

Supplementary material

for “Robust estimation for small domains in business surveys” by Paul A. Smith, Chiara Bocci, Nikos Tzavidis, Sabine Krieg, Marc J.E. Smeets

S1: Additional figures and tables

Table S1 gives the population and sample sizes for the studied retail business population. Figure S1 shows the distribution of the sampling weights arising from the SBS sampling design (excluding the completely enumerated size classes 6-9). See main paper section 2.

Table S1: Population size (upper number) and sample size (lower number) in the strata of the Netherlands Structural Business Survey design used for this study.

Industry	Size class					Total
	1	2	3	4	5	
52110	385	681	275	201	357	1899
	11	65	35	43	76	230
52120	38	37	7	18	10	110
	15	15	3	5	3	41
52200	1913	4171	1313	249	41	7687
	211	259	83	50	17	620
52310	38	48	139	209	64	498
	17	15	30	65	60	187
52321	226	513	217	64	9	1029
	5	16	16	7	6	50
52330	99	69	20	9	2	199
	5	5	4	5	2	21
52413	331	201	27	1	2	562
	7	16	9	1	2	35
52422	2314	3405	657	189	48	6613
	23	77	23	49	30	202
52431	270	728	188	50	11	1247
	6	29	14	7	9	65
52440	1312	1950	605	122	29	4018
	13	45	21	32	18	129
52450	1021	1028	272	58	12	2391
	22	69	31	16	9	147
52460	886	1297	491	169	45	2888
	49	118	68	70	38	343
52470	349	692	232	47	11	1331
	13	37	22	7	4	83
52485	2518	3259	748	121	27	6673
	32	79	34	41	18	204
52491	3566	3747	845	196	60	8414
	65	511	84	68	41	769
52500	1730	727	63	18	7	2545
	52	44	6	5	7	114
52610	1441	433	21	7	4	1906
	288	86	9	5	4	392
52620	4471	2181	234	29	1	6916
	358	175	47	17	1	598
52630	2557	972	95	18	3	3645
	511	110	10	5	3	639
52700	2378	881	130	17	4	3410
	74	91	26	10	4	205
Total	27843	27020	6579	1792	747	63981
	1777	1862	575	508	352	5074

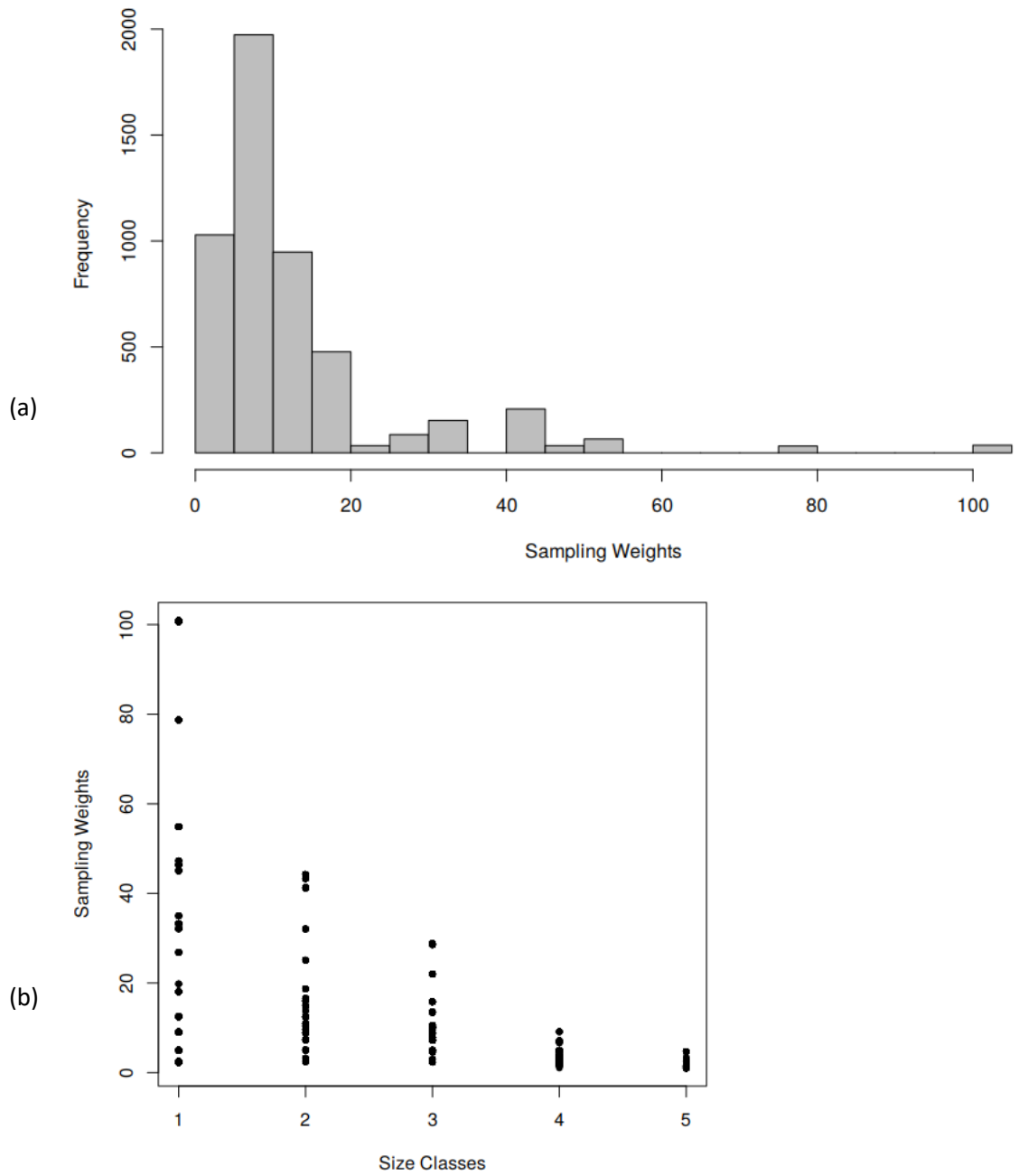


Fig. S1: (a) Histogram of the sampling weights for the design used in this study; (b) distribution of sampling weights across size classes.

We investigate the ignorability of the design empirically by plotting the standardised model residuals under the nested error regression model against the design weights in Figure S2 (see Beaumont & Alavi (2004) Robust generalized regression estimation. *Survey Methodology* **30** 195-208.). The plot shows that large weights are associated with small standardized residuals and there is no obvious association between these two quantities (correlation coefficient =0.0076) which suggests that under the model, the ignorability assumption is not unrealistic.

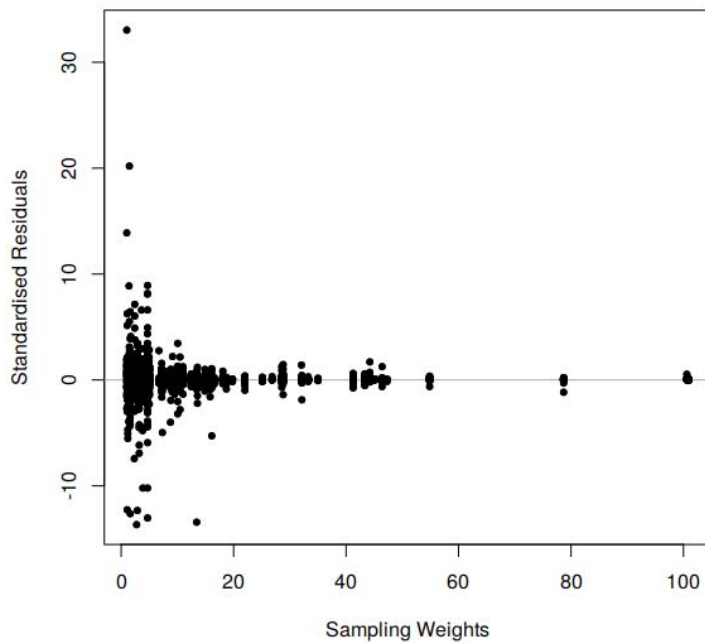


Fig. S2: The standardised model residuals under the nested error regression model plotted against the design weights.

Table S2 shows the AIC, BIC and log-likelihood values which support modelling decisions described in the text.

Table S2: Model fit statistics for EBLUP method (6) and different models.

Model	AIC	BIC	logL
EBLUP (6) & model (4)	29,552.24	29,617.55	-14,766.12
EBLUP (6) & model (5)	29,655.50	29,707.75	-14,819.75
EBLUP (6) & model (4), $\mathbf{e}_{sc} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{e,sc})$	19,501.94	19,593.36	-9,736.97
EBLUP (6) & model (4), $\mathbf{e} = \mathbf{w}\mathbf{p}^T \boldsymbol{\varepsilon}$ with $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon})$	20,990.68	21,055.99	-10,485.34
EBLUP (6) & model (4), $\mathbf{e} = (\mathbf{w}\mathbf{p}^2)^T \boldsymbol{\varepsilon}$ with $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon})$	19,113.68	19,178.98	-9,546.84

Table S3 summarises the results from Figs 3 and 4 of the main paper and Tables S4 and S5 in a form which is easier for comparative reading.

Table S3: Summary information on relative bias (from Table S4) and relative rmse (from Table S5) for the models listed in Table 2 of the main paper. The best-performing models are indicated in bold.

	mean rb	mean abs(rb)	mean rrmse
Direct (HT)	0.02	0.34	9.86
Direct (GREG)	0.49	0.93	3.78
EBLUP	-0.29	3.81	6.92
EBLUP (var = f(sc))	0.71	1.60	3.32
pseudo-EBLUP	0.98	1.68	3.99
Robust synthetic	0.20	1.75	1.95
M-quantile naïve ($b_{\psi} = 1.345$)	-0.19	1.25	1.67
M-quantile bias-adjusted ($b_{\psi} = 1.345$)	-0.33	5.71	9.48
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345$, $b_{\phi} = 1$)	-0.25	0.99	1.64
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345$, $b_{\phi} = 2$)	-0.30	1.02	1.81
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345$, $b_{\phi} = 3$)	-0.37	1.10	2.02
weighted M-quantile naïve ($b_{\psi} = 1.345$)	-0.47	1.25	1.71
weighted M-quantile bias-adjusted ($b_{\psi} = 1.345$)	0.02	0.16	4.16

Table S4: Industry-specific relative bias (rb) (%) of small area point estimators of total *tto*. The best-performing methods are indicated in bold in the summary lines.

Industry	Direct		EBLUP	EBLUP	pseudo-EBLUP	robust synthetic	M-quantile naïve	M-quantile bias-adjusted	M-quantile robustly bias-adjusted			M-quantile naïve	M-quantile bias-adjusted
	HT	GREG							unweighted				
				var = $f(sc)$					$b_\phi = 1$	$b_\phi = 2$	$b_\phi = 3$		
52110	-0.28	-0.03	-1.03	0.26	0.02	1.01	0.64	0.06	0.39	0.38	0.42	0.32	-0.05
52120	-0.59	-1.00	0.07	2.86	1.47	1.16	0.35	0.07	0.11	-0.12	-0.31	0.20	-0.22
52200	0.08	-0.04	0.79	1.91	0.37	1.24	0.29	1.42	0.00	-0.12	-0.15	0.07	0.04
52310	-0.16	-0.04	-1.14	-0.66	-0.77	-0.04	0.44	0.02	0.61	0.60	0.54	0.29	-0.02
52321	-0.27	0.11	-0.62	2.50	1.85	2.71	1.70	0.23	1.26	1.24	1.26	1.48	0.21
52330	-0.24	-0.83	-5.28	1.86	5.84	1.39	1.10	-29.51	1.21	0.95	0.47	0.74	0.11
52413	0.48	0.26	-5.16	5.10	8.44	5.38	2.15	-10.33	-0.10	-1.91	-3.38	1.97	0.21
52422	-0.48	-0.32	2.01	-0.50	-0.62	-0.73	-0.75	7.26	-0.41	-0.01	0.25	-0.99	-0.22
52431	0.19	-0.04	-2.35	-0.92	-1.07	-1.44	-0.81	-0.22	-0.52	-0.22	-0.03	-1.04	-0.50
52440	0.26	0.03	-8.25	0.56	0.24	0.78	0.22	-6.09	-0.49	-1.12	-1.73	-0.09	0.00
52450	0.56	3.00	-1.46	1.59	1.60	2.41	1.70	-9.98	1.55	1.54	1.53	1.28	0.12
52460	0.29	0.07	-4.07	-1.12	-0.76	-1.64	-1.68	-1.16	-1.66	-1.69	-1.72	-1.92	0.10
52470	0.06	-0.26	-2.89	1.75	0.94	2.42	1.52	-2.24	1.30	1.23	1.18	1.22	-0.14
52485	0.33	0.08	-4.41	-1.44	-0.96	-2.01	-1.52	0.13	-0.83	-0.51	-0.53	-1.78	0.13
52491	-0.01	0.31	2.24	0.54	0.33	-0.39	-0.11	4.71	0.33	0.73	1.05	-0.37	0.04
52500	0.90	0.27	8.45	1.88	3.16	-1.25	-1.78	6.10	-1.60	-1.38	-1.15	-2.22	0.65
52610	-0.68	1.16	21.38	-3.57	-2.87	-5.08	-4.58	31.29	-4.20	-3.88	-3.61	-4.92	-0.06
52620	0.31	1.36	-1.86	2.24	1.28	0.98	0.53	0.43	0.58	0.58	0.56	0.27	0.14
52630	-0.48	0.01	0.26	-0.46	0.08	-0.52	-1.27	-0.82	-1.29	-1.25	-1.21	-1.66	-0.06
52700	0.12	-0.06	-2.42	-0.20	0.99	-2.42	-1.87	2.05	-1.31	-1.03	-0.92	-2.15	-0.09
median (rb)	0.07	0.22	-1.30	0.55	0.35	0.37	0.25	0.07	-0.05	-0.12	-0.09	-0.01	0.02
mean (rb)	0.02	0.49	-0.29	0.71	0.98	0.20	-0.19	-0.33	-0.25	-0.30	-0.37	-0.47	0.02
mean abs(rb)	0.34	0.93	3.81	1.60	1.68	1.75	1.25	5.71	0.99	1.02	1.10	1.25	0.16

Table S5: Industry-specific relative rmse (%) of small area point estimators of the total *tto*. The best-performing methods are indicated in bold in the summary lines.

Industry	Direct		EBLUP	EBLUP	pseudo-EBLUP	robust synthetic	M-quantile naïve	M-quantile bias-adjusted	M-quantile robustly bias-adjusted			M-quantile naïve	M-quantile bias-adjusted
	HT	GREG							unweighted				
				$var = f(sc)$					$b_\phi = 1$	$b_\phi = 2$	$b_\phi = 3$		
52110	3.88	1.99	2.71	4.06	1.88	1.12	0.89	2.62	0.80	0.86	0.96	0.78	2.07
52120	10.22	3.31	2.51	4.60	2.76	1.67	1.60	3.29	1.62	1.68	1.75	1.60	3.24
52200	4.33	1.91	2.32	2.96	1.83	1.29	0.50	2.48	0.44	0.50	0.55	0.42	1.86
52310	2.98	1.01	1.99	2.04	1.96	0.39	0.62	0.75	0.74	0.75	0.72	0.53	1.00
52321	12.70	4.13	3.45	4.05	2.99	2.75	1.94	5.34	1.78	1.98	2.19	1.76	4.31
52330	14.78	6.90	8.79	2.92	7.00	2.25	2.63	33.36	3.02	3.66	4.35	2.45	5.59
52413	15.02	4.65	12.45	6.69	11.72	5.40	2.59	11.37	2.33	3.57	4.96	2.43	4.69
52422	10.43	3.43	5.35	1.44	2.35	0.81	0.92	8.95	0.89	1.07	1.35	1.14	3.86
52431	7.85	3.14	4.56	1.66	2.45	1.51	1.18	6.36	1.15	1.27	1.45	1.36	4.14
52440	7.45	2.81	9.91	1.63	1.85	0.88	0.73	8.93	1.21	1.84	2.46	0.74	2.70
52450	16.24	9.24	9.47	4.50	6.66	3.16	2.71	29.21	2.62	2.66	2.73	2.38	9.51
52460	4.53	2.12	4.67	2.13	2.06	1.70	1.75	2.28	1.76	1.82	1.89	1.98	2.14
52470	7.12	3.03	5.03	2.75	2.18	2.48	1.75	6.68	1.65	1.72	1.82	1.49	2.70
52485	11.64	5.62	7.20	2.53	3.10	2.05	1.63	5.92	1.20	1.19	1.37	1.89	6.30
52491	5.68	1.43	4.06	1.43	1.42	0.51	0.48	5.40	0.61	0.91	1.20	0.59	1.40
52500	24.90	9.68	15.12	8.17	11.31	1.39	2.11	12.30	2.26	2.43	2.60	2.48	13.55
52610	16.35	7.84	24.05	4.73	5.98	5.38	4.97	32.39	4.63	4.36	4.14	5.27	5.03
52620	4.71	2.43	4.81	3.32	3.57	1.09	0.86	4.76	0.91	0.93	0.95	0.74	4.77
52630	9.49	2.26	2.68	1.67	2.29	0.71	1.47	2.31	1.49	1.45	1.42	1.83	2.20
52700	6.94	2.42	7.21	3.02	4.51	2.46	2.03	4.99	1.62	1.53	1.60	2.29	2.23
median (rrmse)	8.67	3.28	4.92	2.94	2.61	1.59	1.61	5.66	1.56	1.60	1.68	1.68	3.55
mean (rrmse)	9.86	3.78	6.92	3.32	3.99	1.95	1.67	9.48	1.64	1.81	2.02	1.71	4.16

Table S6: Mean of the bootstrap estimates of the rmse for the robustly bias-adjusted M-quantile estimator with $b_\phi = 1$, and the true (empirical) rmse calculated over the simulations. The corresponding rrmse's are also shown

Industry	True rmse	Mean estimated rmse	True rrmse (%)	Mean estimated rrmse (%)
52110	317.80	360.72	0.80	0.91
52120	18.35	20.33	1.62	1.80
52200	145.28	215.84	0.44	0.65
52310	107.20	102.54	0.74	0.70
52321	114.75	89.91	1.78	1.40
52330	21.47	65.67	3.02	9.24
52413	19.72	20.89	2.33	2.46
52422	243.39	329.97	0.89	1.21
52431	77.58	98.76	1.15	1.47
52440	239.80	359.69	1.21	1.81
52450	336.25	749.83	2.62	5.84
52460	360.98	180.97	1.76	0.88
52470	125.61	126.97	1.65	1.67
52485	306.26	339.63	1.20	1.33
52491	163.40	319.76	0.61	1.18
52500	63.07	71.05	2.26	2.55
52610	179.05	429.75	4.63	11.12
52620	82.96	97.71	0.91	1.07
52630	88.78	54.26	1.49	0.91
52700	70.36	74.92	1.62	1.73
Median	120.18	114.75	1.56	1.43
Mean	154.10	205.46	1.64	2.50

S2: Reduced population

This section gives results from a smaller population with the most extreme outliers removed, as described in section 5.2 of the main paper. Fig. S3 shows the outlier diagnostics for the reduced population relative to the working model, which is the fixed part of (4).

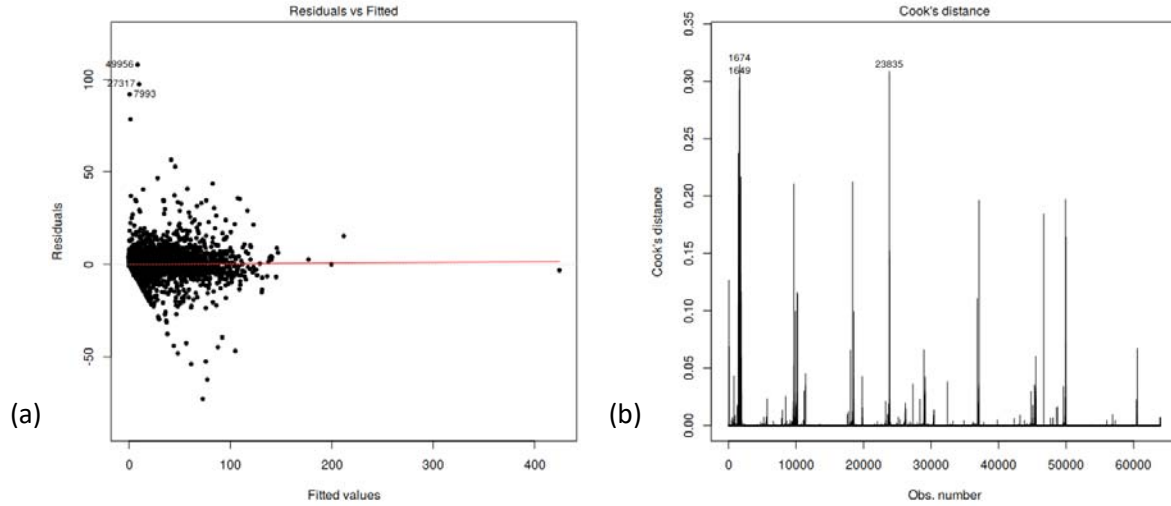


Fig. S3: (a) Residuals and (b) Cook's distance from reduced population.

Table S7: Summary information on relative bias (from Table S8) and relative rmse (from Table S9) for the models listed in Table 2 using the reduced population described in section 5.2. The best-performing models are indicated in bold.

	mean rb	mean abs(rb)	mean rmse
Direct (HT)	0.17	0.28	8.99
Direct (GREG)	0.36	0.75	3.09
EBLUP	-0.15	1.40	2.85
EBLUP (var = f(sc))	0.49	1.43	2.38
pseudo-EBLUP	0.37	1.86	2.53
Robust synthetic	-0.02	1.72	1.89
M-quantile naïve ($b_{\psi} = 1.345$)	-0.30	1.19	1.55
M-quantile bias-adjusted ($b_{\psi} = 1.345$)	-0.92	4.24	6.79
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345$, $b_{\phi} = 1$)	-0.39	0.96	1.54
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345$, $b_{\phi} = 2$)	-0.46	1.00	1.73
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345$, $b_{\phi} = 3$)	-0.55	1.07	1.96
weighted M-quantile naïve ($b_{\psi} = 1.345$)	-0.64	1.27	1.66
weighted M-quantile bias-adjusted ($b_{\psi} = 1.345$)	-0.02	0.13	3.31

Table S8: Industry-specific relative bias (rb) (%) of small area point estimators of total *tto* in the reduced population. The best-performing models are indicated in bold in the summary lines.

Industry	Direct		EBLUP	EBLUP	pseudo-EBLUP	robust synthetic	M-quantile naïve	M-quantile bias-adjusted	M-quantile robustly bias-adjusted			M-quantile naïve	M-quantile bias-adjusted
	HT	GREG							unweighted				
				var = $f(sc)$					$b_\phi = 1$	$b_\phi = 2$	$b_\phi = 3$		
52110	-0.09	0.23	0.34	0.52	0.24	0.49	0.15	0.41	-0.10	-0.10	-0.04	-0.10	0.03
52120	-0.67	-1.05	1.64	1.71	2.13	1.26	0.50	0.23	0.25	0.01	-0.19	0.29	-0.10
52200	0.13	-0.57	1.15	1.77	1.07	1.57	0.62	0.83	0.32	0.19	0.15	0.37	-0.08
52310	0.27	-0.25	0.70	-0.18	-0.32	0.02	0.52	0.55	0.67	0.67	0.62	0.36	0.01
52321	-0.32	1.67	2.70	2.51	2.95	2.83	1.80	0.24	1.33	1.28	1.28	1.53	-0.27
52330	0.26	-0.76	-0.98	1.57	5.99	1.23	0.84	-31.72	0.94	0.58	0.01	0.50	-0.50
52413	0.48	0.11	1.39	5.38	6.51	5.34	2.24	-11.32	0.02	-1.80	-3.33	1.96	-0.04
52422	0.05	0.70	1.50	0.16	-0.11	0.01	0.00	7.44	0.30	0.65	0.86	-0.30	0.03
52431	-0.04	-0.31	-1.31	-1.55	-1.75	-2.09	-1.47	0.86	-1.19	-0.82	-0.58	-1.69	0.06
52440	0.12	-0.06	-1.47	0.45	0.69	0.73	0.15	-5.17	-0.63	-1.28	-1.88	-0.21	-0.14
52450	0.65	-0.14	0.14	0.58	0.92	1.44	0.77	-1.21	0.53	0.50	0.47	0.36	0.06
52460	0.20	-0.26	-1.81	-1.13	-1.13	-1.45	-1.51	-1.60	-1.51	-1.56	-1.61	-1.77	0.01
52470	0.44	0.72	2.02	1.72	1.85	2.12	1.32	1.52	1.07	1.01	0.99	0.98	0.11
52485	0.31	0.11	-0.61	-1.15	-1.42	-1.52	-0.99	1.81	-0.33	-0.03	-0.05	-1.32	-0.12
52491	0.09	1.11	0.96	0.24	-0.28	-0.60	-0.22	3.08	0.19	0.56	0.86	-0.55	0.04
52500	1.25	-0.30	-2.83	1.43	-0.83	-1.33	-1.43	5.53	-1.32	-1.19	-1.06	-2.08	0.40
52610	0.02	5.64	-0.85	-4.94	-6.74	-7.26	-6.25	8.63	-5.76	-5.35	-5.01	-6.82	-0.24
52620	0.22	0.19	-1.38	1.08	-0.02	-0.31	-0.48	0.24	-0.49	-0.54	-0.59	-0.85	0.12
52630	0.07	-0.21	-1.10	-0.20	-0.38	-0.44	-0.96	-0.59	-1.02	-1.00	-0.98	-1.42	0.15
52700	0.05	0.59	-3.20	-0.27	-1.88	-2.44	-1.58	1.81	-1.16	-0.95	-0.89	-2.04	0.01
median (rb)	0.12	0.02	-0.24	0.48	-0.06	0.01	0.07	0.48	-0.04	-0.06	-0.12	-0.25	0.01
mean (rb)	0.17	0.36	-0.15	0.49	0.37	-0.02	-0.30	-0.92	-0.39	-0.46	-0.55	-0.64	-0.02
mean abs(rb)	0.28	0.75	1.40	1.43	1.86	1.72	1.19	4.24	0.96	1.00	1.07	1.27	0.13

Table S9: Industry-specific relative rmse (%) of small area point estimators of the total *tto* in the reduced population. The best-performing models are indicated in bold in the summary lines.

Industry	Direct		EBLUP	EBLUP	pseudo-EBLUP	robust synthetic	M-quantile naïve	M-quantile bias-adjusted	M-quantile robustly bias-adjusted			M-quantile naïve	M-quantile bias-adjusted
	HT	GREG							unweighted				
				$\text{var} = f(sc)$				$b_\phi = 1$	$b_\phi = 2$	$b_\phi = 3$			
52110	3.36	1.38	1.37	1.00	1.04	0.63	0.55	2.18	0.61	0.69	0.77	0.61	1.71
52120	10.51	3.20	2.23	2.48	2.29	1.75	1.61	3.31	1.60	1.65	1.72	1.59	3.27
52200	3.95	1.87	1.77	2.13	1.63	1.59	0.72	2.02	0.52	0.50	0.54	0.54	1.76
52310	2.95	1.00	0.93	0.89	0.90	0.33	0.66	0.88	0.78	0.79	0.76	0.55	0.94
52321	14.36	3.23	3.10	3.25	3.11	2.85	2.00	5.00	1.77	1.94	2.14	1.78	4.28
52330	14.20	5.69	4.04	2.51	6.07	2.09	2.37	35.07	2.75	3.32	4.04	2.26	5.47
52413	14.49	4.15	4.02	5.98	6.75	5.36	2.60	12.33	2.15	3.36	4.80	2.37	4.26
52422	9.38	1.91	2.60	0.93	0.83	0.26	0.48	8.67	0.83	1.23	1.56	0.61	1.94
52431	7.76	2.11	1.80	1.79	1.92	2.12	1.65	4.31	1.52	1.39	1.45	1.87	2.53
52440	7.16	2.26	3.28	1.29	1.16	0.79	0.60	7.69	1.17	1.83	2.45	0.69	2.47
52450	7.40	2.26	1.88	1.55	1.50	1.49	1.02	3.84	1.09	1.29	1.46	0.81	3.09
52460	4.41	1.87	2.16	1.76	1.52	1.49	1.57	2.36	1.61	1.70	1.78	1.83	2.07
52470	7.15	2.23	2.35	2.29	2.10	2.15	1.50	3.70	1.41	1.52	1.65	1.24	2.75
52485	6.93	2.35	2.48	1.70	1.79	1.55	1.14	5.38	0.93	1.14	1.40	1.46	2.48
52491	3.74	1.48	1.64	0.90	0.78	0.65	0.40	3.50	0.43	0.72	1.00	0.66	1.21
52500	23.63	6.38	4.80	6.05	3.52	1.45	1.81	11.94	1.96	2.17	2.38	2.38	13.20
52610	16.88	12.06	8.64	6.24	8.41	7.64	6.72	14.36	6.29	5.92	5.63	7.27	6.79
52620	4.52	1.20	2.10	1.80	1.31	0.45	0.66	2.10	0.71	0.79	0.87	0.98	1.51
52630	9.41	2.55	1.83	1.18	1.56	0.64	1.11	2.27	1.18	1.18	1.18	1.57	2.27
52700	7.61	2.57	3.99	1.79	2.48	2.47	1.73	4.90	1.43	1.41	1.53	2.17	2.20
median (rmse)	7.51	2.26	2.29	1.79	1.71	1.52	1.32	4.08	1.30	1.40	1.54	1.51	2.48
mean (rmse)	8.99	3.09	2.85	2.38	2.53	1.89	1.55	6.79	1.54	1.73	1.96	1.66	3.31

S3: Reduced model

This section gives results from the original, complete population but with models including the reduced set of variables in equation (5). The same range of estimators is used.

Table S10: Summary information on relative bias (from Table S11) and relative rmse (from Table S12) for the models listed in Table 2 using the full population but the simpler model (5). The best-performing models are indicated in bold.

	mean rb	mean abs(rb)	mean rmse
Direct (HT)	0.02	0.34	9.86
Direct (GREG)	0.49	0.93	3.78
EBLUP	-1.75	4.30	7.55
EBLUP (var = f(sc))	0.62	1.51	3.08
pseudo-EBLUP	0.97	1.65	3.98
Robust synthetic	0.26	1.76	1.95
M-quantile naïve ($b_{\psi} = 1.345$)	0.01	1.22	1.60
M-quantile bias-adjusted ($b_{\psi} = 1.345$)	-0.41	5.68	9.49
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345, b_{\phi} = 1$)	-0.11	0.95	1.58
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345, b_{\phi} = 2$)	-0.20	0.98	1.76
M-quantile robustly bias-adjusted ($b_{\psi} = 1.345, b_{\phi} = 3$)	-0.30	1.06	1.98
weighted M-quantile naïve ($b_{\psi} = 1.345$)	-0.41	1.22	1.67
weighted M-quantile bias-adjusted ($b_{\psi} = 1.345$)	0.02	0.15	4.17

Table S11: Industry-specific relative bias (rb) (%) of small area point estimators of total *tto* in the full population using model (5). The best-performing models are indicated in bold in the summary lines.

Industry	Direct		EBLUP	EBLUP	pseudo-EBLUP	robust synthetic	M-quantile naïve	M-quantile bias-adjusted	M-quantile robustly bias-adjusted			M-quantile naïve	M-quantile bias-adjusted
	HT	GREG							unweighted				
				var = $f(sc)$					$b_\phi = 1$	$b_\phi = 2$	$b_\phi = 3$		
52110	-0.28	0.51	-0.84	0.64	-0.24	0.96	0.58	0.17	0.34	0.36	0.43	0.26	-0.05
52120	-0.59	-1.16	1.11	2.26	1.82	1.20	0.47	0.08	0.21	-0.03	-0.24	0.23	-0.22
52200	0.08	-0.50	0.66	1.43	0.38	1.35	0.41	1.43	0.09	-0.05	-0.09	0.10	0.04
52310	-0.16	-0.02	-0.16	-0.60	-0.81	0.05	0.52	0.06	0.67	0.65	0.59	0.30	-0.02
52321	-0.27	1.80	-0.21	2.24	1.88	2.87	1.84	0.37	1.34	1.26	1.27	1.49	0.20
52330	-0.24	-0.76	-9.68	1.48	5.34	1.45	1.35	-29.81	1.35	1.02	0.49	0.83	0.11
52413	0.48	0.26	-11.87	5.12	8.08	5.35	2.32	-10.83	0.05	-1.80	-3.31	1.93	0.21
52422	-0.48	-0.30	-0.04	-0.62	-0.52	-0.60	-0.61	7.40	-0.34	0.01	0.24	-0.97	-0.22
52431	0.19	1.02	-2.52	-1.29	-1.11	-1.32	-0.73	-0.21	-0.51	-0.26	-0.07	-1.09	-0.50
52440	0.26	0.18	-8.38	0.53	0.29	0.93	0.38	-5.95	-0.45	-1.14	-1.82	-0.05	0.00
52450	0.56	0.89	-2.59	1.50	1.67	2.60	1.88	-10.13	1.67	1.62	1.60	1.32	0.10
52460	0.29	-0.31	-3.80	-1.16	-0.70	-1.50	-1.57	-1.04	-1.59	-1.64	-1.67	-1.89	0.11
52470	0.06	0.58	-2.07	1.65	0.82	2.57	1.65	-2.10	1.38	1.28	1.23	1.25	-0.14
52485	0.33	-0.53	-4.45	-1.49	-0.74	-1.87	-1.37	-0.09	-0.75	-0.48	-0.52	-1.77	0.13
52491	-0.01	1.76	1.20	0.37	0.39	-0.34	0.08	4.62	0.47	0.84	1.13	-0.32	0.03
52500	0.90	-0.48	-4.60	1.83	3.18	-1.40	-1.30	5.74	-1.18	-1.07	-0.94	-2.09	0.65
52610	-0.68	4.18	22.54	-3.25	-2.64	-5.04	-4.07	30.77	-3.72	-3.42	-3.18	-4.66	-0.05
52620	0.31	2.56	-2.16	2.26	1.21	0.94	0.82	0.35	0.82	0.76	0.71	0.43	0.13
52630	-0.48	-0.35	-1.12	-0.22	0.17	-0.44	-0.86	-0.75	-0.93	-0.93	-0.92	-1.34	-0.05
52700	0.12	0.53	-6.01	-0.27	0.99	-2.49	-1.60	1.79	-1.12	-0.89	-0.83	-2.06	-0.09
median (rb)	0.07	0.22	-2.12	0.59	0.38	0.49	0.39	0.07	0.07	-0.04	-0.08	0.03	0.01
mean (rb)	0.02	0.49	-1.75	0.62	0.97	0.26	0.01	-0.41	-0.11	-0.20	-0.30	-0.41	0.02
mean abs(rb)	0.34	0.93	4.30	1.51	1.65	1.76	1.22	5.68	0.95	0.98	1.06	1.22	0.15

Table S12: Industry-specific relative rmse (%) of small area point estimators of the total *tto* in the full population using model (5). The best-performing models are indicated in bold in the summary lines.

Industry	Direct		EBLUP	EBLUP	pseudo-EBLUP	robust synthetic	M-quantile naïve	M-quantile bias-adjusted	M-quantile robustly bias-adjusted			M-quantile naïve	M-quantile bias-adjusted
	HT	GREG							unweighted				
				$\text{var} = f(sc)$					$b_\phi = 1$	$b_\phi = 2$	$b_\phi = 3$		
52110	3.88	1.72	2.86	2.22	2.04	1.07	0.85	2.64	0.77	0.85	0.97	0.72	2.08
52120	10.22	3.23	2.56	3.03	2.55	1.70	1.62	3.27	1.62	1.66	1.74	1.60	3.24
52200	4.33	2.06	2.37	2.37	1.76	1.38	0.56	2.49	0.43	0.48	0.53	0.42	1.86
52310	2.98	1.02	1.40	1.95	2.02	0.37	0.67	0.74	0.78	0.78	0.74	0.53	1.00
52321	12.70	3.32	3.30	3.75	2.81	2.90	2.06	5.34	1.83	2.00	2.21	1.76	4.35
52330	14.78	5.80	12.54	2.59	5.96	2.28	2.73	33.71	3.06	3.65	4.32	2.49	5.60
52413	15.02	4.79	15.30	6.83	11.73	5.37	2.71	11.84	2.28	3.49	4.90	2.40	4.70
52422	10.43	3.53	5.39	1.44	2.32	0.67	0.80	9.07	0.85	1.07	1.35	1.12	3.86
52431	7.85	3.45	4.96	1.84	2.30	1.39	1.11	6.38	1.15	1.28	1.45	1.39	4.15
52440	7.45	2.33	10.18	1.63	1.79	1.00	0.77	8.84	1.19	1.86	2.53	0.73	2.71
52450	16.24	9.63	10.69	4.33	6.50	3.27	2.81	29.37	2.67	2.68	2.74	2.41	9.53
52460	4.53	1.92	4.40	2.17	2.02	1.54	1.63	2.20	1.69	1.78	1.85	1.94	2.14
52470	7.12	2.28	4.80	2.75	2.03	2.62	1.85	6.64	1.71	1.75	1.84	1.52	2.71
52485	11.64	3.87	6.72	2.51	3.40	1.90	1.48	5.91	1.14	1.17	1.37	1.87	6.31
52491	5.68	2.19	3.82	1.30	1.50	0.46	0.44	5.33	0.67	0.99	1.27	0.55	1.42
52500	24.90	6.09	16.97	8.42	11.71	1.53	1.69	12.12	1.93	2.19	2.45	2.35	13.52
52610	16.35	8.97	24.97	4.40	5.64	5.34	4.46	31.91	4.16	3.91	3.71	5.01	5.03
52620	4.71	4.42	5.25	3.49	4.34	1.06	1.03	4.81	1.05	1.04	1.04	0.78	4.81
52630	9.49	2.48	2.71	1.48	2.31	0.64	1.05	2.27	1.11	1.13	1.14	1.51	2.19
52700	6.94	2.46	9.84	3.11	4.84	2.53	1.76	4.89	1.44	1.42	1.54	2.20	2.23
median (rmse)	8.67	3.28	5.11	2.55	2.44	1.53	1.55	5.63	1.31	1.54	1.64	1.56	3.55
mean (rmse)	9.86	3.78	7.55	3.08	3.98	1.95	1.60	9.49	1.58	1.76	1.98	1.67	4.17

S4 Investigation of the second tuning constant in the robustly bias-adjusted M-quantile estimator

Table S13: Relative rmse for robustly bias-adjusted M-quantile estimator with tuning parameter for the bias adjustment from 0.25 to 3 in steps of 0.25. The max – min summary row shows the range of the rmse over the considered values of b_ϕ . The minimum rmse over the range of b_ϕ values is highlighted in bold. (This is a version of Table 3 in the main paper, but showing results for all industry domains.)

b_ϕ	52110	52120	52200	52310	52321	52330	52413	52422	52431	52440
0.25	0.84	1.60	0.45	0.65	1.81	2.70	2.30	0.96	1.17	0.79
0.50	0.81	1.61	0.43	0.68	1.75	2.78	2.14	0.95	1.17	0.91
0.75	0.80	1.61	0.43	0.71	1.75	2.90	2.17	0.92	1.16	1.06
1.00	0.80	1.62	0.44	0.74	1.78	3.02	2.33	0.89	1.15	1.21
1.25	0.81	1.63	0.45	0.75	1.83	3.16	2.57	0.90	1.16	1.36
1.50	0.83	1.65	0.47	0.75	1.88	3.32	2.88	0.94	1.19	1.51
1.75	0.84	1.66	0.48	0.75	1.93	3.49	3.22	1.00	1.22	1.67
2.00	0.86	1.68	0.50	0.75	1.98	3.66	3.57	1.07	1.27	1.84
2.25	0.88	1.70	0.51	0.74	2.03	3.83	3.93	1.14	1.31	2.00
2.50	0.91	1.71	0.53	0.73	2.09	4.00	4.28	1.21	1.36	2.16
2.75	0.93	1.73	0.54	0.73	2.14	4.17	4.63	1.28	1.40	2.31
3.00	0.96	1.75	0.55	0.72	2.19	4.35	4.96	1.35	1.45	2.46
max - min	0.16	0.15	0.12	0.10	0.45	1.65	2.82	0.46	0.29	1.67

b_ϕ	52450	52460	52470	52485	52491	52500	52610	52620	52630	52700
0.25	2.65	1.75	1.68	1.50	0.49	2.11	4.88	0.89	1.47	1.89
0.50	2.62	1.75	1.65	1.37	0.50	2.16	4.79	0.90	1.48	1.78
0.75	2.62	1.76	1.64	1.27	0.55	2.21	4.71	0.90	1.49	1.69
1.00	2.62	1.76	1.65	1.20	0.61	2.26	4.63	0.91	1.49	1.62
1.25	2.62	1.77	1.66	1.16	0.68	2.30	4.56	0.91	1.48	1.57
1.50	2.63	1.78	1.68	1.15	0.75	2.34	4.49	0.92	1.47	1.54
1.75	2.64	1.80	1.70	1.16	0.83	2.39	4.42	0.92	1.46	1.52
2.00	2.66	1.82	1.72	1.19	0.91	2.43	4.36	0.93	1.45	1.53
2.25	2.68	1.84	1.74	1.23	0.99	2.47	4.30	0.94	1.44	1.54
2.50	2.70	1.85	1.76	1.27	1.07	2.51	4.24	0.94	1.43	1.56
2.75	2.72	1.87	1.79	1.32	1.14	2.56	4.19	0.95	1.42	1.58
3.00	2.73	1.89	1.82	1.37	1.20	2.60	4.14	0.95	1.42	1.60
max - min	0.12	0.14	0.17	0.35	0.72	0.49	0.74	0.07	0.07	0.37

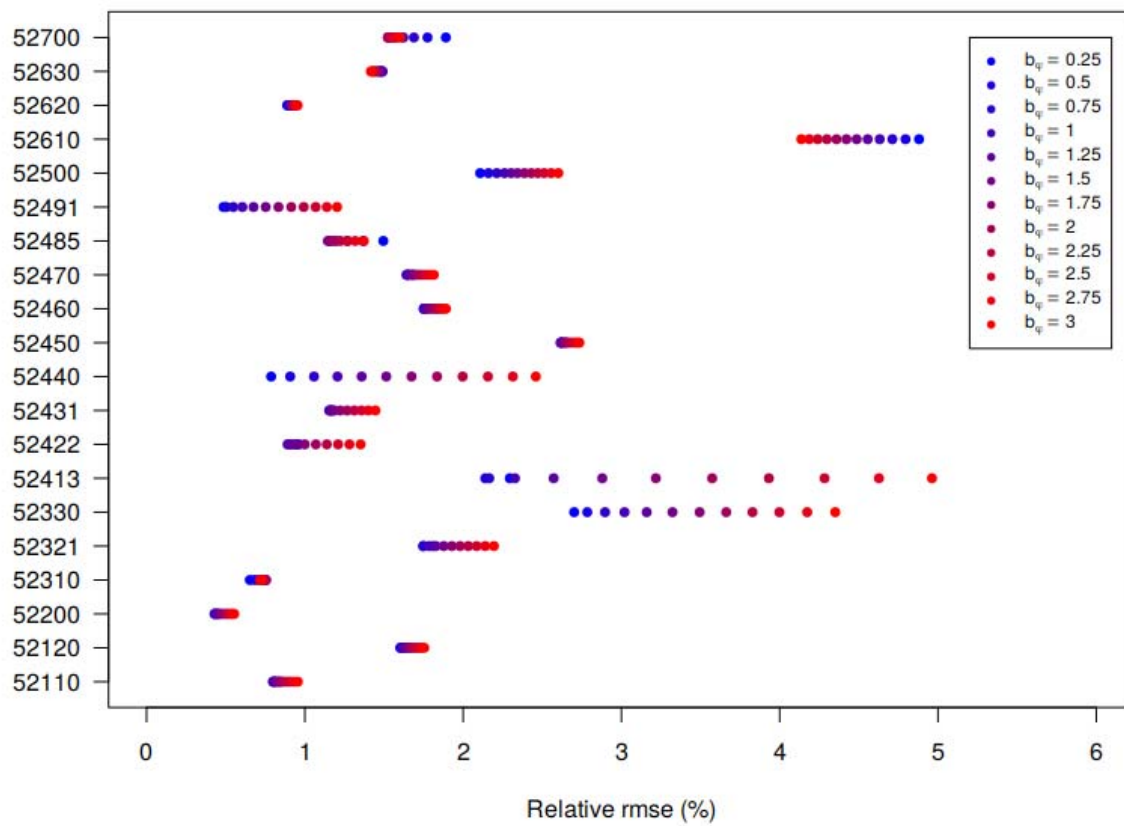


Fig. S4: The effect of changing b_ϕ on the relative rmse of the different industries. Note that some industries are much more affected than others, and the effects do not always go in the same direction.