Hedging demand in long-term asset allocation with an application to carry trade strategies

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Abstract

We derive a closed-form expression for the mean and marginal hedging demand on risky assets in long-term asset allocation problems for individuals with CRRA preferences. Our parametric portfolio policy rule accommodates an arbitrarily large number of state variables for predicting the state of nature, and number of assets in the portfolio. The closed-form expression for the hedging demand is exact under polynomial specifications of the portfolio policy rule and a suitable approximation for unknown smooth parametric portfolio policy rules using Taylor expansions. The hedging demand on risky assets depends positively on the predictability of the risky asset and the persistence of the predictors, and negatively on the degree of investor's relative risk aversion. We illustrate these insights empirically for a basket of currencies by showing the outperformance of rebalancing carry trade strategies over different investment horizons against a short-term (myopic) portfolio.

Keywords: currency carry trade; hedging demand; parametric portfolio policy rules; strategic asset allocation; state variables.

JEL Codes: G11, G15, F31.

Introduction

The choice of an optimal portfolio of assets is a classic problem in financial economics. In a single-period setting the problem is well understood, and analytical solutions for optimal portfolio weights are available in important special cases. In a multiperiod setting the problem is far less tractable. Closed-form solutions for portfolio weights are available in the special cases where investment opportunities are constant or the investor has log utility and hence acts myopically. Interest in long-horizon portfolio choice has recently been stimulated by empirical evidence questioning the validity of the conditions under which the multiperiod problem reduces to a single-period problem.

Expected asset returns vary through time implying that investment opportunities are not constant. The evidence for predictable variation in the equity premium is particularly strong, see early contributions by Campbell and Shiller (1988), Fama and French (1988) and Hodrick (1992), among many others. Predictability of currency markets has also been explored in different contexts, see Fama (1984). In this context, the seminal contributions of Merton (1969, 1971) and Samuelson (1969) have shown that time-varying investment opportunities can have important effects on optimal portfolios for investors with long horizons. More specifically, these authors theorize that if investment opportunities are time varying the portfolio choice of a multiperiod investor can differ from that of a single-period investor because of hedging demands. Investors try to hedge against predictable changes in future investment opportunities. Also, when investment opportunities are time varying investors follow decision rules that map the state of nature, characterized by observable forecasting variables, into the portfolio choice in that state.

In response to these insights, the literature on financial economics has solved the optimal asset allocation problem under different investment horizons using models with realistic assumptions on the predictability of returns. Kandel and Stambaugh (1996), for example, show that weak predictive regressions yield economically significant variations in the portfolio choice of a single-period investor. Balduzzi and Lynch (1997) and Barberis (2000) estimate the multiperiod portfolio choice corresponding to standard predictive regressions and find that multiperiod decisions differ substantially from single-period decisions. Campbell and Viceira (1999) confirm this result by using a vector autoregressive specification and calibrating an approximation of the portfolio and consumption choice of an infinitely-lived investor. Brennan, Schwartz, and Lagnado (1997) obtain a similar conclusion for the long-term asset allocation problem of a continuously rebalancing investor. An interesting alternative initiated by Brandt (1999) and Aït-Sahalia and Brandt (2001) is to directly estimate the portfolio choice from the data. These authors develop nonparametric and semiparametric methods for recovering single-period and multiperiod decision rules based on the maximization of the investor's objective function over different finite investment horizons. More recently, Brandt and Santa-Clara (2006) and Brandt, Santa-Clara and Valkanov (2009) impose specific functional forms to the portfolio policy rule that depend on a set of state variables. These authors obtain consistent estimates of the optimal decision rules applying GMM estimation procedures, see Hansen (1982) and Hansen and Singleton (1982), to the conditional Euler equations obtained from single-period and multiperiod investment problems.

The objective of the current paper is to study the hedging demand on the risky asset in long-term asset allocation problems. We interpret the hedging demand as the difference in the allocation to the risky asset between a multiperiod investment horizon and a single-period investment horizon. We consider two different alternatives for modelling the investors' objective function in a portfolio rebalancing context. In the main text, we consider a multiperiod utility function that is additively time separable and characterized by constant relative risk aversion (CRRA) period utility functions. In this setting a risk-averse investor aims to maximize the expected utility of the present value of the stream of future payoffs of the portfolio over the investment horizon. This objective function shares the functional specification proposed in Merton (1973), however, in our context, investors obtain utility from intermediate future wealth and not from intermediate future consumption[1]. In the appendix, we consider a simpler framework characterized by an individual that maximizes the conditional expected utility of terminal wealth without considering wealth in the intermediate periods.

The second ingredient that allows us to obtain an analytical expression for the hedging demand is the assumption that the individual's portfolio policy rule in each period is parametric and depends polynomially on a set of state variables. This approach to modelling investors' optimal portfolio decision in dynamic settings was initiated by Aït-Sahalia and Brandt (2001), Brandt and Santa-Clara (2006) and Brandt, Santa-Clara and Valkanov (2009), as seminal examples. In particular, we follow Aït-Sahalia and Brandt (2001) and map the predicted state of nature into a set of state variables capturing uncertainty in each period. However, in contrast to these authors, our portfolio policy rule is fully parametric, that is, we propose a parametric specification for the relationship between the optimal weight function and the state of nature predicted by the linear combination of state variables. In order to keep the investment problem analytically tractable, we make several simplifying assumptions that can be also found in Campbell and Viceira (1999). We assume that there are two assets: a riskless asset and a risky asset, the return on the risky asset is predictable in the short-term by a set of state variables that follow a mean-reverting AR(1) process, and the portfolio is rebalanced each period. Both assumptions can be relaxed at the expense of more involved algebra to entertain an arbitrary number of assets in the portfolio and higher persistence in the state variables.

Our main contribution is to derive a closed-form expression for the mean hedging demand and marginal hedging demand on risky assets in long-term asset allocation problems. The mean hedging demand is defined as the average difference between the allocation to the risky

[1]Similar objective functions considering wealth in the individuals' objective function are Gârleanu and Pedersen (2013) and DeMiguel, Martín-Utrera and Nogales (2015).

asset in a long-term asset allocation problem and the corresponding short-term asset allocation. The marginal hedging demand with respect to a state variable measures the difference in the sensitivity of the risky asset to such state variable between the long-term and short-term optimal asset allocations. To obtain analytical expressions for these different characterizations of the hedging demand on a risky asset we apply a multivariate mean-value theorem that exploits the relationship between the short-term Euler equation characterizing the one-period optimal asset allocation problem and a multiperiod Euler equation that characterizes the long-term optimal asset allocation problem. We show that both types of hedging demand are positively related to the exposure of the risky asset to the state variables and the persistence of the state variables, and negatively related to the degree of investor's relative risk aversion. Thus, for very risk-averse individuals, the long-term and short-term optimal allocations to the risky asset are very similar entailing a hedging demand close to zero.

It is worth noting that our closed-form expression for the hedging demand is exact when the state of nature is driven by a linear combination of state variables and the portfolio policy rule is a polynomial expression. Otherwise, our closed-form expression is an approximation. Nevertheless, under mild assumptions, we can use a Taylor rule to approximate an unknown nonlinear portfolio policy function by a high-order polynomial. Similarly, we can approximate a nonlinear function predicting the state of nature by a polynomial of order greater than one. Combining both polynomial expressions we show that our method provides a closed-form expression that can be arbitrarily close to the actual unknown hedging demand obtained under nonlinear specifications of the portfolio policy rule and state of nature.

The main advantage of this approach for solving the multiperiod asset allocation problem and deriving the hedging demand is that there is no need to impose any structure on the dynamics of asset returns and accommodates an arbitrarily large set of state variables and risky assets in the investment portfolio. The model is based on the assumption that investors exhibit

CRRA preferences over a mutiperiod investment horizon. This assumption is standard in the related literature and facilitates the construction of closed-form solutions to the long-term asset allocation problem. Thus, under CRRA preferences, Wachter (2002) and Munk and Sorensen (2004) have obtained closed-form solutions to the consumption and portfolio allocation problem in a continuous time setting. Wachter (2002) solves the optimal portfolio choice problem for an investor with utility over consumption under mean-reverting returns and the assumption that markets are complete. Munk and Sorensen (2004) also work in a continuous-time dynamically complete markets but study, instead, the optimal hedging demand on risky assets for changes in the term structure of interest rates.

We apply these theoretical insights on the hedging demand to study the differences in the optimal allocation to a carry trade strategy between a long-term rebalancing portfolio and the corresponding short-term portfolio. The asset allocation problem involving currency carry trade strategies fits very well our investment problem as there is empirical evidence on the predictability of carry trade returns when the uncovered interest parity (UIP) condition fails, see Fama (1984). In the empirical exercise we consider a portfolio comprised by the three-month U.S. Treasury bill rate acting as risk-free asset and a risky asset given by a basket of currencies of the G10 group reported by Bloomberg. The set of state variables that are used to predict the state of nature and model the dynamics of the portfolio weights are the level and three-month change of the volatility of G10 exchange rates, denoted as as the currency volatility level and currency volatility momentum, the level and three-month change of the U.S. Ted spread, denoted as the U.S. Ted spread level, and U.S. Ted spread momentum, and the three-month change of the Commodity Research Bureau (CRB) Industrial Index. The choice of state variables is motivated by empirical work in Menkhoff et al. (2012) and Bakshi and Panayotov (2013).

Our empirical study uncovers the following findings. First, the mean hedging demand is, in general, positive and increasing on the investment horizon, suggesting that long-term currency carry trades invest more aggressively on the carry trade strategy than myopic strategies. Second, the sign of the marginal hedging demand to these state variables is consistent with the sign of the exposure of the risky asset to the state variables obtained from linear VAR specifications. More specifically, the marginal hedging demand on the U.S. Ted Spread and volatility momentum are negative and the marginal hedging demand on the CRB industrial return is positive. The magnitude of the hedging demand increases with the investment horizon. Third, optimal portfolio allocations constructed with single state variables lead to more aggressive investment into the risky asset than portfolio rules characterized by a vector of state variables. This is due to the presence of cross-correlation between the state variables that have a negative effect on the overall mean hedging demand and the marginal hedging demand to the state variables. Finally, in in-sample portfolio and out-of-sample performance comparisons, we show the outperformance of the long-term investment strategy compared to the short-term strategy using economic and statistical performance measures. Similar results are also found in an unreported exercise - available from the authors upon request - using a currency portfolio obtained from Lustig, Roussanov and Verdelhan (2011) that contains currency prices from 15 developed countries.

Our empirical application is related to recent literature exploring the ability of state variables to predict the risk premium on carry trade returns. Barroso and Santa-Clara (2015), for example, show how to construct optimal currency portfolios based on interest rate variables, momentum, long-term reversal, the current account and the real exchange rate. Bakshi and Panayotov (2013) study the relationship between different variables and the payoffs of rebalanced carry trades using monthly predictive regressions at horizons up to six months. These authors find that a measure of currency volatility is negatively related to the expected currency carry trade returns, while measures of market liquidity and the return on the CRB Industrial

Index are positively related to expected currency carry trade returns. Cenedese, Sarno and Tsiakas (2014) find that high market variance, measured as the variance of the returns to the foreign exchange market portfolio, is negatively related to large future carry trade losses. Lu and Jacobsen (2016) find that equity returns predict carry trade profits obtained from shorting low interest rate currencies, and commodity prices predict profits obtained from longing high interest rate currencies.

The rest of the paper is structured as follows. Section 1 derives a closed-form expression for the hedging demand on the risky asset over different investment horizons. This is done by obtaining analytically the difference between the parameters driving the optimal portfolio allocation for an individual with a one-period investment horizon and an individual with a long-term investment horizon. Section 2 discusses GMM estimation and statistical inference for the parameters characterizing the optimal parametric portfolio rule. Section 3 presents an empirical application comparing the investment strategy of a myopic investor maximizing one-period utility against the investment strategy of an individual maximizing a multiperiod utility function. Section 4 concludes. A mathematical appendix discusses the optimal portfolio allocation when the investor's objective function is defined over terminal wealth. Tables and figures are collected at the end of the paper.

1 Hedging demand in long-term asset allocation

This section derives the main result of the study, namely, a closed-form expression for the hedging demand on a risky asset. To do this, we study the first order conditions of the asset allocation problem for individuals with short-term and long-term investment horizons. We introduce first the assumptions of the model.

1.1 Return predictability and parametric portfolio policy rule

Investors begin life with an exogenous endowment $W_0 \geq 0$. At the beginning of period $t+1$ investors receive income from allocating the wealth accumulated at time t in an investment portfolio offering a return R^p_{t+1}. For simplicity, we consider a portfolio given by a risk-free asset with return R^f_t and a risky asset with excess return given by R_{t+1}; however, the results below can be easily extended to more than one risky asset. The portfolio return is

$$R^p_{t+1}(\alpha_t) = R^f_t + \alpha_t(z_t;\theta)R_{t+1}, \tag{1}$$

where $\alpha_t(z_t;\theta)$ is the optimal allocation to the risky asset. In the spirit of Brandt and Santa-Clara (2006) and Brandt, Santa-Clara and Valkanov (2009), we propose a parametric specification of the portfolio policy rule:

$$\alpha_t(z_t;\theta_k) = f(s_t(z_t;\nu);\widetilde{\nu}_k), \tag{2}$$

with $f(s;\widetilde{\nu}_k)$ reflecting the parametric relationship between the portfolio policy decision α_t and the state of nature $s = s_t(z_t;\nu)$. The parameter vector $\widetilde{\nu}_k$ describes the relationship between the state of nature s and the optimal portfolio allocation and the subscript k reflects the fact that the portfolio policy rule is horizon-specific, that is, the optimal portfolio decision may depend differently on the realized state s under different investment horizons. Following Aït-Sahalia and Brandt (2001), we accommodate a potentially large number of predictors $z_t = (z_{1t},\ldots,z_{nt})^\top$ such that the state of nature at each point in time t can be predicted by a linear combination of the state variables z_t, namely, $s_t(z_t;\nu) = \nu' z_t$, with $\nu = (\nu_1,\ldots,\nu_n)^\top$ a vector that measures the sensitivity of changes in the investment opportunity set to the realization of the predictors. This vector does not depend on the investment horizon. The vector θ_k denotes the combination of parameters ν and $\widetilde{\nu}_k$.

As an illustrative example, let us consider the following polynomial specification of second order $f(s;\widetilde{\nu}_k) = \widetilde{\nu}_{0k} + \widetilde{\nu}_{1k}s + \widetilde{\nu}_{2k}s^2$, with $s = \nu_1 z_1 + \nu_2 z_2$. In this case simple algebra shows that the portfolio policy rule $\alpha_t(z_t;\theta_k)$ is

$$\alpha_t(z_t;\theta_k) = \theta_{0k} + \theta_{1k}z_{1t} + \theta_{2k}z_{2t} + \theta_{1k}^{(2)}z_{1t}^2 + \theta_{2k}^{(2)}z_{2t}^2 + \theta_{12,k}z_{1t}z_{2t}, \tag{3}$$

with $\theta_k = (\theta_{0k}, \theta_{1k}, \theta_{2k}, \theta_{1k}^{(2)}, \theta_{2k}^{(2)}, \theta_{12,k})$, where $\theta_{0k} = \widetilde{\nu}_{0k}$, $\theta_{lk} = \widetilde{\nu}_{1k}\nu_l$, $\theta_{lk}^{(2)} = \widetilde{\nu}_{2k}\nu_l^2$, and $\theta_{12,k} = \widetilde{\nu}_{2k}\nu_1\nu_2$, where $l = 1, 2$.

The state variables are determined by their ability to predict changes in the investment opportunity set. Thus, we assume that $E[R_{t+1} \mid \Im_t] = \eta_{01} + \eta_{11}z_{1t} + \ldots + \eta_{n1}z_{nt}$, with $E[\cdot \mid \Im_t]$ the conditional expectation with respect to the individual's information set $\Im_t = \{z_{1t}, \ldots, z_{nt}\}$. We will use $E_t[\cdot]$ hereafter to denote the conditional expectation. The investment setting is completed by defining the dynamics of the state variables. To do this, we assume the following family of autoregressive processes of order one: $z_{l,t+1} = \mu_l + \phi_l(z_{lt} - \mu_l) + v_{l,t+1}$, with $l = 1, \ldots, n$, and μ_l and $|\phi_l| < 1$ the parameters of the AR(1) model; $v_{l,t+1}$ is a zero-mean error term that is serially uncorrelated and also cross-sectionally uncorrelated with all of the state variables. We will consider a demeaned set of state variables as in Brandt, Santa-Clara and Valkanov (2009). Abusing of notation, we have

$$z_{l,t+1} = \phi_l z_{lt} + v_{l,t+1}, \; l = 1, \ldots, n, \tag{4}$$

where z_t denotes now a set of zero-mean state variables.

1.2 Optimal myopic asset allocation

Let us consider first the portfolio choice of an investor who maximizes the one-period-ahead expected utility of wealth (W_t). Individuals' preferences exhibit constant relative risk aversion (CRRA) and are modelled by a isoelastic utility function. In this context, an investor maximizes

$$E_t\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right], \tag{5}$$

with $\gamma > 0$, $\gamma \neq 1$, denoting the relative risk aversion coefficient. The coefficient γ captures risk aversion, the reluctance to trade wealth for a fair gamble over wealth today. The standard myopic maximization problem is completed by defining the accumulation equation determining the buildup of wealth by the investor over time:

$$W_{t+1} = (1 + R_{t+1}^p(\alpha_t(z_t;\theta_1))W_t. \tag{6}$$

The decision rule of the myopic investor is determined by the vector of parameters $\theta_1^* = (\theta_{01}^*, \theta_{11}^*, \ldots, \theta_{\widetilde{n}1}^*)^\top$. More formally, the Euler equation characterizing the optimal value of the parameter vector θ_1 is

$$E_t\left[\psi_{l,t+1}(\theta_1^*)\right] = 0, \tag{7}$$

with $\psi_{l,t+1}(\theta_1^*) = \alpha_{lt}(z_t;\theta_1^*)R_{t+1}(1+R_{t+1}^p(\alpha_t(z_t;\theta_1^*))^{-\gamma}$, where $\alpha_{lt}(z_t;\theta_1^*)$ denotes the first derivative of the portfolio policy rule α_t with respect to θ_{l1} for $l = 0, 1\ldots, \widetilde{n}$. To simplify and unify notation with the multiperiod horizon case, let $\widetilde{R}_{t+1}(\theta) := 1 + R_{t+1}^p(\alpha_t(z_t;\theta))$ denote the portfolio gross return, and we write the function $\psi_{l,t+1}(\theta_1^*)$ as $\psi_{l,t+1}(\theta_1^*) = \frac{\alpha_{lt}(z_t;\theta_1^*)R_{t+1}}{\widetilde{R}_{t+1}}\widetilde{R}_{t+1}^{1-\gamma}(\theta_1^*)$.

As an aside comment, we should note that for the myopic case the assumption that the conditioning information set contains all the information available to the individual implies that the function $\alpha_{lt}(z_t, \theta_1^*)$ is known at time t. In this case expression (7) is equal to $E_t\left[R_{t+1}\widetilde{R}_{t+1}^{-\gamma}(\theta_1^*)\right] =$

0. This set of conditional Euler equations is the same for the $\widetilde{n}$ parameters defining the vector θ_1^*. Identification of model parameters is achieved through an additional set of restrictions obtained from the conditioning information set, see Hansen (1982) and Hansen and Singleton (1982) for identification of the model parameters through moment conditions. This is discussed in Section 2.

1.3 Hedging demand in rebalancing portfolios

In this section we investigate the portfolio choice of an investor who maximizes the expected utility of wealth over $K \leq \infty$ periods. Assume that the multiperiod utility function is additively time separable and exhibits constant relative risk aversion over wealth in each period. The choice of a CRRA specification is instrumental for deriving a closed-form expression for the hedging demand[2]. In this context, an investor maximizes

$$\sum_{j=1}^{K} \beta^{j-1} E_t \left[\frac{W_{t+j}^{1-\gamma}}{1-\gamma} \right]. \tag{8}$$

The discount factor β measures patience, the willingness to give up wealth today for wealth tomorrow. In this setting the investment portfolio (1) is rebalanced each period and the optimal portfolio allocation is defined by (2) evaluated at each period between t and T. An alternative investment exercise is to replace the objective function (8) by $E_t \left[\frac{W_T^{1-\gamma}}{1-\gamma} \right]$. The appendix illustrates the optimal portfolio allocation exercise in this case.

Let $\widetilde{R}_{t+1:t+j}(\theta_K) := \prod_{i=1}^{j} \widetilde{R}_{t+i}(\theta_K)$ denote the multiperiod gross return on the portfolio. In this multiperiod setting investors' wealth evolves over time as $W_{t+j} = \widetilde{R}_{t+1:t+j}(\theta_K) W_t$, such that the

[2]The methodology developed in this paper can be extended under some further algebra to HARA utility functions of the type $U(W) = \frac{\gamma}{1-\gamma} \left(\frac{a}{\gamma} W \right)^{1-\gamma}$. Extensions to more general HARA functions of the form $U(W) = \frac{\gamma}{1-\gamma} \left(\frac{a}{\gamma} W + b \right)^{1-\gamma}$, with $b \neq 0$, render the mathematical problem in discrete time intractable.

investor's objective function (8) becomes

$$\max_{\{\theta_K \in \Theta\}} \left\{ \sum_{j=1}^{K} \beta^{j-1} \frac{W_t^{1-\gamma}}{1-\gamma} E_t \left[\widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K) \right] \right\},$$

with Θ the parameter set. Simple algebra shows that the first order conditions of this optimization problem with respect to the parameter vector θ_K are

$$E_t \left[\phi_{l,t+K}(\theta_K^*) \right] = 0, \tag{9}$$

with $\phi_{l,t+K}(\theta_K^*) = \sum_{j=1}^{K} \beta^{j-1} \widetilde{\psi}_{l,t+j}(\theta_K^*)$, where $\widetilde{\psi}_{l,t+j}(\theta_K^*) = \left(\sum_{i=1}^{j} \frac{\alpha_{l,t+i-1}(z_{t+i-1};\theta_K^*) R_{t+i}}{\widetilde{R}_{t+i}(\theta_K^*)} \right) \widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K^*)$. For simplicity in the notation we will drop hereafter the stars from the optimal allocations θ_1^* and θ_K^* in (7) and (9), and assume that the parameter vectors θ_1 and θ_K denote the solutions to the first order conditions of the maximization problems (5) and (8), respectively. We can operate with the above expression to obtain $\widetilde{\psi}_{l,t+j}(\theta_K)$, for all $j = 1, \ldots, K$, as a function of $\psi_{l,t+1}(\theta_K)$. To be able to do this, we need the following assumption:

Assumption A.1: The functional specification $f(s; \widetilde{\nu}_k)$ is the same across different investment horizons k.

Assumption A.1 implies that differences in portfolio policy rules across investment horizons are only driven by the parameter vector $\widetilde{\nu}_k$, however, the function $f(s; \widetilde{\nu}_k)$ linking the optimal weight function α_t and the predicted state s is the same across investment horizons. Similar assumptions are found in Brandt and Santa-Clara (2006) and Brandt, Santa-Clara and Valkanov (2009) for multiperiod asset allocation problems.

Under assumption A.1, it follows that $\widetilde{\psi}_{l,t+1}(\theta) = \psi_{l,t+1}(\theta)$, with $\psi_{l,t+1}(\theta)$ defined in (7) and

$\theta \in \Theta$, such that, for $j > 1$, the components of $\phi_{l,t+K}(\theta_K)$ in (9) can be decomposed as

$$\widetilde{\psi}_{l,t+j}(\theta_K) = \psi_{l,t+1}(\theta_K)\widetilde{R}^{1-\gamma}_{t+2:t+j}(\theta_K) + \sum_{i=2}^{j} \frac{\alpha_{l,t+i-1}(z_{t+i-1};\theta_K)R_{t+i}}{\widetilde{R}_{t+i}(\theta_K)} \widetilde{R}^{1-\gamma}_{t+1:t+j}(\theta_K). \tag{10}$$

To simplify notation, we define $\widetilde{R}_{t+2:t+1} = 1$ such that the function $\phi_{l,t+K}(\theta_K)$ is

$$\phi_{l,t+K}(\theta_K) = \psi_{l,t+1}(\theta_K)\sum_{j=1}^{K}\beta^{j-1}\widetilde{R}^{1-\gamma}_{t+2:t+j}(\theta_K) + \sum_{j=2}^{K}\beta^{j-1}\widetilde{R}^{1-\gamma}_{t+1:t+j}(\theta_K)\sum_{i=2}^{j}\frac{\alpha_{l,t+i-1}R_{t+i}}{\widetilde{R}_{t+i}(\theta_K)}.$$

Taking conditional expectations, the multiperiod Euler equation (9) can be written as

$$E_t[\phi_{l,t+K}(\theta_K)] = E_t[\psi_{l,t+1}(\theta_K)]\sum_{j=1}^{K}\beta^{j-1}E_t[\widetilde{R}^{1-\gamma}_{t+2:t+j}(\theta_K)]+ \tag{11}$$

$$\underbrace{\sum_{j=1}^{K}\beta^{j-1}Cov_t[\psi_{l,t+1}(\theta_K), \widetilde{R}^{1-\gamma}_{t+2:t+j}(\theta_K)]}_{d_{1lt}(K)} + \underbrace{\sum_{j=2}^{K}\beta^{j-1}\sum_{i=2}^{j}E_t[\alpha_{l,t+i-1}R_{t+i}]E_t[\widetilde{R}^{1-\gamma}_{t+1:t+j}(\theta_K)\widetilde{R}^{-1}_{t+i}(\theta_K)]+}_{d_{2lt}(K)}$$

$$\underbrace{\sum_{j=2}^{K}\beta^{j-1}\sum_{i=2}^{j}Cov_t[\alpha_{l,t+i-1}R_{t+i}, \widetilde{R}^{1-\gamma}_{t+1:t+j}(\theta_K)\widetilde{R}^{-1}_{t+i}(\theta_K)]}_{d_{3lt}(K)} = 0.$$

The quantity $d_{1t}(K)$ captures the linear dependence between the function defining the first order conditions of the myopic investor and a measure of the CRRA utility provided by the gross returns over the long-term investment horizon; $d_{2t}(K)$ is the main quantity in the analysis of long-term asset allocation. This quantity reflects the contribution of the forecasting variables z_t to the long-term portfolio and, in particular, to the hedging demand on the risky asset. The quantity $d_{3t}(K)$ captures the linear dependence between $\alpha_{l,t+i-1}R_{t+i}$ and a weighted measure of the CRRA utility obtained from the gross return over different investment horizons.

The above expression shows that the multiperiod Euler equation (9) can be written in terms of the Euler equation (7) that defines the short-term asset allocation problem. More formally,

Theorem 1. The multiperiod Euler equation (9) obtained from the maximization problem (8) can be written as

$$E_t[\psi_{t+1}(\theta_K)] = c_t(K), \tag{12}$$

with $\psi_{t+1}(\theta) = (\psi_{0,t+1}(\theta), \psi_{1,t+1}(\theta), \ldots, \psi_{\widetilde{n},t+1}(\theta))^\top$ and $c_t(K) = (c_{0t}(K), c_{1t}(K), \ldots, c_{\widetilde{n}t}(K))^\top$, where $c_{lt}(K) = -\frac{d_{1lt}(K)+d_{2lt}(K)+d_{3lt}(K)}{\sum_{j=1}^{K} \beta^{j-1} E_t[\widetilde{R}^{1-\gamma}_{t+2:t+j}(\theta_K)]}$ for $l = 0, 1, \ldots, \widetilde{n}$.

Proof: The proof of this result follows from rearranging the terms in expression (11).

The quantity $c_t(K)$ can be interpreted as a distance to the short-term Euler equation characterized by $c_t(K) = 0$, and provides an intuitive measure of the contribution of the long-term portfolio to the optimality conditions determining the optimal asset allocation.

1.4 The hedging demand

Hedging demand is the demand for risky assets in a portfolio beyond the demand of a short-term myopic investor. It arises from the need of risk-averse individuals with long-term investment horizons to hedge against variation in expected returns over time. For simplicity, we consider hereafter a linear portfolio policy rule for studying analytically the hedging demand on a risky asset. Linearity of $f(s; \widetilde{\nu}_K)$ simplifies the comparison of the Euler equations (7) and (12). This is so because in the linear case the first derivative of the optimal weight function $\alpha_t(z_t; \theta)$ with respect to θ only depends on the vector of state variables z_t regardless the investment horizon. More formally, $f(s; \widetilde{\nu}_k) = \widetilde{\nu}_{0k} + \widetilde{\nu}_{1k} s$, with $s = \nu' z_t$, and the corresponding portfolio policy rule is

$$\alpha_t(z_t; \theta_k) = \theta_k' x_t, \tag{13}$$

with $x_t = (1\ z_t^\top)^\top$ and $\theta_k = (\theta_{0k}, \theta_{1k}, \ldots, \theta_{\widetilde{n}k})$, where $\theta_{0k} = \widetilde{\nu}_{0k}$ and $\theta_{lk} = \widetilde{\nu}_{1k}\nu_l$ for $l = 1, \ldots, \widetilde{n}$, with $\widetilde{n} = n$ in this case. Nevertheless, when possible, we will also derive expressions for the hedging demand for higher orders of the portfolio policy function $f(s; \widetilde{\nu}_k)$. In fact, it is worth noting that, under mild assumptions, we can use a Taylor rule to approximate an unknown nonlinear portfolio policy function by a high-order polynomial around the predicted state of nature s. Similarly, we can approximate a nonlinear function predicting s by a polynomial of order greater than one. In this scenario, it is not difficult to see that the relevant weight function is $\alpha_t(z_t; \theta_k) = \theta_k' \widetilde{x}_t$, with $\widetilde{x}_t = (1\ z_t^\top\ \ldots (z_t^m)^\top)^\top$ and m denoting a polynomial of high order. The vector $\widetilde{x}_t$ also contains cross-products of the different state variables at higher orders $z_{it}^r z_{jt}^s$, with $r + s \leq m$.

Corollary 1. Under the linear portfolio policy (13), expression $c_t(K)$ in equation (12) simplifies such that

$$d_{2lt}(K) = \sum_{j=2}^{K} \beta^{j-1} \sum_{i=2}^{j} E_t[z_{l,t+i-1} R_{t+i}] E_t[\widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K) \widetilde{R}_{t+i}^{-1}(\theta_K)], \tag{14}$$

and

$$d_{3lt}(K) = \sum_{j=2}^{K} \beta^{j-1} \sum_{i=2}^{j} Cov_t[z_{l,t+i-1} R_{t+i}, \widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K) \widetilde{R}_{t+i}^{-1}(\theta_K)].$$

Proof: The proof of this result is obtained by noting that $\alpha_{l,t}(z_t; \theta_1) = z_{lt}$ in the linear case given by the portfolio policy (13).

We derive now a closed-form expression for the mean hedging demand and the marginal hedging demand on the risky asset with respect to the set of state variables. To do this, we formally introduce these concepts.

Definition. The mean hedging demand is defined as the average difference between the allocation to the risky asset in a long-term asset allocation problem and the corresponding short-term asset allocation: $E[\alpha_t(z_t;\theta_K) - \alpha_t(z_t;\theta_1)]$. The marginal hedging demand measures the difference in the sensitivity of the risky asset to a state variable between the long-term and short-term optimal asset allocations. More formally, let $\theta_{l,1K} := \theta_{lK} - \theta_{l1}$ denote the marginal hedging demand to the state variable z_{lt}, with $l = 1, \ldots, \widetilde{n}$.

With these definitions in place, we can derive the following main results.

Lemma 1. The mean hedging demand for the linear portfolio policy rule (13) is fully captured by the parameter $\theta_{0,1K} := \theta_{0K} - \theta_{01}$ if $E[z_t] = 0$.

Proof: The proof of this result is obtained by noting that $E[\alpha_t(z_t, \theta_K)] = \theta_{0K}$ and $E[\alpha_t(z_t, \theta_1)] = \theta_{01}$ for portfolio policy (13) defined under different investment horizons.

For polynomial portfolio policy rules of higher order, the expression for the mean hedging demand also involves other parameters. More specifically, for a quadratic specification of the policy rule, as in (3), the mean hedging demand is $\theta_{0K} - \theta_{01} + \theta_{1K}^{(2)} - \theta_{11}^{(2)} + \theta_{2K}^{(2)} - \theta_{21}^{(2)} + (\theta_{12,K} - \theta_{12,1})\rho_{12}$, with $V(z_t) = 1$ and $\rho_{12} = E[z_{1t}z_{2t}]$. This expression only depends on the parameters of the weight functions $\alpha_t(z_t;\theta_1)$ and $\alpha_t(z_t;\theta_K)$ and can be consistently estimated, as shown in the next section.

Theorem 2. Under the linear portfolio policy (13) evaluated at different investment horizons $k = 1$ and $k = K$, and with z_t a set of zero-mean state variables, the vector of hedging demands $\theta_{1K} = (\theta_{0,1K}, \ldots, \theta_{n,1K})^\top$ on the risky asset is

$$\theta_{1K} = -\frac{\Omega_t^{-1} c_t(K)}{\gamma E_t\left[R_{t+1}^2 \widetilde{R}_{t+1}^{-\gamma-1}(\bar{\theta})\right]}, \tag{15}$$

with $\Omega_t = x_t x_t^\top$ and $\bar{\theta} = (\bar{\theta}_0, \ldots, \bar{\theta}_n)^\top$ a vector of parameters inside the intervals defined by the vectors θ_1 and θ_K.

Proof: The proof of this result is immediate by applying the mean value theorem to the function $\psi_{t+1}(\theta)$ and taking conditional expectations. Thus, for $l = 0, 1, \ldots, n$, each component of $\psi_{t+1}(\theta)$ satisfies that

$$\psi_{l,t+1}(\theta_K) - \psi_{l,t+1}(\theta_1) = \sum_{\widetilde{l}=0}^{\widetilde{n}} \frac{\partial \psi_{l,t+1}(\theta_{\widetilde{l}.})}{\partial \theta_{\widetilde{l}.}} |_{\theta_{\widetilde{l}.}=\bar{\theta}_{\widetilde{l}}} (\theta_{\widetilde{l}K} - \theta_{\widetilde{l}1}).$$

Furthermore, noting that $\frac{\partial \psi_{l,t+1}(\theta_{\widetilde{l}.})}{\partial \theta_{\widetilde{l}.}} = -\gamma z_{l,t} z_{\widetilde{l},t} R_{t+1}^2 \widetilde{R}_{t+1}^{-\gamma-1}(\theta_{\widetilde{l}.})$, and taking conditional expectations, the former expression is in matrix form equal to

$$E_t[\psi_{t+1}(\theta_K)] - E_t[\psi_{t+1}(\theta_1)] = -\gamma E_t \left[R_{t+1}^2 \widetilde{R}_{t+1}^{-\gamma-1}(\bar{\theta}) \right] \Omega_t \theta_{1K}.$$

Assuming that θ_K and θ_1 are the solutions to the Euler equations (12) and (7), respectively, the preceding expression becomes $c_t(K) = -\gamma E_t \left[R_{t+1}^2 \widetilde{R}_{t+1}^{-\gamma-1}(\bar{\theta}) \right] \Omega_t \theta_{1K}$, and expression (15) immediately follows.

It is important to note that the vector of hedging demands for higher-order specifications of the weight function behaves similarly. Thus, for the quadratic specification (3), we have

$$\theta_{1K} = -\frac{\widetilde{\Omega}_t^{-1} c_t(K)}{\gamma E_t \left[R_{t+1}^2 \widetilde{R}_{t+1}^{-\gamma-1}(\bar{\theta}) \right]}, \tag{16}$$

with $\widetilde{\Omega}_t = \widetilde{x}_t \widetilde{x}_t^{\top}$ and $\widetilde{x}_t = (1 \; z_{1t} \; z_{2t} \; z_{1t}^2 \; z_{2t}^2 \; z_{1t} z_{2t})^{\top}$. The vector $c_t(K)$ is as in Theorem 1.

The sign and magnitude of the hedging demand on a risky asset depend on the conditional expected value of the multiperiod returns, that we assume to be positive, the functional form of the optimal weight function chosen by the portfolio manager, the investment horizon and the degree of investor's relative risk aversion. The quantity $c_t(K)$ is also a major factor influencing the hedging demand. Intuitively, departures of this quantity from zero determine the magnitude of the hedging demand.

The persistence of the state variables is another factor that influences the hedging demand. To gain further intuition on the importance of this factor we assume that the quantities measuring the conditional covariances in (14), namely, d_{1lt} and d_{3lt} are both zero for $l = 0, 1, \ldots, n$, and analyze the closed-form expression derived in Theorem 2 in more detail. In this case, the obtained closed-form expression is an approximation of the actual hedging demand on the risky assets. The accuracy of the approximation depends on the importance of the neglected covariance terms.

Corollary 2. The mean hedging demand on a risky asset with portfolio policy rule $\alpha_t(z_t; \theta_K) = \theta_{0K} + \theta_{1K} z_{1t}$, with $z_{1t} = \phi_1 z_{1,t-1} + v_{1,t+1}$ and $|\phi_1| < 1$, is

$$\theta_{0,1K} = \frac{\eta z_{1t}}{\gamma(1 - z_{1t}^2)} \frac{\sum_{j=2}^{K} \beta^{j-1} \sum_{i=1}^{j} \left(\phi_1^{i-1} - \phi_1^{2(i-1)} z_{1t}^2 - \frac{1-\phi_1^{2i}}{1-\phi_1^2} \sigma_{v_1}^2 \right) E_t[\widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K) \widetilde{R}_{t+i}^{-1}(\theta_K)]}{E_t\left[R_{t+1}^2 \widetilde{R}_{t+1}^{-\gamma-1}(\bar{\theta}) \right] \sum_{j=1}^{K} \beta^{j-1} E_t[\widetilde{R}_{t+2:t+j}^{1-\gamma}(\theta_K)]}. \tag{17}$$

Similarly, the marginal hedging demand with respect to z_{1t} is

$$\theta_{1,1K} = \frac{\eta z_{1t}^2}{\gamma(1 - z_{1t}^2)} \frac{\sum_{j=2}^{K} \beta^{j-1} \sum_{i=1}^{j} \left(\phi_1^{i-1}(\phi_1^{i-1} - 1) + \frac{1-\phi_1^{2i}}{1-\phi_1^2} \frac{\sigma_{v_1}^2}{z_{1t}^2} \right) E_t[\widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K) \widetilde{R}_{t+i}^{-1}(\theta_K)]}{E_t\left[R_{t+1}^2 \widetilde{R}_{t+1}^{-\gamma-1}(\bar{\theta}) \right] \sum_{j=1}^{K} \beta^{j-1} E_t[\widetilde{R}_{t+2:t+j}^{1-\gamma}(\theta_K)]}, \tag{18}$$

with $\sigma_{v_1}^2$ the variance of the error term v_1 driving the dynamics of z_{1t}.

Proof: The proof of this result is immediate from applying the expression in Theorem 2 for $c_t(K)$ with components $c_{lt}(K) = -\frac{d_{2lt}(K)}{\sum_{j=1}^{K} \beta^{j-1} E_t[\widetilde{R}_{t+2:t+j}^{1-\gamma}(\theta_K)]}$. More specifically, for $\alpha_t(z_t; \theta_K)$ given by a single state variable, we obtain $\Omega_t^{-1} = \frac{1}{1-z_{1t}^2}\begin{pmatrix} 1 & -z_{1t} \\ -z_{1t} & 1 \end{pmatrix}$. The component d_{20t} is characterized by the quantity $E_t[R_{t+i}]$ and the component d_{21t} is characterized by the quantity $E_t[z_{t+i-1}R_{t+i}]$. Furthermore, replacing expressions (1) and (4), we obtain $E_t[R_{t+i}] = \eta E_t[z_{t+i-1}] = \eta\phi^{i-1}z_t$. Similarly, we have $E_t[z_{t+i-1}R_{t+i}] = \eta E_t[z_{t+i-1}^2] = \eta\left(\phi^{2(i-1)}z_{1t}^2 + \frac{1-\phi_1^{2i}}{1-\phi_1^2}\sigma_{v_1}^2\right)$, with $\sigma_{v_1}^2$ the variance of the error term in (4). We obtain $d_{20t} = \eta\sum\limits_{j=2}^{K}\beta^{j-1}\sum\limits_{i=2}^{j}\phi_1^{i-1}z_{1t}E_t[\widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K)\widetilde{R}_{t+i}^{-1}(\theta_K)]$ and $d_{21t} = \eta\sum\limits_{j=2}^{K}\beta^{j-1}\sum\limits_{i=2}^{j}\left(\phi^{2(i-1)}z_{1t}^2 + \frac{1-\phi_1^{2i}}{1-\phi_1^2}\sigma_{v_1}^2\right)E_t[\widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K)\widetilde{R}_{t+i}^{-1}(\theta_K)]$. The results in the corollary are obtained by multiplying Ω_t^{-1} by the elements of $d_{2t}(K)$.

This result shows that the hedging demand depends on the ratio η/γ, the persistence of the state variable ϕ_1, the variance of the shocks to the state variables $\sigma_{v_1}^2$, the investment horizon K and the contribution of the conditional expectations $E_t[\widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K)\widetilde{R}_{t+i}^{-1}(\theta_K)]$. In particular, expression (18) shows that the magnitude of the hedging demand is increasing on the exposure of the risky asset to the state variable and decreasing with respect to the relative risk aversion coefficient. The sign of both hedging demands is determined by the sign of η and the persistence parameter ϕ. The effect of the investment horizon on both hedging demands is nonlinear and, in general, suggests a positive and increasing hedging demand with the investment horizon.

2 Econometric analysis of the hedging demand

This section presents suitable methods for estimating the parameter θ_K driving the optimal portfolio weights. An option is to estimate consistently θ_1 obtained from the single-period objective function corresponding to the short-term myopic case, and estimate separately the expression for the hedging demand in (15). Unfortunately, this expression is unfeasible as the parameter vector $\bar{\theta}$ in the denominator is unknown. A suitable alternative is to exploit the

Euler condition (9) and apply the generalized method of moments introduced in Hansen (1982) and Hansen and Singleton (1982).

The moment conditions (9) characterizing the optimal asset allocation in a multiperiod context is

$$\phi_{l,t+K} \equiv E\left[\left(\sum_{j=1}^{K} \beta^{j-1} \widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K) \sum_{i=1}^{j} \frac{\alpha_{lt}(z_t;\theta_K) R_{t+i}}{\widetilde{R}_{t+i}(\theta_K)}\right) U_t\right] = 0, \tag{19}$$

for $l = 0, 1, \ldots, \widetilde{n}$, and all $\Im_t-$measurable functions U_t and all t, $1 \leq t \leq T - K$, with $T > K$ the sample size. Following Giacomini and Komunjer (2005), we assume the existence of a vector of variables U_t^* that are observed at time t and that contain all of the relevant information in the sigma-algebra $\Im_t$. We refer to U_t^* as the information vector. The general requirement on $\{U_t^*\}$ is that it is a strictly stationary and mixing series. In our framework, we consider U_t^* to be proxied by the vector x_t.

Under these assumptions and the parametric portfolio policy rule (13), it follows that $\alpha_{lt}(z_t;\theta_K) = z_{l,t}$ and $\widetilde{n} = n$. Then, the $n + 1$ Euler equations above yield a set of $(n + 1)^2$ unconditional moments given by

$$\phi_{l,\widetilde{l}} \equiv E\left[\left(\sum_{j=1}^{K} \beta^{j-1} \widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K) \sum_{i=1}^{j} \frac{z_{l,t+i-1} R_{t+i}}{\widetilde{R}_{t+i}(\theta_K)}\right) z_{\widetilde{l},t}\right] = 0, \tag{20}$$

indexed by $l, \widetilde{l} = 0, 1, \ldots, n$. Using these estimates of the population quantities, we define

$$g_{it}(\theta_K) = z_{\widetilde{l},t} \sum_{j=1}^{K} \beta^{j-1} \widetilde{R}_{t+1:t+j}^{1-\gamma}(\theta_K) \sum_{i=1}^{j} \frac{z_{l,t+i-1} R_{t+i}}{\widetilde{R}_{t+i}(\theta_K)},$$

with $i = 1, \ldots, (n+1)^2$ an index that accounts for all possible combinations of $l, \widetilde{l} = 0, \ldots, n$. Let $g_t(\theta_K)$ be a vector of dimension $(n + 1)^2 \times 1$ that stacks the variables $g_{it}(\theta_K)$. Condition (20) becomes

$$E[g_t(\theta_K)] = 0.$$

Let $g_N(\theta_K)$ be the empirical counterpart of $E[g_t(\theta_K)]$ that stacks the sample moment conditions $\frac{1}{N}\sum_{t=1}^{N} g_{it}(\theta_K)$, with $N = T - K$ and $i = 1, \ldots, (n+1)^2$. The idea behind GMM is to choose an estimate of θ_K, namely $\widehat{\theta}_K$, so as to make the sample moments $g_N(\widehat{\theta}_K)$ as close to zero as possible. Let $G_N(\theta_K) = g_N^\top(\theta_K)\widehat{V}_N^{-1}g_N(\theta_K)$ with $\widehat{V}_N$ a consistent estimator of the long-run covariance matrix of $\sqrt{N}g_N(\theta_K)$. This matrix is defined as

$$V_0(\theta_K) = \frac{1}{N}\sum_{t=1}^{N}\sum_{s=1}^{N} E[g_t(\theta_K)g_s^\top(\theta_K)], \tag{21}$$

and captures the potential serial correlation in the sequence $g_t(\theta_K)$ due to the presence of the state variables. An estimator of V_0 can be obtained by applying HAC variance estimators. More specifically, let $\Gamma_N(j) = \frac{1}{N}\sum_{t=j+1}^{N} g_t(\theta_K)g_{t-j}^\top(\theta_K)$ be the sample covariance matrix between $g_t(\theta_K)$ and $g_{t-j}(\theta_K)$. A suitable Newey-West HAC estimator is

$$\widehat{V}_N(\widehat{\theta}_K) = \Gamma_N(0) + \sum_{j=1}^{l}\frac{l-j}{j}\left(\Gamma_N(j) + \Gamma_N^\top(j)\right), \tag{22}$$

with l a bandwidth parameter that determines the maximum order of autocorrelation taken into account by the estimator. Using this notation, we obtain an estimator of θ_K as the solution to the minimization problem

$$\widehat{\theta}_K = \underset{\theta\in\Theta}{\arg\min}\; G_N(\theta), \tag{23}$$

with Θ the parameter space for θ_K. In a first stage, to obtain a consistent estimator of θ_K, we use the identity matrix as an initial candidate for $\widehat{V}_N$. In a second stage, the minimization process is repeated replacing the identity matrix by the matrix $\widehat{V}_N(\widetilde{\theta}_K)$. This minimization process is iterated until a satisfactory solution is obtained with $\widehat{\theta}_K$ denoting the estimator of the model parameters obtained in the last step.

Statistical inference for the portfolio weights is obtained by applying asymptotic theory results from GMM estimation to the quantity $g_N(\theta_K)$. To show this, we assume the portfolio returns and state variables to be jointly stationary, and derive the consistency and asymptotic normality of $g_N(\widehat{\theta}_K)$. Standard results for GMM estimators imply that $\sqrt{N} g_N(\widehat{\theta}_K)$ converges in distribution to a $N(0, V_{0K})$ with V_{0K} the asymptotic covariance matrix defined in (21). Using these results, it follows that

$$\sqrt{N}(\widehat{\theta}_K - \theta_K) \xrightarrow{d} N(0, \Omega_{0K}), \tag{24}$$

with $\Omega_{0K} = (D_{0K} V_{0K}^{-1} D_{0K})^{-1}$, where $D_{0K} = E\left[\frac{\partial g_t(\theta)}{\partial \theta}|_{\theta_K}\right]$. We also derive the asymptotic distribution of the hedging demand θ_{1K}. To do this, we note that $\sqrt{N}(\widehat{\theta}_{1K} - \theta_{1K}) = \sqrt{N}(\widehat{\theta}_K - \theta_K) - \sqrt{N}(\widehat{\theta}_1 - \theta_1)$, and using expression (24), we obtain

$$\sqrt{N}(\widehat{\theta}_{1K} - \theta_{1K}) \xrightarrow{d} N\left(0, v' \Sigma_{1K} v\right), \tag{25}$$

with $v = (1 \ \ -1)'$ and $\Sigma_{1K} = \begin{pmatrix} \Omega_{0K} & Cov(\widehat{\theta}_K, \widehat{\theta}_1) \\ Cov(\widehat{\theta}_K, \widehat{\theta}_1) & \Omega_{01} \end{pmatrix}$, where $Cov(\widehat{\theta}_K, \widehat{\theta}_1)$ denotes the asymptotic covariance between the parameter estimators.

3 Hedging demand in long-term rebalancing carry trades

The failure of the uncovered interest rate parity (UIP) condition implies that the risk-free interest rate differential between two countries is not offset by the expected depreciation of the high-yield currency. This is an important consequence for carry trade investors as the expected currency excess return differs from zero and is potentially predictable. Fama (1984) rationalizes the failure of this equilibrium condition by noting the existence of a time-varying risk premium in the foreign currency market that entails profitable strategies. These strategies are denominated currency carry trades and consist of selling low interest rate currencies (the funding currencies) and investing the proceeds in high interest rate currencies (the investment currencies), see Brunnermeier, Nagel and Pedersen (2009) for a discussion of these strategies.

The presence of a time-varying risk premium, see Fama (1984), implies an equilibrium condition between the spot currency price (s_{t+1}) and forward currency price (f_t), both in logs, given by

$$E_t[s_{t+1}] = f_{t,t+1} + E_t[r_{t+1}], \tag{26}$$

with r_{t+1} a risk premium term in the foreign currency market that is unknown at time t. Let z_t denote the set of state variables with power to predict the risk premium r_{t+1}. The excess return R_{t+1} in the carry trade strategy is equal to the risk premium r_{t+1}. Predictability of the excess return on the carry trade strategy implies that

$$r_{t+1} = \eta' z_t + \varepsilon_{t+1}, \tag{27}$$

with η the vector of parameters capturing the exposure of the carry trade return to the state variables and ε_{t+1} a zero-mean error term serially uncorrelated and also uncorrelated to z_t. The currency carry trade model is completed by defining the dynamics of the state variables that, for simplicity, we assume to follow the autoregressive process (4).

This section studies empirically the influence of the investment horizon on the optimal allocation to the currency carry trade strategy. To do this, we use GMM estimation and inference methods proposed in the preceding section. We analyse the one-month, myopic, investment horizon strategy versus different (one-year, five-year and ten-year) investment horizons ($K = 12, 60, 120$ months). The section commences with a description of the variables used in the empirical application.

3.1 Data description

Our data cover the period January 1985 to December 2016. Data are collected from Bloomberg on a monthly frequency on the three-month U.S. Treasury bill rate, the three-month interbank interest rate and the CRB industrial. We also collect from Bloomberg daily observations on the G10 currency exchange rates to create the currency volatility variable. The nominal yield on the U.S. one-month risk-free rate is obtained from Kenneth French's database.

Our investment portfolio comprises the U.S. risk-free rate with return denoted as R_t^f, and a naive currency carry trade portfolio with return R_t. To study the naive carry trade benchmark we rely on the FXFB Crncy strategy reported by Bloomberg, which reports the returns on the currency carry trade strategy constructed from the G10 group of developed economies that comprises Australia, Canada, Switzerland, Germany, the United Kingdom, Japan, Norway, New Zealand, Sweden and the United States. Currencies are quoted against the U.S. dollar. This portfolio is implemented by taking positions in the currency forward market implying, in turn, the possibility of highly leveraged positions[3]. The average annualized carry trade return is 4.76%, the volatility is 8.93%, and the Sharpe ratio is about 0.5. The distribution of the carry trade strategy is left-skewed and heavy tailed.

The investor's information set is described by a set of state variables that are expected to explain the risk premium in (27). We entertain the following variables: 1) level and three-month change of the volatility of G10 exchange rates, see Bakshi and Panayotov (2013), Menkhoff et al. (2012).[4] These variables are hereafter denominated as the currency volatility level and

[3]The choice of this portfolio for illustrating the currency carry trade strategy rather than using single currencies is motivated by three reasons. First, data on this portfolio are widely available through the Bloomberg FXFB Crncy function. Second, this portfolio comprises a combination of currencies providing more generality to our findings than if obtained using a single currency. Finally, this portfolio contains major currencies from the group of G10 countries. These currencies operate under a floating regime with prices driven by demand and supply and, hence, not distorted by government manipulation and other types of external interventions into foreign exchange markets.

[4]We use Bakshi and Panayotov's (2013) description of monthly volatility, which is computed as the square root of the sum of squares of daily log changes in the exchange rates defined against the U.S. dollar over a

currency volatility momentum. 2) level and three-month change of the U.S. Ted spread, see Bakshi and Panayotov (2013). These variables are denominated as U.S. Ted spread level and U.S. Ted spread momentum. 3) Three-month change of the CRB commodity index as in Bakshi and Panayotov (2013). Following Brandt, Santa-Clara and Valkanov (2009), we demean and standardize all the state variables in the optimization process. These variables incorporate market features related to the uncertainty in global currency markets, aggregate liquidity and commodity markets on the long-term asset allocation problem. Figure 1 reports the dynamics of the state variables.

Table 1 shows the estimates of a VAR(1) that describes the dynamics of the currency carry trade return and the state variables, assuming that all the shocks are normally distributed and a constant unconditional variance-covariance matrix of the error term. This exercise is motivated by Campbell and Viceira (1999). These authors estimate the parameters determining the optimal allocation to the risky asset through the estimation of a VAR(1) model. The results confirm empirically the presence of predictability of the carry trade strategy. In particular, we find that the currency carry trade return is significantly and negatively related to the currency volatility momentum and, marginally, to the US Ted spread. It is also positively related to the CRB industrial return. The VAR(1) specification also confirms the autoregressive dynamics for the state variables. Thus, for the U.S. Ted Spread the autoregressive persistence parameter is 0.84 and statistically very significant. Similarly, for the currency volatility momentum the autoregressive parameter is 0.37 and for the three-month CRB industrial return the autoregressive parameter is 0.77. Interestingly, in the VAR(1) specification, the U.S. Ted Spread manifests as a relevant state variable exhibiting predictive ability not only for the carry trade return but also for the remaining state variables in the system. These empirical findings are very much in line with the assumptions in equations (27) and (4) on the relationship between

month, averaged across the G10 currencies.

the risk premium on the carry trade and the state variables, and the autoregressive character of the state variables, respectively. These results are also in the spirit of Bakshi and Panayotov (2013).

For completeness, we also report in Table 2 the correlation between the innovation terms in equations (27) and (4). These correlations play an important role for determining the sign and magnitude of the hedging demand in Campbell and Viceira's framework. The first column shows a strong negative correlation between the innovations to the carry trade return and the innovations to all of the state variables.

3.2 Optimal long-term rebalancing vs. myopic carry trades

We study the optimal portfolio problem of an investor who faces a time-varying investment opportunity set. There are two assets available to the investor: the risk-free asset and the currency carry trade strategy.

In order to assess empirically the properties derived in previous sections we entertain several polynomial specifications for modelling the optimal portfolio weight $\alpha_t(z_t;\theta_k) = f(s_t(z_t;\nu);\widetilde{\nu}_k)$, with $f(s_t(z_t;\nu);\widetilde{\nu}_k) = \widetilde{\nu}_{0k} + \sum_{i=1}^{j} \widetilde{\nu}_{ik} s_t(z_t;\nu)^i$. The baseline case is the linear portfolio rule ($j = 1$). This is the standard parametric portfolio policy proposed in the asset allocation literature, see Campbell and Viceira (1999), Brandt and Santa-Clara (2006) and Brandt, Santa-Clara and Valkanov (2009) as seminal examples. As a robustness exercise, we also consider a parametric portfolio policy given by polynomials of orders $j = 2, 3$.[5] Following Aït-Sahalia and Brandt (2001), the state of nature at each point in time t is predicted by a linear combination of the state variables. In our case, $s_t(z_t;\nu) = \sum_{i=1}^{5} \nu_i z_{it}$, with z_{it} the state variables introduced above. The corresponding myopic allocation is given by $\alpha_t(z_t;\theta_1) = f(s_t(z_t;\nu);\widetilde{\nu}_1)$, with $f(s_t(z_t;\nu);\widetilde{\nu}_1) =$

[5]Polynomials of orders higher than three when the information set contains five state variables imply the estimation of more than 50 parameters. Estimation of such large number of parameters given the available sample size results in noisy parameter estimates that are not statistically significant in most cases.

$\widetilde{\nu}_{01} + \sum_{i=1}^{j} \widetilde{\nu}_{i1} s_t(z_t; \nu)^i$. The estimated mean hedging demand $\widehat{\theta}_{0,1K}$ is computed as the empirical counterpart of $E[\alpha_t(z_t; \theta_K) - \alpha_t(z_t; \theta_1)]$. For the linear case the mean hedging demand is captured by the difference in intercept parameters across specifications. The estimated marginal hedging demand $\widehat{\theta}_{l,1K}$ with $l = 1, \ldots, \widetilde{n}$ is defined as the difference in parameter estimates of the functions $\alpha_t(z_t; \theta_k)$ and $\alpha_t(z_t; \theta_1)$ associated to the state variable z_{lt}.

We present first the results for the linear specification of the portfolio rule. Table 3 studies the mean and marginal hedging demand for a portfolio policy defined by a single state variable and Table 4 the mean and marginal hedging demand for a portfolio policy comprised by a vector of state variables. Estimation of the model parameters is done using the GMM approach discussed in the preceding section and obtained from the sample moment conditions (23). The discount factor is $\beta = 0.95$. Similar results are obtained for small variations of the discount factors. These results are available from the authors upon request. Differences between the short-term and long-term allocation to the carry trade strategy are mainly explained by the role of the hedging demand offered by the currency carry trade strategy over long horizons. In order to empirically assess the predictions of Theorem 2, we study first a long-term asset allocation model in which the carry trade return is a linear function of a single state variable. Table 3 reports the parameter estimates of the hedging demand for the three state variables with power to predict the excess return on the currency return used as single predictors of the risk premium, see first row of Table 1. These variables are the U.S. Ted Spread (Panel A), the currency volatility momentum (Panel B) and the CRB Industrial return (Panel C). The first column of each table reports the value of θ_1 corresponding to the one-period short-term allocation. Columns 2 to 4 report the hedging demand θ_{1K} for different values of K that oscillate between one ($K = 12$) and ten years ($K = 120$). As shown in the VAR analysis, all of these variables are stationary with an autoregressive persistence parameter of 0.84, 0.37 and 0.77, respectively. The relationship between the return on the carry trade and the state variables reflected in η in expression (27) and reported in Table 1 is -0.33, -0.34 and 0.31, respectively.

The estimates in Panel A of Table 3 corroborate the predictions of Corollary 2. As shown in Table 1, the exposure of the currency return to the U.S. Ted spread state variable is $\eta = -0.33$ and the autoregressive parameter is $\phi = 0.84$. These values suggest a negative marginal hedging demand of the currency return to the volatility momentum. This is indeed reflected by the estimates of the hedging demand in Table 3. The myopic allocation is determined by $\theta_{01} = -0.43$. This allocation increases in magnitude monotonically with the investment horizon. The remaining parameters in the first row report estimates of the mean hedging demand. More specifically, column 1 reports θ_{01} and columns 2 to 4 report the mean hedging demand.

The results in Table 4 studying the hedging demand for a portfolio policy comprised by a linear combination of the state variables are similar in spirit to the analysis of Table 3. The signs of the hedging demand parameters match those corresponding to the single state variable exercise, however, the magnitude is much smaller across all parameter estimates. There is no clear pattern on the relationship between the magnitude of the parameters and the investment horizon. These differences can be theoretically assessed using expressions (17) and (18). The presence of additional state variables introduces extra terms in the definition of the hedging demand that have an effect on the mean hedging demand (first row of Table 4) but also on the hedging demands to the state variables (remaining rows in Table 4).

Table 5 studies the sensitivity of the parameters θ_{1K} and θ_{sK} to variation in the CRRA coefficient. As expected, the higher the risk aversion coefficient the lower the mean hedging demand for the currency carry trade asset. This is well known in the long-term asset allocation literature, see Campbell and Viceira (1999) for the infinite-horizon case, and is confirmed by our parameter estimates. This result holds for the mean hedging demand and the marginal hedging demand.

We also assess the economic performance of the different strategies at an investment horizon of five years. This is done in an in-sample exercise where we compute the main summary statistics of the distribution of the portfolio return R^p_{t+1} obtained from plugging-in the optimal values of the portfolio policy rule (13) under different investment horizons and values of the relative risk aversion coefficient. The aim of this exercise is to compare the distributions of the portfolio return between the myopic investment, given by investing over one period, and the long-term investment obtained from a five-year investment horizon. Table 6 reports the main summary statistics for the myopic and long-term portfolios for different risk aversion coefficients $\gamma = 5, 20, 40$ and 100. We also compute the accumulated gross return of a strategy that starts investing one U.S. dollar in the portfolio for the different investment strategies and risk aversion profiles. The terminal wealth of these strategies in annualized terms is reported in the last row of Table 6. Figure 2 reports the dynamics of wealth in both portfolios over the evaluation period. The chart shows staggering differences in the aggregate return over time.

The results provide interesting insights about the performance of each strategy and the role of risk aversion. In general, the long-term investment strategy provides a higher mean return and a higher Sharpe ratio. The difference in mean return and variance decreases monotonically with the relative risk aversion coefficient γ. Thus, for $\gamma = 100$, the gross return of the myopic and long-term strategies become closer indicating that both strategies are similar when investors have low tolerance to risk. The skewness and kurtosis of the long-term portfolio are both smaller than for the myopic case. These findings suggest that the long-term investment portfolio achieves a return distribution with lower risk and higher mean return than the myopic investment. A potential explanation for this result is the flexibility of the long-term optimal portfolio allocation to adapt to changes in economic conditions by increasing/decreasing the exposure to the risky asset with respect to the short-term portfolio.

3.2.1 Certainty equivalent returns

The in-sample performance of these portfolios is evaluated using two related methods: *i*) the certainty equivalent return (CER) for both long-term and short-term strategies and *ii*) differences in expected utility obtained from each investment strategy.

The CER measure is defined as the guaranteed return, in annualized wealth, that would provide an investor with the same expected utility as the given optimal portfolio rule. For a multiperiod utility function, the CER is defined as the return R^{CER} that solves the following equation, see Laborda and Olmo (2017):

$$\sum_{j=1}^{K}\beta^j \frac{W_t^{1-\gamma}}{1-\gamma}\prod_{i=1}^{j}(1+R^{CER})^{1-\gamma} = \sum_{j=1}^{K}\beta^j \frac{W_t^{1-\gamma}}{1-\gamma}E_t\left[\prod_{i=1}^{j}(1+R^p_{t+j+1-i}(\alpha_{t+j-i}))^{1-\gamma}\right]. \tag{28}$$

Because the power utility function is homothetic in wealth, without loss of generality, the initial wealth can be normalized to one. Therefore, we can compute the certainty equivalent return as the annualized return on wealth earned with certainty that provides the investor with the same utility as the optimal portfolio. The above condition becomes

$$E_t\left[\sum_{j=1}^{K}\beta^j\left((1+R^{CER})^{j(1-\gamma)} - \prod_{i=1}^{j}(1+R^p_{t+j+1-i}(\alpha_{t+j-i}))^{1-\gamma}\right)\right] = 0. \tag{29}$$

The empirical counterpart of this expression yields the following equation:

$$\frac{1}{T-K}\sum_{t=1}^{T-K}\left[\sum_{j=1}^{K}\beta^j\left((1+R^{CER})^{j(1-\gamma)} - \prod_{i=1}^{j}(1+R^p_{t+j+1-i}(\alpha_{t+j-i}))^{1-\gamma}\right)\otimes z_t\right] = 0. \tag{30}$$

For simplicity, we report at the end of Table 6 the CER for the unconditional version of the above expression that considers $z_t = 1$. The differences in the annualized certainty equivalent return between the long-term and myopic portfolios vary from 7.01% for $\gamma = 5$ to 1.02% for

$\gamma = 100$. Long-term investors enjoy larger expected utility than myopic investors regardless the investors relative risk aversion.

The difference in sample average utilities across the two methods is reported in the bottom row of Table 6. For the myopic case we compute the average of the realized one-period utility over the investment horizon K. For the long-term investor, we compute the realized expected utility obtained from (8) and divide this quantity by K, with K the investment horizon. The results show an increase in average utility for the long-term portfolio compared to the myopic portfolio. The difference becomes smaller as the investor's risk aversion coefficient decreases.

To illustrate further the results of our empirical study, we plot in Figure 3 the dynamics of the optimal myopic portfolio weight α_t for the currency carry trade strategy (top panel) and the optimal hedging demand corresponding to an investment horizon of one-, five- and ten years (bottom panel). The degree of relative risk aversion is $\gamma = 10$. In all cases the allocation to the carry trade asset is negative around the 1998 Asian crisis and the $2007 - 2009$ subprime crisis. The hedging demand varies significantly with the state variables rather than remaining stable across time. Interestingly, the most important changes in the hedging demand, which is on average positive, correspond to the period starting in the subprime crisis onwards, reflecting a potential change in the investment opportunity set. Consequently, the strategic asset allocation, which can be larger or lower than the myopic one in absolute terms, is on average larger. This finding highlights the complexity of computing the optimal long-term asset allocation.

3.3 Robustness exercises

3.3.1 Parametric portfolio policies of higher order

As a robustness exercise, we report in Tables 7 and 8 the results for parametric portfolio policies given by polynomials of orders two and three. The degree of relative risk aversion is $\gamma = 10, 100$, respectively, and the investment horizon is $K = 60$ periods (five years). In order to be able to estimate the model parameters reliably we impose the restriction that the coefficients associated to the cross products between the variables are zero. By doing so, we restrict the parameter space to 11 coefficients in the quadratic case and 16 coefficients in the cubic case.

Both tables show that the mean hedging demand is positive regardless the specification of the portfolio policy rule and the risk aversion coefficient. However, it is worth noting that for $\gamma = 100$ the difference in hedging demands between the long-term and myopic strategies is significantly smaller than for $\gamma = 10$. This result adds further evidence on the suitability of the currency carry trade as a good hedge against changes in the investment opportunity set. Interestingly, the mean hedging demand of the portfolio policy rules that consider polynomial specifications of orders two and three is smaller than in the linear case. The parameters of the linear and quadratic specifications are of the same sign although the magnitude of the coefficient slightly varies across specifications. The results for the cubic polynomial are also very similar to the linear and quadratic models. The only exceptions are the marginal hedging demands to the U.S. Ted spread and the currency volatility. The inspection of the coefficients associated to higher orders of the state variables does not provide important insights into the dynamics of the portfolio weights. The only exception is the square of the currency volatility momentum. This state variable seems to have additional predictive power to describe the dynamics of the portfolio weights. The portfolio has a positive exposure to the state variable suggesting that increases in the volatility of the volatility momentum predict larger allocations to the currency portfolio. The remaining state variables are not statistically significant and may suggest that the model is overparametrized.

The robustness exercise is concluded by comparing the economic performance of the linear and quadratic portfolio rules. Table 9 shows that the portfolio policy rule that considers a polynomial of order two delivers a higher mean return and final wealth than in the linear case but assuming a higher volatility and kurtosis. The difference in the annualized certainty equivalent return between the long-term and myopic portfolio is also significantly higher for the portfolio policy rule that considers a polynomial of order two, 9.63%, than for the linear case that reports an increment in CER of 4.64%. The difference in sample average realized utility between the long-term and short-term portfolios is also positive but decreases as the level of investor's risk aversion increases from $\gamma = 10$ to $\gamma = 100$.

3.3.2 Out-of-sample exercise

For completeness, we also implement an out-of-sample experiment that solves the asset allocation problem over different horizons every year using a rolling data window until the end of the sample, covering the period January 1985 to December 2016. This exercise aims to assess the robustness of the results to the use of a different rebalancing period. The optimal portfolio is re-estimated on a yearly basis using a rolling data window until the end of the sample. The first portfolio is computed with data from January 1985 to December 2006. The second portfolio is computed with data from January 1986 to December 2007, and similarly until the end of the evaluation period. The out-of-sample period spans December 2006 to December 2016. We assume $\gamma = 10$ and an investment horizon of five years.

Figure 4 plots the dynamics of the out-of-sample parameter estimates describing the marginal hedging demand with respect to the five standardized state variables (the U.S. Ted spread; the level of currency volatility; the three-month change of U.S. Ted spread (U.S. Ted spread momentum); the three-month change of currency volatility (currency volatility momentum); and the three-month CRB industrial return). The results are similar to the previous analyses presented in the paper for monthly rebalancing. The U.S. Ted spread level and momentum parameter signs remain negative and positive, respectively, across the out-of-sample period. On the other hand, the parameters of the currency volatility variables change sign during the latter part of the out-of-sample period, coinciding with the lower currency volatility level after the turbulent subprime period, which constitutes an improvement of the investment opportunity set. This exercise suggests that the ability of the state variables to predict changes in the investment opportunity set remains across different rebalancing schemes and holding periods.

4 Conclusion

This paper derives a closed-form expression for the hedging demand in long-term asset allocation problems. To do this, we assume a portfolio policy rule that depends linearly on a set of state variables. We have differentiated between mean hedging demand and marginal hedging demand. The former type measures the difference in allocation to the risky asset between long-term and short-term portfolios. The latter type measures differences in sensitivity to the state variables between the long and short-term portfolios. Our theoretical and empirical insights show that the mean hedging demand is generally positive and increasing on the investment horizon. In contrast, the marginal hedging demand depends positively on the exposure of the risky asset to the state variables and negatively on the degree of investor's risk aversion. The persistence of the state variables also has a positive effect on the hedging demand. The number of state variables also influences both hedging demands by increasing the complexity of the expression due to the presence of cross correlations between the state variables.

These insights have been tested with data of a currency carry trade portfolio. Our results confirm empirically the existence of a hedging demand for the carry trade strategy satisfying the properties derived theoretically. We show that the currency volatility momentum, the three-month change of the U.S. Ted spread, and the three-month change of the CRB commodity index exhibit predictive ability for the carry trade return, and hence, are suitable candidates to be included in the parametric portfolio policy rule of long-term carry trade strategies.

A similar exercise available from the authors upon request obtains very similar findings for a currency portfolio obtained from Lustig, Roussanov and Verdelhan (2011) that contains currency prices from 15 developed countries.

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Appendix: Investor's objective function on terminal wealth

In this section we investigate the portfolio choice of an investor who maximizes the expected utility of terminal wealth W_T, with $T = t + K$. In this context, an investor maximizes

$$E_t\left[\frac{W_T^{1-\gamma}}{1-\gamma}\right]. \tag{31}$$

There are two possible scenarios to build up wealth using the investment portfolio (1). These scenarios depend on whether the investor decides to rebalance the portfolio or not. If the investor does not rebalance the portfolio then the investor's portfolio decision problem is equivalent to the problem presented in Section 1.2 but replacing 1 by K. In contrast, if the investment portfolio (1) is rebalanced each period wealth accummulates as follows;

$$W_T \equiv W_{t+K} = \prod_{i=1}^{K}(1 + R^p_{t+i}(\alpha_{t+i-1}(z_{t+i-1};\theta_K)))W_t,$$

and the investor's optimization problem (31) becomes

$$\max_{\{\theta_K \in \Theta\}}\left\{\frac{W_t^{1-\gamma}}{1-\gamma}E_t\left[\widetilde{R}^{1-\gamma}_{t+1:t+K}(\theta_K)\right]\right\},$$

with $\widetilde{R}_{t+1:t+K}(\theta_K) := \prod_{i=1}^{K}\widetilde{R}_{t+i}(\theta_K)$ denoting the multiperiod gross return on the portfolio and $\widetilde{R}_{t+i}(\theta_K) = 1 + R^p_{t+i}(\alpha_{t+i-1}(z_{t+i-1};\theta_K))$. Simple algebra shows that the first order conditions of this optimization problem with respect to the parameter vector θ_K are

$$E_t\left[\phi_{l,t+K}(\theta_K^*)\right] = 0, \tag{32}$$

with $\phi_{l,t+K}(\theta_K^*) = \left(\sum_{i=1}^{K}\frac{\alpha_{l,t+i-1}(z_{t+i-1};\theta_K^*)R_{t+i}}{\widetilde{R}_{t+i}(\theta_K^*)}\right)\widetilde{R}^{1-\gamma}_{t+1:t+K}(\theta_K^*)$. For simplicity, hereafter, we drop the stars from the parameter vector θ_K^*. Under Assumption A.1, we can operate with the above expression to obtain $\phi_{l,t+K}(\theta_K)$ as a function of $\psi_{l,t+1}(\theta_K)$. More specifically, $\phi_{l,t+K}(\theta_K)$ in (32) can be decomposed as

$$\phi_{l,t+K}(\theta_K) = \psi_{l,t+1}(\theta_K)\widetilde{R}^{1-\gamma}_{t+2:t+K}(\theta_K) + \sum_{i=2}^{K}\frac{\alpha_{l,t+i-1}(z_{t+i-1};\theta_K)R_{t+i}}{\widetilde{R}_{t+i}(\theta_K)}\widetilde{R}^{1-\gamma}_{t+1:t+K}(\theta_K), \tag{33}$$

with $\psi_{l,t+1}(\theta_K)$ defined in (7). Taking conditional expectations, the multiperiod Euler equation (32) can be written as

$$E_t[\phi_{l,t+K}(\theta_K)] = E_t[\psi_{l,t+1}(\theta_K)]E_t[\widetilde{R}^{1-\gamma}_{t+2:t+K}(\theta_K)]+ \tag{34}$$

$$\underbrace{Cov_t[\psi_{l,t+1}(\theta_K), \widetilde{R}^{1-\gamma}_{t+2:t+K}(\theta_K)]}_{d_{1lt}(K)} + \underbrace{\sum_{i=2}^{K} E_t[\alpha_{l,t+i-1}R_{t+i}]E_t[\widetilde{R}^{1-\gamma}_{t+1:t+K}(\theta_K)\widetilde{R}^{-1}_{t+i}(\theta_K)]+}_{d_{2lt}(K)}$$

$$\underbrace{\sum_{i=2}^{K} Cov_t[\alpha_{l,t+i-1}R_{t+i}, \widetilde{R}^{1-\gamma}_{t+1:t+K}(\theta_K)\widetilde{R}^{-1}_{t+i}(\theta_K)]}_{d_{3lt}(K)} = 0.$$

The quantities $d_{it}(K)$ for $i = 1, 2, 3$ are interpreted as in the main text. The above expression shows that the multiperiod Euler equation (32) can be written in terms of the Euler equation (7) that defines the short-term asset allocation problem. More formally,

Theorem A.1. The multiperiod Euler equation (32) obtained from the maximization problem (31) can be written as

$$E_t[\psi_{t+1}(\theta_K)] = c_t(K), \tag{35}$$

with $\psi_{t+1}(\theta) = (\psi_{0,t+1}(\theta), \psi_{1,t+1}(\theta), \ldots, \psi_{\widetilde{n},t+1}(\theta))^\top$ and $c_t(K) = (c_{0t}(K), c_{1t}(K), \ldots, c_{\widetilde{n}t}(K))^\top$, where $c_{lt}(K) = -\frac{d_{1lt}(K)+d_{2lt}(K)+d_{3lt}(K)}{E_t[\widetilde{R}^{1-\gamma}_{t+2:t+K}(\theta_K)]}$ for $l = 0, 1, \ldots, \widetilde{n}$.

Proof: The proof of this result follows from rearranging the terms in expression (34).

The quantity $c_t(K)$ can be interpreted as a distance to the short-term Euler equation characterized by $c_t(K) = 0$, and provides an intuitive measure of the contribution of the long-term portfolio to the optimality conditions determining the optimal asset allocation.

The following result is adapted to reflect the objective function (31) instead of (8) in the investor's optimization problem.

Corollary A.1. Under the linear portfolio policy (13), expression $c_t(K)$ in equation (35) simplifies such that

$$d_{2lt}(K) = \sum_{i=2}^{K} E_t[z_{l,t+i-1} R_{t+i}] E_t[\widetilde{R}_{t+1:t+K}^{1-\gamma}(\theta_K) \widetilde{R}_{t+i}^{-1}(\theta_K)],$$

and

$$d_{3lt}(K) = \sum_{i=2}^{K} Cov_t[z_{l,t+i-1} R_{t+i}, \widetilde{R}_{t+1:t+K}^{1-\gamma}(\theta_K) \widetilde{R}_{t+i}^{-1}(\theta_K)].$$

Proof: The proof of this result is obtained by noting that $\alpha_{l,t}(z_t; \theta_1) = z_{lt}$ in the linear case given by the portfolio policy (13).

Table 1: Estimation of Vector Autoregressive Model of order one.

	R_{t-1}	$Spread_{t-1}$	σ^c_{t-1}	$Spread_{mom,t-1}$	$\sigma^c_{mom,t-1}$	CRB_{t-1}
R_t	0.03	−0.33	0.15	−0.06	−0.34	0.31
	[0.48]	[−1.80]	[0..81]	[−0.39]	[−2.05]	[2.03]
$Spread_t$	−0.005	0.84	−0.06	−0.06	0.06	−0.01
	[−0.52]	[24.8]	[−1.74]	[−2.00]	[2.11]	[−0.20]
σ^c_t	0.01	0.27	0.51	−0.05	−0.05	−0.09
	[0.10]	[4.90]	[9.34]	[−1.10]	[−0.97]	[−1.89]
$Spread_{mom,t}$	−0.02	−0.08	−0.06	0.54	0.15	0.01
	[−1.09]	[−1.59]	[−1.20]	[11.2]	[3.02]	[0.13]
$\sigma^c_{mom,t}$	−0.01	0.14	−0.12	−0.07	0.37	−0.03
	[−0.30]	[2.09]	[−1.91]	[−1.27]	[6.15]	[−0.60]
CRB_t	0.03	−0.14	0.08	−0.04	−0.05	0.77
	[2.22]	[−3.42]	[1.90]	[−1.19]	[−1.22]	[22.1]

This table describes the dynamics of the currency carry trade return variables and the following standardized state variables: the U.S. Ted spread; the level of currency volatility; the three-month change of U.S. Ted spread (U.S. Ted spread momentum); the three-month change of currency volatility (currency volatility momentum); and the three-month CRB industrial return. T-statistics are reported in squared brackets. Data are collected from Bloomberg on a monthly frequency over the period January 1985 to December 2016 on the three-month U.S. Treasury bill rate, the three-month interbank interest rate and the CRB industrial. The currency volatility variable is constructed using daily observations from Bloomberg on the G10 currency exchange rates.

Table 2: Correlations of state variables and currency carry trade return innovations.

	R_{t-1}	$Spread_{t-1}$	σ^c_{t-1}	$Spread_{mom,t-1}$	$\sigma^c_{mom,t-1}$	CRB_{t-1}
R_t	1					
$Spread_t$	−0.69	1				
σ^c_t	−0.62	0.63	1			
$Spread_{mom,t}$	−0.37	0.12	−0.04	1		
$\sigma^c_{mom,t}$	−0.77	0.43	0.35	0.21	1	
CRB_t	0.80	−0.43	−0.58	−0.14	−0.35	1

This table shows the correlation structure of the innovations in the VAR(1) system. The VAR(1) system describes the dynamics of the currency carry trade return and the following standardized state variables: the U.S. Ted spread; the level of currency volatility; the three-month change of U.S. Ted spread (U.S. Ted spread momentum); the three-month change of currency volatility (currency volatility momentum); and the three-month CRB industrial return. Data are collected from Bloomberg on a monthly frequency over the period January 1985 to December 2016 on the three-month U.S. Treasury bill rate, the three-month interbank interest rate and the CRB industrial. The currency volatility variable is constructed using daily observations from Bloomberg on the G10 currency exchange rates.

Table 3: Effect of the investment horizon on single state variable linear portfolio choices.

$\gamma = 10$	$K = 1$	$K = 12$	$K = 60$	$K = 120$
	θ_{l1}	$\theta_{l,1K}$	$\theta_{l,1K}$	$\theta_{l,1K}$
Panel A: $TedSpread$				
$l = 0$	0.55 [2.32]	4.27 [3.20]	5.86 [10.12]	6.33 [8.70]
$l = 1$	-0.43 [-2.30]	-4.42 [-3.72]	-5.54 [-11.57]	-5.69 [-12.02]
Panel B: σ^c_{mom}				
$l = 0$	0.66 [2.67]	5.27 [4.38]	5.60 [8.35]	6.75 [9.37]
$l = 1$	-0.68 [-3.02]	-5.80 [-4.15]	-6.09 [-6.25]	-4.25 [4.24]
Panel C: CRB Industrial				
$l = 0$	0.78 [3.08]	5.52 [4.38]	6.22 [9.32]	7.20 [10.9]
$l = 1$	0.61 [3.06]	4.46 [6.20]	5.30 [11.2]	5.22 [6.00]

Column 2 reports estimates of the myopic demand and columns 3 to 5 the hedging demand ($\theta_{l,1K} = \theta_{lK} - \theta_{l1}$) for $l = 0, 1$ and different values of K (investment horizons). The relative risk aversion coefficient is $\gamma = 10$ and the discount factor is $\beta = 0.95$. The parameters are estimated under the assumption $\alpha_t(z_t, \theta_K) = \theta_{0K} + \theta_{1K} z_t$, with z_t a standardized state variable following process (4). Panel A considers the U.S. Ted Spread as single state variable. Panel B considers the currency volatility momentum as single state variable, and Panel C the three-month CRB Industrial return. The t-statistics are reported in squared brackets. Data are collected from Bloomberg on a monthly frequency over the period January 1985 to December 2016 on the three-month U.S. Treasury bill rate, the three-month interbank interest rate and the CRB industrial. The currency volatility variable is constructed using daily observations from Bloomberg on the G10 currency exchange rates.

Table 4: Effect of the investment horizon on multiple state variable linear portfolio choices.

$\gamma = 10$	$K = 1$	$K = 12$	$K = 60$	$K = 120$
	θ_{l1}	$\theta_{l,1K}$	$\theta_{l,1K}$	$\theta_{l,1K}$
$l = 0$	0.72 [2.73]	0.85 [5.06]	1.15 [7.79]	0.85 [4.48]
$l = 1$				
$\theta_{Spread,K}$	−0.23 [−1.04]	−0.16 [−1.32]	−0.38 [−2.84]	−0.68 [−2.55]
$\theta_{\sigma^c,K}$	0.46 [1.52]	0.40 [2.04]	0.34 [1.87]	−1.65 [−6.32]
$\theta_{Spread_{mom},K}$	0.08 [0.04]	0.09 [0.51]	0.32 [1.99]	0.37 [2.20]
$\theta_{\sigma^c_{mom},K}$	−0.70 [−2.70]	−0.71 [−3.65]	−0.93 [−4.69]	0.43 [2.09]
$\theta_{\sigma_{CRB},K}$	0.62 [2.06]	0.88 [5.40]	0.74 [4.49]	−0.01 [−0.05]

Column 2 reports estimates of the myopic demand and columns 3 to 5 the hedging demand ($\theta_{l,1K} = \theta_{lK} - \theta_{l1}$) for $l = 0, 1$ and different values of K (investment horizons). The relative risk aversion coefficient is $\gamma = 10$ and the discount factor is $\beta = 0.95$. The parameters are estimated under the assumption $\alpha_t(z_t, \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt}$, with z_t a set of standardized state variables following process (4). The state variables are U.S. Ted spread, the level of currency volatility, the three-month change of U.S. Ted spread (U.S. Ted spread momentum), the three-month change of currency volatility (currency volatility momentum) and the three-month CRB industrial return. T-statistics are reported in squared brackets. Data are collected from Bloomberg on a monthly frequency over the period January 1985 to December 2016 on the three-month U.S. Treasury bill rate, the three-month interbank interest rate and the CRB industrial. The currency volatility variable is constructed using daily observations from Bloomberg on the G10 currency exchange rates.

Table 5: Effect of investor's risk aversion on multiple state variable linear portfolio choices.

$K = 60$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$	$\gamma = 40$	$\gamma = 100$
	θ_{l1}	$\theta_{l,1K}$	$\theta_{l,1K}$	$\theta_{l,1K}$	$\theta_{l,1K}$
$l = 0$	2.22 [9.78]	1.15 [7.79]	0.56 [7.58]	0.17 [7.84]	0.12 [6.06]
$l = 1$					
$\theta_{Spread,K}$	-0.78 [-3.27]	-0.38 [-2.84]	-0.16 [-2.94]	-0.06 [-2.87]	-0.06 [-2.46]
$\theta_{\sigma^c,K}$	0.54 [1.63]	0.34 [1.87]	0.22 [2.67]	0.14 [4.45]	0.11 [5.98]
$\theta_{Spread_{mom},K}$	0.47 [1.83]	0.32 [1.99]	0.18 [2.33]	0.03 [3.47]	0.07 [2.24]
$\theta_{\sigma^c_{mom},K}$	-1.51 [-4.30]	-0.93 [-4.69]	-0.58 [-6.20]	-0.20 [-9.41]	-0.24 [-3.53]
$\theta_{\sigma_{CRB},K}$	1.41 [4.48]	0.74 [4.49]	0.38 [5.43]	0.17 [6.49]	0.09 [2.86]

Column 2 to 6 report estimates of the hedging demand ($\theta_{l,1K} = \theta_{lK} - \theta_{l1}$) for $l = 0, 1$ and different values of γ (relative risk aversion coefficient). The investment horizon is $K = 60$ and the discount factor is $\beta = 0.95$. The parameters are estimated under the assumption $\alpha_t(z_t, \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt}$, with z_t a set of standardized state variables following process (4). The state variables are the U.S. Ted spread, the level of currency volatility, the three-month change of U.S. Ted spread (U.S. Ted spread momentum), the three-month change of currency volatility (currency volatility momentum) and the three-month CRB industrial return. T-statistics are reported in squared brackets. Data are collected from Bloomberg on a monthly frequency over the period January 1985 to December 2016 on the three-month U.S. Treasury bill rate, the three-month interbank interest rate and the CRB industrial. The currency volatility variable is constructed using daily observations from Bloomberg on the G10 currency exchange rates.

Table 6: Comparison of investment performance between $K = 1$ and $K = 60$.

	$\gamma = 5$		$\gamma = 10$		$\gamma = 40$		$\gamma = 100$	
	$K = 1$	$K = 60$	$K = 1$	$K = 60$	$K = 1$	$K = 60$	$K = 1$	$K = 60$
Mean	0.015	0.021	0.008	0.011	0.004	0.006	0.001	0.002
Volatility	0.059	0.084	0.030	0.045	0.015	0.023	0.003	0.007
Skewness	1.85	1.84	1.82	1.62	1.85	1.46	2.14	1.84
Kurtosis	10.1	12.1	9.71	9.27	9.65	6.66	10.8	7.76
Terminal wealth	87.5	395.2	11.0	31.1	3.48	6.28	1.31	1.68
Δ CER		7.01%		4.64%		1.49%		1.02%
Δ EU		0.2193		0.1005		0.0224		0.0049

This table reports summary statistics of the portfolio return $R^p_{t+1} = R^{US}_t + \alpha_t(z_t; \theta_K) R_{t+1}$ with $\alpha_t(z_t; \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt}$, with z_t a set of standardized state variables following process (4). The state variables are given by the U.S. Ted spread, the level of currency volatility, the three-month change of U.S. Ted spread (U.S. Ted spread momentum), the three-month change of currency volatility (currency volatility momentum) and the three-month CRB industrial return. Data are collected from Bloomberg on a monthly frequency over the period January 1985 to December 2016 on the three-month U.S. Treasury bill rate, the three-month interbank interest rate and the CRB industrial. The currency volatility variable is constructed using daily observations from Bloomberg on the G10 currency exchange rates. The table also reports the final wealth of the different investment strategies for $W_0 = 1$; the difference of certainty equivalent, Δ CER, between the myopic and long-term portfolios, see expression (30); and the difference in sample average utility, Δ EU, between the myopic and long-term portfolios. The investment horizon is $K = 60$ (five years) and the discount factor is $\beta = 0.95$.

Table 7: Hedging demand for different investment horizons and polynomial portfolio rules.

$\gamma = 10$	Linear case K=1	Linear case K=60	Polynomial order 2 K=1	Polynomial order 2 K=60	Polynomial order 3 K=1	Polynomial order 3 K=60
	$\theta_{l,1}$	$\theta_{l,1K}$	$\theta_{l,1}$	$\theta_{l,1K}$	$\theta_{l,1}$	$\theta_{l,1K}$
l=0	0.72	0.43	0.75	0.51	0.93	0.41
l=1						
$\theta_{Spread,K}$	-0.23	-0.38	-0.07	-0.26	0.12	-0.41
	(-1.04)	(-2.84)	(-0.26)	(-1.33)	(-0.20)	(-0.16)
$\theta_{\sigma^c,K}$	0.46	0.34	0.19	0.21	-0.50	0.28
	(-1.52)	(-1.87)	(-0.40)	(-0.44)	(-0.94)	(-0.07)
$\theta_{Spread_{mom},K}$	0.08	0.32	0.23	0.09	0.34	0.35
	(-0.04)	(-1.99)	(-1.03)	(-2.42)	(-1.17)	(-0.13)
$\theta_{\sigma^c_{mom},K}$	-0.71	-0.93	-0.61	-0.58	-0.03	-0.47
	(-2.70)	(-4.69)	(-2.15)	(-1.99)	(-0.08)	(-0.10)
$\theta_{\sigma_{CRB},K}$	0.62	0.74	0.64	0.40	0.42	0.30
	(-2.06)	(-4.49)	(-2.28)	(-1.49)	(-1.40)	(-0.39)
$\theta_{Spread^2,K}$			-0.18	0.09	1.58	2.89
			(-0.92)	(-0.23)	(-1.43)	(-0.19)
$\theta_{\sigma^{c^2},K}$			-0.65	-1.52	3.84	0.71
			(-1.58)	(-5.41)	(-1.17)	(-0.16)
$\theta_{Spread_{mom^2},K}$			0.50	-0.21	0.37	0.01
			(-1.21)	(-0.66)	(-0.98)	(-0.61)
$\theta_{\sigma^{c2}_{mom},K}$			1.03	0.75	1.45	0.46
			(-2.91)	(-5.69)	(-2.92)	(-0.22)
$\theta_{\sigma^2_{CRB},K}$			-0.03	0.47	0.43	-0.56
			(-0.09)	(-1.04)	(-0.89)	(-0.02)
$\theta_{Spread^3,K}$					-2.74	-3.57
					(-1.32)	(-0.20)
$\theta_{\sigma^{c^3},K}$					-6.33	-6.53
					(-0.82)	(-0.46)
$\theta_{Spread_{mom^3},K}$					-0.72	-0.30
					(-1.34)	(-1.24)
$\theta_{\sigma^{c3}_{mom},K}$					-1.47	-0.49
					(-1.85)	(-0.13)
$\theta_{\sigma^3_{CRB},K}$					1.11	-1.22
					(-0.78)	(-0.02)

This table reports estimates of the myopic ($K = 1$) and hedging demand ($K = 60$) for $l = 0, 1$; $\gamma = 10$ and $\beta = 0.95$. The policy rule is $\alpha_t(z_t; \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt}$ for the linear case, $\alpha_t(z_t; \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt} + \theta^{(2)}_{1K} z^2_{1t} + \ldots + \theta^{(2)}_{nK} z^2_{nt}$ for the quadratic case and $\alpha_t(z_t; \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt} + \theta^{(2)}_{1K} z^2_{1t} + \ldots + \theta^{(2)}_{nK} z^2_{nt} + \theta^{(3)}_{1K} z^3_{1t} + \ldots + \theta^{(3)}_{nK} z^3_{nt}$.

Table 8: Hedging demand for different investment horizons and polynomial portfolio rules.

$\gamma = 100$	Linear case K=1	Linear case K=60	Polynomial order 2 K=1	Polynomial order 2 K=60	Polynomial order 3 K=1	Polynomial order 3 K=60
	$\theta_{l,1}$	$\theta_{l,1K}$	$\theta_{l,1}$	$\theta_{l,1K}$	$\theta_{l,1}$	$\theta_{l,1K}$
l=0	0.06	0.12	0.07	0.01	0.08	0.81
l=1						
$\theta_{Spread,K}$	-0.03	-0.06	-0.01	-0.01	0.01	-0.35
	(-1.28)	(-2.46)	(-0.50)	(-0.34)	(0.190)	(-0.14)
$\theta_{\sigma^c,K}$	0.07	0.11	0.04	0.18	-0.06	0.11
	(2.21)	(5.98)	(0.96)	(5.35)	(-1.16)	(0.01)
$\theta_{Spread_{mom},K}$	0.02	0.07	0.03	0.02	0.04	0.81
	(0.93)	(2.24)	(1.05)	(2.54)	(1.32)	(0.24)
$\theta_{\sigma^c_{mom},K}$	-0.09	-0.24	-0.08	-0.19	-0.03	-0.66
	(-3.38)	(-3.53)	(-2.31)	(-5.56)	(-0.89)	(-0.17)
$\theta_{\sigma_{CRB},K}$	0.08	0.09	0.08	0.04	0.05	0.89
	(2.44)	(2.86)	(2.72)	(2.98)	(1.43)	(0.63)
$\theta_{Spread^2,K}$			-0.02	-0.01	0.12	4.15
			(-0.89)	(-0.31)	(1.09)	(0.13)
$\theta_{\sigma^{c2},K}$			-0.07	-0.30	0.38	3.25
			(-2.07)	(-2.46)	(1.74)	(0.06)
$\theta_{Spread_{mom^2},K}$			0.06	0.01	0.05	0.43
			(1.37)	(0.55)	(1.39)	(0.11)
$\theta_{\sigma^{c2}_{mom},K}$			0.10	0.06	0.14	1.43
			(2.61)	(3.42)	(2.97)	(0.81)
$\theta_{\sigma^2_{CRB},K}$			0.01	0.06	0.10	-0.37
			(0.36)	(1.60)	(1.38)	(-0.06)
$\theta_{Spread^3,K}$					-0.21	-5.91
					(-1.05)	(-0.13)
$\theta_{\sigma^{c3},K}$					-0.32	-11.26
					(-1.17)	(-0.15)
$\theta_{Spread_{mom^3},K}$					-0.11	-1.13
					(-1.64)	(-0.34)
$\theta_{\sigma^{c3}_{mom},K}$					-0.11	-1.54
					(-1.38)	(-1.42)
$\theta_{\sigma^3_{CRB},K}$					0.20	-0.61
					(1.78)	(-0.24)

This table reports estimates of the myopic ($K = 1$) and hedging demand ($K = 60$) for $l = 0, 1$; $\gamma = 100$ and $\beta = 0.95$. The policy rule is $\alpha_t(z_t; \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt}$ for the linear case, $\alpha_t(z_t; \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt} + \theta^{(2)}_{1K} z^2_{1t} + \ldots + \theta^{(2)}_{nK} z^2_{nt}$ for the quadratic case and $\alpha_t(z_t; \theta_K) = \theta_{0K} + \theta_{1K} z_{1t} + \ldots + \theta_{nK} z_{nt} + \theta^{(2)}_{1K} z^2_{1t} + \ldots + \theta^{(2)}_{nK} z^2_{nt} + \theta^{(3)}_{1K} z^3_{1t} + \ldots + \theta^{(3)}_{nK} z^3_{nt}$.

Table 9: Comparison of investment performance between $K = 1$ and $K = 60$.

	Linear case		Polyn order 2		Linear case		Polyn order 2	
	$\gamma = 10$				$\gamma = 100$			
	$K = 1$	$K = 60$	$K = 1$	$K = 60$	$K = 1$	$K = 60$	$K = 1$	$K = 60$
Mean	0.008	0.011	0.011	0.019	0.001	0.002	0.001	0.002
Volatility	0.03	0.045	0.048	0.088	0.003	0.007	0.004	0.010
Skewness	1.82	1.62	5.8	6.13	2.14	1.84	4.02	4.43
Kurtosis	9.71	9.27	61.15	60.5	10.8	7.76	29.5	7.95
Terminal wealth	11.01	31.16	26.92	237.62	1.31	1.68	1.46	2.03
Δ CER		4.64%		9.63%		1.02%		1.17%
Δ EU		0.1005		0.1063		0.0049		0.0053

This table reports summary statistics of the portfolio return $R_{t+1}^{p} = R_{t}^{US} + \alpha_t(z_t;\theta_K)R_{t+1}$ with $\alpha_t(z_t;\theta_K) = \theta_{0K} + \theta_{1K}z_{1t} + \ldots + \theta_{nK}z_{nt}$ for the linear case and $\alpha_t(z_t;\theta_K) = \theta_{0K} + \theta_{1K}z_{1t} + \ldots + \theta_{nK}z_{nt} + \theta_{1K}^{(2)}z_{1t}^{2} + \ldots + \theta_{nK}^{(2)}z_{nt}^{2}$, with z_t a set of standardized state variables following a linear and a quadratic portfolio policy. The table also reports the final wealth of the different investment strategies for $W_0 = 1$; the difference of certainty equivalent, Δ CER, between the myopic and long-term portfolios, see expression (30); and the difference in sample average utility, Δ EU, between the myopic and long-term portfolios. The investment horizon is $K = 60$ (five years) and the discount factor is $\beta = 0.95$.

Figure 1: Dynamics of state variables

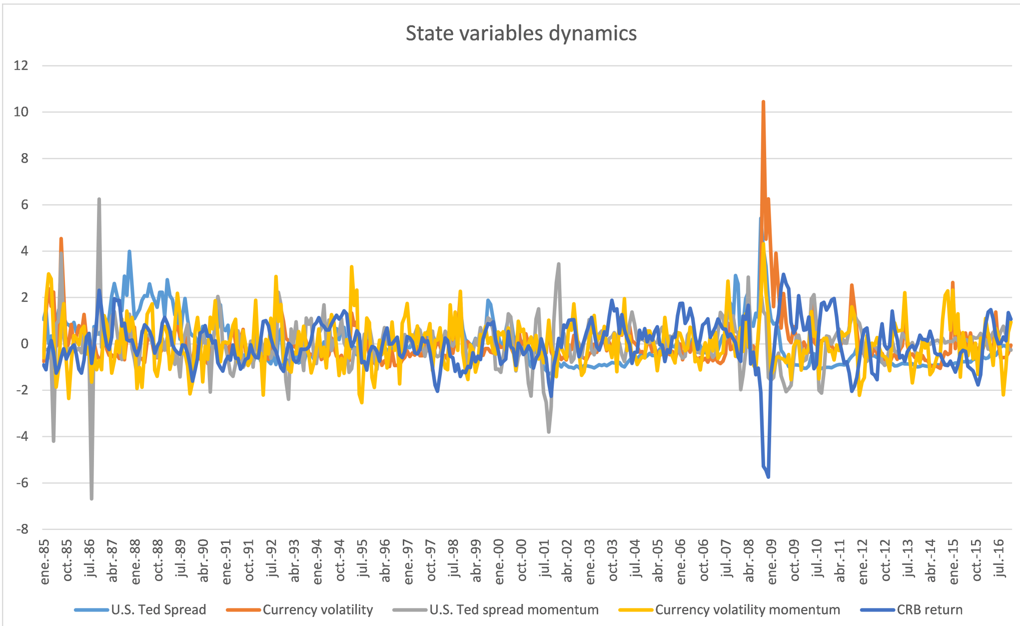


Figure 2: Dynamics of terminal wealth

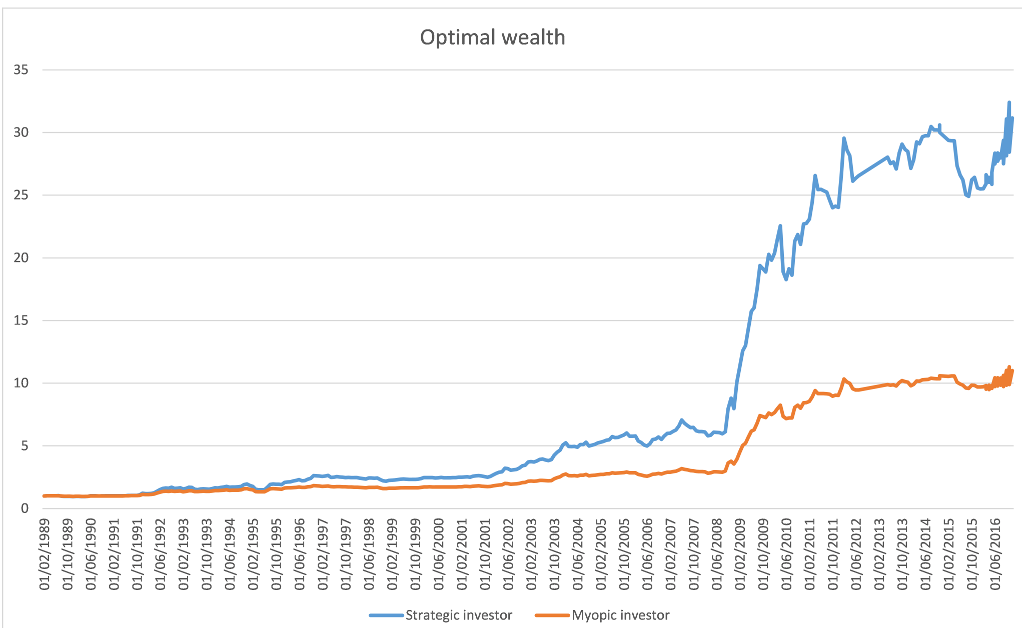
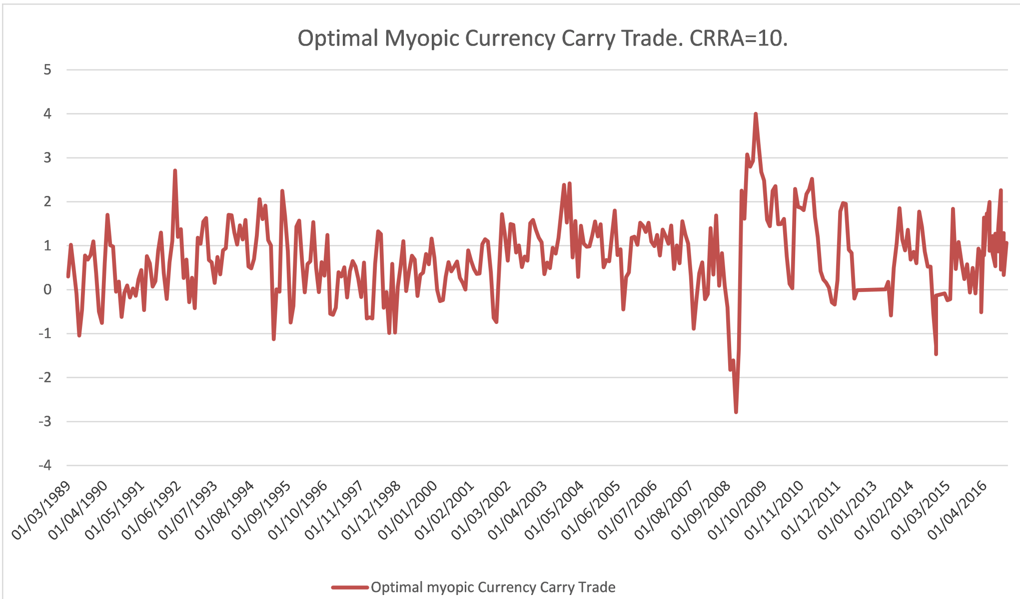


Figure 3: Dynamics of mean hedging demand



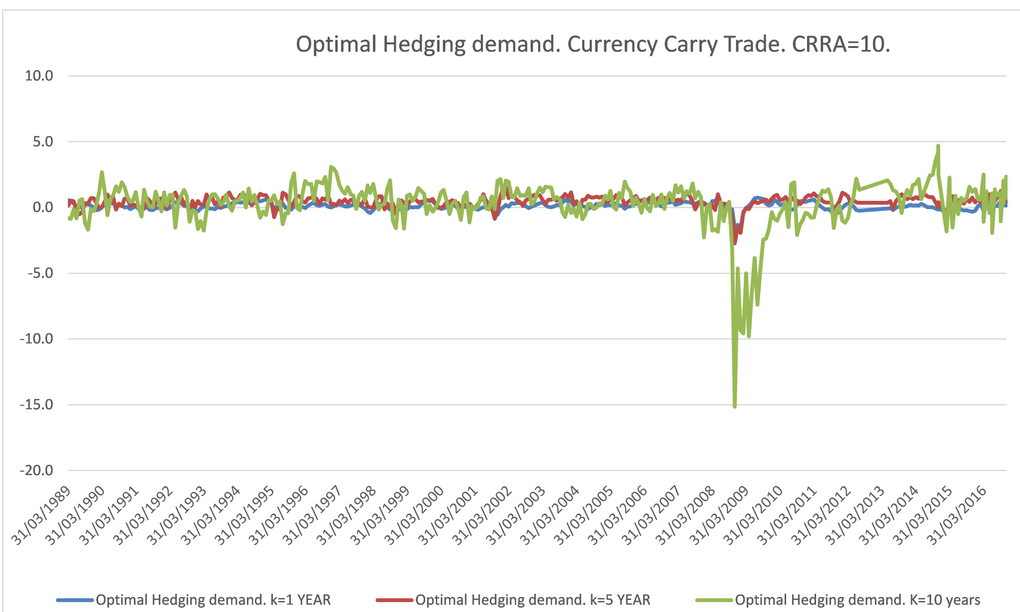


Figure 4: Out-of-sample parameter estimates for the optimal currency carry trade strategy