UNIVERSITY OF SOUTHAMPTON

FACULTY OF SOCIAL, HUMAN & MATHEMATICAL SCIENCES

DIVISION OF SOCIAL STATISTICS AND DEMOGRAPHY

Nonignorable Nonresponse Adjustment using Fully Nonparametric Approach

by

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Abstract

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Nonresponse is an increasingly common problem in surveys. It is a problem because it causes missing data and, more importantly, because such missing data are a potential source of bias for estimates. Most of the methods dealing with nonresponse assume either explicitly or implicitly that the missing values are missing at random (MAR). We consider the situations where the probability to respond may depend on the outcome value even after conditioning on the covariates. For this kind of response mechanism, the missing outcomes are not missing at random (NMAR). The problem of missing data is handled either using fully parametric or semi-parametric approaches. These approaches have some potential issues, for example, strict distributional assumptions, heavy computations, etc.

We propose a fully non-parametric approach; first we postulate *informative individual* response probabilities i.e. the response probability may depend on the values of interest, and it may be specific to each individual. We treat the outcome variable as a fixed constant just like in the design based approach to survey sampling. Then we use an estimating equations approach to define the finite population parameters. Hence the approach is fully non-parametric provided the individual specific response probabilities can be estimated non-parametrically. For longitudinal data it is possible that one can have individual historic response rate and those can be used as an empirical estimator for the individual specific response probability. We utilize this individual historic response rate as an estimator for the unknown response probability. If the unknown response probability is consistently estimated then the proof for consistency of estimators is much easier and much more common. But in our case the historic response rate is unbiased but not consistent because practically we cannot have infinitely many historic time points but we can have many units. We try to prove the asymptotic unbiasedness of estimating equations and further the consistency of estimates but we could not prove it and the reason is discussed in Section 2.4. It provides an interesting investigation of pursing consistency. We develop the associated variance estimator. Being a fully non-parametric and computationally simple method, it can be used as a widely applicable exploratory data analysis technique for

NMAR mechanisms, as long as there exit a response history, in advance of more sophisticated and possibly more efficient modelling methods.

The approach is extended for a longitudinal setting and two types of EEs are defined to estimate parameters that are defined over time, such as the change between two successive time points or the regression coefficients involving outcomes over time. The associated variances estimators using both EEs are also developed.

The *non-parametric estimating equations* (NEE) approach for cross-sectional and longitudinal setting is not unbiased. We therefore develop bias-adjusting NEE approach to adjust the bias in cross-sectional and longitudinal parameter estimates. Another advantage of the bias-adjusting EE approach is that the variance estimator based on bias-adjusting NEE is expected to be less biased as compared to the unadjusted approach. Moreover, Taylor expansion is used to adjust the bias in variance estimate obtained from simple and bias-adjusted NEE approaches.

A comprehensive simulation study is conducted using real and simulated data to assess the performance of NEE and bias-adjusted NEE approaches under various settings for cross-sectional as well as for longitudinal data.

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Declaration of Authorship

I, Zahoor Ahmad
declare that this thesis and the work presented in it are my own and has been generated by me as the result of my own original research.

Title of thesis: Nonignorable Nonresponse Adjustment using Fully Nonparametric Approach

I confirm that:

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Chapter 1

Introduction

Nonresponse is an increasingly common problem in surveys. It is a problem because it causes missing data and, more importantly, because such missing data are a potential source of bias for estimates. In the presence of unit nonresponse, it is often assumed that each unit in the population has an associated probability to respond. Such a response probability is unknown and several methods are proposed to estimate it either explicitly, using response propensity modelling like logistic regression models or implicitly, using response homogeneity groups or more generally calibration (see Sarndal and Lundstrom (2005), for an overview). Once estimates are computed, a commonly used method to deal with unit nonresponse is reweighting: sampling weights of the respondents are adjusted by the inverse of the estimated response probability providing new weights. Estimation of response probabilities typically requires the availability of auxiliary information, either in the form of the value of some auxiliary variables for all units in the originally selected sample or of their population mean or total.

Next, missing data mechanisms concern the relationship between the response indicators and the values of the variables in the corresponding data matrix. Missing data mechanisms are usually categorized into three classes: missing completely at random (MCAR), missing at random (MAR), and missing not at random (MNAR). If the response indicators are unrelated to both the missing outcomes and the set of observed outcomes, the observed outcomes are a random subset of the entire sample. This is referred to as MCAR. If the response indicators depend on the observed outcomes (and other auxiliary variables) but are otherwise unrelated to the missing values, the missing data are said to be MAR. MAR mechanisms are most commonly assumed in statistical analysis including longitudinal data analysis. However, in many situations, the response indicators are related to the missing values, even after controlling for all the observed values, referred to as MNAR. Ignoring the impact of the MNAR mechanism can result in serious bias of inference. Over the years, a variety of models and methods have been developed to account for MNAR mechanisms in longitudinal data analysis.

When covariates are known for every sample unit, a common way to deal with the nonresponse is to postulate a parametric model for the joint distribution of the outcome variable and response indicator given the covariates. Little and Rubin (2002) distinguish between selection models and pattern-mixture models, depending on how the joint distribution is factorized. For fully *parametric* selection models, the likelihood based on all the units, respondents or not, can be used to estimate the parameters of the model for the outcome variable as well as the model for the response probability given the outcome variable (and covariates). Qin et al. (2002) propose a semi-parametric estimation method for the case where the covariates are only known for the respondents. They assume a parametric model on the response mechanism but a non-parametric model on the distribution of the outcome variable and the covariates. Pfeffermann and Sikov (2011) propose a fully parametric estimation approach for NMAR nonresponse, which does not require knowledge of the covariates for the nonrespondents.

Longitudinal data analysis is of great interest in a wide array of disciplines across the medical, economic and social sciences. Cross-sectional data can only provide a snapshot at a single point of time and does not possess the capacity to reflect change, growth, or development. Aware of the limitations in cross-sectional studies, many researchers have advanced the analytic perspective by examining data with repeated measurements. By measuring the same variable of interest repeatedly over time, the change is displayed, and constructive findings can be derived with regard to the significance of pattern revealed. Data with repeated measurements are referred to as longitudinal data. In many longitudinal data designs, subjects are assigned specified levels of a treatment or subjected to other risk factors over a number of time points that are separated by specified intervals. Analysing longitudinal data poses many challenges due to several unique features inherent in such data. The most troublesome feature of longitudinal analysis is missing data in repeated measurements. There is an enormous literature on literature missing data methods in longitudinal studies. We refer the reader to the excellent books by Diggle et al. (2002), Fitzmaurice et al. (2004), Verbeke and Molenberghs (2000), Verbeke and Molenberghs (2005), Molenberghs and Kenward (2007), Daniels and Hogan (2008), Fitzmaurice et al. (2008), and the many references therein. Ibrahim and Molenberghs (2009) provide a review on missing data approaches in longitudinal studies. Most of the literature focuses on maximum likelihood methods of estimation with nonignorable missing longitudinal data, predominantly focusing on mixed-effects models and normally distributed outcomes. A substantial part of the literature also assumes monotone patterns of missingness, where sequences of measurements on some subjects simply terminate prematurely. Approaches using selection models include Diggle and Kenward (1994), Little (1995), and Ibrahim et al. (2001). Approaches based on pattern-mixture models include Little (1994, 1995), Little and Wang (1996), Hogan and Laird (1997), and Thijs et al. (2002). Troxel et al. (1998a) and Troxel et al. (1998b) propose a selection model which is valid for nonmonotone missing data but is intractable for more than three time points.

While discussing the semi-parametric approaches, Kim and Yu (2011) proposed the exponential tilting model and developed a semiparametric estimation procedure for nonignorable missing data. Tang et al. (2014) further extended the idea of Kim and Yu (2011). Zhao and Shao (2015) proposed a pseudo likelihood approach to generalized linear models in the presence of nonignorable missing data, and presented a two-step iteration algorithm to implement the numerical maximization of the pseudo likelihood. Matei and Ranalli (2015) proposed a latent modeling approach to deal with non-ignorable nonresponse in survey sampling. Feder and Pfeffermann (2016) discussed the use of empirical likelihood while dealing with NMAR nonresponse along with informative sampling and indicated that the empirical likelihood approach has the computational advantages over fully parametric approaches.

There is a large literature on the use of estimating equations (EE); see, for example, Godambe (1991a), Liang and Zeger (1995), Hardin and Hilbe (2003) and Zhou et al. (2008). Robins et al. (1994) suggested a semi-parametric approach based on inverse response-probability weighted EE. It is based on the assumption that the probability of nonresponse is either known or can be modelled parametrically. FitzGerald (2002) introduced a weighting method for handling missing data in generalized estimating equations (GEE) analysis. The method relies also on the specification of a parametric nonresponse model.

All the aforementioned NMAR techniques require a parametric model for the response probability, regardless how the outcome variable is modelled. In this thesis we present a *non-parametric EE (NEE)* estimation approach for cross-sectional as well as longitudinal data analysis, where we neither specify a parametric model for the response probability nor the outcome variable. This can provide a useful, flexible alternative to the existing methods. The basic idea can be outlined below in Section 1.2.

1.1 Motivation of the Study

As we discussed above that the problem of NMAR nonresponse is handled either using fully parametric or semi-parametric approaches and these approaches have some potential issues, for example, strict distributional assumptions, heavy computations, etc.

We propose a fully non-parametric approach in the sense that; first we postulate *informative individual* response probabilities i.e. the response probability may depend on the values of interest, and it may be specific to each individual. For example, the actual response at a business could depend on the accounting system, the person responsible for the response, etc. all of which can potentially be related to the size of the business and hence possibly the response variable y of interest, beyond whatever covariates x that are available. Meanwhile, there is bound to be some stability over a limited time period. For such a scenario, one assumes the response probability to be individual (hence, informative) but stable over a given period of time. Each individual response probability is an unknown parameter and the number of parameters increases by increasing the population size. Second we treat the outcome variable as a fixed constant just like in the design based approach to survey sampling. Then we use an estimating equations approach to define the finite population parameters. Hence the approach is fully non-parametric provided the individual specific response probabilities can be estimated non-parametrically.

For longitudinal data it is possible that one can have an individual historic response and this can be used as an empirical estimator for the individual specific response probability. We utilise this individual historic response rate as an estimator for the unknown response probability and develop NEE approach to estimate population parameters for both cross-sectional and longitudinal data under informative nonresponse. We cannot claim that this is the only alternative to the above discussed parametric and semi-parametric approaches while dealing with informative missing data. This is totally a new idea of dealing with informative missing data using a fully non-parametric approach. In this thesis our aim is to develop the basic theory of the NEE approach and to explore it as potential alternative to the many existing approaches.

We extended this approach for the estimation of longitudinal parameters with some additional treatment. The associated variance estimators are given for both settings. The approach is as such applicable to the longitudinal missing data to estimate cross-sectional and longitudinal parameters. The general outline of approach is given below.

1.2 An outline of NEE approach

In this section we outline a new NEE approach to cross-sectional and longitudinal data with MNAR nonresponse, the details of which will be developed in the subsequent Sections. Under the NEE approach to MNAR nonresponse, we do not assume a parametric model of the response probabilities that pertain to all the population units. To accommodate potentially informative missing data, we postulate an individual response probability which may depend on the longitudinal outcomes of interest and covariates specific to each observational unit. The individual response probability can be considered as a propensity of observation that accounts for the initial sample selection mechanism in addition, which may be probability sampling or nonrandom or informative itself. That is, the response indicator is the product of sample inclusion indicator and survey response indicator. The outcome values are also treated non-parametrically as unknown constants, just like in the design-based approach to survey sampling. Under this set-up, the observation propensity is estimated using individual-specific observation history, without involving the others in the population. The approach is applicable whenever there exist historical response/observation indicators. In other words, any unit who never responds will not be included in the estimation

To enable inference regarding the population mean and regression coefficients about the never respondents based on the sometime respondents, we make the following assumption. For cross sectional setting, while considering the population mean, we can assume that the population mean of never-respondents is the same as that of the sometime-respondents, or so conditional on some appropriate auxiliary variables. For regression coefficients, we can assume that the regression model that holds for never-respondents is the same as for the sometime-respondents. For the longitudinal setting, while considering the population mean change parameter, we can assume that the population mean change of never-respondents is the same as that of the population of sometime-respondents, or so conditional on some appropriate auxiliary variables that need to be same for both time points. For change in population regression coefficients, we can assume the same regression model holds for never-respondents and sometime-respondents for each time point.

Let the target of estimation be given as a finite population parameter defined in terms of a *population EE*. For its estimation we use the *observed (respondent) NEE*, where the unknown individual response propensity is replaced by an estimate based on the response history of the same individual. For instance, one may use the observed historic response rate for a unit to estimate its individual response probability, under the assumption that the unknown response probability is "stable" over the given period of time. There can be different assumptions of the exact nature of such stability over time, e.g. stable before the dropout for a unit with monotone missing data pattern, but over the entire history for someone with a nonmonotone pattern. Or, for example, unknown response probability have a trend then still we can make this assumption and assume the model congenial to this assumption. We can fit model like that and put time in to it. The key point is that we are estimating every body individually. We can allow different assumptions for each individuals even. For someone we can allow to fit a linear trend and for some we use the stable assumption if we want. Of course if our assumption is wrong then their is some limitation that the results will not be okay and that can be checked by simulations. We don't have to be stable, its just an easy illustration of NEE to start. Our approach is not like that, if one have stable response then only one can apply this, if one doesn't have stableness one can't apply. No, rather to the contrary, because this approach is completely individual it allows to use different models, different assumption for different individuals. One doesn't need to have a single model to cover every one. This is actually the strength of the flexibility of our approach compared to the existing parametric approaches.

To focus the idea, NEE-based estimators for mean at current wave and two different NEE-based estimators for the *change* between any two waves will be discussed in forthcoming relevant chapters, although the NEE formulation accommodates many other types of analysis, such as estimation of regression coefficients or analysis of variance.

While the estimator of the individual response probability can be unbiased according to the given assumption, it can never be consistent due to the fact that the response history cannot be infinitely long for anyone. Moreover, the plug-in observed NEEs will be somewhat biased if the 'score-term' in the population EE is correlated with the response propensity, as in the case of informative nonresponse. The matter will be considered in forthcoming chapters concerning cross-sectional setting as well as longitudinal settings. There we consider bias in estimates, possible venues for bias adjustment, the associated variance estimation. We illustrate and investigate the performance of the NEE approach under both settings using simulation study. Finally, a summary of the conclusions are given for each chapter.

1.3 Study Achievements

We developed a fully non-parametric estimating equation approach to accommodate potentially informative missing data and we postulate an individual response probability which may depend on the longitudinal outcomes of interest and covariates specific to each observational unit. The individual response probability is estimated using individual historic response. The key point is that we are estimating every body individually. We can allow different assumptions for each individuals even. This is actually the strength of the flexibility of our approach compared to the existing parametric approaches. Currently we assume the stable response assumption and the response probability is estimated using historic response rate. This simple empirical estimator is used to estimate the parameters under informative nonresponse. We also develop the associated variance estimator. Compared to alternative fully or semi-parametric approaches, our approach is simple in construction and easy in computation and does not depend on strict distributional assumptions about the outcome variable, and the explicit/parametric form of the response probability model.

The approach is extended for a longitudinal setting and two types of EEs are defined to estimate parameters that are defined over time, such as the change between two successive time points or the regression coefficients involving outcomes over time. Theoretical properties of both EEs are established.

In our case we have biased estimating equations. We therefore develop bias-adjusted EE approach to reduce the bias in estimates of cross-sectional and longitudinal parameters. Another advantage of bias-adjusted EE approach is that the variance estimator based on bias-adjusting NEE is expected to be less biased as compared to the naïve NEE, because we used bias-adjusted EE to obtain variance and its plug-in estimator and then Taylor expansion is used on this bias-corrected estimator to further correct the bias.

A comprehensive simulation study is conducted to assess the performance of the NEE and bias-adjusted NEE approaches for various simulation settings using real as well as simulated data under both cross-sectional and longitudinal settings.

1.4 Outline of the remaining chapters

The aim of the thesis is to develop a fully non-parametric estimating equations approach for cross-sectional and longitudinal missing data. The theoretical development of the approach and its application is explained in the following chapters.

Chapter 2 cover the NEE approach that is based on the cross-sectional setting. In this Chapter, we postulate the individual specific response probability model for the response indicator and then propose a response probability estimator based on the individual historic response rate. The finite population EEs are developed to define the finite population parameters and then EEs based on unknown response probabilities are provided. The observed EEs based on estimated response probabilities are also defined and this EEs are used to estimate the parameters. The NEE is not unbiased because we are using estimated response probabilities, and hence the bias is also derived for the EEs. We try for the consistency of the estimators, however that can not be proved but an interesting effort is given. The corresponding variances of the estimators are derived and its plug-in estimators are also given. A comprehensive simulation study for the cross-sectional setting is also given using real data as well as simulated data.

In Chapter 3, we extend the NEE approach from the cross-sectional setting to the longitudinal setting and define two NEEs; first, the NEE that uses the individuals who respond at both time points and second, NEE that uses also the individuals who respond at only one of the two time points. We also defined the unknown response probability models for the respective NEEs. To capture different dropout patterns underlying the assumed models, different response probability estimators are suggested. The corresponding variance of estimators are derived and their plug-in estimators are given for both types of NEEs. A simulation study using real data as well as simulated data is also given.

The observed NEEs for cross-sectional and longitudinal settings are not unbiased and the bias is discussed in their respective Chapters. In Chapter 4, we provide the bias-adjusting NEE approach for both cross-sectional and longitudinal settings. Here we define only the observed bias-adjusted NEEs for both settings. The aim of defining the bias-adjusted EE is to reduce the bias in estimates and possibly the bias in variance estimates using different venues for bias adjustment. The other settings for bias-adjusted EEs remain the same as for their corresponding simple NEEs discussed in previous chapters. The variance of estimators are derived and their plug-in estimators are given for bias-adjusted NEEs. The bias in plug-in estimators is further corrected using Taylor expansion of the variance estimator using the simple NEE and the bias-adjusted NEE. The simulation study using real data as well as simulated data is also given for the bias-adjusted NEE approach.

We conclude the main outcomes of the study in Chapter 5. The main findings of simple NEE for cross-sectional and longitudinal settings are discussed and then discussion on comparison of simple and bias-adjusted NEEs for both settings is also covered in this Chapter. The strengths and limitations of the works are also discussed along with some future research directions.

Chapter 2

Non-parametric Estimating Equations Approach for Cross-sectional Data

In the previous chapter we provide a general outline of our approach to handle informative missing data for the cross-sectional and longitudinal setting in Section 1.2. In this chapter we provide the theoretical detail on the approach under the cross-sectional setting. After providing the general outline of NEE approach above in Section 1.2, the set-up given below in Section 2.6.3 explains the model for response probabilities, their estimator and the way forward to use NEE approach for estimation of the finite population mean and regression coefficients. In Section 2.2 we discuss the asymptotic properties of the theoretical estimator and an effort on the consistency of actual estimator is discussed in Section 2.4. The hypothetical estimator is based on the known response probability that actually cannot exist in reality but having the population data we used it to know the reasons for bad performances of the actual estimator that is based on estimated response probabilities. The variance and its plug-in estimator of the actual estimator is given in Section 2.5 followed by a simulation study using real and simulated data.

2.1 NEE approach

Let $U = \{1, ..., N\}$ be the target finite population, and let y_i be the variable of interest, for $i \in U$. Let $\delta_i = 1$ indicate response, in which case one observes y_i, and $\delta_i = 0$ if y_i is missing. Under the NEE approach, y_i is treated as a fixed constant. *Informative missing* is the case provided $\Pr(\delta_i = 1|y_i = y) \neq \Pr(\delta_i = 1|y_i = y')$ for $y \neq y'$. For a *flexible* model that accommodates informative missingness, put

$$\Pr(\delta_i = 1|y_i, x_i) = \pi_i \tag{2.1}$$

i.e. each unit is allowed its own individual response probability. Clearly, the model as such is unidentifiable. Now, suppose that there exists data of response on T occasions, denoted by $(\delta_{i1}, \ldots, \delta_{iT})$ for i. Then, under the assumption π_i is the same on all these T occasions, an unbiased estimator is given by

$$\hat{\pi}_i = \sum_{t=1}^{T} \delta_{it}/T, \tag{2.2}$$

To show that $\hat{\pi}_i$ is an unbiased estimator of π_i, note that $\delta_{it} \sim Bernoulli\,(\pi_i)$ where δ_{it}'s are assumed to be independent of each other for different t's and π_i is unknown response probability that is same for each time point under stable response assumption. Now we have $E\left(\hat{\pi}_i|T, \delta_{iT} = 1\right) = \sum_{t=1}^{T} E\left(\delta_{it}|\pi_i, \delta_{iT} = 1\right)/T = T\pi_i/T = \pi_i$.

The δ_{iT} is the current time point at which the parameters will be estimated but for simplicity below we denote the current time point with δ_i.

The EE-approach is developed based on these estimates of response probabilities for all $i \in U$ to estimate the finite population parameters such as the mean and regression coefficients.

Let θ_0 be a finite population parameter defined as the solution to the following estimating equations

$$H_N\left(\theta\right) = N^{-1} \sum_{i=1}^{N} S_i\left(\theta\right); \quad H_N\left(\theta_0\right) = 0, \tag{2.3}$$

where $S_i()$ is a scalar or vector function with the y-values considered fixed. It is the contribution to the EE from the *i-th* unit. The (unobserved) estimating equations based on the responding units are given by,

$$\tilde{H}_N\left(\theta\right) = N^{-1} \sum_{i=1}^{N} \frac{\delta_i}{\pi_i} S_i\left(\theta\right) \quad and \quad \tilde{H}_N(\tilde{\theta}) = 0, \tag{2.4}$$

where π_i denote the unknown response probability at each time point under stable response assumption and $\tilde{\theta}$ is the theoretical estimator of θ. Now replacing the conditional unbiased estimator $\hat{\pi}_i$ of π_i given $\delta_i = 1$ in (2.4), we have the respondents plug-in NEE,

$$\hat{H}_N\left(\theta\right) = N^{-1} \sum_{i=1}^{N} \frac{\delta_i}{\hat{\pi}_i} S_i\left(\theta\right) \quad and \quad \hat{H}_N(\hat{\theta}) = 0. \tag{2.5}$$

In particular, if only historical responses are used, i.e. excluding the current $\delta_i = 1$ while computing the $\hat{\pi}_i$, one can define $\hat{\pi}_i^{-1} = 0$ if $\sum_{t=1}^{T-1} \delta_{it} = 0$.

The population parameters are defined by the estimating equations (2.3) and the basis of inference is the model for the response indicator given in (2.1). When the census estimating equations (2.3) are the likelihood equations, the estimators obtained by solving these EE with known inclusion probabilities are known in the sampling literature as 'pseudo mle' (pmle). See Binder (1983), Skinner et al. (1989), Pfeffermann (1993), Pfeffermann (1996) and Godambe and Thompson (2009) for discussion with many examples. One might draw an analogy between the observed EE and the pseudo-MLE approach; but the S_i is not necessarily derived from likelihood, and the $\hat{\pi}_i$ is estimated instead of known.

Notice that our approach is fully non-parametric and basis of inference is the model for response and only the δ_i is treated as random variable. The Rubin (1976) theory of nonresponse mechanism may not be completely applied here because we are estimating the response probability using the response rate rather fitting a model. Although we assume an informative model for response given in (2.1), the estimate of the response probability is based on the response rate and if only past responses are used, then $\hat{\pi}_i$ looks no different to MAR-weighting class adjustment. When the current response indicator is used, $\hat{\pi}_i$ still looks like a weighting-class adjustment, but nonresponse can be NMAR by virtue of (2.1).

Below we illustrate the EE approach for estimation of the finite population mean, variance and regression coefficient in following examples.

Example-1 (Estimation of Population Mean): To illustrate the estimating equations approach for estimation of a finite population mean θ, by finite population EE (2.3), the θ can be defined as

$$\sum_{i=1}^{N}(y_i-\theta)=0 \implies \theta_{\mathrm{FP}}=\frac{1}{N}\sum_{i=1}^{N}y_i$$

where the function $y_i-\theta$ is not necessarily derived from the likelihood. To estimate θ using (2.5), the observed estimating equations along with expression of estimator can be written as

$$\sum_{i=1}^{r}\hat{w}_i\left(y_i-\theta\right)=0 \implies \hat{\theta}=\sum_{i=1}^{r}\hat{w}_iy_i/\sum_{i=1}^{r}\hat{w}_i$$

where $\hat{w}_i=\hat{\pi}_i^{-1}$.

Example-2 (Estimation of Population Mean and Variance): To illustrate the estimating equations approach for estimation of the finite population mean and variance, suppose y_i follow normal distribution with mean θ and variance σ^2. Then the density $f_p(y_i)$ for the population can be written as

$$f_p\left(y_i\right)=\frac{1}{\sqrt{2\pi\sigma^2}}exp\left\{-\frac{1}{2}\left(\frac{y_i-\theta}{\sigma}\right)^2\right\}.$$

The log-likelihood function is

$$log(L)=-\frac{N}{2}log(\sigma^2)-\frac{N}{2}log(2\pi)-\frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(y_i-\theta\right)^2.$$

The census parameters θ and σ^2 are defined by the finite population EE given in (2.3), as

$$\sum_{i=1}^{N}\left(y_i-\theta\right)/\sigma^2=0 \implies \theta_{\mathrm{FP}}=\frac{1}{N}\sum_{i=1}^{N}y_i$$

$$\sum_{i=1}^{N}\left\{\left(y_i-\theta\right)^2/\sigma^4-1/\sigma^2\right\}=0 \implies \sigma^2_{\mathrm{FP}}=\frac{1}{N}\sum_{i=1}^{N}\left(y_i-\theta\right)^2,$$

where the functions $\left(y_i-\theta\right)/\sigma^2$ and $\left(y_i-\theta\right)^2/\sigma^4-1/\sigma^2$ are not necessarily derived from the likelihood. To estimate θ and σ^2 using (2.5), the response estimating equations along with expression of estimators can be written as

$$\sum_{i=1}^{r} \hat{w}_i \left(y_i - \theta\right) = 0 \implies \hat{\theta} = \sum_{i=1}^{r} \hat{w}_i y_i / \sum_{i=1}^{r} \hat{w}_i$$

$$\sum_{i=1}^{r} \hat{w}_i \left\{\left(y_i - \theta\right)^2 - \sigma^2\right\} = 0 \implies \hat{\sigma}^2 = \sum_{i=1}^{r} \hat{w}_i \left(y_i - \theta\right)^2 / \sum_{i=1}^{r} \hat{w}_i.$$

Example-3 (Estimation of Regression Coefficients): To illustrate the estimating equations approach for estimation of regression coefficients, suppose the density of the population is

$$f_p\left(y_i|x_i\right) = \frac{1}{\sqrt{2\pi\sigma^2 x_i}} exp\left\{-\frac{1}{2}\left(\frac{y_i - \beta_0 - \beta_1 x_i}{\sigma\sqrt{x_i}}\right)^2\right\}.$$

The log-likelihood function can be written as

$$log(L) = -\frac{N}{2} log(\sigma^2) + \sum_{i=1}^{N} log(1/\sqrt{x_i}) - \frac{N}{2} log(2\pi) - \frac{1}{2\sigma^2}\sum_{i=1}^{N}\left(\frac{y_i - \beta_0 - \beta_1 x_i}{\sqrt{x_i}}\right)^2.$$

The census parameters β_0, β_1 and σ^2 are defined by the finite population estimating equations (2.3) as

$$\sum_{i=1}^{N} \frac{1}{\sigma^2 x_i}\left(y_i - \beta_0 - \beta_1 x_i\right) = 0,$$

$$\sum_{i=1}^{N} \frac{1}{\sigma^2}\left(y_i - \beta_0 - \beta_1 x_i\right) = 0,$$

$$\sum_{i=1}^{N}\left\{\frac{1}{\sigma^4}\left(\frac{y_i - \beta_0 - \beta_1 x_i}{\sqrt{x_i}}\right)^2 - \frac{1}{\sigma^2}\right\} = 0.$$

Now, solving the above equations, the finite population parameters are

$$\beta_{\mathrm{FP}} = \left(\dot{X}_N^\top \dot{X}_N\right)^{-1} \dot{X}_N \dot{Y}_N$$

$$SD\left(\beta_{\mathrm{FP}}\right) = diag\left(\sqrt{\sigma^2\left(\dot{X}_N^\top \dot{X}_N\right)^{-1}}\right)$$

$$\sigma^2_{\mathrm{FP}} = \frac{1}{N}\sum_{U}\left[\frac{1}{x_i}\left(y_i - \beta_0 - \beta_1 x_i\right)^2\right] = \frac{\epsilon_N^\top W_N \epsilon_N}{N},$$

where $\epsilon_N = (y_i - x_i\beta_{FP})$ and $W_N = diag\left[1/x_i\right]$, $X_N = [1 \ x_i]_{N\times 2}$, $\dot{X}_N = W_N X_N$, $\dot{Y}_N = W_N Y_N$

and $\beta_{\text{FP}} = (\beta_0, \beta_1)^\top$. From (2.5), the observed estimating equations can be written as

$$\sum_{i=1}^{r} \frac{\hat{w}_i}{\sigma^2 x_i} (y_i - \beta_0 - \beta_1 x_i) = 0$$
$$\sum_{i=1}^{r} \frac{\hat{w}_i x_i}{\sigma^2 x_i} (y_i - \beta_0 - \beta_1 x_i) = 0$$
$$\sum_{i=1}^{r} \hat{w}_i \left\{ \frac{1}{\sigma^4} \left(\frac{y_i - \beta_0 - \beta_1 x_i}{\sqrt{x_i}} \right)^2 - \frac{1}{\sigma^2} \right\} = 0.$$

Solving above estimating equations, the resulting estimators are

$$\hat{\beta} = \left(\dot{X}_r^\top \dot{X}_r \right)^{-1} \dot{X}_r \dot{Y}_r$$
$$SE\left(\hat{\beta}\right) = diag\left(\sqrt{\sigma^2 \left(\dot{X}_r^\top \dot{X}_r \right)^{-1}} \right)$$
$$\hat{\sigma}^2 = \frac{1}{\sum_r \hat{w}_i} \sum_r \left[\frac{\hat{w}_i}{x_i} (y_i - \beta_0 - \beta_1 x_i)^2 \right] = \frac{\hat{\epsilon}_r^\top \hat{W}_r \hat{\epsilon}_r}{\sum_r \hat{w}_i},$$

where $\hat{\epsilon}_r = (y_i - x_i \hat{\beta})$ and $\hat{W}_r = diag\left[\hat{w}_i / x_i\right]$, $\dot{X}_r = \hat{W}_r X_r$, $\dot{Y}_r = \hat{W}_r\ Y_r$, $X_r = [1\ x_i]_{r \times 2}$ and $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1)^\top$.

2.2 Consistency and CLT of $\tilde{\theta}$

Below we consider first the theoretical properties of the hypothetical estimator $\tilde{\theta}$ that is the solution of EE $\tilde{H}_N(\theta)$ given in (2.4). The $\tilde{\theta}$ is the solution of hypothetical EE in which response probability π_i is known and y is assumed fixed like designed based approach in survey sampling. Our estimator is like an HT estimator and in literature the CLT and consistency of HT estimator is much more common where inclusion probability is known. And similarly if the response probability is obtained using logistic model, then the response probability can be estimated consistently and the CLT and consistency of estimators is also straightforward. We can prove the consistency of $\tilde{\theta}$ using common procedure but we use an approach given by Foutz (1977) because we investigate the consistency of $\hat{\theta}$ using the same approach below in Section 2.4 that is not straightforward. The detail is given in Section 2.4.

The consistency and CLT of $\tilde{\theta}$ is proved below in subsequent two sections.

2.2.1 Consistency of $\tilde{\theta}$

Using the approach of Foutz (1977), the following Lemma (2.2.1) states that there exists a unique consistent solution of $\tilde{H}_N(\theta)$.

Lemma 2.2.1 *With probability going to one as $N \to \infty$, an estimator $\tilde{\theta}$ which satisfies*

(A) The elements of the matrix $\tilde{H}'_N(\theta) = N^{-1} \sum_{i=1}^{N} (\delta_i/\pi_i) \partial S_i(\theta)/\partial \theta$ exist and are continuous on Θ;

(B) $\tilde{H}'_N(\theta) - \tilde{G}_N(\theta)$ *converges to* 0 *in probability, where* $\tilde{G}_N(\theta) = E[\tilde{H}'_N(\theta)]$*;*

(C) The matrix $\tilde{H}'_N(\theta)$ *evaluated at the true parameter* θ_0 *is negative definite with probability converging to one as* $N \to \infty$*;*

(D) The estimating equations $\tilde{H}_N(\theta)$ *are unbiased, i.e. at the true parameter* θ_0*,* $E[\tilde{H}_N(\theta_0)] = H_N(\theta_0) = 0$*;*

exists and is unique. Moreover, $\tilde{\theta}$ *is consistent for the true parameter* θ_0*.*

We investigate consistency for $\hat{\theta}$ below in section 2.4 in detail by following the general idea of Foutz (1977) approach. The first three conditions (A)–(C) for $\tilde{H}_N(\theta)$ can be discussed on the same lines as given below in section 2.4.1. For condition (D), we can write,

$$E[\tilde{H}_N(\theta_0)] = \frac{1}{N}\sum_{i=1}^{N} E\left(\frac{\delta_i}{\pi_i}\right) S_i(\theta_0) = \frac{1}{N}\sum_{i=1}^{N} S_i(\theta_0) = H_N(\theta_0) = 0$$

which implies that $\tilde{H}_N(\theta)$ are unbiased.

2.2.2 CLT of $\tilde{\theta}$

In the literature, a CLT of estimators based on estimating equations is given when y is random. We consider the finite population estimating equations in which $y_i's$ are considered fixed. Below we prove the CLT of theoretical estimator $\tilde{\theta}$ that is solution of the estimation equation given in (2.4). Suppose $W_i = \frac{\delta_i}{\pi_i} S_i(\theta)$ then W_i are non-IID random variable. Below we will prove CLT for scalar $\tilde{\theta}$.

Lemma 2.2.2 *(CLT of* $\tilde{H}_N(\theta)$*): Let* $W_i = \delta_i S_i(\theta)/\pi_i$ *be independently distributed with means* $E(W_i) = \xi_i$ *and variances* σ_i^2 *and with finite third moments. Asymptotically, as* $N \to \infty$*,*

$$Z_N(\theta) = \frac{\tilde{H}_N(\theta) - H_N(\theta)}{\sqrt{Var(\tilde{H}_N(\theta))}} = \frac{\sqrt{N}(\tilde{H}_N(\theta) - H_N(\theta))}{\sqrt{\sum_{i=1}^{N} \sigma_i^2/N}} \xrightarrow{D} N(0,1).$$

Proof : We can write

$$E(W_i) = S_i(\theta) E\left\{(\delta_i/\pi_i)|\pi_i\right\} = S_i(\theta) = \xi_i$$

and

$$Var(W_i) = \left\{\frac{1-\pi_i}{\pi_i} + \frac{V(\pi_i)}{\pi_i^3}\right\} S_i^2(\theta)$$

and let

$$s_N^2 = \sum_{i=1}^{N} Var(W_i) = \sum_{i=1}^{N} \sigma_i^2. \tag{2.6}$$

Using Lyapounov theorem,

$$Z_N(\theta) = \frac{\sqrt{N}(\tilde{H}_N(\theta) - H_N(\theta))}{\sqrt{s_N^2/N}} \xrightarrow{D} N(0,1), \tag{2.7}$$

provided,

$$\left(\sum_{i=1}^{N} E|W_i - \xi_i|^3\right)^2 = o\left[(s_N^2)^3\right]. \quad (\textit{Lehmann}\ 2004,\ pp-97) \tag{2.8}$$

To prove condition (2.8), from corollary 2.7.1 (Lehmann 2004, pp-98), suppose W_i are uniformly bounded, i.e. there exists a constant A such that, $|W_i| \leq A$ for all i. Then we can write

$$\sum_{i=1}^{N} |W_i - \xi_i|^3 \leq 2A \sum_{i=1}^{N} E(W_i - \xi_i)^2.$$

and hence

$$E\left[\sum_{i=1}^{N} |W_i - \xi_i|^3\right] \leq (2As_N^2) \tag{2.9}$$

and therefore left hand side of (2.8) is $\leq 4A^2 s_N^4$.

Assume $Var(W_i) = O(1)$ as $N \to \infty$, then $s_N^2 = O(N)$. Then for any finite constant C, we have $s_N^2 > C$ for large enough N; hence $s_N^2 \to \infty$.

Now from right hand side of (2.9), for $s_N^2 \to \infty$, $4A^2 s_N^4$ is $o[(s_N^2)^3]$ because $4A^2 s_N^4/(s_N^2)^3 \to 0$ as $s_N^2 \to \infty$. This proves (2.8) and hence (2.7).

Lemma 2.2.3 *(CLT of $\tilde{\theta}$): Suppose $W_i' = \partial W_i/\partial\theta$ is uniformly bounded. Let $s_N^2 = \sum_{i=1}^{N} Var(W_i)$. Provided $\tilde{H}_N'(\theta_0) \xrightarrow{p} G$, where $G = E\left[\tilde{H}_N'(\theta)\right] = E\left[\frac{1}{N}\sum_{i=1}^{N} \frac{\delta_i}{\pi_i}\frac{\partial}{\partial\theta}S_i(\theta)\right]$, then asymptotically, as $N \to \infty$,*

$$\frac{\sqrt{N}G(\tilde{\theta} - \theta)}{\sqrt{s_N^2/N}} \xrightarrow{D} N(0,1).$$

Proof : Provided $S_i(\theta)$ is a smooth function of θ, thus $\tilde{H}_N(\theta)$, is analytic, we have by Taylor expansion:

$$\begin{aligned} 0 = \tilde{H}_N(\tilde{\theta}) &= \tilde{H}_N(\theta_0) + \tilde{H}_N'(\theta_0)(\tilde{\theta} - \theta_0) + \frac{1}{2}\tilde{H}_N''(\theta_0)(\theta^* - \theta_0)^2 \\ &= \left[\tilde{H}_N(\theta_0) - H_N(\theta_0)\right] + \tilde{H}_N'(\theta_0)(\tilde{\theta} - \theta_0) + \frac{1}{2}\tilde{H}_N''(\theta_0)(\theta^* - \theta_0)^2 \\ &= Z_N(\theta_0) + \frac{\sqrt{N}\tilde{H}_N'(\theta_0)(\tilde{\theta} - \theta_0)}{\sqrt{s_N^2/N}} + \frac{\sqrt{N}\tilde{H}_N''(\theta_0)(\theta^* - \theta_0)^2}{2\sqrt{s_N^2/N}}, \end{aligned}$$

where θ^* lies between $\tilde{\theta}$ and θ_0. It follows from CLT of $\tilde{H}_N(\theta)$ that, asymptotically,

$$\frac{\sqrt{N}\tilde{H}'_N(\theta_0)(\tilde{\theta}-\theta_0)}{\sqrt{s_N^2/N}} + \frac{\sqrt{N}\tilde{H}''_N(\theta_0)(\theta^*-\theta_0)^2}{2\sqrt{s_N^2/N}} \xrightarrow{D} N(0,1).$$

Provided $\frac{\sqrt{N}\tilde{H}''_N(\theta_0)(\theta^*-\theta_0)^2}{2\sqrt{s_N^2/N}}$ is bounded in probability, we have $\frac{\sqrt{N}\tilde{H}''_N(\theta_0)(\theta^*-\theta_0)^2}{2\sqrt{s_N^2/N}} \xrightarrow{p} 0$ because s_N diverges asymptotically. Provided $\tilde{H}'_N(\theta_0) \xrightarrow{p} G$, it follows from Slutsky's Theorem that

$$\frac{\sqrt{N}G(\tilde{\theta}-\theta)}{\sqrt{s_N^2/N}} \xrightarrow{D} N(0,1) \qquad \square$$

2.3 On the Bias of NEE

The theoretical estimating equations $\tilde{H}_N(\theta)$ is unbiased i.e. $E[\tilde{H}_N(\theta)] = H_N(\theta)$ whereas the observed EE $\hat{H}_N(\theta)$ is biased. Here we examine the bias, given which one can derive bias-adjusted EEs those are discussed in next Chapter 4. The bias can be written as

$$B = E[\hat{H}_N(\theta)] - H_N(\theta) = N^{-1}\sum_{i=1}^{N} S_i(\theta)E\left(\frac{\delta_i}{\hat{\pi}_i} - 1\right). \tag{2.10}$$

To make the notations clear in this section we will denote the current response indicator δ_i by δ_{iT}. The $\hat{\pi}_i = 0 \implies \delta_{iT} = 0$, hence $\hat{\pi}_i$ will not be 0 at current time point for respondents. Now using Taylor expansion around π_i yields

$$\begin{aligned}
B &= N^{-1}\sum_{i=1}^{N} S_i(\theta)\left[E\left(\frac{\delta_{iT}}{\pi_i} - \frac{\delta_{iT}(\hat{\pi}_i-\pi_i)}{\pi_i^2} + \frac{\delta_{iT}(\hat{\pi}_i-\pi_i)^2}{{\pi_i^*}^3}\right) - 1\right] \\
&= N^{-1}\sum_{i=1}^{N} S_i(\theta)\left\{-\frac{E[\delta_{iT}(\hat{\pi}_i-\pi_i)]}{\pi_i^2} + \frac{E[\delta_{iT}(\hat{\pi}_i-\pi_i)^2]}{{\pi_i^*}^3}\right\}, \\
B &= N^{-1}\sum_{i=1}^{N} S_i(\theta)\left\{1 - \frac{E(\delta_{iT}\hat{\pi}_i)}{\pi_i^2} + \frac{E(\delta_{iT}\hat{\pi}_i^2) - 2\pi_i E(\delta_{iT}\hat{\pi}_i) + \pi_i^3}{{\pi_i^*}^3}\right\}
\end{aligned} \tag{2.11}$$

for some π_i^* between π_i and $\hat{\pi}_i$. Moreover, we have

$$\begin{aligned}
E(\delta_{iT}\hat{\pi}_i) &= E\Big(\frac{\delta_{iT}}{T}\sum_{t'=1}^{T}\delta_{it'}\Big) = \frac{1}{T}\sum_{t'=1}^{T-1}E(\delta_{iT}\delta_{it'}) + \frac{1}{T}E(\delta_{iT}^2) \\
&= \frac{T-1}{T}E[E(\delta_{iT}|\pi_{iT})E(\delta_{it'}|\pi_{it'})|\pi_i] + \frac{1}{T}E[E(\delta_{iT}^2|\pi_{it})|\pi_i] \\
&= \frac{T-1}{T}\pi_i^2 + \frac{\pi_i}{T} = \pi_i^2 + \frac{\pi_i(1-\pi_i)}{T} \\
&= \pi_i^2 + V(\hat{\pi}_i)
\end{aligned}$$

and, after some algebra,

$$
\begin{aligned}
E(\delta_{iT}\hat{\pi}_i^2) &= E\big[\frac{\delta_{iT}}{T^2}(\sum_{t'=1}^{T}\delta_{it'})^2\big] \\
&= E\big[\frac{\delta_{iT}}{T^2}\left(\sum_{t'=1}^{T}\delta_{it'}^2 + 2\sum_{t'=t''=1,t'\neq t''}^{T}\delta_{it'}\delta_{it''}\right)\big] \\
&= \frac{1}{T^2}E\left\{\sum_{t'=1}^{T-1}\delta_{iT}\delta_{it'}^2 + \delta_{iT}^3 + 2\sum_{t'=1}^{T-1}\delta_{iT}\delta_{it'}^2 + 2\sum_{t'=t''=1,t'\neq t''}^{T-1}\delta_{iT}\delta_{it'}\delta_{it''}\right\} \\
&= \frac{1}{T^2}\big\{(T-1)E[\delta_{iT}\delta_{it'}^2] + E[\delta_{iT}^3] + 2(T-1)E[\delta_{iT}^2\delta_{it'}] \\
&\quad +2(T-1)(T-2)E[\delta_{iT}\delta_{it'}\delta_{it''}]\big\} \\
&= \frac{1}{T^2}\big\{(T-1)\pi_i^2 + \pi_i + 2(T-1)\pi_i^2 + 2(T-1)(T-2)\pi_i^3\big\} \\
&= \frac{1}{T^2}\pi_i\big[1 + 3(T-1)\pi_i + 2(T-1)(T-2)\pi_i^2\big] \\
&= \frac{1}{T^2}\pi_i\kappa_i,
\end{aligned}
$$

where $\kappa_i = 1 + 3(T-1)\pi_i + (T-1)(T-2)\pi_i^2$. Now from (2.11), we obtain

$$
\begin{aligned}
B &= N^{-1}\sum_{i=1}^{N}S_i(\theta)\left\{-\frac{V(\hat{\pi}_i)}{\pi_i^2} + \frac{\frac{1}{T^2}\pi_i\kappa_i - 2\pi_i(\pi_i^2 + V(\hat{\pi}_i)) + \pi_i^3}{\pi_i^{*3}}\right\} \\
&= N^{-1}\sum_{i=1}^{N}S_i(\theta)\left\{\frac{\pi_i\kappa_i}{T^2\pi_i^{*3}} - \frac{\pi_i(\pi_i^2 + 2V(\hat{\pi}_i))}{\pi_i^{*3}} - \frac{V(\hat{\pi}_i)}{\pi_i^2}\right\} \\
B &= N^{-1}\sum_{i=1}^{N}w_iS_i(\theta), \qquad (2.12)
\end{aligned}
$$

where

$$
w_i = \frac{\pi_i\kappa_i}{T^2\pi_i^{*3}} - \frac{\pi_i(\pi_i^2 + 2V(\hat{\pi}_i))}{\pi_i^{*3}} - \frac{V(\hat{\pi}_i)}{\pi_i^2}. \qquad (2.13)
$$

The coefficients w_i's above are functions of π_i, such that it may depend on $\mathbf{y}_i$, even though the functional form of the dependence is unspecified under the NEE approach. In other words, the term B is not zero as long as the population covariance of w_i and $S_i(\theta)$ is not zero, which is given by $N^{-1}\sum_{i=1}^{N}w_iS_i$ since $N^{-1}\sum_{i=1}^{N}S_i(\theta) = 0$ by definition.

2.4 Consistency of $\hat{\theta}$

We investigate the consistency of $\hat{\theta}$ using the same approach of Foutz (1977). This approach dealt directly with the vector parameter case. Foutz's approach is based on the Inverse Function Theorem given by Rudin (1976). This approach is used for unique consistent solution to the unbiased estimating equations by Pepe et al. (1994) and with similar arguments by Yuan and Jennrich (1998). Wang et al. (1997) referred to Foutz's approach while dealing with asymptotically unbiased estimating equations but they did not supply a proof of the consistency of $\hat{\theta}$.

Before proceeding further, we state the Inverse Function Theorem used by Foutz in Theorem 2.4.1 below for ease of reference. Let the norm $\|M\|$ of a $p \times p$ dimensional matrix to be the least upper bound of all numbers $|Mx|$ (e.g. the eigenvalues or singular values of M, the trace and determinant, etc.), where x ranges over all vectors in $\mathbb{R}^p$ with $|x| \leq 1$, where E is Euclidean space.

Theorem 2.4.1 *(**Inverse Function Theorem**): Suppose h is a mapping from an open set Θ in E^r to E^r, the partial derivatives of h exist and are continuous on Θ, and the matrix of derivatives $h'(\theta^\star)$ has inverse $h'(\theta^\star)^{-1}$ for some $\theta^\star \in \Theta$. Write*

$$\lambda = 1/(4 \left\|h'(\theta^\star)^{-1}\right\|).$$

Use the continuity of the elements of $h'(\theta^\star)^{-1}$ to fix a neighbourhood U_ω of $\theta^\star$ of sufficiently small radius $\omega > 0$ to ensure

$$\left\|h'(\theta) - h'(\theta^\star)\right\| < 2\lambda, \tag{2.14}$$

whenever $\theta \in U_\omega$. Then

(i). for every θ_1, θ_2 in U_ω ,

$$|h(\theta_1) - h(\theta_2)| \geq 2\lambda \left|\theta_1 - \theta_2\right|, \tag{2.15}$$

(ii). the image set $h(U_\omega)$ contains the open neighbourhood with radius $\lambda\omega$ about $h(\theta^\star)$.

The conclusion (i) given in (2.15) ensures that h is one-to-one on U_ω and that h^{-1} is well defined on the image set $h(U_\omega)$. This Inverse Function Theorem is proved in this form by Huzurbazar (1948) at page 193. In our setting, the estimator $\hat{\theta}$ of θ is the solution of estimating equations $\hat{H}_N(\theta) = 0$ for function $\hat{H}_N : \Theta \to \mathbb{R}^p$. From another point of view, the estimator $\hat{\theta}$ is the value of the inverse function $\hat{H}_N^{-1} : \mathbb{R}^p \to \Theta$, evaluated at 0, i.e. $\hat{H}_N^{-1}(0) = \hat{\theta}$. Using the Inverse Function Theorem, it is shown below in Theorem 2.4.2 that $\hat{H}_N^{-1}$ is well defined in an open neighbourhood about 0 with probability going to one, and $\hat{\theta} = \hat{H}_N^{-1}(0)$ is shown to be a consistent estimator of θ.

Foutz (1977) required four conditions to prove the existence of a unique consistent solution to the likelihood equations using the above Inverse Function Theorem, the last condition is unbiasedness of the likelihood equations. In Foutz's proof, this condition is further used to prove that the likelihood equations converge to 0 with probability one. However, this convergence can be proved for asymptotically unbiased estimating equations. In our setting, to prove the existence of a unique consistent solution of $\hat{H}_N(\theta)$, the following five conditions are required from which first three conditions are the same as in Foutz (1977) and last two condition are used instead of the fourth condition of Foutz.

(A). The elements of the matrix $\hat{H}'_N(\theta) = N^{-1} \sum_{i=1}^N (\delta_i/\hat{\pi}_i)\partial S_i(\theta)/\partial\theta$ exist and are continuous on Θ;

(B). $\hat{H}'_N(\theta) - G_N(\theta)$ converges to 0 in probability, where $G_N(\theta) = E[\hat{H}'_N(\theta)]$;

(C). The matrix $\hat{H}'_N(\theta)$ evaluated at the true parameter θ_0 is negative definite with probability converging to one as $N \to \infty$;

(D). The estimating equations $\hat{H}_N(\theta)$ are asymptotically unbiased, i.e. at the true parameter θ_0, $\lim_{N\to\infty} E[\hat{H}_N(\theta_0)] = H_N(\theta_0) = 0$;

(E). The difference $\hat{H}_N(\theta) - E[\hat{H}_N(\theta)]$ converges to 0 with probability one.

The existence of a consistent solution to the asymptotically unbiased estimating equations $\hat{H}_N(\theta)$ and its uniqueness are now given by the following theorem.

Theorem 2.4.2 *There exists $\hat{\theta}_N$ such that*

$$\hat{H}_N(\hat{\theta}_N) = 0 \tag{2.16}$$

with probability going to one as $N \to \infty$, and

$$\hat{\theta}_N \to \theta_0 \tag{2.17}$$

in probability. If $\bar{\theta}_N$ also satisfies (2.16) *and* (2.17) *then $\bar{\theta}_N = \hat{\theta}_N$ with probability going to one as $N \to \infty$.*

Remark: The first three conditions (A)-(C) are used in first part of the proof given below of above Theorem 2.4.2 that is modelled on the proof given by Foutz. Then conditions (D) and (E) are used as an alternative to the last condition of Foutz to prove the convergence of asymptotically unbiased estimating equations to 0 and then we rejoin Foutz's proof to conclude the existence of unique consistent solution to asymptotically unbiased estimating equations. In a nutshell we can say that the first three conditions (A)-(C) are used for the Inverse Function Theorem to show that function $\hat{H}_N$ is one-to-one and this one-to-oneness implies that the inverse function $\hat{H}_N^{-1}$ is well defined. Then the conditions (D) and (E) imply $\hat{H}_N(\theta_0) \to 0$ with probability one and it ensures that 0 can be an image of $\hat{H}_N$ that lies in the image set of $\hat{H}_N$ within a very small radius $\lambda'_N\omega/2$ centered at $\hat{H}_N(\theta_0)$. This determines the existence of a solution to the estimating equations $\hat{H}_N(\theta) = 0$. Further the solution $\hat{\theta}_N$ that is the image of $\hat{H}_N^{-1}(0)$ can be found in the image set of $\hat{H}_N^{-1}$ with sufficiently small radius ω centered at θ_0 which gives us the consistency i.e. convergence of $\hat{\theta} = \hat{H}_N^{-1}(0)$ to θ_0 in probability. By the one-to-oneness of $\hat{H}_N$ on U_ω, any other estimator $\bar{\theta}_N$ of $\hat{H}_N(\theta) = 0$ necessarily lies outside of U_ω with probability going to one and estimator $\bar{\theta}_N$ does not converge to θ_0 and it ensures the uniqueness of $\hat{\theta}_N$. The situation is illustrated in the following figure, which is borrowed from Foutz (1977) but with different notations.

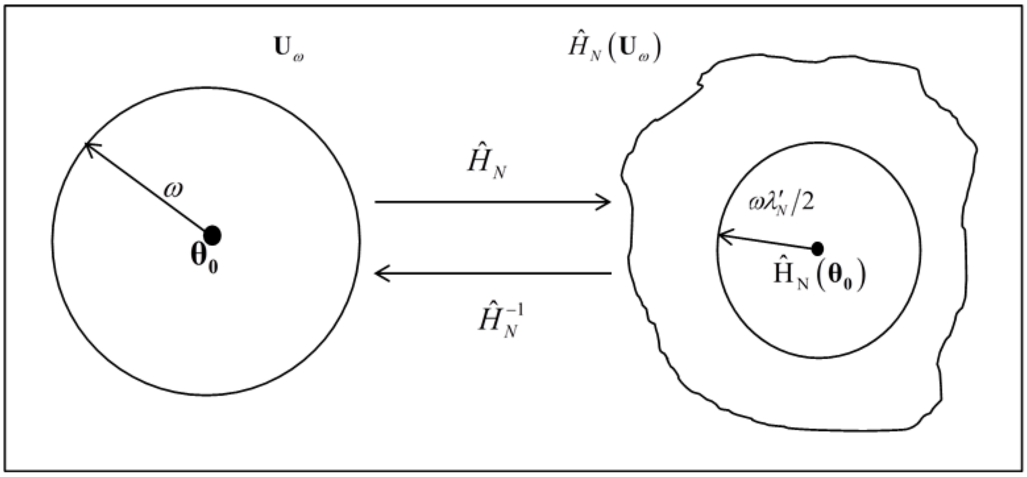

Figure 2.1: The function $\hat{H}_N$ and $\hat{H}_N^{-1}$

Proof (Theorem 2.4.2): By condition (C), the matrix $\hat{H}'_N(\theta)$ evaluated at the true parameter θ_0 is negative definite with probability converging to 1 as $N \to \infty$ and by condition (B) the convergence of $\hat{H}'_N(\theta) - G_N(\theta) \to 0$ in probability ensures that $G_N(\theta_0)$ is negative definite; thus, we may define $\lambda'_N = 1/(4||G_N(\theta_0)^{-1}||)$. Select ω sufficiently small so that

$$||G_N(\theta) - G_N(\theta_0)|| < \lambda'_N/2 \tag{2.18}$$

whenever $|\theta - \theta_0| < \omega$, then uniform convergence of $\hat{H}'_N(\theta) - G_N(\theta) \to 0$ holds i.e.

$$||\hat{H}'_N(\theta) - G_N(\theta)|| < \lambda'_N/2 \tag{2.19}$$

for $|\theta - \theta_0| < \omega$ for θ in an open neighbourhood about θ_0.

Write $\lambda_N = 1/(4||\hat{H}'_N(\theta_0)^{-1}||)$. Whenever $\hat{H}'_N(\theta_0)$ is negative definite and we already have $\lambda'_N = 1/(4||G_N(\theta_0)^{-1}||)$ then using the Continuous Mapping Theorem, the uniform convergence of $\hat{H}'_N(\theta) - G_N(\theta) \to 0$ implies $\lambda_N - \lambda'_N \to 0$ in probability and if $|\theta - \theta_0| < \omega$, then by the triangular inequality,

$$||\hat{H}'_N(\theta) - \hat{H}'_N(\theta_0)|| \leq ||\hat{H}'_N(\theta) - G_N(\theta)|| + ||G_N(\theta_0) - \hat{H}'_N(\theta_0)|| + ||G_N(\theta) - G_N(\theta_0)||$$

Using (2.18) and (2.19), we have

$$||\hat{H}'_N(\theta) - \hat{H}'_N(\theta_0)|| < \lambda'_N/2 + \lambda'_N/2 + \lambda'_N/2 < 2\lambda_N \tag{2.20}$$

with probability going to one as $N \to \infty$. The (2.20) is the main condition similar to (2.14) given in the Inverse Function Theorem to prove that $\hat{H}_N$ is one-to-one. Then similar to (2.15), for every θ_1, θ_2 in U_ω,

$$\left|\hat{H}_N(\theta_1) - \hat{H}_N(\theta_2)\right| \geq 2\lambda_N\,|\theta_1 - \theta_2|\,.$$

This ensures that $\hat{H}_N$ is one-to-one for $U_\omega \to \hat{H}_N(U_\omega) = V_\omega(say)$ with probability approaching one. We can write $\hat{H}_N^{-1}(V_\omega) = U_\omega$ if and only if $\hat{H}_N(U_\omega) = V_\omega$.

Now it can be seen that conditions (A), (B) and (C) are those required by the Inverse Function Theorem to ensures with probability approaching one that $\hat{H}_N$ is a one-to-one function from U_ω onto $\hat{H}_N(U_\omega)$ and that the image set $\hat{H}_N(U_\omega)$ contains the open neighbourhood of radius $\omega\lambda_N$ about $\hat{H}_N(\theta_0)$. Since $\omega\lambda_N - \omega\lambda'_N \to 0$ in probability, $\hat{H}_N(U_\omega)$ also contains the neighbourhood of radius $\omega\lambda'_N/2$ about $\hat{H}_N(\theta_0)$ with probability going to one. The situation is illustrated in Figure 2.1 above.

Foutz (1977) proved that the likelihood equations converges to 0 almost surely when the the likelihood equations are unbiased but we need to prove this for asymptotically unbiased estimating equations. We may see that $0 \in \hat{H}_N(U_\omega)$ with probability going to one by observing that from condition (D), $E[\hat{H}_N(\theta_0)] \to 0$ and from condition (E), $\hat{H}_N(\theta) - E[\hat{H}_N(\theta)] \to 0$ with probability one as $N \to \infty$, then by Slutsky's theorem $\hat{H}_N(\theta_0) \to 0$ with probability going to one as $N \to \infty$ or $|\hat{H}_N(\theta_0) - 0| < \omega\lambda'_N/2$.

Consider the inverse function $\hat{H}_N^{-1} : \hat{H}_N(U_\omega) \to U_\omega$ and $\hat{H}_N^{-1}$ is well defined whenever $\hat{H}_N$ is one-to-one, i.e. with probability going to one. Since $0 \in \hat{H}_N(U_\omega)$ with probability going to one as $N \to \infty$, we may conclude:

(1). the root, $\hat{H}_N^{-1}(0)$, of the estimating equations $\hat{H}_N(\theta)$ exists in U_ω with probability going to one as $N \to \infty$; .

(2). since ω may be taken arbitrarily small, $\hat{H}_N^{-1}(0)$ converges in probability to θ_0;

(3). by the one-to-oneness of $\hat{H}_N$ on U_ω, any other vector of estimators $\bar{\theta}_N$ of roots to $\hat{H}_N(\theta) = 0$, necessarily lies out side of U_ω with probability going to one which means that $\bar{\theta}_N$ does not converge to θ_0 and it ensures the uniqueness of $\hat{\theta}_N$.

The proof is now complete with $\hat{\theta}_N = \hat{H}_N^{-1}(0)$.

The condition (A)-(C) and (E) are discussed later in Section 2.4.1 but the discussion on condition (D) is lengthy and is given in the form of following Lemma 2.4.3.

Lemma 2.4.3 *For estimating equations* $\hat{H}_N(\theta) = N^{-1}\sum_{i=1}^{N}(\delta_i/\hat{\pi}_i)S_i(\theta)$, *let* $w_i = E[\delta_i/\hat{\pi}_i - 1]$ *and a lower triangular matrix of infinite dimensions with elements* $a_{ij} = w_j/\sum_{k=1}^{i} w_k$. *Put*

(i). the function $S_i(\theta)$ *is a null sequence, i.e.* $\lim_{N\to\infty} S_N = 0$.

(ii). for given j, $\lim_{i\to\infty} a_{ij} \to 0$,

(iii). The $\sum_{j=1}^{i}|a_{ij}| = O(1)$,

then the estimating equations $\hat{H}_N(\theta)$ *are asymptotically unbiased, i.e.* $\lim_{N\to\infty} E[\hat{H}_N(\theta_0)] = H_N(\theta_0) = 0$ *at the true parameter* θ_0.

To prove the above Lemma 2.4.3, we need the following Theorem that is given by Knopp (1954) with proof at page 74.

Theorem 2.4.4 (Knopp (1954)) *Let* $(x_0, x_1, ...)$ *be a null sequence and suppose the coefficients* a_{np} *of the system*

$$A = \begin{bmatrix} a_{00} & & & & & & & & \\ a_{10} & a_{11} & & & & & & & \\ a_{20} & a_{21} & a_{22} & & & & & & \\ . & . & . & . & . & & & & \\ . & . & . & . & . & . & & & \\ a_{n0} & a_{n1} & a_{n2} & . & . & . & a_{nn} & & \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \end{bmatrix},$$

satisfy the two conditions:

(a′). Every column contains a null sequence, i.e. for fixed $p \geq 0$

$$a_{np} \to 0 \text{ when } n \to \infty \tag{2.21}$$

(b′). There exists a constant K, such that the sum of the absolute values of the term in any one row is bounded by K, i.e., for every n,

$$|a_{n0}| + |a_{n1}| + ... + |a_{nn}| < K \tag{2.22}$$

Then the sequence formed by the numbers

$$x'_n = a_{n0}x_0 + a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n \tag{2.23}$$

is also a null sequence.

Regarding Lemma 2.4.3, the bias of estimating equations $\hat{H}_N(\theta)$ from (2.11) is

$$B_N = N^{-1}\sum_{i=1}^{N} w_i S_i(\theta), \text{ where } w_i = \frac{\pi_i \kappa_i}{(T-1)^2 \pi_i^{*3}} - \frac{\pi_i(\pi_i^2 + 2V(\hat{\pi}_i))}{\pi_i^{*3}} - \frac{V(\hat{\pi}_i)}{\pi_i^2} \tag{2.24}$$

where w_i is a function of π_i, and π_i is allowed to be unit-specific hence a function of y_i. Since $N^{-1}\sum_{i=1}^{N} S_i(\theta) = 0$ by definition, one would expect $B_N \neq 0$, as long as the population covariance of w_i and S_i is not zero, which is essentially $N^{-1}\sum_{i=1}^{N} w_i S_i$.
Knopp's theorem states that, provided a_i's satisfy certain regularity conditions, $\sum_{i=1}^{N} a_i x_i$ converges to 0 *mathematically*, given any null sequence x_i, as $N \to \infty$.
Applying Knopp's theorem to NEE one can let $a_i = w_i / \sum_{i=1}^{N} w_i$. As long as $w_i \geq 0$ and $w_i = O(1)$, as $N \to \infty$, we have $\sum_{i=1}^{N} w_i/N = O(1)$, and the conditions on a_i's for the convergence of $\sum_{i=1}^{N} a_i x_i$ are satisfied. For condition (ii), assuming $w_i = O(1)$, augments to assume that $Var(\hat{\pi}_i)$ is bounded and π_i^* is bounded away from 0. In other words π_i cannot be arbitrarily close to zero, and there must exist response history for each unit. The condition (iii) also holds because $\sum_{j=1}^{i}(w_j / \sum_{k=1}^{i} w_k) = 1 \implies \sum_{j=1}^{i} a_{ij} = O(1)$.
To apply Knopp's theorem to the plug-in NEE, one needs to turn S_i into a null sequence x_i that is the first condition of Knopp's theorem. Obviously, letting $S_i = x_i$ does not yield a null sequence. However, the sum of $a_i S_i$ remains the same under any permutation of $\{1, .., N\}$, say

$\{1^*, 2^*, ..., N^*\}$, i.e.

$$\sum_{i=1}^{N} a_i S_i = \sum_{i^*=1}^{N^*} a_{i^*} S_{i^*}$$

For any random variable that has a continuous density function, it is almost surely that some realised value can be arbitrarily close to the mean of the distribution, as $N \to \infty$. Thus, it is a mild requirement that $\{y_1, ..., y_N\}$ can be rearranged as $\{y_{1^*}, ..., y_{N^*}\}$, such that S_{i^*} is a null sequence, as $N \to \infty$. But this argument could not work and we explain the reason below.

While the permutation does not affect the conditions on $\{a_1, ..., a_N\}$ for any given N, it affects the transition from $\{a_{1^*}, ..., a_{N^*}\}$ to $\{a_{1^{**}}, ..., a_{N^{**}}, a_{(N+1)^{**}}\}$, where $\{1^*, ..., N^*\}$ is a permutation based on $\{y_1, ..., y_N\}$ and $\{a_{1^{**}}, ..., a_{N^{**}}, a_{N+1^{**}}\}$ that based on $\{y_1, ..., y_N\ , y_{N+1}\}$. There is no way to ensure that we have $(N+1)^{**} = N+1$ and $i^{**} = i^*$ for $i^* = 1^*, ..., N^*$. However, the a_i's in Knopp's theorem need to be arranged in a triangular form in correspondence to the x_i's, i.e.

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \leftrightarrow \begin{bmatrix} a_{11} & & & \\ a_{21} & a_{22} & & \\ \vdots & \cdots & \ddots & \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix} \sim \begin{bmatrix} S_{1^*} \\ S_{2^*} \\ \vdots \\ S_{N^*} \end{bmatrix} \leftrightarrow \begin{bmatrix} a_{1^*1^*} & & & \\ a_{2^*1^*} & a_{2^*2^*} & & \\ \vdots & \cdots & \ddots & \\ a_{N^*1^*} & a_{N^*2^*} & \cdots & a_{N^*N^*} \end{bmatrix}.$$

This can be achieved given any N. However, moving from N to $N+1$, it becomes impossible to simply add another row corresponding to y_{N+1} and S_{N+1} at the bottom of the triangle built on $\{y_1, ..., y_N\}$, as long as S_{N+1} does not happen to be $S_{(N+1)^{**}}$ that is determined on the basis of $\{y_1, ..., y_{N+1}\}$.

As discussed above we could not prove the asymptotic unbiasedness of NEE and hence the consistency. However, this investigation together with below discussed conditions show in a detailed manner that a bias-adjusted NEE discussed in Chapter 4 would have worked.

2.4.1 Discussion of the Conditions

In this section, we discuss the conditions (A)-(E) used to prove above Theorem 2.4.2 in our setting. For all conditions to be satisfied, initially we need to assume that estimated response probability $\hat{\pi}_i \neq 0$. The $\hat{\pi}_i$ is the response rate of a unit including the current response and it cannot be zero for observed units.

Condition-(A)

For condition (A) to be satisfied, we need to assume that $S_i(\theta)$ is a smooth function; meaning that it is infinitely differentiable on Θ and these partial derivatives exist at every point in Θ and are continuous on Θ. In our case of estimation of the population mean, $S_i(\theta) = (y_i - \theta)$. Clearly it is a smooth function for $\theta \in \mathbb{R}$ because it is infinitely differentiable on θ and these partial derivatives exist at every point in θ and are continuous on θ. Similarly for estimation of the variance along with the mean the S function for variance can be $[(y_i - \theta)^2 - \sigma^2]$ and it also a smooth function for $\sigma^2 \geq 0$. Similarly in the case of estimation of regression coefficients with one independent variable, the score functions are $(y_i - \beta_0 - \beta_1 x_i)$, $\big(y_i x_i - \beta_0 x_i - \beta_1 x_i^2\big)$

and $\{(y_i - \beta_0 - \beta_1 x_i)^2 - \sigma^2\}$. These score functions are also smooth for $\beta_0, \beta_1 \in \mathbb{R}$ and $\sigma^2 \geq 0$. Hence, the condition (A) can easily be satisfied for our purposes.

Condition-(B)

Let a matrix $W_i' = (\delta_i/\hat{\pi}_i)S_i'(\theta)$ then we can write

$$\hat{H}_N'(\theta) - G_N(\theta) = N^{-1}\sum_{i=1}^{N} W_i' - N^{-1}\sum_{i=1}^{N} E(W_i').$$

Suppose $w_{i(jk)}' = (\delta_i/\hat{\pi}_i)S_{i(jk)}'(\theta)$ is an element of W_i', then

$$E\left(w_{i(jk)}'\right) = S_{i(jk)}'(\theta)E\left(\frac{\delta_i}{\hat{\pi}_i}\right) < \infty \text{ and } Var\left(w_{i(jk)}'\right) = S_{i(jk)}'^2(\theta)Var\left(\frac{\delta_i}{\hat{\pi}_i}\right) < \infty.$$

Then by Chebyshev's inequality, for each element of W_i', $N^{-1}\sum_{i=1}^{N}[w_{i(jk)}' - E(w_{i(jk)}')]$ converges to 0 in probability. It implies $N^{-1}\sum_{i=1}^{N}[W_i' - E(W_i')] \to 0$ in probability. Hence, $\hat{H}_N'(\theta) - G_N(\theta) \to 0$ in probability and it concludes (B).

Condition-(C)

To satisfy this condition in our setting, we assumed that the matrix $[-G_N(\theta)]$ is positive definite. This assumption can hold for the general case. For estimation of the population mean we have the following estimating equations,

$$\hat{H}_N(\theta) = N^{-1}\sum_{i=1}^{N}\frac{\delta_i}{\hat{\pi}_i}(y_i - \theta) \implies -G_N(\theta) = E\left[-\frac{\partial}{\partial\theta}\hat{H}_N(\theta)\right] = N^{-1}\sum_{i=1}^{N} w_i, \tag{2.25}$$

where w_i is given above in (2.24). From (2.25),$-G_N(\theta)$ is positive. Now for simultaneous estimation of population mean and variance, we can write the estimating equations as

$$\hat{H}_N(\theta) = \frac{1}{N}\sum_{i=1}^{N}\frac{\delta_i}{\hat{\pi}_i}\begin{bmatrix}(y_i - \theta)\\ \\ \{(y_i - \theta)^2 - \sigma^2\}\end{bmatrix} \implies [-G_N(\theta)] = \frac{1}{N}\sum_{i=1}^{N} w_i\begin{bmatrix}1 & 0\\ \\ 2(y_i - \theta) & 1\end{bmatrix}$$

The leading principal minors of $-G_N(\theta)$ are $N^{-1}\sum_{i=1}^{N} w_i$ and $\left(N^{-1}\sum_{i=1}^{N} w_i\right)^2$ and both are positive, hence, the matrix $-G_N(\theta)$ is positive definite.

For estimation of regression parameters with one covariate, we can write the estimating equations as

$$\hat{H}_N(\theta) = \frac{1}{N}\sum_{i=1}^{N}\frac{\delta_i}{\hat{\pi}_i}\begin{bmatrix}\epsilon_i\\ \epsilon_i x_i\\ \epsilon_i^2 - \sigma^2\end{bmatrix} \implies -G_N(\theta) = \frac{1}{N}\sum_{i=1}^{N} w_i\begin{bmatrix}1 & x_i & 0\\ x_i & x_i^2 & 0\\ 2\epsilon_i & 2x_i\epsilon_i & 1\end{bmatrix},$$

where $\epsilon_i = y_i - \beta_0 - \beta_1 x_i$. The first leading principal minor of $-G_N(\theta)$ is,

$$\frac{1}{N}\sum_{i=1}^{N} w_i, \tag{2.26}$$

the second leading principal minor is

$$\begin{aligned}
&\left(\frac{1}{N}\sum_{i=1}^{N} w_i\right)\left(\frac{1}{N}\sum_{i=1}^{N} w_i x_i^2\right) - \left(\frac{1}{N}\sum_{i=1}^{N} w_i x_i\right)^2 \\
&\qquad = \left(\frac{1}{N^2}\sum_{i=1}^{N} w_i\right)\left\{\sum_{i=1}^{N} w_i x_i^2 - \left(\sum_{i=1}^{N} w_i x_i\right)^2 \Big/ \sum_{i=1}^{N} w_i\right\} \\
&\qquad = \left(\frac{1}{N^2}\sum_{i=1}^{N} w_i\right)\left\{\sum_{i=1}^{N} w_i \left(x_i - \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}\right)^2\right\}
\end{aligned} \tag{2.27}$$

and the third leading principal minor is

$$\begin{aligned}
&\left(\frac{1}{N}\sum_{i=1}^{N} w_i\right)\left\{\left(\frac{1}{N}\sum_{i=1}^{N} w_i\right)\left(\frac{1}{N}\sum_{i=1}^{N} w_i x_i^2\right) - \left(\frac{1}{N}\sum_{i=1}^{N} w_i x_i\right)^2\right\} \\
&\qquad = \frac{1}{N^3}\left(\sum_{i=1}^{N} w_i\right)^2\left\{\sum_{i=1}^{N} w_i \left(x_i - \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}\right)^2\right\}.
\end{aligned} \tag{2.28}$$

From (2.26), (2.27) and (2.28), it is clear that all three principal minors are positive. Hence, $[-G_N(\theta)]$ is positive definite. Now pointwise convergence given in condition (B) and positive definiteness of $[-G_N(\theta)]$ implies that the matrix $\hat{H}'_N(\theta)$ evaluated at the true parameter θ_0 is negative definite with probability converging to 1 as $N \to \infty$ and it concludes condition (C).

Condition-(E)

For this condition we need to show that $\hat{H}_N(\theta) - E[\hat{H}_N(\theta)]$ converges to 0 with probability one. Let $w'_i = (\delta_i/\hat{\pi}_i)S_i(\theta)$ then we can write

$$\hat{H}_N(\theta) - E[\hat{H}_N(\theta)] = N^{-1}\sum_{i=1}^{N}\left[w'_i - E(w'_i)\right]. \tag{2.29}$$

To show that (2.29) converges to 0 with probability one, we need to show this for each element of w'_i. Suppose $w_{i(j)}$ is an element of vector w'_i then we can have

$$\begin{aligned}
E|w_{i(j)}|^2 = E[(\delta_i/\hat{\pi}_i)S_{i(j)}(\theta)]^2 = S^2_{i(j)}(\theta)E[(\delta_i/\hat{\pi}_i)]^2 < \infty \text{ and} \\
Var(w_{i(j)}) = S'^2_{i(j)}(\theta)Var(\delta_i/\hat{\pi}_i) = O(1) \implies \sum_{i=1}^{\infty}\frac{Var(w_{i(j)})}{i^2} < \infty.
\end{aligned}$$

Then by Kolmogorov's Strong Law of Large Numbers, $N^{-1}\sum_{i=1}^{N}[w_{i(j)} - E(w_{i(j)})]$ converge to 0 with probability one. It implies $N^{-1}\sum_{i=1}^{N}[w'_i - E(w'_i)] \to 0$ with probability one. Hence,

$\hat{H}_N(\theta) - G_N(\theta) \to 0$ with probability one and it concludes (E).

In summary we made the following assumptions while discussing the conditions required for consistency.

(a) The estimated response probability should not be 0, i.e. $\hat{\pi}_i \neq 0$;

(b) The score function, $S_i(\theta)$ is smooth function for $\theta \in \Theta$;

(c) The weights w_i are bounded, where π_i^* lies between $\hat{\pi}_i$ and π_i,

(d) The score function $S_i(\theta)$ is a null sequence.

(e) For given j, $\lim_{i\to\infty} a_{ij} \to 0$, where $a_{ij} = w_j / \sum_{k=1}^{i} w_k$

From the discussion of conditions (A)-(E) given above which are required for the consistency, the assumption (a) is necessary for all (A)-(E) conditions. The assumptions (a) and (b) imply the condition (A). All the assumptions imply the conditions (B) and (E). The assumptions (a) and (c)-(e) are used to prove the condition (D) stated in Lemma 2.4.3. But we could not prove the condition (D), the reason is discussed above after Theorem 2.4.4.

2.5 Variance of $\hat{\theta}$

We could not prove the consistency of the point estimator $\hat{\theta}$ for the estimating equations $\hat{H}_N(\theta)$. The variance of the point estimator can be obtained using the standard sandwich form based on Taylor expansion of the estimating equations around $E(\hat{\theta})$. Let $\theta' = E(\hat{\theta})$ and $\hat{H}_N(\theta)$ have first derivative at θ'. In addition the second derivative of $\hat{H}_N(\theta)$ exists in a neighbourhood of θ'. By Taylor expansion (Lehmann (2004)), we have

$$0 = \hat{H}_N(\hat{\theta}_N) = \hat{H}_N(\theta') + \hat{H}'_N(\theta')(\hat{\theta}_N - \theta') + \frac{1}{2}(\hat{\theta}_N - \theta')^T \hat{H}''_N(\theta^*)(\hat{\theta}_N - \theta'),$$

where θ^* lies between $\hat{\theta}_N$ and θ' and we write $\hat{\theta}_N$ and $\hat{H}_N(\theta')$ to emphasize that they depend on N but for simple notation below we use $\hat{\theta}$ as the estimator. Provided

$$0 = \hat{H}_N(\theta') + \hat{H}'_N(\theta')(\hat{\theta} - \theta') + o_p(||\hat{\theta} - \theta'||) \tag{2.30}$$

Now, from condition (B) i.e. $\hat{H}'_N(\theta) - G_N(\theta') \xrightarrow{p} 0$, we can write

$$0 = \hat{H}_N(\theta') + G_N(\theta')(\hat{\theta} - \theta') + o_p(||\hat{\theta} - \theta'||) \tag{2.31}$$

Now ignoring the remainder, from (2.31), we have

$$\begin{aligned} G_N(\theta')(\hat{\theta} - \theta') &\approx -\hat{H}_N(\theta') \\ \implies (\hat{\theta} - \theta') &\approx -G_N^{-1}(\theta_0)\hat{H}_N(\theta') \end{aligned} \tag{2.32}$$

From (2.32), the approximate variance of $\hat{\theta}_N$ can be written as

$$Var(\hat{\theta}) = G_N^{-1}(\theta')Var[\hat{H}_N(\theta')]G_N^{-T}(\theta') = G^{-1}(\theta')Var[\hat{H}(\theta')]G^{-T}(\theta') \tag{2.33}$$

where $G(\theta) = E[\hat{H}'_N(\theta)]$. Then

$$G(\theta) = \frac{1}{N}\sum_{i=1}^{N} E\left(\delta_i/\hat{\pi}_i\right)\left\{\frac{\partial}{\partial\theta}S_i(\theta)\right\}$$

and

$$Var[\hat{H}(\theta)] = \frac{1}{N}\sum_{i=1}^{N} Var\left(\delta_i/\hat{\pi}_i\right) S_i(\theta)S_i^T(\theta)$$

with

$$E\left(\delta_i/\hat{\pi}_i\right) \approx 1 - \frac{E(\delta_i\hat{\pi}_i) - \pi_i^2}{\pi_i^2} + \frac{E(\delta_i\hat{\pi}_i^2) - 2\pi_i E(\delta_i\hat{\pi}_i) + \pi_i^3}{\pi_i^3} \stackrel{def}{=} \mu_{1i}, \tag{2.34}$$

$$E\left(\delta_i/\hat{\pi}_i\right)^2 \approx \frac{1}{\pi_i} - \frac{2\left(E(\delta_i\hat{\pi}_i) - \pi_i^2\right)}{\pi_i^3} + \frac{6\left(E(\delta_i\hat{\pi}_i^2) - 2\pi_i E(\delta_i\hat{\pi}_i) + \pi_i^3\right)}{2\pi_i^4} \stackrel{def}{=} \mu_{2i}, \tag{2.35}$$

where $E(\delta_i\hat{\pi}_i) = \pi_i^2 + Var(\hat{\pi}_i)$ and $E(\delta_i\hat{\pi}_i^2) = \pi_i\kappa_i/(T-1)^2$.

The plug-in estimator of variance of $\hat{\theta}_N$ given in (2.33) can be written as

$$\widehat{Var}(\hat{\theta}) = G^{-1}(\hat{\theta})\widehat{Var}[\hat{H}(\hat{\theta})]G^{-T}(\hat{\theta}), \tag{2.36}$$

where

$$G(\hat{\theta}) = \frac{1}{N}\sum_{i=1}^{r} \hat{g}_i S_i'(\hat{\theta}) \tag{2.37}$$

$$\widehat{Var}[\hat{H}(\hat{\theta})] = \frac{1}{N^2}\sum_{i=1}^{r} \hat{v}_i S_i(\hat{\theta})S_i^T(\hat{\theta}), \tag{2.38}$$

and

$$\hat{g}_i = \frac{1}{\hat{\pi}_i}\hat{\mu}_{1i} \tag{2.39}$$

$$\hat{v}_i = \frac{1}{\hat{\pi}_i}\left\{(\hat{\mu}_{2i} - (\hat{\mu}_{1i})^2\right\}. \tag{2.40}$$

2.6 Simulation Study

The NEE approach provides a method for exploring informative nonresponse in the longitudinal setting, which is computationally easy and flexible in specification. From the outset there are several factors that can be expected to affect its performance in a given situation.

First, provided the suitable nonresponse assumptions for all the responding individuals, the NEE estimator should perform better given a *longer* history of response. For instance, in the case of two waves data and we want to estimate the parameter at $T = 2$, where there are two observations of δ_{it} for each individual, there are only two possible histories for each respondent with $\delta_{i2} = 1$, where δ_{i1} is either 1 or 0. The estimate $\hat{\pi}_{it}$ that enters the NEE (2.5) either takes value 1 or 0.5, such that the estimator of θ_T is only based on two weighting classes. Whereas the näive estimator under the MCAR assumption is based on a single weighting class. Clearly, the ability to adjust for potentially informative nonresponse by the NEE approach is rather limited in this case. Thus, a factor that matters in the simulation study will be the length of response history. We estimated the mean and regression coefficients at $T = 3$, 4, 7 and 10.

Next, a relevant factor is the variation of y_{it}'s over time, for each given individual. Take again the estimator $\hat{\pi}_{it}$ that is averaged over all the δ_{it}'s. On the one hand, it is unbiased if the informative response propensity depends only on a scalar summary of the y_{it}'s, in which case it does not matter how volatile the y_{it}'s are over time. On the other hand, intuitively the risk of bias is heightened, when the y_{it}'s are volatile, as compared to the extreme case where $y_{it} \equiv y_i$ is completely static. Moreover, as the variance of the resulting estimator of θ_T increases with more volatile y_{it}'s, it would be interesting to explore if this has any compounding effect together with the heightened risk of bias.

Last but not least, the nonresponse mechanism itself will be a critical factor to the performance of the NEE. That is, if the assumption for unbiased estimation of π_{it}'s is clearly violated, then the NEE estimator may suffer extra bias beyond the inherent bias of the NEE as explained in Section 2.3.

Below we describe first the data used for the simulation, the models that can be used for simulating response history, and then the chosen simulation set-up, including the specific response probability models, the sample size corresponding to overall response rate, and the estimators to be evaluated, before we present and discuss the simulation results.

2.6.1 Real Data

We have available a dataset of real turnover from 16788 firms over three successive years. There are some (about 13%) partially missing turnovers. We impute these values using the R-package *mice*. A summary of the completed population turnover values for each of the three waves are given below. It can be seen that the population distributions are skewed but reasonably stable over time.

Table 2.1: Summary of turnover values over three waves

Wave	$Mean$	Min	Q_3	Max	β_0	β_1
First	11000	1	10211	2008585	11023.06	-0.4041
Second	10749	1	11010	2012973	10770	-0.3758
Third	11747	1	12295	2026659	11764.93	-0.3202

We then increase the number of waves from 3 to 10 for simulations by recycling wave 1 to be wave 4, and wave 2 to be wave 5, wave 3 to be wave 6, and so on. Below we refer to this 10-wave dataset as the *stable* population. In addition, we create a 10-wave *volatile* population where, for a given wave, we permute the turnover values among all the firms within each industrial group. In this way, the population distribution at each wave remains the same, but the values y_{it}'s associated with each individual firm are perturbed quite a lot, and the individual variation of turnover over time is greatly increased.

2.6.2 Simulated Data

As we are dealing with informative missing and in which case the response probability for each time point should depend on the y_{it}. We expect that the EE performs better than the naïve if the response probability varies more across the individuals than for each individual over time. For such data, the natural choice is to generate the data for each time point by creating strata with different stratum y-averages. Furthermore, the data is generated with some increasing/decreasing trend overtime that naturally exist in longitudinal data. The longitudinal data on y_{it} are generated for 10 time points by creating three strata. The multivariate log-normal distribution is used with following mean vectors,

$$(1.20, 0.40, 1.60, 0.80, 2.0, 1.20, 3.40, 1.60, 3.80, 4.0)$$

$$(2.20, 1.40, 2.60, 1.80, 3.0, 2.20, 3.40, 2.60, 4.80, 5.0)$$

$$(4.20, 3.40, 6.60, 4.80, 6.0, 5.20, 7.40, 5.60, 7.80, 8.0)$$

and the variance covariance matrix is obtained using AR(1) covariance structure with $\rho = 0.75$ and $\sigma = 2$. The AR(1) covariance structure allow observations that are further apart to be less strongly correlated.

Below table shows the finite population mean and regression coefficients for all time points. For regression model the covariate x_i is assumed to be same for all time points and $x \sim log - normal(2, 2)$.

Table 2.2: Population mean and regression coefficients for all y's

	$Mean$	Min	Q_3	Max	β_0	β_1
y_{i1}	87.20	0.006	39.80	29045.81	86.86	0.0279
y_{i2}	38.91	0.001	16.61	9528.61	39.49	-0.0486
y_{i3}	808.10	0.000	76.10	556228.50	873.43	-5.39
y_{i4}	108.54	0.003	29.98	18922.10	114.95	-0.5296
y_{i5}	297.92	0.010	98.87	69079.59	328.40	-2.5173
y_{i6}	170.85	0.010	42.60	44158.04	192.37	-1.7778
y_{i7}	1619.30	0.000	179.30	1332815.00	1176.5290	36.56
y_{i8}	275.25	0.010	63.26	180442.27	311.11	-2.9615
y_{i9}	2206.90	0.000	591.20	814161.30	2271.9340	-5.37
y_{i10}	2397.50	0.000	731.90	415204.90	2222.86	14.4251

2.6.3 Response Probability Models

One can consider a range of models: some of which are compatible with the assumptions of the NEEs, while others represent different nonresponse mechanisms which are used to explore the sensitivity of the NEE approach. For $t \geq 2$, let

$$\pi_{it} = \Pr(\delta_{it} = 1 | \delta_{i,t-1}, y_{i1}, ..., y_{iT}, \mathbf{x}_i).$$

One can simulate monotone dropout patterns by increasing the coefficient of $\delta_{i,t-1}$ relatively, to make it the dominating predictor; one can accommodate informative nonresponse as long as the coefficients of $(y_{i1}, ..., y_{iT})$ are not all zero; one can achieve informative but stable response probability if $(y_{i1}, ..., y_{iT})$ is replaced by $\eta_i = \eta(y_{i1}, ..., y_{iT})$ that is a scalar function of them.

More specifically, for the simulation study, we consider the following response probability models,

$$logit(\pi_{it}) = \gamma_0 + \gamma_1\eta_i + \gamma_2 x_i \tag{2.41}$$

$$logit(\pi_{it}) = \gamma_0 + \gamma_1\delta_{i,t-1} + \gamma_2\eta_i + \gamma_3 x_i \tag{2.42}$$

$$logit(\pi_{it}) = \gamma_0 + \gamma_1 y_{it} + \gamma_2 x_i \tag{2.43}$$

$$logit(\pi_{it}) = \gamma_0 + \gamma_1\delta_{i,t-1} + \gamma_2 y_{it} + \gamma_3 x_i \tag{2.44}$$

$$logit(\pi_{it}) = \gamma_0 + \gamma_1 x_i \tag{2.45}$$

$$logit(\pi_{it}) = \gamma_0 + \gamma_1\delta_{i,t-1} + \gamma_2 x_i. \tag{2.46}$$

The model (2.41) is congenial to the estimator (2.2) and the model (2.42) possibly so, the models (2.43) and (2.44) are informative mechanisms that do not necessarily have stable response probabilities for the given population y-values, and the models (2.45) and (2.46) are MAR mechanisms. The values of $\gamma's$ can be chosen to achieve any desired overall response rate, where relatively large coefficients for y_{it} and η_i can add more informativeness to the response mechanism. After simulating the $\pi_{it}'s$, the $\delta_{it}'s$ are generated independently for each t.

2.6.4 Simulation set-up

We carry out simulations separately for the stable, volatile and simulated populations, as described below. The number of simulations are determined on the basis of about 1% CV of the Monte Carlo errors.

We experiment the different response probability models (2.41) - (2.46). The results using different estimator $\hat{\pi}_i$ in (2.2) are largely similar under models (2.42), (2.44) and (2.46) (see Tables D.4 to D.6), compared to those under models (2.41), (2.43) and (2.45), respectively, as long as the coefficient of $\delta_{i,t-1}$ under the former group is not large enough to induce monotone response patterns. Moreover, the model (2.45) yields stable MAR response probability, so that the NEE estimators are nearly unbiased. Below we focus on simulation under the models (2.41), (2.43) and (2.45), where η_i is set to the mean of $y_{i1}, ..., y_{iT}$, and the additional covariate x_i is a random variable from $LogN(2, 2)$.

For each response probability model, we set the γ-coefficients such that the overall response rate is either about 60% or 80%, to be referred to as the *low* or *high* response setting, respectively. As explained in Section (2.12), the bias of the NEE estimators depends on the correlation between w_i and $S_i(\theta_t)$. We can vary the correlation by changing the coefficient γ_1 in (2.41) and (2.43), relative to the other γ-terms, while holding the overall response rate at the required setting. We simulate a *low* correlation scenario, where the population correlation between w_i and $S_i(\theta_t)$ is e.g. -0.01819 at wave 7, and a *high* correlation scenario, where the correlation between w_i and $S_i(\theta_t)$ is -0.05805 at the same wave. The high correlation induces non-negligible bias of the NEE estimators.

Given each response probability model, the response indicators δ_{it}'s are generated independently for all the 10 waves, based on which the NEE estimator $\hat{\theta}_t$ from (2.5) is calculated using $\hat{\pi}_i$ in (2.2), for various T. Given that we have full knowledge of the simulated data, we can calculate the hypothetical estimator using the NEE based on the individual mean of the true response probabilities over time. Including this hypothetical estimator, denoted by $\mathbf{EE}_h$, allows us to understand when a result is due to the empirical property of the observed NEE, and when it is caused by the misspecified nonresponse assumption, i.e. when the data are simulated under the model (2.43) but the estimator is ideal under the model (2.41). Finally, the naïve estimator under the MCAR assumption is included as the baseline estimator for comparison.

2.6.5 Results

The results related to the estimation of mean and regression coefficients under above discussed simulation set-up are obtained and given in sections D.1 and D.2 of Appendix D respectively. These tables also include the results related to the bias adjustment that are discussed in Chapter 4. We discussed below some selective results and we report for each estimator its absolute percent relative bias (APRB), its standard error (SE), and the expected square root of variance estimator (ERSE).

Results for Mean: Table 2.3 shows the results under the model (2.41), in the high response setting and high correlation scenario, for T = 3, 4, 7 and 10. The results are obtained using stable, volatile and simulated data. The stable and volatile data follow the cyclic pattern and due to the cyclic pattern of the population data, the target parameter is the same for $T = 4, 7$ and 10, where $\theta_4 = \theta_7 = \theta_{10} = 11000.5$, so that the differences in the results are mainly caused by the length of response history. Notice that one may e.g. consider the NEE estimator by (2.5) to be based on 3, 6 and 9 weighting classes (of possible values of $\hat{\pi}_i$), respectively, for T = 4, 7 and 10. Moreover, we also obtained the results for $T = 3$ to know that how much NEE is effective for short history. For $T = 3$, the target parameter $\theta_3 = 11747.05$. For simulated data the target parameters for T = 3, 4, 7 and 10 are $\theta_3 = 808.1355$, $\theta_4 = 108.5394$, $\theta_7 = 1619.253$, $\theta_1 0 = 2397.519$,respectively. We observe the followings.

- The hypothetical estimator $\mathbf{EE}_h$ is unbiased for NEE under the model (2.41). The NEE estimators are biased in this high correlation scenario. The bias is nevertheless greatly reduced compared to the naïve estimator, and it decreases as T increases in general. The reason for the latter is that the bias has two causes: the informative nonresponse and the non-linear term $1/\hat{\pi}_i$. With large T and smaller variance of $\hat{\pi}_i$, the contribution of non-linearity to the bias decreases. As explained before, increased volatility of the individual y_{it}'s does not affect the bias of the NEE estimators here.
- Due to differential weighting, the SE of the hypothetical estimator can still be higher than the naïve estimator. The NEE estimators have even higher SEs, as can be expected. The bias-variance trade-off compared to the naïve estimator is clearly affected by the volatility of the individual y_{it}'s, although in these simulations it is still in favour of the NEE estimators for the volatile population.

- The variance of NEE is overestimated especially for small T, for example, when $T = 3$ and 4 whereas this overestimation decreases as T increases. This overestimation problem is address later in Chapter 4.

- Here the SEs are not much larger in volatile population than stable population because when the response is high then even with high correlation the volatility is not much affecting the variance of probability estimator under the congenial model.

Table 2.3: Results under model (2.41), high response and high correlation. Population: stable(Left), volatile(Middle) and simulated(Right)

	T=3, { Stable and Volatile: $\theta_3 = 11747.05$, Simulated: $\theta_3 = 808.1355$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.56	0.00	2.92	11.15	0.01	3.10	23.53	0.02	2.62
SE	43.44	44.44	54.26	27.85	29.97	38.62	5.82	6.05	7.77
ERSE	409.00	43.33	84.53	378.08	31.21	65.43	190.75	6.32	12.33
	T=4, { Stable and Volatile: $\theta_4 = 11000.5$, Simulated: $\theta_4 = 108.5394$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.06	0.01	5.42	15.92	0.00	4.31	21.49	0.01	5.58
SE	39.43	40.33	50.34	40.93	39.60	50.12	0.74	0.79	1.04
ERSE	331.84	39.89	82.45	344.21	39.22	78.17	11.87	0.80	1.65
	T=7, { Stable and Volatile: $\theta_7 = 11000.5$, Simulated: $\theta_7 = 1619.253$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.05	0.00	4.82	12.16	0.00	2.86	24.51	0.01	5.80
SE	38.59	40.09	49.92	41.39	40.53	48.30	12.86	12.91	15.79
ERSE	331.98	39.86	75.34	334.88	40.30	64.47	433.32	12.63	23.35
	T=10, { Stable and Volatile: $\theta_{10} = 11000.5$, Simulated: $\theta_{10} = 2397.519$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.05	0.00	3.25	10.26	0.01	1.56	23.72	0.01	3.84
SE	39.19	40.14	47.02	42.99	41.78	46.87	18.22	18.18	21.10
ERSE	332.06	39.85	60.40	329.53	42.13	54.01	234.88	18.19	27.35

Table 2.4: Results under model (2.43), high response and high correlation. Population: stable(Left), volatile(Middle) and simulated(Right)

	T=7, { Stable and Volatile: $\theta_7 = 11000.5$, Simulated: $\theta_7 = 1619.253$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.43	0.16	4.62	20.45	16.41	15.36	23.41	4.48	10.10
SE	39.39	40.12	49.82	38.96	39.26	74.83	11.78	10.18	25.64
ERSE	332.33	40.54	75.38	332.39	140.31	180.12	428.23	72.65	79.84

Table 2.4 shows the results under the model (2.43), also in the high response setting and high correlation scenario, for $T = 7$ only, as the message is the same for the other choices of T. The results show clearly that the underlying nonresponse assumption needs to be fairly close to the truth, in order for the NEE estimators to perform well. The risk of using the individual average of response probabilities over time is heightened with increasing volatility of the individual y_{it}'s, as can be seen from the bias of $\mathbf{EE}_h$ for the volatile population and from the simulated population that is also volatile to some extent; in contrast, $\mathbf{EE}_h$ remains nearly unbiased for the stable population. Since lack of mean heterogeneity is a potential shortcoming for any parametric

estimation approach in the presence of NMAR mechanisms, more empirical research is worthwhile regarding how to sensibly tailor the individual specification of $\hat{\pi}_i$ under the NEE approach.

Table 2.5: Results under model (2.45), high response and high correlation. Population: stable(Left), volatile(Middle) and simulated(Right)

	T=7, { Stable and Volatile: $\theta_7 = 11000.5$, Simulated: $\theta_7 = 1619.253$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.56	0.00	0.08	0.41	0.05	0.17	4.32	0.03	0.93
SE	104.42	139.62	189.08	109.96	141.35	187.60	80.55	87.35	107.82
ERSE	311.18	141.20	299.91	310.47	140.68	286.32	419.75	88.58	150.85

Table 2.5 shows the results under the model (2.45), also in the high response setting and high correlation scenario, for $T = 7$ only. Moreover, the model (2.45) yields stable MAR response probability, so that the NEE estimators are nearly unbiased and the results are not affected by volatility too.

Table 2.6: Results under model (2.41), by response and correlation. Population: stable (Left), volatile (Right). T=7, { Stable and Volatile: $\theta_7 = 11000.5$, Simulated: $\theta_7 = 1619.253$ }

	low response and low correlation								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	18.87	0.03	4.09	5.50	0.04	1.40	57.70	0.03	10.94
SE	107.97	89.74	104.57	175.88	158.29	189.45	28.71	19.33	21.77
ERSE	436.21	89.70	143.57	396.66	159.09	265.87	579.48	18.97	31.91
	low response and high correlation								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	35.88	0.01	7.33	15.02	0.07	3.08	59.28	0.02	11.48
SE	76.21	59.32	69.53	139.18	113.64	125.52	26.92	18.58	21.18
ERSE	445.35	58.32	102.19	445.93	106.71	154.08	565.99	18.35	32.19
	high response and low correlation								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	14.86	0.01	3.75	8.60	0.02	2.22	19.69	0.00	4.41
SE	33.98	36.17	46.70	53.40	52.66	61.39	11.15	10.89	13.11
ERSE	334.18	37.05	69.48	333.18	52.68	81.68	417.95	10.89	18.66
	high response and high correlation								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.05	0.00	4.82	12.16	0.00	2.86	24.51	0.01	5.80
SE	38.59	40.09	49.92	41.39	40.53	48.30	12.86	12.91	15.79
ERSE	331.98	39.86	75.34	334.88	40.30	64.47	433.32	12.63	23.35

In Table 2.6 the different response rate settings and correlation scenarios are contrasted with each other, for $T = t = 7$. We notice the followings.

- The effects of low response setting on the variance is clear and as expected, where all the SEs are increased, which is more striking for the volatile population when holding the correlation scenario fixed.

- The NEE estimators yield useful bias reduction compared to the naïve estimator in all the cases, even in the low response setting.

- It is interesting to observe how the bias of the NEE estimator varies. The bias is increased in the high correlation scenario, but more so under the low response setting. Moreover, the bias is higher absolutely in the low response setting, where SE is larger and the contribution of non-linear $1/\hat{\pi}_i$ is relatively greater.

Results for Regression Coefficients: Table 2.7 shows the results concerning the regression parameter estimation under the model (2.41) for same settings as above for the case of mean estimation given in Table 2.3. Due to the cyclic pattern of stable and volatile population data, the target regression parameters are also same for $T = 4, 7$ and 10, which are given below in the tables and also for $T = 3$. Similar to the case of mean estimation, the results under model 2.43 and 2.45 are given below in Table 2.8 and 2.9 respectively. Similar to the case of mean estimation above in Table 2.6, the different response rate settings and correlation scenarios are contrasted with each other, for $T = 7$ in Table 2.10 for the estimation of regression parameters. Observing the results concerning estimation of regression coefficients corresponding to the results of mean estimation given above, the discussions remain almost the same except the bias is very high for β_1 for low correlation setting using stable and volatile population.

Table 2.7: Results under model (2.41), high response and high correlation. Population: stable(Left), volatile(Middle) and simulated(Right)

T=3, (stable/volatile, simulated): $\beta_0 = (11764.93, 873.43)$, $\beta_1 = (-0.320, -5.39)$									
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	20.52	0.01	2.95	19.82	0.00	2.25	23.17	0.07	2.46
SE	44.40	45.57	56.65	48.25	45.64	57.10	7.53	8.02	10.23
ERSE	409.91	43.82	85.48	438.51	44.20	85.88	211.62	7.93	15.33
APRB(β_1)	0.76	1.35	3.60	6.81	3.72	4.04	18.85	0.27	1.50
SE	0.11	0.11	0.14	0.12	0.11	0.14	0.30	0.34	0.40
ERSE	0.81	0.11	0.21	0.97	0.11	0.20	5.04	0.32	0.59
T=4, (stable/volatile, simulated): $\beta_0 = (11023.06, 114.95)$ $\beta_1 = (-0.404, -0.529)$									
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	20.03	0.00	5.41	15.90	0.02	4.29	21.04	0.01	5.45
SE	40.60	41.58	51.08	40.25	38.99	48.24	0.92	0.98	1.26
ERSE	333.80	40.32	83.32	351.34	39.69	79.07	13.77	0.98	2.01
APRB(β_1)	6.05	1.52	4.42	3.61	0.32	3.17	14.51	0.17	4.15
SE	0.10	0.10	0.13	0.10	0.09	0.12	0.04	0.04	0.05
ERSE	0.55	0.10	0.19	0.99	0.10	0.19	0.52	0.04	0.08
T=7, (stable/volatile, simulated): $\beta_0 = (11023.06, 114.95)$ $\beta_1 = (-0.404, -0.529)$									
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	20.01	0.02	4.83	12.15	0.01	2.87	22.75	0.07	5.46
SE	38.95	40.21	50.80	41.06	39.90	46.37	37.03	41.90	50.10
ERSE	333.90	40.29	76.06	332.81	40.77	65.28	414.98	37.84	60.84
APRB(β_1)	7.12	0.38	3.45	13.14	0.45	3.04	30.68	0.49	6.66
SE	0.09	0.09	0.12	0.10	0.10	0.12	3.39	3.78	4.47
ERSE	0.55	0.09	0.17	0.47	0.10	0.15	53.43	3.38	5.44
T=10, (stable/volatile, simulated): $\beta_0 = (11023.06, 114.95)$ $\beta_1 = (-0.404, -0.529)$									
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	20.01	0.01	3.26	10.31	0.02	1.55	23.31	0.02	3.76
SE	41.07	42.43	47.63	43.81	42.48	45.85	29.15	30.28	34.93
ERSE	333.96	40.27	60.91	336.79	42.94	54.96	284.58	28.23	40.46
APRB(β_1)	6.92	0.69	2.73	86.21	13.57	1.72	30.03	0.73	4.29
SE	0.09	0.10	0.11	0.17	0.16	0.17	2.10	2.22	2.55
ERSE	0.55	0.10	0.14	0.61	0.15	0.18	17.55	2.04	2.85

Table 2.8: Results under model (2.43), high response and high correlation. Population: stable(Left), volatile(Middle) and simulated(Right)

T=7, (stable/volatile, simulated): $\beta_0 = (11023.06, 114.95)$ $\beta_1 = (-0.404, -0.529)$									
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	20.40	0.15	4.62	20.48	16.41	15.37	23.02	3.24	8.64
SE	39.35	40.12	49.96	39.12	39.33	72.27	34.39	33.57	44.91
ERSE	334.23	40.98	76.13	330.62	138.46	177.04	402.72	77.50	90.94
APRB(β_1)	8.59	1.07	3.32	34.67	20.46	18.50	25.01	7.82	14.28
SE	0.09	0.09	0.12	0.08	0.08	0.13	3.03	2.94	4.47
ERSE	0.54	0.10	0.17	0.44	0.19	0.25	51.61	10.09	11.07

Table 2.9: Results under model (2.45), high response and high correlation. Population: stable(Left), volatile(Middle) and simulated(Right)

T=7, (stable/volatile, simulated): $\beta_0 = (11023.06, 114.95)$ $\beta_1 = (-0.404, -0.529)$									
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	0.63	0.00	0.06	0.52	0.01	0.18	0.11	0.18	0.14
SE	106.41	141.22	192.89	114.54	145.20	187.33	104.59	113.25	143.68
ERSE	314.14	145.71	308.94	309.08	144.46	286.65	370.02	115.09	204.38
APRB(β_1)	8.80	0.07	0.78	6.28	0.12	2.21	0.39	0.23	0.19
SE	0.06	0.08	0.11	0.06	0.08	0.10	2.18	2.62	3.48
ERSE	0.46	0.08	0.17	0.33	0.08	0.16	41.98	2.66	5.05

Table 2.10: Results under model (2.41), by response and correlation. Population: stable (Left), volatile (Right). T=7, { Stable and Volatile: $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$, Simulated: $\beta_0 = 1176.529$, $\beta_1 = 36.56497$ }

	low response and low correlation								
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	19.58	0.02	4.18	5.81	0.03	1.48	75.87	0.06	13.89
SE	112.74	92.45	106.95	188.18	167.45	193.64	33.13	22.56	24.51
ERSE	444.61	91.90	146.47	397.40	162.97	267.54	448.57	21.65	34.75
APRB(β_1)	218.13	0.19	48.81	53.88	0.30	14.00	1.94	0.07	0.42
SE	0.05	0.05	0.05	0.08	0.08	0.10	1.56	1.30	1.44
ERSE	0.55	0.05	0.08	0.34	0.08	0.14	49.32	1.22	1.77
	low response and high correlation								
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	35.93	0.03	7.30	14.60	0.00	3.36	55.34	0.76	12.03
SE	76.90	59.70	69.38	129.11	106.78	122.34	101.85	61.43	63.38
ERSE	448.46	59.14	103.49	421.20	104.32	160.98	553.79	56.13	75.46
APRB(β_1)	42.87	3.75	6.50	7.05	0.29	3.69	70.96	2.17	10.20
SE	0.20	0.17	0.19	0.24	0.21	0.25	9.17	5.51	5.72
ERSE	0.83	0.15	0.25	0.61	0.20	0.33	70.57	4.98	6.71
	high response and low correlation								
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	15.42	0.00	3.87	8.98	0.02	2.24	30.46	0.02	6.57
SE	35.82	38.24	48.10	55.34	54.20	65.71	12.82	11.81	13.95
ERSE	339.17	38.25	71.55	333.78	54.29	84.02	369.42	11.94	20.25
APRB(β_1)	220.46	0.06	56.18	100.96	0.21	25.84	17.81	0.00	4.13
SE	0.02	0.02	0.03	0.03	0.03	0.03	0.29	0.29	0.34
ERSE	0.55	0.02	0.04	0.36	0.03	0.04	41.83	0.29	0.50
	high response and high correlation								
	naïve	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**	**naïve**	**EE$_h$**	**EE**
APRB(β_0)	20.01	0.02	4.83	12.15	0.01	2.87	22.75	0.07	5.46
SE	38.95	40.21	50.80	41.06	39.90	46.37	37.03	41.90	50.10
ERSE	333.90	40.29	76.06	332.81	40.77	65.28	414.98	37.84	60.84
APRB(β_1)	7.12	0.38	3.45	13.14	0.45	3.04	30.68	0.49	6.66
SE	0.09	0.09	0.12	0.10	0.10	0.12	3.39	3.78	4.47
ERSE	0.55	0.09	0.17	0.47	0.10	0.15	53.43	3.38	5.44

2.7 Conclusions

In this chapter we propose a new NEE approach for the estimation of parameters based on cross-sectional data subjected to informative missing mechanisms. The following conclusions emerge from the theoretical investigation and the simulation study.

The NEE approach is easy to compute and flexible in specification of the estimators of individual response probabilities. This makes it a widely applicable technique for exploratory data analysis of cross-sectional missing data mechanisms, based on the observed response history. The results can provide a basis for deciding whether more sophisticated modelling is needed in a given situation.

The NEE estimators are clearly better than naïve estimator, provided sensible choices of the response probability estimator, which can easily accommodate NMAR mechanisms. This is especially the case given low volatility of the individual outcome variables over time, despite considerable variation of the same outcome variable may exist across the population. The ability to vary the nonresponse assumption for different individuals makes it potentially a flexible alternative to standard parametric modelling approach, where the same model parameters are assumed to apply across the population.

The NEE estimator is not exactly unbiased, even when the response probability estimator is unbiased. The bias has two sources: the correlation between e.g. π_i and y_i, and the non-linearity of $1/\hat{\pi}_i$, or the variance of $\hat{\pi}_i$. The variance of $\hat{\pi}_i$ is naturally reduced given longer response history. We identify that the variance is overestimated using NEE approach, we will return to this later on in Chapter 4.

Chapter 3

Non-parametric Estimating Equations Approach for Longitudinal Data

3.1 Introduction

Longitudinal data analysis is of great interest in a wide array of disciplines across the medical, economic and social sciences. Cross-sectional data can only provide a snapshot at a single point of time and does not possess the capacity to reflect change, growth, or development. Aware of the limitations in cross-sectional studies, many researchers have advanced the analytic perspective by examining data with repeated measurements. By measuring the same variable of interest repeatedly over time, the change is displayed, and constructive findings can be derived with regard to the significance of patterns revealed. Data with repeated measurements are referred to as longitudinal data. In many longitudinal data designs, subjects are assigned specified levels of a treatment or subjected to other risk factors over a number of time points that are separated by specified intervals.

Analysing longitudinal data poses many challenges due to several unique features inherent in such data. First, the most troublesome feature of longitudinal analysis is missing data in repeated measurements. In a longitudinal survey, missing observations of the variable of interest frequently occurs. For example, in a clinical trial on the effectiveness of a new medical treatment for disease, patients may be unavailable to a follow-up investigation due to migration or health problems, or some baseline respondents may lose interest in participating at subsequent times. The missing cases may possess specific characteristics and attributes, resulting in the observed sample at later time points to have a different structure to the sample initially gathered. Second, repeated measurements for the same observational unit are usually correlated, due to the fact that they are clustered within each unit. At the same time, an individual's repeated measurements may be subjected to a time-varying systematic process, resulting in serial correlation. Third, longitudinal data may be ordered either in equal or unequal time intervals, where each scenario may call for a different analytic approach. Sometimes, despite an equal-spacing design, some respondents may enter a follow-up investigation after the specified survey date, which creates unequal time intervals for different individuals by chance.

To fix the scope it is helpful to make a distinction between repeated measures in general and the type of longitudinal data considered in this chapter. Repeated measures data represent a wider concept as they sometimes involve a large number of time points and permit changing experimental or observational conditions (West et al. (2007)). The longitudinal data which we consider here can be regarded as a special case of repeated measures data. They are composed of observations for the same subject ordered by a limited number of time points (i.e. *waves*) with predetermined time scale, interval and other related conditions. This is typical of data arising from longitudinal social surveys (Lynn (2009)). In statistics and econometrics, such longitudinal data are often referred to as panel data.

As indicated above, missing data is one of the primary problems to contend with in longitudinal data analysis. Missing data can be due to units that have dropped out (unit or wave nonresponse) or unanswered items(item nonresponse). At each wave longitudinal data are collected in a particular period of time in which the outcome and other relevant variables are recorded sequentially. Therefore, the researcher can only observe responses for those who are available within the duration of follow-up. There arise different situations of missing data. Sometimes the missing data represent a random sample of all cases. Or, missing data do not occur randomly, but the missing-ness can be controlled for with respect to observed variables such as age, gender, and health status. In these situations, missing data can cause loss of efficiency of the analysis but not necessarily bias. In many circumstances, however, the probability of data being missing is related to the missing values of the outcome variable, and failure to account for such missing data can be detrimental to the analysis of the pattern of change over time in the corresponding outcome variable. It is thus essential for the researcher to investigate and understand the nature of the missing data mechanism at hand The missing data mechanisms have already been discussed in Chapter 1.

Below we extend the NEE approach from the cross-sectional setting to the longitudinal setting. Here we want to estimate the change parameters for two successive time points. For this purpose we define two types of estimating equations; first, the EE that uses the individuals who respond at both time points and second, that uses also the individuals who respond at only one of the two time points. The set-up given below in Section 3.2 explains the models for response probabilities, and different response probability estimators for both types of estimating equations and way forward to use both types of EEs for estimation of change parameters. For both types of EEs, different response probability estimators are defined so that different dropout patterns can be captured underlying the assumed models for the unknown response probability. In Section 3.3 we discuss the bias of both types of estimating equations. The variance of change estimators and their plug-in estimators are derived both types of estimating equations and given in Appendix A.The variance of change estimators based on the second type of estimating equations require tedious algebra especially for its covariance term. Finally a simulation study is conducted to estimate the change parameters using both types of estimating equations using real and simulated data. The performance of NEE approach is explored using various simulation settings while comparing both types of EEs along with different response probability estimators.

3.2 Two NEE estimators of change

Under the NEE approach to MNAR nonresponse, we do not assume a parametric model of the response probabilities that pertain to *all* the population units. To accommodate potentially informative missing data, we postulate an *individual* response probability which may depend on the longitudinal outcomes of interest and covariates specific to each observational unit. The outcome values are also treated non-parametrically as unknown constants, just like in the design-based approach to survey sampling. Under this set-up, the observation propensity is estimated using individual-specific observation history, without involving the others in the population. The approach is applicable whenever there exist historical response/observation indicators. In other words, any unit who never responds will not be included in the estimation. To maintain the focus, we shall assume in this chapter that these never-respondents are a completely random sample from the population, without getting into the details of exploring different modelling options for them otherwise.

Let the population $U = \{1, ..., N\}$ be fixed over time points $t = 1, ..., T$, from the most distant ($t = 1$) to the most recent wave ($t = T$). Let y_{it} be a value associated with individual i at time t, for $i \in U$, and $\mathbf{y}_i = (y_{i1}, ..., y_{iT})^T$. Let δ_{it} be the response/observation indicator for y_{it} at time t. For the finite-population change, without losing generality, let us consider

$$\Delta_t = \bar{Y}_t - \bar{Y}_{t-1} = \sum_{i \in U} y_{it}/N - \sum_{i \in U} y_{i,t-1}/N = \sum_{i \in U} (y_{it} - y_{i,t-1})/N$$

for $t \geq 2$. Let $d_{it} = y_{it} - y_{i,t-1}$. The population EE that defines Δ_t is given by

$$H(\Delta_t) = N^{-1} \sum_{i=1}^{N} S_i(\Delta_t); \qquad H(\Delta_t) = 0, \tag{3.1}$$

i.e. $S_i(\Delta_t) = S(\Delta_t; \mathbf{y}_i) = d_{it} - \Delta_t$ is the 'score', so specified that Δ_t is the solution to $H(\Delta_t) = 0$.

Let $r_{it} = \delta_{i,t-1}\delta_{it}$. Let r_{it} and $r_{it'}$ be independent, where $t \neq t'$, given $\mathbf{y}_i$ and relevant covariates $\mathbf{x}_i$. An informative nonresponse probability assumption of r_{it} can be given as

$$p_{it} = \Pr(r_{it} = 1 | \mathbf{y}_i, \mathbf{x}_i) = \Pr(\delta_{i,t-1}\delta_{it} = 1 | \mathbf{y}_i, \mathbf{x}_i), \tag{3.2}$$

An unbiased respondent EE for Δ_t is then given by

$$\tilde{H}(\Delta_t) = N^{-1} \sum_{i=1}^{N} \frac{r_{it}}{p_{it}} S_i(\Delta_t); \qquad \tilde{H}(\tilde{\Delta}_t) = 0,$$

based on the respondents at both waves. However, $\tilde{H}(\Delta_t)$ is not operational because p_{it} is unknown. The observed (respondent) NEE is given by

$$\hat{H}(\Delta_t) = N^{-1} \sum_{i=1}^{N} \frac{r_{it}}{\hat{p}_{it}} S_i(\Delta_t); \qquad \hat{H}(\hat{\Delta}_t) = 0, \tag{3.3}$$

on replacing p_{it} with a suitable estimator $\hat{p}_{it}$ for each respondent with $r_{it} = 1$. The corresponding

NEE-based estimator of Δ_t is simply given by

$$\hat{\Delta}_t = \sum_{i=1}^{N} \frac{r_{it}}{\hat{p}_{it}} d_{it} / \sum_{i=1}^{N} \frac{r_{it}}{\hat{p}_{it}}.$$

In its basic form the estimator $\hat{p}_{it}$ is constructed individually for each observational unit on its own, based on the relevant historic values $r_{i2}, \ldots, r_{iT}$. For example, consider the following three estimators

$$\hat{p}_i = \sum_{t=2}^{T} \frac{r_{it}}{T-1}, \quad \hat{p}_{1i} = \sum_{t=2}^{T_{1i}} \frac{r_{it}}{T_{1i}-1} \quad \text{and} \quad \hat{p}_{2i} = \sum_{t=2}^{T_{2i}} \frac{r_{it}}{T_{2i}-1}, \tag{3.4}$$

where $T_{1i} = \max_{t=2,\ldots,T} \delta_{it} t$ is the most recent time point of response, and $T_{2i} = \max_{t=2,\ldots,T} r_{it} t$ is that of the most recent successive responses. The three estimators are the same if $t = T$, in which case $T_{1i} = T_{2i} = T$, but they may differ if $t < T$. For instance, suppose $T = 6$ and $t = 3$. The estimator $\hat{p}_i$ uses all the five r_{it}'s, which may not be appropriate for the units observed with a monotone missing data pattern, where nonresponse after the dropout point can be irrelevant to the estimation of response propensity before the dropout point. The dropout is dealt with somewhat differently by $\hat{p}_{1i}$ and $\hat{p}_{2i}$. Consider a unit with $r_{i3} = 1$, $\delta_{i4} = \delta_{i6} = 1$ and $\delta_{i5} = 0$, such that $T_{1i} = 6$ and $T_{2i} = 4$, by which $\hat{p}_{1i}$ and $\hat{p}_{2i}$ will differ to each other, even though $\sum_{t=2}^{T_{1i}} r_{it} = \sum_{t=2}^{T_{2i}} r_{it}$ in this case.

Notice that given the form of an estimator $\hat{p}_{it}$, whether it is unbiased depends on the nature of (3.2), which generally varies from one individual to another. For instance, for someone with a nonmonotone missing data pattern, $\hat{p}_i$ is unbiased if the response probability is constant over time, regardless how it depends on $\mathbf{y}_i$ and $\mathbf{x}_i$. Whereas the estimators $\hat{p}_{1i}$ and $\hat{p}_{2i}$ are biased in such a case, unless $T = t$, because the 'stopping' times T_{1i} and T_{2i} are informative otherwise. Of course, there are other situations where $\hat{p}_{1i}$ or $\hat{p}_{2i}$ may be unbiased instead of $\hat{p}_i$, and so on.

Finally, there will be individuals for which all these three estimators are biased, and some other estimator of p_{it} is more appropriate. For instance, one may allow $\hat{p}_{it}$ to depend on t, provided one detects a 'trend' in the r_{it}'s over time for the given individual. The flexibility of the approach here is that it allows one to *vary* the specification of $\hat{p}_{it}$ for each individual and the assumption that is considered most appropriate given the response history of that particular individual, instead of imposing a single parametric form across the population, as under the parametric modelling approach to (3.2).

Now, the NEE (3.3) uses only the completely observed units at both t and $t-1$. This could potentially entail a loss of efficiency. For an alternative that uses the individuals who respond at either one of the two time points, consider the MNAR response probability assumption for each t:

$$\pi_{it} = \Pr(\delta_{it} = 1 | \mathbf{y}_i, \mathbf{x}_i). \tag{3.5}$$

The observed (respondent) NEE can then be given as

$$\begin{cases} \hat{H}(\theta_t) = N^{-1} \sum_{i=1}^{N} \frac{\delta_{it}}{\hat{\pi}_{it}} S_i(\theta_t) \\ \hat{H}(\theta_{t-1}) = N^{-1} \sum_{i=1}^{N} \frac{\delta_{i,t-1}}{\hat{\pi}_{i,t-1}} S_i(\theta_{t-1}) \end{cases} \tag{3.6}$$

where $\theta_t = \bar{Y}_t$ and $\theta_{t-1} = \bar{Y}_{t-1}$, such that $\Delta_t = \theta_t - \theta_{t-1}$, and $S_i(\theta_t) = y_{it} - \theta_t$ is such that that θ_t is the solution to the population EE, $H(\theta_t) = 0$, and similarly for $S_i(\theta_{t-1})$. The $\hat{\pi}_{it}$ is response probability estimator for time t and $\hat{\pi}_{i,t-1}$ that for time $t-1$. Below we refer to (3.6) as $\hat{H}(\theta_t, \theta_{t-1})$.

Having estimated the cross-sectional parameters θ_t and θ_{t-1}, one can derive the corresponding NEE-based estimate of Δ_t as the difference between the two, which is given by

$$\hat{\Delta}_t = \sum_{i=1}^{N} \frac{\delta_{it}}{\hat{\pi}_{it}} y_{it} / \sum_{i=1}^{N} \frac{\delta_{it}}{\hat{\pi}_{it}} \; - \; \sum_{i=1}^{N} \frac{\delta_{i,t-1}}{\hat{\pi}_{i,t-1}} y_{i,t-1} / \sum_{i=1}^{N} \frac{\delta_{i,t-1}}{\hat{\pi}_{i,t-1}}.$$

The variance of this estimator is given as

$$V(\hat{\Delta}_t) = V(\hat{\theta}_t) - 2Cov(\hat{\theta}_t, \hat{\theta}_{t-1}) + V(\hat{\theta}_{t-1}). \tag{3.7}$$

Similarly to p_{it}, one may postulate different non-parametric estimators of π_{it}. We consider the following two estimators in the simulation study later on:

$$\hat{\pi}_i = \sum_{i=1}^{T} \frac{\delta_{it}}{T} \qquad \text{and} \qquad \hat{\pi}_{1i} = \sum_{i=1}^{T_{1i}} \frac{\delta_{it}}{T_{1i}}, \tag{3.8}$$

As with the estimator $\hat{p}_{it}$ above, one can subject the estimator $\hat{\pi}_{it}$ to appropriate modifications, to suit different assumptions of (3.5) across the observational units.

3.3 On the bias of NEE

The NEE $\hat{H}(\Delta_t)$ given by (3.3) or $\hat{H}(\theta_t, \theta_{t-1})$ given by (3.6) is not exactly unbiased, even when $\hat{p}_{it}$ is unbiased for p_{it} or $\hat{\pi}_{it}$ unbiased for π_{it}. Here we examine the bias and explore possible adjustments.

Let $\tau_{it} = E(\hat{p}_{it})$ be the expectation of $\hat{p}_{it}$, where $\tau_{it} = p_{it}$ in case $\hat{p}_{it}$ is unbiased. Via Taylor expansion of $\hat{p}_{it}$ around τ_{it}, provided $\hat{p}_{it} \neq 0$, the bias of NEE (3.3) can be written as

$$\begin{aligned} B_1 &= E[\hat{H}(\Delta_t)] - H(\Delta_t) = N^{-1} \sum_{i=1}^{N} S_i(\Delta_t) E\left(\frac{r_{it}}{\hat{p}_{it}} - 1\right) \\ &= N^{-1} \sum_{i=1}^{N} S_i(\Delta_t) \left\{ (\frac{2p_{it}}{\tau_{it}} - 1) - \frac{E(r_{it}\hat{p}_{it})}{\tau_{it}^2} + \frac{E(r_{it}\hat{p}_{it}^2) - 2\tau_{it}E(r_{it}\hat{p}_{it}) + \tau_{it}^2 p_{it}}{p_i^{*3}} \right\} \end{aligned} \tag{3.9}$$

for some p_i^* between τ_{it} and $\hat{p}_{it}$. Further derivation of the expression (3.9) depends on the choice of $\hat{p}_{it}$. The case of $\hat{p}_{it} = \hat{p}_i$ in (3.4) is given below; the other cases are similar and omitted here. We have

$$E(r_{it}\hat{p}_i) = E(\frac{r_{it}}{T-1} \sum_{t'=2}^{T} r_{it'}) = \frac{1}{T-1} \sum_{t'=2}^{T} p_{it} p_{it'} + \frac{1}{T-1} Var(r_{it}) = p_i^2 + V(\hat{p}_i)$$

where the last expression follows if $p_{it} \equiv p_i$, i.e. when $\hat{p}_i$ is unbiased for p_{it}. Next, after some algebra,

$$E(r_{it}\hat{p}_i^2) = E\big[\frac{r_{it}}{(T-1)^2}(\sum_{t'=2}^{T} r_{it'})^2\big] = \frac{p_i\kappa_i}{(T-1)^2},$$

where $\kappa_i = 1 + 3(T-2)p_i + 2(T-2)(T-3)p_i^2$. Thus, given $\tau_{it} = p_{it}$, the bias (3.9) becomes

$$B_1 = N^{-1}\sum_{i=1}^{N} w_i S_i(\Delta_t),$$

where

$$w_i = \frac{p_i\kappa_i}{(T-1)^2 p_i^{*3}} - \frac{p_i^3}{p_i^{*3}} + (\frac{2p_i}{p_i^{*3}} - \frac{1}{p_i^2})V(\hat{p}_i).$$

The coefficients w_i's above are functions of p_i, such that it may depend on $\mathbf{y}_i$, even though the functional form of the dependence is unspecified under the NEE approach. In other words, the term B_1 is not zero as long as the population covariance of w_i and $S_i(\Delta_t)$ is not zero, which is given by $N^{-1}\sum_{i=1}^{N} w_i S_i$ since $N^{-1}\sum_{i=1}^{N} S_i(\Delta_t) = 0$ by definition.

Consider the two cross-sectional NEEs in (3.6) separately. Let $\tau_{it} = E(\hat{\pi}_{it})$ be the expectation of $\hat{\pi}_{it}$ for wave t. By Taylor expansion, provided $\hat{\pi}_{it} \neq 0$, we obtain

$$\begin{aligned} B_2(t) = E[\hat{H}(\theta_t)] - H(\theta_t) &= N^{-1}\sum_{i=1}^{N} S_i(\theta_t) E\left(\frac{\delta_{it}}{\hat{\pi}_{it}} - 1\right) \\ &= N^{-1}\sum_{i=1}^{N} S_i(\theta_t)\left\{(\frac{2\pi_{it}}{\tau_{it}} - 1) - \frac{E(\delta_{it}\hat{\pi}_{it})}{\tau_{it}^2} + \frac{E(\delta_{it}\hat{\pi}_{it}^2) - 2\tau_{it}E(\delta_{it}\hat{\pi}_{it}) + \tau_{it}^2\pi_{it}}{\pi_i^{*3}}\right\} \end{aligned} \quad (3.10)$$

for some π_i^* between τ_{it} and $\hat{\pi}_{it}$. Similarly as above, in the case of unbiased $\hat{\pi}_i$ in (3.8), we have

$$E(\delta_{it}\hat{\pi}_i) = \pi_i^2 + V(\hat{\pi}_i) \qquad \text{and} \qquad E(\delta_{it}\hat{\pi}_i^2) = \pi_i\kappa_i'/T^2,$$

where $\kappa_i' = 1 + 3(T-1)\pi_i + 2(T-1)(T-2)\pi_i^2$. The corresponding bias (3.10) becomes

$$B_2(t) = N^{-1}\sum_{i=1}^{N} u_i S_i(\theta_t),$$

where

$$u_i = \frac{\pi_i\kappa_i'}{T^2\pi_i^{*3}} - \frac{\pi_i^3}{\pi_i^{*3}} + (\frac{2\pi_i}{\pi_i^{*3}} - \frac{1}{\pi_i^2})V(\hat{\pi}_i)$$

One can obtain $B_2(t-1)$ as the bias of $\hat{H}(\theta_{t-1})$ similarly. Again, B_2 may not be zero if the population covariance of u_i and $S_i(\theta_t)$ is not zero.

The variances of respective change estimators are given in Appendix A

3.4 Simulation Study

The NEE approach provides a method for exploring informative nonresponse in the longitudinal setting, which is computationally easy and flexible in specification. From the outset there are several factors that can be expected to affect its performance in a given situation.

First, provided the suitable nonresponse assumptions for all the responding individuals, the NEE estimator should perform better given a *longer* history of response. For instance, in the case of $T = t = 3$, where there are two observations of r_{it} for each individual, there are only two possible histories for each respondent with $r_{i3} = 1$, where r_{i2} is either 1 or 0. The estimate $\hat{p}_{it}$ that enters the NEE (3.3) either takes value 1 or 0.5, such that the estimator of Δ_t is only based on two weighting classes. Whereas the näive estimator under the MCAR assumption is based on a single weighting class. Clearly, the ability to adjust for potentially informative nonresponse by the NEE approach is rather limited in this case. Thus, a factor that matters in the simulation study will be the length of response history. Therefore, we estimated the change in mean and change in regression coefficients using $T = t = 3$, 4, 7 and 10, and using $t = 4$, 7 when $T = 10$.

Next, a relevant factor is the variation of y_{it}'s over time, for each given individual. Take again the estimator $\hat{p}_t$ that is averaged over all the r_{it}'s. On the one hand, it is unbiased if the informative response propensity depends only on a scalar summary of the y_{it}'s, in which case it does not matter how volatile the y_{it}'s are over time. On the other hand, intuitively the risk of bias is heightened, when the y_{it}'s are volatile, as compared to the extreme case where $y_{it} \equiv y_i$ is completely static. Moreover, as the variance of the resulting estimator of Δ_t increases with more volatile y_{it}'s, it would be interesting to explore if this has any compounding effect together with the heightened risk of bias.

Last but not least, the nonresponse mechanism itself will be a critical factor to the performance of the NEE. That is, if the assumption for unbiased estimation of p_{it}'s is clearly violated, then the NEE estimator may suffer extra bias beyond the inherent bias of the NEE as explained in Section 3.3.

Below we describe first the data used for the simulation, the models that can be used for simulating response history, and then the chosen simulation set-up, including the specific response probability models, the sample size corresponding to overall response rate, and the estimators to be evaluated, before we present and discuss the simulation results.

3.4.1 Data and Response Probability Models

We use stable, volatile and simulated data for estimation of cross-sectional parameters in the previous chapter and the same data sets are used here for the estimation of longitudinal parameters. We estimate the change parameter such as change in mean and change in the regression coefficients. We use the same set of response probability models given in Section 2.6.3 of previous Chapter.

3.4.2 Simulation set-up

We carry out simulations separately for the stable, volatile and simulated population, as described below. The number of simulations are determined on the basis of about 1% CV of the Monte Carlo errors.

We experiment the different response probability models (2.41) - (2.46). The results using different estimators $\hat{p}_{it}$ in (3.4) and $\hat{\pi}_{it}$ in (3.8) are largely similar under models (2.42), (2.44) and (2.46), compared to those under models (2.41), (2.43) and (2.45), respectively, as long as the coefficient of $\delta_{i,t-1}$ under the former group is not large enough to induce monotone response patterns. Alternatively, when clear monotone response patterns are simulated, the estimators $\hat{p}_{1i}$ and $\hat{p}_{2i}$ would outperform $\hat{p}_{it}$, as can be expected; similarly for $\hat{\pi}_{1i}$ compared to $\hat{\pi}_i$. Moreover, the model (2.45) yields stable MAR response probability, so that the NEE estimators are nearly unbiased as seen in the case of cross-sectional setting in the previous chapter. Below we focus on simulation under the models (2.41) and (2.43), where η_i is set to the mean of $y_{i1}, ..., y_{iT}$, and the additional covariate x_i is a random variable from $LogN(2, 2)$.

For each response probability model, we set the γ-coefficients such that the overall response rate is either about 60% or 80%, to be referred to as the *low* or *high* response setting, respectively. As explained in Section (3.3), the bias of the NEE estimators depends on the correlation between w_i and $S_i(\Delta_t)$ or, similarly, that between u_i and $S_i(\theta_t)$. We can vary the correlation by changing the coefficient γ_1 in (2.41) and (2.43), relative to the other γ-terms, while holding the overall response rate at the required setting. We simulate a *low* correlation scenario, where the population correlation between w_i and $S_i(\Delta_t)$ is e.g. -0.0536 at wave 6, and a *high* correlation scenario, where the correlation between w_i and $S_i(\Delta_t)$ is -0.1363 at the same wave. The high correlation induces non-negligible bias of the NEE estimators.

Given each response probability model, the response indicators δ_{it}'s are generated independently for all the 10 waves, based on which the NEE estimator $\hat{\Delta}_t$ from (3.3) is calculated using $\hat{p}_i$ in (3.4), denoted by $\mathbf{EE}(\hat{p}_i)$, for various combinations of (T, t). Given that we have full knowledge of the simulated data, we can calculate the hypothetical estimator using the NEE based on the individual mean of the true response probabilities over time. Including this hypothetical estimator, denoted by **EE_**h, allows us to understand when a result is due to the empirical property of the observed NEE, and when it is caused by the misspecified nonresponse assumption, i.e. when the data are simulated under the model (2.43) but the estimator is ideal under the model (2.41). Finally, the naïve estimator under the MCAR assumption is included as the baseline estimator for comparison. Similarly for the NEE estimator $\hat{\Delta}_t$ from (3.6), calculated using $\hat{\pi}_i$ in (3.8) and denoted by $\mathbf{EE}(\hat{\pi}_i)$, together the corresponding hypothetical estimator $\mathbf{EE}_h$and naïve estimator under the MCAR assumption.

3.4.3 Results

The results related to the estimation of change in mean and change in regression coefficients under above simulation set-up are obtained and given in section E.1 and E.2 of Appendix E, respectively. These tables also include the results related to the bias adjustment that is discussed in next chapter. We discussed below some selective results. In the results below we report for each estimator its absolute percent relative bias (APRB), its standard error (SE), and the expected square root of variance estimator (ERSE).

Estimation of Change in Mean: Table 3.1 shows the results under the model (2.41), in the high response setting and high correlation scenario, for $T = t = 4, 7$ and 10. Due to the cyclic pattern of the stable and volatile population data, the target parameter is the same on all these three occasions, where $\Delta_4 = \Delta_7 = \Delta_{10} = -746.55$, so that the differences in the results are chiefly caused by the length of response history. Notice that one may e.g. consider the NEE estimator by (3.3) to be based on 3, 6 and 9 weighting classes (of possible values of $\hat{p}_i$), respectively, for $T = 4$, 7 and 10. Moreover, for simulated data the target parameters are $\Delta_4 = -699.5961$, $\Delta_7 = 1448.408$ and $\Delta_{10} = 190.6555$. From results of Table 3.1, we observe the followings.

- The hypothetical estimator $\mathbf{EE}_h$is unbiased for both NEEs under the model (2.41). Both the NEE estimators are biased in this high correlation scenario. The bias is nevertheless greatly reduced compared to the naïve estimator, and it decreases as T increases. The reason for the latter is that the bias has two causes: the informative nonresponse and the non-linear term $1/\hat{p}_i$. With large T and smaller variance of $\hat{p}_i$, the contribution of non-linearity to the bias decreases. As explained before, increased volatility of the individual y_{it}'s does not affect the bias of the NEE estimators here.
- Due to differential weighting, the SE of the hypothetical estimator can still be higher than the naïve estimator. The NEE estimators have even higher SEs, as can be expected. The bias-variance trade-off compared to the naïve estimator is clearly affected by the volatility of the individual y_{it}'s, although in these simulations it is still in favour of the NEE estimators for the volatile population.
- While the variance of the NEE estimator by (3.3) is increased dramatically from the stable to the volatile population, that from the NEE (3.6) is about the same for both. As the δ_{it}'s are independent over time, the covariance term in (3.7) is close to zero, so that the NEE (3.6) actually loses efficiency compared to (3.3) for the stable population, despite it ostensibly uses more observations. Moreover, its variance remains about the same for the volatile population and for the simulated population, because the population distribution is about the same over time according to how these population are generated.

Table 3.1: Results under model (2.41), high response and high correlation. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=4, {Stable and Volatile: $\Delta_4 = -746.55$}, {Simulated: $\Delta_4 = -699.5961$ }																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	48.98	0.10	12.70	24.24	0.19	22.13	67.29	0.04	15.51	32.61	0.00	16.24	46.79	0.07	14.48	24.15	0.01	3.09
SE	10.58	10.75	14.86	59.45	61.21	77.01	53.28	47.68	75.34	52.97	52.71	66.78	7.82	9.01	10.00	6.36	6.55	8.02
ERSE	258.14	10.62	22.04	517.54	59.02	102.42	641.63	48.17	110.73	528.60	53.78	91.82	224.39	8.98	14.49	191.23	6.40	11.01
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	51.83	0.02	1.35	25.46	0.44	0.85	61.64	0.27	0.04	29.51	0.40	0.06	48.70	0.01	2.22	24.65	0.04	0.03
SE	10.84	10.93	14.14	56.65	58.61	69.09	62.78	57.42	75.23	53.07	52.75	59.37	16.90	18.00	22.25	12.65	12.73	14.64
ERSE	258.25	10.73	20.15	517.70	58.86	90.18	610.92	58.27	104.88	513.67	53.06	72.75	512.24	18.40	34.50	432.83	12.70	18.72
	T=t=10, {Stable and Volatile: $\Delta_{10} = -746.55$}, {Simulated: $\Delta_{10} = 190.6555$ }, high response and high correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	53.04	0.02	1.46	26.45	0.02	0.14	53.97	0.08	2.48	26.56	0.11	0.17	45.02	0.01	0.75	22.95	0.08	0.07
SE	10.40	10.60	13.51	56.48	58.34	65.25	71.00	65.57	78.79	54.05	53.66	58.06	3.79	3.71	4.62	24.83	24.81	27.33
ERSE	258.41	10.81	18.71	518.04	58.85	75.99	596.17	65.10	98.66	505.98	54.42	64.34	341.29	3.65	6.21	367.54	24.78	31.84

Table 3.2: Results under model (2.43), high response and high correlation. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	61.43	0.85	5.88	41.36	10.23	13.41	42.28	27.69	32.83	40.81	21.77	33.89	32.71	4.81	5.20	8.33	8.34	9.87
SE	22.50	21.71	27.84	57.54	58.27	67.66	247.73	249.15	370.33	59.49	60.00	124.60	14.07	12.73	54.26	7.00	6.71	20.02
ERSE	260.94	21.56	39.15	521.26	60.31	91.60	586.62	324.86	659.95	526.75	216.54	300.99	462.09	85.96	116.66	382.06	54.49	62.46

Table 3.2 shows the results under the model (2.43), also in the high response setting and high correlation scenario, for $T = t = 7$ only, as the message is the same for the other choices of (T, t). The results show clearly that the underlying nonresponse assumption needs to be fairly close to the truth, in order for the NEE estimators to perform well. The risk of using the individual average of response probabilities over time is heightened with increasing volatility of

the individual y_{it}'s, as can be seen from the bias of $\mathbf{EE}_h$for the volatile population; in contrast, $\mathbf{EE}_h$remains nearly unbiased for the stable population. Since lack of mean heterogeneity is a potential shortcoming for any parametric estimation approach in the presence of NMAR mechanisms, more empirical research is worthwhile regarding how to sensibly tailor the individual specification of $\hat{p}_{it}$ (or $\hat{\pi}_{it}$) under the NEE approach.

Table 3.3: Results under model (2.41), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, low response and low correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	112.36	0.53	12.08	47.76	1.26	4.95	96.80	1.36	15.52	44.19	2.30	3.83	156.22	0.05	12.73	62.97	0.03	0.28
SE	103.40	79.53	110.59	151.28	127.65	147.96	447.53	354.59	497.54	254.91	226.17	272.63	57.61	27.15	36.69	27.61	18.43	21.32
ERSE	438.89	77.85	145.58	675.40	131.27	188.97	909.33	368.02	682.58	628.27	233.79	337.46	916.51	28.64	54.06	578.91	18.71	29.44
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, low response and high correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	156.16	0.08	12.80	63.57	0.20	5.33	156.41	1.56	15.16	63.56	0.34	3.29	144.69	0.06	12.43	60.34	0.04	0.38
SE	55.76	36.77	50.58	105.40	82.91	97.92	308.21	205.82	288.36	172.01	142.27	158.61	55.30	28.53	36.90	26.85	18.42	21.13
ERSE	448.59	37.08	68.58	681.29	83.33	133.26	1038.50	210.15	390.31	667.36	140.55	195.84	865.02	28.22	53.07	565.64	18.48	29.19
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, high response and low correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	50.28	0.08	1.79	25.33	0.12	1.40	56.65	0.36	1.39	27.78	0.54	0.21	49.07	0.01	1.23	24.35	0.02	0.05
SE	18.95	19.14	25.31	51.46	54.96	65.86	109.84	110.36	146.68	76.58	77.47	86.58	18.27	17.72	21.51	13.09	12.65	14.30
ERSE	257.57	19.05	35.79	518.37	53.92	82.26	610.77	108.48	203.36	515.95	77.01	103.01	513.97	17.68	33.57	432.02	12.35	17.63
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, high response and high correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	51.83	0.02	1.35	25.46	0.44	0.85	61.64	0.27	0.04	29.51	0.40	0.06	48.70	0.01	2.22	24.65	0.04	0.03
SE	10.84	10.93	14.14	56.65	58.61	69.09	62.78	57.42	75.23	53.07	52.75	59.37	16.90	18.00	22.25	12.65	12.73	14.64
ERSE	258.25	10.73	20.15	517.70	58.86	90.18	610.92	58.27	104.88	513.67	53.06	72.75	512.24	18.40	34.50	432.83	12.70	18.72

In Table 3.3 the different response rate settings and correlation scenarios are contrasted with each other, for $T = t = 7$. Using all three populations data, we observe the followings.

- The effects of low response setting on the variance is clear and as expected, where all the SEs are increased, which is more dramatic for the volatile population when holding the correlation scenario fixed.
- The NEE estimators yield useful bias reduction compared to the naïve estimator in all the cases, even in the low response setting where the NEE estimator $\mathbf{EE}(\hat{p}_i)$ from (3.3) has large bias itself.
- It is intriguing to observe how the bias of the NEE estimator $\mathbf{EE}(\hat{p}_i)$ from (3.3) varies. The bias is increased in the high correlation scenario, but more so under the low response setting. Moreover, the bias is much higher absolutely in the low response setting, where $\mathrm{SE}(\hat{p}_i)$ is larger and the contribution of non-linear $1/\hat{p}_i$ is relatively greater.

The results for $t < T$ are given in Tables from E.8 to E.15 of Appendix E, which lead to few new understandings in these simulations, since estimation for $t < T$ is based on the same $\hat{p}_i$ and $\hat{\pi}_i$ as $t = T$. One can remove the arbitrary difference in Δ_t and Δ_T by e.g. letting $t = T - 3$, given the cyclic data here. The differences in the results would be entirely due to Monte Carlo variation in the simulated δ_{it}'s and δ_{iT}'s. Furthermore for $t < T$, all three estimators (3.4) are different and similarly the two estimators (3.8). The simulated data is not cyclic. To know the performance of different response probability estimators, below we present the results in Table 3.4 for simulated population when $T = 10$ and $t = 7$.

Table 3.4: Results under model (2.41), by response and correlation. Population: stable

	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	$\mathbf{EE}(\hat{p}_{i1})$	$\mathbf{EE}(\hat{p}_{i2})$	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	$\mathbf{EE}(\hat{\pi}_{i1})$
	T=10, t=7, $\Delta_7 = -746.55$, low response and low correlation.								
APRB	144.62	0.03	9.24	18.08	31.64	58.74	0.04	0.05	7.46
SE	57.80	28.97	38.63	40.78	42.48	28.69	19.64	22.38	23.07
ERSE	896.79	29.25	55.08	57.01	55.60	571.84	19.23	25.19	24.11
	T=10, t=7, $\Delta_7 = -746.55$, low response and high correlation.								
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	$\mathbf{EE}(\hat{p}_{i1})$	$\mathbf{EE}(\hat{p}_{i2})$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	$\mathbf{EE}(\hat{\pi}_{i1})$
APRB	137.74	0.10	9.74	18.57	31.85	57.98	0.11	0.14	7.60
SE	53.79	29.02	39.50	41.41	42.42	26.17	18.45	21.12	21.68
ERSE	852.12	28.22	53.88	55.55	53.86	561.83	18.46	24.65	23.41
	T=10, t=7, $\Delta_7 = -746.55$, high response and low correlation.								
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	$\mathbf{EE}(\hat{p}_{i1})$	$\mathbf{EE}(\hat{p}_{i2})$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	$\mathbf{EE}(\hat{\pi}_{i1})$
APRB	43.99	0.00	2.14	5.44	11.24	21.60	0.01	0.01	2.95
SE	18.77	18.03	22.18	22.17	21.25	12.80	12.28	13.74	13.54
ERSE	509.64	17.40	29.71	28.60	25.16	428.54	12.08	14.59	13.49
	T=10, t=7, $\Delta_7 = -746.55$, high response and high correlation.								
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	$\mathbf{EE}(\hat{p}_{i1})$	$\mathbf{EE}(\hat{p}_{i2})$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	$\mathbf{EE}(\hat{\pi}_{i1})$
APRB	48.36	0.01	3.04	6.90	13.26	24.51	0.04	0.05	3.52
SE	16.99	17.70	21.78	21.52	20.35	12.55	12.51	14.37	13.88
ERSE	513.02	18.22	32.88	31.53	27.47	433.33	12.55	15.86	14.41

The results using different estimators $\hat{p}_{it}$ in (3.4) and $\hat{\pi}_{it}$ in (3.8) varies. The estimators $\hat{p}_i$ and $\hat{\pi}_i$ are outperforming than their counterparts, that means the clear monotone response patterns are not emerging in these simulations. Alternatively, when clear monotone response patterns are simulated, the estimators $\hat{p}_{1i}$ and $\hat{p}_{2i}$ would outperform $\hat{p}_{it}$, as can be expected; similarly for $\hat{\pi}_{1i}$ compared to $\hat{\pi}_i$.

The results concerning the estimation of change in regression coefficients are given in Tables E.16 to E.30 of Appendix E for $T = t = 4$, 7, and 10. Observing these tables, the discussions remain almost same as above in the case of change in mean estimation. Therefore, the results for change in regression coefficients are also omitted here.

3.5 Conclusions

In this chapter we propose we extended the NEE approach to estimation based on longitudinal data subjected to informative missing mechanisms. The following conclusions can be made.

The NEE approach is easy to compute and flexible in specification of the estimators of individual response probabilities. This makes it a widely applicable technique for exploratory data analysis of longitudinal missing data mechanisms, based on the observed response history. The results can provide a basis for deciding whether more sophisticated modelling is needed in a given situation.

The NEE estimators are clearly better than naïve estimator, provided sensible choices of the response probability estimator, which can easily accommodate NMAR mechanisms. This is especially the case given low volatility of the individual outcome variables over time, despite considerable variation of the same outcome variable may exist across the population. The ability to vary the nonresponse assumption for different individuals makes it potentially a flexible alternative to standard parametric modelling approach, where the same model parameters are assumed to apply across the population.

Chapter 4

Bias-adjusted Non-parametric Estimating Equations Approach

4.1 Introduction

In Chapter 2, we discussed the NEE approach for the cross-sectional setting; the observed NEE is not unbiased and their bias is also discussed there. Similarly in Chapter 3, we extended the NEE approach to the longitudinal setting and defined two NEEs. The bias in both NEEs is also discussed in this chapter. From the expressions for the biases, a bias-adjusted NEE approach can be developed for cross-sectional as well as for longitudinal settings. We noted from the simulation results of previous chapters that there is need to adjust the bias in estimates and especially the bias in variance estimates that are overestimated in most of cross-sectional and longitudinal cases. As we could not prove the consistency of estimates, therefore, the bias-adjusted NEE becomes more important.

In this Chapter, we define the bias-adjusted NEE approach for both settings to estimate the respective parameters. The bias-adjusted NEE is also not unbiased rather this approach may reduce the bias as compared to the simple NEE approach. It is expected that the bias-adjusted NEEs reduce the bias possibly in parameter estimates and bias in their variance estimates. The variance of estimators along with their plug-in estimators are also derived using the bias-adjusted EEs. Furthermore, we also adjust the bias in plug-in variance estimate using Taylor expansion for simple NEE approach as well as for bias-adjusted NEE approaches under cross-sectional and longitudinal settings.

For simulation results using bias-adjusted NEE approach, every thing remains the same as given in simulation study sections of Chapter 2 and 3 for cross-sectional setting and longitudinal settings, respectively. We compare the results based on bias-adjusted NEE with corresponding results of simple NEEs to asses the effectiveness of bias adjustment in terms of bias and variance.

Based on simulation results, the adjustment of variance estimates using Taylor expansion could not turn out to be more effective. Therefore, we will only make the comparison of variance estimates using NEE with bias-adjusted NEE.

4.2 Bias-Adjusted NEE approach for Cross-sectional Setting

In previous Chapter 2, we discussed the NEE approach to estimate the cross-sectional parameters. The observed NEE is not unbiased and bias in NEEs is already discussed in Section 2.3. Below in section 4.2.1, we define the bias-adjusted NEE approach for cross-sectional setting. The corresponding plug-in variance estimators based on NEE and bias-adjusted NEE are given in Section 4.2.2. The bias correction of these variance estimators using Taylor expansions is discussed in Appendix B.

4.2.1 Bias-adjusted NEE

The bias in estimating equations $\hat{H}_N(\theta)$ has already been discussed in Section 2.12 of Chapter 2, and rewriting the expression for the bias,

$$
\begin{aligned}
B = E[\hat{H}_N(\theta)] - H_N(\theta) &= N^{-1}\sum_{i=1}^{N} S_i(\theta) E\left(\frac{\delta_i}{\hat{\pi}_i} - 1\right) \\
&= N^{-1}\sum_{i=1}^{N} S_i(\theta)\left\{-\frac{E[\delta_i(\hat{\pi}_i - \pi_i)]}{\pi_i^2} + \frac{E[\delta_i(\hat{\pi}_i - \pi_i)^2]}{\pi_i^{*3}}\right\},
\end{aligned}
$$

then the bias-adjusted version of $\hat{H}_N(\theta)$ can be written as,

$$
\hat{H}_{ba}(\theta) = N^{-1}\sum_{i=1}^{N} \delta_i \left(\frac{1}{\hat{\pi}_i} + \frac{E[\delta_i(\hat{\pi}_i - \pi_i)]}{\pi_i^3} - \frac{E[\delta_i(\hat{\pi}_i - \pi_i)^2]}{\pi_i \pi_i^{*3}}\right) S_i(\theta)
$$

Then

$$
\hat{H}_{ba}(\theta) = N^{-1}\sum_{i=1}^{N} \frac{\delta_i}{\hat{\pi}_{i_{ba}}} S_i(\theta) \text{ with } \hat{\pi}_{i_{ba}} = \left(\frac{1}{\hat{\pi}_i} + \frac{\hat{e}_{i1}}{\hat{\pi}_i^3} - \frac{\hat{e}_{i2}}{\hat{\pi}_i \pi_i^{*3}}\right), \tag{4.1}
$$

where $e_{i1} = E[\delta_i(\hat{\pi}_i - \pi_i)] = \pi_i^2 + V(\pi_{i,t})$ and $e_{i2} = E[\delta_i(\hat{\pi}_i - \pi_i)^2] = \pi_i \kappa_i'/T^2 - 2\pi_i e_{i1} + \pi_i^3$, then $\hat{e}_{i1} = \hat{\pi}_i^2 + \hat{V}(\hat{\pi}_{i,t})$ and $\hat{e}_{i2} = \hat{\pi}_i \kappa_i'/T^2 - 2\hat{\pi}_i \hat{e}_{i1} + \hat{\pi}_i^3$ with $\kappa_i' = 1 + 3(T-2)\hat{\pi}_i + 2(T-2)(T-3)\hat{\pi}_i^2$. Let θ^* be the solution to the bias-adjusted EE $\hat{H}_{ba}(\theta)$ such that $\hat{H}_{ba}(\theta^*) = 0$. Further let For bias adjustment, one can drop the last term or one can obtain an approximation to B from replacing π_i^* by $\hat{\pi}_i$. Now that π_i^* lies between $\hat{\pi}_i$ and its expectation π_i, its likely values can be given via the standard error (SE) of $\hat{\pi}_i$, as $\pi_i^* = \hat{\pi}_i \pm \alpha \widehat{SE}(\hat{\pi}_i)$, for chosen α-values and subjected to the range $\pi_i^* \in (0,1)$. One can then estimate θ based on the NEE that is adjusted by the resulting $B(\alpha)$. A grid of α-values will generate accordingly a set of alternative estimates of θ, which provide an indication of the likely range of an unbiased estimator of θ under MNAR nonresponse.

4.2.2 Variance of $\hat{\theta}^\star$

The variance of estimator $\hat{\theta}^\star$ can be obtained using the standard sandwich form based on Taylor expansion of the estimating equations as we already did for $\hat{\theta}$ in Chapter 2. The sandwich variance of $\hat{\theta}^{\star}$ can be written as

$$Var(\hat{\theta}^{*}) = G_{ba}^{-1}(\theta_0)Var[\hat{H}_{ba}(\theta_0)]G_{ba}^{-T}(\theta_0), \tag{4.2}$$

where $G_{ba}(\theta) = E[\hat{H}'_{ba}(\theta)]$. Then

$$G_{ba}(\theta) = \frac{1}{N}\sum_{i=1}^{N} E\left(\delta_i/\hat{\pi}_{i_{ba}}\right)\left\{\frac{\partial}{\partial\theta}S_i(\theta)\right\}$$
$$Var[\hat{H}_{ba}(\theta)] = \frac{1}{N^2}\sum_{i=1}^{N} Var\left(\delta_i/\hat{\pi}_{i_{ba}}\right)\left\{S_i(\theta)S_i^T(\theta)\right\},$$

with $E\left(\delta_i/\hat{\pi}_{i_{ba}}\right) \approx 1$ and $E\left(\delta_i/\hat{\pi}_{i_{ba}}\right)^2 \approx 1/\pi_i$. Now the plug-in estimator of variance of $\hat{\theta}^{*}$ given in (4.2) can be written as

$$\widehat{Var}(\hat{\theta}^{*}) = G_{ba}^{-1}(\hat{\theta}^{*})\widehat{Var}[\hat{H}_{ba}(\hat{\theta}^{*})]G^{-T}(\hat{\theta}^{*}), \tag{4.3}$$

where

$$G_{ba}(\hat{\theta}^{*}) = \frac{1}{N}\sum_{i=1}^{r} \hat{g}_{i_{ba}} S'_i(\hat{\theta}^{*})$$
$$\widehat{Var}[\hat{H}_{ba}(\hat{\theta}^{*})] = \frac{1}{N}\sum_{i=1}^{r} \hat{v}_{i_{ba}} S_i(\hat{\theta}^{*})S_i^T(\hat{\theta}^{*})$$

with $\hat{g}_{i_{ba}} = 1/\hat{\pi}_{i_{ba}}$ and $\hat{v}_{i_{ba}} = 1/\hat{\pi}_{i_{ba}}\left(1/\hat{\pi}_i - 1\right)$.

4.3 Bias-adjusted NEE approach for Longitudinal Setting

In the previous chapter, we discussed NEE approach to estimate the longitudinal parameters. The observed NEEs are not unbiased and the bias in NEEs has already been discussed in Section 3.3. We develop the bias-adjusted NEE approach for both types of NEEs. The corresponding plug-in variance of estimators based on both bias-adjusted EEs are given in Section 4.3.2. The bias correction of these variance estimators using Taylor expansions is discussed in Appendix C.

4.3.1 Bias-adjusted NEE

Given the finite samples, EEs $\hat{H}(\Delta_t)$ and $\hat{H}(\theta_{t,t-1})$ given in (3.3) and (3.6) respectively are not unbiased and the bias of both EEs is examined in Chapter 3. Below we discuss the bias-adjusted NEE approach using both EEs under longitudinal setting.

Bias-adjusted $\hat{H}(\Delta_t)$

Rewriting the bias in estimating equations $\hat{H}(\Delta_t)$ from Section 3.3 we have,

$$\begin{aligned} B_1 = E[\hat{H}(\Delta_t)] - H(\Delta_t) &= N^{-1}\sum_{i=1}^{N} S_i(\Delta_t) E\left(\frac{r_{it}}{\hat{p}_{it}} - 1\right) \\ &= N^{-1}\sum_{i=1}^{N} S_i(\Delta_t)\left\{-\frac{E[r_{it}(\hat{p}_{it}-\tau_{it})]}{\tau_{it}^2} + \frac{E[r_{it}(\hat{p}_{it}-\tau_{it})^2]}{p_i^{*^3}}\right\}, \end{aligned}$$

Then the bias-adjusted version of $\hat{H}(\Delta_t)$ for the case of $\hat{p}_{it} = \hat{p}_i$ in (3.4) is given below; the other cases are similar.

$$\hat{H}_{ba}(\Delta_t) = N^{-1}\sum_{i=1}^{N} r_{it}\left(\frac{1}{\hat{p}_i} + \frac{E[r_{it}(\hat{p}_i - p_i)]}{p_i^3} - \frac{E[r_{it}(\hat{p}_i - p_i)^2]}{p_i^4}\right) S_i(\Delta_t)$$

$$\hat{H}_{ba}(\Delta_t) = N^{-1}\sum_{i=1}^{N} r_{it}\left(\frac{1}{\hat{p}_i} + \frac{\hat{h}_{i1}}{\hat{p}_i^3} - \frac{\hat{h}_{i2}}{\hat{p}_i p_i^{*^3}}\right) S_i(\Delta_t) \qquad (4.4)$$

where

$$\hat{h}_{i1} = \hat{p}_i^2 + \hat{V}(\hat{p}_i),\ \hat{h}_{i2} = \frac{\hat{p}_i\hat{\kappa}_i'}{T^2} - 2\hat{p}_i\hat{h}_{i1} + \hat{p}_i^3$$

$$\text{and } \hat{\kappa}_i' = 1 + 3(T-2)\hat{p}_i + 2(T-2)(T-3)\hat{p}_i^2.$$

Bias-adjusted $\hat{H}(\theta_{t,t-1})$

The $\hat{H}(\theta_{t,t-1})$ is a set of two cross-sectional EEs given in (3.6), the two cross-sectional EEs are for time t and $t-1$. We will first find the bias-adjusted EE for time t. The same can be done for time $t-1$.

The expression of the bias from Section 3.3 can be written as,

$$\begin{aligned} B(t) = E[\hat{H}(\theta_t)] - H(\theta_t) &= N^{-1}\sum_{i=1}^{N} S_i(\theta_t) E\left(\frac{\delta_{it}}{\hat{\pi}_i} - 1\right) \\ &= N^{-1}\sum_{i=1}^{N} S_i(\theta_t)\left\{-\frac{E[\delta_{it}(\hat{\pi}_{it}-\tau_{it})]}{\tau_{it}^2} + \frac{E[\delta_{it}(\hat{\pi}_{it}-\tau_{it})^2]}{\pi_i^{*^3}}\right\}, \end{aligned}$$

Then the bias-adjusted version of $\hat{H}(\Delta_t)$ for the case of $\hat{\pi}_{it} = \hat{\pi}_i$ in (3.8) is given below; the other cases are similar.

$$\hat{H}_{ba}(\theta_t) = N^{-1}\sum_{i=1}^{N} \delta_{it}\left(\frac{1}{\hat{\pi}_i} + \frac{E[\delta_{it}(\hat{\pi}_i - \pi_i)]}{\pi_i^3} - \frac{E[\delta_{it}(\hat{\pi}_i - \pi_i)^2]}{\pi_i^4}\right) S_i(\theta_t)$$

$$\hat{H}_{ba}(\theta_t) = N^{-1}\sum_{i=1}^{N} \delta_{it}\left(\frac{1}{\hat{\pi}_i} + \frac{\hat{q}_{i1}}{\hat{\pi}_i^3} - \frac{\hat{q}_{i2}}{\hat{\pi}_i \pi_i^{*^3}}\right) S_i(\theta_t), \qquad (4.5)$$

where

$$\hat{q}_{i1} = \hat{\pi}_i^2 + \hat{V}(\hat{\pi}_i),\ \hat{q}_{i2} = \frac{\hat{\pi}_i\hat{\kappa}_i'}{T^2} - 2\hat{\pi}_i\hat{q}_{i1} + \hat{\pi}_i^3$$

$$\text{and } \hat{\kappa}_i' = 1 + 3(T-1)\hat{\pi}_i + 2(T-1)(T-2)\hat{\pi}_i^2.$$

Similarly the bias-adjusted EE at time $t-1$ can be obtained by just replacing t with $t-1$ in this section.

For bias adjustment of (3.3), one can obtain an approximation to B_1 by dropping the last term or from replacing p_i^* by $\hat{p}_{it}$. Now that p_i^* lies between $\hat{p}_{it}$ and its expectation τ_{it}, its likely values can be given via the standard error (SE) of $\hat{p}_{it}$, as $p_i^* = \hat{p}_{it} \pm \alpha \widehat{SE}(\hat{p}_{it})$, for chosen α-values and subjected to the range $p_i^* \in (0,1)$. One can then estimate Δ_t based on the NEE that is adjusted by the resulting $B_1(\alpha)$. A grid of α-values will generate accordingly a set of alternative estimates of Δ_t, which provide an indication of the likely range of an unbiased estimator of Δ_t under MNAR nonresponse. Similarly for the NEE (3.6). The adjustment will be illustrated in the simulation study later.

4.3.2 Variance of $\hat{\Delta}_t^*$ using Bias-adjusting EEs

For the longitudinal setting, the variance of the estimator $\hat{\Delta}_t$ is obtained using the standard sandwich form based on Taylor expansion of the simple EEs in Chapter 3. Below we derive the variance of $\hat{\Delta}_t^*$ using both bias-adjusted EEs.

Variance of $\hat{\Delta}_t^*$ using $\hat{H}_{ba}(\Delta_t)$

Here we use the bias-adjusted EE $\hat{H}_{ba}(\Delta_t)$ to derive the variance of $\hat{\Delta}_t^*$. We can write the expression of variance of $\hat{\Delta}_t^*$ from (A.2) as

$$Var_{ba}(\hat{\Delta}_t^*) = G_{ba}^{-1}(\Delta_{0t}) Var[\hat{H}_{ba}(\Delta_{0t})] G_{ba}^{-T}(\Delta_{0t}) \tag{4.6}$$

where $G_{ba}(\Delta_{0t}) = E[\hat{H}_{ba}'(\Delta_{0t})]$ and

$$G_{ba}(\Delta_{0t}) = \frac{1}{N} \sum_{i=1}^{N} \mu_{1i} \left\{ \frac{\partial}{\partial \Delta_{0t}} S_i(\Delta_{0t}) \right\} \tag{4.7}$$

and

$$Var[\hat{H}_{ba}(\Delta_{0t})] = \frac{1}{N} \sum_{i=1}^{N} \mu_{2i} \; S_i(\Delta_{0t}) S_i^T(\Delta_{0t}) \tag{4.8}$$

where for $p_i^\star = p_i$, we have

$$\mu_{1i} = E\left[r_{it} \left(\frac{1}{\hat{p}_i} + \frac{h_{1i}}{p_i^3} - \frac{h_{2i}}{p_i^4} \right) \right] \approx 1 \text{ and } \mu_{2i} = E\left[r_{it} \left(\frac{1}{\hat{p}_i} + \frac{h_{1i}}{p_i^3} - \frac{h_{2i}}{p_i^4} \right) \right]^2 \approx 1/p_i.$$

The plug-in estimator of the variance of $\hat{\Delta}_t^*$ given in (4.6) can be written as

$$\widehat{Var}_{ba}(\hat{\Delta}_t^*) = G_{ba}^{-1}(\hat{\Delta}_t^*) \widehat{Var}[\hat{H}_{ba}(\hat{\Delta}_t^*)] G_{ba}^{-T}(\hat{\Delta}_t^*) \tag{4.9}$$

where

$$G_{ba}(\hat{\Delta}_t^*) = \frac{1}{N}\sum_{i=1}^{r} \hat{g}_{i_{ba}} S_i'(\hat{\Delta}_t^*) \tag{4.10}$$

$$\widehat{Var}[\hat{H}_{ba}(\hat{\Delta}_t^*)] = \frac{1}{N}\sum_{i=1}^{r} \hat{v}_{i_{ba}} S_i(\hat{\Delta}_t^*) S_i^T(\hat{\Delta}_t^*), \tag{4.11}$$

with

$$\hat{g}_{i_{ba}} = \hat{\mu}_{1i}\left(\frac{1}{\hat{p}_i} + \frac{\hat{h}_{i1}}{\hat{p}_i^3} - \frac{\hat{h}_{i2}}{\hat{p}_i p_i^{\star 3}}\right),\ \hat{v}_{i_{ba}} = \left\{(\hat{\mu}_{2i} - (\hat{\mu}_{1i})^2\right\}\left(\frac{1}{\hat{p}_i} + \frac{\hat{h}_{i1}}{\hat{p}_i^3} - \frac{\hat{h}_{i1}}{\hat{p}_i p_i^{\star 3}}\right) \tag{4.12}$$

where the $p_i^\star$ can be replaced according to the chosen adjustment of the NEE.

Variance of $\hat{\Delta}_t^* = \hat{\theta}_t^* - \hat{\theta}_{t-1}^*$ using $\hat{H}_{ba}(\theta_{t,t-1})$

Here the variance of $\hat{\Delta}_t^* = \hat{\theta}_t^* - \hat{\theta}_{t-1}^*$ is derived using the second bias-adjusting EE $\hat{H}_{ba}(\theta_{t,t-1})$ that is a set of two cross-sectional bias-adjusted EEs for each time t and $t-1$. We will discuss for time t and for time $t-1$ it is straightforward. Using $\hat{H}_{ba}(\theta_t)$, for time t, we can write the expression of the variance as

$$Var_{ba}(\hat{\theta}_t^*) = G_{ba}^{-1}(\theta_{0t}) Var[\hat{H}_{ba}(\theta_{0t})] G_{ba}^{-T}(\theta_{0t}).$$

The corresponding plug-in estimator can be written as

$$\widehat{Var}_{ba}(\hat{\theta}_t^*) = G_{ba}^{-1}(\hat{\theta}_t^*) \widehat{Var}[\hat{H}_{ba}(\hat{\theta}_t^*)] G_{ba}^{-T}(\hat{\theta}_t^*), \tag{4.13}$$

where $G_{ba}^{-1}(\theta_{0t})$, $Var[\hat{H}_{ba}(\theta_{0t})]$, $G_{ba}^{-1}(\hat{\theta}_t^*)$ and $\widehat{Var}[\hat{H}_{ba}(\hat{\theta}_t^*)]$ can be written respectively from (4.7), (4.8), (4.10) and (4.11) by replacing r_{it} with δ_{it}, p_i with π_i, Δ_t with θ_t and h_i with q_i. Similarly the variance of θ_{t-1}^* and its plug-in estimator can be written by replacing t with $t-1$ in (4.13) as.

$$\widehat{Var}_{ba}(\hat{\theta}_{t-1}^*) = G_{ba}^{-1}(\hat{\theta}_{t-1}^*) \widehat{Var}[\hat{H}_{ba}(\hat{\theta}_{t-1}^*)] G_{ba}^{-T}(\hat{\theta}_{t-1}^*). \tag{4.14}$$

As we used the approximation in (A.13) for finding the $Cov(\delta_{it}/\hat{\pi}_{i,t}, \delta_{i,t-1}/\hat{\pi}_{i,t-1})$ and in this approximation the $\hat{\pi}_{i,t}$ and $\hat{\pi}_{i,t-1}$ are not in the denominator, we are not correcting the bias of $Cov\left(\hat{H}(\hat{\theta}_t^*), \hat{H}^T(\hat{\theta}_{t-1}^*)\right)$ using bias-adjusted EEs. Then we have,

$$Cov_{ba}(\hat{\theta}_t^*, \hat{\theta}_{t-1}^*) = G_{ba}^{-1}(\theta_{0t}) Cov[\hat{H}(\theta_{0t}), \hat{H}(\theta_{0t-1})] G_{ba}^{-T}(\theta_{0t-1}).$$

and its plug-in estimator is

$$\widehat{Cov}_{ba}(\hat{\theta}_t^*, \hat{\theta}_{t-1}^*) = G_{ba}^{-1}(\hat{\theta}_t^*) \widehat{Cov}[\hat{H}(\hat{\theta}_t^*), \hat{H}(\hat{\theta}_{t-1}^*)] G_{ba}^{-T}(\hat{\theta}_{t-1}^*). \tag{4.15}$$

where the plug-in estimator $\widehat{Cov}[\hat{H}(\hat{\theta}_t^*), \hat{H}(\hat{\theta}_{t-1}^*)]$ is given in (A.19).

Finally we can write

$$\widehat{Var}_{ba}(\hat{\Delta}_t^*) = \widehat{Var}_{ba}(\hat{\theta}_t^* - \hat{\theta}_{t-1}^*) = Var_{ba}(\hat{\theta}_t^*) + Var_{ba}(\hat{\theta}_{t-1}^*) - 2\widehat{Cov}_{ba}(\hat{\theta}_t^*, \hat{\theta}_{t-1}^*). \tag{4.16}$$

where

$$G_{ba}(\hat{\theta}_t^*) = \frac{1}{N}\sum_{i=1}^{r} \hat{g}_{i_{ba}} S_i'(\hat{\theta}_t^*) \tag{4.17}$$

$$\widehat{Var}[\hat{H}_{ba}(\hat{\delta}_t^*)] = \frac{1}{N}\sum_{i=1}^{r} \hat{v}_{i_{ba}} S_i(\hat{\theta}_t^*) S_i^T(\hat{\theta}_t^*), \tag{4.18}$$

$$\widehat{Cov}\left(\hat{H}(\hat{\theta}_t^*), \hat{H}^T(\hat{\theta}_{t-1}^*)\right) = \frac{1}{N^2}\sum_{i=1}^{N} \hat{c}_i \; S_i(\hat{\theta}_t^*) S_i^T(\hat{\theta}_{t-1}^*), \tag{4.19}$$

where $\hat{c}_i = \widehat{Cov}\,(\delta_{it}/\hat{\pi}_{i,t}, \delta_{i,t-1}/\hat{\pi}_{i,t-1})$ is estimator of (A.17) and

$$\hat{g}_{i_{ba}} = \hat{\mu}_{1i}\left(\frac{1}{\hat{\pi}_i} + \frac{\hat{q}_{i1}}{\hat{\pi}_i^3} - \frac{\hat{q}_{i2}}{\hat{\pi}_i \pi_i^{\star 3}}\right),\; \hat{v}_{i_{ba}} = \left\{(\hat{\mu}_{2i} - (\hat{\mu}_{1i})^2\right\}\left(\frac{1}{\hat{\pi}_{i,t}} + \frac{\hat{q}_{i1}}{\hat{\pi}_i^3} - \frac{\hat{q}_{i1}}{\hat{\pi}_i \pi_i^{\star 3}}\right) \tag{4.20}$$

where the $\pi_i^\star$ can be replaced according to the chosen adjustment of the NEE, and $\hat{\mu}_{1i} \approx 1$ and $\hat{\mu}_{2i} \approx 1/\hat{\pi}_i$

Similarly the (4.17) and (4.18) can be written for $\hat{\theta}_{t-1}^*$.

4.4 Simulation Study

In simulation study sections of previous two chapters, we use the simple NEE on stable, volatile and simulated population data to estimate the cross-sectional and longitudinal parameters. Here we use the above discussed bias-adjusted NEE approach for the estimation of both cross-sectional and longitudinal parameters.

To assess the performance of the bias-adjusted NEE under cross-sectional setting, keeping the matter same as given in Section 2.6, further we explore bias adjustment of the NEE (2.5), discussed at the end of Section 4.2.1, we consider the following options: $\mathbf{EE}_i$, simply dropping the term of B in (4.1) involving π_i^*; $\mathbf{EE}_{ii}$, replacing π_i^* by $\hat{\pi}_i$; $\mathbf{EE}(\alpha)$, let $\pi_i^* = \hat{\pi}_i \pm \alpha \hat{\text{SE}}(\hat{\pi}_i)$, for various α-values, and $\pi_i^* \in (0, 1]$.

To assess the performance of bias-adjusted NEE under longitudinal setting, keeping the matter remain same as given in Section 3.4, further we explore bias adjustment of the NEE (3.3), discussed at the end of Section 4.3.1, we consider the following options: $\mathbf{EE}_i$, simply dropping the term of B_1 in (4.4) involving p_i^*; $\mathbf{EE}_{ii}$, replacing p_i^* by $\hat{p}_i$; $\mathbf{EE}(\alpha)$, let $p_i^* = \hat{p}_i \pm \alpha \hat{\text{SE}}(\hat{p}_i)$, for various α-values, and $p_i^* \in (0, 1]$. Similarly for the NEE (3.6).

Here our purpose is to know whether the bias-adjusted NEE approach is improving the results in terms of bias and variance or not. We discuss the performance of NEE for cross-sectional setting in the following Section and then for longitudinal setting in Section 4.4.2.

4.4.1 Results for cross-sectional setting

The results related to the estimation of mean and regression coefficients using stable, volatile and simulated populations data are given in Section D.1 and Section D.2 respectively of Appendix D. The results based on NEE are already discussed in Chapter 2 and here our focus is to compare NEE results with bias-adjusted NEE. The tables given below contain results for NEE and its various adjustments discussed above for cross-sectional setting.

Results for mean: Table 4.1 shows the results under the model (2.41), in the high response setting and high correlation scenario, for $T = 3$, 4, 7 and 10. The values of the population mean for respective T are given in table. In table below, comparing the results based on simple **EE** with bias-adjustments $\mathbf{EE}_i$, $\mathbf{EE}_{ii}$ and $\mathbf{EE}(0.5)$, we notice the following.

- Using the bias-adjustment $\mathbf{EE}_{ii}$, i.e. replacing π_i^* by $\hat{\pi}_i$ improves the results in terms of bias as compared to $\mathbf{EE}_i$ and $\mathbf{EE}(0.5)$. However, in terms of variance $\mathbf{EE}(0.5)$ is similar to the $\mathbf{EE}_{ii}$. But we prefer $\mathbf{EE}_{ii}$ because its improves the results in terms of bias and variance. The adjustment $\mathbf{EE}_{ii}$ works well for all three types of populations.
- The adjustment is more effective for small T, e.g. $T = 3$ and 4 because the response probability estimator is more variable for small T and it is adjusted by the bias-adjustment.
- The notable thing here is that the bias-adjusted NEE approach adjusted the bias in variance estimates quite nicely whereas the Taylor expansion approach is not effective, see Table D.1.

Table 4.2 shows the results under the model (2.43), also in the high response setting and high correlation scenario, for $T = 7$ only, as the message is the same for the other choices of T. The results show clearly that the underlying nonresponse assumption needs to be fairly close to the truth, in order for the NEE and bias-adjusted NEE estimators to perform well. However, under the model (2.43) the bias-adjustments for stable population is still improving the results sufficiently good in terms of bias and variance irrespective of the miss-specified model. The reason is that even modeling the response under strong informative mechanism, i.e. under the model 2.43, the stable population can still indirectly helps to provide stable response assumption and therefore results are improved here.

Table 4.1: Results under model (2.41), high response and high correlation. Population: stable(Left), volatile(Middle) and simulated(Right)

	T=3, { Stable and Volatile: $\theta_3 = 11747.05$, Simulated: $\theta_3 = 808.1355$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.56	0.00	2.92	11.15	0.01	3.10	23.53	0.02	2.62
SE	43.44	44.44	54.26	27.85	29.97	38.62	5.82	6.05	7.77
ERSE	409.00	43.33	84.53	378.08	31.21	65.43	190.75	6.32	12.33
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	16.22	0.90	12.09	11.68	1.12	8.88	17.98	1.76	13.18
SE	61.51	49.68	59.92	49.90	34.18	46.64	8.98	7.05	8.70
ERSE	49.23	55.75	51.80	42.57	42.06	43.28	7.24	8.16	7.60
	T=4, { Stable and Volatile: $\theta_4 = 11000.5$, Simulated: $\theta_4 = 108.5394$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.06	0.01	5.42	15.92	0.00	4.31	21.49	0.01	5.58
SE	39.43	40.33	50.34	40.93	39.60	50.12	0.74	0.79	1.04
ERSE	331.84	39.89	82.45	344.21	39.22	78.17	11.87	0.80	1.65
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	16.79	1.26	12.85	12.96	0.96	10.02	18.11	0.94	13.76
SE	60.26	43.66	57.73	59.88	45.18	56.98	1.26	0.89	1.20
ERSE	49.48	51.87	51.60	50.24	52.10	51.79	1.00	1.04	1.04
	T=7, { Stable and Volatile: $\theta_7 = 11000.5$, Simulated: $\theta_7 = 1619.253$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.05	0.00	4.82	12.16	0.00	2.86	24.51	0.01	5.80
SE	38.59	40.09	49.92	41.39	40.53	48.30	12.86	12.91	15.79
ERSE	331.98	39.86	75.34	334.88	40.30	64.47	433.32	12.63	23.35
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	11.30	3.16	8.45	6.59	2.08	4.92	13.61	3.77	10.18
SE	61.67	44.16	57.75	56.32	45.18	53.30	19.02	14.22	17.97
ERSE	52.84	49.54	53.58	50.67	49.02	51.03	16.25	15.59	16.58
	T=10, { Stable and Volatile: $\theta_{10} = 11000.5$, Simulated: $\theta_{10} = 2397.519$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.05	0.00	3.25	10.26	0.01	1.56	23.72	0.01	3.84
SE	39.19	40.14	47.02	42.99	41.78	46.87	18.22	18.18	21.10
ERSE	332.06	39.85	60.40	329.53	42.13	54.01	234.88	18.19	27.35
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	7.09	2.82	5.09	3.37	1.45	2.39	8.37	3.31	6.01
SE	53.51	44.76	50.99	49.62	46.21	48.39	23.76	20.11	22.75
ERSE	48.81	47.16	49.20	48.21	48.08	48.50	21.96	21.40	22.21

Table 4.2: Results under model (2.43), high response and high correlation. Population: stable(Left), volatile(Middle) and simulated(Right)

	T=7, { Stable and Volatile: $\theta_7 = 11000.5$, Simulated: $\theta_7 = 1619.253$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.43	0.16	4.62	20.45	16.41	15.36	23.41	4.48	10.10
SE	39.39	40.12	49.82	38.96	39.26	74.83	11.78	10.18	25.64
ERSE	332.33	40.54	75.38	332.39	140.31	180.12	428.23	72.65	79.84
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	11.09	2.99	8.24	14.01	15.58	14.61	17.63	8.31	14.40
SE	61.05	44.67	57.28	89.34	73.38	82.71	28.11	25.67	26.71
ERSE	53.10	50.08	53.89	160.03	161.59	161.15	68.36	76.28	71.20

Table 4.3: Results under model (2.45), high response. Population: stable(Left), volatile(Middle) and simulated(Right)

	T=7, { Stable and Volatile: $\theta_7 = 11000.5$, Simulated: $\theta_7 = 1619.253$ }								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.56	0.00	0.08	0.41	0.05	0.17	4.32	0.03	0.93
SE	104.42	139.62	189.08	109.96	141.35	187.60	80.55	87.35	107.82
ERSE	311.18	141.20	299.91	310.47	140.68	286.32	419.75	88.58	150.85
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	0.15	0.12	0.11	0.29	0.16	0.23	2.21	0.67	1.63
SE	258.13	159.54	232.47	254.04	159.02	229.27	132.65	100.14	122.65
ERSE	220.68	182.84	215.92	212.59	179.84	208.53	120.54	110.40	119.23

Table 4.3 shows the results under the model (2.45) in the high response setting for $T = 7$

only. The model (2.45) yields stable MAR response probability, so that the NEE estimators are nearly unbiased and bias-adjustment is not effective as expected, however, bias-adjustment is improving the variance estimates still very nicely.

Table 4.4: Results under model (2.41), by response and correlation. Population: stable (Left), volatile (Right). T=7, { Stable and Volatile: $\theta_7 = 11000.5$, Simulated: $\theta_7 = 1619.253$ }

	low response and low correlation								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	18.87	0.03	4.09	5.50	0.04	1.40	57.70	0.03	10.94
SE	107.97	89.74	104.57	175.88	158.29	189.45	28.71	19.33	21.77
ERSE	436.21	89.70	143.57	396.66	159.09	265.87	579.48	18.97	31.91
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	10.06	0.63	7.93	3.61	0.06	2.86	24.63	5.15	19.33
SE	116.49	99.82	112.39	218.38	179.61	208.00	22.94	20.28	22.91
ERSE	102.37	110.68	106.07	192.35	199.51	197.39	19.10	22.63	20.67
	low response and high correlation								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	35.88	0.01	7.33	15.02	0.07	3.08	59.28	0.02	11.48
SE	76.21	59.32	69.53	139.18	113.64	125.52	26.92	18.58	21.18
ERSE	445.35	58.32	102.19	445.93	106.71	154.08	565.99	18.35	32.19
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	17.19	2.30	13.55	7.25	1.20	5.64	26.18	5.02	20.51
SE	75.99	63.66	74.60	131.78	122.33	129.47	22.35	19.43	22.40
ERSE	63.54	71.08	67.37	118.93	126.41	122.58	18.37	22.06	20.05
	high response and low correlation								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	14.86	0.01	3.75	8.60	0.02	2.22	19.69	0.00	4.41
SE	33.98	36.17	46.70	53.40	52.66	61.39	11.15	10.89	13.11
ERSE	334.18	37.05	69.48	333.18	52.68	81.68	417.95	10.89	18.66
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	8.97	2.20	6.74	5.22	1.41	3.93	10.07	3.51	7.41
SE	58.29	41.10	54.30	70.69	58.51	66.95	15.54	12.16	14.67
ERSE	49.71	46.18	50.09	66.03	64.25	66.26	14.01	13.35	14.16
	high response and high correlation								
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.05	0.00	4.82	12.16	0.00	2.86	24.51	0.01	5.80
SE	38.59	40.09	49.92	41.39	40.53	48.30	12.86	12.91	15.79
ERSE	331.98	39.86	75.34	334.88	40.30	64.47	433.32	12.63	23.35
	EE$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)	**EE**$_i$	**EE**$_{ii}$	**EE**(0.5)
APRB	11.30	3.16	8.45	6.59	2.08	4.92	13.61	3.77	10.18
SE	61.67	44.16	57.75	56.32	45.18	53.30	19.02	14.22	17.97
ERSE	52.84	49.54	53.58	50.67	49.02	51.03	16.25	15.59	16.58

In Table 4.4 the different response rate settings and correlation scenarios are contrasted with each other, for $T = 7$. Comparing the results of simple EE with its adjustments, we notice the followings.

- Bias adjustment is also affected by contrasting the response rate setting and low and high correlation scenarios, especially in the low response and low correlation combination, where the bias is almost entirely caused by the non-linearity, and replacing the p_i^* with $\hat{p}_i$ is more beneficial than other two adjustments.
- Bias adjustment of the NEEs greatly improves the variance estimation again.

The results concerning the estimation of regression coefficients are given in Tables D.13 to D.19 of Appendix D.2. From these results, similar statements can be made as above for the case of mean estimation, hence these results are omitted here.

4.4.2 Results for longitudinal parameters

The results related to the estimation change in mean and change in regression coefficients using the bias-adjusted NEE are given in Section E.1 and Section E.2 respectively of Appendix E. Below we discuss the results using bias-adjusted NEE in comparison with the results based on unadjusted NEE that are already discussed in previous chapter. Here our purpose is to know the effectiveness of bias-adjustments in terms of bias and variance.

Estimating the change in Mean: Table 4.5 shows the results under the model (2.41), in the high response setting and high correlation scenario, for $T = t = 4, 7$ and 10. We notice the followings.

- Considering the bias adjustment, dropping the indeterminable term (involving p_i^* or π_i^*) is not appealing *a priori*. Replacing p_i^* by $\hat{p}_i$ (or π_i^* by $\hat{\pi}_i$) can be problematic due to the variance of $\hat{p}_i$. For instance, in the case of $T = t = 10$, the bias is actually increased with $\mathbf{EE}_h$compared to $\mathbf{EE}(\hat{p}_i)$ because of this. Similarly with *ad hoc* choices of $\mathbf{EE}(\alpha)$, illustrated for $\alpha = 0.5$ here.

- The bias for NEE is high when T is small, e.g. $T = 4$ and the bias-adjustment especially $\mathbf{EE}(0.5)$ is reducing the bias considerably good whereas for large T the bias is already small and bias-adjustments are not much effective.

- An interesting effect of these bias adjustments is that they clearly improve the variance estimation, e.g. when p_i^* is replaced by $\hat{p}_i$. The sandwich variance estimator based on the direct plug-in observed NEE tends to overestimate the variance, sometimes considerably, in these simulations. The sandwich variance estimator derived from the various bias-adjusted observed NEEs performs much better, and the improvement seems not related to the effect on point estimation.

Table 4.5: Results under model (2.41), high response and high correlation. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=4, {Stable and Volatile: $\Delta_4 = -746.55$}, {Simulated: $\Delta_4 = -699.5961$ }																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	48.98	0.10	12.70	24.24	0.19	22.13	67.29	0.04	15.51	32.61	0.00	16.24	46.79	0.07	14.48	24.15	0.01	3.09
SE	10.58	10.75	14.86	59.45	61.21	77.01	53.28	47.68	75.34	52.97	52.71	66.78	7.82	9.01	10.00	6.36	6.55	8.02
ERSE	258.14	10.62	22.04	517.54	59.02	102.42	641.63	48.17	110.73	528.60	53.78	91.82	224.39	8.98	14.49	191.23	6.40	11.01
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$
APRB	8.16	20.67	0.76	14.01	26.26	3.44	12.42	26.54	0.62	17.95	21.58	6.79	4.87	21.78	3.48	8.95	5.42	3.63
SE	19.70	13.09	17.55	91.65	69.67	84.97	101.55	65.39	89.93	81.05	61.46	74.30	11.25	9.24	10.81	9.62	7.33	8.93
ERSE	14.69	14.02	14.71	67.68	66.34	68.94	74.59	71.02	74.63	63.82	61.90	64.35	8.31	9.56	8.89	7.13	7.19	7.32
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	51.83	0.02	1.35	25.46	0.44	0.85	61.64	0.27	0.04	29.51	0.40	0.06	48.70	0.01	2.22	24.65	0.04	0.03
SE	10.84	10.93	14.14	56.65	58.61	69.09	62.78	57.42	75.23	53.07	52.75	59.37	16.90	18.00	22.25	12.65	12.73	14.64
ERSE	258.25	10.73	20.15	517.70	58.86	90.18	610.92	58.27	104.88	513.67	53.06	72.75	512.24	18.40	34.50	432.83	12.70	18.72
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$
APRB	15.45	10.08	6.91	19.02	6.24	9.62	19.49	8.73	9.47	14.67	2.36	7.29	13.82	10.99	5.70	5.44	0.60	2.23
SE	17.16	12.85	15.59	79.38	65.02	73.86	91.61	69.68	82.71	65.11	58.27	61.82	25.78	19.02	24.28	16.65	13.92	15.55
ERSE	13.92	13.67	14.13	68.82	65.97	69.14	78.18	76.78	78.67	60.72	59.18	60.70	19.85	21.42	21.30	14.57	14.20	14.71
	T=t=10, {Stable and Volatile: $\Delta_{10} = -746.55$}, {Simulated: $\Delta_{10} = 190.6555$ }, high response and high correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	53.04	0.02	1.46	26.45	0.02	0.14	53.97	0.08	2.48	26.56	0.11	0.17	45.02	0.01	0.75	22.95	0.08	0.07
SE	10.40	10.60	13.51	56.48	58.34	65.25	71.00	65.57	78.79	54.05	53.66	58.06	3.79	3.71	4.62	24.83	24.81	27.33
ERSE	258.41	10.81	18.71	518.04	58.85	75.99	596.17	65.10	98.66	505.98	54.42	64.34	341.29	3.65	6.21	367.54	24.78	31.84
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$
APRB	14.53	4.58	7.32	9.95	1.30	4.37	15.36	2.06	8.05	6.81	0.02	2.86	12.50	5.21	6.08	2.16	0.86	1.64
SE	16.24	12.36	14.75	69.69	63.82	67.19	89.46	75.68	83.16	59.98	57.88	58.80	5.29	4.33	4.93	28.78	26.94	27.96
ERSE	13.58	13.22	13.74	64.99	64.52	65.50	80.43	79.43	80.77	58.72	58.69	58.92	4.39	4.45	4.51	27.18	27.15	27.47

Table 4.6: Results under model (2.43), high response and high correlation. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	61.43	0.85	5.88	41.36	10.23	13.41	42.28	27.69	32.83	40.81	21.77	33.89	32.71	4.81	5.20	8.33	8.34	9.87
SE	22.50	21.71	27.84	57.54	58.27	67.66	247.73	249.15	370.33	59.49	60.00	124.60	14.07	12.73	54.26	7.00	6.71	20.02
ERSE	260.94	21.56	39.15	521.26	60.31	91.60	586.62	324.86	659.95	526.75	216.54	300.99	462.09	85.96	116.66	382.06	54.49	62.46
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)
APRB	12.54	15.94	3.23	7.36	18.97	2.51	31.05	32.39	31.95	31.45	34.18	32.73	16.83	0.02	10.90	13.41	9.78	11.27
SE	33.49	26.09	30.46	77.41	64.28	72.14	501.89	311.26	433.77	159.70	119.69	140.13	58.31	58.02	55.58	22.33	20.20	20.72
ERSE	28.08	27.84	28.42	70.07	67.49	70.50	469.48	436.11	469.41	264.79	261.26	264.36	97.61	116.41	103.82	56.19	58.31	57.39

Table 4.7 shows the results under the model (2.43), also in the high response setting and high correlation scenario, for $T = t = 7$ only, as the message is the same for the other choices of (T, t). From the results of various adjustments, we can notice that **EE**(0.5) is more effective here in reducing the bias in estimate and reducing the overestimation of variance as compared to the other two adjustments only under stable data. For volatile data, all adjustments are not improving the results. Whereas for simulated data, the results are only improved in terms of bias for $\hat{p}_i$ using the adjustment $\mathbf{EE}_{ii}$. Hence, the results show clearly that the underlying nonresponse assumption needs to be fairly close to the truth, in order for the NEE and its bias-adjustments to perform well.

Table 4.7: Results under model (2.45), high response. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, high response																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	6.52	0.21	2.38	3.94	0.18	0.45	23.50	0.61	3.40	13.07	0.28	0.75	10.05	0.49	0.03	4.83	0.42	0.24
SE	83.59	164.16	185.59	170.25	237.36	277.13	187.31	322.77	372.89	155.28	203.25	240.22	116.81	136.20	172.44	81.00	87.76	101.29
ERSE	208.65	163.62	291.79	470.12	233.26	379.70	541.48	322.87	571.16	486.92	198.49	313.99	489.17	133.11	252.27	418.77	88.85	124.22
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)
APRB	0.40	4.70	0.97	2.23	0.31	1.28	4.42	7.92	0.46	3.25	1.48	1.08	3.17	1.58	1.54	1.34	0.16	0.69
SE	262.83	138.99	223.41	349.25	256.67	309.07	514.48	294.63	441.73	303.51	219.79	268.26	224.32	148.36	197.14	115.46	98.80	107.16
ERSE	198.27	156.55	192.12	298.38	256.99	287.04	393.56	324.64	383.60	249.91	219.83	241.62	177.68	159.16	175.94	106.13	100.92	104.66

Table 4.7 shows the results under the model (2.45), also in the high response setting and high correlation scenario, for $T = 7$ only. The model (2.45) yields stable MAR response probability, so that the NEE estimators are nearly unbiased and bias-adjustment is not effective as expected except for volatile population data, however, bias-adjustment is adjusting the overestimation of variance estimates still very nicely.

Table 4.8: Results under model (2.41), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, low response and low correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	112.36	0.53	12.08	47.76	1.26	4.95	96.80	1.36	15.52	44.19	2.30	3.83	156.22	0.05	12.73	62.97	0.03	0.28
SE	103.40	79.53	110.59	151.28	127.65	147.96	447.53	354.59	497.54	254.91	226.17	272.63	57.61	27.15	36.69	27.61	18.43	21.32
ERSE	438.89	77.85	145.58	675.40	131.27	188.97	909.33	368.02	682.58	628.27	233.79	337.46	916.51	28.64	54.06	578.91	18.71	29.44
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)
APRB	11.01	33.60	0.44	15.73	16.14	6.44	6.64	38.37	3.40	10.66	10.97	3.70	17.98	40.06	3.88	11.43	2.72	4.98
SE	124.07	105.66	117.31	157.92	144.53	152.30	572.02	461.39	535.34	304.32	262.40	286.72	32.06	37.88	34.62	22.18	20.48	21.82
ERSE	94.94	112.00	101.25	144.38	150.84	148.79	444.94	519.30	473.59	261.85	268.33	267.60	24.78	44.97	30.51	19.53	21.62	21.02
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, low response and high correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	156.16	0.08	12.80	63.57	0.20	5.33	156.41	1.56	15.16	63.56	0.34	3.29	144.69	0.06	12.43	60.34	0.04	0.38
SE	55.76	36.77	50.58	105.40	82.91	97.92	308.21	205.82	288.36	172.01	142.27	158.61	55.30	28.53	36.90	26.85	18.42	21.13
ERSE	448.59	37.08	68.58	681.29	83.33	133.26	1038.50	210.15	390.31	667.36	140.55	195.84	865.02	28.22	53.07	565.64	18.48	29.19
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)
APRB	18.79	40.49	4.25	27.05	21.22	12.77	16.81	44.26	2.16	19.72	13.40	8.89	17.60	38.25	3.70	11.12	2.77	4.77
SE	54.77	50.82	52.51	105.88	92.18	101.99	323.27	282.02	305.15	167.79	157.03	162.30	32.54	38.19	35.00	22.39	19.97	21.82
ERSE	45.03	55.80	48.47	90.48	95.76	95.15	258.32	312.67	276.79	153.60	160.55	158.13	24.47	42.81	29.95	19.43	21.30	20.83
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, high response and low correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	50.28	0.08	1.79	25.33	0.12	1.40	56.65	0.36	1.39	27.78	0.54	0.21	49.07	0.01	1.23	24.35	0.02	0.05
SE	18.95	19.14	25.31	51.46	54.96	65.86	109.84	110.36	146.68	76.58	77.47	86.58	18.27	17.72	21.51	13.09	12.65	14.30
ERSE	257.57	19.05	35.79	518.37	53.92	82.26	610.77	108.48	203.36	515.95	77.01	103.01	513.97	17.68	33.57	432.02	12.35	17.63
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)
APRB	14.70	10.55	6.33	15.89	6.25	7.62	17.78	11.50	8.04	13.06	2.75	6.53	14.55	9.45	6.54	5.08	0.42	2.03
SE	31.52	22.63	28.26	76.50	61.66	70.79	182.53	133.29	163.41	94.15	85.73	89.69	24.60	18.91	23.28	15.69	13.90	14.93
ERSE	25.49	24.46	25.61	63.48	60.31	63.43	147.90	141.35	148.06	87.83	86.01	87.60	19.40	21.08	20.86	14.00	13.80	14.16
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, high response and high correlation																	
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$
APRB	51.83	0.02	1.35	25.46	0.44	0.85	61.64	0.27	0.04	29.51	0.40	0.06	48.70	0.01	2.22	24.65	0.04	0.03
SE	10.84	10.93	14.14	56.65	58.61	69.09	62.78	57.42	75.23	53.07	52.75	59.37	16.90	18.00	22.25	12.65	12.73	14.64
ERSE	258.25	10.73	20.15	517.70	58.86	90.18	610.92	58.27	104.88	513.67	53.06	72.75	512.24	18.40	34.50	432.83	12.70	18.72
	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	**EE**(0.5)
APRB	15.45	10.08	6.91	19.02	6.24	9.62	19.49	8.73	9.47	14.67	2.36	7.29	13.82	10.99	5.70	5.44	0.60	2.23
SE	17.16	12.85	15.59	79.38	65.02	73.86	91.61	69.68	82.71	65.11	58.27	61.82	25.78	19.02	24.28	16.65	13.92	15.55
ERSE	13.92	13.67	14.13	68.82	65.97	69.14	78.18	76.78	78.67	60.72	59.18	60.70	19.85	21.42	21.30	14.57	14.20	14.71

In Table 4.8 the different response rate settings and correlation scenarios are contrasted with each other, for $T = t = 7$. we observe the followings.

- The effects of low response setting on the variance is clear and as expected, where all the SEs are increased, which is more dramatic for the volatile population when holding the correlation scenario fixed. Bias adjustment of the NEEs greatly improves the variance estimation again.

- The bias is much higher absolutely in the low response setting, where $\mathrm{SE}(\hat{p}_i)$ is larger and the contribution of non-linear $1/\hat{p}_i$ is relatively greater. Bias adjustment is affected, especially in the low response and low correlation combination, where the bias is almost entirely caused by the non-linearity, and using $\mathbf{EE}(0.5)$ is more beneficial than replacing p_i^* with $\hat{p}_i$.

Furthermore for $t < T$, all three estimators (3.4) are different and similarly the two estimators (3.8). Below we present the results in Table 4.9 for simulated population when $T = 10$ and $t = 7$ to know the affect of bias-adjustment for these additional response probability estimators when $t < T$. The results show that for $\hat{p}_i$ and $\hat{\pi}_i$, the bias-adjustment $\mathbf{EE}(0.5)$ is adjusting the bias nicely where needed. For $\hat{p}_{i1}$ & $\hat{p}_{i2}$ and $\hat{\pi}_{i1}$, bias-adjustment by dropping the interminable term i.e. $\mathbf{EE}_i$ is adjusting the bias nicely. Therefore, we can say that all three adjustments can be useful depending upon the response probability estimators we are using especially when $t < T$.

Table 4.9: Results under model (2.41), by response and correlation. Population: simulated

	T=10, t=7, $\Delta_7 = -746.55$, low response and low correlation.																											
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{p}_{i1})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{p}_{i2})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{\pi}_{i1})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$
APRB	144.62	0.03	9.24	13.28	24.52	1.80	18.08	5.07	32.25	6.72	31.64	8.35	42.91	20.19	58.74	0.04	0.05	7.33	0.69	2.82	7.46	0.58	7.79	4.81
SE	57.80	28.97	38.63	38.07	37.59	38.72	40.78	40.56	39.40	41.02	42.48	42.00	41.75	42.54	28.69	19.64	22.38	22.81	21.75	22.67	23.07	23.22	22.68	23.22
ERSE	896.79	29.25	55.08	29.91	40.48	34.38	57.01	31.60	41.42	35.95	55.60	32.47	41.17	36.40	571.84	19.23	25.19	19.88	21.30	20.92	24.11	19.67	20.94	20.57
	T=10, t=7, $\Delta_7 = -746.55$, low response and high correlation																											
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{p}_{i1})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{p}_{i2})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{\pi}_{i1})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$
APRB	137.74	0.10	9.74	12.93	25.15	1.33	18.57	4.59	32.75	7.25	31.85	8.77	43.05	20.55	57.98	0.11	0.14	7.34	0.84	2.77	7.60	0.64	7.97	4.92
SE	53.79	29.02	39.50	39.16	37.17	39.84	41.41	41.64	38.99	42.00	42.42	42.76	40.53	43.01	26.17	18.45	21.12	21.80	20.31	21.51	21.68	21.99	21.22	21.90
ERSE	852.12	28.22	53.88	28.48	38.51	32.91	55.55	30.11	39.24	34.35	53.86	30.97	38.88	34.70	561.83	18.46	24.65	19.18	20.50	20.18	23.41	18.90	20.08	19.77
	T=10, t=7, $\Delta_7 = -746.55$, high response and low correlation																											
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{p}_{i1})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{p}_{i2})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{\pi}_{i1})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$
APRB	43.99	0.00	2.14	7.06	5.28	1.84	5.44	3.35	8.18	1.64	11.24	3.43	13.11	7.85	21.60	0.01	0.01	2.91	0.02	0.98	2.95	0.37	2.90	2.09
SE	18.77	18.03	22.18	25.48	20.64	23.76	22.17	25.50	20.65	23.74	21.25	23.78	20.32	22.44	12.80	12.28	13.74	14.31	13.56	13.99	13.54	13.96	13.44	13.72
ERSE	509.64	17.40	29.71	20.55	20.10	21.08	28.60	20.21	19.69	20.63	25.16	18.76	18.45	19.05	428.54	12.08	14.59	12.98	13.09	13.13	13.49	12.17	12.28	12.29
	T=10, t=7, $\Delta_7 = -746.55$, high response and high correlation																											
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{p}_{i1})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{p}_{i2})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$	$\mathbf{EE}(\hat{\pi}_{i1})$	$\mathbf{EE}_i$	$\mathbf{EE}_{ii}$	$\mathbf{EE}(0.5)$
APRB	48.36	0.01	3.04	8.00	7.64	1.85	6.90	3.62	10.89	2.23	13.26	3.92	16.06	9.11	24.51	0.04	0.05	3.51	0.16	1.21	3.52	0.40	3.53	2.43
SE	16.99	17.70	21.78	26.02	19.31	23.88	21.52	25.70	19.34	23.56	20.35	23.71	18.96	21.97	12.55	12.51	14.37	15.29	14.19	14.76	13.88	14.53	13.80	14.16
ERSE	513.02	18.22	32.88	21.24	20.65	21.96	31.53	20.89	20.13	21.44	27.47	19.33	18.78	19.67	433.33	12.55	15.86	13.69	13.71	13.84	14.41	12.71	12.77	12.83

The results concerning the estimation of change in regression coefficients are given in Tables E.16 to E.30 of Appendix E.2. From these results, similar statements can be made as above for the case of change in mean estimation.

4.5 Conclusions

In this chapter we propose bias-adjusted NEE approach to estimation based on cross-sectional and longitudinal data subjected to informative missing mechanisms. The following conclusions emerge.

The variance is overestimated using NEE for cross-sectional as well as for longitudinal that can be seen from the results discussed in previous two chapters. The sandwich variance estimator derived from the various bias-adjusted observed NEEs performs much better, and the improvement seems not related to the effect on point estimation.

The different bias-adjustments also improves the results in terms of bias in estimates for both cross-sectional and longitudinal settings especially for small T and in low response setting. For large T, the bias-adjustment is more effective for variance estimator. Different bias-adjustments works well for different settings, for example, $\mathbf{EE}_{ii}$ performs better for cross-sectional estimates, $\mathbf{EE}(0.5)$ works well for longitudinal setting using $\hat{p}_i$ and $\hat{\pi}_i$, and $\mathbf{EE}_i$ performs better when $\hat{p}_{i1}$, $\hat{p}_{i2}$ and $\hat{\pi}_{i1}$ are used for $t < T$ under longitudinal setting.

Chapter 5

Summary and Future Research

5.1 Summary of Research Contributions

So far the problem of NMAR nonresponse is handled either using fully parametric or semi-parametric approaches and these approaches have some potential issues, for example, strict distributional assumptions, heavy computations, etc. We developed a fully non-parametric estimating equation approach to deal with informative missing cross-sectional and longitudinal data.

The main contribution of this thesis starts from Chapter 3 where we developed a fully non-parametric estimating equation approach, where the unknown individual response propensity is replaced by an estimate based the response history of the same individual. As reviewed earlier in the literature, the response probability is usually estimated under some parametric model of response probability. But here we use the observed historic response rate for each unit to estimate its individual response probability, under the assumption that the unknown response probability is individual but "stable" over a given period of time. There can be different assumptions of the exact nature of such stability over time, e.g. stable before the the dropout for a unit with monotone missing data pattern, but over the entire history for someone with a nonmonotone pattern.

While the estimator of the individual response probability can be unbiased according to the given assumption, it can never be consistent due to the fact that the response history cannot be infinitely long for anyone. Moreover, the plug-in observed NEE will be somewhat biased if the 'score-term' in the population EE is correlated with the response propensity, as in the case of informative nonresponse. The observed NEE is used to estimate cross-sectional parameters and longitudinal parameters. We explore the bias in NEE that is further used for bias-adjustment. We also develop the associated variance estimator. Compared to alternative fully or semi-parametric approaches, our approach is simple in construction and easy in computation and does not depend on strict distributional assumptions of the outcome variable, and explicit/parametric form of response probability model.

The NEE approach is extended for longitudinal setting and two types of EEs are defined in Chapter 4 to estimate the longitudinal parameters The bias is also explored for the both EEs and the associated variance estimators are also provided. The NEEs under both cross-sectional and longitudinal setting are not unbiased. We therefore developed bias-adjusting EE approach to adjust the bias in parameter estimates and bias in their variance estimates. See Chapter 5 for more detail.

The performance of proposed approach is assessed while estimating the cross-sectional and longitudinal parameters such as mean, regression coefficients, change in mean and change in regression coefficients using simulations based on real and simulated data. The performance is assessed for different lengths of response history with different response rates and correlations between response propensity and score-term. The performance is also assessed under different response mechanisms, some of which are compatible with the assumptions of our approach, while others represent different nonresponse mechanisms which are used to explore the sensitivity of the NEE approach. For detail see the simulation study sections of Chapters 3, 4 and 5 and their respective conclusions.

Being a computationally simple and flexible method, the NEE approach can be a widely used as exploratory data analysis technique to deal with cross-sectional and longitudinal informative missing data, provided the response history is available for given time window. Moreover, this technique can provide the basis for deciding whether more sophisticated methods of analysis may be necessary.

5.2 Technical Strengths

The present work is advantageous in some aspects when compared to the methods given in literature related to handling informative missing data. The NEE approach is simple in construction and easy in computation and does not depend on strict distributional assumptions of the outcome variable, and explicit/parametric form of response probability model. The strengths can be listed as follows:

1. We developed a fully non-parametric estimating equation approach to accommodate potentially informative missing data and we postulate an individual response probability which may depend on the longitudinal outcomes of interest and covariates specific to each observational unit. The individual response probability is estimated using individual historic response. The key point is that we are estimating every body individually. We can allow different assumptions for each individuals even. This is actually the strength of the flexibility of our approach compared to the existing parametric approaches. Currently we assume the stable response assumption and the response probability is estimated using historic response rate. This simple empirical estimator is used to estimate the parameters under informative nonresponse. We also develop the associated variance estimator. Compared to alternative fully or semi-parametric approaches, our approach is simple in construction and easy in computation and does not depend on strict distributional assumptions about the outcome variable, and the explicit/parametric form of the response probability model.

2. Our NEE approach is general in nature, it accommodates the estimation of cross-sectional and longitudinal parameters. Depending on the situations, any suitable assumptions can be made for individual response probability to apply the NEE. It accommodate almost all types of non-response mechanisms and patterns.

3. The NEE estimators are clearly better that naïve estimator, provided sensible choices of the response probability estimator, which can easily accommodate NMAR mechanisms. The ability to vary the nonresponse assumption for different individuals makes it potentially a flexible alternative to standard parametric modelling approach, where the same model parameters are assumed to apply across the population.

4. An interesting effect of bias adjusted NEE is that it clearly improve the variance estimation. The sandwich variance estimator based on the direct plug-in observed NEE tends to overestimate the variance, sometimes considerably, in these simulations. The sandwich variance estimator derived from the various bias-adjusted observed NEEs performs much better, and the improvement seems not related to the effect on point estimation.

5. The NEE approach is widely applicable technique for exploratory data analysis of longitudinal missing data mechanisms, based on the observed response history. The results can provide a basis for deciding whether more sophisticated modelling is needed in a given situation.

5.3 Future work

We developed NEE approach for cross-sectional and extended it for longitudinal setting. The response probability is estimated using historic response rate assuming that the unknown response probability remains stable over given time window. The following can be considered future work.

1. Using response rate to estimate the unknown response probability, for same number of responses no matter what is the pattern of response over time, same response probability estimate is being used but in future research we will introduce "response patterns weighting" approach in which case each individual unknown response probability will have different individual response probability estimate.

2. The NEE estimator is not exactly unbiased, even when the response probability estimator is unbiased. The bias has two sources: the correlation between e.g. p_{it} and y_{it}, and the non-linearity of $1/\hat{p}_{it}$, or the variance of $\hat{p}_{it}$. The variance of $\hat{p}_{it}$ is naturally reduced given longer response history. To reduce the bias caused by the non-linearity in situations of short history, one could possibly improve the efficiency of $\hat{p}_{it}$ by grouping the units with a similar response pattern *as well as* similar observed y_{it}'s. Therefore, a topic for future is to study empirically how to vary $\hat{p}_{it}$ according to the different historic response patterns, while improving its efficiency based on similar individuals in both senses. The same consideration applies to the choice between the different NEEs, such as (3.3) vs. (3.6).

3. Another more difficult topic that requires further theoretical development is how to adjust the bias caused by the correlation e.g. between p_{it} and y_{it}. The simulation study shows that this is desirable, despite the large reduction of bias compared to the naïve estimator

even without such adjustment, also because it can improve the associated variance estimation. But it is not an easy task, because simply plugging in $\hat{p}_{it}$ (or $\hat{\pi}_{it}$) is problematic due to its variance.

5.4 Limitations

Our technique is at least limited to the extents that can be summarized as follows:

1. The historic response for each individual should be known for given time window.
2. The proposed NEE may not perform better when $T = 2$ for cross-sectional setting and $T = 3$ for longitudinal setting. Some extra treatment is required for short history scenario as discussed above.
3. A longitudinal survey data with missing observations is required to assess the performance of NEE that would be a real data application of the approach.

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Appendix A

Variance of Change Estimator under Longitudinal Setting

A.1 Variance of $\hat{\Delta}_t$

As the both NEEs are not unbiased and therefore not consistent. The variance of $\hat{\Delta}_t$ can be obtained using the standard sandwich form based on Taylor expansion of the estimating equations around $\Delta'_t = E(\hat{\Delta}_t)$ for (3.3) and around $\theta'_t = E(\hat{\theta}_t)$ and $\theta'_{t-1} = E(\hat{\theta}_{t-1})$ for (3.6). The detail is given in the following sections.

A.1.1 Variance of $\hat{\Delta}_t$ using EE $\hat{H}(\Delta_t)$

By Taylor expansion of (3.3) and after simplification, we have

$$\hat{\Delta}_t - \Delta'_t \approx -G^{-1}(\Delta'_t)\hat{H}(\Delta'_t) \tag{A.1}$$

From (A.1), the variance of $\hat{\Delta}_t$ can be approximately given as

$$Var(\hat{\Delta}_t) = G^{-1}(\Delta'_t)Var[\hat{H}(\Delta'_t)]G^{-T}(\Delta'_t), \tag{A.2}$$

where

$$G(\Delta'_t) = E[\hat{H}'(\Delta'_t)] = \frac{1}{N}\sum_{i=1}^{N} E\left(\frac{r_{it}}{\hat{p}_i}\right)\left\{\frac{\partial}{\partial \Delta'_t}S_i(\Delta'_t)\right\}$$

and

$$Var[\hat{H}(\Delta'_t)] = \frac{1}{N}\sum_{i=1}^{N} Var\left(\frac{r_{it}}{\hat{p}_i}\right)S_i(\Delta'_t)S_i^T(\Delta'_t),$$

where

$$E\left(\frac{r_{it}}{\hat{p}_i}\right) \approx 1 - \frac{E(r_{it}\hat{p}_i) - p_i^2}{p_i^2} + \frac{E(r_{it}\hat{p}_i^2) - 2p_iE(r_{it}\hat{p}_i) + p_i^3}{p_i^3} \stackrel{def}{=} \mu_{1i}, \tag{A.3}$$

$$E\left(\frac{r_{it}}{\hat{p}_i}\right)^2 \approx \frac{1}{p_i} - \frac{2\left(E(r_{it}\hat{p}_i) - p_i^2\right)}{p_i^3} + \frac{6\left(E(r_{it}\hat{p}_i^2) - 2p_iE(r_{it}\hat{p}_i) + p_i^3\right)}{2p_i^4} \stackrel{def}{=} \mu_{2i}, \tag{A.4}$$

where $E(r_{it}\hat{p}_i) = p_i^2 + Var(\hat{p}_i)$ and $E(r_{it}\hat{p}_i^2) = p_i\kappa_i/(T-1)^2$. Notice that we used the generic denotations μ_{1i} and μ_{2i} to cover both cases, which will be convenient later on. The plug-in estimator of $Var(\hat{\Delta}_t)$ given in (A.2) can be written as

$$\widehat{Var}(\hat{\Delta}_t) = N^{-1}G^{-1}(\hat{\Delta}_t)\widehat{Var}[\hat{H}(\hat{\Delta}_t)]G^{-T}(\hat{\Delta}_t), \tag{A.5}$$

where

$$G(\hat{\Delta}_t) = \frac{1}{N}\sum_{i=1}^{r} \hat{g}_i S_i'(\hat{\Delta}_t), \text{ where } \hat{g}_i = \frac{1}{\hat{p}_i}\hat{\mu}_{1i} \tag{A.6}$$

$$\widehat{Var}[\hat{H}(\hat{\Delta}_t)] = \frac{1}{N}\sum_{i=1}^{r} \hat{w}_i S_i(\hat{\Delta}_t)S_i^T(\hat{\Delta}_t), \text{ where } \hat{w}_i = \frac{1}{\hat{p}_i}\left\{(\hat{\mu}_{2i} - (\hat{\mu}_{1i})^2)\right\} \tag{A.7}$$

A.1.2 Variance of $\hat{\Delta}_t$ using EE $\hat{H}(\theta_t, \theta_{t-1})$

The variance of $\hat{\Delta}_t = \hat{\theta}_t - \hat{\theta}_{t-1}$ using the observed EE (3.6) can be given as

$$Var(\hat{\Delta}_t) = Var(\hat{\theta}_t) - 2Cov(\hat{\theta}_t, \hat{\theta}_{t-1}) + Var(\hat{\theta}_{t-1}) \tag{A.8}$$

where

$$Var(\hat{\theta}_t) = G^{-1}(\theta_{0t})Var[\hat{H}(\theta_{0t})]G^{-T}(\theta_{0t}), \tag{A.9}$$

and

$$Var(\hat{\theta}_{t-1}) = G^{-1}(\theta_{0t-1})Var[\hat{H}(\theta_{0t-1})]G^{-T}(\theta_{0t-1}), \tag{A.10}$$

where θ_{0t} is the true value of parameter at time t.

The expressions for $Var(\hat{\theta}_t)$ can be obtained similarly as above for $Var(\hat{\Delta}_t)$ given in (A.2) after replacing $\hat{\Delta}_t$ with $\hat{\theta}_t$, Δ_{0t} with θ_{0t}, r_{it} with δ_{it}, p_i with π_i and $\hat{p}_i$ with $\hat{\pi}_{i,t}$. Similarly for $Var(\hat{\theta}_{t-1})$. To obtain $Cov(\hat{\theta}_t, \hat{\theta}_{t-1})$, we proceed as follows. By Taylor expansion of $\hat{H}(\theta_t)$ and after simplification, we have

$$\hat{\theta}_t = \theta_{0t} - G^{-1}(\theta_{0t})\hat{H}(\theta_{0t}),$$
$$\hat{\theta}_{t-1} = \theta_{0t-1} - G^{-1}(\theta_{0t-1})\hat{H}(\theta_{0t-1}).$$

Then, after simplification, we obtain

$$
\begin{aligned}
Cov(\hat{\theta}_t, \hat{\theta}_{t-1}) &= E\left[(\hat{\theta}_t - E(\hat{\theta}_t))(\hat{\theta}_{t-1} - E(\hat{\theta}_{t-1}))^T\right] \\
&= E\left[\left(\left\{\theta_{0t} - G^{-1}(\theta_{0t})\hat{H}(\theta_{0t})\right\} - E\left\{\theta_{0t} - G^{-1}(\theta_{0t})\hat{H}(\theta_{0t})\right\}\right)\right. \\
&\left.\left(\left\{\theta_{0t-1} - G^{-1}(\theta_{0t-1})\hat{H}(\theta_{0t-1})\right\} - E\left\{\theta_{0t-1} - G^{-1}(\theta_{0t-1})\hat{H}(\theta_{0t-1})\right\}\right)^T\right] \\
&= E\left[\left(-G^{-1}(\theta_{0t})\hat{H}(\theta_{0t}) + G^{-1}(\theta_{0t})E[\hat{H}(\theta_{0t})]\right)\right. \\
&\left.\left(-\hat{H}^T(\theta_{0t-1})G^{-T}(\theta_{0t-1}) + E[\hat{H}^T(\theta_{0t-1})]G^{-T}(\theta_{0t-1})\right)\right] \\
&= G^{-1}(\theta_{0t})\left(E[\hat{H}(\theta_{0t})\hat{H}^T(\theta_{0t-1})] - E[\hat{H}(\theta_{0t})]E[\hat{H}^T(\theta_{0t-1})]\right)G^{-T}(\theta_{0t-1}) \\
&= G^{-1}(\theta_{0t})Cov\left(\hat{H}(\theta_{0t}), \hat{H}^T(\theta_{0t-1})\right)G^{-T}(\theta_{0t-1}), \qquad \text{(A.11)}
\end{aligned}
$$

where e.g.

$$G(\theta_{0t}) = \frac{1}{N}\sum_{i=1}^{N} E\left(\delta_{it}/\hat{\pi}_{i,t}\right)\left\{\frac{\partial}{\partial\theta_{0t}}S_i(\theta_{0t})\right\}.$$

Now, we have

$$E\left(\delta_{it}/\hat{\pi}_{i,t}\right) \approx 1 + \frac{Var(\hat{\pi}_{i,t})}{\pi_i^2} - \frac{m_3}{\pi_i^3}, \qquad \text{(A.12)}$$

where

$$E(\delta_{it}\hat{\pi}_{i,t}) = \pi_i^2 + Var(\hat{\pi}_{i,t}) \text{ and } E(\delta_{it}\hat{\pi}_{i,t}^2) = \pi_i\kappa_i'/T^2.$$

Meanwhile

$$Cov\left(\hat{H}(\theta_{0t}), \hat{H}^T(\theta_{0t-1})\right) = \frac{1}{N^2}\sum_{i=1}^{N} Cov\left(\frac{\delta_{it}}{\hat{\pi}_{i,t}}, \frac{\delta_{i,t-1}}{\hat{\pi}_{i,t-1}}\right)S_i(\theta_{0t})S_i^T(\theta_{0t-1})$$

Considering

$$
\begin{aligned}
Cov\left(\frac{\delta_{it}}{\hat{\pi}_{i,t}}, \frac{\delta_{i,t-1}}{\hat{\pi}_{i,t-1}}\right) &\approx Cov\left(\frac{\delta_{it}}{\pi_{it}}(\hat{\pi}_{it} - \pi_{it}), \frac{\delta_{i,t-1}}{\pi_{i,t-1}}(\hat{\pi}_{i,t-1} - \pi_{i,t-1})\right) \\
&= Cov\left(\frac{\delta_{it}}{\pi_{it}}\hat{\pi}_{it}, \frac{\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right) - Cov\left(\delta_{it}, \frac{\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right) \\
&\quad - Cov\left(\frac{\delta_{it}}{\pi_{it}}\hat{\pi}_{it}, \delta_{i,t-1}\right) + Cov\left(\delta_{it}, \delta_{i,t-1}\right) \qquad \text{(A.13)}
\end{aligned}
$$

where $Cov\left(\delta_{it}, \delta_{i,t-1}\right) = 0$. We have

$$Cov\left(\frac{\delta_{it}}{\pi_{it}}\hat{\pi}_{i,t}, \frac{\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right) = E\left(\frac{\delta_{it}\delta_{i,t-1}}{\pi_{it}\pi_{i,t-1}}\hat{\pi}_{i,t}\hat{\pi}_{i,t-1}\right) - E\left(\frac{\delta_{it}}{\pi_{it}}\hat{\pi}_{i,t}\right)E\left(\frac{\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right), \quad (A.14)$$

$$Cov\left(\delta_{it}, \frac{\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right) = E\left(\frac{\delta_{it}\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right) - E\left(\delta_{it}\right)E\left(\frac{\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right) \quad (A.15)$$

$$Cov\left(\frac{\delta_{it}}{\pi_{it}}\hat{\pi}_{i,t}, \delta_{i,t-1}\right) = E\left(\frac{\delta_{it}\delta_{i,t-1}}{\pi_{it}}\hat{\pi}_{i,t}\right) - E\left(\frac{\delta_{it}}{\pi_{it}}\hat{\pi}_{i,t}\right)E\left(\delta_{i,t-1}\right). \quad (A.16)$$

We can write

$$\begin{aligned}
E\left(\frac{\delta_{it}\delta_{i,t-1}}{\pi_{it}\pi_{i,t-1}}\hat{\pi}_{i,t}\hat{\pi}_{i,t-1}\right) &= E(\frac{\hat{\pi}_{i,t}\hat{\pi}_{i,t-1}}{\pi_{it}\pi_{i,t-1}}|\delta_{it}\delta_{i,t-1} = 1)\pi_{it}\pi_{i,t-1} \\
&= E(\hat{\pi}_{i,t}\hat{\pi}_{i,t-1}|\delta_{it}\delta_{i,t-1} = 1) \\
E\left(\frac{\delta_{it}}{\pi_{it}}\hat{\pi}_{i,t}\right) &= E(\hat{\pi}_{i,t}|\delta_{it} = 1) \\
E\left(\frac{\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right) &= E(\hat{\pi}_{i,t-1}|\delta_{i,t-1} = 1) \\
E\left(\frac{\delta_{it}\delta_{i,t-1}}{\pi_{i,t-1}}\hat{\pi}_{i,t-1}\right) &= E(\frac{\delta_{it}\hat{\pi}_{i,t-1}}{\pi_{i,t-1}}|\delta_{i,t-1} = 1)\pi_{i,t-1} \\
&= E(\delta_{it}\hat{\pi}_{i,t-1}|\delta_{i,t-1} = 1) \\
&= E(\hat{\pi}_{i,t-1}|\delta_{it} = 1, \delta_{i,t-1} = 1)E(\delta_{it}) \\
&= \pi_{it}E(\hat{\pi}_{i,t-1}|\delta_{it}\delta_{i,t-1} = 1) \\
E\left(\frac{\delta_{it}\delta_{i,t-1}}{\pi_{it}}\hat{\pi}_{i,t}\right) &= E(\frac{\delta_{i,t-1}\hat{\pi}_{i,t}}{\pi_{it}}|\delta_{it} = 1)\pi_{it} \\
&= E(\delta_{i,t-1}\hat{\pi}_{i,t}|\delta_{it} = 1) \\
&= E(\hat{\pi}_{i,t}|\delta_{i,t-1} = 1, \delta_{it} = 1)E(\delta_{i,t-1}) \\
&= \pi_{i,t-1}E(\hat{\pi}_{i,t}|\delta_{it}\delta_{i,t-1} = 1).
\end{aligned}$$

Then

$$\begin{aligned}
Cov\left(\frac{\delta_{it}}{\hat{\pi}_{i,t}}, \frac{\delta_{i,t-1}}{\hat{\pi}_{i,t-1}}\right) \approx & \left(E[\hat{\pi}_{i,t}\hat{\pi}_{i,t-1}|\delta_{it}\delta_{i,t-1} = 1] - E[\hat{\pi}_{i,t}|\delta_{it} = 1]E[\hat{\pi}_{i,t-1}|\delta_{i,t-1} = 1]\right) \\
& - \pi_{it}\left(E[\hat{\pi}_{i,t-1}|\delta_{it}\delta_{i,t-1} = 1] - E[\hat{\pi}_{i,t-1}|\delta_{i,t-1} = 1]\right) \\
& - \pi_{i,t-1}\left(E[\hat{\pi}_{i,t}|\delta_{it}\delta_{i,t-1} = 1] - E[\hat{\pi}_{i,t}|\delta_{i,t} = 1]\right),
\end{aligned}$$

The expectations used in above covariance can be calculated according to the definition of $\hat{\pi}_{i,t}$ and $\hat{\pi}_{i,t-1}$,

$$E[\hat{\pi}_{i,t}|\delta_{i,t}=1] = E[(\delta_{i,t}+\sum_{t=1}^{T-1}\delta_{i,t})/T|\delta_{it}=1] = (1+(T-1)\pi_{i,t})/T$$

$$\begin{aligned}
E[\hat{\pi}_{i,t-1}|\delta_{i,t-1}=1] &= E[(\delta_{i,t-1}+\sum_{t=2}^{T-1}\delta_{i,t-1})/(T-1)|\delta_{i,t-1}=1] \\
&= (1+(T-2)\pi_{i,t-1})/(T-1) \\
E[\hat{\pi}_{i,t}|\delta_{i,t}\delta_{i,t-1}=1] &= E[(\sum_{t=1}^{T}\delta_{i,t})/T|\delta_{i,t}\delta_{i,t-1}=1] = \pi_{i,t}
\end{aligned}$$

$$E[\hat{\pi}_{i,t-1}|\delta_{i,t}\delta_{i,t-1}=1] = E[(\sum_{t=2}^{T}\delta_{i,t-1})/(T-1)|\delta_{i,t}\delta_{i,t-1}=1] = \pi_{i,t-1}$$

$$E[\hat{\pi}_{it}\hat{\pi}_{i,t-1}|\delta_{it}\delta_{i,t-1}=1] = E[[\hat{\pi}_{i,t}|\delta_{it}\delta_{i,t-1}=1]|\hat{\pi}_{i,t-1}]E[\hat{\pi}_{i,t-1}|\delta_{it}\delta_{i,t-1}=1] = \pi_{it}\pi_{i,t-1}$$

Then

$$\begin{aligned}
Cov\left(\frac{\delta_{it}}{\hat{\pi}_{i,t}}, \frac{\delta_{i,t-1}}{\hat{\pi}_{i,t-1}}\right) &\approx [\pi_{it}\pi_{i,t-1} - (1+(T-1)\pi_{i,t})(1+(T-2)\pi_{i,t-1})/[T(T-1)]] \\
&- \pi_{it}[\pi_{i,t-1} - (1+(T-2)\pi_{i,t-1})/(T-1)] \\
&- \pi_{i,t-1}[\pi_{it} - (1+(T-1)\pi_{i,t})/T] \qquad \text{(A.17)}
\end{aligned}$$

The plug-in estimator of $Var(\hat{\theta}_t - \hat{\theta}_{t-1})$ can thus be written as

$$\widehat{Var}(\hat{\Delta}_t) = \widehat{Var}(\hat{\theta}_t - \hat{\theta}_{t-1}) = \widehat{Var}(\hat{\theta}_t) + \widehat{Var}(\hat{\theta}_{t-1}) - 2\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \qquad \text{(A.18)}$$

where

$$\begin{aligned}
\widehat{Var}(\hat{\theta}_t) &= G^{-1}(\hat{\theta}_t)\widehat{Var}[\hat{H}(\hat{\theta}_t)]G^{-T}(\hat{\theta}_t), \\
\widehat{Var}(\hat{\theta}_{t-1}) &= G^{-1}(\hat{\theta}_{t-1})\widehat{Var}[\hat{H}(\hat{\theta}_{t-1})]G^{-T}(\hat{\theta}_{t-1}), \\
\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) &= G^{-1}(\hat{\theta}_t)\widehat{Cov}\left(\hat{H}(\hat{\theta}_t), \hat{H}^T(\hat{\theta}_{t-1})\right)G^{-T}(\hat{\theta}_{t-1}).
\end{aligned}$$

Moreover,

$$\begin{aligned}
G(\hat{\theta}_t) &= \frac{1}{N}\sum_{i=1}^{r}\hat{q}_i S_i'(\hat{\theta}_t), \text{ with } \hat{q}_i = \hat{\pi}_i^{-1}\hat{\mu}_{1i} \\
\widehat{Var}[\hat{H}(\hat{\theta}_t)] &= \frac{1}{N^2}\sum_{i=1}^{r}\hat{v}_i S_i(\hat{\theta}_t)S_i^T(\hat{\theta}_t), \text{ with } \hat{v}_i = \hat{\pi}_i^{-1}(\hat{\mu}_{2i} - (\hat{\mu}_{1i})^2)
\end{aligned}$$

where $\hat{\mu}_{1i}$ and $\hat{\mu}_{2i}$ are given by (A.3) and (A.4), after replacing r_{it} with δ_{it} and p_i with π_i.

Similarly for $G(\hat{\theta}_{t-1})$ and $\widehat{Var}[\hat{H}(\hat{\theta}_{t-1})]$. Finally,

$$\widehat{Cov}\left(\hat{H}(\hat{\theta}_t), \hat{H}^T(\hat{\theta}_{t-1})\right) = \frac{1}{N}\sum_{i=1}^{N}\widehat{Cov}\left(\frac{\delta_{it}}{\hat{\pi}_{i,t}}, \frac{\delta_{i,t-1}}{\hat{\pi}_{i,t-1}}\right) S_i(\hat{\theta}_t) S_i^T(\hat{\theta}_{t-1}) \tag{A.19}$$

where $\widehat{Cov}\left(\delta_{it}/\hat{\pi}_{i,t}, \delta_{i,t-1}/\hat{\pi}_{i,t-1}\right)$ is estimator of (A.17).

Appendix B

Bias-correction of variance estimator using Taylor expansion under cross-sectional setting

The variance estimator of $\hat{\theta}$ has already been discussed in Chapter 2, based on simple EE. We can also correct the bias of variance estimator of $\hat{\theta}$ directly using Taylor expansion and similarly bias-correction for variance estimator of $\hat{\theta}^{\star}$. We discussed below the bias-correction of variance estimator using Taylor expansion for both scalar and vector $\hat{\theta}$ and then explain that the bias-adjusted variance estimator for $\hat{\theta}^{\star}$ will be obtained with necessary replacements.

B.1 Bias-correction for scalar $\hat{\theta}$ using Taylor expansion

From (2.36), the variance estimator of scalar $\hat{\theta}$ can be written as

$$\widehat{Var}(\hat{\theta}) = G^{-2}(\hat{\theta})\widehat{Var}[\hat{H}(\hat{\theta})],$$

The three terms Taylor expansion for $\hat{\theta}$ around θ_0 is

$$\widehat{Var}(\hat{\theta}) = \widehat{Var}(\theta_0) + \frac{\partial}{\partial\hat{\theta}}\widehat{Var}(\hat{\theta})|_{\hat{\theta}=\theta_0}(\hat{\theta}-\theta_0) + \frac{1}{2}\frac{\partial^2}{\partial\hat{\theta}^2}\widehat{Var}(\hat{\theta})|_{\hat{\theta}=\theta_0}(\hat{\theta}-\theta_0)^2$$

Then

$$Bias\left(\widehat{Var}(\hat{\theta})\right) = E\left[\widehat{Var}(\hat{\theta}) - \widehat{Var}(\theta_0)\right] = \frac{1}{2}D(\theta_0)E(\hat{\theta}-\theta_0)^2 = \frac{1}{2}D(\theta_0)\widehat{Var}(\hat{\theta}),$$

where

$$D(\theta_0) = \frac{\partial^2}{\partial\hat{\theta}^2}\widehat{Var}(\hat{\theta})|_{\hat{\theta}=\theta_0}.$$

The bias-adjusted $\widehat{Var}(\hat{\theta})$ can be written as

$$\widehat{Var}_{bc}(\hat{\theta}) = \widehat{Var}(\hat{\theta}) - \frac{1}{2}D(\hat{\theta})\widehat{Var}(\hat{\theta}) = \widehat{Var}(\hat{\theta})\left[1 - D(\hat{\theta})/2\right], \tag{B.1}$$

where

$$D(\hat{\theta}) = \frac{\partial^2}{\partial\hat{\theta}^2}\widehat{Var}(\hat{\theta}) = \frac{\partial^2}{\partial\hat{\theta}^2}\left\{G^{-2}(\hat{\theta})\widehat{Var}[\hat{H}(\hat{\theta})]\right\}.$$

Below we illustrate the bias correction of the variance estimator using the Taylor expansion for scalar $\hat{\theta}$.

Example-1: In the case of estimation of the population mean θ, the estimating equation for responding units can be written as

$$\hat{H}_N(\theta) = \frac{1}{N}\sum_{i=1}^{N}\frac{\delta_i}{\hat{\pi}_i}(y_i - \theta)$$

and from equations (2.37) and (2.38), we have

$$G(\hat{\theta}) = -\frac{1}{N}\sum_{i=1}^{r}\hat{g}_i \quad \text{and} \quad V(\hat{\theta}) = \frac{1}{N^2}\sum_{i=1}^{r}\hat{v}_i(y_i - \theta)^2,$$

where $\hat{g}_i$ and $\hat{v}_i$ are given in equations (2.39) and (2.40). From (B.1), the bias-adjusted variance of $\hat{\theta}$ is

$$\widehat{Var}_{bc}(\hat{\theta}) = \widehat{Var}(\hat{\theta})\left[1 - D(\hat{\theta})/2\right], \tag{B.2}$$

where

$$\widehat{Var}(\hat{\theta}) = N^{-1}G^{-2}(\hat{\theta})V(\hat{\theta}) = \frac{\sum_{i=1}^{r}\hat{v}_i(y_i - \theta)^2}{\left(\sum_{i=1}^{r}\hat{g}_i\right)^2}$$

and

$$\frac{\partial}{\partial\theta}\widehat{Var}(\hat{\theta}) = \frac{-2\sum_{i=1}^{r}\hat{v}_i(y_i - \theta)}{\left(\sum_{i=1}^{r}\hat{g}_i\right)^2} \implies D(\hat{\theta}) = \frac{\partial^2}{\partial\theta^2}|_{\theta=\hat{\theta}}\widehat{Var}(\hat{\theta}) = \frac{2\sum_{i=1}^{r}\hat{v}_i}{\left(\sum_{i=1}^{r}\hat{g}_i\right)^2}.$$

Finally from (C.5),

$$\widehat{Var}_{bc}(\hat{\theta}) = \frac{\sum_{i=1}^{r}\hat{v}_i(y_i - \hat{\theta})^2}{\left(\sum_{i=1}^{r}\hat{g}_i\right)^2}\left[1 - \frac{\sum_{i=1}^{r}\hat{v}_i}{\left(\sum_{i=1}^{r}\hat{g}_i\right)^2}\right], \tag{B.3}$$

where $\hat{g}_i$ and $\hat{v}_i$ are given in (2.39) and (2.40), respectively. The bias-adjusted variance for bias-adjusted EE can be obtained from (B.3) by replacing $(\hat{\theta}, \hat{g}_i, \hat{v}_i)$ with $(\hat{\theta}^*, \hat{g}_{i_{bc}}, \hat{v}_{i_{bc}})$.

B.2 Bias-correction for vector $\hat{\theta}$ using Taylor expansion

For the estimated variance covariance matrix of a p-dimensional vector $\hat{\theta}$, the three terms of the Taylor expansion of $\widehat{Var}(\hat{\theta})$ for $\hat{\theta}$ around θ_0 can be written as

$$\widehat{Var}(\hat{\theta}) = \widehat{Var}(\theta_0) + \frac{\partial}{\partial\hat{\theta}^T}|_{\hat{\theta}=\theta_0}\widehat{Var}(\hat{\theta})\{(\hat{\theta} - \theta_0)\otimes I_p\} + \frac{1}{2}\frac{\partial^2}{\partial\hat{\theta}\partial\hat{\theta}^T}|_{\hat{\theta}=\theta_0}\widehat{Var}(\hat{\theta})\{(\hat{\theta} - \theta_0)^{\otimes 2}\otimes I_p\},$$

where $\otimes$ denotes the Kronecker product. The bias of $\widehat{Var}(\hat{\theta})$ can be written as

$$E\left[\widehat{Var}(\hat{\theta})-\widehat{Var}(\theta_0)\right]=\frac{1}{2}\frac{\partial^2}{\partial\hat{\theta}\partial\hat{\theta}^T}|_{\hat{\theta}=\theta_0}\widehat{Var}(\hat{\theta})E\{(\hat{\theta}-\theta_0)^{\otimes 2}\otimes I_p\}$$

Then the bias-adjusted $\widehat{Var}(\hat{\theta})$ is

$$\widehat{Var}_{bc}(\hat{\theta})=G^{-1}(\hat{\theta})V(\hat{\theta})G^{-T}(\hat{\theta})-\frac{1}{2}\mathbf{H}(\hat{\theta})E\{(\hat{\theta}-\theta_0)^{\otimes 2}\otimes I_p\}, \tag{B.4}$$

where $\mathbf{H}(\hat{\theta})=\frac{\partial^2}{\partial\hat{\theta}\partial\hat{\theta}^T}\widehat{Var}(\hat{\theta})$.

To find $\mathbf{H}(\hat{\theta})$, the first order partial derivative structure of $\widehat{Var}(\hat{\theta})$ with respect to $\hat{\theta}$ of dimension p can be written as

$$\frac{\partial}{\partial\hat{\theta}^T}\widehat{Var}(\hat{\theta})=\left[\frac{\partial}{\partial\hat{\theta}_1}\widehat{Var}(\hat{\theta})\ \vdots\ \frac{\partial}{\partial\hat{\theta}_2}\widehat{Var}(\hat{\theta})\ \vdots\ \cdots\ \vdots\ \frac{\partial}{\partial\hat{\theta}_p}\widehat{Var}(\hat{\theta})\right].$$

Second order partial derivative of $\widehat{Var}(\hat{\theta})$ with respect to 2-dimensional vector $\hat{\theta}$ can be written as

$$\mathbf{H}(\hat{\theta})=\frac{\partial^2}{\partial\hat{\theta}\partial\hat{\theta}^T}\widehat{Var}(\hat{\theta})=\left[\frac{\partial^2}{\partial\hat{\theta}_1^2}\widehat{Var}(\hat{\theta})\ \vdots\ \frac{\partial^2}{\partial\hat{\theta}_1\partial\hat{\theta}_2}\widehat{Var}(\hat{\theta})\ \vdots\ \frac{\partial^2}{\partial\hat{\theta}_2\partial\hat{\theta}_1}\widehat{Var}(\hat{\theta})\ \vdots\ \frac{\partial^2}{\partial\hat{\theta}_2^2}\widehat{Var}(\hat{\theta})\right].$$

Similarly, it can be extended for p-dimensions. Below we illustrate the bias correction of variance estimators using Taylor expansion for vector $\hat{\theta}$ with two dimensions.
Example-2: To illustrate the bias-correction of the variance estimator for a vector of parameters. Suppose we want to estimate the finite population mean and variance, then the estimating equations can be written as

$$\hat{H}_N(\theta)=\frac{1}{N}\sum_{i=1}^{N}\frac{\delta_{i0}}{\hat{\pi}_i}\begin{bmatrix}(y_i-\theta)\\(y_i-\theta)^2-\sigma^2\end{bmatrix}.$$

From (B.4), the bias-adjusted $\widehat{Var}(\hat{\theta})$ can be written as

$$\widehat{Var}_{bc}(\hat{\theta})=\widehat{Var}(\hat{\theta})-\frac{1}{2}\mathbf{H}(\hat{\theta})E\{(\hat{\theta}-\theta_0)^{\otimes 2}\otimes I_2\}, \tag{B.5}$$

First, to find the estimated variance covariance matrix $\widehat{Var}(\hat{\theta})$, we have

$$G(\hat{\theta})=-\frac{1}{N}\sum_{i=1}^{r}\hat{g}_i\begin{bmatrix}1 & 0\\2(y_i-\hat{\theta}) & 1\end{bmatrix}\approx-\frac{1}{N}\sum_{i=1}^{r}\hat{g}_i\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},$$

since $\frac{1}{N}\sum_{i=1}^{r}\hat{g}_i(y_i-\hat{\theta})$ is almost 0. Then

$$G^{-1}(\hat{\theta})\approx-\frac{1}{N}\begin{bmatrix}1/\sum_{i=1}^{r}\hat{g}_i & 0\\0 & 1/\sum_{i=1}^{r}\hat{g}_i\end{bmatrix}$$

and

$$\widehat{Var}[\hat{H}(\hat{\theta})] = \frac{1}{N^2}\begin{bmatrix} \sum_{i=1}^{r} \hat{v}_i(y_i-\hat{\theta})^2 & \sum_{i=1}^{r} \hat{v}_i(y_i-\hat{\theta})\left((y_i-\hat{\theta})^2-\hat{\sigma}^2\right) \\ \sum_{i=1}^{r} \hat{v}_i(y_i-\hat{\theta})\left((y_i-\hat{\theta})^2-\hat{\sigma}^2\right) & \sum_{i=1}^{r} \hat{v}_i\left((y_i-\hat{\theta})^2-\hat{\sigma}^2\right)^2 \end{bmatrix}.$$

where $\hat{v}_i$ and $\hat{v}_i$ is given (2.39) and (2.40).

Now we can write

$$\widehat{Var}(\hat{\theta}) = G^{-1}(\hat{\theta})\widehat{Var}[\hat{H}(\hat{\theta})]G^{-T}(\hat{\theta}) = \begin{bmatrix} v_{ij} \end{bmatrix}_{2\times 2} = \hat{V} \tag{B.6}$$

with

$$\begin{aligned} v_{11} &= \frac{\sum_{i=1}^{N} \hat{v}_i(y_i-\hat{\theta})^2}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2} \\ v_{12} = v_{21} &= \frac{\sum_{i=1}^{N} \hat{v}_i(y_i-\hat{\theta})\left((y_i-\hat{\theta})^2-\hat{\sigma}^2\right)}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2} \\ v_{22} &= \frac{\sum_{i=1}^{N} \hat{v}_i\left((y_i-\hat{\theta})^2-\hat{\sigma}^2\right)^2}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2}. \end{aligned}$$

Then we can write

$$E\{(\hat{\theta}-\theta_0)^{\otimes 2}\otimes I_2\} = \begin{bmatrix} v_{11}I_2 & \vdots & v_{12}I_2 & \vdots & v_{21}I_2 & \vdots & v_{22}I_2 \end{bmatrix}^T. \tag{B.7}$$

And

$$\mathbf{H}(\hat{\theta}) = \begin{bmatrix} \frac{\partial^2}{\partial\theta^2}\hat{V} & \vdots & \frac{\partial}{\partial\theta\partial\sigma^2}\hat{V} & \vdots & \frac{\partial}{\partial\sigma^2\partial\theta}\hat{V} & \vdots & \frac{\partial^2}{\partial(\sigma^2)^2}\hat{V} \end{bmatrix}, \tag{B.8}$$

where

$$\begin{aligned} &\frac{\partial^2}{(\partial\theta)^2}(v_{11}) = 2\frac{\sum_{i=1}^{N}\hat{v}_i}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2}, \quad \frac{\partial^2}{(\partial\theta)^2}(v_{12}) = 6\frac{\sum_{i=1}^{N}\hat{v}_i(y_i-\hat{\theta})}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2}, \\ &\frac{\partial^2}{(\partial\theta)^2}(v_{22}) = 4\frac{\sum_{i=1}^{N}\left(\hat{v}_i\left((y_i-\hat{\theta})^2-\hat{\sigma}^2\right)+2\hat{v}_i(y_i-\hat{\theta})^2\right)}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2}, \\ &\frac{\partial^2}{\partial\sigma^2\partial\theta}(v_{11}) = 0, \quad \frac{\partial^2}{\partial\sigma^2\partial\theta}(v_{12}) = \frac{\sum_{i=1}^{N}\hat{v}_i}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2}, \quad \frac{\partial^2}{\partial\sigma^2\partial\theta}(v_{22}) = 4\frac{\sum_{i=1}^{N}\hat{v}_i(y_i-\hat{\theta})}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2} \\ &\frac{\partial^2}{(\partial\sigma^2)^2}(v_{11}) = 0, \quad \frac{\partial^2}{(\partial\sigma^2)^2}(v_{12}) = 0 \text{ and } \frac{\partial^2}{(\partial\sigma^2)^2}(v_{22}) = 2\frac{\sum_{i=1}^{N}\hat{v}_i}{\left(\sum_{i=1}^{N}\hat{g}_i\right)^2}. \end{aligned}$$

Using (B.6), (B.7) and (B.8) in (B.5), the bias-adjusted variance estimate can be obtained. The bias-adjusted variance covariance matrix $\widehat{Var}_{bc}(\hat{\theta})$ for the bias-adjusted estimating equations can be obtained from (C.13) by replacing $(\hat{\theta}, \hat{\sigma}^2, \hat{g}_i, \hat{v}_i)$ with $(\hat{\theta}^*, \hat{\sigma}^{2*}, \hat{g}_{i_{bc}}, \hat{v}_{i_{bc}})$.

Example-3: To illustrate the bias-correction of variance estimator for vector of regression coefficients. Suppose we want to estimate the finite population regression coefficients for simple linear regression model with $\epsilon_i \sim N(0, \sigma^2 x_i^\alpha)$, then the estimating equations can be written as

$$\hat{H}_N(\theta) = \frac{1}{N}\sum_{i=1}^{N}\frac{\delta_{i0}}{\hat{\pi}_i}\begin{bmatrix}(y_i - \beta_0 - \beta_1 x_i)/x_i^\alpha \\ x_i(y_i - \beta_0 - \beta_1 x_i)/x_i^\alpha \\ (y_i - \beta_0 - \beta_1 x_i)^2/x_i^\alpha - \sigma^2\end{bmatrix}$$

From (B.4), the bias-corrected $\widehat{Var}(\hat{\theta})$ can be written as

$$\widehat{Var}_{bc}(\hat{\theta}) = \Sigma_{\hat{\theta}}(\hat{\theta}) - \frac{1}{2}\mathbf{H}(\hat{\theta})\hat{E}\{(\hat{\theta} - \theta_0)^{\otimes 2} \otimes I_3\}, \tag{B.9}$$

First to find the estimated variance covariance matrix $\Sigma_{\hat{\theta}}(\hat{\theta})$, we can have

$$G(\hat{\theta}) = -\frac{1}{N}\sum_{i=1}^{r}\tilde{g}_i\begin{bmatrix}1 & x_i & 0 \\ x_i & x_i^2 & 0 \\ 2(y_i - \beta_0 - \beta_1 x_i) & 2x_i(y_i - \beta_0 - \beta_1 x_i) & x_i^\alpha\end{bmatrix} \approx -\frac{1}{N}\sum_{i=1}^{r}\tilde{g}_i\begin{bmatrix}1 & x_i & 0 \\ x_i & x_i^2 & 0 \\ 0 & 0 & x_i^\alpha\end{bmatrix},$$

where $\frac{1}{N}\sum_{i=1}^{r}\tilde{g}_i(y_i - \beta_0 - \beta_1 x_i)$ and $\frac{1}{N}\sum_{i=1}^{r}\tilde{g}_i x_i(y_i - \beta_0 - \beta_1 x_i)$ are almost 0 and $\tilde{g}_i = \hat{g}_i/x_i^\alpha$.

$$\begin{aligned}
&G^{-1}(\hat{\theta}) \\
&\approx -\frac{1}{dN}\begin{bmatrix}\sum_{i=1}^{r}\tilde{g}_i x_i^\alpha \sum_{i=1}^{r}\tilde{g}_i x_i^2 & -\sum_{i=1}^{r}\tilde{g}_i x_i^\alpha \sum_{i=1}^{r}\tilde{g}_i x_i & 0 \\ -\sum_{i=1}^{r}\tilde{g}_i x_i \sum_{i=1}^{r}\tilde{g}_i x_i^\alpha & \sum_{i=1}^{r}\tilde{g}_i x_i^\alpha \sum_{i=1}^{r}\tilde{g}_i & 0 \\ 0 & 0 & \sum_{i=1}^{r}\tilde{g}_i x_i^2 \sum_{i=1}^{r}\tilde{g}_i - \left(\sum_{i=1}^{r}\tilde{g}_i x_i\right)^2\end{bmatrix} \\
&= -\frac{1}{N}\begin{bmatrix}g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ 0 & 0 & g_{33}\end{bmatrix},
\end{aligned}$$

where $d = \sum_{i=1}^{r}\tilde{g}_i x_i^\alpha \sum_{i=1}^{r}\tilde{g}_i x_i^2 \sum_{i=1}^{r}\tilde{g}_i - \sum_{i=1}^{r}\tilde{g}_i x_i^\alpha \left(\sum_{i=1}^{r}\tilde{g}_i x_i\right)^2$.

$$\widehat{Var}[\hat{H}(\hat{\theta})] = \frac{1}{N^2}\begin{bmatrix}\sum_{i=1}^{r}\tilde{v}_i\hat{e}_i^2 & \sum_{i=1}^{r}\tilde{v}_i x_i\hat{e}_i^2 & \sum_{i=1}^{r}\tilde{v}_i\hat{e}_i(\hat{e}_i^2 - \hat{\sigma}^2 x_i^\alpha) \\ \sum_{i=1}^{r}\tilde{v}_i x_i\hat{e}_i^2 & \sum_{i=1}^{r}\tilde{v}_i x_i^2\hat{e}_i^2 & \sum_{i=1}^{r}\tilde{v}_i x_i\hat{e}_i(\hat{e}_i^2 - \hat{\sigma}^2 x_i^\alpha) \\ \sum_{i=1}^{r}\tilde{v}_i\hat{e}_i(\hat{e}_i^2 - \hat{\sigma}^2 x_i^\alpha) & \sum_{i=1}^{r}\tilde{v}_i x_i\hat{e}_i(\hat{e}_i^2 - \hat{\sigma}^2 x_i^\alpha) & \sum_{i=1}^{r}\tilde{v}_i(\hat{e}_i^2 - \hat{\sigma}^2 x_i^\alpha)^2\end{bmatrix},$$

where $\hat{e}_i = (y_i - \hat{\beta}_0 - \hat{\beta}_{1t}x_i)$ and $\tilde{v}_i = \hat{v}_i/x_i^{2\alpha}$. Now we can wrie

$$\widehat{Var}(\hat{\theta}) = G^{-1}(\hat{\theta})\widehat{Var}[\hat{H}(\hat{\theta})]G^{-T}(\hat{\theta}) = \left[v_{ij}\right]_{3\times 3} = \hat{V}$$

wih

$$v_{11} = g_{12}\left(g_{11}\sum_{i=1}^{r}\tilde{v}_i x_i(\hat{e}_i)^2 + g_{12}\sum_{i=1}^{r}\tilde{v}_i x_i^2(\hat{e}_i)^2\right) + g_{11}\left(g_{11}\sum_{i=1}^{r}\tilde{v}_i(\hat{e}_i)^2 + g_{12}\sum_{i=1}^{r}\tilde{v}_i x_i(\hat{e}_i)^2\right)$$

$$v_{12} = g_{21}\left(g_{11}\sum_{i=1}^{r}\tilde{v}_i(\hat{e}_i)^2 + g_{12}\sum_{i=1}^{r}\tilde{v}_i x_i(\hat{e}_i)^2\right) + g_{22}\left(g_{11}\sum_{i=1}^{r}\tilde{v}_i x_i(\hat{e}_i)^2 + g_{12}\sum_{i=1}^{r}\tilde{v}_i x_i^2(\hat{e}_i)^2\right)$$

$$v_{13} = g_{33}\left(g_{11}\sum_{i=1}^{r}\tilde{v}_i(\hat{e}_i)\left((\hat{e}_i)^2 - \hat{\sigma}^2 x_i^{\alpha}\right) + g_{12}\sum_{i=1}^{r}\tilde{v}_i x_i(\hat{e}_i)\left((\hat{e}_i)^2 - \hat{\sigma}^2 x_i^{\alpha}\right)\right)$$

$$v_{22} = g_{22}\left(g_{21}\sum_{i=1}^{r}\tilde{v}_i x_i(\hat{e}_i)^2 + g_{22}\sum_{i=1}^{r}\tilde{v}_i x_i^2(\hat{e}_i)^2\right) + g_{21}\left(g_{21}\sum_{i=1}^{r}\tilde{v}_i(\hat{e}_i)^2 + g_{22}\sum_{i=1}^{r}\tilde{v}_i x_i(\hat{e}_i)^2\right)$$

$$v_{23} = g_{33}\left(g_{21}\sum_{i=1}^{r}\tilde{v}_i(\hat{e}_i)\left((\hat{e}_i)^2 - \hat{\sigma}^2 x_i^{\alpha}\right) + g_{22}\sum_{i=1}^{r}\tilde{v}_i x_i(\hat{e}_i)\left((\hat{e}_i)^2 - \hat{\sigma}^2 x_i^{\alpha}\right)\right)$$

$$v_{33} = g_{33}^2\sum_{i=1}^{r}\tilde{v}_i\left((\hat{e}_i)^2 - \hat{\sigma}^2 x_i^{\alpha}\right)^2.$$

Further,

$$\hat{E}\{(\hat{\theta} - \theta_0)^{\otimes 2} \otimes I_3\} =$$
$$\begin{bmatrix} v_{11}I_3 & \vdots & v_{12}I_3 & \vdots & v_{13}I_3 & \vdots & v_{21}I_3 & \vdots & v_{22}I_3 & \vdots & v_{23}I_3 & \vdots & v_{31}I_3 & \vdots & v_{32}I_3 & \vdots & v_{33}I_3 \end{bmatrix}^T.$$

Now

$$\mathbf{H}(\hat{\theta}) = \begin{bmatrix} \frac{\partial^2}{\partial\hat{\beta}_0^2}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_1\partial\hat{\beta}_0}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_0}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_1^2}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_0\partial\hat{\beta}_1}\hat{V} & \vdots \\ \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_1}\hat{V} & \vdots & \frac{\partial^2}{\partial(\hat{\sigma}_t^2)^2}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_0\partial\hat{\sigma}_t^2}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_1\partial\hat{\sigma}_t^2}\hat{V} & & & \end{bmatrix}.$$

Each element can be obtained as

$$\frac{\partial^2}{(\partial\hat{\beta}_0)^2}(v_{11}) = 2g_{12}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^2\right) + 2g_{11}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_0)^2}(v_{12}) = 2g_{21}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i\right) + 2g_{22}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^2\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_0)^2}(v_{13}) = 6g_{33}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i(\hat{e}_i) + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i(\hat{e}_i)\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_0)^2}(v_{22}) = 2g_{22}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i^2\right) + 2g_{21}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_0)^2}(v_{23}) = 6g_{33}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i(\hat{e}_i) + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i(\hat{e}_i)\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_0)^2}(v_{33}) = 4g_{33}^2\sum_{i=1}^{N}\left(\tilde{v}_i\left((\hat{e}_i)^2 - \hat{\sigma}^2 x_i^{\alpha}\right) + 2\tilde{v}_i(\hat{e}_i)^2\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_1\partial\hat{\beta}_0}(v_{11}) = 2g_{11}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^2\right) + 2g_{12}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i^2 + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^3\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_1\partial\hat{\beta}_0}(v_{12}) = 2g_{21}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^2\right) + 2g_{22}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i^2 + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^3\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_1\partial\hat{\beta}_0}(v_{13}) = 6g_{33}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i(\hat{e}_i) + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^2(\hat{e}_i)\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_1\partial\hat{\beta}_0}(v_{22}) = 2g_{21}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i^2\right) + 2g_{22}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i^2 + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i^3\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_1\partial\hat{\beta}_0}(v_{23}) = 6g_{33}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i(\hat{e}_i) + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i^2(\hat{e}_i)\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_1\partial\hat{\beta}_0}(v_{33}) = 4g_{33}^2\sum_{i=1}^{N}\left(\tilde{v}_i x_i\left((\hat{e}_i)^2 - \hat{\sigma}^2 x_i^{\alpha}\right) + 2\tilde{v}_i x_i(\hat{e}_i)^2\right)$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_0}(v_{11}) = \frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_0}(v_{22}) = \frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_0}(v_{12}) = 0,\ \ \frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_0}(v_{33}) = g_{33}^2\sum_{i=1}^{N}4\tilde{v}_i x_i^{\alpha}(\hat{e}_i),$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_0}(v_{23}) = g_{33}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i^{\alpha} + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i x_i^{\alpha}\right), \frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_0}(v_{13}) = g_{33}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i^{\alpha} + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i x_i^{\alpha}\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_1)^2}(v_{11}) = 2g_{12}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i^3 + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^4\right) + 2g_{11}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i^2 + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^3\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_1)^2}(v_{12}) = 2g_{21}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i^2 + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^3\right) + 2g_{22}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i^3 + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^4\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_1)^2}(v_{13}) = 6g_{33}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i^2(\hat{e}_i) + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^3(\hat{e}_i)\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_1)^2}(v_{22}) = 2g_{22}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i^3 + g_{22}\sum_{i=1}^{N}2\tilde{v}_i x_i^4\right) + 2g_{21}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i^2 + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i^3\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_1)^2}(v_{23}) = 6g_{33}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i^2(\hat{e}_i) + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i^3(\hat{e}_i)\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_1)^2}(v_{33}) = 4g_{33}^2\sum_{i=1}^{N}\left(\tilde{v}_i x_i^2\left((\hat{e}_i)^2 - \hat{\sigma}^2 x_i^{\alpha}\right) + 2\tilde{v}_i x_i^2(\hat{e}_i)^2\right)$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_1}(v_{11}) = 0,\ \ \frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_1}(v_{12}) = 0,\ \ \frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_1}(v_{13}) = g_{33}\left(g_{11}\sum_{i=1}^{N}\tilde{v}_i x_i x_i^{\alpha} + g_{12}\sum_{i=1}^{N}\tilde{v}_i x_i^2 x_i^{\alpha}\right),$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_1}(v_{22}) = 0,\quad \frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_1}(v_{23}) = g_{33}\left(g_{21}\sum_{i=1}^{N}\tilde{v}_i x_i x_i^{\alpha} + g_{22}\sum_{i=1}^{N}\tilde{v}_i x_i^2 x_i^{\alpha}\right),$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2\partial\hat{\beta}_1}(v_{33}) = 4g_{33}^2\sum_{i=1}^{N}\tilde{v}_i x_i x_i^{\alpha}(\hat{e}_i),$$

$$\frac{\partial^2}{(\partial\hat{\sigma}^2)^2}(v_{11}) = 0,\ \ \frac{\partial^2}{(\partial\hat{\sigma}^2)^2}(v_{12}) = 0,\ \ \frac{\partial^2}{(\partial\hat{\sigma}^2)^2}(v_{13}) = 0,\ \ \frac{\partial^2}{(\partial\hat{\sigma}^2)^2}(v_{22}) = 0,$$

$$\frac{\partial^2}{(\partial\hat{\sigma}^2)^2}(v_{23}) = 0 \text{ and } \frac{\partial^2}{(\partial\hat{\sigma}^2)^2}(v_{33}) = 2g_{33}^2\sum_{i=1}^{N}\tilde{v}_i x_i^{2\alpha}.$$

The bias-corrected variance covariance matrix $\widehat{V}ar_{bc}(\hat{\theta})$ for bias-adjusted estimating equations can be obtained from (B.9) by replacing $(\hat{\beta}_0, \hat{\beta}_1, \hat{\sigma}^2, \hat{g}_i, \hat{v}_i)$ wih $(\hat{\beta}_0^*, \hat{\beta}_1^*, \hat{\sigma}^{2*}, \hat{g}_i^*, \hat{v}_i^*)$.

Appendix C

Bias-correction of variance estimator using Taylor expansion under longitudinal setting

The variance estimator $\widehat{Var}(\hat{\Delta}_t)$ has already been discussed in Chapter 3 that is based on simple EEs $\hat{H}(\Delta_t)$ and $\hat{H}(\theta_{t,t-1})$. The variance estimator $\widehat{Var}_{ba}(\hat{\Delta}_t^*)$ is discussed above using bias-adjusted EEs $\hat{H}_{ba}(\Delta_t)$ and $\hat{H}_{ba}(\theta_{t,t-1})$. Each of two variance estimators are basically based on two types of EEs. Here in this section we use the Taylor expansion of both estimators using their respective pairs of estimating equations to correct the bias in these variance estimators.

We discuss below the bias-correction of variance estimator using Taylor expansion for both scalar and vector $\hat{\Delta}_t$ and then explain that the bias-adjusted variance estimator for $\hat{\Delta}^*$ can be obtained with necessary replacements.

C.1 Bias-correction for scalar $\widehat{Var}(\hat{\Delta}_t)$

C.1.1 Bias-correction using $\hat{H}(\Delta_t)$

From (A.5), using EE $\hat{H}(\Delta_t)$, the variance estimator for scalar $\hat{\Delta}_t$ can be written as

$$\widehat{Var}(\hat{\Delta}_t) = N^{-1}G^{-2}(\hat{\Delta}_t)\widehat{Var}[\hat{H}(\hat{\Delta}_t)],$$

From (B.1), the bias-adjusted variance estimator of $\hat{\Delta}_t$ using Taylor expansion can be written as

$$\widehat{Var}_{bcT}(\hat{\Delta}_t) = \widehat{Var}(\hat{\Delta}_t) - \frac{1}{2}D(\hat{\Delta}_t)\widehat{Var}(\hat{\Delta}_t) = \widehat{Var}(\hat{\Delta}_t)\left[1 - D(\hat{\Delta}_t)/2\right], \quad \text{(C.1)}$$

$$\text{where } D(\hat{\Delta}_t) = \frac{\partial^2}{\partial\hat{\Delta}_t^2}\widehat{Var}(\hat{\Delta}_t) = \frac{\partial^2}{\partial\hat{\Delta}_t^2}\left\{N^{-1}G^{-2}(\hat{\Delta}_t)\widehat{Var}[\hat{H}(\hat{\Delta}_t)]\right\}.$$

The above expression for the bias-adjusted variance estimator is in general form. The bias-adjusted variance estimator can be obtained from this expression using simple and bias-adjusted

EEs.

C.1.2 Bias-correction using $\hat{H}(\theta_{t,t-1})$

Here we need to correct the bias of $\widehat{Var}(\hat{\Delta}_t) = \widehat{Var}(\hat{\theta}_t) + \widehat{Var}(\hat{\theta}_{t-1}) - \widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$. Similar to the above section, using two cross-sectional EEs $\hat{H}(\theta_t)$ and $\hat{H}(\theta_{t-1})$, the bias-adjusted estimator for $\widehat{Var}_{\hat{\theta}_t}(\hat{\theta}_t)$ and $\widehat{Var}_{\hat{\theta}_{t-1}}(\hat{\theta}_{t-1})$ can be written from (B.1) as

$$\widehat{Var}_{bcT}(\hat{\theta}_t) = \widehat{Var}(\hat{\theta}_t) - \frac{1}{2}D(\hat{\theta}_t)\widehat{Var}(\hat{\theta}_t) = \widehat{Var}(\hat{\theta}_t)\left[1 - D(\hat{\theta}_t)/2\right], \tag{C.2}$$

and

$$\widehat{Var}_{bcT}(\hat{\theta}_{t-1}) = \widehat{Var}(\hat{\theta}_{t-1}) - \frac{1}{2}D(\hat{\theta}_{t-1})\widehat{Var}(\hat{\theta}_{t-1}) = \widehat{Var}(\hat{\theta}_{t-1})\left[1 - D(\hat{\theta}_{t-1})/2\right], \tag{C.3}$$

where

$$D(\hat{\theta}_t) = \frac{\partial^2}{\partial\hat{\theta}_t^2}\widehat{Var}(\hat{\theta}_t) = \frac{\partial^2}{\partial\hat{\theta}_t^2}\left\{N^{-1}G^{-2}(\hat{\theta}_t)\widehat{Var}[\hat{H}(\hat{\theta}_t)]\right\}.$$

$$D(\hat{\theta}_{t-1}) = \frac{\partial^2}{\partial\hat{\theta}_{t-1}^2}\widehat{Var}(\hat{\theta}_{t-1}) = \frac{\partial^2}{\partial\hat{\theta}_{t-1}^2}\left\{N^{-1}G^{-2}(\hat{\theta}_{t-1})\widehat{Var}[\hat{H}(\hat{\theta}_{t-1})]\right\}.$$

The above expressions for the bias-adjusted variance estimators for time t and $t-1$ are in general form. The bias-adjusted variance estimator can be obtained from this expression using simple and bias-adjusted EEs.

For the bias-adjusted estimator of $\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$ using Taylor expansion, we proceed as follows.

The covariance estimator of scalars $\hat{\theta}_t$ and $\hat{\theta}_{t-1}$ can be written as

$$\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) = N^{-1}G^{-1}(\hat{\theta}_t)G^{-1}(\hat{\theta}_{t-1})\widehat{Cov}(\hat{H}(\hat{\theta}_t)), \hat{H}(\hat{\theta_{t-1}}),$$

The four terms Taylor expansion for $(\hat{\theta}_t, \hat{\theta}_{t-1})$ around $(\theta_{0t}, \theta_{0t-1})$ is

$$\begin{aligned}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) = {} & \widehat{Cov}(\theta_{t0}, \theta_{0t-1}) + \frac{\partial}{\partial\hat{\theta}_t}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})|_{\hat{\theta}_t=\theta_{0t}}(\hat{\theta}_t - \theta_{0t}) \\ & + \frac{\partial}{\partial\hat{\theta}_{t-1}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})|_{\hat{\theta}_{t-1}=\theta_{0t-1}}(\hat{\theta}_{t-1} - \theta_{0t-1}) \\ & + \frac{1}{2}\frac{\partial^2}{\partial\hat{\theta}_t\partial\hat{\theta}_{t-1}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})|_{(\hat{\theta}_t, \hat{\theta}_{t-1})=(\hat{\theta}_{0t}, \hat{\theta}_{0t-1})}(\hat{\theta}_t - \theta_{0t})(\hat{\theta}_{t-1} - \theta_{0t-1}).\end{aligned}$$

Then

$$\begin{aligned}
Bias\left(\widehat{Cov}(\hat\theta_t,\hat\theta_{t-1})\right) &= E\left[\widehat{Cov}(\hat\theta_t,\hat\theta_{t-1}) - \widehat{Cov}(\theta_{t0},\theta_{0t-1})\right] \\
&= \frac{1}{2}D(\theta_{t0},\theta_{0t-1})E((\hat\theta_t-\theta_{0t})(\hat\theta_{t-1}-\theta_{0t-1}) \\
&= \frac{1}{2}D(\theta_{t0},\theta_{0t-1})Cov(\hat\theta_t,\hat\theta_{t-1}),
\end{aligned}$$

where $D(\theta_{t0},\theta_{0t-1}) = \frac{\partial^2}{\partial\hat\theta_t\partial\hat\theta_{t-1}}Cov(\hat\theta_t,\hat\theta_{t-1})|_{(\hat\theta_t,\hat\theta_{t-1})=(\hat\theta_{0t},\hat\theta_{0t-1})}$.

The bias-adjusted $\widehat{Cov}(\hat\theta_t,\hat\theta_{t-1})$ can be written as

$$\begin{aligned}
\widehat{Cov}_{bcT}(\hat\theta_t,\hat\theta_{t-1}) &= \widehat{Cov}(\hat\theta_t,\hat\theta_{t-1}) - \frac{1}{2}D(\hat\theta_t,\hat\theta_{t-1})Cov(\hat\theta_t,\hat\theta_{t-1}) \\
&= \widehat{Cov}(\hat\theta_t,\hat\theta_{t-1})\left[1 - D(\hat\theta_t,\hat\theta_{t-1})/2\right],
\end{aligned}$$

where

$$\begin{aligned}
D(\hat\theta_t,\hat\theta_{t-1}) &= \frac{\partial^2}{\partial\hat\theta_t\partial\hat\theta_{t-1}}\widehat{Cov}(\hat\theta_t,\hat\theta_{t-1}) \\
&= \frac{\partial^2}{\partial\hat\theta_t\partial\hat\theta_{t-1}}\left\{N^{-1}G^{-1}(\hat\theta_t)G^{-1}(\hat\theta_{t-1})\widehat{Cov}(\hat H(\hat\theta_t),\hat H(\hat{\theta_{t-1}}))\right\}.
\end{aligned}$$

Now the bias-adjusted variance of scalar $\hat\Delta_t = \hat\theta_t - \hat\theta_{t-1}$ can be written as

$$\begin{aligned}
Var_{bcT}(\hat\Delta_t) = \widehat{Var}(\hat\theta_t)\left[1 - D(\hat\theta_t)/2\right] - 2\widehat{Cov}(\hat\theta_t,\hat\theta_{t-1})\left[1 - D(\hat\theta_t,\hat\theta_{t-1})/2\right] \\
+ \widehat{Var}(\hat\theta_{t-1})\left[1 - D(\hat\theta_{t-1})/2\right].
\end{aligned} \tag{C.4}$$

Below we illustrate the bias-correction of $\widehat{Var}(\hat\Delta_t)$ with an example.

Example-1: In the case of estimation of the population change $\Delta_t = \theta_t - \theta_{t-1}$, from estimating equation (3.3), we can write

$$\hat H(\Delta_t) = \frac{1}{N}\sum_{i=1}^{N}\frac{r_{it}}{\hat p_i}(d_{it} - \Delta_t)$$

and from equations (A.6) and (A.7), we have

$$G(\hat\Delta_t) = -\frac{1}{N}\sum_{i=1}^{r}\hat g_i \quad \text{and} \quad V(\hat\Delta_t) = \frac{1}{N^2}\sum_{i=1}^{r}\hat v_i(d_{it} - \hat\Delta_t)^2,$$

From (B.1), the bias-adjusted variance of $\hat\Delta_t$ is

$$Var_{bcT}(\hat\Delta_t) = Var(\hat\Delta_t)\left[1 - D(\hat\Delta_t)/2\right], \tag{C.5}$$

where

$$Var(\hat{\Delta}_t) = \frac{\sum_{i=1}^{r} \hat{v}_i (d_{it} - \Delta_t)^2}{\left(\sum_{i=1}^{r} \hat{g}_i\right)^2}$$

and

$$D(\hat{\Delta}_t) = \frac{\partial^2}{\partial \Delta_t^2}|_{\Delta_t = \hat{\Delta}_t} Var(\hat{\Delta}_t) = \frac{2\sum_{i=1}^{r} \hat{v}_i}{\left(\sum_{i=1}^{r} \hat{g}_i\right)^2}.$$

Further bias-correction of (C.5) using bias-adjusting estimating equations can be obtained using $\hat{g}_i$ and $\hat{v}_i$.

For estimating equation (3.6), the bias-adjusted variance of $\Delta_t = \theta_t - \theta_{t-1}$, we have

$$\begin{aligned} Var_{bcT}(\hat{\Delta}_t) &= Var_{bcT}(\hat{\theta}_t - \hat{\theta}_{t-1}) \\ &= Var_{bcT}(\hat{\theta}_t) + Var_{bcT}(\hat{\theta}_{t-1}) - 2Cov_{bcT}(\hat{\theta}_t, \hat{\theta}_{t-1}) \end{aligned} \tag{C.6}$$

where

$$\begin{aligned} Var_{bcT}(\hat{\theta}_t) &= \frac{\sum_{i=1}^{r} \hat{v}_i (y_{it} - \hat{\theta}_t)^2}{\left(\sum_{i=1}^{r} \hat{q}_i\right)^2} \left[1 - \frac{\sum_{i=1}^{r} \hat{v}_i}{\left(\sum_{i=1}^{r} \hat{q}_i\right)^2}\right], \\ Var_{bcT}(\hat{\theta}_{t-1}) &= \frac{\sum_{i=1}^{r} \hat{v}_i (y_{i,t-1} - \hat{\theta}_{t-1})^2}{\left(\sum_{i=1}^{r} \hat{q}_i\right)^2} \left[1 - \frac{\sum_{i=1}^{r} \hat{v}_i}{\left(\sum_{i=1}^{r} \hat{q}_i\right)^2}\right], \\ Cov_{bcT}(\hat{\theta}_t, \hat{\theta}_{t-1}) &= \frac{\sum_{i=1}^{r} \hat{v}_i (y_{it} - \hat{\theta}_t)(y_{i,t-1} - \hat{\theta}_{t-1})}{\left(\sum_{i=1}^{r} \hat{q}_i\right)^2} \left[1 - \frac{\sum_{i=1}^{r} \hat{v}_i}{\left(\sum_{i=1}^{r} \hat{q}_i\right)^2}\right]. \end{aligned}$$

Further bias-correction of (C.6) using bias-adjusting estimating equations can be obtained using $\hat{q}_i$ and $\hat{v}_i$.

C.2 Bias-correction for vector $\widehat{Var}(\hat{\Delta}_t)$

C.2.1 Bias-correction using $\hat{H}(\Delta_t)$

For a p-dimensional vector $\hat{\Delta}_t$, the bias-adjusted variance covariance matrix using Taylor expansion can be written as

$$\begin{aligned} \widehat{Var}_{bc}(\hat{\Delta}_t) &= N^{-1} G^{-1}(\hat{\Delta}_t) \widehat{Var}[\hat{H}(\hat{\Delta}_t)] G^{-T}(\hat{\Delta}_t) - \frac{1}{2}\mathbf{H}(\hat{\Delta}_t) E\{(\hat{\Delta}_t - \hat{\Delta}_{0t})^{\otimes 2} \otimes I_p\} \\ \widehat{Var}_{bc}(\hat{\Delta}_t) &= \widehat{Var}(\hat{\Delta}_t) - \frac{1}{2}\mathbf{H}(\hat{\Delta}_t) E\{(\hat{\Delta}_t - \hat{\Delta}_{0t})^{\otimes 2} \otimes I_p\}, \end{aligned} \tag{C.7}$$

where $\mathbf{H}(\hat{\Delta}_t) = \partial^2 \widehat{Var}(\hat{\Delta}_t) / \partial \hat{\Delta}_t \partial \hat{\Delta}_t^T$. To find $\mathbf{H}(\hat{\Delta}_t)$, the first order partial derivative structure of $\widehat{Var}(\hat{\Delta}_t)$ with respect to $\hat{\Delta}_t$ of dimension p can be written as

$$\frac{\partial}{\partial \hat{\Delta}_t^T} \widehat{Var}(\hat{\Delta}_t) = \left[\tfrac{\partial}{\partial \hat{\Delta}_{t1}} \widehat{Var}(\hat{\Delta}_t) \;\vdots\; \tfrac{\partial}{\partial \hat{\Delta}_{t2}} \widehat{Var}(\hat{\Delta}_t) \;\vdots\; \cdots \;\vdots\; \tfrac{\partial}{\partial \hat{\Delta}_{tp}} \widehat{Var}(\hat{\Delta}_t) \right].$$

The second order partial derivatives of $\widehat{Var}(\hat{\Delta}_t)$ with respect to a 2-dimensional vector $\hat{\Delta}_t$ can be written as

$$\mathbf{H}(\hat{\Delta}_t) = \frac{\partial^2}{\partial\hat{\Delta}_t \partial\hat{\Delta}_t^T}\widehat{Var}(\hat{\Delta}_t)$$
$$= \left[\frac{\partial^2}{\partial\hat{\Delta}_{t1}^2}\widehat{Var}(\hat{\Delta}_t) \ \vdots \ \frac{\partial^2}{\partial\hat{\Delta}_{t1}\partial\hat{\Delta}_{t2}}\widehat{Var}(\hat{\Delta}_t) \ \vdots \ \frac{\partial^2}{\partial\hat{\Delta}_{t2}\partial\hat{\Delta}_{t1}}\widehat{Var}(\hat{\Delta}_t) \ \vdots \ \frac{\partial^2}{\partial\hat{\Delta}_{t2}^2}\widehat{Var}(\hat{\Delta}_t) \right]. \quad \text{(C.8)}$$

Similarly, it can be extended for p-dimensions. The above expression (C.7) for bias-adjusted variance estimator is in general form. The bias-adjusted variance estimator can be obtained from this expression using simple and bias-adjusting EEs.

C.2.2 Bias-correction using $\hat{H}(\theta_{t,t-1})$

Here we need to correct the bias of $\widehat{Var}(\hat{\Delta}_t) = \widehat{Var}(\hat{\theta}_t) + \widehat{Var}(\hat{\theta}_{t-1}) - \widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$. For estimating equations $\hat{H}(\theta_t)$ and $\hat{H}(\theta_{t-1})$, treating $\hat{\theta}_t$ and $\hat{\theta}_{t-1}$ as vectors, the bias-adjusted estimator using Taylor expansion for $\widehat{Var}(\hat{\theta}_t)$ and $\widehat{Var}(\hat{\theta}_{t-1})$ can be written from (B.4) respectively as

$$\widehat{Var}_{bc}(\hat{\theta}_t) = \widehat{Var}(\hat{\theta}_t) - \frac{1}{2}\mathbf{H}(\hat{\theta}_t)E\{(\hat{\theta}_t - \hat{\theta}_{0t})^{\otimes 2} \otimes I_p\}, \quad \text{(C.9)}$$

and

$$\widehat{Var}_{bc}(\hat{\theta}_{t-1}) = \widehat{Var}(\hat{\theta}_{t-1}) - \frac{1}{2}\mathbf{H}(\hat{\theta}_{t-1})E\{(\hat{\theta}_{t-1} - \hat{\theta}_{0t-1})^{\otimes 2} \otimes I_p\}, \quad \text{(C.10)}$$

where

$$\widehat{Var}(\hat{\theta}_t) = G^{-1}(\hat{\theta}_t)\widehat{Var}[\hat{H}(\hat{\theta}_t)]G^{-T}(\hat{\theta}_t)$$

and

$$\widehat{Var}(\hat{\theta}_{t-1}) = G^{-1}(\hat{\theta}_{t-1})\widehat{Var}[\hat{H}(\hat{\theta}_{t-1})]G^{-T}(\hat{\theta}_{t-1}).$$

To proceed for bias-adjusted $\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$, we can write the covariance estimator from (A.11) as

$$\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) = G^{-1}(\hat{\theta}_t)\widehat{Cov}\left(\hat{H}(\hat{\theta}_t), \hat{H}^T(\hat{\theta}_{t-1})\right)G^{-T}(\hat{\theta}_{t-1}) \quad \text{(C.11)}$$

The four terms Taylor expansion of $\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$ for $(\hat{\theta}_t, \hat{\theta}_{t-1})$ around $(\theta_{0t}, \theta_{0t-1})$ can be written as

$$\begin{aligned}\widehat{Cov}(\hat{\theta}_t,\hat{\theta}_{t-1}) &= \widehat{Cov}(\theta_{0t}, \theta_{0t-1}) + \frac{\partial}{\partial\hat{\theta}_t^T}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})|_{\hat{\theta}_t=\theta_{0t}}\{(\hat{\theta}_t - \theta_{0t}) \otimes I_p\} \\ &+ \frac{\partial}{\partial\hat{\theta}_{t-1}^T}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})|_{\hat{\theta}_{t-1}=\theta_{0t-1}}\{(\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_p\} \\ &+ \frac{1}{2}\frac{\partial}{\partial\hat{\theta}_t\hat{\theta}_{t-1}^T}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})|_{(\hat{\theta}_t,\hat{\theta}_{t-1})=(\theta_{0t},\theta_{0t-1})}\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_p\}\end{aligned}$$

where $\otimes$ denotes the Kronecker product. The bias of $\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$ can be written as

$$E\left[\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) - \widehat{Cov}(\theta_{0t}, \theta_{0t-1})\right] = \frac{1}{2}\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1})E\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_p\},$$

Then the bias-adjusted $\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$ is

$$\widehat{Cov}_{bc}(\hat{\theta}_t, \hat{\theta}_{t-1}) = \widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) - \frac{1}{2}\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1})E\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_p\} \tag{C.12}$$

where $\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1}) = \frac{\partial}{\partial\hat{\theta}_t\hat{\theta}_{t-1}^T}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})|_{(\hat{\theta}_t, \hat{\theta}_{t-1})=(\theta_{0t}, \theta_{0t-1})}$.

To find $\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1})$, the first order partial derivative structure of $\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$ with respect to $\hat{\theta}_t$ of dimension p can be written as

$$\frac{\partial}{\partial\hat{\theta}_{t-1}^T}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) = \begin{bmatrix} \frac{\partial}{\partial\hat{\theta}_{(t-1)1}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \\ \cdots \\ \frac{\partial}{\partial\hat{\theta}_{(t-1)2}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \\ \cdots \\ \vdots \\ \cdots \\ \frac{\partial}{\partial\hat{\theta}_{(t-1)p}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \end{bmatrix}^T.$$

Now the partial derivative $\frac{\partial}{\partial\hat{\theta}_{t-1}^T}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$ with respect to 2-dimensional vectors $\hat{\theta}_t$ and $\hat{\theta}_{t-1}$ can be written as

$$\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1}) = \frac{\partial^2}{\partial\hat{\theta}_t\partial\hat{\theta}_{t-1}^T}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) = \begin{bmatrix} \frac{\partial^2}{\partial\hat{\theta}_{t1}\partial\hat{\theta}_{(t-1)1}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \\ \cdots \\ \frac{\partial^2}{\partial\hat{\theta}_{t1}\partial\hat{\theta}_{(t-1)2}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \\ \cdots \\ \frac{\partial^2}{\partial\hat{\theta}_{t2}\partial\hat{\theta}_{(t-1)1}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \\ \cdots \\ \frac{\partial^2}{\partial\hat{\theta}_{t2}\partial\hat{\theta}_{(t-1)2}}\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \end{bmatrix}^T.$$

Similarly, it can be extended for p-dimensions.

Now the bias-adjusted variance of vector $\hat{\Delta}_t = \hat{\theta}_t - \hat{\theta}_{t-1}$ can be written as

$$\begin{aligned}\widehat{Var}_{bc}(\hat{\Delta}_t) &= \widehat{Var}_{bc}(\hat{\theta}_t - \hat{\theta}_{t-1}) = \widehat{Var}_{bc}(\hat{\theta}_t) - 2\widehat{Cov}_{bc}(\hat{\theta}_t, \hat{\theta}_{t-1}) + \widehat{Var}_{bc}(\hat{\theta}_{t-1}) \\ &= \widehat{Var}(\hat{\theta}_t) - \frac{1}{2}\mathbf{H}(\hat{\theta}_t)E\{(\hat{\theta}_t - \hat{\theta}_{0t})^{\otimes 2} \otimes I_p\} - 2\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) \\ &+ \frac{1}{2}\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1})E\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_p\} \\ &+ \widehat{Var}(\hat{\theta}_{t-1}) - \frac{1}{2}\mathbf{H}(\hat{\theta}_{t-1})E\{(\hat{\theta}_{t-1} - \hat{\theta}_{0t-1})^{\otimes 2} \otimes I_p\}\end{aligned}$$

$$\widehat{Var}_{bc}(\hat{\Delta}_t) = \widehat{Var}(\hat{\theta}_t - \hat{\theta}_{t-1}) - \frac{1}{2}\Big\{\mathbf{H}(\hat{\theta}_t)E\{(\hat{\theta}_t - \hat{\theta}_{0t})^{\otimes 2} \otimes I_p\}$$
$$-2\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1})E\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_p\} +$$
$$\mathbf{H}(\hat{\theta}_{t-1})E\{(\hat{\theta}_{t-1} - \hat{\theta}_{0t-1})^{\otimes 2} \otimes I_p\}\Big\}.$$

where $\widehat{Var}(\hat{\theta}_t - \hat{\theta}_{t-1})$ is given in (A.18). The following example illustrates the bias-correction of variance estimators using both types of equation for vector of estimators.

Example-2: To illustrate the bias-correction of variance estimator for vector of parameters. Suppose we want to estimate the finite population chage in mean and variance simultaneously, then using EEs (3.3), we can have

$$\hat{H}(\Delta_t) = \frac{1}{N}\sum_{i=1}^{N}\frac{r_{it}}{\hat{p}_i}\begin{bmatrix}(d_{it} - \Delta_{t1})\\(d_{it} - (\Delta_{t1})^2 - \Delta_{t2}\end{bmatrix}$$

where $\Delta_{t1} = (\theta_t - \theta_{t-1})$ and $\Delta_{t2} = (\sigma_t^2 - \sigma_{t-1}^2)$. From (B.4), the bias-adjusted $Var(\hat{\Delta})$ can be written as

$$\widehat{Var}_{bc}(\hat{\Delta}_t) = \widehat{Var}(\hat{\Delta}_t) - \frac{1}{2}\mathbf{H}(\hat{\Delta}_t)E\{(\hat{\Delta}_t - \Delta_{0t})^{\otimes 2} \otimes I_2\}, \tag{C.13}$$

First to find the estimated variance covariance matrix $\widehat{Var}(\hat{\Delta}_t)$, we can have

$$G(\hat{\Delta}_t) = -\frac{1}{N}\sum_{i=1}^{r}\hat{g}_i\begin{bmatrix}1 & 0\\2(d_{it} - \hat{\Delta}_{t1}) & 1\end{bmatrix} \approx -\frac{1}{N}\sum_{i=1}^{r}\hat{g}_i\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},$$

since $\frac{1}{N}\sum_{i=1}^{r}\hat{g}_i(d_{it} - \hat{\Delta}_{t1})$ is almost 0. Then

$$G^{-1}(\hat{\Delta}_t) \approx -\frac{1}{N}\begin{bmatrix}1/\sum_{i=1}^{r}\hat{g}_i & 0\\0 & 1/\sum_{i=1}^{r}\hat{g}_i\end{bmatrix}$$

and

$$\widehat{Var}[\hat{H}(\hat{\Delta}_t)] = \frac{1}{N^2}\begin{bmatrix}\sum_{i=1}^{r}\hat{v}_iE_{it}^2 & \sum_{i=1}^{r}\hat{v}_iE_{it}\left(E_{it}^2 - \hat{\Delta}_{t2}^2\right)\\\sum_{i=1}^{r}\hat{v}_iE_{it}\left(E_{it}^2 - \hat{\Delta}_{t2}^2\right) & \sum_{i=1}^{r}\hat{v}_i\left(E_{it}^2 - \hat{\Delta}_{t2}^2\right)^2\end{bmatrix}.$$

where $\hat{g}_i$ and $\hat{v}_i$ are given in (2.39) and (2.40) respectively for simple NEE, and $E_{it} = (d_{it} - \hat{\Delta}_{t1})$. Now we can write

$$\widehat{Var}(\hat{\Delta}_t) = N^{-1}G^{-1}(\hat{\Delta}_t)\widehat{Var}[\hat{H}(\hat{\Delta}_t)]G^{-T}(\hat{\Delta}_t) = \begin{bmatrix}v_{ij}\end{bmatrix}_{2\times 2} = \hat{V}$$

with

$$v_{11} = \frac{\sum_{i=1}^r \hat{v}_i E_{it}^2}{\left(\sum_{i=1}^r \hat{g}_i\right)^2}, \ v_{22} = \frac{\sum_{i=1}^r \hat{v}_i \left(E_{it}^2 - \hat{\Delta}_{t2}^2\right)^2}{\left(\sum_{i=1}^r \hat{g}_i\right)^2}$$

$$v_{12} = v_{21} = \frac{\sum_{i=1}^r \hat{v}_i E_{it} \left(E_{it}^2 - \hat{\Delta}_{t2}^2\right)}{\left(\sum_{i=1}^r \hat{g}_i\right)^2}.$$

Then we can write

$$E\{(\hat{\Delta}_t - \Delta_{0t})^{\otimes 2} \otimes I_2\} = \left[\begin{array}{ccccccc} v_{11}I_2 & \vdots & v_{12}I_2 & \vdots & v_{21}I_2 & \vdots & v_{22}I_2 \end{array} \right]^T.$$

And

$$\mathbf{H}(\hat{\Delta}_t) = \left[\begin{array}{ccccccc} \frac{\partial^2}{\partial \hat{\Delta} t1^2}\hat{V} & \vdots & \frac{\partial}{\partial \hat{\Delta} t1 \partial \hat{\Delta} t2}\hat{V} & \vdots & \frac{\partial}{\partial \hat{\Delta} t2 \partial \hat{\Delta} t1}\hat{V} & \vdots & \frac{\partial^2}{\partial \hat{\Delta} t2^2}\hat{V} \end{array} \right],$$

where

$$\frac{\partial^2}{(\partial \hat{\Delta}_{t1})^2}(v_{11}) = 2\frac{\sum_{i=1}^r \hat{v}_i}{\left(\sum_{i=1}^r \hat{g}_i\right)^2}, \ \frac{\partial^2}{(\partial \hat{\Delta}_{t1})^2}(v_{12}) = 6\frac{\sum_{i=1}^r \hat{v}_i E_{it}}{\left(\sum_{i=1}^r \hat{g}_i\right)^2},$$

$$\frac{\partial^2}{(\partial \hat{\Delta}_{t1})^2}(v_{22}) = 4\frac{\sum_{i=1}^r \left(\hat{v}_i \left(E_{it}^2 - \hat{\Delta}_{t2}^2\right) + 2\hat{v}_i E_{it}^2\right)}{\left(\sum_{i=1}^r \hat{g}_i\right)^2},$$

$$\frac{\partial^2}{\partial \hat{\Delta}_{t2} \partial \hat{\Delta}_{t1}}(v_{11}) = 0, \ \frac{\partial^2}{\partial \hat{\Delta}_{t2} \partial \hat{\Delta}_{t1}}(v_{12}) = \frac{\sum_{i=1}^r \hat{v}_i}{\left(\sum_{i=1}^r \hat{g}_i\right)^2},$$

$$\frac{\partial^2}{\partial \hat{\Delta}_{t2} \partial \hat{\Delta}_{t1}}(v_{22}) = 4\frac{\sum_{i=1}^r \hat{v}_i E_{it}}{\left(\sum_{i=1}^r \hat{g}_i\right)^2}, \ \frac{\partial^2}{(\partial \hat{\Delta}_{t2}^2)^2}(v_{11}) = 0,$$

$$\frac{\partial^2}{(\partial \hat{\Delta}_{t2}^2)^2}(v_{12}) = 0 \text{ and } \frac{\partial^2}{(\partial \hat{\Delta}_{t2}^2)^2}(v_{22}) = 2\frac{\sum_{i=1}^r \hat{v}_i}{\left(\sum_{i=1}^r \hat{g}_i\right)^2}.$$

The bias-adjusted variance covariance matrix of vector $\hat{\Delta}_t^*$ for the bias-adjusted estimating equations can be obtained from (C.13) using the respective g_i and v_i.

To estimate the finite population change in the mean and variance using EEs (3.6), first we have

$$\widehat{Var}_{bc}(\hat{\theta}_t) = \widehat{Var}(\hat{\theta}_t) - \frac{1}{2}\mathbf{H}(\hat{\theta}_t)E\{(\hat{\theta}_t - \theta_{0t})^{\otimes 2} \otimes I_2\}. \tag{C.14}$$

Now we can write

$$\widehat{Var}(\hat{\theta}_t) = N^{-1}G^{-1}(\hat{\theta}_t)\widehat{Var}[\hat{H}(\hat{\theta}_t)]G^{-T}(\hat{\theta}_t) = \left[v_{ij}\right]_{2\times 2} = \hat{V}$$

with

$$
\begin{aligned}
v_{11} &= \frac{\sum_{i=1}^{r} \hat{v}_i (y_{it} - \hat{\theta}_t)^2}{\left(\sum_{i=1}^{r} \hat{g}_i\right)^2} \\
v_{12} = v_{21} &= \frac{\sum_{i=1}^{r} \hat{v}_i (y_{it} - \hat{\theta}_t) \left((y_{it} - \hat{\theta}_t)^2 - \widehat{(\sigma_t^2)^2}\right)}{\left(\sum_{i=1}^{r} \hat{g}_i\right)^2} \\
v_{22} &= \frac{\sum_{i=1}^{r} \hat{v}_i \left((y_{it} - \hat{\theta}_t)^2 - \widehat{(\sigma_t^2)^2}\right)^2}{\left(\sum_{i=1}^{r} \hat{g}_i\right)^2}.
\end{aligned}
$$

Then we can write

$$
E\{(\hat{\theta}_t - \theta_{0t})^{\otimes 2} \otimes I_2\} = \left[\begin{array}{ccccccc} v_{11} I_2 & \vdots & v_{12} I_2 & \vdots & v_{21} I_2 & \vdots & v_{22} I_2 \end{array} \right]^T
$$

and

$$
\mathbf{H}(\hat{\theta}_t) = \left[\begin{array}{ccccccc} \frac{\partial^2}{\partial \hat{\theta}_{t1}^2} \hat{V} & \vdots & \frac{\partial}{\partial \hat{\theta}_{t1} \partial \hat{\theta}_{t2}} \hat{V} & \vdots & \frac{\partial}{\partial \hat{\theta}_{t2} \partial \hat{\theta}_{t1}} \hat{V} & \vdots & \frac{\partial^2}{\partial \hat{\theta}_{t2}^2} \hat{V} \end{array} \right],
$$

where the elements of $\mathbf{H}(\hat{\theta}_t)$ can be written from the elements of $\mathbf{H}(\hat{\Delta}_t)$ after replacing d_{it} with y_{it}, Δ_t with θ_t, and using the respective $\hat{g}_i$ and $\hat{v}_i$. The bias-adjusted variance covariance matrix of vector $\hat{\theta}_t^*$ for the bias-adjusted estimating equations can be obtained from (C.13) using the respective g_i and v_i.

Now we have

$$
\widehat{Var}_{bc}(\hat{\theta}_{t-1}) = \widehat{Var}(\hat{\theta}_{t-1}) - \frac{1}{2} \mathbf{H}(\hat{\theta}_{t-1}) E\{(\hat{\theta}_{t-1} - \theta_{0t-1})^{\otimes 2} \otimes I_2\}. \tag{C.15}
$$

The expressions for $\widehat{Var}(\hat{\theta}_{t-1})$, $\mathbf{H}(\hat{\theta}_{t-1})$ and $E\{(\hat{\theta}_{t-1} - \theta_{0t-1})^{\otimes 2} \otimes I_2\}$ can be obtained by replacing θ_t with θ_{t-1} in $\widehat{Var}(\hat{\theta}_t)$, $\mathbf{H}(\hat{\theta}_t)$ and $E\{(\hat{\theta}_t - \theta_{0t})^{\otimes 2} \otimes I_2\}$.

The bias-adjusted variance covariance matrix of vector $\hat{\theta}_{t-1}^*$ for bias-adjusted estimating equations can be obtained from (C.15) using respective g_i and v_i.

For $\widehat{Cov}_{bc}(\hat{\theta}_t, \hat{\theta}_{t-1})$, we have

$$
\widehat{Cov}_{bc}(\hat{\theta}_t, \hat{\theta}_{t-1}) = \widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) - \frac{1}{2} \mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1}) E\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_2\}, \tag{C.16}
$$

Now we can write

$$
\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) = N^{-1} G^{-1}(\hat{\theta}_t) \widehat{Cov}[\hat{H}(\hat{\theta}_t), \hat{H}^T(\hat{\theta}_{t-1})] G^{-T}(\hat{\theta}_{t-1}) = \left[c_{ij} \right]_{2 \times 2} = C
$$

with

$$c_{11} = \frac{\sum_{i=1}^{r} \hat{v}_i (y_{it} - \hat{\theta}_t)(y_{i,t-1} - \hat{\theta}_{t-1})}{S_{\hat{g}_1} S_{\hat{g}_2}}$$

$$c_{12} = \frac{\sum_{i=1}^{r} \hat{v}_i (y_{it} - \hat{\theta}_t) \left((y_{i,t-1} - \hat{\theta}_{t-1}) - \hat{\sigma}^2_{t-1} \right)}{S_{\hat{g}_1} S_{\hat{g}_2}}$$

$$c_{21} = \frac{\sum_{i=1}^{N} \hat{v}_i \left((y_{i,t} - \hat{\theta}_t)^2 - \hat{\sigma}^2_t \right) (y_{i,t-1} - \hat{\theta}_{t-1})}{S_{\hat{g}_1} S_{\hat{g}_2}}$$

$$c_{22} = \frac{\sum_{i=1}^{r} \hat{v}_i \left((y_{it} - \hat{\theta}_t)^2 - \hat{\sigma}^2_t \right) \left((y_{i,t-1} - \hat{\theta}_{t-1}) - \hat{\sigma}^2_{t-1} \right)}{S_{\hat{g}_1} S_{\hat{g}_2}},$$

where $S_{\hat{g}_1} S_{\hat{g}_2} = \sum_{i=1}^{r} \hat{g}_{1i} \sum_{i=1}^{r} \hat{g}_{2i}$ and $\hat{v}_i = \widehat{Cov}(\delta_{it}/\hat{\pi}_{i,t}, \delta_{i,t-1}/\hat{\pi}_{i,t-1})$. Then we can write

$$E\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_2\} = \left[\begin{array}{ccccccc} c_{11} I_2 & \vdots & c_{12} I_2 & \vdots & c_{21} I_2 & \vdots & c_{22} I_2 \end{array} \right]^T$$

and

$$\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1}) = \left[\begin{array}{ccccccc} \frac{\partial^2}{\partial \hat{\theta}_t \partial \theta_{(t-1)}} C & \vdots & \frac{\partial}{\partial \hat{\theta}_t \partial \hat{\sigma}^2_{t-1}} C & \vdots & \frac{\partial}{\partial \hat{\sigma}^2_t \partial \hat{\theta}_{t-1}} C & \vdots & \frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\sigma}^2_{t-1}} C \end{array} \right],$$

where

$$\begin{aligned}
&\frac{\partial^2}{\partial \hat{\theta}_t \partial \hat{\theta}_{t-1}} (c_{11}) = \frac{\sum_{i=1}^{r} \hat{v}_i}{S_{\hat{g}_1} S_{\hat{g}_2}}, \ \frac{\partial^2}{\partial \hat{\theta}_t \partial \hat{\theta}_{t-1}} (c_{12}) = 2 \frac{\sum_{i=1}^{r} \hat{v}_i (y_{i,t-1} - \hat{\theta}_{t-1})}{S_{\hat{g}_1} S_{\hat{g}_2}}, \\
&\frac{\partial^2}{\partial \hat{\theta}_t \partial \hat{\theta}_{t-1}} (c_{21}) = 2 \frac{\sum_{i=1}^{r} \hat{v}_i (y_{it} - \hat{\theta}_t)}{S_{\hat{g}_1} S_{\hat{g}_2}}, \\
&\frac{\partial^2}{\partial \hat{\theta}_t \partial \hat{\theta}_{t-1}} (c_{22}) = 4 \frac{\sum_{i=1}^{r} \hat{v}_i (y_{i,t-1} - \hat{\theta}_{t-1})(y_{it} - \hat{\theta}_t)}{S_{\hat{g}_1} S_{\hat{g}_2}}, \ \frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\theta}_{t-1}} (c_{11}) = 0, \\
&\frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\theta}_{t-1}} (c_{12}) = 0, \ \frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\theta}_{t-1}} (c_{21}) = \frac{\sum_{i=1}^{r} \hat{v}_i}{S_{\hat{g}_1} S_{\hat{g}_2}}, \\
&\frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\theta}_{t-1}} (c_{22}) = 2 \frac{\sum_{i=1}^{r} \hat{v}_i (y_{i,t-1} - \hat{\theta}_{t-1})}{S_{\hat{g}_1} S_{\hat{g}_2}}, \ \frac{\partial^2}{\partial \hat{\sigma}^2_{t-1} \partial \hat{\theta}_t} (c_{11}) = 0, \\
&\frac{\partial^2}{\partial \hat{\sigma}^2_{t-1} \partial \hat{\theta}_t} (c_{12}) = \frac{\sum_{i=1}^{r} \hat{v}_i}{S_{\hat{g}_1} S_{\hat{g}_2}}, \ \frac{\partial^2}{\partial \hat{\sigma}^2_{t-1} \partial \hat{\theta}_t} (c_{21}) = 0, \\
&\frac{\partial^2}{\partial \hat{\sigma}^2_{t-1} \partial \hat{\theta}_t} (c_{22}) = 2 \frac{\sum_{i=1}^{r} \hat{v}_i (y_{it} - \hat{\theta}_t)}{S_{\hat{g}_1} S_{\hat{g}_2}}, \ \frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\sigma}^2_{t-1}} (c_{11}) = 0, \\
&\frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\sigma}^2_{t-1}} (c_{12}) = 0, \frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\sigma}^2_{t-1}} (c_{21}) = 0 \text{ and } \frac{\partial^2}{\partial \hat{\sigma}^2_t \partial \hat{\sigma}^2_{t-1}} (c_{22}) = \frac{\sum_{i=1}^{r} \hat{v}_i}{S_{\hat{g}_1} S_{\hat{g}_2}}.
\end{aligned}$$

The bias-adjusted covariance matrix of vector $\hat{\theta}^*_t$ and $\hat{\theta}^*_{t-1}$ for the bias-adjusted estimating equations can be obtained from (C.16) using the respective $\hat{g}_i$ and $\hat{v}_i$.

Example-3: To illustrate the bias-correction of the variance estimator for vector of regression coefficients. Suppose we want to estimate the change in population regression coefficients for simple linear regression model with $\epsilon_i \sim N(0, d_{\sigma_t^2} x_i^{\alpha})$, then using (3.3), we can write

$$\hat{H}(\Delta_t) = \frac{1}{N} \sum_{i=1}^{N} \frac{r_{it}}{\hat{p}_i} \begin{bmatrix} D_{it}/x_i^{\alpha} \\ D_{it}x_i/x_i^{\alpha} \\ D_{it}^2/x_i^{\alpha} - d_{\sigma_{it}^2} \end{bmatrix}$$

where $d_{\sigma_{it}^2} = (\sigma_t^2 - \sigma_{(t-1)}^2)$ and $D_{it} = d_{it} - (\beta_{0t} - \beta_{0t-1}) - (\beta_{1t} - \beta_{1(t-1)})x_i = d_{it} - d_{\beta_{0t}} - d_{\beta_{1t}} x_i$. From (B.4), the bias-corrected $\widehat{Var}(\hat{\Delta}_t)$ can be written as

$$\widehat{Var}_{bc}(\hat{\Delta}_t) = \widehat{Var}(\hat{\Delta}_t) - \frac{1}{2}\mathbf{H}(\hat{\Delta}_t)\hat{E}\{(\hat{\Delta}_t - \Delta_{0t})^{\otimes 2} \otimes I_3\}. \tag{C.17}$$

First, to find the estimated variance covariance matrix $\widehat{Var}_{bc}(\hat{\Delta}_t)$, we have

$$G(\hat{\Delta}_t) = -\frac{1}{N} \sum_{i=1}^{r} \tilde{g}_i \begin{bmatrix} 1 & x_i & 0 \\ x_i & x_i^2 & 0 \\ 2\hat{D}_{it} & 2x_i\hat{D}_{it} & x_i^{\alpha} \end{bmatrix} \approx -\frac{1}{N} \sum_{i=1}^{r} \tilde{g}_i \begin{bmatrix} 1 & x_i & 0 \\ x_i & x_i^2 & 0 \\ 0 & 0 & x_i^{\alpha} \end{bmatrix},$$

where $\frac{1}{N}\sum_{i=1}^{r} \tilde{g}_i D_{it}$ and $\frac{1}{N}\sum_{i=1}^{r} \tilde{g}_i x_i D_{it}$ are almost 0 and $\tilde{g}_i = \hat{g}_i / x_i^{\alpha}$.

$$G^{-1}(\hat{\theta}_t) \approx -\frac{1}{dN} \begin{bmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix},$$

where

$$\begin{aligned} d &= \sum_{i=1}^{r} \tilde{g}_i x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_i x_i^2 \sum_{i=1}^{r} \tilde{g}_i - \sum_{i=1}^{r} \tilde{g}_i x_i^{\alpha} \left(\sum_{i=1}^{r} \tilde{g}_i x_i \right)^2, \\ g_{11} &= \sum_{i=1}^{r} \tilde{g}_i x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_i x_i^2, \\ g_{12} = g_{21} &= -\sum_{i=1}^{r} \tilde{g}_i x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_i x_i, \\ g_{22} &= \sum_{i=1}^{r} \tilde{g}_i x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_i, \\ g_{33} &= \sum_{i=1}^{r} \tilde{g}_i x_i^2 \sum_{i=1}^{r} \tilde{g}_i - \left(\sum_{i=1}^{r} \tilde{g}_i x_i \right)^2. \end{aligned}$$

Now

$$\widehat{Var}(\hat{\Delta}_t) = \frac{1}{N^2} \begin{bmatrix} S_{wD^2} & S_{wxD^2} & S_{wDD_\alpha} \\ S_{wxD^2} & S_{wx^2D^2} & S_{wxDD_\alpha} \\ S_{wDD_\alpha} & S_{wxDD_\alpha} & S_{wD_\alpha^2} \end{bmatrix},$$

where $S_{wxD^2} = \sum_{i=1}^{r} \tilde{v}_i x_i D_{it}^2$, $S_{wx^2D^2} = \sum_{i=1}^{r} \tilde{v}_i x_i^2 D_{it}^2$, $S_{wD^2} = \sum_{i=1}^{r} \tilde{v}_i D_{it}^2$, $S_{wDD_\alpha} = \sum_{i=1}^{r} \tilde{v}_i \hat{D}_{it}\hat{D}_\alpha$, $S_{wxDD_\alpha} = \sum_{i=1}^{r} \tilde{v}_i x_i \hat{D}_{it}\hat{D}_\alpha$ and $S_{wD_\alpha^2} = \sum_{i=1}^{r} \tilde{v}_i \hat{D}_\alpha^2$, and $\tilde{v}_i = \hat{v}_i / x_i^{2\alpha}$ and $\hat{D}_\alpha = (\hat{D}_{it}^2 - d_{\hat{\sigma}_{it}^2} x_i^{\alpha})$.

Now we can write

$$\widehat{Var}(\hat{\Delta}_t) = G^{-1}(\hat{\Delta}_t)\widehat{Var}[\hat{H}(\hat{\Delta}_t)]G^{-T}(\hat{\Delta}_t) = \Big[v_{ij}\Big]_{3\times 3} = \hat{V}$$

with

$$\begin{aligned}
v_{11} &= g_{12}\left(g_{11}S_{wxD^2} + g_{12}S_{wx^2D^2}\right) + g_{11}\left(g_{11}S_{wD^2} + g_{12}S_{wxD^2}\right)\\
v_{12} &= g_{21}\left(g_{11}S_{wD^2} + g_{12}S_{wxD^2}\right) + g_{22}\left(g_{11}S_{wxD^2} + g_{12}S_{wx^2D^2}\right)\\
v_{13} &= g_{33}\left(g_{11}S_{wDD_\alpha} + g_{12}S_{wxDD_\alpha}\right)
\end{aligned}$$

$$\begin{aligned}
v_{22} &= g_{22}\left(g_{21}S_{wxD^2} + g_{22}S_{wx^2D^2}\right) + g_{21}\left(g_{21}S_{wD^2} + g_{22}S_{wxD^2}\right)\\
v_{23} &= g_{33}\left(g_{21}S_{wDD_\alpha} + g_{22}S_{wxDD_\alpha}\right)\\
v_{33} &= g_{33}^2 S_{wD_\alpha^2},
\end{aligned}$$

Further,

$$\hat{E}\{(\hat{\Delta}_t - \Delta_{0t})^{\otimes 2} \otimes I_3\} = \begin{bmatrix} v_{11}I_3 & \vdots & v_{12}I_3 & \vdots & v_{13}I_3 & \vdots \\ v_{21}I_3 & \vdots & v_{22}I_3 & \vdots & v_{23}I_3 & \vdots \\ v_{31}I_3 & \vdots & v_{32}I_3 & \vdots & v_{33}I_3 & \end{bmatrix}^T.$$

Now

$$\mathbf{H}(\hat{\Delta}_t) = \begin{bmatrix} \frac{\partial^2}{\partial d^2_{\hat{\beta}_{0t}}}\hat{V} & \vdots & \frac{\partial^2}{\partial d_{\hat{\beta}_{1t}}\partial d_{\hat{\beta}_{0t}}}\hat{V} & \vdots & \frac{\partial^2}{\partial d_{\hat{\sigma}_t^2}\partial d_{\hat{\beta}_{0t}}}\hat{V} & \vdots \\ \frac{\partial^2}{\partial d^2_{\hat{\beta}_{1t}}}\hat{V} & \vdots & \frac{\partial^2}{\partial d_{\hat{\beta}_{0t}}\partial d_{\hat{\beta}_{1t}}}\hat{V} & \vdots & \frac{\partial^2}{\partial d_{\hat{\sigma}_t^2}\partial d_{\hat{\beta}_{1t}}}\hat{V} & \vdots \\ \frac{\partial^2}{\partial (d_{\hat{\sigma}_t^2})^2}\hat{V} & \vdots & \frac{\partial^2}{\partial d_{\hat{\beta}_{0t}}\partial (d_{\hat{\sigma}_t^2})^2}\hat{V} & \vdots & \frac{\partial^2}{\partial d_{\hat{\beta}_{1t}}\partial (d_{\hat{\sigma}_t^2})^2}\hat{V} & \end{bmatrix}.$$

Each element can be obtained as

$$\begin{aligned}
\frac{\partial^2}{(\partial d_{\hat{\beta}_{0t}})^2}(v_{11}) &= 2g_{12}\left(g_{11}S_{wx} + g_{12}S_{wx^2}\right) + 2g_{11}\left(g_{11}S_w + g_{12}S_{wx}\right)\\
\frac{\partial^2}{(\partial d_{\hat{\beta}_{0t}})^2}(v_{12}) &= 2g_{21}\left(g_{11}S_w + g_{12}S_{wx}\right) + 2g_{22}\left(g_{11}S_{wx} + g_{12}S_{wx^2}\right)\\
\frac{\partial^2}{(\partial d_{\hat{\beta}_{0t}})^2}(v_{13}) &= 6g_{33}\left(g_{11}S_{wD} + g_{12}S_{wxD}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2}{(\partial d_{\hat{\beta}_{0t}})^2}(v_{22}) &= 2g_{22}\left(g_{21}S_{wx} + g_{22}S_{wx^2}\right) + 2g_{21}\left(g_{21}2S_w + g_{22}S_{wx}\right)\\
\frac{\partial^2}{(\partial d_{\hat{\beta}_{0t}})^2}(v_{23}) &= 6g_{33}\left(g_{21}S_{wD} + g_{22}S_{wxD}\right)\\
\frac{\partial^2}{(\partial d_{\hat{\beta}_{0t}})^2}(v_{33}) &= 4g_{33}^2\left(S_{wD_\alpha} + 2S_{wD^2}\right)
\end{aligned}$$

$$\frac{\partial^2}{\partial d_{\hat{\beta}_{1t}} \partial d_{\hat{\beta}_{0t}}}(v_{11}) = 2g_{11}\left(g_{11}S_{wx} + g_{12}S_{wx^2}\right) + 2g_{12}\left(g_{11}S_{wx^2} + g_{12}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial d_{\hat{\beta}_{1t}} \partial d_{\hat{\beta}_{0t}}}(v_{12}) = 2g_{21}\left(g_{11}S_{wx} + g_{12}S_{wx^2}\right) + 2g_{22}\left(g_{11}S_{wx^2} + g_{12}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial d_{\hat{\beta}_{1t}} \partial d_{\hat{\beta}_{0t}}}(v_{13}) = 6g_{33}\left(g_{11}S_{wxD} + g_{12}S_{wx^2D}\right)$$

$$\frac{\partial^2}{\partial d_{\hat{\beta}_{1t}} \partial d_{\hat{\beta}_{0t}}}(v_{22}) = 2g_{21}\left(g_{21}S_{wx} + g_{22}S_{wx^2}\right) + 2g_{22}\left(g_{21}S_{wx^2} + g_{22}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial d_{\hat{\beta}_{1t}} \partial d_{\hat{\beta}_{0t}}}(v_{23}) = 6g_{33}\left(g_{21}S_{wxD} + g_{22}S_{wx^2D}\right)$$
$$\frac{\partial^2}{\partial d_{\hat{\beta}_{1t}} \partial d_{\hat{\beta}_{0t}}}(v_{33}) = 4g_{33}^2\left(S_{wxD_\alpha} + 2S_{wxD^2}\right)$$

$$\frac{\partial^2}{\partial d_{\hat{\sigma}_{it}^2} \partial d_{\hat{\beta}_{0t}}}(v_{11}) = \frac{\partial^2}{\partial d_{\hat{\sigma}_{it}^2} \partial d_{\hat{\beta}_{0t}}}(v_{22}) = \frac{\partial^2}{\partial d_{\hat{\sigma}_{it}^2} \partial d_{\hat{\beta}_{0t}}}(v_{12}) = 0,$$
$$\frac{\partial^2}{\partial d_{\hat{\sigma}_{it}^2} \partial d_{\hat{\beta}_{0t}}}(v_{33}) = 4g_{33}^2 S_{wx^\alpha D},\ \ \frac{\partial^2}{\partial d_{\hat{\sigma}_{it}^2} \partial d_{\hat{\beta}_{0t}}}(v_{23}) = g_{33}\left(g_{21}S_{wx^\alpha} + g_{22}S_{wxx^\alpha}\right)$$
$$\frac{\partial^2}{\partial d_{\hat{\sigma}_{it}^2} \partial d_{\hat{\beta}_{0t}}}(v_{13}) = g_{33}\left(g_{11}S_{wx^\alpha} + g_{12}S_{wxx^\alpha}\right)$$

$$\frac{\partial^2}{(\partial d_{\hat{\beta}_{1t}})^2}(v_{11}) = 2g_{12}\left(g_{11}S_{wx^3} + g_{12}S_{wx^4}\right) + 2g_{11}\left(g_{11}S_{wx^2} + g_{12}S_{wx^3}\right)$$
$$\frac{\partial^2}{(\partial d_{\hat{\beta}_{1t}})^2}(v_{12}) = 2g_{21}\left(g_{11}S_{wx^2} + g_{12}S_{wx^3}\right) + 2g_{22}\left(g_{11}S_{wx^3} + g_{12}S_{wx^4}\right)$$
$$\frac{\partial^2}{(\partial d_{\hat{\beta}_{1t}})^2}(v_{13}) = 6g_{33}\left(g_{11}S_{wx^2D} + g_{12}S_{wx^3D}\right)$$
$$\frac{\partial^2}{(\partial d_{\hat{\beta}_{1t}})^2}(v_{22}) = 2g_{22}\left(g_{21}S_{wx^3} + g_{22}S_{wx^4}\right) + 2g_{21}\left(g_{21}S_{wx^2} + g_{22}S_{wx^3}\right)$$
$$\frac{\partial^2}{(\partial d_{\hat{\beta}_{1t}})^2}(v_{23}) = 6g_{33}\left(g_{21}S_{wx^2D} + g_{22}S_{wx^3D}\right)$$
$$\frac{\partial^2}{(\partial d_{\hat{\beta}_{1t}})^2}(v_{33}) = 4g_{33}^2\left(S_{wx^2D_\alpha} + 2S_{wx^2D}\right)$$

$$\frac{\partial^2}{\partial d_{\hat{\sigma}^2_{it}}\partial d_{\hat{\beta}_{1t}}}(v_{11}) = 0, \quad \frac{\partial^2}{\partial d_{\hat{\sigma}^2_{it}}\partial d_{\hat{\beta}_{1t}}}(v_{12}) = 0,$$
$$\frac{\partial^2}{\partial d_{\hat{\sigma}^2_{it}}\partial d_{\hat{\beta}_{1t}}}(v_{13}) = g_{33}\left(g_{11}S_{wxx^\alpha} + g_{12}S_{wx^2x^\alpha}\right),$$
$$\frac{\partial^2}{\partial d_{\hat{\sigma}^2_{it}}\partial d_{\hat{\beta}_{1t}}}(v_{22}) = 0, \quad \frac{\partial^2}{\partial d_{\hat{\sigma}^2_{it}}\partial d_{\hat{\beta}_{1t}}}(v_{23}) = g_{33}\left(g_{21}S_{wxx^\alpha} + g_{22}S_{wx^2x^\alpha}\right),$$
$$\frac{\partial^2}{\partial d_{\hat{\sigma}^2_{it}}\partial d_{\hat{\beta}_{1t}}}(v_{33}) = 4g_{33}^2 S_{wxx^\alpha D},$$

$$\frac{\partial^2}{(\partial d_{\hat{\sigma}^2_{it}})^2}(v_{11}) = 0, \quad \frac{\partial^2}{(\partial d_{\hat{\sigma}^2_{it}})^2}(v_{12}) = 0, \quad \frac{\partial^2}{(\partial d_{\hat{\sigma}^2_{it}})^2}(v_{13}) = 0, \quad \frac{\partial^2}{(\partial d_{\hat{\sigma}^2_{it}})^2}(v_{22}) = 0,$$
$$\frac{\partial^2}{(\partial d_{\hat{\sigma}^2_{it}})^2}(v_{23}) = 0 \text{ and } \frac{\partial^2}{(\partial d_{\hat{\sigma}^2_{it}})^2}(v_{33}) = 2g_{33}^2 S_{wx^{2\alpha}}.$$

The bias-corrected variance covariance matrix $\hat{V}ar_{bc}(\hat{\Delta}^*_t)$ for the bias-adjusted estimating equations can be obtained by using the respective $\hat{g}_i$ and $\hat{v}_i$. To find the bias-corrected estimator of the variance for $\hat{\Delta}_t = \hat{\theta}_t - \hat{\theta}_{t-1}$ using EEs (3.6) and Taylor expansion. We need to find

$$\widehat{Var}_{bc}(\hat{\theta}_t - \hat{\theta}_{t-1}) = \widehat{Var}_{bc}(\hat{\theta}_t) - 2\widehat{Cov}_{bc}(\hat{\theta}_t, \hat{\theta}_{t-1}) + \widehat{Var}_{bc}(\hat{\theta}_{t-1}).$$

Suppose θ_t denote the regression parameters for time t and $\epsilon_{it} \sim N(0, \sigma_t^2 x_i^\alpha)$ then we can write

$$\hat{H}(\theta_t) = \frac{1}{N}\sum_{i=1}^{N}\frac{\delta_{it}}{\hat{\pi}_i}\begin{bmatrix}(y_{it} - \beta_{0t} - \beta_{1t}x_i)/x_i^\alpha \\ x_i(y_{it} - \beta_{0t} - \beta_{1t}x_i)/x_i^\alpha \\ (y_{it} - \beta_0 - \beta_1 x_i)^2/x_i^\alpha - \sigma_t^2\end{bmatrix}.$$

From (B.4), the bias-corrected $\widehat{Var}(\hat{\theta}_t)$ can be written as

$$\widehat{Var}_{bc}(\hat{\theta}_t) = \widehat{Var}(\hat{\theta}_t) - \frac{1}{2}\mathbf{H}(\hat{\theta}_t)\hat{E}\{(\hat{\theta}_t - \theta_{0t})^{\otimes 2} \otimes I_3\}, \tag{C.18}$$

First, to find the estimated variance covariance matrix $\widehat{Var}(\hat{\theta}_t)$, we have

$$G(\hat{\theta}_t) = -\frac{1}{N}\sum_{i=1}^{r}\tilde{g}_i\begin{bmatrix}1 & x_i & 0 \\ x_i & x_i^2 & 0 \\ 2(y_{it} - \beta_{0t} - \beta_{1t}x_i) & 2x_i(y_{it} - \beta_{0t} - \beta_{1t}x_i) & x_i^\alpha\end{bmatrix}$$
$$\approx -\frac{1}{N}\sum_{i=1}^{r}\tilde{g}_i\begin{bmatrix}1 & x_i & 0 \\ x_i & x_i^2 & 0 \\ 0 & 0 & x_i^\alpha\end{bmatrix},$$

where $\frac{1}{N}\sum_{i=1}^{r}\tilde{g}_i(y_{it}-\beta_{0t}-\beta_{1t}x_i)$ and $\frac{1}{N}\sum_{i=1}^{r}\tilde{g}_i x_i(y_{it}-\beta_{0t}-\beta_{1t}x_i)$ are almost 0 and $\tilde{g}_i=\hat{g}_i/x_i^{\alpha}$.

$$G^{-1}(\hat{\theta}_t) \approx -\frac{1}{dN}\begin{bmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix},$$

where

$$\begin{aligned}
d &= \sum_{i=1}^{r}\tilde{g}_i x_i^{\alpha}\sum_{i=1}^{r}\tilde{g}_i x_i^2\sum_{i=1}^{r}\tilde{g}_i - \sum_{i=1}^{r}\tilde{g}_i x_i^{\alpha}\left(\sum_{i=1}^{r}\tilde{g}_i x_i\right)^2,\\
g_{11} &= \sum_{i=1}^{r}\tilde{g}_i x_i^{\alpha}\sum_{i=1}^{r}\tilde{g}_i x_i^2,\\
g_{12} = g_{21} &= -\sum_{i=1}^{r}\tilde{g}_i x_i^{\alpha}\sum_{i=1}^{r}\tilde{g}_i x_i,\\
g_{22} &= \sum_{i=1}^{r}\tilde{g}_i x_i^{\alpha}\sum_{i=1}^{r}\tilde{g}_i,\\
g_{33} &= \sum_{i=1}^{r}\tilde{g}_i x_i^2\sum_{i=1}^{r}\tilde{g}_i - \left(\sum_{i=1}^{r}\tilde{g}_i x_i\right)^2.
\end{aligned}$$

Now

$$\widehat{Var}[\hat{H}(\hat{\theta}_t)] = \frac{1}{N^2}\begin{bmatrix} S_{we^2} & S_{wxe^2} & S_{wee_\alpha} \\ S_{wxe^2} & S_{wx^2e^2} & S_{wxee_\alpha} \\ S_{wee_\alpha} & S_{wxee_\alpha} & S_{we_\alpha^2} \end{bmatrix},$$

where $S_{wxe^2}=\sum_{i=1}^{r}\tilde{v}_i x_i e_{it}^2$, $S_{wx^2e^2}=\sum_{i=1}^{r}\tilde{v}_i x_i^2 e_{it}^2$, $S_{we^2}=\sum_{i=1}^{r}\tilde{v}_i e_{it}^2$, $S_{wee_\alpha}=\sum_{i=1}^{r}\tilde{v}_i\hat{e}_{it}\hat{e}_\alpha$, $S_{wxee_\alpha}=\sum_{i=1}^{r}\tilde{v}_i x_i\hat{e}_{it}\hat{e}_\alpha$, $S_{we_\alpha^2}=\sum_{i=1}^{r}\tilde{v}_i\hat{e}_\alpha^2$, and $\tilde{v}_i=\hat{v}_i/x_i^{2\alpha}$ and $\hat{e}_\alpha=(\hat{e}_{it}^2-\hat{\sigma}_{it}^2 x_i^{\alpha})$ with $\hat{e}_{it}=(y_{it}-\hat{\beta}_{0t}-\hat{\beta}_{1t}x_i)$ and $\tilde{v}_i=\hat{v}_i/x_i^{2\alpha}$.

Now we can write

$$\widehat{Var}(\hat{\theta}_t) = N^{-1}G^{-1}(\hat{\theta}_t)\widehat{Var}[\hat{H}(\hat{\theta}_t)]G^{-T}(\hat{\theta}_t) = \left[v_{ij}\right]_{3\times 3} = \hat{V}$$

with

$$\begin{aligned}
v_{11} &= g_{12}\left(g_{11}S_{wxe^2}+g_{12}S_{wx^2e^2}\right)+g_{11}\left(g_{11}S_{we^2}+g_{12}S_{wxe^2}\right)\\
v_{12} &= g_{21}\left(g_{11}S_{we^2}+g_{12}S_{wxe^2}\right)+g_{22}\left(g_{11}S_{wxe^2}+g_{12}S_{wx^2e^2}\right)\\
v_{13} &= g_{33}\left(g_{11}S_{wee_\alpha}+g_{12}S_{wxee_\alpha}\right)
\end{aligned}$$

$$\begin{aligned}
v_{22} &= g_{22}\left(g_{21}S_{wxe^2}+g_{22}S_{wx^2e^2}\right)+g_{21}\left(g_{21}S_{we^2}+g_{22}S_{wxe^2}\right)\\
v_{23} &= g_{33}\left(g_{21}S_{wee_\alpha}+g_{22}S_{wxee_\alpha}\right)\\
v_{33} &= g_{33}^2 S_{we_\alpha^2}.
\end{aligned}$$

Further,

$$\hat{E}\{(\hat{\theta}_t - \theta_{0t})^{\otimes 2} \otimes I_3\} = \begin{bmatrix} v_{11}I_3 & \vdots & v_{12}I_3 & \vdots & v_{13}I_3 & \vdots \\ v_{21}I_3 & \vdots & v_{22}I_3 & \vdots & v_{23}I_3 & \vdots \\ v_{31}I_3 & \vdots & v_{32}I_3 & \vdots & v_{33}I_3 & \end{bmatrix}^T.$$

Now

$$\mathbf{H}(\hat{\theta}_t) = \begin{bmatrix} \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0t-1}}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0t}}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{0t}}\hat{V} & \vdots \\ \frac{\partial^2}{\partial\hat{\beta}_{1t}^2}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1t}}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{1t}}\hat{V} & \vdots \\ \frac{\partial^2}{\partial(\hat{\sigma}_t^2)^2}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial(\hat{\sigma}_t^2)^2}\hat{V} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_{1t}\partial(\hat{\sigma}_t^2)^2}\hat{V} & \end{bmatrix}.$$

Each element can be obtained as

$$\frac{\partial^2}{(\partial\hat{\beta}_{0t})^2}(v_{11}) = 2g_{12}\left(g_{11}S_{wx} + g_{12}S_{wx^2}\right) + 2g_{11}\left(g_{11}S_w + g_{12}S_{wx}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{0t})^2}(v_{12}) = 2g_{21}\left(g_{11}S_w + g_{12}S_{wx}\right) + 2g_{22}\left(g_{11}S_{wx} + g_{12}S_{wx^2}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{0t})^2}(v_{13}) = 6g_{33}\left(g_{11}S_{we} + g_{12}S_{wxe}\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_{0t})^2}(v_{22}) = 2g_{22}\left(g_{21}S_{wx} + g_{22}S_{wx^2}\right) + 2g_{21}\left(g_{21}S_w + g_{22}S_{wx}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{0t})^2}(v_{23}) = 6g_{33}\left(g_{21}S_{we} + g_{22}S_{wxe}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{0t})^2}(v_{33}) = 4g_{33}^2(S_{we_\alpha} + 2S_{we^2})$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0t}}(v_{11}) = 2g_{11}\left(g_{11}S_{wx} + g_{12}S_{wx^2}\right) + 2g_{12}\left(g_{11}S_{wx^2} + g_{12}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0t}}(v_{12}) = 2g_{21}\left(g_{11}S_{wx} + g_{12}S_{wx^2}\right) + 2g_{22}\left(g_{11}S_{wx^2} + g_{12}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0t}}(v_{13}) = 6g_{33}\left(g_{11}S_{wxe} + g_{12}S_{wx^2e}\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0t}}(v_{22}) = 2g_{21}\left(g_{21}S_{wx} + g_{22}S_{wx^2}\right) + 2g_{22}\left(g_{21}S_{wx^2} + g_{22}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0t}}(v_{23}) = 6g_{33}\left(g_{21}S_{wxe} + g_{22}S_{wx^2e}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0t}}(v_{33}) = 4g_{33}^2(S_{wxe_\alpha} + 2S_{wxe^2})$$

$$\frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{0t}}(v_{11}) = \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{0t}}(v_{22}) = \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{0t}}(v_{12}) = 0, \quad \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{0t}}(v_{33}) = 4g_{33}^2 S_{wx^\alpha e},$$
$$\frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{0t}}(v_{23}) = g_{33}\left(g_{21}S_{wx^\alpha} + g_{22}S_{wxx^\alpha}\right), \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{0t}}(v_{13}) = g_{33}\left(g_{11}S_{wx^\alpha} + g_{12}S_{wxx^\alpha}\right)$$

$$\frac{\partial^2}{(\partial\hat{\beta}_{1t})^2}(v_{11}) = 2g_{12}\left(g_{11}S_{wx^3} + g_{12}S_{wx4}\right) + 2g_{11}\left(g_{11}S_{wx^2} + g_{12}S_{wx^3}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{1t})^2}(v_{12}) = 2g_{21}\left(g_{11}S_{wx^2} + g_{12}S_{wx^3}\right) + 2g_{22}\left(g_{11}S_{wx^3} + g_{12}S_{wx4}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{1t})^2}(v_{13}) = 6g_{33}\left(g_{11}S_{wx^2e} + g_{12}S_{wx^3e}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{1t})^2}(v_{22}) = 2g_{22}\left(g_{21}S_{wx^3} + g_{22}S_{wx^4}\right) + 2g_{21}\left(g_{21}S_{wx^2} + g_{22}S_{wx^3}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{1t})^2}(v_{23}) = 6g_{33}\left(g_{21}S_{wx^2e} + g_{22}S_{wx^3e}\right)$$
$$\frac{\partial^2}{(\partial\hat{\beta}_{1t})^2}(v_{33}) = 4g_{33}^2(S_{wx^2e^\alpha} + 2S_{wx^2e^2})$$

$$\frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{1t}}(v_{11}) = 0, \quad \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{1t}}(v_{12}) = 0, \quad \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{1t}}(v_{13}) = g_{33}\left(g_{11}S_{wxx^\alpha} + g_{12}S_{wx^2x^\alpha}\right),$$
$$\frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{1t}}(v_{22}) = 0, \quad \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{1t}}(v_{23}) = g_{33}\left(g_{21}S_{wxx^\alpha} + g_{22}S_{wx^2x^\alpha}\right),$$
$$\frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\beta}_{1t}}(v_{33}) = 4g_{33}^2 S_{wxx^\alpha e}$$

$$\frac{\partial^2}{(\partial\hat{\sigma}_t^2)^2}(v_{11}) = 0, \quad \frac{\partial^2}{(\partial\hat{\sigma}_t^2)^2}(v_{12}) = 0, \quad \frac{\partial^2}{(\partial\hat{\sigma}_t^2)^2}(v_{13}) = 0, \quad \frac{\partial^2}{(\partial\hat{\sigma}_t^2)^2}(v_{22}) = 0,$$
$$\frac{\partial^2}{(\partial\hat{\sigma}_t^2)^2}(v_{23}) = 0 \text{ and } \frac{\partial^2}{(\partial\hat{\sigma}_t^2)^2}(v_{33}) = 2g_{33}^2 S_{wx^{2\alpha}}.$$

The bias-corrected variance covariance matrix $\hat{V}ar_{bc}(\hat{\theta}_t^*)$ for the bias-adjusted estimating equations can be obtained by using the respective $\hat{g}_i$ and $\hat{v}_i$.

For $\widehat{Var}_{bc}(\hat{\theta}_{t-1})$, we have

$$\widehat{Var}_{bc}(\hat{\theta}_{t-1}) = \widehat{Var}(\hat{\theta}_{t-1}) - \frac{1}{2}\mathbf{H}(\hat{\theta}_{t-1})\hat{E}\{(\hat{\theta}_{t-1} - \theta_{0t-1})^{\otimes 2} \otimes I_3\}, \tag{C.19}$$

The expressions for $\widehat{Var}(\hat{\theta}_{t-1})$, $\mathbf{H}(\hat{\theta}_{t-1})$ and $\hat{E}\{(\hat{\theta}_{t-1} - \theta_{0t-1})^{\otimes 2} \otimes I_3\}$ can be obtained by replacing θ_t with θ_{t-1} in $\widehat{Var}(\hat{\theta}_t)$, $\mathbf{H}(\hat{\theta}_t)$ and $\hat{E}\{(\hat{\theta}_t - \theta_{0t})^{\otimes 2} \otimes I_3\}$ that are discussed above for the case of θ_t where θ_t denote the regression parameters at time t. The bias-corrected variance covariance matrix $\hat{V}ar_{bc}(\hat{\theta}_{t-1}^*)$ for the bias-adjusted estimating equations can be obtained by using the respective $\hat{g}_i$ and $\hat{v}_i$.

The bias-corrected $\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$ can be written as

$$\widehat{Cov}_{bc}(\hat{\theta}_t, \hat{\theta}_{t-1}) = \widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) - \frac{1}{2}\mathbf{H}(\hat{\theta}_t, \hat{\theta}_{t-1})\hat{E}\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_3\}, \quad \text{(C.20)}$$

First, to find $\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1})$, we can have

$$G(\hat{\theta}_t) \approx -\frac{1}{N}\sum_{i=1}^{r} \tilde{g}_{1i} \begin{bmatrix} 1 & x_i & 0 \\ x_i & x_i^2 & 0 \\ 0 & 0 & x_i^{\alpha} \end{bmatrix}, \text{ and } G(\hat{\theta}_{t-1}) \approx -\frac{1}{N}\sum_{i=1}^{r} \tilde{g}_{2i} \begin{bmatrix} 1 & x_i & 0 \\ x_i & x_i^2 & 0 \\ 0 & 0 & x_i^{\alpha} \end{bmatrix},$$

where $\tilde{g}_{1i} = \hat{g}_{1i}/x_i^{\alpha}$ and $\tilde{g}_{2i} = \hat{g}_{2i}/x_i^{\alpha}$.

$$G^{-1}(\hat{\theta}_t) \approx -\frac{1}{d_1 N} \begin{bmatrix} g_{111} & g_{112} & 0 \\ g_{121} & g_{122} & 0 \\ 0 & 0 & g_{133} \end{bmatrix},$$

where

$$d = \sum_{i=1}^{r} \tilde{g}_{1i} x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_{1i} x_i^2 \sum_{i=1}^{r} \tilde{g}_{1i} - \sum_{i=1}^{r} \tilde{g}_{1i} x_i^{\alpha} \left(\sum_{i=1}^{r} \tilde{g}_{1i} x_i\right)^2,$$

$$g_{111} = \sum_{i=1}^{r} \tilde{g}_{1i} x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_{1i} x_i^2,$$

$$g_{112} = g_{121} = -\sum_{i=1}^{r} \tilde{g}_{1i} x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_{1i} x_i,$$

$$g_{122} = \sum_{i=1}^{r} \tilde{g}_{1i} x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_{1i},$$

$$g_{133} = \sum_{i=1}^{r} \tilde{g}_{1i} x_i^2 \sum_{i=1}^{r} \tilde{g}_{1i} - \left(\sum_{i=1}^{r} \tilde{g}_{1i} x_i\right)^2.$$

and

$$G^{-1}(\hat{\theta}_{t-1}) \approx -\frac{1}{d_2 N} \begin{bmatrix} g_{211} & g_{212} & 0 \\ g_{221} & g_{222} & 0 \\ 0 & 0 & g_{233} \end{bmatrix},$$

where

$$
\begin{aligned}
d_2 &= \sum_{i=1}^{r} \tilde{g}_{2i} x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_{2i} x_i^{2} \sum_{i=1}^{r} \tilde{g}_{2i} - \sum_{i=1}^{r} \tilde{g}_{2i} x_i^{\alpha} \left(\sum_{i=1}^{r} \tilde{g}_{2i} x_i \right)^2, \\
g_{211} &= \sum_{i=1}^{r} \tilde{g}_{2i} x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_{2i} x_i^{2}, \\
g_{212} &= g_{121} = -\sum_{i=1}^{r} \tilde{g}_{2i} x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_{2i} x_i, \\
g_{222} &= \sum_{i=1}^{r} \tilde{g}_{2i} x_i^{\alpha} \sum_{i=1}^{r} \tilde{g}_{2i}, \\
g_{233} &= \sum_{i=1}^{r} \tilde{g}_{2i} x_i^{2} \sum_{i=1}^{r} \tilde{g}_{2i} - \left(\sum_{i=1}^{r} \tilde{g}_{2i} x_i \right)^2.
\end{aligned}
$$

Now

$$
\widehat{Cov}[\hat{H}(\hat{\theta}_t), \hat{H}^T(\hat{\theta}_{t-1})] = \frac{1}{N^2} \begin{bmatrix} S_{we_t e_{t-1}} & S_{wxe_t e_{t-1}} & S_{we_t e_{\alpha(t-1)}} \\ S_{wxe_t e_{t-1}} & S_{wx^2 e_t e_{t-1}} & S_{wxe_t e_{\alpha(t-1)}} \\ S_{we_{t-1} e_{\alpha t}} & S_{wxe_{t-1} e_{\alpha t}} & S_{we_{\alpha t} e_{\alpha(t-1)}} \end{bmatrix},
$$

where

$$
\begin{aligned}
S_{we_t e_{t-1}} &= \sum_{i=1}^{r} \tilde{v}_i e_{it} e_{i,t-1}, \ S_{wxe_t e_{t-1}} = \sum_{i=1}^{r} \tilde{v}_i x_i e_{it} e_{i,t-1}, \\
S_{we_t e_{\alpha(t-1)}} &= \sum_{i=1}^{r} \tilde{v}_i e_{it} (\hat{e}_{i,t-1}^2 - \hat{\sigma}_{t-1}^2 x_i^{\alpha}), \ S_{wx^2 e_t e_{t-1}} = \sum_{i=1}^{r} \tilde{v}_i x_i^2 e_{it} e_{i,t-1}, \\
S_{wxe_t e_{\alpha(t-1)}} &= \sum_{i=1}^{r} \tilde{v}_i x_i e_{it} (\hat{e}_{i,t-1}^2 - \hat{\sigma}_{t-1}^2 x_i^{\alpha}), \ S_{we_{t-1} e_{\alpha t}} = \sum_{i=1}^{r} \tilde{v}_i \hat{e}_{i,t-1} (\hat{e}_{it}^2 - \hat{\sigma}_t^2 x_i^{\alpha}), \\
S_{wxe_{t-1} e_{\alpha t}} &= \sum_{i=1}^{r} \tilde{v}_i x_i \hat{e}_{i,t-1} (\hat{e}_{it}^2 - \hat{\sigma}_t^2 x_i^{\alpha}) \\
S_{we_{\alpha t} e_{\alpha(t-1)}} &= \sum_{i=1}^{r} \tilde{v}_i (\hat{e}_{it}^2 - \hat{\sigma}_t^2 x_i^{\alpha})(\hat{e}_{i,t-1}^2 - \hat{\sigma}_{t-1}^2 x_i^{\alpha}).
\end{aligned}
$$

And $\hat{e}_{it} = (y_{it} - \hat{\beta}_{0t} - \hat{\beta}_{1t} x_i)$, $\hat{e}_{i,t-1} = (y_{i,t-1} - \hat{\beta}_{0t-1} - \hat{\beta}_{1t-1} x_i)$ and $\tilde{v}_i = \hat{v}_i / x_i^{2\alpha}$ with $\hat{v}_i = \widehat{Cov}(\delta_{it}/\hat{\pi}_{i,t}, \delta_{i,t-1}/\hat{\pi}_{i,t-1})$.

Now we can write

$$
\widehat{Cov}(\hat{\theta}_t, \hat{\theta}_{t-1}) = N^{-1} G^{-1}(\hat{\theta}_t) \widehat{Cov}[\hat{H}(\hat{\theta}_t), \hat{H}^T(\hat{\theta}_{t-1}) G^{-T}(\hat{\theta}_{t-1}) = \Big[c_{ij} \Big]_{3\times 3} = \hat{C}
$$

with

$$
\begin{aligned}
c_{11} &= g_{212}\left(g_{111}S_{we_te_{t-1}} + g_{112}S_{wx^2e_te_{t-1}}\right) + g_{211}\left(g_{111}S_{we_te_{t-1}} + g_{112}S_{we_te_{t-1}}\right)\\
c_{12} &= g_{221}\left(g_{111}S_{we_te_{t-1}} + g_{112}S_{we_te_{t-1}}\right) + g_{222}\left(g_{111}S_{we_te_{t-1}} + g_{112}S_{wx^2e_te_{t-1}}\right)\\
c_{13} &= g_{233}\left(g_{111}S_{we_te_{\alpha(t-1)}} + g_{112}S_{wxe_te_{\alpha(t-1)}}\right)\\
c_{21} &= g_{211}\left(g_{121}S_{we_te_{t-1}} + g_{122}S_{we_te_{t-1}}\right) + g_{212}\left(g_{121}S_{we_te_{t-1}} + g_{122}S_{wx^2e_te_{t-1}}\right)\\
c_{22} &= g_{222}\left(g_{121}S_{we_te_{t-1}} + g_{122}S_{wx^2e_te_{t-1}}\right) + g_{221}\left(g_{121}S_{we_te_{t-1}} + g_{122}S_{we_te_{t-1}}\right)\\
c_{23} &= g_{233}\left(g_{121}S_{we_te_{\alpha(t-1)}} + g_{122}S_{wxe_te_{\alpha(t-1)}}\right)\\
c_{31} &= g_{133}\left(g_{211}S_{we_{t-1}e_{\alpha t}} + g_{212}S_{wxe_{t-1}e_{\alpha t}}\right)\\
c_{32} &= g_{133}\left(g_{221}S_{we_{t-1}e_{\alpha t}} + g_{222}S_{wxe_{t-1}e_{\alpha t}}\right)\\
c_{33} &= g_{133}g_{233}S_{we_{\alpha t}e_{\alpha(t-1)}}.
\end{aligned}
$$

Further,

$$
\hat{E}\{(\hat{\theta}_t - \theta_{0t}) \otimes (\hat{\theta}_{t-1} - \theta_{0t-1}) \otimes I_3\} = \begin{bmatrix} c_{11}I_3 & \vdots & c_{12}I_3 & \vdots & c_{13}I_3 & \vdots \\ c_{21}I_3 & \vdots & c_{22}I_3 & \vdots & c_{23}I_3 & \vdots \\ c_{31}I_3 & \vdots & c_{32}I_3 & \vdots & c_{33}I_3 & \end{bmatrix}^T.
$$

Now

$$
\mathbf{H}(\hat{\theta}_t) = \begin{bmatrix} \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}\hat{C} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}\hat{C} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}\hat{C} & \vdots \\ \frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}\hat{C} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}\hat{C} & \vdots & \frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}\hat{C} & \vdots \\ \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}\hat{C} & \vdots & \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}\hat{C} & \vdots & \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\sigma}^2_{t-1}}\hat{C} & \end{bmatrix},
$$

Each element can be obtained as

$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{11}) = g_{212}\left(g_{111}S_{wx} + g_{112}S_{wx^2}\right) + g_{211}\left(g_{111}S_w + g_{112}S_{wx}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{12}) = g_{221}\left(g_{111}S_w + g_{112}S_{wx}\right) + g_{222}\left(g_{111}S_{wx} + g_{112}S_{wx^2}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{13}) = 2g_{233}\left(g_{111}S_{we_{t-1}} + g_{112}S_{wxe_{t-1}}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{21}) = g_{211}\left(g_{121}S_w + g_{122}S_{wx}\right) + g_{212}\left(g_{121}S_{wx} + g_{122}S_{wx^2}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{22}) = g_{221}\left(g_{121}S_{wx} + g_{122}S_{wx^2}\right) + g_{222}\left(2g_{121}S_w + g_{122}S_{wx}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{23}) = 2g_{233}\left(g_{121}S_{we_{t-1}} + g_{122}S_{wxe_{t-1}}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{31}) = 2g_{133}\left(g_{211}S_{we_t} + g_{212}S_{wxe_t}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{32}) = 2g_{133}\left(g_{221}S_{we_t} + g_{222}S_{wxe_t}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{0(t-1)}}(c_{33}) = 4g_{133}g_{233}S_{we_te_{t-1}}$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{11}) = g_{211}\left(g_{111}S_{wx} + g_{112}S_{wx^2}\right) + g_{212}\left(g_{111}S_{wx^2} + g_{112}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{12}) = g_{221}\left(g_{111}S_{wx} + g_{112}S_{wx^2}\right) + g_{222}\left(g_{111}S_{wx^2} + g_{112}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{13}) = 2g_{233}\left(g_{111}S_{wxe_{t-1}} + g_{112}S_{wx^2e_{t-1}}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{21}) = g_{211}\left(g_{121}S_{wx} + g_{122}S_{wx^2}\right) + g_{212}\left(g_{121}S_{wx^2} + g_{122}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{22}) = g_{221}\left(g_{121}S_{wx} + g_{122}S_{wx^2}\right) + g_{222}\left(g_{121}S_{wx^2} + g_{122}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{23}) = 2g_{233}\left(g_{121}S_{wxe_{t-1}} + g_{122}S_{wx^2e_{t-1}}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{31}) = 2g_{133}\left(g_{211}S_{wxe_t} + g_{212}S_{wx^2e_t}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{32}) = 2g_{133}\left(g_{221}S_{wxe_t} + g_{222}S_{wx^2e_t}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\beta}_{1(t-1)}}(c_{33}) = 4g_{133}g_{233}S_{wxe_te_{t-1}}$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{11}) = \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{12}) = 0$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{13}) = g_{233}\left(g_{111}S_{wx^\alpha} + g_{112}S_{wxx^\alpha}\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{21}) = \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{22}) = 0$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{23}) = g_{233}\left(g_{121}S_{wx^\alpha} + g_{122}S_{wxx^\alpha}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{31}) = \frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{32}) = 0$$
$$\frac{\partial^2}{\partial\hat{\beta}_{0t}\partial\hat{\sigma}^2_{t-1}}(c_{33}) = 2g_{133}g_{233}S_{wx^\alpha e_{t-1}}$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{11}) = g_{211}\left(g_{111}S_{wx} + g_{112}S_{wx^2}\right) + g_{212}\left(g_{111}S_{wx^2} + g_{112}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{12}) = g_{221}\left(g_{111}S_{wx} + g_{112}S_{wx^2}\right) + g_{222}\left(g_{111}S_{wx^2} + g_{112}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{13}) = 2g_{233}\left(g_{111}S_{wxe_{t-1}} + g_{112}S_{wx^2e_{t-1}}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{21}) = g_{211}\left(g_{121}S_{wx} + g_{122}S_{wx^2}\right) + g_{212}\left(g_{121}S_{wx^2} + g_{122}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{22}) = g_{221}\left(g_{121}S_{wx} + g_{122}S_{wx^2}\right) + g_{222}\left(g_{121}S_{wx^2} + g_{122}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{23}) = 2g_{233}\left(g_{121}S_{wxe_{t-1}} + g_{122}S_{wx^2e_{t-1}}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{31}) = 2g_{133}\left(g_{211}S_{wxe_t} + g_{212}S_{wx^2e_t}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{32}) = 2g_{133}\left(g_{221}S_{wxe_t} + g_{222}S_{wx^2e_t}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{0(t-1)}}(c_{33}) = 4g_{133}g_{233}S_{wxe_te_{t-1}}$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{11}) = g_{211}\left(g_{111}S_{wx^3} + g_{112}S_{wx^4}\right) + g_{212}\left(g_{111}S_{wx^2} + g_{112}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{12}) = g_{221}\left(g_{111}S_{wx^2} + g_{112}S_{wx^3}\right) + g_{222}\left(g_{111}S_{wx^3} + g_{112}S_{wx^4}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{13}) = 2g_{233}\left(g_{111}S_{wx^2e_{t-1}} + g_{112}S_{wx^3e_{t-1}}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{21}) = g_{211}\left(g_{121}S_{wx^2} + g_{122}S_{wx^3}\right) + g_{212}\left(g_{121}S_{wx^3} + g_{122}S_{wx^4}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{22}) = g_{221}\left(g_{121}S_{wx^3} + 2g_{122}S_{wx^4}\right) + g_{222}\left(g_{121}S_{wx^2} + g_{122}S_{wx^3}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{23}) = 2g_{233}\left(g_{121}S_{wx^2e_{t-1}} + g_{122}S_{wx^3e_{t-1}}\right)$$
$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{31}) = 2g_{133}\left(g_{211}S_{wx^2e_t} + g_{212}S_{wx^3e_t}\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{32}) = 2g_{133}\left(g_{221}S_{wx^2e_t} + g_{222}S_{wx^3e_t}\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\beta}_{1(t-1)}}(c_{33}) = 4g_{133}g_{233}S_{wx^2e_te_{t-1}}$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{11}) = \frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{12}) = 0$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{13}) = g_{233}\left(g_{111}S_{wxx^\alpha} + g_{112}S_{wx^2x^\alpha}\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{21}) = \frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{22}) = 0$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{23}) = g_{233}\left(g_{121}S_{wxx^\alpha} + g_{122}S_{wx^2x^\alpha}\right)$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{31}) = \frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{32}) = 0$$

$$\frac{\partial^2}{\partial\hat{\beta}_{1t}\partial\hat{\sigma}^2_{t-1}}(c_{33}) = 2g_{133}g_{233}S_{wxx^\alpha e_{t-1}}$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{11}) = \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{12}) = \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{13}) = 0$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{21}) = \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{22}) = \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{23}) = 0$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{31}) = g_{133}\left(g_{211}S_{wx^\alpha} + g_{212}S_{wxx^\alpha}\right)$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{32}) = g_{133}\left(g_{221}S_{wx^\alpha} + g_{222}S_{wxx^\alpha}\right)$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{0(t-1)}}(c_{33}) = 2g_{233}g_{133}S_{wx^\alpha e_t}$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{11}) = \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{12}) = \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{13}) = 0$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{21}) = \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{22}) = \frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{23}) = 0$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{31}) = g_{133}\left(g_{211}S_{wxx^\alpha} + g_{212}S_{wx^2x^\alpha}\right)$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{32}) = g_{133}\left(g_{221}S_{wxx^\alpha} + g_{222}S_{wx^2x^\alpha}\right)$$

$$\frac{\partial^2}{\partial\hat{\sigma}^2_t\partial\hat{\beta}_{1(t-1)}}(c_{33}) = 2g_{233}g_{133}S_{wxx^\alpha e_t}$$

$$\frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{11}) = \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{12}) = \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{13}) = 0$$
$$\frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{21}) = \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{22}) = \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{23}) = 0$$
$$\frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{31}) = \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{32}) = 0 \frac{\partial^2}{\partial\hat{\sigma}_t^2\partial\hat{\sigma}_{t-1}^2}(c_{33}) = g_{133}g_{233}S_{wx^{2\alpha}}$$

The bias-corrected covariance matrix $\widehat{Cov}_{bc}(\hat{\theta}_t^*, \hat{\theta}_{t-1}^*)$ for the bias-adjusted estimating equations can be obtained by using the respective $\hat{g}_i$ and $\hat{v}_i$.

Appendix D

Results for Cross-sectional Setting

D.1 Estimation of Mean

Table D.1: Results under model (2.41), by response and correlation. Population: stable

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\theta_3 = 11747.05$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.78	0.02	4.39	38.02	0.05	4.42	15.57	0.00	1.37	20.56	0.00	2.92
SE	121.46	100.65	137.78	81.09	62.23	83.35	38.87	41.29	51.83	43.44	44.44	54.26
ERSE	540.52	97.68	202.21	549.47	63.73	120.87	411.25	40.49	79.66	409.00	43.33	84.53
ERSE_ba	540.50	97.68	202.20	549.44	63.73	120.86	411.23	40.49	79.66	408.99	43.33	84.52
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	3.87	8.39	1.63	11.07	11.41	6.84	11.34	1.72	8.27	16.22	0.90	12.09
SE	154.70	132.60	149.59	85.67	81.59	85.07	62.04	47.45	59.16	61.51	49.68	59.92
ERSE	127.71	149.62	133.17	68.06	90.65	72.86	50.09	53.16	51.56	49.23	55.75	51.80
ERSE_ba	127.71	149.61	133.17	68.06	90.64	72.86	50.09	53.16	51.56	49.23	55.75	51.80
	T=4, $\theta_4 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	18.88	0.00	0.66	35.99	0.00	3.28	14.89	0.00	3.70	20.06	0.01	5.42
SE	107.27	89.51	117.62	73.40	57.41	69.92	33.70	35.97	47.57	39.43	40.33	50.34
ERSE	436.25	89.35	169.40	445.28	58.16	106.06	334.10	36.98	75.81	331.84	39.89	82.45
ERSE_ba	436.23	89.35	169.38	445.26	58.16	106.05	334.09	36.98	75.81	331.82	39.89	82.45
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	8.15	4.48	5.97	16.91	5.26	12.87	12.42	0.19	9.45	16.79	1.26	12.85
SE	133.88	111.16	128.81	73.26	66.57	72.57	58.79	41.22	55.52	60.26	43.66	57.73
ERSE	109.64	128.20	114.59	60.17	79.40	64.70	48.10	48.56	49.38	49.48	51.87	51.60
ERSE_ba	109.64	128.20	114.58	60.17	79.39	64.70	48.10	48.56	49.37	49.48	51.87	51.60
	T=7, $\theta_7 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	18.87	0.03	4.09	35.88	0.01	7.33	14.86	0.01	3.75	20.05	0.00	4.82
SE	107.97	89.74	104.57	76.21	59.32	69.53	33.98	36.17	46.70	38.59	40.09	49.92
ERSE	436.21	89.70	143.57	445.35	58.32	102.19	334.18	37.05	69.48	331.98	39.86	75.34
ERSE_ba	436.18	89.70	143.56	445.33	58.32	102.18	334.17	37.05	69.48	331.97	39.86	75.34
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	10.06	0.63	7.93	17.19	2.30	13.55	8.97	2.20	6.74	11.30	3.16	8.45
SE	116.49	99.82	112.39	75.99	63.66	74.60	58.29	41.10	54.30	61.67	44.16	57.75
ERSE	102.37	110.68	106.07	63.54	71.08	67.37	49.71	46.18	50.09	52.84	49.54	53.58
ERSE_ba	102.37	110.68	106.06	63.54	71.07	67.36	49.71	46.18	50.08	52.84	49.54	53.58
	T=10, $\theta_{10} = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	18.82	0.00	3.30	35.85	0.01	5.62	14.83	0.01	2.57	20.05	0.00	3.25
SE	105.45	87.69	96.85	76.47	59.46	67.32	34.72	36.94	43.65	39.19	40.14	47.02
ERSE	436.19	89.81	123.09	445.34	58.40	89.75	334.11	37.07	56.38	332.06	39.85	60.40
ERSE_ba	436.17	89.81	123.09	445.32	58.40	89.74	334.10	37.07	56.37	332.04	39.85	60.40
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	7.30	1.95	5.61	12.11	3.79	9.24	5.67	2.14	4.10	7.09	2.82	5.09
SE	103.46	94.25	100.97	73.49	63.63	71.76	49.99	41.57	47.45	53.51	44.76	50.99
ERSE	98.30	102.88	101.04	64.55	67.22	67.17	45.93	43.99	46.07	48.81	47.16	49.20
ERSE_ba	98.30	102.87	101.04	64.55	67.22	67.17	45.93	43.99	46.07	48.81	47.16	49.20

Table D.2: Results under model (2.43), by response and correlation. Population: stable

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\theta_3 = 11747.05$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	21.89	1.22	5.78	39.15	1.39	5.98	15.56	0.57	1.26	21.78	0.82	2.16
SE	112.95	94.89	138.81	76.74	59.19	81.58	35.57	38.12	49.38	44.37	44.69	56.13
ERSE	541.50	107.54	214.48	549.30	70.81	131.28	403.01	39.29	77.47	412.36	44.52	86.23
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	2.48	9.86	0.25	9.44	13.04	5.24	11.28	1.71	8.16	15.54	1.71	11.40
SE	159.96	132.52	153.70	85.65	80.23	84.44	59.29	44.83	56.57	63.11	51.59	61.60
ERSE	137.16	164.45	143.43	76.85	102.61	82.14	48.81	51.36	50.21	50.47	57.27	53.08
	T=4, $\theta_4 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	19.12	0.37	0.29	36.37	0.66	2.57	14.71	0.28	3.62	20.43	0.16	5.28
SE	107.25	89.05	118.13	75.03	58.27	70.68	31.77	34.56	45.63	39.69	40.48	51.10
ERSE	436.74	91.11	171.29	445.47	60.12	108.11	327.18	35.75	73.60	332.32	40.49	82.68
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	7.79	4.89	5.61	16.18	6.03	12.15	12.31	0.33	9.31	16.65	1.12	12.72
SE	134.10	112.00	129.10	73.04	68.76	72.53	57.02	39.26	53.71	60.70	44.52	58.26
ERSE	111.12	130.47	116.19	62.01	81.95	66.64	46.85	46.76	48.03	49.78	52.45	51.94
	T=7, $\theta_7 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	19.12	0.42	3.70	36.35	0.75	6.58	14.71	0.33	3.30	20.43	0.16	4.62
SE	103.31	86.45	102.29	73.02	57.57	67.29	32.25	35.20	44.90	39.39	40.12	49.82
ERSE	436.69	91.62	145.07	445.45	60.54	104.08	327.21	35.89	67.01	332.33	40.54	75.38
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	9.67	0.21	7.55	16.45	1.51	12.82	8.31	1.95	6.13	11.09	2.99	8.24
SE	114.43	98.37	110.16	74.72	61.82	72.98	56.05	40.09	52.11	61.05	44.67	57.28
ERSE	103.75	112.64	107.56	65.35	73.39	69.26	48.51	44.69	48.73	53.10	50.08	53.89
	T=10, $\theta_{10} = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	19.10	0.43	2.89	36.39	0.82	4.80	14.71	0.34	2.12	20.42	0.14	3.09
SE	105.70	88.86	98.70	71.74	56.18	63.75	31.93	34.63	40.80	38.54	39.21	46.04
ERSE	436.68	91.82	124.95	445.57	60.75	91.93	327.18	35.95	54.04	332.30	40.56	60.70
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	6.89	1.53	5.20	11.30	2.95	8.43	5.06	1.76	3.54	6.93	2.67	4.93
SE	104.57	97.29	102.27	70.50	60.34	68.49	47.07	38.71	44.54	52.38	43.88	49.92
ERSE	99.95	104.78	102.77	66.62	69.56	69.31	44.53	42.61	44.59	49.21	47.73	49.64

Table D.3: Results under model (2.45), by response and correlation. Population: stable

	low res			high res		
	T=3, $\theta_3 = 11747.05$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.93	0.08	0.09	0.29	0.04	0.34
SE	249.20	270.83	351.88	139.84	194.24	241.73
ERSE	433.96	267.60	489.34	372.73	197.06	374.42
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.37	0.33	0.25	0.60	0.32	0.52
SE	417.90	320.50	399.50	326.89	209.86	300.83
ERSE	309.75	337.33	319.56	258.18	234.69	256.59
	T=4, $\theta_4 = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.78	0.08	0.09	0.52	0.04	0.28
SE	202.51	218.65	283.45	105.47	139.29	187.94
ERSE	352.88	218.68	432.71	310.96	141.12	302.31
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.20	0.29	0.12	0.55	0.24	0.45
SE	346.84	244.63	328.33	258.36	154.53	235.44
ERSE	277.02	289.25	285.26	208.63	180.76	206.62
	T=7, $\theta_7 = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.70	0.01	0.22	0.56	0.00	0.08
SE	203.05	218.72	274.15	104.42	139.62	189.08
ERSE	352.19	218.19	418.40	311.18	141.20	299.91
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.50	0.04	0.40	0.15	0.12	0.11
SE	342.08	242.67	318.94	258.13	159.54	232.47
ERSE	290.13	274.85	294.19	220.68	182.84	215.92
	T=10, $\theta_{10} = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.73	0.03	0.14	0.56	0.02	0.02
SE	203.37	219.39	268.89	105.82	140.87	179.64
ERSE	353.11	218.76	360.64	311.26	141.59	245.15
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.30	0.09	0.23	0.05	0.05	0.02
SE	320.41	247.41	300.86	226.01	164.10	206.81
ERSE	277.58	263.44	279.23	198.88	176.18	194.90

Table D.4: Results under model (2.42), by response and correlation. Population: stable

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\theta_3 = 11747.05$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	20.84	0.03	4.44	37.93	0.03	4.34	15.56	0.01	1.38	20.55	0.01	2.94
SE	120.02	99.04	136.05	83.74	64.37	84.97	36.54	38.60	50.81	41.86	43.13	54.61
ERSE	540.53	97.70	202.16	549.16	63.67	120.83	411.12	40.48	79.65	408.99	43.33	84.52
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	3.83	8.45	1.60	11.14	11.33	6.91	11.35	1.71	8.28	16.25	0.87	12.11
SE	155.12	129.71	149.44	86.87	83.67	86.36	61.86	46.12	58.76	62.04	49.87	60.42
ERSE	127.67	149.58	133.13	68.05	90.64	72.85	50.10	53.14	51.57	49.22	55.75	51.79
	T=4, $\theta_4 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	18.87	0.01	0.70	35.98	0.00	3.27	14.90	0.01	3.69	20.05	0.00	5.43
SE	106.30	88.32	113.06	76.53	59.18	71.82	35.10	37.37	47.82	38.43	39.62	50.36
ERSE	436.14	89.33	169.21	445.29	58.15	106.09	334.13	36.98	75.81	331.81	39.89	82.44
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	8.20	4.45	6.02	16.90	5.26	12.85	12.41	0.19	9.44	16.80	1.27	12.86
SE	128.74	107.10	123.83	74.52	68.60	74.05	58.77	41.82	55.57	60.53	43.47	57.94
ERSE	109.51	128.07	114.45	60.20	79.41	64.73	48.11	48.56	49.38	49.48	51.86	51.60
	T=7, $\theta_7 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	18.85	0.01	4.10	35.88	0.01	7.33	14.86	0.01	3.75	20.03	0.01	4.82
SE	109.52	91.33	104.60	72.77	56.61	65.84	33.94	36.31	45.92	38.80	39.83	49.66
ERSE	436.06	89.67	143.23	445.25	58.31	102.11	334.15	37.04	69.49	331.93	39.85	75.22
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	10.07	0.64	7.95	17.19	2.31	13.55	8.97	2.21	6.74	11.30	3.18	8.45
SE	116.08	101.18	112.02	73.04	60.81	71.36	57.38	40.63	53.41	61.22	44.34	57.35
ERSE	102.19	110.60	105.89	63.50	71.06	67.32	49.71	46.16	50.08	52.78	49.51	53.53
	T=10, $\theta_{10} = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	18.80	0.01	3.30	35.84	0.01	5.62	14.84	0.00	2.56	20.04	0.01	3.25
SE	108.78	90.47	99.29	76.18	59.07	66.58	34.25	36.47	42.68	38.59	39.98	46.72
ERSE	436.10	89.78	123.25	445.27	58.39	89.75	334.15	37.08	56.32	332.03	39.85	60.27
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	7.29	1.95	5.60	12.11	3.79	9.24	5.65	2.13	4.08	7.09	2.83	5.08
SE	105.41	97.10	103.09	72.56	63.10	70.87	49.10	40.50	46.54	53.28	44.23	50.75
ERSE	98.40	102.94	101.13	64.54	67.19	67.16	45.90	43.98	46.04	48.75	47.14	49.14

Table D.5: Results under model (2.44), by response and correlation. Population: stable

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\theta_3 = 11747.05$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	21.90	1.22	5.80	39.12	1.36	5.97	15.55	0.56	1.26	21.76	0.80	2.18
SE	114.50	95.88	135.95	83.53	64.68	88.02	36.03	39.01	48.95	43.11	43.53	55.00
ERSE	541.72	107.57	214.48	549.22	70.79	131.28	402.98	39.29	77.48	412.26	44.50	86.20
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	2.46	9.89	0.23	9.45	13.02	5.24	11.29	1.70	8.17	15.56	1.68	11.41
SE	155.24	131.43	149.35	90.52	86.84	89.81	59.56	44.43	56.60	62.47	50.38	60.80
ERSE	137.15	164.45	143.43	76.85	102.59	82.13	48.83	51.38	50.22	50.46	57.25	53.06
	T=4, $\theta_4 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	19.11	0.36	0.27	36.35	0.64	2.59	14.71	0.28	3.62	20.43	0.16	5.27
SE	106.70	88.64	117.78	72.83	56.94	72.84	32.20	34.92	46.07	40.58	41.69	52.69
ERSE	436.74	91.12	171.42	445.41	60.10	108.04	327.18	35.75	73.61	332.34	40.49	82.71
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	7.75	4.91	5.59	16.20	6.02	12.17	12.31	0.33	9.31	16.64	1.11	12.71
SE	134.59	111.11	129.35	76.06	68.98	75.49	57.58	39.45	54.26	62.70	45.51	60.19
ERSE	111.20	130.55	116.27	61.96	81.91	66.59	46.85	46.76	48.03	49.81	52.47	51.97
	T=7, $\theta_7 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	19.09	0.40	3.72	36.36	0.75	6.56	14.71	0.33	3.31	20.45	0.17	4.61
SE	106.40	88.93	105.24	72.95	57.36	69.30	31.70	34.35	44.38	37.87	38.47	49.43
ERSE	436.70	91.61	145.13	445.41	60.55	104.07	327.20	35.89	67.03	332.38	40.54	75.45
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	9.69	0.24	7.57	16.44	1.50	12.80	8.32	1.96	6.14	11.08	2.98	8.24
SE	117.82	100.47	113.43	77.19	63.13	75.33	56.00	39.26	51.91	61.36	43.36	57.41
ERSE	103.74	112.54	107.54	65.35	73.41	69.27	48.51	44.69	48.73	53.12	50.08	53.92
	T=10, $\theta_{10} = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	19.12	0.45	2.88	36.35	0.79	4.82	14.71	0.34	2.12	20.44	0.16	3.08
SE	107.72	89.63	99.80	72.56	57.11	66.45	31.86	34.78	41.03	38.50	39.10	44.81
ERSE	436.64	91.84	124.88	445.43	60.73	91.83	327.16	35.95	54.00	332.34	40.56	60.78
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	6.88	1.52	5.18	11.32	2.97	8.45	5.06	1.76	3.54	6.92	2.65	4.92
SE	105.93	97.96	103.55	73.40	62.19	71.37	47.10	39.02	44.64	50.73	43.10	48.40
ERSE	99.92	104.75	102.74	66.56	69.51	69.24	44.51	42.60	44.58	49.25	47.75	49.68

Table D.6: Results under model (2.46), by response and correlation. Population: stable

	low res			high res		
	T=3, $\theta_3 = 11747.05$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.99	0.00	0.22	0.18	0.10	0.43
SE	243.15	265.19	343.07	138.26	187.80	230.52
ERSE	433.17	267.00	491.32	368.24	189.30	365.50
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	0.19	0.42	0.08	0.65	0.43	0.58
SE	406.38	314.12	388.60	308.99	202.66	284.73
ERSE	311.04	338.08	320.80	253.09	228.84	251.20
	T=4, $\theta_4 = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.65	0.06	0.05	0.51	0.03	0.18
SE	198.57	214.06	283.04	100.66	132.27	179.75
ERSE	350.62	216.54	430.74	307.35	135.93	295.14
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	0.34	0.14	0.25	0.41	0.16	0.32
SE	346.97	244.50	328.26	250.11	147.06	227.07
ERSE	275.72	286.78	283.79	205.00	176.14	202.57
	T=7, $\theta_7 = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.65	0.06	0.24	0.46	0.03	0.12
SE	201.21	216.46	280.53	102.02	133.65	178.96
ERSE	351.25	216.87	423.38	307.04	134.53	276.22
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	0.50	0.09	0.41	0.19	0.15	0.15
SE	353.35	243.14	328.85	243.14	152.29	218.98
ERSE	293.27	275.88	297.08	206.85	173.10	202.19
	T=10, $\theta_{10} = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.73	0.03	0.13	0.48	0.00	0.05
SE	200.41	215.70	259.08	99.97	131.99	164.04
ERSE	352.28	217.74	356.73	307.17	134.02	211.57
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	0.28	0.08	0.22	0.09	0.06	0.06
SE	307.01	240.07	288.68	197.62	153.88	183.23
ERSE	275.74	262.32	277.37	178.75	164.40	175.84

Table D.7: Results under model (2.41), by response and correlation. Population: volatile

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\theta_3 = 11747.05$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	12.25	0.03	3.33	26.31	0.01	3.12	10.94	0.01	1.14	11.15	0.01	3.10
SE	160.42	137.19	197.57	90.95	73.57	99.76	43.15	44.99	60.31	27.85	29.97	38.62
ERSE	522.82	136.47	282.74	531.70	74.47	149.12	403.03	43.62	90.37	378.08	31.21	65.43
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	1.45	5.76	0.16	7.89	8.14	4.88	8.44	1.11	6.18	11.68	1.12	8.88
SE	227.65	186.43	218.86	110.73	95.82	107.49	76.58	55.22	71.50	49.90	34.18	46.64
ERSE	179.97	207.34	187.17	92.40	110.94	96.82	61.74	61.35	62.33	42.57	42.06	43.28
	T=4, $\theta_4 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	7.98	0.01	0.32	19.68	0.01	1.34	11.16	0.00	2.89	15.92	0.00	4.31
SE	161.35	140.59	183.58	104.52	84.66	111.69	45.22	44.87	58.83	40.93	39.60	50.12
ERSE	408.11	140.90	276.05	435.19	86.11	161.45	337.71	45.62	90.38	344.21	39.22	78.17
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	3.04	2.85	2.09	9.14	3.85	6.86	9.37	0.22	7.18	12.96	0.96	10.02
SE	215.77	169.71	205.85	126.89	105.15	122.19	73.63	53.17	68.73	59.88	45.18	56.98
ERSE	179.79	202.35	186.84	104.54	122.76	109.39	62.59	62.18	63.44	50.24	52.10	51.79
	T=7, $\theta_7 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	5.50	0.04	1.40	14.63	0.01	3.34	8.60	0.02	2.22	12.16	0.00	2.86
SE	175.88	158.29	189.45	125.92	103.79	119.59	53.40	52.66	61.39	41.39	40.53	48.30
ERSE	396.66	159.09	265.87	424.25	103.25	159.86	333.18	52.68	81.68	334.88	40.30	64.47
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	3.61	0.06	2.86	8.06	0.79	6.37	5.22	1.41	3.93	6.59	2.08	4.92
SE	218.38	179.61	208.00	132.39	115.34	127.74	70.69	58.51	66.95	56.32	45.18	53.30
ERSE	192.35	199.51	197.39	117.68	126.10	121.56	66.03	64.25	66.26	50.67	49.02	51.03
	T=10, $\theta_{10} = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	3.58	0.04	0.92	11.90	0.01	2.16	7.35	0.04	1.21	10.26	0.01	1.56
SE	198.89	180.73	203.24	143.04	117.83	127.74	57.85	57.23	62.82	42.99	41.78	46.87
ERSE	382.10	178.43	247.33	416.19	115.80	150.92	330.25	58.31	74.39	329.53	42.13	54.01
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	2.06	0.43	1.62	4.75	1.37	3.64	2.69	1.05	1.93	3.37	1.45	2.39
SE	220.09	197.51	213.31	133.40	125.86	131.11	66.37	62.10	64.68	49.62	46.21	48.39
ERSE	201.66	207.27	205.74	125.64	131.33	128.80	66.74	66.66	67.06	48.21	48.08	48.50

Table D.8: Results under model (2.43), by response and correlation. Population: volatile

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\theta_3 = 11747.05$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	21.66	13.39	20.06	39.19	21.60	30.45	15.51	10.85	11.78	21.74	13.20	11.56
SE	111.43	107.58	265.98	79.55	71.84	224.22	34.81	39.48	144.28	44.94	45.67	114.63
ERSE	541.20	261.78	584.69	549.61	239.26	515.48	402.79	156.16	331.01	412.19	136.43	251.21
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	17.48	21.78	18.14	23.77	34.58	25.50	8.41	13.37	9.40	5.27	13.90	7.16
SE	369.21	208.01	342.07	314.35	175.75	290.71	232.31	113.15	205.70	167.74	101.43	151.29
ERSE	369.48	417.64	383.41	325.95	376.28	339.44	236.87	228.22	236.98	182.23	187.45	184.60
	T=4, $\theta_4 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	22.31	15.58	16.97	36.46	23.75	24.15	14.72	11.71	10.84	20.43	14.62	12.45
SE	92.52	91.42	196.23	71.45	66.47	170.78	31.97	36.44	102.21	39.57	40.10	90.97
ERSE	434.11	217.53	421.10	440.85	205.41	373.82	327.11	123.64	235.91	332.33	124.58	205.83
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	13.61	19.96	14.52	17.73	29.45	19.50	8.00	12.50	8.90	8.29	14.17	9.69
SE	268.75	151.61	248.19	235.97	132.10	217.51	159.66	80.37	140.67	131.00	79.70	117.23
ERSE	275.99	309.29	286.48	247.40	284.23	257.78	175.16	166.39	174.82	158.66	159.26	160.03
	T=7, $\theta_7 = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	22.29	17.83	16.21	37.95	29.70	26.91	14.73	12.82	11.90	20.45	16.41	15.36
SE	91.48	92.80	179.11	75.53	71.12	143.12	31.61	35.94	75.26	38.96	39.26	74.83
ERSE	434.38	223.65	360.19	466.72	247.54	336.53	327.10	120.35	169.40	332.39	140.31	180.12
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	13.61	18.21	14.46	23.25	28.98	24.56	10.64	12.41	11.13	14.01	15.58	14.61
SE	242.08	147.42	221.05	180.93	131.06	167.02	99.23	69.64	89.33	89.34	73.38	82.71
ERSE	265.13	276.64	271.89	271.48	290.52	279.08	145.67	143.66	145.66	160.03	161.59	161.15
	T=10, $\theta_{10} = 11000.5$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	22.31	20.40	19.44	36.48	30.03	28.20	14.68	14.30	13.98	20.44	16.93	16.40
SE	89.90	92.27	175.03	71.99	69.70	131.98	32.13	36.98	78.48	38.86	38.81	59.54
ERSE	434.33	269.58	370.68	440.86	240.68	307.41	327.09	159.27	197.58	332.34	130.18	153.44
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	18.26	20.11	18.69	26.08	29.01	26.93	13.63	14.09	13.77	15.76	16.42	16.10
SE	221.42	157.88	203.51	158.73	124.87	147.84	92.30	77.34	85.47	65.20	59.47	62.10
ERSE	306.51	314.24	311.87	261.73	272.84	267.46	181.61	181.31	181.74	142.01	143.44	143.03

Table D.9: Results under model (2.45), by response and correlation. Population: volatile

	low res			high res		
	T=3, $\theta_3 = 11747.05$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	1.22	0.09	0.18	1.41	0.02	0.15
SE	264.13	298.96	369.19	171.01	233.89	296.64
ERSE	410.52	299.55	534.08	353.32	231.46	445.14
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.36	0.45	0.21	0.74	0.49	0.48
SE	436.82	336.85	418.00	400.39	257.31	368.74
ERSE	337.17	364.58	347.58	307.01	279.26	305.15
	T=4, $\theta_4 = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.25	0.06	0.19	0.22	0.04	0.16
SE	355.80	250.83	336.74	273.62	162.53	249.08
ERSE	289.71	301.23	298.22	220.70	192.99	218.78
	T=7, $\theta_7 = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.19	0.03	0.02	0.41	0.05	0.17
SE	214.76	228.15	297.90	109.96	141.35	187.60
ERSE	363.54	233.72	436.46	310.47	140.68	286.32
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.02	0.00	0.03	0.29	0.16	0.23
SE	376.99	253.25	349.74	254.04	159.02	229.27
ERSE	310.58	288.09	313.31	212.59	179.84	208.53
	T=10, $\theta_{10} = 11000.5$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.81	0.05	0.22	0.54	0.01	0.12
SE	214.09	244.60	304.84	139.28	187.84	237.71
ERSE	333.76	243.90	426.00	287.54	189.10	321.85
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.42	0.14	0.34	0.24	0.11	0.18
SE	373.20	275.34	347.51	293.79	220.06	270.27
ERSE	323.09	298.77	323.70	264.03	237.03	259.12

Table D.10: Results under model (2.41), by response and correlation. Population: simulated

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\theta_3 = 808.1355$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	57.02	0.00	0.42	61.64	0.07	2.39	19.63	0.02	4.73	23.53	0.02	2.62
SE	12.93	8.80	10.93	13.52	9.28	12.04	5.54	5.52	7.10	5.82	6.05	7.77
ERSE	248.42	8.87	16.35	253.44	9.37	17.39	185.07	5.50	11.13	190.75	6.32	12.33
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	25.12	9.20	18.19	23.90	12.92	16.56	18.35	1.41	14.01	17.98	1.76	13.18
SE	9.61	11.05	10.07	10.46	12.13	11.03	8.32	6.45	8.02	8.98	7.05	8.70
ERSE	7.71	12.30	8.71	8.06	13.16	9.15	6.73	7.34	7.02	7.24	8.16	7.60
	T=4, $\theta_4 = 108.5394$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	44.52	0.01	6.52	51.69	0.03	6.07	17.63	0.01	5.58	21.49	0.01	5.58
SE	1.79	1.34	1.63	1.66	1.21	1.47	0.69	0.70	0.90	0.74	0.79	1.04
ERSE	15.55	1.35	2.45	15.86	1.23	2.25	11.59	0.69	1.44	11.87	0.80	1.65
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	23.54	2.79	18.32	25.47	5.02	19.57	15.80	2.54	12.12	18.11	0.94	13.76
SE	1.71	1.58	1.69	1.46	1.42	1.48	1.11	0.78	1.05	1.26	0.89	1.20
ERSE	1.44	1.85	1.54	1.19	1.69	1.31	0.92	0.92	0.95	1.00	1.04	1.04
	T=7, $\theta_7 = 1619.253$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	57.70	0.03	10.94	59.28	0.02	11.48	19.69	0.00	4.41	24.51	0.01	5.80
SE	28.71	19.33	21.77	26.92	18.58	21.18	11.15	10.89	13.11	12.86	12.91	15.79
ERSE	579.48	18.97	31.91	565.99	18.35	32.19	417.95	10.89	18.66	433.32	12.63	23.35
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	24.63	5.15	19.33	26.18	5.02	20.51	10.07	3.51	7.41	13.61	3.77	10.18
SE	22.94	20.28	22.91	22.35	19.43	22.40	15.54	12.16	14.67	19.02	14.22	17.97
ERSE	19.10	22.63	20.67	18.37	22.06	20.05	14.01	13.35	14.16	16.25	15.59	16.58
	T=10, $\theta_{10} = 2397.519$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	37.23	0.01	5.69	53.59	0.01	7.93	20.61	0.00	3.08	23.72	0.01	3.84
SE	63.98	48.81	51.75	37.60	26.91	29.79	17.58	16.99	19.18	18.22	18.18	21.10
ERSE	314.99	48.71	63.26	311.83	27.11	41.60	233.89	16.78	23.40	234.88	18.19	27.35
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	12.19	3.94	9.30	16.92	5.59	12.84	6.63	2.84	4.70	8.37	3.31	6.01
SE	52.93	51.35	52.55	31.90	28.36	31.48	20.87	18.69	20.20	23.76	20.11	22.75
ERSE	50.95	54.65	52.77	28.82	30.93	30.40	19.67	19.46	19.90	21.96	21.40	22.21

Table D.11: Results under model (2.43), by response and correlation. Population: simulated

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\theta_3 = 808.1355$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	51.44	15.46	20.78	49.33	9.22	13.02	17.23	0.95	3.41	22.37	1.93	6.26
SE	11.87	9.68	69.50	10.77	8.73	50.50	5.15	5.13	21.64	5.74	5.51	22.70
ERSE	235.86	78.75	158.46	231.34	65.54	115.66	180.95	30.77	46.95	188.76	31.14	46.59
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	4.06	29.40	8.57	5.72	22.20	0.61	15.01	0.23	11.37	20.93	1.93	16.44
SE	98.24	57.70	90.39	68.48	46.12	63.34	28.28	23.22	25.88	27.93	24.89	25.99
ERSE	103.75	121.38	108.04	78.71	98.20	82.68	37.25	41.90	38.16	36.11	43.14	37.50
	T=4, $\theta_4 = 108.5394$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	51.90	16.29	10.74	57.26	15.74	9.94	21.34	6.48	1.57	31.52	7.58	1.38
SE	1.63	1.30	2.69	1.67	1.31	2.28	0.76	0.75	1.39	1.00	0.93	1.42
ERSE	15.65	3.97	5.56	15.75	3.40	4.69	11.85	1.98	2.93	12.79	1.84	2.73
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	5.68	21.32	0.85	8.36	21.24	2.91	7.36	4.40	4.19	11.12	5.83	6.83
SE	3.14	2.69	2.99	2.54	2.32	2.45	1.76	1.35	1.63	1.67	1.35	1.59
ERSE	3.94	5.18	4.19	3.30	4.42	3.52	2.31	2.49	2.36	2.01	2.28	2.09
	T=7, $\theta_7 = 1619.253$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	58.45	23.69	14.30	62.02	17.70	6.80	21.16	2.45	1.98	23.41	4.48	10.10
SE	26.61	21.48	84.53	27.53	20.92	66.28	11.77	11.23	24.60	11.78	10.18	25.64
ERSE	563.88	173.43	201.22	570.00	163.70	172.58	421.26	51.94	62.87	428.23	72.65	79.84
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	2.01	21.39	6.31	7.58	15.41	2.62	7.69	0.75	5.17	17.63	8.31	14.40
SE	96.00	85.62	91.06	69.59	69.95	67.53	28.26	24.35	26.48	28.11	25.67	26.71
ERSE	163.42	199.11	172.44	140.41	181.42	150.19	54.72	58.27	56.07	68.36	76.28	71.20
	T=10, $\theta_{10} = 2397.519$											
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	50.80	31.60	26.72	57.73	30.30	23.65	21.53	9.12	6.77	20.46	0.43	3.69
SE	37.02	33.47	67.54	38.49	32.75	62.53	17.70	17.75	30.34	15.77	14.38	27.09
ERSE	308.88	135.33	162.81	309.11	126.96	151.52	230.80	64.06	75.98	226.88	65.16	74.88
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	21.23	28.95	23.41	16.20	26.96	19.09	4.09	7.15	5.39	7.53	3.11	5.69
SE	79.15	64.56	74.40	73.60	59.34	69.29	34.23	29.77	32.33	30.66	26.40	29.03
ERSE	137.65	149.63	142.26	125.26	138.89	130.24	68.50	70.47	69.55	66.13	69.59	67.83

Table D.12: Results under model (2.45), by response and correlation. Population: simulated

	low response			high response		
	T=3, $\theta_3 = 808.1355$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	1.48	0.11	0.15	0.19	0.16	0.80
SE	123.70	126.03	168.17	66.96	69.75	95.11
ERSE	197.95	127.72	248.12	178.52	71.70	156.77
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	1.10	0.36	0.85	0.81	0.94	0.79
SE	196.91	154.20	188.95	128.91	84.42	118.30
ERSE	155.51	171.09	160.90	109.21	104.46	109.42
	T=4, $\theta_4 = 108.5394$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	0.94	0.15	0.21	1.26	0.02	0.41
SE	7.87	8.19	10.48	4.69	5.45	7.47
ERSE	12.52	8.15	16.00	10.86	5.45	11.85
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.68	0.12	0.54	1.18	0.10	0.92
SE	12.73	9.09	12.08	10.18	6.27	9.29
ERSE	10.21	10.71	10.53	8.30	7.38	8.25
	T=7, $\theta_7 = 1619.253$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	6.66	0.36	1.97	4.32	0.03	0.93
SE	225.58	198.08	235.59	80.55	87.35	107.82
ERSE	501.61	197.70	336.11	419.75	88.58	150.85
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	4.06	0.97	3.28	2.21	0.67	1.63
SE	275.98	219.83	261.61	132.65	100.14	122.65
ERSE	243.33	245.31	248.67	120.54	110.40	119.23
	T=10, $\theta_{10} = 2397.519$					
	naïve	**EE**$_h$	**EE**	**naïve**	**EE**$_h$	**EE**
APRB	1.23	0.16	0.43	1.59	0.01	0.34
SE	146.87	148.58	173.02	76.99	84.25	95.00
ERSE	246.65	147.95	225.50	216.52	83.44	114.97
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	0.76	0.31	0.62	0.75	0.27	0.55
SE	198.16	163.57	188.34	105.11	92.73	100.47
ERSE	180.08	176.01	181.83	101.45	98.05	100.92

D.2 Estimation of Regression Coefficients

Table D.13: Results under model (2.41), by response and correlation. Population: stable

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\beta_0 = 11764.93$, $\beta_1 = -0.3202386$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	21.55	0.03	4.52	37.90	0.00	4.37	16.14	0.02	1.43	20.52	0.01	2.95
SE	123.98	102.07	143.46	83.41	64.46	84.18	41.57	43.77	54.48	44.40	45.57	56.65
ERSE	549.11	100.08	207.10	551.00	64.56	122.56	415.35	41.78	81.92	409.91	43.82	85.48
APRB(β_1)	322.71	0.25	97.35	6.35	5.40	5.31	310.69	0.43	22.65	0.76	1.35	3.60
SE	0.05	0.05	0.07	0.24	0.16	0.21	0.02	0.02	0.03	0.11	0.11	0.14
ERSE	0.70	0.05	0.11	1.21	0.17	0.31	0.69	0.02	0.04	0.81	0.11	0.21
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	3.94	8.66	1.66	11.10	11.35	6.87	11.64	1.76	8.51	16.25	0.86	12.12
SE	161.96	137.93	156.39	86.44	82.54	85.82	64.82	50.04	61.94	64.39	51.51	62.72
ERSE	130.35	153.71	136.08	69.05	91.88	73.90	51.21	54.83	52.83	49.80	56.37	52.39
APRB(β_1)	18.63	161.27	11.32	1.99	9.95	2.68	218.78	40.01	158.73	8.64	3.54	6.65
SE	0.09	0.07	0.09	0.22	0.21	0.22	0.03	0.03	0.03	0.16	0.12	0.15
ERSE	0.07	0.08	0.07	0.17	0.23	0.18	0.03	0.03	0.03	0.12	0.14	0.13
	T=4, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	19.59	0.02	0.69	35.97	0.01	3.28	15.47	0.01	3.81	20.03	0.00	5.41
SE	114.01	93.64	124.12	76.44	59.46	75.44	35.17	37.33	49.97	40.60	41.58	51.08
ERSE	444.64	91.54	173.11	448.25	58.94	107.54	339.13	38.19	77.95	333.80	40.32	83.32
APRB(β_1)	218.16	0.40	2.95	40.33	1.49	1.34	221.09	0.10	53.38	6.05	1.52	4.42
SE	0.05	0.05	0.06	0.19	0.16	0.19	0.02	0.02	0.03	0.10	0.10	0.13
ERSE	0.55	0.05	0.09	0.82	0.15	0.27	0.55	0.02	0.04	0.55	0.10	0.19
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	8.34	4.61	6.13	16.91	5.25	12.86	12.73	0.19	9.70	16.77	1.27	12.84
SE	140.95	116.69	135.77	79.18	71.07	78.48	61.42	43.20	58.12	60.27	45.29	57.91
ERSE	111.78	131.64	116.96	61.07	80.48	65.65	49.21	50.11	50.62	50.04	52.41	52.17
APRB(β_1)	82.40	67.73	58.52	18.70	10.02	13.60	182.45	0.16	138.81	11.05	3.58	8.39
SE	0.08	0.06	0.07	0.20	0.18	0.20	0.03	0.02	0.03	0.15	0.11	0.14
ERSE	0.06	0.07	0.06	0.15	0.20	0.16	0.03	0.03	0.03	0.12	0.12	0.12
	T=7, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	19.58	0.02	4.18	35.93	0.03	7.30	15.42	0.00	3.87	20.01	0.02	4.83
SE	112.74	92.45	106.95	76.90	59.70	69.38	35.82	38.24	48.10	38.95	40.21	50.80
ERSE	444.61	91.90	146.47	448.46	59.14	103.49	339.17	38.25	71.55	333.90	40.29	76.06
APRB(β_1)	218.13	0.19	48.81	42.87	3.75	6.50	220.46	0.06	56.18	7.12	0.38	3.45
SE	0.05	0.05	0.05	0.20	0.17	0.19	0.02	0.02	0.03	0.09	0.09	0.12
ERSE	0.55	0.05	0.08	0.83	0.15	0.25	0.55	0.02	0.04	0.55	0.09	0.17
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	10.27	0.61	8.11	17.17	2.27	13.53	9.23	2.28	6.94	11.30	3.18	8.46
SE	117.65	104.53	113.83	76.03	64.17	74.49	60.49	41.99	56.25	62.90	44.63	58.89
ERSE	104.28	113.45	108.19	64.40	72.05	68.27	51.00	47.59	51.47	53.38	50.05	54.13
APRB(β_1)	123.97	1.38	97.98	20.27	0.48	15.28	134.59	32.20	101.35	7.51	2.49	5.59
SE	0.06	0.05	0.06	0.21	0.18	0.20	0.03	0.02	0.03	0.14	0.11	0.14
ERSE	0.06	0.06	0.06	0.16	0.18	0.17	0.03	0.03	0.03	0.12	0.12	0.12
	T=10, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	19.51	0.02	3.44	35.90	0.03	5.57	15.39	0.02	2.66	20.01	0.01	3.26
SE	111.48	91.91	102.84	78.03	60.54	68.81	36.31	38.51	45.81	41.07	42.43	47.63
ERSE	444.52	92.01	126.01	448.56	59.22	90.83	339.08	38.28	57.99	333.96	40.27	60.91
APRB(β_1)	217.26	0.45	41.92	43.66	3.88	2.65	220.06	0.21	38.64	6.92	0.69	2.73
SE	0.05	0.05	0.05	0.21	0.17	0.19	0.02	0.02	0.02	0.09	0.10	0.11
ERSE	0.55	0.05	0.06	0.81	0.15	0.22	0.55	0.02	0.03	0.55	0.10	0.14
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	7.54	2.05	5.80	12.07	3.75	9.19	5.83	2.21	4.22	7.09	2.83	5.09
SE	109.36	100.77	106.90	74.46	65.72	72.85	52.47	43.50	49.84	53.53	45.75	51.23
ERSE	100.38	105.30	103.27	65.42	68.14	68.07	47.18	45.33	47.38	49.28	47.64	49.67
APRB(β_1)	93.11	22.70	72.01	11.16	0.53	7.40	85.04	31.88	61.73	4.90	2.49	3.71
SE	0.06	0.05	0.06	0.20	0.18	0.20	0.03	0.02	0.03	0.12	0.11	0.12
ERSE	0.05	0.05	0.05	0.16	0.17	0.17	0.03	0.02	0.03	0.11	0.11	0.11

Table D.14: Results under model (2.43), by response and correlation. Population: stable

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\beta_0 = 11764.93$, $\beta_1 = -0.3202386$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	22.74	1.27	6.00	39.09	1.39	6.03	16.14	0.59	1.30	21.72	0.79	2.19
SE	119.20	100.32	139.05	84.59	64.77	91.93	37.52	40.25	51.28	44.17	44.48	57.27
ERSE	550.14	110.06	219.72	551.14	71.65	133.36	406.83	40.56	79.70	413.45	45.06	87.29
APRB(β_1)	339.29	17.02	117.47	1.61	2.10	13.63	310.87	11.27	20.68	8.02	4.02	5.64
SE	0.05	0.05	0.07	0.25	0.18	0.25	0.02	0.02	0.03	0.14	0.13	0.16
ERSE	0.70	0.05	0.11	1.25	0.17	0.32	0.69	0.02	0.04	0.89	0.12	0.23
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	2.49	10.25	0.21	9.37	13.08	5.17	11.57	1.76	8.38	15.57	1.67	11.42
SE	156.50	134.61	151.20	94.95	90.38	94.08	61.56	46.68	58.74	65.23	52.17	63.48
ERSE	140.19	169.68	146.83	78.12	104.22	83.48	49.92	52.99	51.45	51.09	57.99	53.73
APRB(β_1)	0.44	183.06	30.61	12.08	17.51	12.28	218.25	39.58	157.04	8.81	5.91	7.42
SE	0.09	0.07	0.08	0.24	0.24	0.24	0.03	0.02	0.03	0.18	0.15	0.17
ERSE	0.08	0.08	0.08	0.18	0.24	0.19	0.03	0.03	0.03	0.13	0.16	0.14
	T=4, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	19.88	0.39	0.23	36.35	0.65	2.59	15.26	0.29	3.72	20.39	0.14	5.30
SE	109.93	91.17	119.17	75.98	59.60	74.21	34.16	36.81	46.49	40.55	41.44	52.87
ERSE	445.27	93.39	175.43	448.21	60.92	109.47	331.81	36.91	75.69	334.20	40.92	83.62
APRB(β_1)	221.77	4.67	8.60	41.78	3.73	0.26	218.33	4.27	52.27	9.24	0.05	3.64
SE	0.05	0.05	0.06	0.19	0.16	0.20	0.02	0.02	0.02	0.09	0.09	0.12
ERSE	0.55	0.05	0.09	0.80	0.15	0.26	0.55	0.02	0.04	0.54	0.09	0.19
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	7.87	5.09	5.67	16.21	6.02	12.18	12.61	0.33	9.54	16.67	1.14	12.74
SE	136.15	111.80	130.88	76.72	71.31	76.26	57.10	40.80	54.03	63.25	45.49	60.63
ERSE	113.47	134.08	118.78	62.82	82.98	67.50	47.94	48.23	49.24	50.37	53.05	52.55
APRB(β_1)	76.66	73.77	52.86	17.75	12.24	12.49	181.02	2.03	136.81	11.16	2.14	8.23
SE	0.08	0.06	0.07	0.21	0.19	0.21	0.03	0.02	0.03	0.14	0.10	0.14
ERSE	0.06	0.07	0.07	0.15	0.20	0.16	0.03	0.03	0.03	0.12	0.12	0.12
	T=7, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	19.79	0.38	3.85	36.37	0.76	6.57	15.28	0.35	3.39	20.40	0.15	4.62
SE	113.46	93.54	109.26	75.44	59.22	69.26	33.56	35.89	45.99	39.35	40.12	49.96
ERSE	445.20	93.86	148.51	448.43	61.40	105.36	331.90	37.07	69.02	334.23	40.98	76.13
APRB(β_1)	220.78	4.42	44.71	44.13	5.86	4.25	218.58	5.10	49.31	8.59	1.07	3.32
SE	0.05	0.05	0.06	0.20	0.16	0.19	0.02	0.02	0.02	0.09	0.09	0.12
ERSE	0.55	0.05	0.08	0.81	0.15	0.25	0.55	0.02	0.04	0.54	0.10	0.17
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	9.94	0.28	7.78	16.45	1.51	12.81	8.53	2.01	6.30	11.08	2.99	8.24
SE	122.78	102.79	118.26	76.90	63.89	75.05	57.53	40.64	53.49	61.27	44.83	57.48
ERSE	105.90	115.45	109.90	66.22	74.35	70.17	49.80	46.08	50.10	53.64	50.59	54.44
APRB(β_1)	119.80	2.77	93.87	18.02	3.06	13.09	124.57	28.35	92.04	7.26	2.76	5.33
SE	0.07	0.05	0.06	0.20	0.17	0.20	0.03	0.02	0.03	0.14	0.11	0.13
ERSE	0.06	0.06	0.06	0.16	0.18	0.17	0.03	0.02	0.03	0.12	0.12	0.12
	T=10, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	19.91	0.50	2.91	36.41	0.84	4.78	15.27	0.37	2.15	20.40	0.15	3.08
SE	111.01	91.73	102.40	71.38	56.03	66.34	32.04	34.70	39.95	41.31	42.55	49.33
ERSE	445.31	94.14	128.21	448.46	61.59	92.99	331.87	37.12	55.53	334.22	41.00	61.35
APRB(β_1)	222.04	5.94	35.51	43.69	5.25	1.30	218.52	5.31	31.31	7.57	2.07	3.91
SE	0.05	0.05	0.05	0.20	0.17	0.19	0.02	0.02	0.02	0.09	0.09	0.10
ERSE	0.55	0.05	0.07	0.79	0.15	0.22	0.55	0.02	0.03	0.53	0.10	0.13
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	7.01	1.52	5.28	11.28	2.93	8.41	5.16	1.79	3.61	6.92	2.66	4.92
SE	109.55	99.15	106.92	73.70	61.74	71.56	45.65	38.55	43.30	55.93	46.83	53.39
ERSE	102.25	107.36	105.21	67.48	70.49	70.20	45.74	43.92	45.86	49.74	48.24	50.17
APRB(β_1)	86.65	16.30	65.60	9.91	1.14	6.18	75.36	25.78	52.79	6.28	3.59	4.97
SE	0.06	0.05	0.06	0.20	0.18	0.19	0.02	0.02	0.02	0.12	0.10	0.11
ERSE	0.05	0.05	0.05	0.16	0.17	0.17	0.02	0.02	0.02	0.11	0.11	0.11

Table D.15: Results under model (2.45), by response and correlation.
Population: stable

	low res			high res		
	T=3, $\beta_0 = 11764.93$, $\beta_1 = -0.3202386$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	1.14	0.02	0.17	0.37	0.01	0.36
SE	254.91	276.14	351.66	144.89	199.99	248.04
ERSE	437.70	276.29	501.83	374.06	203.71	385.13
APRB(β_1)	27.47	0.56	0.37	6.07	0.30	6.77
SE	0.12	0.14	0.18	0.08	0.11	0.14
ERSE	0.65	0.14	0.25	0.66	0.11	0.21
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.34	0.43	0.20	0.63	0.34	0.54
SE	415.53	321.62	397.75	334.63	214.73	308.33
ERSE	316.50	347.56	327.04	264.56	241.88	263.31
APRB(β_1)	18.13	5.50	12.52	10.96	6.66	9.60
SE	0.23	0.16	0.21	0.19	0.12	0.17
ERSE	0.17	0.17	0.17	0.15	0.13	0.15
	T=4, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.90	0.04	0.02	0.63	0.01	0.21
SE	209.39	224.45	286.64	109.58	146.66	193.62
ERSE	357.57	224.70	445.51	314.22	145.94	313.53
APRB(β_1)	16.13	0.14	2.65	8.77	0.14	2.83
SE	0.10	0.11	0.14	0.06	0.08	0.11
ERSE	0.46	0.11	0.21	0.46	0.08	0.17
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.34	0.27	0.23	0.47	0.18	0.37
SE	353.18	245.16	333.91	266.72	158.58	243.07
ERSE	284.12	298.29	292.95	215.58	187.44	213.76
APRB(β_1)	12.11	1.83	8.86	6.19	2.48	4.94
SE	0.18	0.12	0.17	0.15	0.09	0.13
ERSE	0.14	0.14	0.14	0.12	0.10	0.12
	T=7, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.77	0.09	0.26	0.63	0.00	0.06
SE	203.42	216.98	284.56	106.41	141.22	192.89
ERSE	354.44	222.66	441.40	314.14	145.71	308.94
APRB(β_1)	15.19	0.64	4.56	8.80	0.07	0.78
SE	0.09	0.10	0.14	0.06	0.08	0.11
ERSE	0.46	0.11	0.22	0.46	0.08	0.17
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.54	0.09	0.44	0.13	0.11	0.09
SE	359.78	244.16	334.76	262.81	163.68	236.87
ERSE	304.17	286.96	308.52	226.83	188.58	222.20
APRB(β_1)	10.35	1.92	8.03	1.53	1.56	0.98
SE	0.18	0.12	0.17	0.15	0.09	0.13
ERSE	0.15	0.14	0.15	0.13	0.10	0.12
	T=10, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.76	0.11	0.34	0.67	0.06	0.02
SE	214.27	228.85	265.96	111.73	148.49	182.90
ERSE	355.40	223.05	358.29	314.22	146.00	242.17
APRB(β_1)	14.66	1.42	5.34	9.41	0.96	0.18
SE	0.10	0.11	0.13	0.06	0.08	0.10
ERSE	0.46	0.11	0.18	0.46	0.08	0.13
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.57	0.25	0.48	0.09	0.02	0.05
SE	310.51	250.43	293.28	222.56	171.62	205.74
ERSE	277.56	267.04	279.83	199.29	180.42	195.95
APRB(β_1)	9.44	3.95	7.58	1.08	0.19	0.51
SE	0.16	0.12	0.15	0.12	0.09	0.11
ERSE	0.14	0.13	0.14	0.11	0.10	0.11

Table D.16: Results under model (2.41), by response and correlation. Population: volatile

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\beta_0 = 11764.93$, $\beta_1 = -0.3202386$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	12.79	0.03	3.54	25.73	0.02	4.00	14.44	0.00	1.27	19.82	0.00	2.25
SE	165.84	140.50	193.53	106.31	84.78	118.62	46.88	46.52	61.07	48.25	45.64	57.10
ERSE	536.63	140.01	289.47	555.80	83.86	170.24	428.87	47.40	95.84	438.51	44.20	85.88
APRB(β_1)	169.13	0.19	70.78	33.90	5.73	3.17	246.19	0.04	16.12	6.81	3.72	4.04
SE	0.08	0.07	0.10	0.30	0.21	0.30	0.02	0.02	0.03	0.12	0.11	0.14
ERSE	0.72	0.07	0.15	1.35	0.21	0.42	0.72	0.02	0.05	0.97	0.11	0.20
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	1.34	6.02	0.03	6.34	8.92	3.54	10.09	1.60	7.42	13.93	1.49	10.41
SE	220.77	184.69	212.66	132.15	114.03	128.11	74.27	57.16	70.20	64.17	53.48	62.32
ERSE	183.62	212.57	191.14	105.91	127.45	110.98	63.01	65.97	64.40	51.52	58.61	53.90
APRB(β_1)	18.99	104.79	31.23	13.97	0.57	10.84	165.86	34.59	120.73	14.13	0.18	11.11
SE	0.13	0.09	0.12	0.32	0.29	0.31	0.04	0.03	0.04	0.15	0.13	0.15
ERSE	0.11	0.11	0.11	0.26	0.32	0.27	0.03	0.03	0.03	0.12	0.14	0.12
	T=4, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	8.45	0.08	0.44	19.67	0.02	1.39	11.59	0.00	2.99	15.90	0.02	4.29
SE	166.98	145.25	187.75	110.54	89.36	116.54	47.70	46.93	61.24	40.25	38.99	48.24
ERSE	420.35	144.67	282.69	444.42	87.91	164.10	346.49	47.00	92.68	351.34	39.69	79.07
APRB(β_1)	81.67	0.97	11.38	7.34	1.43	2.42	141.46	0.05	35.56	3.61	0.32	3.17
SE	0.08	0.08	0.10	0.46	0.33	0.43	0.02	0.02	0.03	0.10	0.09	0.12
ERSE	0.72	0.07	0.15	1.23	0.32	0.57	0.73	0.02	0.05	0.99	0.10	0.19
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	2.99	3.04	2.02	9.19	3.81	6.91	9.63	0.23	7.40	12.93	0.94	9.99
SE	219.17	173.59	209.55	131.90	110.69	127.10	75.09	56.00	70.53	57.83	43.65	54.97
ERSE	183.71	207.99	191.11	106.41	125.06	111.34	63.96	63.97	64.93	50.83	52.70	52.39
APRB(β_1)	20.05	39.13	11.81	4.74	1.89	3.93	118.09	0.05	90.52	9.06	0.59	7.04
SE	0.13	0.09	0.12	0.46	0.43	0.45	0.04	0.03	0.04	0.14	0.11	0.14
ERSE	0.11	0.11	0.11	0.38	0.47	0.40	0.03	0.03	0.03	0.12	0.13	0.12
	T=7, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	5.81	0.03	1.48	14.60	0.00	3.36	8.98	0.02	2.24	12.15	0.01	2.87
SE	188.18	167.45	193.64	129.11	106.78	122.34	55.34	54.20	65.71	41.06	39.90	46.37
ERSE	397.40	162.97	267.54	421.20	104.32	160.98	333.78	54.29	84.02	332.81	40.77	65.28
APRB(β_1)	53.88	0.30	14.00	7.05	0.29	3.69	100.96	0.21	25.84	13.14	0.45	3.04
SE	0.08	0.08	0.10	0.24	0.21	0.25	0.03	0.03	0.03	0.10	0.10	0.12
ERSE	0.34	0.08	0.14	0.61	0.20	0.33	0.36	0.03	0.04	0.47	0.10	0.15
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	3.79	0.06	3.01	8.09	0.80	6.39	5.33	1.41	4.00	6.60	2.09	4.93
SE	220.76	186.10	210.86	134.34	118.87	129.91	75.10	62.83	71.31	54.04	43.58	51.13
ERSE	193.96	203.42	199.42	118.59	127.22	122.52	67.80	66.16	68.11	51.29	49.60	51.65
APRB(β_1)	37.41	2.63	29.60	8.91	0.41	7.18	61.79	15.59	46.51	7.44	2.26	5.38
SE	0.12	0.10	0.11	0.29	0.24	0.27	0.04	0.03	0.04	0.13	0.11	0.13
ERSE	0.11	0.10	0.11	0.24	0.25	0.25	0.04	0.03	0.04	0.12	0.12	0.12
	T=10, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	3.80	0.03	0.91	12.07	0.05	2.09	7.57	0.01	1.30	10.31	0.02	1.55
SE	202.55	185.34	209.55	142.49	118.02	129.71	59.46	58.52	65.61	43.81	42.48	45.85
ERSE	395.24	183.38	252.06	426.16	117.65	153.64	339.50	59.98	76.42	336.79	42.94	54.96
APRB(β_1)	118.21	2.01	31.95	212.79	21.14	2.59	327.29	0.39	57.60	86.21	13.57	1.72
SE	0.09	0.09	0.11	0.44	0.38	0.41	0.03	0.03	0.03	0.17	0.16	0.17
ERSE	0.51	0.09	0.13	0.75	0.35	0.45	0.54	0.03	0.04	0.61	0.15	0.18
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	2.11	0.37	1.64	4.69	1.31	3.57	2.82	1.14	2.04	3.36	1.44	2.38
SE	227.20	203.17	220.14	134.47	129.54	132.38	69.44	64.76	67.65	48.57	44.94	47.39
ERSE	205.87	212.91	210.34	127.90	133.66	131.11	68.45	68.43	68.81	49.08	48.97	49.38
APRB(β_1)	77.78	8.38	60.92	27.60	0.62	15.27	125.53	49.84	91.03	10.61	2.20	3.81
SE	0.12	0.10	0.11	0.42	0.41	0.42	0.04	0.03	0.04	0.18	0.17	0.18
ERSE	0.11	0.11	0.11	0.38	0.40	0.39	0.04	0.04	0.04	0.16	0.16	0.16

Table D.17: Results under model (2.43), by response and correlation. Population: volatile

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\beta_0 = 11764.93$, $\beta_1 = -0.3202386$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	25.89	15.63	23.08	39.15	21.63	29.98	16.10	11.16	12.13	21.80	13.21	11.54
SE	102.59	98.23	250.44	81.45	73.85	222.75	37.33	42.48	144.52	45.00	45.76	111.85
ERSE	551.36	261.74	582.45	553.42	239.03	511.43	411.78	161.05	341.35	419.16	137.48	249.10
APRB(β_1)	355.58	237.78	381.16	30.85	20.03	12.96	287.15	201.54	222.89	42.19	2.01	15.62
SE	0.05	0.05	0.13	0.21	0.19	0.52	0.02	0.02	0.08	0.10	0.11	0.31
ERSE	0.76	0.13	0.31	1.33	0.55	1.22	0.73	0.09	0.19	0.91	0.36	0.72
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	19.68	25.30	20.55	23.15	34.15	24.92	8.64	13.77	9.66	5.22	13.89	7.13
SE	347.10	199.69	321.41	309.98	177.53	286.88	229.71	115.66	203.94	161.63	101.72	145.94
ERSE	368.59	417.26	382.55	324.59	373.57	337.79	243.40	235.64	243.82	181.20	187.11	183.65
APRB(β_1)	375.88	406.74	372.82	3.67	17.65	6.22	164.00	252.42	180.95	47.03	7.12	37.11
SE	0.20	0.10	0.18	0.73	0.42	0.67	0.13	0.06	0.11	0.50	0.26	0.44
ERSE	0.21	0.22	0.21	0.79	0.89	0.82	0.13	0.13	0.13	0.53	0.51	0.53
	T=4, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	23.16	15.95	17.49	36.39	23.73	24.17	15.30	12.06	11.10	20.40	14.66	12.51
SE	93.26	91.44	197.21	70.62	65.69	172.42	34.09	38.54	110.72	39.19	39.75	93.38
ERSE	447.14	222.97	438.96	450.35	209.15	384.35	335.59	127.44	243.65	338.75	126.26	208.52
APRB(β_1)	229.43	165.80	194.00	14.89	21.72	28.19	195.36	155.22	143.89	9.32	37.17	41.77
SE	0.04	0.05	0.10	0.20	0.18	0.50	0.02	0.02	0.06	0.10	0.09	0.16
ERSE	0.76	0.11	0.23	1.31	0.58	1.05	0.74	0.07	0.13	0.91	0.18	0.31
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	14.03	20.56	14.96	17.79	29.43	19.55	8.12	12.83	9.06	8.40	14.23	9.78
SE	273.42	148.65	252.01	238.00	133.66	219.43	172.17	85.46	152.02	132.85	83.62	119.12
ERSE	286.58	321.84	297.67	254.19	291.14	264.74	180.06	171.16	179.85	161.00	161.84	162.41
APRB(β_1)	169.21	225.00	174.42	31.07	27.89	30.09	105.94	166.66	117.83	49.26	43.31	45.96
SE	0.15	0.08	0.14	0.69	0.38	0.64	0.09	0.05	0.08	0.22	0.14	0.20
ERSE	0.16	0.16	0.16	0.71	0.81	0.73	0.10	0.09	0.10	0.24	0.24	0.24
	T=7, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	23.08	18.15	16.39	36.50	28.09	25.09	15.27	13.16	12.21	20.48	16.41	15.37
SE	94.94	95.92	174.57	71.15	67.55	151.00	32.87	37.36	78.63	39.12	39.33	72.27
ERSE	437.98	228.51	359.27	438.17	230.93	337.03	327.70	123.97	175.09	330.62	138.46	177.04
APRB(β_1)	209.21	164.49	147.31	50.64	34.76	30.60	179.78	155.90	144.69	34.67	20.46	18.50
SE	0.04	0.05	0.10	0.17	0.17	0.28	0.02	0.02	0.04	0.08	0.08	0.13
ERSE	0.43	0.13	0.20	0.65	0.35	0.56	0.41	0.07	0.09	0.44	0.19	0.25
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	13.62	18.52	14.52	20.80	27.96	22.24	10.90	12.74	11.41	14.02	15.59	14.62
SE	232.97	146.53	213.40	199.00	131.63	182.41	102.60	73.12	92.69	85.87	71.21	79.62
ERSE	266.06	281.96	273.47	258.22	277.78	266.00	150.28	148.42	150.37	157.25	158.82	158.38
APRB(β_1)	118.91	170.46	128.02	23.45	34.44	25.96	128.99	151.23	135.02	15.35	18.41	16.92
SE	0.13	0.08	0.12	0.36	0.24	0.33	0.06	0.04	0.05	0.15	0.12	0.14
ERSE	0.16	0.16	0.16	0.42	0.44	0.43	0.08	0.08	0.08	0.22	0.22	0.22
	T=10, $\beta_0 = 11023.06$, $\beta_1 = -0.4041232$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	23.07	20.87	19.83	36.55	30.09	28.23	15.21	14.72	14.39	20.46	16.96	16.41
SE	93.81	95.54	177.87	71.70	69.28	136.32	33.12	37.97	77.97	39.45	39.59	62.07
ERSE	449.05	277.12	379.29	449.61	245.35	313.93	336.66	164.20	203.31	338.67	132.69	155.85
APRB(β_1)	800.47	783.88	750.71	199.36	167.01	163.54	687.15	670.13	655.11	87.25	97.17	96.44
SE	0.04	0.05	0.09	0.26	0.25	0.32	0.02	0.02	0.04	0.08	0.08	0.10
ERSE	0.64	0.14	0.20	0.73	0.41	0.53	0.61	0.09	0.11	0.53	0.20	0.23
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	18.57	20.53	19.03	26.11	29.05	26.96	14.01	14.49	14.16	15.77	16.43	16.12
SE	221.28	165.86	204.24	165.55	128.37	153.74	91.23	76.74	84.68	67.90	62.02	64.70
ERSE	312.90	321.46	318.67	266.48	277.03	272.21	186.77	186.76	187.05	144.31	145.83	145.37
APRB(β_1)	709.99	781.73	721.99	156.12	165.78	159.10	638.11	660.11	644.77	97.13	96.46	96.55
SE	0.12	0.09	0.11	0.35	0.31	0.34	0.05	0.04	0.05	0.11	0.10	0.11
ERSE	0.17	0.17	0.17	0.46	0.47	0.46	0.10	0.10	0.10	0.21	0.22	0.22

Table D.18: Results under model (2.45), by response and correlation. Population: volatile

	low res			high res		
	T=3, $\beta_0 = 11766.36$, $\beta_1 = -0.3459978$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	1.17	0.13	0.05	1.39	0.04	0.16
SE	268.06	300.19	380.42	182.36	247.96	315.40
ERSE	419.26	308.76	554.98	360.21	240.17	456.11
APRB(β_1)	27.57	2.40	2.11	26.61	0.86	2.76
SE	0.13	0.15	0.20	0.10	0.14	0.17
ERSE	0.70	0.16	0.29	0.70	0.13	0.25
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.53	0.35	0.37	0.67	0.47	0.42
SE	450.36	346.10	431.03	421.05	275.26	388.91
ERSE	348.93	379.64	360.16	313.65	287.53	312.27
APRB(β_1)	21.00	4.10	15.03	13.56	8.67	8.59
SE	0.25	0.18	0.23	0.23	0.15	0.21
ERSE	0.19	0.19	0.19	0.17	0.16	0.17
	T=4, $\beta_0 = 11025.77$, $\beta_1 = -0.4526413$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.42	0.08	0.04	0.21	0.02	0.06
SE	224.91	243.53	315.08	117.45	150.46	203.46
ERSE	350.17	235.21	460.07	314.58	153.63	323.96
APRB(β_1)	5.95	0.62	0.54	2.85	0.28	0.87
SE	0.10	0.12	0.15	0.06	0.08	0.11
ERSE	0.70	0.12	0.23	0.70	0.08	0.18
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.11	0.14	0.07	0.02	0.11	0.01
SE	379.29	274.86	360.69	276.99	168.69	253.05
ERSE	293.38	307.76	302.46	223.83	197.22	222.24
APRB(β_1)	3.70	1.04	2.64	0.34	1.54	0.05
SE	0.19	0.13	0.18	0.15	0.09	0.14
ERSE	0.15	0.15	0.15	0.12	0.11	0.12
	T=7, $\beta_0 = 11025.77$, $\beta_1 = -0.4526413$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.91	0.01	0.18	0.52	0.01	0.18
SE	199.93	217.51	278.77	114.54	145.20	187.33
ERSE	363.82	212.32	418.80	309.08	144.46	286.65
APRB(β_1)	11.20	0.07	2.46	6.28	0.12	2.21
SE	0.09	0.11	0.14	0.06	0.08	0.10
ERSE	0.33	0.11	0.22	0.33	0.08	0.16
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.43	0.05	0.33	0.37	0.13	0.29
SE	352.59	242.38	327.80	249.00	162.46	225.89
ERSE	287.60	269.32	291.52	213.78	183.57	210.17
APRB(β_1)	5.82	0.88	4.49	4.42	1.55	3.47
SE	0.19	0.12	0.17	0.14	0.09	0.12
ERSE	0.15	0.14	0.15	0.12	0.10	0.12
	T=10, $\beta_0 = 11025.77$, $\beta_1 = -0.4526413$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.86	0.00	0.24	0.54	0.00	0.05
SE	221.13	249.88	318.11	146.33	195.77	247.82
ERSE	343.35	250.78	439.81	294.03	194.07	326.54
APRB(β_1)	47.46	0.31	11.82	25.18	0.16	2.34
SE	0.11	0.13	0.17	0.08	0.11	0.14
ERSE	0.48	0.13	0.24	0.48	0.11	0.18
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.44	0.17	0.35	0.14	0.04	0.09
SE	391.16	284.71	364.02	306.47	228.69	281.91
ERSE	333.53	309.82	334.52	265.89	238.02	261.16
APRB(β_1)	23.04	8.93	17.88	6.20	2.02	4.11
SE	0.21	0.15	0.20	0.17	0.13	0.16
ERSE	0.18	0.16	0.18	0.15	0.13	0.14

Table D.19: Results under model (2.41), by response and correlation. Population: simulated

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\beta_0 = 873.4317$, $\beta_1 = -5.392867$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	62.51	0.05	1.38	61.74	0.20	2.46	25.06	0.03	5.34	23.17	0.07	2.46
SE	16.63	11.38	14.24	18.79	12.58	15.97	7.89	7.65	9.72	7.53	8.02	10.23
ERSE	283.14	11.29	20.78	282.54	12.23	22.31	214.28	7.66	15.14	211.62	7.93	15.33
APRB(β_1)	107.63	0.24	19.53	64.20	2.01	3.73	75.49	0.18	11.37	18.85	0.27	1.50
SE	0.38	0.31	0.39	0.96	0.62	0.79	0.18	0.18	0.23	0.30	0.34	0.40
ERSE	5.30	0.31	0.61	6.93	0.54	0.93	4.49	0.18	0.36	5.04	0.32	0.59
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	24.85	12.04	17.57	23.85	12.96	16.49	21.73	1.03	16.60	17.63	1.84	12.88
SE	12.56	14.31	13.16	13.68	16.34	14.46	11.08	8.82	10.79	11.82	9.25	11.46
ERSE	9.75	15.64	11.01	10.35	17.04	11.76	8.79	10.06	9.29	9.02	10.15	9.47
APRB(β_1)	20.67	39.08	10.04	23.13	14.05	15.55	60.13	3.41	45.17	14.75	2.09	10.51
SE	0.40	0.38	0.40	0.66	0.82	0.71	0.27	0.21	0.26	0.46	0.37	0.44
ERSE	0.32	0.42	0.34	0.43	0.74	0.49	0.22	0.24	0.23	0.34	0.39	0.36
	T=4, $\beta_0 = 114.9522$, $\beta_1 = -0.5296455$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	49.24	0.08	6.12	51.43	0.08	6.02	22.78	0.03	6.75	21.04	0.01	5.45
SE	2.26	1.71	2.17	2.13	1.57	1.88	0.98	0.97	1.26	0.92	0.98	1.26
ERSE	18.18	1.68	3.08	18.54	1.54	2.79	13.90	0.95	1.96	13.77	0.98	2.01
APRB(β_1)	105.54	0.13	0.64	48.24	0.93	4.66	91.43	0.14	25.61	14.51	0.17	4.15
SE	0.05	0.05	0.06	0.09	0.06	0.08	0.02	0.02	0.03	0.04	0.04	0.05
ERSE	0.52	0.05	0.10	0.71	0.06	0.11	0.44	0.02	0.05	0.52	0.04	0.08
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	24.37	4.51	18.87	25.40	5.04	19.50	19.33	2.71	14.89	17.81	0.92	13.51
SE	2.29	2.08	2.26	1.84	1.82	1.87	1.50	1.09	1.44	1.54	1.08	1.47
ERSE	1.81	2.33	1.93	1.47	2.10	1.62	1.20	1.25	1.25	1.22	1.27	1.27
APRB(β_1)	36.40	27.69	26.84	23.82	5.79	17.90	78.64	5.65	60.45	14.36	0.80	10.67
SE	0.07	0.06	0.07	0.07	0.08	0.08	0.04	0.03	0.03	0.06	0.04	0.06
ERSE	0.06	0.07	0.06	0.06	0.09	0.06	0.03	0.03	0.03	0.05	0.05	0.05
	T=7, $\beta_0 = 1176.529$, $\beta_1 = 36.56497$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	75.87	0.06	13.89	55.34	0.76	12.03	30.46	0.02	6.57	22.75	0.07	5.46
SE	33.13	22.56	24.51	101.85	61.43	63.38	12.82	11.81	13.95	37.03	41.90	50.10
ERSE	448.57	21.65	34.75	553.79	56.13	75.46	369.42	11.94	20.25	414.98	37.84	60.84
APRB(β_1)	1.94	0.07	0.42	70.96	2.17	10.20	17.81	0.00	4.13	30.68	0.49	6.66
SE	1.56	1.30	1.44	9.17	5.51	5.72	0.29	0.29	0.34	3.39	3.78	4.47
ERSE	49.32	1.22	1.77	70.57	4.98	6.71	41.83	0.29	0.50	53.43	3.38	5.44
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	31.10	5.65	24.71	26.35	5.74	20.84	14.84	5.18	10.99	12.80	3.77	9.53
SE	25.33	23.40	25.38	60.13	62.04	62.25	16.17	12.99	15.44	57.23	43.67	55.41
ERSE	21.53	25.75	23.22	44.32	63.28	49.63	14.80	14.50	15.13	40.06	45.43	42.22
APRB(β_1)	0.48	3.70	0.49	26.05	3.04	19.91	9.47	2.92	7.11	16.05	3.52	12.08
SE	1.51	1.39	1.49	5.42	5.59	5.61	0.41	0.32	0.39	5.08	3.93	4.93
ERSE	1.36	1.43	1.40	3.93	5.63	4.41	0.37	0.35	0.37	3.57	4.07	3.77
	T=10, $\beta_0 = 2222.861$, $\beta_1 = 14.42511$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	42.34	0.03	6.50	52.22	0.16	8.01	28.67	0.01	4.15	23.31	0.02	3.76
SE	79.02	60.93	65.38	67.35	45.48	48.81	23.37	21.36	24.66	29.15	30.28	34.93
ERSE	364.26	60.18	78.03	379.57	43.78	60.53	286.39	21.44	30.08	284.58	28.23	40.46
APRB(β_1)	35.66	0.30	7.53	72.71	2.81	6.19	83.91	0.04	13.13	30.03	0.73	4.29
SE	2.38	2.16	2.33	5.17	3.38	3.56	0.52	0.51	0.60	2.10	2.22	2.55
ERSE	16.69	2.12	2.69	23.48	3.17	4.14	14.18	0.51	0.74	17.55	2.04	2.85
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	13.99	4.21	10.76	16.89	5.68	12.88	8.90	3.79	6.36	8.18	3.26	5.87
SE	67.34	64.23	66.70	50.17	46.91	50.24	27.05	23.64	26.18	38.42	33.34	37.17
ERSE	62.98	67.86	65.20	42.63	48.01	45.38	24.75	24.74	25.20	32.36	32.76	33.01
APRB(β_1)	17.16	1.33	14.32	16.57	3.68	11.74	28.39	11.49	20.56	10.05	3.33	7.18
SE	2.40	2.32	2.37	3.58	3.47	3.61	0.67	0.57	0.64	2.80	2.42	2.72
ERSE	2.37	2.42	2.40	2.94	3.41	3.15	0.61	0.59	0.61	2.26	2.35	2.32

Table D.20: Results under model (2.43), by response and correlation. Population: simulated

	low res and low corr			high res and low corr			low res and high corr			high res and high corr		
	T=3, $\beta_0 = 873.4317$, $\beta_1 = -5.392867$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	55.99	15.20	22.42	48.72	8.29	11.49	21.90	0.72	4.15	21.96	2.28	6.49
SE	15.25	12.47	72.61	14.08	11.21	54.00	7.18	6.98	25.18	7.08	6.87	22.53
ERSE	267.67	82.93	170.33	255.95	68.14	121.25	208.39	33.21	52.84	209.17	31.65	49.96
APRB(β_1)	95.08	13.54	38.22	41.05	3.89	6.57	65.68	0.38	12.17	17.56	6.35	10.97
SE	0.36	0.35	1.59	0.59	0.45	1.64	0.16	0.17	0.39	0.28	0.29	0.62
ERSE	5.13	1.73	3.70	5.98	1.74	3.66	4.41	0.45	0.80	4.92	0.79	1.21
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	4.18	32.01	9.07	7.31	20.48	2.16	18.16	0.07	13.84	21.06	2.23	16.60
SE	102.37	59.28	94.42	72.64	50.01	67.25	31.90	27.03	29.52	27.69	24.87	25.76
ERSE	110.50	132.43	115.58	82.09	101.43	86.11	40.29	47.01	41.72	38.47	45.90	39.96
APRB(β_1)	4.50	56.97	13.43	27.18	1.45	21.36	55.55	1.74	42.33	24.98	7.66	20.59
SE	2.58	1.27	2.27	2.32	1.37	2.13	0.54	0.39	0.49	0.75	0.66	0.70
ERSE	2.67	2.58	2.66	2.40	2.76	2.49	0.64	0.65	0.64	0.91	1.08	0.95
	T=4, $\beta_0 = 114.9522$, $\beta_1 = -0.5296455$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	57.17	17.57	12.69	56.95	16.33	10.65	27.62	8.07	2.03	31.37	7.91	1.77
SE	1.98	1.62	3.32	2.09	1.67	2.98	1.16	1.12	1.98	1.23	1.14	1.72
ERSE	18.28	4.83	7.21	18.45	4.25	5.86	14.30	2.67	3.95	14.90	2.37	3.47
APRB(β_1)	123.98	40.19	42.88	53.29	26.56	23.37	110.51	34.15	11.63	28.77	12.72	8.11
SE	0.05	0.05	0.10	0.10	0.08	0.11	0.03	0.03	0.04	0.05	0.05	0.07
ERSE	0.52	0.13	0.21	0.73	0.14	0.19	0.45	0.06	0.09	0.56	0.08	0.12
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	4.91	24.67	0.18	7.41	21.81	2.03	9.12	5.87	5.25	10.56	6.16	6.32
SE	4.09	3.14	3.85	3.37	2.93	3.24	2.41	1.94	2.25	2.04	1.66	1.93
ERSE	5.04	6.48	5.33	4.11	5.50	4.39	3.06	3.37	3.14	2.57	2.92	2.67
APRB(β_1)	8.79	74.20	17.41	8.92	32.58	13.24	34.68	30.59	19.17	1.69	11.65	1.71
SE	0.13	0.09	0.11	0.11	0.11	0.11	0.06	0.04	0.05	0.08	0.07	0.08
ERSE	0.16	0.18	0.16	0.13	0.18	0.14	0.07	0.08	0.07	0.09	0.10	0.09
	T=7, $\beta_0 = 1176.529$, $\beta_1 = 36.56497$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	77.64	31.42	19.25	62.67	22.27	11.99	32.52	5.97	0.24	23.02	3.24	8.64
SE	29.46	23.93	84.14	79.84	55.07	87.87	13.82	12.62	29.48	34.39	33.57	44.91
ERSE	439.72	166.58	202.73	525.74	160.47	185.55	368.69	62.78	75.67	402.72	77.50	90.94
APRB(β_1)	2.98	4.27	5.40	60.88	5.56	6.71	17.63	4.10	0.45	25.01	7.82	14.28
SE	1.43	1.36	6.73	7.11	4.86	8.30	0.30	0.30	0.88	3.03	2.94	4.47
ERSE	49.04	15.78	18.99	67.01	18.09	18.90	41.92	1.88	2.42	51.61	10.09	11.07
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	3.29	29.38	8.71	2.11	20.85	2.69	8.22	1.71	4.80	15.85	6.97	12.74
SE	98.77	83.13	92.77	95.06	86.42	92.45	33.19	29.75	31.29	49.65	41.98	48.14
ERSE	167.76	194.71	174.28	142.10	180.69	151.66	65.27	70.72	67.21	73.58	82.68	76.91
APRB(β_1)	7.65	1.47	7.35	21.66	1.12	16.33	4.37	2.06	2.48	22.82	12.08	19.19
SE	7.78	6.85	7.21	8.35	8.60	8.28	1.02	0.89	0.94	4.88	4.27	4.72
ERSE	17.00	17.79	17.26	14.99	19.51	16.09	2.22	2.26	2.22	9.25	10.45	9.67
	T=10, $\beta_0 = 2222.861$, $\beta_1 = 14.42511$											
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	59.42	36.98	30.76	56.59	28.83	22.31	29.53	12.79	9.67	20.05	0.28	3.50
SE	41.85	38.61	76.39	71.64	61.68	91.19	22.43	21.94	38.18	33.61	28.16	39.34
ERSE	358.15	154.92	187.01	378.29	158.02	188.79	280.58	77.58	92.04	279.61	76.60	89.43
APRB(β_1)	62.57	35.76	25.46	72.65	49.03	41.20	82.11	34.88	25.02	25.81	2.67	6.95
SE	1.34	1.42	2.89	5.55	4.79	6.55	0.50	0.54	0.88	2.70	2.21	2.83
ERSE	16.49	6.18	7.50	23.54	11.02	13.02	14.14	1.76	2.15	17.47	4.73	5.65
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	24.07	33.74	26.66	14.89	25.58	17.78	6.13	10.24	7.82	7.29	2.94	5.47
SE	89.91	72.59	84.47	102.43	87.81	98.03	43.13	37.44	40.77	43.36	38.25	41.63
ERSE	157.47	171.57	162.67	155.84	173.25	162.07	81.99	84.97	83.53	78.14	82.16	80.16
APRB(β_1)	13.17	33.43	17.29	33.39	45.24	36.29	13.46	27.34	18.72	11.59	6.16	9.37
SE	3.35	2.83	3.13	7.11	6.48	6.88	1.01	0.86	0.95	3.10	2.73	2.99
ERSE	6.82	6.94	6.87	10.81	12.07	11.25	1.95	1.97	1.96	4.87	5.11	4.99

Table D.21: Results under model (2.45), by response and correlation. Population: simulated

	low res			high res		
	T=3, $\beta_0 = 873.4317$, $\beta_1 = -5.392867$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.58	0.27	0.65	2.68	0.39	1.14
SE	152.07	151.17	197.52	87.79	90.27	124.38
ERSE	229.48	147.08	276.44	209.17	90.71	189.96
APRB(β_1)	4.81	0.02	2.93	14.35	1.47	1.76
SE	2.49	2.87	3.87	1.60	1.88	2.48
ERSE	4.24	2.83	5.80	4.27	1.85	3.75
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.33	0.92	0.41	0.15	1.77	0.22
SE	226.00	183.64	218.16	158.62	112.76	148.23
ERSE	171.69	195.13	178.56	128.31	129.29	130.15
APRB(β_1)	1.15	5.38	0.03	8.21	4.87	4.96
SE	5.23	3.44	4.79	3.31	2.20	3.05
ERSE	4.08	3.75	4.03	2.58	2.44	2.57
	T=4, $\beta_0 = 114.9522$, $\beta_1 = -0.5296455$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.12	0.11	0.23	0.47	0.10	0.35
SE	9.57	10.21	12.96	6.99	7.89	10.68
ERSE	14.47	10.21	19.96	12.77	7.51	16.05
APRB(β_1)	0.25	0.04	1.30	3.23	0.57	1.68
SE	0.23	0.29	0.39	0.16	0.19	0.25
ERSE	0.41	0.29	0.63	0.41	0.18	0.38
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.22	0.24	0.22	0.73	0.18	0.60
SE	15.86	11.17	15.01	14.05	9.03	12.98
ERSE	12.69	13.36	13.08	10.99	10.14	11.02
APRB(β_1)	1.37	1.37	1.30	3.61	1.12	2.91
SE	0.56	0.33	0.50	0.34	0.21	0.31
ERSE	0.45	0.39	0.44	0.27	0.24	0.26
	T=7, $\beta_0 = 1176.529$, $\beta_1 = 36.56497$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	5.12	0.17	1.28	0.11	0.18	0.14
SE	203.04	204.69	246.83	104.59	113.25	143.68
ERSE	367.24	204.92	364.05	370.02	115.09	204.38
APRB(β_1)	4.42	1.02	2.28	0.39	0.23	0.19
SE	14.18	14.93	17.42	2.18	2.62	3.48
ERSE	42.70	15.40	21.88	41.98	2.66	5.05
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	2.11	1.11	1.72	0.06	0.27	0.07
SE	298.81	221.96	280.93	179.13	131.77	165.39
ERSE	262.09	257.20	266.24	158.39	144.67	157.25
APRB(β_1)	2.57	2.57	2.25	0.04	0.29	0.10
SE	19.08	17.23	18.28	4.53	3.12	4.12
ERSE	19.60	19.28	19.50	3.88	3.38	3.81
	T=10, $\beta_0 = 2222.861$, $\beta_1 = 14.42511$					
	naïve	**EE_h**	**EE**	**naïve**	**EE_h**	**EE**
APRB(β_0)	0.03	0.05	0.20	1.27	0.07	0.28
SE	173.04	174.87	210.06	100.43	108.61	123.05
ERSE	295.29	173.38	275.98	267.56	110.30	150.17
APRB(β_1)	0.36	0.88	0.55	6.78	0.03	1.83
SE	4.30	4.84	5.84	2.12	2.49	2.88
ERSE	14.92	4.83	7.20	14.24	2.48	3.54
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB(β_0)	0.16	0.23	0.17	0.70	0.16	0.51
SE	244.43	195.55	231.40	134.31	120.86	129.15
ERSE	215.51	209.50	217.79	132.07	129.61	132.04
APRB(β_1)	0.40	1.18	0.10	3.94	1.32	2.94
SE	6.67	5.63	6.30	3.23	2.82	3.07
ERSE	6.16	5.83	6.10	3.09	2.97	3.06

Appendix E

Results for Longitudinal Setting

E.1 Estimation of Change in Mean

Table E.1: Results under model (2.41), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=4, {Stable and Volatile: $\Delta_4 = -746.55$}, {Simulated: $\Delta_4 = -699.5961$ }

	low response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	106.57	0.08	37.62	44.72	0.43	34.78	101.88	2.28	35.95	44.62	0.54	21.51	157.25	0.03	47.43	63.20	0.03	8.66
SE	104.98	80.73	120.04	154.01	128.92	168.12	406.97	312.35	483.58	237.59	206.95	261.83	30.38	14.03	22.19	14.18	9.40	12.17
ERSE	438.71	78.22	162.02	675.10	131.57	223.72	958.29	318.38	661.59	647.51	203.91	348.22	402.35	13.77	28.11	254.76	9.30	16.52
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	11.63	50.25	21.85	13.49	33.67	18.71	10.99	48.27	20.80	5.79	22.37	10.78	6.41	67.20	22.54	14.31	15.08	4.83
SE	139.31	112.07	131.09	189.49	160.78	179.12	566.63	447.40	531.62	307.12	247.79	285.40	18.38	23.39	20.03	11.79	11.74	12.02
ERSE	101.31	114.44	106.33	152.05	162.55	157.29	413.53	466.04	433.88	239.26	250.21	245.89	12.74	22.30	15.32	8.69	11.61	9.80
	low response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	147.25	0.12	46.74	59.73	0.38	53.77	167.02	1.10	52.67	67.29	0.33	37.57	146.81	0.00	46.74	61.23	0.08	8.87
SE	57.14	38.01	61.64	104.82	81.63	103.74	272.40	179.25	291.46	149.54	122.05	157.75	25.70	13.56	20.67	12.95	8.99	11.74
ERSE	448.40	37.25	84.77	681.20	83.58	144.05	1077.61	169.59	390.30	682.33	117.64	199.16	382.12	13.81	27.25	250.25	9.33	16.48
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	8.99	65.00	23.84	12.12	54.52	22.20	11.23	73.22	27.43	5.47	40.18	14.75	7.00	65.42	22.79	13.86	15.17	4.45
SE	71.47	57.74	67.22	110.19	97.43	107.40	333.21	274.31	315.23	179.18	150.19	168.95	17.74	21.32	19.09	11.60	11.20	11.74
ERSE	52.65	60.07	55.39	85.97	100.24	92.31	242.47	278.06	255.55	134.88	145.50	139.99	12.51	21.20	14.95	8.70	11.50	9.79
	high response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	47.53	0.03	13.04	24.07	0.22	19.61	50.40	0.34	15.01	25.41	0.34	13.59	46.86	0.03	11.81	23.41	0.03	2.37
SE	18.63	19.16	26.95	49.80	52.80	65.39	91.83	90.89	129.38	65.33	66.88	82.98	8.49	8.52	9.86	6.24	6.06	7.42
ERSE	257.47	19.09	40.13	518.51	54.05	92.94	620.64	87.86	193.80	521.85	66.47	110.54	224.72	8.37	14.45	190.16	6.08	10.61
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	6.83	20.66	1.67	9.86	22.93	1.09	5.14	22.89	3.45	8.96	16.79	1.54	8.32	19.49	0.29	9.33	4.38	4.16
SE	36.38	23.55	32.16	79.18	59.47	72.75	174.31	113.46	154.15	102.46	77.75	92.91	10.79	9.25	10.49	8.83	6.87	8.21
ERSE	27.29	25.36	27.10	63.05	60.65	63.60	132.19	122.70	131.24	82.26	77.94	81.61	8.09	9.68	8.76	6.93	6.99	7.11
	high response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	48.98	0.10	12.70	24.24	0.19	22.13	67.29	0.04	15.51	32.61	0.00	16.24	46.79	0.07	14.48	24.15	0.01	3.09
SE	10.58	10.75	14.86	59.45	61.21	77.01	53.28	47.68	75.34	52.97	52.71	66.78	7.82	9.01	10.00	6.36	6.55	8.02
ERSE	258.14	10.62	22.04	517.54	59.02	102.42	641.63	48.17	110.73	528.60	53.78	91.82	224.39	8.98	14.49	191.23	6.40	11.01
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	8.16	20.67	0.76	14.01	26.26	3.44	12.42	26.54	0.62	17.95	21.58	6.79	4.87	21.78	3.48	8.95	5.42	3.63
SE	19.70	13.09	17.55	91.65	69.67	84.97	101.55	65.39	89.93	81.05	61.46	74.30	11.25	9.24	10.81	9.62	7.33	8.93
ERSE	14.69	14.02	14.71	67.68	66.34	68.94	74.59	71.02	74.63	63.82	61.90	64.35	8.31	9.56	8.89	7.13	7.19	7.32

Table E.2: Results under model (2.41), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }

	low response and low correlation																	
	naïve	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)
APRB	112.36	0.53	12.08	47.76	1.26	4.95	96.80	1.36	15.52	44.19	2.30	3.83	156.22	0.05	12.73	62.97	0.03	0.28
SE	103.40	79.53	110.59	151.28	127.65	147.96	447.53	354.59	497.54	254.91	226.17	272.63	57.61	27.15	36.69	27.61	18.43	21.32
ERSE	438.89	77.85	145.58	675.40	131.27	188.97	909.33	368.02	682.58	628.27	233.79	337.46	916.51	28.64	54.06	578.91	18.71	29.44
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	11.01	33.60	0.44	15.73	16.14	6.44	6.64	38.37	3.40	10.66	10.97	3.70	17.98	40.06	3.88	11.43	2.72	4.98
SE	124.07	105.66	117.31	157.92	144.53	152.30	572.02	461.39	535.34	304.32	262.40	286.72	32.06	37.88	34.62	22.18	20.48	21.82
ERSE	94.94	112.00	101.25	144.38	150.84	148.79	444.94	519.30	473.59	261.85	268.33	267.60	24.78	44.97	30.51	19.53	21.62	21.02
	low response and high correlation																	
	naïve	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)
APRB	156.16	0.08	12.80	63.57	0.20	5.33	156.41	1.56	15.16	63.56	0.34	3.29	144.69	0.06	12.43	60.34	0.04	0.38
SE	55.76	36.77	50.58	105.40	82.91	97.92	308.21	205.82	288.36	172.01	142.27	158.61	55.30	28.53	36.90	26.85	18.42	21.13
ERSE	448.59	37.08	68.58	681.29	83.33	133.26	1038.50	210.15	390.31	667.36	140.55	195.84	865.02	28.22	53.07	565.64	18.48	29.19
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	18.79	40.49	4.25	27.05	21.22	12.77	16.81	44.26	2.16	19.72	13.40	8.89	17.60	38.25	3.70	11.12	2.77	4.77
SE	54.77	50.82	52.51	105.88	92.18	101.99	323.27	282.02	305.15	167.79	157.03	162.30	32.54	38.19	35.00	22.39	19.97	21.82
ERSE	45.03	55.80	48.47	90.48	95.76	95.15	258.32	312.67	276.79	153.60	160.55	158.13	24.47	42.81	29.95	19.43	21.30	20.83
	high response and low correlation																	
	naïve	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)
APRB	50.28	0.08	1.79	25.33	0.12	1.40	56.65	0.36	1.39	27.78	0.54	0.21	49.07	0.01	1.23	24.35	0.02	0.05
SE	18.95	19.14	25.31	51.46	54.96	65.86	109.84	110.36	146.68	76.58	77.47	86.58	18.27	17.72	21.51	13.09	12.65	14.30
ERSE	257.57	19.05	35.79	518.37	53.92	82.26	610.77	108.48	203.36	515.95	77.01	103.01	513.97	17.68	33.57	432.02	12.35	17.63
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	14.70	10.55	6.33	15.89	6.25	7.62	17.78	11.50	8.04	13.06	2.75	6.53	14.55	9.45	6.54	5.08	0.42	2.03
SE	31.52	22.63	28.26	76.50	61.66	70.79	182.53	133.29	163.41	94.15	85.73	89.69	24.60	18.91	23.28	15.69	13.90	14.93
ERSE	25.49	24.46	25.61	63.48	60.31	63.43	147.90	141.35	148.06	87.83	86.01	87.60	19.40	21.08	20.86	14.00	13.80	14.16
	high response and high correlation																	
	naïve	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)
APRB	51.83	0.02	1.35	25.46	0.44	0.85	61.64	0.27	0.04	29.51	0.40	0.06	48.70	0.01	2.22	24.65	0.04	0.03
SE	10.84	10.93	14.14	56.65	58.61	69.09	62.78	57.42	75.23	53.07	52.75	59.37	16.90	18.00	22.25	12.65	12.73	14.64
ERSE	258.25	10.73	20.15	517.70	58.86	90.18	610.92	58.27	104.88	513.67	53.06	72.75	512.24	18.40	34.50	432.83	12.70	18.72
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	15.45	10.08	6.91	19.02	6.24	9.62	19.49	8.73	9.47	14.67	2.36	7.29	13.82	10.99	5.70	5.44	0.60	2.23
SE	17.16	12.85	15.59	79.38	65.02	73.86	91.61	69.68	82.71	65.11	58.27	61.82	25.78	19.02	24.28	16.65	13.92	15.55
ERSE	13.92	13.67	14.13	68.82	65.97	69.14	78.18	76.78	78.67	60.72	59.18	60.70	19.85	21.42	21.30	14.57	14.20	14.71

Table E.3: Results under model (2.41), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=10, {Stable and Volatile: $\Delta_{10} = -746.55$}, {Simulated: $\Delta_{10} = 190.6555$ }

	low response and low correlation																	
	naïve	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)
APRB	113.67	0.21	2.90	46.81	0.48	0.03	81.16	1.06	5.47	37.58	0.31	0.56	48.19	0.32	3.15	22.83	0.72	1.26
SE	102.96	77.62	103.15	157.96	132.72	147.02	520.23	413.73	556.41	300.83	267.19	298.40	48.58	28.49	37.64	52.85	37.50	41.79
ERSE	439.42	78.05	137.55	675.51	131.28	164.71	846.47	417.72	739.92	604.86	264.29	329.83	594.85	28.22	45.28	483.97	37.27	49.52
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	17.04	20.68	7.22	12.94	4.03	6.20	11.75	21.64	3.43	7.49	2.79	3.10	9.96	16.63	3.72	4.55	4.06	2.43
SE	118.29	96.72	110.28	149.93	145.69	148.21	642.89	513.29	597.57	310.21	295.78	303.07	39.43	38.77	38.28	42.20	41.04	42.08
ERSE	93.28	105.96	98.81	138.94	144.42	142.65	501.20	559.06	528.92	282.50	291.10	288.45	32.14	39.94	34.65	38.81	41.33	40.69
	low response and high correlation																	
	naïve	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)
APRB	159.94	0.25	2.21	64.92	0.10	0.20	136.40	1.94	0.40	54.56	1.83	1.67	84.53	0.53	2.45	36.57	1.08	1.41
SE	57.85	38.12	47.90	104.84	82.55	94.55	354.17	244.79	311.46	191.04	158.79	172.14	27.77	18.08	22.46	49.99	36.55	41.37
ERSE	448.71	37.10	62.31	681.47	83.28	113.50	1016.26	238.08	401.94	657.51	157.70	191.70	566.57	17.88	28.86	475.80	35.44	48.48
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	24.17	23.83	11.05	18.94	5.39	9.01	24.02	18.79	12.25	13.92	1.06	7.23	15.33	18.68	6.61	3.51	4.65	2.07
SE	51.60	48.47	49.45	98.57	92.19	96.56	341.30	308.71	324.51	174.82	171.29	173.16	23.89	23.21	23.00	42.33	40.35	41.95
ERSE	42.67	50.98	45.70	88.62	92.51	92.08	279.78	321.74	296.90	166.00	172.38	170.13	20.46	25.24	22.00	37.14	39.50	39.02
	high response and low correlation																	
	naïve	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)
APRB	51.22	0.07	1.36	25.59	0.14	0.02	44.48	0.55	0.78	23.09	0.88	0.91	20.87	0.17	0.83	11.56	0.31	0.38
SE	18.35	18.80	23.94	51.26	54.51	61.11	120.82	123.68	148.08	87.37	88.81	94.78	19.82	17.91	21.82	24.66	24.10	26.11
ERSE	257.55	19.08	32.54	518.42	53.89	69.58	606.35	122.82	201.05	512.85	85.43	100.78	338.55	17.35	27.19	363.15	23.54	28.86
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	14.32	4.78	7.20	9.01	1.16	3.99	12.34	4.53	5.96	4.77	1.41	1.46	7.82	2.89	4.07	2.69	0.03	1.77
SE	28.66	22.39	25.99	65.56	59.97	63.00	175.39	140.22	159.55	98.28	94.21	96.10	24.71	20.95	23.02	27.06	25.91	26.51
ERSE	24.37	23.42	24.45	59.85	59.02	60.07	156.71	151.36	156.73	92.23	92.26	92.47	21.45	21.42	21.67	25.50	25.58	25.74
	high response and high correlation																	
	naïve	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)	**naïve**	**EE**_h	**EE**($\hat{p}_i$)	**naïve**	**EE**_h	**EE**($\hat{\pi}_i$)
APRB	53.04	0.02	1.46	26.45	0.02	0.14	53.97	0.08	2.48	26.56	0.11	0.17	45.02	0.01	0.75	22.95	0.08	0.07
SE	10.40	10.60	13.51	56.48	58.34	65.25	71.00	65.57	78.79	54.05	53.66	58.06	3.79	3.71	4.62	24.83	24.81	27.33
ERSE	258.41	10.81	18.71	518.04	58.85	75.99	596.17	65.10	98.66	505.98	54.42	64.34	341.29	3.65	6.21	367.54	24.78	31.84
	EE_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	14.53	4.58	7.32	9.95	1.30	4.37	15.36	2.06	8.05	6.81	0.02	2.86	12.50	5.21	6.08	2.16	0.86	1.64
SE	16.24	12.36	14.75	69.69	63.82	67.19	89.46	75.68	83.16	59.98	57.88	58.80	5.29	4.33	4.93	28.78	26.94	27.96
ERSE	13.58	13.22	13.74	64.99	64.52	65.50	80.43	79.43	80.77	58.72	58.69	58.92	4.39	4.45	4.51	27.18	27.15	27.47

Table E.4: Results under model (2.43), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=4, {Stable and Volatile: $\Delta_4 = -746.55$}, {Simulated: $\Delta_4 = -699.5961$ },

	low response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	162.42	0.54	61.98	76.63	11.87	72.80	56.60	158.71	36.42	77.26	9.06	72.00	120.67	18.41	86.33	52.50	13.61	28.89
SE	112.37	61.29	95.99	106.28	84.28	111.43	557.66	438.19	711.80	103.28	94.64	312.73	108.28	68.07	198.10	11.55	9.28	72.75
ERSE	439.58	58.58	119.05	684.21	91.32	159.52	895.18	488.78	1095.54	691.65	312.07	728.57	361.27	118.71	401.36	237.88	76.61	174.85
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	22.65	83.31	38.06	35.03	73.42	43.89	29.12	40.47	31.95	69.35	72.33	70.26	75.18	91.50	79.49	17.57	33.34	22.20
SE	98.69	99.26	96.87	122.28	103.77	117.52	831.09	649.31	782.36	477.73	227.97	405.51	254.60	166.44	232.25	110.16	57.78	94.41
ERSE	76.10	93.82	81.29	100.49	116.31	107.01	673.03	781.67	712.15	489.24	488.53	498.35	245.60	284.18	259.71	118.10	125.19	121.75
	low response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	164.59	0.24	61.68	77.30	11.37	73.78	56.51	164.88	35.45	77.00	11.04	73.54	121.05	15.71	79.89	48.25	8.33	20.01
SE	107.53	56.66	89.53	105.48	82.35	107.06	539.52	419.51	686.81	104.66	95.35	301.65	61.10	37.30	173.73	10.89	8.82	57.79
ERSE	436.73	54.89	111.02	681.35	89.13	155.05	899.24	483.94	1101.11	689.41	307.85	709.13	357.24	105.15	398.20	231.83	65.70	135.86
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	21.09	83.46	37.02	34.25	74.16	43.53	27.39	40.28	30.51	71.61	73.04	72.08	66.39	86.08	71.62	6.44	25.23	12.02
SE	92.47	92.20	90.53	115.88	100.26	112.09	804.58	625.54	756.46	459.97	223.02	390.60	234.45	138.58	210.70	85.27	49.03	73.62
ERSE	70.71	85.96	75.31	95.82	111.68	102.46	675.92	785.26	715.32	477.69	478.93	486.88	243.78	279.52	257.26	94.28	104.16	98.03
	high response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	54.08	0.17	19.47	30.74	5.10	28.09	3.64	89.07	10.66	29.65	2.89	34.27	46.68	1.23	14.63	18.35	1.79	0.76
SE	26.10	25.62	35.44	51.92	55.12	69.59	238.40	272.49	421.14	50.06	56.36	184.43	10.66	8.97	95.86	5.42	5.22	24.23
ERSE	257.08	26.18	56.31	518.67	55.50	95.93	559.39	319.07	699.89	524.73	206.31	451.58	227.09	47.44	203.35	184.29	31.57	54.28
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	1.56	28.49	7.42	0.36	31.06	8.40	14.50	9.12	12.88	39.57	33.57	37.67	0.26	20.77	5.85	11.06	1.34	6.55
SE	47.46	31.33	42.03	85.22	62.93	77.91	580.70	356.56	511.30	306.31	143.88	249.54	146.14	70.67	125.47	32.17	26.11	28.63
ERSE	38.34	35.68	38.08	65.88	63.39	66.34	471.61	444.46	470.76	336.83	302.39	328.86	131.90	130.63	133.87	44.17	47.98	45.30
	high response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	61.40	0.83	21.61	41.53	10.44	36.59	39.94	98.33	16.39	41.26	7.48	28.24	48.36	1.67	7.81	17.93	4.05	4.09
SE	22.68	22.05	30.23	59.82	60.71	76.96	239.37	227.73	381.09	58.54	59.26	150.01	9.77	8.49	76.02	5.40	5.28	19.18
ERSE	260.94	21.50	44.99	521.33	60.48	104.19	594.67	283.13	700.58	527.20	184.84	346.44	229.02	37.07	147.41	184.18	25.24	39.48
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	2.17	31.99	7.94	1.05	41.68	10.10	5.32	20.86	9.89	19.37	30.39	23.07	10.90	15.99	3.27	15.58	1.86	10.56
SE	39.34	27.13	35.25	91.45	69.92	84.83	516.77	324.49	458.64	229.66	131.89	192.00	114.01	56.73	98.49	24.85	20.81	22.33
ERSE	30.33	28.66	30.27	69.09	68.13	70.45	459.25	457.89	466.28	268.27	255.49	265.75	95.01	95.66	96.85	31.87	35.04	32.83

Table E.5: Results under model (2.43), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }

	low response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	161.74	0.25	18.69	76.80	10.40	17.62	77.56	55.01	73.89	77.41	41.63	76.63	99.96	12.24	24.25	44.45	8.22	9.14
SE	121.95	66.10	85.06	107.10	84.89	103.29	550.77	477.93	753.37	104.93	100.13	270.91	197.14	108.76	152.84	19.83	15.95	47.51
ERSE	438.69	58.53	98.57	684.52	91.39	141.10	898.74	572.57	1222.83	691.45	362.65	655.84	764.70	157.48	218.36	508.40	108.31	132.08
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	13.85	48.85	1.05	14.61	33.95	0.34	74.34	72.01	74.15	75.95	77.24	76.18	3.14	44.77	12.77	1.40	10.85	5.59
SE	84.70	92.68	84.11	112.42	97.61	107.82	932.98	613.02	848.62	397.69	215.00	332.47	144.28	175.61	147.04	52.30	48.05	49.28
ERSE	66.65	87.62	72.55	99.13	105.80	104.06	771.21	841.09	815.24	497.47	482.51	501.26	165.47	247.61	185.69	112.36	124.92	118.11
	low response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	165.51	0.72	19.25	77.15	9.67	17.30	78.94	56.16	78.23	76.25	40.55	75.46	101.66	6.93	18.69	37.57	0.26	0.27
SE	106.85	56.46	77.36	105.01	82.71	103.34	538.91	466.13	716.36	106.17	100.96	280.30	135.86	67.89	111.64	18.28	14.56	35.76
ERSE	437.29	54.96	92.92	682.14	89.19	139.96	893.29	570.82	1213.90	688.83	362.97	649.38	746.29	148.35	201.92	487.06	94.06	111.17
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	14.17	49.90	1.15	15.77	33.88	1.16	78.82	76.84	78.55	74.43	76.33	74.80	3.80	39.55	6.47	8.26	1.42	3.91
SE	78.54	83.16	77.31	112.55	96.97	108.07	889.09	588.02	807.57	410.02	221.38	343.54	104.73	130.02	106.94	38.62	36.11	36.71
ERSE	63.17	83.83	68.88	97.04	103.63	102.06	766.45	836.73	810.24	494.80	481.24	498.76	156.48	241.75	177.50	94.40	106.09	99.77
	high response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	54.43	0.63	4.91	30.80	4.47	6.75	38.16	28.07	30.83	31.50	13.29	25.37	43.49	3.55	2.72	12.70	7.61	9.06
SE	26.11	25.53	33.72	48.63	52.03	61.50	202.59	240.49	378.67	50.96	57.23	130.81	19.80	16.55	60.07	8.72	8.01	21.70
ERSE	257.11	26.19	48.06	518.79	55.48	84.02	589.32	301.76	685.25	524.44	196.79	300.42	503.27	86.47	122.23	397.86	55.63	66.45
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	12.28	14.44	3.58	10.52	11.66	2.26	29.92	30.10	30.42	23.81	25.89	24.71	16.45	4.39	9.60	13.31	8.86	10.79
SE	41.89	30.90	37.52	71.46	58.05	66.03	526.18	303.58	450.62	179.11	118.83	152.65	63.38	64.24	61.08	24.03	21.94	22.42
ERSE	34.73	34.02	35.00	65.28	62.26	65.26	474.99	416.72	469.52	257.59	248.23	254.98	100.38	125.48	108.33	59.14	62.08	60.77
	high response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	61.43	0.85	5.88	41.36	10.23	13.41	42.28	27.69	32.83	40.81	21.77	33.89	32.71	4.81	5.20	8.33	8.34	9.87
SE	22.50	21.71	27.84	57.54	58.27	67.66	247.73	249.15	370.33	59.49	60.00	124.60	14.07	12.73	54.26	7.00	6.71	20.02
ERSE	260.94	21.56	39.15	521.26	60.31	91.60	586.62	324.86	659.95	526.75	216.54	300.99	462.09	85.96	116.66	382.06	54.49	62.46
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	12.54	15.94	3.23	7.36	18.97	2.51	31.05	32.39	31.95	31.45	34.18	32.73	16.83	0.02	10.90	13.41	9.78	11.27
SE	33.49	26.09	30.46	77.41	64.28	72.14	501.89	311.26	433.77	159.70	119.69	140.13	58.31	58.02	55.58	22.33	20.20	20.72
ERSE	28.08	27.84	28.42	70.07	67.49	70.50	469.48	436.11	469.41	264.79	261.26	264.36	97.61	116.41	103.82	56.19	58.31	57.39

Table E.6: Results under model (2.43), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=10, {Stable and Volatile: $\Delta_{10} = -746.55$}, {Simulated: $\Delta_{10} = 190.6555$ }

	low response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	161.83	0.45	5.89	76.99	10.14	12.11	49.71	38.09	46.80	76.65	56.75	76.12	64.66	73.64	52.13	20.10	10.24	22.91
SE	117.38	62.69	75.41	105.42	82.91	96.40	575.15	526.15	797.82	104.49	101.05	238.78	219.61	174.75	339.83	52.26	47.30	107.04
ERSE	438.43	58.41	88.26	684.45	91.51	121.94	877.26	621.21	1310.32	691.80	389.52	558.56	558.59	282.65	610.12	485.80	228.83	312.11
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	20.99	28.74	7.70	6.79	17.23	3.03	47.34	44.83	47.08	75.26	76.88	75.68	58.74	39.40	55.80	25.07	24.95	23.19
SE	74.92	80.55	74.63	100.34	94.34	98.30	1005.78	653.92	901.40	302.83	220.45	267.33	441.72	265.52	391.84	132.00	101.43	118.09
ERSE	63.10	80.90	68.58	97.40	102.11	101.10	851.62	845.04	884.04	476.30	478.42	481.70	400.48	444.93	424.48	265.40	276.76	272.45
	low response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	164.70	0.43	6.20	76.69	8.72	10.62	52.41	31.59	51.18	76.50	58.98	78.27	64.95	125.96	140.11	15.88	19.14	15.43
SE	110.13	59.58	73.84	101.87	80.32	93.55	543.55	489.99	755.38	107.33	104.16	234.68	140.81	104.25	246.73	49.74	41.90	85.79
ERSE	436.26	54.76	82.89	681.68	89.23	120.27	877.68	608.72	1275.33	689.12	382.74	542.65	555.19	227.97	484.14	477.90	192.48	249.12
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	20.95	29.03	7.50	8.85	15.95	1.26	51.73	49.81	51.47	77.91	78.73	78.03	161.19	112.40	151.57	24.51	8.40	20.66
SE	74.18	78.44	73.46	97.98	90.99	95.69	956.07	621.42	855.21	296.64	217.22	262.12	321.31	186.68	285.63	102.93	81.21	93.35
ERSE	59.15	75.58	64.23	95.09	99.63	98.80	831.82	828.58	863.57	463.63	466.12	469.08	314.86	373.07	338.38	210.68	225.93	218.76
	high response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	54.23	0.41	0.96	30.84	4.25	4.99	21.15	29.19	4.25	29.25	18.60	25.63	26.50	49.10	59.22	5.83	10.74	14.03
SE	27.00	26.44	33.43	51.46	54.76	61.99	248.17	311.78	455.97	49.95	56.60	129.86	31.74	26.21	76.78	25.76	22.98	40.15
ERSE	257.07	26.18	42.50	518.45	55.50	71.48	559.34	380.99	769.36	524.61	247.22	327.08	345.06	109.93	169.20	370.09	112.05	130.94
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	12.36	7.46	5.05	4.05	6.12	0.98	1.32	5.11	3.00	25.77	25.54	25.78	66.20	53.99	62.78	23.46	11.02	18.95
SE	39.05	31.36	35.85	66.86	60.68	64.07	618.06	385.95	529.10	162.96	123.67	143.45	100.00	65.60	88.13	44.64	39.48	41.94
ERSE	32.63	32.35	32.92	61.68	60.89	61.91	564.02	491.43	552.63	297.36	292.32	295.82	122.92	148.17	131.63	115.99	122.41	119.66
	high response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	61.27	0.69	1.47	40.80	9.70	10.95	4.79	25.21	11.35	41.17	37.33	46.36	20.52	44.49	53.73	7.74	15.00	19.86
SE	22.21	21.80	25.09	55.50	56.04	64.36	262.97	259.58	346.17	60.19	60.43	104.76	21.94	19.20	65.30	20.18	18.39	34.44
ERSE	260.93	21.53	33.52	521.42	60.27	77.62	584.90	326.37	540.61	527.30	217.84	265.81	315.41	95.44	145.97	352.85	101.03	115.99
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	12.69	8.23	4.91	0.20	12.21	6.18	12.87	11.01	12.04	46.68	46.43	46.41	59.86	50.19	56.81	30.07	17.11	25.03
SE	28.35	24.49	26.44	69.07	62.88	66.43	416.64	325.97	376.55	116.75	104.58	109.22	87.94	53.81	76.32	38.21	33.98	35.91
ERSE	25.92	26.48	26.36	66.37	65.98	66.94	430.62	414.65	430.69	245.95	247.11	247.05	105.56	123.94	112.43	103.13	108.51	106.25

Table E.7: Results under model (2.45), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=4, {Stable and Volatile: $\Delta_4 = -746.55$}, {Simulated: $\Delta_4 = -699.5961$ }, low response																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	12.34	0.20	6.51	5.30	0.28	2.40	29.51	2.73	12.57	13.43	0.79	3.00	3.90	0.59	0.88	1.75	0.00	0.27
SE	198.02	230.40	252.44	315.69	341.71	425.73	488.92	644.87	660.51	334.68	375.38	451.92	191.88	205.66	235.30	126.85	129.27	149.93
ERSE	297.75	231.70	336.88	553.40	347.21	579.23	642.59	636.83	831.38	532.38	377.58	627.81	241.82	203.11	306.66	195.91	125.82	208.68
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	4.29	7.52	5.17	0.36	2.86	1.07	5.58	16.20	8.35	2.46	4.99	0.27	0.31	1.56	0.15	0.34	0.44	0.09
SE	294.32	232.57	276.86	521.27	383.06	477.21	772.70	605.21	726.43	561.46	406.37	511.40	268.23	218.66	254.63	179.43	139.77	166.00
ERSE	211.84	233.32	221.00	388.91	382.89	394.53	522.61	575.29	545.16	419.90	410.37	425.38	190.08	215.82	199.95	138.97	140.18	141.68
	T=t=4, {Stable and Volatile: $\Delta_4 = -746.55$}, {Simulated: $\Delta_4 = -699.5961$ }, high response																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	6.02	0.99	4.45	5.06	1.39	2.76	31.61	3.60	12.37	17.31	1.60	2.85	0.43	0.87	0.85	0.01	0.51	0.05
SE	84.33	166.91	144.35	170.45	235.79	278.14	219.33	420.82	380.46	190.72	261.37	298.90	95.02	103.05	136.36	65.58	68.43	85.46
ERSE	209.52	167.06	206.09	470.48	234.71	392.39	488.08	438.97	555.06	460.74	264.08	435.32	194.97	99.27	198.09	176.06	66.82	117.89
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	3.36	4.93	3.83	2.18	2.97	2.54	0.37	16.93	5.58	5.87	4.81	2.01	1.51	0.62	1.23	0.21	0.10	0.13
SE	195.75	125.25	172.84	378.86	243.43	330.46	523.38	327.90	459.65	408.92	264.72	356.31	181.65	120.22	161.43	111.25	79.74	98.87
ERSE	143.15	126.18	139.99	286.94	243.36	276.43	385.77	340.74	377.41	319.16	273.09	308.01	134.75	125.95	134.06	88.21	80.71	86.43
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, low response																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	11.25	0.66	0.51	5.80	0.74	0.14	23.85	0.22	4.45	12.88	1.03	1.66	18.05	0.70	1.37	7.93	0.30	0.50
SE	198.91	235.84	265.26	328.42	359.61	425.52	453.14	545.67	637.83	297.65	321.77	386.68	413.64	319.63	413.42	235.06	207.99	234.36
ERSE	296.71	230.83	372.38	553.93	347.18	554.73	759.02	544.84	911.71	573.20	326.49	525.38	711.85	308.72	547.56	502.42	198.39	263.99
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	4.04	3.20	2.44	1.70	0.79	0.82	2.40	12.27	0.68	2.32	3.48	0.37	3.22	6.09	1.14	2.48	0.09	1.42
SE	316.35	231.28	291.80	502.03	396.27	460.99	761.91	545.03	702.82	464.27	357.98	422.60	464.30	392.08	438.75	249.90	230.96	240.95
ERSE	237.54	255.11	249.55	409.27	387.64	410.25	578.77	611.91	606.85	387.90	367.76	388.86	358.00	421.16	381.80	213.74	221.65	218.86
	T=t=7, {Stable and Volatile: $\Delta_7 = -746.55$}, {Simulated: $\Delta_7 = 1448.408$ }, high response																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	6.52	0.21	2.38	3.94	0.18	0.45	23.50	0.61	3.40	13.07	0.28	0.75	10.05	0.49	0.03	4.83	0.42	0.24
SE	83.59	164.16	185.59	170.25	237.36	277.13	187.31	322.77	372.89	155.28	203.25	240.22	116.81	136.20	172.44	81.00	87.76	101.29
ERSE	208.65	163.62	291.79	470.12	233.26	379.70	541.48	322.87	571.16	486.92	198.49	313.99	489.17	133.11	252.27	418.77	88.85	124.22
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	0.40	4.70	0.97	2.23	0.31	1.28	4.42	7.92	0.46	3.25	1.48	1.08	3.17	1.58	1.54	1.34	0.16	0.69
SE	262.83	138.99	223.41	349.25	256.67	309.07	514.48	294.63	441.73	303.51	219.79	268.26	224.32	148.36	197.14	115.46	98.80	107.16
ERSE	198.27	156.55	192.12	298.38	256.99	287.04	393.56	324.64	383.60	249.91	219.83	241.62	177.68	159.16	175.94	106.13	100.92	104.66
	T=t=10, {Stable and Volatile: $\Delta_{10} = -746.55$}, {Simulated: $\Delta_{10} = 190.6555$ }, low response																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	12.07	0.17	0.63	5.94	0.80	0.90	29.15	1.63	4.19	15.18	1.31	2.69	76.38	10.38	9.36	29.25	0.47	2.75
SE	195.71	228.94	284.94	320.94	346.15	394.39	495.78	638.08	760.43	338.93	377.16	428.18	276.08	309.03	392.52	243.54	246.52	277.69
ERSE	298.42	231.42	410.58	551.18	345.64	469.51	663.69	628.63	1126.96	538.69	371.07	518.81	370.08	312.67	565.21	384.75	245.33	322.10
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	3.54	2.13	2.13	0.20	1.30	0.40	1.03	8.55	1.50	0.94	2.63	1.98	6.47	25.30	1.21	8.76	1.28	5.33
SE	349.28	236.87	316.98	434.07	382.65	411.44	946.37	627.62	853.08	484.03	409.02	452.12	482.76	327.00	437.43	299.40	271.95	286.94
ERSE	266.88	264.86	276.74	388.28	381.93	390.21	727.75	705.80	752.33	423.63	410.30	424.10	367.96	362.90	381.17	272.60	272.98	275.35
	T=t=10, {Stable and Volatile: $\Delta_{10} = -746.55$}, {Simulated: $\Delta_{10} = 190.6555$ }, high response																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB	6.68	0.45	0.61	4.02	0.05	0.45	27.21	0.95	0.97	15.56	1.31	0.97	34.81	1.28	5.76	16.57	0.77	2.73
SE	85.42	167.13	205.45	163.67	228.09	260.73	237.99	410.78	516.19	190.89	249.02	271.43	147.02	185.46	241.91	124.67	135.03	147.62
ERSE	208.47	163.44	328.39	470.11	234.15	316.56	502.46	402.09	781.56	467.36	245.18	334.27	279.68	179.69	343.95	338.74	134.15	161.24
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB	2.78	3.27	1.02	1.23	0.23	0.59	8.12	6.79	3.26	1.97	1.42	0.19	13.66	3.54	9.19	4.75	3.22	3.31
SE	290.88	157.71	244.82	300.26	253.15	276.84	713.38	410.20	606.54	309.50	265.27	286.27	318.61	204.92	276.30	155.69	146.78	150.71
ERSE	227.63	174.67	218.66	275.14	256.69	268.72	548.91	438.72	530.41	288.83	268.89	282.37	250.98	216.62	245.51	147.27	145.98	146.93

Table E.8: Results under model (2.41), by response and correlation. Population: stable

	T=10, t=4, $\Delta_4 = -746.55$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	158.22	0.17	9.89	15.53	28.37	2.66	20.72	5.41	37.76	7.82	58.85	36.67	63.26	47.87	62.81	0.95	0.96	19.68	4.25	9.97	23.70	10.23	23.99	17.18
SE	60.68	39.76	54.61	59.35	54.03	56.65	56.67	61.71	55.52	58.80	58.47	58.82	58.77	58.34	111.31	87.92	103.62	107.77	100.92	105.69	105.46	106.18	104.83	105.99
ERSE	452.81	40.47	67.65	47.12	54.14	50.01	68.39	48.15	54.26	50.80	57.54	46.04	50.52	47.94	683.40	84.91	114.53	90.27	94.21	93.67	101.19	85.12	89.53	88.06
	T=10, t=4, $\Delta_4 = -746.55$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	159.25	0.05	9.74	15.93	28.17	2.90	20.74	5.62	37.68	7.76	58.92	36.49	63.34	47.85	63.54	1.20	0.67	19.71	4.47	9.84	24.26	10.49	24.51	17.59
SE	56.01	36.70	48.09	52.73	47.01	50.14	48.81	53.35	47.99	50.75	51.41	51.37	51.64	51.14	102.84	79.72	93.43	97.66	91.44	95.51	96.92	98.04	96.19	97.61
ERSE	448.59	37.09	62.24	43.19	49.46	45.82	62.91	44.10	49.52	46.51	52.42	41.91	46.00	43.65	680.57	83.25	113.49	88.60	92.50	92.06	99.70	83.35	87.81	86.35
	T=10, t=4, $\Delta_4 = -746.55$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	51.39	0.15	2.93	8.83	7.92	2.26	7.24	3.94	11.56	2.30	20.24	12.71	21.09	16.94	25.65	0.14	0.10	9.21	1.07	4.14	10.05	3.81	9.93	7.25
SE	18.61	19.37	23.94	28.61	22.19	25.96	23.69	28.10	22.13	25.58	21.65	23.20	21.46	22.28	51.14	54.74	64.40	69.65	63.04	66.67	62.43	65.60	62.04	63.80
ERSE	257.56	19.07	30.89	23.58	22.43	23.54	29.74	23.00	21.77	22.88	21.87	19.08	19.03	19.09	518.10	53.89	69.56	59.85	59.03	60.07	59.63	53.35	53.48	53.66
	T=10, t=4, $\Delta_4 = -746.55$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	53.06	0.04	2.77	9.10	7.67	2.45	7.14	4.16	11.40	2.17	20.32	12.61	21.16	16.96	26.05	0.43	0.40	10.55	0.80	4.94	11.09	4.11	10.90	7.93
SE	10.29	10.57	12.66	15.07	11.78	13.72	12.65	14.98	11.83	13.67	11.62	12.41	11.52	11.95	58.36	59.44	71.73	77.48	70.14	74.28	69.29	72.67	68.79	70.81
ERSE	258.38	10.81	17.98	13.28	12.70	13.35	17.33	13.00	12.35	13.01	12.54	10.78	10.80	10.82	517.72	58.84	76.05	65.01	64.51	65.51	65.54	58.31	58.73	58.85
	T=10, t=7, $\Delta_7 = -746.55$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	144.62	0.03	9.24	13.28	24.52	1.80	18.08	5.07	32.25	6.72	31.64	8.35	42.91	20.19	58.74	0.04	0.05	7.33	0.69	2.82	7.46	0.58	7.79	4.81
SE	57.80	28.97	38.63	38.07	37.59	38.72	40.78	40.56	39.40	41.02	42.48	42.00	41.75	42.54	28.69	19.64	22.38	22.81	21.75	22.67	23.07	23.22	22.68	23.22
ERSE	896.79	29.25	55.08	29.91	40.48	34.38	57.01	31.60	41.42	35.95	55.60	32.47	41.17	36.40	571.84	19.23	25.19	19.88	21.30	20.92	24.11	19.67	20.94	20.57
	T=10, t=7, $\Delta_7 = -746.55$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	137.74	0.10	9.74	12.93	25.15	1.33	18.57	4.59	32.75	7.25	31.85	8.77	43.05	20.55	57.98	0.11	0.14	7.34	0.84	2.77	7.60	0.64	7.97	4.92
SE	53.79	29.02	39.50	39.16	37.17	39.84	41.41	41.64	38.99	42.00	42.42	42.76	40.53	43.01	26.17	18.45	21.12	21.80	20.31	21.51	21.68	21.99	21.22	21.90
ERSE	852.12	28.22	53.88	28.48	38.51	32.91	55.55	30.11	39.24	34.35	53.86	30.97	38.88	34.70	561.83	18.46	24.65	19.18	20.50	20.18	23.41	18.90	20.08	19.77
	T=10, t=7, $\Delta_7 = -746.55$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	43.99	0.00	2.14	7.06	5.28	1.84	5.44	3.35	8.18	1.64	11.24	3.43	13.11	7.85	21.60	0.01	0.01	2.91	0.02	0.98	2.95	0.37	2.90	2.09
SE	18.77	18.03	22.18	25.48	20.64	23.76	22.17	25.50	20.65	23.74	21.25	23.78	20.32	22.44	12.80	12.28	13.74	14.31	13.56	13.99	13.54	13.96	13.44	13.72
ERSE	509.64	17.40	29.71	20.55	20.10	21.08	28.60	20.21	19.69	20.63	25.16	18.76	18.45	19.05	428.54	12.08	14.59	12.98	13.09	13.13	13.49	12.17	12.28	12.29
	T=10, t=7, $\Delta_7 = -746.55$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	48.36	0.01	3.04	8.00	7.64	1.85	6.90	3.62	10.89	2.23	13.26	3.92	16.06	9.11	24.51	0.04	0.05	3.51	0.16	1.21	3.52	0.40	3.53	2.43
SE	16.99	17.70	21.78	26.02	19.31	23.88	21.52	25.70	19.34	23.56	20.35	23.71	18.96	21.97	12.55	12.51	14.37	15.29	14.19	14.76	13.88	14.53	13.80	14.16
ERSE	513.02	18.22	32.88	21.24	20.65	21.96	31.53	20.89	20.13	21.44	27.47	19.33	18.78	19.67	433.33	12.55	15.86	13.69	13.71	13.84	14.41	12.71	12.77	12.83

Table E.9: Results under model (2.41), by response and correlation. Population: volatile

	T=10, t=4, $\Delta_4 = -746.55$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	51.00	1.50	8.52	5.11	22.27	1.50	14.43	1.13	26.56	7.53	31.54	22.52	34.41	26.60	26.77	3.05	3.85	2.51	6.00	0.85	12.10	7.90	12.42	10.07
SE	502.24	420.76	542.59	620.43	510.06	578.67	560.51	640.23	524.08	596.93	534.31	556.51	531.92	543.07	295.11	265.16	293.03	306.24	289.21	298.27	294.05	298.90	293.45	295.81
ERSE	834.05	422.63	741.04	507.64	541.98	530.16	739.04	511.39	536.14	530.71	556.95	443.62	473.13	456.95	605.27	267.97	337.84	288.24	295.94	293.99	305.58	269.69	277.52	274.65
	T=10, t=4, $\Delta_4 = -746.55$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	139.73	3.12	5.68	17.77	22.23	5.85	15.29	8.63	30.54	3.51	49.08	29.10	53.01	39.22	56.81	1.45	1.42	13.95	1.32	7.09	14.14	4.70	14.41	9.90
SE	351.32	246.94	334.15	372.88	316.25	351.81	341.72	380.41	323.92	359.09	339.18	347.08	338.05	341.99	182.13	153.06	173.37	177.54	172.62	175.00	175.85	177.57	175.50	176.46
ERSE	1006.58	238.18	398.88	281.81	312.55	296.40	401.22	286.25	312.37	299.45	330.59	267.25	289.34	277.03	657.54	157.32	191.29	165.59	171.92	169.72	178.86	158.72	164.14	162.14
	T=10, t=4, $\Delta_4 = -746.55$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	42.07	0.30	3.51	6.27	7.56	0.81	7.05	2.14	10.54	2.99	17.62	11.55	18.34	14.95	21.17	0.02	0.28	5.19	0.74	2.02	6.30	2.32	6.24	4.64
SE	127.17	126.84	149.26	172.74	142.63	158.86	147.99	169.86	142.58	156.81	143.79	150.93	143.47	146.43	91.36	91.01	96.52	99.20	96.56	97.48	95.09	96.81	95.10	95.67
ERSE	600.24	126.36	197.00	155.88	149.20	155.30	190.55	152.23	145.10	151.26	145.33	128.62	128.35	128.61	512.22	87.50	103.07	94.41	94.53	94.69	94.59	87.60	87.84	87.83
	T=10, t=4, $\Delta_4 = -746.55$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	54.43	0.45	2.54	9.22	6.38	2.47	6.84	4.27	10.19	2.11	19.60	12.03	20.21	16.42	27.09	0.45	0.60	6.00	0.78	2.07	7.71	2.65	7.50	5.67
SE	65.59	60.62	77.44	88.74	73.88	82.07	76.89	87.31	73.82	81.13	72.26	75.98	71.98	73.71	54.29	54.17	61.83	64.33	61.63	62.78	60.93	62.72	60.89	61.60
ERSE	591.64	65.16	94.44	77.95	76.34	78.01	91.76	76.38	74.61	76.29	74.99	67.27	67.45	67.42	505.96	54.36	64.24	58.61	58.58	58.81	58.68	54.21	54.37	54.40
	T=10, t=7, $\Delta_7 = -746.55$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	84.41	4.83	12.55	4.71	27.01	3.80	18.97	1.86	31.81	10.27	26.63	9.88	36.73	18.07	37.52	1.15	0.12	8.46	2.61	3.81	8.17	1.00	9.40	4.87
SE	484.05	412.28	502.92	570.61	487.42	533.33	506.68	571.33	491.71	535.30	489.97	536.52	483.88	509.98	270.82	245.88	271.27	281.79	270.41	275.34	270.89	277.45	270.82	273.29
ERSE	873.43	400.71	687.15	476.11	512.14	497.44	672.26	472.16	503.93	491.25	601.93	437.64	467.43	453.57	615.42	253.13	317.00	271.42	279.25	276.93	292.88	255.85	262.69	260.43
	T=10, t=7, $\Delta_7 = -746.55$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	149.17	0.15	9.70	15.42	26.93	2.66	19.42	6.05	35.29	6.87	32.59	7.29	44.94	20.11	60.83	0.23	0.36	13.67	2.71	6.39	13.04	1.75	14.43	7.92
SE	330.24	227.34	288.23	314.38	289.68	299.02	295.98	321.03	296.36	306.08	287.42	301.86	290.50	292.67	179.57	150.41	167.33	170.82	166.05	168.69	170.45	172.59	169.70	171.24
ERSE	1009.73	237.79	389.72	278.74	311.03	293.40	388.00	280.61	309.65	294.14	351.98	264.14	291.38	276.08	656.91	157.19	190.70	165.28	171.79	169.44	179.92	158.52	163.98	162.00
	T=10, t=7, $\Delta_7 = -746.55$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	55.81	0.36	2.87	9.99	7.65	2.77	7.21	5.04	11.40	1.83	14.74	3.98	17.58	10.01	28.11	0.21	0.24	6.27	0.72	2.42	6.63	1.43	6.70	4.51
SE	122.70	123.98	149.15	174.49	140.81	159.77	148.16	171.82	140.75	158.04	141.67	158.04	137.57	148.45	82.77	82.94	88.54	91.84	88.06	89.76	86.67	88.82	86.49	87.45
ERSE	603.28	122.11	184.38	148.23	143.64	147.98	177.20	144.18	139.64	143.68	154.27	129.92	127.40	129.55	512.73	84.92	100.10	91.56	91.57	91.80	92.23	85.13	85.21	85.30
	T=10, t=7, $\Delta_7 = -746.55$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	58.98	0.18	2.20	9.52	5.18	2.71	6.51	4.66	9.09	1.83	14.58	4.76	16.27	10.47	28.96	0.19	0.05	6.84	0.24	2.71	7.08	1.56	6.97	4.88
SE	72.11	67.09	79.67	85.66	79.40	81.92	80.05	85.64	79.74	82.13	79.31	83.04	79.43	80.63	55.35	54.86	60.83	62.68	60.82	61.53	60.19	61.56	60.25	60.69
ERSE	593.08	64.87	91.32	76.64	75.98	76.90	88.56	74.99	74.23	75.13	79.03	68.74	68.48	68.86	505.35	54.25	64.09	58.48	58.47	58.69	59.06	54.40	54.46	54.57

Table E.10: Results under model (2.41), by response and correlation. Population: simulated

	T=10, t=4, $\Delta_4 = -699.5961$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	145.68	0.04	8.93	14.07	25.37	2.39	18.87	4.82	34.03	7.20	53.46	33.09	57.43	43.42	58.97	0.10	0.02	8.54	1.06	3.49	10.28	2.88	10.44	7.31
SE	32.99	16.75	21.97	21.72	21.85	21.99	23.12	23.01	22.85	23.19	25.33	24.40	25.39	24.91	15.76	10.92	12.43	12.66	12.10	12.58	12.72	12.56	12.63	12.68
ERSE	399.43	15.67	28.85	16.27	21.80	18.51	30.20	17.33	22.35	19.51	26.02	18.35	21.65	19.85	254.05	10.62	14.11	10.99	11.80	11.56	12.88	10.72	11.48	11.20
	T=10, t=4, $\Delta_4 = -699.5961$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	138.99	0.13	9.42	13.75	25.93	1.94	19.38	4.31	34.46	7.77	53.15	33.33	57.00	43.42	58.41	0.08	0.10	8.67	1.27	3.50	10.56	3.06	10.74	7.54
SE	26.16	14.15	19.66	19.76	18.39	19.95	20.91	21.26	19.63	21.31	22.13	21.78	21.98	22.03	13.07	9.27	11.01	11.59	10.47	11.33	11.16	11.22	11.03	11.23
ERSE	379.47	14.23	27.22	14.56	19.59	16.74	28.42	15.57	20.01	17.65	23.40	16.12	18.96	17.45	249.61	9.55	13.28	10.01	10.71	10.55	11.75	9.62	10.32	10.08
	T=10, t=4, $\Delta_4 = -699.5961$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	44.06	0.08	1.84	7.64	5.39	2.28	5.32	3.72	8.42	1.39	15.91	9.65	16.50	13.22	21.68	0.02	0.02	3.43	0.04	1.25	3.64	0.80	3.56	2.64
SE	10.10	9.34	11.50	13.64	10.30	12.51	11.51	13.62	10.40	12.50	10.62	11.32	10.45	10.95	7.13	6.88	7.61	7.95	7.53	7.75	7.51	7.70	7.49	7.59
ERSE	226.90	9.36	16.08	11.22	10.89	11.46	15.56	11.07	10.66	11.25	11.16	9.32	9.47	9.46	190.54	6.67	8.21	7.23	7.26	7.30	7.37	6.66	6.73	6.72
	T=10, t=4, $\Delta_4 = -699.5961$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	48.51	0.03	2.73	8.64	7.82	2.33	6.88	3.90	11.28	2.08	19.38	12.15	20.25	16.17	24.49	0.07	0.11	4.29	0.11	1.68	4.40	1.01	4.36	3.15
SE	8.95	9.33	11.16	13.43	9.92	12.29	10.92	13.19	9.80	12.02	10.03	10.77	9.88	10.38	6.32	6.36	7.15	7.66	7.06	7.37	6.81	7.06	6.78	6.92
ERSE	228.45	8.85	16.41	10.45	10.08	10.81	15.79	10.32	9.81	10.57	10.39	8.29	8.41	8.42	192.44	6.27	8.24	6.93	6.90	7.01	7.04	6.21	6.28	6.29
	T=10, t=7, $\Delta_7 = 1448.408$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	144.62	0.03	9.24	13.28	24.52	1.80	18.08	5.07	32.25	6.72	31.64	8.35	42.91	20.19	58.74	0.04	0.05	7.33	0.69	2.82	7.46	0.58	7.79	4.81
SE	57.80	28.97	38.63	38.07	37.59	38.72	40.78	40.56	39.40	41.02	42.48	42.00	41.75	42.54	28.69	19.64	22.38	22.81	21.75	22.67	23.07	23.22	22.68	23.22
ERSE	896.79	29.25	55.08	29.91	40.48	34.38	57.01	31.60	41.42	35.95	55.60	32.47	41.17	36.40	571.84	19.23	25.19	19.88	21.30	20.92	24.11	19.67	20.94	20.57
	T=10, t=7, $\Delta_7 = 1448.408$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	137.74	0.10	9.74	12.93	25.15	1.33	18.57	4.59	32.75	7.25	31.85	8.77	43.05	20.55	57.98	0.11	0.14	7.34	0.84	2.77	7.60	0.64	7.97	4.92
SE	53.79	29.02	39.50	39.16	37.17	39.84	41.41	41.64	38.99	42.00	42.42	42.76	40.53	43.01	26.17	18.45	21.12	21.80	20.31	21.51	21.68	21.99	21.22	21.90
ERSE	852.12	28.22	53.88	28.48	38.51	32.91	55.55	30.11	39.24	34.35	53.86	30.97	38.88	34.70	561.83	18.46	24.65	19.18	20.50	20.18	23.41	18.90	20.08	19.77
	T=10, t=7, $\Delta_7 = 1448.408$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	43.99	0.00	2.14	7.06	5.28	1.84	5.44	3.35	8.18	1.64	11.24	3.43	13.11	7.85	21.60	0.01	0.01	2.91	0.02	0.98	2.95	0.37	2.90	2.09
SE	18.77	18.03	22.18	25.48	20.64	23.76	22.17	25.50	20.65	23.74	21.25	23.78	20.32	22.44	12.80	12.28	13.74	14.31	13.56	13.99	13.54	13.96	13.44	13.72
ERSE	509.64	17.40	29.71	20.55	20.10	21.08	28.60	20.21	19.69	20.63	25.16	18.76	18.45	19.05	428.54	12.08	14.59	12.98	13.09	13.13	13.49	12.17	12.28	12.29
	T=10, t=7, $\Delta_7 = 1448.408$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	48.36	0.01	3.04	8.00	7.64	1.85	6.90	3.62	10.89	2.23	13.26	3.92	16.06	9.11	24.51	0.04	0.05	3.51	0.16	1.21	3.52	0.40	3.53	2.43
SE	16.99	17.70	21.78	26.02	19.31	23.88	21.52	25.70	19.34	23.56	20.35	23.71	18.96	21.97	12.55	12.51	14.37	15.29	14.19	14.76	13.88	14.53	13.80	14.16
ERSE	513.02	18.22	32.88	21.24	20.65	21.96	31.53	20.89	20.13	21.44	27.47	19.33	18.78	19.67	433.33	12.55	15.86	13.69	13.71	13.84	14.41	12.71	12.77	12.83

Table E.11: Results under model (2.43), by response and correlation. Population: stable

	T=10, t=4, $\Delta_4 = -746.55$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	162.64	0.68	10.81	15.57	30.10	2.25	18.65	8.49	36.39	5.26	64.83	42.02	69.52	53.42	76.13	9.19	11.10	7.84	16.25	2.00	37.93	24.48	38.22	31.39
SE	112.33	61.78	78.04	79.83	80.68	78.41	81.83	83.72	83.66	82.20	91.57	88.37	92.53	89.57	111.51	88.03	106.77	112.33	103.25	109.50	107.82	109.50	106.85	108.79
ERSE	438.68	58.43	89.26	64.11	77.36	68.76	91.06	65.74	76.82	69.90	85.86	69.48	77.11	72.65	684.59	91.51	122.08	97.46	102.15	101.17	111.17	94.44	99.35	97.63
	T=10, t=4, $\Delta_4 = -746.55$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	164.56	0.24	10.51	16.26	29.92	2.72	18.45	9.14	36.36	4.86	65.20	42.05	69.93	53.65	77.74	9.45	11.74	7.73	17.06	2.38	39.27	25.46	39.54	32.56
SE	110.84	60.08	78.01	78.61	81.11	77.82	81.64	82.29	84.31	81.42	90.66	87.36	91.77	88.60	106.69	83.79	100.37	105.79	96.85	103.09	100.84	102.25	99.98	101.72
ERSE	436.77	54.86	84.26	60.80	73.98	65.34	86.12	62.34	73.22	66.37	81.61	66.16	73.55	69.22	681.74	89.27	120.61	95.38	99.93	99.10	108.90	92.02	96.91	95.22
	T=10, t=4, $\Delta_4 = -746.55$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	54.08	0.30	2.86	9.60	8.16	2.66	5.75	6.30	10.43	0.43	22.75	14.97	23.67	19.30	30.80	4.23	4.73	4.40	5.89	0.68	15.82	9.69	15.70	13.06
SE	27.19	25.76	31.08	35.77	29.79	33.05	31.38	35.88	30.16	33.26	30.40	32.45	30.11	31.22	50.17	53.17	62.20	67.52	60.74	64.48	59.16	61.95	58.81	60.36
ERSE	257.03	26.14	41.10	32.01	31.23	32.11	40.13	31.64	30.69	31.63	31.69	27.72	27.58	27.68	518.88	55.49	71.50	61.69	60.91	61.93	61.83	55.47	55.62	55.79
	T=10, t=4, $\Delta_4 = -746.55$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	61.40	0.86	3.74	9.63	9.32	2.17	6.39	6.69	11.38	0.64	25.29	16.63	26.29	21.47	41.25	10.03	11.39	0.62	12.68	6.61	24.93	17.60	24.75	21.63
SE	23.62	23.00	27.22	30.53	26.67	28.60	27.35	30.66	26.85	28.71	26.57	28.31	26.34	27.26	61.14	61.88	73.18	78.23	71.94	75.40	71.21	74.23	70.82	72.56
ERSE	260.98	21.53	32.84	25.73	25.74	25.99	32.59	25.80	25.57	25.95	26.88	23.37	23.27	23.35	521.10	60.28	77.62	66.38	66.00	66.95	67.26	59.85	60.36	60.44
	T=10, t=7, $\Delta_7 = -746.55$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	161.88	0.26	11.05	14.67	28.66	1.63	17.77	8.68	34.08	4.76	38.65	12.09	51.81	25.48	75.49	8.94	11.07	7.77	16.25	2.02	32.98	17.29	35.24	25.34
SE	115.85	62.44	74.03	73.07	78.78	72.96	76.95	75.92	80.93	75.81	79.41	75.46	83.49	76.94	104.39	82.08	99.20	104.21	96.08	101.68	100.48	103.86	98.54	102.22
ERSE	439.63	58.70	88.26	63.66	76.27	68.21	89.16	64.64	75.43	68.76	83.03	62.73	73.23	66.65	684.63	91.45	121.82	97.25	101.92	100.95	115.00	95.41	99.76	98.57
	T=10, t=7, $\Delta_7 = -746.55$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	164.71	0.33	11.45	14.74	29.34	1.44	18.30	8.63	34.89	5.09	39.68	12.68	53.05	26.31	76.13	8.17	10.07	9.29	15.30	0.76	32.46	16.30	34.72	24.59
SE	109.57	57.18	72.73	72.40	76.53	72.11	75.27	74.90	78.66	74.58	77.68	74.97	80.89	75.93	96.95	76.03	91.82	97.90	87.69	94.77	92.18	96.29	89.54	94.24
ERSE	436.05	54.58	81.94	59.28	71.30	63.55	83.08	60.41	70.75	64.29	77.25	58.47	68.32	62.11	681.65	89.25	120.43	95.21	99.76	98.93	113.19	93.20	97.47	96.37
	T=10, t=7, $\Delta_7 = -746.55$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	54.10	0.24	3.35	8.75	8.13	1.99	6.02	5.72	10.24	0.85	16.13	6.00	19.06	11.63	31.07	4.40	5.12	4.01	6.31	1.07	14.93	8.07	15.26	11.83
SE	28.66	27.73	32.69	36.40	32.31	34.16	32.96	36.45	32.63	34.33	31.66	33.98	31.61	32.55	52.16	55.55	65.26	71.01	63.53	67.72	62.82	66.84	61.97	64.53
ERSE	257.15	26.21	40.16	31.62	31.02	31.74	39.27	31.26	30.51	31.29	34.04	28.20	27.86	28.21	519.37	55.50	71.37	61.61	60.85	61.85	63.80	56.50	56.20	56.71
	T=10, t=7, $\Delta_7 = -746.55$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	61.33	0.79	3.80	9.28	8.85	1.96	6.20	6.63	10.74	0.57	18.08	7.00	21.22	13.17	41.47	10.27	11.79	1.22	12.89	7.10	23.86	15.85	24.06	20.24
SE	21.79	21.24	25.04	28.98	23.83	26.71	25.31	29.18	24.15	26.94	24.01	26.64	23.38	25.11	57.49	57.73	66.93	72.33	65.09	69.30	65.56	69.39	64.54	67.25
ERSE	261.02	21.63	33.40	26.18	25.96	26.38	33.15	26.23	25.77	26.32	28.71	23.65	23.48	23.71	521.25	60.28	77.64	66.38	65.99	66.95	69.52	61.02	61.04	61.50

Table E.12: Results under model (2.43), by response and correlation. Population: volatile

	T=10, t=4, $\Delta_4 = -746.55$, low response and low correlation.																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	62.61	36.64	54.87	53.06	56.44	53.95	57.43	55.66	58.97	56.50	60.36	59.07	60.67	59.63	76.15	50.80	71.32	70.49	71.38	70.93	75.74	75.39	75.68	75.55
SE	560.13	506.42	758.62	943.24	663.70	847.96	752.84	932.45	662.00	839.17	674.92	742.24	660.30	706.11	107.85	104.49	240.33	314.75	218.40	273.40	207.73	247.09	202.79	224.92
ERSE	898.62	611.34	1318.17	872.93	848.85	898.73	1270.18	849.77	820.79	870.83	880.74	682.97	704.28	697.67	691.90	392.43	584.21	491.12	486.21	494.85	492.74	433.46	438.51	438.29
	T=10, t=4, $\Delta_4 = -746.55$, low response and high correlation																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	53.68	31.29	46.67	44.23	49.17	45.41	49.94	48.09	51.40	48.93	51.27	50.01	51.51	50.53	77.05	51.96	71.41	69.58	71.89	70.52	78.16	77.56	78.16	77.79
SE	553.27	502.28	770.31	982.46	626.28	874.62	765.53	974.23	631.23	867.66	649.55	724.75	629.36	686.55	108.08	104.67	226.14	282.27	211.99	250.77	194.13	225.49	191.30	207.63
ERSE	894.72	594.93	1289.61	855.63	831.78	880.69	1246.94	835.21	805.77	855.68	858.79	667.74	690.34	682.73	689.05	385.34	550.63	469.95	471.92	475.32	479.36	423.11	429.41	428.33
	T=10, t=4, $\Delta_4 = -746.55$, high response and low correlation																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	5.63	20.01	29.00	34.53	26.97	31.45	28.03	33.57	26.10	30.48	19.65	22.95	19.34	21.10	30.22	32.52	42.21	44.07	42.41	42.88	41.55	43.42	41.68	42.21
SE	250.33	324.18	489.09	669.51	401.52	570.49	476.62	649.94	394.92	554.64	366.00	429.46	354.24	395.15	53.02	60.09	130.05	156.93	127.17	140.98	113.79	132.78	112.97	121.26
ERSE	559.81	386.27	761.49	563.13	481.25	548.06	715.55	535.38	459.08	520.11	454.43	390.36	375.22	385.42	524.90	257.59	339.13	310.41	306.45	308.95	299.21	276.30	273.78	275.23
	T=10, t=4, $\Delta_4 = -746.55$, high response and high correlation																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	40.88	27.47	35.12	33.56	35.25	34.39	37.11	35.86	37.14	36.51	41.38	40.87	41.39	41.09	41.65	29.26	36.02	34.51	36.14	35.37	42.02	41.49	42.02	41.74
SE	237.11	241.88	324.39	413.83	291.37	363.00	317.35	402.38	288.03	353.89	288.93	318.86	285.48	301.54	58.47	58.63	102.77	116.08	102.27	107.69	96.34	106.60	96.48	100.01
ERSE	594.60	327.32	557.61	440.38	410.43	436.66	538.50	427.39	397.13	422.92	407.93	357.52	354.19	357.53	527.04	220.01	268.11	247.69	248.56	248.73	246.58	229.07	229.85	229.84
	T=10, t=7, $\Delta_7 = -746.55$, low response and low correlation.																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	72.60	38.04	55.70	51.42	59.32	53.52	57.96	54.27	60.44	56.05	59.10	55.00	61.61	56.99	77.01	51.00	68.80	66.47	69.51	67.69	72.29	70.77	72.51	71.54
SE	548.00	481.74	768.39	959.56	637.46	862.26	757.44	945.40	634.99	849.51	704.25	853.79	626.65	777.85	106.34	103.61	238.67	307.75	216.58	269.42	210.18	261.96	197.72	233.24
ERSE	893.43	579.39	1177.53	794.86	797.57	821.30	1142.88	780.25	780.03	803.28	1011.33	715.80	724.41	734.57	691.12	377.36	544.22	462.11	463.16	467.35	493.46	429.21	432.48	433.84
	T=10, t=7, $\Delta_7 = -746.55$, low response and high correlation																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	77.28	43.74	63.91	59.27	68.13	61.51	63.71	58.97	67.78	61.27	68.28	64.22	71.03	66.12	77.04	51.08	68.43	65.48	69.48	67.01	72.70	70.77	73.11	71.72
SE	536.19	471.77	707.42	888.50	613.39	794.62	712.02	889.55	620.01	797.08	657.71	793.34	605.57	722.62	106.77	103.41	214.24	270.53	197.47	239.08	192.23	233.04	184.22	210.01
ERSE	890.12	578.31	1197.95	806.00	798.96	831.14	1165.25	791.77	780.54	813.30	1026.36	724.80	727.64	742.65	688.66	376.71	530.78	455.09	459.48	461.08	486.54	425.65	430.59	430.69
	T=10, t=7, $\Delta_7 = -746.55$, high response and low correlation																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	37.33	12.74	17.20	11.08	20.38	14.43	20.18	14.68	22.84	17.67	25.08	20.67	26.86	23.03	31.55	15.37	20.80	18.89	20.96	20.06	24.61	23.35	24.65	24.13
SE	193.77	230.54	348.72	456.77	300.48	397.02	335.23	434.38	295.16	379.47	304.77	379.56	280.36	338.22	50.07	55.75	107.89	126.75	106.20	115.36	98.24	113.18	97.60	104.02
ERSE	589.64	301.58	546.47	421.72	384.99	415.30	516.19	402.89	368.44	396.15	440.62	358.14	335.51	352.76	524.46	201.64	257.27	236.13	234.75	235.83	236.07	218.14	217.05	217.79
	T=10, t=7, $\Delta_7 = -746.55$, high response and high correlation																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	45.30	25.62	34.37	31.68	35.08	33.16	36.70	34.52	37.11	35.71	40.23	38.72	40.42	39.52	41.21	28.00	35.24	33.77	35.30	34.62	38.48	37.60	38.45	38.10
SE	253.55	248.67	333.19	392.75	315.96	358.79	328.97	385.28	313.48	353.13	314.92	357.39	306.02	333.14	60.25	60.56	99.78	110.14	99.69	103.56	93.34	102.40	93.45	96.58
ERSE	587.50	307.21	476.29	390.20	379.17	390.54	463.72	383.19	372.41	383.00	414.14	352.41	345.86	352.22	526.32	212.09	257.13	238.10	239.56	239.35	243.32	226.17	227.05	226.99

Table E.13: Results under model (2.43), by response and correlation. Population: simulated

	T=10, t=4,$\Delta_4 = -699.5961$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB	80.59	30.66	49.28	39.01	61.02	43.91	56.02	46.20	66.26	50.82	72.47	66.39	74.69	68.91	66.61	42.54	51.10	47.37	52.28	49.38	57.48	55.03	57.78	56.32
SE	218.76	165.07	257.11	308.33	234.14	281.89	261.72	312.20	241.05	285.73	238.32	254.77	235.61	245.43	17.75	15.67	75.24	95.07	69.55	84.32	65.78	79.42	64.21	71.87
ERSE	362.56	195.30	396.30	268.28	293.13	282.14	397.96	272.66	294.58	285.32	307.40	239.19	257.47	247.53	268.94	131.69	190.39	161.53	167.77	165.31	173.40	151.07	155.76	154.02
	T=10, t=4,$\Delta_4 = -699.5961$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB	137.55	27.89	50.13	28.30	74.17	38.66	61.04	39.03	83.01	49.38	98.68	82.48	104.16	89.54	81.54	35.35	41.16	32.98	43.45	37.42	52.36	46.09	53.04	49.47
SE	238.43	136.49	202.33	202.76	221.85	201.40	216.41	219.20	232.38	216.34	240.77	240.63	243.60	238.86	17.48	13.35	48.56	54.63	48.46	51.15	46.85	52.91	46.70	49.35
ERSE	437.36	155.85	252.15	186.80	252.87	204.66	270.08	200.01	260.79	217.39	285.67	225.03	256.52	237.09	285.43	109.55	139.50	119.65	130.89	124.98	138.70	120.54	128.00	124.59
	T=10, t=4,$\Delta_4 = -699.5961$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB	121.30	24.85	36.35	18.11	46.64	27.60	41.33	23.35	50.80	32.69	62.80	49.16	65.41	56.20	52.46	12.89	14.63	8.57	15.15	12.15	20.20	14.95	20.33	18.08
SE	103.47	60.51	78.27	72.11	83.78	74.96	81.22	75.33	86.25	78.01	94.21	90.77	95.71	92.04	12.79	10.05	20.67	22.18	20.64	21.15	21.28	22.79	21.38	21.74
ERSE	362.97	84.49	104.58	86.88	109.08	94.55	109.00	90.93	112.33	98.49	129.57	110.48	123.55	116.27	237.88	55.36	64.85	57.69	61.31	59.73	68.02	61.00	64.02	62.80
	T=10, t=4,$\Delta_4 = -699.5961$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB	120.64	26.22	37.15	19.89	45.48	29.07	41.08	24.12	48.74	33.13	58.51	45.35	60.70	52.36	48.21	9.27	10.46	4.65	10.83	8.15	14.89	9.72	14.96	12.85
SE	65.68	39.11	59.74	57.56	62.76	58.40	61.53	59.63	64.25	60.29	68.70	68.38	69.59	68.07	10.54	8.45	16.92	17.90	16.99	17.18	17.37	18.33	17.49	17.60
ERSE	356.34	72.82	95.33	80.02	97.29	86.33	98.44	82.95	99.54	89.13	113.21	97.31	107.70	102.06	231.77	45.10	52.20	46.62	49.37	48.22	54.20	48.76	51.13	50.20
	T=10, t=7, $\Delta_7 = 1448.408$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB	101.62	28.08	47.13	32.89	61.04	39.75	55.07	40.87	67.80	47.64	65.56	51.83	75.66	58.25	64.11	32.59	37.51	32.88	38.37	35.51	44.18	40.24	44.70	42.46
SE	317.46	229.41	404.02	487.66	357.73	444.60	411.09	492.30	369.83	449.98	403.09	466.09	380.33	432.83	31.86	26.99	96.92	112.86	94.40	103.67	94.60	108.38	93.36	100.24
ERSE	951.10	334.93	662.17	460.18	530.01	489.23	671.05	473.62	543.71	502.31	643.54	474.99	548.99	502.98	596.70	215.85	278.66	243.76	257.19	251.19	279.42	247.00	258.52	253.61
	T=10, t=7, $\Delta_7 = 1448.408$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB	140.26	27.09	46.92	24.86	69.40	35.37	56.81	34.53	77.56	45.06	70.41	48.45	86.83	58.70	73.71	28.92	32.27	25.49	33.48	29.34	40.20	34.16	40.98	37.58
SE	340.59	217.48	331.32	340.53	352.64	334.32	354.87	369.74	370.13	360.19	362.77	372.82	376.66	365.58	32.13	24.64	85.95	97.28	84.45	90.61	90.38	102.72	88.81	95.39
ERSE	1065.57	322.46	479.30	363.68	506.61	402.66	522.07	395.86	533.17	435.01	538.82	419.71	545.16	457.69	616.72	208.94	251.29	218.55	235.72	227.61	266.29	233.57	248.35	241.76
	T=10, t=7, $\Delta_7 = 1448.408$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB	99.19	12.18	20.08	3.99	28.55	12.39	24.06	8.17	31.85	16.45	32.66	17.75	38.32	25.51	39.66	5.48	6.00	1.54	6.18	4.32	8.93	4.76	9.01	7.37
SE	167.47	91.13	110.08	100.02	117.21	104.81	113.85	103.86	120.67	108.58	117.82	108.59	122.58	112.90	19.55	15.79	31.13	32.75	31.20	31.54	31.79	33.46	31.91	32.19
ERSE	743.82	129.57	158.56	131.22	165.19	143.37	164.47	136.69	169.59	148.70	172.87	145.11	173.28	156.14	493.09	85.38	97.42	87.84	92.56	90.71	100.23	90.78	95.12	93.48
	T=10, t=7, $\Delta_7 = 1448.408$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB	93.83	12.02	19.05	4.07	25.91	12.03	22.15	7.39	28.47	15.23	29.59	15.82	34.13	23.12	33.43	1.58	1.63	2.50	1.73	0.11	3.80	0.10	3.84	2.37
SE	100.59	56.16	79.31	76.07	82.19	77.36	81.20	78.08	83.90	79.26	85.00	82.08	87.23	83.09	15.85	13.18	24.53	25.78	24.59	24.83	24.95	26.23	25.04	25.25
ERSE	709.19	124.58	153.13	128.22	157.03	139.12	157.58	132.42	160.30	143.16	167.64	142.27	166.68	152.28	472.51	76.99	85.91	77.80	81.69	80.22	87.82	79.81	83.44	82.11

Table E.14: Results under model (2.45), by response and correlation.

	T=10, t=4, $\Delta_4 = -746.55$, low response. Population: **stable**																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	12.36	0.46	2.10	0.37	3.00	1.24	2.98	1.03	4.14	2.01	6.53	4.98	6.80	5.75	5.86	0.53	0.54	0.38	0.54	0.13	1.41	0.49	1.46	1.01
SE	198.63	236.48	306.33	389.03	243.10	347.15	300.56	377.92	246.68	338.45	252.70	282.83	244.75	267.56	334.68	361.49	428.15	475.04	414.26	448.32	408.82	432.63	405.32	419.16
ERSE	296.82	228.14	436.44	283.09	259.16	289.01	415.82	273.94	251.13	278.14	266.09	205.76	207.82	208.47	551.76	345.48	477.67	391.57	381.83	392.80	389.56	339.79	341.79	342.54
	T=10, t=4, $\Delta_4 = -746.55$, high response. Population: **stable**																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	6.37	0.27	1.69	0.88	3.57	0.47	2.43	0.14	3.95	1.33	4.39	3.20	4.71	3.78	4.32	0.22	0.76	0.04	0.83	0.47	1.42	0.86	1.44	1.19
SE	82.71	160.68	195.19	276.58	148.04	232.42	185.02	259.93	144.71	219.15	136.04	164.85	129.76	149.54	165.20	227.83	272.16	314.77	262.35	289.69	240.68	263.93	238.49	250.24
ERSE	209.44	166.70	303.47	215.16	167.39	205.95	277.75	199.95	156.71	190.81	154.06	129.89	119.51	126.12	470.41	234.88	312.85	272.50	255.03	266.39	248.86	226.66	219.48	223.36
	T=10, t=7, $\Delta_7 = -746.55$, low response. Population: **stable**																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	11.89	0.05	0.77	4.13	2.36	2.47	0.32	3.03	3.11	1.38	1.97	1.19	4.10	0.35	4.75	0.74	1.98	3.55	1.44	2.69	0.67	2.08	0.31	1.31
SE	203.67	243.15	276.49	329.76	249.64	302.00	270.74	319.89	248.07	294.15	255.36	296.54	239.64	275.02	320.79	343.09	406.73	452.38	393.78	426.27	385.45	416.30	379.04	398.71
ERSE	297.64	232.25	393.98	261.85	253.72	268.88	372.22	251.36	244.16	257.12	331.97	231.62	227.07	235.76	550.17	344.97	476.57	392.70	384.56	394.20	413.18	351.68	348.52	353.28
	T=10, t=7, $\Delta_7 = -746.55$, high response. Population: **stable**																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	6.62	0.13	0.95	1.84	2.67	0.37	1.61	1.01	3.15	0.36	2.51	0.31	3.53	1.45	5.91	2.19	2.69	1.81	2.93	2.32	3.47	2.81	3.60	3.19
SE	84.14	165.90	194.89	264.76	162.61	226.45	184.26	250.49	154.90	214.12	160.21	209.77	142.51	182.80	162.62	226.15	269.19	313.69	258.15	287.49	246.28	276.38	241.27	258.69
ERSE	209.04	165.02	292.25	210.81	169.35	202.45	271.40	197.58	158.12	189.02	217.47	165.74	139.44	159.01	470.18	233.51	320.93	277.12	256.77	270.22	267.80	238.24	225.88	233.44
	T=10, t=4, $\Delta_4 = -746.55$, low response. Population: **volatile**																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	29.64	0.44	0.86	6.52	7.81	2.88	1.95	6.06	9.06	2.09	12.78	7.36	14.21	9.95	12.88	1.06	2.87	6.24	1.89	4.37	0.95	1.60	1.20	0.18
SE	504.25	652.56	785.48	976.35	653.56	879.16	770.41	957.44	651.63	861.35	656.09	731.19	637.57	693.00	354.33	397.09	440.39	496.17	422.64	464.30	416.65	442.12	412.60	427.58
ERSE	639.39	631.24	1116.95	730.75	683.95	747.48	1071.63	709.06	655.79	720.34	672.68	520.09	524.04	526.28	530.61	376.53	532.52	433.97	419.28	434.09	424.75	369.41	369.70	371.63
	T=10, t=4, $\Delta_4 = -746.55$, high response, Population: **volatile**																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	31.74	1.85	0.66	12.40	6.58	6.07	1.90	9.52	8.55	3.35	12.78	6.26	14.00	9.70	17.28	0.99	1.85	5.72	1.52	3.39	1.58	1.59	1.66	0.34
SE	223.49	428.16	536.76	774.62	396.53	645.77	495.35	710.09	379.50	593.57	357.59	434.62	341.46	393.92	195.62	266.47	306.35	352.00	296.84	325.10	278.35	305.07	275.80	289.25
ERSE	487.50	438.12	803.74	563.77	428.68	538.44	738.63	525.26	401.73	499.98	399.18	336.07	309.80	326.74	460.28	263.84	349.92	306.52	288.59	300.08	284.08	258.42	249.96	254.58
	T=10, t=7, $\Delta_7 = -746.55$, low response. Population: **volatile**																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	21.38	4.63	0.84	6.91	4.26	3.94	1.27	4.85	6.00	1.87	3.63	2.18	7.14	0.64	12.43	0.30	0.86	1.66	1.49	0.28	3.57	1.45	3.88	2.61
SE	446.92	535.80	629.90	758.44	561.67	691.61	618.27	738.89	559.29	675.78	586.16	686.76	545.00	634.27	288.53	312.07	348.88	388.65	338.60	365.80	333.95	360.67	328.72	345.33
ERSE	757.94	547.30	927.71	617.67	600.70	634.37	889.52	599.31	578.79	612.37	779.35	544.18	533.97	553.87	572.27	326.27	441.24	364.71	358.83	366.62	385.48	328.90	327.12	330.79
	T=10, t=7, $\Delta_7 = -746.55$, high response. Population: **volatile**																							
	naïve	**EE**$_h$	**EE**($\hat{p}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{p}_{i2}$)	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**($\hat{\pi}_i$)	**EE**_i	**EE**_ii	**EE**(0.5)	**EE**($\hat{\pi}_{i1}$)	**EE**_i	**EE**_ii	**EE**(0.5)
APRB	23.41	0.81	3.99	1.04	5.70	1.74	5.65	0.78	7.27	3.47	8.42	4.24	9.41	6.54	12.29	0.83	0.03	2.14	0.06	0.80	2.22	0.36	2.13	1.52
SE	192.41	320.49	376.47	518.50	300.16	440.69	353.21	480.84	289.77	410.87	319.09	416.93	277.60	363.61	152.66	195.46	220.34	253.01	213.45	233.57	205.62	227.61	202.34	214.54
ERSE	541.78	324.64	577.30	412.52	327.49	396.13	524.77	381.49	307.21	365.89	429.26	325.49	273.85	312.84	486.47	198.30	269.83	234.24	218.95	229.13	227.02	202.87	193.90	199.44

Table E.15: Results under model (2.45), by response and correlation.

	T=10, t=4,$\Delta_4 = -699.5961$, low response. Population: **simulated**																											
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)				
APRB	1.82	2.52	2.57	3.45	1.90	3.01	1.85	2.65	1.41	2.26	1.38	2.26	1.14	1.82	1.08	0.68	0.09	0.06	0.06	0.09	0.49	0.51	0.45	0.51				
SE	204.73	215.35	260.38	314.81	228.44	286.61	253.66	304.21	227.11	277.75	233.77	252.83	229.48	242.72	121.81	123.65	134.27	145.39	130.46	139.02	129.04	133.21	128.49	130.79				
ERSE	253.29	211.27	382.07	254.44	248.62	261.76	363.07	245.15	238.77	250.94	258.21	201.35	206.46	204.99	198.24	127.35	160.39	135.16	136.11	136.84	134.69	118.43	120.87	120.12				
	T=10, t=4, $\Delta_4 = -699.5961$, high response. Population: **simulated**																											
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)				
APRB	0.56	0.26	1.02	0.78	1.22	0.89	1.11	0.88	1.33	0.98	1.39	1.34	1.44	1.34	0.42	0.06	0.06	0.12	0.13	0.02	0.30	0.20	0.31	0.25				
SE	92.62	102.13	129.68	160.57	116.69	143.09	127.96	156.25	116.57	140.19	115.52	125.98	113.92	120.03	63.09	66.01	74.88	79.09	74.51	76.51	72.46	75.41	72.37	73.57				
ERSE	198.25	100.78	165.63	126.51	115.60	124.86	156.90	121.89	112.18	120.21	112.69	98.81	97.69	98.48	177.35	67.25	81.49	74.31	73.88	74.23	73.77	68.00	67.73	67.93				
	T=10, t=7, $\Delta_7 = 1448.408$, low response. Population: **simulated**																											
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)				
APRB	20.55	0.64	3.51	0.36	5.47	1.94	4.77	1.53	6.64	3.15	6.69	3.45	8.16	5.06	8.79	0.47	0.38	0.89	0.55	0.14	1.47	0.30	1.57	1.00				
SE	415.15	323.25	435.55	517.42	387.50	474.34	438.36	515.98	394.52	474.76	421.92	478.69	396.48	448.35	227.79	200.03	218.22	223.05	218.01	220.10	220.08	223.10	220.02	221.20				
ERSE	726.32	314.24	606.16	406.70	405.25	420.20	587.04	399.63	398.92	411.63	518.85	367.59	374.63	378.12	504.73	200.62	242.23	211.66	218.28	216.03	224.66	198.96	204.65	202.68				
	T=10, t=7, $\Delta_7 = 1448.408$, high response. Population: **simulated**																											
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)				
APRB	10.56	0.06	1.33	0.68	1.99	0.46	2.14	0.27	2.65	1.33	3.45	1.77	3.82	2.72	5.12	0.12	0.01	0.77	0.06	0.27	0.73	0.05	0.74	0.49				
SE	111.75	135.42	163.67	206.63	148.89	182.20	160.01	201.21	145.96	177.73	147.72	175.82	140.87	159.74	80.46	88.13	94.81	98.94	94.79	96.30	91.70	94.76	91.80	92.76				
ERSE	491.47	134.35	231.12	174.75	154.89	171.20	219.79	167.96	148.97	164.24	183.44	147.01	135.35	144.30	419.76	89.47	106.52	98.16	97.60	98.02	97.16	90.08	89.73	89.98				

E.2 Estimation of Change in Regression Coefficients

Table E.16: Results under model (2.41), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=4, {Stable and Volatile: $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$}, {Simulated: $\Delta_4(\beta_0) = 114.9522$, $\Delta_4(\beta_1) = -0.5296$ }

	low response and low correlation																	
	naïve	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$
$APRB_{\Delta(\beta_0)}$	110.98	0.63	80.95	44.40	1.29	77.56	112.97	2.27	90.53	44.60	0.55	43.83	164.25	0.04	106.49	66.02	0.02	3.64
SE	111.60	81.86	179.86	160.68	133.24	168.05	447.61	334.80	723.61	240.30	208.31	271.52	37.63	17.85	45.87	17.06	11.53	14.34
ERSE	453.88	80.29	252.28	687.17	134.81	267.90	999.02	326.45	1025.51	664.27	209.02	419.86	451.75	17.31	56.28	285.58	11.61	21.47
$APRB_{\Delta(\beta_1)}$	402.04	0.72	346.55	164.60	3.91	343.19	295.42	7.07	327.40	116.74	0.98	159.49	209.11	0.04	169.96	87.22	0.06	16.85
SE	0.05	0.04	0.08	0.07	0.06	0.09	0.19	0.17	0.35	0.11	0.11	0.14	1.00	0.52	1.21	0.49	0.35	0.44
ERSE	0.32	0.04	0.12	0.88	0.07	0.14	1.06	0.16	0.50	1.02	0.10	0.22	7.46	0.52	1.53	5.55	0.36	0.67
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$APRB_{\Delta(\beta_0)}$	44.13	100.14	58.30	58.75	64.39	62.34	55.84	109.24	69.15	30.35	38.75	34.01	46.24	137.17	69.58	24.10	14.53	13.18
SE	205.45	167.41	194.86	184.88	162.03	176.92	825.96	675.75	783.55	313.55	257.98	294.05	37.30	48.94	40.91	12.47	14.59	13.30
ERSE	154.54	184.01	164.35	171.08	200.14	181.38	628.10	748.30	668.13	270.63	308.95	285.25	24.75	46.02	30.17	10.03	16.29	11.90
$APRB_{\Delta(\beta_1)}$	238.76	406.97	278.91	310.19	275.41	307.68	299.74	358.95	304.54	152.13	137.14	148.65	98.13	205.06	126.57	16.95	31.50	3.75
SE	0.10	0.07	0.09	0.10	0.08	0.09	0.45	0.31	0.40	0.18	0.13	0.16	1.06	1.26	1.13	0.41	0.43	0.42
ERSE	0.08	0.08	0.08	0.09	0.10	0.10	0.34	0.34	0.34	0.15	0.16	0.15	0.70	1.22	0.84	0.34	0.49	0.39
	low response and high correlation																	
	naïve	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$
$APRB_{\Delta(\beta_0)}$	144.34	0.43	95.31	59.40	0.05	112.87	163.58	0.66	112.78	66.28	0.17	75.61	146.26	0.15	91.93	61.13	0.04	2.95
SE	60.65	39.27	93.04	106.26	83.05	100.26	257.06	166.72	401.70	140.46	113.50	154.02	36.85	18.61	43.94	18.87	12.91	15.62
ERSE	450.68	38.15	134.00	684.27	84.49	156.72	1100.82	173.16	616.61	695.54	119.50	233.52	424.39	17.83	53.57	278.41	12.18	22.16
$APRB_{\Delta(\beta_1)}$	695.83	76.67	493.51	243.72	37.90	2.77	395.00	21.93	145.32	139.70	19.20	2.86	145.65	2.32	89.07	60.98	0.92	2.43
SE	0.46	0.22	0.50	0.32	0.24	0.30	1.18	0.61	2.00	0.54	0.38	0.50	1.92	0.91	2.16	0.92	0.61	0.76
ERSE	1.04	0.15	0.43	1.44	0.23	0.41	3.22	0.60	2.74	1.78	0.37	0.66	10.99	0.76	2.19	6.80	0.54	0.93
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$APRB_{\Delta(\beta_0)}$	45.10	121.15	64.50	70.72	96.91	79.88	58.29	142.01	79.10	45.35	68.17	53.02	37.53	118.58	58.89	23.88	13.05	13.23
SE	106.25	87.45	100.60	100.75	97.07	100.53	462.93	373.73	437.30	176.32	145.79	166.08	35.67	46.73	39.24	13.50	15.91	14.46
ERSE	81.63	97.92	86.92	87.21	116.11	96.46	374.87	455.44	400.74	148.85	175.26	158.13	23.84	43.07	28.94	10.36	16.86	12.32
$APRB_{\Delta(\beta_1)}$	229.03	629.57	330.22	76.35	33.80	44.12	39.96	232.54	30.31	38.23	13.88	21.37	34.15	115.65	55.79	24.27	12.19	13.58
SE	0.42	0.55	0.44	0.30	0.29	0.30	2.30	1.88	2.17	0.53	0.51	0.51	1.70	2.33	1.89	0.63	0.78	0.69
ERSE	0.23	0.35	0.26	0.22	0.30	0.25	1.66	2.07	1.78	0.44	0.54	0.47	0.97	1.80	1.18	0.43	0.74	0.52
	high response and low correlation																	
	naïve	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$
$APRB_{\Delta(\beta_0)}$	49.67	0.06	18.87	24.54	0.21	33.10	53.04	0.03	25.08	26.65	0.05	21.65	53.94	0.00	20.49	26.85	0.02	3.90
SE	19.62	20.20	34.31	49.69	52.37	64.48	96.96	92.97	157.72	69.39	69.93	91.57	10.99	11.03	14.44	8.28	8.13	9.63
ERSE	262.18	19.66	52.75	524.71	55.65	111.37	639.42	90.58	255.10	534.62	68.51	139.55	258.82	11.09	21.04	216.17	7.95	15.55
$APRB_{\Delta(\beta_1)}$	225.65	0.23	94.36	112.09	1.03	166.69	190.02	0.01	98.75	96.14	0.07	86.20	111.15	0.03	70.17	57.02	0.01	0.78
SE	0.01	0.01	0.02	0.03	0.03	0.03	0.05	0.05	0.08	0.04	0.04	0.05	0.24	0.27	0.37	0.20	0.20	0.25
ERSE	0.30	0.01	0.03	0.88	0.03	0.06	1.02	0.05	0.13	1.02	0.04	0.07	4.84	0.28	0.55	4.45	0.20	0.41
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$APRB_{\Delta(\beta_0)}$	5.13	28.70	4.90	1.51	30.26	9.24	2.10	34.77	11.64	2.63	21.76	4.17	5.48	31.17	5.35	21.28	0.85	13.89
SE	45.22	29.92	40.49	76.30	57.36	71.22	209.64	137.16	187.07	114.97	83.68	103.85	14.77	13.79	14.79	10.55	8.94	10.22
ERSE	34.97	34.31	35.29	68.51	72.43	71.53	169.00	166.92	170.93	96.77	95.90	97.81	11.06	14.68	12.35	8.93	10.37	9.63
$APRB_{\Delta(\beta_1)}$	10.43	137.47	33.32	32.70	149.38	63.30	21.44	131.83	53.40	4.30	84.70	26.29	30.02	87.27	46.66	35.02	10.57	20.68
SE	0.02	0.02	0.02	0.04	0.03	0.04	0.11	0.07	0.10	0.06	0.04	0.06	0.42	0.34	0.40	0.30	0.23	0.28
ERSE	0.02	0.02	0.02	0.04	0.04	0.04	0.09	0.09	0.09	0.05	0.05	0.05	0.31	0.36	0.33	0.26	0.27	0.27
	high response and high correlation																	
	naïve	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{p}_i)$	**naïve**	**EE_h**	$\mathbf{EE}(\hat{\pi}_i)$
$APRB_{\Delta(\beta_0)}$	48.67	0.05	15.83	24.48	0.12	34.28	67.17	0.35	18.88	32.69	0.01	18.54	45.71	0.01	19.93	23.63	0.03	2.32
SE	11.73	11.58	18.17	56.42	58.21	67.70	54.51	49.02	90.47	54.13	53.03	66.19	9.79	11.43	14.50	7.69	8.14	9.75
ERSE	259.14	11.00	28.05	519.38	59.44	119.07	655.07	48.88	143.13	539.01	54.31	109.09	247.22	11.27	20.53	211.92	8.04	15.50
$APRB_{\Delta(\beta_1)}$	86.09	9.40	38.34	50.68	14.15	10.37	69.76	19.70	57.16	34.22	9.23	14.30	35.17	0.82	15.54	19.00	0.13	1.63
SE	0.09	0.07	0.10	0.15	0.15	0.19	0.15	0.14	0.25	0.15	0.14	0.17	0.43	0.50	0.60	0.32	0.35	0.40
ERSE	0.45	0.06	0.10	0.95	0.15	0.29	1.89	0.12	0.36	1.39	0.14	0.27	5.98	0.45	0.79	5.07	0.33	0.59
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$APRB_{\Delta(\beta_0)}$	9.22	25.96	1.29	5.82	32.28	4.29	14.24	32.97	0.63	20.24	21.63	8.98	2.34	28.63	7.15	18.01	1.95	11.28
SE	23.31	16.18	21.06	76.84	61.46	73.07	118.06	79.13	106.20	78.60	59.98	72.94	15.94	13.42	15.51	11.03	8.98	10.56
ERSE	18.27	18.39	18.56	70.50	76.76	74.67	93.57	95.12	95.43	69.45	72.56	71.99	11.34	13.90	12.36	9.08	10.30	9.70
$APRB_{\Delta(\beta_1)}$	5.51	50.98	19.25	7.10	16.90	1.67	41.55	64.77	47.77	7.30	17.82	10.70	2.40	22.05	5.45	15.12	1.85	9.20
SE	0.11	0.10	0.11	0.21	0.17	0.20	0.32	0.22	0.29	0.20	0.16	0.19	0.65	0.56	0.63	0.44	0.37	0.42
ERSE	0.06	0.08	0.06	0.17	0.19	0.18	0.24	0.24	0.24	0.17	0.18	0.18	0.43	0.54	0.47	0.34	0.40	0.37

Table E.17: Results under model (2.41), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=7, {Stable and Volatile: $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$}, {Simulated: $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$ }

	low response and low correlation																	
	naïve	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$	**naïve**	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$	**naïve**	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$
$\text{APRB}_{\Delta(\beta_0)}$	117.74	0.17	18.03	48.31	0.25	6.88	99.96	1.95	26.99	42.23	0.29	6.91	186.14	0.48	21.32	73.21	0.18	13.65
SE	102.61	75.35	129.70	157.13	130.62	145.77	488.92	382.08	596.35	275.42	240.30	284.76	140.46	53.58	76.14	59.82	36.18	37.14
ERSE	453.80	79.81	182.30	686.92	134.58	221.95	918.82	376.67	842.41	627.25	239.37	406.89	720.82	51.70	94.43	486.31	33.88	47.92
$\text{APRB}_{\Delta(\beta_1)}$	424.04	0.42	96.81	179.19	2.66	14.72	42.43	1.13	12.54	18.64	0.15	2.91	81.73	1.14	8.81	37.59	0.76	7.04
SE	0.04	0.04	0.06	0.07	0.06	0.07	0.21	0.19	0.30	0.12	0.12	0.15	11.45	4.71	6.38	4.81	3.01	3.11
ERSE	0.32	0.04	0.10	0.88	0.07	0.11	0.79	0.19	0.45	0.80	0.12	0.21	83.00	4.46	7.84	59.86	2.87	3.95
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	7.91	49.80	3.46	31.23	14.91	21.26	1.92	61.59	12.75	24.69	7.51	16.80	18.12	71.52	1.07	30.27	5.63	21.78
SE	145.05	122.41	137.72	158.73	141.02	151.65	668.10	559.56	634.01	324.74	271.90	303.07	60.10	93.66	67.63	35.54	37.51	36.44
ERSE	114.96	150.11	125.19	155.02	169.59	162.51	530.81	688.98	577.58	289.18	304.97	299.55	44.18	99.19	56.31	30.81	38.25	34.34
$\text{APRB}_{\Delta(\beta_1)}$	8.25	218.29	45.82	105.89	75.48	69.52	0.72	27.70	6.02	11.00	3.57	7.34	32.94	10.03	21.56	16.84	5.14	11.35
SE	0.08	0.06	0.07	0.09	0.07	0.08	0.38	0.27	0.34	0.18	0.14	0.16	5.16	7.54	5.75	3.05	3.13	3.09
ERSE	0.06	0.07	0.07	0.08	0.09	0.08	0.30	0.33	0.31	0.16	0.15	0.16	3.73	7.83	4.71	2.68	3.21	2.95
	low response and high correlation																	
	naïve	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$	**naïve**	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$	**naïve**	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$
$\text{APRB}_{\Delta(\beta_0)}$	153.09	0.37	12.24	62.31	0.62	15.26	157.16	0.10	17.84	63.67	0.47	13.27	133.47	1.22	9.25	57.19	0.25	11.74
SE	60.50	38.29	61.32	109.29	85.37	92.81	319.38	212.15	347.62	170.93	139.80	163.16	255.55	104.28	148.46	101.17	61.43	64.16
ERSE	451.01	38.07	85.83	684.67	84.40	151.27	1027.32	213.78	477.17	660.75	142.61	227.74	889.30	86.83	143.12	547.11	56.56	75.44
$\text{APRB}_{\Delta(\beta_1)}$	690.81	81.11	140.63	247.49	46.55	24.12	142.76	16.85	37.00	55.08	7.56	0.80	173.24	3.45	20.12	68.12	0.71	11.48
SE	0.45	0.21	0.29	0.32	0.25	0.27	1.03	0.73	1.12	0.55	0.45	0.53	23.04	9.38	13.48	9.02	5.45	5.67
ERSE	1.04	0.15	0.27	1.42	0.22	0.39	2.51	0.68	1.40	1.49	0.40	0.66	112.49	7.85	13.19	69.75	5.00	6.68
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	23.13	53.23	7.52	53.66	16.76	38.13	17.56	61.54	2.07	42.52	8.87	29.78	23.51	46.03	8.96	26.05	5.64	18.56
SE	65.90	62.64	63.52	98.25	86.91	95.87	382.79	338.43	365.56	180.91	157.75	171.13	107.94	191.84	126.58	60.45	63.37	62.81
ERSE	54.45	74.16	59.64	90.93	104.24	98.56	304.01	407.09	332.39	164.16	177.35	171.23	61.97	170.57	82.02	44.63	63.40	51.70
$\text{APRB}_{\Delta(\beta_1)}$	26.91	267.96	77.42	5.61	28.08	11.02	12.00	66.24	23.11	10.36	6.34	4.77	18.43	63.70	1.28	27.08	4.54	19.01
SE	0.26	0.33	0.27	0.28	0.25	0.27	1.23	1.06	1.17	0.60	0.50	0.56	9.83	17.31	11.52	5.33	5.64	5.55
ERSE	0.15	0.27	0.18	0.24	0.27	0.25	0.88	1.17	0.96	0.46	0.51	0.48	5.69	15.57	7.53	3.94	5.62	4.57
	high response and low correlation																	
	naïve	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$	**naïve**	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$	**naïve**	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$
$\text{APRB}_{\Delta(\beta_0)}$	52.37	0.03	6.57	26.12	0.02	13.96	60.01	0.42	7.48	29.86	0.13	11.94	66.77	0.02	6.53	32.93	0.04	7.38
SE	18.67	19.05	29.42	53.09	55.58	63.01	116.99	115.42	172.39	78.60	78.57	93.10	21.67	20.28	26.22	14.51	13.78	16.68
ERSE	262.27	19.60	44.03	524.15	55.60	108.15	614.25	111.77	254.23	515.26	79.37	128.57	413.13	19.94	39.77	368.66	14.19	24.38
$\text{APRB}_{\Delta(\beta_1)}$	237.99	0.15	26.17	119.41	0.03	63.43	29.24	0.19	3.18	14.64	0.07	5.87	3.42	0.08	8.98	1.79	0.05	0.16
SE	0.01	0.01	0.02	0.03	0.03	0.03	0.06	0.06	0.09	0.04	0.04	0.05	1.40	1.31	1.80	0.92	0.89	1.01
ERSE	0.30	0.01	0.02	0.88	0.03	0.06	0.80	0.06	0.14	0.80	0.04	0.07	48.11	1.24	2.70	45.06	0.86	1.23
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	27.34	8.24	17.23	41.14	0.52	28.60	31.67	10.02	19.92	32.92	2.89	22.68	30.97	12.20	19.28	16.94	5.13	11.56
SE	37.46	24.63	33.45	74.75	57.84	68.56	215.83	152.03	193.72	106.41	90.02	98.76	28.36	23.47	27.61	19.55	15.27	18.05
ERSE	29.76	29.98	30.46	74.52	69.88	75.26	174.13	173.89	177.69	102.07	98.64	101.99	21.94	27.50	24.40	17.62	17.29	18.00
$\text{APRB}_{\Delta(\beta_1)}$	121.15	42.97	75.03	190.25	1.15	131.82	15.04	5.59	9.30	16.36	1.20	11.25	14.13	9.42	11.05	0.06	1.01	0.09
SE	0.02	0.01	0.02	0.04	0.03	0.04	0.12	0.08	0.10	0.06	0.05	0.05	2.16	1.58	1.99	1.10	0.99	1.05
ERSE	0.02	0.02	0.02	0.04	0.04	0.04	0.09	0.09	0.10	0.05	0.05	0.05	1.73	1.80	1.81	1.04	1.03	1.05
	high response and high correlation																	
	naïve	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$	**naïve**	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$	**naïve**	**EE_h**	**EE**$(\hat{p}_i)$	**naïve**	**EE_h**	**EE**$(\hat{\pi}_i)$
$\text{APRB}_{\Delta(\beta_0)}$	51.51	0.14	7.34	25.67	0.10	15.49	60.66	0.63	15.36	29.59	0.27	14.84	44.50	1.15	7.21	22.49	0.34	5.68
SE	11.84	11.35	16.17	57.90	59.12	67.07	67.27	62.30	87.98	55.52	54.92	62.36	48.39	63.44	65.46	36.36	41.11	49.66
ERSE	259.27	11.15	24.41	519.82	59.43	115.64	606.22	59.94	128.93	509.42	54.08	91.22	503.12	53.44	82.07	415.69	37.26	60.83
$\text{APRB}_{\Delta(\beta_1)}$	96.09	14.12	7.91	41.16	1.42	7.49	79.38	14.29	11.90	37.28	5.56	0.84	60.64	2.92	2.09	30.66	0.89	6.02
SE	0.11	0.08	0.09	0.15	0.15	0.17	0.38	0.36	0.42	0.27	0.25	0.26	4.49	5.76	6.06	3.27	3.65	4.42
ERSE	0.46	0.06	0.09	0.95	0.15	0.27	1.42	0.27	0.48	1.12	0.20	0.30	64.93	4.89	7.64	53.56	3.33	5.43
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	27.91	6.89	17.85	45.62	1.16	31.80	40.02	0.25	27.86	38.29	6.56	26.81	25.85	5.61	16.72	12.71	4.21	8.69
SE	19.63	14.35	17.89	78.30	61.82	72.47	107.63	79.98	97.41	71.34	59.58	66.33	66.79	56.81	67.22	57.08	43.03	53.75
ERSE	16.06	16.62	16.59	78.79	74.51	79.96	89.98	92.05	92.10	70.13	66.89	70.17	41.37	60.44	47.87	39.83	44.91	42.57
$\text{APRB}_{\Delta(\beta_1)}$	16.16	24.75	4.36	17.72	1.24	12.34	9.00	27.80	1.07	9.86	1.39	4.98	19.57	18.91	9.10	15.36	2.88	10.26
SE	0.09	0.09	0.09	0.19	0.16	0.18	0.46	0.40	0.44	0.28	0.26	0.27	6.20	5.24	6.24	5.06	3.85	4.77
ERSE	0.06	0.08	0.07	0.19	0.18	0.19	0.31	0.38	0.33	0.22	0.24	0.23	3.85	5.63	4.46	3.54	4.02	3.80

Table E.18: Results under model (2.41), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=10, {Stable and Volatile: $\Delta_{10}(\beta_0) = 11023.06$, $\Delta_{10}(\beta_1) = -0.4041$}, {Simulated: $\Delta_{10}(\beta_0) = 2222.861$, $\Delta_{10}(\beta_1) = 14.4251$ }

	low response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
$\text{APRB}_{\Delta(\beta_0)}$	119.27	0.22	3.79	48.73	0.17	13.22	91.29	1.78	2.57	40.38	1.24	6.66	224.79	5.22	66.79	92.91	1.25	47.92
SE	102.39	76.37	111.60	160.88	132.56	138.09	533.15	418.61	614.32	304.00	268.24	305.55	61.78	36.27	47.79	61.92	45.78	47.87
ERSE	454.21	79.81	158.48	687.28	134.51	189.15	885.46	429.75	858.05	623.33	271.30	386.41	712.19	35.86	65.07	544.45	45.39	68.85
$\text{APRB}_{\Delta(\beta_1)}$	429.67	1.48	2.69	180.17	1.79	50.59	116.21	1.60	1.80	53.74	1.36	9.21	62.78	0.40	14.40	29.42	0.03	4.78
SE	0.04	0.04	0.06	0.07	0.07	0.07	0.22	0.21	0.32	0.14	0.13	0.15	2.94	1.77	2.13	2.36	1.97	2.07
ERSE	0.32	0.04	0.09	0.88	0.07	0.10	0.81	0.22	0.47	0.78	0.14	0.20	25.57	1.70	2.95	21.50	1.90	2.64
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	25.48	25.01	15.40	31.83	1.96	22.68	19.34	34.85	9.37	18.41	0.48	12.34	106.14	66.78	83.48	117.58	5.06	85.09
SE	126.43	105.99	118.99	143.56	137.28	140.20	705.66	571.88	660.44	333.39	296.69	317.18	51.07	52.00	49.07	49.62	46.32	48.79
ERSE	102.33	129.91	111.12	148.12	155.52	153.20	552.78	689.29	598.71	309.15	318.50	316.80	43.69	59.73	47.85	48.39	51.86	51.25
$\text{APRB}_{\Delta(\beta_1)}$	76.94	119.94	40.98	126.61	0.66	89.57	29.25	44.99	14.64	25.22	0.67	16.84	31.33	2.61	22.56	10.62	3.20	7.19
SE	0.07	0.05	0.06	0.08	0.07	0.07	0.41	0.28	0.36	0.17	0.15	0.16	2.32	2.19	2.21	2.12	2.02	2.10
ERSE	0.06	0.06	0.06	0.08	0.08	0.08	0.32	0.35	0.33	0.16	0.16	0.16	1.97	2.39	2.12	2.10	2.16	2.17
	low response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
$\text{APRB}_{\Delta(\beta_0)}$	156.38	0.59	12.20	64.30	0.08	19.86	140.33	1.55	12.42	57.12	0.44	14.11	450.17	23.19	17.68	172.63	9.12	46.47
SE	58.37	37.49	54.30	106.22	83.59	89.41	349.03	239.64	327.93	189.40	158.76	176.76	118.70	59.89	68.37	80.38	56.41	58.45
ERSE	451.09	38.06	71.49	684.02	84.38	134.81	1037.74	242.64	458.07	669.79	160.22	214.01	728.53	48.48	65.34	548.99	51.45	75.37
$\text{APRB}_{\Delta(\beta_1)}$	656.11	67.18	50.57	225.74	24.78	9.39	47.58	42.40	97.80	22.05	12.15	12.31	161.86	5.29	4.21	66.49	2.21	6.38
SE	0.43	0.21	0.23	0.32	0.24	0.26	1.24	1.03	1.44	0.66	0.59	0.66	10.20	5.04	5.63	5.55	3.79	3.92
ERSE	1.01	0.15	0.23	1.42	0.22	0.34	1.75	0.92	1.77	1.16	0.54	0.72	38.87	4.09	5.24	27.18	3.33	4.47
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	40.55	22.39	27.23	46.43	5.23	33.41	38.52	19.51	26.26	31.88	5.91	22.68	77.02	40.15	47.81	126.32	0.72	88.31
SE	59.84	53.24	56.97	95.33	85.44	92.33	367.81	316.28	347.33	186.14	173.29	180.58	58.28	82.40	62.98	58.70	57.57	58.78
ERSE	46.52	61.70	50.85	93.97	98.00	98.56	300.77	381.87	326.40	175.65	183.45	180.84	37.48	75.21	44.49	52.11	57.36	55.84
$\text{APRB}_{\Delta(\beta_1)}$	22.54	135.47	12.61	9.94	10.54	2.08	101.31	107.25	99.18	10.54	13.99	11.39	32.11	28.62	18.81	16.70	3.92	10.72
SE	0.22	0.25	0.22	0.27	0.25	0.27	1.61	1.36	1.52	0.70	0.65	0.68	4.57	6.84	5.09	3.87	3.87	3.92
ERSE	0.14	0.22	0.16	0.24	0.26	0.25	1.16	1.44	1.25	0.60	0.63	0.62	2.74	6.26	3.40	3.18	3.60	3.43
	high response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
$\text{APRB}_{\Delta(\beta_0)}$	53.54	0.02	12.50	26.64	0.04	10.22	45.43	0.43	10.93	23.22	0.12	6.09	84.18	0.29	18.55	37.40	0.31	29.84
SE	19.56	19.76	26.40	51.33	54.06	58.97	129.28	128.30	179.49	88.13	88.72	98.18	28.47	24.16	31.66	33.69	30.83	33.05
ERSE	262.34	19.64	40.53	524.32	55.61	87.74	624.83	126.46	256.74	525.38	87.99	115.16	460.90	23.65	43.61	435.65	31.55	45.13
$\text{APRB}_{\Delta(\beta_1)}$	243.31	0.04	56.07	121.92	0.04	47.50	64.41	0.60	15.08	33.09	0.19	8.93	5.83	0.00	1.53	2.74	0.02	1.74
SE	0.01	0.01	0.01	0.03	0.03	0.03	0.07	0.07	0.10	0.05	0.05	0.05	0.62	0.58	0.80	0.74	0.73	0.80
ERSE	0.30	0.01	0.02	0.89	0.03	0.05	0.79	0.07	0.14	0.79	0.05	0.06	16.85	0.57	1.13	17.27	0.75	1.12
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	29.68	0.33	20.71	24.37	5.94	16.77	26.44	0.07	18.31	14.66	4.17	9.86	34.02	10.56	25.45	69.83	17.63	49.33
SE	32.63	24.16	29.24	66.00	56.68	62.00	226.13	156.86	200.81	103.72	97.51	100.25	37.42	29.85	34.22	34.75	32.57	33.79
ERSE	28.13	27.58	28.64	69.61	66.43	69.66	183.45	177.33	185.42	102.33	101.67	102.56	31.25	33.14	32.32	36.81	36.60	37.37
$\text{APRB}_{\Delta(\beta_1)}$	136.04	1.80	94.42	114.10	26.54	78.45	37.79	1.48	25.98	21.66	5.91	14.57	3.00	0.86	2.16	4.14	0.93	2.89
SE	0.02	0.01	0.02	0.04	0.03	0.03	0.12	0.08	0.11	0.06	0.05	0.05	1.00	0.73	0.89	0.86	0.78	0.82
ERSE	0.02	0.01	0.02	0.04	0.04	0.04	0.10	0.09	0.10	0.05	0.05	0.05	0.82	0.81	0.83	0.91	0.88	0.92
	high response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
$\text{APRB}_{\Delta(\beta_0)}$	52.81	0.05	12.87	26.13	0.26	11.51	52.80	0.65	16.86	25.19	1.14	8.44	72.18	7.24	0.66	38.23	2.43	26.21
SE	11.70	11.21	15.77	57.44	58.69	65.30	72.93	68.04	87.97	55.29	54.68	59.00	23.28	28.46	32.19	35.19	36.14	38.96
ERSE	259.47	11.23	22.81	519.57	59.47	93.32	608.17	67.03	122.30	515.79	55.43	72.93	434.90	23.72	34.86	422.85	34.96	52.35
$\text{APRB}_{\Delta(\beta_1)}$	97.19	14.34	5.02	38.67	0.21	5.17	106.83	22.93	4.77	55.81	11.68	6.01	51.49	1.61	7.50	26.75	0.77	3.58
SE	0.11	0.08	0.09	0.15	0.15	0.17	0.37	0.37	0.43	0.22	0.22	0.24	2.13	2.61	2.97	2.17	2.31	2.51
ERSE	0.46	0.06	0.08	0.95	0.15	0.22	1.19	0.31	0.45	0.93	0.20	0.24	22.85	2.19	3.33	20.83	2.23	3.19
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	29.84	1.13	20.94	26.93	7.02	18.69	34.92	6.55	25.18	17.65	7.25	12.34	19.82	19.09	9.28	68.43	11.42	46.92
SE	19.48	13.67	17.54	72.75	62.30	68.60	105.04	82.28	95.37	61.82	58.42	60.10	33.23	27.91	33.18	42.18	37.62	40.42
ERSE	15.39	15.46	15.82	73.76	70.86	74.08	91.74	90.73	92.76	64.64	63.94	64.77	19.09	28.29	22.01	41.29	40.88	41.93
$\text{APRB}_{\Delta(\beta_1)}$	11.31	16.06	2.61	11.14	3.15	7.54	19.27	21.13	6.73	1.12	6.10	3.38	23.89	5.45	15.40	8.79	2.85	5.60
SE	0.09	0.09	0.09	0.18	0.16	0.18	0.47	0.42	0.45	0.24	0.24	0.24	3.09	2.56	3.07	2.70	2.43	2.60
ERSE	0.06	0.07	0.06	0.18	0.17	0.18	0.34	0.39	0.36	0.22	0.22	0.22	1.81	2.63	2.08	2.52	2.57	2.59

Table E.19: Results under model (2.43), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=4, {Stable and Volatile: $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$}, {Simulated: $\Delta_4(\beta_0) = 114.9522$, $\Delta_4(\beta_1) = -0.5296$ }

	low response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
$\text{APRB}_{\Delta(\beta_0)}$	165.76	0.55	105.74	78.20	12.20	127.34	60.19	160.84	115.75	78.81	9.52	113.93	125.26	16.95	139.15	54.48	12.68	20.86
SE	116.01	61.78	158.84	106.23	83.62	104.74	558.59	435.67	929.72	108.61	99.63	292.42	118.91	73.63	288.39	14.92	12.13	70.43
ERSE	444.90	59.22	224.61	689.99	92.48	169.45	913.95	499.46	1383.96	704.49	317.90	645.81	404.34	124.66	537.75	265.92	80.12	159.28
$\text{APRB}_{\Delta(\beta_1)}$	305.64	8.93	284.61	149.26	29.63	371.11	70.84	313.11	82.10	103.49	41.88	221.77	152.36	0.59	169.70	70.15	2.98	19.27
SE	0.04	0.04	0.07	0.08	0.08	0.11	0.30	0.39	0.65	0.08	0.10	0.30	1.25	0.93	4.17	0.43	0.38	1.71
ERSE	0.32	0.04	0.12	0.89	0.09	0.19	1.06	0.46	1.04	1.08	0.30	0.69	6.97	2.59	9.12	5.35	1.86	4.07
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	41.34	144.32	66.07	85.87	112.52	95.04	52.82	170.22	76.60	101.38	101.88	103.25	117.15	150.76	125.55	2.12	30.04	9.51
SE	173.07	157.90	166.44	108.31	101.72	106.69	1032.39	877.38	990.23	411.09	225.34	361.54	357.07	249.21	329.97	98.48	58.97	87.12
ERSE	139.66	176.93	150.33	98.77	129.95	108.48	832.00	1031.71	894.02	416.37	475.58	438.92	325.08	389.99	346.75	103.97	124.67	110.62
$\text{APRB}_{\Delta(\beta_1)}$	200.68	342.08	231.97	377.55	297.97	360.18	48.33	157.14	4.88	298.47	156.89	261.72	140.86	184.17	152.07	9.64	33.10	1.88
SE	0.10	0.06	0.09	0.13	0.10	0.12	0.97	0.55	0.82	0.51	0.23	0.41	5.86	3.33	5.16	2.69	1.33	2.27
ERSE	0.08	0.07	0.08	0.13	0.13	0.13	0.76	0.64	0.72	0.53	0.45	0.51	5.84	6.10	6.00	2.80	2.81	2.83
	low response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
$\text{APRB}_{\Delta(\beta_0)}$	162.00	0.06	97.65	76.12	11.14	127.34	54.19	168.63	100.11	77.64	11.61	111.23	119.64	13.98	119.38	47.42	7.03	11.99
SE	111.93	60.09	159.74	108.57	85.39	104.14	563.33	440.04	930.15	104.28	96.00	269.36	67.45	41.48	243.52	14.34	11.57	58.30
ERSE	439.20	55.77	216.83	685.05	90.06	164.52	916.08	491.67	1366.15	702.57	312.67	628.65	394.14	109.40	518.69	256.25	68.25	126.81
$\text{APRB}_{\Delta(\beta_1)}$	687.74	86.35	508.28	227.68	15.02	32.97	6.78	123.53	298.63	40.85	36.19	8.58	107.96	5.20	29.77	39.72	7.23	9.32
SE	0.46	0.22	0.51	0.32	0.24	0.29	1.84	1.32	3.28	0.29	0.25	0.70	1.78	0.98	5.98	0.61	0.48	1.59
ERSE	1.02	0.15	0.54	1.46	0.23	0.41	2.72	1.41	4.16	1.84	0.78	1.55	10.15	2.78	14.17	6.05	1.75	3.57
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	33.33	135.65	58.12	84.46	112.07	93.94	34.08	157.56	59.03	97.39	99.48	99.61	97.04	130.31	105.61	6.80	20.61	0.66
SE	174.29	157.27	167.76	104.36	103.13	104.18	1019.10	885.52	982.58	378.76	210.52	332.82	319.60	198.30	290.02	79.66	52.04	70.89
ERSE	134.33	167.27	144.08	94.02	125.46	103.85	820.15	1019.56	882.10	406.39	465.03	428.59	314.15	369.36	333.70	85.48	104.69	91.29
$\text{APRB}_{\Delta(\beta_1)}$	256.59	637.13	352.44	105.50	7.74	74.25	356.29	275.79	332.48	17.81	11.84	13.80	11.42	50.56	4.41	30.26	1.74	21.85
SE	0.45	0.55	0.47	0.28	0.28	0.28	3.59	3.14	3.46	0.97	0.56	0.85	8.01	4.80	7.21	2.23	1.33	1.97
ERSE	0.29	0.43	0.32	0.23	0.31	0.25	2.52	3.13	2.70	1.02	1.17	1.07	8.64	10.09	9.15	2.35	2.74	2.48
	high response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
$\text{APRB}_{\Delta(\beta_0)}$	56.36	0.14	15.73	31.75	5.05	38.59	3.58	91.88	23.83	30.80	3.00	29.99	54.09	0.66	21.47	20.95	2.47	7.10
SE	27.94	26.82	48.51	51.10	53.67	65.21	271.25	306.48	467.04	50.82	56.75	183.56	13.65	11.48	103.16	6.91	6.61	23.81
ERSE	261.73	26.92	78.84	525.08	57.15	113.44	574.91	329.20	738.10	537.32	212.58	436.29	262.08	48.99	212.21	208.63	32.79	51.13
$\text{APRB}_{\Delta(\beta_1)}$	256.09	0.79	82.18	144.70	22.91	191.89	9.11	330.18	56.28	110.87	8.05	122.39	113.60	4.15	59.26	44.14	8.67	14.22
SE	0.01	0.01	0.02	0.03	0.03	0.03	0.14	0.17	0.25	0.03	0.03	0.10	0.28	0.28	1.66	0.17	0.17	0.49
ERSE	0.30	0.01	0.04	0.89	0.03	0.06	1.01	0.18	0.38	1.06	0.11	0.24	4.87	0.82	3.32	4.40	0.58	0.98
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	16.80	31.08	3.23	6.46	36.33	14.44	76.26	13.62	54.73	17.99	30.44	21.42	0.97	30.30	9.20	22.12	2.85	15.76
SE	64.78	42.02	57.73	76.30	58.40	71.51	601.06	411.15	543.68	291.30	142.99	242.84	148.67	78.08	130.57	29.65	25.73	27.16
ERSE	52.15	51.17	52.65	70.45	74.54	73.47	482.99	489.99	492.35	312.11	303.58	312.85	133.00	141.10	137.82	39.11	46.59	41.37
$\text{APRB}_{\Delta(\beta_1)}$	60.13	149.81	0.82	55.73	177.16	87.36	216.71	67.77	152.28	91.38	121.15	98.16	20.30	76.87	35.91	45.82	3.99	32.59
SE	0.03	0.02	0.03	0.04	0.03	0.04	0.32	0.22	0.29	0.16	0.08	0.13	2.82	1.20	2.30	0.69	0.49	0.60
ERSE	0.03	0.03	0.03	0.04	0.04	0.04	0.25	0.25	0.26	0.17	0.16	0.17	2.33	1.98	2.24	0.83	0.82	0.82
	high response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
$\text{APRB}_{\Delta(\beta_0)}$	60.81	0.64	11.90	41.29	10.35	44.01	45.19	98.44	63.26	42.36	8.81	3.05	47.90	1.85	11.92	17.50	4.41	8.79
SE	23.98	22.77	41.17	57.10	57.92	67.43	241.57	230.11	381.72	62.89	63.60	140.70	11.70	10.57	78.95	6.39	6.37	19.13
ERSE	262.08	22.12	64.05	523.20	60.97	120.89	607.57	287.71	708.37	536.78	186.74	324.84	252.78	37.87	155.94	204.11	25.88	39.22
$\text{APRB}_{\Delta(\beta_1)}$	156.81	32.24	55.12	73.84	8.82	4.09	440.19	86.93	111.72	92.51	161.19	237.54	43.84	3.74	0.71	13.89	7.65	12.31
SE	0.15	0.11	0.18	0.17	0.17	0.19	0.66	0.60	1.07	0.15	0.16	0.38	0.50	0.44	2.15	0.26	0.28	0.55
ERSE	0.49	0.08	0.17	1.01	0.16	0.30	1.65	0.64	1.71	1.33	0.42	0.79	6.00	0.94	4.26	4.83	0.65	1.08
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	25.18	29.54	9.75	0.90	43.41	12.17	154.07	0.64	117.84	42.56	8.97	28.11	9.41	21.14	0.78	22.81	5.21	16.83
SE	53.27	36.17	48.07	75.61	61.91	72.24	479.59	340.26	438.48	204.46	125.29	174.94	116.49	59.20	101.29	23.74	20.70	21.77
ERSE	41.93	41.78	42.50	71.77	78.62	76.09	448.55	488.35	466.52	242.42	247.62	245.84	99.04	102.16	101.72	30.41	35.37	31.91
$\text{APRB}_{\Delta(\beta_1)}$	4.25	82.02	19.81	19.73	11.06	8.24	402.32	8.67	283.36	370.61	216.65	309.90	24.09	11.18	13.92	25.81	9.44	19.98
SE	0.19	0.17	0.18	0.21	0.18	0.20	1.41	0.93	1.26	0.56	0.32	0.48	3.24	1.61	2.79	0.67	0.57	0.62
ERSE	0.09	0.12	0.10	0.18	0.20	0.19	1.12	1.16	1.15	0.59	0.57	0.58	2.76	2.73	2.80	0.81	0.94	0.85

Table E.20: Results under model (2.43), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=7, {Stable and Volatile: $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$}, {Simulated: $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$ }

									low response and low correlation									
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB$_{\Delta(\beta_0)}$	165.34	0.27	13.80	78.29	10.65	4.38	76.26	62.13	64.84	81.35	44.49	39.72	139.90	19.37	43.50	60.72	12.83	1.38
SE	122.35	65.70	103.02	108.82	85.90	94.43	536.83	463.39	761.97	113.28	108.19	258.86	191.24	99.04	190.27	24.46	19.58	45.22
ERSE	444.07	59.04	124.35	690.24	92.70	157.81	905.51	577.47	1228.79	690.40	364.04	568.72	536.00	162.13	292.64	409.61	109.51	124.53
APRB$_{\Delta(\beta_1)}$	301.55	4.80	88.37	148.34	24.65	18.35	1.16	19.44	14.92	15.59	11.96	12.02	1.55	0.41	21.15	1.53	0.03	0.41
SE	0.04	0.04	0.06	0.09	0.09	0.10	0.24	0.42	0.79	0.09	0.10	0.27	3.72	3.34	10.43	1.21	1.09	3.96
ERSE	0.32	0.04	0.10	0.88	0.09	0.15	0.84	0.58	1.54	0.86	0.40	0.64	54.40	15.08	20.30	47.47	10.19	11.96
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	25.09	60.97	8.06	43.69	28.66	27.67	39.39	105.21	50.11	15.33	64.52	25.76	13.69	88.16	26.25	12.75	7.80	5.53
SE	104.85	112.02	103.29	99.18	90.20	97.05	880.14	662.64	826.48	352.51	213.48	305.85	198.47	213.95	193.33	49.58	46.27	46.88
ERSE	81.58	120.24	90.51	98.22	113.49	106.20	758.48	962.47	825.46	418.39	448.76	434.40	211.09	310.24	232.59	105.81	121.13	111.42
APRB$_{\Delta(\beta_1)}$	28.18	184.14	52.48	52.54	101.53	26.45	25.08	12.81	19.31	7.55	18.88	9.20	31.01	22.24	25.60	0.59	2.41	0.13
SE	0.09	0.06	0.08	0.11	0.09	0.10	1.28	0.54	1.02	0.41	0.24	0.33	11.66	11.06	10.95	4.45	4.03	4.11
ERSE	0.07	0.06	0.07	0.11	0.11	0.11	1.13	0.83	1.05	0.56	0.51	0.54	16.73	20.87	17.91	10.77	11.26	11.00
									low response and high correlation									
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB$_{\Delta(\beta_0)}$	162.00	0.01	12.16	76.57	9.89	6.10	74.20	61.74	60.11	77.20	41.78	33.32	98.60	11.86	23.19	37.17	4.20	3.39
SE	106.25	56.83	92.66	106.50	83.54	91.90	537.35	467.92	762.97	105.79	99.99	247.20	182.85	94.57	162.77	42.31	34.72	52.06
ERSE	440.03	55.87	117.76	685.27	90.28	156.08	880.25	568.56	1201.49	682.74	361.86	556.93	681.37	156.83	234.54	449.58	100.73	114.09
APRB$_{\Delta(\beta_1)}$	709.20	92.06	142.85	214.72	13.62	11.58	195.74	65.21	109.70	73.15	30.29	19.81	110.06	3.92	3.22	38.84	8.30	17.57
SE	0.47	0.22	0.30	0.32	0.24	0.26	1.32	1.05	1.67	0.44	0.38	0.54	12.23	6.27	12.41	3.79	3.07	4.78
ERSE	1.04	0.16	0.29	1.45	0.23	0.39	2.45	1.22	2.61	1.52	0.72	1.04	88.11	17.86	22.39	57.28	11.04	11.74
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	26.50	58.03	9.50	46.29	27.57	29.91	31.64	105.49	43.70	6.90	59.60	18.31	2.00	54.84	9.05	13.22	0.29	8.04
SE	95.58	100.55	93.59	96.77	87.75	94.61	890.06	648.79	832.42	335.82	208.46	291.37	149.04	190.64	154.81	55.73	49.33	54.04
ERSE	77.42	113.97	85.88	95.51	110.55	103.56	742.43	934.47	806.73	413.29	443.45	428.87	160.54	292.75	186.41	89.47	108.03	96.82
APRB$_{\Delta(\beta_1)}$	30.45	271.21	80.24	39.55	8.02	23.93	71.45	169.83	87.89	4.36	29.13	11.87	25.64	35.77	12.95	29.20	13.38	23.11
SE	0.28	0.35	0.29	0.27	0.25	0.26	1.92	1.54	1.80	0.65	0.51	0.59	10.64	15.27	11.38	5.01	4.59	4.90
ERSE	0.17	0.29	0.19	0.24	0.28	0.26	1.66	2.24	1.80	0.78	0.87	0.81	16.17	30.92	19.11	9.41	11.45	10.22
									high response and low correlation									
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB$_{\Delta(\beta_0)}$	56.82	0.55	7.35	31.89	4.54	9.28	39.93	29.80	9.08	33.11	14.43	6.67	53.48	6.18	13.07	14.78	10.10	14.61
SE	28.14	27.38	41.82	51.47	54.22	62.40	204.75	238.73	412.23	52.52	58.67	131.67	30.65	24.16	60.18	13.91	12.23	23.67
ERSE	261.76	26.92	60.76	525.32	57.21	109.61	591.04	311.48	725.30	524.39	202.64	303.43	416.39	83.89	120.44	355.51	55.19	68.28
APRB$_{\Delta(\beta_1)}$	258.05	2.53	29.25	145.36	20.56	42.26	19.72	15.08	3.53	16.16	6.74	2.84	17.04	3.10	3.84	6.65	1.11	2.14
SE	0.01	0.01	0.02	0.03	0.03	0.03	0.10	0.13	0.22	0.03	0.03	0.07	2.28	1.86	8.19	1.11	1.00	3.02
ERSE	0.30	0.01	0.03	0.89	0.03	0.06	0.79	0.16	0.38	0.84	0.11	0.16	50.92	12.66	17.31	44.76	7.95	9.81
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	30.35	9.43	19.19	36.73	4.26	24.03	35.93	13.70	23.26	8.35	15.12	0.94	31.83	0.89	22.96	20.53	13.90	17.10
SE	51.39	37.53	46.52	74.29	56.54	68.10	560.52	316.70	487.87	180.68	118.32	154.22	66.10	62.17	62.62	26.72	23.47	24.73
ERSE	41.62	42.87	42.73	76.02	71.63	76.80	481.56	457.69	489.19	252.21	248.52	252.27	100.16	115.80	105.22	61.86	63.27	62.64
APRB$_{\Delta(\beta_1)}$	134.54	49.24	83.60	170.34	22.81	111.23	16.48	7.72	10.39	4.78	7.19	1.01	13.86	0.86	8.70	4.09	2.26	2.88
SE	0.03	0.02	0.02	0.04	0.03	0.04	0.30	0.16	0.26	0.10	0.06	0.08	8.77	8.85	8.37	3.37	3.07	3.12
ERSE	0.02	0.02	0.02	0.04	0.04	0.04	0.25	0.24	0.26	0.13	0.13	0.13	14.64	17.67	15.56	8.94	9.19	9.08
									high response and high correlation									
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB$_{\Delta(\beta_0)}$	60.89	0.70	9.38	41.24	10.39	6.37	42.52	27.48	31.17	42.35	23.93	16.08	30.73	6.99	15.42	6.31	9.57	12.14
SE	23.20	21.88	31.91	59.79	61.09	68.64	241.09	240.35	386.73	61.02	61.00	123.97	39.52	53.95	69.77	24.75	29.26	40.32
ERSE	262.08	22.11	48.95	523.61	60.96	116.89	584.16	323.85	675.79	522.64	215.51	291.00	437.05	89.74	122.60	362.35	57.35	73.19
APRB$_{\Delta(\beta_1)}$	163.46	35.34	31.53	68.43	2.04	16.28	68.59	0.51	6.22	35.30	5.20	0.13	37.55	0.42	3.99	12.96	5.71	9.07
SE	0.15	0.11	0.13	0.17	0.16	0.18	0.51	0.46	0.64	0.22	0.21	0.27	3.49	4.76	8.83	2.20	2.60	4.16
ERSE	0.49	0.08	0.12	1.02	0.16	0.29	1.27	0.54	1.01	1.06	0.37	0.49	56.13	13.17	17.30	46.32	8.24	9.97
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	33.54	7.80	21.81	38.48	8.93	23.66	64.77	5.43	48.92	2.59	20.96	9.30	29.01	8.01	22.26	15.58	12.17	13.46
SE	39.02	29.26	35.38	79.33	63.70	73.80	515.10	310.69	451.61	156.48	118.33	138.50	74.24	68.50	72.09	47.39	36.50	43.81
ERSE	33.50	35.16	34.53	79.84	75.99	81.16	457.11	451.65	467.55	251.99	252.26	253.35	92.97	119.84	100.90	60.81	64.55	62.75
APRB$_{\Delta(\beta_1)}$	2.40	48.66	16.69	31.34	9.16	23.44	32.72	13.85	20.02	8.60	1.02	3.95	19.80	7.15	12.15	14.14	8.23	11.26
SE	0.13	0.14	0.13	0.21	0.17	0.19	0.77	0.59	0.70	0.31	0.26	0.29	9.15	9.41	8.93	4.80	3.91	4.45
ERSE	0.08	0.11	0.09	0.19	0.19	0.20	0.71	0.75	0.72	0.41	0.43	0.42	13.92	18.27	15.16	8.66	9.19	8.92

Table E.21: Results under model (2.43), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right). T=t=10, {Stable and Volatile: $\Delta_{10}(\beta_0) = 11023.06$, $\Delta_{10}(\beta_1) = -0.4041$}, {Simulated: $\Delta_{10}(\beta_0) = 2222.861$, $\Delta_{10}(\beta_1) = 14.4251$ }

	low response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB$_{\Delta(\beta_0)}$	165.59	0.36	11.55	77.64	9.48	10.86	52.20	39.22	23.49	79.70	58.75	47.15	12.44	183.07	125.61	85.47	144.12	111.60
SE	120.01	63.52	80.38	111.54	87.61	93.79	584.74	536.46	801.66	106.87	103.57	222.89	284.19	226.16	413.48	61.71	56.91	121.20
ERSE	444.37	59.17	99.36	690.11	92.82	141.23	896.25	632.93	1264.49	705.64	397.31	524.56	675.18	345.21	681.95	552.98	262.72	323.07
APRB$_{\Delta(\beta_1)}$	299.72	3.06	14.89	145.64	21.08	20.97	38.83	85.72	80.48	51.11	6.31	7.09	27.06	27.94	35.62	20.45	23.45	22.46
SE	0.04	0.04	0.06	0.08	0.08	0.09	0.25	0.36	0.62	0.09	0.11	0.18	6.60	7.01	13.32	1.73	1.84	3.80
ERSE	0.33	0.04	0.09	0.88	0.09	0.12	0.83	0.44	1.11	0.87	0.29	0.38	21.12	11.39	23.39	19.86	8.33	10.43
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	41.67	26.33	27.62	38.12	4.15	24.72	57.14	33.79	41.91	32.89	56.87	39.99	241.71	121.34	191.60	51.71	182.16	76.87
SE	80.91	91.16	79.94	99.47	89.29	96.62	960.76	674.76	884.50	278.63	206.37	248.05	495.13	367.36	455.50	147.82	113.15	133.20
ERSE	67.21	97.35	74.56	101.17	106.56	106.09	798.56	918.15	856.56	437.91	455.82	448.93	444.49	587.71	483.80	266.71	294.89	279.06
APRB$_{\Delta(\beta_1)}$	37.68	103.70	15.33	84.38	24.76	54.43	146.03	9.01	112.30	25.30	2.53	15.29	33.37	44.19	34.12	20.32	25.39	21.20
SE	0.08	0.05	0.07	0.10	0.09	0.09	0.93	0.45	0.76	0.22	0.17	0.19	17.69	10.94	15.40	4.57	3.65	4.12
ERSE	0.07	0.06	0.07	0.10	0.10	0.10	0.81	0.63	0.77	0.35	0.34	0.34	16.55	16.90	16.88	9.31	9.48	9.40
	low response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB$_{\Delta(\beta_0)}$	161.44	0.34	12.81	75.98	8.90	11.68	40.74	43.99	33.83	76.50	58.87	50.16	535.63	165.41	147.49	270.15	74.01	1.66
SE	110.24	59.32	72.14	108.36	84.64	89.46	558.47	506.84	751.30	105.49	101.86	218.02	232.75	172.05	346.32	78.27	66.59	109.28
ERSE	439.62	55.94	92.18	685.11	90.35	139.77	891.17	618.20	1225.86	701.38	389.55	515.18	706.48	304.37	576.83	548.45	232.75	277.34
APRB$_{\Delta(\beta_1)}$	679.74	81.21	63.79	220.18	12.64	3.78	148.82	128.71	181.95	93.23	79.73	78.44	159.35	64.02	86.49	66.62	29.07	21.07
SE	0.47	0.22	0.25	0.32	0.24	0.25	1.08	1.06	1.51	0.35	0.34	0.52	16.17	13.43	24.55	5.32	4.57	7.27
ERSE	1.02	0.16	0.24	1.44	0.23	0.34	1.57	1.15	2.30	1.18	0.69	0.94	35.88	20.13	39.91	26.72	13.47	17.13
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	42.81	24.34	28.76	39.22	3.49	25.70	67.25	22.84	52.12	37.16	59.22	43.57	366.49	247.54	270.03	109.88	83.85	59.91
SE	72.93	80.91	71.93	94.92	86.03	92.13	910.61	631.74	833.91	273.37	199.38	243.13	412.99	302.94	381.37	125.49	104.45	116.57
ERSE	62.34	90.00	69.12	98.89	103.95	103.80	776.25	893.53	832.60	431.32	448.15	441.95	373.34	526.54	411.61	228.20	257.34	240.69
APRB$_{\Delta(\beta_1)}$	7.39	148.74	26.80	24.16	1.99	11.58	198.29	171.43	190.23	76.14	79.83	77.26	60.18	135.77	71.78	11.31	24.85	16.47
SE	0.24	0.28	0.24	0.26	0.24	0.25	1.83	1.29	1.67	0.62	0.48	0.57	29.46	20.60	27.15	8.60	6.85	7.87
ERSE	0.15	0.24	0.17	0.24	0.26	0.26	1.48	1.66	1.56	0.78	0.80	0.80	25.30	33.78	27.64	13.97	15.27	14.58
	high response and low correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB$_{\Delta(\beta_0)}$	56.57	0.27	13.72	32.03	4.35	6.07	20.75	31.64	49.87	30.71	19.76	16.13	52.08	234.87	290.66	85.37	80.85	133.45
SE	26.40	25.58	35.36	55.13	57.51	63.18	259.10	321.21	500.42	52.54	59.29	128.96	43.79	36.80	102.40	29.84	26.95	49.64
ERSE	261.78	26.94	52.10	525.23	57.25	89.33	575.38	392.51	885.59	538.20	254.71	325.01	425.88	138.80	215.21	420.08	127.14	146.82
APRB$_{\Delta(\beta_1)}$	256.88	1.19	61.73	145.89	19.56	29.01	31.08	47.18	73.85	44.43	27.96	22.52	25.27	5.66	10.96	9.82	6.29	8.43
SE	0.01	0.01	0.02	0.03	0.03	0.03	0.13	0.17	0.27	0.03	0.03	0.07	2.32	2.24	7.27	1.24	1.19	2.57
ERSE	0.30	0.01	0.03	0.89	0.03	0.05	0.79	0.21	0.48	0.88	0.14	0.17	18.97	7.86	15.01	18.49	5.97	7.19
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	32.27	0.37	22.61	20.53	1.67	12.76	78.24	24.42	63.80	11.75	17.67	14.19	360.67	234.86	326.11	206.53	100.54	170.06
SE	43.58	31.83	39.14	69.77	60.89	66.03	687.70	400.79	588.92	156.87	124.37	140.45	136.43	84.46	119.05	56.00	48.45	52.25
ERSE	37.08	38.04	38.02	71.19	68.13	71.27	608.03	540.33	606.97	294.13	293.45	294.53	153.79	188.76	164.32	128.23	136.78	132.70
APRB$_{\Delta(\beta_1)}$	148.25	1.88	103.35	97.02	7.50	60.59	117.18	34.84	95.08	15.87	24.93	19.58	19.42	5.23	15.07	12.02	7.35	9.97
SE	0.02	0.02	0.02	0.04	0.03	0.04	0.38	0.21	0.32	0.08	0.07	0.07	10.52	5.26	8.84	2.92	2.57	2.71
ERSE	0.02	0.02	0.02	0.04	0.04	0.04	0.33	0.29	0.33	0.16	0.16	0.16	10.75	10.86	11.00	6.53	6.67	6.61
	high response and high correlation																	
	naïve	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)	**naïve**	**EE_h**	**EE**($\hat{p}_i$)	**naïve**	**EE_h**	**EE**($\hat{\pi}_i$)
APRB$_{\Delta(\beta_0)}$	60.89	0.72	14.20	41.22	10.40	1.62	2.68	26.87	44.23	41.40	38.00	36.70	145.53	281.36	351.23	128.85	130.67	192.51
SE	23.66	22.42	28.23	59.16	59.77	65.86	259.15	255.40	380.19	59.99	60.37	103.08	52.66	35.92	98.97	36.96	31.49	48.89
ERSE	262.04	22.05	40.35	523.45	60.95	94.68	594.63	332.28	619.88	536.22	221.55	265.97	406.64	127.31	198.00	405.52	117.78	135.64
APRB$_{\Delta(\beta_1)}$	153.20	28.58	12.93	70.07	2.45	4.52	85.67	44.54	36.70	48.07	22.66	17.60	46.11	23.16	32.15	19.55	15.86	20.10
SE	0.15	0.11	0.12	0.16	0.15	0.16	0.46	0.42	0.58	0.16	0.16	0.21	3.96	2.60	8.17	2.69	2.23	3.51
ERSE	0.49	0.08	0.11	1.01	0.16	0.23	0.95	0.50	0.81	0.87	0.34	0.40	21.63	9.00	16.85	19.85	6.81	8.18
	EE_i	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)	**EE_i**	**EE_ii**	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	33.12	0.99	23.23	18.01	3.08	9.20	64.38	28.37	54.00	34.10	37.18	35.47	432.98	286.82	392.76	270.55	167.87	229.52
SE	32.59	27.74	30.13	72.68	63.63	68.84	493.10	329.62	431.89	114.72	103.03	107.40	128.72	78.53	113.94	53.95	48.13	50.99
ERSE	29.05	30.97	30.02	74.97	72.29	75.39	457.59	436.47	460.32	244.71	246.91	246.45	138.36	169.18	148.58	118.88	126.27	122.87
APRB$_{\Delta(\beta_1)}$	8.16	26.79	3.11	12.78	4.46	7.54	26.18	47.21	31.21	12.73	17.75	15.68	44.73	21.78	38.54	25.42	19.72	22.19
SE	0.11	0.12	0.12	0.18	0.16	0.17	0.70	0.53	0.63	0.22	0.21	0.21	11.20	5.75	9.72	3.91	3.46	3.68
ERSE	0.08	0.09	0.08	0.18	0.18	0.19	0.62	0.64	0.63	0.36	0.37	0.37	11.26	12.71	11.92	7.14	7.52	7.34

Table E.22: Results under model (2.45), by response and correlation. Population: stable(Left), volatile(Middle), simulated(right)

	T=t=4, {Stable and Volatile: $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$}, {Simulated: $\Delta_4(\beta_0) = 114.9522$, $\Delta_4(\beta_1) = -0.5296$ }, low response																	
	naïve	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**
$\text{APRB}_{\Delta(\beta_0)}$	12.00	0.21	13.82	4.48	0.78	1.54	36.69	3.07	42.82	15.51	0.11	3.07	0.50	0.80	0.48	0.32	0.07	0.25
SE	204.14	233.93	328.18	335.14	359.10	454.37	516.51	655.94	831.63	355.15	392.66	502.26	236.63	236.31	363.46	143.98	142.47	191.08
ERSE	304.45	238.86	433.27	555.47	354.55	665.25	661.49	652.38	995.78	543.66	387.15	721.23	296.54	235.75	456.14	228.86	145.96	274.53
$\text{APRB}_{\Delta(\beta_1)}$	60.16	0.13	53.30	23.93	3.04	1.37	159.15	11.31	134.84	74.43	0.75	4.55	4.91	2.83	16.58	6.01	0.31	6.06
SE	0.08	0.12	0.13	0.15	0.18	0.22	0.23	0.33	0.39	0.17	0.20	0.25	3.37	4.46	5.93	2.35	2.75	3.65
ERSE	0.30	0.12	0.17	0.78	0.18	0.33	1.02	0.33	0.48	0.99	0.20	0.37	4.55	4.36	7.90	4.22	2.79	5.55
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$\text{APRB}_{\Delta(\beta_0)}$	10.70	15.59	11.92	1.54	2.25	0.49	36.36	46.14	38.89	4.29	6.70	1.41	1.33	0.15	0.98	0.05	0.54	0.07
SE	374.35	305.39	355.49	531.55	409.57	497.78	944.69	772.10	899.15	599.66	447.68	557.36	394.74	347.80	381.84	220.19	176.52	207.91
ERSE	267.59	309.81	282.73	421.74	454.52	439.56	614.62	707.05	648.46	456.19	489.20	474.98	275.86	336.60	295.02	170.84	193.59	179.99
$\text{APRB}_{\Delta(\beta_1)}$	23.56	65.15	36.76	31.42	3.10	19.63	67.39	158.60	97.97	43.40	18.96	22.73	23.75	14.01	20.66	3.87	7.86	4.83
SE	0.16	0.12	0.15	0.27	0.20	0.25	0.47	0.36	0.44	0.32	0.22	0.29	7.35	5.40	6.71	4.86	3.27	4.29
ERSE	0.11	0.12	0.11	0.22	0.22	0.22	0.31	0.32	0.31	0.24	0.24	0.25	5.23	5.28	5.28	3.90	3.63	3.84
	T=t=4, {Stable and Volatile: $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$}, {Simulated: $\Delta_4(\beta_0) = 114.9522$, $\Delta_4(\beta_1) = -0.5296$ }, high response																	
	naïve	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**
$\text{APRB}_{\Delta(\beta_0)}$	6.98	0.43	10.91	3.08	1.86	1.80	35.60	0.65	27.31	20.36	1.15	2.29	5.16	0.06	5.42	2.92	0.02	0.05
SE	86.21	168.35	153.56	176.54	243.65	298.89	233.32	448.13	417.96	199.43	272.08	345.98	122.87	128.96	198.52	81.86	84.04	112.66
ERSE	210.81	171.57	208.82	473.21	242.67	491.74	498.57	456.61	606.22	469.76	272.54	539.94	240.36	128.98	287.52	206.40	85.78	178.58
$\text{APRB}_{\Delta(\beta_1)}$	36.50	2.17	53.25	16.44	9.14	8.41	137.76	2.84	103.71	78.93	4.23	8.61	26.34	0.02	20.51	14.68	0.20	2.42
SE	0.05	0.09	0.08	0.10	0.13	0.17	0.13	0.25	0.23	0.11	0.15	0.19	2.14	2.75	3.49	1.51	1.75	2.28
ERSE	0.31	0.10	0.11	0.77	0.13	0.27	1.01	0.25	0.32	0.99	0.15	0.30	4.55	2.75	4.74	4.23	1.79	3.59
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$\text{APRB}_{\Delta(\beta_0)}$	11.69	10.66	11.35	1.88	1.77	1.97	15.90	31.92	20.78	9.46	5.83	4.42	4.47	5.83	4.88	1.94	0.92	1.08
SE	203.55	133.92	181.72	398.76	256.99	352.61	569.38	357.19	503.20	474.34	296.23	415.38	246.11	177.44	226.25	144.29	102.17	130.08
ERSE	142.62	130.31	141.03	341.07	299.18	334.21	414.26	379.30	409.82	375.89	333.98	369.15	183.36	195.67	189.89	121.68	120.74	123.14
$\text{APRB}_{\Delta(\beta_1)}$	54.64	52.75	54.16	6.67	8.52	8.03	57.07	122.07	77.25	37.59	22.23	17.69	9.36	24.32	14.52	14.98	1.68	9.33
SE	0.11	0.07	0.10	0.22	0.14	0.20	0.31	0.19	0.27	0.26	0.16	0.23	4.30	3.15	3.95	3.08	2.00	2.71
ERSE	0.08	0.07	0.08	0.19	0.17	0.19	0.22	0.20	0.22	0.21	0.18	0.20	3.08	3.23	3.14	2.48	2.33	2.46
	T=t=7, {Stable and Volatile: $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$}, {Simulated: $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$ }, low response																	
	naïve	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**
$\text{APRB}_{\Delta(\beta_0)}$	12.87	0.67	4.65	4.84	0.74	3.93	22.39	1.34	10.97	10.10	0.35	2.44	18.76	0.12	3.12	6.88	0.01	1.37
SE	209.80	242.62	308.90	338.95	363.60	429.08	490.55	576.09	742.44	313.26	334.18	434.50	333.21	334.60	512.41	200.73	203.40	254.67
ERSE	305.48	237.85	443.04	556.63	354.96	686.41	761.25	561.96	1017.56	569.77	335.36	667.60	440.79	329.38	718.37	366.68	206.27	357.13
$\text{APRB}_{\Delta(\beta_1)}$	63.42	2.59	1.59	21.90	7.50	26.20	16.82	0.66	3.38	8.81	0.24	1.72	13.24	1.87	12.62	6.09	0.15	1.13
SE	0.08	0.12	0.15	0.15	0.18	0.21	0.21	0.29	0.36	0.14	0.17	0.22	19.48	22.13	37.02	14.90	15.68	17.79
ERSE	0.30	0.12	0.22	0.78	0.18	0.35	0.77	0.28	0.51	0.80	0.17	0.34	42.18	22.42	59.74	42.39	15.30	21.08
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$\text{APRB}_{\Delta(\beta_0)}$	1.20	9.38	2.70	7.58	0.59	5.92	4.22	21.53	7.12	6.94	0.67	4.88	8.20	0.92	5.81	2.58	1.00	1.92
SE	357.97	264.00	335.58	510.02	387.40	468.30	852.13	643.38	802.01	545.45	371.30	489.04	615.16	440.09	563.44	303.19	232.31	277.46
ERSE	274.92	326.98	295.52	465.88	448.52	474.24	630.93	746.45	677.67	451.02	429.77	458.39	473.10	524.89	490.72	259.34	258.42	264.06
$\text{APRB}_{\Delta(\beta_1)}$	29.22	27.13	13.85	45.78	12.19	36.40	5.25	10.59	0.86	5.73	0.10	3.67	19.21	10.34	15.80	1.51	1.55	1.19
SE	0.20	0.12	0.17	0.27	0.19	0.24	0.45	0.30	0.41	0.28	0.18	0.25	51.64	30.91	43.89	18.97	17.72	18.24
ERSE	0.14	0.14	0.14	0.24	0.22	0.24	0.34	0.34	0.34	0.24	0.21	0.24	43.93	37.83	42.68	18.94	18.78	18.87
	T=t=7, {Stable and Volatile: $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$}, {Simulated: $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$ }, high response																	
	naïve	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**
$\text{APRB}_{\Delta(\beta_0)}$	6.61	0.90	3.59	5.70	1.72	0.84	23.81	1.57	4.93	13.70	1.54	1.73	1.75	0.17	0.47	0.57	0.23	0.35
SE	87.40	169.50	210.30	180.83	250.26	318.34	194.75	331.97	429.25	159.58	205.84	280.02	150.22	175.44	242.70	107.88	116.88	139.63
ERSE	210.99	171.64	336.31	472.73	240.88	529.52	539.64	335.64	666.17	483.75	204.72	423.82	416.02	178.05	370.45	369.62	117.17	206.29
$\text{APRB}_{\Delta(\beta_1)}$	34.73	4.41	16.80	29.13	8.27	3.44	12.41	0.86	2.70	7.18	0.82	0.90	0.47	0.12	0.10	0.09	0.10	0.21
SE	0.05	0.09	0.12	0.10	0.14	0.18	0.10	0.18	0.23	0.09	0.11	0.15	2.89	4.17	5.69	2.29	2.74	3.43
ERSE	0.31	0.10	0.19	0.78	0.13	0.30	0.79	0.18	0.36	0.80	0.11	0.23	42.11	4.21	8.88	41.97	2.72	5.09
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$\text{APRB}_{\Delta(\beta_0)}$	0.43	6.87	1.94	1.64	2.49	0.36	2.89	10.66	0.95	7.76	0.76	4.65	0.86	0.27	0.67	0.63	0.19	0.48
SE	290.35	152.23	250.92	422.74	271.98	366.74	585.85	319.12	508.29	383.76	232.39	327.99	310.40	196.88	277.39	172.44	128.82	154.06
ERSE	221.06	182.11	218.49	384.35	313.74	369.54	443.17	381.70	440.38	312.62	263.53	302.25	241.76	238.59	248.41	160.54	147.64	158.31
$\text{APRB}_{\Delta(\beta_1)}$	0.60	33.59	7.85	9.59	11.72	2.82	1.43	5.73	0.60	4.09	0.42	2.45	0.06	0.05	0.10	0.37	0.09	0.29
SE	0.16	0.08	0.14	0.24	0.15	0.20	0.32	0.17	0.28	0.21	0.13	0.18	7.64	4.39	6.66	4.41	3.08	3.86
ERSE	0.12	0.10	0.12	0.21	0.17	0.21	0.24	0.21	0.24	0.17	0.14	0.17	5.99	5.25	5.93	3.94	3.47	3.83
	T=t=10, {Stable and Volatile: $\Delta_{10}(\beta_0) = 11023.06$, $\Delta_{10}(\beta_1) = -0.4041$}, {Simulated: $\Delta_{10}(\beta_0) = 2222.861$, $\Delta_{10}(\beta_1) = 14.4251$ }, low response																	
	naïve	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**
$\text{APRB}_{\Delta(\beta_0)}$	11.82	0.73	1.90	5.47	0.24	1.01	29.64	1.19	0.45	16.08	2.04	0.07	140.23	41.98	40.59	31.66	4.12	5.49
SE	204.38	234.13	304.63	346.54	371.27	425.91	507.83	648.26	836.17	345.91	381.13	476.07	357.79	389.95	524.81	280.44	280.03	324.16
ERSE	304.99	239.89	457.95	555.72	354.59	601.80	686.86	644.21	1210.36	551.48	380.03	683.45	462.21	386.93	742.91	448.10	282.83	450.22
$\text{APRB}_{\Delta(\beta_1)}$	58.94	3.74	17.12	28.52	1.10	4.74	56.40	0.46	5.08	31.23	2.96	1.61	7.01	0.54	0.41	1.45	0.98	0.77
SE	0.08	0.12	0.16	0.15	0.18	0.21	0.23	0.34	0.44	0.16	0.19	0.25	7.02	9.33	12.88	5.52	6.52	7.63
ERSE	0.30	0.12	0.25	0.78	0.18	0.31	0.79	0.33	0.65	0.76	0.19	0.36	17.30	9.37	19.84	17.77	6.79	10.87
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$\text{APRB}_{\Delta(\beta_0)}$	4.98	2.19	3.57	2.36	0.45	1.66	6.85	10.71	3.51	3.29	1.26	1.52	46.29	17.40	43.91	8.49	1.51	7.48
SE	365.55	250.41	336.46	491.97	395.22	455.41	1007.19	659.87	926.39	583.77	428.66	524.44	622.19	440.84	575.49	372.39	307.16	345.16
ERSE	286.36	314.81	305.53	453.68	431.31	455.44	755.77	822.80	805.38	508.21	471.05	507.36	464.70	526.86	497.66	345.10	337.64	349.03
$\text{APRB}_{\Delta(\beta_1)}$	32.65	1.89	24.89	10.99	2.86	7.58	26.54	14.71	15.30	8.55	0.69	4.50	1.59	3.89	0.65	0.38	1.13	0.52
SE	0.22	0.12	0.19	0.26	0.20	0.23	0.56	0.34	0.50	0.31	0.22	0.27	17.11	10.43	14.82	9.00	7.21	8.20
ERSE	0.17	0.15	0.17	0.24	0.22	0.23	0.42	0.41	0.43	0.27	0.24	0.26	13.73	12.41	13.59	8.94	8.24	8.76
	T=t=10, {Stable and Volatile: $\Delta_{10}(\beta_0) = 11023.06$, $\Delta_{10}(\beta_1) = -0.4041$}, {Simulated: $\Delta_{10}(\beta_0) = 2222.861$, $\Delta_{10}(\beta_1) = 14.4251$ }, high response																	
	naïve	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**	**naïve**	**EE_h**	**EE($\hat{p}_i$)**	**naïve**	**EE_h**	**EE($\hat{\pi}_i$)**
$\text{APRB}_{\Delta(\beta_0)}$	7.30	0.29	0.50	4.39	0.08	0.92	27.46	1.03	5.69	15.53	1.20	2.69	27.26	9.04	34.22	31.90	13.83	10.97
SE	86.96	169.87	247.56	179.03	246.30	295.08	252.70	426.22	585.28	193.63	249.36	316.23	206.40	242.47	356.86	167.28	176.65	194.26
ERSE	210.47	170.53	409.78	473.14	242.39	434.83	513.99	411.40	947.06	476.41	251.80	444.33	378.19	244.07	561.45	401.85	176.24	239.23
$\text{APRB}_{\Delta(\beta_1)}$	38.06	1.37	1.39	23.04	0.22	4.67	43.03	1.70	8.81	24.45	1.96	4.15	3.37	0.87	2.10	2.61	0.61	0.22
SE	0.05	0.09	0.14	0.10	0.14	0.16	0.13	0.23	0.32	0.10	0.14	0.17	4.20	5.44	8.22	3.41	3.88	4.37
ERSE	0.31	0.09	0.23	0.77	0.13	0.24	0.78	0.23	0.52	0.75	0.14	0.25	16.03	5.50	13.05	16.87	3.90	5.49
	EE_i	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**	**EE_i**	**EE_ii**	**EE(0.5)**
$\text{APRB}_{\Delta(\beta_0)}$	3.79	4.94	1.65	2.47	0.46	1.60	18.17	4.98	11.79	6.88	1.60	4.43	26.87	44.72	30.43	1.52	17.75	5.01
SE	351.06	170.21	297.76	362.95	272.85	323.67	807.65	435.19	692.05	399.31	288.81	350.84	473.38	284.52	413.05	207.63	192.92	199.51
ERSE	270.01	208.42	264.77	348.78	305.06	336.81	631.45	510.79	622.34	356.94	313.24	345.27	374.82	338.40	378.22	210.41	209.03	210.72
$\text{APRB}_{\Delta(\beta_1)}$	21.20	23.80	9.78	12.70	2.40	8.17	28.42	7.84	18.37	10.76	2.42	6.89	0.66	3.30	1.44	0.91	0.70	0.29
SE	0.20	0.09	0.17	0.20	0.15	0.18	0.45	0.24	0.38	0.22	0.16	0.19	11.16	6.34	9.63	4.79	4.32	4.54
ERSE	0.15	0.12	0.15	0.19	0.17	0.19	0.35	0.28	0.34	0.20	0.17	0.19	8.70	7.63	8.71	4.81	4.70	4.78

Table E.23: Results under model (2.41), by response and correlation. Population: stable

	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response and low correlation.																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	161.37	0.13	11.06	39.44	24.08	26.15	2.89	32.75	30.83	18.78	22.90	9.15	48.19	6.04	65.40	0.28	19.09	45.12	4.87	32.36	3.17	19.18	10.49	8.23
SE	58.91	37.81	52.38	58.47	50.57	55.32	54.20	60.58	51.99	57.21	55.06	61.48	52.76	57.87	112.71	87.31	90.73	95.46	88.78	92.96	90.99	92.05	90.79	91.45
ERSE	458.71	41.09	77.96	50.78	67.12	55.47	79.57	51.97	67.03	56.50	78.69	53.14	64.03	56.89	689.20	86.26	135.67	95.77	99.99	100.31	121.88	92.20	96.52	95.97
APRB$_{\Delta(\beta_1)}$	283.74	3.01	16.23	33.03	100.35	12.25	31.99	20.53	111.04	1.81	77.25	17.89	136.56	44.20	118.13	0.52	36.66	96.25	6.59	68.35	12.14	36.81	34.55	14.11
SE	0.03	0.03	0.05	0.07	0.04	0.06	0.05	0.07	0.04	0.06	0.05	0.06	0.04	0.05	0.08	0.08	0.09	0.10	0.09	0.09	0.09	0.09	0.09	0.09
ERSE	0.33	0.04	0.08	0.06	0.05	0.06	0.08	0.06	0.05	0.06	0.06	0.05	0.04	0.05	0.89	0.08	0.12	0.10	0.10	0.10	0.11	0.09	0.09	0.09
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response and high correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	156.24	0.65	11.76	39.97	22.56	26.71	3.53	33.15	29.33	19.24	21.73	9.83	46.26	5.19	62.75	1.33	20.63	46.87	6.35	34.01	1.53	20.96	8.79	9.95
SE	61.82	39.90	54.18	59.56	53.27	56.75	56.22	61.73	54.68	58.80	57.12	61.18	56.69	58.74	108.73	83.80	91.02	97.01	87.14	94.01	90.36	93.03	88.64	91.70
ERSE	451.25	38.07	71.56	46.55	61.64	50.87	73.03	47.64	61.55	51.81	72.37	48.77	58.76	52.23	684.86	84.40	134.82	93.99	98.02	98.58	120.57	90.35	94.58	94.15
APRB$_{\Delta(\beta_1)}$	702.94	88.55	68.34	5.99	158.61	29.58	94.59	14.66	181.64	52.78	180.90	89.92	246.87	133.45	276.94	71.47	50.58	27.85	56.77	41.68	78.21	57.82	79.05	70.41
SE	0.49	0.23	0.25	0.23	0.28	0.24	0.26	0.24	0.29	0.25	0.29	0.26	0.32	0.28	0.33	0.25	0.26	0.27	0.26	0.27	0.27	0.28	0.26	0.27
ERSE	1.06	0.15	0.24	0.14	0.23	0.16	0.24	0.15	0.23	0.16	0.26	0.16	0.22	0.17	1.44	0.23	0.34	0.24	0.26	0.26	0.31	0.23	0.25	0.25
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response and low correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	53.71	0.24	12.62	29.92	0.35	20.89	8.57	25.65	2.76	16.73	3.83	10.84	10.76	3.09	26.61	0.20	10.53	24.71	6.29	17.10	0.25	10.66	2.10	4.84
SE	19.00	19.42	27.68	34.80	23.98	31.01	27.28	34.29	23.77	30.54	25.55	31.14	23.31	28.05	50.94	53.14	56.77	63.01	54.87	59.42	55.76	59.63	54.62	57.40
ERSE	262.31	19.62	40.21	27.95	27.53	28.48	39.40	27.62	26.95	28.02	34.36	25.44	24.61	25.55	524.71	55.60	87.87	69.67	66.44	69.71	74.67	62.51	61.14	62.69
APRB$_{\Delta(\beta_1)}$	244.05	1.07	56.74	137.23	1.60	95.35	37.96	117.35	15.78	76.03	19.24	48.77	52.11	12.96	121.45	1.15	49.41	116.14	28.68	80.43	0.99	49.99	10.09	22.90
SE	0.01	0.01	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
ERSE	0.30	0.01	0.02	0.02	0.01	0.02	0.02	0.01	0.01	0.02	0.02	0.01	0.01	0.01	0.89	0.03	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response and high correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	52.76	0.12	13.04	30.04	1.26	21.12	9.06	25.83	1.79	17.03	3.12	11.27	9.73	3.62	26.25	0.17	11.38	26.78	6.90	18.55	0.38	11.37	2.24	5.13
SE	12.14	11.71	15.75	19.10	13.81	17.35	15.61	18.89	13.90	17.16	14.84	17.33	13.86	15.96	60.63	61.64	66.74	72.99	64.91	69.45	65.12	68.39	64.73	66.54
ERSE	259.42	11.23	22.77	15.36	15.46	15.80	22.37	15.23	15.15	15.60	19.69	14.20	13.89	14.37	519.71	59.47	93.39	73.80	70.89	74.12	79.88	66.60	65.48	67.00
APRB$_{\Delta(\beta_1)}$	98.57	15.59	5.38	11.76	16.82	2.62	10.59	7.00	21.78	2.35	27.70	12.65	33.38	20.73	39.09	1.93	5.15	9.62	4.86	6.74	2.42	7.99	1.48	4.48
SE	0.10	0.08	0.08	0.09	0.09	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.15	0.15	0.16	0.18	0.16	0.17	0.16	0.17	0.16	0.16
ERSE	0.46	0.06	0.09	0.06	0.07	0.07	0.09	0.06	0.07	0.06	0.08	0.06	0.07	0.06	0.95	0.15	0.22	0.18	0.18	0.18	0.19	0.16	0.16	0.16
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response and low correlation.																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	161.37	0.16	11.04	39.51	24.20	26.18	2.80	32.75	31.04	18.74	22.96	9.10	48.24	6.10	65.20	0.02	19.13	45.25	4.77	32.44	3.10	19.28	10.46	8.31
SE	66.54	42.75	58.37	63.55	60.00	60.71	60.03	65.65	60.79	62.53	63.97	67.94	64.04	65.48	113.31	89.16	94.27	99.74	90.21	96.99	94.94	96.94	93.33	95.96
ERSE	458.51	41.04	77.80	50.70	67.12	55.40	79.39	51.88	67.04	56.43	78.63	53.12	64.05	56.88	689.42	86.23	135.70	95.78	100.02	100.33	121.85	92.19	96.54	95.96
APRB$_{\Delta(\beta_1)}$	283.69	1.43	11.72	39.60	97.44	17.71	28.35	25.91	108.74	2.64	73.53	12.85	133.38	39.90	116.47	1.14	39.64	100.64	4.87	71.97	10.00	39.39	32.67	16.46
SE	0.03	0.04	0.05	0.07	0.04	0.06	0.05	0.06	0.04	0.06	0.04	0.06	0.04	0.05	0.08	0.08	0.09	0.10	0.09	0.09	0.09	0.09	0.09	0.09
ERSE	0.33	0.04	0.08	0.06	0.05	0.06	0.08	0.06	0.05	0.05	0.06	0.05	0.04	0.05	0.88	0.08	0.12	0.10	0.10	0.10	0.11	0.09	0.09	0.09
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response and high correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	156.73	0.27	11.57	39.83	22.89	26.56	3.37	33.04	29.62	19.11	22.12	9.46	46.63	5.56	63.82	0.37	20.22	46.75	5.60	33.75	2.56	20.08	9.95	9.00
SE	58.24	38.84	54.77	60.13	54.65	57.32	56.27	61.88	55.61	58.89	57.54	61.97	58.09	59.36	102.06	79.99	85.82	90.41	83.28	88.10	86.69	88.75	85.42	87.69
ERSE	451.23	38.06	71.45	46.50	61.72	50.84	73.01	47.65	61.66	51.83	72.46	48.86	58.96	52.33	684.66	84.38	134.68	93.97	97.99	98.55	120.44	90.32	94.55	94.11
APRB$_{\Delta(\beta_1)}$	682.77	72.67	43.55	32.91	135.44	3.71	68.55	13.90	159.75	25.49	151.70	59.79	221.86	104.02	239.23	42.40	21.52	1.18	28.56	12.35	50.91	29.44	53.70	42.30
SE	0.47	0.22	0.24	0.23	0.27	0.23	0.25	0.24	0.28	0.24	0.28	0.26	0.30	0.27	0.31	0.24	0.25	0.26	0.24	0.26	0.25	0.26	0.25	0.26
ERSE	1.04	0.15	0.24	0.14	0.22	0.16	0.24	0.15	0.22	0.16	0.26	0.16	0.22	0.17	1.42	0.22	0.34	0.24	0.26	0.25	0.31	0.23	0.25	0.24
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response and low correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	53.49	0.06	12.71	29.96	0.49	20.95	8.69	25.71	2.57	16.82	3.62	11.06	10.55	3.30	26.70	0.05	10.28	24.37	6.10	16.80	0.67	10.01	2.36	4.31
SE	19.44	19.11	26.67	33.39	23.36	29.79	26.32	32.76	23.30	29.29	25.12	30.44	23.16	27.47	51.35	53.81	57.90	64.22	55.92	60.60	56.13	59.88	55.23	57.71
ERSE	262.28	19.63	40.81	28.32	27.74	28.82	39.97	27.98	27.17	28.37	34.81	25.73	24.78	25.82	524.32	55.60	87.43	69.51	66.45	69.59	74.43	62.43	61.15	62.63
APRB$_{\Delta(\beta_1)}$	243.09	0.30	57.07	137.32	1.03	95.56	38.43	117.55	15.00	76.36	18.33	49.67	51.20	13.86	122.06	0.16	47.86	114.12	27.41	78.64	3.40	46.48	11.73	19.96
SE	0.01	0.01	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.02	0.01	0.01	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
ERSE	0.30	0.01	0.02	0.02	0.01	0.02	0.02	0.02	0.01	0.02	0.02	0.01	0.01	0.01	0.88	0.03	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response and high correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	52.89	0.04	12.82	29.83	1.06	20.90	8.81	25.58	2.02	16.77	3.29	11.13	9.93	3.47	26.70	0.18	10.83	26.16	6.41	17.96	1.04	10.61	2.81	4.42
SE	12.27	11.91	14.94	17.80	13.77	16.26	14.94	17.78	13.83	16.23	14.88	17.46	13.93	16.02	58.48	59.34	63.36	68.99	62.10	65.73	61.47	64.38	61.18	62.65
ERSE	259.55	11.25	22.85	15.41	15.51	15.86	22.45	15.29	15.21	15.65	19.78	14.26	13.97	14.43	519.80	59.48	93.57	73.90	70.95	74.21	79.91	66.63	65.50	67.02
APRB$_{\Delta(\beta_1)}$	96.76	14.01	1.82	15.50	13.24	6.26	6.66	10.87	17.54	1.53	22.89	7.43	29.20	15.74	43.88	4.73	8.22	16.51	5.37	11.62	0.59	6.60	1.88	2.29
SE	0.11	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.08	0.09	0.09	0.09	0.09	0.15	0.16	0.17	0.18	0.16	0.18	0.16	0.17	0.16	0.17
ERSE	0.46	0.06	0.09	0.06	0.07	0.07	0.08	0.06	0.07	0.07	0.08	0.06	0.07	0.06	0.96	0.15	0.22	0.18	0.18	0.18	0.19	0.16	0.16	0.16

Table E.24: Results under model (2.41), by response and correlation. Population: volatile

	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response and low correlation.																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	48.80	0.22	3.31	13.08	30.55	5.71	6.98	9.86	32.75	2.31	17.90	1.08	36.82	8.64	24.46	0.34	6.04	15.65	0.31	10.82	1.50	6.15	4.40	2.32
SE	513.88	425.55	598.66	686.25	564.13	642.73	606.71	695.29	567.62	650.77	606.88	690.08	580.42	645.40	301.02	271.49	299.24	325.21	292.89	309.91	298.98	313.66	295.25	304.87
ERSE	868.85	434.08	862.08	555.80	694.25	602.07	869.93	563.03	688.55	606.89	848.39	566.21	644.79	599.09	620.91	274.82	387.79	311.49	322.39	319.49	351.67	294.72	305.89	301.76
APRB$_{\Delta(\beta_1)}$	89.27	1.24	37.46	0.70	126.79	14.26	44.49	5.72	128.58	20.28	66.17	25.50	127.95	41.44	47.53	2.80	13.20	39.95	6.82	27.49	6.01	14.58	16.25	5.18
SE	0.22	0.22	0.32	0.41	0.28	0.36	0.32	0.40	0.28	0.35	0.30	0.37	0.28	0.33	0.14	0.14	0.16	0.18	0.15	0.16	0.15	0.16	0.15	0.16
ERSE	1.04	0.22	0.48	0.32	0.35	0.33	0.47	0.32	0.34	0.33	0.43	0.30	0.31	0.31	1.00	0.14	0.20	0.16	0.17	0.17	0.18	0.15	0.15	0.15
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response and high correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	146.06	1.41	7.65	33.09	22.99	21.13	0.49	27.11	29.00	14.62	22.70	5.24	44.11	8.04	58.52	0.10	12.84	30.40	4.73	21.29	2.28	12.35	6.32	4.75
SE	348.44	238.72	326.38	367.72	316.49	346.35	332.61	374.65	320.83	352.58	341.99	376.87	333.02	357.11	198.72	164.38	169.57	177.47	166.26	172.66	174.25	176.97	173.24	175.14
ERSE	1028.95	243.08	463.03	303.66	384.00	329.35	471.14	309.85	383.40	334.50	459.49	313.55	364.93	333.14	670.24	159.93	214.13	175.61	183.18	180.73	199.41	169.33	176.31	173.77
APRB$_{\Delta(\beta_1)}$	276.90	61.45	16.47	41.93	84.80	15.61	39.43	19.95	103.66	6.54	87.82	20.95	153.27	50.74	111.32	32.09	5.08	10.48	7.60	1.80	21.25	7.18	22.58	15.06
SE	1.60	1.03	1.51	1.62	1.54	1.56	1.58	1.71	1.58	1.64	1.60	1.69	1.60	1.64	0.74	0.58	0.62	0.61	0.63	0.62	0.65	0.64	0.65	0.64
ERSE	2.72	0.98	1.81	1.24	1.69	1.35	1.87	1.28	1.69	1.39	1.79	1.27	1.59	1.37	1.61	0.56	0.71	0.61	0.65	0.63	0.69	0.60	0.63	0.62
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response and low correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	42.46	0.79	11.86	27.48	0.59	19.32	8.43	23.65	1.85	15.70	2.66	9.85	8.74	3.25	21.09	0.90	7.40	16.02	5.33	11.21	0.72	7.33	0.10	3.64
SE	129.40	126.73	178.95	225.98	156.75	200.42	178.72	225.27	157.13	199.84	166.81	200.99	155.45	181.73	89.15	88.94	95.63	100.41	95.21	97.42	94.17	97.13	94.11	95.23
ERSE	617.52	130.16	259.80	186.47	182.17	188.84	254.69	184.25	178.73	185.97	219.34	168.21	163.83	168.68	524.38	90.18	117.23	104.35	103.91	104.68	107.36	97.41	97.49	97.74
APRB$_{\Delta(\beta_1)}$	151.35	2.60	41.65	98.73	0.55	69.03	29.16	84.73	9.26	55.83	11.01	34.54	33.79	10.64	75.28	3.43	27.32	59.14	19.36	41.44	2.77	27.03	0.42	13.55
SE	0.07	0.07	0.10	0.12	0.08	0.11	0.10	0.12	0.08	0.11	0.09	0.11	0.08	0.10	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
ERSE	1.02	0.07	0.14	0.10	0.10	0.10	0.14	0.10	0.10	0.10	0.12	0.09	0.09	0.09	1.00	0.05	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response and high correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	53.22	0.69	16.00	33.97	5.74	24.28	11.92	29.43	2.48	19.99	0.22	14.35	5.83	6.41	26.49	0.17	7.40	16.72	6.14	11.35	0.05	7.39	0.45	3.10
SE	71.54	65.55	86.76	104.20	79.65	94.41	86.19	102.91	79.50	93.48	81.76	93.03	78.66	86.46	56.27	55.64	60.79	63.54	60.33	61.86	60.16	62.12	59.91	60.91
ERSE	602.86	66.13	121.00	90.97	89.79	91.93	118.70	89.93	88.35	90.66	103.71	83.11	82.20	83.50	515.43	55.03	72.53	64.27	63.54	64.38	65.98	59.69	59.50	59.86
APRB$_{\Delta(\beta_1)}$	39.70	1.51	7.89	16.37	1.79	11.88	6.01	14.88	0.40	10.19	5.70	1.73	9.05	2.11	26.58	6.88	3.38	0.68	2.95	2.42	5.61	2.46	5.78	4.44
SE	0.18	0.16	0.20	0.23	0.18	0.21	0.20	0.23	0.19	0.21	0.19	0.21	0.19	0.20	0.15	0.14	0.14	0.15	0.14	0.15	0.14	0.15	0.14	0.14
ERSE	1.59	0.16	0.26	0.20	0.21	0.21	0.26	0.20	0.20	0.20	0.23	0.19	0.19	0.19	1.25	0.13	0.17	0.15	0.15	0.15	0.15	0.14	0.14	0.14
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response and low correlation.																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	83.29	0.50	4.83	29.24	33.15	18.15	1.26	23.58	36.34	12.33	20.76	3.79	45.26	7.38	38.67	1.86	7.94	20.46	1.14	14.04	2.20	7.89	5.49	2.72
SE	525.93	419.16	595.54	674.15	564.42	634.88	609.00	692.85	569.81	650.37	614.39	696.08	582.79	651.97	301.60	270.68	298.98	327.58	289.18	310.94	295.91	310.61	291.64	301.81
ERSE	884.93	411.98	823.11	528.77	652.70	571.97	833.72	536.92	646.78	577.66	806.88	536.37	604.37	566.55	614.25	259.50	369.93	295.56	303.82	302.64	334.15	278.89	288.07	285.17
APRB$_{\Delta(\beta_1)}$	36.23	0.80	3.12	15.30	14.24	9.52	0.07	12.24	15.83	6.51	8.61	2.99	19.57	2.45	17.93	0.73	3.68	9.76	0.41	6.58	1.03	3.78	2.59	1.27
SE	0.21	0.21	0.32	0.39	0.28	0.35	0.32	0.39	0.28	0.35	0.30	0.36	0.28	0.33	0.14	0.14	0.15	0.17	0.15	0.16	0.15	0.16	0.15	0.15
ERSE	0.78	0.20	0.45	0.31	0.32	0.31	0.44	0.30	0.31	0.31	0.40	0.28	0.28	0.28	0.80	0.13	0.19	0.16	0.15	0.16	0.17	0.14	0.14	0.14
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response and high correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	150.75	1.88	15.38	45.53	22.98	31.43	7.19	38.51	29.46	23.89	18.56	13.86	44.73	1.43	61.74	0.90	16.33	35.55	7.69	25.57	0.31	16.43	4.05	8.04
SE	371.33	255.18	347.84	382.73	345.30	364.32	351.01	385.84	349.20	367.13	356.44	384.26	353.53	367.67	197.57	163.26	174.64	180.31	174.71	176.56	177.12	179.13	177.13	177.58
ERSE	998.61	241.46	456.00	299.39	379.87	324.86	462.63	304.83	379.01	329.28	450.84	308.31	360.07	327.74	650.32	159.24	211.83	174.01	181.90	179.18	197.50	167.85	174.96	172.30
APRB$_{\Delta(\beta_1)}$	114.75	14.36	9.12	7.39	28.46	0.56	13.41	3.94	32.18	4.42	32.73	16.07	44.63	24.13	44.95	6.45	0.18	6.29	2.55	2.63	6.91	0.83	8.43	4.28
SE	1.01	0.76	0.97	1.09	0.92	1.03	0.98	1.11	0.93	1.04	1.01	1.10	0.97	1.05	0.54	0.46	0.48	0.50	0.47	0.49	0.48	0.49	0.48	0.48
ERSE	2.30	0.70	1.31	0.86	1.05	0.92	1.32	0.86	1.04	0.92	1.23	0.84	0.96	0.88	1.44	0.42	0.56	0.45	0.48	0.47	0.51	0.43	0.45	0.44
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response and low correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	59.72	1.35	14.94	35.46	0.74	24.71	10.32	30.45	2.81	19.91	3.74	13.47	11.91	4.38	29.65	0.41	7.15	17.06	5.06	11.44	0.62	7.27	1.52	2.79
SE	124.99	123.69	172.51	216.99	153.59	192.68	170.64	213.98	152.83	190.15	156.58	189.18	146.28	170.67	85.83	85.33	95.33	100.70	95.14	97.31	92.11	95.21	92.40	93.17
ERSE	606.34	125.75	255.59	182.62	176.68	184.62	250.17	180.20	173.16	181.55	215.55	164.25	158.38	164.35	511.58	87.50	114.86	101.91	101.18	102.13	104.81	94.90	94.80	95.16
APRB$_{\Delta(\beta_1)}$	29.09	0.62	7.35	17.67	0.07	12.28	5.04	15.16	1.68	9.88	1.95	6.64	6.13	2.12	14.52	0.17	3.60	8.59	2.51	5.77	0.28	3.66	0.76	1.43
SE	0.06	0.07	0.09	0.12	0.08	0.10	0.09	0.12	0.08	0.10	0.08	0.10	0.08	0.09	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
ERSE	0.80	0.07	0.14	0.10	0.09	0.10	0.14	0.10	0.09	0.10	0.12	0.09	0.08	0.09	0.80	0.05	0.06	0.05	0.05	0.05	0.06	0.05	0.05	0.05
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response and high correlation																							
	naïve	**EE**$_h$	**EE**$(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	**EE**$_h$	**EE**$(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	**EE**$(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
APRB$_{\Delta(\beta_0)}$	58.98	0.72	18.71	37.87	8.22	27.48	14.52	33.30	4.79	23.11	1.19	16.93	4.55	8.30	28.69	0.52	8.44	18.24	7.22	12.56	0.55	8.40	0.20	3.83
SE	71.63	66.14	85.96	104.20	79.42	93.92	85.96	103.37	80.21	93.49	79.98	91.10	77.40	84.55	55.60	55.15	60.14	63.21	59.63	61.33	59.06	60.90	58.97	59.76
ERSE	588.93	66.36	121.19	90.85	89.83	91.86	118.74	89.74	88.36	90.52	103.67	82.83	82.07	83.26	501.62	55.20	72.72	64.39	63.68	64.52	66.15	59.81	59.64	59.99
APRB$_{\Delta(\beta_1)}$	48.32	5.39	3.71	16.03	4.74	9.51	0.51	12.47	7.23	6.15	9.35	0.97	14.54	4.49	24.90	3.18	0.27	4.21	0.05	1.76	3.31	0.13	3.42	2.03
SE	0.32	0.32	0.38	0.41	0.36	0.39	0.38	0.41	0.36	0.40	0.37	0.40	0.36	0.39	0.23	0.22	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
ERSE	1.24	0.28	0.43	0.31	0.35	0.33	0.42	0.31	0.35	0.32	0.38	0.29	0.33	0.30	1.03	0.19	0.24	0.21	0.22	0.21	0.22	0.20	0.20	0.20

Table E.25: Results under model (2.41), by response and correlation. Population: simulated

	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
	T=10, t=4, $\Delta_4(\beta_0) = 114.9522$, $\Delta_4(\beta_1) = -0.5296$, low response and low correlation.																							
$\text{APRB}_{\Delta(\beta_0)}$	152.64	0.00	9.56	36.11	24.22	23.71	1.74	29.80	30.76	16.71	22.82	7.49	47.28	6.84	61.91	0.00	10.01	21.50	6.49	15.10	0.41	10.03	2.28	4.15
SE	35.87	19.21	24.24	22.88	26.00	23.59	25.67	24.26	26.75	25.00	29.67	28.51	29.69	29.19	18.04	12.96	14.42	15.18	13.71	14.83	14.70	15.14	14.24	14.95
ERSE	448.05	19.42	33.80	17.51	31.19	20.80	35.72	18.51	31.68	21.88	40.91	21.83	31.93	25.19	284.48	13.09	19.95	13.73	15.00	14.65	18.45	13.67	14.77	14.43
$\text{APRB}_{\Delta(\beta_1)}$	197.54	0.04	3.79	32.99	57.93	16.23	15.56	23.36	66.80	5.61	48.44	6.09	87.24	25.71	83.83	0.28	14.85	33.01	7.13	23.29	1.24	14.89	5.41	6.17
SE	0.92	0.59	0.74	0.77	0.73	0.75	0.76	0.79	0.75	0.78	0.83	0.85	0.82	0.83	0.53	0.43	0.47	0.49	0.45	0.48	0.47	0.49	0.47	0.48
ERSE	7.25	0.62	1.14	0.65	0.90	0.73	1.17	0.67	0.90	0.74	1.20	0.71	0.88	0.78	5.47	0.43	0.63	0.48	0.50	0.50	0.58	0.46	0.48	0.48
	T=10, t=4, $\Delta_4(\beta_0) = 114.9522$, $\Delta_4(\beta_1) = -0.5296$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	139.46	0.08	9.95	36.06	21.31	23.71	2.34	29.73	27.51	16.78	21.08	7.79	43.11	6.05	58.76	0.15	9.38	20.50	6.11	14.26	0.66	9.36	2.38	3.68
SE	35.39	18.65	22.10	19.40	24.25	20.88	23.94	21.25	25.47	22.74	28.03	26.25	27.83	27.30	18.57	13.02	14.13	14.86	13.47	14.54	14.65	14.96	14.32	14.84
ERSE	420.37	18.23	31.26	14.85	28.64	18.28	33.21	15.93	29.09	19.42	38.40	19.52	29.27	22.88	277.77	12.33	18.98	12.72	14.06	13.68	17.51	12.72	13.85	13.50
$\text{APRB}_{\Delta(\beta_1)}$	149.90	1.50	10.28	38.02	22.40	24.84	1.89	31.00	29.21	17.17	22.93	7.61	45.79	7.08	63.94	1.43	8.44	20.02	5.30	13.49	2.13	8.30	3.70	2.35
SE	1.75	0.85	1.00	0.85	1.16	0.93	1.08	0.93	1.23	1.01	1.24	1.10	1.34	1.17	0.92	0.62	0.64	0.64	0.63	0.65	0.68	0.67	0.68	0.68
ERSE	10.08	0.77	1.22	0.60	1.21	0.74	1.29	0.64	1.23	0.78	1.48	0.78	1.23	0.91	6.62	0.54	0.75	0.52	0.60	0.56	0.71	0.53	0.59	0.57
	T=10, t=4, $\Delta_4(\beta_0) = 114.9522$, $\Delta_4(\beta_1) = -0.5296$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	61.91	0.01	14.23	31.15	2.19	22.39	10.18	27.09	1.08	18.33	2.69	12.48	9.80	4.52	29.29	0.04	4.94	10.65	4.30	7.17	0.01	4.94	0.24	1.90
SE	15.35	13.14	16.14	17.76	15.03	17.04	16.34	18.11	15.25	17.31	16.72	18.82	15.61	17.77	10.79	9.74	10.86	11.75	10.53	11.26	10.78	11.33	10.60	11.02
ERSE	283.01	13.35	25.88	15.14	17.34	16.50	25.91	15.35	17.25	16.61	24.49	15.58	16.45	16.45	225.12	9.44	13.60	10.98	10.95	11.17	12.15	10.26	10.35	10.43
$\text{APRB}_{\Delta(\beta_1)}$	194.77	0.04	43.11	101.68	2.46	71.81	29.42	87.65	12.86	57.94	13.62	37.96	40.25	11.31	93.83	0.15	16.91	36.79	14.06	24.87	0.01	16.89	1.21	6.66
SE	0.33	0.32	0.40	0.47	0.36	0.44	0.40	0.47	0.36	0.44	0.40	0.47	0.36	0.44	0.24	0.23	0.26	0.29	0.25	0.27	0.25	0.27	0.25	0.26
ERSE	5.55	0.32	0.66	0.39	0.42	0.42	0.65	0.39	0.41	0.41	0.60	0.38	0.38	0.40	4.68	0.23	0.34	0.27	0.27	0.27	0.30	0.25	0.25	0.25
	T=10, t=4, $\Delta_4(\beta_0) = 114.9522$, $\Delta_4(\beta_1) = -0.5296$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	48.30	0.09	10.73	26.72	0.93	18.38	6.97	22.72	3.73	14.50	4.33	9.12	10.85	2.02	24.47	0.05	4.47	9.99	3.58	6.67	0.15	4.46	0.53	1.66
SE	10.54	11.31	14.50	16.37	11.89	15.66	14.55	16.64	11.92	15.80	14.44	17.07	12.39	15.83	7.88	7.99	9.16	10.51	8.67	9.76	8.65	9.43	8.46	9.00
ERSE	252.54	11.15	22.69	12.40	14.22	13.68	22.55	12.58	13.99	13.72	20.89	12.78	12.97	13.41	213.94	7.90	12.53	9.67	9.39	9.77	10.71	8.77	8.68	8.86
$\text{APRB}_{\Delta(\beta_1)}$	45.73	0.72	9.17	24.56	1.67	16.46	5.46	20.54	4.38	12.60	5.34	7.29	11.16	0.56	23.76	0.57	3.80	9.16	2.90	5.93	0.69	3.77	1.09	1.06
SE	0.44	0.47	0.57	0.62	0.48	0.61	0.57	0.63	0.48	0.61	0.56	0.65	0.49	0.61	0.31	0.31	0.37	0.42	0.34	0.39	0.34	0.38	0.33	0.36
ERSE	5.90	0.44	0.82	0.45	0.55	0.50	0.82	0.46	0.54	0.50	0.76	0.46	0.49	0.49	5.05	0.31	0.47	0.37	0.37	0.37	0.41	0.33	0.34	0.34
	T=10, t=7, $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	189.28	0.14	8.65	38.03	32.36	24.67	0.42	31.10	40.12	16.76	29.21	6.33	59.65	10.15	74.10	0.10	10.34	21.57	7.60	15.13	0.09	10.35	1.61	4.29
SE	87.16	42.51	47.15	42.08	53.24	44.66	49.82	44.57	55.60	47.21	59.20	54.54	61.05	56.88	39.74	27.15	28.01	28.25	27.49	28.21	29.27	28.99	28.99	29.23
ERSE	650.67	38.30	63.97	33.58	58.84	40.07	67.45	35.62	59.98	42.22	76.09	42.16	60.77	48.52	455.55	25.50	34.66	26.09	28.30	27.64	32.95	26.17	28.14	27.51
$\text{APRB}_{\Delta(\beta_1)}$	33.59	0.66	17.34	30.82	13.27	23.37	14.75	27.73	11.10	20.51	5.63	15.63	4.72	9.84	18.70	0.35	2.08	4.89	2.68	2.88	0.29	2.12	0.13	0.37
SE	5.58	3.13	3.43	3.31	3.46	3.40	3.57	3.47	3.57	3.55	3.90	3.86	3.86	3.89	2.75	2.01	2.13	2.12	2.10	2.13	2.17	2.14	2.15	2.16
ERSE	66.85	2.83	4.78	2.61	3.80	3.04	4.91	2.73	3.85	3.15	5.05	3.08	3.84	3.42	53.84	1.85	2.29	1.92	2.04	2.00	2.21	1.90	2.00	1.97
	T=10, t=7, $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	127.56	1.61	11.89	36.44	17.75	24.89	4.87	30.65	23.55	18.52	16.90	10.37	37.86	2.65	54.76	0.38	8.54	17.59	6.48	12.31	0.39	8.59	0.73	3.75
SE	239.06	112.48	116.00	88.39	146.96	102.10	125.76	97.17	154.52	111.32	154.89	127.37	172.88	141.01	104.33	67.39	67.89	64.94	67.47	66.87	72.95	69.82	72.50	71.82
ERSE	858.45	87.07	113.63	52.29	131.44	68.01	121.33	56.90	133.24	73.01	140.62	71.98	132.84	87.87	540.90	57.89	70.13	51.26	60.97	56.21	70.05	53.72	61.74	58.07
$\text{APRB}_{\Delta(\beta_1)}$	161.42	4.41	3.92	32.43	29.78	18.83	4.73	25.13	36.67	10.89	31.17	0.02	54.46	15.01	65.30	1.27	7.33	17.21	5.22	11.43	1.96	7.17	3.10	1.79
SE	21.34	10.05	10.30	7.88	13.03	9.09	11.18	8.67	13.70	9.92	13.74	11.32	15.37	12.52	9.25	5.98	5.96	5.68	5.95	5.86	6.42	6.13	6.40	6.31
ERSE	108.26	7.84	10.47	4.76	11.90	6.20	11.17	5.19	12.06	6.67	12.95	6.59	12.02	8.03	68.77	5.10	6.18	4.52	5.38	4.96	6.18	4.74	5.45	5.12
	T=10, t=7, $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	73.88	0.03	16.82	36.17	3.38	26.12	12.18	31.58	0.47	21.50	2.83	14.67	10.78	5.45	34.71	0.04	4.70	10.07	4.38	6.63	0.08	4.72	0.16	1.62
SE	23.60	18.98	23.33	25.34	21.45	24.53	23.93	26.06	21.89	25.18	24.88	27.59	22.99	26.30	16.38	14.45	15.40	16.18	15.04	15.75	15.56	15.91	15.42	15.73
ERSE	443.54	19.31	36.77	21.40	25.19	23.52	37.00	21.80	25.13	23.78	35.30	22.41	24.07	23.79	380.75	13.79	18.62	15.57	15.74	15.88	17.04	14.75	15.00	15.02
$\text{APRB}_{\Delta(\beta_1)}$	36.17	0.01	7.74	18.01	0.46	12.84	5.34	15.64	2.33	10.45	2.46	6.81	7.29	2.05	17.19	0.01	2.50	5.39	2.25	3.59	0.02	2.52	0.12	0.92
SE	0.48	0.44	0.56	0.63	0.50	0.60	0.57	0.64	0.51	0.61	0.58	0.67	0.52	0.63	0.35	0.33	0.35	0.38	0.34	0.37	0.35	0.37	0.35	0.36
ERSE	41.90	0.46	0.90	0.53	0.59	0.57	0.90	0.54	0.58	0.57	0.84	0.53	0.55	0.56	41.87	0.32	0.45	0.38	0.37	0.38	0.41	0.35	0.35	0.36
	T=10, t=7, $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	46.89	0.24	11.87	27.59	0.89	19.36	8.34	23.82	1.78	15.71	2.70	10.54	8.83	3.52	23.17	0.39	4.11	8.40	3.67	5.69	0.50	4.19	0.33	1.84
SE	53.97	65.00	70.45	73.19	61.92	72.87	69.80	73.48	61.63	72.62	69.52	76.03	62.01	73.41	37.09	40.01	44.87	47.87	43.41	46.35	43.26	45.80	42.57	44.47
ERSE	490.67	54.99	85.46	46.14	64.85	52.86	84.47	46.49	62.58	52.61	79.17	47.34	56.26	51.56	412.40	36.19	48.35	39.02	41.06	40.36	44.03	36.68	38.08	37.67
$\text{APRB}_{\Delta(\beta_1)}$	52.81	1.08	7.93	24.72	5.55	16.05	3.53	20.04	8.69	11.51	8.45	5.75	16.09	1.64	27.92	1.06	3.13	8.24	2.53	5.10	1.46	2.98	1.74	0.23
SE	4.82	5.82	6.30	6.55	5.55	6.52	6.27	6.62	5.52	6.53	6.30	6.93	5.61	6.67	3.34	3.60	4.10	4.39	3.95	4.24	3.97	4.21	3.90	4.08
ERSE	62.71	4.98	7.85	4.24	5.91	4.85	7.76	4.28	5.71	4.83	7.27	4.35	5.14	4.73	52.85	3.23	4.29	3.47	3.66	3.59	3.93	3.27	3.40	3.36

Table E.26: Results under model (2.43), by response and correlation. Population: stable

	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	165.87	0.74	11.44	41.60	26.54	27.53	6.06	37.74	30.41	22.92	28.57	4.84	55.43	10.89	78.35	10.01	10.31	37.55	4.70	24.16	14.69	8.36	22.28	2.94
SE	118.57	63.05	81.25	82.13	89.77	81.05	84.37	85.41	91.81	84.15	92.26	90.94	98.25	90.70	110.62	87.33	94.76	101.20	89.57	97.93	94.67	97.32	92.45	95.96
ERSE	444.68	59.25	98.42	66.63	96.58	73.92	101.96	69.00	96.50	76.07	105.17	74.50	97.22	81.08	689.91	92.82	141.33	101.22	106.61	106.14	129.29	99.33	104.63	103.45
$\text{APRB}_{\Delta(\beta_1)}$	299.78	2.97	15.55	36.65	104.39	14.52	25.40	30.68	109.32	6.52	89.91	27.42	152.04	55.15	147.91	21.35	20.94	85.44	25.78	54.90	35.11	16.58	58.58	7.56
SE	0.04	0.04	0.07	0.09	0.06	0.08	0.06	0.09	0.06	0.07	0.06	0.07	0.05	0.06	0.08	0.08	0.09	0.10	0.09	0.09	0.09	0.09	0.09	0.09
ERSE	0.33	0.04	0.09	0.07	0.06	0.07	0.09	0.07	0.06	0.06	0.07	0.06	0.05	0.05	0.89	0.09	0.12	0.10	0.10	0.10	0.11	0.10	0.10	0.10
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	161.39	0.45	12.89	42.89	24.27	28.84	7.69	39.16	27.93	24.38	26.69	6.25	52.71	9.32	76.27	9.00	11.32	38.72	3.74	25.26	13.65	9.55	21.33	1.81
SE	112.41	59.45	73.19	72.80	84.70	72.30	76.74	76.39	86.72	75.78	84.19	81.42	92.33	81.83	111.01	85.96	92.37	97.39	88.95	94.89	94.66	97.22	92.97	95.94
ERSE	439.26	55.81	93.28	63.24	91.61	70.14	96.69	65.51	91.47	72.19	99.72	70.74	92.02	76.91	685.20	90.36	140.17	99.07	104.08	103.99	127.26	96.85	101.91	100.94
$\text{APRB}_{\Delta(\beta_1)}$	689.11	81.63	53.26	20.82	140.86	14.68	82.10	3.02	168.27	40.76	161.64	72.89	228.33	115.61	215.27	8.92	6.68	23.50	7.51	12.79	9.97	9.38	9.41	2.79
SE	0.44	0.21	0.24	0.23	0.27	0.24	0.25	0.24	0.28	0.25	0.29	0.27	0.30	0.28	0.32	0.24	0.26	0.28	0.25	0.27	0.27	0.28	0.26	0.28
ERSE	1.02	0.16	0.24	0.15	0.24	0.17	0.25	0.15	0.24	0.17	0.26	0.16	0.23	0.18	1.45	0.23	0.34	0.24	0.26	0.26	0.31	0.24	0.25	0.25
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	56.66	0.39	13.63	32.14	0.32	22.50	11.11	29.60	1.34	19.94	5.74	9.60	13.08	1.53	31.88	4.29	6.29	20.73	1.88	12.97	5.41	5.53	7.29	0.30
SE	27.55	26.41	37.47	45.46	33.97	41.13	37.81	46.08	34.06	41.58	35.54	42.20	33.19	38.46	53.32	55.95	61.64	68.73	58.81	64.71	59.78	63.94	58.55	61.57
ERSE	261.84	26.97	51.83	36.83	37.67	37.75	51.56	36.90	37.25	37.65	46.11	34.87	34.60	35.18	524.93	57.25	89.09	71.07	68.06	71.16	76.31	64.23	63.00	64.44
$\text{APRB}_{\Delta(\beta_1)}$	257.29	1.77	61.35	147.70	2.01	102.87	49.62	135.78	9.55	90.91	28.03	43.13	62.89	5.85	145.38	19.54	29.77	97.70	8.21	61.30	24.81	26.32	34.07	0.83
SE	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03
ERSE	0.30	0.01	0.03	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.89	0.03	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	60.91	0.70	14.19	33.12	0.98	23.22	12.05	31.05	0.39	21.08	5.96	9.88	13.28	1.52	41.17	10.28	1.90	18.40	2.89	9.53	12.15	0.41	14.15	6.28
SE	23.74	22.58	28.82	32.98	27.94	30.66	29.28	33.57	28.26	31.17	28.82	32.40	28.06	30.33	62.04	61.98	68.12	74.52	65.53	70.96	67.31	71.75	65.54	69.29
ERSE	262.13	22.14	39.93	28.82	30.94	29.83	40.09	29.13	30.79	30.01	36.44	27.99	28.73	28.46	523.63	60.97	94.91	75.07	72.32	75.47	81.51	68.02	67.03	68.49
$\text{APRB}_{\Delta(\beta_1)}$	163.99	38.26	18.97	4.14	34.83	8.19	25.92	2.91	40.45	15.15	51.33	28.62	60.78	40.84	76.77	8.56	5.31	1.05	3.54	3.22	10.25	2.48	9.36	7.56
SE	0.15	0.11	0.12	0.12	0.13	0.12	0.12	0.12	0.13	0.12	0.13	0.12	0.13	0.13	0.18	0.17	0.18	0.19	0.17	0.18	0.17	0.18	0.17	0.18
ERSE	0.49	0.08	0.11	0.08	0.10	0.09	0.11	0.08	0.10	0.09	0.11	0.08	0.09	0.08	1.01	0.16	0.23	0.18	0.18	0.19	0.20	0.17	0.17	0.17
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	166.00	0.48	11.64	41.86	26.41	27.75	6.30	38.01	30.16	23.17	28.58	4.80	55.34	10.91	78.41	10.00	10.39	37.62	4.56	24.24	14.70	8.34	22.23	2.95
SE	114.33	61.38	79.31	81.33	87.75	79.68	82.13	84.09	89.97	82.37	93.07	93.25	98.11	92.19	112.74	88.36	95.16	98.91	93.33	96.97	99.11	100.93	98.16	99.96
ERSE	446.05	59.51	96.85	65.64	95.60	72.89	99.95	67.74	95.14	74.74	103.46	73.28	95.73	79.78	689.91	92.83	141.16	101.08	106.44	105.99	128.93	99.10	104.40	103.22
$\text{APRB}_{\Delta(\beta_1)}$	301.88	4.02	11.42	45.60	104.01	20.84	21.91	38.61	109.22	12.04	90.45	26.58	153.75	55.03	146.18	20.27	22.08	85.51	23.20	55.54	33.94	16.86	56.47	6.81
SE	0.04	0.04	0.06	0.08	0.05	0.07	0.06	0.08	0.05	0.07	0.05	0.07	0.05	0.06	0.09	0.09	0.09	0.10	0.09	0.09	0.09	0.09	0.09	0.09
ERSE	0.33	0.04	0.09	0.07	0.06	0.07	0.09	0.07	0.06	0.06	0.07	0.06	0.05	0.05	0.89	0.09	0.12	0.10	0.10	0.10	0.11	0.10	0.10	0.10
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	161.31	0.34	13.40	43.30	23.52	29.30	8.23	39.62	27.25	24.88	26.20	6.69	52.16	8.86	76.19	9.00	11.28	38.58	3.63	25.17	13.63	9.50	21.18	1.83
SE	120.46	63.06	75.10	75.81	82.28	74.89	78.32	78.97	84.73	77.99	87.08	84.85	92.36	85.15	104.04	80.74	86.27	92.52	82.08	89.31	85.96	88.14	85.01	86.99
ERSE	438.64	55.70	91.42	61.52	87.74	68.06	94.74	63.79	87.92	70.14	97.39	68.45	87.81	74.21	685.66	90.36	139.71	98.72	103.63	103.60	126.84	96.43	101.39	100.49
$\text{APRB}_{\Delta(\beta_1)}$	677.27	70.31	52.44	17.34	124.38	16.49	82.60	7.63	153.01	43.83	164.42	80.74	216.19	121.48	203.18	6.62	9.36	23.21	13.52	13.74	0.62	17.80	0.04	5.95
SE	0.46	0.21	0.24	0.23	0.25	0.23	0.25	0.24	0.27	0.24	0.28	0.26	0.30	0.27	0.32	0.24	0.26	0.27	0.25	0.26	0.26	0.27	0.26	0.26
ERSE	1.04	0.16	0.25	0.15	0.23	0.17	0.25	0.16	0.24	0.18	0.25	0.16	0.23	0.18	1.46	0.23	0.34	0.25	0.26	0.26	0.31	0.24	0.25	0.25
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	56.59	0.33	13.32	31.75	0.06	22.15	10.75	29.14	1.61	19.53	6.02	9.23	13.31	1.21	31.88	4.28	6.40	20.95	1.93	13.13	5.53	5.43	7.39	0.41
SE	28.05	27.25	35.02	41.76	32.63	38.06	35.37	42.15	32.72	38.41	34.08	39.63	32.37	36.48	51.05	53.89	58.16	64.55	56.27	60.89	57.03	61.55	55.59	58.98
ERSE	261.77	26.95	52.25	37.18	38.07	38.10	52.02	37.27	37.62	38.01	46.06	34.98	34.88	35.32	525.12	57.25	89.18	71.12	68.11	71.22	76.28	64.24	63.04	64.46
$\text{APRB}_{\Delta(\beta_1)}$	257.04	1.49	59.66	145.49	3.43	100.93	47.70	133.21	11.03	88.69	29.57	41.04	64.17	4.05	145.33	19.44	30.33	98.79	8.52	62.13	25.37	25.79	34.55	1.35
SE	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03
ERSE	0.30	0.01	0.03	0.02	0.02	0.02	0.03	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.89	0.03	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	60.87	0.69	14.02	32.80	1.00	22.97	11.88	30.72	0.35	20.83	6.02	9.72	13.25	1.40	41.07	10.26	1.57	17.87	3.07	9.11	12.29	0.11	14.20	6.51
SE	22.77	21.59	28.31	33.52	26.55	30.66	28.49	33.87	26.61	30.90	28.10	32.49	26.97	30.01	59.15	59.75	66.83	74.08	63.70	70.06	63.11	66.73	62.20	64.73
ERSE	262.07	22.07	40.29	29.07	31.08	30.06	40.47	29.39	30.94	30.24	36.98	28.31	28.86	28.74	523.14	60.95	94.61	74.94	72.26	75.36	81.28	67.92	66.98	68.40
$\text{APRB}_{\Delta(\beta_1)}$	145.38	22.96	0.11	23.23	15.39	10.86	6.77	16.18	20.89	3.91	32.83	10.77	41.40	22.73	73.60	6.79	1.31	9.63	1.18	4.34	8.54	1.20	7.01	6.05
SE	0.14	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.12	0.11	0.12	0.11	0.12	0.11	0.16	0.15	0.17	0.19	0.17	0.18	0.17	0.18	0.16	0.17
ERSE	0.48	0.08	0.11	0.08	0.10	0.09	0.11	0.08	0.10	0.09	0.10	0.08	0.09	0.08	1.00	0.16	0.23	0.18	0.18	0.19	0.20	0.17	0.17	0.17

Table E.27: Results under model (2.43), by response and correlation. Population: volatile

	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	60.83	34.89	25.94	14.32	46.07	19.55	30.57	19.65	47.04	24.46	29.36	23.80	31.35	26.60	78.28	52.46	41.36	26.46	50.97	33.92	47.95	35.34	53.59	41.62
SE	546.50	496.31	780.38	944.76	645.94	866.07	765.09	927.72	637.26	849.55	746.03	891.78	661.44	818.76	106.80	104.36	220.48	277.79	203.25	246.35	188.49	221.82	183.23	203.32
ERSE	914.29	623.23	1236.66	785.42	916.79	843.62	1208.96	770.40	887.38	824.26	1086.21	716.23	786.54	753.93	704.17	400.63	532.20	443.30	460.32	454.19	462.69	398.76	415.93	408.43
$\text{APRB}_{\Delta(\beta_1)}$	60.19	0.32	3.95	48.69	28.73	20.33	14.36	27.29	36.20	5.22	21.67	11.02	51.67	4.38	106.80	80.20	67.44	51.81	88.86	59.08	75.73	63.04	88.21	68.83
SE	0.31	0.45	0.79	1.18	0.60	0.96	0.76	1.12	0.58	0.92	0.62	0.84	0.53	0.71	0.08	0.10	0.20	0.25	0.19	0.22	0.18	0.21	0.17	0.19
ERSE	1.06	0.53	1.35	0.99	0.76	0.93	1.26	0.94	0.73	0.88	0.90	0.70	0.60	0.67	1.08	0.36	0.48	0.44	0.42	0.43	0.41	0.38	0.37	0.37
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	60.43	36.39	30.12	18.52	49.86	23.77	34.62	23.75	50.92	28.56	32.98	28.05	33.77	30.51	77.78	51.84	39.97	24.05	50.13	32.04	47.73	34.55	53.56	41.11
SE	563.85	505.45	727.72	878.14	631.15	805.36	730.18	882.65	628.64	808.37	693.09	830.77	619.20	760.48	112.71	108.38	224.31	275.09	212.25	247.03	198.36	227.53	194.65	211.15
ERSE	918.16	611.23	1218.38	774.01	897.20	830.71	1205.70	767.20	873.16	819.66	1096.11	719.73	776.70	755.71	702.69	392.74	519.37	434.90	451.67	445.55	454.30	392.68	409.32	402.13
$\text{APRB}_{\Delta(\beta_1)}$	41.09	16.73	44.97	64.39	49.89	54.58	32.97	44.84	52.49	38.75	24.71	22.07	20.72	23.44	38.35	36.13	32.05	48.12	27.49	39.10	15.02	9.95	13.43	12.77
SE	1.87	1.60	2.39	2.83	2.10	2.61	2.39	2.82	2.08	2.61	2.21	2.56	2.04	2.38	0.30	0.29	0.55	0.67	0.51	0.60	0.50	0.58	0.49	0.53
ERSE	2.82	1.83	3.55	2.30	2.71	2.46	3.49	2.27	2.64	2.42	3.00	2.03	2.26	2.13	1.88	1.03	1.32	1.12	1.17	1.15	1.18	1.02	1.07	1.05
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	6.36	22.68	23.88	23.71	27.14	23.62	23.86	24.36	25.98	23.90	18.85	23.22	14.84	20.89	30.75	33.13	30.33	27.06	32.34	28.66	31.87	29.59	32.97	30.64
SE	257.69	329.33	534.76	733.08	415.34	629.23	519.97	715.15	403.66	612.73	463.76	616.79	387.32	534.81	53.54	60.80	139.04	171.65	133.49	152.68	121.43	144.69	119.20	131.00
ERSE	575.64	400.89	903.45	618.70	544.05	616.58	871.21	600.29	524.49	596.07	720.28	520.14	456.61	511.33	537.17	265.48	343.25	309.92	308.02	309.76	304.25	278.03	277.46	278.12
$\text{APRB}_{\Delta(\beta_1)}$	19.33	88.82	96.91	100.59	108.16	97.69	95.45	101.28	102.48	97.22	74.46	93.89	58.81	83.34	110.85	125.48	115.76	104.66	123.51	109.91	119.94	112.29	124.19	115.63
SE	0.14	0.18	0.29	0.41	0.22	0.35	0.28	0.40	0.22	0.34	0.25	0.34	0.21	0.29	0.03	0.03	0.08	0.09	0.07	0.08	0.07	0.08	0.06	0.07
ERSE	1.01	0.22	0.50	0.34	0.29	0.34	0.48	0.33	0.28	0.33	0.39	0.28	0.24	0.28	1.06	0.14	0.19	0.17	0.17	0.17	0.16	0.15	0.15	0.15
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	44.45	28.88	21.38	13.20	26.65	17.43	22.92	15.05	27.69	19.11	26.60	22.06	26.56	24.43	43.04	29.46	25.44	20.76	25.98	23.42	31.32	27.71	31.51	29.73
SE	236.16	239.49	371.77	490.69	314.30	426.43	369.23	483.98	314.07	421.89	329.30	406.94	299.47	364.25	59.49	59.51	103.36	114.69	103.15	107.60	95.34	104.47	95.51	98.63
ERSE	606.54	333.77	634.80	463.78	433.00	464.67	611.49	449.91	420.19	450.15	497.50	390.73	377.38	391.38	537.06	223.59	265.66	244.55	246.86	246.33	244.54	226.02	227.89	227.45
$\text{APRB}_{\Delta(\beta_1)}$	411.14	136.23	46.87	26.43	72.95	13.93	13.47	63.08	39.66	20.42	34.43	30.13	39.65	6.60	81.92	57.43	71.68	89.93	72.75	77.59	78.56	96.98	79.90	84.29
SE	0.66	0.61	0.91	1.18	0.79	1.03	0.90	1.17	0.79	1.02	0.85	1.02	0.79	0.93	0.15	0.15	0.26	0.28	0.26	0.27	0.24	0.27	0.25	0.25
ERSE	1.67	0.74	1.41	1.06	1.00	1.06	1.39	1.05	0.99	1.05	1.19	0.96	0.94	0.96	1.33	0.48	0.59	0.54	0.55	0.54	0.55	0.50	0.51	0.51
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	76.65	42.35	42.81	29.33	67.34	35.41	36.76	21.30	64.30	28.44	12.39	11.86	48.89	0.14	81.34	54.64	44.76	29.25	54.83	36.98	52.93	41.26	57.91	47.02
SE	544.69	475.85	717.72	863.31	619.96	793.14	722.61	866.58	624.36	796.66	735.49	889.47	638.94	811.93	109.37	105.41	214.94	271.05	197.40	240.06	196.79	238.72	187.69	215.38
ERSE	902.74	586.19	1170.63	742.46	873.18	799.23	1164.76	740.69	857.56	793.97	1144.05	746.14	806.15	784.43	690.03	378.71	502.42	417.28	433.51	427.89	463.11	394.88	408.13	403.55
$\text{APRB}_{\Delta(\beta_1)}$	0.10	11.20	4.32	0.71	5.17	2.18	3.86	0.47	3.43	2.06	5.84	3.49	2.67	4.98	14.71	9.50	7.10	2.48	10.51	4.77	12.91	10.32	14.52	11.46
SE	0.24	0.41	0.76	1.20	0.56	0.95	0.72	1.14	0.54	0.91	0.59	0.89	0.48	0.72	0.08	0.10	0.21	0.27	0.21	0.23	0.19	0.23	0.19	0.20
ERSE	0.84	0.61	1.49	1.13	0.85	1.05	1.39	1.06	0.80	0.99	1.07	0.84	0.68	0.79	0.87	0.41	0.54	0.50	0.49	0.49	0.47	0.44	0.43	0.43
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	76.12	45.61	43.34	28.44	69.50	35.16	39.64	23.50	66.93	30.89	9.72	17.27	49.56	3.93	77.34	51.79	38.53	22.64	48.05	30.59	47.60	35.49	52.34	41.49
SE	563.41	500.05	774.86	939.28	622.16	860.92	772.10	935.01	631.78	856.84	771.38	941.17	649.99	856.65	107.21	103.33	209.24	255.68	195.61	229.99	198.00	233.01	191.62	213.39
ERSE	881.61	577.60	1158.47	735.30	851.98	789.32	1151.14	732.46	835.39	782.94	1128.95	737.02	787.68	773.04	682.90	375.66	486.39	408.06	425.44	418.70	450.03	386.50	400.44	395.17
$\text{APRB}_{\Delta(\beta_1)}$	201.71	95.11	107.35	75.66	157.54	90.19	102.24	70.66	148.66	85.23	120.53	90.17	153.19	104.30	72.49	39.01	31.75	22.87	36.21	27.60	30.89	22.67	33.42	27.13
SE	1.25	1.05	1.64	1.91	1.50	1.77	1.64	1.90	1.50	1.77	1.63	1.89	1.51	1.75	0.49	0.45	0.59	0.64	0.57	0.61	0.54	0.58	0.53	0.56
ERSE	2.48	1.36	2.50	1.66	2.11	1.79	2.43	1.63	2.02	1.74	2.37	1.64	1.89	1.72	1.52	0.76	0.96	0.81	0.86	0.84	0.87	0.74	0.78	0.76
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	37.20	12.07	5.11	6.10	14.36	0.36	6.25	5.40	15.89	0.59	1.54	18.09	11.59	9.31	32.51	15.88	10.43	4.00	12.14	7.60	15.64	10.72	16.45	13.46
SE	194.53	229.18	384.88	528.66	300.45	453.27	374.04	514.60	294.04	440.72	340.46	455.82	290.55	393.97	51.37	57.86	109.05	130.84	106.65	117.74	100.55	116.70	99.65	106.87
ERSE	591.00	310.72	688.92	477.70	435.02	478.61	664.62	464.30	421.08	463.72	571.92	420.40	383.62	416.28	524.43	207.67	266.55	239.47	238.10	239.76	243.16	221.56	221.12	221.86
$\text{APRB}_{\Delta(\beta_1)}$	18.34	5.48	1.91	4.03	6.76	0.97	2.55	3.59	7.58	0.42	1.29	9.85	5.47	5.30	15.83	7.38	4.62	1.32	5.51	3.16	7.21	4.69	7.64	6.10
SE	0.10	0.12	0.21	0.28	0.16	0.24	0.20	0.28	0.15	0.24	0.18	0.24	0.15	0.21	0.03	0.03	0.06	0.07	0.06	0.06	0.05	0.06	0.05	0.06
ERSE	0.79	0.16	0.37	0.26	0.23	0.25	0.36	0.25	0.22	0.25	0.30	0.22	0.20	0.22	0.84	0.11	0.14	0.13	0.12	0.13	0.13	0.12	0.12	0.12
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	43.31	25.63	17.38	6.53	25.79	12.13	19.14	8.34	27.46	13.91	8.97	5.73	19.42	2.08	41.71	28.97	25.54	20.99	26.06	23.55	29.17	25.51	29.33	27.58
SE	245.26	239.33	344.78	437.73	303.39	386.93	343.48	436.09	303.66	385.31	319.85	394.16	293.41	352.68	55.72	55.72	96.61	106.98	96.44	100.43	92.27	101.43	92.31	95.56
ERSE	582.77	306.13	560.64	415.96	399.86	419.22	554.76	413.00	394.53	415.16	496.70	385.47	368.03	385.36	522.47	211.29	251.18	231.24	233.65	233.07	237.68	219.78	221.56	221.16
$\text{APRB}_{\Delta(\beta_1)}$	71.02	20.69	2.08	16.46	14.03	6.76	2.86	15.35	14.25	5.82	7.19	8.99	14.77	0.45	35.39	14.45	12.00	8.61	11.99	10.74	11.82	8.40	11.75	10.57
SE	0.51	0.46	0.62	0.78	0.55	0.69	0.62	0.78	0.55	0.69	0.58	0.70	0.54	0.63	0.23	0.23	0.26	0.28	0.26	0.27	0.25	0.26	0.25	0.26
ERSE	1.27	0.56	0.96	0.71	0.70	0.72	0.94	0.70	0.68	0.71	0.82	0.64	0.63	0.64	1.07	0.38	0.44	0.40	0.41	0.41	0.41	0.37	0.38	0.38

Table E.28: Results under model (2.43), by response and correlation. Population: simulated

	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
	T=10, t=4, $\Delta_4(\beta_0)=114.9522$, $\Delta_4(\beta_1)=-0.5296$, low response and low correlation.																							
$\text{APRB}_{\Delta(\beta_0)}$	90.27	31.51	44.65	29.00	73.66	35.89	50.76	35.16	76.93	41.95	62.54	46.93	81.59	53.78	73.41	44.90	36.39	26.24	41.88	31.48	44.79	36.41	47.87	40.71
SE	265.37	187.06	319.53	366.47	285.79	344.11	330.12	382.16	289.85	356.98	323.17	369.83	301.77	345.93	20.93	18.57	67.72	78.51	66.68	72.71	62.26	71.86	61.46	66.52
ERSE	424.82	215.74	462.34	293.37	387.56	321.80	482.56	305.44	389.10	332.91	470.10	305.62	364.70	327.76	307.86	146.06	173.86	144.68	165.53	152.91	170.28	144.06	158.86	150.59
$\text{APRB}_{\Delta(\beta_1)}$	146.58	51.21	98.50	67.35	165.76	80.76	111.06	81.08	170.11	93.69	127.00	95.48	171.97	109.07	122.58	70.55	50.19	25.22	65.52	37.94	70.57	50.92	79.06	60.77
SE	3.23	3.09	6.01	8.07	4.80	6.99	5.96	7.96	4.76	6.90	5.49	7.34	4.65	6.34	0.49	0.53	1.39	1.68	1.35	1.51	1.22	1.44	1.21	1.31
ERSE	6.44	4.12	9.96	6.82	6.82	6.88	9.78	6.70	6.66	6.76	8.99	6.25	5.97	6.24	5.54	2.79	3.51	3.11	3.23	3.15	3.21	2.87	2.97	2.91
	T=10, t=4, $\Delta_4(\beta_0)=114.9522$, $\Delta_4(\beta_1)=-0.5296$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	135.55	25.06	31.61	5.70	77.19	17.15	39.66	12.86	82.98	24.63	62.02	34.31	94.82	46.47	81.78	34.81	23.67	10.90	29.87	17.54	34.19	23.03	37.83	28.83
SE	264.31	149.43	202.80	200.76	250.63	199.89	212.03	213.13	253.22	210.43	228.03	231.01	257.25	226.66	23.45	17.49	47.29	51.04	48.49	48.82	43.80	47.71	44.19	45.36
ERSE	490.24	166.78	267.15	183.27	305.54	207.58	286.05	194.33	308.18	218.52	316.34	216.47	302.81	238.76	317.67	118.16	128.07	107.14	127.04	114.80	128.33	108.53	123.19	114.86
$\text{APRB}_{\Delta(\beta_1)}$	136.97	2.72	0.22	24.32	36.93	13.33	11.04	14.91	46.81	3.40	40.54	12.19	69.39	24.74	84.84	28.03	17.18	4.04	22.76	11.01	29.21	17.67	32.41	23.78
SE	4.74	2.53	4.90	5.04	5.71	4.95	5.13	5.28	5.86	5.18	5.35	5.58	5.73	5.42	1.13	0.79	1.26	1.34	1.26	1.29	1.28	1.34	1.28	1.30
ERSE	14.01	4.65	7.37	5.21	9.09	5.93	7.66	5.41	9.11	6.13	8.06	5.77	8.57	6.42	7.65	2.77	3.07	2.54	2.97	2.71	3.12	2.62	2.96	2.77
	T=10, t=4, $\Delta_4(\beta_0)=114.9522$, $\Delta_4(\beta_1)=-0.5296$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	124.60	23.63	8.30	16.33	33.96	4.55	12.16	12.79	36.99	0.86	27.48	2.77	46.84	14.67	54.62	12.74	5.43	3.25	7.33	1.67	11.20	3.39	12.30	7.84
SE	124.10	71.27	74.33	63.01	89.73	68.10	77.32	65.92	92.13	71.02	88.25	77.51	99.57	82.24	14.61	11.73	22.03	23.31	22.18	22.42	22.79	24.20	22.76	23.21
ERSE	402.68	88.85	91.45	72.17	117.14	83.04	95.32	75.40	119.93	86.40	111.63	89.34	128.70	100.26	266.08	59.10	64.35	55.79	61.68	58.68	67.70	59.33	64.35	61.95
$\text{APRB}_{\Delta(\beta_1)}$	153.10	16.92	6.59	27.02	46.73	11.28	12.02	22.06	50.47	6.09	32.18	2.15	62.27	14.11	70.47	9.71	1.12	15.18	3.00	7.40	7.64	4.80	10.07	2.12
SE	1.38	0.94	1.41	1.46	1.48	1.42	1.44	1.49	1.52	1.45	1.55	1.62	1.60	1.57	0.43	0.36	0.59	0.65	0.59	0.61	0.60	0.65	0.60	0.62
ERSE	7.00	2.09	2.53	2.03	2.57	2.19	2.57	2.07	2.60	2.23	2.77	2.28	2.71	2.42	5.35	1.41	1.60	1.44	1.49	1.47	1.63	1.47	1.52	1.50
	T=10, t=4, $\Delta_4(\beta_0)=114.9522$, $\Delta_4(\beta_1)=-0.5296$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	119.07	24.55	8.13	14.81	28.46	3.52	11.11	11.99	30.79	0.62	23.18	0.68	38.61	11.84	47.41	8.36	2.21	5.34	3.48	0.95	6.51	0.35	7.24	3.66
SE	68.57	40.91	52.74	47.29	61.04	49.72	54.01	48.66	62.10	51.01	60.13	55.78	66.22	57.56	14.01	11.22	18.08	19.32	17.90	18.51	18.17	19.15	18.22	18.46
ERSE	393.40	76.82	80.15	63.50	96.51	72.30	82.71	65.70	98.31	74.54	94.16	75.76	104.34	84.31	256.16	47.80	52.18	45.60	49.57	47.68	54.10	47.74	51.23	49.65
$\text{APRB}_{\Delta(\beta_1)}$	104.22	5.75	10.93	31.37	3.95	20.99	7.71	28.35	6.52	17.86	3.14	16.91	13.84	6.60	39.38	2.20	7.52	14.50	6.76	10.30	4.10	10.55	3.71	6.64
SE	1.77	1.00	1.44	1.35	1.63	1.39	1.47	1.39	1.65	1.42	1.65	1.59	1.76	1.61	0.58	0.45	0.61	0.63	0.61	0.62	0.62	0.64	0.62	0.63
ERSE	10.04	2.26	2.48	1.89	2.86	2.15	2.55	1.95	2.91	2.21	2.88	2.26	3.09	2.50	6.02	1.32	1.48	1.28	1.39	1.34	1.52	1.33	1.43	1.38
	T=10, t=7, $\Delta_7(\beta_0)=1176.529$, $\Delta_7(\beta_1)=36.56497$, low response and low correlation.																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	129.75	27.25	39.94	19.05	79.35	28.09	48.86	27.18	85.66	36.47	79.78	58.26	103.59	67.36	88.08	43.88	34.53	23.91	38.61	29.66	45.27	36.25	47.60	41.13
SE	330.02	230.69	380.67	422.24	367.17	402.55	392.62	439.24	375.10	416.53	434.74	502.75	399.06	467.54	35.30	30.10	90.23	102.39	88.74	95.42	88.93	100.99	87.87	93.94
ERSE	609.71	311.59	595.63	401.55	536.06	439.51	619.92	417.87	540.30	454.14	705.79	477.32	546.49	505.32	457.83	204.77	239.57	207.96	227.43	216.35	243.38	213.81	228.02	220.51
$\text{APRB}_{\Delta(\beta_1)}$	7.96	28.08	12.77	8.30	8.52	11.04	13.67	9.74	10.10	12.21	16.66	16.21	13.60	16.83	0.46	8.73	10.29	12.58	8.70	11.45	9.67	11.74	8.76	10.69
SE	9.41	9.50	20.24	23.84	20.39	21.78	20.68	24.37	20.78	22.25	22.45	27.15	21.93	24.42	1.58	1.54	6.46	7.24	6.47	6.72	6.60	7.39	6.62	6.86
ERSE	56.83	27.32	37.51	31.19	36.06	32.64	38.61	32.28	36.82	33.65	42.93	36.41	38.81	37.22	49.75	17.75	20.44	18.93	19.33	19.15	20.96	19.50	19.79	19.66
	T=10, t=7, $\Delta_7(\beta_0)=1176.529$, $\Delta_7(\beta_1)=36.56497$, low response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	90.82	15.85	23.21	5.35	54.73	13.22	29.99	11.91	58.99	19.80	47.75	29.90	68.28	37.63	71.21	33.18	24.90	15.74	28.27	20.72	33.50	25.65	35.49	29.92
SE	476.22	252.64	400.78	398.83	476.88	396.98	430.63	433.35	495.22	428.61	485.17	481.73	537.62	478.80	121.50	83.23	112.34	113.46	113.32	112.64	120.65	122.66	121.03	121.24
ERSE	1130.91	331.43	558.62	376.58	654.17	429.20	608.33	408.79	677.78	462.76	695.23	482.17	714.94	536.72	589.55	204.36	228.13	191.26	219.46	203.46	245.36	208.63	231.92	219.59
$\text{APRB}_{\Delta(\beta_1)}$	245.15	48.35	56.93	14.98	127.11	33.62	70.15	26.04	138.67	45.53	114.48	67.19	169.37	87.97	79.82	20.22	10.98	0.28	14.62	5.88	22.02	11.71	24.32	17.34
SE	37.95	16.99	33.61	29.83	45.65	31.38	36.21	32.47	47.50	33.95	46.28	44.12	53.89	44.67	10.87	7.41	9.75	9.59	9.91	9.65	10.60	10.54	10.69	10.53
ERSE	150.53	36.56	47.22	35.61	72.89	42.03	51.70	38.90	75.99	45.60	69.36	52.16	85.22	59.34	74.83	21.62	23.19	19.63	22.76	21.00	25.42	21.78	24.48	23.06
	T=10, t=7, $\Delta_7(\beta_0)=1176.529$, $\Delta_7(\beta_1)=36.56497$, high response and low correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	126.02	16.28	1.59	22.92	29.04	11.44	5.54	19.38	32.11	7.71	20.56	4.43	41.35	7.39	49.42	8.44	2.68	3.97	3.51	0.01	6.62	0.46	7.12	4.14
SE	157.16	83.75	93.17	84.48	108.69	87.94	96.14	87.40	110.96	90.83	105.18	97.17	116.29	100.14	23.72	19.03	31.71	33.45	31.80	32.20	32.50	34.20	32.55	32.96
ERSE	544.64	128.02	140.63	112.25	158.96	124.72	144.64	115.65	161.55	128.11	158.23	128.17	166.12	139.59	411.38	86.10	95.23	85.89	90.46	88.47	97.16	88.11	92.20	90.49
$\text{APRB}_{\Delta(\beta_1)}$	31.44	4.35	9.22	20.23	6.25	14.07	7.94	18.70	5.10	12.67	2.38	11.40	0.92	6.27	14.24	0.24	2.00	4.09	2.23	2.63	0.96	2.94	1.15	1.56
SE	4.69	3.43	6.66	6.66	6.91	6.61	6.75	6.77	6.99	6.71	7.04	7.16	7.25	7.03	1.69	1.43	2.95	3.11	2.97	2.98	2.99	3.16	3.01	3.02
ERSE	62.60	13.31	13.96	11.75	14.99	12.82	14.26	12.03	15.19	13.08	15.54	13.29	15.88	14.22	50.08	8.67	9.69	8.93	9.22	9.11	9.84	9.10	9.36	9.27
	T=10, t=7, $\Delta_7(\beta_0)=1176.529$, $\Delta_7(\beta_1)=36.56497$, high response and high correlation																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\tilde{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\tilde{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	91.20	15.61	0.98	19.03	19.06	9.23	3.31	16.74	20.77	6.93	13.18	6.18	26.98	3.37	32.29	4.22	0.45	4.03	0.78	1.26	2.52	1.67	2.71	0.93
SE	144.86	86.53	97.58	87.25	105.49	92.80	99.00	89.07	106.75	94.41	106.13	98.16	111.95	102.39	40.79	34.83	42.86	44.58	42.23	43.67	42.87	44.50	42.30	43.63
ERSE	648.61	138.09	150.63	105.33	169.21	121.87	154.14	108.48	170.87	125.00	169.35	122.75	176.74	138.78	440.88	85.94	94.74	82.65	88.78	86.24	95.79	84.31	89.86	87.65
$\text{APRB}_{\Delta(\beta_1)}$	101.39	4.32	10.87	31.22	6.84	21.15	7.92	28.54	9.37	18.34	2.65	17.63	16.22	7.54	36.29	4.21	9.28	15.25	8.81	11.58	6.30	11.97	6.02	8.47
SE	9.98	6.09	8.56	7.83	9.03	8.23	8.66	7.98	9.13	8.35	9.09	8.65	9.44	8.88	3.50	2.96	3.96	4.12	3.90	4.03	3.94	4.08	3.89	4.00
ERSE	83.66	15.92	15.91	11.83	18.82	13.67	16.36	12.23	19.10	14.07	18.28	13.96	19.98	15.76	56.17	9.56	10.24	9.01	9.69	9.41	10.44	9.25	9.88	9.63

Table E.29: Results under model (2.45), by response and correlation.

	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$. Population: **stable**																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	12.48	0.24	1.42	4.48	2.75	3.07	0.52	3.65	3.32	2.22	1.72	1.63	4.73	0.07	5.97	0.49	2.70	4.55	2.00	3.58	1.20	3.00	0.71	2.06
SE	208.60	242.88	313.09	368.57	265.03	342.06	310.20	366.85	263.43	339.61	301.63	360.74	263.10	330.85	333.49	357.34	418.02	480.49	393.69	445.50	397.11	444.20	380.29	417.56
ERSE	304.35	239.46	443.84	278.74	312.12	297.96	439.58	276.84	303.44	294.20	421.51	271.28	275.58	281.94	556.59	355.40	607.19	455.45	430.01	456.66	517.30	408.42	393.58	409.34
$\text{APRB}_{\Delta(\beta_1)}$	60.89	0.44	16.41	33.21	0.35	24.76	11.58	28.84	4.08	20.20	0.87	16.81	13.36	7.95	26.04	2.86	14.63	21.39	13.45	17.75	8.93	16.49	7.74	12.35
SE	0.08	0.13	0.17	0.22	0.13	0.19	0.16	0.21	0.12	0.18	0.15	0.19	0.12	0.16	0.15	0.17	0.21	0.25	0.19	0.23	0.19	0.22	0.18	0.20
ERSE	0.30	0.12	0.24	0.16	0.15	0.16	0.23	0.15	0.14	0.15	0.21	0.14	0.12	0.14	0.78	0.18	0.32	0.24	0.22	0.23	0.26	0.21	0.19	0.20
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	6.90	0.11	1.32	6.55	4.21	3.94	0.19	4.37	4.49	2.11	3.18	0.05	5.42	1.58	4.14	0.33	1.90	3.82	1.16	2.76	0.99	2.48	0.67	1.65
SE	87.74	169.36	252.76	363.47	164.52	306.62	234.93	336.37	161.30	284.22	194.13	274.18	148.84	232.08	171.13	235.57	280.70	350.53	256.69	309.93	255.54	298.00	243.67	273.22
ERSE	210.39	170.35	417.89	274.62	209.38	268.89	383.86	255.78	198.22	249.59	301.84	209.94	164.72	202.02	472.56	241.58	446.50	353.52	303.75	340.26	344.42	290.67	265.74	282.75
$\text{APRB}_{\Delta(\beta_1)}$	36.31	0.31	7.18	34.37	20.60	20.63	0.53	23.20	22.15	11.35	15.90	0.34	27.17	7.70	21.76	1.48	9.56	19.46	5.85	13.92	4.78	12.50	3.19	8.18
SE	0.05	0.09	0.14	0.20	0.09	0.17	0.13	0.19	0.09	0.16	0.11	0.15	0.08	0.13	0.09	0.13	0.16	0.20	0.14	0.17	0.14	0.17	0.14	0.15
ERSE	0.31	0.09	0.23	0.15	0.12	0.15	0.21	0.14	0.11	0.14	0.17	0.12	0.09	0.11	0.78	0.13	0.25	0.20	0.17	0.19	0.19	0.16	0.15	0.16
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	10.19	2.54	4.09	7.67	1.06	6.03	3.57	7.46	1.77	5.67	1.11	5.24	3.13	3.31	8.79	4.12	3.81	3.60	2.92	3.72	4.75	4.33	4.25	4.56
SE	204.87	233.88	300.12	356.91	256.53	329.56	292.77	347.75	255.22	321.08	275.04	320.88	255.05	297.66	335.16	360.23	404.89	473.89	381.91	434.82	387.93	427.24	378.02	404.53
ERSE	300.20	233.58	435.59	273.33	305.26	292.15	421.85	266.71	296.46	283.86	381.75	250.20	266.44	261.55	556.05	355.04	657.79	481.98	438.77	480.10	528.61	415.40	397.97	415.82
$\text{APRB}_{\Delta(\beta_1)}$	53.20	10.46	28.98	49.43	7.38	39.14	26.34	48.78	3.25	37.45	9.94	30.27	6.71	20.16	42.20	18.84	17.86	18.73	12.01	18.42	21.34	20.17	18.25	21.00
SE	0.08	0.12	0.15	0.20	0.12	0.18	0.15	0.19	0.12	0.17	0.13	0.17	0.12	0.15	0.15	0.18	0.20	0.25	0.19	0.22	0.19	0.22	0.18	0.20
ERSE	0.30	0.12	0.23	0.15	0.14	0.16	0.22	0.14	0.14	0.15	0.19	0.13	0.12	0.13	0.78	0.18	0.34	0.25	0.22	0.25	0.27	0.21	0.20	0.21
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	6.32	1.42	0.64	4.82	3.63	2.73	0.61	3.19	4.25	1.30	2.20	1.07	4.78	0.57	4.13	0.61	2.01	4.23	0.93	3.00	0.31	1.93	0.40	1.04
SE	88.00	168.31	236.37	330.82	169.41	281.96	228.62	321.14	164.23	273.08	200.17	281.97	153.59	238.59	173.89	238.43	285.63	354.28	259.68	314.76	261.98	307.76	247.25	281.33
ERSE	210.94	171.90	399.55	264.47	207.55	259.70	378.82	252.63	196.89	246.73	317.06	219.07	168.97	210.37	473.87	242.31	444.59	352.39	303.08	339.20	353.46	294.11	264.54	285.11
$\text{APRB}_{\Delta(\beta_1)}$	33.29	6.94	3.89	25.73	17.56	14.67	2.47	17.38	20.77	7.37	10.98	5.85	23.84	2.68	21.53	3.13	10.30	21.69	4.88	15.35	1.59	9.88	1.93	5.29
SE	0.05	0.09	0.13	0.18	0.09	0.16	0.13	0.18	0.09	0.15	0.11	0.16	0.08	0.13	0.09	0.13	0.16	0.20	0.14	0.18	0.15	0.17	0.14	0.16
ERSE	0.31	0.10	0.22	0.15	0.11	0.14	0.21	0.14	0.11	0.14	0.18	0.12	0.09	0.12	0.78	0.13	0.25	0.20	0.17	0.19	0.20	0.16	0.15	0.16
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, low response. Population: **volatile**																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	38.37	4.85	12.54	6.81	19.70	9.46	14.89	9.05	21.35	11.73	18.81	12.41	23.58	15.36	19.47	4.50	1.21	3.79	4.59	1.14	5.35	1.76	6.79	3.66
SE	521.70	668.94	798.42	942.80	683.42	873.67	798.29	949.78	675.46	876.50	778.66	944.96	667.35	861.25	345.88	383.27	495.76	624.21	427.41	554.21	441.84	517.40	414.27	475.85
ERSE	659.94	649.85	1213.17	757.28	822.57	806.74	1198.71	750.26	798.22	794.47	1134.57	725.58	717.88	751.15	542.92	386.61	707.11	518.25	471.45	516.04	573.07	447.64	425.34	447.24
$\text{APRB}_{\Delta(\beta_1)}$	164.33	15.68	39.26	7.79	66.71	24.27	52.28	23.22	75.25	38.15	71.85	44.17	87.22	58.11	88.33	16.11	0.75	23.97	14.59	10.16	20.20	2.62	26.08	12.44
SE	0.23	0.34	0.42	0.52	0.34	0.47	0.41	0.51	0.33	0.46	0.39	0.49	0.32	0.44	0.16	0.19	0.26	0.33	0.22	0.29	0.23	0.27	0.21	0.25
ERSE	1.02	0.33	0.65	0.42	0.41	0.43	0.63	0.40	0.40	0.41	0.57	0.37	0.35	0.38	0.99	0.20	0.37	0.27	0.24	0.27	0.30	0.23	0.21	0.23
	T=10, t=4, $\Delta_4(\beta_0) = 11023.06$, $\Delta_4(\beta_1) = -0.4041$, high response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	34.66	1.09	2.10	14.98	8.30	8.36	0.06	12.99	10.15	6.26	6.42	5.89	14.09	0.52	19.38	0.02	4.97	11.31	2.99	7.68	0.46	5.25	0.31	2.48
SE	232.38	434.39	594.01	843.53	413.76	714.51	566.73	805.86	402.90	681.81	488.88	691.25	376.01	583.77	191.47	259.09	341.78	444.70	304.34	385.81	296.46	356.93	281.29	321.99
ERSE	498.57	448.85	1019.12	672.46	523.23	660.04	967.22	643.05	497.35	627.78	806.94	556.78	429.50	535.00	470.25	272.36	503.63	399.77	344.87	385.08	389.07	328.09	300.02	319.23
$\text{APRB}_{\Delta(\beta_1)}$	134.30	4.11	8.17	58.61	32.16	32.57	0.33	50.61	39.41	24.26	24.80	23.16	54.47	1.88	75.31	0.06	19.46	44.29	11.73	30.05	1.79	20.48	1.17	9.67
SE	0.12	0.24	0.33	0.47	0.23	0.40	0.31	0.45	0.22	0.38	0.27	0.38	0.21	0.32	0.10	0.14	0.19	0.25	0.17	0.22	0.16	0.20	0.16	0.18
ERSE	1.01	0.25	0.57	0.37	0.29	0.37	0.54	0.36	0.27	0.35	0.45	0.31	0.24	0.30	0.99	0.15	0.28	0.22	0.19	0.21	0.22	0.18	0.17	0.18
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, low response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	19.75	1.66	3.26	11.62	9.93	7.85	2.06	10.89	11.04	6.89	4.34	4.37	13.95	0.41	10.23	0.39	4.49	9.44	1.23	6.88	1.40	5.26	0.21	3.28
SE	492.74	581.03	793.30	941.75	630.60	871.66	792.54	941.98	635.91	870.88	752.92	907.89	635.67	832.00	311.97	330.90	398.15	479.60	367.28	434.05	384.52	439.93	365.19	408.56
ERSE	758.72	558.94	1069.69	669.77	740.07	714.96	1055.60	663.20	719.13	703.89	1010.62	649.98	656.59	674.83	570.10	334.88	579.15	433.23	407.27	434.00	477.48	380.74	371.43	382.64
$\text{APRB}_{\Delta(\beta_1)}$	15.31	0.76	4.10	11.91	3.52	7.85	2.83	10.52	4.52	6.55	2.13	4.50	7.36	1.15	8.40	0.25	3.01	6.35	1.41	4.50	0.80	3.39	0.06	1.95
SE	0.20	0.29	0.39	0.50	0.30	0.44	0.39	0.49	0.30	0.44	0.35	0.44	0.29	0.39	0.14	0.16	0.20	0.25	0.18	0.22	0.18	0.22	0.17	0.20
ERSE	0.77	0.28	0.55	0.36	0.36	0.37	0.54	0.35	0.34	0.36	0.49	0.32	0.30	0.33	0.80	0.17	0.30	0.22	0.20	0.22	0.24	0.19	0.18	0.19
	T=10, t=7, $\Delta_7(\beta_0) = 11023.06$, $\Delta_7(\beta_1) = -0.4041$, high response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**.i	**EE**.ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**.i	**EE**.ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**.i	**EE**.ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	23.00	1.22	0.18	6.75	4.23	3.33	2.12	3.94	5.59	0.79	8.71	4.51	9.80	6.69	13.69	2.02	1.06	4.76	0.05	2.63	1.95	1.18	2.66	0.62
SE	200.39	335.19	462.32	632.59	344.98	543.99	449.61	617.61	336.57	529.84	395.81	543.30	316.61	464.32	163.06	209.54	272.08	345.34	245.35	302.98	241.02	285.20	230.05	259.44
ERSE	538.69	329.38	741.26	494.03	399.92	487.06	702.70	472.44	380.96	463.49	574.18	403.70	328.76	390.91	483.62	204.61	370.84	294.34	254.38	283.88	293.06	246.20	224.69	239.71
$\text{APRB}_{\Delta(\beta_1)}$	12.05	0.76	0.09	3.39	2.43	1.58	1.28	1.92	3.12	0.26	4.71	2.50	5.31	3.65	7.22	1.13	0.46	2.40	0.07	1.28	1.11	0.53	1.48	0.41
SE	0.11	0.18	0.25	0.35	0.19	0.30	0.25	0.34	0.18	0.29	0.22	0.30	0.17	0.25	0.09	0.11	0.15	0.19	0.13	0.17	0.13	0.16	0.13	0.14
ERSE	0.79	0.18	0.41	0.27	0.22	0.27	0.39	0.26	0.21	0.25	0.32	0.22	0.18	0.21	0.81	0.11	0.21	0.16	0.14	0.16	0.16	0.14	0.12	0.13

Table E.30: Results under model (2.45), by response and correlation.

	T=10, t=4, $\Delta_4(\beta_0) = 1176.529$, $\Delta_4(\beta_1) = 36.56497$, low response. Population: **simulated**																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	0.31	0.13	0.95	0.48	1.94	0.71	1.10	0.66	2.09	0.87	1.81	1.50	2.63	1.64	1.21	0.97	0.18	0.50	0.70	0.15	0.81	0.38	1.10	0.60
SE	235.27	235.79	322.83	371.55	283.73	348.55	322.37	371.58	282.17	348.16	335.51	393.12	296.08	364.27	149.53	149.51	171.50	191.37	165.70	180.16	167.70	182.15	163.37	173.94
ERSE	294.66	236.35	459.56	286.90	341.86	310.61	457.09	286.47	336.17	308.62	462.38	295.17	317.65	310.93	230.31	147.14	222.33	172.85	175.39	176.35	197.17	160.57	163.73	163.45
$\text{APRB}_{\Delta(\beta_1)}$	6.75	2.60	5.11	1.55	12.20	1.85	7.16	1.63	13.06	4.40	9.24	4.31	14.43	6.81	11.19	6.72	2.15	2.20	4.26	0.15	5.83	2.95	6.95	4.49
SE	3.42	4.49	6.62	8.88	5.35	7.64	6.38	8.44	5.24	7.32	5.92	7.38	5.21	6.59	2.55	2.94	3.67	4.41	3.44	3.98	3.38	3.84	3.26	3.57
ERSE	4.58	4.47	10.22	7.04	6.32	6.90	9.66	6.68	6.10	6.57	8.12	5.75	5.47	5.71	4.23	2.81	4.82	3.87	3.57	3.80	3.92	3.30	3.17	3.26
	T=10, t=4, $\Delta_4(\beta_0) = 1176.529$, $\Delta_4(\beta_1) = 36.56497$, high response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	6.45	1.19	0.62	1.52	2.42	0.45	1.42	0.56	2.89	0.42	2.61	0.91	3.60	1.77	3.50	0.65	0.44	1.58	0.03	0.94	0.49	0.34	0.68	0.13
SE	125.77	132.59	183.72	238.60	149.53	209.98	184.11	239.64	150.41	210.51	169.11	212.26	148.80	188.81	79.36	81.85	92.78	99.62	92.29	95.46	91.22	96.25	90.92	93.16
ERSE	242.17	130.15	285.05	190.92	176.46	193.49	282.47	190.23	173.84	191.94	241.72	171.31	157.25	171.05	206.88	86.34	116.30	101.44	100.98	101.76	105.37	93.90	93.76	94.15
$\text{APRB}_{\Delta(\beta_1)}$	29.77	3.16	1.59	13.17	6.43	7.03	1.86	8.87	8.52	3.21	8.36	0.44	12.65	4.21	16.63	1.99	3.25	8.72	1.50	5.55	1.33	2.64	2.11	0.33
SE	2.17	2.82	4.09	5.68	3.08	4.84	4.02	5.57	3.05	4.75	3.57	4.81	2.94	4.14	1.49	1.74	1.98	2.19	1.96	2.06	1.91	2.07	1.90	1.97
ERSE	4.58	2.77	6.73	4.48	3.67	4.40	6.54	4.37	3.56	4.28	5.37	3.71	3.11	3.63	4.24	1.81	2.57	2.22	2.15	2.20	2.26	2.00	1.96	1.99
	T=10, t=7, $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$, low response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	16.98	1.55	5.47	7.94	3.35	6.72	4.19	6.72	2.38	5.48	1.39	4.40	0.07	2.90	6.25	0.71	2.42	3.59	2.03	2.89	1.06	1.94	1.01	1.40
SE	317.99	327.09	463.76	543.95	405.54	503.55	460.68	537.77	401.01	499.49	450.41	521.64	408.47	485.82	202.82	204.69	232.55	256.33	225.38	242.59	229.08	248.81	222.96	237.34
ERSE	432.82	325.74	645.22	427.26	482.17	450.72	642.04	423.42	471.09	446.56	602.66	404.93	430.91	421.75	367.34	206.30	292.59	240.95	243.85	244.47	269.23	227.10	228.72	229.71
$\text{APRB}_{\Delta(\beta_1)}$	11.83	2.71	7.98	9.22	8.31	8.42	6.76	7.95	7.31	7.18	5.90	7.73	6.34	6.62	4.01	1.95	3.88	4.72	3.79	4.14	2.92	3.61	2.97	3.11
SE	19.51	22.10	32.24	41.19	29.33	36.06	30.60	37.67	28.72	33.61	28.34	32.29	27.87	29.99	14.10	14.88	16.52	17.18	16.51	16.74	16.10	16.60	16.11	16.26
ERSE	42.45	22.54	46.38	36.53	33.38	35.81	41.85	34.01	31.97	33.47	34.31	29.97	29.27	29.66	42.78	15.43	19.68	18.36	18.26	18.31	18.13	17.00	16.91	16.95
	T=10, t=7, $\Delta_7(\beta_0) = 1176.529$, $\Delta_7(\beta_1) = 36.56497$, high response																							
	naïve	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{p}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{p}_{i2})$	**EE**_i	**EE**_ii	**EE**(0.5)	**naïve**	$\mathbf{EE}_h$	$\mathbf{EE}(\hat{\pi}_i)$	**EE**_i	**EE**_ii	**EE**(0.5)	$\mathbf{EE}(\hat{\pi}_{i1})$	**EE**_i	**EE**_ii	**EE**(0.5)
$\text{APRB}_{\Delta(\beta_0)}$	2.37	0.58	0.56	0.50	0.51	0.54	0.22	0.08	0.50	0.09	0.81	0.76	0.67	0.80	0.99	0.23	0.08	0.30	0.06	0.17	0.06	0.20	0.05	0.11
SE	145.93	171.54	258.97	351.15	193.58	303.81	249.98	337.11	192.55	292.21	230.88	306.74	191.37	266.55	105.68	114.45	130.36	143.56	128.02	135.60	124.23	133.18	122.82	127.72
ERSE	416.62	179.49	399.05	265.88	238.77	268.04	379.12	255.54	231.11	257.05	336.81	236.11	209.56	234.09	369.88	117.76	162.86	141.94	137.77	141.13	144.51	128.68	126.26	128.22
$\text{APRB}_{\Delta(\beta_1)}$	0.86	0.35	0.62	0.76	0.56	0.68	0.30	0.24	0.48	0.26	0.54	0.60	0.47	0.56	0.33	0.16	0.03	0.08	0.14	0.02	0.02	0.08	0.00	0.04
SE	2.80	4.21	6.46	9.04	4.58	7.70	6.12	8.52	4.49	7.27	5.48	7.57	4.35	6.46	2.30	2.76	3.13	3.53	3.07	3.29	2.93	3.20	2.89	3.03
ERSE	42.11	4.24	9.97	6.76	5.66	6.67	9.32	6.38	5.41	6.29	8.10	5.73	4.82	5.58	41.97	2.74	4.04	3.52	3.34	3.46	3.49	3.12	3.00	3.08