Influencing the dynamics of correlated opinions in the voter model on multiplex networks

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Processes of opinion formation have found much attention in the recent literature on complex systems and networks [2]. Whereas many studies have considered the spread of opinions on a *single* network, recognition of, in particular, the multi-relational structure of online activities [6] has led to an increased interest in spreading processes on *multi-layer* networks, where ignoring the existence of different types of connections can have important consequences for our ability to model such systems [7, 5]. Here, building on previous work on opinion propagation in the voting dynamics [4, 8] we are interested in maximizing influence on a multi-layer network. Different to previous work [9], we study influence maximization for multiple correlated opinions, where opinions about different issues spread on different layers of a multiplex network. As an example, consider how people discuss different topics with their friends in offline and online social networks. Yet, holding a particular opinion about one topic may change the individuals' susceptibility to adopt opinions about other topics, thus effectively coupling different network layers through opinion correlations. Studying influence maximization in such settings allows for a gradual control of the population, as has been observed in real-world scenarios [3]. For instance, we can first prime a population by spreading a particular attitude, which then allows for an easier propagation of our desired opinion. In this paper we formalize a model of such correlated opinion dynamics on multiplex networks, develop an algorithm for optimizing influence on controlled opinions, and provide analysis of different scenarios of optimal control in the presence of an adversary.

Model description. We propose a generalisation of the voting dynamics [1] to multiplex networks (see Figure 1 - left). More specifically, consider a network with two layers. Suppose in layer $\alpha=0$ each agent $i\in N$ decides between opinions $s_i^0 \in \{A,B\}$, and in layer $\alpha=1$ between $s_i^1 \in \{a,b\}$. After initialising the states s_i^α at random, consider the following update step. First, randomly pick a layer α and an agent i. Second, randomly pick an incoming edge $j\to i$. Such edge may come either from a neighbouring agent j or from an external influencer. These influencers have in-strength p_A^i, p_B^i on layer $\alpha=0$ and p_a^i, p_b^i on layer $\alpha=1$. Furthermore, their state is fixed to their index. If $s_i^\alpha \neq s_j^\alpha$, agent i will change her opinion to s_j^α with probability $p(s_i^\alpha \to s_j^\alpha | s_i^{1-\alpha})$. These probabilities encode the correlation between the layers. For instance, if we have $p(s_i^0 \to s_j^0 | s_i^1)$ but $p(s_i^1 \to s_j^1 \perp s_i^0)$ then layer $\alpha=0$ depends on layer $\alpha=1$, but the latter evolves independently. At the same time, the probabilities can be biased, e.g. $p(A\to B|a) >> p(B\to A|a)$, which can either help forming a consensus or induce a cycle between the opinion states (see Figure 1 - center). Given the probabilities and the network structure, we can optimise the external influences p_A^i, p_a^i against a fixed choice of p_B^i, p_b^i by an adversary. An example for the well-mixed case is in Figure 1 - right. In our talk we will present similar results for more realistic network structures.

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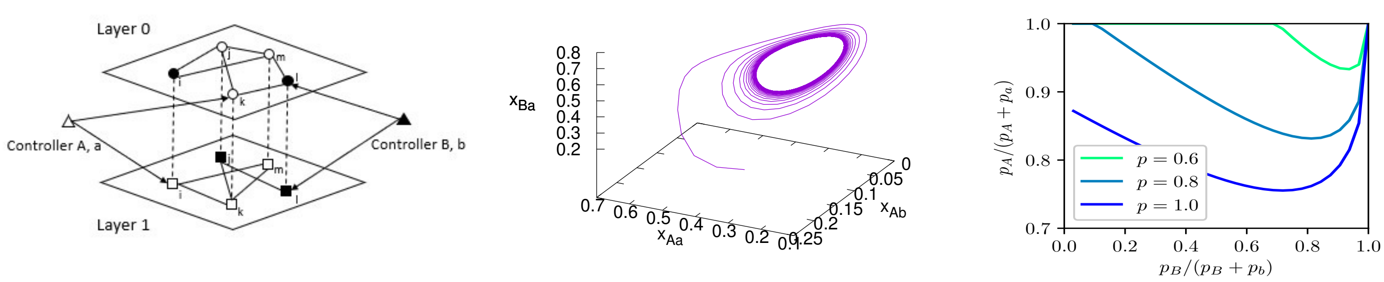

FIG. 1. Left: a graphical illustration of our multiplex model with correlated opinions. Center: an example of a limit cycle found in the well-mixed case for $p(A\to B|a) = .24$, $p(A\to B|b) = .05$, $p(B\to A|a) = .07$, $p(B\to A|b) = .90$, $p(a\to b|A) = .06$, $p(a\to b|B) = .67$, $p(b\to a|A) = .97$, and $p(b\to a|B) = .25$. The axes x_{Aa}, x_{Ab}, x_{Ba} represent the concentration of the respective opinion state and $x_{Bb} = 1 - x_{Aa} + x_{Ab} + x_{Ba}$. Right: optimal allocation of resources across the two layers in the well-mixed case, where the influencers' budgets $p_A + p_a$ and $p_B + p_b$ are the same, and the objective is maximising opinion A, i.e. $x_A = x_{Aa} + x_{Ab}$. Note that the switching probabilities are $p(s_i^\alpha \to s_j^\alpha | s_i^{1-\alpha}) = .5$ except for $p(B\to A|a) = p$, thus there is an advantage in targeting both layers in most cases.