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Construction of Full Residual Stress Depth Profile in Glass Using the Knowledge of Surface Stress

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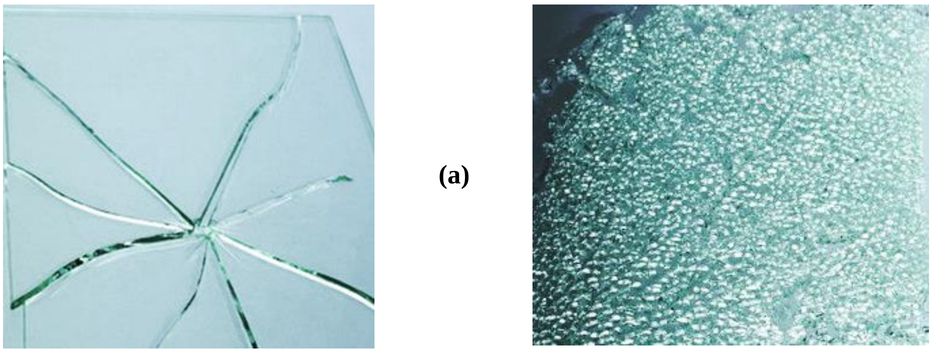
This paper presents the development of a simple method to determine the full residual stress depth profile in architectural (i.e. construction sector) glass. The proposed model requires only the knowledge of the surface residual stress of glass, which can be known from the glass manufacturer or can be measured using a Scattered-Light-Polariscope (SCALP), as input. The requirement of through-thickness force equilibrium and the knowledge of parabolic shape of the residual stress depth profile are used to uniquely determine the residual stress depth profile in any given glass panel. Unlike the complex models reported in the literature, the proposed technique does not require modelling the complex multi-physics phenomenon of the generation of residual stress or the use of complex computational models. The residual stress predictions from the proposed model were validated against experimental results. The paper also presents a sensitivity analysis in order to justify the accuracy of the proposed model even after the possible errors/inaccuracies in the only input data (i.e. surface stress) of the model was incorporated in the analyses.

Keywords: Analysis, Design, Glass, Residual stress, Structures.

1. Introduction

Residual stresses are developed in architectural glass owing to the differential cooling experienced by glass during the manufacturing processes. Annealed glass (i.e. basic float glass) is manufactured by slow cooling of the molten ingredients (silica, lime, soda and other minor ingredients). During the cooling of hot glass, the surface regions of the glass panels cool and solidify first. The subsequent cooling and shrinking of the inner regions of the glass panels cause tensile residual stresses in the mid-thickness regions of the glass panels. The self-equilibrium requirement of the residual stresses means that compression residual stresses developed in the surface regions of glass panels as a means of balancing the tensile stresses in the mid-thickness regions. In thermally-strengthened glass (i.e. heat-strengthened glass and tempered (toughened) glass) high magnitudes of residual stresses are developed due to the purposely employed rapid cooling of the hot glass during the thermal treatment processes.

The magnitude of the surface compression residual stresses governs the tensile strength of a given glass panel. Typically, commercially available annealed, heat-strengthened and tempered glass possess surface compression residual stresses of magnitudes 3-8 MPa, 25-50 MPa and 80-150 MPa, respectively. The magnitudes of the residual stresses also govern the fracture behavior of architectural glass. Annealed glass usually fractures into large shards, whereas tempered glass shatters into small dices of few millimeters (see Fig. 1). Usually, glass with high surface residual stress, such as fully tempered glass, are strong, but they are brittle with no post-fracture load resistance.



(a) (b)

Fig. 1 Fracture (failure) behaviour of (a) annealed glass, (b) tempered glass

The effects of residual stresses are critical to the structural behavior of glass structures. Therefore, the effects of residual stresses must be included in the stress analysis/structural design of glass structures. However, the effects of residual stresses are not explicitly included in current industrial practice of glass design. The residual stresses are ignored in the stress analysis of annealed glass structures. In thermally-strengthened glass, stress analysis are first carried out without including the residual stress and then the maximum design surface tensile stresses are compared against the surface compressive residual stress in the glass as a means of ensuring structural safety. Unavailability of a reliable computational tool for incorporating the effects of residual stresses in glass structural design is one of the reasons for not explicitly incorporating the residual stresses in current structural design of glass structures.

The complexity of the process of generating of residual stress in glass means that development of a modelling technique which mimics the exact mechanisms is virtually impossible. The difficulty of knowing the relevant thermal, material and mechanical parameters and the computational complexity of the analyses means that the existing modelling methods are too complex and impractical to be used in stress analysis of glass structures. However, reliable methods, such as Scattered-Light-Polariscopes (SCALP) (SCALP 2015) based experimental techniques are available to experimentally characterize the stabilized residual stress states (i.e. the stress states after the cooling processes and the all thermal and mechanical changes had happened) in glass.

This paper presents the development of a simple method that requires only the knowledge of the surface residual stress, which is usually available from the glass manufacturer or can be determined using a Scattered-Light-Polariscope, for modelling the full residual stress depth profile in architectural glass. Unlike methods reported in the literature, the proposed model uses the self-equilibrium characteristic of the residual stress distribution. The predictions form the proposed model were validated against experimental results, including the effects of possible error/uncertainty in the surface residual stress value used in the analysis.

2. Proposed residual stress depth profile model

In flat (i.e. float) glass panels, the magnitudes of the longitudinal and transverse components of the residual stresses in any plane, far away from the edges of the panel, are largely equal. The magnitudes of the in-plane shear and through-thickness stresses in the glass panels are small compared to the longitudinal and transverse components of the residual stress distribution (Pourmoghaddam & Schneider 2018; Castellini et al. 2012). Therefore, usually only the knowledge of one in-plane direct stress component of the residual stress distribution is sufficient to represent the residual stress distribution in glass panels.

Analytical and computational models (e.g. Nielsen et al. 2010; Daudeville & Carre 1998), as well as residual stress data measured using SCALPs (e.g. Aben et al. 2015; Aben et al. 2010; Zaccaria & Overend 2015) suggest that the residual stress depth profile in glass is parabolic with respective to the median plane of the glass specimen. This knowledge of parabolic shape of the residual stress depth profile together with the self-balance (i.e. static force equilibrium) characteristics of the residual stress distribution, it is possible determine the equation of parabola that can represent the residual stress depth profile in a given glass specimen.

Fig. 2 shows a typical illustration of the residual stress depth profile in a given glass specimen. *X*-axis of Fig. 2 represents the value of the residual stress whilst the *Y*-axis represent the thickness direction of the glass specimen. The parabola is symmetric with respect to the mid-thickness plane (i.e. y = 0) of the glass specimen. The equation of this parabola can be written as:

$$x(y) = ay^2 + x(y = 0) \qquad (1)$$

(where $x(y)$ is the residual stress at distance y from y=0 axis and a is a constant)

The above equation has two knowns: (1) a and (2) $x(y=0)$. The knowledge of the surface stress (i.e. $x(y=t)$, where t is half-thickness of the glass specimen) and through-thickness force balance over a unit width of the glass specimen (i.e. $\int_{-t}^{t} x(y)\text{x}1\, dy = 0$) can be used to uniquely determine the two unknowns in Equation (1) (i.e. a and $x(y=0)$). After a and $x(y=0)$ for a given glass specimen are known, the full residual stress depth profile of the glass specimen can be uniquely determined by using Equation (1).

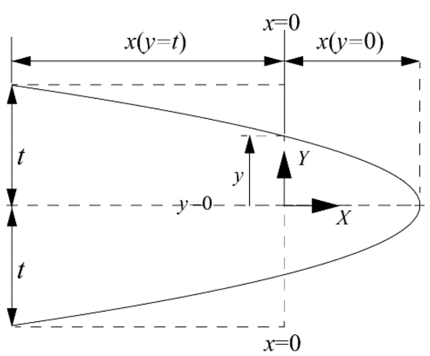


Fig. 2 Parabolic shape of the residual stress depth profile

3. Applications of the proposed model

The proposed residual stress depth profile model (i.e. Eq (1) can be used to predict the full residual stress depth profile in all types of glass (i.e. annealed, heat-strengthened and fully-tempered glass). Table 1 shows the surface residual stress values measured using a SCALP for a few selected glass test specimens. Figure 3 (solid lines) shows the full residual depth profile predictions from the proposed model for each glass specimen.

Table 1: Surface residual stress values of three glass test specimens

Glass specimen No:	Reference	Glass type	Thickness of the test specimen (mm)	Surface (compression) residual stress (MPa)
1	Balan & Achintha 2015	Annealed	10	5.6
2	Aben et al. 2010	Heat-strengthened	6	60
3	Balan & Achintha 2015	Tempered	10	95.6

4. Validation of the model

The predictions from the proposed residual stress depth profile model were compared against the experimentally measured residual stress data. Fig. 3 also shows the comparisons between the model predicted and the measured residual stress data for the same glass test specimens considered in Table 1. The experimental stress data shown in Fig.3 were obtained using SCALP. In the experiments the stresses were measured from both surfaces in order to construct the full residual stress depth profile. However, in thick glass specimens (e.g. 10 mm thick glass), the stress data was measured only up to a thickness where it was possible to measure the stresses reliably (usually up to 3-4 m deep from the surface). The results shown in Fig. 3 suggest that the proposed parabolic model accurately predicts the residual stress depth profiles in all glass test specimens. Although for brevity, only the results for three selected glass test specimens are shown in this paper, comparisons between the model predictions and experimentally measured residual stress data were carried out of a range of glass test specimens, including different glass types and thicknesses. The comparisons between the model predictions and the experimental results for all other test specimens investigated in the present study were qualitatively similar to the results shown in Figure 3.

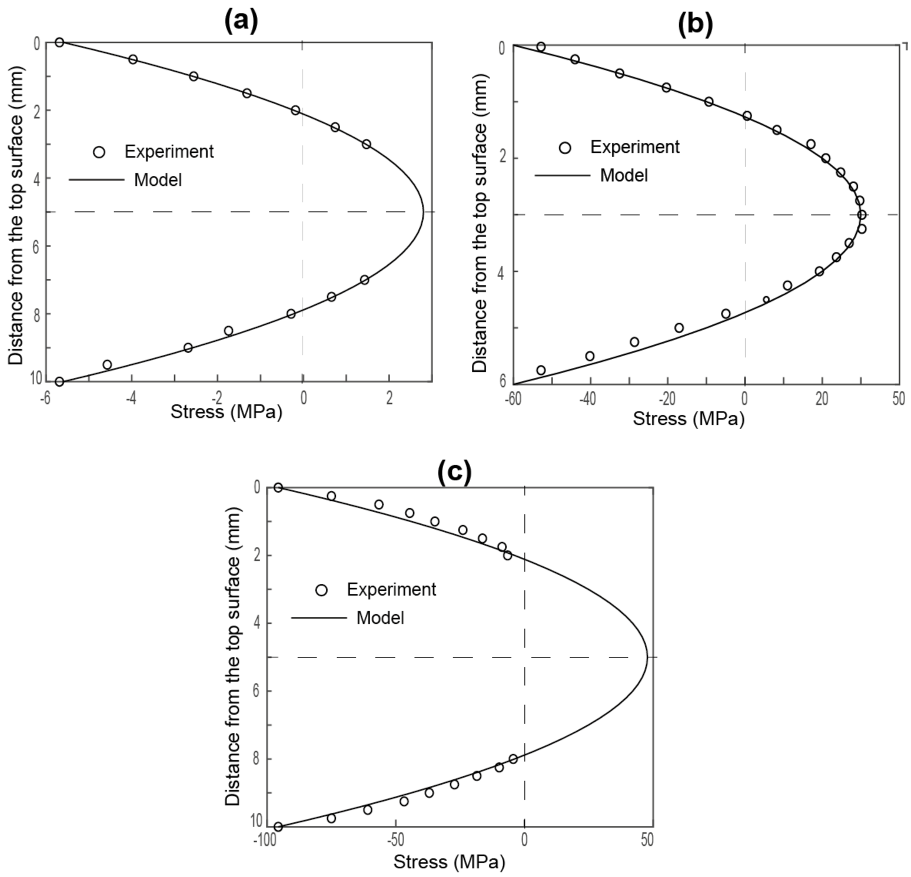


Fig. 3 Comparisons between the experimentally measured and the model predicted residual stress depth profiles in (a) 10 mm thick annealed, (b) 6 mm thick heat-strengthened and (c) 10 mm thick tempered glass test specimens

5. Effects due to the error/uncertainty in the surface stress used in the model

The results (Section 4) of the comparisons between the measured residual stresses and the predictions from the model proposed in the present study (i.e. Eq. (1)) suggest that the proposed model can accurately determine the full residual stress depth profile, if the surface residual stress of the given glass specimen is known. As stated previously, the surface residual stress data used in the present study (Fig. 3) were measured using SCALP. Errors are inevitably present in the experimentally measured residual stress data, for example, stress measurements from SCALP depend on the assumption of the photoelastic constant which itself may depend on the thermal and mechanical processes during glass manufacturing and/or thermal strengthening. Furthermore, successive stress measurements at a given surface location of a glass specimen can differ. Manufactures of SCALP usually expect ±5% error in the measured stresses (e.g. SCALP, 2015). In addition to the inevitable inaccuracy/uncertainty associated with the measured residual stress values, other factors such as non-uniform stress across the glass surfaces due to non-uniform cooling can also introduce additional uncertainties.

A sensitivity analysis was carried in the present study in order to investigate the effects due to possible inaccuracy in the surface residual stress value used in the models. It was decided to investigate the effects of error within the range of ±10% (this is twice the error range quoted by the SCALP manufacturers). The predictions based on the assumed surface stress values of (σ_s–10%σ_s), σ_s and (σ_s+10%σ_s) (σ_s is surface stress obtained from SCALP experiments) were carried out for all glass test specimens considered in the present study. For brevity, only the results for the three glass test specimens considered in Table 1 are presented below. The comparisons between the model predictions and the experimental results for all other test specimens investigated in the present study were qualitatively similar to the results shown in this paper.

5.1 10 mm thick annealed glass test specimen

Table 2 shows the surface stress values assumed in the proposed parabolic model (i.e. Eq. (1)) in order to the determine the full residual stress depth profiles corresponding to different assumed values of the surface residual stress in 10 mm thick annealed glass test specimen.

Table 2: Assumed surface residual stress values for 10 mm thick annealed glass test specimen

Assumed percentage error in the surface stress	Assumed surface compression value assumed in the parabolic model (MPa)
-10%	5.04
0	5.6
+10%	6.16

Fig. 4 shows the model predictions for the residual stress depth profile in 10 mm thick annealed glass test specimen based on the analysis with assumed surface compression values shown in Table 2. The figure also shows the experimentally measured residual stress data for the same 10 mm thick annealed glass test specimen.

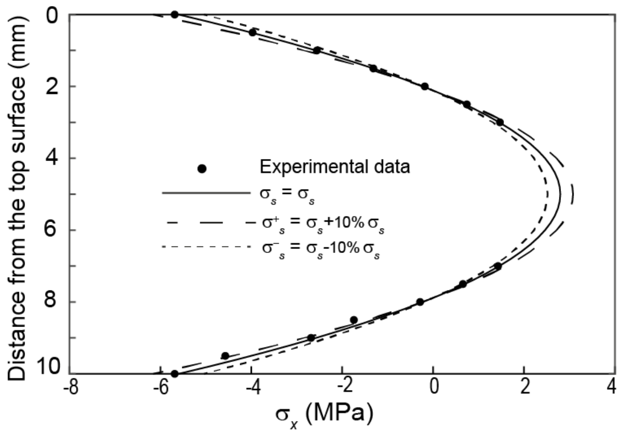


Fig. 4 Predicted residual stress depth profile based on different assumed surface stress values and the measured residual stress data for 10 mm thick annealed glass test specimen

5.2 6 mm thick heat-strengthened glass test specimen

Table 3 shows the surface stress values assumed in the proposed parabolic model (i.e. Eq. (1)) in order to the determine the full residual stress depth profiles corresponding to different assumed values of the surface residual stress in 6 mm thick heat-strengthened glass test specimen.

Fig. 5 shows the model predictions for the residual stress depth profile in 6 mm thick heat-strengthened glass test specimen based on the analysis with assumed surface compression values shown in Table 3. The figure also shows the experimentally measured residual stress data for the same 6 mm thick heat-strengthened glass test specimen.

Table 3: Assumed surface residual stress values for 6 mm thick heat-strengthened glass test specimen

Assumed percentage error in the surface stress	Assumed surface compression value assumed in the parabolic model (MPa)
-10%	54
0	60
+10%	72.6

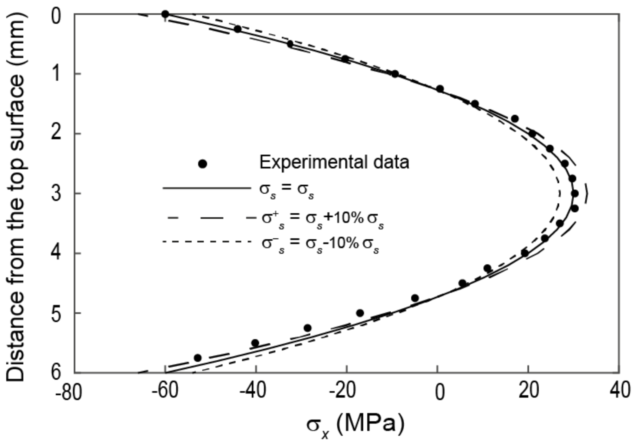


Fig. 5 Predicted residual stress depth profile based on different assumed surface stress values and the measured residual stress data for 6 mm thick heat-strengthened glass test specimen

5.3 10 mm thick tempered glass test specimen

Table 5 shows the surface stress values assumed in the proposed parabolic model (i.e. Eq. (1)) in order to the determine the full residual stress depth profiles corresponding to different assumed values of the surface residual stress in 10 mm thick tempered glass test specimen.

Table 4: Assumed surface residual stress values for 10 mm thick tempered glass test specimen

Assumed percentage error in the surface stress	Assumed surface compression value assumed in the parabolic model (MPa)
-10%	86.04
0	95.6
+10%	105.16

Fig. 6 shows the model predictions for the residual stress depth profile in 10 mm thick tempered glass test specimen based on the analysis with assumed surface compression values shown in Table 4. The figure also shows the experimentally measured residual stress data for the same 10 mm thick tempered glass test specimen.

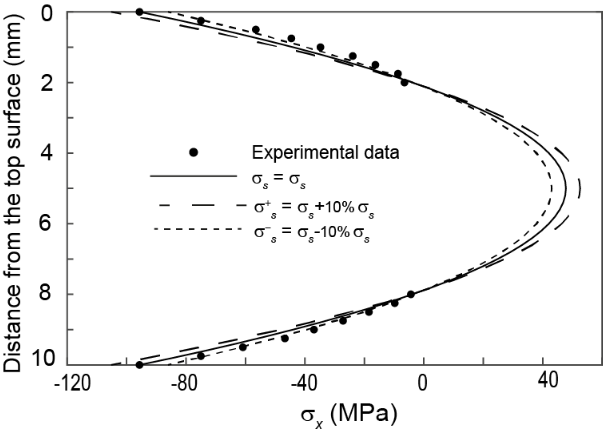


Fig. 6 Predicted residual stress depth profile based on different assumed surface stress values and the measured residual stress data for 10 mm thick tempered glass test specimen

5.4 Discussion

The comparisons between the experimentally measured residual stress data and the predictions from the proposed model shown in Figs. 4-6 suggest that the predictions based on the assumed surface residual stress values within the chosen range (i.e. within 10% higher and lower than the actually measured value) still agree with the experimental results. The chose error range (i.e. ±10%) is twice the error/uncertainty range identified by the SCALP manufacturer. Thus, the results suggest that the proposed parabolic residual stress model can accurately estimate the residual stress depth profile in glass panels even after any possible inaccuracy in the surface stress used in the model was considered.

6. Conclusions

A simple closed-form solution, which requires only the knowledge of the surface residual stress, which is usually available from the glass manufacturer or can be determined accurately using a scattered-light-polariscope, was determined for predicting full residual stress depth profile in architectural glass. The proposed residual stress depth profile model was validated against the measured residual stress depth profiles reported in the literature for all types of architectural glass. The results of a sensitivity analysis which includes possible error/uncertainty in the surface residual stress used in the model suggest that the proposed model can still accurately predict the full residual stress depth profile even after errors/inaccuracies in the only input data (i.e. surface stress) was taken into account in the analysis.

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