Towards the development of a digital twin for structural dynamics applications

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Abstract

A digital twin is a powerful new concept in computational modelling that aims to produces a one-to-one mapping of a physical structure, operating in a specific context, into the digital domain. This technology could therefore provide improved and robust decision making for asset management. Although the applications of digital twins vary, this paper focuses on digital twins for structural dynamic systems. A key consideration in developing a digital twin is in the construction of a workflow that defines decisions and interactions within the modelling framework. This process will generally be bespoke to specific applications, however key principles will apply. Furthermore, a workflow will provide a methodology for identifying poor predictive performance and systematically improving predictions via optimal decision making. In this paper a three storey building structure is introduced as a case study in order to motivate the challenges and technologies required of a digital twin. The context of this case study is to develop a digital twin of the building structure that consistently predicts the acceleration response of the three floors given an unknown structural state, caused by a contact nonlinearity between two floors. This reflects realistic challenges for a digital twin in that the physical twin will degrade with age, and its response may change under various loading scenarios, unforeseen in the initial model development phase. Key elements within a potential workflow for this application are discussed. These include indicating when model updating schemes become problematic and how augmenting physics-based models with a data-based component can provide information about poor predictive performance. These techniques are linked to hybrid testing, as a potential method for improving model development based on the physical structure in a controlled offline manner. Finally, the impact of these procedures are discussed for model based control methods in terms of vibration attenuation performance, but also robustness against model uncertainties and external disturbances. The workflow and key technologies investigated in this specific case study are expected to outline the general processes that apply to digital twins more broadly, and provide a clearer understanding of how a digital twin should be implemented.

Keywords: Digital twin; validation; hybrid testing; robust control

1 Introduction

In recent years the concept of a digital twin has been widely cited within industry and academia as a promising novel technology [1, 2]. Although there is a wide range of different opinions within the literature about the definition of a digital twin, the general concept is to create a digital-based representation (or a virtualisation) of a structure that aims to provide a one-to-one mapping with a physical structure throughout the operational life of the physical twin. In order to achieve this aim a key element of a digital twin will be that the physics-based models it is constructed from can 'learn' and improve over time, in an automatic manner, leading to online model development and improved predictive capabilities. This clearly is a challenging task as current validated models are typically improved in an offline setting with a high level of modeller input. This paper seeks to motivate the challenges with conventional model updating-based approaches, the need for a data-based component to augment the physics-based model, and how this can be used to guide model development, with reference to hybrid testing techniques. These processes have considerable implications for model based control, as any online 'learning' from data will be coupled with the controller dynamics. In addition, as there are multiple methods for achieving model improvements for a digital twin, all associated with a different level of cost, a formal decision process will need to be stated such that optimal decisions are made.

This paper motivates the challenges in developing a digital twin via an experimental case study of a three storey building structure. The context for developing a digital twin is to predict the acceleration response at each of the three floors over the operational period of the structure. The structure has a contact nonlinearity (here denoted as the bumper mechanism) between the middle and top floors which only comes into contact at particular forcing levels. Here it is assumed that the initial dataset from the structure was obtained in a scenario when the bumper mechanism did not come into contact, and thus the system behaves linearly. Model development is performed in this region, as no knowledge about a nonlinearity is assumed at this stage. The developed model is validated and subsequently used in two operational conditions, one where the bumper mechanism is not in contact, and another where the bumper mechanism makes contact. This scenario therefore reflects unforeseen structural behaviour, which is expected to be encountered in an operational digital twin, as often all physics are not known, or cannot be modelled, prior to the structure being used in operation.

The outline of the paper is as follows. Section 2 introduces the case study, providing an overview of the digital twin in this context. The problems associated with conventional model updating approaches to model validation are then demonstrated in Section 3 leading to the development of a data-augmented physics-based model in Section 4. The idea utilising hybrid testing such that model development can be performed is discussed in Section 5, along with the associated challenges. The implications for model based control methods are then highlighted in Section 6. A central concept to developing a digital twin will be the formalised decision processes, which are discussed in Section 7 before final conclusions are made in Section 8.

2 Overview of the digital twin

A digital twin aims to provide accurate predictions of quantities of interest throughout the operational life of a structure. This virtualisation is constructed from physics-based models that 'learn' from operational data such that the model improves with time. This section outlines a case study in order to motivate challenges in constructing a digital twin, highlighting potential solutions and required technologies, with consideration of the decision processes in a digital twin.

The case study considers an aluminium, experimental, three storey building structure (the physical twin) and seeks to develop a digital twin that predicts the acceleration response of the three floors throughout the structure's operational phase. A schematic of the structure, shown in Figure 1, indicates that a bumper mechanism, constructed from two aluminium blocks, is located between the middle and top floors of the structure. This bumper mechanism creates a contact nonlinearity when specific initial conditions and excitation are applied. The imagined scenario considers that the structure is designed to operate under a random excitation at a consistent forcing level. However, in the design and construction phase of the physical twin it is assumed that the bumper mechanism will not come into contact, and therefore the system can be assumed and modelled as a linear system. This was confirmed as a modelling decision as initial data from the structure indicated that the bumper mechanism did not come into contact. As is common in an industrial setting, an engineering modelling team assumes that a simpler linear computer model is satisfactory for this scenario. This assumption is confirmed by the linear model's performance on a validation dataset, and reflects the extra cost in developing a nonlinear model, which often requires a more complex validation procedure and more computational resource. In the operational phase, under the same band-limited white noise forcing level the bumper mechanism comes into contact, creating a harsh nonlinearity. This case study reflects these common real world scenarios whereby unforeseen physical behaviour, not initially captured in the computer model, reduces predictive performance and, given the event, this new physical behaviour must be incorporated into the digital twin.

2.1 Experimental Data

Experimental data were collected for the physical twin, depicted in Figure 1, subject to a 25.6Hz band-limited random excitation (F_s) at the first floor, applied by an electrodynamic shaker. Data were recorded at a sampling frequency of 51.2Hz at three accelerometers at each of the floors ($\{\hat{y}_i\}_{i=1:3}$). Three 20 second datasets were obtained; a training dataset (dataset one) where the bumper mechanism *did not* come into contact, a validation dataset (dataset two)

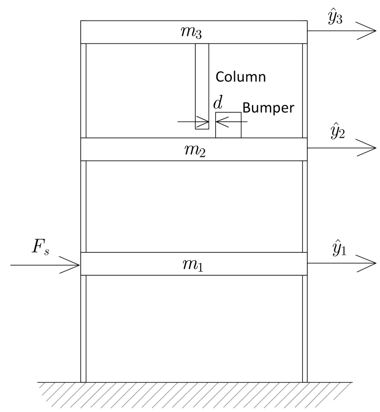

Figure 1: A schematic of the three storey structure (physical twin) detailing the bumper mechanism, shaker attachment, and accelerometer positioning.

where the bumper mechanism *did not* come into contact, and a validation dataset (dataset three) where the bumper mechanism *did* come into contact. These three dataset reflect the scenarios described in Section 2.

2.2 Initial Modelling

The initial modelling was performed on the training dataset (dataset one), where the bumper mechanism *did not* come into contact. Even through the physical twin is a nonlinear system, the initial data from the physical twin did not include observations of this nonlinearity, and hence a linear model was developed. Given the frequency response functions of the initial dataset, displayed in Figure 2, the system was modelled as a three degree-of-freedom structure,

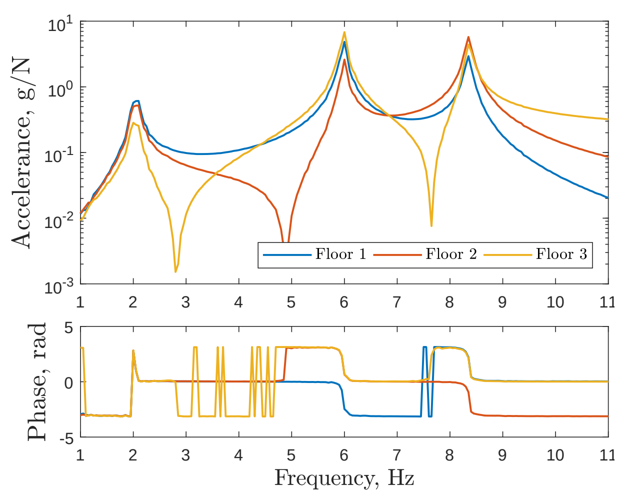

Figure 2: Frequency response functions between the first floor and the accelerations from each of the three floors.

$$
\begin{aligned}
\ddot{y}_1 &= (F - k_1 y_1 - k_2(y_1 - y_2) - c_1 \dot{y}_1 - c_2(\dot{y}_1 - \dot{y}_2))/m_1 \\
\ddot{y}_2 &= (k_2(y_1 - y_2) - k_3(y_2 - y_3) + c_2(\dot{y}_1 - \dot{y}_2) - c_3(\dot{y}_2 - \dot{y}_3))/m_2 \\
\ddot{y}_3 &= (k_3(y_2 - y_3) + c_3(\dot{y}_2 - \dot{y}_3))/m_3
\end{aligned} \tag{1}
$$

where $\{m_i\}_{i=1:3}$ are the masses, $\{c_i\}_{i=1:3}$ are the damping coefficients and $\{k_i\}_{i=1:3}$ are the stiffness coefficients for each of the three floors (indexed by i). Additionally the force, displacement, velocity and acceleration terms are denoted as, F, $\{y_i\}_{i=1:3}$, $\{\dot{y}_i\}_{i=1:3}$ and $\{\ddot{y}_i\}_{i=1:3}$ respectively. The physics-based model selected here is analytical,

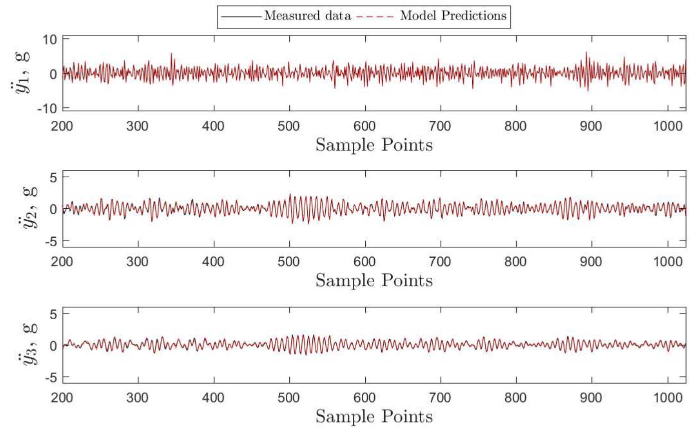

Figure 3: MCMC acceleration predictions on dataset two.

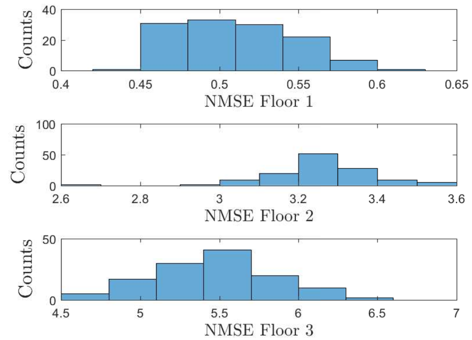

Figure 4: NMSEs for the MCMC acceleration predictions on dataset two.

however the principles and techniques discussed are applicable to more complex model forms, such as finite element (FE) or multi-physics models.

The parameters of the model in Equation 1 were identified from the dataset one using Markov Chain Monte Carlo (MCMC) with a joint Gaussian likelihood (the product of the Gaussian likelihood for each floor) with a fixed noise variance $\sigma_n^2 = 3 \times 10^{-3}$, reflecting engineering judgement of the sensors and Gaussian priors for the mass, stiffness, and damping terms. Four chains were run in parallel, with a burn-in of 2500 samples before obtaining a further 10000 samples, such that the $\hat{R}$ statistics could be measured to check convergence[1]. The MCMC output posterior predictive distributions for acceleration at each floor are presented for dataset two (no bumper contact) in Figure 3, where the normalised mean squared errors (NMSEs) are shown in Figure 4. The low NMSEs for each floor indicate that the model is 'valid' for its given context, i.e. on an unseen test dataset the errors are low indicating the model will predict well for a similar loading scenario. There is no indication that the digital twin will fail in operation given the evidence so far, as typically assumed in most engineering contexts.

The 'validated' digital twin model is left operating and during operation it receives dataset three, where the bumper makes contact. The predictive performance, shown in Figures 5 and 6, is poor, reflecting the fact that the initial model does not model the harsh contact nonlinearity. This problem motivates the following questions for developing a digital twin (explored in next sections):

- How does a digital twin account for missing physics?
- What does the digital twin do when predictive performance is bad?
- What is the best way in improving the digital twin given poor predictive performance?
- How will this effect the control of the structure?

3 The problem of model updating in a digital twin

One conventional approach to overcoming poor predictive performance is to utilise model updating [4]. The understanding here it that the parameters of the model may have diverged from the real structures, due to degradation, environmental factors etc. The predictive discrepancy may lead an engineer to decide that the model-form may be correct and the parameters need recalibrating. This section discusses the implications of this approach, motivating the need for inferring missing physics, as well as updating parameter sets.

The system model used in this work is quite simple (for the purpose of demonstration) but this is usually not the case in real applications. The system model of a full-scale structure are normally one or more complicated FE models possibly with nonlinear elements involved. Updating the system model can be computationally expensive and it also

[1]For more information on Bayesian statistics the reader is referred to [3]

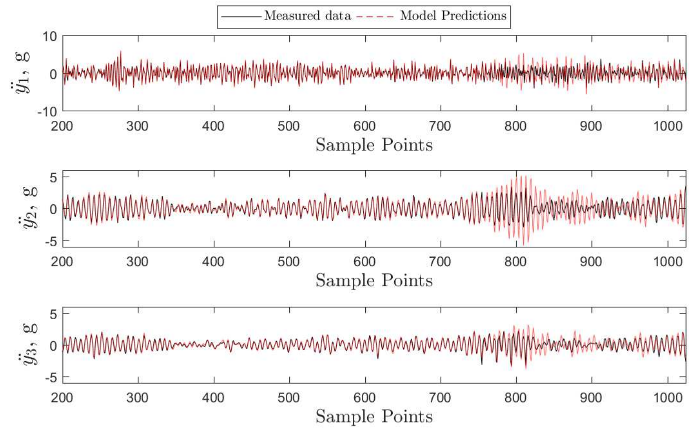

Figure 5: MCMC acceleration predictions on dataset three.

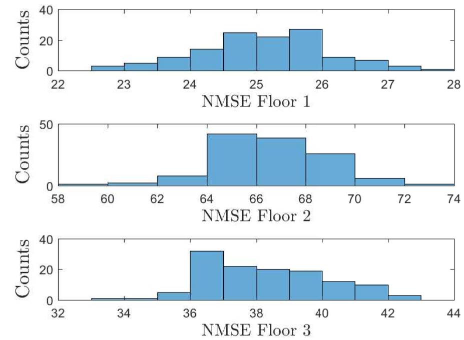

Figure 6: NMSEs for the MCMC acceleration predictions on dataset three.

Table 1: Identification results of modal parameters.(f:natural frequency; dr: damping ratio; MPV: most probable value; COV: coefficient of variation=posterior variance/MPV)

	Dataset 1				Dataset 2				Dataset 3			
	f (Hz)		dr (%)		f (Hz)		dr (%)		f (Hz)		dr (%)	
	MPV	COV	MPV	COV	MPV	COV	MPV	COV	MPV	COV	MPV	COV
Mode 1	1.915	0.021	7.347	0.2139	2.018	0.005	2.315	0.1055	1.939	0.034	6.056	0.2662
Mode 2	5.878	0.003	1.830	0.1140	5.953	0.004	1.384	0.1604	6.038	0.005	3.188	0.0830
Mode 3	8.322	0.001	0.875	0.1193	8.373	0.001	0.938	0.2280	8.507	0.004	1.906	0.2506

unnecessary to update the system model when there is no significant change in the real structure. To address the above concern, an intermediate model can be introduced when constructing the digital twin. The intermediate model can be a surrogate version of the system model whose parameters can be efficiently identified from the measured data. Divergence analysis can then be conducted to determine whether the system model should be updated or not by investigating divergence in the parameters of the intermediate model. This also allows the digital twin to be run in a real-time manner.

In this case study, a modal model is used as an intermediated model where the modal parameters (e.g., natural frequencies and damping ratios) are investigated for divergence analysis. Table 1 summaries the identified natural frequencies and damping ratios with the associated identification uncertainties of the modal modes of the structure based on the previously mentioned datasets using [5]. In order to fully account for the posterior distribution of the identified modal parameters, the Hellinger distance is used to quantify the discrepancies between the training (dataset one) and testing (dataset two and three) datasets. The results are calculated to be 0.668 and 0.997. If a updating threshold is set as the Hellinger distance larger than 0.9 (say), the system model then needs to be updated when dataset three is encountered.

However, it should be noted that model updating is not sufficient to resolve the issue from dataset three, even though a large divergence has occurred. When there are missing physics encountered (e.g., the bumping mechanism in dataset three is not involved in the system model), model updating will lead to biased estimation of system parameters and the predictive performance of the digital twin will not generally be improved. Model improvement is needed in this context where a data-augmented model can be trained to capture the missing physics. Additional decision making is also demanded to decide when model updating or model improvement is needed. These issues will be discussed in the following sections.

4 Data-augmented modelling

The addition of a data-based component to a physics-based model can be termed data-augmented modelling. This is similar to the concept of inferring model discrepancy, the mismatch between observational data and a model output, given the 'true' parameters are known [6]. Within the literature various techniques exist for modelling model discrepancy with regression models [6, 7, 8]. In this case study a Gaussian Process (GP) model[2] is inferred between the output of linear model and data for dataset one. The aim of augmenting the physics-based model with a Gaussian Process (GP) model is two fold. Firstly, the addition of a GP model will reduce the residual between predictions and observational data. Secondly, the GP model is probabilistic with an associated uncertainty inferred from the training dataset. This is particularly informative as the uncertainty increases for inputs 'far' from the training set (given the inferred hyperparameters). Essentially, the GP model provides a measure of how different the data are and whether new behaviour, different to that observed in training, has occurred. This is particularly informative in a digital twin scenario, as the GP model will inform when an engineer can expect the digital twin to have poor predictive performance, without reference to measured observations. This information can then be used to mitigate this behaviour, or inform model development.

In this case study the model discrepancy for each floor is expect to contain dynamic information, meaning that the GP model is trained using lagged outputs from the linear model and lagged input forces (where the forcing is expected to be known at time t_n) i.e. $\{\ldots, \ddot{y}_i(t_n-3), \ddot{y}_i(t_n-2), \ddot{y}_i(t_n-1), \ldots, \ddot{F}(t_n-3), \ddot{F}(t_n-2), \ddot{F}(t_n-1), \ddot{F}(t_n)\}$; where the outputs from the linear model are the averaged output prediction from the MCMC samples. The number of lags used in training each GP model were selected from the autocorrelation of the residual between the linear model predictions and training observations, identifying correlation occurred up to around ten lags for each response. The three GPs were modelled using zero mean and Matérn 3/2 covariance functions, and the training dataset was composed of sample points 200 to 400 from dataset one, such that the transients were removed.

The predictions of the data-augmented model for dataset two and three are shown in Figures 7 and 8 respectively. The NMSEs for the mean prediction on dataset two were $\{3.672,\ 2.426,\ 1.107\}$ and were $\{30.084,\ 39.837,\ 21.054\}$ for data set three. This demonstrates an improved predictive performance for floors two and three (over both the MCMC predictions), however the NMSE increases for the first floor when compared to the MCMC results. This is likely due to a lack of information about the dynamics contained within the residual of the response of the first floor due to the positioning of the force. More informatively, the standard deviation of the prediction provides a measure of confidence in the predictions and whether new phenomena have occurred. In Figure 7 (no bumper contact) the standard deviation of the predictions is relatively low, and close to zero and almost constant for the middle and top floors. This changes in Figure 8 where the uncertainty increases around the point where contact between the bumper mechanism was made. This increase in variance indicates the presence of epistemic uncertainty, and can be used to extract parts of the observational loading that are informative for identifying the physics of the nonlinearity. This can be informative in conducting hybrid testing based experiments.

5 Hybrid testing

Real-time hybrid testing is an experimental framework that can be used to assess the dynamics of an assembly by isolating a component of interest (the *physical substructure*) and testing it under realistic, dynamic boundary conditions, by imposing the reactions calculated from a model of the rest of the assembly (the *numerical substructure*) through one or more actuators [10]. As the numerical substructure lends itself quite naturally to represent a digital twin, hybrid testing can be used to explore design modifications or to improve the model of complex, possibly nonlinear elements in a controlled environment and without having to build a full prototype or intervene on the physical twin.

Fig. 9 shows the concept of hybrid testing as applied to the three-storey building a new test case. The real assembly, subjected to base excitation $\ddot{y}_G$, can be partitioned into the physical substructure, which includes the bumper as the feature to be investigated in detail together with the second and third floors, while the first floor and its columns can be reduced to the numerical substructure, as it can reasonably be assumed that a validated digital twin would be available for these linear components. The digital twin would be implemented in a real-time digital controller, where the theoretical displacement of the second floor y_{2n}, corresponding to the force F_m, can be effectively exchanged and measured at the interface between the substructures and sent to a servo-drive and actuator as the target boundary condition to be imposed. In the case of perfect control, the actual displacement y_{2p} of the physical side of the interface would match the numerical displacement y_{2n}, and the hybrid assembly could be considered dynamically equivalent to the real one — within the uncertainty boundaries of the numerical substructure.

However, because of the intrinsic dynamics of the actuator, and the limited bandwidth offered by the PID controllers usually implemented in off-the-shelf servo-drives, the physical displacement y_{2p} can have a different amplitude and be shifted in time with respect to y_{2n}. This error in the applied boundary condition can lead to a

[2] In keeping with the required brevity of this paper the reader is referred to [9] for more details on Gaussian Process regression.

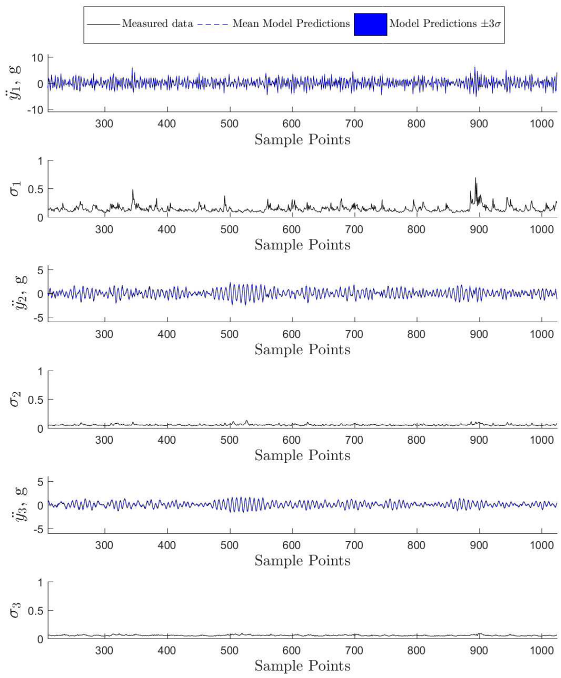

Figure 7: Data augmented model predictions on dataset two (no bumper contact) and predictive standard deviations.

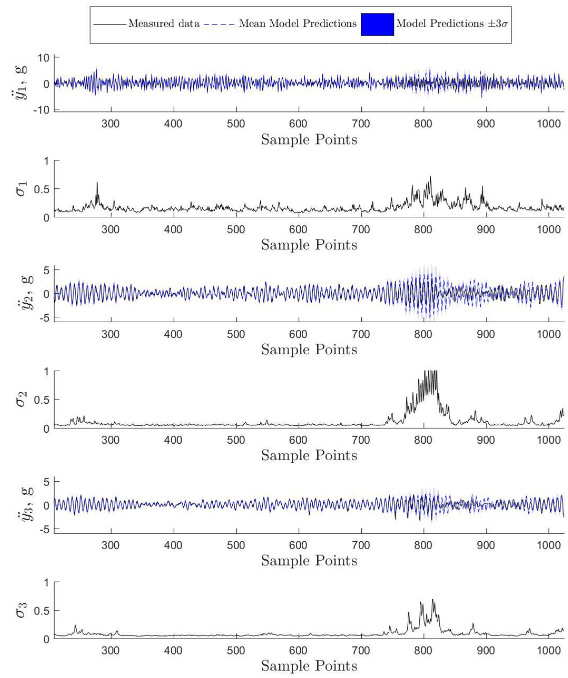

Figure 8: Data augmented model predictions on dataset three (bumper contact) and predictive standard deviations.

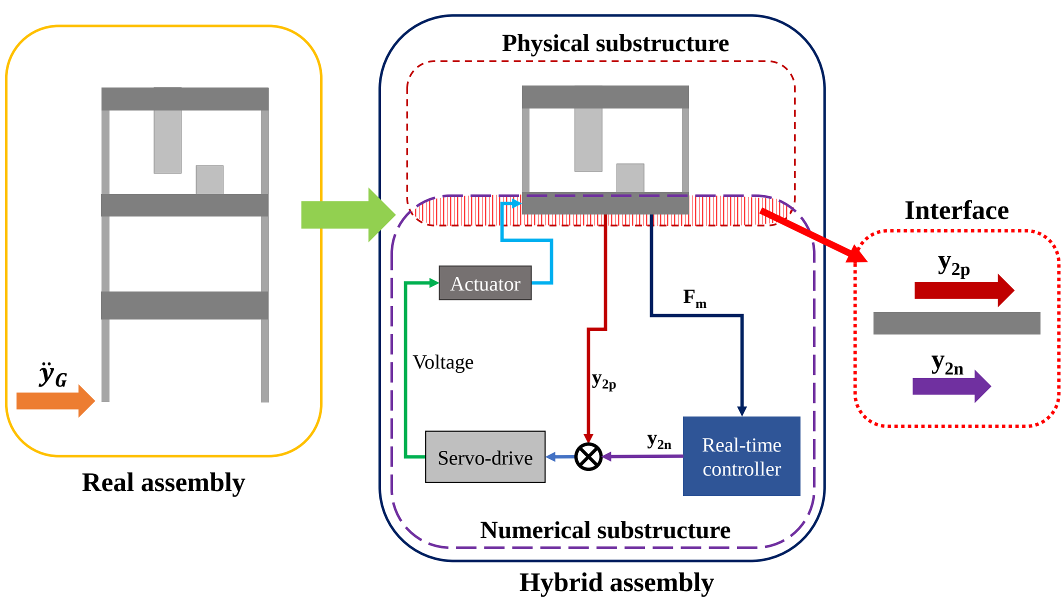

Figure 9: Schematic overview of real-time hybrid testing applied to the three-storey building test case.

non-representative hybrid assembly, and even dangerous instabilities [11].

In order to highlight both the potential and challenges associated to hybrid testing in the context of digital twins, a simplified simulation of the setup of Fig. 9 was carried out with the aim of identifying the threshold in the base excitation at which the bumper mechanism would be activated. Eqs. 1 can be adapted to the hybrid testing nomenclature and written as

$$m_1\ddot{y}_{1n} + c_1(\dot{y}_{1n} - \dot{y}_G) + c_2(\dot{y}_{1n} - \dot{y}_{2n}) + k_1(y_{1n} - y_G) + k_2(y_{1n} - y_{2n}) = 0 \tag{2a}$$

$$m_2\ddot{y}_{2p} + c_3(\dot{y}_{2p} - \dot{y}_{3p}) + k_3(y_{2p} - y_{3p}) - F_{nl}(y_r) = F_m \tag{2b}$$

$$m_3\ddot{y}_{3p} + c_3(\dot{y}_{3p} - \dot{y}_{2p}) + k_3(y_{3p} - y_{2p}) + F_{nl}(y_r) = 0, \tag{2c}$$

where $y_r = y_{2p} - y_{3p}$ is the relative displacement between floors 2 and 3, and $F_{nl}(y_r)$ represents the nonlinear force introduced by the bumper.

Eqs. (2b) and (2c) describe the dynamics of the second and third floor, and will be used in the following purely to simulate the response of the physical substructure (in a real hybrid test, y_{2p}, y_{3p} and F_m would simply be measured). Eq. (2a), on the other hand, represents the model of the numerical substructure. By comparison with Eqs. (1), it can be easily shown that the force F_m between the physical substructure and the actuator coincides with the interaction between the second and first floors, and can be expressed as

$$-F_m = c_2(\dot{y}_{2n} - \dot{y}_{1n}) + k_2(y_{2n} - y_{1n}) \tag{3}$$

In an actual hybrid test, Eqs. (2a) and (3) would be integrated within the real-time controller in order to calculate the theoretical displacement y_{2n} of the second floor to use as target signal to the servo-drive.

For the sake of simplicity, the bumper nonlinearity was modelled in the simulations as a bilinear spring as follows

$$F_{nl}(y_r) = \begin{cases} 0 & \text{if } y_r \leqslant d \\ k_{nl} y_r & \text{if } y_r > d, \end{cases}$$

where $k_{nl} = 900$ N/mm represents the stiffness of the bumper, and $d = 0.5$ mm the gap width. The base excitation y_G was a version of the El Centro earthquake scaled in time in order to excite the first three bending modes of the structure. Its peak amplitude was varied in the simulations to identify the level that brings the bumper into contact and activates the nonlinear response of the building. For a qualitative assessment of the effect of an imperfect control system on the results, an amplitude error $a_{err} = \pm 0.1$ was deliberately introduced at the interface with the actuator so that the physical displacement would differ from the numerical one ($y_{2p} = (1 + a_{err})\, y_{2n}$).

The simulation results are shown in Fig. 10. With perfect control, the bumper nonlinearity would be activated at a peak excitation amplitude of 1.05 mm. It can also be seen that, with errors higher than +7% in the actuation amplitude, this hybrid testing set-up would yield a nonlinear response at all excitation levels, thereby completely misrepresenting the dynamics of the real assembly. However, below this threshold, even relatively large errors in the actuator amplitude would yield an acceptable estimation of the critical excitation amplitude.

This simple analysis illustrates how hybrid testing can provide experimental data representative of the whole physical twin without having to modify the physical structure and in controlled conditions, thanks to the real-time interaction between the component of interest and the digital twin of the rest of assembly. It can also be used to selectively target the physics of complex components in the context of the larger assembly, as the effects of the harsh nonlinearity introduced by the bumper mechanism can be measured and then propagated to the numerical substructure without modelling approximations. At the same time, this analysis highlights the need for active research in order to address the mutual influence between the uncertainties introduced by the hybrid test control and those associated with the digital twin itself.

6 Impact on control

A key feature that distinguishes a digital twin from a simple model is the connection to its physical twin through an exchange of data in real time. The physical twin sends the data that the sensors have gathered to the digital twin, which in turn manipulates the data and sends back some scheduling signals. This arrangement can be investigated as a feedback control system, as shown in Figure 11. In general, the feedback loop wants to follow a reference r, which generates an error e that the controller manipulates to create a control signal u. The control signal is then used to drive the actuators that exert a control force x on the structure, which responds with an output y. Sensors

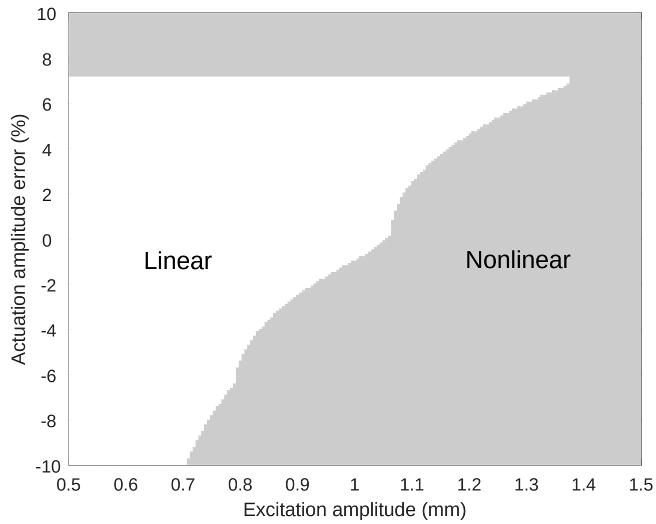

Figure 10: Identification of the threshold of nonlinear behaviour and dependence on the accuracy of the control and actuation system.

are then used to measure the output and observers can be used to estimate the other states of the system, which are then fed back to close the loop. In practice, as it is shown in 11, the actuator, the structure and the sensors are subject to potential faults, their model parameters may be uncertain and there can be disturbances that are not known beforehand, or that are not measured.

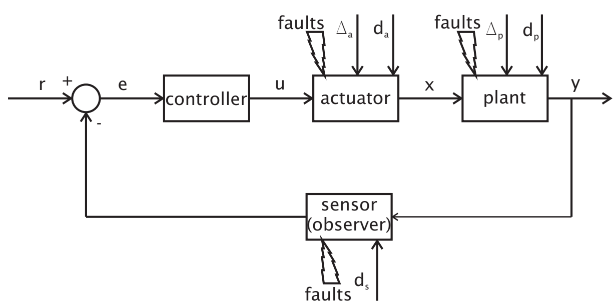

Figure 11: General block diagram of a feedback control system, in which the actuators, the structure and the sensors are affected by potential faults, parameter uncertainties Δ and un-modelled disturbances d.

All these uncertainties, or differences, between the digital and physical twin can have an impact on the performance, but also on the stability of the control loop. Changes in the dynamics of the structure can be addressed by the data-augmented modelling, as discussed in Section 4, which can then be used to update, or to redesign, the controller. Hence, the controller would need to adapt itself to the new plant dynamics. Although this could lead to a better performance, it remains necessary for the controller to be robust against any other uncertainties and to be tolerant of failures in each of the components of the control loop.

The objectives of the controller design, for this case study, are the disturbance rejection, which is given by the primary force F_s (see Figure 1), and the robustness of the controller against uncertainties. The metric used to judge the performance of a controller is the time-averaged kinetic energy of the three storey structure [12], which is defined as,

$$T = \lim_{t \to +\infty} \frac{1}{t} \int_0^t \frac{1}{2} \sum_{i=1}^{3} m_i \dot{y}_i^2(t') \mathrm{d}t'. \tag{4}$$

The control action is given by a secondary force F_a on the third floor of the three storey structure and it is assumed that the accelerations of each of the three storeys are measured. Different control architectures are investigated, one that is model based and one that is non-model based.

A direct velocity feedback (VFC) [13] on the third floor of the structure has been chosen as the non-model based control method. In this case, the velocity of the structure is measured at the control location and is fed back to the controller, which amplifies the signal and then applies an equal but opposite force on the structure at the same location, damping out the vibration. This control strategy does not need the knowledge of the plant dynamics, hence it is potentially robust against changes in the dynamic behaviour of the structure, however, it may give a non optimal performance since it reduces the velocity at a single location instead of the entire kinetic energy of the structure.

A linear quadratic regulator (LQR) [14] has been chosen as a full state control method that is model based. The LQR solves the linear quadratic Gaussian problem that minimises a cost function that is a trade off between the performance of the controller and the control effort required to achieve that performance. The control action, in this case, is still applied only on the third floor of the structure, but the matrix of feedback gains is calculated to achieve a global performance of vibration reduction, hence considering all the states of the system. This method can give a better performance than the previous one, however, if the dynamics of the structure changes during the operation, the control performance could degrade, or worse, the controller could become unstable.

Figure 12 shows a comparison among these control strategies for the data set one (bumper not in contact) in terms of kinetic energy for the same control effort requirements. The LQR strategy performs slightly better than the VFC in this scenario, giving a better reduction of the kinetic energy. If the plant dynamics changes, however, the LQR feedback gains are not designed to adapt to these changes and could lead to a worse than expected performance.

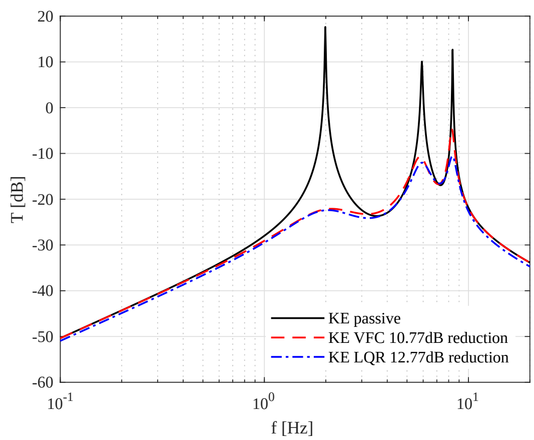

Figure 12: Time-averaged kinetic energy of the 3-storey building with and without a control action on the third floor. The control gains of the Velocity Feedback Controller and the Linear Quadratic Regulator are designed in terms of equal control effort. The kinetic energy reduction is calculated as the integral of the frequency dependent kinetic energy.

The design parameters of the LQR controller, are computed for the nominal system and usually stay the same throughout the asset operation even though the plant dynamics could change. Instead of using a LQR controller, one could use another model based control method, such as model predictive control (MPC). MPC would still be liable to the same problems faced by the LQR, however, the internal model of MPC can be made adaptive and it could be updated by the data-augmented model. Adaptive MPC can be implemented to achieve an optimal performance, however, its robustness against uncertainties and potential faults needs to be compared against the robustness and performance of the non-model based control methods, such as VFC.

7 Decisions in a digital twin

Decision-making is a key process required for digital twins. In order to remain a useful tool for the end-user, digital twins must continuously adapt so that they are representative of the physical twins. In the three-storey building structure case study, data-augmented modelling allows the digital twin to identify when a decision may be required as an inflation in the prediction variance is observed. Upon seeing this trigger caused by the previously unseen nonlinear condition, the digital twin should decide whether to perform one of the following actions:

- Update the model parameters.
- Learn an improved data-augmented model.
- Learn missing physics online from observed data.
- Advise to learn missing physics offline with hybrid testing.

Decision theory provides a formal framework for comparing courses of action [15]. In order to make a decision, one must evaluate the expected utilities associated with each action. In the context of the three storey building structure, each possible action has a different computational and financial cost whilst also offering varying improvements in predictive performance and thus value of information. In general, the value of information provided by the improvement in predictive performance is dependent on the utility gain achieved in the high-level decision-making for which the digital twin is implemented. Retaining the uncertainties associated with the models is important as it allows robust decisions to be made; if the uncertainty is high conservative actions will be preferred, however, with lower uncertainties cost-optimal decisions can be made.

Furthermore, the cost of poor predictive performance is significant if a model based control design is utilised. In this context, the digital twin is required to predictive accurately to avoid instabilities in the controller and sub-optimal control. As a result, the uncertainty associated with any prediction must be used in deciding how the model interacts with the controller, potentially leading to more robust control.

8 Conclusions

Developing a digital twin poses several challenges which require optimal decision making. This paper has explored the required decision making by a digital twin by investigating a three storey building structure. This case study has motivated different approaches to overcoming poor predictive performance, such as model updating, data-augmented a physics-based model and improvement of the model offline through hybrid testing. Due to the coupled nature of a digital twin and controller, any model based controller design with be significantly impacted by any poor predictive performance, unless mitigated for in the design stage.

Realising a digital twin will require each of these stages to be formally combined using decision theory, based on cost optimal decisions making that will improve predictive performance and controllability. This is a valuable area of further research, and will help develop a workflow required for developing more general digital twins.

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