On the potential of a functionally graded acoustic black hole using multi-material additive manufacturing

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ABSTRACT

Acoustic Black Holes (ABHs) have been demonstrated as an effective and particularly lightweight passive vibration control solution. They are able to achieve a high level of structural damping with a thin layer of concentrated damping material by introducing a reduction in the structural wave speed. This is generally achieved by introducing a geometrical taper into the structure and the performance is maximised with a long taper that reaches a small tip height. This design approach introduces a potential weakness into the structure and, therefore, this paper explores an alternative method of achieving a reduction in the wave speed. Instead of a geometrical taper, the potential of realising the ABH effect using a functionally graded structural feature that can be achieved through multi-material additive manufacturing is investigated. Requirements on the gradient of the material properties are first investigated and then a design optimisation strategy is presented to enable the practical realisation within the constraints of a commercially available multi-material additive manufacturing process.

1. INTRODUCTION

Acoustic Black Holes (ABH) are structural features that have been shown to provide a significant level of vibration control [1–3]. Generally, ABHs are realised by introducing a reducing thickness profile into a structure, which results in a decreasing wave speed. In the theoretical limit, where the thickness of the tapering structure reduces to zero, the wave speed reaches zero and there is no reflection from the boundary. In practice, the taper cannot reach zero thickness and so the wave speed does not reach zero, however, the reduced wave speed means that through the addition of a small amount of damping material it is possible to achieve a significant level of vibration control. Resultingly, ABHs are able to achieve a significant level of structural vibration control without increasing the structural mass. This is clearly an attractive property from the perspective of noise and vibration control and ABHs have, therefore, received significant research interest [3].

Despite the potentially significant performance and weight benefits of ABHs, the tapering thickness profile of conventional ABHs results in a trade-off between performance and strength [4]. In particular, since the ABH relies on focusing the vibrational energy into the thin part of the taper, there is also a need to consider the structural fatigue. One solution to this structural strength limitation is the compound ABH, which is realised via a double-layered hollow tapered beam, and has been shown to increase the ABH effect whilst increasing the structural stiffness and strength compared to a conventional ABH taper design [4].

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Another area of recent research in ABH design has explored the use of multi-material 3D printing to enhance the ABH effect by introducing both a decreasing thickness and elastic modulus into the ABH [5]. This work demonstrated that by decreasing the elastic modulus along the length of the ABH, it was possible to achieve a significant increase in the ABH effect and a resulting decrease in the reflection coefficient from the ABH taper compared to a conventional ABH beam termination. The functionally-graded ABH proposed in [5], however, still utilises a reducing thickness profile and, therefore, may have limitations in certain applications where the strength of the ABH is critical. Building on the idea of functionally-graded ABHs, this paper presents an initial investigation into the potential of realising the ABH effect via functionally-grading the material properties of a beam termination without a simultaneous decrease in its thickness.

Section 2 presents a theoretical investigation into how the elastic modulus of a beam termination can be graded in order to decrease the structural wave speed and reduce the reflection coefficient in a beam. The potential performance of the functionally-graded ABHs is also compared to a conventional ABH realised via a decreasing thickness profile, which provides an indication of the suitability of this design approach. In practice, it is not possible to have unconstrained control over the material properties and Section 3 investigates how the functional grading of the elastic modulus can be optimised when a discrete number of material property values are realisable, as is the case in the multi-material additive manufacturing processes utilised in [5] . Finally, Section 4 provides discussion and concluding remarks.

2. THEORETICAL REQUIREMENTS OF A FUNCTIONALLY GRADED ABH

As noted in the introduction, ABHs are generally implemented by varying the geometrical properties of the structure to achieve a reduction in the wave speed. The wave speed in a beam can be expressed in terms of the geometrical and material properties as [2, 6]

$$c_f(x) = \left(\frac{E(x)h(x)^2\omega^2}{12\rho(x)} \right)^{(1/4)}, \tag{1}$$

where x is the coordinate position, E is the elastic modulus, h is the thickness, ω is the angular frequency and ρ is the density of the beam. From Equation 1 it can be seen that the wave speed is proportional to the square root of the beam height, such that a decrease in the wave speed can be introduced by a gradually decreasing thickness profile, as is typical in structural ABH design. However, it can also be seen that the wave speed could alternatively be controlled by modifying the elastic modulus or the density along the length of the beam. In practice, this could be achieved using additive layer manufacturing techniques, including the use of multi-material methods [7] or by controlling the infill [8].

In this section, the requirements in terms of grading the elastic modulus along the length of a beam termination to reach comparable performance to an ABH realised via a decreasing thickness profile will be explored. In the first instance, this has been investigated using the geometrical acoustics, or first order WKB approximation [1, 2]. This approach offers a simple means of investigating the behaviour of ABHs, but essentially assumes a smoothly varying profile and, therefore, neglects reflections from the junction between the ABH and the beam sections [9]. However, this modelling approach has been shown to provide indicative results on average and, therefore, provides a useful first step.

According to the first order WKB approximation, the reflection coefficient from a beam termination can be calculated by integrating the wavenumber over the length of the termination to give

$$R = \exp\left(2 \int_0^{l_{ABH}} \mathrm{Im}(k(x))\mathrm{d}x \right), \tag{2}$$

where l_{ABH} is the length of the ABH termination, coordinate location 0 is defined as the junction between the ABH termination and the beam and the wavenumber can be expressed following equation Equation 1 as

$$k(x) = \frac{\omega}{c_f(x)} = \left(\frac{12\rho(x)\omega^2}{E(x)h(x)^2}\right)^{(1/4)}. \tag{3}$$

It should be noted that due to an alternative definition of the coordinate system, Equation 2 presents a modified version of the reflection coefficient to that usually defined in the literature [1, 2, 10]; this modification aims to avoid confusion when, rather than truncating a decreasing thickness profile, the termination is defined in terms of a functionally-graded material profile starting at the junction between the beam and the termination.

In the case of the geometrically tapered ABH, which will be used as a reference point in the following analysis, the thickness profile is defined here according to a power law profile given by

$$h(x) = (h_0 - h_{tip})\left(\frac{l_{ABH} - x}{l_{ABH}}\right)^{\mu} + h_{tip}, \tag{4}$$

where h_{tip} is the height of the tip of the ABH taper, h_0 is the height of the beam and μ is the power law, which defines the rate at which the taper decreases in height. The ABH with a power law profile has been extensively investigated in the literature [2, 3, 6, 10–13] and it has been shown that the reflection coefficient from the power-law ABH termination is minimised by maximising the length of the taper and minimising the tip height. The power-law, meanwhile, must be optimised to reach a trade-off between the modal density and reflection from the junction between the beam and the taper [6]. This trade-off means that the optimal power depends on the taper length and tip height, as investigated in [6].

Following the investigation presented in [6], and to provide a point of reference for the functionally graded ABHs, the geometrically tapered ABH has been defined with the properties detailed in Table 1. In comparison to [6], where the ABH is constructed from aluminium, it has been assumed here that the beam and the geometrically tapered ABH have been constructed from the VeroWhitePlus rigid photopolymer based 'Digital Material' provided by Stratasys and utilised for the realisation of a functionally graded and geometrically tapered ABH in [5]. It is worth noting that the inherent damping of this material is relatively high and, therefore, there is no need to add an additional layer of damping material to the ABH. In the following two subsections, the variation in the elastic modulus required to match the performance of the geometrically tapered ABH defined according to the parameters presented in Table 1 will be investigated.

Table 1: Beam and geometrically tapered ABH properties.

Parameter	h_0	h_{tip}	l_{ABH}	μ	E_0	ρ_0	η
Value	0.01 m	0.5×10^{-3} m	70×10^{-3} m	4	2.5 GPa	1160 kgm^{-3}	0.1

2.1. Functionally graded elastic modulus

As shown above, it is possible to control the reflection coefficient in a beam by grading the elastic modulus. This has previously been considered in [5] in combination with a geometrical taper, however, it is also possible to achieve a reduction in the wave speed by decreasing the elastic modulus but maintaining a constant thickness profile. To match the performance of the thickness profile defined according to Equation 4, the elastic modulus should vary along the length of the constant height ABH termination as

$$E(x) = \frac{h(x)^2}{h_0^2}E_0, \tag{5}$$

where E_0 and h_0 are the elastic modulus and height of the beam respectively, as defined in Table 1. If the beam termination parameters are defined according to those specified in Table 1, then the resulting graded elastic modulus calculated according to Equation 5 and the corresponding reflection coefficient, calculated according to Equation 2, are shown in Figure 1. From the right-hand plot it can be seen that the required elastic modulus ranges from 2.5 GPa to 6.25 MPa and this results in a reflection coefficient that matches the geometrically tapered ABH, as shown by the results in the left-hand plot. It can be seen from these results that both the geometrical and graded elastic modulus ABHs show a decrease in the reflection coefficient as frequency increases and offer a significant reduction in the reflected wave.

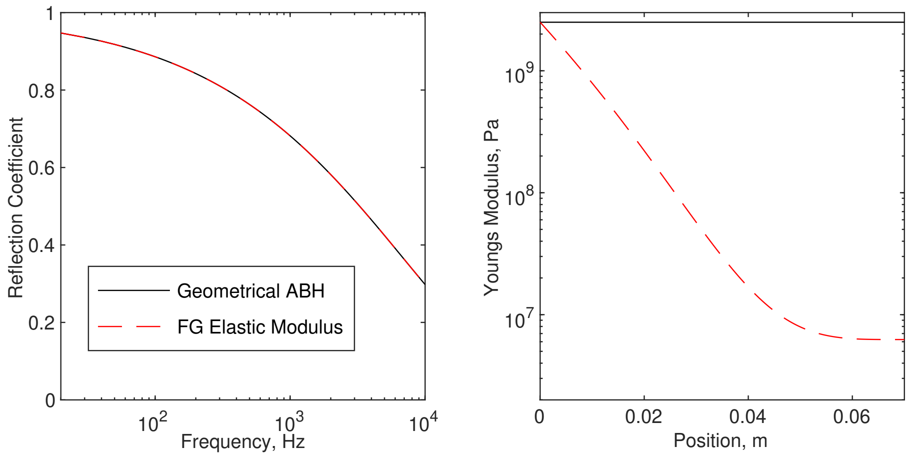

Figure 1: **The reflection coefficient over frequency (left) and the elastic modulus over the length of the termination (right) for a geometrically tapered ABH and the functionally graded elastic modulus ABH.**

3. OPTIMISATION OF A REALISABLE FUNCTIONALLY GRADED ABH

The previous section has shown how the elastic modulus in a functionally graded ABH termination should be varied so that it is able to achieve the same level of reflection coefficient as a geometrically tapered ABH. However, it is important to highlight that it was assumed in the previous section that the elastic modulus varies smoothly along the length of the termination, which may not be possible in practice. Additionally, by utilising the first order WKB approximation, the results in the previous section neglect the effect of reflections from the junction between the ABH and the beam sections, which results in a trade-off in practice. Therefore, this section will utilise a more complete numerical model to consider the design of potentially realisable graded elastic modulus ABH.

One approach to realising a graded elastic modulus is the use of multi-material additive manufacturing, as previously explored in [5]. To this end, Stratasys provide a series of 'Digital Materials' that offer a range of properties and these will be utilised here to consider the design of a realisable graded elastic modulus ABH. Table 2 details the properties of the range of materials that are available between the two extremes of the relatively rigid VeroWhitePlus (Material 0) and the rubber-like TangoPlus (Material 9). It can be seen from Table 2 that the range of the elastic modulus offered by this multi-material manufacturing process covers that required according to the results presented in Figure 1, however, this does not consider the discrete nature of the 'Digital Materials'.

To begin to investigate the effect of discontinuities in the graded material properties, a numerical model of the beam has been implemented in Comsol using Finite Elements. In this case the material properties in Table 2 have been used directly and the length of each material in the termination section has been optimised using a topological optimisation based on a pattern search process [14]. The

Table 2: Properties of the Stratasys Digital materials [5].

Material	0	1	2	3	4	5	6	7	8	9
E (GPa)	2.5	2	1.6	1.3	1.0	0.8	0.6	0.4	0.2	0.002
ρ (kgm^{-3})	1160									
η	0.1									

objective of the optimisation procedure was to minimise the broadband average reflection coefficient. Figure 2 shows the resulting reflection coefficient for the standard beam, the geometrical ABH and the ABH implemented using a graded elastic modulus. From these results it is initially worth highlighting that the peaks and nulls in the reflection coefficient are due to standing waves that occur in practice, but are not accounted for in the model based on the first order WKB method. In general, it can be seen from the results presented in Figure 2 that the graded elastic modulus ABH is able to outperform the geometrically tapered ABH. The broadband average reflection coefficient is 0.63 for the geometrical ABH and 0.62 for the graded elastic modulus ABH. Although this broadband performance increase is relatively small, it is to note that at frequencies above around 2 kHz, the graded elastic modulus termination does achieve a relatively constant reflection coefficient and offers a general reduction in the reflection coefficient of around 20%.

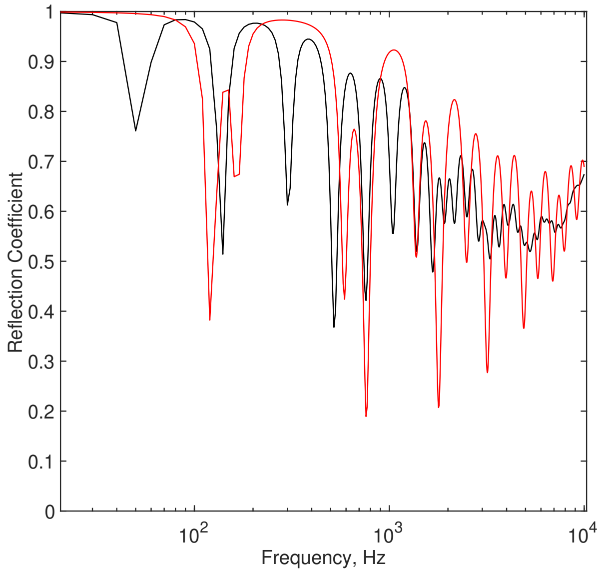

Figure 2: **The reflection coefficient over frequency for a flat beam (black), a geometrically tapered ABH (red) and a functionally graded elastic modulus ABH (blue).**

4. CONCLUSIONS

This paper has presented an initial investigation into the potential of realising the ABH effect via grading the material properties of a beam termination alone. This attempts to go beyond previous work, which has investigated the potential of enhancing the performance of a geometrically tapered ABH by also grading the elastic modulus. It has initially been demonstrated how the elastic modulus should be graded in order to match the performance achieved by a geometrically tapered ABH and although the range of material properties is potentially achievable, this requires the material properties to be smoothly varied. Therefore, a numerical model has been implemented in order to investigate the achievable performance when a discrete set of material properties are available. This study has provided an initial demonstration that the ABH realised with discrete changes in the elastic modulus is able to achieve comparable properties to a conventional geometrical ABH. Future work is clearly required to validate these findings experimentally, extend the proposed concepts to more advanced structures and investigate the potential of controlling the wave speed by varying the density of the structure.

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