# Low-Complexity Joint Power Allocation and Trajectory Design for UAV-Enabled Secure Communications with Power Splitting 

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#### Abstract

An unmanned aerial vehicle (UAV)-aided secure communication system is conceived and investigated, where the UAV transmits legitimate information to a ground user in the presence of an eavesdropper (Eve). To guarantee the security, the UAV employs a power splitting approach, where its transmit power can be divided into two parts for transmitting confidential messages and artificial noise (AN), respectively. We aim to maximize the average secrecy rate by jointly optimizing the UAVs trajectory, the transmit power levels and the corresponding power splitting ratios allocated to different time slots during the whole flight time, subject to both the maximum UAV speed constraint, the total mobility energy constraint, the total transmit power constraint, and other related constraints. To efficiently tackle this non-convex optimization problem, we propose an iterative algorithm by blending the benefits of the block coordinate descent (BCD) method, the concave-convex procedure (CCCP) and the alternating direction method of multipliers (ADMM). Specially, we show that the proposed algorithm exhibits very low computational complexity and each of its updating steps can be formulated in a nearly closed form. Besides, it can be easily extended to the case of three-dimensional (3D) trajectory design. Our simulation results validate the efficiency of the proposed algorithm.


Index Terms—Physical layer security, UAV, artificial noise, trajectory design, power allocation.

## I. Introduction

Unmanned aerial vehicular (UAV) communications have recently attracted growing research interests in both academia and industry [1]-[9], given their unique benefits of prompt ondemand deployment, low latency as well as agility and flexibility. Since UAVs are generally expected to operate at a higher

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altitude than conventional cellular base stations (BSs), the line-of-sight (LoS) component dominates the air-to-ground/ground-to-air channels in many practical scenarios [10]. Hence UAVaided $\operatorname{LoS}$ links tend to have better channel quality than typical terrestrial channels, which often suffer from severe fading and shadowing effects. However, unfortunately the UAV-aided LoS links suffer from an increased eavesdropping probability [11] due to the open broadcast nature of wireless channels. From this perspective, the LoS propagation of UAVs becomes a double-edged sword, since additionally the terrestrial communications are also potentially exposed to malicious UAVs. Therefore, the delicate handling of the underlying security issues holds the key to unlocking the potential of UAV-aided communications.

Recently, physical layer security has drawn significant attention in UAV-enabled communication systems as a promising technique of protecting legitimate transmissions against eavesdropping attacks and also as a complement of conventional encryption techniques [12], [13]. Focusing on resource allocation/management for secrecy communication performance maximization, a range of physical layer security (PLS) techniques have been considered in the literature, such as UAVmounted BSs [14]-[18], UAV-enabled relaying [19] and UAVassisted cooperative jamming [2], [20]-[24], etc. In particular, a single-UAV communication system was investigated in [14], where the UAV sends confidential information to a legitimate ground user (Bob) in the presence of a groundbased eavesdropper, and the secrecy rate is maximized by jointly allocating the UAV's transmit power and optimizing its flight trajectory. The authors of [15] have considered a scenario of multiple users and maximized the minimum secrecy rate for ensuring fairness among the users. By contrast, the authors of [16] considered coordinated multi-point (CoMP) reception of the legitimate users and three-dimensional (3D) trajectory optimization in the presence of multiple malicious eavesdroppers. In [17], the total transmit power of the UAVmounted BS was minimized through joint beamforming optimization. As a further development, the authors of [19] studied the security problems of UAV-aided relaying systems and judiciously shared the transmit power between the source and the UAV.

Furthermore, in addition to exploiting the agile maneuverability of the UAVs for improving their secrecy performance, UAVs can also be employed as cooperative friendly jammers [25] that are able to send artificial noise (AN) (can be viewed as external interference) to assist the legitimate users
[2], [20]-[24]. Specifically, in [2], a dual-UAV-aided secure communication scheme has been proposed, where a second UAV was employed to jam a number of eavesdroppers on the ground. In [20], the impact of the UAV's jamming power and position on the outage probability and intercept probability have been examined. In order to improve the secrecy rate, in [21] a mobile UAV-aided jammer was harnessed for opportunistically interfering with the potential eavesdroppers. The authors of [22] studied the associated secrecy vs. energy efficiency maximization problem, where multiple source UAVs and jamming UAVs work cooperatively to serve the ground users. In [23], AN-beamforming and cooperative jamming were utilized, whilst only relying on location and statistical channel state information (CSI) of the eavesdroppers, where imperfect CSI knowledge of the link between the UAVaided jammer and the destination was considered. Finally, the authors of [24] considered the worst-case secrecy rate maximization problem by taking into account the uncertainty of Eves location.

Against the above backdrop, we investigate a UAV-aided secure communication system, where the UAV transmits legitimate information to a ground-user Bob in the presence of a ground-based Eve. In contrast to prior studies, we conceive a power-splitting aided secure transmission scheme for protecting the UAV's communications. Explicitly, the UAV divides its transmit power into two parts, where a portion $\rho$ of the signal power is used for transmitting confidential messages to Bob, while the remaining portion $1-\rho$ is devoted to transmitting AN to interfere with Eve's reception. Note that in the conventional secrecy communication systems operating without UAV-assiatnce, injecting AN has been widely applied for improving the secrecy transmission rates of Bob [26]-[28]. By relying on this power-splitting approach and exploiting the nimble mobility of the UAV, we aim for jointly optimizing the trajectory of the UAV and the communication/jamming power levels over time for maximizing the average secrecy rate of the UAV-Bob link, subject to the maximum UAV speed constraint, the total propulsion energy constraint, the total transmit power constraint, and other related constraints. To solve the resultant highly non-convex optimization problem efficiently, we propose a low-complexity iterative algorithm by combining the benefits of the block coordinate descent (BCD) method [29], the concave-convex procedure (CCCP) method [30] and the alternating direction method of multipliers (ADMM) [31].

Specifically, in order to address the related optimization variable coupling issues, we propose to decompose the original problem into two subproblems, i.e. the power allocation subproblem and the trajectory optimization subproblem, by applying the BCD method. The resultant subproblems, although much simplified compared to the original problem, they still remain non-convex. Therefore, by exploiting the fact that the underlying non-convex parts admit a difference-ofconvex (DC) structure, we propose to transform them into more tractable forms with the aid of first-order approximations. We first show that a nearly closed-form optimal solution of the approximated power allocation subproblem can be devised by resorting to its Lagrangian dual problem. Then, by tactfully
introducing auxiliary variables, the approximated trajectory optimization subproblem can be iteratively and globally solved by the ADMM method, and we demonstrate that each updating step therein can also be expressed in closed form. Given the fact that the existing algorithms suitable for solving joint power and trajectory optimization problems usually involve standard convex solvers, such as CVX [32], the proposed algorithm exhibits a very attractive and unique feature, namely that the optimization can be formulated almost in closed form, thus imposing a low computational complexity. Furthermore, the proposed algorithm is proved to be monotonically convergent. Our numerical results show the benefits of the power spitting approach proposed.

The main contributions of this treatise are as follows:

1) We formulate a joint power and trajectory optimization problem for a UAV-aided secure communication system relying on a power splitting approach for improving the secrecy performance.
2) To solve this challenging optimization problem, we propose a low-complexity iterative algorithm and show that each step in the proposed algorithm can be represented in a nearly closed form. Fyrthermore, we illustrate that the proposed algorithm can be easily extended to the case of 3D trajectory design.
3) We provide comprehensive numerical results for characterizing the efficiency of the proposed algorithm and the power splitting approach advocated. We then demonstrate the impact of the key system parameters on the average secrecy rate. In particular, we show that by appropriately splitting the transmit power of the UAV, the overall system performance can be substantially improved as compared to that without power splitting. Furthermore, compared to the existing algorithms using CVX, the running time of the proposed algorithm is at least 30 times lower.
This paper is structured as follows. In Section II, we introduce the considered UAV-enabled secure communication system and formulate the joint optimization problem. In Section III, we propose an efficient iterative algorithm to solve the considered problem with very low complexity and guaranteed convergence. Simulation results are presented in Section IV to show the effectiveness of our proposed algorithm and conclusions are drawn in Section V.
Notations: Scalars, vectors and matrices are respectively denoted by lower case, boldface lower case and boldface upper case letters. For a matrix $\mathbf{A}, \mathbf{A}^{T}$ denote its transpose. For a vector $\mathbf{a},\|\mathbf{a}\|$ represents its Euclidean norm. $|\cdot|$ denotes the absolute value of any real or complex scalar. $\mathbb{R}^{m \times n}$ denotes the space of $m \times n$ real matrices. The set difference is defined as $\mathcal{A} \backslash \mathcal{B} \triangleq\{x \mid x \in \mathcal{A}, x \notin \mathcal{B}\} .[x]^{+} \triangleq \max (x, 0)$.

## II. System model and Problem formulation

In this section, we introduce the system model and formulate the optimization problem of interest.

## A. System Model

We consider a secure communication system where a UAV transmits confidential information to Bob in the presence of
a potential Eve, as shown in Fig. 1. In order to improve the security of the UAV-Bob link, the UAV also sends jamming signals (through injecting AN) to interfere Eves signal reception and increase the secrecy capacity.

Without loss of generality, we consider a 3D Cartesian coordinate system with Bob and Eve located at $(0,0,0)$ and $(L, 0,0)$, respectively, i.e., Bob and Eve are both on the ground with a distance of $L$ meters (m). For simplicity, we focus on the UAVs operation during a finite duration of $N$ seconds (s) and ignore its take-off and landing phases. We further assume that the UAV is flying at a fixed altitude $H$, which is considered as the minimum altitude that is required for terrain or building avoidance. ${ }^{1}$ Then, the time interval $N$ is discretized into $T$ equally spaced time slots, i.e., $N=T \delta_{t}$, where $\delta_{t}$ denotes the elemental slot length that is chosen to be sufficiently small. Thus, the time-varying trajectory of the UAV $(x(t), y(t), H)$ over the considered time period can be approximated by the $T$-length sequence $(x[i], y[i], H)$, $i \in \mathcal{T} \triangleq\{1, \cdots, T\}$, where $(x[i], y[i])$ denotes the UAVs $x-y$ coordinate at time slot $i$. Furthermore, let $\left(x_{1}, y_{1}\right)$ and $\left(x_{T}, y_{T}\right)$ denote the initial and final locations of the UAV and let $V_{\max }$ denote the maximum UAV speed, then we have the following mobility constraints:

$$
\begin{gather*}
x[1]=x_{1}, y[1]=y_{1}, x[T]=x_{T}, y[T]=y_{T}  \tag{1}\\
\frac{\sqrt{(x[i]-x[i+1])^{2}+(y[i]-y[i+1])^{2}}}{\delta_{t}} \leq V_{\max }, \forall i \in \mathcal{T} \backslash T \tag{2}
\end{gather*}
$$

Besides, the UAV's mobility is also constrained by its energy budget. Specifically, the energy consumed by the UAV engine at time slot $i$ is in proportion to the square of the velocity at this time slot and according to (2), the energy consumed by the UAV engine at time slot $i$, denoted as $E_{\text {mov }}[i]$, can be expressed as [33], [34]

$$
\begin{equation*}
E_{\mathrm{mov}}[i]=\kappa\left((x[i]-x[i+1])^{2}+(y[i]-y[i+1])^{2}\right), \tag{3}
\end{equation*}
$$

where we have $\kappa=0.5 M \delta_{t}$ and $M$ denotes the UAVs mass, including its payload. Thus, we have the following energy constraint for the mobility of the UAV:

$$
\begin{equation*}
\sum_{i=1}^{T-1} E_{\mathrm{mov}}[i] \leq E_{\mathrm{tr}} \tag{4}
\end{equation*}
$$

where $E_{\text {tr }}$ is the total mobility energy stored at the UAV, i.e., the UAVs energy budget.

We assume that the UAV altitude $H$ is sufficiently large such that the LoS components dominate the channels of the UAV-Bob and UAV-Eve links, which is consistent with the channel models in the Third Generation Partnership Project (3GPP) TR 36.777 specification [35] (e.g., larger than 40 m in the rural macro with aerial vehicles (RMa-AV) scenario and 100 m in the urban macro with aerial vehicles (UMaAV ) scenario) and the channel measurement results in [36]. Therefore, we consider LoS channel models and the channel

[^0]

Fig. 1: The considered UAV-enabled secure communication system.
power gains of the UAV-Bob and UAV-Eve links at time slot $i$ follow the free-space path loss model ${ }^{2}$ given by [4], [14]

$$
\begin{equation*}
g_{\mathrm{I}}[i]=\frac{\gamma_{0}}{d_{\mathrm{I}}^{2}[i]}, g_{\mathrm{E}}[i]=\frac{\gamma_{0}}{d_{\mathrm{E}}^{2}[i]}, \tag{5}
\end{equation*}
$$

where $\gamma_{0}$ is the power gain at the reference distance of 1 m which depends on the carrier frequency and the antenna gains at the transmitter and receiver, $d_{\mathrm{I}}[i]$ and $d_{\mathrm{E}}[i]$ denote the distances from the UAV to Bob and Eve at time slot $i$, respectively, which can be expressed as

$$
\begin{align*}
d_{\mathrm{I}}[i] & =\sqrt{x^{2}[i]+y^{2}[i]+H^{2}}  \tag{6}\\
d_{\mathrm{E}}[i] & =\sqrt{(x[i]-L)^{2}+y^{2}[i]+H^{2}}
\end{align*}
$$

Let $p[i]$ denote the transmit power of the UAV at time slot $i$, we divide it into two parts where a portion of $p[i] \rho[i]$ is used for information transmission and the other $p[i](1-\rho[i])$ is utilized for transmitting AN to block Eve from successfully recovering the confidential information, where $\rho[i]$ is the power splitting ratio which satisfies

$$
\begin{equation*}
0 \leq \rho[i] \leq 1, \forall i \in \mathcal{T} \tag{7}
\end{equation*}
$$

Note that the AN can be eliminated by Bob but not necessarily by Eve [41]. The transmit power levels $\{p[i]\}$ are constrained by the limitation of both average power and peak power, which can be expressed as follows:

$$
\begin{gather*}
\frac{1}{T} \sum_{i=1}^{T} p[i] \leq \bar{P}  \tag{8}\\
0 \leq p[i] \leq P_{\max }, \forall i \in \mathcal{T} \tag{9}
\end{gather*}
$$

where $\bar{P}$ and $P_{\max }$ denote the average and peak power budgets, respectively. Equivalently, the average power constraint (8) can be rewritten as

$$
\begin{equation*}
\sum_{i=1}^{T} p[i] \leq P \tag{10}
\end{equation*}
$$

where $P=T \bar{P}$ represents the total power available during the whole flight. Then, the signal-to-noise ratio (SNR) of the UAV-Bob link at time slot $i$ is given by

$$
\begin{equation*}
\operatorname{SNR}_{\mathrm{I}}[i] \triangleq \frac{\gamma_{0} p[i] \rho[i]}{d_{\mathrm{I}}^{2}[i] \sigma^{2}} \tag{11}
\end{equation*}
$$

[^1]where $\sigma^{2}$ is the additive white Gaussian noise (AWGN) variance at the receiver of Bob. Similarly, the signal-to-interference-plus-noise ratio (SINR) of the UAV-Eve link at time slot $i$ can be expressed as
\[

$$
\begin{align*}
\operatorname{SINR}_{\mathrm{E}}[i] & \triangleq \frac{\gamma_{0} p[i] \rho[i]}{d_{\mathrm{E}}^{2}[i]\left(\frac{\gamma_{0}(1-\rho[i]) p[i]}{d_{\mathrm{E}}^{[ }[i]}+\sigma^{2}\right)}  \tag{12}\\
& =\frac{\gamma_{0}[i] \rho[i]}{\gamma_{0}(1-\rho[i]) p[i]+\sigma^{2} d_{\mathrm{E}}^{2}[i]} .
\end{align*}
$$
\]

Based on (11) and (12), the secrecy rate of the UAV-Bob link at time slot $i$ is given by [42]

$$
\begin{equation*}
R_{\mathrm{S}}[i] \triangleq\left[\log \left(1+\operatorname{SNR}_{\mathrm{I}}[i]\right)-\log \left(1+\operatorname{SINR}_{\mathrm{E}}[i]\right)\right]^{+} \tag{13}
\end{equation*}
$$

and the average secrecy rate can be written as $R_{\mathrm{as}}(\{x[i], y[i], p[i], \rho[i]\}) \triangleq \frac{1}{T} \sum_{i=1}^{T} R_{\mathrm{s}}[i]$.

## B. Problem Formulation

To this end, our objective is to maximize the average secrecy rate $R_{\text {as }}$ subject to the UAVs mobility constraints in (1), (2) and (4), and the average and peak transmit power constraints in (10) and (9). Therefore, we can formulate the following optimization problem:

$$
\begin{align*}
& \max _{\{x[i], y[i], p[i], \rho[i]\}} R_{\mathrm{as}}(\{x[i], y[i], p[i], \rho[i]\})  \tag{14}\\
& \text { s.t. (1), (2), (4), (7), (9) and (10), }
\end{align*}
$$

where the optimization variables include the UAV's trajectory $\{x[i], y[i]\}$, the transmit power levels $\{p[i]\}$ and the power splitting ratios $\{\rho[i]\}$.

Problem (14) is difficult to address due to the following two reasons. First, the operator $[\cdot]^{+}$makes the objective function of problem (14) non-smooth. Second, the variables $\{p[i], \rho[i], x[i], y[i]\}$ are tightly coupled in the objective function, which makes problem (14) highly non-convex. Besides, even with fixed trajectory $\{x[i], y[i]\}$ and without $[\cdot]^{+}$, the variables $\{p[i]\}$ and $\{\rho[i]\}$ are still coupled in the objective function, therefore problem (14) is potentially more complex than the one considered in [14]. In the next section, instead of using the existing convex solvers such as CVX [32], we exploit the special structure of problem (14) and propose an efficient algorithm to tackle it with low complexity by blending the benefits of the BCD method, the CCCP method and the ADMM method.

## III. Proposed Low-Complexity Algorithm

First, in order to handle the non-smoothness of the objective function of (14), we can simply ignore the operator $[\cdot]^{+}$in the objective function since if the secrecy rate is negative at an arbitrary time slot, say $l$, we can always let the corresponding transmit power $p[l]$ be 0 such that $R_{\mathrm{s}}[l]=0$ is satisfied. Therefore, ignoring the operator $[\cdot]^{+}$causes no loss of optimality for problem (14), and we can obtain the following equivalent problem:

$$
\begin{align*}
& \max _{\{x[i], y[i], p[i], \rho[i]\}} \bar{R}_{\mathrm{as}}(\{x[i], y[i], p[i], \rho[i]\})  \tag{15}\\
& \text { s.t. (1), (2), (4), (7), (9) and (10) }
\end{align*}
$$

where

$$
\begin{align*}
& \bar{R}_{\mathrm{as}}(\{x[i], y[i], p[i], \rho[i]\}) \\
& \qquad  \tag{16}\\
& \qquad \frac{1}{T} \sum_{i=1}^{T}\left(\log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{d_{\mathrm{I}}^{2}[i] \sigma^{2}}\right)\right. \\
& \left.\quad-\log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{\gamma_{0}(1-\rho[i]) p[i]+\sigma^{2} d_{\mathrm{E}}^{2}[i]}\right)\right)
\end{align*}
$$

Then, it can be observed that the constraints of problem (15) are all convex, and the optimization variables are only coupled in the objective function. Thus, we can apply the BCD method to solve this problem by dividing the optimization variables into two blocks (i.e., $\{p[i], \rho[i]\}$ and $\{x[i], y[i]\}$ ) and optimizing them in an alternative manner. Specifically, with fixed trajectory, the power allocation subproblem can be expressed as

$$
\begin{gather*}
\max _{\{p[i], \rho[i]\}} \bar{R}_{\mathrm{as}}(\{p[i]\},\{\rho[i]\})  \tag{17}\\
\text { s.t. (7), (9) and (10) }
\end{gather*}
$$

while by fixing the transmit power levels and power splitting ratios, the trajectory optimization subproblem can be written as

$$
\begin{array}{r}
\max _{\{x[i], y[i]\}} \bar{R}_{\mathrm{as}}(\{x[i], y[i]\})  \tag{18}\\
\text { s.t. (1), (2) and (4). }
\end{array}
$$

In other words, we can handle problem (15) by solving subproblems (17) and (18) iteratively, which yields a highquality suboptimal solution, as detailed in the following two subsections.

## A. Solving the Power Allocation Subproblem

In this subsection, we focus on problem (17) and propose to first convert it into a convex problem through proper transformation and approximation. Then, an efficient algorithm is presented to solve the resulting convex problem by employing the Lagrange duality method, where the basic idea is to build some complicated constraints into objective functions and then solve the dual problem instead of the original problem. It is worth mentioning that in [28], a similar power allocation/splitting optimization problem was considered (with a static transmitter and an additional energy harvesting constraint at the Eve), where both optimal and suboptimal solution were obtained and the approaches therein can also be employed to solve problem (17). However, the optimal solution in [28] requires a onedimensional search to find the optimal power splitting ratio for each $i$ while the suboptimal solution relies on an iterative procedure to alternately optimize the transmit power allocations and power splitting ratios, which both exhibit relatively high computational complexity. Note that compared with these existing methods, the proposed algorithm can provide lower computational complexity and by combining with the ellipsoid method, the proposed algorithm can also be easily extended to solve the problems in [28].

To proceed, we introduce two groups of auxiliary variables $a[i]$ and $b[i]$, which satisfy

$$
\begin{equation*}
a[i]=p[i] \rho[i], \quad b[i]=p[i](1-\rho[i]) \tag{19}
\end{equation*}
$$

As a result, problem (17) can be equivalently reformulated as

$$
\begin{align*}
\max _{\{a[i], b[i]\}} & \sum_{i=1}^{T} g_{i}(a[i], b[i])  \tag{20a}\\
\text { s.t. } & a[i]+b[i] \leq P_{\max }, a[i] \geq 0, b[i] \geq 0, \forall i \in \mathcal{T},  \tag{20b}\\
& \sum_{i=1}^{T}(a[i]+b[i]) \leq P \tag{20c}
\end{align*}
$$

where

$$
\begin{align*}
g_{i}(a[i], b[i]) \triangleq & \log \left(1+\frac{\gamma_{0} a[i]}{d_{\mathrm{I}}^{2}[i] \sigma^{2}}\right) \\
& -\log \left(1+\frac{\gamma_{0} a[i]}{\gamma_{0} b[i]+\sigma^{2} d_{\mathrm{E}}^{2}[i]}\right) \tag{21}
\end{align*}
$$

Although problem (20) is much simplified as compared with problem (17), it is still a non-convex problem which cannot be solved efficiently in general. However, it can be readily seen that $g_{i}(a[i], b[i])$ can be viewed as the subtraction of two concave terms, i.e., $\log \left(1+\frac{\gamma_{0} a[i]}{d_{1}^{2}[i] \sigma^{2}}\right)+\log \left(\gamma_{0} b[i]+\sigma^{2} d_{\mathrm{E}}^{2}[i]\right)$ and $\log \left(\gamma_{0} b[i]+\sigma^{2} d_{\mathrm{E}}^{2}[i]+\gamma_{0} a[i]\right)$, or equivalently, (20a) can be expressed in a DC form. Therefore, by employing the CCCP method [30], [43], [44], the lower bound of (20a) can be obtained as

$$
\begin{equation*}
\sum_{i=1}^{T} g_{i}(a[i], b[i]) \geq \sum_{i=1}^{T} \hat{g}_{i}\left(a[i], b[i] ; a_{f}[i], b_{f}[i]\right) \tag{22}
\end{equation*}
$$

where $\left\{a_{f}[i], b_{f}[i]\right\}$ is the given feasible solution of problem (17) ${ }^{3}$ and

$$
\begin{align*}
& \hat{g}_{i}\left(a[i], b[i] ; a_{f}[i], b_{f}[i]\right) \triangleq \log \left(1+\frac{\gamma_{0} a[i]}{d_{\mathrm{I}}^{2}[i] \sigma^{2}}\right) \\
& -\log \left(\gamma_{0} b_{f}[i]+\sigma^{2} d_{\mathrm{E}}^{2}[i]+\gamma_{0} a_{f}[i]\right)+\log \left(\gamma_{0} b[i]+\sigma^{2} d_{\mathrm{E}}^{2}[i]\right) \\
& -\frac{\gamma_{0}}{\gamma_{0} b_{f}[i]+\sigma^{2} d_{\mathrm{E}}^{2}[i]+\gamma_{0} a_{f}[i]}\left(a[i]-a_{f}[i]+b[i]-b_{f}[i]\right) . \tag{23}
\end{align*}
$$

Note that the equality in (22) holds when $a[i]=a_{f}[i]$ and $b[i]=b_{f}[i]$. Consequently, problem (20) can be approximated by the following convex problem:

$$
\begin{align*}
\max _{\{a[i], b[i]\}} & \sum_{i=1}^{T} \hat{g}_{i}\left(a[i], b[i] ; a_{f}[i], b_{f}[i]\right)  \tag{24}\\
\text { s.t. } & (20 \mathrm{~b}) \text { and (20c). }
\end{align*}
$$

Then, we note that without the total power constraint (20c), the other constraints in problem (24) are separable over different time slots $i \in \mathcal{T}$. Inspired by this observation, we introduce a Lagrange multiplier (dual variable) $\lambda \geq 0$ to (20c) and define the partial Lagrangian associated with problem (24) as [45]

$$
\begin{align*}
\mathcal{L}(\{a[i]\},\{b[i]\}, \lambda)= & \sum_{i=1}^{T} \hat{g}_{i}\left(a[i], b[i] ; a_{f}[i], b_{f}[i]\right) \\
& -\lambda \sum_{i=1}^{T}(a[i]+b[i])+\lambda P \tag{25}
\end{align*}
$$

[^2]With (25), the dual function, denoted by $d(\lambda)$, can be written as [45]

$$
\begin{gather*}
d(\lambda) \triangleq \max _{\{a[i], b[i]\}} \mathcal{L}(\{a[i]\},\{b[i]\}, \lambda)  \tag{26}\\
\text { s.t. (20b). }
\end{gather*}
$$

Let $\{a[i](\lambda)\}$ and $\{b[i](\lambda)\}$ denote an optimal solution of problem (26) with fixed $\lambda$. It is not difficult to see that, if $\{a[i](0), b[i](0)\}$ satisfy the total power constraint (20c), then $\{a[i](0), b[i](0)\}$ is optimal for problem (24), since when $\lambda=0$, problem (26) becomes a relaxed version of problem (24) without the total power constraint (20c) and if (20c) is automatically satisfied in this case, the only possibility is that $\{a[i](0), b[i](0)\}$ is optimal. Otherwise, we need to increase $\lambda$ to enhance the dominance of $-\lambda \sum_{i=1}^{T}(a[i]+b[i])+\lambda P$ in $\mathcal{L}(\{a[i]\},\{b[i]\}, \lambda)$ and force $\{a[i](\lambda), b[i](\lambda)\}$ to satisfy (20c).

Since problem (24) is convex and strong duality [45] holds, we have $p^{\mathrm{opt}}=d\left(\lambda^{\mathrm{opt}}\right) \leq d(\lambda)$ for any $\lambda \geq 0$, where $p^{\mathrm{opt}}$ is the optimal objective value of problem (24) and $\lambda^{\text {opt }}$ denotes the optimal dual variable. Hence, in order to solve problem (24), we can instead solve the following dual problem:

$$
\begin{equation*}
\min _{\lambda \geq 0} d(\lambda) \tag{27}
\end{equation*}
$$

Since $d(\lambda)$ is a convex function with respect to $\lambda$ and $P-\sum_{i=1}^{T}(a[i]+b[i])$ is a subgradient of $d(\lambda)$ [46, pp. 12], we can infer that if $\left\{a[i]\left(\lambda^{\mathrm{opt}}\right), b[i]\left(\lambda^{\mathrm{opt}}\right)\right\}$ satisfies (20c) and $\lambda^{\mathrm{opt}}\left(\sum_{i=1}^{T}\left(a[i]\left(\lambda^{\mathrm{opt}}\right)+b[i]\left(\lambda^{\mathrm{opt}}\right)\right)-P\right)=0$, then $\left\{a[i]\left(\lambda^{\mathrm{opt}}\right), b[i]\left(\lambda^{\mathrm{opt}}\right)\right\}$ is an optimal solution of problem (24).

To this end, our main focus is on solving the dual problem (27) and this can be conducted by using the Bisection method [45] with the aid of the subgradient $P-\sum_{i=1}^{T}(a[i]+$ $b[i])$. We summarize the proposed Lagrange duality method in Algorithm 1, where Steps 1-4 check whether or not $\{a[i](0), b[i](0)\}$ is the optimal solution, Steps 5-10 represent the Bisection method to solve the dual problem (27) globally. Note that in Step 9, we increase $\lambda$ when the subgradient $P-\sum_{i=1}^{T}(a[i]+b[i])$ is positive and decrease $\lambda$ otherwise, so as to find the optimal dual variable. In the following, we show that problem (26) can be solved globally in closed form with given $\lambda$.

```
Algorithm 1 Proposed Algorithm for Solving Problem (24)
    Let \(\lambda \leftarrow 0\) and solve problem (26) to obtain \(\{a[i](0), b[i](0)\}\).
    if \(\sum_{i=1}^{T}(a[i](0)+b[i](0)) \leq P\) then
        output \(\{a[i](0), b[i](0)\}\) and exit the algorithm.
    end if
    \(\lambda_{l} \leftarrow 0\), find \(\lambda_{r}\) such that that \(\sum_{i=1}^{T}\left(a[i]\left(\lambda_{r}\right)+b[i]\left(\lambda_{r}\right)\right) \leq P\).
    repeat
        \(\lambda \leftarrow\left(\lambda_{l}+\lambda_{r}\right) / 2\).
        Obtain \(\{a[i](\lambda), b[i](\lambda)\}\) by solving problem (26).
        if \(\sum_{i=1}^{T}(a[i](\lambda)+b[i](\lambda))<P\) then \(\lambda_{r} \leftarrow \lambda\), else \(\lambda_{l} \leftarrow \lambda\).
        end if
    until \(\left|\sum_{i=1}^{T}(a[i](\lambda)+b[i](\lambda))-P\right|\) is less than a certain
    threshold
    output \((\{a[i](\lambda), b[i](\lambda)\}\).
```

It is readily seen that problem (26) can be divided into $T$ independent subproblems for each time slot $i$. Since each subproblem can be solved similarly, we only need to focus on
one particular subproblem, and the corresponding optimization problem can be expressed as (the time slot index is omitted here for simplicity)

$$
\begin{align*}
\max _{a, b} & \tilde{g}(a, b)  \tag{28}\\
\text { s.t. } & a+b \leq P_{\max }, a \geq 0, b \geq 0
\end{align*}
$$

where $\tilde{g}(a, b) \triangleq \hat{g}\left(a, b ; a_{f}, b_{f}\right)-\lambda(a+b)$. It can be observed that problem (28) is convex and there are only two optimization variables. With fixed $a, \tilde{g}(a, b)$ is a strictly concave function with respect to $b$ since $\log (1+x)(x \geq 0)$ is strictly concave. In what follows, we show how problem (28) can be efficiently solved with low complexity.

First, we recast problem (28) as the following equivalent two-tier maximization problem:

$$
\begin{equation*}
\max _{0 \leq a \leq P_{\max }-b} \max _{0 \leq b \leq P_{\max }-a} \tilde{g}(a, b) \tag{29}
\end{equation*}
$$

For given $a$, the optimal $b$ (it is unique since $\tilde{g}(a, b)$ is strictly concave with fixed $a$ ), denoted as $\bar{b}(a)$, can be obtained by resorting to the first-order optimality condition of the inner maximization problem, i.e.,

$$
\begin{equation*}
\frac{d \tilde{g}(a, b)}{d b}=\frac{\gamma_{0}}{\gamma_{0} b+\sigma^{2} d_{\mathrm{E}}^{2}}-\frac{\gamma_{0}}{\gamma_{0} b_{f}+\sigma^{2} d_{\mathrm{E}}^{2}+\gamma_{0} a_{f}}-\lambda=0 \tag{30}
\end{equation*}
$$

and we can obtain the stationary point of $\tilde{g}(a, b)$ as $b_{s}=$ $1 / C_{b}-\sigma^{2} d_{\mathrm{E}}^{2} / \gamma_{0}$, where $C_{b} \triangleq \lambda+\frac{\gamma_{0}}{\gamma_{0} b_{f}+\sigma^{2} d_{\mathrm{E}}^{2}+\gamma_{0} a_{f}}$.

Since the inner maximization problem is a univariate convex problem with a bound constraint, its optimal objective value must be attained either on the boundary of the constraint or at the stationary point $b_{s}$. To be specific, the optimal solution of the inner maximization problem can be obtained by

$$
\bar{b}(a)=\left\{\begin{array}{l}
0, \text { if } b_{s} \leq 0  \tag{31}\\
b_{s}, \text { if } 0<b_{s}<P_{\max }-a \\
P_{\max }-a, \text { otherwise }
\end{array}\right.
$$

Substituting $\bar{b}(a)$ into the objective function of the outer maximization problem of (29), it can be recast as follows with $a$ as the only variable:

$$
\begin{align*}
& \max _{a} \bar{g}(a)  \tag{32}\\
& \text { s.t. } a+\bar{b}(a) \leq P_{\max }, a \geq 0
\end{align*}
$$

where $\bar{g}(a) \triangleq \tilde{g}(a, \bar{b}(a))$. As discussed above, for a univariate optimization problem with a bound constraint, the optimal objective value must be attained at either the endpoints of the bound interval or some feasible stationary point of the objective function. Accordingly, the optimal value of problem (32) must be attained either at the point that satisfies $\frac{d \bar{g}(a)}{d a}=0\left(0<a<P_{\max }-\bar{b}(a)\right)$, or $a \in\left\{0, P_{\max }-\bar{b}(a)\right\}$. Therefore, our basic idea to solve problem (32) is to search over all stationary points and boundary points and then choose the one that achieves the maximum objective value.

Next, we solve problem (32) by considering the above mentioned two cases. By taking the derivative of $\bar{g}(a)$ with respect to $a$, we have

$$
\begin{align*}
\frac{d \bar{g}(a)}{d a}= & \frac{\gamma_{0}}{\gamma_{0} a+\sigma^{2} d_{\mathrm{I}}^{2}}+\frac{\gamma_{0} \frac{d \bar{b}(a)}{d a}}{\gamma_{0} \bar{b}(a)+\sigma^{2} d_{\mathrm{E}}^{2}}  \tag{33}\\
& -\left(\frac{\gamma_{0}}{\gamma_{0} b_{f}+\sigma^{2} d_{\mathrm{E}}^{2}+\gamma_{0} a_{f}}+\lambda\right)\left(1+\frac{d \bar{b}(a)}{d a}\right)
\end{align*}
$$

1) Case I $\left(0<a<P_{\max }-\bar{b}(a)\right)$ : According to (31), we need to further consider the following two cases: $\bar{b}(a)=b_{s}$ or $\bar{b}(a)=0$. For both cases, we have $\frac{d \bar{b}(a)}{d a}=0$. By plugging $\frac{d \bar{b}(a)}{d a}=0$ into (33) and letting (33) equal to 0 , we have $\frac{\gamma_{0}}{\gamma_{0} a+\sigma^{2} d_{1}^{2}}=\frac{\gamma_{0}}{\gamma_{0} b_{f}+\sigma^{2} d_{\mathrm{E}}^{2}+\gamma_{0} a_{f}}+\lambda$. Accordingly, $a$ can be obtained by

$$
\begin{equation*}
a=\frac{\gamma_{0} b_{f}+\sigma^{2} d_{\mathrm{E}}^{2}+\gamma_{0} a_{f}}{\gamma_{0}+\lambda\left(\gamma_{0} b_{f}+\sigma^{2} d_{\mathrm{E}}^{2}+\gamma_{0} a_{f}\right)}-\frac{d_{\mathrm{I}}^{2} \sigma^{2}}{\gamma_{0}} \tag{34}
\end{equation*}
$$

2) Case II $\left(a \in\left\{0, P_{\max }-\bar{b}(a)\right\}\right)$ : In this case, $a$ can take on two possible values, i.e., $a=0$ or $a=P_{\max }-\bar{b}(a)$. If $a=0$, we have $b=\bar{b}(0)$, otherwise, if $a=P_{\max }-\bar{b}(a)$, this implies that $\bar{b}(a)=P_{\text {max }}-a$ and $\frac{d \bar{b}(a)}{d a}=-1$. Consequently, we have $\frac{d \bar{g}(a)}{d a}=\frac{\gamma_{0}}{d_{1}^{2} \sigma^{2}+\gamma_{0} a}-\frac{\gamma_{0}}{d_{\mathrm{E}}^{2} \sigma^{2}+\gamma_{0}\left(P_{\max }-a\right)}=0$, which can be further simplified to a linear equation and its solution can be easily obtained by

$$
\begin{equation*}
a=\frac{\left(\sigma^{2}\left(d_{\mathrm{E}}^{2}-d_{\mathrm{I}}^{2}\right)+\gamma_{0} P_{\max }\right)}{2 \gamma_{0}} \tag{35}
\end{equation*}
$$

Then, by checking the abovementioned four sub-cases and discarding those do not satisfy the case conditions $0<a<$ $P_{\text {max }}-\bar{b}(a)$ or $a \in\left\{0, P_{\max }-\bar{b}(a)\right\}$, we can obtain several feasible solutions of problem (28). Consequently, problem (28) can be globally solved in closed form by choosing the feasible solution that achieves the maximum objective value.

Together with Algorithm 1, the approximated power allocation subproblem (24) can be efficiently solved and with the optimized $\{a[i], b[i]\}$, we can easily obtain $\{p[i], \rho[i]\}$ according to (19).

## B. Solving the Trajectory Optimization Subproblem

In this subsection, we focus on solving the trajectory optimization subproblem (18) with fixed $\{p[i], \rho[i]\}$. Note that although the constraints of problem (18) are convex, its objective function is non-concave with respect to $\{x[i], y[i]\}$ and it cannot be solved optimally in general. In order to resolve the difficulty caused by the non-concave objective function, we introduce two sets of auxiliary variables $\{u[i]\}$ and $\{t[i]\}$, which satisfy

$$
\begin{gather*}
u[i] \geq x^{2}[i]+y^{2}[i]+H^{2}, \forall i \in \mathcal{T},  \tag{36}\\
t[i] \leq(x[i]-L)^{2}+y^{2}[i]+H^{2}, \forall i \in \mathcal{T} . \tag{37}
\end{gather*}
$$

As a result, we have the following equivalent optimization problem:

$$
\begin{align*}
\max _{\{x[i], y[i], u[i], t[i]\}} & \tilde{R}_{\mathrm{as}}(u[i], t[i])  \tag{38}\\
& \text { s.t. (1), (2), (4), (36) and (37), }
\end{align*}
$$

where

$$
\begin{align*}
\tilde{R}_{\mathrm{as}}(u[i], t[i]) \triangleq & \sum_{i=1}^{T}\left(\log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{u[i] \sigma^{2}}\right)\right. \\
& \left.-\log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{\gamma_{0}(1-\rho[i]) p[i]+\sigma^{2} t[i]}\right)\right) \tag{39}
\end{align*}
$$

We note that constraints (36) and (37) in problem (38) must be satisfied with equality at optimality since otherwise, we can always slightly decrease $u[i]$ and increase $t[i]$ such that a larger objective value can be achieved without violating any constraint. Therefore, problem (18) and problem (38) are equivalent.
It can be observed that the term $\log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{u[i] \sigma^{2}}\right)$ in $\tilde{R}_{\text {as }}(u[i], t[i])$ and the term $(x[i]-L)^{2}+y^{2}[i]$ in (37) are convex with respect to $u[i]$ and $\{x[i], y[i]\}$, respectively. Therefore, although $\tilde{R}_{\mathrm{as}}(u[i], t[i])$ is non-concave and constraint (37) is non-convex, they can be expressed in DC forms and problem (38) can be addressed by employing the CCCP method. Specifically, we propose to approximate problem (38) to a convex one and then present an ADMM-based algorithm to solve it globally. First, the proposed algorithm assumes a given solution $\left\{x_{f}[i], y_{f}[i], u_{f}[i], t_{f}[i]\right\}$ in the previous BCD iteration which is feasible to (38). Then, by employing the firstorder Taylor approximation, we construct the lower bounds for $(x[i]-L)^{2}+y^{2}[i]+H^{2}$ and $\log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{u[i] \sigma^{2}}\right)$ as follows:

$$
\begin{align*}
-x_{f}^{2}[i] & +2 x_{f}[i] x[i]-2 x[i] L+L^{2}-y_{f}^{2}[i]  \tag{40}\\
& +2 y_{f}[i] y[i]+H^{2} \leq(x[i]-L)^{2}+y^{2}[i]+H^{2} \\
\log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{u[i] \sigma^{2}}\right) \geq & \log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{u_{f}[i] \sigma^{2}}\right) \\
& -\frac{\gamma_{0} p[i] \rho[i]\left(u[i]-u_{f}[i]\right)}{u_{f}^{2}[i] \sigma^{2}+\gamma_{0} p[i] \rho[i] u_{f}[i]} \tag{41}
\end{align*}
$$

Similarly, we also approximate the second term in (39), i.e., $\varpi(t[i]) \triangleq \log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{\gamma_{0}(1-\rho[i]) p[i]+\sigma^{2} t[i]}\right)$, and obtain the following upper bound:

$$
\begin{align*}
& \log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{\gamma_{0}(1-\rho[i]) p[i]+\sigma^{2} t[i]}\right) \\
& =\log \left(\gamma_{0} p[i]+\sigma^{2} t[i]\right)-\log \left(\gamma_{0}(1-\rho[i]) p[i]+\sigma^{2} t[i]\right) \\
& \leq \log \left(\gamma_{0} p[i]+\sigma^{2} t_{f}[i]\right)+\frac{\sigma^{2}\left(t[i]-t_{f}[i]\right)}{t_{f}[i] \sigma^{2}+\gamma_{0} p[i]} \\
& \quad-\log \left(\gamma_{0} p[i](1-\rho[i])+\sigma^{2} t[i]\right) . \tag{42}
\end{align*}
$$

Note that although replacing $\varpi(t[i])$ by its upper bound in (42) is mathematically unnecessary since it is already a convex function, it will be clear later that with this approximation, the resulting problem is easier to handle. Moreover, we will show in the simulation results that even with such additional approximation, the performance achieved by the proposed lowcomplexity algorithm is similar to that achieved by using the CVX solver. After the above mentioned approximations, it is not difficult to see that the original non-concave objective func-
tion $\tilde{R}_{\text {as }}(u[i], t[i])$ and non-convex constraint (37) in problem (38) can be approximated by

$$
\begin{align*}
& t[i] \leq-x_{f}^{2}[i]+2 x_{f}[i] x[i]-2 x[i] L+L^{2} \\
& \quad-y_{f}^{2}[i]+2 y_{f}[i] y[i]+H^{2}, \forall i \in \mathcal{T}  \tag{43}\\
& \check{R}_{\mathrm{as}}(u[i], t[i]) \\
& \triangleq \sum_{i=1}^{T}\left(-\frac{\gamma_{0} p[i] \rho[i]\left(u[i]-u_{f}[i]\right)}{u_{f}^{2}[i] \sigma^{2}+\gamma_{0} p[i] \rho[i] u_{f}[i]}\right. \\
& \left.\quad-\frac{\sigma^{2}\left(t[i]-t_{f}[i]\right)}{t_{f}[i] \sigma^{2}+\gamma_{0} p[i]}+\log \left(\gamma_{0} p[i](1-\rho[i])+\sigma^{2} t[i]\right)\right) \tag{44}
\end{align*}
$$

respectively. ${ }^{4}$ Therefore, problem (38) can be approximated as the following convex problem:

$$
\begin{align*}
\max _{\{x[i], y[i], u[i],} & t[i]\}  \tag{45}\\
& \check{R}_{\mathrm{as}}(u[i], t[i]) \\
& \text { s.t. (1), (2), (4), (36) and (43). }
\end{align*}
$$

It is noteworthy that the approximation in (42) will not affect the convergence of the proposed algorithm [47], since the resultant objective function $\breve{R}_{\text {as }}(u[i], t[i])$ is a global lower bound of $\tilde{R}_{\text {as }}(u[i], t[i])$ and the first-order behaviors of $\check{R}_{\text {as }}(u[i], t[i])$ and $\tilde{R}_{\mathrm{as}}(u[i], t[i])$ are the same at the point of approximation.

Subsequently, we develop a low-complexity ADMM-based algorithm to globally solve problem (45) efficiently. By exploiting the special structure of problem (45), we show that by tactfully introducing auxiliary variables, it can be efficiently solved and each step in the proposed ADMM method can be carried out in closed form. For completeness, a brief introduction of the ADMM method is provided in Appendix A. It can be seen that problem (45) is not in the standard form of problem (66), therefore, it is difficult to directly apply the ADMM method. The main difficulties lie in: 1) how to partition the optimization variables of problem (45) into two groups, as in the ADMM framework, 2) how to decompose each group problem for much easier implementation. To proceed, we introduce four redundancy copies of the variables $\{x[i], y[i]\}$ to help address the abovementioned difficulties, i.e.,

$$
\begin{align*}
& x[i]=\bar{x}[i], y[i]=\bar{y}[i], x[i]=\tilde{x}[i], y[i]=\tilde{y}[i],  \tag{46a}\\
& x[i]=\hat{x}[i], y[i]=\hat{y}[i], \hat{x}[i]=\ddot{x}[i], \hat{y}[i]=\ddot{y}[i], \forall i \in \mathcal{T} . \tag{46b}
\end{align*}
$$

Then, due to the introduction of (46), constraints (2), (4), (36) and (43) are modified as follows without loss of optimality:

$$
\begin{gather*}
(x[i]-\bar{x}[i+1])^{2}+(y[i]-\bar{y}[i+1])^{2} \leq V_{\max }^{2} \delta_{t}^{2}, \forall i \in \mathcal{T} \backslash T,  \tag{47}\\
\sum_{n=1}^{T-1}\left((\ddot{x}[i]-\ddot{x}[i+1])^{2}+(\ddot{y}[i]-\ddot{y}[i+1])^{2}\right) \leq \frac{E_{\text {tr }}}{\kappa},  \tag{48}\\
u[i] \geq \tilde{x}^{2}[i]+\tilde{y}^{2}[i]+H^{2}, \forall i \in \mathcal{T}  \tag{49}\\
t[i] \leq-\tilde{x}_{f}^{2}[i]+2 \tilde{x}_{f}[i] x[i]-2 \tilde{x}[i] L+L^{2} \\
\quad-\tilde{y}_{f}^{2}[i]+2 \tilde{y}_{f}[i] \tilde{y}[i]+H^{2}, \forall i \in \mathcal{T} . \tag{50}
\end{gather*}
$$

Next, by dualizing and penalizing the equality constraints in (46) to the objective function, we can obtain the augmented Lagrangian (AL) function of problem (45), which is given by

$$
\begin{align*}
& L_{\delta}(\mathcal{Q}, \mathcal{U})=\check{R}_{\mathrm{as}}(u[i], t[i])-\frac{\delta}{2} \sum_{i=1}^{T}\left(\left(x[i]-\bar{x}[i]-\lambda_{x_{i}} / \delta\right)^{2}\right. \\
& +\left(y[i]-\bar{y}[i]-\lambda_{y_{i}} / \delta\right)^{2}+\left(x[i]-\tilde{x}[i]-\eta_{x_{i}} / \delta\right)^{2} \\
& +\left(y[i]-\tilde{y}[i]-\eta_{y_{i}} / \delta\right)^{2}+\left(x[i]-\hat{x}[i]-\omega_{x_{i}} / \delta\right)^{2} \\
& +\left(y[i]-\hat{y}[i]-\eta_{y_{i}} / \delta\right)^{2}+\left(\hat{x}[i]-\ddot{x}[i]-\theta_{x_{i}} / \delta\right)^{2} \\
& \left.+\left(\hat{y}[i]-\ddot{y}[i]-\theta_{y_{i}} / \delta\right)^{2}\right) \tag{51}
\end{align*}
$$

where $\mathcal{Q} \triangleq\{x[i], y[i], \bar{x}[i], \bar{y}[i], \hat{x}[i], \hat{y}[i], \tilde{x}[i], \tilde{y}[i], \ddot{x}[i], \ddot{y}[i]$, $u[i], t[i]\}, \delta$ is the penalty parameter, $\mathcal{U} \triangleq\left\{\lambda_{x_{i}}, \lambda_{y_{i}}, \eta_{x_{i}}, \eta_{y_{i}}\right.$, $\left.\omega_{x_{i}}, \omega_{y_{i}}, \theta_{x_{i}}, \theta_{y_{i}}\right\}, \quad\left\{\lambda_{x_{i}}, \lambda_{y_{i}}\right\}, \quad\left\{\eta_{x_{i}}, \eta_{y_{i}}\right\}, \quad\left\{\omega_{x_{i}}, \omega_{y_{i}}\right\}$ and $\left\{\theta_{x_{i}}, \theta_{y_{i}}\right\}$ are the dual variables associated with the constraints in (46), respectively. Accordingly, we have the following AL problem:

$$
\begin{align*}
& \max _{\mathcal{Q}} L_{\delta}(\mathcal{Q}, \mathcal{U})  \tag{52}\\
& \text { s.t. }(1),(46)-(50) .
\end{align*}
$$

Note that in (51), we have dualized and penalized the equality constraints in (46), and moreover added some constant terms in order to complete the square terms. The added constants will not affect the optimality of the AL problem (52).

To solve problem (52), we need to divide the primal variables $\mathcal{Q}$ into two groups (corresponding to $\mathbf{x}$ and $\mathbf{z}$ in Appendix A), as the ADMM can diverge for certain pathological problems when the number of blocks is larger than two [48]. For this purpose and to facilitate parallel implementation, we group the variables $\mathcal{Q} \backslash\{\ddot{x}[i], \ddot{y}[i]\}$ according to the parity of their corresponding time slot indices, while the variables $\{\ddot{x}[i], \ddot{y}[i]\}$ are handled in one group since they all appear in constraint (48). Besides, we also classify these variables into three different types according to the forms of their corresponding optimization subproblems, as shown in Fig. 2. In the following, we elaborate the details on how to solve these subproblems efficiently.


Fig. 2: Grouping and classification of the optimization variables.

1) Group 1: The Type 1 subproblem is involved with variables $\{x[2 i], \bar{x}[2 i+1], y[2 i], \bar{y}[2 i+1]\}$ and the corresponding optimization problem can be expressed as

$$
\begin{align*}
& \underset{x[2 i], \bar{x}[2 i+1], y[2 i], \bar{y}[2 i+1]}{ } L_{\delta, 1} \\
& \text { s.t. }(x[2 i]-\bar{x}[2 i+1])^{2}+(y[2 i]-\bar{y}[2 i+1])^{2} \leq V_{\max }^{2} \delta_{t}^{2}, \tag{53}
\end{align*}
$$

where $L_{\delta, 1} \triangleq-\frac{\delta}{2}\left(\left(x[2 i]-\bar{x}[2 i]-\frac{\lambda_{x_{2 i}}}{\delta}\right)^{2}+(y[2 i]-\bar{y}[2 i]-\right.$ $\left.\frac{\lambda_{y_{2 i}}}{\delta}\right)^{2}+\left(x[2 i]-\tilde{x}[2 i]-\frac{\eta_{x_{2 i}}}{\delta}\right)^{2}+\left(y[2 i]-\tilde{y}[2 i]-\frac{\eta_{y_{2 i}}}{\delta}\right)^{2}+(x[2 i]-$ $\left.\hat{x}[2 i]-\frac{\omega_{x_{2 i}}}{\delta}\right)^{2}+\left(y[2 i]-\hat{y}[2 i]-\frac{\omega_{y_{2 i}}}{\delta}\right)^{2}+(x[2 i+1]-\bar{x}[2 i+$ $\left.\left.1]-\frac{\lambda_{x_{2 i+1}}}{\delta}\right)^{2}+\left(y[2 i+1]-\bar{y}[2 i+1]-\frac{\lambda_{y_{2 i+1}}}{\delta}\right)^{2}\right)$. Problem (53) is a quadratically constrained quadratic programming (QCQP) problem with only one constraint, therefore, it can be globally solved and the detailed derivation of its optimal solution is relegated to Appendix B. Note that for each time slot $i$, the corresponding variables can be optimized in parallel.

The Type 2 subproblem involves the optimization of $\{\tilde{x}[2 i+$ $1], \tilde{y}[2 i+1], u[2 i+1], t[2 i+1]\}$, which can be written as

$$
\begin{align*}
& \max _{\tilde{x}[2 i+1], \tilde{y}[2 i+1], u[2 i+1], t[2 i+1]} L_{\delta, 2} \\
& \text { s.t. } u[2 i+1] \geq \tilde{x}^{2}[2 i+1]+\tilde{y}^{2}[2 i+1]+H^{2} \text {, } \\
& t[2 i+1] \leq-\tilde{x}_{f}^{2}[2 i+1]+2 \tilde{x}_{f}[2 i+1] \tilde{x}[2 i+1]  \tag{54}\\
& +L^{2}-2 \tilde{x}[2 i+1] L-\tilde{y}_{f}^{2}[2 i+1] \\
& +2 \tilde{y}_{f}[2 i+1] \tilde{y}[2 i+1]+H^{2},
\end{align*}
$$

where $L_{\delta, 2} \triangleq-a[2 i+1] u[2 i+1]-\frac{\sigma^{2} t[2 i+1]}{t_{f}[2 i+1]+\gamma_{0} p[2 i+1]}+$ $\log \left(\gamma_{0}(1-\rho[2 i+1]) p[2 i+1]+\sigma^{2} t[2 i+1]\right)-\frac{\delta}{2}((x[2 i+$ $\left.\left.1]-\tilde{x}[2 i+1]-\frac{\omega_{x_{2 i+1}}}{\delta}\right)^{2}+\left(y[2 i+1]-\tilde{y}[2 i+1]-\frac{\omega_{y_{2 i+1}}}{\delta}\right)^{2}\right)$ and $a[2 i+1]=\frac{\gamma_{0} p[2 i+1] \rho[2 i+1]}{u_{f}^{2}[2 i+1] \sigma^{2}+\gamma_{0} p[2 i+1] \rho[2 i+1] u_{f}[2 i+1]}$. It can be observed that problem (54) is a convex problem with two quadratic constraints. Although there is no closed-form solution for such kind of optimization problems in general, we show that it can be efficiently solved in closed form by exploiting its special structure and the details are provided in Appendix C. It is important to note that this closed-form solution is easier to obtain when the term $\varpi(t[i])$ in (39) is approximated by its upper bound in (42); otherwise, the derivation would be very involved as the gradient of $\varpi(t[i])$ with respect to $t[i]$ is $\frac{1}{\gamma_{0} p[i]+\sigma^{2} t[i]}-\frac{1}{\gamma_{0}(1-\rho[i]) p[i]+\sigma^{2} t[i]}$ and the Lagrangian function associated with problem (76) would be a quadratic function of $t[i]$, thus rendering it difficult to obtain the optimal dual variable.

The Type 3 subproblem involves the optimization of $\{\hat{x}[2 i+$ $1], \hat{y}[2 i+1], \ddot{x}[i], \ddot{y}[i]\}$ and we can obtain the following problem:

$$
\begin{align*}
& \quad \max _{\{\hat{x}[2 i+1], \hat{y}[2 i+1], \ddot{x}[i], \ddot{y}[i]\}} L_{\delta, 3} \\
& \text { s.t. (1), } \sum_{n=1}^{T-1}\left((\ddot{x}[i]-\ddot{x}[i+1])^{2}+(\ddot{y}[i]-\ddot{y}[i+1])^{2}\right) \leq \frac{E_{\mathrm{tr}}}{\kappa}, \tag{55}
\end{align*}
$$

where $L_{\delta, 3} \triangleq-\frac{\delta}{2} \sum_{n=1}^{T}\left(\left(\hat{x}[i]-\ddot{x}[i]-\frac{\theta_{x_{i}}}{\delta}\right)^{2}+(\hat{y}[i]-\ddot{y}[i]-\right.$ $\left.\left.\frac{\theta_{y_{i}}}{\delta}\right)^{2}+\left(x[i]-\hat{x}[i]-\frac{\eta_{x_{i}}}{\delta}\right)^{2}+\left(y[i]-\hat{y}[i]-\frac{\eta_{y_{i}}}{\delta}\right)^{2}\right)$. Similar to problem (53), problem (55) is also a convex QCQP problem with only one constraint and strong duality holds for this problem. Therefore, it can be globally solved in closed form and the details are presented in Appendix D.
2) Group 2: The Type 1 subproblem in group 2 can be obtained by changing the time slot indices in problem (53) from $2 i$ and $2 i+1$ to $2 i+1$ and $2 i+2$, respectively. Therefore, it can be solved by resorting to Appendix B, the details are not shown here for brevity. Similarly, the Type 2 subproblem can be obtained by changing the time slot indices in problem
(54) and it can be efficiently solved according to Appendix C. Besides, the Type 3 subproblem is given by

$$
\begin{equation*}
\max _{\{\hat{x}[2 i], \hat{y}[2 i]\}} L_{\delta, 3} \tag{56}
\end{equation*}
$$

Since problem (56) is an unconstrained convex problem, its global optimal solution can be easily obtained by (resorting to the first-order optimality condition)

$$
\begin{align*}
\hat{x}^{\mathrm{opt}}[2 i] & =\frac{\ddot{x}[2 i]+x[2 i]}{2}+\frac{\theta_{x_{2 i}}-\eta_{x_{2 i}}}{2 \delta},  \tag{57}\\
\hat{y}^{\mathrm{opt}}[2 i] & =\frac{\ddot{y}[2 i]+y[2 i]}{2}+\frac{\theta_{y_{2 i}}-\eta_{y_{2 i}}}{2 \delta} .
\end{align*}
$$

Finally, the dual variables can be updated by

$$
\begin{align*}
& \lambda_{x_{i}}=\lambda_{x_{i}}+\delta(\bar{x}[i]-x[i]), \quad \lambda_{y_{i}}=\lambda_{y_{i}}+\delta(\bar{y}[i]-y[i]), \\
& \eta_{x_{i}}=\eta_{x_{i}}+\delta(\hat{x}[i]-x[i]), \quad \eta_{y_{i}}=\eta_{y_{i}}+\delta(\hat{y}[i]-y[i]), \\
& \omega_{x_{i}}=\omega_{x_{i}}+\delta(\tilde{x}[i]-x[i]), \omega_{y_{i}}=\omega_{y_{i}}+\delta(\tilde{y}[i]-y[i]), \\
& \theta_{x_{i}}=\theta_{x_{i}}+\delta(\ddot{x}[i]-\hat{x}[i]), \quad \theta_{y_{i}}=\theta_{y_{i}}+\delta(\ddot{y}[i]-\hat{y}[i]) . \tag{58}
\end{align*}
$$

Overall, the proposed algorithm to solve problem (45) is summarized in Algorithm 2. Note that if the 3D trajectory optimization is considered, we can similarly introduce auxiliary variables for the altitudes of the UAV and solve the resulting subproblems accordingly without much difficulty.

```
Algorithm 2 Proposed ADMM Method for Problem (45)
    Let \(\mathcal{U}=\mathbf{0}\), set a threshold \(\epsilon\) and the penalty parameter \(\delta\).
    repeat
        Update the variables in group 1 by solving subproblems (53),
        (54) and (55).
        Change the time slot indices in subproblems (53) and (54) and
        update the variables in group 2 by solving subproblems (53),
        (54) and (56).
        Update the dual variables according to (58)
        Calculate the primal residual \(\mathbf{r}\) and dual residual \(\mathbf{s}\) using (69).
    until \(\max (\|\mathbf{r}\|,\|\mathbf{s}\|)<\epsilon\).
    output \(\{x[i], y[i]\}\).
```


## C. Overall Algorithm and Analysis

To summarize, the proposed algorithm can find a suboptimal solution of problem (15) by applying the BCD method, i.e., the power allocation subproblem (17) and the trajectory optimization subproblem (18) are solved alternately in an iterative manner. The detailed steps of the proposed algorithm are listed in Algorithm 3. Furthermore, regarding to the convergence of Algorithm 3, we have the following proposition:

Proposition 1. The sequence of the objective values generated by Algorithm 3 is guaranteed to converge.

Proof. Since problems (17) and (18) are equivalent to problems (20) and (38), respectively, and the latter two can be approximated by problems (24) and (45) through the first-order approximations, we can infer that the solution obtained in the $(t-1)$-th iteration of Algorithm 3 , denoted by $\left\{p^{t-1}[i], \rho^{t-1}[i], x^{t-1}[i], y^{t-1}[i]\right\}$, is feasible to problem (15). Besides, due to the fact that Algorithm 1 and Algorithm 2 can obtain the optimal solutions of problems (24) and (45), respectively, it
can be readily seen that $\bar{R}_{\text {as }}\left(\left\{x^{t}[i], y^{t}[i], p^{t}[i], \rho^{t}[i]\right\}\right) \geq$ $\bar{R}_{\text {as }}\left(\left\{x^{t-1}[i], y^{t-1}[i], p^{t-1}[i], \rho^{t-1}[i]\right\}\right)$. Together with the fact that the objective value of problem (15) is upper bounded by a certain value due to the power constraints (8) and (9), we conclude that the sequence $\left\{\bar{R}_{\mathrm{as}}\left(\left\{x^{t}[i], y^{t}[i], p^{t}[i], \rho^{t}[i]\right\}\right)\right\}$ guarantees to converge. This completes the proof.

```
Algorithm 3 Proposed Algorithm for Problem (15)
    Initialize \(\mathcal{Q},\{p[i], \rho[i]\}\) and set a threshold \(\tau\).
    repeat
        Solve problem (24) using Algorithm 1 with fixed trajectory
        and obtain \(\{p[i], \rho[i]\}\).
        Solve problem (45) using Algorithm 2 with fixed \(\{p[i], \rho[i]\}\)
        and obtain \(\mathcal{Q}\).
        \(\mathcal{Q}_{f} \leftarrow \mathcal{Q},\left\{p_{f}[i], \rho_{f}[i]\right\} \leftarrow\{p[i], \rho[i]\}\).
    until The fractional increase of the objective value of problem
    (15) is below the threshold \(\tau\).
    output \(\{p[i], \rho[i], x[i], y[i]\}\).
```

Besides, Algorithm 3 exhibits very low computational complexity and the detailed analysis is presented as follows. As mentioned in Section III-A, since the power allocation subproblem is divided into $T$ independent subproblems and each subproblem is solved efficiently in closed form, the worst-case complexity of Algorithm 1 is $\mathcal{O}\left(N_{B} T\right)$, where $N_{B}$ denotes the number of iterations required by the Bisection method. For Algorithm 2, we can see that its complexity is dominated by solving problem (55) using Gaussian eliminations. Since the complexity of solving one instance of problem (55) scales with $T$, the complexity of Algorithm 2 can be expressed as $\mathcal{O}\left(N_{A} N_{B} T\right)$, where $N_{A}$ denotes the number of ADMM iterations. In summary, the complexity of Algorithm 3 can be expressed as $\mathcal{O}\left(N_{\mathrm{BCD}}\left(N_{A} N_{B} T+N_{B} T\right)\right)$, where $N_{\mathrm{BCD}}$ represents the number of BCD iterations. Note that the complexity of the conventional algorithm in [14] is $\mathcal{O}\left(N_{\mathrm{BCD}} T^{3.5}\right)$, therefore, the proposed Algorithm 3 exhibits a much lower complexity ${ }^{5}$ and it will be shown in Section IV that Algorithm 3 can achieve a similar performance with that of the conventional algorithm using existing convex solvers.

## D. Extension to 3D Trajectory Design

In this subsection, we extend the proposed algorithm to the case of 3D trajectory design, where the UAV's time-varying altitudes are also treated as optimization variables. In this case, problem (15) becomes:

$$
\begin{align*}
& \max _{\{x[i], y[i], z[i], p[i], \rho[i]\}} \bar{R}_{\mathrm{as}}^{3 \mathrm{D}}(\{x[i], y[i], z[i], p[i], \rho[i]\}) \\
& \text { s.t. }(1),(4),(7),(9) \text { and }(10), \\
& z[1]=z_{1}, z[T]=z_{T}, z_{\min } \leq z[i] \leq z_{\max }, \forall i \in \mathcal{T},  \tag{59}\\
& \quad\left((x[i]-x[i+1])^{2}+(y[i]-y[i+1])^{2}\right. \\
& \left.\quad+(z[i]-z[i+1])^{2}\right)^{\frac{1}{2}} / \delta_{t} \leq V_{\max }, \forall i \in \mathcal{T} \backslash T,
\end{align*}
$$

where $\bar{R}_{\mathrm{as}}^{3 \mathrm{D}}(\{x[i], y[i], z[i], p[i], \rho[i]\}) \triangleq \frac{1}{T} \sum_{i=1}^{T}(\log (1+$ $\left.\left.\frac{\gamma_{0} p[i] \rho[i]}{d_{13}^{2}[i] \sigma^{2}}\right)-\log \left(1+\frac{\gamma_{0} p[i] \rho[i]}{\gamma_{0}(1-\rho[i]) p[i]+\sigma^{2} d_{\mathrm{E3}}^{2}[i]}\right)\right), d_{\mathrm{IB}}^{2}[i] \triangleq x[i]^{2}+$

[^3]$y[i]^{2}+z[i]^{2}, d_{\mathrm{E} 3}^{2}[i] \triangleq(x[i]-L)^{2}+y[i]^{2}+z[i]^{2}, z_{\text {min }}$ and $z_{\text {max }}$ denote the UAVs minimally and maximally allowed altitudes, respectively, while $z_{1}$ and $z_{T}$ are the initial and final altitudes of the UAV, respectively. Let $\bar{R}_{\text {as }}^{3 \mathrm{D}}\left(\left\{x[i], y[i], z_{\min }, p[i], \rho[i]\right\}\right)$ denote the average secrecy rate with the altitudes of the UAV fixed to $z[i]=z_{\min }, \forall i \in \mathcal{T}$, then we have the following proposition:
Proposition 2. For all possible trajectories $\{x[i], y[i], z[i]\}$ of problem (59), we have
\[

$$
\begin{align*}
& \bar{R}_{\mathrm{as}}^{3 \mathrm{D}}\left(\left\{x[i], y[i], z_{\mathrm{min}}, p[i], \rho[i]\right\}\right) \\
& \quad \geq \bar{R}_{\mathrm{as}}^{3 \mathrm{D}}(\{x[i], y[i], z[i], p[i], \rho[i]\}) \tag{60}
\end{align*}
$$
\]

Proof. Please refer to Appendix E.
Proposition 2 implies that the UAV will try to fly as low as possible to obtain a better performance according to (60), thus we can safely ignore the constraints $z[i] \leq z_{\text {max }}, \forall i \in \mathcal{T}$ without loss of optimality.

We first consider the special case when the initial and final altitudes of the UAV are set to $z_{\min }$, i.e., $z[1]=z[T]=z_{\min }$. According to Proposition 2, we can directly set the altitude of the UAV at each time slot to $z_{\text {min }}$, then it can be readily seen that problem (59) reduces to problem (15) in this case and can be efficiently solved by Algorithm 3.

For the general case when the initial and final altitudes of the UAV are not necessarily at $z_{\text {min }}$, we have an addition sequence of optimization variables, i.e., $\{z[i]\}$, in problem (59) as compared to those in problem (15). Since the power allocation subproblem is similar to the 2D case, we mainly focus on the trajectory optimization subproblem in the sequel. In particular, similar to the idea in (46), we further introduce auxiliary variables $\{\bar{z}[i], \tilde{z}[i], \hat{z}[i], \ddot{z}[i]\}$ which satisfy $\bar{z}[i]=$ $\tilde{z}[i]=\hat{z}[i]=\ddot{z}[i]=z[i], \forall i \in \mathcal{T}$. Accordingly, constraints (47)-(50) are modified as follows:

$$
\begin{align*}
& (x[i]-\bar{x}[i+1])^{2}+(y[i]-\bar{y}[i+1])^{2} \\
& +(z[i]-\bar{z}[i+1])^{2} \leq V_{\max }^{2} \delta_{t}^{2}, \forall i \in \mathcal{T} \backslash T,  \tag{61}\\
& \sum_{n=1}^{T-1}\left((\ddot{x}[i]-\ddot{x}[i+1])^{2}+(\ddot{y}[i]-\ddot{y}[i+1])^{2}\right.  \tag{62}\\
& \left.+(\ddot{z}[i]-\ddot{z}[i+1])^{2}\right) \leq \frac{E_{\mathrm{tr}}}{\kappa}, \\
& u[i] \geq \tilde{x}^{2}[i]+\tilde{y}^{2}[i]+\tilde{z}^{2}[i], \forall i \in \mathcal{T},  \tag{63}\\
& t[i] \leq-\tilde{x}_{f}^{2}[i]+2 \tilde{x}_{f}[i] x[i]-2 \tilde{x}[i] L+L^{2}-\tilde{y}_{f}^{2}[i] \\
& +2 \tilde{y}_{f}[i] \tilde{y}[i]-\tilde{z}_{f}^{2}[i]+2 \tilde{z}_{f}[i] \tilde{z}[i], \forall i \in \mathcal{T} . \tag{64}
\end{align*}
$$

We note that the grouping/classification of the variables and the optimization of these variables can be similarly conducted as those in the 2D case with proper modifications. Take the 3D version of problem (54) as an example, by following the derivation in Appendix C, problem (76) becomes

$$
\begin{aligned}
& \max _{\tilde{x}, \tilde{y}, \tilde{z}, t} L_{\delta, 2, z} \\
& \text { s.t. } t \leq-\tilde{x}_{f}^{2}+2 \tilde{x}_{f} \tilde{x}+L^{2}-2 \tilde{x} L-\tilde{y}_{f}^{2} \\
&+2 \tilde{y}_{f} \tilde{y}-\tilde{z}_{f}^{2}+2 \tilde{z} \tilde{z} \\
& \tilde{z} \geq z_{\min }
\end{aligned}
$$

where $L_{\delta, 2, z} \triangleq-a\left(\tilde{x}^{2}+\tilde{y}^{2}+\tilde{z}^{2}\right)-\frac{\sigma^{2} t}{t_{f}+\gamma_{0} p}+\log \left(\gamma_{0}(1-\rho) p+\right.$ $\left.\sigma^{2} t\right)-\frac{\delta}{2}\left(\left(x-\tilde{x}-\frac{\omega_{x}}{\delta}\right)^{2}+\left(y-\tilde{y}-\frac{\omega_{y}}{\delta}\right)^{2}+\left(z-\tilde{z}-\frac{\omega_{z}}{\delta}\right)^{2}\right)$ and $\omega_{z}$ is similarly defined as $\omega_{x}$ and $\omega_{y}$. The corresponding Lagrange function can be expressed as $L_{\delta, 2, z, \tilde{\mu}} \triangleq L_{\delta, 2, z}+\tilde{\mu}\left(-\tilde{x}_{f}^{2}+\right.$ $2 \tilde{x}_{f} \tilde{x}+L^{2}-2 \tilde{x} L-\tilde{y}_{f}^{2}+2 \tilde{y}_{f} \tilde{y}-\tilde{z}_{f}^{2}+2 \tilde{z}_{f} \tilde{z}-t$ ), where $\tilde{\mu}$ is the Lagrangian dual variable. By checking the first-order optimality condition of problem (65), we can see that the optimal solutions of $\tilde{x}, \tilde{y}$ and $t$ are the same as (77), while the optimal $\tilde{z}$ can be expressed as $\tilde{z}^{\text {opt }}(\tilde{\mu})=\max \left(\frac{\delta z-\omega_{z}+2 \tilde{\mu} \tilde{z}_{f}}{2 a+\delta}, z_{\text {min }}\right)$. If the solution $\left\{\tilde{x}^{\text {opt }}(0), \tilde{y}^{\text {opt }}(0), \tilde{z}^{\text {opt }}(0), t^{\text {opt }}(0)\right\}$ satisfies the constraint of problem (65), then it is the optimal solution, otherwise, the optimal dual variable $\tilde{\mu}^{\text {opt }}$ satisfies $t(\tilde{\mu})=$ $-\tilde{x}_{f}^{2}+2 \tilde{x}_{f} \tilde{x}(\tilde{\mu})+L^{2}-2 \tilde{x}(\tilde{\mu}) L-\tilde{y}_{f}^{2}+2 \tilde{y}_{f} \tilde{y}(\tilde{\mu})-\tilde{z}_{f}^{2}+$ $2 \tilde{z}_{f} \tilde{z}(\tilde{\mu})$. If $\tilde{z}^{\text {opt }}=z_{\text {min }}$, the optimal solution of $\tilde{\mu}$ is the same as that in the 2 D case with $H^{2}$ replaced by $-\tilde{z}_{f}^{2}+2 \tilde{z}_{f} z_{\min }$. On the other hand, if $\tilde{z}^{\text {opt }}(\tilde{\mu})=\frac{\delta z-\omega_{z}+2 \tilde{\mu} \tilde{z}_{f}}{2 a+\delta}$, then we have $\tilde{\mu}^{\text {opt }}=\left(-b_{\tilde{\mu}}^{3 \mathrm{D}}+\sqrt{\left(b_{\tilde{\mu}}^{3 \mathrm{D}}\right)^{2}-4 a_{\tilde{\mu}}^{3 \mathrm{D}} c_{\tilde{\mu}}^{3 \mathrm{D}}}\right) /\left(2 a_{\tilde{\mu}}^{3 \mathrm{D}}\right)$, where $a_{\tilde{\mu}}^{3 \mathrm{D}}=\frac{4\left(\tilde{x}_{f}-L\right)^{2}+4 \tilde{y}_{f}^{2}+4 \tilde{z}_{f}^{2}}{2 a+\delta}, b_{\tilde{\mu}}^{3 \mathrm{D}}=\frac{\sigma^{2} a_{\mu}^{3 \mathrm{D}}}{\sigma^{2} t_{f}+\gamma_{0} p}+d_{\tilde{\mu}}^{3 \mathrm{D}}$, $c_{\tilde{\mu}}^{3 \mathrm{D}}=\frac{\sigma^{2} d_{\tilde{\mu}}^{3 \mathrm{D}}}{\sigma^{2} t_{f}+\gamma_{0} p}-1$ and $d_{\tilde{\mu}}^{3 \mathrm{D}}=-\tilde{x}_{f}^{2}-\tilde{y}_{f}^{2}-\tilde{z}_{f}^{2}+L^{2}+$ $\frac{\gamma_{0}(1-\rho) p}{\sigma^{2}}+\frac{2\left(\tilde{x}_{f}-L\right)\left(\delta x-\omega_{x}\right)+2 \tilde{y}_{f}\left(\delta y-\omega_{y}\right)+2 \tilde{z}_{f}\left(\delta z-\omega_{z}\right)}{2 a+\delta}$. By comparing the objective values achieved by these two cases, we can solve the 3D version of problem (54) efficiently. The details of the other optimization steps are not elaborated here for brevity.

To summarize, we can conclude that the proposed lowcomplexity algorithm can be easily extended to the case of 3D trajectory design and its convergence can also be guaranteed in the 3D case similar to the analysis in Proposition 1.

## IV. Simulation results

In this section, we provide numerical results to evaluate the performance of our proposed low-complexity algorithm (i.e., Algorithm 3). For comparison, we also provide the performance of the following three benchmark schemes:

- The fixed trajectory (FT) scheme: the transmit power levels and power splitting ratios are jointly optimized, while the UAV is assumed to fly from $\left(x_{1}, y_{1}\right)$ to $\left(x_{T}, y_{T}\right)$ straightly at a constant speed.
- The naive power splitting (NPS) scheme: running Algorithm 3 with fixed $\rho[i]=0.5, \forall i$.
- The without AN scheme: running Algorithm 3 with fixed $\rho[i]=1, \forall i$.
In our simulations, the channel bandwidth, the noise power spectrum and the channel power gain are set to 20 MHz , $N_{0}=-169 \mathrm{dBm} / \mathrm{Hz}$ and $\gamma_{0}=-36 \mathrm{~dB}$, respectively, and the carrier frequency is set at 5 GHz . Hence, the reference SNR at a distance of 1 m is $\frac{\gamma_{0}}{\sigma^{2}}=60 \mathrm{~dB}$. The nominal system configuration is defined by the following choice of parameters unless otherwise specified: $L=100 \mathrm{~m}, H=100 \mathrm{~m}$, $V_{\max }=12 \mathrm{~m} / \mathrm{s}, M=4 \mathrm{~kg}, N=125 \mathrm{~s}, \delta_{t}=0.5 \mathrm{~s},\left(x_{1}, y_{1}\right)=$ $(-200 \mathrm{~m},-150 \mathrm{~m}),\left(x_{T}, y_{T}\right)=(1000 \mathrm{~m},-150 \mathrm{~m}), \bar{P}=0$ dBm and $P_{\max }=4 \bar{P}$. The total mobility energy stored at the UAV $E_{\text {tr }}$ is set to 19.40 kJ . The simulations are implemented in MATLAB R2016b and carried out on a workstation with Intel Xeon(R) CPU E5-2640 running at 2.6 GHz and with 128 GB RAM.

1) Convergence property: First, we illustrate in Fig. 3 the convergence of our proposed Algorithm 2 and 3. From Fig. 3 (a), it is observed that the outer BCD iteration of Algorithm 3 is monotonically convergent and it needs about 10 iterations to obtain the steady performance. Besides, in Fig. 3 (b) and (c), we plot the primal and dual residuals $\|\mathbf{r}\|$ and $\|\mathbf{s}\|$ versus the number of ADMM iterations in Algorithm 2. As can be seen, Algorithm 2 can converge well within 2000 iterations. Although this number is relatively large as compared with the number of outer BCD iterations, the complexity is low since each updating step in Algorithm 2 is very simple, this will be verified in the following results.


Fig. 3: Convergence behavior of the proposed Algorithm 2 and 3.
2) Performance and complexity: In Fig. 4 and Table I, we respectively investigate the average running time required by Algorithm 3 to complete one outer BCD iteration and the achieved objective value (i.e., the average secrecy rate) in bits $/ \mathrm{s} / \mathrm{Hz}$ by Algorithm 3. For comparison, we also provide the performance achieved by replacing Algorithm 1 and 2 in steps 3 and 4 of Algorithm 3 by using the CVX solver [32]. From Fig. 4, we observe that the running time required by the proposed algorithm is significantly less than that required by using CVX. The running time increases with the increasing of $T$, however, it increases much slower for the proposed algorithm. This is consistent with the complexity analysis in Section III-C and it shows that the proposed algorithm design is more scalable. Besides, we observe from Table I that the average secrecy rate achieved by the proposed algorithm and that by CVX is almost identical. In certain cases, such as $T=240$, the performance of the proposed algorithm is even better. This is because the CVX solver uses a successive approximation heuristic method to solve convex optimization problems involving $\log (\cdot)$ functions, which may lead to certain performance loss due to precision issues.

TABLE I: Achieved Average Secrecy Rate Comparison

|  | Numbers of time slots $T$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 200 | 220 | 240 | 260 | 280 | 300 |
| Using CVX | 2.2019 | 2.9532 | 3.2711 | 3.5375 | 3.7640 | 3.9580 |
| Algorithm 3 | 2.2019 | 2.9532 | 3.2721 | 3.5375 | 3.7640 | 3.9580 |

3) Average secrecy rate versus the Bob-Eve distance L: In Fig. 5, we plot the average secrecy rates achieved by the


Fig. 4: Running time comparison.
considered schemes under various values of $L$. First, it is observed that the proposed algorithm achieves the best performance among the considered schemes. Second, the achieved average secrecy rates by all the considered schemes increases with $L$, which is expected since it is more difficult for Eve to intercept the communications between Bob and the UAV when $L$ is large. Similarly, since transmitting AN is less important under larger $L$, the performance of the without AN scheme approaches that of the proposed algorithm with the increasing of $L$. Besides, we observe that the performance of the NPS scheme is better than that of the FT scheme. This is due to the fact that optimizing the UAV's trajectory under the considered simulation setup enables the UAV to fly close to Bob and away from Eve to achieve higher secrecy rate, while the performance gain offered by optimizing the power splitting ratios $\{\rho[i]\}$ is not that pronounced.


Fig. 5: Average secrecy rate versus $L$.
4) Average secrecy rate versus the total flight time $N$ : In Fig. 6, we investigate the average secrecy rate versus various values of $N$. As can be seen, the performance of all the considered schemes improves with the increasing of $N$, except for the FT scheme. This is because with increasingly large $T$, the UAV is able to hover over its favorable locations for a longer time, which leads to higher secrecy rate. However, if the


Fig. 6: Average secrecy rate versus $N$.


Fig. 7: Average secrecy rate versus $\bar{P}$.
the trajectories of the proposed algorithm and the NPS scheme are almost identical, which implies that the UAV's trajectory is not sensitive to the power splitting ratios under the considered simulation setups.
7) Trajectories under various values of $N$ : In Fig. 9, we show the trajectories of the UAV by employing different schemes when $N=103 \mathrm{~s}$ and $N=125 \mathrm{~s}$. We observe that when the flight time is long enough (i.e., $N=125 \mathrm{~s}$ ), the UAV can fly close to Bob and Eve, while when $N=103$ s , the UAV has to head back to the final location before it can reach its most favorable location. Besides, similar to the results in Fig. 8, the trajectories of the proposed algorithm and the without AN scheme are different owing to the difference in the ability of transmitting AN signals.


Fig. 8: Trajectories of the UAV under various values of $E_{\text {th }}$.

## V. Conclusion

In this work, we proposed a power splitting approach to secure the UAV communication against a potential ground Eve, by enabling the UAV to transmit confidential information and AN simultaneously. By exploiting the power splitting capability of the UAV and its controllable mobility, we maximized the average secrecy rate by jointly optimizing the


Fig. 9: Trajectories of UAV using different algorithms.

UAVs trajectory, the transmit power levels and the power splitting ratios over time. An iterative algorithm with very low complexity was proposed to solve the considered optimization problem with guaranteed convergence. Numerical results showed the effectiveness of our proposed algorithm. It is worth noting that the proposed algorithm and the underlying techniques that are employed can be extended to other joint power and trajectory optimization problems for UAV-enabled communication systems.

## Appendix A

## BRIEF INTRODUCTION TO ADMM

To illustrate the idea of the ADMM, let us consider the following convex optimization problem:

$$
\begin{array}{rl}
\min _{\mathbf{x} \in \mathbb{R}^{n \times 1}, \mathbf{z} \in \mathbb{R}^{m \times 1}} & f(\mathbf{x})+g(\mathbf{z})  \tag{66}\\
& \text { s.t. } \mathbf{A x}+\mathbf{B z}=\mathbf{c}, \mathbf{x} \in \mathcal{C}_{1}, \mathbf{z} \in \mathcal{C}_{2}
\end{array}
$$

where $f(\cdot): \mathbb{R}^{n \times 1} \mapsto \mathbb{R}$ and $g(\cdot): \mathbb{R}^{m \times 1} \mapsto \mathbb{R}$ are convex functions, $\mathcal{C}_{1} \in \mathbb{R}^{n \times 1}$ and $\mathcal{C}_{2} \in \mathbb{R}^{m \times 1}$ are non-empty convex sets, $\mathbf{A} \in \mathbb{R}^{p \times n}, \mathbf{B} \in \mathbb{R}^{p \times m}, \mathbf{c} \in \mathbb{R}^{p \times 1}$. Assume that problem (66) is feasible and strong duality holds.

The ADMM solves problem (66) by resorting to the following AL problem:

$$
\begin{align*}
& \min _{\mathbf{x} \in \mathbb{R}^{n \times 1}, \mathbf{z} \in \mathbb{R}^{m \times 1}} L_{\rho}(\mathbf{x}, \mathbf{z}, \boldsymbol{\lambda})  \tag{67}\\
& \text { s.t. } \mathbf{x} \in \mathcal{C}_{1}, \mathbf{z} \in \mathcal{C}_{2}
\end{align*}
$$

where $L_{\rho}(\mathbf{x}, \mathbf{z}, \boldsymbol{\lambda})=f(\mathbf{x})+g(\mathbf{z})+\boldsymbol{\lambda}^{T}(\mathbf{A} \mathbf{x}+\mathbf{B} \mathbf{z}-\mathbf{c})+\frac{\rho}{2} \| \mathbf{A} \mathbf{x}+$ $\mathbf{B z}-\mathbf{c} \|^{2}, \boldsymbol{\lambda}$ denotes the dual variable and $\rho$ is the penalty parameter. Then, the ADMM iterates over the following three steps:

$$
\begin{align*}
& \mathbf{x}_{k+1}=\underset{\mathbf{x} \in \mathbb{R}^{n \times 1}}{\arg \min } L_{\rho}\left(\mathbf{x}, \mathbf{z}_{k}, \boldsymbol{\lambda}_{k}\right)  \tag{68a}\\
& \mathbf{z}_{k+1}=\underset{\mathbf{z} \in \mathbb{R}^{m \times 1}}{\arg \min } L_{\rho}\left(\mathbf{x}_{k+1}, \mathbf{z}, \boldsymbol{\lambda}_{k}\right)  \tag{68b}\\
& \boldsymbol{\lambda}_{k+1}=\boldsymbol{\lambda}_{k}+\rho\left(\mathbf{A} \mathbf{x}_{k+1}+\mathbf{B} \mathbf{z}_{k+1}-\mathbf{c}\right) \tag{68c}
\end{align*}
$$

where $k$ denotes the iteration index. The convergence criterion of the ADMM can be expressed as $\left\|\mathbf{r}_{k+1}\right\| \leq \epsilon$ and $\left\|\mathbf{s}_{k+1}\right\| \leq$
$\epsilon$, where $\mathbf{r}_{k+1}$ and $\mathbf{s}_{k+1}$ denote the primal residual and dual residual in the $(k+1)$-th iteration, which are defined as

$$
\begin{align*}
& \mathbf{r}_{k+1}=\mathbf{A} \mathbf{x}_{k+1}+\mathbf{B} \mathbf{z}_{k+1}-\mathbf{c} \\
& \mathbf{s}_{k+1}=\rho \mathbf{A}^{T} \mathbf{B}\left(\mathbf{z}_{k+1}-\mathbf{z}_{k}\right) \tag{69}
\end{align*}
$$

It can be seen that the ADMM alternately performs one iteration of primal variables updates, i.e., (68a) and (68b), and one step of outer subgradient update for the dual variable, i.e., (68c). It converges to the global optimum of problem (66) under relatively loose conditions, for two-block convex separable problems. For more details, please refer to [49].

## Appendix B

Optimal Solution to Problem (53)
It can be readily seen that problem (53) is convex and strong duality holds, therefore, it can be solved by resorting to the dual problem. Specifically, the Lagrangian function of problem (53) is given by $L_{1}=L_{\delta, 1}-\mu\left((x[2 i]-\bar{x}[2 i+1])^{2}+\right.$ $\left.(y[2 i]-\bar{y}[2 i+1])^{2}-V_{\max }^{2} \delta_{t}^{2}\right)$, where $\mu$ denotes the Lagrangian multiplier. Then, by setting $\frac{\partial L_{1}}{\partial \bar{x}[2 i+1]}=0$, we have

$$
\begin{equation*}
\bar{x}^{\mathrm{opt}}[2 i+1]=\frac{1}{2 \mu+\delta}\left(2 \mu x[2 i]+\delta x[2 i+1]-\lambda_{x_{2 i+1}}\right) \tag{70}
\end{equation*}
$$

Substituting (70) into $L_{1}$ and taking the partial derivative of $L_{1}$ with respect to $x[2 i]$, we can obtain

$$
\begin{align*}
& -\delta\left(x[2 i]-\bar{x}[2 i]-\frac{\lambda_{x_{2 i}}}{\delta}+x[2 i]-\hat{x}[2 i]-\frac{\eta_{x_{2 i}}}{\delta}+x[2 i]\right. \\
& \left.-\tilde{x}[2 i]-\frac{\omega_{x_{2 i}}}{\delta}\right)-\frac{2 \mu \delta}{2 \mu+\delta}\left(x[2 i]-x[2 x+1]+\frac{\lambda_{x_{2 i+1}}}{\delta}\right)=0 \tag{71}
\end{align*}
$$

Based on (71), the optimal $x^{\mathrm{opt}}[2 i]$ can be expressed as

$$
\begin{align*}
x^{\mathrm{opt}}[2 i]= & \frac{1}{3 \delta+\frac{2 \mu \delta}{2 \mu+\delta}}\left(\delta(\bar{x}[2 i]+\hat{x}[2 i]+\tilde{x}[2 i])+\lambda_{x_{2 i}}\right. \\
& \left.+\eta_{x_{2 i}}+\omega_{x_{2 i}}+\frac{2 \mu \delta}{2 \mu+\delta}\left(x[2 i+1]-\frac{\lambda_{x_{2 i+1}}}{\delta}\right)\right) \tag{72}
\end{align*}
$$

Similarly, we have

$$
\begin{equation*}
\bar{y}^{\mathrm{opt}}[2 i+1]=\left(2 \mu y[2 i]+\delta y[2 i+1]-\lambda_{y_{2 i+1}}\right) /(2 \mu+\delta), \tag{73}
\end{equation*}
$$

and

$$
\begin{align*}
y^{\mathrm{opt}}[2 i]= & \frac{1}{3 \delta+\frac{2 \mu \delta}{2 \mu+\delta}}\left(\delta(\bar{y}[2 i]+\hat{y}[2 i]+\tilde{y}[2 i])+\lambda_{y_{2 i}}\right. \\
& \left.+\eta_{y_{2 i}}+\omega_{y_{2 i}}+\frac{2 \mu \delta}{2 \mu+\delta}\left(y[2 i+1]-\frac{\lambda_{y_{2 i+1}}}{\delta}\right)\right) \tag{74}
\end{align*}
$$

Then, it is not difficult to see that if $\bar{x}^{\text {opt }}[2 i+1], x^{\text {opt }}[2 i]$, $\bar{y}^{\text {opt }}[2 i+1]$ and $y^{\text {opt }}[2 i]$ satisfy $\left(x^{\text {opt }}[2 i]-\bar{x}^{\text {opt }}[2 i+1]\right)^{2}+$ $\left(y^{\mathrm{opt}}[2 i]-\bar{y}^{\mathrm{opt}}[2 i+1]\right)^{2} \leq V_{\max }^{2} \delta_{t}^{2}$ when $\mu=0$, then this is the optimal solution. Otherwise, we substitute (70), (72), (73) and (74) into $(x[2 i]-\bar{x}[2 i+1])^{2}+(y[2 i]-\bar{y}[2 i+1])^{2}=V_{\max }^{2} \delta_{t}^{2}$
(due to the complementary slackness). By solving this equation with respect to $\mu$, we have $\mu^{\mathrm{opt}}=\left(\sqrt{A}-3 \delta^{2}\right) /(8 \delta)$, where

$$
\begin{align*}
A= & \frac{\delta^{2}}{V_{\max }^{2} \delta_{t}^{2}}\left(\left(\delta(\hat{x}[2 i]+\tilde{x}[2 i]+\bar{x}[2 i])+\omega_{x_{2 i}}+\eta_{x_{2 i}}\right.\right. \\
& \left.+\lambda_{x_{2 i}}+3 \lambda_{x_{2 i+1}}-3 \delta x[2 i+1]\right)^{2}  \tag{75}\\
& +\left(\delta(\hat{y}[2 i]+\tilde{y}[2 i]+\bar{y}[2 i])+\omega_{y_{2 i}}+\eta_{y_{2 i}}\right. \\
& \left.\left.+\lambda_{y_{2 i}}+3 \lambda_{y_{2 i+1}}-3 \delta y[2 i+1]\right)^{2}\right) .
\end{align*}
$$

Substituting $\mu^{\mathrm{opt}}$ back into (70), (72), (73) and (74), we can obtain the optimal solution of problem (53).

## Appendix C

## Optimal Solution to Problem (54)

For notational simplicity, in this appendix, we ignore the time slot index $2 i+1$ in the variables $\{\tilde{x}[2 i+1], \tilde{y}[2 i+$ 1], $x[2 i+1], y[2 i+1], u[2 i+1], t[2 i+1], \rho[2 i+1], p[2 i+$ $\left.1], \omega_{x_{2 i+1}}, \omega_{y_{2 i+1}}\right\}$ without loss of generality. First, we can observe that $L_{\delta, 2}$ is a decreasing function with respect to $u$, therefore the optimal $u$, denoted as $u^{\text {opt }}$, must satisfy $u^{\mathrm{opt}}=\tilde{x}^{2}+\tilde{y}^{2}+H^{2}$. By substituting $u^{\mathrm{opt}}$ into $L_{\delta, 2}$, we obtain $L_{\delta, 2}=-a\left(\tilde{x}^{2}+\tilde{y}^{2}+H^{2}\right)-\frac{\sigma^{2} t}{t_{f}+\gamma_{0} p}+\log \left(\gamma_{0}(1-\rho) p+\sigma^{2} t\right)-$ $\frac{\delta}{2}\left(\left(x-\tilde{x}-\frac{\omega_{x}}{\delta}\right)^{2}+\left(y-\tilde{y}-\frac{\omega_{y}}{\delta}\right)^{2}\right)$. Hence, problem (54) becomes

$$
\begin{align*}
& \max _{\tilde{x}, \tilde{y}, t} L_{\delta, 2} \\
& \quad \text { s.t. } t \leq-\tilde{x}_{f}^{2}+2 \tilde{x}_{f} \tilde{x}+L^{2}-2 \tilde{x} L-\tilde{y}_{f}^{2}+2 \tilde{y}_{f} \tilde{y}+H^{2} \tag{76}
\end{align*}
$$

Since problem (76) is convex, we can globally solve it by resorting to its dual problem. The corresponding Lagrange function for problem (76) can be expressed as $L_{\delta, 2, \tilde{\mu}} \triangleq$ $L_{\delta, 2}-\tilde{\mu}\left(t+\tilde{x}_{f}^{2}-2\left(\tilde{x}_{f}-L\right) \tilde{x}-L^{2}+\tilde{y}_{f}^{2}-2 \tilde{y}_{f} \tilde{y}-H^{2}\right)$, where $\tilde{\mu}$ is the dual variable.

By checking the first-order optimality condition, we can express the optimal solution of problem (76) as a function of $\tilde{\mu}$, i.e.,

$$
\begin{align*}
\tilde{x}^{\mathrm{opt}}(\tilde{\mu}) & =\frac{\delta x-\omega_{x}+2 \tilde{\mu}\left(\tilde{x}_{f}-L\right)}{2 a+\delta} \\
\tilde{y}^{\mathrm{opt}}(\tilde{\mu}) & =\frac{\delta y-\omega_{y}+2 \tilde{\mu} \tilde{y}_{f}}{2 a+\delta}  \tag{77}\\
t^{\mathrm{opt}}(\tilde{\mu}) & =\frac{\sigma^{2} t_{f}+\gamma_{0} p}{\tilde{\mu}\left(\sigma^{2} t_{f}+\gamma_{0} p\right)+\sigma^{2}}-\frac{\gamma_{0}(1-\rho) p}{\sigma^{2}}
\end{align*}
$$

If the solution $\left\{\tilde{x}^{\text {opt }}(0), \tilde{y}^{\text {opt }}(0), t^{\text {opt }}(0)\right\}$ automatically satisfies the constraint of problem (76), then it is the optimal solution, otherwise, we can see that the optimal dual variable $\tilde{\mu}^{\text {opt }}$ satisfies

$$
\begin{align*}
t^{\mathrm{opt}}\left(\tilde{\mu}^{\mathrm{opt}}\right)= & -\tilde{x}_{f}^{2}+2\left(\tilde{x}_{f}-L\right) \tilde{x}^{\mathrm{opt}}\left(\tilde{\mu}^{\mathrm{opt}}\right) \\
& +L^{2}-\tilde{y}_{f}^{2}+2 \tilde{y}_{f} \tilde{y}^{\mathrm{opt}}\left(\tilde{\mu}^{\mathrm{opt}}\right)+H^{2} \tag{78}
\end{align*}
$$

Substituting (77) into (78) and solving the resulting quadratic equation, we obtain
$\tilde{\mu}^{\mathrm{opt}}=\left(-b_{\tilde{\mu}}+\sqrt{b_{\tilde{\mu}}^{2}-4 a_{\tilde{\mu}} c_{\tilde{\mu}}}\right) /\left(2 a_{\tilde{\mu}}\right)$, where $a_{\tilde{\mu}}=$ $\frac{4\left(\tilde{x}_{f}-L\right)^{2}+4 \tilde{y}_{f}^{2}}{2 a+\delta}, b_{\tilde{\mu}}=\frac{\sigma^{2} a_{\tilde{\mu}}}{\sigma^{2} t_{f}+\gamma_{0} p}+d_{\tilde{\mu}}, c_{\tilde{\mu}}=\frac{\sigma^{2} d_{\tilde{\mu}}}{\sigma^{2} t_{f}+\gamma_{0} p}-$ 1 and $d_{\tilde{\mu}}=-\tilde{x}_{f}^{2}-\tilde{y}_{f}^{2}+L^{2}+H^{2}+\frac{\gamma_{0}(1-\rho) p}{\sigma^{2}}+$ $\frac{2\left(\tilde{x}_{f}-L\right)\left(\delta x-\omega_{x}\right)+2 \tilde{y}_{f}\left(\delta y-\omega_{y}\right)}{2 a+\delta}$.

## Appendix D <br> Optimal Solution to Problem (55)

The Lagrangian function of problem (55) can be expressed as $L_{3}=L_{\delta, 3}-\phi \sum_{i=1}^{T-1}\left((\ddot{x}[i]-\ddot{x}[i+1])^{2}+(\ddot{y}[i]-\ddot{y}[i+1])^{2}\right)+\phi \frac{E_{\mathrm{tr}}}{\kappa}$. By setting $\frac{\partial L_{\delta, 3}}{\partial \hat{x}[2 i+1]}=0$ and $\frac{\partial L_{\delta, 3}}{\partial \hat{y}[2 i+1]}=0$, we have
$\hat{x}^{\mathrm{opt}}[2 i+1]=\frac{1}{2}\left(\ddot{x}[2 i+1]+\frac{\theta_{x_{2 i+1}}}{\delta}+x[2 i+1]-\frac{\eta_{x_{2 i+1}}}{\delta}\right)$,
$\hat{y}^{\mathrm{opt}}[2 i+1]=\frac{1}{2}\left(\ddot{y}[2 i+1]+\frac{\theta_{y_{2 i+1}}}{\delta}+y[2 i+1]-\frac{\eta_{y_{2 i+1}}}{\delta}\right)$.
Substituting (79) and (80) into $L_{3}$ and letting $\frac{\partial L_{3}}{\partial \ddot{x}[2 i+1]}=0$ and $\frac{\partial L_{3}}{\partial \ddot{x}[2 i]}=0$, we can obtain the following equations:

$$
\begin{gather*}
\left(-4 \phi-\frac{\delta}{2}\right) \ddot{x}[2 i+1]+2 \phi(\ddot{x}[2 i+2]+\ddot{x}[2 i])  \tag{81}\\
+\frac{\delta}{2}\left(x[2 i+1]-\frac{\eta_{x_{2 i+1}}+\theta_{x_{2 i+1}}}{\delta}\right)=0 \\
-(\delta+4 \phi) \ddot{x}[2 i]+\delta \hat{x}[2 i]-\theta_{x_{2 i}}+2 \phi(\ddot{x}[2 i+1]+\ddot{x}[2 i+1])=0 . \tag{82}
\end{gather*}
$$

Similarly, for $\ddot{y}[2 i+1]$ and $\ddot{y}[2 i]$, we have

$$
\begin{gather*}
\left(-4 \phi-\frac{\delta}{2}\right) \ddot{y}[2 i+1]+2 \phi(\ddot{y}[2 i+2]+\ddot{y}[2 i]) \\
+\frac{\delta}{2}\left(y[2 i+1]-\frac{\eta_{y_{2 i+1}}+\theta_{y_{2 i+1}}}{\delta}\right)=0 \\
-(\delta+4 \phi) \ddot{y}[2 i]+\delta \hat{y}[2 i]-\theta_{y_{2 i}}+2 \phi(\ddot{y}[2 i+1]+\ddot{y}[2 i+1])=0 . \tag{84}
\end{gather*}
$$

Together with $\ddot{x}[1]=x_{1}, \ddot{x}[T]=x_{T}, \ddot{y}[1]=y_{1}$ and $\ddot{y}[T]=y_{T}$, we can employ the Gaussian elimination [50] (a celebrated algorithm in linear algebra for solving a system of linear equations) to solve the above equations for a given dual variable $\phi$ and the optimal $\phi$ can be found by using the Bisection method.

## Appendix E <br> Proof of Proposition 2

Let $g_{i}^{3 \mathrm{D}}(x[i], y[i], z[i], p[i], \rho[i]) \triangleq \log \left(1+\frac{\gamma_{0} a[i]}{d_{13}^{2}[i] \sigma^{2}}\right)-$ $\log \left(1+\frac{\gamma_{0} a[i]}{\gamma_{0} b[i]+\sigma^{2} d_{\mathrm{EB}}^{2}[i]}\right)$ (recall that $a[i]=p[i] \rho[i]$ and $b[i]=$ $p[i](1-\rho[i])$ as defined in (19)), then by taking the partial derivative of
$g_{i}^{3 \mathrm{D}}(x[i], y[i], z[i], p[i], \rho[i])$ with respect to $z[i]$, we have ${ }^{6}$
$\frac{\partial g_{i}^{3 \mathrm{D}}}{\partial z[i]}=\left\{\begin{array}{l}2 z[i]\left(f\left(d_{\mathrm{I}}[i]^{2}\right)-f\left(\frac{\gamma_{0} b[i]}{\sigma^{2}}+d_{\mathrm{E} 3}[i]^{2}\right)\right), \text { if } g_{i}^{3 \mathrm{D}} \geq 0, \\ 0, \text { otherwise } .\end{array}\right.$
where $f(x) \triangleq \frac{1}{\gamma_{0} a[i] / \sigma^{2}+x}-\frac{1}{x}$. Note that $f(x)$ is a monotonically increasing function since $\frac{\partial f(x)}{\partial x}=-\frac{1}{\left(\gamma_{0} a[i] / \sigma^{2}+x\right)^{2}}+$ $\frac{1}{x^{2}}>0$. Besides, we have $d_{\mathrm{I} 3}[i]^{2}<\gamma_{0} b / \sigma^{2}+d_{\mathrm{E} 3}[i]^{2}$ when $g_{i}^{3 \mathrm{D}}>0$. Therefore, $\frac{\partial g_{i}^{3 \mathrm{D}}}{\partial z[i]}$ satisfies $\frac{\partial g_{i}^{3 \mathrm{D}}}{\partial z[i]}<0$, if $g_{i}^{3 \mathrm{D}}>0$, and $\frac{\partial g_{i}^{3 \mathrm{D}}}{\partial z[i]}=0$ otherwise. This implies that $\frac{\partial g_{i}^{3 \mathrm{D}}}{\partial z[i]} \leq 0$ is always satisfied, which means that $g_{i}^{3 \mathrm{D}}$ will decrease or remain unchanged with the increasing of $z[i]$. Thus, we can see that (60) holds, which completes the proof.

[^4]
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[^0]:    ${ }^{1}$ The proposed algorithm can also be extended to 3D trajectory optimization, which will become clear later.

[^1]:    ${ }^{2}$ There may exist other UAV-to-ground communication scenarios in practice, where the UAV-Bob/Eve channels follow the probabilistic LoS channel model [37]-[39] or the Rician channel model [40]. How to optimize the UAVs trajectory, the transmit power levels and the power splitting ratios with low complexity under these channel models and further investigation into the multi-antenna case are interesting topics that are left for future works.

[^2]:    ${ }^{3}$ In the following, the subscript $f$ is used to denote the feasible variable obtained in the previous BCD iteration.

[^3]:    ${ }^{5}$ Since $T$ is usually on the order of several hundreds, thus $T^{3.5} \gg$ $N_{A} N_{B} T$.

[^4]:    ${ }^{6}$ In the following, $g_{i}^{3 \mathrm{D}}(x[i], y[i], z[i], p[i], \rho[i])$ is abbreviated as $g_{i}^{3 \mathrm{D}}$ for notational simplicity.

