resulting in stock prices temporarily departing from fundamental values. Jegadeesh (1990) and Jegadeesh & Titman (1993) examined short-term reversals based on one week to one month returns. The study reported similar results to De Bondt & Thaler (1985), namely that selecting stocks based on returns in the previous week or month generated significant abnormal returns.

In an international study, Rouwenhorst (1998) reviewed securities between the period 1980 and 1995 and found that winners outperformed medium term losers after correcting for risk by more than 1 percent per month. However, Chui, Titman, & Wei (2000) noted that the Japanese market did not exhibit momentum. Griffin, Ji, and Martin (2003) examined momentum strategies in 40 countries around the world and found profitable momentum strategies in North America, Europe and Latin America. Griffin, Ji, and Martin (2003) also confirmed Chui, Titman, & Wei (2000) findings that momentum strategies were not significantly profitable in Asia. In a later study, Lesmond, Schill, & Zhou (2004) re-examined the profitability of executing a momentum strategy using US data for the period January 1980 to December 1998 and found that the momentum profits were illusory. Stocks which generated large momentum returns were those which were associated with disproportionately high trading costs, resulting in no profits. The results suggest that markets have since become efficient in pricing winners and losers.

Similar results have also been reported for other market attributes. Dubois & Louvet (1996) showed that the day of the week effect had disappeared in the US. Similarly, Schwert (2002) showed that size, value, weekend, and dividend yield effects seem to have weakened or disappeared after papers which highlighted them were published. Schwert concluded that the disappearance of these anomalies had coincided with practitioners setting up investment vehicles (hedge funds) to implement the strategies implied by these academic papers and exploit them to the extent that strategies are no longer profitable.

Anthropological studies also show that there are cultural biases that influence decision marking behaviour. Henrich (2002) noted that individuals when faced with incomplete information individuals rely heavily on biased cultural transmissions and do not perform a cost-benefit analysis in their decision-making behaviour. Financial market studies also support this view of significant cultural bias towards holding locally listed securities compared to holding a portfolio of international investments, even though returns on international investments may be higher than domestic markets. Thomas, Warnock and Wongswan (2004)) found that US investors held only 14% in international securities even though these securities accounted for 54% of the world market capitalisation; French & Porteba (1991) noted that investors in US, Japan, and UK allocated 94%, 98%, and 82% of their respective equity investment, to domestic shares; Campbell & Kraussl (2007) found that investors in US, UK and Japan hold a greater proportion of domestic securities than would have been the case in the mean-variance world. Campbell & Kraussl (2007, p. 1240) suggest that investors perceive a greater risk in investing abroad than there is in investing in domestic markets: "*The risk from foreign equity investment is higher than is currently captured using the standard deviation of the historical returns and the mean-variance framework". * Ke, Ng, & Wang, 2010 (2010) also report a bias towards home equity holdings by fund managers and a lack of international portfolio diversification.

Similarly, Barberis, Huang and Santos (2001, p. 2) utilise prospect theory to explain asset prices where the *investor derives direct utility not only from consumption but also from changes in the value of his financial wealth.* The relative value of an individual's wealth (the individual's reference point) and gains/ losses to the individual's wealth were noted to be the key determinants of the utility function. The investors experienced changing risk aversion where risk preferences changed over time as a function of investment performance, requiring higher risk premiums for holding stocks.

Shleifer and Vishny (1997) argue that there are limits to arbitrage which restrict the opportunities for mispriced assets to exist. Limits to arbitrage arise due to agency problems that exist between arbitrageurs, market specialists, and wealthy investors (who are the capital providers). These agency issues result in traders taking less than optimum positions in different trades, and as a consequence there are limitations in achieving market efficiency. Shleifer and Vishny (1997) argue that these limitations arise when asset mispricing worsens and arbitrageurs are less than willing to allocate additional investor capital (given that their performance is measured by investors who allocate money based on past returns of arbitrageurs). Money managers therefore choose not to fully exploit apparent mispricing due to these agency costs and have limited effectiveness in bringing asset prices to their fair values.

4.3.4 Summary CAPM

In summary, the behavioural finance based empirical evidence suggests that CAPM lacks the ability to fully explain how assets are priced. It could be argued therefore that the CAPM story so far is that empirical evidence is weak. The emergence of behavioural finance as an alternative view on how assets are priced and individuals make economic decisions is persuasive. The weakness of behavioural finance is that there is no unified theoretical framework for discussion, although there is empirical evidence that suggests that decision making process do not follow CAPM. One proposition is that assets pricing is between the two extremes of linear pricing (CAPM) and take into account the qualitative behaviour of individual's decision-making processes.

Harrison & Kreps (1978) noted that although investors have complete information from the outset they still arrive at different subjective assessments. *Investors must turn to public information, such as prices and trading volumes, to discover what their fellow investors know and how they will react to incoming information* (Harrison & Kreps, 1978, p. 335). If investors are not to undervalue securities they must consider the beliefs, preferences of their fellow investors which will be aggregated in the future prices.

Lo (2005) proposed an alternative view, Adaptive Expectations Hypothesis (AEH) to reconcile the efficient markets hypothesis and behavioural finance literature. Lo states that markets are not efficient all of the time and that individual's evolutionary decision making and learning are contributory factors . Lo suggests (2005) that market efficiency evolves and that market efficiency and inefficiency coexists. The AEH is a qualitative and descriptive framework. The *"Adaptive Markets Hypothesis can be viewed as a new version of the efficient market hypothesis, derived from evolutionary principles"* (Lo A. , 2005, p. 18). These principles are based on the following key ideas:

i. Individuals act in their own self-interest.
ii. Individuals make mistakes.
iii. Individuals learn and adapt.
iv. Competition drives adaptation and innovation.
v. Natural selection shapes market ecology.
vi. Evolution determines market dynamics.

The AEH would suggest that any empirical findings could be explained either using the traditional market efficiency or a behavioural explanation. Shefrin (2005) on the other hand suggests that stochastic discount factors in determining prices includes a behavioural component, where market sentiment is used as a proxy. The next section reviews factor-based models.

4.4 Factor-Based Models

The Arbitrage Pricing Models (APMs) (Ross, 1976), have been an alternative to CAPM. In APM's the primary focus is on identifying factors that determine asset price returns, (for example Chen, Roll, & Ross (1986). These models are one-period models where asset returns are explained as a linear function of a common factor structure or factor risks to which the securities are exposed. There are common factors which drive stock returns that are not firm specific. Arbitrage pricing does not rely on measuring market performance. Rather it relates the price of a security to these fundamental factors driving it. Common factors driving asset returns may include (but not limited to) macroeconomic factors such as GDP, interest rates, inflation, changes in long-term bond yields and oil prices. In notation form (Campbell, Lo, & Mackinlay, 1997, p. 220):

$$R_i = a_i + b'_i f + \epsilon_i \qquad \text{EQ. 4-7}$$

Where

R_i Represents return on security i

a_i Represents intercept of the factor model

b_i Represents a (K x 1) vector of factor sensitivities for security i

f Represents a (K x 1) vector of common factor realizations

ϵ_i Represents the disturbance term

The arbitrage pricing model assumes investors are risk averse with homogenous expectations. Three key steps are involved in the implementation of an arbitrage pricing model:

(i) Identification of fundamental factors
(ii) Determination of security exposure to these factors
(iii) Pricing of the factors compared to a risk-free asset.

The arbitrage pricing model is essentially the law of one price where if a security is over/under priced then a factor must also be over/under priced. The focus therefore is on identifying factors that have been priced (or not priced) in a security. Chen, Roll, & Ross (1986) identified a number of macroeconomic factors including inflation, industrial production, oil prices and consumption. The study found that a number of these variables were significant in explaining stock returns. Industrial production, changes in risk premium and changes in yield curves were significant. Others such as consumption, stock market index, and oil price index had no effect on stock returns.

A key criticism of Arbitrage Pricing theory is that it provides no guidance on what these factors should be or the number of factors that are required to explain asset returns. The factors are required to be empirically determined by calibrating subsets of securities to identify the factors and there is no guarantee that all relevant factors have been identified. Connor and Korajzycj (1988) used principal component analysis to identify factors and found limited sensitivity in explaining returns when the number of factors was increased beyond five. In addition to macroeconomic factors, empirical studies show that fundamental company information such as market size; leverage and price to earnings ratio are significant in explaining variations in stock returns. The next section discusses these factor models.

4.4.1 Market Size, Leverage and Firm Earnings

Banz (1981) examined the relationship between total market value of a firm's shares and its return using a linear regression predictor model.

$$E(R_i) = \gamma_0 + \gamma_1\beta_1 + \gamma_2[(\emptyset_i - \emptyset_m)\emptyset_m] \qquad \text{EQ. 4-8}$$

Where

$E(R_i)$	Represent expected return on security i
γ_0	Represents expected return on a zero-beta portfolio
γ_1	Represents expected market risk premium
$\emptyset_i$	Represents market value of security i
$\emptyset_m$	Represents Average Market value
γ_2	Represents the constant measuring of the contribution of $\emptyset_i$to the expected return of security i

In a fifty-year study of stock returns from 1925 to 1976 Banz (1981) found that, in addition to risk, firm size was a significant factor in explaining stock returns. Banz (1981) showed that logarithm of a firm's market value was inversely related to stock returns. Firms with lower market capitalization had higher than average returns given the firms' beta/risk estimates and firms with high market capitalization had lower than average returns given their beta estimates. Keim (1983) confirmed Banz findings in a study of seasonal returns. Specifically, Keim showed abnormal January returns and an inverse relationship between size and returns, with a size premium of no less than 2.5% per month over the period 1963–1979 for the US stocks. Similar findings were also reported by Lamoureux & Sanger (1989) who noted a size premium of 2.0% per month for NASDAQ stocks and 1.7% for NYSE/Amex stocks over the period 1973–1985 and, Fama & French (1992) who reported that stocks in the smallest size deciles outperformed the largest by 0.63% per month. These studies confirmed size as a significant explanatory variable. International studies have also reported consistent results of a size premium. Dijk (2011) noted that small firms outperformed large firms in 18 out of 19 countries with monthly size premium in ranges from 0.13% for the Netherlands, 1.18% for UK to 5.06% for Australia.

In a similar model to Banz, Bhandari (1988) used debt to equity ratio as a proxy for leverage and after controlling for beta and firm size effects found that stock returns were positively related to debt to equity ratio. Bhandari (1988) noted that firms with high leverage (high debt/equity ratios) had higher average returns than firms with low leverage for the period 1948-1979. The higher returns persisted after size and beta were included as explanatory variables. Similarly, Rosenberg, Reid, & Lanstein (1985) earlier noted that book to market equity ratio had more substantive predictive power of asset returns in the US market. Chan, Hamao, & Lakonishok (1991) reported similar results in the Japanese market where book to market ratio and cash flow yield had significant impact on returns.

Given these anomalous findings, where variables other than the security risk-return relationships explained the security returns, Fama & French (1992) developed a three-factor (1992), (1993) and updated it to a five factor model (2015), to explain security returns. The next section discusses these models.

4.4.2 Three and Five Factor Security Models

Fama & French (1992), (1993) constructed a three-factor regression CAPM model and found that monthly performance of a diversified portfolio of U.S. stocks could be explained by only three factors: portfolio's exposure to the market, to small-cap stocks and price-to book-ratio.

The three-factor regression took the following form:

$$R_{jt} - R_{ft} = a_j + b_j \left(R_{mt} - R_{ft}\right) + sjSMB_t + hjHML_t + ejt \qquad \text{EQ. 4-9}$$

Where

R_{jt}	Represents the return to portfolio j for month t
R_{ft}	Represents the T-Bill return for month t
R_{mt}	Represents the return to the CRSP value weighted index for month t

SMB_t Represents the realization on a capitalization-based factor portfolio that buys small cap stocks and sells large cap stocks

HML_t Represents the realization on a factor portfolio that buys high book-to-market stocks and sells low book-to-market stocks

sj and hj Represents coefficients that measure the sensitivity of the portfolio's return to the small-minus-big and high-minus-low factors, respectively.

The three factor model is in essence the traditional CAPM model plus two additional factors: (i) a factor for market capitalisation of small stocks less large stocks (to take into account risks associated with holding small stocks or size) and (ii) a factor for high book-to-market stocks less low book-to-market stocks (or stocks with a high dividend yield compared to the price of the dividend yielding stock). In otherwords, according to Fama and French, three factors, risk, size and value explain stock returns. Fama & French (1992), (1993) postulated that portfolios of value stocks will have a high value for h while growth portfolios will have a negative h. In addition, Fama & French suggested that sj will be negative for large cap portfolios and small cap portfolios will have a large positive value for s. The study found that size and value factors had positive returns whereas value stocks had higher returns than growth stocks. In addition, small stocks had higher returns than large stocks with small value stocks having the highest returns of all.

The three-factor model has been one of the key models over the past two decades that described the cross-section of security returns. However, subsequent studies have shown that profitability and investments profiles of companies also determine security returns where profitability is positively correlated to returns and investments having a negative correlation. Novy-Marx (2013, p. 1) noted that "*gross profitability as measured by gross profits-to-assets, has the same power as book-to-market in predicting average returns",* and that profitable firms have significantly higher returns compared to unprofitable firms. Titman, Wei, & Xie, (2004) found that firms which substantially increased capital investments had higher returns later on. Fama & French (2006) noted that book-to-market equity, expected profitability and expected investment rates explained equity returns.

Given these findings of information content in other fundamental variables, Fama & French (2015) expanded the three-factor to a five-factor model adding profitability and investment to the earlier three factor model above. The five-factor model provided a better explanation of security returns compared to the earlier three-factor model. However, Fama & French (2015), noted that the value factor became redundant when investment and profitability were included in the model. In other words, the five-factor model reduced to a four-factor model.

The factor models discussed up to now have considered firm variable attributes that are predominantly market-related and to the wider economy. For example, risk, price-volatility, market-to-book ratio have been included as variables in pricing assets. These models do not consider firm-specific variables such as dividends or a firm's earnings. As noted in Section 4.1 dividend policies and firm related variables matter, given the circular relationship to prices where a firm's surplus is available for distribution as dividends to equity holders. Dividends and earning-based models are discussed next.

4.5 Dividend, Earnings and Price-Based Models

The dividend-based model[5] assumes that firms distribute earnings surpluses to shareholders as dividends and a security therefore can be priced based on future expected dividends (Gordon, 1962), as follows:

$$V_o = \frac{D_1}{1+k} + \frac{D_2}{(1+k)^2} + \cdots + \frac{D_H + P_H}{(1+k)^H} \qquad \text{EQ. 4-10}$$

Where

V_o Represents value of the stock

D_1, D_2 Represents dividends to be received at the end of periods 1 and 2

D_H Represents dividends to be received at the end of the holding period

k Represents required rate of return of stock

P_H Represents expected stock selling price at the end of the holding period

With the assumption of a constant growth rate in dividends the model translates to:

$$P_t = \frac{D_{t-1}}{(k-g)_{t-1}} \qquad \text{EQ. 4-11}$$

[5] It should also be noted that dividend models, although included as a heuristic model is an exception as dividends also feature in fundamental earnings models.

Where

P Represents price of stock

D Represents current dividend rate and D>0

k Represents required rate of return

g Represents growth rate

However, there are a number of inherent limitations with the dividend model. The problem with the dividend model is that there has been a general decline in companies paying dividends in recent years. Fama & French (2001) in a twenty-year study from 1978 to 2001 found that the number of companies in US paying dividends declined from 66.5% to 20.8% and that zero-dividend payers were more profitable, and had higher market capitalisation to book value ratio. The requirement to pay dividends restricts the model's usefulness to mature and stable companies. Early and growth stage companies (for example technology firms) either utilize cash for future growth or have negative cash flows for considerable periods and are therefore unlikely to pay dividends. In addition, companies in financial distress would also not pay dividends.

The dividend model also has algebraic limitations. The model implies that stock price is expected to grow at the same rate as dividends and assumes that growth rate is less than the cost of capital. This is unrealistic, as a company's growth rate fluctuates and there could also be periods of negative growth. In addition, growth rate must be less than the required rate of return otherwise the denominator becomes negative, resulting in a negative stock price. When growth rate is equal to the required rate of return then stock price results in infinity. The dividend model therefore is sensitive to values where growth rates are close to the required rate of return.

A contradiction to the dividend model is the Modigliani & Miller theorem ((1958)), which states that a firm's capital structure is irrelevant and value depends not only on future dividends but also on all future cash flows. Empirical evidence also does not support the view that dividend models can be used to predict prices. Goetzmann & Jorion (1993) found no strong evidence that dividend yields can be used to predict stock returns. Wolf (2000) replicated Goetzmann & Jorion study using three different data sets (NYSE for period 1947 to 1986, S&P 500 data from 1947 to 1995 and S&P 500 Index data), to consider return predictability over horizons of 1, 12, 24, 36 and 48-month periods. Wolf (2000, p. 29) confirmed Goetzmann & Jorion's (1993) findings and concluded that there is *"no convincing case for the predictability of stock returns from dividend yields".* Similarly, Campbell & Shiller (2005, p. 176) noted that *"dividend-price ratio has done a poor job as a forecaster of future dividend growth rates".* Their study explored the dividend-price relationship for predictability of future dividend growth rates using dividend and price data from 1872 to 2000. Cochrane (2005, p. 39) noted *"that there is not a shred of evidence that higher market price dividend ratios are associated with higher subsequent dividend growth".*

However, there are studies which show that dividend information provides significant price signals. Foerster and Sapp (2005) studied the share price of, Bank of Montreal, a consistent dividend paying Canadian company, over a 120-year period from 1871 to 2003. Their study found that dividend-based models performed well in explaining actual prices. Arnott and Asness (2003) examined dividend pay-out ratio and its relationship to subsequent 10-year real earnings growth for US stocks for a 130-year period from 1871 to 2001. The study showed that companies with higher dividend pay-out ratios had higher earnings growth and, earnings growth was slowest when dividend pay-out ratios were low. The results contradict market expectations that low dividend pay-outs are a sign of strong future earnings to come. In an international study involving eleven countries,[6] Gwilym, Seaton, Suddason and Thomas (2006) explored the role of dividend pay-out ratios in explaining future real dividend growth and returns. The study confirmed Arnott & Asness' results that higher dividend pay-out ratios resulted in higher subsequent earnings growth, but not higher real dividend growth. Gwilym, Seaton, Suddason and Thomas (2006) concluded that dividend pay-out ratios had limited information content for future stock returns. McManus, Gwilym and Thomas (2004) examined the relationship between stock returns and dividend yield in the UK stock market using dividend pay-out ratio. The study found that in explaining returns dividend, pay-out ratio conveyed additional signalling information to dividend yields. Lewellen (2004) in a study on NYSE index returns found that dividend yields had strong predictive power compared to earnings-to-price and book-to-market ratios.

In summary therefore, although previous literature shows that dividends have information content, there is no evidence that dividend models can form the basis of a model that is able to explain price efficiency. The usefulness of dividend-based pricing models is limited due to their singular information content and the declining relevance of dividends to the current

[6] France, Germany, Greece, Italy, Japan, Netherlands, Portugal, Spain, Switzerland, UK and US

market environment, where fewer companies are now paying dividends. Prices are impacted by multiple events on a continuous daily basis, whereas dividend events occur less frequently (at most on a quarterly basis).

Dividends, therefore, as the sole basis for modelling price efficiency will not be effective due to a lack of inclusion of additional price-related variables. However, where dividends have been combined with other fundamental variables there is evidence of return predictability. The next section reviews models which have combined these variables.

4.5.1 Dividend-Price Ratio, Earnings Growth, and Price-Earnings Ratio

Campbell & Vuolteenaho (2004) present a three-way decomposition of the stock market dividend yield into rationally expected long-run dividend growth, subjective risk premium on the market and a mispricing term due to a divergence between the objective (i.e., rational) and subjective (i.e., irrational) growth forecast, as follows:

$$\frac{D}{P} = -G^{e,OBJ} + P^{e,SUBJ} + (G^{e,OBJ} - G^{e,SUBJ}) \qquad \text{EQ. 4-12}$$

Where

G Represents the long-term growth rate of dividends

P Represents the long-term discount rate

Using a log linear valuation framework of Campbell & Shiller (1988) to allow for time-varying discount rates and dividend growth rates and a vector auto-regression (VAR) system, empirical estimates of the three dividend components were constructed. Campbell and Vuolteenaho (2004) found positive correlation between high inflation and rationally expected long-run real dividend growth. In addition, the study found that inflation was uncorrelated with the subjective risk premium. However, Campbell and Vuolteenaho (2004) noted that inflation was highly correlated with mispricing, supporting the Modigliani & Cohn (1979) view that investors formed subjective growth forecasts by extrapolating past nominal growth rates without adjusting for changes in inflation. Consistent with the Modigliani & Cohn (1979) hypothesis, Campbell and Vuolteenaho (2004) found that the level of inflation explained almost 80% of the time-series variation in stock-market mispricing.

Binsbergen & Koijen (2010) utilized a latent-variables approach within a present-value model to estimate the time series of expected returns and expected dividend growth rates of the aggregate stock market. The price-dividend ratio and dividend growth rates were utilised to obtain predictors for future returns and dividend growth rates. The study found that both returns and dividend growth rates were predictable with R-squared values ranging from 8.2-8.9 percent for returns to 13.9-31.6 percent for dividend growth rates.

Ang & Bekaert (2007) utilised a regressions model using US, United Kingdom, France, and Germany data to examine the predictive power of the dividend and earnings yields for forecasting excess returns, cash flows, and interest rates. The study found that dividend yields predicted excess returns only at short horizons together with the short rate and did not have any long-horizon predictive power. Ang & Bekaert (2007) noted that long horizons critically depended on the choice of standard errors. With standard Hansen-Hodrick (1980) or Newey–West (1987) standard errors, there was evidence for long horizon predictability. However, this predictability, disappeared when corrected for heteroscedasticity and when removing the moving average structure in the error terms.

Price-to-earnings ratio studies have also shown that prices provide information about earnings ahead of time (for example, (Beaver, Lambert, & Morse, 1980), (Beaver, Lambert, & Ryan, 1987)) and that earnings captured events that impact security prices with a lag. For example, low P/E ratios may signal undervalued stocks and portfolios of low P/E stocks should yield excess returns. Basu (1977) studied returns from 1957 to 1971 and found that stocks with low price to earnings ratios (stock undervalued) earned significantly higher returns than stocks with high earnings/price ratios (stock overvalued).

Ferreira and Santa-Clara (2011) utilized the following regression to forecast separately the three components of stock market returns, dividend–price ratio, earnings growth, and price-to-earnings ratio growth to exploit the different time series persistence of the components, using US, UK and Japan data:

$$r_{t+1} = \alpha + \beta dp_t + \varepsilon_{t+1} \qquad \text{EQ. 4-13}$$

$$\mu_s = u_s^{gm} + u_s^{ge} + u_s^{dp}$$

Where

u_s^{ge} Represents expected earnings growth

u_s^{dp} Represents expected dividend price ratio

u_s^{gm} Represents expected price-earnings ratio

Ferreira and Santa-Clara (2011) forecasted earnings growth using long-run historical average (20-year moving average) because earnings growth is close to unpredictable in the short-run but has a low-frequency predictable component. The study found evidence in favour of return predictability in the sample tested and produced statistically and economically significant gains for investors, with better out-of-sample performance than the historical mean or standard regressions. The next section discusses volatility models.

4.6 Stochastic and Volatility Models

Risk is the primary factor that impacts the price of a security in CAPM and assets are priced in relation to other securities. In comparison, securities are priced directly in the contingent claims model. The investor is considered risk-neutral and risk can be priced. This strand of literature has led to the two key developments in securities pricing, pricing assets in continuous time and the significance of volatility as a fundamental factor in pricing. There is a wide body of literature on contingent claims and state prices which have formed the basis of general asset pricing and model development in financial markets, specifically pricing of derivative securities such as options and pricing securities in continuous time.

Risk-neutral pricing (Harrison & Kreps, 1979) assumes that arbitrage opportunities do not exist and that the price (payoff) for a portfolio of securities must be the same as the sum of the prices of individual securities in the portfolio, otherwise arbitrage opportunities would exist for traders to make risk-free profits. Two assets with the same risk and identical payoffs therefore will have the same market price and where assets are mispriced arbitrageurs will move in for the riskless profit opportunities. The absence of arbitrage opportunities ensures that a set of positive state prices exists and hence a positive stochastic discount factor exists. State pricing of securities has then been extended to pricing in discrete and in continuous time. The next section examines pricing assets in continuous time.

4.6.1 Stochastic Models and Pricing Assets in Continuous Time

The development of asset pricing models in continuous time can be traced to papers by Merton (1971), (1973). He demonstrated that a Brownian motion or Wiener process described the pricing process and that stock prices were independent increments, where the conditional distribution of future state of stock prices depended only on the current state. Asset pricing in continuous time is then expressed as follows (Merton R. C., 1973, p. 873):

$$\frac{dP_i}{P_i} = \propto_i \ dt + \ \sigma_{\ i} dz_i \qquad \text{EQ. 4-14}$$

Where

P_i Represents current stock price at a given point in time, i

dP_i Represents change in stock price P in i

$\propto_i$ Represents the instantaneous return expected return on common stock in time, i

$\sigma_{\ i}$ Represents the instantaneous variance of the return with a mean of zero and is normally distributed in time, i

dz_i Represents the Wiener process – a stochastic process with independent increments in time, i

Merton (1973, p. 873) describes the asset pricing process as Ito processes; *"are not differentiable while continuous".*

Pricing assets in continuous time has been the building block for a number of pricing processes, in particular in pricing of derivative assets such as options. These studies noted stylised facts about price distributions as well as mathematical models. For example, Duffie, Pan, & Singleton (2000) derive affine jump-diffusion model for option prices with stochastic volatility and jumps. In their analysis of 5,000 consecutive daily returns from 3rd July, 1962, to 8th June, 1982 for 100 firms, Lau, Lau and Wingender (1990) noted that the return series had a higher kurtosis than the normal distribution and existence of returns' skewness.

The continuous time models suggest that a key measure in pricing assets is variance of return, $\sigma_{\ i}$ and volatility, σ^2. Non-parametric techniques such as ARCH and GARCH (and variations of these) models have been widely employed to understand prices, price distribution and volatility. The next section discusses ARCH and GARCH Models.

4.6.2 ARCH and GARCH Models

The ARCH (Autoregressive conditional heteroscedasticity) and GARCH (generalized autoregressive conditional heteroscedasticity) and their various extensions have been the most widely used techniques to capture several important stylised features in financial markets. The ARCH (Engle R. , 1982) and GARCH (Bollersev, 1986) models deal with volatility clustering where the size of returns is time-varying. The ARCH model lets the weights of the volatility parameters be estimated, as follows:

Given for example, a regression model:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + \varepsilon_t \qquad \text{EQ. 4-15}$$

Where

Y_t Represents the dependant variable

X_{2t} Represents the independent variables

ε Represents the error term

Model variance is determined by:

$$\sigma_i^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \qquad \text{EQ. 4-16}$$

Where

σ_i^2 Volatility has two components: a constant and last period's news about volatility modelled as last period's squared residual (the ARCH term).

ε_t Is heteroscedastic, conditional on ε_{t-1}. By considering ε_t more efficient estimates of the parameters $\beta_1,\ \beta_2, \beta_3$ in the regression model below are then obtained.

Similarly, variance could be determined by any number of lagged periods (Pindyck & Rubinfield, 1998, p. 286):

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots\ldots.\ . + \alpha_p \varepsilon_{t-p}^2 \qquad \text{EQ. 4-17}$$

The autoregressive terms (in the term ARCH) are autoregressive in squared returns where future period's volatility is conditional upon information in the current period, and volatility is non-constant or heteroscedastic.

The GARCH is a variation of the ARCH model where the variance of the error term has three components: a constant, last period's volatility (the ARCH term) and last period's variance (the GARCH term).

$$\sigma_i^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 \sigma_{t-1}^2 \quad \text{EQ. 4-18}$$

In the GARCH model the variance is dependent upon all past volatilities but with geometrically declining weights that are positive. In other words, the best predictor of future variance is a weighted average of the long-run average variance. There is a significant body of empirical work of volatility related observations of asset returns. The next section discusses some of these papers.

4.6.3 Volatility Studies

Bansal, Kiku, Shaliastovich, & Yaron (2014), in their study of macroeconomic volatility, found that volatility risk carried a positive and economically significant risk premium that explained the level and cross-sectional variation in expected returns, and a positive compensation was received for volatility risk. The estimation of the volatility process for underlying returns and the analysis of implied volatilities for prices of derivatives assets (such as options) of the underlying assets have provided support to the view that volatilities and correlations vary over time and are not constant. However, the validity of volatility measures depends upon specific distributional assumptions.

The availability of higher frequency data has permitted modelling of asset return distributions, at weekly and daily horizons (cf. earlier frequencies of quarterly and monthly intervals). For example, Lo & Mackinlay (1988) studied weekly portfolio returns and show that the returns did not follow a random walk for the 1216 weeks and were positively correlated. French, Schwert, & Stambaugh (1987) found that the distribution of logarithmic monthly standard deviations constructed from the daily returns within the month was close to a normal distribution. Andersen, Bollerslev, Diebolds and Ebens (2001) studied daily equity return volatility and correlation obtained from high-frequency intraday transaction prices on thirty actively traded stocks included in the Dow Jones Industrial over a five-year period with an artificially constructed five-minute return horizons. Andersen, Bollerslev, Diebolds and Ebens (2001) found that unconditional distributions of the variances and co-variances for all thirty stocks were leptokurtic (returns are around mean values compared to normal distributions resulting in higher peaks, which leads to thick tails on both sides) and highly skewed to the right, while the logarithmic values of realised variances deviations and daily correlations all appeared approximately normal. They also found that realized volatility is an unbiased and highly efficient estimator of return volatility.

Ghysels, Santa-Clara and Valkanov (2005) utilize volatility measure, MIDAS (mixed data sampling) to predict returns. The MIDAS approach mixed daily and monthly market return data from 1928 to 2000 data to estimate the conditional variance of the stock market. Ghysels, Santa-Clara and Valkanov (2005) found a positive and statistically significant relation between risk and return with the MIDAS estimator explaining approximately 40% of the variation of realized variance in the subsequent month. The explanatory power compared favourably to GARCH models. Using GARCH models, Ghysels, Santa-Clara and Valkanov (2005) confirm the findings of French, Schwert and Stambaugh (1987) and (1993) of a positive but insignificant g coefficient in the risk-return trade-off. Ghysels, Santa-Clara and Valkanov (2005) noted that the MIDAS estimator was a better forecaster of the stock market variance than rolling window or GARCH estimators.

Two key themes arise from volatility literature with respect to pricing: (i) Volatility is a significant factor in pricing assets. This factor is consistent throughout from the early papers by Markowitz (1952) on portfolio theory to current literature; (ii) it is important to determine the appropriate measurement and time horizon to determine volatility. Although the standard deviation has been the generally accepted measure for volatility, literature suggests (for example, (2011), stochastic volatility models. It is also not clear from literature what is the appropriate time horizon, daily, weekly, monthly or annualised for measuring volatility.

We have now discussed CAPM-based factor models where risk is the fundamental factor that is priced relative to other securities. We have also dividend models and contingent claims model where volatility is a fundamental variable that impacts prices, and securities are priced directly. Linear and non-linear models are discussed next.

4.7 Linear and Non-Linear Models

4.7.1 Equity Valuation Models

Stochastic equity models have been developed to determine prices. The Bakshi & Chen (2005) model and General Equity Valuation Model (GEVM) (Dong & Hishleifer, 2004)) (also referred to as the Bakshi & Chen-Dong model)) are partial equilibrium models based on the earnings process. These models assume that a firm's earnings, as measured by earnings-per-share, grows at a stochastic rate and the expected earnings growth rate follows a mean-reverting process.

$$S_t = \int_t^{\infty} s(t, \tau; G, R, Y)_{d_\tau} \qquad \text{EQ. 4-19}$$

Where

S_t Represents the time-t price of a claim that pays $Y(t+\tau)$ at a future date $t+\tau)$

G_t Represents the rate of growth in firm's earnings per share

$Y(t+\tau)$ Represents the firm's earnings per share at t.

R_t Represents the rate of interest

The model makes the following assumptions:

- Dividend equals a fraction of earnings per share plus noise, and adjusted earnings per share follows Ito process
- The expected adjusted EPS growth follows a mean-reverting stochastic process based on analysts' forecasts.
- The economy's pricing kernel is consistent with Vasicek (1977) term structure of interest rates, where the instantaneous interest rate follows a mean reverting stochastic process.

The earnings structure captures the firm's growth cycle with three parameters: the long-run EPS growth rate, the rate at which current expected EPS growth reverts to its long-run mean, and the volatility of expected EPS growth. These assumptions therefore limit the model's overall performance to predict stocks which exhibit mean reverting earnings.

Bakshi & Chen (2005) examined the empirical performance of the model by pricing price the S&P 500 index, the 30 stocks in the Dow Jones Industrial Average, and a sample of 20 technology stocks. Bakshi & Chen (2005) acknowledged the limitations of the model's poor performance for high-tech growth firms, where dispersion in model pricing-errors was much higher for growth-oriented technology stocks than for blue-chip stocks. The model's out-of-sample error ranged from 8.17% to 23.87%. The Bakshi & Chen (2005) model also assumes that a fixed proportion of earnings (plus some noise) will be paid out as dividends to shareholders. A key finding, however, was that pricing errors were serially correlated across stocks with long cycles of high/low errors. This result suggests the existence of variables associated with the model's earnings dynamics that are important in the market's valuation of stocks, but which are missing from the model. A key limitation of the Bakshi & Chen model is the requirement that earnings are positive; otherwise there are difficulties in calibrating the variable and testing it empirically (due to estimation requirements).

The General Equity Valuation Model (GEVM) (Dong & Hishleifer, 2004) allows for inclusion of positive and negative earnings and is an enhancement of Bakshi & Chen (2005) model. Dong & Hishleifer (2004) noted that the Bakshi & Chen (2005) model did a better job of pricing indices than individual stocks. In addition, prices from the Bakshi & Chen (2005) model were found to have volatility unrelated to market prices (which was not noted in the GEVM model), and in all cases prices were lower than actual prices. By contrast, GEVM exhibited lower pricing errors. There are, however, limited empirical studies to confirm the robustness of the Bakshi & Chen (2005) and GEVM models (Dong & Hishleifer, 2004).

The Bakshi & Chen (2005) and GEVM models (Dong & Hishleifer, 2004) illustrate that there are number of gaps in fundamental models which limit the effectiveness of these models in determining market prices. Specifically, fundamental models have primarily relied on financial statement data. However, financial statement information appears at most quarterly, with formal financial statements published on an annual basis, whereas prices in financial markets are continuously determined. A wider source of information than financial statements is required. Financial statement data needs to be supplemented with additional sources of fundamental information that is available at a higher level of frequency.

Empirical evidence also shows that value of accounting information declines over time and trading based on non-financial statement data information increases. Dontoh, Radhakrishnan, & Ronen (2004) studied 34,070 firms and found that the value relevance of accounting information declined over time and non-information based trading increase over time. The R^2 of regressions of stock price on accounting information declined over time and this was increased by non-information based trading.

4.7.2 Clean-Surplus Model

Ohlson (1995) and Feltham & Ohlson (1995) developed a Linear Information Model, also referred to as Clean-Surplus Model, where price is a function of: (i) present value of expected dividends discounted at a risk free rate, (ii) abnormal earnings which follow a stochastic process and (iii) a variable representing information that is uncorrelated with accounting information. The dividend stream is then re-expressed as the book value of firm and equation reduced to:

$$P_t = y_t + \alpha_1 x_t^a + \alpha_2 V_t \qquad \text{EQ. 4-20}$$

Where

P_t	Represents price in time t
y_t	Represents net book value in time t
x_t^a	Represents abnormal earnings in time t which follows a stochastic process
V_t	Represents information other than abnormal earnings in time t which follows a stochastic process
α_1 & α_2	Represents valuation coefficients

The Feltham & Ohlson (1995) model makes the additional assumption that financial assets held by the firm are zero and abnormal operating earnings and net operating assets evolve in a linear fashion. Ohlson (1995) and Feltham and Ohlson (1995) however do not provide any guidance on how V_t should be determined other than denoting that V_t has yet to be captured in current financial statements (but impacted future abnormal earnings). This has been noted in a number of studies, for example, Morel (2003), Ota (2002), Callen & L Segal (2005). V_t is unobservable (Ota, 2002)) or very difficult to observe because of its inherent properties, resulting in difficulties in empirically testing the model. As a result, the variable V_t has not been included in empirical testing. Ota (2002) noted that x_t^a, the source of abnormal earnings, is monopoly rents.

Empirical results show that the Ohlson (1995) model has limited empirical validity, with indifferent performance when compared to a naïve or a traditional security valuation model. Ota (2002) tested three variations of an empirically testable Ohlson (1995) model for its ability to explain stock prices and predict future stock returns using data for 274 stocks for the period 1965-1998 in the Japanese market. The R^2ranged from 0.40 to 0.48 and the relative ability of the model to predict future returns ranged from 0.55 to 0.44 for the 8 years,1991 to 1998. In addition, the explanatory power declined over time with a higher R^2 being recorded at the beginning of test sample period than at the end. However, on implementing a buy and hold strategy, all three models produced positive returns over a 50-month sample test period, with returns ranging from 8.6% to 17.6%. These results provide general support for the Ohlson (1995) model.

Karathanassia (2003) employed data from the Athens Stock Exchange for the period 1993 to 1998, using a combination of time series and panel data analysis. This study compared performance of the Ohlson (1995) model to a Karathanassia model, where price was determined as a function of dividends, growth in dividends (or earnings), variability of earnings, leverage and size. Karathanassia found that both models had similar explanatory power. Similarly, in a US study (Morel, 2003), Morel estimated the earnings dynamic and valuation equation using OLS for each firm separately employing a sample of 735 firms covering a study period from 1962-1966. Morel (Morel, 2003) found the Ohlson (1995) model to be empirically problematic with potential multi-collinearity problems. The signs and values of the OLS coefficients in both the estimated earnings dynamic and valuation equation were inconsistent with sign predictions of the Ohlson (Ohlson, 1995) model. The parameters of the model did not yield internally consistent results, with parameters for the earnings equation being significantly different from corresponding parameters in the valuation equation.

Callen and Segal (2005) explored the predictive ability of a year ahead from earnings. They found that the signs of the estimated valuation regression coefficients conformed to the theoretical predictions of the Feltham and Ohlson (1995) model for almost all empirical variations of the model (panel data techniques, non-parametric estimation, reverse regressions and portfolio regressions). However, they also found that neither the Feltham and Ohlson (1995) nor the Ohlson (1995) models were more accurate than a naïve earning's model in predicting equity prices one year ahead. They concluded that the models were unlikely to perform well in explaining security prices given that many fundamental issues likely to affect security prices (e.g. bankruptcy costs, taxes) were excluded from the models.

In summary, empirical testing of the Ohlson (1995) model been undertaken employing a number of statistical measures, each differing in how the underlying model variables are specified and how the test results are measured. The results show a model specification issue, where, for example, model variables cannot be directly identified, and model ineffectiveness, i.e. indifference in performance when compared to naïve models. The next section discuses logit models.

4.7.3 Logit Models

Ou and Penman (1989) utilised a logit regression technique to predict the changes in earnings and returns one year ahead and show that a firm's fundamental information was included in financial statements. According to Ou and Penman (1989, p. 296) "*stock prices deviate from fundamental values from time to time and only slowly gravitate towards fundamental values and an analysis of published financial statements would discover values not reflected in prices*". Sixty-eight financial statement variables[7], were identified from a survey of financial accounting and financial analysis texts based on their ability to predict earnings a year ahead and the direction of earnings, as represented by earnings per share. The sample included industrial companies traded on NYSE and AMEX for the period from 1965 to 1984.

A general logit model was estimated, based on data pooled over firms and time, to determine the probability of future earnings increase as follows:

$$Pr_{it} = [1 + \exp(-\theta X_{it})]^{-1} \qquad \text{EQ. 4-21}$$

Where

[7] p304-5, Table 2. (Ou & Penman, 1989). Variables included inventory turnover, return on opening/closing equity, debt-to-equity ratio, dividend per share, Net Profit Margin

Pr Represents the logit estimate, an assessment of probability of a one-year ahead earnings increase given the accounting attributes, where > 0.5 represented a probability of future increase in earnings.

X_{it} Represents the set of accounting variables in the annual financial statements for firm i in fiscal year t

θ Represents the set of estimated coefficient weights applied to the accounting variables

Two independent sample periods were used to determine model parameters; data from 1965-72 and 1973-1977 for two models. Model output from 1965-1972 (hereafter, Model 1) data was then tested on the 1973-1977 out-of-sample-period, and model output from 1973-1977 (hereafter Model 2) was tested on 1973-1983 out-of-sample-period. All variables with a p-value of less than 0.10 were then removed from the predictor variables set. Ou and Penman (1989) noted that although the predictor variables were related measures only 6 variables featured in both models out of the 16 in Model 1 and 18 in Model 2.

The logit model output formed the basis for implementing a buy and hold investment strategy from 1973 to 1983 where securities were held for a period of two-years before the positions being closed. These positions were determined on 1 April (three months after the fiscal year end for US companies), the assumption being that information was publicly available after 31 March. An arbitrary probability cut-off of Pr, 0.6, for long positions and Pr, 0.4, for short positions was used as the basis for investment decision making. Ou and Penman (1989, p. 309) viewed that *"values in the vicinity of 0.5 probably don't indicate the direction of earnings changes very well".*

Model 1 correctly predicted 67% of earnings increase and 66% of earnings decreases. Similarly, Model 2 predicted 66% of earnings increase and 67% of earnings decreases. With the exception of one year (1983 produced a negative return), the long positions for years from 1973 to 1982 produced positive returns. The constructed portfolios generated positive returns of 16.85% which Ou and Penman (1989) could not attribute to risk as measured by market beta. This led to Ou and Penman (1989, pp. 310,318) to conclude:

"*Pr_{it} identifies price reversals as well as earnings turning points: high values of Pr_{it} are associated with prior price declines followed by price increases and low values of Pr_{it} are* *associated with prior price increases followed by price declines and …. …that the predicted returns cannot be explained by return-based risk measures".*

The Ou and Penman (1989) logit model, with refinements, has been replicated in a number of studies and these have produced mixed results. Holthausen and Larcker (1992) studied predicted excess returns instead of earnings and utilised the 68 variables used in the Ou and Penman (1989) study to predict returns on stocks listed on US exchanges. Similar to Ou and Penman (1989), logit models were estimated for two time periods, 1973-1977 and 1978-1982, and predicted over the out-of sample periods 1978-1982 and 1983-1988, respectively. Three return measures were predicted: market-adjusted returns, excess returns computed using the Capital Asset Pricing Model (CAPM) and size-adjusted returns.

Holthausen and Larcker (1992) found that a trading strategy based on a model that directly predicted excess returns performed better than the Ou and Penman (1989) model and earned significant abnormal returns.

> *"The returns to the hedge strategy are positive in virtually every year from 1978 to 1988 regardless of the excess return measure used….The results suggest that a trading strategy based on a model which predicts excess returns directly is able to earn significant abnormal returns in the 1978-1988 period. We find it surprising that a statistical model, derived without consideration of any economic foundations, can earn excess returns of the magnitude documented here ".* (Holthausen & Larcker, 1992, pp. 384,408,410)

In a replica study, Greig (1992) reported consistent results to Ou and Penman (1989). Greig (1992) however suggested that the results were not as robust when controlling for size using an alternative definition (i.e., where size was defined as difference between the firm-specific monthly return and the monthly return on the size deciles relative to the industry for which the firm is a member) and that the probabilities were more a proxy for firm size than firm accounting variables. Similarly, Greig (1992) explained Holthausen and Larcker (1992) results by the size variable, when the proxy for absolute size was replaced by log of market value of equity.

Abarbanell & Bushee (1997, p. 6) also report similar findings on accounting variables:

"Our results thus reinforce the findings of Ou and Penman (1989)*, with a difference that the relations we study are drawn from economic intuition rather than from a statistical process"*

Gerlach, Bird, and Hall (2002) compared the logit model with Markov Chain Monte Carlo techniques on UK, Australian and US financial statement data using 57 fundamental variables. Gerlach, Bird, and Hall (2002) noted that the logit model performed as well as the Bayesian model. In a Swedish market study, Skogsvik and Skogsvik (2010) utilised a logit model to predict returns on equity. The study noted substantial and consistent positive returns in excess of CAPM that were not explained by risk or size proxies. Charitou & Panagiotides (1999) applied the Ou and Penman (1989) model to the UK market and also found significant average excess return of 21.46 per cent over the period 1991 to 1995.

Van Canegham, Van Campenhout and Van Uytbergen (2002) conducted a study of the Belgian market and found that although earnings-based variables were predictable on the basis of financial statement data, trading strategies based on this information did not yield abnormal returns (as proxies for unexpected earnings lacked value relevance to stock prices). These results suggest that although the Ou and Penman (1989) model is promising, additional variable calibration and investment strategy refinement is required to improve effectiveness of the logit model. For example, returns are gross and do not take into account transaction costs. Holthausen and Larcker (1992) showed that model effectiveness can be improved by also including variables with marginal information content. In addition, the Greig (1992) study shows size is a significant explanatory variable that was not included in the models.

Similar to firm-specific variables, macroeconomic information has also been related to security prices given the relationships microeconomic decision-making behaviour of individuals on consumption and investments which in turn influences aggregate demand and production decisions of firms. There have been a number of studies utilising macroeconomic data. The next section discusses macroeconomic models.

4.7.4 Macroeconomic Models

There is evidence that macroeconomic data reveal information on the future expectation of a firm’s returns. DeStefano (2004) examined whether movements in economic factors could explain broad movements in stock returns over the business cycle and found that stock returns decrease throughout economic expansions and become negative during the first half of recessions. Chen, Roll and Ross (1986) noted that a number of economic variables are significant in explaining expected stock returns, including industrial production, yield curve and inflation rate.

Lev and Thiagarajan (1993) identified 12 macroeconomic variables and utilised cross-sectional regressions to examine incremental value relevance of fundamental variables, conditioned for various states of the economy. Macroeconomic variables including, Consumer Price Index for Inflation, Annual change in real GNP for level of economic activity and Annual change in Business Inventory for Level of Business Activity, were utilised to model for different economic conditions.

$$R_i = a + b_o \Delta PTE_i + \sum_{j=1}^{12} b_j S_{ji} + v_i \qquad \text{EQ. 4-22}$$

Where

R_i Represents 12-month excess return of firm I determined by subtracting realised returns as determined by a market model.

ΔPTE_i Represents the annual change in pre-tax earnings times one minus last year's effective tax rate.

S_{ji} Represents 12 identified fundamental signals

Lev and Thiagarajan (1993) reported statistically significant findings that the variables explained approximately 70% of earnings with respect to excess returns. Campbell and Yogo (2006) used a regression model to predict stock return with the short-term nominal interest rate and the long-short yield spread as predictor variables.

$$r_t = \alpha + \beta x_{t-1} + \mu_t$$
$$x_t = \gamma + \rho x_{t-1} + e_t \qquad \text{EQ. 4-23}$$

Where

r_t Represents excess stock return in period t; computed as annual, quarterly and annual return on respective indices index less a risk-free rate as denoted by 1-month Treasury bill, for monthly returns, and 3-month Treasury bill rate for quarterly and annual returns

β Represents the unknown co-efficient of interest

ρ Represents the unknown degree of persistence invariable x_t

x_{t-1} Represents a variable (for example dividends-price ratio) observed in t-1 with the ability to predict returns r_t

e_t, μ_t Represents error terms that are assumed to be normally distributed

In the sample period 1952–2002, the study found strong evidence that these variables predict returns. In addition, Campbell and Yogo (2006) used annual, quarterly, and monthly frequency U.S. data for dividend-price and smoothed earnings-price ratios to predict annual S&P500 Index returns, and annual, quarterly and monthly returns for NYSE/AMEX index. The study noted that earnings-price ratio reliably predicted returns at all frequencies in the sample period 1926–2002. The dividend-price ratio however predicted returns only at annual frequency and not quarterly and monthly frequencies.

In summary, the study found "*reliable evidence for predictability with the earnings-price ratio, the T-bill rate, and the yield spread"* (Campbell & Yogo, 2006, p. 52) but weak evidence for predictability with the dividend-price ratio. *"The most popular and economically sensible candidates for predictor variables (such as the dividend-price ratio, earnings-price ratio, or measures of the interest rate) are highly persistent"* (Campbell & Yogo, 2006, p. 56).

Lettau and Ludvigson (2001) found that fluctuations in the consumption-wealth ratio were strong predictors of both real stock returns and excess returns over Treasury bill rate, using U.S. quarterly stock market data. The study found consumption-wealth ratio was a better forecaster of future returns at short and intermediate horizons than dividend yield or dividend pay-out ratio and the greatest predictive power for returns was over business cycle frequencies ranging from one to five quarters

Guo (2006) utilised a regression model to confirm Lettau and Ludvigson's (2001) findings that the consumption-wealth ratio in conjunction with a measure of aggregate stock market volatility exhibited substantial out-of-sample forecasting power for excess stock market returns, generating returns of higher mean and lower volatility than a buy-and-hold strategy.

"The out-of-sample predictability of stock market returns was both statistically and economically significant (Guo, 2006, p. 667)", suggesting that stock return predictability was not inconsistent with rational pricing. Guo (2006) found that in conjunction with the consumption-wealth ratio, stock market volatility also had strong forecasting power for returns. *"My results thus suggest that stock return predictability is not consistent rational pricing."* (Guo, 2006, p. 667).

A logical extension would be models that combined fundamental analysis of financial statement and macroeconomic variables. The next section discusses this.

4.7.5 Combined Macroeconomic and Financial Statement Variables

Abarbanell and Bushee (1998) used a regression model with a collection of nine firm-related fundamental signals (from a study by Lev and Thiagarajan (1993) related to changes in inventories, accounts receivables, gross margins, selling expenses, capital expenditures, effective tax rates, inventory methods, audit qualifications, and labour force sales productivity) and found evidence that these fundamental signals variables provided information on future earnings and information that was not priced in securities.

The primary objective was to determine whether the information contained in fundamental signals about future earnings is fully exploited in earnings' revisions. The results suggested that analysts' forecast revisions fail to impound all information about future earnings contained in fundamental signals. These results confirmed Lev and Thiagarajan (1993) findings. Portfolios based on these signals earned an annual average cumulative size-adjusted abnormal return of 13.2 percent for a sample period 1974-1988 and were concentrated around subsequent earnings announcements. However, the returns did not persist beyond twelve months. They found little support for the notion that fundamental signals capture information about earnings multiple years ahead. This contradicts Ou and Penman's (1989) results where positions were held for 24 months.

Further sample testing over a holdout sample period 1989 to 1993 suggested that the strategy continued to generate abnormal returns subsequent to fundamental signals being discussed in the literature. In addition, their strategy did not generate large losses in any given year.

Conrad, Cooper and Kaul (2003) constructed a series of portfolios based on fifteen variables that were found in previous literature to explain firm performance. The study found strategies that performed best were those based on firm characteristics most commonly reported in the literature. The annualised returns ranged from 5% to 9%, with book-to-market and cash-flow related variables producing the maximum profits. In addition to regression models, machine learning techniques have also been utilised to predict returns and is discussed next.

4.7.6 Neural Networks and Support Vector Machines (SVM)

Kim (2003) employed SVM with a Gaussian radial basis function as the kernel function to predict the direction of change in the daily Korea composite stock price index (KOSPI) with 12 technical indicators as input variables. The results were compared with backward-propagation (BP) neural network and case-based reasoning (CBR). The SVM model outperformed both BP and CBR.

Similarly, Huang, Nakamori, & Wang (2005) compared the performance of SVM in forecasting the weekly movement direction of NIKKEI 225 with four other models; random walk, linear discriminant analysis, quadratic discriminant analysis and neural network models. Model inputs were S&P500 Index and USD/JPY exchange rates. The SVM had the highest forecasting accuracy (70%), compared to the random walk model (50%) and neural network model (69%). In addition, when the SVM was combined with the other four models the combined model had an improved forecasting accuracy of 75%, as the weakness in one model was offset by the strength of another. Huang, Nakamori, & Wang (2005) noted that *"different classification methods typically have access to different information and therefore produce different forecasting results"* (Huang, Nakamori, & Wang, 2005, p. 2520). For example, SVM's minimise structural risks and quadratic discriminant analysis had a higher hit ratio. Cao & Tay (2002) also reported similar findings when comparing SVM performance to neural networks in financial time series forecasting of futures contracts listed on Chicago Mercantile exchange.

Neural networks have also been developed to predict stock returns. Refenes, Bentz, Bunn, Burgess and Zapranisa (1997) developed a discount least squares to model stock returns as a function of six fundamental stock and macroeconomic variables. The study used a recency weighting procedure to take into account economic structural changes that tend to occur slowly over time as the economic environment evolves

$$\frac{\Delta S}{S} = f(\frac{\Delta LR}{LR}, \frac{\Delta SR}{SR}, \frac{\Delta EPS}{EPS}, \frac{\Delta \$}{\$}, \frac{\Delta S}{S}, PER) \qquad \text{EQ. 4-24}$$

Where

LR Represents long-term interest rates and thirty-day change

SR Represents short-term interest rates and thirty-day change

EPS Represents earnings per share

$\$$ Represents FX Rate US$/French Francs

PER Represents price-to-earnings ratio

The study used daily closing prices of CAC-40 stocks (Paris stock Exchange), consisting of 1025 observations, where the first 800 was used as training set and the remaining for testing. The results showed that discounted least squares were an efficient procedure for "weakly" non-stationary data series; a data series with properties where mean and variance are constant irrespective of the time dimension (Box & Jenkins, 1976). Refenes, Bentz, Bunn, Burgess and Zapranisa (1997) concluded that models must be re-estimated regularly to make use of the most recent data, but the modelling must be conducted in a way that retains the integrity of the sample size.

4.8 Conclusion Related to Modelling Methods in Financial Markets

The questions raised have been as follows: What determines equity prices? Are markets and individuals rational in the decision-making process to price assets? Are modelling methodologies in financial market effective in pricing assets? The finance literature review above suggests that an array of model methods have been used to attempt to answer these questions.

Studies from other disciplines show that individuals do not have rational expectations, as suggested by economic theory and CAPM's linear pricing. Dempsey (2013, p. 7) summarise: *"in choosing to attribute CAPM rationality to the markets, we are imposing a model of rationality that is firmly contradicted by the empirical evidence of academic research."*

In other words, the world of pricing in financial markets cannot be explained by individual consumption and consumption-based models alone, even though the framework in economic theory is sound. The existence of rational individuals and individuals with naturally occurring cognitive biases distort the efficiency of market participants' ability to aggregate information from various sources and correctly interpret complex information signals to determine prices under uncertainty. The behavioural finance literature provides a collection of studies that suggest that markets are not entirely efficient. This literature however fails to provide a cohesive framework to provide guidance to asset pricing.

The asset pricing models on the other hand suggest that securities can either be priced directly in a linear (or non-linear) fashion using CAPM, dividends models, contingent claims models or priced in relation to other assets (as suggested by factor models). For example, arbitrage pricing can be used, where factors would be identified that are prices or using contingent claims models. These linear and non-linear models utilise single-stage methodologies where all variables are input regardless of importance.

The general impression is that evidence appears to be in favour of stock return predictability, although it is not entirely unambiguous given the existence of contradictory findings. It is also clear that factors that are attributable to future consumption and macroeconomic factors are relevant to prices. For example, dividends (Lewellen, 2004), consumption-wealth ratio (Lettau & Ludvigson, 2001), industrial production (Chen, Roll, & Ross, 1986) have all been found to predict stock returns. The tone in the literature is perhaps best summarised in Lettau and Ludvigson (2001, p. 842):

> *''It is now widely accepted that excess returns are predictable by variables such as dividend-price ratios, earnings-price ratios, dividend-earnings ratios, and an assortment of other financial indicators"*

In addition to consumption factors, firm-related factors such as profitability (Novy-Marx, 2013) and firm size (Dijk, 2011) also have price-related information. The predictability of returns however does not imply that strategies could be implemented to benefit from predictability. Cochrane (2005, p. 390) probably best echoes this sentiment of findings of information content in fundamental variables, predictability and tradability:

"Variables including dividend/price ratio and term premium can in fact predict substantial amounts of stock return variation. This phenomenon occurs over business cycle and longer horizons. Daily, weekly and monthly stock returns are still close to unpredictable and "technical" systems for predicting such movements are still close to useless after transaction costs."

Cochrane (2005) suggests that at best the price of an asset is a judicious combination of relative pricing and absolute pricing where relative and absolute pricing are considered two extreme approaches to asset pricing. Assets therefore could be mispriced and present trading opportunities for the shrewd investors. Bossaerts (2002, p. x) suggests that, at best, asset pricing theory is *"persuasive" and believed to be correct…Yet there is little evidence that* *the theory explains the past, let alone predicts the future.* On the other hand, evidence from behavioural finance suggests the need for a more contemporary paradigm and framework to study market efficiency, rather than consumption-based models. Lo (2004) proposes the Adaptive Expectations Hypothesis as a framework for markets which acknowledges that there will be times when markets are not efficient as the market evolves.

In summary, empirical results suggest that that there is no consensus view that markets are price efficient. Although there is a wide body of evidence which show that returns are predictable, the proponents of efficient markets suggest that returns are well explained by risks, where higher return assets are correlated with higher volatility measures. The dynamic landscape, where equity prices are compounded with information and where studies use diverse modelling techniques, adds to this complexity, making the information signals less clear.

Racetrack betting markets on the other hand are simple financial markets. In these markets, information is revealed at the end of the race which can be used to effectively interpret price signals. This contrasts with equity markets with its complexity of micro and macroeconomic linkages, requiring analysis of financial statement data. In addition, racetrack betting research studies are firm advocates of multi-stage modelling techniques.

The next section discusses modelling methods employed in racetrack betting market studies.

Chapter 5: Modelling Methods in Racetrack Betting Market Studies

The main aim of this chapter is to detail the modelling techniques employed in racetrack betting markets and provide a comparative view to modelling methodologies in financial markets, and therefore the potential to apply to studies in financial markets.

This chapter is organised as follows: Section 5.1 discusses FLB, a well-documented deviation in market efficiency and a key factor in the development of early racetrack betting models to exploit the perceived inefficiencies in these markets. Section 5.2 then discusses modelling methods employed in racetrack betting market studies. Modelling methods that have been developed and deployed in racetrack betting markets have been grouped into "early" and "contemporary" betting models to provide a general timeline of the betting model development process. The early models primarily tested for weak-form market efficiency utilising prices as the input variables and established optimal betting strategies. The contemporary betting models tested for semi-strong form market efficiency and utilised fundamental information, as well as prices as input variables. In addition, these approaches used refined betting strategies. These are discussed in section 5.3. Section 5.4 provides a summary of the key findings from the racetrack betting market literature.

The FLB, where long-shots are over bet and favourites under bet studies, has been one of the most researched phenomenon in racetrack betting literature. The next section discusses the FLB.

5.1 The FLB

A number of explanations have been put forward over the past fifty years for these findings. These explanations have been in terms of individual utility behaviour and risk preferences, bookmaker behaviour and profit protection. The findings have also formed the basis for developing wagering strategies to exploit profitable opportunities from the these perceived weak-from market inefficiencies. The next sections discuss these issues.

5.1.1 Individual Utility and Risk Behaviour Explanation

Griffiths (1949) noted a systematic undervaluation of short-odd horses and an overvaluation of long-odd horses in the US betting market. Griffiths (1949) suggested there was an inherent psychological bias in the market participants' assessment of the probability attached to the different outcomes, resulting in a divergence from true winning percentages. Weitzman (1965) studied 12,000 races over a ten period from 1954-1963 on four New York racetracks and offered a utility explanation where the average bettor *"possesses greater propensity toward risk-bearing"* (Weitzman, 1965, p. 26) (i.e. are risk-lovers). Rosett (1965) studied whether racetrack bettors made a rational choice when faced with risky alternatives. Using Weitzman's data (Weitzman, 1965) he noted that *"sophistication and rationality can probably be expected from people who must frequently make choices"* (Rosett, 1965, p. 604). Ali (1977) estimated utility functions over wealth of racetrack bettors and found an increasing absolute risk function, confirming that bettors took more risks as wealth declined. Ali (1977) confirmed Weitzman's (1965) finding that the FLB was caused by bettors being risk loving.

Golec and Tamarkin (1998, p. 223) noted that bettors were better characterised by *"utility functions that go beyond mean and variance of return".* Specifically, they suggest that bettors prefer to trade-off negative expected return and variance for positive skewness. Golec and Tamarkin (1998, p. 208) explain that the bettors considered an *"evening of bets"* rather than one bet in isolation, and derived "*entertainment value which could explain the overall negative returns*". The utility was derived from holding a long-shot ticket as it was more pleasurable to pick a long-shot to win over a favourite. Hausch, Ziemba Rubinstein (1981, p. 1438) offer *"luck, entertainment and ego of gamblers"* as an explanation of why punters prefer to bet on long-shots than favourites. Going to the racetrack was seen as a similar utility activity as going to the opera or owning a boat for pleasure. Rhoda, Olson, and Rappaport (1999) find the presence of both risk-averse and risk-loving bettors when analysing 939 races at Philadelphia (PA) and Garden State (NJ) Parks over a recent twelve-year period. However, as the day progressed risk-loving bettors dominated betting and risk-averse behaviour dominating early in the wagering period.

Asch, Malkiel, & Quandt (1984) noted the fact that bets are placed on different horses with different expected returns does not result from irrational bettor behaviour. Rather, it *"is simply a consequence of bettors having different utility functions and selecting different points among available opportunities characterised by different objective and subjective winning probabilities"* (Asch, Malkiel, & Quandt, 1984, p. 174). The only contradictory evidence to the FLB was noted in the Hong Kong markets. Busche and Hall (1988) studied the Hong Kong betting market from 1981 to 1987 for 2,653 races and found no evidence that Hong Kong racetrack bettors preferred risk, and these bettors were either risk neutral or risk averse. Busche and Hall (1988) rationalise that representative bettor's expected utility derived from the gamble exceeded the expected utility of certain and greater wealth of not gambling. In addition, average betting volumes in the Hong Kong racetrack were higher when compared to US markets. In a later study of 13,000 races run on 18 Japanese racetracks Walls & Busche (2003) found evidence consistent with the Hong Kong study where there was no evidence of skewness-preference betting behaviour on racetracks with high betting volume. Skewness preference was limited to tracks with low betting turnover. Walls & Busche (2003) noted that bettors at tracks with high turnover placed bets as if they were maximising betting returns, whereas bettors at low-turnover tracks may trade-off returns for *"consumption of a beer, a hot dog and the excitement of occasionally hitting the long-shot"* (Walls & Busche, 2003, p. 44).

A possible explanation is the presence of professional bettors as well as casual bettors in these markets where there are high betting volumes. These professional bettors utilise sophisticated modelling techniques and as a result are better informed on the horses' true winning probabilities than the general public. The biases in the general public's betting behaviour as shown in the odds are then exploited by these professional bettors thereby moving the odds closer to objective winning probabilities of the horses. Adams, Rusco, & Walls (2002) noted that professionals' participation caused final track odds to converge to the level implied by the horses' true win probabilities when there is a high volume of betting. A similar observation was made by Smith, Paton, & Williams (2006) in a study of person-to-person exchange betting markets who found that races with higher than average betting volume (Class 3) and races with very high betting volume (Class 4) had a lower degree of bias, compared to races with low (Class 1) and moderate (Class 2) betting volumes.

Handicapping experts also appear to exhibit a FLB. Snyder (Snyder, 1978) analysed handicapper odds as published by the various form guides and newspapers for 846 races in the US market (Arlington Park, Chicago) and found that the experts exhibited greater FLB than the general betting public. In summary, these studies provide explanations based on individual utility and risk preferences to describe bettor behaviour and the existence of FLB. The next section explores the supply-side (bookmaker behaviour) explanations for FLB.

5.1.2 Bookmaker Profit and Risk Management

Shin (1991), (1992) provided a bookmaker-based explanation for the FLB. He suggested that prices are set by bookmakers to protect against those bettors who may have insider information on the possible winners. Bookmarkers therefore provide shorter odds on the favourites and overstate the long-shots (to protect themselves from large winning bets on longshots). Shin describes the market participants as having different of information levels on the possible states that could be realized *"the insiders who know the state that will be realised, the bookmakers who knows the true probability distribution over the states of the world, but who does not know the realisation of the true state"* (Shin H. S., 1992, p. 1180). In a subsequent study of the bookmaker market, Williams & Paton (1997) reviewed forecast prices from The Sporting Life and bookmaker starting prices for 510 races from the 1992 UK season. Their results supported Shin's (1991), (1992) explanation, that the FLB arises because insiders may possess superior information which confronts the bookmakers. They conclude that *"insider trading is a likely explanation of the positive correlation between the sum of prices and the number of runners* (Williams & Paton, 1997, p. 157)".

Schnytzer, Lamers, & Makropoulou (2010, p. 537) suggest that it is the presence of insider trading that makes *"bookmakers' odds deviate from winning probabilities*". They examined 4,017 races from the 1997–1998 Australian horse racing season for the bookmaker market. Using a calculated variable, "Plunge Weights", determined from changes in the opening bookmaker odds as the market progress, Schnytzer, Lamers, & Makropoulou (2010) measured the extent of insider trading for each of the races in the sample. The study found that although the opening price was a significant variable, the Plunge Weights were also significant for prediction of insider trading behaviour and influenced profits and losses; *"the presence of insider trading therefore could not be ignored"*.

In summary, empirical results from bookmaker market studies supports the existence of FLB. These studies suggest that the most likely explanations for the FLB include a combination of factors: the utility functions of bettors and their predisposition for risk; the risk and profit management of book makers and the explanation offered by Peirson & Blackburn (2003), namely that of market structure and customer product preferences. However, studies also suggest that FLB declines and non-existent where betting volumes are high. A logical extension of the existence of a FLB has been studies to identify profitable betting opportunities that could result from mispricing. Profitability across betting pools, across markets, tipster information, betting strategies have all been studied in this regard. The next sections discuss these.

5.1.3 Profitable Betting Opportunities arising from Mispricing

Asch & Quandt (1987) investigated the exacta bet, where the bettor picks the horses that will finish first and second, and the daily double where the bettor picks the winners of two consecutive races. They examined 705 races in the US market (Meadowlands racecourse). Asch & Quandt (1987) found that the daily doubles were substantially more profitable than the exacta bets, at $52.54 compared to $41.38. In addition, the daily doubles payoff was higher than betting on winners for two individual races. The study suggests that racetrack betting markets are not efficient given the availability of betting strategies that could be exploited to generate consistent profits.

5.1.4 Tipster Information

Figlewski (1979) and Bird & McCrae (1987) show that the betting public discount tipster information effectively. Figlewski (1979) analysed published handicapper information for New York (Belmont: 1977 season) racing racetracks for 14 tipsters to determine whether published forecasts had any information content or were completely discounted by the betting public. Figlewski (1979) utilised a conditional logit (CL) model to estimate probabilities using track odds and tipster information as input variables. The model was estimated using 143 races and tested on 46 out of sample races. The out of sample tests revealed that market odds were efficient. Price variables contributed significantly to explaining the winning probabilities and tipster information was effectively discounted in the market odds offered. He found that the market had discounted subjective information contained in the published predictions of handicapper forecasts and although valuable information was produced by the tipsters, using their information would not improve the accuracy of odds forecasts significantly.

Similarly, Bird & McCrae (1987) studied tipster information published in Melbourne newspapers on the morning of each race and developed a betting strategy based on subjective information derived from these tipsters. Bird & McCrae (1987) confirmed Figlewski's (1979) findings that none of the strategies produced positive returns and concluded that the market was efficient, as expert information was incorporated in bookmaker odds.

However, a later study in the UK market (Smith M. A., 2003) found that tipster information had significant impact on prices from max/mean to starting prices. This suggested that knowledge of tipsters' selections (e.g., those of "Winsome") was a useful predictor of large contractions in price, with the prospect of potentially large potential arbitrage opportunities.

In parallel with efficiency studies across betting pools and tipster information, racetrack wagering strategies, betting models and systems have also developed. This development can be grouped into two distinct phases: (i) the early betting models and tests for weak-form market efficiency have primarily been in the tote markets. The early betting models have been based on the premise that given the odds, what needs to determined is a wagering strategy and how much to bet given the prices. For example, Harville (1973) model, Isaac's wagering strategy (1953), Kelly (1956) all determine amounts to bet given the winning probabilities. Weak form efficiency tests have been an extension of the FLB to exploit these inefficiencies. For example, Dr Z's technical system (Hausch, Ziemba, & Rubinstein, 1981) which have primarily focussed on a single variable; market odds. (ii) Later models have been tests for semi-strong from efficiency, where the focus has been on multiple variables and based on the premise that winning probabilities can be objectively determined. These models include analysis of fundamental variables and prices, where updated modelling techniques and wagering strategies take into account the complexity of variable processing. The Bolton and Chapman (1986) study could be considered the beginning phase of this strand of the literature. They utilised a CL to model involving multiple fundamental variables, such as past horse performance and past records of the jockey and trainer to estimate winning probabilities, and then deployed wagering strategies based on the logit model output. The next section discusses the early betting models and strategies.

5.2 Weak-From Efficiency - Early Betting Models and Wagering Strategies

The literature suggests that a number of betting strategies are available to a race goer and these strategies are dependent on whether the true winning probabilities are known to bettors. The next section discusses the betting strategies available to the bettor and early probability models developed.

5.2.1 Isaacs Strategy

Isaacs (1953) suggested an algorithmic betting strategy for the tote market. This strategy assumes that the bettor knows perfectly the true win odds of the horses in a race taking into account the effect of one's bets on the odds, and the amount bet by the public. Isaacs (1953, p. 310) refers to this as *"taking advantage of the collective error of the crowd in appraising the probabilities as registered by the amounts they bet".* The amount bet according to Isaacs is determined as follows (Isaacs, 1953, p. 313):

$$x_i = \gamma_t \sqrt{p_i s_i} - s_i \qquad for\ i = t, \ldots\ldots, n \qquad \text{EQ. 5-1}$$

Where

x_i Represents the amount to bet on horse i

p_i Represents the winning probability for horse i

s_i Represents the amount to bet on horse i by the betting public.

γ_t Represents the ratio of amount bet by the public and the winning probability and is determined as follows

$$\gamma_t^2 = Q\sum_{j=1}^{t-1} s_j \,/\, \left(1 - Q\sum_{j=t}^{n} p_j\right) \qquad \text{EQ. 5-2}$$

Where

$0 < \ Q < 1$ Represents a factor or track-take.

The strategy, however, assumes that expected value maximising bettor is the last bettor, as any subsequent bettor will change the odds and therefore change the optimal amount to be bet. Hence, there are operational issues with the implementation of Isaac's optimal betting strategy. In practice, the flurry of last minute betting would suggest that operationalising Isaacs (1953) would encounter problems in calibrating the correct amounts to be wagered (Bolton & Chapman, 1986).

5.2.2 Rosner Strategy

Rosner (1975), an extension of Isaac's (1953) tote wagering strategy, formulated the strategy into *"two clear problems facing the bettor"*: how to handicap and how to bet given the handicap. When the probability is known the amount bet on each horse could then be determined as follows:

$$\gamma_i = P_i - \beta P_i \,/\, (\beta - f) \qquad \textit{for } i = 1, \ldots\ldots.., s$$

$$\gamma_i = 0 \qquad \textit{for } i = s+1, \ldots\ldots.., k \qquad \text{EQ. 5-3}$$

$$\gamma_i = \beta(1 - f) \,/\, (\beta - f) \qquad \textit{for } i = k+1$$

Where

γ_i – Represents fractions bet on horse i

P_i – Represents probability on horse i

f – Represents Track – take, a fixed percentage before winnings are allocated

β = Represents sum of the probabilities $\sum_{s+1}^{k} P_i$

However, the strategy does not take into consideration bets placed by other bettors. As a consequence, returns on bets may be over or underestimated. One's own bets are small relative to the total amount bet by the public and the optimality criterion is that of maximizing the expected log return.

Bolton and Chapman (1986) found that the Rosner strategy performed worse than a random betting strategy and average returns were worse than Isaac's strategy with average returns of -37.4%. Initial capital declined from $1,000 to $95.63.

5.2.3 Kelly Strategy

The Kelly (1956) strategy is probably the most well-known betting strategy discussed in financial market literature. This strategy "*maximises the expected value of the logarithm of capital"* (Kelly, 1956, pp. 925, 926) and is independent of bettor behaviour: *"nothing to do with the value function which he attached to his money"*

The Kelly (1956) strategy is given by

$$G_{max} = \sum_{1}^{t} p(s) log p(s) \alpha_s + (1 - p_t) log \frac{1 - p_t}{1 - \sigma_t} \qquad \text{EQ. 5-4}$$

Where

G Represents the exponential rate of growth of capital

$p(s)$ Represent the probability

α_s Represents the odds paid – the number of dollars returned for a one-dollar bet (including that in dollar)

Where

t Represents the smallest index which gives $\frac{1-p_t}{1-\sigma_t}$ its minimum positive value.

The Kelly strategy (1956) and its fractional variations (referred to as fractional Kelly where the amount bet is less than the full Kelly amount), has been a core strategy deployed in racetrack betting studies.

The main advantages Maclean, Ziemba, & Thorp (2010) of the Kelly strategy is that it maximises the rate of asset growth where the expected time taken to a reach a preassigned goal is asymptotically least. In addition, the bettor never risks ruin. The main disadvantage Maclean, Ziemba, & Thorp (2010, p. 7) is that the Kelly strategy may suggest large wagers. *"The bets may be a large fraction of current wealth when the wager is favourable and the risk of loss is very small".* As a result, fractional Kelly in certain instances is a preferred.

The Kelly criterion (1956) has been utilised in the racetrack betting literature as the preferred wagering strategy (for example, Sung and Lessman (2012), Johnson, Jones and Tang (2006), Edelman (2003), (Gu, Huang, & Benter, 2003)). The Kelly strategy determines how much to be bet on each horse to maximise the expected log payoff across all potential winners. Breiman (1961) and Thorp (1971) showed the Kelly strategy to be optimal as it maximised the asymptotic rate of increase in wealth with a zero probability of ruin. The sizes of bets are larger where the probability of winning is greater (for the same expected return) and when the expected return is higher for the same winning probability.

The fractional Kelly strategy is a modification of the Kelly strategy. The fractional Kelly criterion for optimal capital growth is the generally accepted model for placing optimal bets.

$$f^* = \frac{bp - q}{b} \qquad \text{EQ. 5-5}$$

Where

$\mathrm{f}^* -$ Represents the fraction of the initial capital

$\mathrm{p} -$ Represents the winning probability

$\mathrm{q} -$ Represents the losing probability of $1 - \mathrm{p}$

$\mathrm{b} -$ Represents the net odds received for the wager

The next section discusses the early probability models.

5.2.4 Harville Model

Harville (1973) developed a linear probability model that calculated probabilities based on the public odds as revealed from betting patterns in the tote market, *"to identify win, place (first or second), or show (first, second, or third)"* (Harville, 1973, p. 312).

$$P_{ij} = \frac{P_i P_j}{1 - P_i} \qquad \text{EQ. 5-6}$$

Where

P_i Represents the probability for horse P_i finishing first

P_j Represents the probability for horse P_j finishing second

Subsequent studies (1981) however found that the Harville (1973) model had flaws as it could not be *"derived from first principles using individual horses running times"* (Hausch, Ziemba, & Rubinstein, 1981, p. 1439). In addition, the Harville model (1973) had the "*'Silky Sullivan' problem where some horses generally either win or finish out-of-the-money*" (Hausch, Ziemba, & Rubinstein, 1981, p. 1439). The Harville (1973) formula was also noted to over-estimate true probabilities of finishing second or third. This was also confirmed in a later study by Lo & Bacon-Shone (1994) who noted that the Harville (1973) model systematically overestimated the ranking probabilities of finishing second and third for favourites and underestimate those for longshots. Benter (1994) also confirmed these findings and noted significant discrepancies in conditional probabilities for second and third positions.

5.2.5 Henery Model

Henery (1985) suggested that bets be placed only in those races where the odds offered are greater than the bookmakers' advantage, similar to Kelly. First, the empirical loss probability, q_j , or the *"fraction of horses which lose for the given odds"* (Henery R. J., 1985, p. 344) has to be determined. This is given by

$$q_j = 1 - p_j \qquad \text{EQ. 5-7}$$

Where

p_j Is the win probability.

In determining the overall loss function Henery (1985) suggests that the bookmaker's advantage in a given race (or "book over-round") and starting price odds offered by the bookmakers be taken into account, and bet only on those races where the odds offered are more generous. The book over-round is calculated as follows:

$$R = \sum \frac{1}{1+X} - 1 \qquad \text{EQ. 5-8}$$

Where

R Is the bookmaker advantage

X Are the winning odds

Henery (1985) suggests that as a practical betting strategy to bet on horses with high winning probabilities (or low odds) and that the bookmaker advantage must be taken into account.

5.2.6 "Dr Z's" System

Hausch, Ziemba, & Rubinstein (1981) (HZR) use the Harville (1973) formula to calculate probabilities and develop a strategy for betting in the place and show markets, based on amounts bet on the win pool. A divergence in expected returns between the win and show pool would suggest profitable betting opportunities.

Hausch, Ziemba, & Rubinstein (1981) developed a non-linear model (HZR system) to test for efficiency of place and show pool compared to the win pool, relying only current price data. Win probabilities were first estimated based on the monies bet on different horses in the win pool and the Harville (Harville, 1973) model utilised to predict ranking probabilities. The probabilities were then compared with actual amounts bet to determine profitable betting opportunities. The HZR system yielded a return of 48.6% and increased final wealth increased by $1,216 to $3,716 when implemented over a 9-day period during the summer 1980 racing season at Exhibition Park in Vancouver. Ninety races were examined and twenty-two theoretical bets placed, confirming the existence of market inefficiency.

Lo, Bacon-Shone, & Busche (1995) extended the HZR system and proposed a model of computing ranking probabilities that closely approximated those based on the Henery (Henery R. , 1981) model. The Lo, Bacon-Shone, & Busche approach assumes independent normal distributions for the running times in contrast to the HZR system which assumed independent exponential distribution of running times.

$$\pi_i\pi_j\pi_k \equiv P\ (horse\ i\ wins, j\ finishes\ second\ and\ k\ third) = \frac{\pi_i\pi_j\pi_k}{(1-\pi_i)(1-\pi_i-\pi_j)} \qquad \text{EQ. 5-9}$$

Using data sets from the United States and Hong Kong, Lo and Bacon- Shone show improved profit over the HZR system for lower levels of risk using final betting data (assuming zero computational costs).

However, Sung and Johnson (2007) cast doubt on the reliability of rank order finishing data. Sung and Johnson (2007) noted that rank order finish data beyond position two could not be relied upon.

They noted that there are incentives for jockeys to secure a poorer finish position on non-winning horses than they might be able to achieve. This will have the effect of reducing the public's perception of the ability of the horse, which could result in higher odds being available for the owner on the horse in subsequent races. Lessman, Sung, & Johnson (2009) provides further evidence of the unreliability of the rank ordered finishing positions.

5.2.7 Logit Model

Asch, Malkiel, & Quandt (1984) used a logit model and price-related variables to determine market efficiency.

$$W_{ij} = \beta' x_{ij} + \mu_{ij} \qquad \text{EQ. 5-10}$$

Where

W_{ij} Represents a measure of "winningness" of the horse - the higher the value of W_{ij} the higher the winning probability of the horse

β' Represents a vector of coefficients as determined by the logit model

x_{ij} Represents a vector of observable price variables

μ_{ij} Represents a vector of error terms

Asch, Malkiel, & Quandt's (1986) pricing model showed net positive returns on both place and show bets and on horses with highest winning probabilities, suggesting that all price information as indicated by the win-pool was not efficiently reflected in the place and show pool. However, in a subsequent recalibration of the model Asch, Malkiel, & Quandt (1986) noted bettors would be unlikely be able to consistently generate excessive profits, and that bettors on the whole were rational and markets efficient. In addition, Asch, Malkiel, & Quandt's model had operational implementation limitations as final odds were not known until after betting had closed.

Studies show that prices are a good predictor of race results. Johnson, Jones, & Tang (2006) constructed a CL model to show that prices incorporate complex information not directly discernible by the betting public. Bookmaker odds data from 1200 "flat" races at 41 different racetracks in UK over the period April 1998 to June 1998 were included in this study. Johnson, Jones, & Tang (2006) utilised time sequences of prices: Variables employed included price volatility, final odds, odds movements from start to market close and a predictor variable to capture late changes in betting (to allow for asymmetric information). The study found that prices had tradable information and captured significantly more information than closing odds alone, suggesting that the price curve model could be exploited to earn positive returns. A Kelly strategy yielded positive returns. However, Johnson, Jones, & Tang (2006) noted that implementation required significant effort in terms of real-time data capture and that the small positive growth rate of wealth did not merit the potential risks.

In summary, racetrack betting studies suggest that the market exhibits weak form efficiency and that bettors would unlikely have profitable bets. The next section discusses contemporary betting models and semi-strong form efficiency.

5.3 Semi-Strong Form Efficiency - Contemporary Betting Models

The contemporary betting models are based on the premise that objective probabilities of horses' winning abilities are predictable from fundamental variables and prices. A statistical model that objectively determines these winning probabilities could therefore exploit divergences in public estimates of odds. Profitable wagering strategies could then be deployed to maximise returns by betting on those horses where model probability estimates are a better guide to winning probabilities than final odds offered in the bookmaker or tote market.

In the racetrack betting literature, the CL model (McFadden, 1974), a linear regression model, has been the primary model for estimating probabilities from fundamentals as this takes into account *"competition within a race"*. The CL models in later studies have been supplemented with non-linear modelling techniques; for example, classification and regression techniques (CART) such as Support Vector Machines and Random Forests to identify information in fundamental variables that have not been appropriately priced. The next sections discuss the CL and non-linear models.

5.3.1 Linear Models - Logistic Regression – The CL and Probit Models

The logit model is a choice modelling linear regression that has been used in number of qualitative studies, where the dependent variable is expressed as a binary outcome, 1 or 0. For example, whether an individual would prefer to travel by bus or car, given the individual's attributes such as disposable income, cost of travel and related attributes that influence the individual's decision-making process. The output is then expressed as probability between 0 and 1 of whether the outcome will occur. In this case the individual making the choice of transportation mode and the respective probabilities.

The advantage of the logit model is that it transforms the problem of predicting probabilities to that of predicting the odds of events occurring. The probabilities sum to one over all choice alternatives and guarantee that probabilities estimated will always lie between 0 and 1. This allows a ready interpretation of the selective probabilities in terms of the relative winning probabilities.

The CL model (McFadden, 1974) is an extension of the logit model where all individuals are subject to the different alternatives before making the choice. In other words, it is choice-specific. The CL model has been the mainstay for constructing fundamental racetrack models to forecast winning probabilities as this statistical technique takes into account competition between horses within a race.

In the CL model (McFadden, 1974) the variables or choices are modelled in terms of characteristics of the alternatives rather than attributes of the individuals. Compared to the binary logit, the McFadden logit model takes into classification relative to the remaining runners in the population sample and is given by:

$$Prob\ (Y = 1\ |x) = \frac{\exp(x'\beta)}{\sum_{j=1}^{J} \exp(x'\beta)} \qquad \text{EQ. 5-11}$$

Where

β_K is estimated using maximum likelihood parameters.

The choice probability is relative to other runners in a particular race. The maximum likelihood function is used to estimate variable coefficients which yield consistent parameter estimators and are asymptotically efficient with minimum covariance. The likelihood function is expressed as:

$$f(y_1, \ldots\ldots. y_n|\theta) = \prod_{i=1}^{n} f(y_i|\theta) = L(\theta|y) \qquad \text{EQ. 5-12}$$

And the log of the likelihood function given by

$$\ln l(\theta|y) = \sum_{i=1}^{n} \ln f(y_i|\theta) \qquad \text{EQ. 5-13}$$

Statistical/Econometric software, for example SPSS, STATA, LIMDEP have in-built functions for logit regression using maximum likelihood, and where output is normally the log-odds ratio. The R-squared function is then utilised as an effective measure of goodness of fit of the logit model where

R-squared value closer to 1 (compared to zero) indicates that the model is better able to explain the outcome.

Bolton & Chapman (1986) utilised ten jockey and horse-related fundamental variables to predict winning probabilities and develop a profitable wagering system. The specific form of the multinomial logit model employed in Bolton & Chapman, 1986 was, as follows:

$$U_h = \theta_1 LIFE\%WIN_h + \theta_2 AVESPRAT_h + \theta_3 W/RACE_h + \theta_4 LSPEDRAT_h + \theta_5 JOCK\%WIN_h + \theta_6 JOCK\#WIN_h + \theta_7 JMISDATA_h + \theta_8 WEIGHT_h + \theta_9 POSTPOS_h + \theta_{10} NEWDIST_h + \varepsilon_h \quad \text{EQ. 5-14}$$

Where

U_h	Represents overall value of the horse h
$LIFE\%WIN$	Represents percentage of races won in the past two years and a proxy for overall winning potential
$AVESPRAT$	Represents average speed rating for the last four races for each horse and a proxy for the overall competitive level
$W/RACE$	Represents winnings ($000's) per race in the current year – a proxy for overall winning potential
$LSPEDRAT$	Represents track-adjusted speed rating for the previous race in which the horse ran – a proxy for recent competitive level component of past performance
$WEIGHT$	Represents overall carrying weight of the horse
$NEWDIST$	Represents horse running at a new distance
$POSTPO$	Represents post position
$JOCK\%WIN$	Represents percentage of winning rides in the current year
	Represents number of winning rides in the current year
$JMISDATA$	Represents value of one when jockey variables are missing, and zero otherwise.

Two hundred race observations for 3-year olds from 5 different US race tracks were utilised to estimate the model coefficients. Due to the limited sample size, a data explosion technique to a depth of three was employed. This effectively increased the sample size to 597. The data explosion with a depth of three works as follows; the CL is deployed, winner is identified and then removed from the sample. The CL model is then redeployed, winner is identified (would be the horse finishing second)) and then removed from the sample, before redeploying the CL for the third time (to identify the horse finishing third). Total sample therefore is equal to 200 + 199 + 198 = 597.

The model results were mixed and the betting strategies yielded negative returns, except, when long-shots were excluded and a modest profit of 5% was then reported. Bolton and Chapman (1986) concluded that that it was most likely due to the limited dataset (200 races) used to determine model parameters. They argued that a larger sample size would probably needed to assess the effectiveness of the logit model.

Bolton & Chapman (1986) attempted to employ a variety of wagering strategies. However, they concluded that Isaac's strategy would perform considerably worse than a random betting strategy. Four separate wagering strategies on holdout samples were implemented. Rosner's Wagering Strategy (1975) led to average returns of -14.1%; a Multiple Units Strategy led to average return of -21.8% and a Single Bet Per Race Strategy led to average return was 3.1% with returns ranging from 3.1% to 38.7%.

Performance was also much worse when true winning probabilities were known, with significant variations noted with modest probability changes. Isaacs wagering strategy produced an average return of -39.5% with returns ranging from -2.6% to -65.9% and a weighted average return of -27.8%. Bolton and Chapman (1986, p. 1052) concluded that the Isaacs strategy was unlikely to be profitable unless winning probabilities were highly accurate as the *"wagers will significantly lower the odds"*.

Chapman (1994) replicated the CL model using a much larger dataset of 2000 Hong Kong races from September 1985 to November 1991 and increased the number of independent variables from 10 to 20. The independent variables included constructs of transformed fundamental variables, including, for example, recency-weighted mean of past lengths beaten with distance normalized, recency-weighted estimated strength of other horses in this horse's past races where recent performance were weighted more than older performance in variable, regressions on all past performances of the horses or jockeys. A full history of each horse's past performance was used to determine the values of these constructs of fundamental variables. This contrasts with the Bolton and Chapman (1986) study where the independent variables were determined using a few races. Chapman (1994, p. 175) noted that 19 of the variables were statistically significant at the 5% level and that the *"signs on the estimated fundamental horse race handicapping weights are plausible and consistent with horse race handicapping theory and principles".* Sample size was exploded to a depth of two. Chapman (1994, p. 180) concluded that "*positive returns at the track are achievable with a sophisticated pure fundamental multinomial logit horse race handicapping model"* and that *"higher expected return may be achievable by including the log of the public's win probabilities"* when the number of variables was increased from 20 to 21 to include market prices 78.6% of the total variance was explained by the fundamental variables and 21.4% explained by public estimates. The study reported positive returns to a single unit bets strategy and returns in excess of 20% when long-shots with less than 4% winning probability were excluded. The Chapman (1994) study also suggests that prices, in addition to the fundamental variables, are significant in identifying winning probabilities.

Benter (1994), extended the logit modelling technique and implemented a two-stage CL model where fundamental probabilities were first calculated and then combined with log prices as a second step to determine final winning probabilities. The logic was that combining market prices took into account the possibility that fundamental information exists that could not be practically incorporated into a model. In addition, the existence of significant inside information will be reflected in the prices and this also needs to be considered. Benter (1994, p. 187) noted that for fundamental models to compete *'the public's opinion must be taken into account by a fundamental handicapping model, as the betting public is sophisticated'.* Benter (1994) included individual horse-specific variables as well as variables that impacted all horses, such as wet or dry surface conditions, to determine fundamental probabilities.

The final probability therefore was a combined probability, reflecting the judgement of two experts: the public and the fundamental model, as follows (Benter, 1994, p. 187):

$$c_i = \frac{\exp(\propto f_i + \beta\pi_i)}{\sum \exp(\alpha f_j + \beta \pi_j)} \qquad \text{EQ. 5-15}$$

(for $j = 1$ to N)

Where

c_i Represents combined probability estimate

f_i Represents log 'out of sample' fundamental probability estimate

π_i Represents log of public's implied probability estimate

Benter (1994) found that the combined model had a higher R^2 (0.1245) than public estimates (R^2 =0.1218) indicating that public had not fully utilised all available information.

The advantage of the Benter (1994) methodology compared to Bolton & Chapman (1986) is that the two-step implementation lends itself to application in a live betting environment, since the model will only require updating for a single variable (price), to determine final probabilities. Benter (1994,

p. 187) reported that *"Four of the five seasons resulted in net profits, the loss incurred during the losing season being approximately 20% of starting capital".* Bolton & Chapman's (1986) study, on the other hand, requires model calibration for fundamental variables and prices together, and this may not be practical in a live environment. However, both data sets suggest positive returns and the ability to extract tradable information, regardless of the variable processing methodology.

The question arises as to what should be the preferred methodology for modelling fundamental variables and prices. Intuitively there may be a slight preference for the Benter (1994) approach given the evidence of practical implementation. However, the two-step results show that the improvements in R^2 (0.1245) are marginal (0.1218); an overall improvement of 2%. In addition, given the advancements in computer processing speeds the difference between one-step and two-step approach may not be relevant.

In a subsequent study, Sung and Johnson (2007) compared the effectiveness of a single and two-step approach in calibrating fundamental variables and prices and found that both approaches confirmed that markets were semi-strong form inefficient. However, the two-step model captured more fundamental information and a betting strategy based on the predictions of the two-stage model resulted in significantly larger profits (17.3%) compared to a profit of 0.96% from a one-stage model.

The final probability in the two-stage model for horse I in race j is determined as follows:

$$P_{ij}^{e} = \frac{\exp(\propto \ln(p_{ij}^{s}) + \gamma \ln(p_{ij}^{f}))}{\sum_{i=1}^{n_j} \exp(\propto \ln(p_{ij}^{s}) + \gamma \ln(p_{ij}^{f}))} \qquad \text{EQ. 5-16}$$

Where $\propto$ and γ are parameters estimated using maximum likelihood procedures.

However, a drawback of the two-stage model is that model estimation requires sample data to be split into two; one to estimate the fundamental model coefficients and the second to overlay the fundamental model with prices to take into account the subtle relationships between fundamental variables and prices. The disadvantage is that the sample size on which the fundamental model is estimated is halved. The next section provides a summary of key principles of effective CL modelling that have emerged in the literature.

5.3.2 Key Principles of Effective CL Modelling

A number of key principles have emerged with respect to modelling in racetrack betting markets, in addition to the deployment of a CL model to take into account the within competition element in a race.

It is clear that transformed variables, as shown in the Chapman (1994) study, are able to extract tradable information. Subsequent studies have also confirmed the advantage of variable constructs and transformations. Edelman (2003, p. 107) utilised differences in weight-corrected performances and noted that the models produced statistically significant forecast values compared to bookmakers' predictions, producing *'clear out-of-sample profit'*. Variables included changes in adjusted beaten lengths for the same horse moving from race i to j and change in carried weight associated with each of the races that were run between 1,000 metres and 1,200 meters. Weighted least-squares was used to determine the variables over 1309 races prior to the conditional logistic regression being deployed. Edelman found that restricting runners with favourable values of the "Competitive Form" variable that were two-to-one or shorter produced profits of twenty-seven percent.

Lessmann, Sung, & Johnson (2007) successfully used a race-wise standardisation procedure, whereby the continuous variables were standardised to zero mean and standard deviation of one before applying a forecasting model, to avoid numerical difficulties with different value ranges. The second key principle that emerges is the separation of prices from fundamental variables in the model inputs yields a better performance, as noted in the comparative study by Sung & Johnson (2007). This was also confirmed in a later study by Sung & Lessman (2012) who found that prices "*masked"* the effect of fundamental variables, as prices were dominant predictors. A third principle is the emergence of multi-staged processing of variables: *"a first stage model is developed to process fundamental variables to produce a score which reflects a runner's ability based on this fundamental information* (Sung & Lessman, 2012, p. 170)*"*

The next key developments have been enhancement of CL models by combining non-linear techniques to extract information from fundamental variables to improve model performance. It is clear that there are limitations to the linear logit models. In particular, these models cannot capture non-linear relationships between the dependent and independent variables, unless explicitly specified as input variables. The maximum likelihood estimation technique also has its drawbacks. Vapnik (1995) noted a risk of over-fitting high dimensional data. Edelman (2003) found that applying maximum likelihood method to win-loss outcomes resulted in information loss with relative performance of remaining runners in each race being ignored. Therefore, there is the likelihood that less obvious, but potentially important relationships, may possibly be ignored.

The CL model assumes that error terms in estimated horse performance, although independently and identically distributed, "*follow a negative double-exponential distribution rather than normal distribution."* (Edelman, 2003, p. 1043). This results in the CL model having discrepancies when data is exploded for the second and third time for estimating the second and third finishing position respectively.

Machine learning algorithms and classification techniques to enable capture of non-linear relationships amongst variables have been considered in racetrack betting literature. These have been employed as a first step, prior to implementation of the CL model. These techniques have included Support Vector Machines, Model Stacking and Random Forests and are discussed below. However, prior to discussing non-linear techniques, a variation of the CL model, the conditional probit is discussed.

5.3.3 Conditional Probit Model

The conditional probit model is a variation of the CL model with a difference in the underlying assumption of the distribution in error terms. The probit model assumes that the error term is normally distributed whereas the logit model assumes that the error terms are exponentially distributed. The advantage of the conditional probit over CL is that the error specification allows correlations between the errors. The disadvantage is the relative computing intensity required. The CL model is relatively simple and the resulting likelihood function, even for a large number of choices, is produced with limited computing processing power. By comparison, the conditional probit model involves multiple integrals and takes a lot longer (in terms of computing time) for convergence to occur to yield estimates. The failure to converge within a reasonable number of iterations has motivated model researchers to use logit methods.

With a normal distribution assumption, the winning probability is of the form:

$$P_i = \int_{-\infty}^{\infty} \prod_{k \neq i} \varphi\,(u + \theta_{ti} + \theta_{tk})\emptyset\,(u)du \qquad \text{EQ. 5-17}$$

Where

$\varphi(.)$ and $\emptyset\,(.)$ Represents standard normal distribution function and density function respectively

and the expected return per dollar wagered is given by

$$Expected\ Return_i = P_i\ x\ Payoff_i - 1.0 \qquad \text{EQ. 5-18}$$

Where

P_i Represents estimated win probability for horse i

$Payoff_i$ Represents the amount returns for a one-dollar wager on that horse

Gu, Huang, & Benter (2003) employed a conditional probit model on Hong Kong racetrack betting data (1998-2000 racing season) and found that the conditional probit model *"enjoyed a modest but significant advantage over the CL model"*. These results confirm that the normal distribution assumption provides a better fit than the negative double exponential distribution of the error term of the logit model.

The study however noted that computation involved 10,000 iterations and the algorithm took a few hours to run. This significantly reduces the model's operational capability and therefore would have implementation issues in a live racetrack betting environment (given, that the model estimation took longer than the twenty minutes between races). A total of 6,726 Hong Kong races were analysed. The results showed that the actual number of wins was close to public estimates with no obvious biases.

Similar to earlier studies, Gu, Huang, & Benter (2003) observed that the log odds were a significant variable, confirming that public were sophisticated in the use of available fundamental information and correctly assessing differences in horses' abilities to determine winning probabilities. The R^2 for log odds was 0.1321 of the total R^2 of 0.1392. However, the incremental R^2 for the remaining fundamental variables (.0071), albeit a small contribution, was sufficient to provide the basis for a highly profitable wagering strategy, suggesting that the public did not fully utilise available fundamental information. The next section discusses the non-linear technique, Support Vector Machines.

5.3.4 Support Vector Machines

The Support Vector Machine (SVM) is a supervised-learning classification technique that finds a linear decision function through a non-linear mapping in a high-dimensional maximum margin feature space hyper-planes, called support vectors. The support vector machines construct a separating hyper-plane in that space, one which maximizes the margin between the two data sets, effectively a non-linear data classifier. The SVM learning method as a data mining technique has previously been utilised in studies to forecast financial time series (for example (Cao & Tay, 2002). SVM's have primarily been deployed as a first stage model to classify the horses into two groups in racetrack betting markets, in accordance with a pre-determined criterion, prior to a second-stage CL (Edelmen, 2007), (Lessman, Sung, & Johnson, 2009).

The SVM is based on the principle of Structural Risk Minimization Principle and seeks to minimize an upper bound of the generalization error (cf. neural networks which minimise empirical risks). SVMs therefore result in a better generalization than neural network learning machines (which minimizes training error) and avoids the problem of over-fitting of data.

Given a set of data points, SVM approximates the function using the following form:

$$f(x) = w.\emptyset(x) + b \quad \text{EQ. 5-19}$$

And where

$\emptyset$ Represents the high dimensional space which is nonlinearly mapped from input space x

And

w and b are estimated by minimising the regularised risk function

$$y(x) = sgn\,(\,(w^*.x) + b^* \quad \text{EQ. 5-20}$$

Where

w^* and b^* are determined by

$$min_{w,b,\delta}\,\frac{1}{2}\,\|w\|^2 + C\sum_{i=1}^{M}\delta_i \quad \text{EQ. 5-21}$$

$$s.t.: y_i\left((w\,.x_i) - b\right) \geq 1 - \delta_i, i = 1\,, \ldots \ldots .\,.\,, M$$

Where

$$1 \leq i \leq M$$

$$\frac{1}{l}\sum_{i=1}^{l} L_\varepsilon(y_i, f(x_i\,))$$

From an implementation perspective, training in SVM is equivalent to solving a linear constrained quadratic problem, with the number of variables limited to twice that of the number of training data points. In SVM, the solution to the problem is only dependent on a subset of training data points which are referred to as support vectors. Using only support vectors, the same solution can be obtained as using all the training data points.

Edelman (2007) utilised a regression-based SVM methodology to model the relationships between fundamental variables associated with horses' most recent performances and factors relating to the current race (e.g., prize money, weight carried), and horses' finish position. In other words, variables prior to the most recent past were excluded as it was assumed to be reflected in the previous performance. The resulting model was based on 200 races in Metropolitan races in Australia. The fundamental model was then combined with closing market prices using a CL model in the second stage. A Kelly betting strategy generated positive returns increasing wealth by 500% over the holdout sample of 100 races. Edelman noted that ordinary regression methods on the same data did not produce a profitable model, in comparison to regression-based SVM. Edelman's (2007) study utilised rank ordered finishing data during the model building process. However, subsequent studies question the validity of rank order finishing data used in the in the Edelman study. Sung and Johnson's (2007, p. 170) empirical findings that rank order finish data beyond position two cannot be relied upon as there may be a *"motivation for jockeys to secure a poorer finish position on non-winning horses than they might be able to achieve"*.

In a similar study, Lessman, Sung, & Johnson (2007) used a search engine inspired Normalised Discounted Cumulative Gain (NDCG) approach and employed Least-Squares-based SVM, without rank-ordering the horses with respect to the finishing positions for stage 1. This was used to develop fundamental variable model. The second stage included the deployment of the CL and adding normalised final prices. The empirical analysis was based on races run at Goodwood racetrack in UK between May 1995 and August 2000. The dataset included 556 races which was partitioned into 200 races for constructing the Least-squares SVM and 200 races for running the stage 2 condition logit with a final holdout sample of 156 races. A Kelly wagering strategy, without reinvestment was then deployed, based on the final model probabilities. The Kelly strategy yielded a return of 10.96% (cf the benchmark two-stage CL model, which produced a return of 1.75%). With reinvestments, the Least-squares SVM methodology yielded 112.20%. However, in comparison the two-stage logit model had negative returns of 16.53%, suggesting that the Least-squares SVM offered a significant improvement over the two-stage logit model.

In a later study, using the same data set and methodology (this time using SVM with CL in a two-stage model) Lessman, Sung, & Johnson (2009) confirmed that the SVM-classifier in a two-stage model was a significant improvement on the two-stage CL model. Returns of 30.58% without reinvestment and 642.65% with reinvestment were achieved.

A drawback with SVM is that it is a machine-learning algorithm and acknowledged as being "black box". This makes it difficult to pin the sources of improved performances: *"resulting coefficients of* *an SVM model are virtually impossible to interpret on their own, containing no discernible information about which input variables are the most influential, nor which combinations of variables interact"* (Edelmen, 2007, p. 333). The next section discusses Random Forests combined with CL model (RF/CL)

5.3.5 Random Forests

Random forest is a prediction model based on classification and regression trees (CART) methodology, where decision tree models or a collection of nodes are constructed and arranged in a hierarchical fashion. The nodes are then randomized by means of bootstrapping in a random subspace procedure to achieve diversity. Random forests (Breiman, 2001) overcome the opacity of SVM's, which are regarded as somewhat of a "black box".

"A random forest is a classifier consisting of a collection of tree-structures $\{h(x, \theta_k), k = 1, \ldots..\}$ *where the* $\{\theta_k\}$ *are independent identically distributed random vectors and each tree casts a unit vote for the most popular class at input x"* (Breiman, 2001, p. 6).

To construct a Random Forest model each decision tree is derived from a bootstrap sub-sample of size N, drawn from the original training dataset S with replacement. The decision tree is then grown from an independent bootstrap resample until all nodes contain observations no more than a pre-specified maximal node size. These trees when constructed only utilise a subset of all available variables. A single overall prediction is obtained by taking a weighted average over the tree predictions from the forest. The final classification decision is obtained by a "*majority vote*" on all the classification trees.

The key advantage of random forest is the bootstrap method (bagging) resolves data over-fitting problem, where there is enhanced in-sample performance of data but poor out-of-sample performance. Bagging has been shown to be able to effectively reduce prediction variances for unstable prediction rules as it averages results from bootstrap resamples and hence reduces the prediction variance. Random Forest is also able to process large data sets, therefore providing a fast and scalable solution. Lessmann, Sung, & Johnson (2010) adapted the random forests classifier to predict race outcomes and found that predictions derived from the model outperformed those from the two-stage CL model, which was used as benchmark and yielded higher profits with a Kelly strategy.

Lessmann, Sung, & Johnson (2010, p. 519) cite the *"ability to identify the importance of individual variables and being able to discern nonlinear interactions between variables"* as the edge the CART process has over CL models (for which non- linear variables must be specifically identified).

The empirical analysis was based on a dataset of 1000 races run at Hong Kong racetracks between 1st January 2005 and 26th December 2006. A set of forty performance related fundamental variables from previous races and preferences relating to the current race (e.g., distance, going, etc.) were included as input variables. A three-stage forecasting process was implemented using random forests and the CL model

First, a subset of R1 races (first 500) was drawn from the dataset to build a random forest forecasting model using only fundamental variables. A specific value of 3 was selected to define a splitting rule to determine which runners should enter which sub-node to maximize the homogeneity within the sub-nodes. This branching continued until all nodes contained only winning or losing horses, leading to a hierarchical, tree-like structure with terminal or leaf nodes. The final tree was then converted into a set of rules that facilitated prediction.

Prices were included in the second stage and random forest used to produce an ability forecast for the horses in the remaining R – R1races (501-1000). Finally, the coefficients of a CL regression model were estimated by means of maximum likelihood function over the same R – R1 second stage races, considering two independent variables: market odds and the random forest based ability forecasts.

Returns were generated by employing a Kelly-betting strategy over the holdout test races. This was used as the primary indicator of forecasting accuracy and then detailed analysis was performed to identify the sources of profitability. The benchmark for performance was a two-stage conditional model used in a previous study. The Kelly-wagering strategy (without reinvestment) based on the two-stage random forest model yielded a significantly higher return compared to the two-stage CL model (20.26% and 8.84%, respectively). The P-value of 0.0085 was significant and greater than for the CL model p-value of 0.1354. Lessmann, Sung, & Johnson (2010) concluded that the two-stage random forest model succeeded in outperforming the market by extracting information from the fundamental variables that had not been fully discounted in market prices by the betting public. The (adjusted) R^2-statistic of the random forest model exceeded the two-stage CL model by 1.6% (0.1296 c.f. 0.1276).

However, it is important to note that both methodologies correctly identified a similar number of true winners (125 and 126, respectively), number of bets on winning horses (230 and 234) and successful bet rates (5.1%). Where the models disagreed was on the relative importance of fundamental variables, when these were ranked. For example, the top two ranked variables in the random forest model were variables relating to the age of the horse (No. 1) and post position bias (No.2). By contrast the CL model regarded "today's" race condition and variables relating to the jockey's past performance as the top variables to identify winners and losers. The age variable was ranked last by the CL model.

Table 5-1 below details the differences in variable ranking between Random Forests /CL and the two-stage CL model.

Table 5-1 Difference Between RF and CL Ranking

(Lessmann, Sung, & Johnson, 2010, p. 532)

Table 4
Ranking of variable categories according to the normalized mean importance score.

	RF	CL	DR[a]
Variables related to the horse's age	1	9	8
Post position bias variable	2	5	3
Conditions of today's race	3	1	2
Variables related to a horse's most recent performance on and off the track	4	8	4
Variables related to a jockey's past performance	5	2	3
Variables related to a horse's past performance	6	4	2
Variables related to days since last ran	7	3	4
Variables related to the weight carried by the horse in both current and past races	8	7	1
Horse's specific preferences (e.g., blinkers, track, distance, surface, jockey)	9	6	3

[a] Difference in rank.

Although the models produced similar outcomes, the random forests methodology combined with the CL model, predicted a higher percentage of losing horses compared to the two-stage CL/CL model which had a higher probability of winning horses (59.3%). This suggests that the CL/CL model potentially put more capital at risk as greater funds would be bet on these losing horses, hence a lower return in comparison to the RF/CL.

It is clear however from the table above that the degree of importance the two models place on the fundamental variables are not inconsistent, based on variable rankings. The next section discusses a model using Stacking in stage 1 and CL in stage 2.

5.3.6 Model Stacking

Model Stacking is a machine learning methodology for combining models to achieve greater predictive accuracy and to reduce the generalization error rate (Wolpert, 1992) . The aim here is to obtain a higher level of out-of-sample generalisation accuracy compared to learning/training sample accuracy, using a cross-validation technique.

Lessmann S. , Sung, Johnson, & Ma (2012) developed an ensemble learning technique for the racetrack betting markets using CL and stacked regression as a pooling mechanism and found that that stacked regression achieved a better return performance compared to the two-stage logit model. A library of base forecast models were first developed utilising modelling techniques that had previously identified in literature. These were then combined in a second stage to take into account the "*strength and diversity*" (Lessmann S. , Sung, Johnson, & Ma, 2012, p. 164) of the individual models for optimum results. The idea being that a model's inherent weakness is compensated by the strength of another model when multiple models are combined.

Model weights were then determined using the Newbold and Granger (1974) approach as follows:

$$w_s = \sum_{s=1}^{T}\sum_{i=1}^{N}(W_i - p_i^s)^2 \,/ \sum_{i=1}^{N}(W_i - p_i^s)^2 \qquad \text{EQ. 5-22}$$

Where

w_s Represents the base forecast weights

T Represents the size of the model pool

p_i^s Represents the winning probability estimate of model s for runner i

W_i Represents the winningness index

Each model then represented an "independent variable" for the second-stage. A model weighting was given dependent on the contribution to the forecast predictions determiend by a generalised CL stacking model, as follows:

$$p_j^i = exp\left(\sum_{s=1}^{s}\beta_s \,.\, \gamma_{is}^j\right) / \sum_{i=1}^{m_j} exp\left(\sum_{s=1}^{s}\beta_s \,.\, \gamma_{is}^j\right) \qquad \text{EQ. 5-23}$$

Where

p_j^i Represents the winning probability for runner i in race *j*

s Represents the number of base models in the library

β Represents vector of coefficients which measure the relative importance of input variables

γ_{is}^j Represents the prediction of the base model s

m_j Represents the number of runners in race j

The model combination procedure involved starting with the most accurate base prediction model and then *"adding one additional prediction to the ensemble assessed by means of a Log-Likelihood-Ratio(LLR) test"* (Lessmann S. , Sung, Johnson, & Ma, 2012, p. 167) for increase in R^2 as follows*:*

$$R^2 = \frac{LL^{CL}}{LL^0} \quad \text{EQ. 5-24}$$

Therefore, base models that did not improve predictability, as determined by change in R^2, were discarded. The study was based on a dataset of 4276 horseraces run in Hong Kong between 6th September 1998 and 8th July 2008 and 40 independent variables were employed. Eight base prediction methods, using regression and classification techniques were deployed. Table 5-2 below details the prediction methods used to develop the base model.

Table 5-2 Base Prediction Methods

Prediction Method	
1	Multivariate linear regression
2	Support vector regression
3	Random forest regression
4	Stochastic gradient boosting
5	Logistic regression
6	Support vector classification
7	Random forest classification
8	AdaBoost

571 individual forecast models were then produced, as detailed in Table 5-3, below.

Table 5-3 Individual Base Forecast Models

(Lessmann S. , Sung, Johnson, & Ma, 2012, p. 169)

Table 2
Summary of individual forecasting models.

Modelling objective	Prediction method	Models per method	Meta-parameters
Discrete choice model			
Probability of winning a race	Conditional logit	1	N.a.
Regression models			
Runners' normalized finishing position	Multivariate linear regression	1	N.a.
	Support vector regression	270 ($5*6*9$)	Error-insensitive tube around (ε) [$\log_2(\varepsilon) = -5, -4, \ldots, -1$] Regularization parameter (C) [$\log_2(C) = -5, -4, \ldots, 0$] Width of radial basis kernel (σ) [$\log_2(\sigma) = -14, -13, \ldots, -6$]
	Random forest regression	25 ($5*5$)	No. of decision trees [100,250,500,1000,2500] No. of variables selected per split [3,6,12,18,24]
	Stochastic gradient boosting	8	No. of trees within ensemble [10,50,100,200,500,1000,1500,2000]
Classification models			
Probability of runner being a winner AND Probability of runner finishing within top- three	Logistic regression	2	N.a.
	Support vector classification	198 ($2*9*11$)	Regularization parameter (C) [$\log_2(C) = 4, 5, \ldots, 14$] Width of radial basis kernel (σ) [$\log_2(\sigma) = -8, -7.5, \ldots, -4$]
	Random forest classification	50 ($2*5*5$)	No. of decision trees [100,250,500,1000,2500] No. of variables selected per split [3,6,12,18,24]
	AdaBoost	16 ($2*8$)	No. of trees within ensemble [10,50,100,200,500,1000,1500,2000]

The CL stacking model was then employed to combine the models. The likelihood-ratio based model selection produced the highest returns of 20.3% compared to the two-stage CL benchmark model, which produced a return of 10.84%. Market odds were identified as a significant variable. The simple weighted average model underperformed the two-stage CL benchmark model, with a rate of return of - 8.4%. Lessmann S. , Sung, Johnson, & Ma (2012, p. 164) concluded that combining models *"statistically and economically accurate forecasts which are superior"* to single stage models and that *"previous studies which employ a single forecasting model to examine betting market efficiency have overestimated the degree to which information is discounted in market prices."* This finding therefore suggests a modelling process where base models are developed first and then combined in the next stage of the modelling development. Such an approach is shown to be superior to a single step linear model.

5.4 Summary and Key Findings Relating to Modelling Winning Probabilities in Racetrack Betting Markets

A racegoer requires the ability to objectively determine a horse's winning probability and to develop a betting strategy to optimise returns. In addition, the persistence of the favourite-longshot bias in racetrack betting markets suggests that prices must consider the behaviour of market participants;

market makers who set the prices and bettors who may place bets in a biased manner. Studies in racetrack betting suggest that there is opacity with respect to fundamental information and bettors are not able to discern potentially tradable price information. This therefore suggests that markets are not semi-strong form efficient. The Kelly criterion (or fractional Kelly) is shown in the literature to be the optimal wagering strategy to exploit difference in predicted winning probabilities and the odds-implied probabilities. This strategy maximises the logarithmic growth of capital, compared to a basic strategy of a simple unit bet

Modelling methods in racetrack betting have evolved from an analysis of markets odds to determining odds objectively from fundamental variables. These models have included both linear and non-linear modelling techniques, with a final step of the CL model, where prices are input as a final variable.

The contemporary models include CL/CL, SVM/CL, RF/CL, SVR /CL, MS /CL where the first stage identifies "tradable" fundamental information. The second stage CL considers direct competition within a race (as this is a key element in horse races). Prices are taken into account, as public estimates of winning probabilities are shown to incorporate valuable information concerning winning probabilities A consistent finding of these models in racetrack betting models has been the success in identifying information inefficiency and models showing positive returns.

Classification and learning techniques do not consider direct competition within a race and this is a key element in horse races. As a result, effective implementation of CART requires combining these non-linear approaches with the CL models to take into account the element of within race competition.

Support Vector Machines and Random Forests have the advantage of being able to process non-linear information, compared to logit models where non-linear variables require specification. These methodologies have yielded significantly higher returns compared to the two-stage CL model. However, support vector machines are virtually "black boxes" and therefore there is limited traceability to identify the sources of model performance.

On the other hand, the strength of the Random Forest procedure is the ability to process large sets of data efficiently and they have a built-in bootstrap procedure. Over fitting of data when using random forest technique is therefore a non-issue given that bootstrapping will optimise the variable coefficients. However, a puzzling fact has been that the random forest procedure ranks the importance of variables significantly differently to the CL model. Intuitively one would expect that fundamental variable rankings would hold (relatively) regardless of the model deployed.

The attractiveness of securing larger returns implies that support vector machines and random forests should be the preferred modelling approach when compared to the CL model. There is however, a lack of collaborative empirical studies in racetrack betting to confirm the superiority of CART models over the CL model. It has also been acknowledged in Lessmann, Sung, & Johnson (2009) that more empirical analysis utilising other databases is required to confirm the stronger performance of the random forest methodology or the SVM methodology, compared to the CL models.

CL models, on the other hand have been the most consistent performers for prediction analysis, with studies extending over a number of decades confirming their predictive ability. In addition, these studies were conducted in different racetrack environments (Hong Kong: (Gu, Huang, & Benter, 2003), (Chapman, 1994), (Australia: (Edelman, 2003), UK: (Lessmann S. , Sung, Johnson, & Ma, 2012) confirming the validity of the CL methodology.

A significant feature in the development process for race track betting models has been identification of the order in which variables are calibrated by racetrack betting models and the importance of these variables. Previous studies show that fundamental variables should be processed prior to including prices (given that odds have been identified as dominant predictors). Processing fundamental variables prior to inclusion with price variables maintains a level of independence when determining model coefficients and ensures that information content in fundamental variables is not "overwhelmed" by market prices (in contrast to the single stage model where fundamental variables are calibrated alongside prices). The fundamental variables are then added as a variable in the second stage of the two-step model.

What is clear therefore is that the performance of multi-staged processing models is consistently better than single-stage models in which fundamental variables and prices are simultaneously calibrated. This is illustrated by the two-step CL model (where prices are incorporated at a later stage of model calibration), and in the ensemble learning methodologies (where variables are pre-processed utilising a CART technique prior to input into a CL model. Where variables are pre-processed, returns are generally highly significant (for example (Edelman, 2003), (Lessmann, Sung, & Johnson, 2007)) indicating both variable pre-processing and staged modelling will improve information extraction from fundamental and price variables.

The two-stage model requires that the training data set is split, which immediately halves the data available for calibrating the model variables. This could be a potential drawback as a larger data set is required to implement two-stage models. However, one could potentially achieve a similar result and extract maximum information form fundamental variables by ensemble learning techniques such as the Bayesian Model averaging technique. These do not require in-sample data to be further split and reduce sample sizes. Model coefficients could be independently determined on the same data set and then combined using a model averaging technique. CL could then be deployed in a racetrack betting model to take into account within race competition, given that the primary objective is to identify potential winners and winning probabilities.

In summary, therefore, three clear modelling principles emerge from racetrack betting literature that could be applicable to financial markets for testing price efficiency:

First a multi-staged modelling process is the preferred approach, as this extracts maximum information from fundamental variables and prices. Variables matter and an effective modelling process must take into account both fundamental variables and prices. Fundamental variables and prices determine the true winning probability of horses. Variable pre-processing adds information. Prices have to be included in a model to allow for deficiencies in the fundamental model, given that all available information is unlikely to be calibrated by the model (for example, some information is revealed in real time in the market which could not be included in fundamental data (e.g. how fit the horse is for today's race)). In addition, the information held by the sophisticated betting public can only be discerned from market prices and betting behaviour. Therefore, clearly identifying and optimising the relevant fundamental variables and including prices is integral to an effective modelling process.

Second, the statistical model of choice for prediction is CL as this takes into account the relative strength of competition within the race, maintaining the relationship amongst the runners. Although ensemble learning techniques enhance the forecasting accuracy of the winning probability, the variables are required to be processed in a CL model as a final step. In addition, the CL model has been validated across different time periods and environments and this is not the case with ensemble learning techniques.

Finally, an effective wagering strategy needs to be implemented to optimise payoffs. The preferred approach in the literature is the Kelly strategy as this maximises the log growth of wealth. The next section discusses the research methodology which will be applied throughout the rest of this thesis.

Chapter 6: Research Methodology

The main aim of this chapter is to develop and present the appropriate research design and methodology that could be adapted from racetrack betting markets and applied to test for semi-strong form efficiency in financial markets. This chapter therefore details the research design and methodology adopted to examine price efficiency of fundamental information in financial markets. This chapter is organised as follows; Section 6.1 discusses the research design principles and paradigms to identify the appropriate methodology for evaluating real world data. Section 6.2 formulates the hypothesis and the research questions to be answered. Section 6.3 details the research data collection for analysis. Sections 6.4 to 6.16 describe in detail the modelling methodology, explaining modelling differences and refinements that are needed to apply racetrack modelling techniques to financial markets for empirical analysis, including calculations for determining the dependent variables and independent variables. Sections 6.17 and 6.18 details model performance measures employed to validate the empirical results and hypothesis. Finally, section 6.19 provides a summary for this section.

6.1 Real World Research Design and Paradigms

Real world research with real data in a naturalistic setting provides a much richer insight into outcomes of decision making under uncertainty, compared to experiments in a laboratory environment (Bruce & Johnson, 2003). However, the diverse ways of viewing and interacting with the surrounding environment and the different beliefs, influence the approach and conduct of research. These diverse views or philosophical assumptions are, in essence, a system of beliefs or paradigms that provide a guide as well as context in which research is to be organised.

Research paradigms in literature have generally been viewed from two philosophical perspectives. Ontology; what is reality and how existence can be understood or Epistemology; what is valid knowledge and how can this knowledge be obtained. These two views are underpinned by two positions; positivism and interpretivism. Positivism is associated with the process of analysis and drawing conclusions from observations made or quantitative research. Interpretivism on the other hand is associated with how individuals behave in society or the surrounding environments or quantitative research. Burrell and Morgan (1979) identified four mutually exclusive sociological research paradigms which provide a framework for seeing and making sense of the social world and how these views of the social world are positioned. These paradigms Burrell and Morgan (1979, p.

23) state are *"very basic meta-theoretical assumptions, which underwrite the frame of reference, mode of theorising and modus operandi of the social theorists who operate within them".*

Figure 6-1 (Burrell & Morgan, 1979, p. 22), below illustrates principal dimensions for social theorizing and viewing reality.

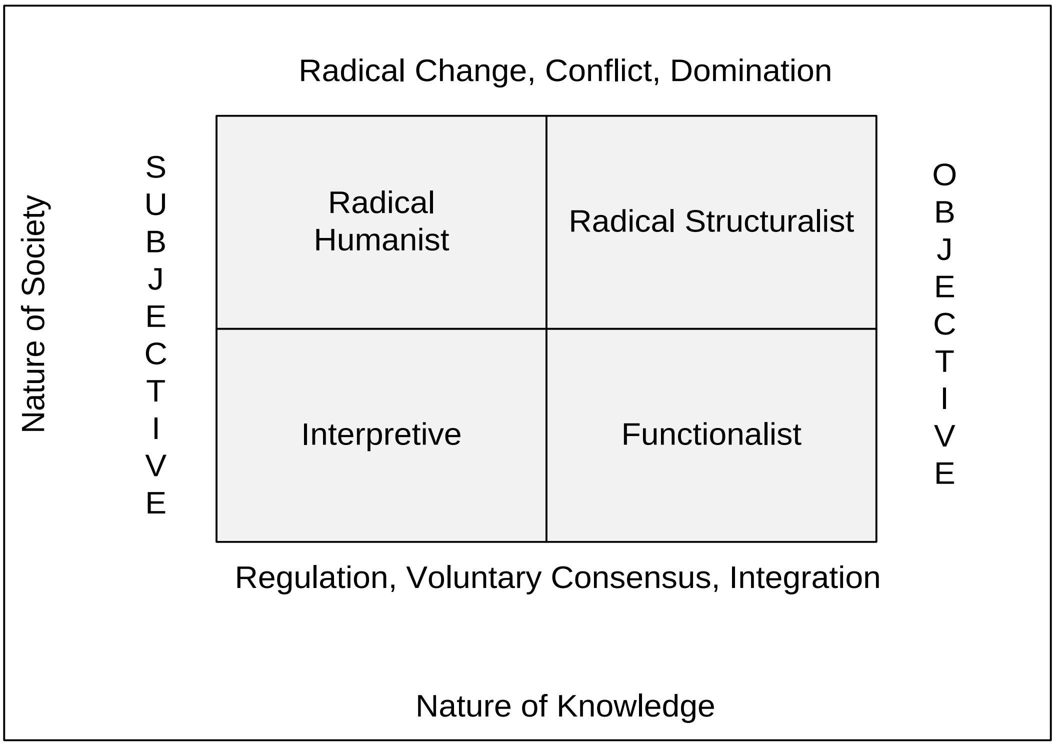


Figure 6-1 Principal Dimensions for Social Theorizing and Viewing Reality

Figure 1 suggests that knowledge and society and can be viewed subjectively as well as objectively. Burrell and Morgan (1979) suggest there are two distinct views and interpretations of the nature of society; regulatory view, which concerns with maintaining the status quo, social order, consensus, social integration and cohesion, solidarity, need satisfaction and actuality bringing to unity. The other view is radical change sociology, which focuses on structural conflict, modes of domination, contradiction, emancipation, deprivation, and potentiality leading to radical change. These subjective and objective assumptions on the nature of society, and the dimensions of radical change versus regulation result in the creation of the four mutually exclusive paradigms (as shown in Figure 1 above); Radical Humanist, Radical Structuralist, Interpretive and Functionalist.

The *radical humanist* paradigm, according Burrell and Morgan (1979), is a sociology of radical change from a subjectivist standpoint and this perspective places central emphasis upon human consciousness. Ardalan (2003, p. 729) suggests that in the radical humanist paradigm "*reality is socially created and sustained"*, where "*order that prevails in society is regarded as instruments of* *ideological domination"*. Radical humanist researchers seek to change the social world through a change in consciousness. Solipsism, French existentialism, anarchist individualism and critical theory are the schools of thought associated with radical humanist. The *radical structuralist* paradigm, according Burrell and Morgan (1979), is a sociology of radical change from an objectivist standpoint and is therefore concrete. The radical structuralists concentrate upon structural relationships within a realist social world where society is viewed as a potentially dominating force. Scientific methods are utilised to find order that prevails in the phenomenon. According to Ardalan (2003, p. 729) "*radical humanist and radical structuralist research in academic finance is non-existent"*

The *interpretive paradigm,* according Burrell and Morgan (1979, p. 28), is concerned with *"understanding the world as it is, to understand the fundamental nature of the social world at the level of subjective experience"* is a more subjective. Here, the world is seen as a process created by individuals that believe there are *"shared multiple realities that are sustained and change* (Ardalan, 2003, p. 725)*".* The interpretive paradigm believes that scientific knowledge is socially constructed and sustained and the significance only understood within its immediate social context. Interpretive finance enables researchers to examine aggregate market behaviour together with ethical, cultural, political and social issues and is based on the beliefs that no universally valid rules of finance and financial management exist.

The *functionalist paradigm* assumes society has a concrete existence following a certain order and these assumptions lead to objectivity resulting in true explanatory and predictive knowledge of reality. According Burrell and Morgan (1979, p. 26) functionalist paradigm provide *"explanations of the status quo, social order, consensus, social integration, solidarity, need satisfaction and actuality*" and *"generates regulative sociology".* In other words, how well are relationships in the real world explained and thereby answering the questions whether the world works in the manner way it has been described. The functionalist paradigm assumes that scientific theories can be assessed objectively by reference to empirical evidence, and these observations are independent of the researcher. The functionalist paradigm seeks to provide rational explanations of social affairs and emphasises the importance of understanding order, equilibrium and stability in society and the way in which these can be maintained. The functionalist view has been the dominant paradigm in finance/racetrack betting research given the two markets are regarded as places of concrete reality where individual behaviour is determined by the economic/betting environment. In both markets, price observations can be made to understand the relationships to the fundamental variables. A number of explanations result from this observation; the model perfectly explains the relationship between prices and the input variables and therefore describes the way in which the asset markets work. The model does not explain the relationships between prices and input variables suggesting that the model most likely requires to be improved in order to explain asset prices. The alternative explanation could also be that the model does describe the way financial asset markets behave, and that markets are inefficient, presenting opportunities.

Financial market research, as part of social science research, is based on the assumptions of the nature of the social world. Financial market and racetrack betting research almost exclusively adheres to the functionalist paradigm. Ardalan (2003) also confirms this view that finance research is predominantly from a functionalist point of view. In summary, the radical structuralist and radical humanist paradigms could be viewed as qualitative research methods as this involve understanding of human behaviour and the reasons that govern human behaviour. The functionalist paradigm on the other hand could be viewed as a quantitative approach as the research methodology requires concrete observations and measurements to test hypothesis, compared to a qualitative approach which may not involve statistical models or a collection of large sets of empirical observations where conclusions made are subjective.

The organising principle of this paper, and as noted in chapter 2 is based on the thesis that the paradigms of racetrack betting and financial markets could be viewed from the same prism, although these are two independent markets. Financial markets are organised around the principle that the existences of these markets facilitate an efficient allocation of economic resources and capital in society. Racetrack betting markets, on the other hand, are organised around gambling and provide a framework to understand behaviour and decision-making processes of individuals within the society. The hypothesis is that applying methodologies from racetrack betting will better explain how assets are priced and markets behave. The resulting methodology however is required to produce an objective outcome given the nature of financial markets, how assets are priced in these markets, the nature of real data in these markets and the functions financial markets perform in society. An interpretive view from of principles from racetrack betting market may not achieve or translate into the aim of better able to explain prices in financial markets, as this would be subjective and reduce validity. Substantive evidence is required to support the hypothesis that racetrack betting methodologies are applicable to financial markets, and given the functionalist form of these markets. This leads to the conclusion that most suitable and effective paradigm in which to conduct this research is a functionalist one, given the nature of financial markets and the existence of real world data. The functionalist paradigm and the objectives of this thesis in turn suggest that a quantitative design is the most appropriate methodology to achieve the aims, given the requisite is an analysis of real data. The next section discusses approaches to research design to determine the appropriate methodology for conducting research with real data.

6.1.1 Research Design

There are two fundamental research designs for experiments in the real world; fixed research designs which depend on quantitative data and statistical methods for making general assertions and flexible research designs that rely on qualitative data (Robson C. , 2002). Fixed research design is also referred to as quantitative research and similarly flexible research designs are referred to as qualitative research methods.

Although most disciplines of study utilise qualitative and quantitative methodologies, fixed research design in general have been used in management fields such as finance and economics and have almost exclusively been employed in racetrack betting market studies. Fixed design is also a standard approach in disciplines where "hard" or factual datasets are readily available for econometric or mathematical analysis. In comparison, flexible research designs are common in sociological studies to understand, for example, behavioural characteristics of a population or a phenomenon. Data sets for qualitative studies may be considered 'soft' or subjective as these are more unique requiring a relatively more resource intensive data gathering exercise such as interviews and questionnaires and interpretation of the data prior to any data analysis

Fixed research designs are regarded as scientific, requiring substantial amounts of pre-specification, development of a conceptual framework and extensive pilot studies to establish feasibility. A clear specification is needed to conduct the research, where tried and tested steps and procedures are followed. The scientific view is that knowledge is objective, gained from direct observation and largely based on quantitative data derived from the use of strict rules and procedures. Hypotheses are tested against these facts.

Robson (2002) discussed the comparative attributes of the two research designs and a summary of is provided in Table 6-1 below. Column 3 specifies the attributes of the research design for this thesis.

Table 6-1 Flexible and Fixed Research Design Attributes

	1	2	3
	Flexible Research Design	**Fixed Research Design**	**Attributes of this Thesis**
Data Attributes			
Data Type Collected	'Soft' data	'Hard' data	Hard Data
Collection Methodology	Questionnaire, Interviews	Readily Available Data sets	Readily Available Data sets
Population Sample Size	Generally, a small sample size	Large	Large
Data Analysis Process & Results			
Data Analysis	Statistical Package / other	Statistical Package	Statistical Package
Results Interpretation	Subjective	Objective	Objective
	Nature of Inquiry	Interpretative Positivism	Positivist
	Inductive through creativity and critical reflection	Deductive through inferences from data	Deductive through inferences from data
Research Findings	Outcome a Theory	Results are measurable	Results are measurable
	Focus on developing a new theory	Focus on testing a new theory	Focus on testing a new theory
	Broad generalisations from specific observations	numerical information derived from statistical interpretations of data	numerical information derived from statistical interpretations of data
Traditional Research Fields			
General Fields of Study	Sociological Studies; Behavioural, population, cultural demographics, study of a phenomenon	Finance, economics, Racetrack, sciences	Finance, economics, Racetrack

It is clear from the attributes detailed in Table 6-1 that this research could be firmly described as one of a fixed research design. The population sample size is both large and a real data set is employed. In addition, inferences will be made from the statistical outputs that will be produced, compared to theorising and making qualitative generalisations from the observations. The results are also measurable and deduced from data. In summary, the study of applying racetrack methodology to financial markets requires a functionalist view of the world and a quantitative design is the most suitable approach. The next section details the research question and hypothesis

6.2 Research Question and Hypothesis

It is postulated that modelling publicly available fundamental information in financial markets, by deploying a racetrack betting market multi-stage methodology, will confirm the existence of market inefficiency in the prices of UK equities. Financial market efficiency states that prices represent the best estimate of security returns and all publicly available information is priced in the security. Trading strategies therefore will not realise abnormal returns after taking into account transaction and holding costs. Consequently, the objective of this research is to determine whether market participants are efficient in processing publicly available fundamental information using techniques established in racetrack betting markets.

The null hypothesis is postulated as follows:

6.2.1 Hypothesis (H_0):

Hypothesis (H_0): Multi-stage racetrack betting modelling methodology when applied to the UK equities market will exhibit semi-strong form market efficiency for publicly available information.

There is a gap in finance literature with respect to modelling fundamental information using a multi-stage modelling methodology. In financial markets, a single-step linear and non-linear modelling methodologies have been utilised to calibrate fundamental variables. For example, Fama & French (2015) utilise a single-stage time series regression technique to test the significance of five-factor model variables. Abarbanell & Bushee (1998) utilised a single-stage regression to test the significance of fundamental variables. Similarly, Lettau & Ludvigson, (2001)) used a single step dynamic least squares technique to test for efficiency and return predictability for macroeconomic factors. This study attempts to bridge this gap and uses a multi-stage modelling technique to test for price efficiency. The hypothesis being a multi-stage modelling methodology will better explain behaviour of security prices and identify the true extent of market efficiency for fundamental information in comparison to a single-stage model.

Market efficiency has been defined in section 3.1. The hypothesis therefore is that in an efficient market it is not possible to make economic profits by trading based on publicly available fundamental information. Security returns should not be predictable from fundamental information and returns greater than an acceptable benchmark would suggest the existence of market inefficiency. This would be true if returns continue to be positive after information has been in the public domain for some time. For example, macroeconomic data is released on a monthly and quarterly basis, and company financial information annually, with quarterly updates. Therefore, fundamental information remains constant between release dates. Security prices, on the other hand, change continuously as trades occur daily. It is therefore expected that markets would have correctly analysed and priced fundamental information, exhibiting semi-strong form efficiency. The existence of 'abnormal' return opportunities will confirm the existence of persistence in market inefficiency.

Market efficiency and return predictability studies have predominantly been based on portfolio construction approach, where portfolios are first created (Abarbanell & Bushee, 1998) (Fama & French, 2015) based on fundamental variables and compared to actual portfolio returns, to confirm the validity of fundamental variables as significant factors. This study, however, takes a novel approach where the direction of security prices is first predicted; prices either going up or down. Daily portfolios are then constructed and trades executed based on model predictions. The returns are then compared to performance benchmarks to measure market efficiency.

Choosing a suitable period for forecasting returns is a consideration to test for semi-strong efficiency. Multiple trading horizons are available for study in financial markets: intra-day, daily, weekly, fortnightly, monthly or annual. From a trading view, the return horizon should be sufficiently long so that the returns generated are greater than transaction costs incurred. From a prediction perspective, the forecasting horizon should be short enough to enable capture of tradable information inefficiency which one would expect to exist for a limited duration. Model effectiveness as a result depends on both the trading and forecasting horizon view. Market efficiency suggests (for example, (Mehra & Prescott, 1985), (Siegel, 2002) that security returns include time-varying premiums and risk-returns adjust accordingly. Studies in financial market have utilised a number of horizon periods, for example Lettau and Ludvigson (2001) used quarterly forecast periods; Bansal, Kiku, Shaliastovich, & Yaron (2014) used annual forecast periods for macroeconomic data; Fama & French (2015) turnover portfolios on annual basis, although returns were measured monthly. This thesis examines monthly returns where a security is bought daily and sold after a one month holding period to test for model effectiveness and semi-strong form efficiency. The monthly frequency is also the earliest when certain macroeconomic information is released and therefore provides the ideal timeframe to study the impact of fundamental information on monthly returns. Studies suggest that fundamental factors influence long-term returns more, than short-term returns (for example (Siegel, 2002), (Cochrane J. H., 2005), (Campbell, Lo, & Mackinlay, 1997) given the cyclical nature of business and economies. The distinction between short and long-term market efficiency of fundamental variables, however, appears blurred.

In summary therefore, this study fills a gap in literature by extending the single-stage modelling methodology and implements a multi-stage model, adapted from racetrack betting markets, to test for semi-strong form efficiency of prices. In addition, the study uses a novel approach where market efficiency is tested by predicting the direction of price movement and then implementing a trading strategy, compared to directly predicting returns. The next section discusses research data, collection and sampling methodology followed by a discussion of the modelling process that will be utilised to validate the assertions.

6.3 Research Data – Security Sample Selection

A judgemental sample of 25 securities was selected (details of selection criteria are below) from those listed on the London Stock Exchange and included the FTSE-100 index; and represents 25% of the securities in the FTSE-100 index. The sample size of 25 is considered sufficient to support the hypothesis of applicability of multi-stage modelling method to financial markets, given that methodology would be applied to each individual security and therefore repetitive. In addition, the implementation of the multi-stage methodology is a time-consuming exercise.

The empirical analysis is based on a sample of 25 securities. The question therefore arises whether the sample population is adequate to confirm validity of the hypothesis. Although the sample size is 25 securities, the population is homogenous and a representative sample FTSE-100 traded securities. A conclusion on market efficiency therefore could be made with a reasonable degree of confidence for securities traded on exchanges where daily traded volumes are significant. Specific samples from other international exchanges, however, would support the results and findings of this study and could be contemplated for future research. Each security is modelled independently using a consistent methodology. The length of the data series is also sufficiently long to validate model robustness. For example, each model includes 2,850 trading days (7 years) of data for model development. The length of data series is also relatively longer compared data to studies in finance literature (for example, Abarbanell & Bushee (1998) used data from 1974-1988, Fama & French (2015) used data from 1963-2013 where monthly returns or 12 annual data points were used in the study). The data series includes periods of economic growth as well as the recessions to provide robustness to the results of the modelling process.

The FTSE-100 is a market-capitalisation weighted index of UK-listed companies and UK's leading index. The FTSE-100 is one of the most observed indices internationally with a total market capitalisation in excess of £1.7 trillion[8]. The attributes of the FTSE-100 index as at 31 July 2015 are detailed in Table 6-2 below.

[8] Source: FTSE-100 factsheet as at 31 July 2015.

Table 6-2 FTSE-100 Index Attributes[9]

FTSE-100 Index Attributes	
Number of Securities	101
Net Market capitalisation (£m)	1,709,781
Constituent Sizes by Market capitalisation: *(£m)*	
Average	16,929
Largest	113,134
Smallest	1,097
Median	8,888
Weight of Largest Constituent (%)	6.62
Top 10 Holdings (% Index Market Capitalisation)	38.39

A strict criterion is set and monitored by the publishers of the Index, London Stock Exchange, for inclusion in the FTSE-100 index. Securities included in the FTSE-100 must meet a liquidity criterion such that the securities can be swiftly bought and sold with minimal influence on the underlying security's prices as a result of the trading decision. In addition, these securities are subject to regulation. These securities have low bid-ask spreads of less than 1 percent and therefore have low transaction costs. There is also a minimum percentage requirement that 25% of the securities must be in public hands and freely available for trading.

A subjective stratified sampling was used to select securities representative of the FTSE-100, given that the total population of securities is 101.

The following two criteria were used to determine the final population:

1. A minimum historical data from 2000 onwards (15 years) was set as a requirement for fundamental information, as a long history of data is required to be available to perform effective empirical analysis and calculate underlying input variables. Although FTSE-100 index has a long history of existence (since 1984), securities are added and deleted from the index on a quarterly basis, therefore not all the securities have been included in the FTSE 100 since inception or a have long history of publicly available information. For example, Experian PLC,

[9] Source: FTSE-100 factsheet as at 31 July 2015.

became a FTSE-100 company from 2006. This stock therefore has short history of data and was excluded.

2. The second criterion was to ensure that the population was representative of a number of industries and sectors in the UK economy and FTSE-100; to reduce the element of bias in the sample towards a specific industry and therefore to ensure the wider applicability of the overall conclusions of this empirical analysis. This process would then support modelling methodology conclusions drawn irrespective of industry.

Table 6-3 below provides details of the securities included in the sample and represents 25% of the securities in the FTSE-100 index.

Table 6-3 List of FTSE-100 Sample Securities

	Security Code	Security Name	Security Type – Financial / Non-Financial	Market[10] Capitalisation (£Billions)	Number of Free Float[11] Shares(Billions)	Daily Average Trading Volume (3-Month Average 2015)[12](Thousands)
1	ARMS	Arms Holding	Non-Financial	12.58	1.37	4,717
2	BRBY	Burberry	Non-Financial	6.35	0.429	1,467
3	DGE	Diageo	Non-Financial	43.72	2.43	5,062
4	GSK	Glaxo-Smith-Kline	Non-Financial	67.08	4.69	9,043
5	KGF	Kingfisher	Non-Financial	8.51	2.27	7,036
6	MKS	Marks & Spencer	Non-Financial	8.60	1.49	4,654
7	NXT	Next	Non-Financial	11.70	0.138	384
8	SBRY	J Sainsbury Plc	Non-Financial	4.72	1.23	7,880
9	TSCO	Tesco	Non-Financial	15.73	7.80	22,199
10	BARC	Barclays Plc	Financials	44.93	12.54	37,690
11	HSBA	HSBC Holdings PLC	Financials	105.26	17.88	23,900
12	IHG	Intercontinental	Non-Financial	5.82	0.200	713
13	JMAT	Johnson Matthey	Non-Financial	5.51	0.194	559
14	MRW	Morrison	Non-Financial	3.94	1.97	7,530
15	RB	Reckitt Benckiser	Non-Financial	41.87	0.619	1,150
16	BA	British Aerospace	Non-Financial	14.01	1.71	7,250
17	AZN	Astra Zeneca	Non-Financial	53.58	1.24	2,429
18	ULVR	Unilever PLC	Non-Financial	77.72	2.81	2,650
19	PSON	Pearson PLC	Non-Financial	9.27	0.794	2,719
20	BT	BT Group PLC	Non-Financial	37.93	7.68	15,461
21	ABF	Associated British	Non-Financial	25.35	0.321	722
22	GKN	GKN PLC	Non-Financial	4.96	1.46	5,468
23	PSN	Persimmon PLC	Non-Financial	6.54	0.300	1,114
24	WOS	Wolseley PLC	Non-Financial	11.18	247	670
25	BDEV	Barratt	Non-Financial	6.35	0.976	5,083

The liquidity, volume of free-float securities, market capitalisation and regulatory protection, all combined, provide an ideal venue for modelling security prices and testing the racetrack modelling methodology. These securities are unlikely to be, as strongly influenced by external factors such as

[10] Source: Yahoo Finance
[11] Source: Yahoo Finance
[12] Source: Yahoo Finance

illiquidity, insider trading or concentration of securities in the hands of a few shareholders. In addition, the available liquidity in these securities and multiple trade execution venues provide a trader with opportunities to quickly execute trading decisions. These trades are available for execution in real-time between willing buyers and sellers. The next section discusses data requirements and source of this information to develop the multi-stage methodology and test the hypothesis.

6.3.1 Data Requirements and Sources

The data requirements for modelling in racetrack betting are fundamental variables and prices or odds. The equivalent data requirements in financial markets to test the multi-stage modelling hypothesis are security prices, company financial and macroeconomic data. There are well-established providers in financial markets (for example, Compustat, Worldscope, DataStream and Bloomberg) who make the services available via a subscription service. However, data length and granularity may differ amongst data providers for fundamental information. Ulbricht & Weiner (2005) compared 650 variables in Compustat and Worldscope databases and found that Worldscope coverages included 25% more securities and although both databases would lead to comparable results variables needed to be *"treated with care"* to avoid impacting the quality of results. The possible likelihood of variations in fundamental data would therefore suggest that an element of cross-validation will be required as part of the data gathering process. CRSP has more detailed data is available for US securities compared to UK securities. Web search engines such as google and yahoo finance now also provide limited financial data such as daily security prices and dividends and do not require a subscription fee. The next section discusses prices.

6.3.1.1 Security Prices

Security prices could be described as market odds and are available on a daily basis. Yahoo finance was selected as the source for price information based on cost and ease of online data access. In addition, price data is homogenous across all financial information providers and there is a limited risk of inaccuracies in price data. The next section discusses company financial information.

6.3.1.2 Company Financial Data

Company financial data could be described as security-specific variables that directly influence security prices and are most closely associated with horse-specific data in racetrack betting such as previous wins, jockey information etc. However, unlike racetrack betting where fundamental information on horse performance are available after every race, company financial information is released to the market on an annual basis via published annual statement, and quarterly updates provided. There are many service providers that offer company financial data like Reuters and Fame, including the sources noted above. However, in each case, variables are usually adjusted for subsequent events that may have occurred a year or two later. These adjustments may be related to, for example, compliance with IFRS, errors discovered in financial statements. However, from a price perspective, the market would not have known of these facts until subsequent corrections were made or highlighted via a substantive piece of investigative financial journalism. Financial information therefore was extracted directly from company websites as released in the year of publication.

The nature of fundamental variables and the complex compilation process of these data at information source mean that there is a lag between the information release date and the period for which the variables are applicable. Any analysis of the fundamental variables therefore must consider the lag in variables to avoid inclusion of information that may have been previously publicly released. In addition, release dates for the information vary from period to period. The actual information release date was obtained for the individual companies. Variables were then made effective from the day after the actual information release date. For example, ARMS Holding Plc which has a balance date of 31 December released its earnings for 2013 on 4th February 2014. The variables therefore were effective from 5th February 2014.

This contrasts to early papers in finance where assumptions were made concerning when data was available for public consumption. For example, Fama & French (1992) assumed six-months after financial year end, Basu (1983) assumed a 3-month after financial year end which was the SEC filing date deadline, the recent study of five factor model (2015) also assumed a six-month lag where variables were effective in June and companies in the sample had December financial period ends. The next section discusses macroeconomic data.

6.3.1.3 Macroeconomic Data

Macroeconomic data is general economic and industry information which influences security prices and impacts all securities to varying degrees. In racetrack betting, this would be similar to fundamental variables such as track conditions, weather that would impact all horses in the race. The source of industry data is Office of National Statistics (ONS) which publishes macroeconomic information on a monthly, quarterly and annual basis. For example, Gross Domestic Product (GDP) is published on a quarterly basis. Consumer Price Index (CPI) as a measure of Inflation published monthly. Retail turnover information which includes Food Stores, Non-Food Stores and Non-Store

Retailing and is a gauge of consumer spending strength as well as a component of GDP growth is published monthly.

The available frequencies of data therefore vary according to the type of variable. Similar to financial markets there is a lag between period end dates and availability of information for public consumption. For example, ONS releases monthly retail data by the 20th of the month following the period to which the data relates. The effective date for retail data in the model therefore was from the 21st or the day after the release date. Table 6-4 below details the publishing frequency of the variables and the average number of observations.

Table 6-4 Variable Frequency

	Variable	Frequency	Average Number of Observations Across Sample Period (Years 2005-2015)
1	Financial statement	Annual	11 annual observations
2	Macroeconomic and Industry	Monthly	132 monthly observations
3	Security Prices	Daily	2,850 daily observations

6.3.1.4 Fundamental Variable Frequency Mismatch

Table 6-4 shows that financial statement information remains a constant from one release date to another given the non-existence of daily information frequency when compared to prices. One alternative would be to utilise quarterly earnings announcements or transform annual data to equivalent daily observations. There are well documented studies, however, which show that markets are not efficient in processing quarterly announcements (for example, (Ball R. , 1978), (Ball R. , 1992), (Bernard & Thomas, 1990)) on post earnings announcement adrift (PEAD). Bernard & Thomas (Bernard & Thomas, 1990) found that security prices failed to reflect fully the implications of current earnings for future earnings in the first four days after quarterly earnings announcement and that,

"stock prices partially reflect a naive earnings expectation: that future earnings will be equal to earnings for the comparable quarter of the prior year" (Bernard & Thomas, 1990, p. 338), suggesting market inefficiency.

A similar result was reported in an earlier study (Bernard & Thomas, 1989) where a strategy based quarterly earnings announcement anomaly produced an abnormal return of 18 during the first quarter subsequent to the earnings announcement. Foster, Olsen, & Shevlin (1984) also reported similar findings where abnormal returns of 25% was noted 60 days after earning announcements. These findings suggest the persistence of abnormal returns after earnings announcements.

Unlike racetrack betting information, financial statement and macroeconomic data have seasonal and cyclical properties due to the nature of economic events. For example, energy companies have relatively higher revenues in winter due to increased seasonal demands for energy products. Quarterly financial statement data would therefore include permanent and transitory components. A methodology would therefore need development to separate these components to confirm variable significance. In addition, data transformation will require behavioural assumptions of the independent variable from one information release date to the next. For example, does the firm's annual earnings patterns exhibit a linear or non-linear behaviour? By comparison, annual financial statement includes would not require behavioural assumptions (for example, seasonality). Annual financial statement data was preferred over quarterly earnings, as the latter would require additional model development considerations, in addition to data gathering requirements. The inclusion of quarterly earnings announcements as independent variables is therefore proposed as a future extension to this research. The inability of markets to correctly process quarterly information announcements suggests that annual results have price-related information. Benhardt & Miao (2004, p. 339) suggest that while new information will arrive over time and new information may be acquired by others "*his information will become stale, but not valueless".* The next section discusses sample size and period.

6.3.2 Sample Size and Period

The number of observations available for modelling has a bearing on the empirical analysis that is feasible. In general, a larger sample size is preferred to minimize the probability of errors and to improve results from which conclusions can be drawn. Two questions therefore arise: what should be the appropriate length of data with respect to the number of observations for the analysis? What time period should be included for data gathering for the analysis to be effective?

The sample period selected was for 11 years from 2005 to 2015. It is assumed that this sample period exhibited a level of uniformity, with respect to prices and fundamental information. The sample period represents a time where securities trading has matured into multiple trading venues and global market participants. The earlier periods (early 90's or before) were not representative of this period as the internet-based trading environments were in their infancy. In addition, periods prior to 2000 had more instances of missing data, with securities being listed and delisted from the exchange. For example, Burberry was listed and included in the FTSE-100 index in 2002. Prior to 2002 publicly available historical information was not available.

Consequently, the data set comprised a total of 2,850 daily price observations per security (on average) for the period 2005 to 2015. Although the final data set was for a 11-year period a longer of series of data is required to enable calculation of certain input variables such as 3-year rate of change and volatility which require previous period data. The next section discusses the data split for sample data.

6.3.2.1 Data sets – Training Set and Out-of-Sample

The data set was split into two independent subsets for model build (training set) and out-of-sample testing. For determining the model, data for the period 2005 to 2011 was used to estimate the logit regression model coefficients. This represented, on average, 1,816 daily price observations over a 7-year period for each security (approximately 64% of the total sample population). The remaining 36% of data (on average 1,044 observations) for the period 2012 to 2015 was then further split into two equal periods for model validation (2012-2013) and out-of-sample testing (2014-2015). This is comparable with sampling split in racetrack betting literature. For example, Sung & Johnson (2007) used 1110 races (66%) to develop the model and 565 (34%) races as a holdout sample, Lessman, Sung, & Johnson (2009) used 400 races (72%) to develop the model and 156 races (28%) as a holdout sample and (Lessmann S. , Sung, Johnson, & Ma (2012) used 2,780 races (65%) to build the model and 35% as holdout sample. The next section discusses the criteria adopted for determining whether to include a particular piece of fundamental information in the model as well as any assumptions made with respect to the variable.

6.3.3 Criteria for including Fundamental Information Inclusion

6.3.3.1 Prices

In racetrack betting markets, single odds are available that can be included in a model. However, in financial markets multiple prices variables are available per security on a continuous basis. Each trading day at close of market the following prices are available for each security:

i. Daily opening price
ii. Daily closing price
iii. Daily high
iv. Daily low.

Intra-day prices are also available. Finance literature (for example, Andersen, Bollerslev, Diebolds and Ebens (2001), Ghysels, Santa-Clara and Valkanov (2005)) suggests that price volatility is a significant prices variable. The above prices were used as the basis for calculating monthly volatility and used as model input variables. The next section discusses the criteria for financial and macroeconomic data.

6.3.3.2 Financial and Macroeconomic Data

Financial market literature has identified a wide range of significant fundamental variables in predicting security returns. For example, Ball & Brown (1968) showed the importance of profit and loss statements, Abarbanell & Bushee (1998), Lewellen (2004) showed the value of financial statement ratios, and Banz (1981), Chan, Chan, Jegadeesh, & Lakonishok (2006) have shown the value of earnings quality.

Similarly, there have been a number of studies on macroeconomic data and return predictability. For example, Moskowitz & Grinblatt (1999) show a link between industry momentum and stock returns. The relationship of economic activity to security prices is also well documented in literature for example, (Chen, Roll, & Ross, 1986), (Shanken & Weinstein, 2006), (Siegel, 2002). Subrahmanyam (2010) in a review of cross-sectional predictors of stock returns identified more than fifty variables in literature. However, it is beyond the scope of this paper to effectively model all the variables that have been identified as significant predictors. The data set therefore has been limited to subset of fundamental variables to validate applicability of multi-stage modelling methodologies to financial markets. Table 6-5 below details the variables included in this analysis, with references to appropriate literature where these variables were previously identified as having information content.

Table 6-5 Fundamental Variables

	Variable Name	Available Frequency of data	Variable Type	Literature Reference
1	Net Sales / Turnover	Quarterly / Annual	Financial Statement	Abarbanell & Bushee (1997); Lewellen (2004)
2	Operating Profit Margin	Quarterly / Annual	Financial Statement	Abarbanell & Bushee, (1997); Lewellen, (2004)
3	Dividends Per Share	Quarterly / Annual	Financial Statement	Campbell & Vuolteenaho (2004)
4	Retail Turnover[13](Food Stores, Non-Food Stored and Total Retail)	Monthly	Macroeconomic	Chen, Roll, & Ross (1986)
5	Manufacturing and Services Turnover[14]	Monthly	Macroeconomic	Chen, Roll, & Ross (1986)
6	Consumer Price Index (CPI)	Monthly	Macroeconomic	Chen, Roll, & Ross (1986)
7	Prices	Daily	Security Specific	Andersen, Bollerslev, Diebolds and Ebens (2001)

It could be argued that sample population may not truly reflect the UK economy as companies which operate internationally are included. UK, however, like other OECD economies is an open economy with multiple trading partners. Therefore, these companies have significant earnings from international operations, as well as operations in the domestic economy. Estimated global trade flows[15] are detailed in Table 6-6 below:

Table 6-6 UK Trade Flows by Country / Area

Country / Area	%
EU	69.96
USA	16.49
Japan	7
	93.45
Rest of the World	6.55
Total	100

[13] This data represents a more granular level of data then the Chen, Roll & Ross (1986) study.
[14] This data represents a more granular level of data then the Chen, Roll & Ross (1986) study.
[15] Source: Bank of England - Aggregated trade flows in manufactured goods.
http://www.bankofengland.co.uk/statistics/Pages/iadb/notesiadb/effective_exc.aspx

The trade flows possibly suggest that macroeconomic data for EU, US and Japan should be included to better model security prices, in addition to UK macroeconomic data. It is, however, not the intent to include all the permutations and combinations of macroeconomic variables as number of variables will be quite large. The aim is to demonstrate the effectiveness of a multi-stage modelling methodology using a limited set of variables. Macroeconomic variables of trading blocs and other countries may be considered for future research to improve model predictability. The level of earning exposure also differs among companies. For example, Barratt Developments (housebuilder) and Morrison (supermarket chain) have very minimal revenue exposure to international economies due to lack of international operations. These companies, however, import raw materials. For example, Morrison imports a significant amount of produce from EU. On the other hand, companies such as British Aerospace and Astra Zeneca generate a significant revenue from international operations. The next section discusses the adaptation process of the multi-stage racetrack betting modelling methodology to financial markets.

6.4 Modelling Methodology - Adapting Racetrack Betting Techniques to Financial Markets

Literature in racetrack betting suggests a *"modelling technique which captures the full information content of fundamental and market-generated variables"* (Sung & Johnson, 2007, p. 44) and a wagering strategy be subsequently implemented. Effective models developed for racetrack betting follow a number of key steps to achieve full information extraction:

i. Independent variable optimisation using variable constructs and transformations. For example, Chapman (1994) used regression for variable estimates, Edelman (2003) utilised differences in weight-corrected performances, Lessmann, Sung, & Johnson (2007) used a race-wise standardisation procedure whereby the continuous variables were standardised to zero mean and standard deviation of one.
ii. A first stage linear/non-linear statistical model to extract fundamental information from fundamental variables followed by a CL model where market odds are included as a final variable in a multi-stage process, or,
iii. Developing first stage base forecast models where these are combined using model stacking (Lessmann S. , Sung, Johnson, & Ma, 2012) and conditional logit.
iv. Finally, a Kelly wagering strategy to optimize payoff and minimize losses.

It is clear from the racetrack betting literature that a multi-stage modelling process is required. In addition, a number of alternatives are available to develop forecast models: linear/linear modelling using CL (Benter, 1994), (Sung & Johnson, 2007); non-linear/linear modelling using CL (Edelmen, 2007), (Lessman, Sung, & Johnson, 2009); developing base forecast models and combining these models using CL (Lessmann S. , Sung, Johnson, & Ma, 2012).

The aim of this study is to effectively apply a racetrack betting multi-stage methodology to financial market data. Application of all available racetrack betting modelling methodologies to financial market data has not been considered but could be contemplated as areas of further subsequent research. However, any adaptation of racetrack betting techniques to financial markets require a careful consideration of the relative complexities of financial markets to take into account any modelling implications that may impact final results. The dynamics of the racetrack betting and financial markets are different and methodologies are not directly transferable. Adaptations will be required to be made to multi-stage racetrack betting methodology for its application to financial markets.

A key difference between racetrack betting and financial markets is the timing and frequency of available data. In racetrack betting all fundamental information of previous races are available to be refreshed prior to each new race. Most recent fundamental information is therefore available to be combined with market odds as a second stage variable. By contrast, the frequency and availability of data in financial markets differs: Prices are available daily on a continuous basis, financial statement data is released annually and macroeconomic data available monthly. Modelling financial market data using racetrack betting methods therefore needs to take into account that prices are combined with fundamental information that is relatively 'stale'. This would suggest that fundamental information released to the market are reflected in prices and profitable trading opportunities should be non-existent. Markets should therefore be semi-strong form efficient.

From a modelling perspective combining two sets of fundamental information, financial statement and macroeconomic data, in a single model suggests a probable risk of intrinsic fundamental information in financial statement data being 'overwhelmed' by macroeconomic variables given the relatively higher frequency of availability of the latter compared to annual financial statement data. The availability of fundamental information data at differing frequencies suggests that base models need to be developed first and then combined as a second stage.

The appropriate methodology therefore as a minimum could be summarised as requiring the following steps:

1. Develop base forecast models for financial statement, macroeconomic data and price volatility, using training data set.

2. Combine the base forecast models at the next stage in an independent validation data set where the input variables are the outputs of the base forecast models.

3. The second-stage models are then tested in the out-of-sample data set.

4. Finally, implementing a Kelly strategy.

Two questions remain to be answered; what should be the variable constructs and transformations to optimise the input variables for fundamental information and prices? What are the appropriate statistical methodologies for developing base models and combining base models? The next sections discuss variable optimisation, using variables constructs and transformations, after which appropriate statistical methodologies are discussed. A key element is the standardisation of data for inputs into the model. This process is discussed next.

6.5 Data Standardisation

Standardisation of data is required to ensure that variables measured in different scales contribute equally to the analysis and are in proportion to one another. For example, companies report revenues in £millions, £billions whereas security prices are normally displayed in units of £1. A standardised data would mean a high likelihood that variable coefficients signs would be consistent with data interpretation. Pindyck & Rubinfield (1998, p. 586) suggest for example *"logarithmic transformations are used as a means of removing growth over time in variance of the data".*

Economic and financial market studies have long used natural log transformations as a suitable methodology to explore relationships amongst economic variables. A log function has the effect of data appearing more linear in comparison to raw data, which will show significant volatility and therefore making it difficult to discern any patterns emerging in the data. For example, a log function of operating profit, which can be subject to significant swings, will present a 'smoother' line compared to the sharp fluctuations if raw data was to be utilised. Similarly, market prices as traded will show a higher level of price volatility in comparison to a log of prices. In addition, log variables have convenient mathematical properties. For example, percentage changes are calculated as first difference of the log variables, (given that log of first difference approximates percentage changes and rates of change are second differences). All variables therefore as a first step transformed into log variables which equates to the function:

$$f(x) = \ln(x) \qquad \text{EQ. 6-1}$$

Where,

x Represents the independent fundament variable

ln Represents the natural log function

$x > 0$

For $x \leq 0$, $\ln(x)$ is undefined.

The transformation of security prices transformation into a log function is straight forward given that prices are > 0. However, unlike fundamental variables in racetrack betting where all values are positive, company financial and economic data include regular occurrences of negative amounts. This arises because firms make profits as well as losses and, similarly, economies experience periods of positive and negative growths. An initial transformation to a positive value is therefore required prior to a log conversion given the added complication of zero and negative values being undefined in a log function.

To enable a log transformation, a positive value was added to all the values in the particular independent variable data series such that all values in the series were made positive. The general principle followed was to add a value equal to the highest absolute negative value, plus either 1 or 0.1, so that the value of the lowest variable was greater than zero. The minimum value in these data series therefore was either 0.1 or 1, to enable a subsequent log transformation.

For example, if a company incurred net operating losses and values in the series were in £millions or £thousands, then the constant added to the data series was equal to the absolute value of the highest loss amount plus 1 to the total data series; training and out of sample data. For economic data series where values are normally in single digits the amount added was equal to the highest absolute negative value plus 0.1 to ensure that all values in the series were greater than zero. Arguably, there will be a loss of information when transforming raw data into a log function in this manner. The transformation results in mean of the variables change. The variance however remains unchanged. It is assumed therefore that this loss of information from data transformation would be less than the information gained to forecast the dependent variable.

The next sections discuss the variables constructs and transformation process. Racetrack betting suggests a number of methodologies: race-wise standardisation of variables (Lessmann, Sung, & Johnson, 2010), regression analysis (Chapman, 1994), analysis of variance (Edelman, 2003). Similarly, econometric modelling suggests a number of choices to identify linear and non-linear trends in data. For example, moving average, where sequential averages are calculated at different points in time, is a well-established process to understand trends in data. This approach is discussed in econometric text books (for example, (Mills & Markellos, 2011)). Refenes, Bentz, Bunn, Burgess and Zapranisa (Refenes, Bentz, Bunn, Burgess, & Zapranisa, 1997) suggest a recency-weighted variable procedure such as Discounted Least Squares, whereby learning is biased towards more recent observations with long term effects experiencing exponential decay through time. However, the standard measuring techniques for understanding behaviour of data are percentage change, rate of change and volatility, and these are clearly understood transformations without requiring a need for a complex explanation. These standard measuring variable constructs are used as input variables. It is also important to understand the nature and properties of financial market variables with respect to modelling behaviour, specifically, the time series properties of data. The next section discusses this.

6.6 Time Series Properties of Financial Data

Data in financial markets have time series properties, where data is non-stationary with changing means and variances, and "*long term memory*" (Granger, 1981). Research shows that time series data in financial markets are complex, evolving in a non-linear inherently noisy fashion. In addition, the distribution of data changes over time (Cao & Tay, 2002), (Refenes, Bentz, Bunn, Burgess, & Zapranisa, 1997)). The non-stationary and inherent noisiness implies that information is not complete to be to able fully capture the relationship between future prices and the past from the historical behaviour. Box & Jenkins (1976) suggest a stationarity test to determine whether the data is affected by time, where if the data has the same joint probability distribution irrespective of the time horizon, the data is considered stationary. Mean and variance are then constant and data set exhibiting linear properties.

The evolving nature of financial data suggests that model coefficient values (and therefore variable significance) may not be the same if the variable was calibrated in a different sample period. For example, earlier studies showed that dividends had predictive information but recent studies downgrade dividends to having no predictive information. Financial and economic data has seasonal and cyclical properties due to the nature of economic events. In addition, the relationship of fundamental variables to prices may change over time. Recent data therefore could provide more information than distant historical data, as the distribution of financial time series can change over time. This could lead to a gradual evolution in the nature of relationship between fundamental variables and prices.

One of the key aspects of economic data is that variables evolve in the long run as changes in preferences, technologies and demographics take place. As changes in economic factors occur, structural transformations in the economy take place which alters firm, industry and national competitive advantages. For example, the arrival of the internet changed distribution and business models of companies in a number of industries (e.g., financial services and retailing). Oil price shocks at periodic intervals in the last four decades have resulted in a change in demand patterns for goods due to inflation (for example (Wang, Wu, & Yang, 2013), (Jebabil, Arouri, & Teulon, 2014)). These structural evolutions change both the characteristics of determinants of time series and relationships between independent and dependant variables. These changes also occur at varying paces over time as the economic environment evolves and trend behaviour can be upward or downward, exponential or approximately linear. The structural relationship between price and its determinants therefore changes over time as the economic environment evolves.

Modelling processes therefore must take into account the time series properties of data, while at the same time being able to independently determine the variable coefficients for fundamental variables and prices. Model specifications must allow for time varying parameters such that recent observations are weighted more heavily than older observations. Alternatively, models must be refreshed on a periodic basis to consider gradually changing input-output relationships.

The regression of time series data therefore requires consideration of the possibility of omitted variable bias, model parameter stability and stationarity of data, to ensure model effectiveness. These are discussed next.

6.6.1 Omitted Variable Bias

Omitted variable bias is widely discussed in econometric texts (for example, (Greene, 2012), (Gujarati, 1999)) and refers to the problem where a significant has been incorrectly excluded from a model resulting in incorrect parameter specifications. The omitted variable would therefore impact the relationships between dependent variable and explanatory variables, most likely leading to incorrect model specifications. Here the distinction must be drawn from variables in the initial sample set and subsequently omitted from the model development process, and those variables not included in the initial sample population, given the numerous variables previously identified in literature.

In this paper, financial statement, macroeconomic and price variables are first independently modelled to develop base models and therefore the risk that a significant variable has not been included. The base models are however combined at the second stage of the model development. This process considers the interaction of the three significant variables to determine the final model parameters. It is therefore unlikely that the final model will exhibit omitted variable bias from the sample of variables modelled. Literature (for example, (Fama E. , 1970) suggests that in an efficient market prices would reflect available information. The risk of omitted variable bias is, therefore, further reduced as prices are included as independent variables. The next section discusses model parameter stability.

6.6.2 Model Parameter Stability

Stable model parameters would suggest that model variables are constant across the sample population and the model does not have “breaks”. In other words, model parameters are stable and therefore a good representation of the sample period. Economies, however, experience business cycles with growth and recessionary periods. As a result, explanatory variables that may be significant during growth phases of the economic cycle may not be relevant during recessions, possibly leading to breaks in model parameters. For example, dividends may not feature as a significant element of company policy during periods of negative economic growth.

The data sample utilised for the model build includes periods of growth as well as recessions, as shown in Figure 6-2 below. Therefore, there is a potential for breaks in model parameters around the recessionary period.

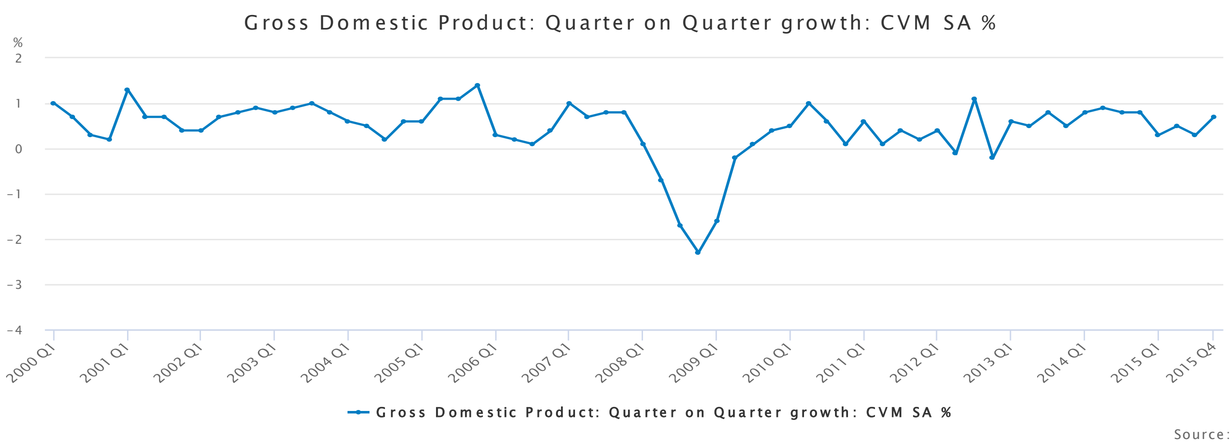


Figure 6-2 GDP Growth UK: 2000-2016 – Source: ONS

A model re-estimation and sub-sampling process was followed to reduce the risk of parameter stability. Four time-series sub-samples, which included every fourth observation in the sample period, were created for model re-estimation. As a final step, model parameters were re-estimated over the total in-sample period, taking into consideration only those variables identified as significant in each sub-sample. The logic being that the sub-samples would consider any possible structural breaks in the model properties. The next section discusses stationarity.

6.6.3 Stationarity Tests of Independent Variables

A time-series variable exhibits stationarity property (or weakly stationary) if the series fluctuates about its mean with a constant variance and does not vary with respect to time[16].

$$E(X_t) = \mu$$
$$var\ (X_t) = \partial^2 \qquad \text{EQ. 6-2}$$

Where

E Represents the expectations operator

X_t Represents the independent variable with respect to time, t

μ Represents the mean

∂^2 Represents the variance of the independent variable.

In other words,

"the joint probability distribution of any set of k observations ….is the same regardless of the origin, t in the timescale" (Greene, 2012, p. 953).

Stationarity would suggest that a linear regression would identify economically significant relationships. In contrast, non-stationarity would suggest a stochastic trend and exhibit relationships in line with a random walk model, and a high likelihood of 'spurious' results. A linear combination (such as first differences) of non-stationary data, however, may suggest that the non-stationary variable in fact exhibits stationary properties (Engle & Granger, 1987). The unit root test is generally employed to evaluate the level of the non-stationarity a data series may exhibit whether the trend is stochastic. For example, if dividend follows a linear process it follows that prices will

[16] Standard textbook definition, for example, (Greene, 2012), (Pindyck & Rubinfield, 1998).

also follow a linear process. In other words, the price-dividend relationship can be transformed and shown that prices and dividends are cointegrated and a stationary linear combination. Literature suggests a number of tests to confirm stationarity of data which are well discussed in econometric texts (for example, (Gujarati, 1999), (Greene, 2012), (Mills & Markellos, 2011)). These include Dickey-Fuller (DF) test (Dickey & Fuller, 1979), the Augmented (DF) test, the Phillips-Perron test (Phillips & Perron, 1988). The Phillips & Perron test, however, makes weaker assumptions than the DF and augmented DF statistics and is generally considered more reliable (Fabozzi, Focardi, Rachev, & Arshanapalli, 2014). The augmented DF test was deployed to test for stationarity of the independent variables and is now explained[17]. Given a random walk model with drift:

$$Y_t = \mu + \rho Y_{t-1} + \epsilon_t \qquad \epsilon_t \sim IID\ (0, \sigma^2) \qquad \text{EQ. 6-3}$$

The previous period's observation is then represented by:

$$Y_{t-1} = \mu_{t-1} + (\rho - 1)Y_{t-2} + \epsilon_{t-1} \qquad \epsilon_{t-1} \sim IID\ (0, \sigma^2) \qquad \text{EQ. 6-4}$$

And change in observed variable is represented by:

$$\Delta Y_t = Y_t - Y_{t-1} \qquad \text{EQ. 6-5}$$

By direct substitution

$$Y_t = \sum_{i=0}^{\infty}(\mu + \epsilon_{t-\mathrm{i}}) \qquad \text{EQ. 6-6}$$

Where

Y_t Represents the variable observed, say price, at the beginning of time, t

μ Represents a constant

ρ Represents the coefficient parameter for Y_t

ϵ Represents the error term that is independently and identically distributed (IID) with a mean of zero and variance of σ^2

t Represents the time

Δ Is the first difference operator

[17] Adapted from Greene (2012).

In other words, the process of the observed variable generation is a function μ. On the other hand, where $Y_t - Y_{t-1} = \mu + \varepsilon_t$ the series is assumed to be stationary and integrated to the order one. DF (1979) derived the distribution for ρ that holds when $\rho = 1$ and generated statistics for the F-test (referred to as tau test) critical values. In other words, the hypothesis whether ρ is significantly different form one is tested and determined as follows:

$$DF_T = \frac{\rho - 1}{Estimated\ Standard\ Error\ (\rho)} \qquad \text{EQ. 6-7}$$

Where,

DF_T Represents DF tau critical values.

Literature (for example (Porteba, 1988) , (Balvers, Wu, & Gilliland, 2000)) has also previously confirmed that the existence of mean reversion tendency of stock prices. The next section discusses long -range dependency.

6.6.4 Long-Range Dependency (R/S) Tests

It is now accepted in literature that certain phenomena show long memory, since the analysis of reservoir levels of river Nile by Hurst (1951) where he developed the "Hurst" exponent to test water levels. Hurst (1951) showed there was linear relationship between historical water levels and future levels which was determined by the rescaled range of water levels over time and its standard deviation.

Hurst (1951) noted that naturally occurring empirical data was well represented by:

$$E = \left[\frac{R(n)}{S(n)}\right] = Cn^H \ as\ n \to \infty \qquad \text{EQ. 6-8}$$

Where

E Represents the expectations operator

R Represents the rescaled range of time series values

S Represents the standard deviation of the time series values

n Represents the number of observations

C Represents a constant term

H Represents the Hurst Exponent where

$H < 0.5$ Represents data has no trend and is mean reverting – White Noise

$H \sim 0.5$ Represents data has a random walk

$H > 0.5$ Represents data has persistence and trending

Long-term memory (or long-range dependency) suggests a high likelihood of return predictability. Mandelbrot (1971), (1972) expanded Hurst's findings to economic and financial time series data. Empirical studies for long range dependency, however, appeared much later in finance literature, for example, (Willinger, Taqqu, & Teverovsky, 1999), (Cajueiro & Tabak, 2008). The Hurst exponent was calculated for the fundament variables to test long range dependency. The next sections discuss variable transformations.

6.7 Variable Transformations – Percentage and Rate of Change

Macroeconomic and financial statement information are monthly and annual time series data, respectively. Invariably trends are possibly identifiable from the historical data set. Percentage change and rate of change have been used to identify general trends and interpret data for economic and financial analysis. For example, GDP announcements by ONS, turnover and profit announcements are all explained to the market as relative percentage changes and emerging trends. These variable transformations are therefore not opaque to the market. Percentage change and rate of change have been used as input variables for data modelling in this study. The percentage change has been calculated as first log difference of the independent variables as follows:

$$\Delta x_{to} = \ln x_{to} - \ln x_{t-1} \qquad \text{EQ. 6-9}$$

Where

Δx_{to} Is the one period percentage change in variable x at t_0 from $t-1$

t Represents time

Following from the equation above, the rates of change in the underlying variable are then calculated as follows:

$$Rate\ of\ \Delta x_{to} = \Delta x_{to}\ /\ \Delta x_{t-1} \qquad \text{EQ. 6-10}$$

Where

$Rate\ of\ \Delta x_{to}$ Represents the one period percentage rate of change in Δx_{to} at t_0 from $t-1$ as determined in equation 6-9.

Δx_{t-1} Represents the one period percentage rate of change in Δx_{to} for the immediately preceding period, as determined in equation 6-9.

Time series data also have a lagged impact where decisions made now may not have an impact on prices immediately but in the future. The lagged impact of variables on prices is well-known in economic analysis (e.g., (Gujarati, 1999)). For example, consumers because of changes in consumer earnings power through increases/decreases in wages do not change their consumption behaviour patterns immediately after changes in price of goods. Similarly, trade unions may negotiate a wage increase, however the rises may be incremental and over a several periods. Changes in demand resulting from technological changes, for example a new generation of computers with new features, may be slow as existing computers are still as good. These factors therefore have a delayed impact on company earnings even though the economic decisions had been made much earlier. Recent studies (Fama & French, 2015) have shown that financial statement information content after a 3-year lag declines. A three-year lagged effect was calculated for input variables.

Table 6-7 below gives an example of how these calculations have been determined for percentage change and rate of change, and taking into account the lagged effect. Table 6-7 shows the results of variable calculations for security Associated British Foods (ABF) which is listed on the London Stock Exchange and a constituent of FTSE-100. Column C details the company's annual financial year end date.

Table 6-7 Percentage Change and Rate of Change Calculations

A	B	C	D	E	F	G	H	I	J	K	L	M
							Percentage ChangeΔx_{to}			Rate of Change Δx_{to}		
Row No.	Security	Year End	Annual Earnings release date	Effective Date	Turnover £millions	ln	1-year	2-year	3-year	1-year	2-year	3-year
1	ABF	18-Sep-99	08-Nov-99	09-Nov-99	4299	8.3661						
2	ABF	16-Sep-00	09-Nov-00	10-Nov-00	4406	8.3907	0.0246					
3	ABF	15-Sep-01	06-Nov-01	07-Nov-01	4418	8.3934	0.0027	0.0273		0.1106		
4	ABF	14-Sep-02	05-Nov-02	06-Nov-02	4545	8.4218	0.0283	0.0311	0.0556	10.4199	1.1376	
5	ABF	13-Sep-03	04-Nov-03	05-Nov-03	4909	8.4988	0.0770	0.1054	0.1081	2.7184	3.3928	1.9427
6	ABF	18-Sep-04	10-Nov-04	11-Nov-04	5165	8.5497	0.0508	0.1279	0.1562	0.6598	1.2135	1.4451
7	ABF	17-Sep-05	08-Nov-05	09-Nov-05	5622	8.6344	0.0848	0.1356	0.2127	1.6678	1.0605	1.3613
8	ABF	16-Sep-06	07-Nov-06	08-Nov-06	5996	8.6988	0.0644	0.1492	0.2000	0.7597	1.1001	0.9406
9	ABF	15-Sep-07	06-Nov-07	07-Nov-07	6800	8.8247	0.1258	0.1902	0.2750	1.9537	1.2751	1.3749
10	ABF	13-Sep-08	04-Nov-08	05-Nov-08	8235	9.0161	0.1915	0.3173	0.3817	1.5217	1.6679	1.3879

Column D represent the day of the actual release date of the financial results and Column E represents the actual date from which the variable is effective, which is a day after the earnings release date. For example, for the financial year ending 13 September 2008, the annual financial results were released on 4 November 2008. The effective date of the independent variable is from 5 November 2008, until the next release date of the annual financial results. It could therefore be said that some price-related fundamental information has already been included in prices on 4 November 2008. However, this also runs the risk of including information in the model before it was released to the market. Therefore, all fundamental variables were made effective the day after earnings release date.

Column F represents the fundamental variable, annual turnover for ABF for the financial period and Column G represents the log transformation. Columns H, I and J represent 1-year, 2-year and 3-year percentage change in turnover and have been determined as follows for the variable effective date 5 Nov 2008:

1-year, 9.0161 less 8.8247 = 0.1915 (19.15% change in turnover over a 1-year period)

2-year, 9.0161 less 8.6988 = 0.3173 (31.73% change in turnover over a 2-year period)

3-year, 9.0161 less 8.6344 *= 0. 0.3817 (38.17% change in turnover over a 3-year period)*

Similarly, Columns K, L and M represent 1-year, 2-year and 3-year rate of change in turnover and has been determined as follows for the variable effective date 5 Nov 2008

1-year, 0.1915 divided by 0.1258 *= 1.5217 (152% change in turnover rate compared to the previous 1-year period)*

2-year, 0.3173 divided by 0.1902 *= 1.6679 (166% change in turnover rate compared to the previous 2-year period)*

3-year, 0.3817 divided by 0.2750 *= 1.3879 (138% change in turnover rate compared to the previous 3-year period)*

The same methodology was used to determine percentage change and rates of change for the categories of fundamental information, except that macroeconomic variables are available monthly and prices are available daily. The criteria for the number of lags is therefore correlated to the available variable frequency. As a result, the variables differ with respect to number of lags for calculation. Table 6-8 below details the fundamental variable lags.

Table 6-8 Fundamental Variable Lags

	Variable	Calculation Lag Period
1	Financial statement	3 to 5 years
2	Macroeconomic	2 months
3	Prices	20-days (monthly)

The next section discusses volatility transformation.

6.8 Variable Transformation - Volatility

Volatility is an accepted measure for responsiveness of price movements away from mean values and an established risk measure to explain price performances. There have been a number of studies which either utilise volatility as an input variable or explain price performances using volatility measures. Sharpe (1966), (1994) ratio uses volatility as the denominator to determine portfolio performance as a measure of risk. Black & Scholes' (Black & Scholes, 1973) option pricing model includes standard deviation as input variables to determine option prices. Shiller (1981, p. 434) noted that stock prices exhibited volatility far in excess of the amount which could be explained by future dividends *"five times higher than the upper bound allowed by our measure of the observed variability of real dividends"*. Cochrane (2005, p. 396) suggest that "*excess volatility is exactly the same thing as return predictability"*

Volatility has also been included in this analysis as an input variable. The standard deviations of the independent variables were determined using the following formula:

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{n}} \qquad \text{EQ. 6-11}$$

Where

σ Represents the standard deviation and measure of volatility

μ Represents the sample mean of the underlying independent input variable.

n Represents the number of observations in the population

A volatility measure of the rate of change and percentage change variables was then determined. This is illustrated in Table 6-9 below, continuing the example in Table 6-7.

Table 6-9 Volatility Calculation Variables

A	B	C	D	E	F	G	H	I	J	K	L
						Volatility in Percentage Change			Volatility in Rate of Change		
Row No.	Security	Year End	Annual Earnings release date	Effective Date	Turnover £millions	1-year	2-year	3-year	1-year	2-year	3-year
1	ABF	18-Sep-99	08-Nov-99	09-Nov-99	4299						
2	ABF	16-Sep-00	09-Nov-00	10-Nov-00	4406						
3	ABF	15-Sep-01	06-Nov-01	07-Nov-01	4418	0.0110					
4	ABF	14-Sep-02	05-Nov-02	06-Nov-02	4545	0.0128	0.0019		5.1547		
5	ABF	13-Sep-03	04-Nov-03	05-Nov-03	4909	0.0244	0.0372	0.0263	3.8508	1.1276	
6	ABF	18-Sep-04	10-Nov-04	11-Nov-04	5165	0.0131	0.0113	0.0241	1.0293	1.0897	0.2488
7	ABF	17-Sep-05	08-Nov-05	09-Nov-05	5622	0.0170	0.0038	0.0283	0.5040	0.0765	0.0419
8	ABF	16-Sep-06	07-Nov-06	08-Nov-06	5996	0.0102	0.0068	0.0063	0.4541	0.0198	0.2104
9	ABF	15-Sep-07	06-Nov-07	07-Nov-07	6800	0.0307	0.0205	0.0375	0.5970	0.0875	0.2172
10	ABF	13-Sep-08	04-Nov-08	05-Nov-08	8235	0.0329	0.0636	0.0534	0.2160	0.1964	0.0065

The 1 -year volatility, 0.0329, for variable effective date, 5 November 2008 is determined as Table 6-10 below.

Table 6-10 Volatility Calculation Example

Security	Effective Date	1-Year Percentage Change (x)	$\boldsymbol{\mu}$	$(\boldsymbol{x}-\boldsymbol{\mu})^2$
ABF	07-Nov-07	0.1258	0.15865	0.0010791
ABF	05-Nov-08	0.1915	0.15865	0.0010791
			$\sum(x-\mu)^2$	0.0021582
			n	2
			$\sqrt{\frac{\sum(x-\mu)^2}{n}}$	0.0329

6.9 Final Data Set – Independent Variables

In summary, the following transformation process was applied to each category of independent variable to capture maximum price sensitive information.

- Percentage Change
- Rate of Change
- Volatility of the percentage and rate of change variables

These variables are not opaque to the markets and information available in these variables are not expected to be significant. There are number of other linear and non-linear variable constructs that are available to be included in the model, examples of which have been discussed earlier. However, the primary focus is to explore to what extent a multi-stage racetrack betting methodology can be effectively applied to financial markets, rather than to produce the perfect forms of each fundamental variable. Other variables constructs could possibly be contemplated for future research. The existing variable transformations, as per above, result in 24 independent variables being created to measure for example company turnover for one year. However, these variables are highly correlated and include both complementary and shared information. The variables, percentage change and rate of change are perfectly linearly correlated. Two issues arise when conducting the analysis; a large number of variables created and a high level multi-collinearity between the variables. The next section discusses multi-collinearity.

6.10 Multi-Collinearity of Data and Stepwise Regression

Multicollinearity occurs when two or more variables are highly, but not perfectly, correlated. Software packages also have difficulty in processing variables that are highly correlated. Although it is possible to obtain variable coefficient estimates, statistical inferences would be difficult to make, even though although the transformed variables are anticipated expected to include information content. The issue of multi-collinearity has been discussed in literature. For example, Greene (2012, p. 129) (also (Pindyck & Rubinfield, 1998)) highlights the following problems faced by researchers where there is a high degree of multi-collinearity of data:

i. *Small changes in data producing wide swings in parameter estimates.*
ii. *Variable coefficients having high standard errors and high R^2 for the regression.*
iii. *Coefficients may have wrong or implausible magnitudes.*

A number of choices have been proposed to reduce multi-collinearity in data. These include decreasing the number of independent variables or using diagnostic tools to identify multi-collinearity in data. Pindyck & Rubinfield (1998) suggests a sequential addition or removal of variables, such as a Stepwise-Regression to identify significance of variables and remove highly correlated predictors from the model. A step-wise regression was run as the next stage to identify significant variables and remove those that were highly correlated. A stepwise regression is a linear regression procedure where variables are individually processed and included in the model if the variable is significant. Software packages have two options available for stepwise regression: Forward Selection Stepwise Regression, which starts with no variables in the model, testing the addition of each variable (the variable is included if the model is improved) and repeating this process until none of the variables improve the model. The alternative is the Backward Elimination Stepwise Regression. This process begins by including all variables in the model and then variables are deleted if the model improves by them being deleted. This procedure is then repeated until the model can no longer be improved.

The Backward Elimination Stepwise Regression was selected as the statistical procedure to reduce multi-collinearity and the number of variables. The Backward Elimination Stepwise Regression was preferred as this takes into account all the input variables in the variable sample population to begin with, compared to Forward Selection methodology which includes variable incrementally. Forward Selection procedure may not include a significant variable that may appear later in the variable population for inclusion. Statistical software, IBM SPSS was used to perform the backward stepwise multiple linear regression and a significance level of 5% (F-statistic) was set as the benchmark for inclusion of a variable.

As part of diagnostics testing, however, model correlation statistics were considered to determine the extent of multi-collinearity of independent variables. Normality tests of data, however, was not performed, as literature (Mills & Markellos (2011) deals with returns and price distributions extensively) already shows that prices do not exhibit normal distribution and are characterised by fat tails, peakedness and negative skewness. Normality test therefore was not considered based on previous consistent findings in literature.

Although racetrack betting literature uses non-linear techniques for variable processing, such as SVM, the primary reason for preferring the linear regression approach is that the procedure is not a "black box" and significant variables could be explained linearly with underlying economic principles. As an extension, and for future research SVM and other non-linear techniques could also be employed for variable analysis. One of the primary objective of this thesis, however, is to validate the multi-stage variable processing methodology utilised in racetrack betting markets.

The linear regression is a standard econometric tool to model relationships between multiple independent predictor variables and dependent variable. The regression model is a standard treatment in econometric text books (for example (Gujarati, 1999), (Greene, 2012))

Formally the multiple regression equation is stated as

$$Y_0 = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \cdots \ldots \ldots \ldots . \beta_n X_n + \varepsilon_i \qquad \text{EQ. 6-12}$$

Where

$\mathrm{Y_0}$ − is the dependent variable

β_0 − is a constant

$\beta_1 \ldots \ldots \beta_n$ − are the constant coefficients or parameters associated with independent variables $\mathrm{X}_{1 \ldots\ldots}\mathrm{X_n}$

$\mathrm{X}_{1 \ldots\ldots}\mathrm{X_n}$ − are the independent predictor variables

ε_i − is the random error noise term that is unobserved

The parameters of the models are derived using the Ordinary Least Squares method which minimizes the error term, ε_i or the Residual Sum of Squares (RSS). The next question is what are the appropriate statistical methodologies that should be used for model estimation. Final probabilities of security prices must be determined, as these are inputs to the Kelly strategy where amounts bet is a function of probabilities. It is therefore apparent that a logit model is the appropriate methodology to determine the probabilities given its success in racetrack betting markets. However, the following additional question arises with respect to securities:

Is pricing of securities a competitive event where all securities are in the 'same race'? For example, two securities included in the current sample are: GSK, pharmaceutical company, and Barclays, a bank; are these in the 'same race? One would argue that a pharmaceutical company and a bank are in two different industries, servicing different clientele and therefore not in the 'same race'. A similar argument could be made for British Aerospace which is in the Aerospace & Defence industry. It is therefore proposed that a logit model rather than a CL model should be implemented. The next section discusses the logit model and justification for the logit model (cf. the CL model).

6.11 Logit or CL Model

In racetrack betting, competition within race is a key factor in model calibration, given that only the winning horse (and for some bets, second and third place) secures a return. Bets on the remaining horses lose. The CL model takes into account competition within a race when determining final winning probability. The racegoer then places bets on horses where calculated probabilities are greater than those probabilities implied by market odds, and discards the rest of the field. There are also opportunities available to make bets which win by selecting losing horses, via online betting exchanges similar to trading in financial markets where short as well as long position can be traded. For example, two securities, A and B, with final probabilities 99% and 1% will potentially yield a symmetrical payoff, except that security A will be a long strategy and security B a short strategy. The availability of symmetrical payoff in financial markets therefore suggests that a security with a 99% probability and another with 1% probability can lead to winning trades. All securities therefore offer the opportunity for winning bets given that security prices can only go either up or down. The most important criterion therefore is the ability to correctly classify winners (prices going up) and losers (prices as going down) in financial markets.

The non-linear machine learning technique, SVM, could also be used for the classifying securities into winners and losers. However, as previously noted in earlier studies (Edelmen, 2007), (Lessmann, Sung, & Johnson, 2010), SVM are 'black boxes' and there is difficulty in interpreting variable coefficients even though SVM show superior performance. SVM's could be contemplated for modelling fundamental financial market data in future research. The next section discusses the logit model.

6.12 Logit Model

The logit model is a probability model of the form:

Let Y_i be the binary response variables where

$$\mathrm{Y_i} = \begin{cases} 1 - \text{upward movement in price} \\ 0 - \text{downward movement in price} \end{cases}$$

EQ. 6-13

$$E(X_i/Y_i) = \pi_i = \frac{e^{(\beta_0+\beta_1 X_1)}}{1 + e^{(\beta_0+\beta_1 X_1)}}$$

$\mathrm{X_i}$ − is the observed value of explanatory variable for security i

$\mathrm{X} = (\mathrm{X_1} \ldots\ldots\ldots\ldots \mathrm{X_K})$ set of explanatory variables

Consider a security S which has a vector K of observed attributes (for example Dividend Yield, return on Equity, Revenues) associated with it denote

$$x_h = [x_{h1}, x_{h2}, x_{h3}, \ldots \ldots \ldots x_{hk}]$$ EQ. 6-14

In addition, each security is impacted by industry and macroeconomic variables characterised by a vector of M attributes

$$y_h = [y_{h1}, y_{h2}, y_{h3}, \ldots \ldots \ldots y_{hk}]$$ EQ. 6-15

A general specification of the model for the security pricing process is given by

$$P_h = p\ (X, Y)$$ EQ. 6-16

Where

X, Y are the relevant security fundamental, macroeconomic and industry variables for all securities in a market. As discussed earlier the differences in timing and available frequency of financial statement and macroeconomic data suggests that base models are required to be independently developed. This avoids financial statement variables being “overwhelmed” by macroeconomic data, should these variables be included in a single model. The base models are then required to be combined into a single model as a second-stage. The next section discusses combining base models.

6.13 Combining Base Models – Financial Statement and Macroeconomic Data

Lessmann S. , Sung, Johnson and Ma (2012) develop a methodology for combining model based predictions in competitive events using an ensemble learning technique, stacked regressions for pooling, first proposed by Breiman (1996). The study found that stacked regression achieved a better return performance compared to a two-stage logit model.

In the Lessmann S. , Sung, Johnson and Ma (2012, p. 164) a library of diverse base models was developed. The study showed that combining models using stack produced results that were *"statistically and economically accurate forecasts which are superior to those generated by a highly challenging benchmark model",* the CL model. Combining models reduces model uncertainty and improves prediction rate by increasing diversity, specially if the model variables are not correlated and independent. In addition the relative forecasting ability of individual models change over time. Models may work in some periods and not well in other periods.

The base models in the Lessmann S. , Sung, Johnson and Ma (2012) study included a diverse number of modelling procedures involving a number of base models that were developed using multiple methodologies; linear regression, support vector regression, support vector classification, Random Forest Classification, Adaboost, Stochastic gradient boosting. However, in this study there are three base logit models that are relatively homogenous in characteristics and were developed as a result of timing differences in data availability. Model stacking as technique for combining models therefore may not be appropriate, given the population of models is limited to 3 compared to 571 models in Lessmann S. , Sung, Johnson and Ma (2012) study. The logit model was therefore also used as the basis for combining the base models.

6.14 Modelling Summary

The steps in the modelling methodology can now be summarised as follows:

(i) Construction, transformation and optimisation of independent variables

(ii) Development of the following independent base forecast models using logit:

a. Financial statement model

b. Macroeconomic model

c. Price model

(iii) Combining the base forecast models developed in step (ii) above to determine final model probability.

(iv) Implementation of Kelly strategy

The modelling methodology is therefore linear and includes non-linear independent variables.

The next section details the calculation methodology for dependent variables.

6.15 Dependent Variable Calculations

Let $Y_{1\ldots\ldots.}Y_n$ be the dependent variables and predictions of security returns where prices are forecasted as a movement of prices either going up or down. Let the upward price movement be denoted as 1 and downward movement be denoted by 0. The dependent variables therefore can be stated as vector:

$$Y_1 \ldots \ldots \ldots . Y_n = \binom{1}{0} \qquad \text{EQ. 6-17}$$

Where

Y Represents a security and there are 1 to n securities

1 Represents an upward change in price in security n

0 Represents a downward change in price in security n

However, in calculating the dependent variable, two assumptions are required: transactions costs incurred for trading and interest costs. The next section discusses transaction costs.

6.15.1 Transaction Costs

Racetrack betting markets returns are net after track-take. However, in financial markets the buying and selling prices quoted are at gross prices and do not include transaction costs. Transaction costs therefore must be included to reflect the true return and profitability on a trade.

Transaction costs also vary, depending on volumes traded, type of security (for example, FTSE 100 v FTSE 250 security), the liquidity of the security and the venue (derivatives versus exchanges) in which the security is traded. Security returns in the analysis have been modelled on a gross basis, as well as on a net basis after transaction costs.

It is assumed that transaction costs are equal to 0.25% per trade or a total cost of 0.5% for a buy and sell trade. It is also assumed that minimum trade value is £2,500. At a trade value of £2,500 brokerage costs incurred is estimated to be £12.50 and comparable to brokers in the market place. The transaction costs are conservative as with a higher frequency of trades brokerage costs declines to £5, independent of the gross value of trade. Table 6-11 below, details indicative transaction costs at different trade volumes and differing transaction cost percentages. Based on the existing brokerage rates offered by the brokers[18] and comparative broker costs it is unlikely that the transaction costs will be as high as £5,000 or higher. For example, £1,000,000 trade executed with online broker, TD Waterhouse would incur a total cost £65.95 for a regular trader. This is also a reasonable assumption as it is highly unlikely that a trader would enter into a single trade in the market place and reveal its position. The trader would most likely execute a series of smaller trades to ensure that the final price received/paid is not influenced by a large trade. For example, a trader with an order to execute a trade of £100m would highly likely execute 100 trades of a £1m each compared to a single trade of £100m to avoid the risk of either driving up the price (if the trade is a buy) or driving the price down (if the trade is a sell), to the trader's detriment.

Table 6-11 Transaction Costs at Different Trade Values

	Transaction Costs - £				TD[19] Waterhouse Costs	Barclays[20]	Hargreaves Lansdown[21]
Gross Trade Value	**0.20%**	**0.25%**	**0.30%**	**0.50%**			
1,000	2	2.5	3	5	£12.50 (<10 trades per month)	£11.95 (<10 trades per month)	£11.95 (<10 trades per month)
2,500	5	6.25	7.5	12.5			
5,000	10	12.5	15	25	£8.95 (10-19 Trades per month)	£8.95 (10-19 Trades per month)	£8.95 (10-19 Trades per month))
10,000	20	25	30	50			
50,000	100	125	150	250	£5.95 (20 or more trades per month)	£5.95 (20 or more trades per month)	£5.95 (20 or more trades per month)
100,000	200	250	300	500			
1,000,000	2,000	2,500	3,000	5,000			

The next section discusses interest costs.

[18] Source for comparison: TD Waterhouse
[19] Source: TD Waterhouse Broker website
[20] Source: Barclays Broker website
[21] Source: Hargreaves Lansdown broker website

6.15.2 Interest Costs

Interest holding costs may be incurred when securities are bought and sold. These costs are however a function how trades have been funded; using own capital or borrowed funds. It is assumed that interest costs are zero. However, interest earned has been used as one of the performance benchmark measures and therefore does not need to be taken into account in return calculations.

6.15.3 Return Calculation Methodology – Dependent Variables

Given that a security is bought and sold, the payoff for a security, dependent variable, Y_n, can defined as follows:

$$Y_n = P^S - P^B \qquad \text{EQ. 6-18}$$

Where

Y_n Represents return on security n

$If\ \ Y_n > 0, then\ Y_n = 1\ and\ \ if\ Y_n < 0, then\ Y_n = 0.$

$P^B = \ Security\ Buying\ Price\ \ = Security\ Purchase\ Price + Transaction\ Costs$

$$P^B = \ln(P^B)$$

$P^S = \ Security\ Selling\ Price\ \ = Security\ Selling\ Price + Transaction\ Costs$

$$P^S = \ln(P^S)$$

The return on a security therefore reduces to:

$Y_n = \ \ln(Security\ Selling(Buying)\ Price) - \ \ln(Security\ Buying(Selling)\ Price)$

The return on security is effectively expressed in percentage terms (or close approximation), rather than actual amounts. The percentage gain or loss on a long trade position is then determined by subtracting buying price from selling price

$$YL_1 = ln\left(P^S_{t+n(CLOSING)}\right) - \ ln\left(P^B_{t(OPENING)}\right) \qquad \text{EQ. 6-19}$$

A fundamental underlying assumption regarding prices is that the buying and selling prices are realized in an orderly market between willing buyers and sellers. Monthly return is defined as decisions made today, t_0, based on all information available until t_o, to buy/sell when market opens the next day, t_{+1}, and sell /buy at market close at the end of the day 20, $t_{+20.}$ It is assumed that there are 20 trading days per month. The monthly trade therefore, will be a trade opened at the beginning of the month, for example 1 April (based on all information available up close of market of the previous month, 31 March) and closed at close of market on at the end of the month, 30th April.

Monthly return therefore is defined as:

$$Monthly\ Return \qquad Y_{L20} = P^{S}_{t+20(CLOSING)} - P^{B}_{t+1(OPENING)} \qquad \text{EQ. 6-20}$$

Where

Y_{L20} Represents one-day gain/loss or one-day return on a long position where Y_{L20} is > 0 for a gain and Y_{L20} < 0 for a loss.

Table 6-12 below, provides an example calculation.

Table 6-12 Calculation of Dependent Variables – Gross Returns

		Market Prices (In pence)		Log		Gain/Loss	Dependent Variable
	Date	Open	Close	ln	ln	YL_1	YL_1
t_o	25/03/2009	335.50	323.50	5.8156	5.7792		
t_1	26/03/2009	323.25	321.00	5.7784	5.7714	-0.0442	0
t_2	27/03/2009	321.00	308.00	5.7714	5.7301	-0.0483	0
t_3	30/03/2009	306.00	307.25	5.7236	5.7277	-0.0438	0
t_4	31/03/2009	310.25	313.00	5.7374	5.7462	0.0226	1
t_5	01/04/2009	313.50	315.00	5.7478	5.7526	0.0152	1
t_6	02/04/2009	319.50	324.00	5.7668	5.7807	0.0329	1

In Table 6-12 above the value of the dependant variable, $YL_{1,}$ at t_3 is calculated as follows:

Closing Market Price at $\mathbf{t_3}$ = 306.00

$\ln(307.25)$ is = 5.7277

Opening Market Price at $\mathbf{t_2}$ = 321.00.

$\ln(321.00)$ is = 5.7714

Therefore, Gain or Loss at t_3 is given, by $t_3 - t_2$ = 5.7277 less 5.7714 = -0.0438, a loss.

As this is a negative value $Y_{L1,}$ at t_3 = 0.

Similarly, in Table 7-14 above the value of the dependant variable, $Y_{L1,}$ at t_4 is calculated as follows:

Closing Market Price at $\mathbf{t_4}$ = 313.00

$\ln(313.00)$ is = 5.7462

Opening Market Price at $\mathbf{t_3}$ = 306.00.

$\ln(306.00)$ is = 5.7236

Therefore, Gain or Loss at t_4 is given, by $t_4 - t_3$ = 5.7462 less 5.7236 = 0.226, a gain.

As this is a positive $Y_{L1,}$ at t_4 = 1.

6.15.3.1 Returns Net of Transaction Costs

The dependant variable in the model is on a gross basis and therefore gross returns have been modelled. However, for the purposes of determining final returns, transaction costs should be taken account.

Transaction costs vary depending on the market in which the trade has been executed, size of trade, and the frequency of trading. For example, TD Waterhouse, a low cost online trading platform charges the following amounts[22]:

- £5.95 if 20 trades or more are executed per quarter, otherwise the rate is £12.50.
- A surcharge of £30 if trade value is between \$100k to £500k
- A surcharge of £60 if trade value is between \$500k to £1m
- Rates are negotiable if trade value is greater than £1m.

Trading online is now a mature market and standard trading costs are now of a similar structure across the service provides over the sample period. Barclays, for example charges a rate of between £5.95 and £11.95[23].

The net gain/loss for $\mathbf{t_3}$ has been determined as follows:

Net purchase Price = $\mathbf{t_2}$ = 321.00 x (1+ 0.25%) = 321.8025p per share.

A 1,000 shares trade would therefore cost:

[22] Source: TD Watehouse website, url: http://www.tddirectinvesting.co.uk/rates-and-charges
[23] Source: Barclays website, url: https://www.barclaysstockbrokers.co.uk/

Shares Gross value 321p x 1,000 shares = £3,210

Transaction costs 321p x 1000 *0.25% = £8.025

Total Cost = £3,218.025 or £3,218.03

Net Sale Price = $\mathbf{t_3}$ = 306.00 x (1- 0.25%) = 305.235p per share.

A 1,000 shares trade would therefore cost:

Shares Gross value 306p x 1,000 shares = £3,060

Transaction costs 306p x 1000 *0.25% = £7.65

Total Selling Price = £3,052.35

Net Gain/(Loss) = £3,052.35 - £3,218.03 = (£165.68)

Net % Return – ln (3052.35) – ln (3218.03) = 8.02367 – 8.0765

= - 0.0528 or - 5.28%

The net return for this trade is therefore negative 5.28%. The next sections discuss Kelly strategy and performance measures.

6.16 Kelly Strategy

The Kelly criterion maximises expected logarithmic growth rate of expected wealth while minimising the probability of ruin. Thorp (1997) adapted and formulated the Kelly strategy for stock markets. The maximum fraction of wealth bet to maximise growth is given by

$$f^{*} = p - q \qquad \text{EQ. 6-21}$$

Where

f* represents fraction of wealth bet

p Represents the winning probability >0.5

q = 1 - p

In other words, the fraction of wealth bet is equivalent to the winning edge, where

$Winning\ Edge = \ Win\ Probability\ (P) - \ Loss\ Probability (Q)$

The expected growth in wealth then is

$$m \equiv f^* \qquad \text{EQ. 6-22}$$

Where,

m Represents the wealth expectations operator.

Thorp (1997), however, states that the wager must consider the wins and losses in determining amount bet *"where a unit wager wins b with probability p>0 and loses a with probability q ………… then if m > 0, f* > 0, f* = m/ab."* (Thorp, 1997, p. 7).

Substituting EQ 6-21 into *f* = m/ab,* the Kelly formula simplifies to the following:

$$f^* = \frac{p-q}{ab} \qquad \text{EQ. 6-23}$$

Where

b Represents the unit winnings with probability >0

a Represents the unit losses with probability q (or 1-p)

Then, if the assumption is made that a unit b is represented by win probability and unit a is represented by loss probability, then equation 6-23 can rewritten (and the fraction of wealth bet as):

$$f^* = \frac{p-q}{pq} \qquad \text{EQ. 6-24}$$

The Kelly strategy is now demonstrated.

Assume 2 securities, A and B with winning probabilities of 12% and 65%, respectively and a $200 initial wealth. Table 6-13 below shows the fraction of wealth bet on the two securities for a full Kelly and fractional Kelly strategy.

Table 6-13 Capital Allocation to Security– Kelly Strategy

	A	B	C	D	E	
	Calculated Probability	Winning Probability (P)	Losing Probability Q = 1 – P)	Edge (P – Q)	D / (B x C) Kelly %	Kelly Allocation
A	0.12	0.88*	0.12	0.88 - 0.12 = 0.76	7.20	85% (7.20 / 8.52)
B	0.65	0.65	0.35	0.65 - 0.35 = 0.30	1.32	15% (1.32 / 8.52)
					8.52	1

*winning probability is equal to 1 less 0.12 as the strategy implemented is a short.

Security A will therefore have more than five times the amount allocated given the relatively higher winning probability and edge.

In a full Kelly strategy, the capital will be allocated as follows:

Security A: 85% x 200 = £170

Security B: 15% x 200 = £30.

In a fractional Kelly, a predetermined fraction of the wealth is bet. Assuming a 50% fractional Kelly, allocation of initial wealth capital will be allocated as follow:

Security A: 85% x 200 x 50% = £85

Security B: 15% x 200 x 50% = £15

£100 will remain as funds held. The next section discusses model and return performance measures.

6.17 Model Performance Measures and Benchmark

The R^2 is a generally accepted statistical measure to evaluate the performance of a regression model and how effectively the model explains the prediction of the dependent variable. The R^2 is the percentage of variation in the dependent variable (Y price prediction) that is explained by a regression model and is given by the following general formula[24]:

$$R^2 = \frac{RSS}{TSS} = \frac{\sum(\hat{Y}_i - \bar{Y})^2}{\sum(Y_i - \bar{Y})^2} \qquad \text{EQ. 6-25}$$

[24] Pindyck & Rubinfield (1998)

Where

RSS Represents the explained variation in Y

TSS Represents the total variation in Y

Y Represents the observes values

$\bar{Y}$ Represents the mean of the observed values

$\hat{Y}$ Represents the predicted values

The R^2 is expressed as a percentage, where a value of 0 suggests that the regression model is a poor fit and does not explain any of the variation in the dependent variable and 1 suggests that the model fully explains the variations in the dependent variable. The R^2 measure, however, is for ordinary least squares regression models and not for logistic regression which utilises the maximum likelihood function to determine model estimates. A number of equivalent R^2measures have been developed to evaluate the goodness-of-fit of logit models. The McFadden R^2 (1974) measures the improvement in the fitted model compared to the null model. Cox-Snell (1989) and Nagelkerke (1991) are refinements of the McFadden R^2 and also measure the improvement in the final model from the base model. The McFadden (1974), Cox-Snell (1989) and Nagelkerke (1991) R-Squares will be calculated as comparative measures for goodness of fit. In addition, Akaike Information Criterion (AIC) (Akaike, 1973) will be used. The next section discusses McFadden R-square.

6.17.1 McFadden R-Square

The logit model is estimated by maximizing the likelihood function where the value of log likelihood closer to zero suggests a better relative model fit. McFadden (1974) proposed the likelihood ratio index (also known as R^2) to measure the comparative performance and effectiveness of competing models. The likelihood ratio index has since been widely used in empirical literature to measure comparative performance of competing models. For example, Benter (1994) compared the value of public information to fundamental information, Sung & Johnson (2007) compared the effectiveness of one-stage and two stage model and Lessmann S. , Sung, Johnson, & Ma (2012) for increase in R^2 and model improvement. The likelihood ratio index will be employed in this study to measure the effectiveness of the final model to the base model to determine the usefulness of a multi-stage modelling process. The McFadden R^2 is computed as follows (Greene, 2012, p. 573):

$$\text{McFadden } R^2 = 1 - \frac{(lnL_1)}{(lnL_0)} \qquad \text{EQ. 6-26}$$

Where

lnL_1 Represents the log likelihood of the final model estimated.

lnL_0 Represents the log likelihood of the base model estimated or the model with no parameters

The computed value of R^2 is between 0 and 1 and suggests the level of improvement from the base model to the final model. The next section discusses the Cox-Snell r-square measure.

6.17.2 Cox-Snell R-Square

The Cox-Snell measure is similar to the McFadden R-square, except ratio of the likelihoods reflects the improvement of the full model over the intercept model

$$\text{Cox-Snell } R^2 = 1 - \frac{(lnL_0)}{(lnL_1)}^{2/N} \quad \text{EQ. 6-27}$$

Where,

lnL_1 Represents the log likelihood of the final model estimated.

lnL_0 Represents the log likelihood of the base model estimated or the model with no parameters

N Represents the number of observations in the data set.

6.17.3 Nagelkerke R-Square

The Nagelkerke R-Square is a refinement of Cox-Snell measure.

$$\text{Nagelkerke } R^2 = \frac{\text{Cox−Snell } R^2}{R^2_{max}}$$

EQ. 6-28

Where,

$$R^2_{max} = 1 - (lnL_0)^{2/N}$$

Yet another measure for model performance is model confidence set (Hansen, Lunde, & Nason, 2011), however, this analysis is a model selection process from a selection of final models. Although base models are produced these are interim steps to a final stage single model. Hansen, Lunde, & Nason (2011) also suggests to use Aikake information criterion (AIC) or Bayesian information Criterion (BIC) for model confidence of regression models. The BIC and AIC produce similar statistic result and therefore one was selected. AIC was included in final results. The next section discusses AIC.

6.17.4 Akaike Information Criterion

A similar measure for determining best model fit is the AIC and determined as follows (Greene, 2012, p. 573):

$$AIC = -2\ lnL_1 + 2K \quad \text{EQ. 6-29}$$

Where

K Represents the number of parameters in the model

A lower value for AIC represents a better model fit.

Monthly returns were then calculated to enable measurement of model performance. The monthly security return is defined as follows:

$$R_n = \frac{Y_{L/S}}{Security\ Opening\ Cost\ Position} \quad \text{EQ. 6-30}$$

Where

R_n Represents percentage return on security n

$Y_{L/S}$ Represents is the return for the period as determined in section 7.13.3.4

Security Opening Cost Position Represents the opening security selling cost (for short positions) or opening buying price (for long positions).

A comparison of model security returns was then made against a benchmark to evaluate model performance. The security returns were measured against the following benchmarks:

i. One-month Libor interest rate. In other words, comparing the returns to security against holding the funds in cash and interest earned.
ii. A FTSE-100 index benchmark. The FTSE-100 index was used as a benchmark with a buy and hold strategy (i.e. a strategy of buying and holding the FTSE 100 index). The FTSE-100 is an industry accepted benchmark used by investment managers to monitor and measure return performance.
iii. A naïve buy-and hold strategy of the specific security in the sample population.

iv. Lastly, a weighted average buy-and hold strategy where holdings in each security whose winning probability has been predicted by the model, was a simple weighted average of the calibrated probabilities as shown in Table 6-14 below (for comparison to the Kelly strategy)

Table 6-14 Capital Allocation to Security– Simple Weighted Average

	A	B	C	D	E
	Calculated Probability	Losing Probability (1 – P)	Winning Probability (P)	Allocation %	Calculation
A	0.12	0.12	0.88*	57.5%	(0.88 / 1.53) = 57.5%
B	0.65	0.35	0.65	42.5%	(0.65 / 1.53) =42.5%
		TOTAL	1.53**	100%	

*winning probability is equal to 1 less 0.12 as the strategy implemented is a short.

** Sum of probabilities to determine final portfolio allocation given two securities, A and B, as shown in column E.

Table 6-14 shows that the calculated probability for security A is 0.12. In other words, there is a 12% probability of price for security A rising or 88% probability (1 - 0.12) that price will decline. The strategy for security A would therefore be a short. Similarly, for security B there is a 65% probability of price for security B rising or 35% probability (1 - 0.65) that price will decline. The strategy for security B would therefore be a long. Security A and B have relatively similar asset allocation in the weighted average method in comparison to the Kelly approach where Security A has a significantly higher allocation. The next section discusses the performance measures for the model output.

6.18 Security Returns Statistics

The overall model returns performance was reviewed using the following descriptive statistical measures:

i. Mean and median
ii. Return volatility
iii. Skew, where a positive skew indicates a return >0 more likely with a mean return higher than median.

A positive mean and median returns would suggest that the model-based trading decisions would highly likely result in positive returns. The return volatility or standard deviation away from the mean indicates the level of variation in mean returns. A low return volatility would be preferred to a higher volatility. In addition to the descriptive measures the Sharpe ratio was calculated to evaluate model performance. The Sharpe ratio ((1966), (1994) is a common measure for evaluating portfolio performance. This was employed as a criterion to measure whether returns were positive after taking into account the relative risks. The Sharpe ratio is an accepted risk-return measurement criterion to evaluate performances in both financial markets and racetrack betting markets (for example, (Lessmann S. , Sung, Johnson, & Ma, 2012) .

The Sharpe ratio, S_i, is calculated as follows:

$$S_i = \frac{\bar{R}_i - \overline{R_f}}{\sigma_i} \qquad \text{EQ. 6-31}$$

$\bar{R}_i$ Represents the daily average return on security i over a given time horizon.

R_f Represents the daily average risk free rate (or benchmark return) over a given time horizon

σ_i Represents the daily standard deviation of the rate of return of security i over a given time horizon

A positive Sharpe ratio would indicate that higher returns can be earned without additional higher risks. The Sharpe ratio therefore is effectively interpreted as an information ratio (Grinold & Kahn, 1994) indicating the model's ability to capture return sensitive information without the additional risks.

Finally, there is also the general risk of data snooping. White (2000)suggests that empirical results could be impacted by data snooping where a data set is used more than once for model inferences. The risk of the results in this study being impacted by data snooping, however, is minimal because each security in the sample population is modelled independently utilising, independent sets of data series. The conclusions are then drawn from the portfolio of securities modelled, rather than an individual security. Therefore, the modelling process includes adequate safeguards against data snooping.

6.19 Research Methodology Summary

The aim of this section was multi-fold:

1. To develop an and present the appropriate research design for this real-world research project, selecting from the wide-ranging research methodologies available. The research methodology

selected, consistent with previous racetrack betting and financial research, adheres to the functionalist paradigm and a quantitative methodology.

2. To develop the hypothesis to be tested: namely, whether a multi-stage modelling methodology from racetrack betting markets could employed to test for semi-strong form efficiency in financial markets.
3. To develop an effective methodology to adapt from racetrack betting markets to financial markets. The methodology for calculation of dependent and independent variables and steps for model development were outlined; a multi-stage process.
4. Finally, the performance measures used for testing the hypothesis were discussed.

The research method can also be equivalently described as broadly following econometric analysis and proceeding along the following lines:

(i) Determination of a hypothesis
(ii) Collection of Data
(iii) Identifying the appropriate statistical technique for testing the hypothesis
(iv) Estimating the parameters of the statistical model from the data collected
(v) Using the model specified for testing hypothesis
(vi) Evaluating the results and reaching a conclusion by accepting or rejecting the hypothesis.

The next section presents the empirical results followed by a discussion and analysis of the results.

Chapter 7: Empirical Results

The empirical results are divided into four parts.

Part 1

Part I details portfolio strategies and benchmark returns against which portfolio performance has been measured. Section 7.1 details the benchmark returns against which performance of the portfolio of securities was measured and Section 7.2 details the portfolio strategies employed for measuring performance.

Part II

Part II details for an example security, ABF, in Section 7.3, the calculation methodology followed to determine the final results. Section 7.4 and section 7.5 presents the results for two additional securities. This section continues the earlier example in the Methodology chapter, Section 7.1

Part III

Part III discusses the model fit and predictability statistics in Sections 7.6 to 7.10. Section 7.6 presents the model fit statistics and Section 7.7 details the model coefficients. Sections 7.8 and 7.9 then presents the results of stationarity and long-range dependency tests. Finally, section 7.10 discusses the model predictability statistics.

Part IV

Part IV presents the main return results for the sample portfolio of securities in section 7.11. Section 7.11 presents the returns on the total portfolio. The next section discusses Libor returns benchmark.

PART I – Portfolio Strategies and Benchmark

7.1 Returns Benchmarks

The one-month Libor (London Interbank Offer Rate) was the benchmark risk-free interest rate, against which performance of the portfolio of securities was measured. The monthly Libor rate was selected as the duration is the same as the holding period of the securities traded in the sample population. The data series was obtained from the online Federal Reserve Bank of St

Louis - Economic Research database. Table 7-1 below presents the summary returns statistics for the one-month Libor rate for the in-sample, validation and out-of-sample periods. The statistical functions in Microsoft Excel were used to determine the summary results.

Table 7-1 Month Libor – Summary Monthly Returns Statistics

	1-Month Libor		
	In-Sample Returns - Period 2005-2011	Validation Sample Returns - Period 2012-2013	Out-of-Sample Returns - Period 2014-2015
Mean Per Annum Return	3.228%	0.554%	0.500%
Median Return	4.588%	0.494%	0.504%
Standard Deviation	2.279%	0.095%	0.009%
Skewness	-15.661%	121.741%	-89.754%
30-day Libor Rate (per annum):			
Minimum	0.50%	0.487%	0.482%
Maximum	6.75%	0.778%	0.514%
Value of £1 Invested at Mean Per Annum Return	£1.25 [1 *(1+3.228%)^^{7years}]	£1.01 [1 *(1+0.554%)^^{2years}]	£1.01 [1 *(1+0.500%)^^{2years}]

The "Value of £1 Invested" in Table 7-1 represents £1 at the end of each sample period. In addition to the monthly Libor rate, security returns were compared to:

i. The FTSE-100 benchmark

ii. A naïve buy-and-hold strategy for the security

The FTSE-100 index was bought and sold after monthly holding periods, same as the security. The FTSE-100 monthly index was used given that the securities in the sample are a composition of the index. An underperformance of the sample portfolio against the FTSE-100 would suggest that a trader would be better off trading the index and that the multi-stage modelling methodology is not effective in extracting information not priced by the market.

The execution of a naïve buy-and-hold strategy is identical to the FTSE-100 benchmark strategy except the trades are in the corresponding security. A naïve buy-and-hold strategy of the sample security would also suggest a similar conclusion of ineffectiveness of a multi-stage modelling methodology.

Table 7-2 below presents the summary returns for the FTSE-100 index.

Table 7-2 FTSE-100 – Summary Returns Statistics

	FTSE-100					
	Training Sample Returns - Period 2005-2011		Validation Sample Returns - Period 2012-2013		Out-of-Sample Returns - Period 2014-2015	
	Gross	Net	Gross	Net	Gross	Net
Mean	0.18%	-0.02%	0.66%	0.46%	-0.51%	-0.71%
Median	1.07%	0.87%	0.94%	0.74%	-0.63%	-0.83%
Standard Deviation	5.06%	5.06%	3.24%	3.24%	3.38%	3.38%
Skewness	-128.70%	-128.70%	-58.03%	-58.02%	-20.788%	-20.79%
Cumulative Returns	327.36%	-37.64%	345.34%	240.94%	-260.044%	-361.44%
No of Trades	1825	1825	522	522	507	507
Minimum	-32.03%	-32.23%	-10.27%	-10.47%	-11.023%	-11.22%
Maximum	15.58%	15.38%	9.40%	9.20%	7.64%	7.44%
FTSE Index Range over sample period	4,853.4 – 5,572.3		5,572.3 – 6,550.7		6,550.7 – 5,990.4	

The FTSE-100 returns shown above are gross and net of transaction costs, which were calculated at 0.10% per index trade. Although the FTSE-100 gross returns were positive, transaction costs have had a significant impact on gross returns, resulting in net returns being negative. The FTSE-100 had high levels of volatility and negative skewness, indicating that negative returns were more likely than positive returns. The overall profile of returns on FTSE-100 over the sample periods suggests that a trader would have been better off holding cash than trading FTSE 100 indices, except for the validation sample period where the returns are highly positive. The next section discusses portfolio strategies.

7.2 Portfolio Strategies

Table 7-3 below explains the portfolio strategies.

Table 7-3 Portfolio Strategies Explained

	Strategy definitions	**Types of Positions in Security**	**Strategy Based Model Probability**	**How is a £1 allocated Across securities**
1	FTSE Benchmark Index	Long	X	N/A

	Strategy definitions	Types of Positions in Security	Strategy Based Model Probability	How is a £1 allocated Across securities
2	Buy-and Hold Strategy	Long	X	Divided by 25 securities
3	Final Model Probabilities Strategy	Short and Long - Based on Model Probability	✓	Divided by 25 securities
4	Weighted-Average Portfolio Allocation Strategy	Short and Long - Based on Model Probability	✓	Weighting based on Probability
5	Kelly Strategy	Short and Long - Based on Model Probability	✓	Based on Kelly Formula

1. The FTSE-100 benchmark is as per Table 7-2.
2. The Buy-and Hold Strategy is as explained in section 7-1 except that funds were equally allocated across the portfolio of sample securities to determine portfolio returns.
3. Final Model Probabilities Strategy equally allocated funds across the portfolio of securities. The strategy, however was based on final model probabilities.
4. The Weighted-Average Portfolio Allocation Strategy. A portfolio-based approach was employed to allocate funds where securities with higher probabilities had higher fund allocation. This is explained with an example in Table 6-14 (Section 6-17).
5. Kelly Strategy. The Kelly strategy uses the Kelly formula to allocate funds across the sample portfolio of securities. This is explained with an example in Table 6-13 (Section 6-16).

The next section provides a detailed modelling illustration for the ABF security, a security in the sample population, and continues the example from variable transformations in sections 6-7 and 6-8. The multi-stage modelling methodology requires an initial development of base forecast models and then combining the base models as a next stage. Three base models were developed utilising financial statement, macroeconomic and price data. The next section discusses the development of the base financial statement model for security ABF.

PART II – Calculation Methodology for an Example Security - ABF

7.3 Security ABF – Base Financial Statement Model

The financial statement variables – turnover, net profit margin and dividends per share – were first transformed to include percentage change, rate- of-change and volatility variables of the base information. A data file was then created for each security that was split into three independent data sets; a training set which was used for developing the base logit model, a validation set for combining base models at the next stage and an out-of-sample data set for testing the validated model. The training set included data from the period 1st January 2005 to 31st December 2011. The validation sample included data from 1st January 2012 to 31st December 2013. The out-of-sample data set was for the period 1st January 2014 to 31st December 2015. Although the training data set was from 2005, additional lagged variables before 2005 were also needed. For example, percentage change and rate of change calculations required data for previous years for variables in the year 2005. Actual data utilised therefore were from 2001 to enable the development of a data set from 2005 onwards. Table 7-4 below provides a sample extract of the data file for logit modelling.

Table 7-4 Sample Extract Data File

A	B	C	D	E	F	G	H	I	Column J and onwards
	DEPENDANT VARIABLE				INDEPENDENT VARIABLE				
Security	Trade Open Date	Trade Close Date	Gross Return	Dependent Variable (Price directional movement, 1, 0)	Independent Variable Effective date (+1 day)	Turnover	Turnover (Log Variable)	Series of Pre-Determined Variables: Percentage Change; Rate of Change and Volatility	
ABF	03/01/2005	28/01/2005	-0.027416861	0	10/11/2004	5165	8.549660382	0.050834847	0.659828398
ABF	04/01/2005	31/01/2005	-0.033879462	0	10/11/2004	5165	8.549660382	0.050834847	0.659828398

Column A is the security identifier, ABF. Columns B to E provide details for the dependent variables, where column D is the Gross Return on security ABF and E is the value as input into the model. Security ABF was bought on 3rd January 2005 (Trade Open Date) and sold on 28th January 2005, realising a loss of -0.027416861. Another trade was then executed the next day, 4th January 2005 and closed after a month on 31st January, and so on. In other words, trades were made daily and closed after a holding period of one month (20 business days). The value for the dependent value in column E is 0, representing a downward movement in price or a loss on a long position.

Columns F to J and onwards represent the related independent variables for the trades executed. Column F denotes the effective date of the independent variables. In the above example, ABF released its annual results on 9th November 2004. The variables therefore became effective from 10th November 2004 until 9th November 2005, when the next set of annual results would have been released. Column G represents the fundamental variable turnover and column H represents the natural log of turnover. Columns I, J and onwards represent the percentage change, rate of change and the volatility in these variables as discussed in sections 7.7 and 7.8., and therefore include variables transformations for Net Profit Margin and Dividends Per Share. The independent variables remain static for the next twelve months given that these are financial statement data. The calculations of daily returns, however, change.

As a first step, a step-wise linear regression was then run to reduce multi-collinearity within variables and exclude those where the F-values were greater than 0.05, to ensure that only significant variables were included in the logit model. SPSS v22.0 was used for this task and for generating outputs. The regression reduced the total number of independent variables to 18; these are detailed Table 7-5 below, including the level of significance and t-values.

Table 7-5 Step-Wise Regression Results: Financial Statement Variables – Security ABF

	Variable ID	Variable Description	Variable Significance	t-value
1	Turnover_A20_2	Percentage change in turnover	3.077	.002
2	Turnover_B34_2	Rate of change in turnover	-7.249	.000
3	Profit Margin_B13_2	Rate of change in profit margin	-6.945	.000
4	Profit Margin_B20_2	Rate of change in profit margin	-2.670	.008
5	Profit Margin_V2Y_B27_2	2-year volatility in rate of change profit margin	-4.310	.000
6	Dividends_A6_0	Percentage change in dividends	10.065	.000
7	Dividends_A13_2	Percentage change in turnover	8.329	.000
8	Dividends_B13_3	Rate of change in dividends	5.209	.000

	Variable ID	Variable Description	Variable Significance	t-value
9	Dividends_B27_2	Rate of change in dividends	-6.928	.000
10	Dividends_V2Y_A6_2	2-year volatility in percentage change in dividends	9.271	.000
11	Dividends_V2Y_A13_3	2-year volatility in percentage change in dividends	9.357	.000
12	Dividends_V2Y_B13_1	2-year volatility in rate of change in dividends	1.911	.056
13	Dividends_V2Y_B13_2	2-year volatility in rate of change in dividends	-3.146	.002
14	Dividends_V2Y_B27_6	2-year volatility in rate of change in dividends	-6.084	.000
15	Dividends_V3Y_B13_2	3-year volatility in rate of change in dividends	5.376	.000
16	Dividends_V3Y_B27_6	3-year volatility in rate of change in dividends	7.241	.000
17	Dividends_V5Y_B13_2	5-year volatility in rate of change in dividends	2.834	.005
18	Dividends_V5Y_B34_6	5-year volatility in rate of change in dividends	-4.157	.000

Variable ID represents the naming convention used for the input variables where the variables with suffix A represent percentage change, suffix B represent rate of change and suffix V represent variable volatility. The numbers represent the variable lag. For example, Dividends_V2Y_B13_2 represents 2-year volatility in rate of change in dividends per share. The stepwise regression results show that Turnover, Net Profit Margin and Dividends were all significant variables after the initial stage.

The base financial statement logit model was then built using the 18 variables, above, as inputs and retested in subsets of the training sample to reduce model overfitting. Table 7-6 below presents the final logit model output and includes variable coefficients, associated p-values and statistical significance of the results. GRETL software was used for logit model analysis. Similar to regression analysis, a p-value of less than 10% was the pre-set benchmark for variable inclusion in the final logit model.

Table 7-6 ABF Model Statistics – Logit – Financial Statement Variables

Model 1: Logit, using observations 1-1814

Dependent variable: Y20_ABF

Standard errors based on Hessian

	Coefficient	*Std. Error*	*z*	*p-value*	
Turnover_B34_2	−0.223141	0.0556422	−4.0103	<0.0001	***
Dividends_A6_0	0.618366	0.0988099	6.2581	<0.0001	***
Dividends_A13_2	0	0	2.4633	0.0138	**
Dividends_B27_2	−0.12006	0.0189978	−6.3196	<0.0001	***
Dividends_V2Y_A6_2	3.6485	0.485277	7.5184	<0.0001	***
Dividends_V2Y_A13_3	5.30234	1.03581	5.1190	<0.0001	***
Dividends_V2Y_B27_6	−0.0188271	0.00350038	−5.3786	<0.0001	***
Dividends_V3Y_B27_6	0.0178183	0.00290778	6.1278	<0.0001	***
Dividends_V5Y_B34_6	−0.0130245	0.00156005	−8.3488	<0.0001	***

Mean dependent var	0.534730	S.D. dependent var	0.498930
McFadden R-squared	0.059062	Adjusted R-squared	0.051879
Log-likelihood	−1178.985	Akaike criterion	2375.970
Schwarz criterion	2425.500	Hannan-Quinn	2394.246

Number of cases 'correctly predicted' = 1130 (62.3%)

f(beta'x) at mean of independent vars = 0.499

Likelihood ratio test: Chi-square (9) = 148.009 [0.0000]

The logit results for ABF show that dividends and turnover were highly significant with p-values of less than 1%. Profit margin, however, was not significant. The training-sample population had 1814 observations of which 1,130 (62.3%) were correctly predicted. McFadden R-squared was 5.9%. The high Chi-square value (148.009) suggests that the results are statistically significant for the base financial statement model. Model probabilities were then calculated based on model output using the formula:

$$Probability = \exp(x)\,/(1+\exp(x)) \qquad \text{EQ. 7-1}$$

where,

x represents the values of independent variable multiplied by the model coefficients.

Table 7-7 below provides an example calculation of model probability.

Table 7-7 ABF Final Probability Calculation example– Financial Statement Variables

Input Variable	A	B	C
	Model Input Variable	Model Coefficients	B x C
Turnover_B34_2	0	-0.22314	0
Dividends_A6_0	0.7532114	0.618366	0.46576
Dividends_A13_2	2.0626887	5.94E-15	1.22E-14
Dividends_B27_2	-0.2418192	-0.12006	0.029033
Dividends_V2Y_A6_2	0.159308	3.648497	0.581235
Dividends_V2Y_A13_3	0.0169987	5.30234	0.090133
Dividends_V2Y_B27_6	96.827612	-0.01883	-1.82298
Dividends_V3Y_B27_6	91.803776	0.017818	1.635785
Dividends_V5Y_B34_6	51.938686	-0.01302	-0.67648
		Sum	**0.302486**
	D	Exponential (sum)	1.353219
	E	(1+Exp (sum))	2.353219
	D/E	**Final Probability**	**0.57505**

The final probability of 57.5% would therefore be a buy signal for that date. A trading strategy was then deployed where a probability >0.5 was a buy signal and a probability < 0.5, a sell signal for the security. Table 7-8 below details the training sample return results for security ABF on a gross and net of transaction costs basis for the base financial statement model.

Table 7-8 ABF Returns –Training Sample - Financial Statement Variables

Returns Summary – Financial Statement Variables						
Security - ABF	Gross Returns	Net Returns	Months	Years	Sample Period	No of Trades
Training Sample	2638.4%	1731.4%	84	7	2005 - 2011	1814

Figure 7-1 below provides a graphical representation of the ABF gross returns for training and out-of-sample period for first stage model.

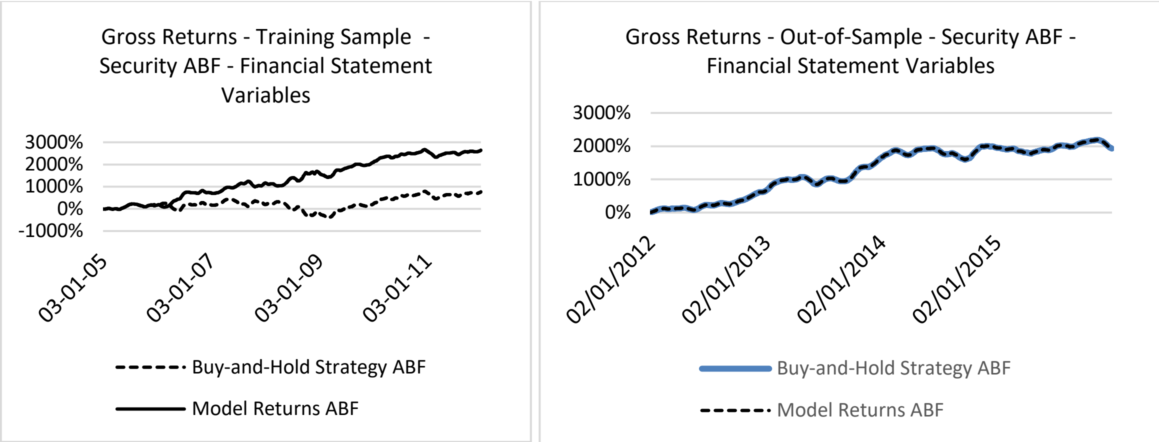


Figure 7-1 Gross Returns – Training and Out-of-Sample - ABF

The return profile of the training sample suggests that the financial statement variables included in the logit model are effective explanatory variables to predict returns for security ABF over the training sample period return period. The out-of-sample prediction, however, for the stage 1 financial statement model does not outperform the naïve model. This concludes the development and analysis of the financial statement variables base model. The next section discusses the development of the base macroeconomic model for ABF.

7.3.1 ABF – Base Macroeconomic Model

The process for the development and analysis of the macroeconomic data was identical to the development of the base financial statement model. A stepwise regression preceded the development of the base logit model for macroeconomic variables. The logit model was then re-tested in sub-samples of the training data after which the model was rerun in the full training sample to determine the final specifications of the model. Table 7-9 presents the logit model results.

Table 7-9 ABF Model Statistics – Logit Results – Macroeconomic Variables

Model 1: Logit, using observations 1-1814

Dependent variable: Y20_ABF

Standard errors based on Hessian

	Coefficient	*Std. Error*	*z*	*p-value*	
Retailing_B6_2	0.0243126	0.00527995	4.6047	<0.0001	***
Retailing_B13_2	−0.00144216	0.000362686	−3.9763	<0.0001	***
Retailing_B20_3	0.00304631	0.000796638	3.8240	0.0001	***
Retailing_B27_2	0.188722	0.0274002	6.8876	<0.0001	***
Retailing_B34_1	−0.0211567	0.00524876	−4.0308	<0.0001	***
Retailing_V2M_A13_2	0.00227233	0.000538964	4.2161	<0.0001	***
Retailing_V2M_B20_1	0.0383416	0.00883732	4.3386	<0.0001	***
Foodstores_A34_2	0.0555002	0.00867862	6.3950	<0.0001	***
Foodstores_B27_2	0.00175174	0.000296827	5.9015	<0.0001	***
Foodstores_V2M_B6_2	−0.00393863	0.00154837	−2.5437	0.0110	**
NonFoodStores_A13_2	0.00224249	0.000710657	3.1555	0.0016	***
NonFoodStores_B27_3	1.42612e-06	4.36175e-07	3.2696	0.0011	***
NonFoodStores_V2M_B20_6	−1.78298e-09	7.05362e-010	−2.5278	0.0115	**
Manufacturing_A13_2	−0.0792026	0.0115605	−6.8512	<0.0001	***
Manufacturing_B6_6	−2.54716e-07	2.75513e-08	−9.2452	<0.0001	***
Manufacturing_B13_2	−0.0376103	0.0073997	−5.0827	<0.0001	***
Manufacturing_B13_5	4.88082e-06	1.0047e-06	4.8580	<0.0001	***
Manufacturing_V2M_A13_2	−0.035082	0.00424956	−8.2554	<0.0001	***
Manufacturing_V2M_A20_2	−0.00198613	0.000347701	−5.7122	<0.0001	***
Manufacturing_V2M_B6_3	−6.13002e-06	9.88185e-07	−6.2033	<0.0001	***
Manufacturing_V2M_B13_13	1.21158e-07	2.6089e-08	4.6440	<0.0001	***
Services_A20_2	−0.0235589	0.00606047	−3.8873	0.0001	***
Services_A27_5	−152.573	28.6638	−5.3229	<0.0001	***
Services_B6_2	0.0327433	0.00386949	8.4619	<0.0001	***
Services_B13_2	−0.0104135	0.00134808	−7.7247	<0.0001	***
Services_B27_1	−0.0880006	0.00923854	−9.5254	<0.0001	***

	Coefficient	Std. Error	z	p-value	
Services_B34_3	−6.53023e-05	1.07255e-05	−6.0885	<0.0001	***
Services_V2M_B6_2	−0.0386715	0.00656261	−5.8927	<0.0001	***
Services_V2M_B13_13	3.80501e-07	1.10466e-07	3.4445	0.0006	***
Services_V2M_B20_3	2.82078e-08	4.60571e-09	6.1245	<0.0001	***
Services_V3M_B13_2	0.0246776	0.00289248	8.5317	<0.0001	***
Turnover_A27_2	0.00019215	4.32965e-05	4.4380	<0.0001	***
Turnover_A27_6	577.363	66.0046	8.7473	<0.0001	***
Turnover_B20_6	1.95381e-05	2.84281e-06	6.8728	<0.0001	***
Turnover_B27_3	2.03033e-09	4.2822e-010	4.7413	<0.0001	***
Turnover_B34_3	−6.6046e-05	1.2711e-05	−5.1960	<0.0001	***
Turnover_V2M_B27_6	1.23545e-010	2.52504e-011	4.8928	<0.0001	***
CPI_A6_2	0.000322893	0.00012086	2.6716	0.0075	***
CPI_B6_6	0.00019824	3.09669e-05	6.4017	<0.0001	***
CPI_B20_2	−0.00926047	0.00288736	−3.2072	0.0013	***
CPI_B27_2	−0.0509948	0.0154456	−3.3016	0.0010	***
CPI_B34_6	0.00162649	0.000419353	3.8786	0.0001	***
CPI_V2M_B6_2	−0.163505	0.0270797	−6.0379	<0.0001	***
CPI_V2M_B6_6	0.000158987	3.27154e-05	4.8597	<0.0001	***
CPI_V3M_B34_6	0.000468775	0.000115992	4.0414	<0.0001	***

Mean dependent var	0.534179	S.D. dependent var	0.498968
McFadden R-squared	0.373341	Adjusted R-squared	0.337431
Log-likelihood	−785.2833	Akaike criterion	1660.567
Schwarz criterion	1908.215	Hannan-Quinn	1751.947

Number of cases 'correctly predicted' = 1398 (77.1%)

f(beta'x) at mean of independent vars = 0.499

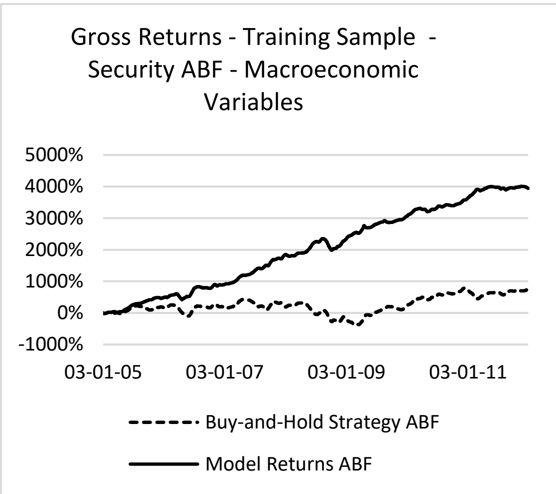
Likelihood ratio test: Chi-Square (45) = 935.688 [0.0000]

The logit results show that all base macroeconomic variables – Retail Sales (Food-stores, non-food stores and total retail), Consumer Price Index, Manufacturing and Services Turnover – were significant at less than 1%. The variable ID definitions are similar in meaning to financial statement variables except that macroeconomic variables are monthly. Volatility variables are therefore monthly. For example, CPI_V3M_B34_6 represents 3-month volatility in rate of change in CPI. The results for ABF also indicate that the macroeconomic model is a relatively better fit in comparison with the financial statement model. McFadden R-squared was 37.6% compared to 5.9% for the financial statement model and training sample prediction rate was 77.1% compared to 62.3%. The higher Chi-square value also suggests that the results are statistically significant for the macroeconomic model. Table 7-10 below details the training sample return results for security ABF on a gross and net of transaction costs basis for the base macroeconomic model.

Table 7-10 ABF Returns – Training Sample – Macroeconomic Variables

Returns Summary – Macroeconomic Variables						
Security - ABF	Gross Returns	Net Returns	Months	Years	Sample Period	No of Trades
Training Sample	4824.3%	3917.3%	84	7	2005 - 2011	1814

Figure 7-2 below provides a graphical representation of the ABF gross returns for the training sample for macroeconomic variables.



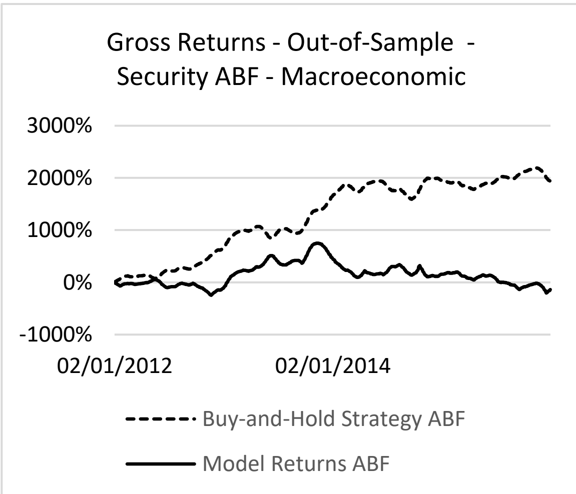


Figure 7-2 Gross Returns – Training Sample - ABF

The returns profile of the training sample also suggests that macroeconomic variables included in the logit model can explain returns for security ABF over the return period. The out-of-sample prediction, however, for the stage 1 macroeconomic model does not outperform the naïve model. This concludes the development and analysis of the financial statement variables base model. The next section discusses the development of the base price volatility model for ABF.

7.3.2 Example Security Analysis – ABF – Pricing Base Model

An identical process to the financial statement and macroeconomic base model developments was followed for the pricing model, where a stepwise regression preceded the development of a logit model. The price logit model was then re-tested in sub-samples of the training sample data set to determine the final logit model for pricing. Monthly price volatilities were the input variables calculated from opening, high, low and closing prices based on the example in section 6.9. Table 7-11 below presents the summary results.

Table 7-11 ABF Model Statistics – Logit Results – Price Variables – Monthly Volatility

Model 3: Logit, using observations 1-1813

Dependent variable: Y20_ABF

Standard errors based on Hessian

	Coefficient	*Std. Error*	*z*	*p-value*	
V20D_A4_1	0.196302	0.0438455	4.4771	<0.0001	***
V20D_A5_2	−0.0145051	0.00273146	−5.3104	<0.0001	***
V20D_A6_4	−3665.94	749.279	−4.8926	<0.0001	***
V20D_A7_4	3074.34	849.082	3.6208	0.0003	***
V20D_A13_4	5478.91	999.126	5.4837	<0.0001	***
V20D_A14_1	0.254004	0.0583738	4.3513	<0.0001	***
V20D_A14_2	0.000416391	7.43087e-05	5.6035	<0.0001	***
V20D_A17_1	−0.722568	0.121983	−5.9235	<0.0001	***
V20D_A18_0	−390.039	72.4858	−5.3809	<0.0001	***
V20D_A19_3	61.7862	11.0454	5.5938	<0.0001	***
V20D_A20_2	0.002885	0.000426079	6.7710	<0.0001	***
V20D_A21_4	−3651.25	462.539	−7.8939	<0.0001	***
V20D_A23_2	0.00197714	0.000715873	2.7619	0.0057	***
V20D_A23_4	3386.37	478.598	7.0756	<0.0001	***
V20D_A32_1	0.218192	0.0503214	4.3360	<0.0001	***
V20D_A39_2	−0.00171046	0.000280525	−6.0974	<0.0001	***
V20D_A39_6	−5875.63	896.808	−6.5517	<0.0001	***
V20D_B1_0	0.145886	0.0486464	2.9989	0.0027	***

	Coefficient	Std. Error	z	p-value	
V20D_B1_6	−1.93e-07	3.43461e-08	−5.6193	<0.0001	***
V20D_B2_0	−0.0437437	0.018755	−2.3324	0.0197	**
V20D_B3_0	0.206707	0.0708767	2.9164	0.0035	***
V20D_B3_4	−0.00143812	0.000575371	−2.4995	0.0124	**
V20D_B4_3	−9.48267e-09	1.80747e-09	−5.2464	<0.0001	***
V20D_B8_1	−0.0193697	0.00259438	−7.4660	<0.0001	***
V20D_B8_6	4.30655e-07	7.40808e-08	5.8133	<0.0001	***
V20D_B9_2	0.00367642	0.00155324	2.3669	0.0179	**
V20D_B10_4	−0.00212216	0.000672747	−3.1545	0.0016	***
V20D_B12_2	0.000445325	0.000225996	1.9705	0.0488	**
V20D_B12_4	0.0131536	0.00264431	4.9743	<0.0001	***
V20D_B13_1	−0.00433492	0.001165	−3.7210	0.0002	***
V20D_B14_1	−0.0112522	0.0015815	−7.1149	<0.0001	***
V20D_B15_1	−0.00243352	0.00113208	−2.1496	0.0316	**
V20D_B16_0	−0.429823	0.0554164	−7.7562	<0.0001	***
V20D_B16_1	0.00753036	0.00124922	6.0280	<0.0001	***
V20D_B16_2	−0.00251858	0.0011773	−2.1393	0.0324	**
V20D_B17_3	−1.01468e-06	3.34443e-07	−3.0339	0.0024	***
V20D_B17_4	0.0113954	0.00239392	4.7601	<0.0001	***
V20D_B18_0	−0.14164	0.0224953	−6.2964	<0.0001	***
V20D_B18_2	−0.0143524	0.00272798	−5.2612	<0.0001	***
V20D_B21_2	0.00148268	0.000589333	2.5159	0.0119	**
V20D_B21_3	−0.000161786	5.19873e-05	−3.1120	0.0019	***
V20D_B21_6	6.33843e-06	1.9778e-06	3.2048	0.0014	***
V20D_B22_1	−0.0029247	0.00141012	−2.0741	0.0381	**
V20D_B23_1	0.00891713	0.00171842	5.1891	<0.0001	***
V20D_B23_4	−0.00624147	0.00110957	−5.6251	<0.0001	***
V20D_B24_2	−0.0136138	0.00266676	−5.1050	<0.0001	***
V20D_B25_5	2.3057e-06	1.02226e-06	2.2555	0.0241	**
V20D_B27_3	−9.25877e-06	2.67121e-06	−3.4661	0.0005	***

	Coefficient	Std. Error	z	p-value	
V20D_B27_4	0.00718076	0.00208379	3.4460	0.0006	***
V20D_B30_0	0.0793197	0.0284357	2.7894	0.0053	***
V20D_B32_3	−2.38105e-06	4.68688e-07	−5.0803	<0.0001	***
V20D_B37_0	−0.1735	0.0425592	−4.0767	<0.0001	***
V20D_B39_1	0.00859855	0.00139866	6.1477	<0.0001	***
V20D_B39_2	−0.0090734	0.00133465	−6.7984	<0.0001	***
V20D_C2_3	76.8086	19.6867	3.9015	<0.0001	***
V20D_C2_4	−6251.23	1003.51	−6.2293	<0.0001	***
V20D_C8_1	−0.531737	0.0955541	−5.5648	<0.0001	***
V20D_C15_2	−0.00141353	0.000347425	−4.0686	<0.0001	***
V20D_C18_0	0.643625	0.0942455	6.8292	<0.0001	***
V20D_C18_4	−0.00835974	0.00139805	−5.9796	<0.0001	***
V20D_C23_1	0.000565412	0.000272872	2.0721	0.0383	**
V20D_C24_2	0.00103426	0.000417261	2.4787	0.0132	**
V20D_C24_4	0.0106328	0.00187555	5.6692	<0.0001	***
V20D_C24_6	−5.05134e-08	1.0512e-08	−4.8053	<0.0001	***
V20D_C25_0	−0.0752855	0.0283877	−2.6520	0.0080	***
V20D_C25_2	0.00010506	4.90961e-05	2.1399	0.0324	**
V20D_C26_2	−0.0145267	0.00351753	−4.1298	<0.0001	***
V20D_C27_1	0.00411999	0.00152245	2.7062	0.0068	***
V20D_C27_2	−0.00131828	0.00023739	−5.5532	<0.0001	***
V20D_C27_4	−0.00312138	0.000682011	−4.5767	<0.0001	***
V20D_C28_1	0.00613603	0.00189664	3.2352	0.0012	***

Mean dependent var	0.535025	S.D. dependent var	0.498909
McFadden R-squared	0.528103	Adjusted R-squared	0.471404
Log-likelihood	−590.9206	Akaike criterion	1323.841
Schwarz criterion	1714.536	Hannan-Quinn	1468.009

Number of cases 'correctly predicted' = 1569 (86.5%)

f(beta'x) at mean of independent vars = 0.499

Likelihood ratio test: Chi-square (71) = 1322.61 [0.0000]

The predictability in the training sample, R^2 and significance of Chi-square values would suggest that the monthly price volatility model is the most effective of the three base models for security ABF. McFadden R-squared was 52.8 % compared to 37.6% and 5.9% for macroeconomic and financial statement models, respectively. Training sample prediction rate was also higher at 86.5% compared to 77.1% and 62.3% for macroeconomic and financial statement models, respectively. Table 7-12 below details the training sample return results for security ABF on a gross and net of transaction costs basis for the base macroeconomic model.

Table 7-12 ABF Returns - Training Sample – Price Variables

Returns Summary – Macroeconomic Variables						
Security - ABF	Gross Returns	Net Returns	Months	Years	Sample Period	No of Trades
Training	6313.4%	5406.9%	84	7	2005 - 2011	1814

Figure 7-3 below provides a graphical representation of the ABF gross returns for training sample for price variables.

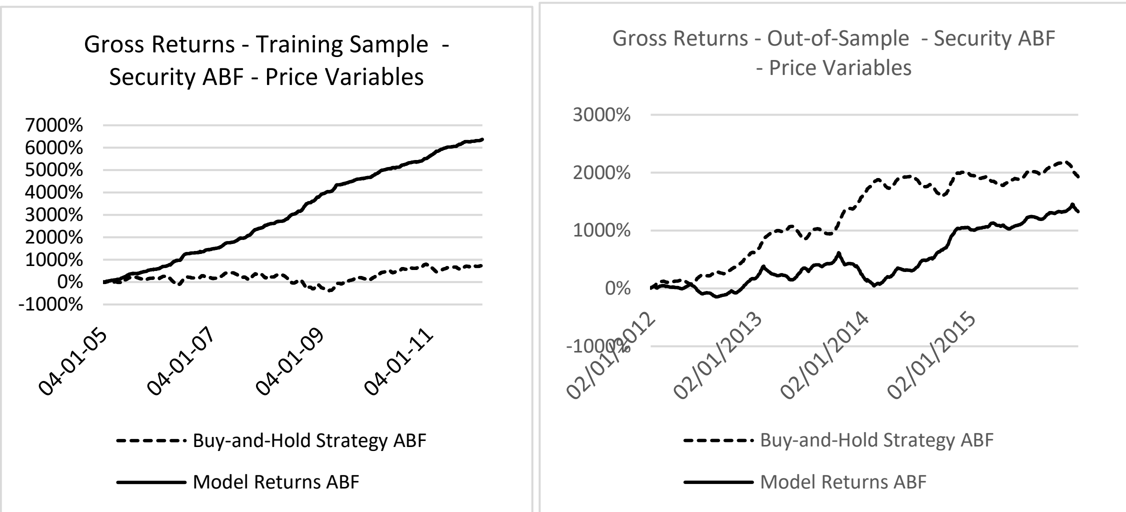


Figure 7-3 Gross Returns – Training and Out-of-Sample - ABF

The return profile of the training sample suggests that the pricing variables included in the logit model can explain returns for security ABF over the return period. The out-of-sample prediction, however, for the stage 1 price model also does not outperform the naïve model. This concludes the development and analysis of the pricing variables base model.

Each base fundamental model has it strengths and weaknesses even though input variables are independent (although correlated). A core strength of the financial statement variables is that these variables are unique to the specific security. However, the weakness of the financial statement variables is the available frequency of data which are released annually compared to the predicted dependent variable, prices, which change daily. Similarly, a core strength of macroeconomic variables is that this set of independent variables provides a linkage of the security to the overall economic environment within which the company operates. However, macroeconomic variables are applicable to all securities where individual securities may be impacted differently. In addition, these independent variables have a monthly frequency in comparison to the predicted variable which changes daily. Lastly, current price variables provide a guide to the future direction of prices. Their key strength, therefore, is that price are market expectations of future prices. However, multiple information signals are compounded in prices, making it complex and "problematic" to discern. Racetrack betting methodology suggests that to improve predictability and combine the strengths of the base models, these base models should be combined in a separate validation sample and the final model tested in the third out-of-sample data set for a true out-of-sample prediction. The next section discusses the final prediction model and out-of-sample testing.

7.3.3 Final Security Prediction Model

The out-of-sample data were divided into two equal samples of 521 trades to facilitate development of the prediction model and the other model for testing. The logit model was utilised as the basis for combining the base models where the input variables were simply the probability outputs of the base models. Benter (1994) (also Sung & Johnson, 2007) suggest that the log variables of the base models provide a better fit at a later stage of the combined model. The log variables of the final probabilities of the base models were therefore included as inputs for the second stage of the combined model, as well as the base probabilities – a total of six variables. Table 7-13 below presents the specifications of the final logit model.

Table 7-13 Logit Results ABF – Final Model

Model 3: Logit, using observations 1-522

Dependent variable: Y20_ABF

Standard errors based on Hessian

	Coefficient	*Std. Error*	*z*	*p-value*	
Probability ABF_Macroeconomic Variables	1.45301	0.243064	5.9779	<0.0001	***
Probability ABF_Prices_Monthly	0.78972	0.253953	3.1097	0.0019	***
LNProbability ABF_Macro	−0.0413866	0.0127981	−3.2338	0.0012	***
LNProbability_ABF_Prices_Monthly	0.00455175	0.00101879	4.4678	<0.0001	***
Mean dependent var	0.706897		S.D. dependent var	0.455622	
McFadden R-squared	0.088494		Adjusted R-squared	0.075826	
Log-likelihood	−287.8185		Akaike criterion	583.6369	
Schwarz criterion	600.6676		Hannan-Quinn	590.3074	

Number of cases 'correctly predicted' = 385 (73.8%)

f(beta'x) at mean of independent vars = 0.456

Likelihood ratio test: Chi-square (4) = 55.8861 [0.0000]

The results suggest that, for security ABF, macroeconomic and price variables were significant in the final model. Financial statement variables were not significant. The R^2 for the final model is 8.8% and the predictability rate over the validation period 73.8%. Although the R^2 is lower in comparison to the base models, the value of the log-likelihood of the final model suggests that the final model is the best fit. Table 7-14 below presents the R^2 values and the level of improvement in the second stage of the combined model.

Table 7-14 ABF Model-Fit Statistics – Final Model Comparison to Base models

	Financial Statement Model	Macroeconomic Model	Pricing Volatility	Final Model
Log-likelihood ratio	-1178.985	-785.2833	-590.9206	-287.8185
Relative improvement in Final Model from Base Model, based on changes in likelihood ratio	75.6%	63.3%	51.3%	
Workings: 1 - $\frac{Likelihood\ Ratio\ fo\ Final\ Model}{Likelihood\ Ratio\ of\ Base\ Model}$	1 -$\frac{-287.8185}{-1178.985}$	1 -$\frac{-287.8185}{-785.2833}$	1 -$\frac{-287.8185}{-590.9206}$	
Akaike Criterion	2375.970	1660.567	1323.841	583.6369

It is clear from the log-likelihood values that the final model is the most effective with respect to model fit, based on the model statistics. The Aikake criterion also confirms that the final model is a better fit compared to the base models. Table 7-15, below, presents the returns summary for the final model.

Table 7-15 ABF Returns Summary

Returns Summary – Final Model							
Security - ABF	Gross Returns	Net Returns	Months	Years	Sample Period	No of Trades	Predictability Rate
Validation Sample	1711.5%	1450.5%	24	2	2012 - 2013	521	73.8%
Out-of-sample	209.3%	-50.7%	24	2	2014 -2014	521	54.3%

The results of the final model show positive out-of-sample gross returns. However, the returns are negative after transaction costs. Table 7-16 below compares the return results to a naive buy-and-hold strategy and the FTSE-100 index returns over the sample periods.

Table 7-16 ABF Returns Comparison

	Model ABF		FTSE 100		Buy & Hold Strategy	
	Gross	Net	Gross	Net	Gross	Net
Validation Sample	1711.5%	1450.5%	345.3%	240.9%	1645.8%	1384.8%
Out of Sample	209.3%	-50.7%	-239.4%	-343.4%	284.0%	24.0%

Although the returns are high in the validation sample period the model returns are below a naïve buy-and-hold strategy. The returns are however considerably higher than the FTSE-100 benchmark in both the sample periods. Table 7-17 below details the returns statistics for the validation and out-of-sample periods. Table 7-17 presents final model returns profile and statistics for security ABF for the validation sample and out-of-sample period. The returns are then compared to the FTSE benchmark and a naïve buy-and-hold strategy for the same period. The three base models were combined and final model determined in an independent data set (validation sample) to the model build stage (in-sample). Returns were then predicted in the third data set, the out-of-sample, period.

Table 7-17 ABF Security Returns Statistics

	Validation Sample (2012-2014)						Out-of-Sample (2014-2015)					
	Gross Returns			Net Returns			Gross Returns			Net Returns		
	ABF – Model Returns	FTSE-100	ABF - Buy & Hold (Naive Strategy)	ABF – Model Returns	FTSE-100	Buy & Hold Strategy ABF	ABF – Model Returns	FTSE-100	ABF - Buy & Hold (Naive Strategy)	ABF – Model Returns	FTSE-100	ABF - Buy & Hold (Naive Strategy)
Mean	3.279%	0.662%	3.153%	2.779%	0.462%	2.653%	0.402%	-0.462%	0.546%	-0.098%	-0.662%	0.046%
Median	3.344%	0.938%	3.288%	2.844%	0.738%	2.788%	0.251%	-0.494%	0.594%	-0.249%	-0.694%	0.094%
Standard Deviation	5.499%	3.239%	5.573%	5.499%	3.239%	5.573%	6.261%	3.371%	6.250%	6.261%	3.371%	6.250%
Sample Variance	0.302%	0.105%	0.311%	0.302%	0.105%	0.311%	0.392%	0.114%	0.391%	0.392%	0.114%	0.391%
Kurtosis	30.563%	57.080%	38.737%	30.563%	57.080%	38.737%	16.117%	5.790%	16.023%	16.117%	5.790%	16.023%
Skewness	3.824%	-58.018%	-1.969%	3.824%	-58.018%	-1.969%	26.175%	-22.887%	19.976%	26.175%	-22.887%	19.976%
Range	32.202%	19.670%	34.548%	32.202%	19.670%	34.548%	36.095%	18.660%	36.095%	36.095%	18.660%	36.095%
Minimum	-13.324%	-10.272%	-15.670%	-13.824%	-10.472%	-16.170%	-15.288%	-11.023%	-15.288%	-15.788%	-11.223%	-15.788%
Maximum	18.878%	9.398%	18.878%	18.378%	9.198%	18.378%	20.807%	7.638%	20.807%	20.307%	7.438%	20.307%
Cumulative Returns	1711.475%	345.339%	1645.821%	1450.475%	240.939%	1384.821%	209.276%	-240.116%	284.015%	-50.724%	-344.116%	24.015%
Number of Trades	522	522	522	522	522	522	520	520	520	520	520	520
Sharpe Ratio	47.588%			42.133%			13.803%			9.012%		

The results suggest a positive Sharpe ratio where the FTSE 100 is the benchmark. The returns however suggest a higher level of return volatility, comparable to the buy-and-hold strategy benchmark. Figures 7-4 and 7-5 detail the cumulative returns over the sample periods compared to the benchmark. [Note that the Libor returns have not been included in the graphs that follow as the returns are low in comparison to the security returns, other than being positive and close to 0% in the axis scale throughout the sample period.

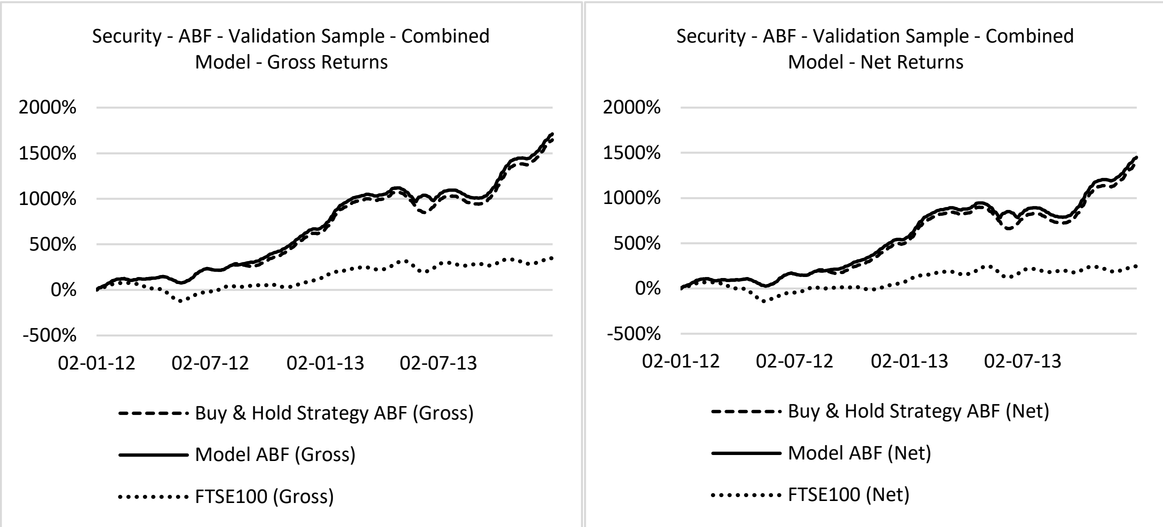


Figure 7-4 Gross and Net Cumulative Returns – Validation Sample – ABF

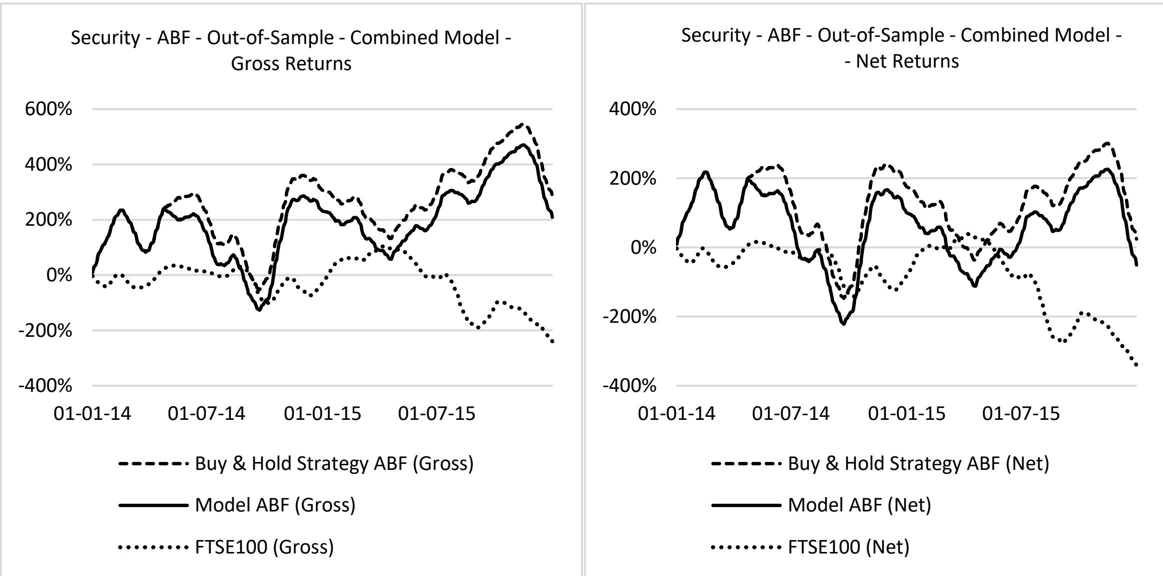


Figure 7-5 Gross and Net Cumulative Returns – Out-of- Sample – ABF

This concludes the modelling analysis for security ABF. An identical methodology was followed for the remaining securities in the sample population. The return results of the individual models over the training sample period (Figures 7-1, 7-2 and 7-3) and the returns for the second stage combined model (Figure 7-4) are comparable for equivalent periods of data, except that Figures 7-1, 7-2 and 7-3 are for a longer time-period. The combined model outperforms the FTSE-100 and the buy-and-hold strategy.

The results also outperform the FTSE-100 over the out-of-sample period but underperforms the naïve buy-and-hold strategy. One possible reason for underperformance could be that the model has been overfitted. Each base model, however, was recalibrated using training sub-sample data to confirm significance of model variables prior to determining the final base model. Therefore, although, there is the likelihood of model overfitting existing, it is expected to be minimal, given the re-sampling and recalibration process followed for base model development.

An additional reason could be that a fundamental variable information may not have been included in the current sample of independent variables to improve the model predictability. For example, security ABF predominantly operates in the food and agricultural industry. Soft commodity prices (non-metal) would highly likely be significant variables. Studies also (for example, (Rossi, 2012), (Bhardwaj, Gorton, & Rouwenhorst, 2015) suggest that commodity prices and equity markets are correlated. Therefore, including commodity prices may potentially improve model performance and could be contemplated for future research as an added series of variables.

Although the combined model underperformance for security ABF over the out-of-sample period may appear to suggest that the multi-stage racetrack betting modelling methodology is not really effective in extracting price-related tradable information to outperform the benchmark model, such as a naïve buy-and-hold strategy, this is not necessarily the outcome for all the securities in the sample population, as noted in Table 9-1 (Appendix).

The return results for two additional securities, SBRY (Sainsbury Plc) and BRBY (Burberry Plc) are presented to demonstrate that multi-stage modelling methodology is effective, and yield well above benchmark returns. The next sections detail the summary returns statistics for securities, SBRY (Sainsbury Plc) and BRBY (Burberry Plc) for the validation sample and out-of-sample periods.

7.4 Security Returns SBRY

Table 7-18 below presents the return results for Sainsbury.

Table 7-18 SBRY Returns Summary

Returns Summary – Final Model							
Security – SBRY	Gross Returns	Net Returns	Months	Years	Sample Period	No of Trades	Predictability Rate
Validation Sample	919.18%	658.18%	24	2	2012 - 2013	521	67.6%
Out-of-Sample	907.88%	647.88%	24	2	2014 -2014	521	60.9%

The results of the final model show a positive return. Table 7-19 below compares the result to a naive buy-and-hold strategy for security SBRY and to the FTSE-100 benchmark over the sample periods.

Table 7-19 SBRY Returns Comparison

	Model SBRY		FTSE 100		Buy & Hold Strategy	
	Gross	Net	Gross	Net	Gross	Net
Validation Sample	919.18%	658.18%	345.3%	240.9%	353.39%	92.39%
Out-of-Sample	907.88%	647.88%	-239.4%	-343.4%	-892.25%	-1152.25%

Figures 7-6 and 7-7 present the cumulative returns for Sainsbury.

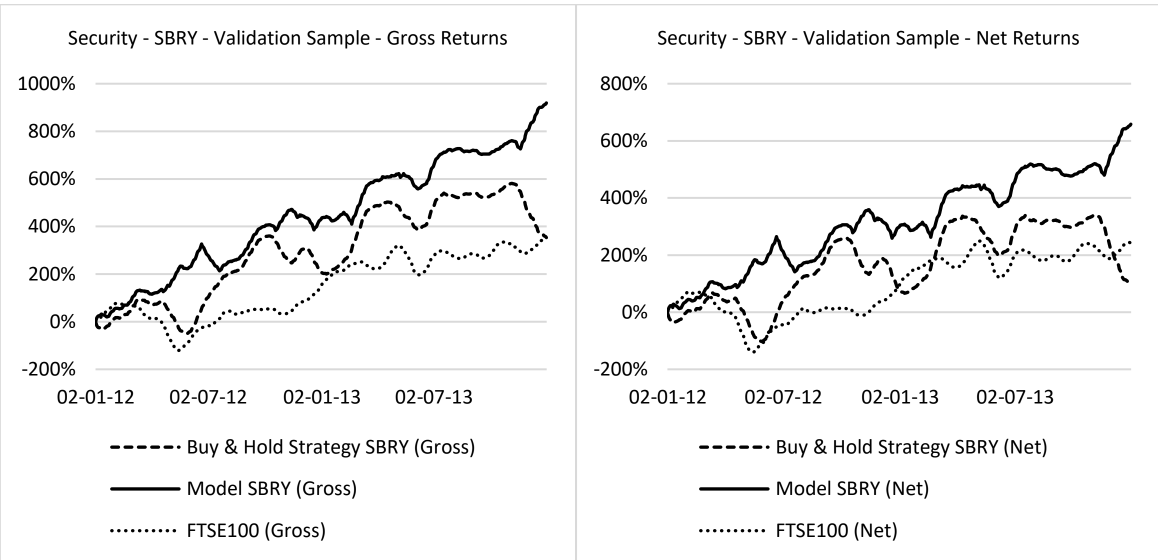


Figure 7-6 Gross and Net Cumulative Returns – Validation Sample – SBRY

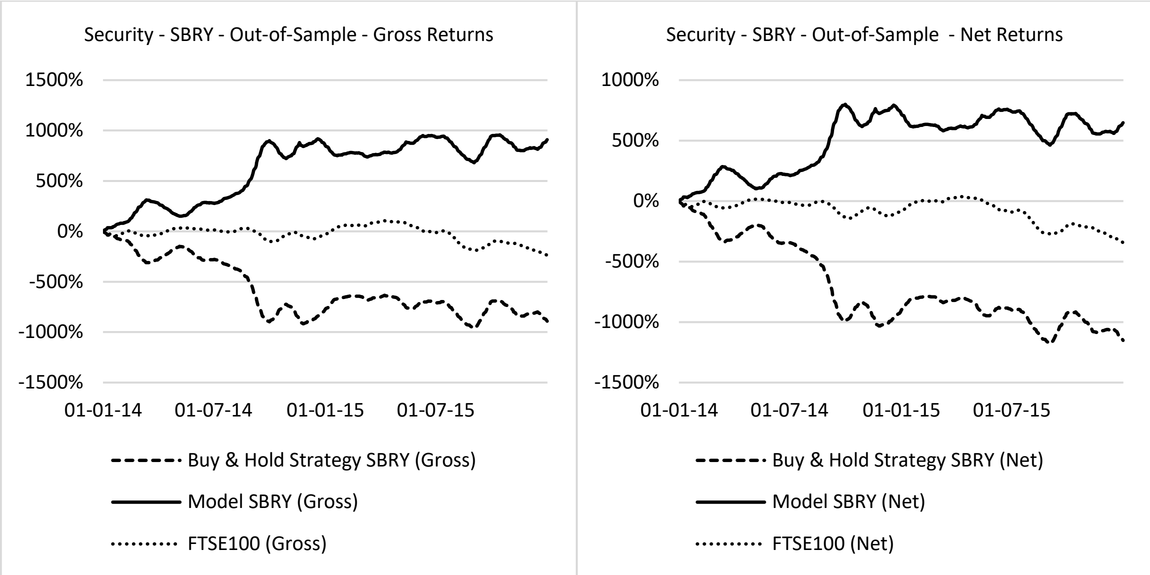


Figure 7-7 Gross and Net Cumulative Returns – Out-of- Sample – SBRY

The results show that the multi-stage modelling methodology can provide profitable buy and sell signals based on SBRY returns. The section details the returns for security BRBY.

7.5 Security Returns BRBY

Table 7-20 below presents the return results for security BRBY (Burberry).

Table 7-20 BRBY Returns Summary

Returns Summary – Final Model							
Security - BRBY	Gross Returns	Net Returns	Months	Years	Sample Period	No of Trades	Predictability Rate
Validation Sample	550.0%	289.0%	24	2	2012 - 2013	521	67.6%
Out-of-sample	367.6%	107.6%	24	2	2014 -2014	521	60.9%

The results of the final model show a positive return. Table 7-21 below compares the results to a naive buy-and-hold strategy and to the FTSE-100 benchmark returns over the sample periods.

Table 7-21 BRBY Returns Comparison

	Model BRBY		FTSE 100		Buy & Hold Strategy	
	Gross	Net	Gross	Net	Gross	Net
Validation Sample	550.0%	289.0%	345.3%	240.9%	270.5%	9.5%
Out-of-sample	367.6%	107.6%	-239.4%	-343.4%	-563.7%	-823.7%

Figures 7-8 and 7-9 present the cumulative returns for BRBY.

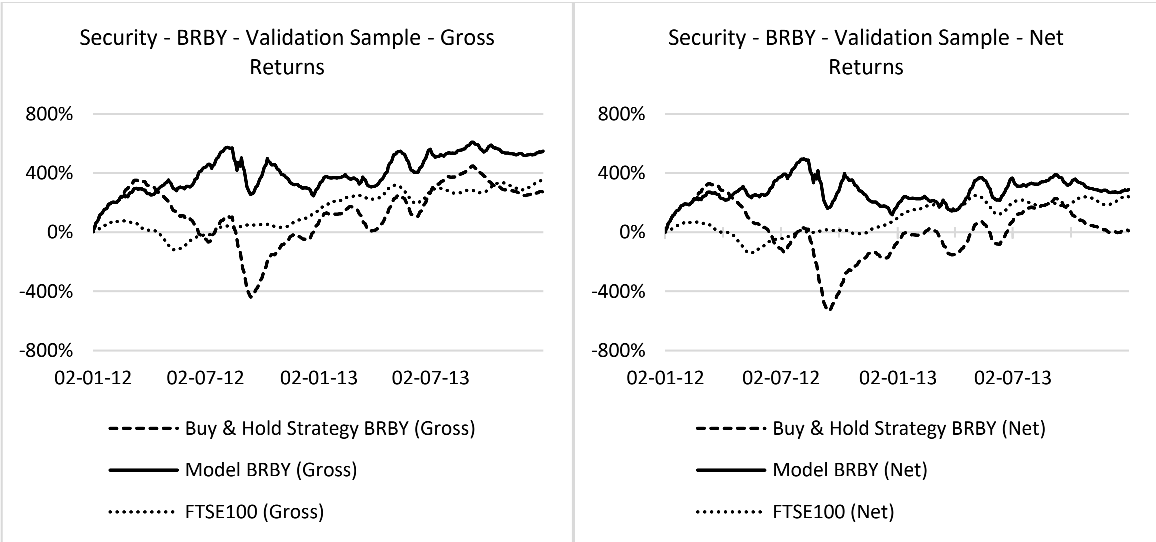


Figure 7-8 Gross and Net Cumulative Returns – Validation Sample – BRBY

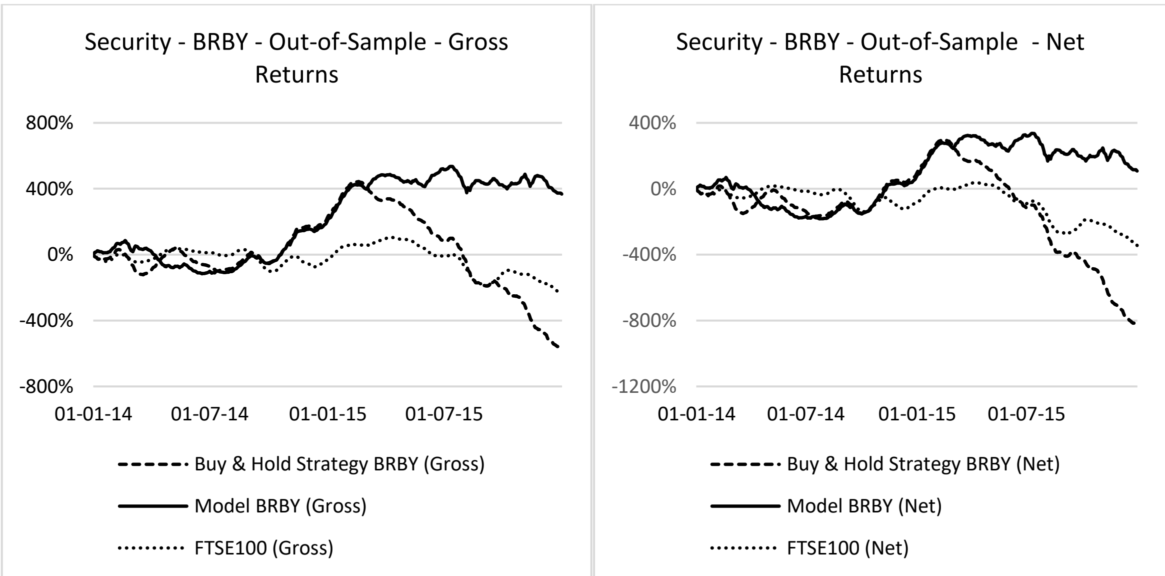


Figure 7-9 Gross and Net Cumulative Returns – Out-of- Sample – BRBY

These results also suggest that a multi-stage modelling methodology is able to confirm that not all securities are efficiently priced, given the sizeable positive returns. The next section, Part III, discusses the model-fit and returns for the portfolio of securities.

PART III – Statistical Results

This section presents the statistical results. Section 7.6 and Table 7-22 presents the model-fit statistics for each security model and Log-likelihood ratios and Akaike criterion are detailed for the base and final models. For the final model, the following additional statistics are presented for each security:

(i) The Chi-square and McFadden R-square
(ii) Cox & Snell and Nagelkerke R-Squares
(iii) Level of significance of the fundamental variables.

Sections 7.7 to 7.9 and Tables 7-23,7-25,7-26 then presents model coefficients, stationarity and long-range dependency test results. Finally, section 7-10 and Table 7-27 presents model predictability rates and returns for each security in the validation and out-of-sample periods for a £1 invested without reinvestment. The next section discusses model fit statistics.

7.6 Model Fit Statistics

Table 7-22 below presents the model fit statistics. The model-fit statistics for the individual securities highlight a consistent trend whereby the financial statement model has the highest log-likelihood ratio (or relatively not the best fit), followed by an improved model-fit for the macroeconomic model and a further enhanced fit for the price model. This trend for model-fit appears to correspond to the relative frequency of data available for calibration in the base models, possibly suggesting that the logit model and input variables can extract a higher amount of price-related information as the frequency of data increases. The model-fit statistics, however, clearly show that the final models were the best fits, demonstrating a significant improvement from the base models and suggesting the effectiveness of a multi-stage modelling methodology. The Akaike criterion values also show a similar trend where the final models have the lowest scores (or best fit). Chi-square values for the individual models are also significant.

The final models show that fundamental variables are highly significant and a p-value of <1% was noted in most instances. The number of fundamental variables that were significant, however, varies from security to security. For example, prices were the only significant variable for security BRBY. In contrast, for security PSN all fundamental variables were noted to be significant at <1%. McFadden R2 ranged from 1% to 18.7%, and an outlier of 1 for BA was noted. Although the R2 statistics may suggest that the final combined models may not be as effective, the ultimate indicator of model success is return predictability in unseen data, the out-of-sample period. The next section discusses model coefficients and functional form.

Table 7-22 MODEL FIT STATISTICS – ALL SAMPLE SECURITIES

		Log-Likelihood Ratios for Models				Akaike Information Criterion (AIC) for Models				Chi-square (d.f.)	McFadden R-Square	Significance of Variables – Final Model (p-value)			Cox & Snell R-Square	Nagelkerke R-Square
No.	SECURITY ID	Financial Statement	Macroeconomic	Prices	Final	Financial Statement	Macroeconomic	Prices	Final	Final		Financial Statement	Macroeconomic	Prices	Final	
1	ABF	-1178.99	-785.28	-590.92	-287.82	2375.97	1660.57	1323.84	587.48	52.0391 (4)	8.24%	<1%	<1%	<1%	24.13%	32.18%
2	ARMS	-1081.15	-882.44	-667.29	-339.43	2188.30	1828.88	1446.59	691.40	19.6872 (2)	1.60%	<1%	n/a	<5%	6.71%	8.94%
3	AZN	-1152.63	-939.71	-849.08	-347.95	2323.27	1939.42	1774.15	701.90	21.726 (3)	3.03%	<1%	n/a	<1%	6.34%	8.45%
4	BA	-1135.83	-932.88	820.17	0.00	2299.67	1909.76	1706.34	0.00		100.00%	<1%	<1%	<1%	75.00%	100.00%
5	BARC	-1089.15	-936.38	-646.99	-295.09	2214.30	1926.75	1445.99	600.18	110.963 (5)	15.82%	<1%	<5%	<1%	22.56%	30.08%
6	BDEV	-1130.25	-831.00	-261.08	-293.24	2280.51	1747.99	800.16	592.49	6.02174 (3)	1.02%	n/a	<1%	<1%	23.11%	30.81%
7	BRBY	-1067.08	-1006.57	-519.31	350.38	2164.15	2053.15	1224.63	704.75	20.1244 (2)	2.79%	n/a	n/a	<1%	4.29%	5.72%
8	BT	-1067.32	-1676.50	-352.05	-303.02	2150.64	3388.99	988.11	614.03	19.0191 (4)	2.95%	<1%	<1%	<5%	20.17%	26.90%
9	DGE	-1110.60	-949.12	-313.87	-302.80	2241.21	1954.25	925.73	611.60	56.1171 (3)	8.48%	<1%	<1%	n/a	20.24%	26.99%
10	GKN	-1153.56	-862.72	-327.87	-307.07	2329.11	1787.43	947.75	620.14	112.609 (4)	16.59%	<1%	<1%	n/a	18.92%	25.23%
11	GSK	-1214.47	-964.51	-653.76	-355.91	2440.95	1987.03	1475.51	715.81	6.2392 (2)	0.86%	n/a	<1%	<10%	2.28%	3.04%
12	HSBA	-1175.21	-908.46	-282.44	-338.91	2372.42	1866.93	848.88	683.82	74.7457 (3)	10.58%	<1%	n/a	<1%	8.40%	11.21%
13	IHG	-1034.15	-799.97	-296.88	-314.42	2092.29	1659.93	833.76	636.84	35.6858 (4)	5.37%	<1%	<1%	n/a	16.61%	22.15%

		Log-Likelihood Ratios for Models				Akaike Information Criterion (AIC) for Models				Chi-square (d.f.)	McFadden R-Square	Significance of Variables – Final Model (p-value)			Cox & Snell R-Square	Nagelkerke R-Square
No.	**SECURITY ID**	**Financial Statement**	**Macroeconomic**	**Prices**	**Final**	**Financial Statement**	**Macroeconomic**	**Prices**	**Final**	**Final**		**Financial Statement**	**Macroeconomic**	**Prices**	**Final**	
14	JMAT	-1204.45	-771.02	-205.98	-324.15	2412.89	1644.03	675.96	656.30	33.9646 (4)	4.98%	<1%	<5%	<5%	13.44%	17.92%
15	KGF	-1088.39	-912.03	184.21	-316.89	2202.78	1874.07	734.41	643.77	62.9672 (5)	9.04%	<1%	<1%	<10%	15.82%	21.09%
16	MKS	-1125.73	-892.71	-745.34	327.31	2267.45	1861.43	1598.69	662.62	54.1243	7.64%	<1%	n/a	<1%	12.39%	16.52%
17	MRW	-1117.99	-993.61	-562.58	-346.83	2265.97	2037.22	1299.16	701.65	28.8885 (4)	4.00%	<1%	n/a	<1%	5.46%	7.28%
18	NXT	-1177.96	-997.53	-600.87	-265.05	2367.91	2039.06	1337.74	536.10	26.8731 (3)	4.84%	<5%	<1%	n/a	32.08%	42.77%
19	PSN	-1063.66	-863.17	-510.63	-322.16	2155.32	1808.35	1201.27	652.33	33.2017 (4)	4.90%	<1%	<1%	<1%	13.20%	17.60%
20	PSON	-1171.85	-1079.18	711.60	-353.73	2365.69	2186.37	1559.20	711.46	12.4701 (2)	1.73%	<1%	<1%	n/a	3.05%	4.07%
21	RB	-1213.17	929.60	-595.58	-320.85	2430.33	1943.21	1347.16	649.70	37.0375 (4)	5.46%	<1%	<1%	<1%	14.53%	19.37%
22	SBRY	-1161.54	-778.39	-610.00	-289.35	2337.08	1646.78	1377.99	590.70	127.189 (6)	18.02%	<1%	<1%	<10%	24.25%	32.33%
23	TSCO	-1168.33	-840.82	-694.26	-310.24	2352.67	1781.63	1514.53	630.49	102.231 (5)	14.15%	<1%	<1%	<1%	17.93%	23.91%
24	UVLR	-1192.06	-956.62	-604.77	-322.43	-2396.11	-1973.24	1375.55	652.86	64.5482 (4)	10.54%	<1%	<1%	<5%	14.01%	18.68%
25	WOS	-1166.53	-869.94	-311.85	-301.35	2345.07	1799.88	865.70	610.69	72.3691 (4)	10.72%	<1%	<1%	<5%	21.53%	28.71%

7.7 Model Coefficients and Functional Form

The equation below details the logit model where the security models is presented as a linear function:

$$Y_i = f_i + lnf_i + m_i + lnm_i + p_i + lnp_i \qquad \text{EQ. 7-2}$$

Where

Y_i Represents the probability for security i as determined in equation where the final probability is then calculated as follows (from Equation 7-1):

$$\frac{\exp(Y_i)}{1 + \exp(Y_i)}$$

f Represents financial statement model coefficients

lnf Represents model coefficients of the natural log function of the financial statement model

m Represents macroeconomic model coefficients

lnm Represents model coefficients of the natural log function of the macroeconomic model

p Represents price model coefficients

lnp Represents model coefficients of the natural log function of the price model

Table 7-23 below details the logit model coefficients for the security models. The largely positive model coefficients for f and lnf suggests an overall positive correlation to financial statement variables; turnover, net operating margin and dividends to prices. This relationship, however, is not consistent across all securities. For example, Morrison's (MRW) and Sainsbury (SBRY) (Retail supermarkets) have positive out-of-sample returns of 27.2% and 25.9% respectively. The model coefficients suggest that the main driver for MRW returns are financial statement variables whereas for SBRY macroeconomic variables are the key factors. Similarly, Tesco (TSCO) which had out-of-sample returns of 47.6% has negative model coefficient for financial statement and macroeconomic variables.

Table 7-23 Model Coefficients

No.	SECURITY ID	f	lnf	m	lnm	p	lnp
1	ABF			1.209	-0.087	0.861	0.004
2	ARMS		-1.503				-0.042
3	AZN	1.953	1.222			-0.427	-0.013
4	BA	-18.488	-19.143	-0.095	18.517	-3.309	0.128
5	BARC	1.006	-0.565	0.793	0.119		1.138
6	BDEV			0.885		0.922	-0.028
7	BRBY					0.406	0.062
8	BT	-4.625	0.248	4.092			0.012
9	DGE	0.778	0.028		-0.050		
10	GKN		1.016	1.771	-0.046		
11	GSK	-4.08837E+16	-0.020				
12	HSBA	0.899	0.175			0.548	
13	IHG	0.562	1.275	1.072	-0.046		
14	JMAT		0.101	1.056	-0.094		-0.009
15	KGF	1.202	0.050	1.349	-0.060		0.001
16	MKS	2.412	-0.665			-1.835	0.017
17	MRW	0.878	1.086			-0.613	0.148
18	NXT	-0.461		1.751	-0.440		-0.023
19	PSN	1.409					0.026
20	PSON	0.763			0.827		
21	RB		-1.177	0.661	-0.043	-0.937	
22	SBRY	-3.410	0.457	2.090	-0.341	0.706	-0.084
23	TSCO		-0.888	-2.030	-0.032	-0.553	0.013
24	UVLR		-0.194	1.119	0.803		-0.011
25	WOS			2.274	-0.033	-1.126	0.057

The next section discusses model correlation and stationarity test statistics.

7.8 Stationarity Test Results

The augmented Dickey-Fuller tests was performed to confirm whether model variables were significant. Table 7-24 below shows the DF t-statistic critical values which are negative, and the more negative the DF test statistic, the stronger the evidence for rejecting the null hypothesis of a unit root. Gretl Software was used for DF statistics. A sample extract of GRETL statistical output in included in Section 9.2 in the Appendix.

Table 7-24 Augmented Dickey and Fuller Test - Critical values[25]

Critical values for Dickey–Fuller *test*.				
	Sample size			
Level of Significance	25	50	100	∞
0.01	-4.38	-4.15	−4.04	−3.96
0.025	−3.95	−3.80	−3.69	−3.66
0.05	−3.60	−3.50	−3.45	−3.41
0.10	−3.24	−3.18	−3.15	−3.13

Table 7-25 below presents these the results as performed on the model coefficients as detailed in Table 7-23. The test results are highly significant at <1% and consistent across all the sample securities for all categories of independent variables; financial macroeconomic and prices. These results suggest that the independent variables are not stochastic, rather have mean-reversion tendency (i.e. fluctuates around mean). The linear relationships determined therefore are economically significant. Mean-reversion tendencies have been well noted in literature (for example, (Balvers, Wu, & Gilliland, 2000), (Porteba, 1988).

Model correlation statistics for the final model variables are presented in the Appendix, Table 9-2. The value of correlation statistics ranges between -1 and 1, where 1 suggests a direct linear relationship and -1 suggests an inverse linear relationship. The values in Table 9-2 shows that there is an element of correlation across the fundamental variables. The degree of correlation, however is relatively low and meaningful conclusions could not be drawn from the correlation statistics. The next section discusses long-range dependency tests.

[25] Source: (Greene, 2012, p. 989)

Table 7-25 Final Model Variable Stationarity Tests - Augmented Dickey and Fuller Test Results

(Test Parameters – No Constant Plus Trend; Variable First Differences)

No.	SECURITY ID	f	lnf	m	lnm	p	lnp
1	ABF			-22.7446	-22.7559	-18.8196	-22.5351
2	ARMS		-22.7866				-22.7633
3	AZN	-5.47349	-5.41813			-23.1464	0.618016
4	BA	-22.785	-12.4648	-22.7496	-22.7947	-12.307	-22.7203
5	BARC	-22.7928	-22.7827	-22.7397	-22.7391		-5.55224
6	BDEV			-22.7404		-19.4433	-13.9017
7	BRBY					-10.1255	-12.3089
8	BT	-22.8322	-22.8107	-22.748			-16.0305
9	DGE	-22.8508	-22.8325		-22.7403		
10	GKN		-22.7448	-22.7394	-22.7389		
11	GSK	constant	-22.6723				
12	HSBA	-22.5402	-22.7227			-24.5068	
13	IHG	-22.7751	-22.7726	-22.7454	22.7406		
14	JMAT		-22.8037	-22.7455	-22.7391		-17.1937
15	KGF	-22.7492	-22.7988	-22.7394	-22.7408		-9.59386
16	MKS	-22.7512	-22.751			-22.8819	-22.8481
17	MRW	-22.8591	-22.7993			-23.2988	-13.8645
18	NXT	-22.802		-22.7481	-22.7428		-6.95911
19	PSN	-22.7205					-20.4348
20	PSON	-22.7649			-22.7421		
21	RB		-22.8188	-22.7606	-22.7382	-11.1136	
22	SBRY	-22.772	-22.8832	-22.7426	-22.7392	-18.26	-11.0956
23	TSCO		-9.55953	-22.744	-22.7393	-15.2331	-4.02217
24	UVLR		-22.7506	-22.7378	-22.7382		-22.7964
25	WOS			-22.762	-22.7404	-19.1138	-22.8166

7.9 Long-Range Dependency Tests Results

Gretl Software was used to calculate the Hurst components for long-range dependency tests. A sample extract of GRETL statistical output for Hurst component is in included in Appendix, Section 9.4. Statistical formula for Hurst component is also detailed in Section 9.4.

Table 7-26 below provides detailed long-range dependency tests for model variables. The out-of-sample results suggests a high degree of long-range dependency in fundamental variables and confirms the existence of market inefficiency and validates the multi-stage modelling methodology. It must be noted, however, that the long-range dependency tests were performed on the transformed final input variables which are logit model outputs (compared to initial stage variable transformations) available at the final step of the multi-stage modelling process. For example, **lnf**, is a final stage input variable that is not available at the initial variable stage. In other words, the Hurst component tests support the multi-stage modelling methodology and the hypothesis that profitable trading opportunities are possible as there is long range dependency in the final input variables.

These results are consistent with earlier studies by Willinger, Taqqu, & Teverovsky (1999) who find empirical evidence of long-range dependence in stock price returns with Hurst values of around 0.6. The long-range dependency was test for independent variables and Hurst values ranged from 0.5 to 0.9 and median value of 0.76, suggesting a higher level of long-range dependency at fundamental variable level. These results, however, appear to contradict Cajueiro & Tabak (2008) who found long-range dependency in world indices returns but not stock returns. It should also be noted that the long-range dependency tests were performed at the fundamental variables rather than stock returns.

The next section discusses model predictability statistics. The returns are based on the probability outputs of the final models and a trading strategy of buying a security where probability was >0.5 and shorting a security where the model probabilities were <0.5.

Table 7-26 Final Model Variable Long Range Dependency Tests -Hurst Exponents

No.	SECURITY ID	f	lnf	m	lnm	p	lnp
1	ABF			0.7604	0.7936	0.8473	0.6864
2	ARMS	0.7413					0.8113
3	AZN	1.1094	0.7472			0.7827	0.7197
4	BA	0.8881	0.6218	0.7440	0.7801	0.8890	0.8451
5	BARC	1.3892	1.2385	0.7090	0.7124		0.8350
6	BDEV			0.8269		0.6960	0.6804
7	BRBY					0.7685	0.8128
8	BT	0.8487	0.9118	0.7957			0.8253
9	DGE	0.7186	0.7191		0.7963		
10	GKN		0.6639	0.7297	0.6908		
11	GSK	0.5420	0.5037				
12	HSBA	0.7497	0.8908			0.7886	
13	IHG	0.7736	0.7462	0.7136	0.7640		
14	JMAT	0.7516		0.8932	0.5784		0.7694
15	KGF	0.8698	0.8021	0.6641	0.7863		0.8252
16	MKS	0.7701	0.7693			0.8042	0.7941
17	MRW	0.7907				0.6765	0.6879
18	NXT	0.7546		0.8163	0.8117		0.7751
19	PSN	0.8604					0.6740
20	PSON	1.0050			0.7302		
21	RB		0.9492	0.6741	0.5956	0.6237	
22	SBRY	1.1114	0.5276	0.7463	0.7001	0.8308	0.7537
23	TSCO		0.7830	0.8610	0.7346	0.8515	0.8788
24	UVLR		0.8437	0.7304	0.7506		0.7480
25	WOS			0.7679	0.6677	0.8091	0.8074

7.10 Model Predictability Statistics

Table 7-27 below presents the model predictability statistics. A predictability rate of >50% was recorded for 20 out 25 security models, suggesting that 80% of the security models would most likely yield positive gross returns. The highest out-of-sample model predictability rate was for security, BA which was 85%; which also recorded the highest validation sample predictability rate of 100%. This result appears consistent with the model-fit statistics for security, BA. The lowest out-of-sample predictability rate was for security GSK (Glaxo Smith-Kline) which was at 43.95% compared to a validation sample predictability rate of 55.2%. The results show that should a £1 be equally invested across all the securities and trade based on the model probabilities the portfolio would yield a gross return of 483.27% and a net return of 22.91%. In other words, a £1 would be worth approximately £3.23 (1 x (1+222.91%) at the end of the sample period, after taking into account transaction costs.

The next section presents the return results for the total sample of securities based on a Kelly strategy. The Kelly strategy considers the relative edge for each security, after final probabilities have been determined. The Kelly edge for each security was calculated and then funds allocated across the portfolio of securities, based on edge offered.

Table 7-27 MODEL PREDICTABILITY STATISTICS - ALL SAMPLE SECURITIES

		Validation Sample			Out-of-sample		
No.	**SECURITY**	**Predictability**	**Cumulative Gross**	**Cumulative Net**	**Predictability**	**Cumulative Gross**	**Cumulative Net**
1	ABF	73.80%	1711.48%	1450.48%	54.30%	209.28%	-50.72%
2	ARMS	60.90%	1048.77%	787.77%	53.36%	28.32%	-232.68%
3	AZN	55.70%	247.21%	-13.79%	54.70%	-53.11%	-314.11%
4	BA	100%	2379.60%	2119.10%	85.06%	1228.25%	967.25%
5	BARC	67.40%	1850.83%	1589.83%	46.92%	-369.66%	-629.66%
6	BDEV	74.50%	2474.10%	2213.10%	62.28%	890.98%	630.98%
7	BRBY	56.90%	550.01%	289.01%	54.91%	367.57%	107.57%
8	BT	69.70%	1267.85%	1006.85%	50.58%	832.07%	572.07%
9	DGE	71.30%	841.80%	580.80%	54.70%	214.59%	-46.41%
10	GKN	75.10%	1622.19%	1361.19%	57.03%	573.58%	313.58%
11	GSK	55.2%	278.44%	17.44%	43.95%	-360.75%	-621.75%
12	HSBA	69.00%	845.79%	584.79%	50.67%	-743.95%	-1003.95%
13	IHG	64.90%	818.55%	557.55%	59.34%	855.32%	594.32%
14	JMAT	64.90%	1071.13%	810.13%	52.40%	278.91%	17.91%
15	KGF	64.60%	973.21%	712.21%	65.32%	1260.24%	1000.24%
16	MKS	63.60%	733.17%	472.17%	47.59%	-115.18%	-375.18%
17	MRW	57.70%	693.31%	432.31%	63.90%	939.95%	679.95%
18	NXT	77.60%	1671.78%	1410.78%	56.35%	188.92%	-72.08%
19	PSN	68.40%	1822.23%	1561.23%	60.85%	791.53%	531.53%
20	PSON	58.40%	474.15%	213.15%	63.44%	1295.14%	1035.14%
21	RB	65.10%	777.30%	516.30%	51.55%	50.89%	-209.11%
22	SBRY	67.60%	919.18%	658.18%	60.89%	907.88%	647.88%
23	TSCO	66.70%	558.79%	297.79%	64.42%	1451.22%	1191.22%
24	UVLR	62.30%	474.38%	213.38%	59.31%	798.37%	537.37%
25	WOS	68.20%	1124.39%	863.39%	60.62%	561.38%	301.38%
Total			**27,229.64%**	**20,705.14%**		**12081.74%**	**5572.74%**
Return on £1 without reinvestment (Divided by 25 Securities in Sample Population) =						**483.27%**	**222.91%**

PART IV – Portfolio Results

This section continues to present a series of tables and graphs. The section presents the portfolio results.

7.11 Portfolio Returns

Table 7-28 presents the final returns and compares these to the benchmarks for the validation sample period. Figures 7-10 and 7-11 then presents the gross and net cumulative returns for the validation sample period. Table 7-29 presents the returns statistics for the out-of-sample period. Table 7-30 presents the number of months there with positive returns. Finally, Table 7-31 presents the return statistics by security of the out-of-sample period.

The out-of-sample return statistics suggest a significantly superior returns performance when compared to the benchmark suggesting at least two conclusions. First the multi-stage modelling methodology is effective in extracting tradable price information and a gross return of 483.27% (Net:222.91%) was recorded. Allocating funds based on relative probabilities also improves portfolio performance as noted from the returns performance on the Weighted-Average Portfolio Allocation methodology. However, the Kelly strategy is significantly better as a methodology for portfolio allocation based on the portfolio performance. The Kelly strategy had a gross return of 1,275.74% (Net: 1,014.74%). The Kelly strategy and models are positive in comparison to the benchmark, suggesting that the models correctly predict the price direction at least more than 50% of the time to yield the positive returns. The Kelly strategy had the highest daily mean net returns of 1.94% and a median of 2.66% in comparison to the benchmark which had both negative mean and median returns. The cumulative of 1,014% over the out-of-sample period also suggests that a leveraged strategy would yield positive returns. Figures 7-10 and 7-11 show that portfolio returns were positive throughout the sample period and well above all the benchmark returns.

The portfolio returns show that the Kelly has been the best performing strategy. In the first year, Kelly reported 12 consecutive months with positive returns. In the second year, Kelly reported five consecutive months with positive returns before a loss was reported and eight of the twelve months reported positive returns. Although the second year reported positive returns these returns were lower in comparison to the first-year returns. This would be anticipated given that variables in financial markets are continuous and models would need to be refreshed to maintain the levels of return predictability of year 1. Figures 7-12 and 7-13 present the gross and net cumulative returns for the out-of-sample period. The next section discusses the empirical results and the findings with respect to previous literature.

Table 7-28 RETURNS STATISTICS – VALIDATION SAMPLE PERIOD

	GROSS CUMULATIVE RETURNS					NET CUMULATIVE RETURNS				
	Kelly	Weighted-Average Portfolio Allocation	Final Model Probability	Naïve Buy-And-Hold Strategy	FTSE-100 Index	Kelly	Weighted-Average Portfolio Allocation	Final Model Probability	Naïve Buy-And-Hold Strategy	FTSE-100 Index
Mean	5.25%	2.34%	2.09%	1.54%	0.66%	4.75%	1.84%	1.59%	1.04%	0.46%
Standard Error	0.15%	0.10%	0.10%	0.14%	0.14%	0.15%	0.10%	0.10%	0.14%	0.14%
Median	5.03%	2.56%	2.23%	1.98%	0.94%	4.53%	2.06%	1.73%	1.48%	0.74%
Standard Deviation	3.42%	2.28%	2.24%	3.31%	3.24%	3.42%	2.28%	2.24%	3.31%	3.24%
Sample Variance	0.12%	0.05%	0.05%	0.11%	0.10%	0.12%	0.05%	0.05%	0.11%	0.10%
Kurtosis	16.36%	23.85%	45.21%	79.26%	57.08%	16.36%	23.85%	45.20%	79.26%	57.08%
Skewness	36.16%	-36.27%	-40.35%	-56.28%	-58.02%	36.16%	-36.27%	-40.35%	-56.28%	-58.02%
Range	22.88%	13.84%	14.55%	22.38%	19.67%	22.88%	13.84%	14.55%	22.38%	19.67%
Minimum	-5.52%	-4.81%	-5.59%	-11.01%	-10.27%	-6.02%	-5.31%	-6.09%	-11.51%	-10.47%
Maximum	17.36%	9.02%	8.97%	11.36%	9.40%	16.86%	8.52%	8.47%	10.86%	9.20%
Total Returns	2,743.09%	1223.56%	1089.19%	805.96%	345.34%	2,482.09%	962.56%	828.21%	544.98%	240.94%
No. of Trades	522	522	522	522	522	522	522	522	522	522
Sharpe Ratio	134.20%	73.73%	63.57%	26.69%	n/a	125.43%	60.58%	50.19%	17.62%	n/a

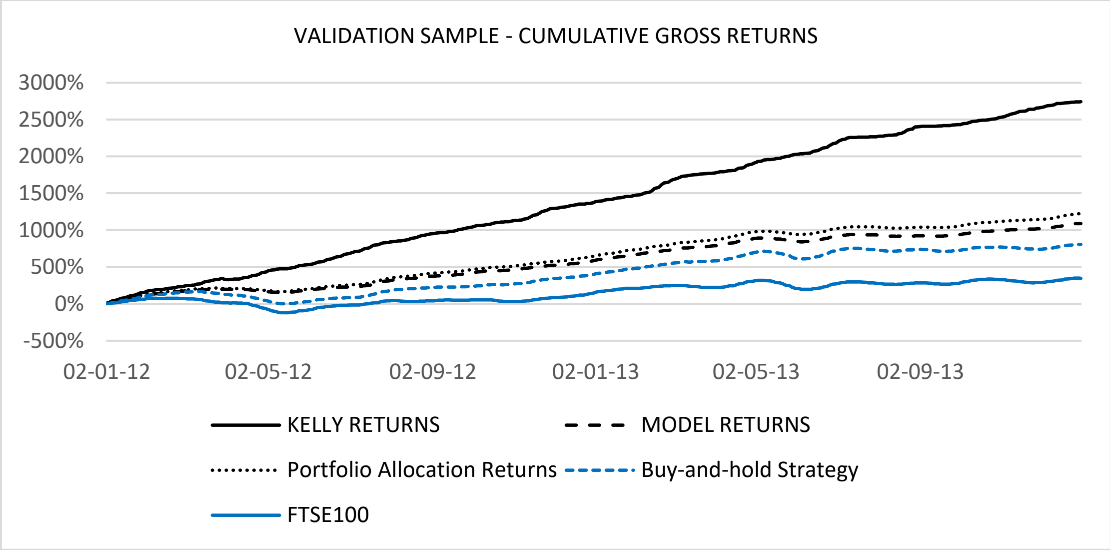


Figure 7-10 Gross Cumulative Returns – All Securities – Validation Sample

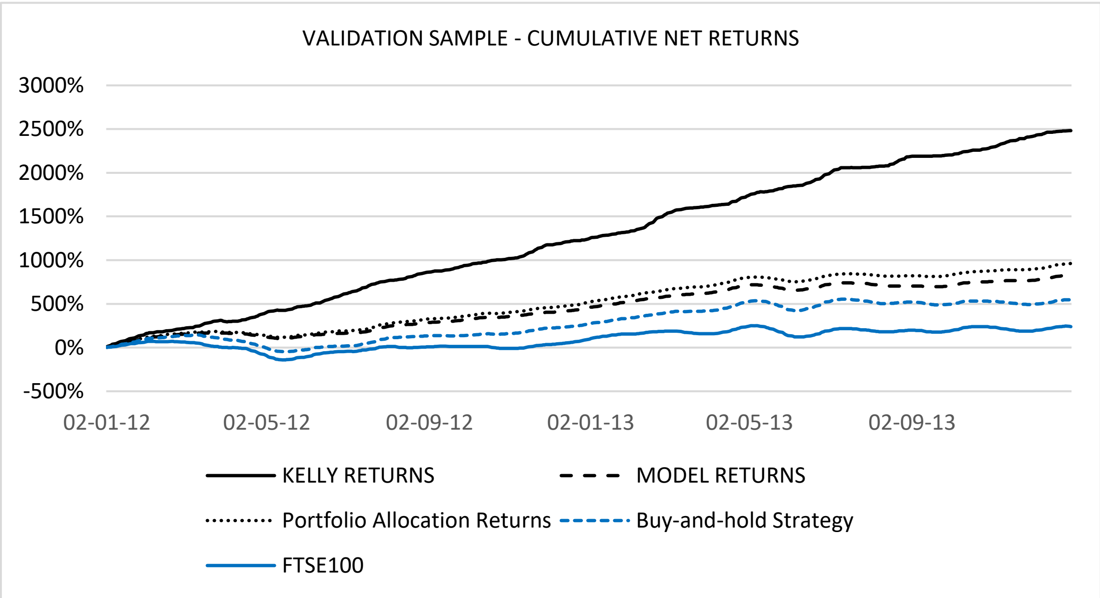


Figure 7-11 Net Cumulative Returns – All Securities – Validation Sample

Table 7-29 RETURNS STATISTICS – OUT-OF-SAMPLE PERIOD

	GROSS CUMULATIVE RETURNS					NET CUMULATIVE RETURNS				
	Kelly	Weighted-Average Portfolio Allocation	Final Model Probability	Naïve Buy-And-Hold Strategy	FTSE-100 Index	Kelly	Weighted-Average Portfolio Allocation	Final Model Probability	Naïve Buy-And-Hold Strategy	FTSE-100 Index
Mean	2.44%	0.94%	0.93%	-0.36%	-0.48%	1.94%	0.44%	0.43%	-0.86%	-0.68%
Standard Error	0.24%	0.07%	0.07%	0.15%	0.15%	0.24%	0.07%	0.07%	0.15%	0.15%
Median	2.66%	0.83%	0.78%	-0.71%	-0.51%	2.16%	0.33%	0.28%	-1.21%	-0.71%
Standard Deviation	5.40%	1.65%	1.65%	3.42%	3.38%	5.40%	1.65%	1.64%	3.42%	3.38%
Sample Variance	0.29%	0.03%	0.03%	0.12%	0.11%	0.29%	0.03%	0.03%	0.12%	0.11%
Kurtosis	174.77%	63.44%	80.82%	-1.02%	2.87%	174.77%	63.44%	81.50%	-0.94%	2.87%
Skewness	-13.47%	41.68%	52.15%	17.95%	-22.39%	-13.47%	41.68%	52.43%	17.85%	-22.39%
Range	41.58%	10.08%	10.70%	18.82%	18.66%	41.58%	10.08%	10.70%	18.82%	18.66%
Minimum	-16.40%	-3.71%	-4.10%	-10.29%	-11.02%	-16.90%	-4.21%	-4.60%	-10.79%	-11.22%
Maximum	25.18%	6.37%	6.60%	8.53%	7.64%	24.68%	5.87%	6.10%	8.03%	7.44%
Total Returns	1,275.74%	493.06%	483.27%	-188.20%	-252.14%	1,014.74%	232.06%	222.91%	-448.56%	-356.54%
No. of Trades	522	522	522	522	522	522	522	522	522	522
Sharpe Ratio	54.18%	86.71%	85.59%	3.58%	n/a	48.63%	68.49%	67.50%	-5.15%	n/a

Table 7-30 MONTHS WITH POSITIVE RETURNS

	GROSS CUMULATIVE RETURNS				NET CUMULATIVE RETURNS			
	VALIDATION SAMPLE		OUT-OF-SAMPLE		VALIDATION SAMPLE		OUT-OF-SAMPLE	
	2012	2013	2014	2015	2012	2013	2014	2015
Kelly	12	12	12	8	12	12	12	6
Weighted-Average	11	10	9	11	11	10	6	9
Final Model Probability	11	10	9	11	11	9	6	9
Naïve Buy-And-Hold	9	9	5	3	9	8	5	3
FTSE-100 Index	8	8	5	4	6	8	4	4

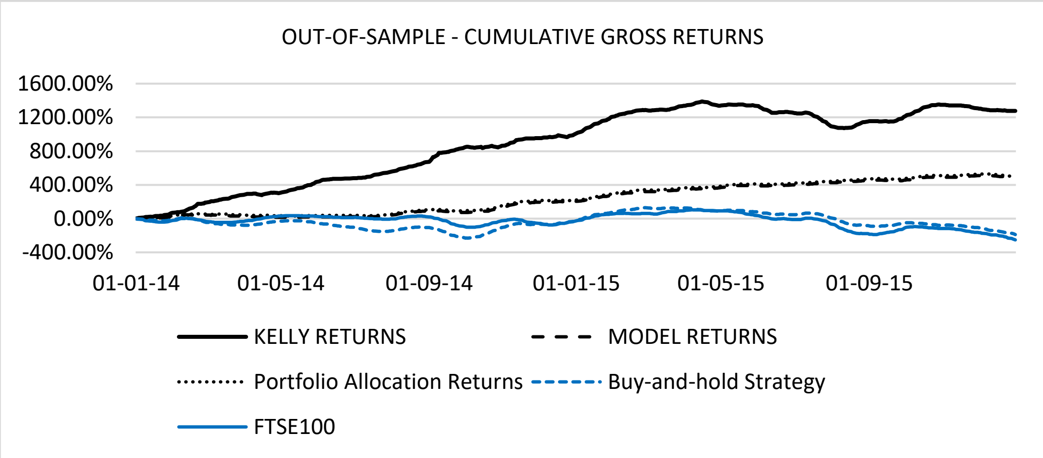


Figure 7-12 Gross and Net Cumulative Returns – All Securities – Out-of-Sample Period

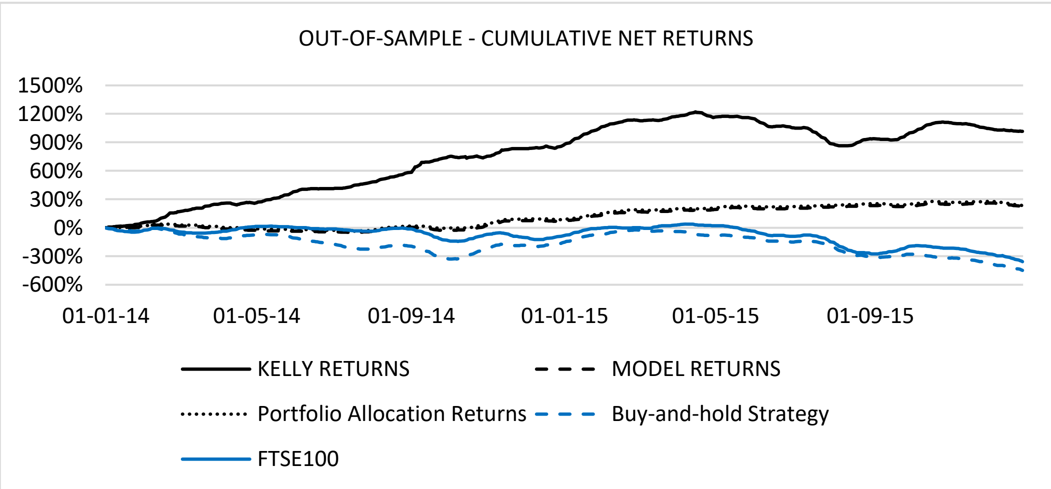


Figure 7-13 Gross and Net Cumulative Returns – All Securities – Out-of-Sample Period

Table 7-31 Net Returns Statistics – Out-Of-Sample Period by Security

Security ID	Kelly	Weighted-Average Portfolio Allocation	Final Model Probability	Naïve Buy-And-Hold Strategy	Model Net Returns - Predictability Rate
ABF	0.00%	-1.41%	-2.03%	1.0%	47.9%
ARM	-65.63%	-14.36%	-9.31%	-15.3%	51.3%
AZN	-0.01%	-13.16%	-12.56%	1.3%	48.9%
BA	834.82%	53.73%	38.69%	0.3%	76.1%
BARC	59.36%	-29.61%	-25.19%	-41.7%	46.2%
BDEV	0.01%	29.13%	25.24%	25.2%	59.0%
BRBY	0.00%	1.56%	4.30%	-32.9%	51.7%
BT	0.01%	23.41%	22.88%	3.5%	60.8%
DGE	-73.51%	-1.48%	-1.86%	-17.6%	51.1%
GKN	0.05%	15.62%	12.54%	-36.6%	56.5%
GSK	0.00%	-22.49%	-24.87%	-24.9%	42.1%
HSBA	0.00%	-36.00%	-40.16%	-36.2%	33.1%
IHG	0.01%	17.42%	23.77%	1.9%	53.6%
JMAT	0.00%	-4.05%	0.72%	-30.5%	48.3%
KGF	-0.01%	34.19%	40.01%	-23.6%	62.1%
MKS	-0.01%	-19.92%	-15.01%	-18.5%	43.5%
MRW	0.03%	32.03%	27.20%	-47.2%	61.3%
NXT	0.00%	-3.48%	-2.88%	-2.9%	52.5%
PSN	0.00%	20.17%	21.26%	21.3%	58.3%
PSON	0.07%	39.99%	41.41%	-56.7%	61.2%
RB	0.00%	-6.69%	-8.36%	9.5%	48.5%
SBRY	283.92%	40.97%	25.92%	-46.1%	57.5%
TSCO	-0.35%	41.56%	47.65%	-71.7%	60.0%
UVLR	-24.03%	23.35%	21.49%	1.2%	55.0%
WOS	0.02%	11.59%	12.06%	-11.3%	55.4%
TOTAL	**1014.74%**	**232.06%**	**222.91%**	**-448.56%**	

Chapter 8: Conclusion

This chapter discusses the empirical results with respect to findings in literature and identifies areas for possible further research.

8.1 Discussion and Analysis of Results

The era of return predictability, which suggests that security returns have predictable components (for example, (Ghysels, Santa-Clara, & Valkanov, 2005), (Lettau & Ludvigson, 2001), and behavioural finance, which provides empirical evidence that excess of market returns could be earned (for example, (De Bondt & Thaler, 1985), (Rouwenhorst, 1998), (Barberis, Huang, & Santos, 2001), is now well established in finance literature. Consequently, the theory of market efficiency and the random walk model, although remaining a useful tool to characterise the behaviour of security prices in financial markets, has its failings.

Market efficiency and the random walk model (Fama E. , 1970), where the premise that markets are efficient and prices follow a stochastic process such as martingale, has been the 'workhorse' for empirical studies in financial markets. An efficient market suggests that above-market security returns would simply be a compensation for additional risk-bearing that cannot be diversified. Statistical modelling for prediction would be useless as net returns would not exceed market returns, given that information has been priced. An index portfolio would therefore be sufficient (Malkiel B. G., 2003).

The empirical results of this study, which could be could be grouped into studies on return predictability, suggests that not all fundamental information is priced. This study therefore, questions the notion whether markets are truly efficient, or, are markets simply dysfunctional such that trading opportunities arise? The general expectation would be that the prices of FTSE-100 securities which are heavily traded, where daily traded volumes (per the sample population) ranged from 384,000 to 38million, would exhibit characteristics of market efficiency. Sixty percent of the securities in the sample population as well as the overall portfolio, however, produced net positive returns. Market efficiency proponents would argue that positive returns are simply rewards for additional risks associated with the securities. These returns were, however, significant in comparison to benchmark portfolios.

Why are the returns demonstrably higher than those reported in previous studies and compared to benchmark? If markets were efficient why is a multi-stage modelling methodology able to extract information that is not priced by financial markets?

One possible explanation is that a multi-stage modelling methodology captures information that is not discernible to other market participants. Price discovery in financial markets is a continuous and complex process where information is revealed over extended periods. It is likely that a multi-stage methodology is better suited to untangling various market signals and discerning price behaviour from the information revealed. Efficient markets would suggest that security prices would reflect information embedded in macroeconomic and financial statement data, given that the latter and former are relatively “stale” news compared to prices. The multi-stage modelling methodology, however calibrates each category of information independently and combines the base models in the stage following. The results suggest that fundamental information models when combined as a second stage provide a better predictive ability. Although prices capture macroeconomic and financial information, the multi-stage modelling methodology gains additional information not priced by financial markets. This incremental information arises from the interaction of base fundamental models and most likely obscure to financial market participants. The incremental model confidence statistics, as indicated by the AIC measure in Table 7-21, provides support to this argument. The final models have higher model-fit statistics compared to the base models.

Another possible explanation is that a multi-stage modelling reduces the effect of multicollinearity. Sung & Johnson (2007, p. 57) noted that a two-stage model allowed *“these fundamental variables to compete for importance….and reduce the problems resulting from multicollinearity”.* The correlation statistics are presented in Table 9-6 (Appendix) and possibly suggests a reduced degree of multicollinearity.

Critics, however, would argue that the sample population is limited (25 securities) to conclude on this study. The repetitive nature of the modelling methodology employed and the relative consistency of the empirical results, however, supports the view and rejects the null hypothesis that UK equities markets is efficient. Each security was also individually modelled, using on average 2,850 days of trading data. Models were then combined in an independent data set of 522 trading days and tested over an out-of-sample trading data set of 522 days. The length of data series is also comparable to financial market studies. For example, Lettau & Ludvigson (2001) utilised predicted quarterly returns and utilised data from 1952 to 1998 (44 years or a maximum of 184 data points). The base model in their analysis used data from 1952-1968 (14 years or 56 data points). The data series for this study is significantly longer for the model estimation sample and for the model prediction sample. The results of the stationarity tests (augmented Dickey and Fuller tests – refer to Appendix – Tables 9-3 and 9-4) are highly significant and support these findings of return predictability. In addition, the results of the long-range dependency tests, presented in Table 9-5 are significant and relatively consistent across the sample population.

The results of this study also suggest that the logit model is an effective tool for modelling financial statement data and prediction analysis; this is consistent with studies by Ou and Penman (1989), Charitou and Panagiotides (1999), Gerlach, Bird and Hall (2002), and Skogsvik and Skogsvik (2010). These results are also consistent with literature on fundamental information and significance of fundamental variables, based on the out-of-sample returns, model-fit statistics and significance of variables reported in Tables 7-22 to 7-25. For example, dividends were significant as well as turnover and net profit margin in the financial statement logit models. The finding that dividends are significant variables is consistent with previous studies (for example, Lewellen (2004) and McManus, Gwilym and Thomas (2004)). Similarly, the finding that earnings were noted to be significant is consistent with previous research (for example, Lambert and Morse (1980), Jegadeesh and Lakonishok (2006) and Ferreira and Santa-Clara (2011)). Macroeconomic information was also noted to be significant and consistent with earlier studies by Chen, Roll and Ross (1986) that included industrial production and inflation rate as input variables, and studies by Lev and Thiagarajan (1993) and Abarbanell and Bushee (1998) who combined macroeconomic and financial statement variables. Similarly, the finding that price volatility is a significant variable is consistent with earlier findings by Ghysels, Santa-Clara and Valkanov (2005). The return results in this study, however, are significantly higher than those demonstrated in previous studies. This confirms the feasibility of applying a multi-stage racetrack betting methodology technical system to financial markets and contradicts the assertion by Cochrane (2005, p. 390) that "*monthly stock returns are still close to unpredictable and "technical" systems for predicting such movements are still close to useless after transaction costs."*.

The cumulative returns are positive throughout the sample period and higher than benchmark portfolios. The positive Sharpe ratio suggests that the model extracts tradable information, confirming the model's abilities to capture return-sensitive information without incremental risks. The results therefore suggest that a multi-staged methodology could potentially identify fundamental information that has not been priced by the market and provide profitable opportunities to be exploited by an astute investor. Markets are therefore not efficient with respect to publicly available information for all equity securities, based on the portfolio returns statistics. In addition, the current interest rate environment (low Libor rates over the sample period) would suggest that a leveraged portfolio strategy would yield further positive returns for a trader.

The concept of efficient markets dominates academia and practice in industry. There is, however, no consensus on the debate whether markets are efficient. Perhaps the concept of market efficiency needs to be redefined considering empirical evidence to date. Grossman & Stiglitz (1980) suggest that information efficiency of markets is a near impossibility as information is costly and traders required compensation, otherwise competitive markets breakdown. The results in this paper appear to provide support to the alternative hypothesis, the Adaptive Expectations Hypothesis (Lo, 2004) that markets are not efficient all the time, and efficiency and inefficiency coexist.

The hypothesis, whether a multi-stage modelling methodology (from racetrack betting market studies) when applied to the UK equities market will reveal publicly available fundamental information that is not priced, was tested. The out-of-sample returns profile suggests that UK equity markets are not able to discern and correctly price fundamental information for all the securities, and rejects the hypothesis that UK equity markets are efficient, based on the sample population tested.

The market inefficiencies identified are, however, with respect to the UK equities market only. It remains to be seen whether these findings are also applicable to other international equity markets. The correlation of equity returns in global markets would probably suggest a high likelihood of the existence of semi-strong inefficiencies in other markets, and a multi-stage modelling methodology could possibly identify these inefficiencies. In addition, it is entirely plausible that a multi-stage modelling methodology could be extended to other asset markets, (e.g., fixed income securities, foreign exchange or commodities trading) which would further confirm the robustness of a multi-stage modelling technique. Similarly, extensions could also be made to study asset returns for periods other than monthly (e.g., weekly and daily returns).

This empirical analysis utilised a linear/linear multi-stage modelling methodology. The racetrack betting market literature, however, shows multiple combinations of multi-stage modelling methodology; for example, SVM/Logit, Model Stacking/Logit. Further empirical analysis in international markets, other asset markets and return periods would confirm the robustness of a multi-stage modelling methodology and the assertion that sports betting markets and financial markets could be viewed from the same prism. The next section concludes this thesis.

8.2 Conclusion

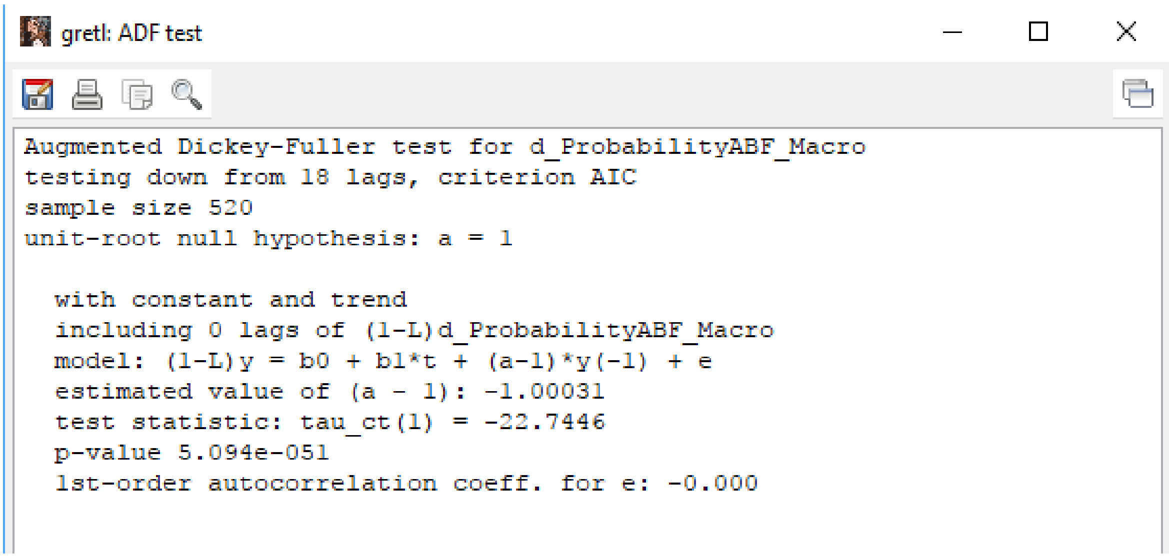
The main contribution of this paper is the application of a multistage methodology to model fundamental information in financial markets. There are gaps in finance literature and modelling fundamental information to test for market efficiency. Competing paradigms describe price efficiency in financial markets. The CAPM-based asset pricing models provide a sound economic framework but limited empirical support on efficiency of prices. These models compete with behavioural finance which provides empirical evidence on market inefficiencies but lack an underlying framework to explain asset price behaviour. Single-stage models underpin the price efficiency testing methodology for fundamental information in financial markets. Perhaps a multi-stage modelling methodology provides a better framework to model prices in financial markets and confirm the true extent of efficiency in these markets.

This paper demonstrates with empirical evidence that a multi-stage methodology could be applied to model fundamental information in financial markets and test for inefficiency. A research design adapting the principles from racetrack betting markets is detailed and then tested on a sample of 25 equity securities to test for semi-strong form efficiency of fundamental information in the UK equities market, and determine the extent to which publicly available fundamental information is priced. A Kelly trading strategy based on final model probabilities produced positive net returns after transaction costs, suggesting the effectiveness of a multi-stage modelling methodology. The empirical results therefore support the view that the null hypothesis can be rejected; confirming that UK equities market is not semi-strong form efficient in pricing publicly available fundamental information for all securities.

The results confirm the applicability of a racetrack betting multi-stage modelling methodology to the wider financial markets. Further empirical analysis in other international markets, asset markets and return periods, however, is required to validate the robustness of a multi-stage modelling methodology and confirm the extent to which markets do not price publicly available information. This paper presents a collection of models and modelling methods that have been developed to test efficiency of fundamental information in financial markets, and then demonstrates the evolution of the model development process from betting strategies to complex multi-stage modelling methodology in racetrack betting markets. These multi-stage racetrack betting models have successfully demonstrated the ability to confirm the existence of market inefficiencies in the betting markets. The similarities and differences in the two markets show that techniques in racetrack betting can be applied to the wider and more complex financial markets, and viewed from the same prism.

Chapter 9: Appendix

9.1 Dickey Fuller Test Extract




```
Augmented Dickey-Fuller test for d_ProbabilityABF_Macro
testing down from 18 lags, criterion AIC
sample size 520
unit-root null hypothesis: a = 1

  with constant and trend
  including 0 lags of (1-L)d_ProbabilityABF_Macro
  model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + e
  estimated value of (a - 1): -1.00031
  test statistic: tau_ct(1) = -22.7446
  p-value 5.094e-051
  1st-order autocorrelation coeff. for e: -0.000
```

Figure 9-1 Dickey Fuller Test Extract

9.2 Correlation Test Results

Table 9-2 below presents the correlation matrix for the final model variables for each security model.

Table 9-1 Correlation Matrix - Final Model Variables

CORRELATION MATRIX - FINAL MODEL VARIABLES							
Security ID	**Fundamental Variable**	**f**	**lnf**	**m**	**lnm**	**p**	**lnp**
ABF	lnp			.419	-.089	-.243	1.000
	lnm			-.055	1.000	.169	-.089
	p			-.560	.169	1.000	-.243
	m			1.000	-.055	-.560	.419
ARMS	lnf		1.000				-.154
	lnp		-.154				1.000
AZN	f	1.000	.767			-.361	.128
	p	-.361	.217			1.000	-.219
	lnf	.767	1.000			.217	-.051
	lnp	.128	-.051			-.219	1.000
BA	lnf	.209	1.000	.019	-.165	.209	.075
	lnm	-.444	-.165	.382	1.000	.426	-.547
	lnp	.772	.075	-.717	-.547	-.234	1.000
	f	1.000	.209	-.943	-.444	-.480	.772
	m	-.943	.019	1.000	.382	.498	-.717
	p	-.480	.209	.498	.426	1.000	-.234
BARC	lnf	-.660	1.000	.600	-.382		-.469
	lnm	.529	-.382	-.537	1.000		-.057
	lnp	.584	-.469	.013	-.057		1.000
	f	1.000	-.660	-.638	.529		.584
	m	-.638	.600	1.000	-.537		.013
BDEV	lnp			.459		-.174	1.000
	m			1.000		-.383	.459

CORRELATION MATRIX - FINAL MODEL VARIABLES							
Security ID	Fundamental Variable	f	lnf	m	lnm	p	lnp
	p			-.383		1.000	-.174
BRBY	lnp					.029	1.000
	p					1.000	.029
BT	lnf	-.716	1.000	.935			.199
	lnp	-.223	.199	.335			1.000
	f	1.000	-.716	-.805			-.223
	m	-.805	.935	1.000			.335
DGE	lnf	-.090	1.000		-.378		
	lnm	.241	-.378		1.000		
	f	1.000	-.090		.241		
GKN	lnf		1.000	.274	-.261		
	lnm		-.261	-.026	1.000		
	m		.274	1.000	-.026		
GSK	lnf	.916	1.000				
	f	1.000	.916				
HSBA	lnf	-.097	1.000			.544	
	f	1.000	-.097			-.609	
	p	-.609	.544			1.000	
IHG	lnf	-.131	1.000	.626	-.348		
	lnm	.617	-.348	-.559	1.000		
	f	1.000	-.131	-.688	.617		
	m	-.688	.626	1.000	-.559		
JMAT	lnf		1.000	.526	-.559		.047
	lnm		-.559	-.244	1.000		-.026
	lnp		.047	.310	-.026		1.000
	m		.526	1.000	-.244		.310
KGF	lnf	-.397	1.000	.684	-.548		.048
	lnm	.322	-.548	-.449	1.000		-.315

CORRELATION MATRIX - FINAL MODEL VARIABLES							
Security ID	**Fundamental Variable**	**f**	**lnf**	**m**	**lnm**	**p**	**lnp**
	lnp	-.047	.048	.270	-.315		1.000
	f	1.000	-.397	-.553	.322		-.047
	m	-.553	.684	1.000	-.449		.270
MKS	lnf	.763	1.000			-.043	-.164
	lnp	.164	-.164			-.394	1.000
	f	1.000	.763			-.615	.164
	p	-.615	-.043			1.000	-.394
MRW	lnf	-.149	1.000			.330	-.175
	lnp	.694	-.175			-.606	1.000
	f	1.000	-.149			-.867	.694
	p	-.867	.330			1.000	-.606
NXT	lnm	.450		-.028	1.000		-.083
	lnp	-.121		.218	-.083		1.000
	f	1.000		-.630	.450		-.121
	m	-.630		1.000	-.028		.218
PSN	lnp	.434					1.000
	f	1.000					.434
PSON	lnm	.850			1.000		
	f	1.000			.850		
RB	lnf		1.000	.769	.040	.397	
	lnm		.040	-.060	1.000	.254	
	m		.769	1.000	-.060	-.089	
	p		.397	-.089	.254	1.000	
SBRY	lnf	-.764	1.000	.752	-.548	.372	-.294
	lnm	.522	-.548	-.743	1.000	.282	-.182
	lnp	.419	-.294	.248	-.182	-.696	1.000
	f	1.000	-.764	-.544	.522	-.513	.419
	m	-.544	.752	1.000	-.743	-.192	.248

CORRELATION MATRIX - FINAL MODEL VARIABLES							
Security ID	**Fundamental Variable**	**f**	**lnf**	**m**	**lnm**	**p**	**lnp**
	p	-.513	.372	-.192	.282	1.000	-.696
TSCO	lnf		1.000	.705	.039	.227	-.279
	lnm		.039	-.223	1.000	.515	-.208
	lnp		-.279	.112	-.208	-.371	1.000
	m		.705	1.000	-.223	-.324	.112
	p		.227	-.324	.515	1.000	-.371
UVLR	lnm		-.102	.738	1.000		-.136
	lnp		.079	.045	-.136		1.000
	lnf		1.000	.240	-.102		.079
	m		.240	1.000	.738		.045
WOS	m			1.000	-.314	-.638	.473
	p			-.638	.436	1.000	-.312
	lnm			-.314	1.000	.436	-.290
	lnp			.473	-.290	-.312	1.000

9.3 Long Range Dependency Tests (R/S)

The R/S (rescaled range) statistic is determined by the range of partial sums of deviations of a time series from its mean rescaled by its standard deviation and calculated as follows ((Campbell, Lo, & Mackinlay, 1997, p. 62) also (Willinger, Taqqu, & Teverovsky, 1999, p. 2) :

$$R/S \equiv \frac{1}{S_n}[\underset{1\leq k\leq n}{\text{Max}} \sum_{j=1}^{k}(r_j - \widetilde{r_n}) - \underset{1\leq k\leq n}{\text{Min}} \sum_{j=1}^{k}(r_j - \widetilde{r_n})] \qquad \text{EQ. 9-1}$$

Where,

The first term in brackets is the maximum (over k) of the partial sums of the first k deviations of r_j from the sample mean and is >0.

The second term in brackets is the minimum (over k) of the partial sums of the first k deviations of r_j from the sample mean and is <0.

$\{r_1, r_2, r_{3,} \dots \dots .. r_n\}$ Represents a set of observation.

$\widetilde{r_n}$ Represents the sample mean $\frac{1}{n}\sum_j r_j$

$$S_n \equiv \left[\frac{1}{n}\sum_j (r_j - r_n)^{\,2}\right]^{\frac{1}{2}}$$

S_n Represents the standard deviation estimator

Model variables were tested for long-term memory and Hurst Exponents estimated using GRETL software. Below is a sample extract for security WOS. The Hurst exponent is the slope of the linear equation y=mx + c. The sample population is rescaled to determine x, y values. Log (size) are x-values and log (RS) y-values to determine coefficient m. The Hurst Exponent critical values are noted in section 6.6.4. The estimated Hurts exponent for WOS is 0.809136, suggesting persistence in the fundamental variable prices.

gretl: Hurst exponent

```
Rescaled range figures for Probability_WOS_PRICES_Monthly
(logs are to base 2)

  Size      RS(avg)    log(Size)     log(RS)
   522       100.63       9.0279      6.6529
   261       58.584       8.0279      5.8724
   130       42.511       7.0224      5.4098
    65       24.719       6.0224      4.6276
    32       13.134       5.0000      3.7152
    16       6.4732       4.0000      2.6945
     8       3.3429       3.0000      1.7411

Regression results (n = 7)

                   coeff    std. error
   Intercept    -0.47881     0.24828
       Slope     0.80914     0.039149

Estimated Hurst exponent = 0.809136
```

Figure 9-2 Long Range Dependency Test – Hurst Estimate - Extract

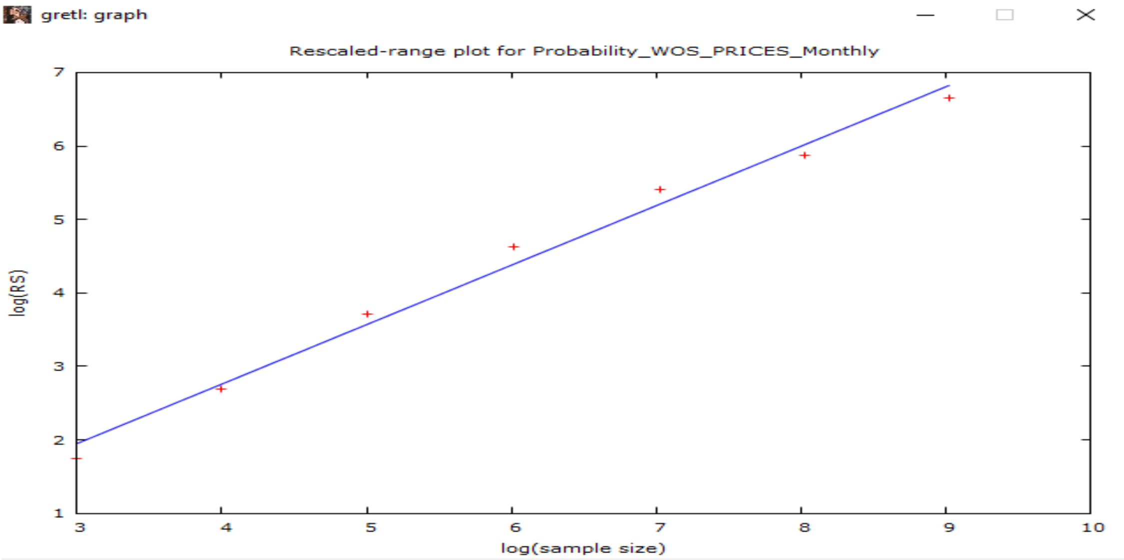


Figure 9-3 Long Range Dependency Test – Hurst Estimate -Slope Plot

Chapter 10: References

Abarbanell, J. S., & Bernard, V. L. (1992). Tests of Analysts' Overreaction/Underreaction to Earnings Informations as an Explanation for Anomalous Stock Price Behaviour. *Journal of Finance*, 1181-1207.

Abarbanell, J. S., & Bushee, B. J. (1997). Fundamental Analysis, Future Earnings and Stock Prices. *Journal of Accounting Research*, 1-24.

Abarbanell, J. S., & Bushee, B. J. (1998, January). Abnormal Returns to a Fundamental Analysis Strategy. *The Accounting Review, 73*(1), 19-45.

Adams, B. R., Rusco, F. W., & Walls, D. W. (2002). Professional Bettors, Odds-Arbitrage Competition, And Betting Market Equilibruim. *Singapore Economic Review*, 111-127.

Akaike, H. (1973). Information Theory and an Extension of teh Maximum Likelihood Principle. *Second International Symposium in Information Theory.* Budapest: Akademiai Kiado.

Ali, M. M. (1977). Probability and Utility Estimates for Racetrack Bettors. *Journal of Political Economy*, 803-815.

Ali, M. M. (1979, March). Some Evidence of Efficiency of a Speculative Market. *Econometrica, 47*(2), 387-392.

Amihud, Y., & Mendelson, H. (1980). Dealership Market: Market Making With Inventory. *Journal of Financial Economics*, 31-53.

Andersen, T. G., Bollerslev, T., Diebolds, F., & Ebens, H. (2001). The distribution of realized stock return volatility. *Journal of Financial Economics*, 43-76.

Ang, A., & Bekaert, G. (2007). Stock Return Predictability: Is it There? *The Review of Financial Studies*, 651-707.

Ardalan, K. (2003). Money and academic finance: The role of paradigms. *International Journal of Social Economics*, 720-740.

Ariely, D. (2006). *Predictably Irrational.*

Arnott, R. D., & Asness, C. S. (2003, Jan / Feb). Surprise Higher Earnings = Higher Earnings Growth. *59*(1), 70-87.

Arrow, K. J., & Debreu, G. (1954). Existence of an equilibrium for a competitive economy. *Econometrica*, 265–290.

Asch, P., & Quandt, R. E. (1987, August). Efficiency and Profitability in Exotic Bets. *Economica, 54*(215), 289-298.

Asch, P., Malkiel, B. G., & Quandt, R. E. (1984, April). Market Efficiency in Racetrack Betting. *Journal of Business, 57*(2), 165-175.

Asch, P., Malkiel, B. G., & Quandt, R. E. (1986, Jan). Market Efficiency in Racetrack Betting: Further Evidence and Correction. *Journal of Business, 59*(1), 157-160.

Bachelier, L. (1900). Théorie de la Spéculation. *Annales Scientifiques de l École Normale Supérieure* .

Bakshi, G., & Chen, Z. (2005). Stock Valuation in Dynamic Economies. *Journal of Financial Markets*, 111-151.

Ball, R. (1978). Anomalies in Relationships Between Securities' Yields and Yield-Surrogates. *Journal of Financial Economics, 6*, 103-126.

Ball, R. (1992). The earnings-price anomaly . *Journal of Accounting and Economics* , 319-345.

Ball, R., & Brown, P. (1968). An Empirical Evaluation of Accounting Income Numbers. *Journal of Accounting Research*, 159-178.

Balvers, R., Wu, Y., & Gilliland, E. (2000). Mean Reversion across National Stock Markets and Parametric Contrarian Investment Strategies. *Journal of Finance*, 745-772.

Bansal, R., Kiku, D., Shaliastovich, I., & Yaron, A. (2014). Volatility, the Macroeconomy, and Asset Prices. *THE JOURNAL OF FINANCE*, 2471-2511.

Banz, R. (1981). The Relation Between Return and Market Value of Common Stocks. *Journal of Financial Economics, 9*, 3-18.

Barberis, N., Huang, M., & Santos, T. (2001, Feb). Prospect Theory and Asset Prices. *Quarterly Journal of Economics, CXVI*(1), 1-53.

Basu, S. (1977, June). Investment Performance of Common Stocks In Relation to Their Price-Earnings Ratios: A Test of Market Efficiency. *Journal of Finance, 32*, 663-682.

Basu, S. (1983). The Relationship Between Earnings Yield, Market Value and Return for NYSE Common Stocks: Further Evidence. *Journal of Financial Economics, 12*, 129-156.

Beaver, W. H., McNichols, M. F., & Wang, Z. Z. (2015). The Information Content of Earnings Announcements: New Insights from Intertemporal and Cross-Sectional Behavior . *Working Paper No. 3338, Stanford Graduate School of Business*.

Beaver, W., Lambert, R., & Morse, D. (1980). The Information Content of Security Prices. *Journal of Accounting and Economics*, 3-28.

Beaver, W., Lambert, R., & Ryan, S. (1987). The Information Content of Security Prices: A Second Look. *Journal of Accounting and Economics*, 139-157.

Benhardt, D., & Miao, J. (2004, February). Informed Trading When Information Becomes Stale. *Journal of Finance, LIX*(1), 339-390.

Benter, W. (1994). Computer Based Horse Race Handicapping and Wagering Systems: A Report. In D. B. Hausch, V. S. Lo, & W. T. Ziemba (Eds.), *Efficiency of Racetrack Betting Markets* (pp. 183-98). London: Academic Press.

Bernard, V. L., & Thomas, J. K. (1989). Post-Earnings-Announcement Drift: Delayed Price Response or Risk Premium? . *Journal of Accounting Research*, 1-36.

Bernard, V. L., & Thomas, J. K. (1990). Evidence That Stock Prices Do Not Fully Reflect The Implications Of Current Earnings For Future Earnings. *Journal of Accounting and Economics*, 305-340.

Bhandari, C. L. (1988). Debt/Equity Ratios and Expected Common Stock Returns: Empirical Evidence. *Journal of Finance, 43*, 507-28.

Bhardwaj, G., Gorton, G., & Rouwenhorst, G. (2015). Facts and Fantasies about Commodity Futures Ten Years Later . *Yale ICF Working Paper No. 15-18* , 1-29.

Binsbergen, J., & Koijen, R. (2010). Predictive Regressions: A Present-Value Approach. *Journal of Finance*, 1439-1471.

Bird, R., & McCrae, M. (1987). Tests of the Efficiency of Racetrack Betting Using Bookmaker Odds. *Management Science*, 1552-1562.

Bird, R., Gerlach, R., & Hall, A. (2001). The Prediction of Earnings Movement Using Accounting data: An update and Extension of Ou and Penman. *Journal of Asset Management*, 180-195.

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 637-654.

Black, F., Jensen, M. C., & Scholes, M. (1972). The Capital Asset Pricing Model: Some Empirical Tests . In M. C. Jensen, *Studies in Theoy of Capital Markets.* New York: Praeger Publishers Inc.

Blume, M. E., & Friend, I. (1973). A New Look at the Capital Asset Pricing Model. *The Journal of Finance*, 19-33.

Bollersev, T. (1986). Generalized Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics*, 307-327.

Bolton, R. N., & Chapman, R. G. (1986, August). Searching for Positive Returns at The Track: A Multinomial Logit Model for Handicapping Horse Races. *Management Science, 32*(8), 1040-1060.

Bossaerts, P. (2002). *The Paradox of Asset Pricing.* Princeton and Oxford: Princeton University Press.

Box, G. E., & Jenkins, G. M. (1976). *Time Series Analysis: Forecasting and Control.* San Francisco: Holden Day.

Breedan, D. (1979). An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities. *Journal of Financial Economics*, 265-296.

Breiman, L. (1996). Stacked Regressions. *Machine Learning*, 49-64.

Breiman, L. (2001). Random Forests. *Machine Learning*, 5-32.

Bruce, A. C., & Johnson, J. E. (2000). Investigating the Roots of The Favourite Longshot Bias: An Analysis of Decision Making by Supply and Demand Side Agents in Parallel Betting Markets. *Journal of Behavioural Decision Making, 13*(4), 413-430.

Bruce, A. C., & Johnson, J. E. (2000). Investigating the Roots of The Favourite Longshot Bias: An Analysis of Decision Making by Supply and Demand Side Agents in Parallel Betting Markets. *Journal of Behavioural Decision Making, 13*(4), 413-430.

Bruce, A. C., & Johnson, J. E. (2001). Efficiency Characteristics of a Market for State Contingent Claims. *Applied Economics, 33*, 1751-1754.

Bruce, A., & Johnson, J. (1996). Decision making under risk: Effect of complexity on performance. *Psychological Reports*, 67-76.

Bruce, A., & Johnson, J. (2003). Investigating Betting Behaviour: A Critical Discussion of Alternative Methodological Approaches. In L. V. Williams, & L. V. Williams (Ed.), *The Economics of Gambling* (pp. 224-246). London: Routledge.

Burrell, G., & Morgan, G. (1979). *Sociological Paradigms and Organizational Analysis.* London: Heinemann Educational Books Ltd.

Busche, K., & Hall, C. D. (1988). An Exception to the Risk Preference Anomaly. *The Journal of Business*, 337-346.

Busche, K., & Hall, C. D. (1988, July). An Exception to The Risk Preference Anomaly. *Journal of Business, 61*(3), 337-346.

Cain, M., Peel, D., & Law, D. (2003). The Favourite-Long shot bias and the Gabriel and Marsden Anomaly. In L. V. Williams (Ed.), *The Economics of Gambling* (pp. 2-14). London: Routledge.

Cajueiro, D. O., & Tabak, B. M. (2008). Testing for long-range dependence in world stock markets. *Chaos, Solitons and Fractals*, 918–927.

Callen, J., & L Segal, D. (2005). Empirical Tests of the Feltham-Ohlson (1995) Model. *Review of Accounting Studies, 10*, 409-429.

Campbell, J. Y. (1987). Stock Returns and the Term Structure. *Journal of Financial Economics*, 373-399.

Campbell, J. Y. (2000, August). Asset Pricing at The Millenium. *The Journal of Finance, LV*(4), 1515-1568.

Campbell, J. Y., & Shiller, R. J. (1988). Stock Prices, Earnings and Expected Dividends. *The Journal of Finance*, 661-676.

Campbell, J. Y., & Shiller, R. J. (1998, Fall). The Dividend-Price Ratio and Expectation of Future Dividends and Discount Factors. *The Review of Financial Studies, 1*(3), 195-228.

Campbell, J. Y., & Shiller, R. J. (2005). Valuation Ratios and the Long Run Stock Market Outlook: An Update. In R. H. Thaler, *Advances in Behavioural Finance, Volume II* (pp. 173-201). Princeton University.

Campbell, J. Y., & Yogo, M. (2006). Efficient Tests of Stock Return Predictability. *Journal of Financial Economics*, 27-60.

Campbell, J., & Hentschel, L. (1992). No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of Financial Economics*, 281-318.

Campbell, J., & Vuolteenaho, T. (2004). Inflation Illusion and Stock Prices. *American Economic Review*, 19-23.

Campbell, J., Lo, A. W., & Mackinlay, A. C. (1997). *The Econometrics of Financial Markets.* Princeton, New Jersey: Princeton University Press.

Campbell, R. A., & Kraussl, R. (2007). Revisiting the Home Bias Puzzle: Downside Equity Risk. *Journal of International Money and Finance*, 1239-1260.

Cao, L. J., & Tay, F. E. (2002). Modified Support Vector Machines in Financial Time Series Forecasting. *Neurocomputing*, 847-861.

Chan, K., Chan, L. K., Jegadeesh, N., & Lakonishok, J. (2006). Earnings Quality and Stock Returns. *Journal of Business*, 1041-1082.

Chan, L. K., Hamao, Y., & Lakonishok, J. (1991). Fundamentals and Stock Returns in Japan. *Journal of Finance*, 1739-1764.

Chapman, R. G. (1994). Still Searching for Positive Returns At The Track: Empirical Results From 2000 Hong Kong Races. In D. B. Hausch, V. S. Lo, & W. T. Ziemba (Eds.), *Efficiency of Racetrcak Betting Markets* (pp. 173-181). London: Academic Press.

Charitou, A., & Panagiotides, G. (1999). Financial analysis, future earnings and cash flows, and the prediction of stock returns: Evidence for the UK. *Accounting and Business Research*, 281-303.

Chen, N.-f., Roll, R., & Ross, S. A. (1986, July). Economic Forces and The Stock Market. *Journal of Business, 59*(3), 383-403.

Chui, A., Titman, S., & Wei, J. K. (2000). Momentum, ownership structure, and financial crises: An analysis of Asian stock markets. *Working Paper, University of Texas, Austin*.

Clarke, B. (2003). Comparing Bayes Model Averaging and Stacking When Model Approximation Error Cannot Be Ignored. *Journal of Machine Learning Research*, 683-712.

Cochrane, J. (2007). The Dog That Did Not Bark : A Defense of Return Predictability. *Review of Financial Studies, 20*(5), 1-43.

Cochrane, J. H. (1999). New Facts in Finance. *Economic Perspectives Federal Reserve Bank of Chicago*, 36-58.

Cochrane, J. H. (2005). *Asset Pricing* (Revised Edition ed.). Princeton and Oxford: Princeton University Press.

Cochrane, J. H. (2011). Presidential Address: Discount Rates. *The Journal of Finance*, 1047-1108.

Connor, G., & Korajzycj, R. A. (1988). Risk and Return in an Equilibrium APT - Application of a New Test Methodology. *Journal of Financial Economics*, 255-289.

Conrad, J., Cooper, M., & Kaul, G. (2003). Value versus Glamour. *Journal of Finance, LVIII*(5), 1969-1995.

Coval, J. D., & Shumway, T. (2005, February). Do Behavioural Biases Affect Prices ? *Journal of Finance, LX*(1), 12420.

Cox, D. R., & Snell, E. J. (1989). *The Analysis of Binary data.* London: Chapman and Hall.

Cox, J. C., Ross, S. A., & Rubinstein, M. (1979). Option Pricing: A Simplified Approach. *Journal of Financial Economics*, 229-263.

De Bondt, W. F., & Thaler, R. (1985, July). Does the Stock Market Overreact. *Journal of Finance, XL*(3), 793-805.

De Bondt, W. F., & Thaler, R. H. (1987, July). Further Evidence on Investor Overreaction and Stock Market Seasonality. *Journal of Finance, XLII*(3), 557-581.

Dempsey, M. (2013). The Capital Asset Pricing Model (CAPM):The History of a Failed Revolutionary Idea in Finance? *Abacus*, 7-23.

DeStefano, M. (2004). Stock Returns and the Business Cycle. *The Financial Review*(39), 527-547.

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the Estimators for Autoregressive Time Series With a Unit Root . *Journal of the American Statistical Association*, 427-431 .

Dijk, M. A. (2011). Is Size Dead? A Review of the Size Effect in Equity Returns. *Journal of Banking and Finance*, 1-12.

Dong, M., & Hishleifer, D. (2004). A Generalized Earnings-Based Stock Valuation Model. *Working Paper*, 1-44.

Dontoh, A., Radhakrishnan, S., & Ronen, J. (2004, Winter). The Declining Value-Relevance of Accounting Information and Non-Information-based Trading: An Empirical Analysis. *Contemporary Accounting Research, 21*(4), 795-812.

Dubois, M., & Louvet, P. (1996). The day-of-the-week effect: International evidence. *Journal of Banking and Finance*, 1463-1484.

Duffie, D. (1996). *Dynamic Asset Pricing Theory* (Second Edition ed.). Princeton, New Jersey: Princeton University Press.

Duffie, D., & Zame, W. (1989). The Consumption-Based Capital Asset Pricing Model. *Econometrica*, 1279-1297.

Duffie, D., Pan, J., & Singleton, K. (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 1343-1376.

Easley, D., & O'Hara, M. (1992). Time and The Process of Security Price Adjustment. *Journal of Finance*, 577-605.

Edelman, D. (2003). A Competitive Horse-race Handicapping Algorithm Based on Analysis of Covariance. In L. V. Williams (Ed.), *The Economics of Gambling* (pp. 106-113). London: Routledge.

Edelmen, D. (2007). Adopting Support Vector Machine Methods for Horserace Odds Prediction. *Ann Oper Research*, 325-336.

Engle, R. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 987-1007.

Engle, R. F., & Granger, C. W. (1987). Co-Integration and Error Correction: Representation, Estimation, and Testing. *Econometrica*, 251-276.

Fabozzi, R. J., Focardi, S. M., Rachev, S. T., & Arshanapalli, B. G. (2014). *The Basics of Financial Econometrics.* New Jersey: John Wiley & Sons.

Fama, E. (1970, March). Efficient Capital Markets: A Review of Theory and Empirical Work. *Journal of Finance, XXV*(2), 383-417.

Fama, E. F. (1965). The Behavior of Stock-Market Prices. *Journal of Business*, 34-105.

Fama, E. F. (1991, December). Efficient Capital Markets II. *Journal of Finance, XLVI*(5), 1575-1617.

Fama, E. F., & French, K. (1993). Common Risk Factors in the Returns of Stocks and Bonds. *Journal of Financial Economics*, 3-56.

Fama, E. F., & French, K. (2001). Disappearing Dividends: Changing Firm Characteristics or Lower Propensity to Pay. *Journal of Financial Economics*, 3-43.

Fama, E. F., & French, K. R. (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance, XLVII*(2), 427-465.

Fama, E. F., & MacBeth, J. D. (1973). Risk, Return, and Equilibrium: Empirical Tests. *The Journal of Political Economy*, 607-636.

Fama, E., & French, K. (1993). Common risk factors in the returns on stocks. *Journal of Financial Economics*, 3-56.

Fama, E., & French, K. (2004, Summer). The Capital Asset Pricing Model: Theory and Evidence. *Journal of Economic Perspectives, 18*(3), 25-46.

Fama, E., & French, K. (2006). Profitability, investment and average returns. *Journal of Financial Economics*, 491-518.

Fama, E., & French, K. (2015). A five-factor Asset Pricing Model. *Journal of Financial Economics*, 1-22.

Feltham, G., & Ohlson, J. (1995). Valuation and Clean Surplus Accounting for Operating and Financial Activities. *Contemporary Accounting Research*, 689-732.

Ferreira, M., & Santa-Clara, P. (2011). Forecasting Stock Market Returns: The Sum of The Parts is more Than the Whole. *Journal of Financial Economics*, 514-537.

Figlewski, S. (1979, February). Subjective Information and Market Efficiency in a Betting Market. *Journal of Political Economy, 87*(1), 75-88.

Foerster, S. R., & Sapp, S. (2005, Fall). The Dividend Discount Model in the Long-Run: A Clinical Study. *Journal of Applied Finance, 15*(2), 55-75.

Foster, G., Olsen, C., & Shevlin, T. (1984). Earnings Releases, Anomalies, and the Behavior of Security Returns . *The Accounting Review*, 574-603.

Frank, k. (n.d.).

French, K. R., & Roll, R. (1986). The Arrival of Infromation and Reaction of Traders. *Journal of Financial Economics*, 5-26.

French, K. R., Schwert, G., & Stambaugh, R. (1987). Expected Stock Returns and Volatility. *Journal of Financial Economics*, 3-29.

French, K., & Porteba, J. (1991). Investor Diversification and International Equity Markets. *American Economic Review*, 222-226.

Gabriel, P. E. (1991). An examination of market efficiency in British racetrack betting Errata and corrections. *Journal of Political Economy*, 657-59.

Gabriel, P. E., & Marsden, J. R. (1990, August). An Examination of Market Efficiency in British Racetrack Betting. *Journal of Political Economy, 98*(4), 874-885.

Gerlach, R., Bird, R., & Hall, A. (2002). Bayesian Variable Selection in Logistic Regression: Predicting Companies Earnings Direction. *Australia and New Zealand Journal of Statistics*, 155-168.

Ghysels, E., Santa-Clara, P., & Valkanov, R. (2005). There is a Risk-Return trade-off After All. *Journal of Financial Economics*, 509-548.

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance*, 1779-1801.

Glosten, L., & Milgrom, P. (1985). Bid, Ask and Transaction Prices in a Specialist Market with Hetereogenous Informed Traders. *Journal of Financial Economics*, 71-100.

Goetzmann, W. N., Ingersoll Jr, J. E., Spiegel, M., & Welch, I. (2002). Sharpening Sharpe Ratios. *NBER Working Paper Series*, Working Paper 9116.

Goetzmann, W., & Jorion, P. (1993). Testing the Predictive Power of Dividend Yields. *Journal of Finance* , 663-679.

Golec, J., & Tamarkin, M. (1998). Bettors love skewness, not risk, at the horse track. *Journal of Political Economy*, 205-225.

Golec, J., & Tamarkin, M. (1998, February). Bettors Love Skewness, Not Risk, at the Horse Track. *Journal of Political Economy, 106*(1), 205-225.

Gordon, M. J. (1962). *The Investment, Financing, and Valuation of the Corporation.* Homewood, IL: R. D. Irwin.

Gramma, M. G., McKinneya, C. N., & Owens, D. H. (2008). The efficiency of exotic wagers in racetrack betting. *Applied Economics*, 89-97.

Granger, C. (1981). Some Properties of Time Series Data and their Use In Econometris Model Specifications. *Journal of Econometrics*, 121-130.

Greene, W. H. (2012). *Econometric Analysis.* Pearson Education Limited.

Greig, C. A. (1992). Fundamental Analysis and Subsequent Stock Returns. *Journal of Accounting and Economics*, 413-442.

Grether. (1980). Bayes' Rule as a Descriptive Model: The Representativeness Heuristic. *The Quarterly Journal of Economics*, 537-557.

Griffin, J. M., Ji, S., & Martin, S. (2003). Global Momentum Strategies: A Portfolio Perspective. *Journal of Finance*, 2515-47.

Griffith, R. (1949). Odds Adjustment by American Horse Race Bettors. *American Journal of Psychology*, 290-294.

Grinold, R., & Kahn, R. N. (1994). *Active Portfolio Management: Quantitative Theory and Applications.* McGraw Hill Professional.

Grossman, S. J., & Stiglitz, J. E. (1980). On the Impossibility of Informationally Efficient Markets. *The American Economic Review,*, 393-408.

Gu, M., Huang, C., & Benter, W. (2003). Multinomial Probit Models for Competitive Horse Racing. *Working paper, Chinese University of Hong Kong*.

Gujarati, D. (1999). *Essentials of Econometrics.* Irwin/McGraw-Hill.

Guo, H. (2006). On the Out-of Sample Predictability of Stock Market Returns. *Journal of Business*, 645-670.

Gwilym, O. a., Seaton, J., Suddason, K., & Thomas, S. (2006, Jan/Feb). International Evidence on the Dividend Payout Ratio, Earnings, Dividends and Returns. *Financial Analysts Journal, 62*(1), 36-53.

Hansen, P. R., Lunde, A., & Nason, J. M. (2011). The Model Confidence Set. *Econometrica*, 453-497.

Harrison, J., & Kreps, D. (1978). Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations. *Quarterly Journal of Economics, XCII*(2), 323-336.

Harrison, M., & Kreps, D. (1979). Martingales and Arbitrage in Multiperiod Securities Markets. *Journal of Economic Theory, 20*, 381-408.

Harsbrouck, J. (2007). *Empirical Market Microstructure: The Institutions, Economics, and Econometrics of Securities Trading.* New York: Oxford University Press.

Harville, D. A. (1973, June). Assigning Probabilities to the Outcomes of Multi-Entry Competition. *Journal of The American Statistical Association, 68*(342), 312-316.

Hausch, D. B., Ziemba, W., & Rubinstein, M. (1981). Efficiency of the Market for Racetrack Betting. *Management Science, 27*(12), 1435-52.

Hausch, D., & Ziemba, W. (1990, January). Arbitrage Strategies for Cross-Track Betting on Major Horse Races. *Journal of Business, 63*(1), 61-78.

Henery, R. (1981). Permutation Probabilities as Models for Horse Races. *Journal Royal Statistical Society B*, 86-91.

Henery, R. J. (1985). On the Average Probability of Losing Bets on Horses with Given Starting Price Odds. *Journal of the Royal Statistical Society*, 342.

Henrich, J. (2002). Decision-making, cultural transmission and adaptation in economic anthropology. In J. Ensminger, *Theory in Economic Anthropology* (pp. 251-295). Walnut Creek, CA: AltaMira Press.

Ho, T., & Stoll, a. H. (1981). Optimal dealer pricing under transactions and return uncertainty. *Journal of Financial Economics*, 47-73.

Holthausen, R. W., & Larcker, D. F. (1992). The Prediction of Stock Returns Using Financial Statement Information. *Journal of Accounting and Economics*, 373-411.

Huang, W., Nakamori, Y., & Wang, S.-Y. (2005). Forecasting Stock Market Movement Direction with Support Vector Machine. *Computer & Operations Research*, 2513-2522.

Hurst, H. (1951). Long-term storage capacity of reservoirs. *Transactions of the American Society of Civil Engineers*, 770–799.

Isaacs, R. (1953, May). Optimal Horse Race Bets. *The American Mathematical Monthly, 60*(5), 310-315.

Jaffe, J., & Westerfield, R. (1985). The Week-End Effect in Common Stock Returns: The International Evidence. *The Journal of Finance*, 433–454.

Jebabil, I., Arouri, M., & Teulon, F. (2014). On the effects of world stock market and oil price shocks on food prices: An empirical investigation based on TVP-VAR models with stochastic volatility. *Energy Economics*, 66-98.

Jegadeesh, N. (1990). Evidence of Predictable Behaviour of Security Returns. *Journal of Finance, 45*, 881-98.

Jegadeesh, N., & Titman, S. (1993, March). Returns to Buying Winners and Losers: Implications for Stock Market Efficiency. *Journal of Finance, 48*(1), 65-91.

Jensen, M. (1978). Some Anomalous Evidence Regarding Market Efficiency. *Journal of Financial Economics, 6*(2), 95-101.

Johnson, J., & Bruce, A. (1998). Risk Strategy under Task Complexity: A multivariate Analysis of Behaviour in a Naturalistic Setting. *Journal of Behavioural Decision Making*, 1-17.

Johnson, J., Jones, O., & Tang, L. (2006, June). Exploring Decision Makers Use of Price Information in a Speculative Market. *Management Science, 52*(6), 897-908.

Kahneman, D., & Tversky, A. (1973). Availability: A heuristic forJudging Frequency and Probability. *Cognitive Psychology, 5*, 207-232.

Kahneman, D., & Tversky, A. (1979, March). Prospect Theory: An Analysis of Decision Under Risk. *Econometrica, 47*(2), 263-292.

Kahneman, D., & Tversky, A. (1984). Choices, Values and Frames. *American Psychologist, 39*, 341-350.

Karathanassia, G. A. (2003). An Empirical Investigation of the Traditional and the Clean Surplus Valuation Models. *Managerial Finance*, 55-66.

Kazemi, H. B. (1991). The Multi-Period CAPM and the Valuation of Multi-Period Stochastic Cash Flows. *Journal of Financial and Quantitative Analysis*, 223-231.

Ke, D., Ng, L., & Wang, Q. (2010). Home bias in Foreign Investment Decisions. *Journal of International Business Studies*, 960-979.

Keim, D. (1983). Size Related Anomaly and Stock Return Seasonality: Further Empirical Evidence. *Journal of Financial Economics, 12*(1), 13-32.

Keim, D., & Stambaugh, R. (1984). A Further Investigation of the Weekend Effect in Stock Returns. *Journal of Finance, 39*(3), 819-40.

Kelly, J. (1956, July). A New Interpretation of Information Rate. *The Bell System Technical Journal*, 917-926.

Kim, K.-Y. J. (2003). Financial time series forecasting using support vector machine. *Neurocomputing*, 307-319.

Kothari, S. P., Shanken, J., & Sloan, R. G. (1995). Another Look at the Cross-section of Expected Stock Returns. *The Journal of Finance*, 185–224 .

Lakonishok, J., & Levi, M. (1982). Weekend Effects on Stock Returns: A Note. *The Journal of Finance*, 883–889, .

Lamont, O. (1998, October). Earnings and Expected returns. *Journal of Finance*(5), 1563-1587.

Lamoureux, C. G., & Sanger, G. (1989). Firm size and turn-of-the-year effects in the OTC/Nasdaq market. *Journal of Finance*, 1219-1245.

Lau, A. H.-L., Lau, H.-S., & Wingender, J. R. (1990). The Distribution of Stock Returns: New Evidence Against the Stable Model. *Journal of Business & Economic Statistics*, 217-223 .

Lesmond, D. A., Schill, M. J., & Zhou, C. (2004). The Illusory Nature of Momentum Profits. *Journal of Financial Economics, 71*, 349-380.

Lessman, S., Sung, M.-C., & Johnson, J. E. (2009). Identifying winners of competitive events: A SVM-based classification model for horse prediction. *European Journal of Operational Research*, 569-577.

Lessmann, S., Sung, M., & Johnson, J. (2007). A new method for predicting the outcome of speculative events. *Discussion Papers in Centre for Risk Research, CRR-07-03, Southampton: University of Southampton*.

Lessmann, S., Sung, M., & Johnson, J. E. (2010). Alternative methods of predicting competitive events: an application in horserace betting markets. *International Journal of Forecasting*, 518-536.

Lessmann, S., Sung, M.-C., Johnson, J. E., & Ma, T. (2012). A new methodology for generating and combining statistical forecasting models to enhance competitive event prediction. *European Journal of Operational Research*, 163-174.

Lessmann, S., Sung, M.-C., Johnson, J. E., & Ma, T. (2012). A new methodology for generating and combining statistical forecasting models to enhance competitive event prediction. *European Journal of Operational Research*, 163-174.

Lettau, M., & Ludvigson, S. (2001). Consumption, Aggregate Wealth and Expected Stock Returns. *Journal of Finance*, 815-849.

Lev, B., & Thiagarajan, S. R. (1993). Fundamental Information Analysis. *Journal of Accounting Research, 31*(2), 190-215.

Lewellen, J. (2004). Predicting Returns With Financial Ratios. *Journal of Financial Economics, 74*, 209-235.

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics*, 13-37.

Lo, A. (2004). The adaptive markets hypothesis. *Journal of Portfolio Management*, 15-29.

Lo, A. (2005). Reconciling Effcient Markets with Behavioral Finance:The Adaptive Markets Hypothesis.

Lo, A. W., & Mackinlay, C. A. (1988). Stock Prices Do Not Follow Random Walk: Evidence from a Simple Specification Test. *Review of Financial Studies, 1*, 41-66.

Lo, V. S., & Bacon-Shone, J. (1994). A Comparison Between Two Models for Predicting Ordering Probabilities in Multiple-Entry Competitions . *Journal of the Royal Statistical Society. Series D (The Statistician)*, 317-327 .

Lo, V. S., Bacon-Shone, J., & Busche, K. (1995). The Application of Ranking Probability Models to Racetrack Betting. *Management Science*, 1048-1059.

Losey, R., & Talbot, J. J. (1980, September). Back on The Track With the Efficient Market Hypothesis. *Journal of Finance, 35*(4), 1039-43.

Luenberger, D. (1998). *Investment Science.* New York: Oxford University Press.

Maclean, L. C., Thorp, E. O., & Ziemba, W. T. (2010). Good an Bad Properties of the Kelly Criterion. *Working Paper*, 1-11.

Maclean, L. C., Ziemba, W., & Blazenko, G. (1992, November). Growth Versus Security in Dynamic Investment Analysis. *Management Science, 38*(11), 1562-1585.

Maclean, L. C., Ziemba, W., & Thorp, E. (2011). *Kelly Capital Growth Investment Criterion : Theory and Practice.* Singapore: World Scientific & Imperial College Press.

Madhavan, A., & Smidt, S. (1993). An Analysis of Changes in Specialist Inventories and Quotations . *The Journal of Finance*, 1595-1628.

Magill, M. J., & Constantinedes, G. M. (1976). Portfolio Selection with Transaction Costs. *Journal of Economic Theory*, 245-263.

Malhotra, N. K. (1982). Information Load and Consumer Decision Making. *Journal of Consumer Research*, 419-430.

Malkiel, B. (1992). Efficient Market Hypothesis. In P. Newman, M. Milgate, & J. Eatwell, *New Palgrave Dictionar of Money and Finance.* London: Macmillan.

Malkiel, B. G. (2003, Winter). The Efficient Market Hypothesis and Its Critics. *Journal of Economic Perspectives, 17*(1), 59-82.

Mandelbrot, B. B. (1971). When Can Price be Arbitraged Efficiently? A Limit to the Validity of the Random Walk and Martingale Models . *The Review of Economics and Statistics*, 225-236.

Mandelbrot, B. B. (1972). Statistical Methodology for Nonperiodic Cycles: From the Covariance to R/S Analysis. *Annals of Economic and Social Measurement*, 259 - 290.

Markowitz, H. (1952, March). Portfolio Selection. *Journal of Finance, 7*(1), 77-91.

McFadden, D. (1974). Conditional Logit Analysis of Qualititative Behaviour. In P. Zarembka, *Frontiers in Econometrics* (pp. 105-142). New York: Academic Press.

McManus, I., Gwilym, O. A., & Thomas, S. (2004, Nov/Dec). The Role of Payout Ratio and in the Relationship between Stock Returns and Dividend Yield. *Journal of Business, Finance and Accounting, 31*, 1355-1387.

Mehra, R., & Prescott, E. (1985). The Equity Premium Puzzle. *Journal of Monetary Economics*, 145-164.

Merton, R. (1992). *Continuous-Time Finance.* Cambridge: Blackwell Publishers.

Merton, R. C. (1971). Optimum consumption and portfolio rules in a continuous time model. *Journal of Economic Theory*, 373–413.

Merton, R. C. (1973). An Intertemporal Capital Asset Pricing Model. *Econometrica*, 867-887.

Mills, T., & Markellos, R. N. (2011). *The Econometric Modelling of Financial Time Series.* UK: Cambridge University Press.

Modigliani, F., & Cohn, R. A. (1979). Inflation and the stock market. *Financial Analysts Journal* , 24–44.

Modigliani, F., & Miller, M. H. (1958). The Cost of Capital, Corporation Finance and the Theory of Investment. *The American Economic Review*, 261-297 .

Morel, M. (2003). Endogenous Parameter Time Series Estimation of the Ohlson Model: Linear and Non Linear Analysis. *Journal of Business, Finance & Accounting*, 1341-1362.

Moskowitz, T. J., & Grinblatt, M. (1999, August). Do Industries Explain Momentum? *Journal of Finance, 54*(4), 1249-1290.

Nagelkerke, N. J. (1991). A note on a general definition of the coefficient of determination. *Biometrika*, 691-692.

Newbold, P., & Granger, C. (1974). Experience with forecasting univariate time series and the combination of forecasts. *Journal of the Royal Statistical Society: Series A (General)*, 131–165.

Novy-Marx, R. (2013). The Other Side of Value:The Gross Profitability Premium. *Journal of Financial Economics*, 1-28.

O'Hara, M. (1995). *Market Microstructure Theory.* Oxford University Press.

Ohlson, J. A. (1995). Earnings, Book Values and Dividends in Equity Valuation. *Contemporary Accounting Research, 11*(2), 661-687.

Ota, K. (2002). A Test of the Ohlson (1995) Model: Empirical Evidence From Japan. *The International Journal of Accounting*, 157-182.

Ou, J. A., & Penman, S. H. (1989). Accounting Measurement, Price-Earnings Ratio, and the Information Content of Security. *Journal of Accounting Research*, 111-144.

Ou, J. A., & Penman, S. H. (1989, November). Financial Statement Analysis and the Prediction of Stock returns. *Journal of Accounting and Economics, 11*(4), 295-329.

Ou, J., & Penman, S. (1989, November). Financial Statement Analysis and the Prediction of Stock returns. *Journal of Accounting and Economics, 11*(4), 295-329.

Patz, D. (1989). UK Analysts' Earnings Forecasts. *Accounting and Business Research, 19*(75), 267-275.

Peirson, J., & Blackburn, P. (2003). Betting at British Racecourses: A Comparison of the Efficiency of Betting with Bookmakers and at the Tote. In e. L. Williams, *Economics of Gambling* (pp. p. 30-42). London: Routledge.

Phillips, P. C., & Perron, P. (1988). Testing for a Unit Root in Time Series Regression . *Biometrika*, 335-346.

Pindyck, R. S., & Rubinfield, D. L. (1998). *Econometric Models and Economic Forecasts.* Singapore: McGraw-Hill.

Plott, C. R., Wit, J., & Yang, W. C. (2003). Parimutuel Markets as Information Aggregation Devices: experimental results. *Economic Theory*, 311-351.

Pontiff, J., & Schall, L. D. (1998). Book to Market Ratio as Predictors of Market Returns. *Journal of Financial Economics*, 141-160.

Porteba, J. M. (1988). Mean reversion in stock prices: Evidence and Implications. *Journal of Financial Economics*, 27-59.

Raftery, A. E., & Zheng, Y. (2003). Discussion: Performance of Bayesian Model Averaging. *Journal of American Statistical Association*, 931-938.

Raftery, A. E., Madigan, D., & Hoeting, J. (1997). Bayesian Model Averaging for Linear Regression Models. *Journal of the American Statistical Association*, 179-191.

Rangvid, J., Schmeling, M., & Schrimpf, A. (2014). Dividend Predictability Around the World. *Journal of Financial And Quantitative Analysis*, 1255-1277.

Refenes, A., Bentz, Y., Bunn, D., Burgess, A., & Zapranisa, A. (1997). Financial Times Series Modelling With Discounted Least Squares Backpropagation. *Neurocomputing, 14*, 123-138.

Reilly, F. K., & Brown, K. C. (1997). *Investment Analysis and Portfoli Management.* Fort Worth: Dryden Press.

Rhoda, K. L., Olson, G., & Rappaport, J. (1999). Risk Preferences and Information Flows In Racetrack Betting Markets. *The Journal of Financial Research.*, 265-28.

Robson, C. (2002). *Real World Research.* Singapore: Blackwell.

Robson, C. (2002). *Real World Research.* Blackwell.

Roll, R. (1977). A Critique of The Asset Pricing Theory's Tests Part 1: On Past and Potential Testability of the Theory. *Journal of Financial Economics, 4*, 129-176.

Rosenberg, B., Reid, K., & Lanstein, R. (1985). Persuasive evidence of market inefficiency. *The Journal of Portfolio Management*, 9-16.

Rosett, R. N. (1965). Gambling and Rationality. *Journal of Political Economy*, 595-607.

Rosner, B. (1975, May). Optimal Allocation of Resources in a Pari-Mutuel Setting. *Management Science - Theory Series, 21*(9), 997-1006.

Ross, S. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory* , 341-360.

Ross, S. M. (1996). *Stochastic Processes.* New York: John Wiley & Sons.

Rossi, B. (2012). The Changing Relationship Between Commodity Prices and Equity Prices in Commodity Exporting Countries. *IMF Economic Review*, 533–569.

Rouwenhorst, K. G. (1998). International Momentum Strategies. *Journal of Finance*, 267-284.

Salanié, B., & Jullien, B. (2000). Estimating preferences under risk: the case of racetrack bettors. *Journal of Political Economy*, 503-530.

Sauer, R. D. (1998, December). The Economics of Wagering Markets. *Journal of Economic Literature, XXXVI*, 2021-2064.

Schnytzer, A., Lamers, M., & Makropoulou, V. (2010). The Impact of Insider Trading in a Bookmakers' Horse Betting Market. *International Journal of Forecasting*, 537-542.

Schwert, G. W. (2002). Anomalies and Market Efficiency. *Simon School of Business Working Paper No FR02-13*.

Shanken, J., & Weinstein, M. I. (2006). Economic Forces and The Stock Market Revisited. *Journal of Empirical Finance, 13*, 129-144.

Sharpe, W. F. (1964). Capital Asset Prices: A Theory Of Market Equilibrium Under Conditions Of Risk*. *The Journal of Finance*, 425-442.

Sharpe, W. F. (1966). Mutual Fund Performance. *Journal of Business*, 119-138.

Sharpe, W. F. (1994). The Sharpe Ratio. *Journal of Portfolio Management*, 49-58.

Shefrin, H. (2005). *A Behavioral Approach to Asset Pricing.* Elsevier Academic Press.

Shefrin, H., & Statman, M. (2000). Behavioural Portfolio Theory. *Journal of Finance and Quantitative Analysis*, 127-151.

Shiller, R. J. (1981). Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? *American Economic Review*, 421-436.

Shin, H. S. (1991). Optimal Betting Odds Against Insider Traders. *The Economic Journal*, 1179-1185.

Shin, H. S. (1992, March). Prices of State Contingent Claims With Insider Traders and The Favourite Longshot Bias. *Economic Journal, 102*(41), 426-435.

Shin, H. S. (1992). Prices of State Contingent Claims with Insider Traders, and the Favourite-Longshot Bias. *The Economic Journal*, 426-435.

Shleifer, A., & Vishny, R. W. (1997). The Limits of Arbitrage. *Journal of Finance, 52*, 32-55.

Siegel, J. J. (2002). *Stocks for The Long Run* (3rd Edition ed.). McGraw-Hill.

Singleton, K. J. (2006). *Empirical Dynamic Asset Pricing .* Princeton University Press.

Singleton, K. J. (2006). *Empirical Dynamic Asset Pricing - Model Specification and Econometric Assessment.* Princeton and Oxford: Princeton University Press.

Skogsvik, S., & Skogsvik, K. (2010). Accounting-Based Probabilistic Prediction of ROE, The Residual Income Valuation Model and the Assessment of Mispricing in the Swedish Stock Market. *ABACUS*, Vol 46, 387-418.

Smith, M. A., & Williams, L. V. (2010). Forecasting Horserace Outcomes: New Evidence on Odds Bias in UK Betting Markets. *International Journal of Forecasting*, 543-550.

Smith, M. A., Paton, D., & Williams, L. V. (2006). Market Efficiency in Person-to-Person Betting . *Economica*, 673-689.

Snyder, W. W. (1978). Horse Racing: Testing the Efficient Markets Model. *The Journal of Finance*, 1109-1118.

Snyder, W. W. (1978, September). Horse Racing: Testing The Efficient Markets Model. *Journal of Finance, 33*(4), 1109-1118.

Stoll, H. (1978). The Supply of Dealer Services In Securities Markets. *Journal of Finance*, 1133–1151 .

Subrahmanyam, A. (2010). The Cross-Section of Expected Stock Returns: What Have We Learnt from the Past Twenty-Five Years of Research? *European Financial Management*, 27- 42.

Sung, M., & Johnson, J. (2007). Comparing The Effectiveness Of One And Two-Step Conditional Logit Model For Predicting Outcomes In a Speculative Market. *The Journal of Prediction Markets*, 43-59.

Sung, M., & Johnson, J. (2007). COMPARING THE EFFECTIVENESS OF ONE AND TWO-STEP CONDITIONAL LOGIT MODELS FOR PREDICTING OUTCOMES IN A SPECULATIVE MARKET. *The Journal of Prediction Markets*, 43-59.

Sung, M.-C., & Johnson, J. E. (2005). Searching for semi-strong form inefficiency in the UK racetrack betting market. In L. V. Williams (Ed.), *Information Efficiency in Financial and Betting Markets* (pp. 179-192). London: Cambridge University Press.

Sung, M.-C., & Lessman, S. (2012). Save the best for last? The treatment of dominant predictors in financial forecasting. *Expert Systems with Applications*, 11898–11910.

Sung, M.-C., Johnson, J. E., & Dror, I. E. (2009). Complexity as a Guide to Understanding Decision Bias: A Contribution to the Favorite-Longshot Bias Debate. *Journal of Behavioral Decision Making*, 318-337.

Tay, F. E., & Cao, L. (2002). Modified Support Vector Machines in Financial Time Series Forecasting. *Neurocomputing*, 847-861.

Thaler, R. H. (1999). Mental Accounting Matters. *Journal of Behavioral Decision Making, 12*(3), 183-206.

Thaler, R. H., & Johnson, E. (1990). Gambling with the House Money and Trying to Break Even: The Effects of Prior Outcomes on Risky Choice. *Management Science*, 643-660.

Thaler, R., & Ziemba, W. T. (1988). Anomalies Parimutuel Betting Markets:Racetracks and Lotteries. *Journal of Economic Perspectives*, 161–174.

Thomas, C., Warnock, F. E., & Wongswan, J. (2004). The Performance of International Portfolios. *International Finance Discusion Papers #817, Board Governors of the Federal Reserve*.

Thorp, E. O. (1997, June). The Kelly Criterion in Blackjack, Sports Betting and Stock Market. *10th International Conference on Gambling and Risk Taking (Also republished as Chapter 9 in Handbook of Asset and Liability Management edited by Zenios, S.A; Ziemba, W; 2006)*, (pp. 1-44).

Titman, S., Wei, K., & Xie, F. (2004). Capital investments and stock returns. *Journal of Financial and Quantitative Analysis*, 677–700.

Tversky, A., & Kahneman, D. (1992). Advances in Prospect Theory : Cumulative Representation of Uncertainty. *Working paper ?*

Ulbricht, N., & Weiner, C. (2005). Worldscope Meets Compustat: A Comparison of Financial Databses. *SFB 649 Discussion Paper 2005-064, Schoolf Of Business and Economics, Hunboldt-Universitat zu, Berlin, Germany*, pp. 1-31.

Van Canegham, T., Van Campenhout, G., & Van Uytbergen, S. (2002). Financial Statement and the Prediction of Stock Returns in a Small Capital Market: The Case of Belgium. *Brussels Economic Review*, 65-90.

Vapnik, V. N. (1995). *The Nature of Statistical Learning Theory.* Springer.

Vasicek, O. (1977). An Equilibrium Characterisation of the Term Structure. *Journal of Financial Economics*, 177-188.

Walls, D., & Busche, K. (2003). Breakage, Turnover, and betting market efficiency: new evidence from Japanese horse tracks. In L. V. Williams, *The Economics of Gambling* (pp. 43-66). Routledge: London.

Walls, W., & Busche, K. (2003). Breakage, Turnover and Betting Market Efficiency: New Evidence from Japanese Horse Tracks. In L. V. Williams, & L. V. Williams (Ed.), *The Economics of Gambling* (pp. 43-66). London: Routledge.

Wang, Y., Wu, C., & Yang, L. (2013). Oil price shocks and stock market activities: Evidence from oil-importing and oil-exporting countries. *Journal of Comparative Economics*, 1220–1239.

Weber, E., Bockenholt, U., Hilton, D., & Wallace, B. (1993). Determinants of Diagnostic Hypothesis Generation: Effects of Information, Base Rates, and Experience. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 1151-1164 .

Weitzman, M. (1965). Utility Analysis and Group Behavior: An Empirical Study . *Journal of Political Economy*, 18-26.

White, H. (2000). A Reality Check for Data Snooping. *Econometrica*, 1097-1126.

Williams, L. V. (2005). Semi-strong and Strong form Information Efficiency in Betting Markets. In L. V. Williams, *Information Efficiency in Financial and Betting Markets* (pp. 123-155). Cambridge: Cambridge University Press.

Williams, L. V., & Paton, D. (1997, January). Why is there a Favourite Longshot Bias in British Racetrack Betting Markets. *Economic Journal, 107*(440), 150-158.

Willinger, W., Taqqu, M. S., & Teverovsky, V. (1999). Stock market prices and long-range dependence. *Finance and Stochastics*, 1-13.

Wolf, M. (2000, January). Stock Returns and Dividend Yields Revisited. *Journal of Business and Economic Statistics, 18*(1), 18-30.

Wolpert, D. H. (1992). Stacked generalization. *Neural Networks,*, 241-259.

Ziemba, W. T. (2007). Good and Bad Properties of the Kelly Criterion. In R. E. Ziemba, & W. T. Ziemba, *Scenarios for Risk Management and Global Investment Strategies* (pp. 29-36). Chichester: John Wiley & Sons.